ERITREAN GRADE EIGHT STUDENTS' UNDERSTANDING OF ALGEBRAIC VARIABLES.

YOSIEF TEKIE SIUM

Submitted in the partial fulfillment of the requirements for the degree of M.Phil. in the School of Science and Mathematics Education, University of the Western Cape.

Supervisor: Professor Cyril Julie

DECLARATION

I declare that *Eritrean grade eight students' understanding of algebraic variables* is my own work and that all the sources I have used or quoted have been indicated and acknowledged by complete references.

YOSIEF TEKIE

Signature: ----------
August 2003.

Acknowledgement

I am personally indebted to Professor C. Julie, my excellent supervisor. As a scholar of massive intellect and immeasurable motivational influence, he took me through a process of rigorous, empowering and captivating intellectual craftsmanship. In fact, he taught me the power of perseverance, the power of visioning, the power of dealing with obstacles in the pursuance of goals and the power of excellence in doing things. Without his active listening, sustainable forbearances, useful guidance, continued availability and immense sacrifice, the completion of this piece of work could not have been possible.

I would like to express my thanks to secondary school teachers and students of Gash-Barka zone for their contribution in organising and collecting the data for the study.

I am grateful to thank all my family members. They were instrumental in creating an environment in which I could unleash my potential, recognize and optimise my competencies in a way that makes positive and sustainable growth in human life. They always reminded me to account for my blessings from the ancestors.

Key words:

Eritrea

Secondary schools

Students

Mathematics education

Algebra

Variables

Concept

Understanding

Students' misconceptions

Abstract

This study investigated Eritrean grade eight students' understanding of algebraic variables. A complete count survey of all secondary schools of one province was done and the one that took part in the study was Gash-Barka province.

The study adopted the test and framework developed by Kuchemann (1980). Children's responses and the items themselves were classified into "levels of understanding". Items of the test were classified into four levels based on the name that can be given to letters in solving a problem and the structural complexity of the item. These items were used to classify the Eritrean grade eight students' performances into five "levels of understanding".

The present study produced results that showed that 72.6 % of the students dealt with letters in algebraic expressions and equations as objects. Whilst 3.7 % of the students were able to regard letters as specific unknowns, only 0.2 % of the students were able to consider letters as generalized numbers or variables. That is, almost all (95.9 %) of the tested Eritrean grade eight students were unable to cope consistently with items that can properly be called algebra, that is, items where the use of letters as unknown numbers cannot be avoided.

Comparisons by school and gender were done to see if there were relationships among the levels of understanding and the two variables. The findings showed that there was no significant relationship among the levels of understanding and gender of the students. However, the comparison by school showed that there was significant relationship between schools and levels of understanding. The Pearson chi-square test showed that the relationship between the level of understanding and gender was not significant, whereas the relationship between levels of understanding and school was statistically significant at 0.05 level of significance.

Table of contents

Chapter Four
Data presentation, Data analysis and Discussion

Chapter Five
Conclusion and Recommendations

Chapter One

Introduction

1.1 The development of the education system in Eritrea

This study was conducted in Eritrea, which is located in the northern horn of Africa. Eritrea is bordered on the northeast and east by the Red Sea, on the south by Ethiopia, on the southeast by the Republic of Djibouti and on the west and northwest by the Democratic Republic of Sudan. Eritrea has a coastal line of about 1000 kilometers along the west coast of the Red Sea. The area of Eritrea is about 123,300 square kilometers. The population of Eritrea is estimated to be 3.7 million (Ministry of Education, 2001a:10). Eritrea is a multiethnic and multilingual society. There are nine ethnic groups, each with its own spoken language.

Eritrea had been ruled successively by different colonizers for about a century: Italian (1890-1941), British (1942-1952) and Ethiopian (1962-1991). The Eritrean education system is inherited from a variety of foreign sources. As a result, the curriculum in different historical periods reflected the ideological interest of the colonizing power. Destafano (1998:71-73) provides a useful summary of the colonial period in relation to the formal introduction of western education in Eritrea as follows:

1. A formal European style of education was introduced into Eritrea during the Italian colonial period. The purpose of education was to indoctrinate the Eritreans with respect to the Italian culture and civilization. Schools were opened for Eritreans to become troops, interpreters, clerks, telephone operators and typists. Eritreans were allowed to learn up to grade four only.

2. With the British invasion and defeat of Italians in east Africa, the British Military Administration sought to provide a level of education to train Eritreans as

functionaries in the administration in order to reduce costs. Hence, the level of education in Eritrea increased by opening more primary schools and the first middle school was opened in 1947. In 1949 five middle schools were established and by 1950 English became the medium of instruction.

3. Between 1952-1962, Eritrea was federated with Ethiopia, the education system being under the control of the Eritrean government. During this time, there was a remarkable advance in education and the first two secondary schools were opened in 1956 in Asmara, which is the capital city of Eritrea. By 1964, after the annexation of Eritrea, Amharic (the official language of Ethiopia) became the medium of instruction in Eritrea.

Eritrea obtained its de facto independence in 1991 after thirty years of war and its official independence in May 1993 after a UN supervised referendum in which over 99% of the people voted in favor of independence (Ministry of Education, 1999:1). The country was in dire need of reconstruction and rehabilitation as the economy and infrastructure had collapsed and social services had disintegrated. Its human resources development was greatly hampered during the struggle as its youth were persecuted and displaced. The quality of education had deteriorated so much that there was a crisis in the system.

The building of a national education system has been one of the most important tasks since independence. In this regard, an effort has been made by the Ministry of Education (MoE) to gradually replace the colonial education system and associated ideologies with an authentic Eritrean system.

The general objectives of the Eritrean education system, as outlined in the Government's Macro Policy, are:

- to produce a population equipped with the necessary skills, knowledge and culture for a self-reliant and modern economy;

- to develop self-consciousness and self-motivation in the population to fight poverty, disease and all the attendant causes of backwardness and ignorance;
- and to make basic education available to all (Ministry of Education, 1995:5).

The structure of the existing Eritrean education system is based on a 5-2-4-4 model, that is, five years of primary school, two years of middle school, four years of secondary school, and four years of university education. The medium of instruction for primary school is the mother tongue. However, the medium of instruction from middle school onwards is English, which is a foreign language.

In Eritrea, mathematics education has a central part in the curriculum of different areas of studies. One of the aims of teaching mathematics in Eritrean secondary schools is to provide students with tools, which they can apply in other subjects (Ministry of Education, 1999). Mathematics is regarded as a basic subject to be taught at all levels of school. Accordingly, the main goals of mathematics education in Eritrean school system are:

- to develop mathematical skill among students, which enable them to function in all practical affairs of life;
- to deepen their appreciation of the importance and role of mathematics in society;
- to develop in pupils a positive attitude towards the subject and thus enjoy learning it;
- to enable everyone to master mathematics in accordance with his/her abilities and prepare the capable ones for higher education;
- to provide pupils with mathematical tools which they can use in other subjects (Ministry of Education, 2001b:1).

1.2 Motivation of the study

There is a widespread public concern about the results being achieved in mathematical education at present. The Eritrean Secondary School Certificate Examination (ESSCE) results show that the percentage of failures is high and increasing.

In most cases, mathematics is one of the subjects with the highest failure rate at secondary school level in Eritrea. For instance, in a nation-wide study that was conducted in 2001 to evaluate grade 8 students' performance in mathematics, it was found that more than two-thirds of the students performed poorly. The performance of the students was categorized as "Good", "Moderate" and "Poor" depending on the following scores: above 75 %, between 50 % and 75 %, and below 50 % respectively. Of the students tested, 72.3 % were in poor category. Nearly a quarter (24.3 %) of the students were in moderate category and only 3.4 % were in good category (Ministry of Education, 2001c:15). The performance of the students was also analyzed with respect to two topics: namely Algebra and Geometry. In this study it was found that students performed better in Geometry than in Algebra with overall performance difference of 10.3 % (Ministry of Education, 2001c: 30).

However, as pointed out by Oyedeji (1992), the basic skill underlying all scientific and technological skills is control of the tools of mathematical structures. Algebra itself is a major area to consider in the relationship between mathematics and subjects which use it. One might, as argued by Selkirk (1982:204), characterize much of science as the attempt to understand natural phenomena by the construction of algebraic models of them. Moreover, the concepts of variable and function are the building blocks of algebra (Davidenko, 1997). Nevertheless, the notion of variable is one that gives students more than a little trouble.

Lack of understanding in mathematics can also be so easily, and so devastatingly, demonstrated that students frequently experience feelings of discomfort and react accordingly. Most often teachers underestimate the difficulty students have with concepts and skills. However, as pointed out by Richards (1982:66), children's problems with learning mathematics are in their difficulties in forming conceptual structures.

Carey (1992), citing the National Council of Teachers of Mathematics (NCTM), points out that learning to communicate mathematically is one of the goals of a mathematics curriculum. According to Carey (1992) one way to communicate mathematical ideas is through the use of symbols. Hung (1997) also asserts that one task of education is to bring about changes in students' interpretation of mathematical symbols.

1.3 Statement of the problem

The experience of teachers and a wide range of empirical research inform that children have great difficulty in understanding the algebra of generalized arithmetic (Graham and Thomas, 2000). They also stated that there are a number of conceptual obstacles to progress in algebra and one of the most important of these is the failure to understand the concept of variable. According to Graham and Thomas (2000) the concept of variable is all too rarely discussed in many classrooms where algebra is presented and yet it underpins all that students learn.

From my experience most students, as well as teachers, have negative feelings about errors and approach them as unfortunate events that need to be eliminated and possibly avoided at all times. Most often incorrect answers to a question posed by a teacher in a class are rejected and ignored until the correct one is produced. Teachers try to assign tasks that "good" students should be able to complete without making errors.

In Eritrea, teachers enter a class with predefined specific objectives. Teachers expect each student to define, compute, solve, prove, etc. at the end of a particular lesson. In such context, learning is enhanced when correct responses are rewarded and incorrect ones are not encouraged and are regarded to happen due to lack of retention and attention. As a result, students and teachers are not invited to see errors in a positive way. However, as pointed out by Borasi (1996:40), students' misconceptions are an inevitable and integral part of learning and a valuable source of information about the learning process. Borasi (1996:40) goes on to say that misconceptions are clues that researchers and teachers should take advantage of for uncovering what a student really knows and how s(he) has constructed such knowledge.

In my experience, one of the most difficult aspects of algebra for students, and one that most teachers seriously underrate, is the meaningful acquisition of the algebraic notation. Students are supposed to use symbolic algebra to represent situations and to solve problems. In doing so, students should be able to use letters or other symbols. The present study addresses one of the difficulties that students have in learning mathematics. It focuses on the misconceptions that students have in understanding algebraic variables.

1.4 Purpose of the study

In my experience of teaching I have noticed that students have problems in understanding mathematics. Research studies have shown that students have problems in understanding algebraic variables. Therefore, this study deals only with problems that students have in understanding the concept of algebraic variables.

One of the most widely accepted ideas within the mathematics education community is that students should understand mathematics. The goal of many research and implementation efforts in mathematics education has been to promote learning with understanding (Hiebert and Carpenter, 1992:65). According to them, understanding is a fundamental aspect of learning and that models of learning must attend to issues of understanding.

The purpose of the study, therefore, was to:

- Investigate students' understanding of algebraic variables.

1.5 Research question

As mentioned above, the purpose of the study was to investigate students' understanding of algebraic variables. In pursuance of the aim of the study, an answer was sought to the following question:

- What are Eritrean students' understandings of algebraic variables?

1.6 Significance of the study

The researcher is not aware of any study in Eritrea specifically concerned with the analysis of grade 8 students' understanding of algebraic variables. Therefore, it is hoped that the findings of the present study might:

- Help to determine invalid conceptions that the students hold about algebraic variables.
- Contribute to efforts aimed at identifying the difficulties that students have in interpreting algebraic variables.
- Serve as a resource in devising teaching methodologies in the Eritrean context when dealing with algebraic variables.
- Provide base line data for further research surrounding the concept of algebraic variables.

1.7 Limitations of the study

- There were financial and time constraints to reach all secondary schools in Eritrea. As a result, only secondary schools of one province that the researcher could access easily were chosen for the study.
- Since the data were collected in secondary schools of one province, the results of the study could not necessarily be generalized to all schools in the country.

- The study used a test as an instrument for data collection. However, understanding of the concept of algebraic variables may not be evidenced by test performance alone since the correct answers may be the result of incorrect understanding.
- Even though a standardized test was used, a comparison by grades was not done to see whether or not the problem persists with students at higher levels (grades) of secondary school.

1.8 Organization of the study

This chapter has dealt with the background of the study and addressed the statement of the problem.

Chapter 2 deals with the literature review of students' understanding of algebraic variables. It focuses on students' misconception of algebraic variables and the framework developed by Kuchemann (1980) in investigating students' understanding of algebraic variables.

Chapter 3 provides details on the methods and instrument used to collect the data for the study. It also contains the procedures for collecting the data.

Chapter 4 includes the data presentation, analysis and discussion of the study. In this chapter the data collected, using the test as an instrument, are presented and analyzed.

Chapter 5 discusses the overall conclusions and recommendations of the study.

Chapter Two

Literature Review

2.1 Introduction

This chapter deals with the literature on students' understanding of algebraic variables. It describes the definition and historical background of variables. It focuses on students' misconceptions of algebraic variables in relation to their origins. It deals with students' interpretation of letters in algebra and the way they conceive the letters when dealing with algebraic expressions and equations. It also discusses the framework for students' understanding of algebraic variables, which was developed by Kuchemann (1980).

2.2 Definition and historical backgrounds of variables

2.2.1 Definition of variable

The meaning of a variable is variable (Schoenfeld and Arcavi, 1988) and the notion of variable is not straightforward as it has a wide variety of uses in algebra. Defining a variable is extremely difficult and using the term differently in different contexts makes it hard for students to understand. Based on literature, Schoenfeld and Arcavi (1988:421-422) have listed ten different definitions of variable. These include:

1. Latin- variabilis: 'changeable'

2. a. A quantity that may assume any one of a specified set of values
   b. A symbol in a mathematical formula representing a variable: placeholder

3. Variable quantities ... are such as are supposed to be continually increasing or decreasing: and so do by the motion of their said increase or decrease, generate lines, areas or solidities.

4. A quantity or force which throughout a mathematical calculation or investigation, is assumed to vary or is capable of varying in value.

5. A variable is a symbol that can be replaced by any element of some designated set of numbers (or other quantities) called the domain of the variable. Any member of the set is the value of the variable. If the set has only one member, the variable becomes a constant. If a mathematical sentence contains two variables related in such a way that when replacement is made for the first variable the value of the second variable is determined, the first variable is called the independent variable, and the second is called the dependent variable.

6. A general purpose term in mathematics for an entity, which takes various values in any particular context. The domain of the variable may be limited to a particular set of numbers or algebraic quantities.

7. Variables, which are usually represented by letters, represent an empty space into which an arbitrary element (or its symbol) from a fixed set can be substituted....Variables are useful in two ways: they make it easy to state laws, and the solution of a problem expressed in terms of variables yields the result for arbitrarily many individual cases without new calculations, by mere substitution.

8. [A] variable is a letter or a string of letters used to stand for a number.... At any particular time, a variable will stand for one particular number, called the value of the variable, which may change from time to time.... The value of a variable may change millions of times.... [W]e will associate with each variable a *window box*. The associated variable is engraved on the top of each box, and inside is a strip of paper with the *present value* of the variable written on it. The variable is a name for the number that currently appears inside.

9. A variable is a named entity possessing a value that may change during execution of the programme. A variable is associated with a specific memory location and the variable's value is the content of that memory location.

10. [A] any symbol whose meaning is not determinate is called a variable, and the various determinations of which its meaning is susceptible are called *values* of the variable. The value may be any set of entities, propositions, functions, classes or relations, according to circumstances. If a statement is made about 'Mr. A and Mr. B,' 'Mr. A' and 'Mr. B' are variables whose values are confined to men. A variable may either have a conventionally assigned range of values, or may (in the absence of any indication of the range of values) have as the range of its values all determinations which render the statement in which it occurs significant.

Leinhardt, Zaslavsky and Stein (1990) gave two interpretations of a variable. They note that the first interpretation of a variable is "a relatively static one, which emphasizes a variable as a tool for generalization or for describing patterns"; while the other is a more dynamic, "which in essence captures the variability and simultaneous changes in one variable in comparison to another" (Leinhardt et al., 1990:22).

Freudenthal, cited in Kaput (1991:67), describes two different forms of numerical variation that can be expressed algebraically: namely polyvarent name and variable object. According to Kaput (1991) polyvarent name, on one hand, involves membership in a class, as when a letter is used to describe a general rule that holds to all particular instances of values of the letter. For instance, in commutative property of addition, $a + b = b + a$, the letters a and b are polyvarent names. Variable object, on the other hand, involves systematic variation as when one writes as x approaches 0, $x \to 0$. Here x is a variable object. Kaput (1991:67) goes on to say, "variable objects, because they seem to supply some of the variation themselves, seem easier to understand than polyvarent names, which also came later in the history of mathematics".

By including virtually every literal symbol in their definition of variable, as pointed out by Philipp (1992a), mathematicians, mathematics educators, and textbook writers, have operationalized a definition that discriminates too little to be of any use to students learning algebra. Philipp (1992a) goes on to say that it would not be productive to redefine variable, but educators need to come to terms with different ways variables are used in mathematical contexts so that students can be given an opportunity to reflect on these many different uses.

2.2.2 Historical background of variables

Philipp (1992a) points out that the notion of a variable representing a varying quantity was first introduced by the inventors of the infinitesimal calculus, Gottfried Wilhelm Leibnitz (1646-1716) and Sir Isaac Newton (1643-1727). This concept of a variable was closely related to the development of the concept of function. In fact, it was Leibnitz who introduced the terms function and variable (Kline, cited in Philipp 1992a). Leinhardt et al. (1990) also assert that the historical development of the variable concept is closely connected with the development of the function concept.

Bergamini, cited in Leinhardt et al. (1990:42), noted that the development of the Cartesian coordinate system captured and made manageable the changing relationships between interconnected quantities and, in the process gave rise to the ideas of variables and functions.

> If an x and a y can be related through an equation or graph, they are called "variables": that is, one changes in value as the other changes in value. The two have what is known as a functional relationship; the variable whose change of value comes about as a result of the other variable's change of value is called a "function" of that other variable (Leinhardt et al. 1990: 42).

The close link between the concept of variable and the concept of function continued throughout the first half of the twentieth century, as can be seen in the following representative definition: "Related numbers that change together are called variables. When one variable depends on another for its value, we say that it is a function of the other" (Upton, 1936:239, cited in Philipp 1992a).

However, later the definition of a variable has changed as that of a function changed (Leinhardt et al., 1990). The modern way of defining a function emphasizes the set theoretic form that mostly depends on the notion of a domain (Leinhardt et al. 1990; Schoenfeld and Arcavi, 1988). Dolciani et al., cited in Philipp (1992a), defined variable as, "a symbol which may represent any of the numbers of a specified set, called the replacement set or domain of the variable". Consequently, variable was no longer associated with a function but became associated with a set.

Philipp (1992a) points out that the mathematics reform movement in the late 1950s and early 1960s brought about a considerable change in the definition of a variable, a change that continues to dominate today. In response to the search for unifying concepts in the mathematics curriculum, the concept of a variable was taught in its most general form right from the start, resulting in all literal symbols being referred to as variables.

