Title

An Ethnomethodological analysis of learners' ways of working in a high-stakes Grade 12 Mathematics National Senior Certificate (NSC) Examination: The case of Trigonometry

A thesis submitted in fulfilment of the requirements for the degree of Doctor Philosophiae in the Faculty of Education, University of the Western Cape

Marius Derick Simons

Supervisors: Prof. C. Julie

Prof. L. Holtman

ABSTRACT

In South Africa the National Senior Certificate (NSC) examination is the capping external examination taken at the culmination of twelve years of schooling. Levels of success in the examination offer examinees access to a variety of career options. High levels of success in the mathematics examination are a pre-requisite for entry into studies linked to so-called elite careers. However, performance of examinees in the NSC Mathematics examination is not of a requisite standard and only a few examinees achieve results that fall within the high levels of the achievement bands.

In order to give mathematics teachers and others insight into performance in the NSC Mathematics examination, various forms of feedback are provided. One purpose in doing so is to provide teachers with an understanding of the examinees' ways of working in order for them to adjust their classroom practice to address mistakes displayed in the work of the examinees. The feedback provided is primarily of a superficial kind with the mere listing of such mistakes. The purpose of this study was to investigate whether or not it is possible to analyse the production of the responses of examinees in the NSC mathematics examinations more meaningfully.

In recent years the production of mathematical work has received much attention, primarily shaped by a form of ethnomethodology which brought to the fore the textures of such mathematical work. Although ethnomethodological approaches to the study of the use and production of knowledge in domains such as laboratory work in science follows a context-immersed approach, for mathematics a documentary analytical approach is normally adopted. This is in a sense forced upon researchers because of the somewhat private nature of the work of mathematicians. The production of responses in the NSC mathematics examination is highly private and individualised and the only access to how these responses are produced is the written work of the examinees. The study reported here analysed this written work for Trigonometry from an ethnomethodological perspective. It was found that the strategies and tactics used by examinees are highly driven by the context within which the high-stakes examination is situated. The results of this study demonstrated that examinees ways of working exhibit the general structure of the practice that is commonly found in mathematicians discourse practices. To exploit the tactics and strategies used by examinees is to provide learners with the produced work of past examinees for them to analyse in a manner similar to what was done in the study.

Keywords:

- Ethnomethodology
- Assessment
- High-stakes examinations
- Feedback
- Trigonometry
- Resistance and Accommodation

Declaration

I declare that 'An Ethnomethodological analysis of learners' way of working in a high-stakes Grade 12 Mathematics National Senior Certificate (NSC) Examination: The case of Trigonometry' is my own work; that it has not been submitted before for any degree or examination in any other university, and that all the sources I have used or quoted have been indicated and acknowledged as complete references.

Marius Derick Simons

Signed: ………………………………………………………….. May 2016

Dedication

To my family: I thank you for the patience and moral support you have availed to me when I spent so much time away from home. May God reward you for your steadfastness in all things throughout your lives.

Acknowledgements

I am grateful to the following people for their support and guidance in completing this thesis:

- My supervisors, Professor Cyril Julie and Professor Lorna Holtman for their invaluable time, effort and guidance throughout this study
- A special acknowledgement to Professor Cyril Julie for believing in me and through our discussions, teaching not only academic but general life lessons, which I will take with me on my journey wherever life leads me
- My colleagues in the LEDIMTALI Project for their comments and critique of this work
- To Professors Martin Goedhart and Tom Koole for your support, guidance and hospitality, during my time at Groningen University
- My dear wife, Yolande and my children Bevan, Brent and Brady for their understanding throughout this project
- A special acknowledgement to my wife for your support and patience during the six months I spent in Europe working on my thesis; I know it was difficult and I will forever be grateful to you for the sacrifices you made
- My sisters for their moral support to my family during my six months stay in Europe
- Above all I give glory to God the Almighty for His unending loving kindness; that He has been with me every step of my journey through thick and thin all my life; it is with His Spirit and the understanding with which He blessed me that I could complete this project

TABLE OF CONTENT

Table of figures

Table of tables

List of acronyms

AMESA	Association for Mathematics Education of South Africa
CAPS	Curriculum Assessment Policy Statement
DBE	Department of Basic Education
LEDIMTALI	Local Evidence-Driven Improvement of Mathematics Teaching and Learning Initiative
LTSM	Learning and Teaching Support Material
MER	Mathematics Education Research
NEPA	National Education Policy Act
NSC	National Senior Certificate
WCED	Western Cape Education Department

CHAPTER ONE

STATEMENT OF THE PROBLEM

1.1 Introduction

In this chapter a general introduction is followed by an explanation of the context and the focus of the study. The statement of the problem is then discussed and the research question presented. A discussion of the significance of the study and the chapter outline is provided. The chapter concludes with definitions of terms and a presentation of the organization of the study for the chapters that follow.

1.2 Background and motivation

Assessment according to the Curriculum Assessment Policy Statement (CAPS) (2010, p. 101) "is a continuous planned process of identifying, gathering and interpreting information about the performances of learners." According to Webb (1993), in any assessment it is as crucial to consolidate the information collected or observed as it is to give comprehensive feedback on it. The Department of Basic Education (DBE) provides various kinds of feedback on the NSC high-stakes Mathematics examination every year. One reason for this is to improve results. The feedback reports comprise the technical report, school performance report and the subject report, each of which I shall discuss briefly.

The technical report gives an analysis of the results at national, provincial and district levels. This includes full-time and part-time students. The technical report reviews the performance of the schooling system in terms of poverty indicators, referred to as quintiles (National Senior Certificate Report 2013). Another report is the school Performance Report, and this one provides information on overall performance as well as performance over the last three years. It concerns school managers, district officials and provincial officials (DBE, 2013).

There is also a subject report which is part of the ongoing drive to improve feedback to schools and districts on the data forthcoming from the national high-stakes mathematics examinations. The report prioritizes all gateway subjects which include subjects that show high enrolment and those regarded as subjects of importance such as Mathematics. Together with the subject report teachers are provided with a question analysis per question paper (National Senior Certificate Subject Report, 2013). The feedback reports provided by the DBE which I want to elaborate on form a diagnostic report of learners' responses to items in the examination.

The diagnostic report provides valuable information that could be used to improve teaching and learning. The diagnostic analysis identifies errors and misconceptions which could be used by teachers, subject advisors, curriculum planners and other education support officials in their planning for the next school year. Furthermore, the report provides suggestions for the improvement of teaching and learning which include teaching methodology and the use of learning and teaching support materials (LTSM).

Figures 1.1 and 1.2 give an indication of the diagnostic report provided by the DBE. Figure 2.1 presents an example of the November 2014 Mathematics examination for the first question in Trigonometry, Question 5 (The diagnostic report, DBE, 2015, p. 127).

QUESTION 5: TRIGONOMETRY

Common errors and misconceptions

(a) Candidates experienced difficulty in selecting and applying the correct trigonometric ratio in a right-angled triangle. They did not understand the instruction: 'Show, by calculation, that $x = 60°$ '.

(b) Many candidates did not make the relationship between the sides and angles of the triangles. Hence they did not know when to use the sine and cosine formulae. It was very disappointing that some candidates used the sine ratio in the cosine formula:

$AD^2 = AP^2 + DP^2 - 2(AP)(DP)\sin D\hat{P}A$. This formula is given in the information sheet. Some candidates did not understand that PA bisectd $D\hat{P}C$ and used the incorrect angle in their calculation. Some candidates assumed that $\hat{D} = 90°$ or used the Theorem of Pythagoras when the triangle was not right-angled.