In a study of mathematics textbooks published between the late 1950s and the early 1980s, it was found that almost every textbook either explicitly or implicitly defined a variable as consisting of a symbol standing as a referent for a set consisting of at least two elements (Philipp, 1992a). According to Philipp, almost all uses of literal symbols were variables. He states, "the literal symbol x in the statement $x + 3 = 7$ is a variable, because x represents any of the elements of the set in the unstated but implicitly assumed domain, be it the real numbers, the rational numbers, the integers, the natural numbers and so forth" (Philipp, 1992a:557). In this regard, as long as that domain has at least two elements, x is a variable. Therefore, the only non-variables would be either numerals or symbols standing for specific numbers, such as the base of the natural logarithm, *e*; the speed of light, c; and pie, π.

2.3 The many uses of literal symbols

> *It is often said that mathematics is a symbolic language.... The symbol of mathematics, like the letters or characters in other languages, form the written language of mathematics* (Usiskin, cited in Rubenstein and Thompson 2001:265).

The development of a concise, powerful symbolic language is one of the greatest contributions of mathematics and the concept of variable lies at the heart of mathematical language (Hirsch and Lappan, 1989). Variables are the basic tool for expressing generalizations. An understanding of the concept of variable is fundamental to students' success with algebra. Yet this concept is more sophisticated than is realized and is often the stumbling block in students' algebraic development (Leitezel, 1989:29). According to Rubenstein and Thompson (2001), symbolism is one of the common features of mathematics and mathematical symbols are that by which writing mathematics and communicating mathematical meaning are brought about.

Philipp (1992a) notes that much of the difficulty students encounter with variables may be related to their inability to recognize the correct role of literal symbols. In line with this, Graham and Thomas (2000:266) highlight, "...one reason that algebra is hard is the wide variety of ways in which letters- or 'literal symbols'- have been used in algebra and the sophisticated and multi-faceted nature of the concept of variable". They also state, "This flexibility is important in mathematics, while important in mathematics, does indeed make life very hard for students, who often adopt their own multiple interpretations of variables which do not correspond to the meanings conventionally used in mathematics" (Graham and Thomas, 2000:266).

The differing uses of letters in algebra also represent a difficult challenge for children to fulfill. In algebra the same letter can be used to represent different numbers within different situations; different letters in the same situation can represent the same number; a letter can also represent a whole class of numbers; and, what seems the most difficult part in all this is that these letters can represent unknown or unspecified numbers.

Algebra involves the use of letters, along with formula rules for operating on these letters. Letters are used in algebra in several different ways; however, as argued by Kieran (1991), two uses predominate. According to Kieran (1991) one of these is the use of letters to represent a range of values, as in the expression of a general solution or the generalization of a number patterns (e.g., $3t + 6$); the other use of letters as unknowns is in equation solving, say for example, $n + 5 = 17$.

Philipp (1992a) points out that context is important in determining the role of a literal. For instance, consider the exponential equation $A = Pe^{kt}$, used to determine the amount of money one would have after time t if P dollars, deposited into an account, were compounded continuously at interest rate k per unit time. This equation, as pointed out by Philipp (1992a), uses literal symbols in three different ways: as constants (e), as quantities that vary (t, A), and as parameters (P, k).

Moreover, as argued by Philipp (1992a), students must not only learn to work with many types of literal symbols in one problem, as in the earlier example involving the exponential ($A = Pe^{kt}$), but they must also learn that a given literal symbol may take on more than a single role within a given problem. For instance, in solving the equation

$2x + 3x - 8 = 25$, at first glance this approach appears to treat a literal symbol as unknown. However, closer inspection indicates that implicit in this solution is the statement $2x + 3x = 5x$ regardless of the value of x, which involves using literal symbols as generalized numbers.

In addition to being used as constants, as parameters, and as varying quantities, literal symbols are also used as unknowns, as generalized numbers, and as abstract numbers (Philipp, 1992a). An unknown involves the use of literal symbol when the goal is to solve an equation. Generalized numbers refer to the use of literal symbols when all replacement values of the literal symbols will result in a true statement, as with identities or other properties of numbers (commutative, distributive and so on).

According to Philipp (1992a), literal symbols can be used as:

- Labels: f, y in $3f = 1y$ (3 feet in 1 yard)
- Constants: e, c
- Unknowns: x in $5x - 3 = 8$
- Generalized numbers: a, b in $a + b = b + a$
- Varying quantities: x, y in $y = 9x - 2$
- Parameters m, b in $y = mx + b$
- Abstract symbols e, x in $e * x = x$ (e is an identity for the operation $*$)

Our day-to-day numeration system uses symbols for numbers, commonly referred to as numerals (Speer, Hayes and Brahier 1997). In addition to using these numerals, as argued by Speer et al. (1997), an algebraic system also needs open-ended symbols, known as variables, to represent quantities in different way. These variables often occur in such contexts as formulas, equations and properties of numbers.

The use of letters to represent a range of values is far more neglected in the teaching prealgebra than their use as unknowns (Kieran, 1991). Nevertheless, literal symbols may be used to generalize patterns, to stand for unknowns or constants, and to represent the parameters of a situation (Speer et al., 1997).

As students have little experience in using algebraic symbolism as a tool with which to think about and to express general relations, they encounter difficulty with these uses of letters. In a study on students' conceptions of generalization and justification, Lee and Wheeler, cited in Kieran (1991), note that only 8 percent of the students they interviewed could use algebraic notation for such problems as the following:

> A girl multiplies a number by 5 and then adds 12. She then subtracts the original number and divides the result by 4. She notices that the answer she gets is 3 more than the number started with. She says, "I think that the same thing would happen, whatever number I started with". Using algebra, shows that the girl is right.

In this study most of the students worked out numerical examples and concluded that the girl was right from these examples. Students do not appear to see algebra as generalized arithmetic and they even do not believe that arithmetic can be generalized.

Graham and Thomas (2000) suggest that the use of literal symbols to generalize arithmetic relationships is too sophisticated to be useful as an introduction to the idea of variable. According to them, students find difficulty with this more sophisticated view of the role of symbols (for example, the notion of expressing generality from tables of values) but they are more likely to achieve this if they have a firm foundation of 'letters as placeholder'. In other words, children need to see letters as capable of representing numbers before they begin to use letters to generalize patterns of numbers.

English and Warren (1998) point out that patterning activities offer a meaningful introduction to early algebraic ideas. However, they added, they can present difficulties for students who lack the requisite skills and knowledge of processes. According to them therefore, flexible, articulate thinking and an understanding of equivalence are particularly important in the students' success with this approach.

English and Warren (1998) offered the following recommendations for ensuring that these prerequisites are in place.

- Provide experiences in verbalizing numerical relationships and relating these relationships to the symbolic form.
- Help students progress from a recursive approach to an explicit approach.
- Provide experiences in completing a pattern and in detecting patterns within patterns.
- Provide supplementary experiences in simple manipulation of algebraic expressions.
- Relate algebraic expressions to concrete contexts.
- Balance the use of visual patterns and tables of data.

As students progress from year to year in mathematics, as argued by Rosnick (1981), the letters they use, like the concepts they are learning, become increasingly abstract and ambiguous to them. According to Rosnick (1981), it is important to stay aware of the difficulties that students are having in trying to understand labels, variables, constants, parameters, and all the rest of the uses of letters.

2.4 Students' misconceptions of variables

A number of research studies have shown that the interpretation of algebraic expressions, particularly of the letters used in the algebraic code, is not an easy matter for many children (Kuchemann, 1980; Oliver 1989). From a range of different perspectives and emphases these studies have found that the majority of 15-year-olds appear to be unable to interpret algebraic letters as generalized numbers or even as specific unknowns.

Large-scale studies have documented that a great proportion of students ignore the letters, replace them by numerical values, or regard them as short hand of names or measurement labels. For instance, a study conducted by Kuchemann (1980) revealed that most students in a large sample were unable to cope with items that required interpreting letters as generalized numbers or specific unknowns.

Kuchemann (1981) studied different students' conceptions of letters in algebra with a sample of 3000 British students (13-15 year olds). The study produced results that showed that 73% of the 13 year olds, 59% of the 14 year olds, and 53% of the 15 year olds, dealt with letters in expressions and equations as objects; few were able to consider letters as specific unknowns; and fewer as generalized numbers or variables. Letter as object, according Kuchemann (1982), refers to the use of a letter as a shorthand for an object or as an object in its own right, rather than as unknown or general number.

Moreover, the study showed that students' misunderstanding of letters seem to be reflected in their approach to symbolizing the relevant relationships in problem solutions. In this respect Kuchemann (1981) reported that all students were asked the question:

> Blue pencils cost 5 pence each and red pencils cost 6 pence each. I buy some blue and some red pencils and altogether it costs me 90 pence. If b is the number of blue pencils bought, and r is the number of red pencils bought, what can you write down about b and r?

It was found that the percentages of correct responses within each age group were 2%, 11%, and 13% respectively. The most common response was $b + r = 90$. This mistake suggests a strong students' tendency to conceive letters as labels denoting specific sets, which seems to be a result of the students' attempt to accommodate their previous arithmetic experience with letters (which emphasizes on like and unlike terms) to the new assigned letters within an algebraic context.

The fact that students view a letter as an object, as pointed out by Vergnaud (1985:32), is typically reinforced by the practice of algebra, where one adds monomials: the letters do not behave as numbers. There is clearly a shift to handling signifiers as physical objects. This is the source of many misunderstandings in the students' mind, particularly because there is no sign for multiplication: so 3c is viewed as '3 something'.

A considerable research effort has been made to investigate the underlying reasons in children's difficulties to cope with the new use of letters introduced in the study of algebra. Perhaps, as argued by Del Arte (1996), the most immediate explanatory frame is provided by children's arithmetic experience in elementary school. For instance, whilst in arithmetic children have experienced that letters can denote measurements, for example 10m to denote 10 meters, in algebra such expression may denote 'ten times an unspecified number'. Traditionally, children's experience with letters in elementary school is restricted to equations such as A = l x w, which seems to reinforce the arithmetic use of letters as labels (l for length and w for width). This interpretation of letters as measurement labels seems to explain the students' tendency to treat numerical variables as if they stood for objects rather than numbers.

The use of letters as objects is a very effective way of reducing the difficulty of certain algebra problems (Kuchemann, 1980, 1981, 1982; Bell, Costello & Kuchemann 1983). However, as argued by Bell et al. (`1983), the continuing tendency to regard letters as symbols for objects rather than numbers appears to be a significant stumbling block in learning algebra.

Studies have shown that symbolizing the relationships in a problem situation is not only difficult for the novice students. Clement, Lockhead and Monk (1981) investigated the responses of 150 freshman-engineering students to the Students-Professors problem:

> Write an equation using the variables S and P to represent the following statement: there are six times as many students as professors at this university. Use S for the number of students and P for the number of professors.

The results showed that only 63% of these students could correctly solve the problem, where 68% of the errors consisted of the reversal situation (6S = P instead of 6P = S).

Philipp (1992a) notes that interviews with individual students (preservice teachers and high school students) have indicated that many students who have written the reversed equations have been treating the literal symbols as labels instead of as quantities. This error comes from the fact that P refers to "professor" and not to the "number of professors," S to "student" and not to the "number of students." So the equals sign is interpreted as a correspondence rather than an equality.

Rosnick (1981) points out that students need to develop a better understanding of the concepts of variable and equation. More specifically, pupils should be able to distinguish between different ways in which letters can be used in equations. They should learn to distinguish when letters are used as labels referring to concrete entities or, alternately, as variables standing abstractly for some number or number of things.

Booth (1988:27) reported the response of fifteen-year-old Peter when asked what the y refers to in the task "add 3 to 5y." Peter responded that y "could be anything.., a yacht. Could be yogurt. Or a yam". The symbol was seen as a label for something, the name of which begins with the letter y, not as a symbol representing a quantity that could be manipulated. In line with this, Philipp (1992b) points out that using initial letters could confound the symbolizing issue in algebra.

Students may not use symbols to represent the intended referent, yet a shift in referent occurs even when the students state the relationship themselves (Kinzel, 1999). Rosnick cited by Kinzel (1999) characterized students' use of symbols as labels for "broad, undifferentiated concepts". For example, a student may have written "b = books" but then shifted between interpreting b as the number of books, the value of a book, the value of the total number of books, and so on, while working on the task.

Herscovics and Kieran (1980) note that the conceptual difficulties involved in learning algebra are greater and more widespread than is commonly believed. Asking twelve-to-seventeen-year old students if different solutions would be obtained from

$$7w + 22 = 109$$

and $$7n + 22 = 109$$

Wagner (1977), cited in Hercovics and Kieran (1980), received quite a variety of answers: "The solution of the first one is greater than the second one because w comes later than n in the alphabet"; "can't tell until both equations are solved"; "of course, the solution is the same". Those students who gave the first and the second answers were unable to understand the fact that different letters in the same situation can represent the same number. In other words, it seems quite clear that these students did not realize that the value of the unknown was independent of the letter used. Wagner (1981) also found similar results with even older high school students with average age of 16 years. The students seemed to react to such exercises in one of two ways. Either they would accept the change of variable name and state that the letter makes no difference as long as the numbers stayed the same, or they would regard the change of variable name as producing a completely new problem.

Wagner (1983) points out that the confusion between the linear order of the alphabet and the linear ordering of whole numbers is a common mistake for students who are just being introduced to literal symbols. She noted that when a teacher asked a student how to represent an unknown integer x when added with 1, the response was "y" without hesitation. This is because the letter next to x is y. MacGregor and Stacey (1997) also found that pre-algebra students had a tendency to assign values to algebraic variables, often turning to other symbol systems to supply that meaning. For instance, when given h as a symbol for the height of a boy, some students assigned h the value of 8, since h is the eighth letter of the alphabet. That is, some students associate letters with numbers according to the position in the alphabet. MacGregor and Stacey (1997) suggest that this is due to the students' experience of puzzles and translation into codes activities. This indicates that students bring ideas from other domains (not necessarily mathematical domains) into the realm of algebra.

As discussed earlier, children find great difficulty in understanding algebra. For a child meeting algebra for the first time, as pointed out by Tall and Thomas (1991), there are various obstacles that must be confronted and resolved. These include: the *parsing obstacle*, the *expected answer obstacle*, the *lack of closure obstacle*, and the *process-product obstacle*.

The *parsing obstacle* is the difficulty of unraveling the sequence in which the algebra must be processed, conflicting with the sequence of the natural language. It is easily noticed in various ways. For instance, a child may consider that ab means the same as a + b, because s(he) reads the symbol ab as a and b, and interprets it as a + b. Or the child may read the expression 2 + 3a from left to right as 2 + 3 giving 5, and consider the whole expression to be the same as 5a.

Booth (1988) reports on data collected for strategies and errors in secondary mathematics project, which was conducted in the United Kingdom from 1980 to 1983 with eighth- to tenth-grade students who had started studying Algebra in seventh grade. Booth (1988) concludes that students tend to view the notation as a label for some concrete object and are thus unwilling to accept algebraic expressions as answers. Kieran (1992) also asserts that students' tendencies to interpret algebraic letters as specific unknowns contribute to their difficulties with viewing expressions as objects.

The fact that children become accustomed to working in mathematical environments where they solve problems by producing a numerical 'answer', as pointed by Tall and Thomas (1991), leads to the expectation that the same will be true for an algebraic expression. According to them, whilst an arithmetic expression such as 2 + 3 is successfully interpreted as an invitation to compute the answer 5, the algebraic expression 3 + 2a cannot be calculated until the value of a is known. This unfulfilled and erroneous expectation was termed as the *expected answer obstacle*. This obstacle causes a related difficulty, which is termed as the *lack of closure obstacle*, in which the child experiences discomfort attempting to handle an algebraic expression which represents a process that s(he) cannot carry out.

Another closely related dilemma is the *process-product obstacle*, caused by the fact that an algebraic expression such as 2 + 3a represents both the *process* by which the computation is carried out and also the *product* of that process. To a child who thinks only in terms of process, as argued by Tall and Thomas (1991), the symbols 3(a + b) and 3a + 3b are quite different, because the first requires the addition of a and b before multiplication of the result by 3, but the second requires each of a and b to be multiplied by 3 and then the result is added.

Gray and Tall (1994) explain that the cognitive complexity of "process-concept duality" can be replaced by the notational convenience of the "process-product ambiguity"; we can set aside the difficult task of specifying whether we intend the process or concept through a flexible use and interpretation of the notation. However, this ambiguity is rarely discussed explicitly in mathematics classrooms, and it therefore remains an implicit awareness for those who are able to shift their attention and a mystery to those who are not. We can think of $2a + 3$ either as the process of multiplying a number by 2 and adding 3 or as the result of that process. The expression itself can indicate the operation to be performed on objects, that is, actual quantities; or it can be operated on as an object itself, that is, used as input into another operation, such as adding expressions (Kinzel, 1999).

MacGregor and Stacey (1997) point out that the difficulties that students are having in learning to use algebraic notation have several origins, including:

- Intuitive assumptions and sensible, pragmatic reasoning about an unfamiliar notation;
- Analogies with symbol systems used in everyday life, in other parts of mathematics or in other school subjects;
- Interference from new learning in mathematics;
- Poorly-designed and misleading teaching materials.

According to Stacey and MacGregor (1997) students' first attempts to interpret and use algebraic letters are usually based on sensible reasoning and draw on a range of previous experiences. They are often thoughtful attempts to make sense of a new notation. Students are told that in algebra, letters stand for numbers. However, they see letters used with other meanings. Letters are used in many contexts, both within and outside of mathematics, as abbreviated words or as labels: "p.6" means "page 6"; and $\angle ABC$ labels an angle in a geometric figure, with the letters A, B, and C denoting points. Quantities are frequently denoted by the initial letters of their names- m as the "mass" and t as the "time taken and so on.

New learning in mathematics could also contribute to the difficulty in learning to use algebraic notation for adult students. Students write x times 4 as x^4, because they think of exponents as an instruction to multiply. Stacey and MacGregor (1997) point out that some students believe that any letter alone stands for 1. One likely cause of this belief, as argued by Stacey and MacGregor (1997), is a misunderstanding of what teachers mean when they say "x without a coefficient means 1x". The student gets a vague message that the letter x by itself is something to do with one. Other sources of confusion for older students are the facts that the power of x is 1 if no index is written ($x = x^1$) and that $x^0 = 1$.

Bell, Costello, and Kuchemann (1983:134) suggest that younger children have a limited understanding of equal signs. Many seven to eight-year-old children do not interpret '=' as 'is the same as' but rather as 'makes' or 'gives'. Thus, children who can solve 4 + = 7 may not be able, at a certain stage, to make any sense of 7 = 4 + . The greater difficulty of the second form persists with some pupils into the secondary school years. Students' interpretation of equations, as noted by Stacey and MacGregor (1997), can be influenced by prior experiences in arithmetic. Their background of arithmetic has been built on a foundation in which the equals sign means "gives" or "makes," as in " 3 plus 5 gives 8".

In a mathematical equation, the signals for ordering are not those of ordinary language. They include parenthesis and more subtle signals that must be deduced from knowledge of formal rules for the precedence of operations. Natural-language rules are of no help in reading mathematical expressions. Another obstacle arising from a false analogy with ordinary language is students' expectation that any procedures they can think about or talk about can be written in simple algebra (Stacey and MacGregor, 1997). As a result, they have difficulty in generating formulas from number patterns and tables.

Traditionally, as pointed out by English and Warren (1998), a students' first encounter with a variable has been in an equation, where it represents an unknown. Edwards (2000) points out that a focus on solving equations, which is often presented too early, can mask the true nature of the concept of variable. Finding solutions to first degree equations clearly involves finding the one value for the variable, out of infinitely many possibilities, that makes the open sentence true; but the idea of "infinitely many possibilities" is often lost on beginning algebra students.

Stacey and MacGregor (1997) also assert that students bring a variety of experiences to their interpretation of beginning algebra. In sum, these experiences include the following:

- The many uses of letters in other contexts;
- Operations implied in composite symbols, such as 5 ½ , 53, and vii;
- Reading the equals sign as "makes" or "gives" and using it to link parts of a calculation;
- Features of natural language, such as indicators of temporal sequence, that students assume carry over into the formal language of algebra; and
- New learning, such as the concept of powers and its notation, can destabilize old knowledge that is not secured.

2.5 A framework for students' understanding of variables

In investigating English students' understanding of algebraic variables, Kuchemann (1980:49) has identified six different ways in which children interpret the letters in generalized arithmetic. These six categories (meanings given to letters) include:

1. Letter evaluated
2. Letter not used
3. Letter used as object
4. Letter used as a specific unknown
5. Letter used as a generalized number
6. Letter used as a variable

Letter evaluated. This category applies to responses where the letter is assigned a numerical value from the outset.

Letter not used. Here the children ignore the letter, or at best acknowledge its existence without giving it a meaning.

Letter used as an object. The letter is regarded as shorthand for an object or as an object in its own right.

Letter used as a specific unknown. Here children regard a letter as a unique (specific) but unknown number, and they can operate on it directly.

Letter as a generalized number. The letter is seen as representing, or at least as being able to take, several values rather than just one.

Letter used as a variable. The letter is seen as representing a range of unspecified values, and a systematic relationship is seen to exist between two such sets of values.

Kuchemann (1981:104) pointed out that the interpretation of letters that children chose to use depended in part on the nature of the questions and also on the questions' complexity. Generally, Kuchemann remarked, "The first three categories indicate a low level response, and it can be argued that for children to have any real understanding of even the beginnings of algebra they need to be able to cope with items that require the use of a letter as specific unknown, at least when the structure of such items is simple" (p.105). Despite this fact, most children (13, 14 and 15 year olds) were not able to do this consistently and used one of the first three interpretations instead. Very few children reached the high degree of understanding required to interpret a letter as a variable.

Apart from these six categories, Kuchemann (1980:64) classified children's responses and the items themselves into four different categories what in his term is called "levels of understanding." The four levels are:

Level 1. The items at level 1 are purely numerical, or they have a simple structure and can be solved by using (treating) the letters as objects.

Level 2. The clear difference between these items and those of level 1 is their increased complexity, though the letters still only have to be evaluated.

Level 3. This level signals a major step forward, in that for the first time letters are genuinely being used as unknown numbers (specific unknown). Children at this level can only cope with specific unknowns when the structure of the item is simple.

Level 4. At this level children can cope with items that require specific unknowns and which have a complex structure. They can also engage with items that require the use of letters as general numbers and as variables.

The items at levels 1 and 2 can be solved without having to operate on letters as unknowns, whereas at levels 3 and 4 the letters have to be treated as specific unknowns, generalized numbers or variables. The difference between level 1 and level 2, and between level 3 and level 4 is essentially a matter of complexity (Kuchemann, 1981:69).

Based on two criteria, namely the structural complexity of the items and the meaning that can be given to the letters, Kuchemann (1980) has developed a procedure for investigating students' understanding of algebraic variables as discussed above. In investigating Eritrean students' understanding of algebraic variables, the present study uses this procedure basically as a framework, though the students' performance is classified into five 'levels of understanding. The way the performances are classified will be discussed in chapter 4.

2.6 Conclusion

The concept of variable is central to mathematics teaching and learning in junior and senior high schools. Understanding the concept of a variable provides the basis for the transition from arithmetic to algebra and is necessary for the meaningful learning of all advanced mathematics. The role of variables is imperfectly understood. Though it is the building block for all abstractions in mathematics, its meaning escapes many students. The meaning of a variable is variable; using the term differently in different contexts can make it hard for students to understand.

Students have problems in performing arithmetic operations on algebraic expressions. Perhaps this is partially due to the difficulties that they have in conceiving a letter as representing a number. Children's interpretations of letters are classified into six categories. They are described as letter evaluated, letter not used (or sometimes ignored), letter as object, letter as specific unknown, letter as generalized number and letter as variable. Broadly speaking, as argued by Costello (1991:37), the last three categories describe legitimate ways in which algebraic letters are used, in three main activities of solving equations, expressing general laws of arithmetic, and studying functions and their properties. The first three categories indicate ways in which children may interpret letters to avoid the formal theoretical understanding implicit in the topic.

As noted by Oliver (1989:25), "From a constructivist point of view, pupils' misconceptions are never arbitrary or altogether unreasonable". Students' misinterpretations of algebraic letters have some origins. Students frequently, as argued by MacGregor and Stacey (1997), base their interpretations of letters and algebraic expressions on intuition and guessing, on analogies with other symbol systems they know, or on a false foundation created by misleading teaching materials. They are often unaware of the general consistency of mathematical notation and the power that this provides. Their misinterpretations lead to difficulties in making sense of algebra and may persist for several years if not recognized and corrected.

This chapter has dealt with the misconceptions and difficulties that students are having in learning the concept of algebraic variables. The next chapter will focus on the methods of research that were used to investigate the problem stated.

Chapter Three

Research Methodology

3.1 Introduction

In the previous chapter, the literature review concerning the many uses of algebraic letters, students' misconceptions of variables, conceptual difficulties in algebra and students' interpretation of literal symbols were discussed. The literature review was used to reveal sources of data for the researcher and to help develop new ideas and approaches to answer the research question.

This chapter deals with the details of research approaches underpinning this study. It examines the research methods and techniques employed in data collection of the present study. The relevant data collected through the instrument [Algebra test developed by Kuchemann (1980)] employed in this study are the main source of information to answer the question under investigation.

3.2 Research methods

Begle (1980:7) defines research as "disciplined or systematic inquiry concerning a certain event or events in an effort to further and/or verify knowledge". According to Begle (1980) research encompasses systematic investigation and experimentation based on hypotheses generated from previous studies, conjectures, and experience. Theories are built and revised in light of new knowledge that facilitates understanding the why, how, and what of mathematics education. Practical applications of those new or revised theories are then made.

Some disciplined inquiries belong to the category of quantitative, empirical studies, for example, surveys and controlled experiments. Other disciplined inquiries belong to the category that includes logical analyses of writings of the past, philosophical treatments, and organized personal experiences, for example, historical studies, philosophical studies, and case studies (Begle, 1980:7).

According to Goodman and Alder (1985), cited in Fraenkel and Wallen (1993:408), the rationale for "choosing one methodology over the other is connected to the nature of the subject studied and the underlying goals of the research". Likewise, Bell (1993:6) points out that the nature of the research inquiry and the type of information required, influences both the approach the researcher adopts and the methods of data collection used.

This study falls within the exploratory and descriptive research paradigm. It is exploratory in that there were no baseline data to fall upon this being the first attempt in Eritrea. The choice for descriptive research was to gather relevant data that could serve as basis for a more in-depth study of issues surrounding algebraic variables.

Descriptive research, as pointed out by Gay (1981:153), involves collecting data in order to test hypotheses or answer questions concerning the current status of the subject of study. According to Wolpert (1981) descriptive research is sometimes assigned an inferior status particularly by over enthusiastic proponents of experimental research. This is unfortunate because descriptive research does serve several important functions viz.:

- It can provide useful information for decision-making.
- It could provide baseline data upon which hypotheses can be posited for testing through an experimental research.
- It may provide a substitute for experimental research in situations where economic, logistic, or ethical considerations make experimental research impossible (Wolpert, 1981).

3.3 Survey research method

There is a general lack of consensus as to what constitutes pure empirical research methodology as distinct from mere data-collecting techniques. However, there is general agreement that surveys constitute a distinctive empirical social research method.

A survey is an attempt to collect data from members of a population in order to determine the current status of that population with respect to one or more variables (Gay, 1981:155). Johnson (1980:21) also asserts that survey research includes status, or correctional studies. Determining the current status of the population, according to Gay (1981:155), may involve assessment of a variety of types of information such as attitudes, opinions, characteristics, and demographic information.

According to Johnson (1980:21) survey research is generally undertaken to provide a descriptive picture of a situation without attempting to relate cause and effect. The major purpose is to establish norms and baseline data for consideration by researchers and practitioners in making their decisions, to help raise relevant questions, or to identify needed research (Johnson, 1980:21).

Survey as a research strategy in social sciences has a number of features. Steinberg and Philcox (1983:132) suggest that a survey has the following features.

- It is a distinctly quantitative method that involves the planned collection of data from or about subjects in a standardized format, as a guide to future actions, or with the intent of correlating relationships between variables.
- Surveys are usually conducted at a given time or over a given period.
- The coverage of a survey can range from a few cases being studied intensively, to a large-scale survey in which the emphasis falls on general enumeration.
- The subjects being studied can be either individual persons or a wide range of larger entities, such as groups and organizations of different kinds. In most cases, the subjects represent a predefined universe from which a representative sample is

drawn to facilitate generalizations. On the other hand, however, volunteers can be arbitrarily selected for specific surveys.

Advantages of survey

The survey research method has a number of meritorious features as far as social research studies are concerned, viz.:

- It provides an overall perspective and facilitates generalized conclusions being drawn of human behavior.
- Its methodology provides for the rigorous testing and examination of complex situations concerning a number of hypotheses and involving several variables.
- It permits the testing of logical explanation as well as empirical verification.
- It also permits the replication of the study among subsets of a sample or with different populations at later stages (Blalock, 1970; Babbie, 1973).

Wiersma (1980:16) points out that some surveys are limited to determining the status quo. But whatever the case, Wiersma (1980) adds, survey research deals more with questions of what is, rather than why it is so. Basically, surveys deal with research questions of "what is?" with, possibly, some emphasis on attempting to explain what is. Therefore, the rationale behind choosing a survey is that the present study deals with the question "what is Eritrean students' understanding of algebraic variables"?

3.4 Data collection

There are numerous approaches to data collection in educational research, but for surveys the data collection generally falls into three categories: the personnel interview, the written questionnaire, and controlled observation (Wiersma, 1980:141). Controlled observation in its simplest form consists of collecting data from such sources as school records. It also includes collecting data using measuring instruments such as achievement tests.

A very important part of any research project is the choice of instrument by which the data are obtained (Behr, 1983:115). The present study used a test as data-gathering technique. Kerlinger (1973) cited by Behr (1983:115) defines a test as a systematic procedure in which the testee "is presented with a set of constructed stimuli to which he responds, the responses enabling the tester to assign the testee a numeral or set of numerals from which inferences can be made about the testee's possession of whatever the test is supposed to measure".

The use of tests to measure things like knowledge and ability is common. The fact that tests can have a role to play in research stems from three features of tests. Denscombe (1998:51) suggests the following characteristics that tests have in common.

- A test procedure relies on giving the same questions or the same tasks to all people. It is, in effect, administered under controlled conditions. In other words, any variation in the responses given to the test is the result of differences in task being measured, not differences in the nature of the test itself.
- In accord with experiment, there is a strong emphasis on rigorous observations and meticulous measurement as key facets of the approach. Tests involve a process of classifying, categorizing and enumerating specific traits, such as people's knowledge or skills.
- Tests are generally designed to produce results which allow comparison to be made.

Achievement tests measure the current status of individuals with respect to proficiency in a given area of knowledge or skill (Gay, 1981:124). Standardized achievement tests are carefully developed to include measurement of objectives common to many school systems. Standardized tests, as pointed out by Gay (1981:124), measure knowledge of facts, concepts, and principles.

Using a standardized test has a number of advantages and disadvantages. Denscombe (1998) suggests the following advantages and disadvantages of using standardized test as a test instrument.

Advantages of standardized test

- It is faster to use standardized test than to devise a new one.
- The standardized test will be of better quality, having been devised and checked out by experts.
- Standardized tests tend to be well tried and have a good reputation.
- Standardized tests often offer an 'objective' referent point that allows comparison within groups, between groups and over time.
- They allow wider generalizations and more certain interpretations from the results.

Disadvantages of standardized test

- They can be expensive to purchase.
- Some tests have restricted accessibility. Only those with an appropriate professional qualification are allowed to administer certain tests, a restriction aimed at making sure that proper procedures are followed and that the tests are not used or interpreted in a cavalier fashion, bringing them into disrepute.
- It is difficult to assess the extent to which the test results reflect the respondent's experience of the test situation itself rather than the specific ability or aptitude under investigation.

Standardized tests are tests that have been thoroughly trialed with a representative sample of the appropriate population (Denscombe, 1998:51). This trialing, as argued by Denscombe (1998), not only establishes the reliability of the test, but also allows the producers of the test to supply objective benchmarks against which the individual user of the test can compare his/her results. This means that the person using the test has a predefined expectation of what the 'normal' results might look like. Taking these advantages into account, the researcher makes use of a standardized test that was developed by Kuchemann (1980) in investigating students' understanding of algebraic variables to collect data.

The clarity and validity of the test was checked prior to application of the test. In general terms, validity refers to the degree to which a test succeeds in measuring what it has set out to measure (Kaplan, 1987:254). By content validity is meant how well the test succeeds in covering the field with which the test is concerned. This type of validity is not established statistically, but depends on the opinion of informed persons (Mulder, 1982:217). In regard to this, my supervisor Professor Cyril Julie have checked it and advised me to replicate the test in the Eritrean context.

3.5 Sampling

Sampling is the process of selecting a number of individuals for a study in such a way that the individuals represent a larger group from which they were selected (Gay, 1981:85). The concept of sampling, as pointed out by Burns (2000:82), involves taking a portion of the population, making observations on this smaller group and then generalizing the findings to the larger population. Generalization is a necessary scientific procedure since rarely it's possible to study all members of a defined population.

Due to financial and time constraints, it was not possible to reach all Eritrean grade eight students and include them in this study. Thus only one district was selected. The selected district is the one called Gash-Barka. In Eritrea, there are six provinces (zones). These are Anseba, Debubawi-Keih-Bahri, Debub, Gash-Barka, Maekel, and Semenawi-Keih-Bahri. The distribution of students in these provinces is not uniform. The following table depicts distribution of the students among these six districts in academic year 2000/2001.

Zone	Totals 8-11			Grade 8		
	Total	Male	Female	Total	Male	Female
Anseba	5678	3879	1799	2335	1490	845
Deb-Keih-Bahri	596	408	188	249	175	74
Debub	20036	14466	5570	7472	5103	2369
Gash-Barka	4070	2967	1103	1524	1062	462
Maekel	30771	16546	14225	11161	5697	5464
Sem-Keih-Bahri	2800	2089	711	1192	863	329
Total	63951	40355	23596	23933	14390	9543

Table 3a. Secondary education enrolment: Grades and gender by zone (Ministry of Education, 2001a:65).

As can be seen from table 3a, provinces like Maekel and Anseba have more students. Being a novice researcher it was difficult to include and manage provinces with such a big number of students in the study, though I live in Maekel. Thus Gash-Barka was selected purposely for two reasons. First, this province has a number of students that could be manageable financially and mentally. Second, I have been assigned to and worked in two schools of Gash-Barka as a mathematics teacher. Hence, I am familiar with the schools and staff members of the schools located in this province.

There are five secondary schools in Gash-Barka. These are Agordat Secondary School, Daeros Secondary School, Duta Secondary School, Jebel-Hamid Secondary School, and Sagem Secondary School. A complete count survey was made and all grade eight students of each school participated in the study. The following table shows the number of students involved in the study from each school.

School	Grade 8 Students		
	Male	Female	Total
Agordat	314	108	422
Daero	245	122	367
Duta	365	117	482
Jebel-Hamid	171	41	212
Sagem	186	104	317
Total	1281	492	1773

Table 3b. Gash-Barka grade eight students: gender by school.

3.6 Pilot survey and administering the test

3.6.1 Pilot survey

As mentioned earlier, the algebra test developed by Kuchemann (1980) was used to analyze Eritrean students' understanding of algebraic variables. This test was trialed among many students and hence standardized. It is the test that was applied in investigating students' understanding of algebraic variables among 3000 students in England (Kuchemann, 1981). Nonetheless, the researcher conducted a pilot survey for the following two reasons.

- To check the reliability of the test in the Eritrean context.
- To identify possible procedural problems that could arise in the main study and prepare remediation for these problems.

To this end, a class of 60 grade 8 students was randomly selected from a school called Adi-Tekelezan. Based on the data obtained from this class, the coefficient of reliability was calculated as 0.923. As the majority of standardized tests worth using have reliability coefficient greater than 0.9 (Mulder, 1982:215), the test proved to be reliable in the Eritrean context.

Concerning the procedure for computing the reliability coefficient, the test-retest was not used. One of the major problems connected to test-retest method is the time-span that elapses between the two administrations of the test (Mulder, 1982:209). According to him if the test is repeated too soon, then the performance of the students will improve since some testees will even remember the answers to certain items. And if the time elapsed is too long, then the testee may have devoted so much attention to work of a completely different nature that his/her knowledge of the field of testing could have receded with his/her performance becoming consequently weaker.

As the numbers of items in each question was not uniform, split-half method also was not used. Because the data for each item was not always readily available, the Kuder and Richardson KR-20 formula was also not used. However, the Kuder and Richardson KR-21 formula, for which the details of each item are not necessary, was applied. This formula, as pointed out by Mulder (1982:214), is much simpler to apply, but always yields an answer smaller than that given by KR-20.

The pilot survey yielded the following two issues. First, the pilot survey proved the test to be stressful. In Eritrea, students are accustomed to being informed at least one day ahead whenever they have a test. Due to the fact that the students had not been informed ahead about the test, the students were in stress. This helped the researcher on how to deal with regard to this problem while administering the test in the main study. This was very important, because the stress could have a negative effect on the study. Second, it helped the researcher to fix the time required for the students to attempt each and every item in the test.

3.6.2 Administering the test

As soon as I arrived in Eritrea in December 2002, I asked the Human Resources Development (HRD) office of the University of Asmara and the Ministry of Education of the state of Eritrea for a supporting letter to the schools which participated in the study (see appendices II and III). A copy of the cooperative letter was handed over to the principals of each school. I briefed the principals and teachers who took part in the study about the purpose of the study.

As mentioned earlier, the test was conducted in five secondary schools of Gash-Barka zone. An arrangement had been made beforehand with the principals of each school. Consultation with the principals of each school resulted in agreement as to when the testing would take place, under what conditions, and with what assistance from school personnel. Accordingly, the test was conducted during school hours. All students of the same school took the test at the same time. All grade eight students (with the exception of those who were absent for that day for several reasons) did the test. Besides, all grade eight teachers of each school took part in the study by invigilating the test. And the teachers were very cooperative.

An effort was also made to insure ideal testing conditions. The testing took place in all schools at different times, but under identical condition. The test was conducted in the same way as that of a regular one. However, to overcome the stress that could result due to this unexpected test, the teachers encouraged the students to attempt each and every question. The students were encouraged to write what they understood and not to worry about the results. The time allowed for the test was one hour and twenty minutes. This decision was based on the pilot survey.

Sierpinska, Kilpatrick, Blacheff, Howson, Sfard and Steinbring (1993) point out that considerations of ethics in research in mathematics education involves matters such as informed consent, confidentiality, and accurate portrayal of situations and persons involved in the research. After the papers were collected, I expressed my gratitude for the principals and teachers of these schools for their cooperation and contribution to the present study. Besides students' results are not announced and are used only for the study purpose.

3.7 Data presentation and analysis

For determining the specific content of the test and for making sense of the results two criteria were chosen just in the same way as was done by Kuchemann (1980). Based on these two criteria, namely the structural complexity of the items, and the meaning that can be given to the letters, the items were classified.

This study, being a replication of the study in investigating students' understanding of variables in the Eritrean context the procedure, developed by Kuchemann (1980) in investigating students' understanding of algebraic variables in England was used. As discussed in the previous chapter, children's uses of letters are classified into six categories. They are described as "letter evaluated", "letter not used" (or, sometimes ignored), "letter as object", "letter as specific unknown", "letter as generalized number" and "letter as variable". Apart from these six categories, children's responses and the items themselves, are classified into different 'levels of understanding'. The items are classified into four levels according to their difficulties. Based on whether the students are able to cope with about one-third of the items assigned to each level, the students' performances are classified into five 'levels of understanding'.