(c) Candidates assumed that $\hat{D} = 90°$ and concluded that $y = 60°$. Some candidates substituted incorrectly into the sine rule when trying to calculate the size of y. Some candidates first calculated the size of $\hat{D}$ by using the sine rule. But instead of using the obtuse angle of 126,25°, they used 53,75°. These candidates did not realise that the ambiguous case of the sine rule applied to ΔDAP.

(d) The notations at the angles: y and x confused some candidates to believe that $y = 2x$. They believed that the double lines at y meant that y was double the size of x. This was not the case.

(e) Candidates rounded off their answers to whole numbers instead of working to 2 decimal places.

Figure 1.1: Diagnostic report, errors and misconceptions

The errors and misconceptions are followed by suggestions for teaching such as presented in Figure 1.2 below (DBE, 2015, p 127).

Suggestions for improvement

(a) The solution of right-angled triangles is taught in Grade 10. Learners should be reminded that using trigonometric ratios to solve right-angled triangles is an acceptable method in Grade 12. They need not only use the sine or cosine formulae in the examinations.

(b) Learners should be able to identify the sides of a right-angled triangle in relation to the required angle. A good starting point would be to write out the relationship as $\sin x = \frac{CP}{AP}$ and then proceed.

(c) Learners should be able to distinguish when they need to use sin and when they need to use $\sin^{-1}$ functions on the calculator.

(d) When in doubt, learners should be encouraged to refer to the information sheet for the correct formula. It is unacceptable for learners to write any formula that is given in the information sheet incorrectly.

(e) Learners should interpret 'Show, by calculation, that $x = 60°$ 'as 'calculate the size of x and the answer you should arrive at is 60° '.

(f) Teachers need to discuss the prerequisites for using the sine and cosine formulae.

Figure 1.2: Diagnostic report, suggestions for improvement

The reports mentioned in the foregoing section sum up the final judgement of the quality of learners' work in mathematics. According to Black and William (1998, p. 6) feedback to teachers and learners should steer away from comparing learners' performances and report on the quality of their work instead. It should also provide learners with the necessary advice on what they can do to improve their performance. To further assist learners to perform better, and in preparation for the next NSC examination the feedback gained from the diagnostic report allows for the implementation of other support strategies, which I discuss in Chapter Two. The diagnostic analysis identifies mistakes, errors and misconceptions which can be used by teachers, subject advisors, curriculum planners and other education support officials. Furthermore, it provides suggestions for the improvement of teaching and learning, which include teaching methodology and the use of learning and teaching support materials (LTSM). It also deals with the analysis of learners' performance in individual questions in the National Senior Certificate (NSC) mathematics examination.

These reports sum up the final judgement of the quality of learners' work in mathematics. According to Black and William (1998, p. 6), "Feedback to any teacher and pupil should be about the particular qualities of his or her work, with advice on what he or she can do to improve, and should avoid comparisons with other pupils".

Although a number of studies have been conducted to identify and analyse the numerous factors that affect academic performance in mathematics and in various other fields of study, the problem of poor performance persists in our schools. However, research has shown that this problem is a general one. Some factors identified by researchers include students' effort and previous schooling (Siegfried & Fels, 1979). Anderson & Benjamen (1994) identified parents and family income. Devadoss & Foltz (1996) found that self-motivation and learners' age as well as learning preferences are factors in this regard. In feedback reports of research efforts notably those of the DBE's diagnostic report, much has been done to improve poor performance in mathematics. However, neither any of the studies conducted nor any of the feedback arising from these, focussed on how responses are actually produced in high-stakes examination settings with a particular non-interest in what was done right and wrong.

According to Lave (1993) knowledge and learning will be found distributed throughout the complex structure of person-acting-in-setting. How learners respond to the mathematics in the high-stakes examination is further emphasized through Knorr Cetina's (1999) argument that reported knowledge should rather be seen in the context of something that is being produced. Julie (1998, p.54) sees this as the production of mathematical artefacts which consist of "accompanying bits" that could be things that could become visible in the process of reaching the anticipated objective. Lave (1988) states that knowledge always undergoes construction and transformation in use. It is therefore the aim of this study to make visible the learners' ways of working in order to magnify the space of knowledge in action and to bring into view that which is seen but unnoticed, rather than just simply to observe what was done wrong or right. The textures of learners' ways of working will give a better indication of how the mathematics is produced. Thus my interest was to investigate the procedures learners employ to bring about their responses to the items in the high-stakes NSC mathematics examination setting.

1.3 Research Question

Exploring the textures of the learners' ways of working with the mathematics might expose how learners grapple with what was set out for them to accomplish in the mathematics examination. Thus to pursue my objective of interpreting and analysing the procedures learners employ to produce responses in a high-stakes mathematics examination setting I investigated the learners' ways of working with trigonometry in the Grade 12 high-stakes mathematics examinations. With this aim in mind the research question pursued is:

"What are the textures of ways of working exhibited by learners' work in trigonometry in the Grade 12 high-stake examinations?"

1.4 Significance of the study

This study is significant for three reasons. Firstly, it has the potential to contribute some insights to current literature on the ways of working with and understanding trigonometry in a high-stakes examination setting. For this research to have an impact on mathematics teaching and learning there is a need for it to be relevant to teachers so that teachers are enabled to create an opportunity for themselves to use responses to examination questions as classroom practice.

Secondly therefore, this research suggests a method of teaching that relies on using the work learners produce in high-stakes examinations or tests as a mathematical object for teaching and learning. This research model involves a type of mathematical object for the teaching and learning of trigonometry that will reduce the feeling of irrelevance which seems to stem from an unfulfilled expectation that research should provide reliable and relevant rules for action, rules that can be put to immediate use (Kennedy, 1997, p. 10). Hence this study shows relevance and fulfilled expectations by providing the reader with an ethnomethodological analysis of examinees' ways of working in a high-stakes examination setting, which brings to the surface that which is seen (textures) and in many cases goes unnoticed.

Thirdly, this research gives a rendition of how ethnomethodological constructs can be used to bring out underlying features of examinees' ways of working which look promising in learners' understanding of trigonometry. Thus in the case of this research the mathematical knowledge became visible through practical action and reasoning in situ. Lastly, the study shows an important link between the "in situ-ness" from an ethnomethodological perspective and agency in terms of the dialectic of resistance and accommodation, which was a crucial feature in the analysis of examinees' responses.

1.5 Chapter outline

The thesis consists of seven chapters. In Chapter One the introduction, background and motivation of the study are provided. In addition, the research question is presented as well as an overview of the study's theoretical underpinnings.

The second chapter gives an overview of the literature regarding assessment in mathematics education and the important features related to it. This includes feedback, high-stakes examinations – in particular the NSC examination – and trigonometry.

Chapter Three focuses on the theoretical underpinning related to overall context of analysis. The chapter presents a bricolage framework consisting of the theoretical framework and a conceptual framework, which were used to guide the theory in action. The work of Garfinkel (1967) and Pickering (1995) are amongst those discussed in this section.

Chapter Four explains the research design and qualitative methodology used in this study. It explains the relevance of the documentary method of analysis as it is linked to ethnomethodology. The methodology chapter highlights the importance of the setting, in this case the examination room. The validity and reliability concerns are presented in this chapter as well as the ethical issues that were considered in the research.

Chapter Five presents an analysis of the data selected. It gives an exposition of the visible textures of abandonment. The chapter shows how conceptual practice corresponds with the analytical framework developed from the work of Garfinkel (1967) and Pickering (1995), which suggests that all scientific practice takes on a particular form of interplay between resistance and accommodation. The data analysis chapter is further underpinned by the role played by endeavours to solve the examination problems in mathematics that emerge from the setting of a high-stakes mathematics examination.