Based on the students' response to the items of the test and the raw scores the data is discussed in terms of students' conceptions, misconceptions, approaches, strategies and use of algebraic variables with facilities (percentages). Finally, the data is analyzed in terms of empirical themes and literature review quantitatively.

Chapter Four

Data presentation, Data analysis and Discussion

4.1 Introduction

This chapter deals with the data presentation, data analysis and a discussion of the present study. It discusses firstly, the classification of the Eritrean grade eight students' performances into different 'levels of understanding'. It secondly focuses on the overall performance on the items for the different levels. Thirdly, it describes the strategies and interpretation of letters used by students. Finally, this chapter compares the students who took part in the study by both school and gender.

Kuchemann (1981) analyzed children's understanding of algebra as part of the CSMS project. He classified children's responses and the items themselves, as discussed in chapter 2, into four "levels of understanding". The present study adopted these classifications in order to define the students' level of understanding.

4.2 Eritrean students' levels of understanding

In this section, we discuss the criteria of a student for being at a particular level of understanding and the number of Eritrean students and percentage of these students who are assigned at the five different "levels of understanding". As was discussed in chapter 3, items of the test were classified into four levels based on the name that can be given to the letters in solving a problem and the structural complexity of the item. Accordingly items from the test that correlated well with each other are classified into 4 "levels of understanding."

In investigating English students' understanding of algebraic variables, Kuchemann (1981) selected 30 of 51 items of the algebra test (see appendix I). The 30 items were then classified into 4 “levels of understanding” based on their correlation. Accordingly 6 items were assigned to level 1, 7 items were assigned to level 2, 8 items were assigned to level 3, and 9 items were assigned to level 4. Likewise, the present study classified the 30 items (see tables 4.3.1, 4.3.2, 4.3.3 and 4.3.4 on pages 47, 49, 51, and 52 respectively) in the same way and these items are used to classify the Eritrean grade eight students' performance into five "levels of understanding".

As the students' responses to the items in the test are discussed with regard to the students' level of understanding, it is important to note that throughout this chapter the levels of students' understanding are defined and, therefore, should be understood as follows:

Level 0. Students at this level do not make a coherent attempt at the easiest items of the test (level 1 items).

Level 1. Lower level of understanding where students are able to cope with items that require treating a letter as an object. Besides, students at this level could cope with items where the letters had to be evaluated or not used.

Level 2. Students at this level of understanding are able to cope with items of the test that involve letters to be evaluated, not used or to be regarded as objects when the structure of the items is complex.

Level 3. Students at this level of understanding are able to treat a letter as specific unknown. The students at level 3 are able to cope with items on the test that require treating a letter as specific unknown when the structure is simple.

Level 4. This is the highest level of understanding of algebraic variables. Students at this level of understanding are able to cope with items that require treating a letter as specific unknown when the structure of the items is complex. Students at this level of understanding are also able to treat a letter as a generalized number or variable.

Eritrean students, as was done by Kuchemann (1981), are described as “being at” a given level if they correctly answered about two thirds of the items at that and no higher level. The criteria were 4/6, 5/7, 5/8 and 6/9 items correct for levels 1 to 4 respectively. A student who did not answer 4 or above correctly of the 6 items assigned to level 1 was described as being at level 0. If a student reached the criterion for level 1, then s(he) would be rechecked for criterion of level 2. If s(he) did not reach the criterion at level 2, then s(he) would be described as being at level 1. However, if the student reached the criterion at level 2, then s(he) would be again checked for the criterion of level 3. The analogy of assigning a student to a particular level of understanding follows the same procedure.

In other words, a student is assigned to level 1 if and only if the student reaches the criterion at level 1 but not the criterion for level 2 or other higher levels of understanding. But a student who has reached a particular level of understanding is checked if he can reach the criterion for the next higher level until s(he) fails to reach a criterion for a particular higher level of understanding. Accordingly, the students are classified into five "levels of understanding".

It should be noticed that, for various reasons, students at a lower level of understanding might cope with some of the items at a higher level of understanding without reaching the criterion for that particular level. For example, a student who is at level 1 may cope with some items which are assigned to level 2 or above without reaching the criterion for level 2. However, a student who is described as level 1 could cope with some items of level 3 without reaching the criterion for that level and so on.

Likewise, a student who is assigned to a higher level of understanding might not cope with some items of the lower levels of understanding. A student who is assigned to level 4 might not cope with some items of level 2. In other words, as far as a student fulfilled the criterion for a particular level of understanding, he would be assigned to that level provided that s(he) had also fulfilled the criterion for the preceding lower levels of understanding. The following table shows the number and percentage of students who are assigned to the five different levels of understanding.

Table 4.2 Number and percentage of students' at each level of understanding.

	Level 0	Level 1	Level 2	Level 3	Level 4	Total
Number of students	416	1006	281	66	4	1773
Percentage	23.5	56.7	15.9	3.7	0.2	100

As can be seen from table 4.2, 23.5 % of the students could not make a coherent attempt at level 1 items. These students are not even able to cope with items that require treating a letter as an object and items that need evaluation or ignorance of the letters. These students are unable to deal with such items even when the structure of the items is simple (not complex). In other words, the students under these levels are unable to make a coherent attempt at level 1 items.

Most of the students (56.7 %) fall into level 1. This clearly indicates that most of the students who took part in the study were unable to cope consistently with items that require treating a letter as an object or evaluated when the structure is complex. Students at this level do not conceive a letter as being capable of representing a specific unknown and most often, they do not treat the letter as a specific unknown but rather they ignore it or evaluate it when it appears in some expressions.

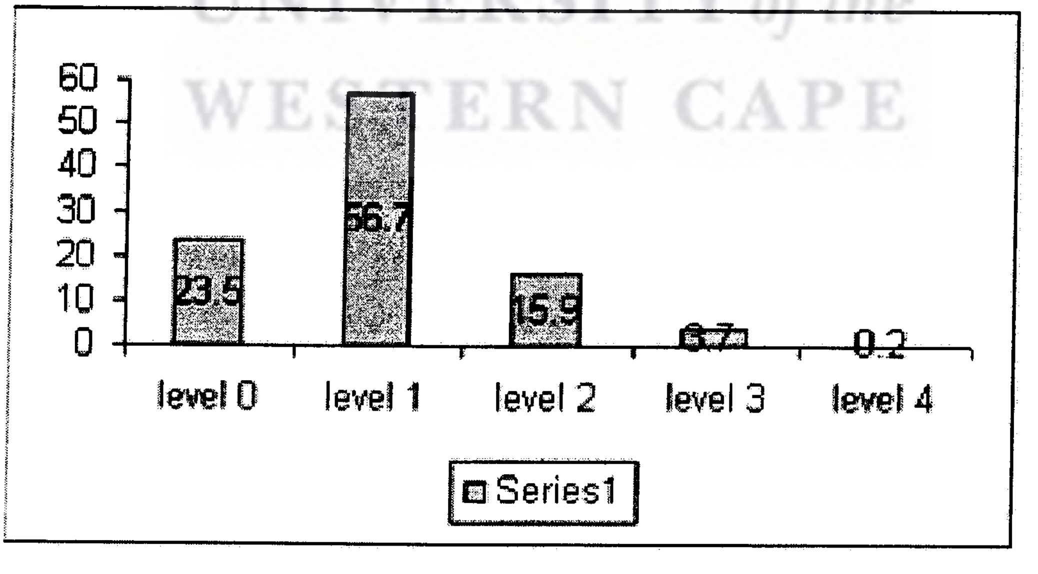
Nearly sixteen percent (15.9 %) of the students were at level 2. These students are capable of dealing with items that involve letters to be evaluated or treated as object. They can also consistently cope with such items when the structure of the items is complex. However, these students still do not cope with items that involve treating a letter as a specific unknown, generalized number or variable. Similar to that of students at level 1, the students at this level hardly cope with items that require treating a letter as a specific number.

Fewer than four percent (3.7 %) reached the level of understanding that requires treating a letter as a specific unknown when the structure of the item is simple. This shows that almost all the tested students are incapable of treating a letter as a specific unknown. These students hardly cope with items that require treating a letter as generalized number or variable. Only 0.2 % of the students (4 out of 1773 students) have reached the highest level of understanding.

The standard of the students about understanding of algebraic variables has proved to be extremely poor. It is evident from the table that there are more students at level 4 than level 3. The difference in facility is 3.5 %. This shows that there were more students who could cope with items that require treating a letter as a specific unknown than those who were able to deal with items that need to treat a letter as generalized number. This supports the fact that students are able to conceive a letter as being capable of representing a specific unknown before they realize that it is capable of taking some values. In other words, students first understand a letter as a specific unknown and then as a generalized number.

The following diagram depicts the graphical representation of the students at each level of understanding.

Figure 4.1 Graphical representation of percentage of students at each algebraic level.



4.3 Overall performance on the items for the different levels

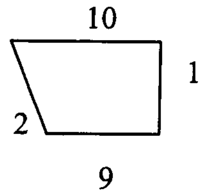
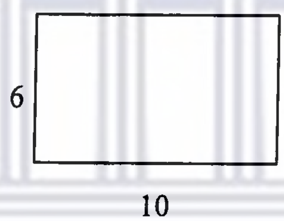
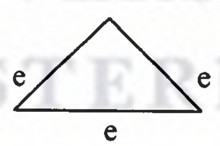
In this section, we discuss the overall students' performances on the items for the different levels and summarize the items that belong to the different difficulty levels. In all cases, the number and percentage of students who gave the correct response to each item of the different levels are displayed together with the interpretation of the letters that deemed adequate to respond each item correctly.

4.3.1 Level 1 items

The items for this level are shown in table 4.3.1 with their percentages. The items were very easy. The items at level 1 are purely numerical (items 8(i) and 20), or they have a simple structure and can be solved by using the letters as objects (items 9(i) and 17(i)), or by evaluating the letters (item 5(i)), or by not using the letters at all (item 4(i)).

The following table shows the items assigned to level 1 with the percentages of the correct responses given by Eritrean students and the interpretation of letters deemed to be adequate to answer the items correctly.

Table 4.3.1 Level 1 items.

STUDENTS		Item	Interpretation of letters
Number	Percentage		
1596	90 %	(20)  P = ……….	No letters involved
1575	89 %	9 (i) 2a + 5a =……..	Object
1501	85 %	5(i) What can you say about a if a + 5 = 8?	Evaluated
1472	83 %	8 (i)  A= …….	No letters involved
1192	67 %	4(i) If a + b = 43, a + b + 2 = ……	Letters not used
1085	61 %	17(i)  P = …….	Object

N.B. The percentage of the students who gave the correct answer for each item within each level is obtained by dividing the number of students who answered each item correctly by the total number of the students who took part in the study i.e. 1773.

As can be seen from table 4.3.1, the item with the highest percentage is item 20, whereas the one with the lowest facility is 17(i). Item 20 is purely numerical and what students need to answer correctly is that they have to recall the fact that the perimeter of a quadrilateral is the sum of the lengths of the four sides. Item 8(i) is also numerical and students need to recall the fact that the area of a rectangle is the length of the rectangle multiplied by its width. Only 83 % of the students gave the correct answer for this item.

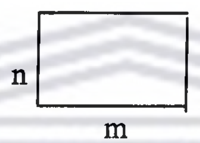
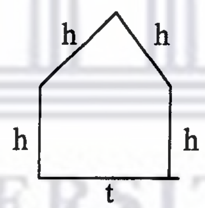
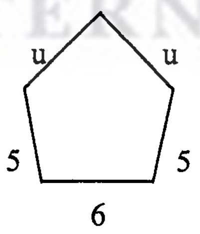
Both items 9(i) and 17(i) involve one letter and in order to solve the two items, the letters in both cases can be treated as objects. The letters in each item can be grouped so that a correct answer is obtained. However, item 17(i) has proved to be more difficult than item 9(i). The difficulty of 17(i) may partially be explained by the fact that students need to recall what a perimeter of triangle is (how to find the perimeter of a triangle) before grouping the 3e's together. Generally speaking, items assigned to level 1 are easier than level 2 items.

The highest percentage of the correct answer obtained at level 1 items is 90 %, whereas the lowest is 61 %. The overall difference between the one that is proven as the most difficult and the one that is proved as the simplest of the items at this level is 29 %. This in particular indicates that item 20 proved to be the simplest of all items at this level. And item 17(i) was the most difficult item of the items assigned to level 1.

4.3.2 Level 2 items

As can be seen from table 4.3.2, the difference between these items and those of level 1 is their increase in complexity. However, the letters only have to be evaluated (7(i) and 7(ii)) or used as objects (8(ii), 9(iv), 17(ii) and 17(iii)).

Table 4.3.2 Items assigned to level 2.

STUDENTS		Item		Interpretation of letters
Number	Percentage			
1135	64 %	19(i)	A shape with 57 sides has ….diagonals give rule “take 3”	No letters involved
1097	62 %	7(i)	What can you say about u if u = v +3 and v = 1	Letter evaluated
1011	57 %	8(ii)	 A =..........	Object
710	40 %	7(ii)	What can you say about m if m = 3n + 1 and n = 4	Letter evaluated
638	36 %	9(iv)	2a + 5b + a =	Object
606	34 %	17(ii)	 P = ...	Object
410	23 %	17(iii)	 P =..........	Object

The one with the highest percentage of the items assigned to level 2 is purely numerical, whereas the one with the lowest percentage requires treating letters as objects. Item 17(iii) is also similar to item 17(ii) in that it requires treating letters as objects. However, item 17(iii) proved to be more difficult than item 17(ii). This difficulty could be partially explained due to the fact that item 17(iii) involves both numerals and letters, whereas item 17(ii) involves only letters. As a result, there was more temptation to closure in item 17(iii) than was in item 17(ii).

Both items 7(i) and 7(ii) can be solved by evaluating the letters. Nevertheless, 7(ii) was more difficult than 7(i) with an overall difference of 22 %. The two items are essentially of the same sort and thus can be solved in a similar way. Here one might argue that there is some difficulty that students encounter with item 7(ii) as the difference in percentage between the two items is large. It might be the case that some students do not understand 3n as "3 multiplied by a certain number n". As was claimed by Vergnaud (1985) the source of this misunderstanding is because there is no sign for multiplication: so 3n is viewed as ' 3 something'.

The highest percentage for the correct answer of the items assigned to level 2 is 64 %, whereas the lowest is 23 %. The overall difference between the one that is proved as the most difficult and the one that is proved as the simplest of the items at this level is 41 %. This in particular indicates that item 19(i) proved to be the simplest of all items at this level. Item 17(iii) was the most difficult item assigned to level 2.

4.3.3 Level 3 items

The items for this level are shown in table 4.3.3. As can be seen from the table, almost all items at this level involve treating letters as specific unknowns with the exception of items 9(ii) and 11. Item 9(ii) can be solved by treating the letters as objects provided that the temptation to closure is overcome. Moreover, Question 11 involves generalized numbers. This item was included, as pointed out by Kuchemann (1980), due to the fact that the item correlated much with the items that are assigned to level 3.

Table 4.3.3 Level 3 items.

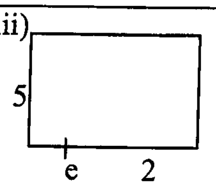
STUDENTS		Item	Other responses		Interpretation of letters
Number	Percentage				
532	30 %	19(ii) A shape with k sides has…… diagonals			Specific object
458	26 %	9(ii) $2a + 5b =$	7ab	32 %	Object
432	24 %	9(viii) $3a - b + a =$			Specific unknown
286	16 %	(10) What can you say about r if $r = s + t$ and $r + s + t = 30$	$r = 10$	32 %	Specific unknown
263	15 %	3(ii) 4 added to n can be written as $n + 4$. Add 4 onto 3n.	7n 7	36 % 16 %	Specific unknown
182	10 %	4(iii) If $e + f = 8$ $e + f + g = \ldots.$	12 9 8g	23 % 6 % 11 %	Specific unknown
70	4 %	17(iv) Part of this figure is not drawn…. n sides altogether P =	32, 34, etc	15 %	Specific unknown
31	2 %	(11) What can you say about c if $c + d = 10$ and c is less than d.	$c = 4$	42 %	Generalized number

As can be seen from above table, the items with the highest and lowest percentages of the items at this level are items 19(ii) and 11 respectively. The highest and lowest facilities are 30 % and 2 % respectively with the overall difference of 28 %. This shows that the items at this level are more difficult than those items which are assigned to levels 1 and 2.

4.3.4 Level 4 items

The items at this level involve treating letters as specific unknowns as in 3(iii), 9(v), 14 and 8(iii). It includes items which require treating letters as generalized numbers as in 12 (ii), 16 and 18(i). It also includes items which require treating letters as variables as in 15 and 2. This level represents items which are more difficult than items assigned to any other level. As can be seen from the table that follows, the two items with the highest and lowest percentages of the items assigned to level 4 are 3(iii) and 18(i) respectively. The highest and lowest percentages are 10 % and 0.05 % with overall differences of 9.95 %.

Table 4.3.4 Level 4 items.

STUDENTS		Item	Incorrect responses	Interpretation of letter(s)
Number	**Percentage**			
186	10 %	3(iii) Multiply n + 5 by 4.	4 x n + 5 3 % n + 20 12 % 20 15 % 20n 20 %	Specific unknown
186	10 %	9(v) (a – b) + b =	a – 2b 16 %	Specific unknown
108	6 %	(14) Cakes cost cents each and biscuits b cents each. If …, 4c + 3b stands for?	4 cakes 12 % & 3 biscuits	Specific unknown (generalized number)
66	4 %	(16) b blue pencils cost 5 cents each and r red pencils 6 cents each. …… (i.e. 5b + 6r = 90)	b + r = 90 5 %	Specific unknown or generalized number
31	1.7 %	12(ii) Is the following always, never, or sometimes (when) true? L + M + N = L + P + N	Never 46 %	Generalized number
27	1.5 %	(15) If $(x + 1)^3 + x = 349$ is true when x = 6, what value of x makes $(5x + 1)^3 + 5x = 349$ true?		Generalized number or variable
18	1 %	8(iii)  A =………		Specific unknown
2	0.11 %	(2) Which is larger, 2n or n + 2? Explain.	2n 24 % Equal 34 % n + 2 15 %	Variable
1	0.05 %	18(i) Basic wage		Specific unknown or generalized number

The items at levels 1 and 2 can be solved without having to operate on letters as unknowns; instead the letters can be evaluated, not used, or regarded as objects. At levels 3 and 4 the letters have to be treated as specific unknowns, generalized numbers or variables (Kuchemann, 1981).

The difference between level 1 and level 2, and between level 3 and level 4, as pointed out by Kuchemann (1981), is essentially a matter of complexity. For example, in the level 1 item, "Find a if $a + 5 = 8$", the letter can be evaluated immediately, by recalling a familiar number bond, whereas in the level 2 item "Find u if $u = v + 3$ and $v = 1$", it is first necessary to cope with the first ambiguous statement. Whilst in the perimeter item 17(i) the objects being collected together are all the same type (3e), in 17(ii) (level 2) the objects are different and the answer ($P = 4h + t$) cannot be closed. Likewise whilst the level 3 item "Add 4 onto 3n" essentially involves just a single operation, in the level 4 item "Multiply $n + 5$ by 4" the operations +5 and x4 have to be coordinated.

The hierarchal difficulty level of the items is maintained in the present study. The items assigned to level 1 proved to be simpler than items assigned to other levels. The most difficult items are those which are assigned to level 4. The facilities for the correct responses of items assigned to item 1 range from 61% to 90%. The facilities for the correct answers of the items assigned to levels 2, 3 and 4 range 23 % - 64 %, 2 % - 30 % and 0.05 % - 10 % respectively.

4.4 Students' strategies and interpretation of letters

In this section, we discuss the strategies and interpretation of letters used by the students in dealing with the items of the Algebra test. This topic has six subtopics. The first three subsections of this topic deal with the methods and strategies used by the students in order to reduce the difficulty that may arise from treating a letter as a specific unknown, a generalized number, or a variable. The other three subsections deal with the legitimate interpretation of letters when they are used in Algebra. However, all subsections are dependent on each other and strongly related and thus discussed accordingly.

4.4.1 Evaluation strategy

This strategy involves evaluation of letters when dealing with algebraic expressions and equations. This is one of the various strategies, which students use in order to avoid operating on a specific unknown. This is a strategy by which students avoid operating on a specific unknown by simply giving the unknown a value. The category also refers to items where children are asked to find a specific value for the unknown but where it is not necessary to manipulate the letter first. This applies to items 5(i), 7(i) and 7(ii) in the table below but not to item 10.

Table 4.4.1 Students' responses to item 5(i), 7(i), 7(ii) and 10.

5(i) (Level 1)		7(i) (Level 2)		7(ii) (Level 2)		10 (Level 3)	
What can you say about a if $a + 5 = 8$		What can you say about u if $u = v + 3$ and $v = 1$		What can you say about m if $m = 3n + 1$ and $n = 4$		What can you say about r if $r = s + t$ And $r + s + t = 30$	
$a = 3$	85 %	$u = 4$	62 %	$m = 13$	40 %	$r = 15$ $r = 30 - s - t$	16 % 2 %
$a = 13$	3 %	$u = 2$	8 %			$r = 10$	32 %

N.B. The percentage of the students who gave the correct and incorrect answers for each item is obtained by dividing the number of students who gave the correct and incorrect responses by the total number of the students who took part in the study i.e. 1773.

As can be seen from table 4.4.1, 5(i) is answered correctly by 85 % of the students. Here, all the students need to do, as suggested by Kuchemann (1980), is that of recalling a familiar number-bond or count from 5 until they reach 8. Ninety-two percent of the students at level 1 gave the correct answer $a = 3$, whereas 93 % of the students at level 2 answered the item correctly. Almost all (93 %) of the students at level 3 and all students at level 4 wrote the correct answer. The variation in percentage from level 1 to level 2 was 1 %, whereas there was no change in percentage from level 2 to level 3. This indicates that the students at these levels (levels 2 and 3) were able to cope with the item equally consistently.

The following table depicts the responses given to item 5(i) by the students at each level of understanding.

Table 4.4.1a Students' responses to item 5(i) at each level of understanding.

	a = 3	a = 13
Level 0	60 %	13 %
Level 1	92 %	0.5 %
Level 2	93 %	0 %
Level 3	93 %	0 %
Level 4	100 %	0 %

N.B. The percentage of the students who gave the correct and incorrect answers for each item at each level of understanding is obtained by dividing the number of students who gave the correct and incorrect responses respectively by the total number of the students at each level of understanding. For example, the percentage of the students at level 0 who gave the correct and incorrect answers for each item is obtained by dividing the number of students who gave the correct and incorrect answers respectively at that level by the total number of students assigned to level 0 i.e. 416. As displayed in table 4.2 of section 4.2, the number of students assigned to levels 0, 1, 2, 3 and 4 are 416, 1006, 281, 66 and 4 respectively.

Both parts of Question 7 are more difficult (level 2). As can be seen from table 4.4.1, item 7(i) was answered correctly by 62 % of the students, whereas only 40 % of the students produced the correct answer m =13 for item 7(ii). With regard to item 7(i) (see table 4.4.1b), it was found that 56 % of the students at level 1 answered the item correctly. Almost all students at level 2 (97 %) and level 3 (95 %) produced the correct answer, u = 4. All students at level 4 wrote the correct answer. The variation in percentage was very large (41 %) from level 1 to level 2, compared to the variation (-2

%) from level 2 to level 3. It was also large when compared with the variation of 5 % from level 3 to level 4. Overall, 8 % of the students wrote the wrong answer u = 2. This error was observed rarely (10 %) with the students who are described as being at level 1. But it was hardly observed with students at levels 2 and 3.

Table 4.4.1b Students' responses to items 7(i) and 7(ii) at each level.

Levels of understanding	Responses to item 7(i)		Item 7(ii)
	u = 4	u = 2	m = 13
Level 0	46 %	9 %	21 %
Level 1	56 %	10 %	31 %
Level 2	97 %	0.7 %	89 %
Level 3	95 %	3 %	88 %
Level 4	100 %	0 %	100 %

As mentioned earlier, 40 % of the students answered item 7(ii) correctly. Thirty-one percent of the students at level 1 gave the correct answer m = 13 for the item. Of the students at level 2, 89 % answered the item correctly and 88 % of the students at level 4 produced the correct answer. All the students at level 4 gave the correct response.

With regard to the two items of Question 7, there was a variation in percentage (22%) from level 7(i) to 7(ii) (see table 4.4.1). This shows that the students were unable to cope with the two items with equal consistency. Though it is difficult to argue that all of the students have difficulty with the nature of these items, no one could expect such items to be mastered by Eritrean grade eight students (especially by those students at lower levels of understanding). This is because in the Eritrean mathematics curriculum, such items appear to be an integral part of relations and functions in grade 9 and such items are exercised and mastered within grade 9.

The increase in difficulty of these two items in relation to item 5(i) is probably mainly due to the fact that the items involve two unknowns rather than one. This makes the first equation in each item ambiguous in the sense that they are true for more than one pair of values; however, as argued by Kuchemann (1981), this ambiguity is resolved as the second equation is reached.

Item 10 can be solved by substituting r for (s + t) in the second equation, but this involves handling a letter as an unknown, which puts the item into the "letter as specific unknown" category. Here r can be evaluated from r + r = 30 after the substitution has been made. Only 16 % of the students gave the correct answer r = 15, while 32 % of the students wrote r = 10 (see table 4.4.1). Most of the students who gave the incorrect answer r = 10, as was seen from their work on the paper, have avoided this category by evaluating r directly from the second equation (10 + 10 + 10 = 30). These students tried to reduce the difficulty by assigning equal values for each letter, but this strategy led them to an incorrect response. This finding is consistent with the one that is reported by Kuchemann (1980).

The percentage of the students who gave the correct answer for Question 10 was half of those who wrote the incorrect response r = 10. As can be seen from table 4.4.1c, only 13 % of the students at level 1 and 22 % of the students at level 2 answered the item correctly. Whist 62 % of the students at level 3 gave the correct answer, three-quarter of the students at level 4 wrote the required response. The variation in percentage was large (40 %) from level 2 to level 3, compared to the change (13 %) from level 3 to level 4. This shows that those students at levels 3 and 4 cope with Question 10 with similar consistency than those students at levels 2 and 3. In other words, the consistency of the students at levels 3 and 4 in relation to coping with Question 10 is more than that of the students at levels 2 and 3.

Table 4.4.1c Students' responses to Question 10 at each level

	r = 15	r = 10	r =30 – s - t
Level 0	13 %	29 %	0.72 %
Level 1	13 %	36 %	1.3 %
Level 2	22 %	28 %	3.9 %
Level 3	62 %	6 %	14 %
Level 4	75 %	0 %	25 %

The wrong answer r = 10 was seen frequently with students at levels 1 and 2. But, it was rarely observed with those who were at level 3 and it was not seen with the students at level 4 at all. Of the students at levels 1 and 2, 36 % and 28 % gave the incorrect response r = 10 for Question 10. Students at level 3 rarely (14 %) produced the wrong response, r = 30 – s - t. One-fourth of the students at level 4 gave such incorrect reply (r = 30 – s – t) which would have been a correct answer if the equation (r = s + t) had not been stated.

4.4.2 Avoidance strategies

Item 4(i) can be solved by not using the letters. The item involves two unknowns. However, nothing needs to be done to these unknowns. These unknowns can be eliminated by a matching strategy, which focuses attention on +2 by which the left hand side of the equations differs and which is then applied to 43. As can be seen from table 4.4.2, 67 % of the students produced the correct answer 45, whereas 4.5 % gave the incorrect answer 41, which seems to be obtained by subtracting 2 from 43 instead of adding.

Table 4.4.2 Students' responses to Question 4

4(i) (Level 1)		4(iii) (Level 3)	
If a + b = 43 a + b + 2 =. ………		If e + f = 8, e + f + g =…	
45	67 %	8 + g	10 %
41	4.5 %	15	0.3 %
		12	23 %
		8g	11%
		9	6 %

A further analysis shows (see table 4.4.2a) that three quarter of the students at level 1 answered the item correctly. Eighty-two percent of the students at level 2 wrote the correct response. Almost all (94 %) of the students at level 3 and all (100 %) of the students at level 4 also produced the correct one. The variation in percentage from level 1 to level 2 was 7 %, whereas the change in percentage from level 2 to level 3 was 12 %. An increase of 6 % was also observed from level 3 to level 4. The change in percentage

Table 4.4.2a Students' response to item 4(i) at each level.

	45	41
Level 0	34 %	9 %
Level 1	75 %	3 %
Level 2	82 %	2.5 %
Level 3	94 %	4.5 %
Level 4	100 %	0 %

from level 2 to level 3 was twice that of the change from level 3 to level 4. It was almost twice that of the change in percentage from level 1 to level 2. This reveals that students at levels 3 and 4 cope with item 4(i) in more similar ways than those students who are at levels 2 and 3.

As can be seen from table 4.4.2, item 4(iii) can also be solved by matching and the letters e and f can be avoided in this way. Nevertheless, children still have to operate with g, which puts the item into the "letter as specific unknown" category. Many students tried to resolve this difficulty by evaluating g in a quite interesting way but this led them to wrong answers. Accordingly, 23 % of the students gave the answer 12 (4 + 4 + 4 = 12), 11 % produced 8g and 6 % wrote 9 (evaluating the letter g as 1). Only 0.3 % of the students answered 15 (evaluating the letter g as 7).

It was claimed that students who are just being introduced to literal symbols have a tendency to associate the linear order of the alphabets with the linear ordering of whole numbers (MacGregor and Stacey, 1997; Wagner, 1983). However, this is not evident in the present study. Perhaps this is because the mother tongue is the medium of instruction in primary school. The Eritrean mathematics textbooks use Geeze alphabets at primary school level and English at middle and secondary schools. Thus the students, being exposed to different alphabets, have nothing to associate the order of the alphabets with the order of whole numbers.

As can be seen from table 4.4.2, only one-tenth of the students gave the correct answer, 8 + g, for item 4(iii). Seven percent of the students at levels 1 and 18 % of the students at level 2 gave the correct answer. Whilst 83 % of the students at level 3 could cope with the item, all the students at level 4 produced the correct response.

Table 4.4.2b Students' responses to item 4(iii) at each level.

	8 + g	12	15	8g	9
Level 0	2 %	15 %	0.24 %	8 %	5 %
Level 1	7 %	27 %	0.39 %	12 %	7 %
Level 2	18 %	25 %	0 %	13 %	7 %
Level 3	83 %	9 %	0 %	1.5 %	1.5 %
Level 4	100 %	0 %	0 %	0 %	0 %

Twenty-three percent of the students gave the incorrect response 12. Of the students at levels 1 and 2, 27 % and 25 % wrote this answer respectively. Nine percent of the students at level 3 produced such a wrong answer. Twelve percent of the students at level

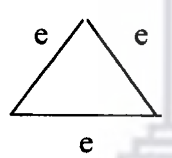
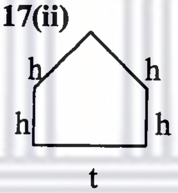
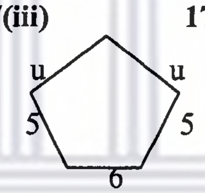
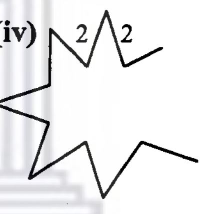
1 and 13 % of the students at level 2 wrote the wrong answer 8g, whereas each of the students at levels 1 and 2 rarely (7 %) gave the incorrect response 9.

4.4.3 Strategies of translation and abbreviation

This category applies to the first three parts of Question 17 but not to 17(iv). For example, 17(i) can be solved by thinking of 'e' simply as a name or label for each of the sides, when the task becomes one of simply collecting the three 'e' s together. Sixty-one percent of the students answered this item correctly, while only 4 % could cope with the item 17(iv), where the given letter has to be regarded as a number.

Table 4.4.3 correct responses to Question 17

What can you write about the perimeter of each of these figures?

17(i)	17(ii)	17(iii)	17(iv)
			
3e 61 %	4h + t 34 % 4h + 1t	2u + 16 23 % u + u + 16 2.u + 2.5 + 1.6	2n 4 % n2

As can be seen from table 4.4.3, 61 % of the students produced the correct response for item 17(i). Thirty-four percent and 23 % of the students correctly answered items 17(ii) and 17(iii) respectively. The two items are similar in that both items involve letters. Item 17(iii) proved to be more difficult than 17(ii). There was more students' temptation to closure in item 17(iii) than is in item 17(ii). This could be due to the fact that item 17(iii) involve numerals in addition to the letters, whereas 17(ii) involves only letters. As a result, 10 % of the student wrote 18u (combining 16 with 2u). Besides, 7 % of the students wrote 22 evaluating u as 3 because the "opposite side" (the side which is not adjacent to the sides whose length is u units) is 6.

As can be seen from table 4.4.3a, 68 % of the students at level 1 wrote the correct answer (p =3e) for item 17(i). Almost all (92 %) of the students at level 2 and 88 % of the students at level 3 answered the item correctly. Item 17(ii) was answered by 34 % of the students. Twenty-seven percent of the students at level 1 answered this item correctly. Whilst 88 % of the students at level 2 gave the correct answer 4h + t, only 68 % of the students at level 3 wrote the correct response. It was observed that the difference in percentage was large. For instance, the change in percentage from level 1 to level 2 was 61%.

Table 4.4.3a Students' correct responses to Question 17.

	Item 17(i)	Item 17(ii)	Item 17(iii)	Item 17(iv)
Level 0	20 %	11 %	5 %	1.5 %
Level 1	68 %	27 %	15 %	2.38 %
Level 2	92 %	88 %	66 %	5.33 %
Level 3	88 %	68 %	73 %	33.33 %
Level 4	75 %	75 %	75 %	75 %

With regard to item 17(iii), 23 % of the students produced the correct answer, 2u + 16. Whilst only 15 % of the students at level 1 gave the correct answer, 66 % of the students at level 2 answered the item correctly. Seventy-three percent of the students at level 3 wrote the correct response. The variation in percentage from level 1 to level 2 was very large (51 %), compared to the change (7 %) from level 2 to level 3 and that of level 3 to level 4 (2 %).

Even though it was not possible to classify all responses given to the first three parts of Question 17, some of the students do not seem to understand these items. Twelve percent of the students omitted the question altogether. Six percent of the students gave responses such as triangle, pentagon, and so on, whereas 3 % wrote the formulae for the area of a triangle and rectangle. This indicates that there were students who misunderstood the question. Perhaps those students who wrote the formulae for the area were unable to differentiate the concept of perimeter from that of area. Moreover, those who wrote names of the polygons misunderstood the question and this could be partially explained due to the difficulty that the students have in English language because it is a foreign language for them.

The interpretation of the letters as objects also works successful in some of the parts of Question 9 as shown in table 4.4.3b, in which students were asked to simplify algebraic expressions. A letter as an object can be used to cope successfully with the items 9(i) and 9(iv). Here no content is given, but students can solve item 9(iv) by inventing one, for example by interpreting the expression as '2 apples and 5 bananas and another apple'; or the letters can be regarded as objects in their own right: '2 a's and 5 b's and another a, which makes 3a 's and 5b's together.' However, 89 % of the students gave the correct answer 7a for the item 9(i), whereas 35% produced the correct answer, 3a + 5b, for item 9(iv). The decrease in percentage of the students who gave the correct answer for the item 9(iv) could partially be explained due to the fact that the item involves two letters. Though it was not possible to classify all the students' responses by percentage, many of the students wrote answers such as, 'unlike terms', 'impossible' and '8ab'.

Table 4.4.3b Students' correct responses to parts of question 9

9(i) (Level 1)		9(iv) (Level 2)		9(viii) (Level 3)		9(v) (Level 4)	
2a + 5a =		2a + 5b + a =		3a - b + a =		(a - b) + b =	
7a	89 %	3a + 5b	35 %	4a - b	24 %	a	10 %

Items 9(viii) and 9(v) were answered correctly by 24 % and 10 % of the students respectively. The decrease in percentage of the students who gave the correct answers to items 9(viii) and 9(v), compared to items 9(i) and 9(iv), can be expressed partially due to the fact that treating letters as objects cannot work with the former two items. For instance, 3a's take away one b doesn't make any sense, unless b is of the same "quality' as a, in other words, a number. A further analysis was made to see how the students responded to these items. The following table depicts the correct responses given by the students at each level.

Table 4.4.3c Correct responses to parts of question 9 at each level.

	Item 9(i)	Item 9(iv)	Item 9(viii)	Item 9(v)
Level 0	66 %	19 %	10 %	8 %
Level 1	95 %	28 %	20 %	8 %
Level 2	97 %	70 %	49 %	20 %
Level 3	100 %	91 %	76 %	20 %
Level 4	100 %	100 %	100 %	25 %

Almost all students at level 1 (95 %) and level 2 (97 %) answered item 9(i) correctly. All students at levels 3 and 4 also gave the correct response to the item. The variation in percentage from one level to the proceeding level was small and this item proved to be easy. Students at levels 1 and above were able to cope with the item consistently.

Thirty-five percent of the students answered item 9(iv) correctly (see table 4.4.3b). Whilst 28 % of the students at level 1 produced the correct answer 3a + 5b, 70 % of the students at level 2 answered the item correctly. Ninety-one percent of the students at level 3 gave the correct response, whereas all of the students at level 4 wrote the required answer. The variation in percentage from level 1 to level 2 was large (42 %) as compared to the change in percentage (21 %) from level 2 to level 3. The change in percentage from level 2 to level 3 was also nearly twice that of the change from level 3 to 4.

One-fifth of the students at level 1 gave the correct response 4a - b for item 9(viii), whereas 49 % of the students at level 2 wrote the correct answer. Besides, 76 % of the students at level 3 and all students at level 4 answered the item correctly. With respect to item 9(v), students at level 1 rarely (8 %) gave the required answer, a. Whilst one-fifth of each of the students at levels 2 and 3 answered the item correctly, a quarter of the students at level 4 wrote the correct response. This shows that the item was difficult even for those students who were assigned to level 4.

Using a letter as an object, which amounts to reducing the letter's meaning from something quite abstract to something far more concrete and real, enabled many children to give correct answers to items which they would have not coped with had they had to use the intended meaning of the letter. In other words, using letters as objects enables students to solve items which they would not have been able to cope otherwise. However, this reduction in meaning often occurred when it was not appropriate. It was claimed by Kuchemann (1981) that this happens particularly with items that involve objects (cabbages, wages, cakes, pencils) but only where it is essential to distinguish between the objects themselves and their number.

The students confronted with Question 6 below, a quarter (25 %) of the students interpreted the expression 8c + 6p as 8 cabbages and 6 pumpkins. Some answers, like "The cost" were ambiguous. The students were also asked for the total number of vegetables bought (c + p). Only 0.1 % of the students gave the correct answer, whereas most of the students (54 %) wrote the answer 14. Though it was observed that many students omitted the question in the present study, the reduction of the letter's meaning in question 6 was consistent with the one reported by Kuchemann (1981) above.