Chapter Six consists of the findings based on the results of the data analysis. This chapter shows how ethnomethodological constructs such as reflexivity bring an understanding to the examinees' ways of working in solution-seeking pursuance.

Chapter Seven is the discussion chapter which relates the research questions to the findings of the study. This chapter presents a methodological reflection as well as the limitations related to the investigation. Implications of the study and suggestions for further research around this topic are presented. Suggestions are made regarding the use of such analyses for the improvement of teaching and learning of Mathematics, as well as concluding remarks.

CHAPTER TWO

ASSESSMENT IN MATHEMATICS EDUCATION

2.1 Introduction

This chapter provides an overview of the broad domain of the study by discussing assessment in mathematics education. It presents a comprehensive discussion of the inclusive features of the issues related to assessment such as high-stakes examinations, in particular the National Senior Certificate examination (NSC). Feedback is a key element of this study, particularly as it applies in mathematics education and specifically as it relates to the NSC examination. The mathematical content under investigation is trigonometry and therefore the chapter provides a broad overview of it.

2.2 Assessment in mathematics education

Niss (1998) defines the term 'assessment' as a means to identify and appraise students' knowledge, insight, understanding, skills, achievement, performance and capability in Mathematics. Assessment thus addresses and brings into view the outcomes of mathematics teaching at the student level, and it plays a crucial part in the understanding of how teaching and learning in mathematics can be improved. These visible outcomes involve the recognition and use of mathematical terms, mathematical procedures, use of mathematical strategies, the understanding of mathematical concepts and the way in which learners formulate and solve problems. Dietel, Herman and Knuth (1991, p.1) state that assessment can be defined as a method from which teachers stand to gain a better understanding of students' existing mathematical knowledge and skills. With this definition they infer that assessment can be a tool used by teachers to make subjective but important decisions on students' performance based on their observations. The concept of current mathematical knowledge, which is always changing, gives teachers the opportunity to make comparative judgements over a longer period. According to Herman and Knuth (1991, p1) research done in assessment provides useful information which may affect decisions about grades, advancement, placement, instructional needs and curriculum, thus making assessment in mathematics education an integral part of educational research with specific objectives to assist in the improvement of mathematics. It is therefore important that assessment practices be in tune with current curriculum content and goals and that these practices become increasingly informative to teaching initiatives and adjustments.

In mainstream thinking the basic assumption about assessment and its purpose is that learners possess attributes like knowledge, understanding, skills and ability – attributes that are discoverable and measurable. This implies that an underlying truth about their ways of working may be discovered (Morgan, 1999). However, Morgan (1999) advocates an alternative to assessment, which takes into account the socially situated nature of behaviour. This trajectory includes theories of discourse and communication and the sociological analysis of the roles of education and assessment within a society or in a social context. These lines of enquiry become an important feature in the later discussion of the study.

Although educators have accepted some version of constructivist epistemology in relation to the ways in which learners make sense of their experiences, Kress (1989) still sees assessment as a concept that works within a traditional paradigm in which meaning resides within the text. Kress (1989) sees the text as something, which provides a reading position that is unproblematic and natural. To narrow down what is clearly a very broad discussion on assessment this study focuses on assessment only in mathematics education, from a South African school's curriculum perspective, and in particular, on the external NSC high-stakes examination as an assessment tool.

According to the Curriculum and Assessment Policy Statement (Department of Basic Education, 2010), (CAPS) “assessment is a continuous planned process of identifying, gathering and interpreting information about the performance of learners, using various forms of assessment”. The recent launch of the Foundations for Learning Campaign in South Africa, with its focus on standardised assessment, showed that there appears to be much emphasis on external assessment in the South African School context (Meier, 2011). Meier (2011) reiterates the defining features of assessment in terms used also by Herman and Knuth (1991, p1) when referring to assessment as a method by which to award grades or marks, to evaluate the curriculum, to decide whether to review it or not. This indicates that assessment can help convince the policy-makers, the public and the educators that change is needed in the education system.

In South Africa, the Grade 12 final marks are calculated according to performance in two different assessment domains: formative and summative assessment, yet the way in which such forms of assessment function in schools in South Africa is distinctive. To see this it is important to consider the nature and purpose served by these two forms of assessment in terms of their weighting towards the final mark. The final result of Grade 12 Mathematics is made up of school-based assessment and a final high-stakes external examination, both of which include summative assessment. This study probes how the summative assessment (or the final high-stakes external examination) is played out in this context. The inter-relationship between summative assessment tests and the formative assessment school-based assessment tasks as the final mark for mathematics consists of 25% school-based assessment and 75% of the NSC examination written at the end of the school year makes up the total mark for Mathematics.

The 25% mark weighting includes the March Control test, June examination and September trial examinations, class tests and assessment tasks. However, this study’s focus is the NSC summative assessment. It is within this context that evidence of achievements is collected and evaluated. The findings are recorded then used to get an understanding of what strategies to apply in order to assist in the development of learners in their knowledge of mathematics. The 75% weighting is a further indication as to why it is so important for Grade 12 learners to perform well in this examination. The CAPS document explains this process as a way to improve the teaching and learning of mathematics and ultimately, to improve results (p. 101). What this shows is that the implemented curriculum learners experience in the classroom is influenced by the nature of the external NSC examination for which they are prepared.

2.2.1 External assessment

Rey (2010) defines external assessment as assessment that involves analysing student results through a centralised system using methods aimed at generating comparative evaluations of student performance at a national, regional, or international level. In South Africa external assessment is controlled by the State (Department of Basic Education). It makes decisions concerning the nature, scheduling and timing of the examination and its administration – scoring, coding and reporting. The regulations governing the administration of the NSC external examination are drafted in the Regulations Pertaining to the Conduct, Administration and Management of the National Senior Certificate (2014). The main purpose of the current use of external assessment in South Africa is to assess and steer the education system. Nagy (2000) sees the purpose served by external examinations from an historical perspective, and the measurement theories behind the roles of such assessment as a tool that identifies gate-keeping, accountability, and instructional diagnosis. Kellaghan and Madaus (2003) identify seven common characteristics of external examinations claiming that they:

- share common characteristics as institutions external to the school
- have an overseeing function

- are based on a prescribed curriculum
- involve the administration of common tests at a given time
- serve to certify qualifications
- are voluntary in most contexts
- make results public in everyone's interests.

Furthermore, the purpose of such organisation and administration according to Rey (2010) is for a number of additional reasons such as to provide teachers and other education officials with important feedback, consisting of comparative evaluations, a diagnostic analysis of the state of the education system as well as objective and effective information about learners' attainment of learning. In view of this Murphy and Broadfoot (1995) predict that these types of high-stakes examinations will continue to fulfil their function well into the future. The external examination is comprised of characteristic distinguishing features that make it a high-stakes examination.

2.3 High-stakes examination

The primary function of external examinations is firstly to sort and classify students according to innate ability. More importantly, the test results are used to make decisions that have serious consequences, for schools, learners and the education system. The extent of consequences attached to high-stakes examinations is what gives it its distinguishing features. Many researchers have examined the impact of high-stakes examinations on things such as dropout rates and emotional impact on learners. For example, Griffin and Heidorn (1996) investigated the relation between dropout rates and behaviour in students at 75 high schools. They found that low achievers did not contribute to the increase in dropout rates of these schools. However, the investigation showed that failure amongst high achievers contributed slightly to an increase in the dropout rate. Furthermore it found that academically stronger students are more susceptible to the causal stigma attached to high-stakes examinations. Griffin and Heidorn (1996, p. 249) further showed through their investigation that these students may experience a substantial drop in self-esteem, or they may feel embarrassed in front of their peers. Such experiences might be especially critical for students with a proven record of academic success. To further emphasise the effect and consequences surrounding high-stakes examinations Madaus (1991, p. 229) states that the disadvantages of high-stakes examinations are that they can be extremely stressful and have a negative effect on students' personality, which ultimately influences their self-concept and self-esteem.

Schmidt (1999) studied the impact of stress on students who repeatedly failed standardised tests, specifically on those students who would not have been expected to fail. According to their interview data, failing such important tests damages students' self-belief, has a negative impact on their relationship with their parents and alters their future educational and career plans.

Literature studies in educational research show that high-stakes examinations, as a tool for educational assessment, have a negative impact on learning and motivation to learn (Kluger and Denisi, 1996; Harlen and Crick, 2002), affecting learning through memorization and reducing creativity and innovation. A study done by Barksdale, Ladd & Thomas (2000) which involved interviewing teachers and parents in two states in the United States of America (USA) about their opinion and experience of high-stakes testing seems to confirm that high-stakes testing may harm learners' health by causing excessive anxiety and stress. Wheelock, Bebell, and Haney (2000) found evidence of causal factors when they asked learners to make a drawing of themselves taking a high-stakes examination. From the analysis of these drawings they found what emerged were issues such as anxiety, anger, and pessimism, particularly among older students. It is clear from the foregoing research outcomes that the pressures attached to high-stakes examinations are in most cases caused by the fear of failing. Thus high-stakes examinations can be associated with uncontrolled and unintended consequences.

Despite the negativity associated with examinations, the argument made by Choi (1999) is that high-stakes examinations are nonetheless widely accepted and that at some point of measurement they become unavoidable in the endeavour either for educational attainment or for employment, and also because of the public confidence in examinations as a fair and objective selection mechanism. More importantly, and crucial to this study is that the literature referred to provides some idea of the consequences and situatedness of high-stakes examinations as well as the pressures experienced by examinees who participate in them. This becomes a very important aspect later in this study.

2.4 National Senior Certificate (NSC) Examination

The NSC examination written by Grade 12 learners in South Africa fits the characteristics of the external high-stakes examination. Although external assessments serving slightly different purposes are done throughout the schooling system the one that stands out and that is relevant to this study in terms of the consequences attached to it, is the National Senior Certificate (NSC) high-stakes examination. Jacobs, Mhakure, Fray, Holtman & Julie (2014) point out that examinations serve a variety of purposes. In their classification of different examinations operative in the South African schooling system their study clearly shows that the NSC examination is heavily weighted in terms of external procedures.

The NSC represents a high point in schooling in South Africa. The NSC examination is the most important examination South African learners write at the culmination of 12 to 13 years of schooling, and much of the teaching in the last three years of schooling is geared towards preparing learners for success in this examination. It is still by far the most popular determinant of access to higher education and, to a lesser extent, to the world of work. Hence, by the standards of any educational system and teaching-learning community, the NSC examination is an unavoidable yet a controversial form of assessment. All its developmental features are external (paper setting, marking and moderating), which apply to all subjects offered in grade twelve, including mathematics.

Most young people are encouraged to aspire towards it. Jacobs et al (2014) show the examinee as the major beneficiary. The outcome of the NSC examination is a public event in which the Minister of Basic Education makes public the results. Various print media and online media are involved in the publication and/or the disclosure of the results.

Over the last decade in South Africa hundreds of students have been enrolled in initiatives such as learnerships, internships and employment schemes in attempts to make them employable (Kraak 2013). However these efforts have been marginal as, for the most part, they have involved learners who did not complete their schooling or who failed their Grade 12 NSC examination. Theoretically, the assumption is that education can facilitate access to the economy by absorbing young people in search of work. The emphasis on passing the NSC examination as an entry point to the job market or to tertiary education has established its status as an essential requirement for South African students. The consequence of not achieving an NSC certificate implies the exclusion of a number of adults and youth from the formal economy (Brown, Lauder and Ashton 2008). The issues mentioned above make the planning, administering and organisation of the NSC examination crucial to its credibility and importance as a measuring tool.

2.4.1 Planning the National Senior Certificate (NSC) Examination

In South Africa nine provinces write the same NSC examination centrally governed by the National Education Policy Act (NEPA) of 1996 (Act No 27 of 1996, Annexure G, Section 2). Each province administers examinations in accordance with national regulations (Department of Basic Education, 2005), and subordinate provincial legislation. Everyone who participates in the setting and administering or implementation of NSC examination and assessment strategies has the responsibility to ensure that it follows and adheres to the requirements of validity, reliability and fairness. The result of the NSC examination is a public measure of learners' performance which includes the quality of teaching and learning, and how well the system is doing (Umalusi Summary Report, 2004). It is important that public confidence be maintained in the NSC examination because of the consequences attached to it.

The officials of the DBE have the responsibility therefore to constantly evaluate the effectiveness of the internal organisational measures, which include compliance, validation and monitoring procedures. Failure to adhere to or to comply with the rules which regulate the NSC examination will be countered with the relevant legislation that constitutively governs the process. These validity and reliability compliant strategies form part of what makes the NSC examination so unique. Earlier in this chapter, I referred to the results of the NSC examination, which are used to identify, gather and interpret information about the performance of learners. This information is compiled so as to provide important feedback to all relevant role players.

2.5 Feedback from high-stakes examination in South Africa

Hattie and Timperley (2007, p.86) suggest a model of feedback which emphasises that the main purpose of feedback "is to reduce discrepancies between current understandings and performance and a goal". According to them effective feedback must answer three questions: 'Where am I going?' (Goals), 'How am I going?' (What progress is being made towards the goal?) and, 'Where to next?' (What do I need to do to improve?) From a South African perspective the level at which the feedback operates will determine how effectively answers to these questions will improve performance and reduce the gap of inconsistencies between the knowing and goal orientated performances.

High-stakes examinations in South-African education function as an instrument of education policy. Provinces and districts use feedback from high-stakes examinations as an accountability measure for students and schools. Provinces and districts use the outcomes of a high-stakes examination to determine the impact of intervention programs offered in schools. Politicians use the feedback from high-stakes examinations as a measuring tool to decide on the quality of the curriculum offered. It is therefore crucial that the information gained and feedback given from the high-stakes examination and in the case of this study, information on learners' ways of working with trigonometry, be precise, relevant and purposeful.

The Department of Basic Education (DBE) provides various other kinds of feedback related to the NSC high-stakes examination as noted in Chapter One. One of the feedback reports the DBE provides and which I shall elaborate on is a diagnostic report.

2.5.1 Support strategies generated from feedback from the diagnostic report

Schools that underperform are put through intervention programmes to assist them to improve their results. Schools that do not obtain a 70% average pass in the NSC examination become part of the District Improvement Plan (DIP) and are targeted with intervention programmes linked to their administration and academic programs. The key intervention strategies are:

- tutoring of learners over weekends and holidays by subject specialist teachers with a record of high learner achievement
- a Telematics Project which broadcasts lessons via satellite in selected subjects after school and over weekends

- a study guideline supplied to all Grade 12 learners with advice on how and what to study for each subject, the structure of exam papers and how to manage study time. (Media Release, Minister of Education, Western Cape 12 July, 2012).

In this study I link intervention with the suggestions from the diagnostic analysis of the DBE. These interventions are a more direct approach and are aimed at getting learners to perform better in the NSC examination. **A**dherence to the suggestions given in the diagnostic analysis is represented in Figure 2.3 which shows remedial measures and responsibilities at each level in the education sector. Figure 2.3 is an example of mathematics remedial measures.