Figure 4.2 Question 6

> Cabbages cost 8 Nacfa each and pumpkins cost 6 Nacfa each. If c stands for the number of cabbages bought and p stands for the number of pumpkins bought, what does 8c + 6p stand for? ....
>
> What is the total number of vegetables bought? ....

Likewise, the same confusion arose in Question 16 below. Most of the students omitted the question. It seems as if the students did not understand that the question required treating the letters as specific unknowns. Only 4 % of the students gave the correct answer 5b + 6r = 90. And 5 % of the students wrote the b + r = 90. (See figure 4.3).

Figure 4.3 Students' responses to Question 16

> Blue pencils cost 5 cents each and red pencils cost 6 cents each. I buy some blue and some red pencils and altogether it costs me 90 cents. If blue is the number of blue pencils bought, and if r is the number of red pencils bought, what can you write down about b and r?

Responses to Question 16	Percentage
5b + 6r = 90	4 %
Two correct pairs, of (6, 10), (12, 5), (18, 0), (0, 15).	5 %
b + r = 90	0.2 %
6b + 10r = 90 or 12b + 5r = 90	0.3 %

4.4.4 Letter as specific unknown

The preceding three sections described ways of avoiding generalized arithmetic, by not using the letters as unknown numbers. Using a letter as specific unknown requires conceiving a letter as being capable of representing a numerical value. Treating a letter as an object, evaluating letters or not using letters do not, in most cases, enable students to solve items involving specific unknowns.

The use of a letter as a specific unknown has already been mentioned for items 17(iv) (n sided figure), 10 (r = s + t and r + s + t =), 4(iii) (e + f = 8, e + f + g =), 9(viii) (simplify (a - b) + b). This usage is also required to solve 3(ii) and (3iii).

Table 4.4.4 Students' responses to part of Question 3.

3(ii) (Level 3)		3(iii) (Level 4)	
Add 4 onto 3n		Multiply n + 5 by 4.	
3n + 4	15 %	4n + 20 or 4 (n + 5)	10 %
		4n +5 or 4 x n + 5	3 %
		n + 20 or n + 5 x 4	12 %
7n	36 %	20	15 %
7	16 %	20n	20 %

Only 15 % of the students answered item 3(ii) correctly. It may seem surprising that 3(ii) was found quite difficult. The required answer, 3n + 4, appears to be very simple. It seems that the item was unsatisfactory to the students due to lack of closure. Nothing was really to be done besides putting the operation "+" between the 3n and the 4, but pupils have to recognize that this is all that can be done to combine the elements, since n is unknown. Many students seemed unwilling to accept this and instead gave the answer 7n or just 7 in which the elements that were meaningful (3 and 4) were "properly" combined but the letter was simply left as it was or ignored entirely.

There are two main ways, as argued by (Costello, 1991) of avoiding dealing with a letter in an algebraic expression: one is to keep it in but ignore it and the other is simply to lose it altogether. The answers 7n and 7 belong to the category letter not used and the same applies in 3(iii) to answers n + 20 and 20. Sometimes, one or other of these procedures gives the right answer. For instance, this approach is sufficient to answer 3(i) (Add 4 onto n + 5) correctly, as far as the letter is retained in the answer. In other words, item 3(i) can be answered correctly by keeping the n but ignoring it without conceiving the letter as a specific unknown (i.e. $n + 5 + 4 = n + 9$).

The items 3(ii) and 3(iii) both involve specific unknowns, but 3(iii) is more difficult because of its structural complexity. Only 10 % of the students produced the correct response, 4n + 20. The operation x 4 has to be applied to both elements of the expression n + 5, but many students just attached the operation to the expression as a whole. This led them to ambiguous answers such as 4 x n + 5 and n + 5 x 4. In Eritrean mathematics curriculum, brackets are introduced in grade 6. Students learn how to use brackets in computing, in most cases, some numerical values and simplifying expressions. Fifteen percent of the students wrote 4n + 5 or n + 20. Concerning these ambiguous answers, it seems that those students who gave such answers do not understand the significance of brackets.

Moreover, the Eritrean mathematics textbooks present most often simplifying of algebraic expressions in the form of 4(n +5) instead of the way the item was posed in the test. Thus, the difficulty of the item could partially be explained due to the difference the way the item is posed in the test and presented and taught in the Eritrean curriculum.

As can be seen from table 4.4.4a, twelve percent of the students at level 1 and 25 % of the students at level 2 wrote the required answer for item 3(ii). Seventy-nine percent of the students at level 3 gave the correct response. The variation in percentage from level 2 to level 3 was large (54 %), compared to the change (13 %) from level 1 to level 2.

Table 4.4.4a Students' responses to item 3(ii) at each level.

	3n + 4	7n	7
Level 0	5 %	32 %	18 %
Level 1	12 %	39 %	17 %
Level 2	25 %	37 %	11 %
Level 3	79 %	8 %	6 %
Level 4	75 %	0 %	0 %

As can be seen from table 4.4.4b, of the students at levels 1, 2 and 3, 7 %, 24 %, and 48 % gave the correct answer for item 3(iii) respectively. However, all the students at level 4 produced the correct response. Students at level 3 and below were not able to cope consistently with the item, whereas all the students at level 4 were found to cope with this item with consistency. Fifteen percent of the students gave the incorrect answer 20 and 20 % wrote the wrong answer n + 20. The following table shows students' responses to item 3(iii).

Table 4.4.4b Students' responses to item 3(iii) at each level.

	4n + 20	4n + 5	n + 20	20	20n
Level 0	3 %	2 %	13 %	17 %	16 %
Level 1	7 %	4 %	11 %	16 %	23 %
Level 2	24 %	3 %	21 %	11 %	16 %
Level 3	48 %	5 %	20 %	4.5 %	8 %
Level 4	100 %	0 %	0 %	0 %	0 %

4.4.5 Letter as generalized number

Here the letter is seen as representing several values rather than representing a unique number. Items 11 and 12(ii) in table 4.4.5 require treating the letters as taking several values in turn.

Table 4.4.5 Students' responses to items 11 and 12(ii).

11 (Level 3)		12(ii) (Level 4)	
What can you say about c if c + d = 10 and c is less than d?		L + M + N = L + P + N is Always Sometimes (when) Never true	
c < 5 c = 1, 2, 3, 4 (systematic list) c = 10 - d	2 % 3 % 4 %	Sometimes when M = P	2 %
c = 4	42 %	Sometimes Never	17 % 46 %

In Question 11, 42 % of the students found just one value (c = 4) for the letter c. Three percent of the students gave a systematic list of counting numbers (c = 1, 2, 3, 4). Four percent of the students gave a general response c = 10 - d that might be correct if the statement "c is less than d" was not stated. In other words, this response is correct in the absence of the second statement (c is less than d). Likewise, only 2 % of the students gave a correct response. The following table shows students' responses to item 11.

Table 4.4.5a Students' responses to Question 11 at each level.

	c < 5	c = 1, 2, 3, 4	c = 10 - d	c = 4
Level 0	0.72 %	1.2 %	3.6 %	30.5 %
Level 1	0.79 %	3 %	3.7 %	44 %
Level 2	1.78 %	6 %	7.8 %	50 %
Level 3	21.21 %	12 %	4.5 %	38 %
Level 4	25 %	0 %	50 %	0 %

Below two percent of the students at levels 1 and 2 gave the correct answer. Besides 21.21 % of the students at level 3 and 25 % of the students at level 4 wrote the required answer. Twelve percent of the students at level 3 wrote systematic list of counting numbers (c = 1, 2, 3, 4). Half of the students at level 4 gave c = 10 - d. Moreover, 44 %, 50 % and 38 % of the students at levels 1, 2 and 3 gave the wrong answer c = 4. This shows the tendency of avoiding letters by evaluation is strong even with the students at level 3.

Students can solve item 12(ii) if they realize that M and P can each take on many values and that some of these values may coincide. However, almost all the students do not realize that the two letters can have equal values. Thus, 46 % of the students gave the wrong answer "Never." This shows that the students who wrote such an answer were not able to conceive the possibility of two different letters having the equal values. For these students different letters always represent different numbers. Only 2 % of the students gave the correct conditional response "Sometimes when M = P". Seventeen percent of the students wrote "sometimes" without giving the conditional response. The following table depicts students' responses at each level to item 12(ii).

Table 4.4.5b Students' responses to item 12(ii) at each level

	Sometimes (M = P)	Sometimes	Never
Level 0	0.24 %	16 %	33 %
Level 1	0.89 %	16 %	46 %
Level 2	3.2 %	18 %	57 %
Level 3	15 %	18 %	67 %
Level 4	50 %	25 %	25 %

As can be seen from table 4.4.5b, 3.2 % of the students at level 2 gave the correct answer. Fifteen percent of the students at level 3 wrote the correct answer. Half of the students at level 4 answered the item correctly. Among the students at levels 1, 2, and 3, 46 %, 57 % and 67 % wrote "Never". Most interestingly, the percentage of the students who gave such response increased uniformly but then decreased to 25 % at level 4. This indicates that these students could not perceive the fact that two different letters could stand for the same number in the same situation and the misconception persisted with the students assigned to higher levels of understanding. To these students, different letters do not represent the same number (equal numbers). However, it cannot be concluded that whether these students are conceiving the two letters as numbers or not, because students might consider the two letters as if they stand for two different objects.

In searching for underlying causes of the misconception that different letters stand for different numbers, Oliver (1989) conducted an interview with ten students randomly chosen. From the interviews, Oliver found that for four students answering "Never" to this question, the literal symbols did not represent numbers, but they represent names of objects. For these pupils, different letters represent different objects, which can never be the same. A further four students viewed the literal symbols in the same question as representing unique, unknown values, from which then it follows that different symbols necessarily represent different values.

Items 11 and 12(ii) were more difficult than many of the specific unknown items on this test, and this is because students can handle specific unknowns before they conceive letters as generalized numbers. However, it is perhaps more fruitful to regard these two ways of interpreting letters as complementary, as it seems likely that in the course of many algebra tasks children will flip from one interpretation to the other, depending on which is momentarily more convenient (Kuchemann, 1981). For example, children might solve item 16 (blue and red pencils) by treating b and r as specific unknowns, but then realize the answer $5b + 6r = 90$ is true for several values.

4.4.6 Letter as variable

The use of a letter as a variable requires more than having to operate on letters as specific unknown and conceiving a letter as capable of having a number of possible values in turn. It goes beyond conceiving a letter as being capable of representing one or alternatively, more than one values simultaneously. It requires understanding the relationship that one value is having with the other value. In particular, students who are able to use a letter as a variable must describe the effect of one value on the other. A student who reached this stage would be able to discuss meaningfully the effect of one variable in relation to the other. One of the items that involve such relation is Question 13.

Figure 4.4 Question 13.

$a = b + 3$. What happens to a if b is increased by 2?...........

$f = 3g + 1$. What happens to f if g is increased by 2?...........

A key feature of both parts of Question 13 seemed to be that they involved a relationship between relations, which according Collis is called "second-order relation" (Kuchemann, 1980). In case of the first part of the question, this is the simplest kind of relation between all pairs of values and can be described as "a is always 3 more than b." That is, the increase or decrease in value of b does not change the relation between a and b, and a is 3 bigger than b. As a result, the increase of b by 2 results in the increase of a by 2.

But in case of the second part of the question, the increase of g by 2 results in the increase of f by 6. Likewise, the decrease of g by 2 results in the decrease of f by 6. It can be concluded that the increase or decrease in value of g by a certain number results in the increase or decrease of f by "three times the number." This item is much more difficult as it needs not only to understand the effect of the change in one value in relation to the other, but also requires to further understand how they change. Confronted with this question, only two students, out of 1773 students, were able to answer the first part of the question correctly. Besides, only one student gave the required response for the second part. In most cases, the students did not seem to understand the question at all. Many students evaluated a and f by substituting 2 for the values of b and g in the first and second parts of the question respectively. Few students replied simply "a increases" and "f increases" for the first and second parts of the question respectively.

As mentioned in chapter 2, there are two ways of interpreting a variable. The first interpretation of a variable is a relatively static one that emphasizes a variable as a tool for generalizations or defining patterns. The other is more dynamic which in essence captures the variability and simultaneous changes in one variable in comparison to another.

Letters, as argued by Kuchemann (1980), are used as variables when a second-order relation is established between them. The distinction between variables defined in this way, and letters used merely as specific unknowns or generalized numbers, can be illustrated by the different meanings that accrue to the relationship, $5b + 6r = 90$, between the number of blue and red pencils in Question 16. With the letters regarded as specific unknowns, the relationship is simply a statement that is true for a particular pair of numbers. This statement is essentially static. When the letters are regarded as generalized numbers, $5b + 6r = 90$ becomes a statement that is true for several pairs of numbers (0,15 6,10 12,5 18,0). This shows that b and r can change, but does not indicate itself how they change, for which it is necessary to compare the change in some way. The first relationship can be stated as "r decreases as b increases". However, the analysis can be taken a step further to establish a relationship like "the increase in b is greater than the corresponding decrease in r." For example, "6 is greater than 0" by more than "10 is less than 15." Or alternatively "0 is less than 6" by more than "15 is greater than 10." This is a second-order relation and describes the degree to which a change in one of the unknowns in $5b + 6r = 90$ produces a change in the other.

The relevance of a second-order relation can best be explained in Question 2 (which is larger 2n, or n + 2). In order to see some relationship between the two expressions, it is important to observe what happens to 2n and n + 2 when specific values are chosen for n, for example n = 3 and n = 6. This gives the pairs 6,5 and 12,8 for 2n and n + 2, from which the most obvious conclusion, which holds for each pair is that "2n is larger than n + 2." According to Kuchemann (1980) such relation is called a first-order relation. However, the analysis can be taken a step further by considering what happen as n changes. Such analysis helps to see a more complex relationship between 2n and n + 2, of the sort "as n increases, the difference between 2n and n + 2 increases" (12 – 8 > 6 – 5), or "as n increases, the increase in 2n is greater than the increase in n + 2" (12 – 6 > 8 – 5). These are second-order relations, whose significance lies in the fact that they open up the possibility that for some smaller values of n the difference between 2n and n +2 may be decreased to zero (n = 2) or even reversed (n < 2). Even though it is not expected that students go precisely through these steps to solve Question 2, students who are able to cope with complex relations of this sort are likely to consider the possible effect of n on the relative size of 2n and n + 2. However, students with less processing capacity will go for something simpler and more direct (Kuchemann, 1980).

Question 2 (which is larger, 2n or n + 2?) proved to be one of the most difficult items: only 2 students out of 1773 students gave a correct conditional response. Thirty-four percent of the students gave the incorrect response "equal." Twenty-four percent of the students wrote 2n is greater than n + 2. Some of the students who gave this answer tried to provide a reason such as "it is multiply", whereas others gave numerical examples. Fifteen percent of the students gave the response "n + 2 is greater than 2n" and to these students, as was seen in most cases from their explanation, n + 2 is the same as 3n and hence greater than 2n. The following table shows the replies of the students for Question 2.

Table 4.4.6 students' responses to Question 2.

Which is the larger, 2n or n + 2?	
2n	24 %
equal	34 %
n + 2	15 %

None of the other items on the test involves second-order relations. However, the coordination required to solve Question 15 is equally complex. In order to solve this Question, students need to realize that a set x can equally be represented by 5x. Furthermore, students require dealing with the resulting transformation on the values of x, which is ÷5, the inverse of x. The question was answered correctly by only 1.52 % of the students.

Figure 4.5 Question 15.

If the equation $(x + 1)^3 + x = 349$ is true when $x = 6$, then what values of x makes the equation $(5x + 1)^3 + 5x = 349$ true? $x =$ ....

4.5 Comparison of students' performance by school and by gender

In this section, we compare the performances of the Eritrean grade eight students by school and by gender. Pearson chi-square test is used to see if there is a relationship between level of understanding and school. Moreover, the same is also done with the level of understanding and gender.

4.5.1 Students' performance by school

In the previous section, we have seen that the performance of the students was extremely poor. It is also important to see whether the performance is equally consistent with the five schools or not. In this section, we shall see the performance of the five schools. Moreover, the study attempts to make a comparison of performance among the students of the five schools.

However, as the number of the students in each school is not the same the analysis could not give the right image as to which school performed well. In order to make such comparisons, it is important to consider the levels of understanding within school. The following table shows the percentages of students at different levels within each school.

Table 4.5.1 Percentage of students at each algebra level by school.

	School 1 (%) (S1)	School 2 (%) (S2)	School 3 (%) (S3)	School 4 (%) (S4)	School 5 (%) (S5)
Level 0	28.9	15.5	26.7	8.5	31
Level 1	53.6	59.1	58.1	58.9	54.5
Level 2	14.2	19.9	12.9	24.1	12.1
Level 3	3.3	5.2	2.3	8	1.7
Level 4	0	0.3	0	0.5	0.7

More than a quarter of the students are at level 0 in schools 1, 3 and 5 i.e. S1, S3 and S5. School 4 (S4) has the lowest percentage of students at level 0, whereas S5 has the highest percentage of the students who are unable to make a coherent attempt at the easiest items of the test (level 1 items).

Most of the students in each school were at level 1. Above half of the students in each school were at a level where solving an item, in its simple structure, requires evaluation of a letter or treating a letter as an object. S2 has the highest facility (59.1 %), whereas S1 has the lowest facility in which 53.6 % of the students were at level 1.

Nearly a quarter of students in S4 reached the level of understanding that enables them to cope with level 2 items, whereas only 12.1 % of the students of S5 secondary school reached this level. The facilities for the three secondary schools S1, S2 and S3 were 14.2 %, 19.9 % and 12.9 % respectively.

In all schools, the facilities of the students who reached at level 3 were below one-tenth. It is more likely that a school with more students at level 2 have more students in the next level. Students of S4 obtained the highest facility (8 %), whereas below two percent (1.7 %) of the students of school 5 (S5) reached a level of understanding where a letter has to be regarded as a specific unknown.

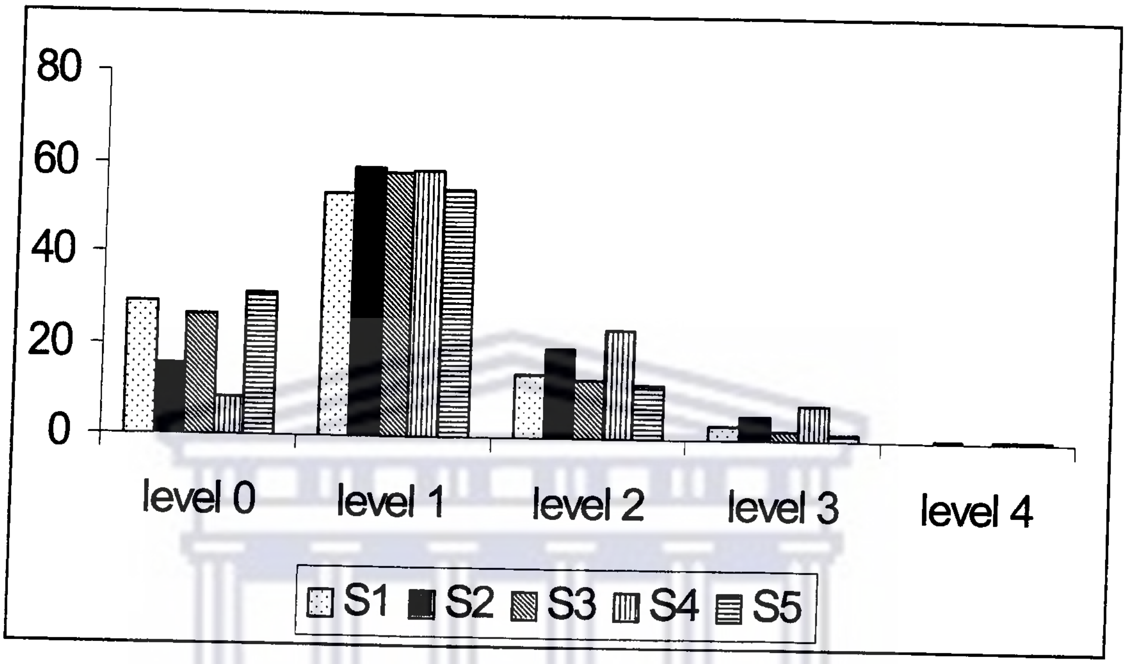
Understanding of algebraic variables involves at least treating a letter as specific unknown. It needs conceiving a letter as being capable of representing a numerical value. The concept of algebraic variables at its higher level involves treating a letter as generalized number or variable. In order to make comparison of the students' performance of the five Eritrean secondary schools, cumulative percentage of students' levels of understanding is required. The following table depicts the cumulative percentage of the students at each algebra level by school.