Grade 12	Subject	Identified weakness	Remedial Measures & Responsibility at each Level in the Sector				Resources	Time Frames
			DBE	PED	District	Teachers		
Grade 12	Maths	Functions and graphs Calculus Analytical geometry learners lack the application of the equation $(x-a)^2 + (y-b)^2 = r^2$ Probability Euclidean Geometry Learners seems to lack the application of similarity of two triangles	Provide Self-study Guides Monitor implementation of CAPS Provide guidance and support for subject advisors.	Provide textbooks aligned to CAPS Develop notes and distribute. Subject advisors should assist with teaching of content to learners. Monitor learners' classwork books.	Print notes and distribute. Distribute Self-study Guides. Subject advisors should assist with teaching of content to learners. Monitor	Teach learners to master formulae. Train learners to use different methods to solve problems. How to sketch a graph and vice versa to derive its equation if it is drawn Use previous question papers to teach the application of calculus. Teachers are encouraged to use various methods to find the turning point and axis of symmetry. Learners need to be taught	Textbook Siyavula Grade 11 textbook Scientific calculator Mathematical instruments Exemplar papers for Grade 12 Revise using November 2014 Grade 12 Maths papers. Self-study Guides; and CAPS document for Grades 10, 11 and 12	Jan. to Dec. 2015

						thoroughly on how to find the equation of the circle with the centre of (0; 0) and when the centre is out the origin of the Cartesian plane. Ensure that probability problems of Grades 10 and 11 are done when explaining counting principles are done in grade 12. Learners are encouraged to know the theorems so that they can give reasons for their statements.		

Figure 2.3: Remedial measures and responsibilities at each level (Diagnostic Report 2014, p. 217)

Part of the support teachers receive from the DBE is the examination guideline which gives teachers some idea of how to structure their method of teaching and their revision program. Figure 2.4 gives an idea of support materials given to teachers in 2014 as guidelines for preparing learners for trigonometry. I am presenting the guidelines for trigonometry since it was the focus of my research. Hence, the discussion that follows the support strategies will deal with trigonometry.

TRIGONOMETRY

1. The reciprocal ratios cosec θ, sec θ and cot θ can be used by candidates in the answering of problems but will not be explicitly tested.
2. The focus of trigonometric graphs is on the relationships, simplification and determining points of intersection by solving equations, although characteristics of the graphs should not be excluded.

Figure 2.4: Examination guidelines for trigonometry

The feedback provided for teachers and curriculum advisers in the diagnostic report is supposed to assist in their planning for the following year. Similarly, the feedback aims to give teachers an overall idea of which types of questions need more emphasis and for which questions learners need more exposure.

Relevant to this study is feedback given on summative assessment in which contextual features of high-stakes examinations and the knowledge of learners' understanding are treated as valid, notably in the South African context. The problem of poor performance persists in our schools despite the number of studies which have used assessment data to give feedback to all relevant role players for the purpose of identifying and analysing the numerous factors affecting academic performance in mathematics and in various other fields of study. However, research has shown that this problem is widespread. Some factors identified by researchers include effort and prior schooling experience of students (Siegfried & Fels, 1979). Anderson and Benjamin (1994) identify parents' and family income. Devadoss and Foltz (1996) isolate factors such as self-motivation and learners' age as well as learning preferences.

The purpose of this investigation was to establish whether or not it is possible to analyse the production of the responses of examinees in the NSC mathematics examinations more meaningfully. Exploring the textures of the examinees' ways of working with the mathematics high-stakes examinations presented the possibility of uncovering how learners grapple with what was set out for them to accomplish in the mathematics examination. Thus, to pursue the research objective I analysed and interpreted the procedures examinees employ to produce responses in a high-stakes mathematics examination setting.

2.6 Trigonometry

The issue under investigation in this study concerns the examinees' ways of working in the high-stakes NSC mathematics examination, in particular in the subject of trigonometry. The Greek origin of the word trigonometry refers to the science of measuring ("metron") triangles ("trigonon") and this has generally been the primary traditional concern of the subject (Antippa, 2001). Trigonometry, as a branch of mathematics that deals with the relationships of sides and angles in triangles forms an important background to the solution of problems in many disciplines (Orhun, 2010). Trigonometry is frequently used in mathematical explanations and definitions of new ideas and concepts. For example, trigonometric ratios are used to describe the relationship of angles and sides in a right-angled triangle. Research studies reveal that many students have not developed clear concepts in trigonometry and that some of them use algebraic notation informally (Maharaj, 2008).

Trigonometry is an important school subject not only for mathematics but also for fields like engineering, astronomy, physics, architecture etc. The subject is seen as systematic calculations of length of sides and measures of angles for any triangle, from sufficient given information (Raj and Nega, 2011). Trigonometry requires a number of sophisticated mathematical concepts. These involve the association of numbers with sides of a triangle representing length, the measure of the angle, ideas of exponential functions of a triangle and symbolic notation (Raj and Nega, 2011). Real world problems involving trigonometry are common in engineering, physics, construction and design. So for learners to pursue these fields of study, they must develop a basis of trigonometric knowledge in school. In terms of mathematics, it is one of the fundamental topics in the transition to advanced mathematics and its applications. A firm understanding of trigonometric functions is required in calculus and analysis (Demir, 2012). Hence, trigonometry has an important place in the mathematics curriculum in many countries even though its meaning may change from country to country at secondary school level (Delice & Roper, 2006). Trigonometry is a compulsory topic studied by all Grade 10 to 12 learners who do Mathematics in South Africa.

2.6.1 Trigonometry in the Curriculum and Assessment Policy Statement

The Curriculum Assessment Policy Statement (CAPS) provides a specification of content progression for trigonometry and other areas of mathematics from Grade 10 to 12. It presents guidelines in terms of what content to teach and the time allocation for each content area. The specification of content focuses on concepts and skills from Grade 10 to Grade 12. Table 2.1 which follows shows the progression of the content for trigonometry from Grade 10 to 12 (Department of Basic Education, 2011).

8. TRIGONOMETRY		
GRADE 10	**GRADE 11**	**GRADE 12**
(a). Definitions of trigonometric ratios $sin\theta$, $cos\theta$ and $tan\theta$ in right angled triangles (b). Extend the definitions of $cos\theta$ and $tan\theta$ to $0° \leq \theta \leq 360°$. (c). Define and use values of the trigonometric ratios (without the use a calculator for the ratios$\theta \epsilon \{0°, 30°, 45°, 60°, 90°\}$. (d). Define the reciprocals of trigonometric ratios.	(a). Derive and use the identities: $tan\theta = \frac{sin\theta}{cos\theta}$ and $sin^2\theta + sin^2\theta = 1$. (b). Derive the reduction formulae. (c). Determine the general solution and / or specific solutions of trigonometric equations. (d). Establish the sine, cosine and area rules.	(a). Prove and use of the compound angle and double angle identities.
Solve problems in two dimensions	Solve problems in two dimensions	

Table 2.1: Curriculum Assessment Policy Statement, FET Phase

The content shown in Table 2.1 is spread over a period of three years from Grade 10 to Grade 12. The CAPS provides both an outline and clarification of the substance of the content. Orhun (2010) states that university students in their first year perform badly in the operations of trigonometric expressions such as addition, subtraction, multiplication and division. First-year students demonstrate difficulties in the multiplication of $sinx.sinx$. Orhun (2010) argues that this may be due to a deficiency in emphasis in the acquisition of addition, subtraction, multiplication and division of trigonometric functions in the secondary school curriculum. This is evident in Table 2.1 which shows trigonometry recommended in the 2011 South African curriculum. Although the CAPS gives this overview of the content progression, it is still the work of teachers to ensure that the learners can follow the content in its ‘progression’ through the years of teaching. However, Weber (2005), Brown (2005) and Challenger (2009) argue that students have a fragmented and incomplete understanding of trigonometry.

2.6.2 Learners' understanding of trigonometry

Brown (2005) developed a model of students' understanding of trigonometric functions which included only the trigonometry of angles in degrees. According to Brown's model students may consider the sine of an angle in different ways: (1) as a ratio, (2) as a distance and (3) as a coordinate. The model showed that students who developed a strong understanding of trigonometric functions were able to coordinate these different components. Khalloufi-Mouha (2009) is of the opinion that an understanding of trigonometric functions requires students to realise:

- the relationship between the length of an arc and the angle that is subtended by the arc
- the relationship between the real line and the unit circle
- the co-variation between a point moving on the unit circle and its projection in one of the axes and
- the idea of graphs of a function.

In general, trigonometry in the South African education context is first learnt in Grade 10 as ratios in right-angled triangles. In this Phase learners deal with the angles in degrees smaller than 90°. Then, in a unit circle model this scope expands to any angle and angles in radians. Progression and transformation of learners' understanding from radians as an angle measure to real numbers as the domain of the trigonometric functions, completes the picture. Students who are introduced to the subject through triangle trigonometry from the beginning have to relate triangular images to numerical relationships; they have to cope with ratios and they have to manipulate the symbols that are involved in such relationships. Thomson (2008) comments that to teach triangle trigonometry before circle trigonometry could also lead to students' understanding of trigonometric functions as taking the right-angled triangles and not measures.

Learners who develop a strong trigonometric understanding from the early grades are able to make transitions among these trigonometric concepts. However, research shows that students experience difficulties and have an incomplete understanding regarding the angle measure in degrees (Martinez-Sierra, 2008, Brown 2005). Also, in a study done by Weber (2005), students encounter learning as a trajectory of procedures. These trajectories are based on their physical constructions: process is based on repeated procedures, and procept is based on their reflections on the processes. Furthermore Kendal and Stacey (1997) who compared two methods of teaching trigonometry, found that teaching the ratio method first promoted a better understanding of the trigonometric functions than teaching the unit circle method as a starting point. The studies by Weber (2005), Kendal, Stacey (1997) and others reveal that students' understanding is highly related to aspects emphasized in the lesson design. Commenting on the lesson design, Presmeg (2006) emphasizes the importance of connecting old knowledge with new knowledge when teaching trigonometry to give ample time for learners to move into knowledge that is more complex. Lesson structures include assessment strategies to test understanding and therefore it is important to note that the results of investigations into students' understanding depend largely on how the assessment is done (Demir 2012).

Orhun (2002) studied the difficulties learners encountered when using trigonometry to solve problems. In the study it was found that the learners did not develop the concept of trigonometry and that they made some mistakes. In a study done by Thomson (2008), it appears that trigonometry is especially difficult for middle school and secondary school students in the United States of America (USA). He claims that the difficulty lies in an incoherence of foundational meanings developed in the lower grades. Weber (2005) states that learning trigonometric ideas is difficult for students and the causes of these difficulties seem to be multifaceted and interrelated. Challenger (2009) asserts that students claim that trigonometry is difficult and that they 'hate' it. They see trigonometry as a complicated section of mathematics and are confused as to whether they should apply triangle trigonometry, circle trigonometry and/or analytical trigonometry. Weber (2008, p. 144) emphasizes the point that "despite the importance of trigonometry and students' potential difficulties in learning it, relatively little research has focused on this subject". Brodie (2010) argues that teachers should consider their learners' prior knowledge in order to understand what they know and thus how they would be able to accommodate new knowledge. This can be done by allowing learners to write baseline assessment tasks. Hence, although we may recognise learners' difficulties in trigonometry this recognition actually involves learner assessment, so as to understand their current and prior knowledge of the topic and to gain insight into their difficulties with the work. Challenger (2009) emphasized the same issue, in terms of "What you get is what you teach." It is important to note the results of investigations into students' understanding depend largely on how assessment is done.

Gür (2009) suggests that misconceptions relate to a concept that produces a mathematical object and symbol. According to Gür this lies in sine being a concept and a symbol. A further misconception is related to processes, the ability to use operations such as representing the result of $sin30°$ and the value of $sin30°$. Some misconceptions are related to concept and process, where $sinx$ is both a function and a value. Lochead and Mestre (1998), Ryan and Williams (2000) explain that learners' misconceptions and errors grow from learning complexities. Delice (2002) establishes students' skills when working with trigonometry by identifying the levels for measurement of their knowledge. The research shows that students have misconceptions and learning complexities which are ascribed to the fact that before learning the trigonometric concepts, students learn certain concepts – pre-learning concepts – incorrectly or defectively.

Several researchers have found that errors and misconceptions displayed by students in their attempts to solve mathematical problems contribute to their poor performance in their learning of mathematics (Brodie, 2005; Davis, 1984; Drews, 2005; Foster, 2007; Hatano, 1996; Luneta & Makonye, 2010; Nesher 1987; Olivier, 1989; Orton, 1983a; Orton, 1983b; Ryan & Williams, 2000 and Smith, DiSessa, & Rosehelle, 1993). The idea that students develop 'misconceptions,' has been the basis of much empirical research on learning mathematics and science in the last fifteen years. Smith et al. (1993) emphasize the point that "misconceptions arise from students' prior learning, either in the classroom (especially for mathematics) or from their interaction with the physical and social world". Although in certain cases and under certain conditions the students were found to have either misconceptions or else a profound understanding, it can be claimed that this understanding relied on the focus of the teaching.

Students' first encounters with trigonometric functions may be the problem with operations that cannot be evaluated algebraically. In a comparative study of English and Turkish students' performance in trigonometry, Delice (2002) found that Turkish students were good at the algebraic aspects of trigonometry like simplifying trigonometric expressions while English students were good at trigonometric applications in real life situations. They observed that differences in mathematics curricula and in teaching methods in the two countries were the reason for this performance difference.

One of the points raised in the diagnostic report of 2014 provided by the BDE, and based on the NSC results, is that the trigonometry section is a cause for concern as candidates performed poorly in questions that tested basic knowledge (DBE, 2015, p. 127). Assessment in trigonometry as well as the type of feedback given to teachers and learners on how students go about producing solutions to trigonometric problems, will allow for better insight into an understanding of poor performance in trigonometry. This will provide valuable information, derived as it is from the difficulties learners encounter and their level of understanding of the subject.

2.6.3 Previous research on learning and performance in trigonometry

Trigonometry requires a number of sophisticated mathematical concepts such as the association of numbers with the sides of a triangle representing length and the measure of the angle, concepts regarding exponential functions of a triangle and symbolic notation (Raj and Nega 2011). Real world problems involving trigonometry are common in engineering, physics, construction and design. For learners to pursue these fields of study they must develop a background of trigonometric knowledge in school.

Trigonometry is an area of mathematics with which students struggle (Gür, 2009). In examining the difficulties faced by learners when using trigonometry for solving problems Orhun (2002) found that the learners did not develop the concept of trigonometry and that they made some mistakes. Gür (2009) suggests three generalized misconceptions relating to trigonometry. The first is that many misconceptions centre around a concept that produces both a mathematical object and symbol. For example, sine is a concept and a symbol of trigonometry. The second type of misconception is related to processes, for example, as in representing the results of calculation of $sin30^0$ and the value of $sin30^0$. And the third common kind of misconception relates to precepts that involve the ability to think of mathematical operations and objects, for example, $sinx$ is both a function and a value (Gür, 2009).