Table 4.5.1a Cumulative percentage of students at each algebra level by school

	School 1 (S1)	School 2 (S2)	School 3 (S3)	School 3 (S4)	School 3 (S5)
Level 1 or above	71.1 %	84.5 %	73.3 %	91.5 %	69 %
Level 2 or above	17.5 %	25.4 %	15.2 %	32.6 %	14.5 %
Level 3 or above	3.3 %	5.5 %	2.3 %	8.5 %	2.4 %
Level 4	0 %	0.3 %	0 %	0.5 %	0.7 %

The performance of the students in each school can also be described graphically as follows:

Figure 4.6 Graphical representation of students' performance of the five schools in percentage.



Students of each school are inclined towards the lower level of understanding of algebraic variables (level 1). There is a greater percentage of students in S5 than there is in other schools at level 0. The school with the lowest percentage of students at level 0 is S4. There is also less percentage of students in S5 than there is in others at level 2. The school with the highest percentage of students at level 2 is S4.

The Pearson Chi-square was calculated in order to test whether there is a relationship between the school and level of understanding. The Pearson Chi-square was obtained as 0. The following table shows the Chi-square test for the relationship between the schools and levels of understanding.

Table 4.5.1b Chi-square test for the schools and levels of understanding.

	df	Asymp. Sig. (2-sided)
Pearson Chi-square	16	.000

Table 4.5.1b shows that the Pearson Chi-square is less than 0.05. It can be concluded that the relationship between schools and levels of understanding is significant at 0.05 level of significance. The highest level of understanding (level 4) was disregarded due to the fact that within each cell the expected count was less than 5. However, in regard to level 3, it is found that the percentage of students in S4 is more than the percentage of students in other schools at this level. The percentage of students in S5 is less than the percentage of students in other schools. These differences are not due to chance, rather it is attributed to the differences in schools. In other words, the differences in levels of understanding are accounted for the differences among the students of the five schools.

4.5.2 Students' Performance by gender

The following table displays the percentage of the students at each level of understanding within each gender.

Table 4.5.2 Percentage of students at each algebra level by gender

	Male (%)	Female (%)
Level 0	22.9	25
Level 1	55.9	58.9
Level 2	17.1	12.6
Level 3	3.8	3.5
Level 4	0.3	0

As can be seen from table 4.5.2, most of the students in both genders lie within the lower level of understanding (level 1). Twenty-five percent of the female students were not able to cope persistently with level 1 items, whereas 22.9 % of the male students were unable to solve items that require to be evaluated or treated as an object. About 4 % of the male students were able to cope with items that require treating a letter as a specific unknown, whereas only 0.3 % of these students were capable of handling a letter as a generalized number or variable. However, 3.5 % of the female students were able to cope with items that require treating a letter as specific unknown and no female student has reached the level of understanding whereby students are able to handle a letter as generalized number or variable in algebraic equations and expressions.

The table below shows the chi-square test for the relationship between the gender and level of understanding.

Table 4.5.2a Chi-square test for the relationship between gender and level of understanding.

	df	Asymptote Sig. (2-sided)
Pearson Chi-square	4	0.113

As can be seen from table 4.5.2a, a Pearson chi-square is calculated in order to see whether there is a relationship between the genders and levels of understanding. It is obtained as 0.113. It is found that there is no significant relationship between the gender and levels of understanding at 95 % confidence interval, since 0.113 > 0.05. However, the findings show that there is more inclination of students towards level 1 in both genders. There are more female students than male students at levels 0 and 1, whereas there are more male students than female students at levels 2, 3 and 4 respectively. Nevertheless, these differences are by chance and are not attributed to the gender differences.

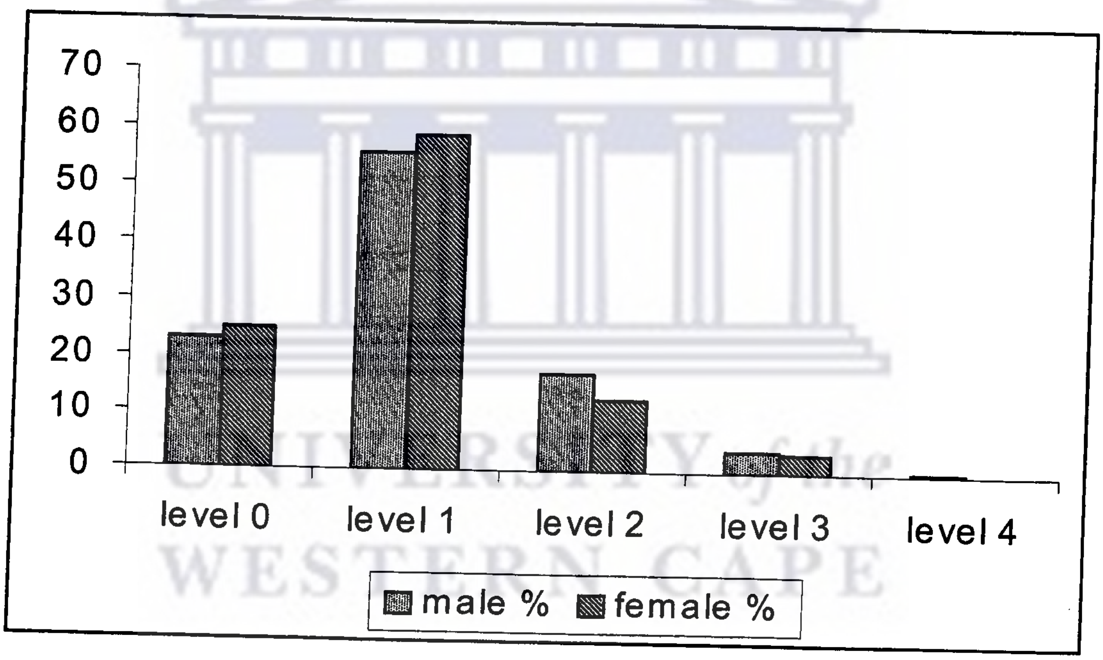
Table 4.5.2b Cumulative percentage of students at each algebra level by gender.

	Male (%)	Female (%)
Level 1 or above	77.1	75
Level 2 or above	21.2	16.1
Level 3 or above	4.1	3.5
Level 4	0.3	0

A quarter of the female students were unable to make a coherent attempt at level 1 items, whereas 22.9 % of the male students were unable to cope with level 1 items consistently. The findings show that almost all students in both genders are unable to cope with items that require treating a letter as a specific unknown. Male students were hardly able to treat a letter as a generalized number or a variable, whereas no single female student was able to cope with such items at all. Therefore, one might conclude that the students' level of understanding of algebraic variables was extremely poor for both genders.

The percentage of the students at each level of understanding with respect to their gender can be described graphically as follows:

Figure 4.7 Graphical representation for students' levels of understanding of both genders in percentage



4.6 Conclusion

This chapter has dealt with the data presentation, data analysis and discussion of the present study. The research question was used as a theme to discuss the collected data. The main topics were Eritrean students' levels of understanding, levels of difficulty of the items, students' strategies and interpretation of letters used by the students in dealing with the items of the test, the legitimate interpretation of letters in algebra and comparison of the students' performances on the test by school and by gender. The next chapter will focus on the conclusion of the study and gives some recommendations that emanate from the study and which are assumed to be useful for the improvement of students' understanding of algebraic variables.

Chapter Five

Conclusion and Recommendations

This chapter presents the conclusion based on the findings of the study and the related literature reviewed. Conclusions are drawn from both the literature study and empirical research. Recommendations, emanating from the study are then proposed that can be useful in laying a foundation for the teaching and learning of the concept of variables and thereby promote students' understanding of algebraic variables.

5.1 Conclusion

The legitimate use of symbols and teacher's perceptions of their status may be different from the pupils' ideas of what is happening. There are many strategies which, as pointed out by Costello (1991), pupils adopt which may or may not be valid, but which frequently circumvent the need for any awareness of the meaning or formal structure which lies behind the symbolism.

Finding the specific numerical value of an unknown or an expression is often a perfect valid activity (Costello, 1991). Sometimes, though, it is not; but it is not entirely surprising that some students fall into inappropriate evaluation. Rather than write 8 + g for item 4(iii) (if e + f = 8, then e + f + g =...), many of the Eritrean grade eight students provide a numerical answer. The most common incorrect answer was 12, given by 23 % of these students. Another common incorrect answer was 9, given by 6 % the students. The perception may be, as argued by Costello (1991), that there is a pattern to be discovered, or a code to be cracked but surely the answer must be a number. The association of the linear orders of alphabets with the linear orders of the whole numbers is not evident with the Eritrean grade eight students in this study.

The two main ways of avoiding dealing with a letter in an algebraic expression, as discussed in chapter 4, are to keep it in but ignore it, and to lose it altogether. Some students use such strategies with considerable consistency, apparently content with the proportion of right answers. Though it sometimes works with items such as 3(i) (add 4 onto n + 5), it doesn't work in most cases when a letter is intended to represent a specific unknown. The Eritrean grade eight students appear to use such a strategy when the letter needs to be regarded as specific unknown, for example, in cases of items 3(ii) (add 4 onto 3n) and 3(iii) (multiply n + 5 by 4). Rather than write 3n + 4 for item 3(ii), 36 % of the students reply 7n (Keep n but ignore it). Sixteen percent of the students wrote 7 (lose n altogether). Likewise, instead of writing 4n + 20, 20 % of the students write 20n and 15 % of these students gave 20 as an answer (see table 4.4.4).

Eritrean grade eight students have the tendency to regard letters as representing objects, rather than generalised numbers or specific unknowns. Sometimes such a perception of a letter's meaning, as a concrete object rather than something quite abstract, allows students to cope with statements and expressions which would otherwise be inaccessible to them. However, it breaks down when it is essential to distinguish between the objects themselves and the number of objects.

In general, the Eritrean grade eight students performed relatively better on the items that require treating letters as objects than those that require handling letters as specific unknowns or generalised numbers. Eighty-nine percent of these students gave the correct answer, 7a, for item 9(i) (2a + 5a = ...), where treating the letter as object is sufficient to answer the item correctly. Likewise, 61 % of the students wrote the correct answer for item 17(i), though it needs recalling the fact that the perimeter of a triangle is the sum of the lengths of the three sides (see table 4.4.3).

Sometimes concrete representations of letters are used to explain basic rules for algebraic manipulation. Adding 2a + 5b + a, as in item 9(iv), can be translated as '2a's and 5b's plus one more a', where a and b represent some concrete objects or stand as objects in their own right. This may appear helpful initially, though it scarcely provides the activity with meaning or justification. However, such perception will produce a stumbling block later if the letter necessary stands for the number of objects rather than objects.

In other words, the use of letters as objects is a very effective way of reducing the difficulty of certain algebra problems. Nevertheless, the continuing tendency to regard letters as symbols for objects rather than numbers appears to be a significant stumbling block in learning algebra.

Wrongly interpreting an algebraic letter as the name of an object is a well-known and serious obstacle to writing expressions and equations in certain contexts. Confronted with Question 6 (see figure 4.2), 25 % of the Eritrean grade eight students interpreted the expression 8c + 6p as 8 cabbages and 6 pumpkins, though the expression represents the total cost of the vegetables bought. Besides, the students are asked to write the total number of vegetables bought. Instead of writing the correct expression, c + p, 54 % of the students gave 14. It is evident in the present study that these students conceive the letters as representing some concrete objects rather than something quite abstract, in this case, the cost of cabbages and pumpkins.

In items involving generalised numbers, many students clearly want to attach a single value to each letter. For example, whilst only 2 % of the students gave the correct answer, $c < 5$, for Question 11, 42 % of the students answered $c = 4$ (see table 4.4.5). The evidence that is found from this study also shows that the Eritrean grade eight students hardly have an understanding of the use of letters as specific unknown or generalised number. Confronted with item 12 (ii) (when is $L + M + N = L + P + N$), 46 % of the students answered 'Never'. Such an item is very difficult for the students who see the letters as concrete objects since they will probably think that two different objects can never be the same. Only 2 % of the students answered the item correctly. It can be concluded that the Eritrean grade eight students are hardly capable of conceiving the fact that M and P can each take on many values and that some of these values may coincide.

The item which Kuchemann (1980) considers best tests the use of a letter as a variable is Question 2 (which is larger, 2n or n + 2?). Only 0.11 % of the Eritrean grade eight students (2 out of 1773 students) answered the question correctly. Thirty-four percent of the students wrote '2n equals n + 2'. Fifteen percent wrote n + 2 was larger for a reason like 'because n + 2 is the same as 3n'. Nearly a quarter (24 %) of the students wrote 2n was larger than n + 2 for a reason like 'because it is multiply'. Others made an inference from a single value. However, rather than describing a single relationship here, it is necessary to explain how one relationship depends on another. The answer depends on whether n is greater or less than 2.

A great number of the Eritrean grade eight students were able to interpret letters as specific unknowns rather than as generalised numbers. This supports the fact that students are able to conceive letters as representing specific unknowns before they conceive letters as being capable of representing generalised numbers. Nevertheless, the majority of the students either treated letters as concrete objects or ignored them.

In attempting to investigate Eritrean grade eight students understanding of algebraic variables, the present study produced results that showed that 72.6 % of the students dealt with letters in algebraic expressions and equations as objects. Only 3.7 % of the students were able to regard letters as specific unknowns and 0.2 % of the students were able to consider letters as generalised numbers or variables (see table 4.2). That is, almost all (95.9 %) of the Eritrean grade eight students were unable to cope consistently with items that can be properly called algebra, that is, items where the use of letters as unknown numbers cannot be avoided. The finding of the present study is worse than the one reported by Kuchemann (1981) which is discussed in chapter 2.

Comparisons by school and gender were done to see if there were relationships among the levels of understanding and these two variables. The findings show that there is no relationship between level of understanding and gender of the students. However, the comparison by school shows that there is significant relationship between schools and levels of understanding. The Pearson chi-square test shows that the relationship between the level of understanding and gender is not significant, whereas the relationship between level of understanding and school is significant at 0.05 level of significance. Thus one might conclude that the differences in performances of the students among the two genders is by chance, whereas the differences in performances of the students among the five schools is not by chance but it is attributed to the differences among the schools. Factors such as different approaches to beginning algebra, teaching materials, teaching styles or the learning environment, as suggested by MacGregor and Stacey (1997), might have a powerful effect on performance of students of different schools. All students can learn and use concepts of algebra, but the manner in which the material is presented and explored may affect the way students see variables. As Speer et al. (1997:308) state, "Students may see variables as arbitrary marks on a paper or a powerful tool for examining patterns and making generalisations".

The role of a variable is imperfectly understood by the Eritrean grade eight students. Although it is the building block for all abstractions in mathematics, its meaning escapes almost all of these students. The elusive nature of the concept of variable and the fact that students often have trouble with it can be attributed to the variable meaning of a variable.

Learning the concept of variable is not as simple as writing a definition on the board. Nevertheless, teachers who themselves have internalised the variable concept seem to pay little attention to it. That is, once teachers have mastered it, they forget how hard it was and just what went into the development of their understanding. As Freudenthal (1983:469) states:

> I have observed, not only with other people but also with myself…, that sources of insight can be clogged by automatisms. One finally masters an activity so perfectly that the question of how and why [students don't understand them] is not asked anymore, cannot be asked anymore and is not even understood anymore as a meaningful and relevant question.

In mathematics, the possibility of doing generalized arithmetic is a powerful and exciting one. In particular, this could be so if a sense of the power and purpose of algebra can genuinely be communicated to pupils and here there are real difficulties. Assurances that it will make sense sometimes or be useful later on are notoriously unsatisfying. The symbolism, when it is introduced, needs to have an immediate meaning and a local purpose. In other words, expressions involving letters need to be used in a helpful way at the time, not in a clumsy or unnecessary way.

Algebra ideas can often be conveyed by a visual approach that emphasizes understanding and meaning rather than symbol manipulation (Speer et al, 1997). As Byers cited by Herscovics and Kieran (1980:579) notes, "Teaching for understanding requires that the continuity of mathematical content be demonstrated to the student during, and prior to, the introduction of new mathematical form".

A focus on solving equations can mask the true nature of the concept of variable. If students are asked to solve 3x - 2 = 10, in this sense x can hardly be thought of as varying. For the same problem, a different approach that might help students in constructing the concept of variable is to ask students to "find a value of x for which 3x - 2 = 10." Then ask, "Can you find a value of x for which '3x - 2' equals 13? Equals 16? Equals 17? Such sets of questions, as argued by Edwards (2000), highlight the true nature of the variable x while still providing the experience in solving simple equations.

Another way to help students understand this concept is to begin the study of a variable with formulas and tables of values. For example, once middle school students have been exposed to the formulas for the area and perimeter of a rectangle, they can work with these formulas to explore the effects of varying the length and width of the rectangle.

5.2 Recommendations

The analysis reveals that almost all the Eritrean grade eight students hardly understand algebraic variables. In order to promote students' understanding of algebraic variables based on this study, some recommendations are forwarded. The recommendations are clustered in two themes that include:

- recommendations on the teaching of algebraic variables
- recommendations for further research

5.2.1 Recommendations for the teaching of algebraic variables

Since variable is such a multi-faceted concept which requires an extensive concept-image (Tall and Vinner, 1981), it is not to be expected that one piece of work would provide all the experiences that students need to build a better understanding. The various uses of the letters in algebra have to be constructed over time, often in a non-hierarchical manner. Based on this study I would like to recommend the following points:

- It is important to stay aware of the difficulties that students are having in trying to understand the different uses of letters in algebra.

- Teachers need to be aware that pupils who appear to perform adequately on a limited range of tasks may be misled into relying on a strategy which is naive or inadequate or illogical and which may quickly break down in more complicated situations (Costello, 1991).

- Teachers need to help students appreciate that algebra is a special language that has its own conventions and uses familiar symbolism in new ways. In order to meet the challenges of dealing with the prior knowledge that students bring to their study of algebra, as suggested by Stacey and MacGregor (1997), teachers need to use algebraic notation more often, emphasise that letters in algebraic expressions stand for number, not for names of things and appreciate that students come to algebra with rich prior experiences of symbol systems.

- The teaching and learning of mathematics should place emphasis on relational understanding rather than on that of instrumental understanding [rules without understanding, as noted by Skemp (1976)]. Skemp states that relational understanding is to do with connections between concepts, rather than between symbols as when we use rules. This kind of understanding may be needed in any application to new or unfamiliar situations specifically when the students learn the familiar ways of using letters in new ways.

- It seems sensible to base the teaching given to students at levels 1 and 2 on meanings for the letters that these students readily understand. The use of letters as objects conflicts with the eventual aim of using letters to represent number of objects. However, teachers have to play a great role in making students aware of such conflicts and see the need to reorganise their thinking and thereby move towards a higher level.