In investigating students' understanding of sine and cosine Brown (2006, p. 228) devised a framework called trigonometric connection. Her study indicated that many students had an incomplete or fragmented understanding of the three major ways in which to view sine and cosine: as coordinates of a point on the unit circle; as a horizontal and vertical distance that constitutes graphical entailments of those coordinates; and as ratios of sides of a reference triangle. Delice (2002) identified levels for measurement of students' knowledge and for defining students' skills when working with trigonometry. The research showed that students have misconceptions and learning complexities, which are attributed to the fact that before learning the trigonometric concepts, the students learn significant concepts – pre-requisite concepts – incorrectly or defectively.

With the South African mathematics curriculum in mind De Villiers and Jugmohan's (2012) research indicates that learners have little understanding of the underlying principles of trigonometry. Furthermore, Chauke (2013) points out that the Grade 12 mathematics students in the Gauteng province had difficulty with trigonometric functions in the end-of-year examination. Thomson (2008) aptly notes that trigonometry is notoriously difficult for middle-school and secondary school students in the United State of America (USA). He claims that the difficulty lies in an incoherence of foundational meanings developed in the lower grades through to Grade 10.

Although the research explored in the foregoing section is primarily concerned with the difficulties learners experience when doing trigonometry, it does not address how examinees actually go about producing the responses that they submit in high-stakes mathematics examination settings. This particular "research gap" forms the cornerstone of the research study.

2.7 Conclusion

Many studies have investigated and reported on different factors that affect teaching, learning and poor performance in mathematics. The recurrence of poor performance in this subject in South Africa calls for a concerted effort to take measures that will improve this situation. According to Makgato (2007) the poor performance of South African students in mathematics at secondary school level is a major concern for various stakeholders. One important step in an endeavour to find solutions to the problem of poor performance would be to undertake investigations that will help inform stakeholders. Research investigations into learners' ways of working are important because these help to identify the problems that need to be resolved.

This discussion has laid out a broad focus of the domain of the study in terms of assessment. The issues at stake such as high-stakes examinations and in particular the NSC examination provide an idea of the social backdrop against which accounts of mathematics performance statistics may be read. The discussion hones in on the central objective which concerns feedback on the NSC mathematics examination. It elaborates on how feedback is given, and on what types of feedback are given to all role-players, with the aim of improving performance in mathematics. It is evident in a review of the literature that research feedback about examinees' responses is very limited. It does not pay any attention to the circumstances and environment in which these accounts are created. In addition, the discussion shows that the focus of the feedback is solely on what the examinees did right or wrong and not how they went about the process of producing and constructing the mathematics to give rise to an accomplishment in its totality. One of the methods that this research will use to bring out the important textures of learners' production of mathematics is an ethnomethodological analysis.

The next chapter deals with another aspect of the literature, which focuses on the construction of the theoretical machinery.

CHAPTER 3

THEORETICAL ISSUES

3.1 Introduction

The theoretical framework draws on Garfinkel's ethnomethodology and discusses ethnomethodology and all its relevant constructs such as social order, reflexivity, accountability and ethnomethodological indifference. Educational theories, according to Roth (2013), seem to theorise the 'before and after' knowledge in a mathematics curriculum unit – what is prescribed in a curriculum unit in a description of the knowledge to be acquired. The same applies when reporting on mathematically produced work, in the instance of what the question requires and what knowledge is needed to perform the mathematical manipulations. The continuation of what was offered in the process of doing mathematics is the knowledge gained from the mathematical experience. The reason for selecting ethnomethodology is that it explores the transitive nature of mathematical work. According to Roth (2013) the direct approach to mathematically produced work articulates the way in which things are done and the relation between the subject of meaning-making and the object being acted upon (Roth 2013).

A case for sociological reasoning emerged in the analysis and structure of the study. This chapter is therefore concerned with ethnomethodology as a branch of sociological investigation, in particular, how ethnomethodology may contribute fundamentally to perspectives that challenge assumptions, through an analysis within the field of mathematics education.

This chapter elaborates on the notion of human and disciplinary agency so as to provide the scope of the main requirements for data collection and to integrate it into the application of the research design of the primary research. The discussion is about the co-existence of the afore-mentioned agencies and the case of non-human (material) agency in mathematics education research. Particular attention will be given to the dialectic of resistance and accommodation (Pickering 1995) because of the important role it plays in the analysis of the data. The collaboration between sociology and science in the production of scientific knowledge exerts a fundamental influence in this study. Through ethnomethodological constructs and science the objective is to show a co-existence in which sociological understanding plays an important role in the production of scientific knowledge. The chapter examines this collaborative relationship involving the visible manoeuvrings, or observable practical action mathematics-production undergoes to reveal unique textures. These textures form the focus of the study.

An exposition of previous work concerning the notion of resistance and accommodation will follow the theoretical underpinning. This conceptual view of the study presents concepts, assumptions, expectations, beliefs and theories that support and inform the research (Miles & Huberman, 1994). It gives a visual or written explanation of what the research intentions are in terms of constructing a conceptual framework. This framework assists in maintaining a sense of the topic, which will create an opportunity to articulate a view of the meaning of the data analysis (Hall, 2004).

There has been growing acknowledgement of the point that the learning context strongly affects what students learn. Early research in mathematics education viewed knowledge as being merely a property of individual mindfulness. The realisation that knowledge is produced in situations (Lave, 1988; Wenger, 1998; Lerman, 2000) thus requires us to move beyond an exploration of learning which relies solely on the psychology of the mind. These authors suggest we consider the setting and its social relationships such as cultural locality, the discursive frameworks available in the setting and the social and political environment in which it exists. Furthermore, we need to know how the setting influences the functions to generate and construct knowledge, particularly within the field of mathematics.

Mathematics education, specifically the production of mathematics in the NSC high-stakes examination, has been of overwhelming political interest in the sense of prescriptive power, authority and the legitimation of knowledge (Hardy and Cotton, 2000; Klein, 2002). Mathematics education research is therefore not neutral with respect to the social impact of mathematical knowledge. It explicitly acknowledges the politics of methodology and its impact on mathematics production and the teaching and learning thereof such as ethnomethodology.

3.2 Ethnomethodology

Ethnomethodology is the study of the ways in which ordinary people use commonsense knowledge, procedures, and considerations to gain an understanding of everyday situations (Garfinkel, 1967). According to Lynch (1993), the word ethnomethodology literally means "folk investigation of the principles or procedures of a practice". Thus ethnomethodology is the study of practical action and practical reasoning (Garfinkel, 1991), having developed as a particular theoretical conception of social phenomena (Coulon, 1995). Livingstone (1987) refers to ethnomethodology as common, everyday naturally occurring, mundane methods that people use to produce and manage the common everyday activities of the everyday social world.

Ethnomethodology's objective is to extract the social facts from the practical social actions. The process of extracting social facts from practical social actions is clarified in Durkheim's (1938) description of sociological studies when he established the objective reality of social facts as sociology's fundamental principle. However, Garfinkel's (1967) re-specification of this gave ethnomethodology its own particular foundation in the field of sociological research.

> For ethnomethodology the objective reality of social facts, in that and just how it is every society's locally, endogenously produced, naturally organised, reflexively accountable, ongoing practical achievement, being everywhere, always, only, exactly and entirely, members' work, with no timeouts, and with no possibility of evasion, hiding out, passing, postponement, or buy-outs, is thereby sociology's fundamental phenomena. Every topic of detail and every topic of order is to be discovered and is discoverable, and is to be respecified and is respecifiable as only locally and reflexively produced naturally accountable phenomena of order. (Garfinkel, 1991, p. 11).

Where Garfinkel uses terms such as "locally" and "endogenously" he refers to the fact that social order is produced and made visible locally, in situ (in the situation). 'Endogenous' refers to the actors themselves who produce, conduct or act in such a way that produces and makes visible the social facts. In turn the social conditions (of high-stakes examinations) determine what actors (examinees) do. All the properties of social order are made visible locally with the result that social scientists and ethnomethodologists can see what participants in society do. The organisation of social order occurs in its setting naturally (Liberman, 2012).