5.2.2 Recommendations for further studies

Understanding the concept of a variable is fundamental to success with algebra. Research studies indicate that there are different ways of building and promoting the concept of algebraic variables. English and Warren (1998) point out that patterning approach can provide a useful base in introducing the variable. Researches also indicate that graphic calculators and computers play an important role in improving students' understanding of algebraic variables (Graham and Thomas, 2000).

The researcher is not aware of any study in Eritrea specifically concerned with the analysis of students' understanding of algebraic variables. Therefore studies that are relevant to the Eritrean context should be conducted.

Some of the topics suggested for possible research are listed below:

- The teaching of algebraic variables
- Mathematics curriculum evaluation in relation to algebraic variables

5.3 Concluding remarks

This study has focused on a conceptual theme that Eritrean grade eight students' have in understanding the algebraic variables. It is hoped that this research will help teachers to see more clearly the diverse conceptual demands of seemingly commonplace activities in school algebra. The research has attempted to describe the difficulties that the students are having and the way that these students treat the literal symbols when dealing with algebraic expressions and equations.

The research has also identified a number of different meanings that can be given to the letters in generalised arithmetic, the choice of which may depend to a large degree on students' levels of understanding (Kuchemann, 1981). The research has found that Eritrean grade eight students frequently tackle algebraic expressions and equations with methods that have little or nothing to do with what has been taught. Perhaps this is because mathematics teaching is often seen as an initiation into rules and procedures, which are often seen by students as meaningless. Students' methods and their levels of understanding need to be taken far more into account, though it is difficult in practice.

BIBLIOGRAPHY

Babbie, E. R. (1973). *Survey research methods*. Belmont: Wadsworth Publishing Corporation.

Begle, E. G. (1980). Why do research? In Richard J. Shumay (Ed.), *Research in mathematics education* (pp. 3-19). Reston: National Council of Teachers of Mathematics.

Behr, A. L. (1983). *Empirical research methods for the human sciences*. Durban: Butterworths Publishers.

Bell, A. W., Costello, J., and Kuchemann, D. (1983). *A review of research in mathematical education,* part A. Windsor: NEFR-Nelson.

Bell, J. (1993). *Doing your research project: A guide for first time researchers in education and social science*. Buckingham: Open University Press.

Blalock, H. M. (1970). *An introduction to social research*. Englewood Cliffs: Prentice-Hall.

Booth, L. (1988). Children's difficulties in beginning algebra. In Arthur F. Coxford (Ed.), *The ideas of algebra, K-12* (1988 Yearbook, pp. 20-32). Reston: National Council of Teachers of Mathematics.

Borasi, R. (1996). *Reconceiving mathematics instruction: A focus on errors*. Norwood: Ablex Publishing Corporation.

Burns, R. B. (2000). *Introduction to research methods*. London: SAGE Publications.

Carey, D. A. (1992). Students' use of symbols. *Arithmetic Teacher*, 40(3), 184-186.

Clement, J., Lockhead, J., and Monk, G. (1981). Translation difficulties in learning mathematics. *American Mathematical Monthly*, 88(4), 286-290.

Costello, J. (1991). *Teaching and learning mathematics 11-16*. London: Routledge.

Davidenko, S. (1997). Building the concept of function from students' everyday activities. *Mathematics Teacher*, 90(2), 144-149.

Del Arte, E. (1996). *Children's notions of algebraic expressions*. [Online]. Available http://www.ilce.edu.mx

Denscombe, M. (1998). *The good research guide: For small-scale social research projects*. Buckingham: Open University Press.

Destafano, J. (1998). *Joint survey on the state high school education in Eritrea*. Unpublished paper presented at a workshop, February 1998, University of Asmara, Asmara.

Edwards, T. G. (2000). Some 'big ideas' of algebra in the middle grades. *Mathematics Teaching in the Middle School*, 6(1), 26-30.

English, L. D., and Warren, E. A. (1998). Introducing the variable through pattern exploration. *Mathematics Teacher*, 91(2), 166-170.

Fraenkel, J. R., and Wallen, N. E. (1993). *How to design and evaluate research in education*. New York: McGraw-Hill Inc.

Freudenthal, H. (1983). *The didactical phenomenology of mathematics structures*. Dordrecht: Reidel.

Gay, R. J. (1981). *Educational research: Competencies for analysis and application*. Columbus: Merrill Publishing Company.

Graham, A. T., and Thomas, M. O. (2000). Building a versatile understanding of algebraic variables with a graphic calculator. *Educational Studies in Mathematics*, 41(3), 265-282.

Gray, E. M., and Tall, D. O. (1994). Duality, ambiguity, and flexibility: A 'proceptual' view of simple arithmetic. *Journal for Research in Mathematics Education*, 25(2), 116-140.

Herscovics, N., and Kieran, C. (1980). Constructing meaning for the concept of equation. *Mathematics Teacher*, 73(8), 572-580.

Hiebert, J., and Carpenter, T. P. (1992). Learning and teaching with understanding. In Douglas A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65-100). New York: Macmillan Publishing Company.

Hirsch, C. R., and Lappan, G. (1989). Implementing the standards: Transition to high school mathematics. *Mathematics Teacher*, 82(8), 614-618.

Hung, D. W. (1997). Meanings, contexts, and mathematical thinking: The meaning-context model. *Journal of Mathematical Behaviour*, 16(4), 311-324.

Johnson, D. C. (1980). Types of research. In Richard J. Shumay (Ed.), *Research in mathematics education* (pp. 20-28). Reston: National Council of Teachers of Mathematics.

Kaplan, R. M. (1987). *Basic statistics for the behavioral sciences*. Boston: Allyn and Bacon Inc.

Kaput, J. J. (1991). Notations and representations as mediators of constructive processes. In Ernst V. Glasersfeld (Ed.), *Radical constructivism in mathematics education* (pp. 53-74). Dordrecht: Kluwer Academic Publishers.

Kieran, C. (1991). Research into practice: Helping to make the transition to algebra. *Arithmetic Teacher*, 38(7), 49-51.

Kieran, C. (1992). The learning and teaching of school algebra. In Douglas A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 390-419). New York: Macmillan Publishing Co.

Kinzel, M. T. (1999). Understanding of algebraic notation from the students' perspective. *Mathematics Teacher*, 92(5), 436-442.

Kuchemann, D. E. (1980). *The understanding of generalized arithmetic by secondary school children*. Unpublished doctoral dissertation, Chelsea College, University of London, London.

Kuchemann, D. E. (1981). Algebra. In K. Hart (Ed.), *Children's understanding of mathematics: 11-16* (pp. 102-119). London: John Murray.

Kuchemann, D. E. (1982). Object lessons in algebra? *Mathematics Teaching*, 98, 47-51.

Leinhardt, G., Zaslavsky, O., and Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research*, 60(1), 1-64.

Leitzel, J. R. (1989). Critical considerations for the future of algebra instruction. In S. Wagner and C. Kieran (Eds.), *Research issues in the learning and teaching of algebra* (pp. 25-32). Reston: Lawrence Erlbaum.

MacGregor, M., and Stacey, K. (1997). Students' understanding of algebraic notation: 11-15. *Educational Studies in Mathematics*, 33(1), 1-19.

Ministry of Education. (1995). *The development of education: National report 1995*. Asmara: Ministry of Education.

Ministry of Education. (1999). *Education brief.* Asmara: Ministry of Education.

Ministry of Education. (2001a). *Eritrea: Basic education statistics 2000/01*. Asmara: Ministry of Education.

Ministry of Education. (2001b). *Curriculum framework for secondary school mathematics*. Unpublished document, Asmara.

Ministry of Education. (2001c). *Mathematics model test 2001 for grade 8*. Asmara: Ministry of Education.

Mulder, J. C. (1982). *Statistical techniques in education*. Pretoria: HAUM Educational Publishers.

Oliver, A. (1989). Different letters stand for different numbers. *Pythagoras*, 20, 25-28.

Oyedeji, O. P. (1992). Effect of instruction based on a validated learning hierarchy in mathematics on student learning outcomes. *International Journal of Mathematical Education in Science and Technology*, 23(2), 267-272.

Philipp, R. A. (1992a). The many uses of algebraic variables. *Mathematics Teacher*, 85(7), 557-561.

Philipp, R. A. (1992b). A study of algebraic variables: Beyond the student-professor problem. *Journal of Mathematical Behavior*, 11(2), 161-176.

Richards, P. (1982). Difficulties in learning mathematics. In M. Cornelius (Ed.), *Teaching mathematics* (pp.59-80). London: Croom Helm.

Rosnick, P. (1981). Some misconceptions concerning the concept of variable. Are you careful about defining your variables? *Mathematics Teacher*, 74(6), 418-420.

Rubenstein, R. N., and Thompson, D. R. (2001). Learning mathematical symbolism: Challenges and instructional strategies. *Mathematics Teacher*, 94(4), 265-271.

Schoenfeld, A. H., and Arcavi, A. (1988). On the meaning of variable. *Mathematics Teacher*, 81(6), 420-427.

Selkirk, K. (1982). Links between mathematics and other subjects. In M. Cornelius (Ed.), *Teaching mathematics* (pp.186-213). London: Croom Helm.

Sierpinska, A., Kilpatrick, J., Blacheff, N., Howson, A.G., Sfard, A., and Steinbring, H. (1993). What is research in mathematics education, and what are its results? *Journal for Research in Mathematics Education*, 24(3), 274-278.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, 77, 20-26.

Speer, W. R., Hayes, D. T., and Brahier, D. J. (1997). Becoming very-able with variables. *Teaching Children Mathematics*, 3(6), 305-314.

Stacey, K., and MacGregor, M. (1997). Ideas about symbolism that students bring to algebra. *Mathematics Teacher*, 90(2), 110-113.

Steinberg, M. B., and Philcox, S. E. (1983). *Research methods for higher degrees*. Cape Town: U.C.T. Publishers.

Tall, D., and Thomas, M. (1991). Encouraging versatile thinking in algebra using the computer. *Educational Studies in Mathematics*, 22(2), 125-147.

Tall, D. O., and Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, 12(2), 151-169.

Vergnaud, G. (1985). Understanding mathematics at the secondary-school level. In A. Bell, B. Low, and J. Kilpatrick (Eds.), *Theory, research and practice in mathematical education: Working group reports and collected papers*, 5th international congress in mathematical education (pp. 27- 45). Nottingham: University of Nottingham.

Wagner, S. (1981). Conservation of equation and function under transformations of variable. *Journal for Research in Mathematics Education*, 12(2), 107-118.

Wagner, S. (1983). What are these things called variables? *Mathematics Teacher*, 76(7), 474-479.

Wiersma, W. (1980). *Research methods in education: An introduction*. Illinois: F.E. Peacock Publishers.

Wolpert, E. M. (1981). *Understanding research in education: An introductory guide*. Iowa: Kendal/Hunt publishing company.

Appendix I

Algebra test

Name:............................. .... School:...................... Class:............

Date:.............................. Sex.........................

---

1. Fill in the gaps: $x \rightarrow x+2$ $x \rightarrow 4x$

(i) $6 \rightarrow$ (iii) $3 \rightarrow$

(ii) $r \rightarrow$

2. Which is the larger, 2n or n+2? ......

Explain:...

3. <u>4 added to n</u> can be written as <u>n+4.</u> <u>n multiplied by 4</u> can be written as <u>4n.</u>

Add 4 onto each of these: Multiply each of these by 4:

(i) n+5 (ii) 3n (iii) n+5 (iv) 3n

....

4. If a + b = 43 if n – 246 = 762 if e + f = 8

(i) a + b + 2 = ... (ii) n – 247 = ... (iii) e + f + g = ...

5. (i) What can you say about a if a + 5 = 8? ....

(ii) What can you say about b if b + 2 is equal to 2b ....

6. Cabbages cost 8 Nacfa each and pumpkins cost 6 Nacfa each. If c stands for the number of cabbages bought and p stands for the number of pumpkins bought,

(i) what does 8c + 6p stand for? ....

(ii) what is the total number of vegetables bought? ....

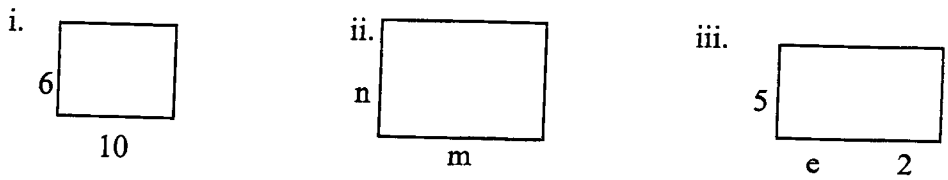
7. (i) What can you say about u if u = v + 3

and v = 1 ....

(ii) What can you say about m if $m = 3n + 1$

and $n = 4$ ....

8. What are the areas of these shapes?



9. <u>a + 3a</u> can be written more simply as <u>4a.</u>

Write these more simply, where possible:

(i) $2a + 5a =$ ....

(ii) $2a + 5b =$ ....

(iii) $(a + b) + a =$ ....

(iv) $2a + 5b + a =$ ....

(v) $(a - b) + b =$ ....

(vi) $3a - (b + a) =$ ....

(vii) $a + b + a - b =$ ....

(viii) $3a - b + a =$ ....

(ix) $(a - b) + (a - b) =$ ....

10. What can you say about r if $r = s + t$

and $r + s + t = 30$ ....

11. What can you say about c if $c + d = 10$

and c is less than d ....

12. When are the following true – always, never, or sometimes?

Underline the correct answer:

(i) $A + B + C = C + A + B$ Always. Never. Sometimes, when ....

(ii) $L + M + N = L + P + N$ Always Never Sometimes, when ....

13. $a = b + 3$. What happens to a if b is increased by 2? ....(i)

$f = 3g + 1$. What happens to f if g is increased by 2? ....(ii)

14. Cakes cost c cents each and biscuits cost b cents each.

If I buy 4 cakes and 3 biscuits,

what does $4c + 3b$ stand for? ....

15. If the equation $(x + 1)^3 + x = 349$ is true when $x = 6$, then what value of x makes the equation $(5x + 1)^3 + 5x = 349$ true? $x =$ ....

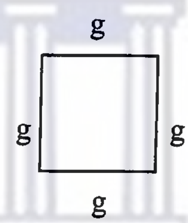
16. Blue pencils cost 5 cents each and red pencils cost 6 cents each. I buy some blue and some red pencils and altogether it costs me 90 cents.

If b is the number of blue pencils bought, and

if r is the number of red pencils bought,

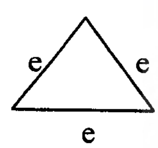
what can you write down about b and r? ....

17. This square has sides of length g.
So, for its perimeter, we can write $p = 4g$.



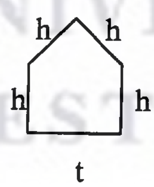
What can we write for the perimeter of each of these shapes?

(i)



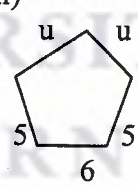
$p =$ ....

(ii)



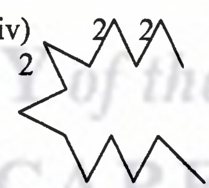
$p =$ ....

(iii)



$p =$ ....

(iv)



part of this figure is not drawn. There are n sides altogether, all of length 2.

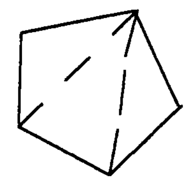
$p =$ ....

18. Azeb's basic wage is 20 Nacfa per week. She is also paid another 2 Nacfa for each hour of overtime that she works.

(i) If h stands for the number of hours of overtime that she works, and if w stands for her <u>total</u> wage (in Nacfa), write down an equation connecting w and h: ....

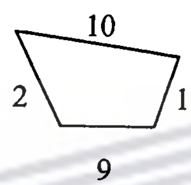
(ii) What would Azeb's total wage be if she worked 4 hours of overtime? ....

19. In a shape like this you can work out the number of diagonals by <u>taking away 3</u> from the number of sides.



A shape with 5 sides has 2 diagonals;
(i) a shape with 57 sides has .... diagonals;
(ii) a shape with k sides has .... diagonals.

20. Work out the perimeter of this shape.



21. If Abraham has A marbles and Petros has P marbles, what could you write for the number of marbles they have altogether? ....

22. Write down the smallest and the largest of these: (i) Smallest (ii) Largest

n+1, n+4, n-3, n, n-7, where n ϵ R ....

23. you can feed any number into this machine: $\rightarrow \boxed{+\,10} \rightarrow \boxed{\text{x } 5} \rightarrow$

Can you find another machine that has the same overall effect? $\rightarrow \boxed{\text{x} \ldots} \rightarrow \boxed{+ \ldots} \rightarrow$

Appendix II

ዩኒቨርሲቲ ኣስመራ   جامعة اسمرا

UNIVERSITY OF ASMARA

ERITREAN HUMAN RESOURCES DEVELOPMENT PROJECT (EHRD)

PROJECT COORDINATING UNIT (PCU)

Ref. No. HRD\4/4847/02   DATE: 16 DEC 2002

To:
Zoba Gash Barka
Ministry of Education
Barentu

Dear Sir/Madam,

The bearer of this letter, **Ato Yosief Tekie Sium**, is one of the students placed by the EHRD Office at the **University of Western Cape** to do his Masters degree in **Education**.

Ato Yosief is presently back in Asmara to collect data/information for his thesis work titled: ***"Eritrean Students' Understanding of Algebraic Variables"***. We have come to learn that, to complete his research project successfully, he would definitely need to have access to your organization's data/information base.

I take this opportunity to request you to assist him in his research endeavour.

I thank you for your time and kind consideration.

Awet N'hafash!

Tewelde Zerom, PhD.
Manager, EHRD-PCU
University of Asmara

18/12/2002

cc.: Mehari Tewolde
Monotoring & Evaluation Officer, EHRD Project
University of Asmara

Mailing Address: University of Asmara, P. O. Box 1220, Asmara, Eritrea
Tel: 291-1-119035, 291-1-161926
Fax: 291-1-124300, 291-1-162236
E-mail: hrdpcu@asmara.uoa.edu.er

Appendix III

ሃገረ ኤርትራ
ሚኒስትሪ ትምህርቲ

دولة ارتريا
وزارة التعليم

THE STATE OF ERITREA
Ministry of Education

ዕለት 11/12/2002 التاريخ

ቁ. መወከሲ Ref. المرجع

ናብ፥ ሓለፍቲ ክፍልታት ሚኒስትሪ ትምህርቲ
ሓለፍቲ ኣብያተ-ጽሕፈት ጨንፈር ትምህርቲ ዞባታት
ሓለፍቲ ኣብያተ-ጽሕፈት ጨንፈር ትምህርቲ ንኡስ ዞባታት
ሓለፍቲ ኣብያተ-ትምህርትን ሱፐርቫይዘራትን

ጉዳይ፥ መጽናዕቲ ንምክያድ ምትሕብባርኩም ምሕታት

መምህር [illegible] ሓደ ካብቶም ኣብ ደቡብ ኣፍሪቃ ብደረጃ ማስትረይት ዲግሪ ትምህርቶም ዝከታተሉ ዘለዉ ኣባላት ሚኒስትሪ ትምህርቲ እዩ። ኣብዚ እዋን'ዚ ድማ ምስ ትምህርቲ ዝተኣሳሰረ መጽናዕቲ ንምክያድ ናብ ኤርትራ መጺኡ ኣሎ።

ብዚ መሰረት፡ መጽናዕቲ ኣብ ምክያድ ዘድሊ ምትሕግጋዝ ክትገብሩሉ ብትሕትና ንሓትት።

ዓወት ንሓፋሽ!!

ኣብረሃም ተኽለ
ዳይረክተር
ጨንፈር ምምሕዳር ...

MINISTRY OF EDUCATION