Central to the approach adopted in this study is the local natural setting in which examinees interacted with the problem text on the examination question paper. This study argues that in some way the local natural setting has an influence on the production of mathematical work. It is therefore important to acknowledge the fundamental principles of ethnomethodology in the establishment and coherence of the theoretical principles of the study. Unavoidably the examination room (setting) contained all its relevant components (desk, pen, paper and scientific calculator) in terms of what was needed for the examinee to engage in meaningful interaction. Hence, in all practical action and reasoning the examination room 'enacted on' the objective reality of social facts.

Durkheim's (1938) use of the term "social facts" refers to facts with special characteristics. They consist of ways of acting, thinking and feeling. Durkheim (1938) insists that social facts are ways of functioning, which proves to be a fitting but flexible concept, clearly defining of the features of social structures.

However, according to Garfinkel (1967) social facts do not impose on the objective reality; the characteristics of the latter are not stable. Garfinkel (1991) did not see social order as something that progresses naturally or that proceeds from those social facts without complication. He drew attention to the work that goes into the production of social order, underscoring how social order is "made to work" in the actions and interactions of people. The social facts, which emerge from the detailed actions and experiences of what people do is of primary concern to ethnomethodologists (Garfinkel 1967). In the pursuance of solutions to mathematical problems, the details of what and the examinee does or how, (ways of working) is of importance. The unseen causality of social facts was part of the sense-making process of doing mathematical work. It is not only about how individuals participate in rational social behaviour, but also about the way in which they are viewed by other actors who engage in it. Social order is not merely something that is just there, and social action is not simply determined by it. The concepts of social order and social action cannot be approached independently as they occur in a cyclical process. The constitutive characteristics of social facts and social action are an unavoidable part of mathematical production.

However, to ethnomethodology producing social order and social action is part of everyday activities. It implies two things. Firstly, it emphasises that social action happens as people are going about everyday practical actions. Dourish and Button (1998) explain this by referring to the work of a plumber. "When I call a plumber and ask him to fix a leaking pipe, my concern is to avoid a pool of water on the floor, not to reproduce a pattern of social interaction based on contracts and wage-labour" (p. 4).

Secondly, the emphasis on practical social action attracts attention to how participants do it and how the production and understanding of the social action is formed "for practical purposes" (Dourish and Button 1998, p.4). They help us to create accounts objectively to accomplish what we set ourselves out to achieve. In the process of solution-seeking to a mathematical problem, the objective which is set by the problem text drives the action taken from it. The practical social action exists in all thinking processes, and in the choice of mathematical skills selected to accomplish what was set out for the examinee to achieve. Garfinkel (1967) establishes ethnomethodology as an empirical study of investigation into situated practices. Garfinkel develops a study of human conduct which simply refers to the situated accomplishments of the people whose local practices assemble repeated scenes of action that makes up a stable society (Lynch 1993). Garfinkel (1991, p. 4) sees these repeated scenes of human social action as "reflexively accountable".

3.2.1 Reflexive accountability

In terms of mathematical work, the meaning of any visible practical action becomes evident by investigating the various ways in which participants solve mathematical problems. Hence, the visibly accomplished interactions are reflexive; they shape and are shaped by localised settings. Reflexivity in this instance refers to the "self-explicating property of ordinary actions" (Have, 2004, p. 20). The reflexive nature of all interaction refers to the ability of practices to both describe and constitute a social framework (Coulon, 1995). Reflexivity, in this sense, is not a conscious process. Participants (examinees) generate a constantly emerging social reality through the dialectic production of meaning in interaction.

Reflexivity is a part of the social action that accepts and acknowledges the conditions of its production and makes the action something to be observed and recognised (Coulon, 1995). Similarly, the implicit assumption, that is, the idea that any communicative act be recognised as intentional will rest upon those presuppositions that emanate from "shared social knowledge" constitutively (Ramsden, 1998, p. 48). Coulon (1995) explains the notion of reflexivity by means of a study done by Wieder (1974) on the centre for rehabilitation of drug addicts. In Wieder's (1974) study, he shows how the words "You know I won't snitch" can generate different elements within a spoken context.

According to Wieder (1974), the utterance becomes part of the unobservable circumstances of the social order that exist in the institution. All social action is enabled by the fact that inmates do not snitch. Hence, as soon as it is made visible, the social world becomes a constitutive part of what they have described. Garfinkel (1959) uses constitutive expectancies as the rules operative in all social situations. These constitutive expectancies include:

1) actors who are expected to choose a set of alternatives regardless of desires, circumstances, plans, interests, or the consequences of his or her choice
2) acceptance of the obligation that they are binding upon all other actors in the situation.

Thus these expectancies form the all-round conditions that mutually coordinate actions (Rogers, 1983). And thus, observable accomplishments are gained through reflexive, presupposed, situated orderliness. The presupposition of reflexivity shapes the methodology implied by ethnomethodological studies. According to Filmer (1972) this interpretation of presupposition creates the reflexive, incarnate character that makes the formal properties of everyday social interactions such a recondite phenomenon. Filmer (1972) argues that it is what makes them inaccessible to positivistic professional sociologists.

Garfinkel (1967) notes that, "when engaged in everyday activities, people are not concerned with discussing practical actions in a self-reflexive fashion. They recognize, demonstrate and make observable for each other the rational character of their actual, and that means their occasional, practices while respecting that reflexivity is an unalterable and unavoidable condition of their inquiries" (Garfinkel, 1967, p. 8).

Coulon (1995) refers to Garfinkel (1967, p. 1) when he states that "the methods people use for making the same activities visible, rational and reportable for all practical purposes" is, in fact, indicative of the need for accountability.

In ethnomethodology the approach to accountability extends beyond its ordinary meaning to draw attention to the ways in which members make sense of what they do. Accountability is an important property of social action and cannot be removed from it (Garfinkel, 1967). Ethnomethodology sees the ways in which participants make sense of what they do in their everyday life and in the course of producing accounts in their sense-making activities as acts of performing intelligible initiatives whose agents are accountable and made visible to others as these acts unfold (Coulon, 1947). The social facts according to Roth (2013) are not seen as abstract things but rather as the performances for which the actors are held accountable. Ethnomethodological studies assume social structures to be accountably produced and made visible to the actor, and to which they themselves react. Therefore, the ways people act subsequently describes when they are held to account for their action. In my opinion, accountability within a social order can be described as fragments of accomplishments which bring about observable achievement.

Hence, what the practical order and arrangement of interaction consists of in any organisational setting, and how that order and arrangement is intentionally produced, is concretely to be found in the natural and reflexive accountability of members. As a general concern, this notion of people's ways of doing things, hence their socially accountable action became one of the critical analytical features of ethnomethodological studies, which concerns itself with a particular form of investigation. Therefore a build-up of accounts of social action, within specific circumstances of its production gives others the means to recognise, observe and report on it.

Garfinkel (1967) sees the description of an account as part of the world in which it occurs. Whatever account people use will actively organise the sense or intelligibility of the world. Hence, the account produces the world. Figure 3.1 represents such processes in solution pursuance when dealing with mathematical problems, to ultimately produce an acceptable practical achievement. What is given in the problem text allows the examinee to make a decision on what direction to choose in the mathematical solution-seeking path. The given and known information reflexively indicates to the examinee how to use this information to navigate the mathematical discovery. In the same instance the solution-seeking pursuit contributes reflexively to the understanding of the problem text. Through this process, an account builds reflexively on a further account, the one contributing to the other.

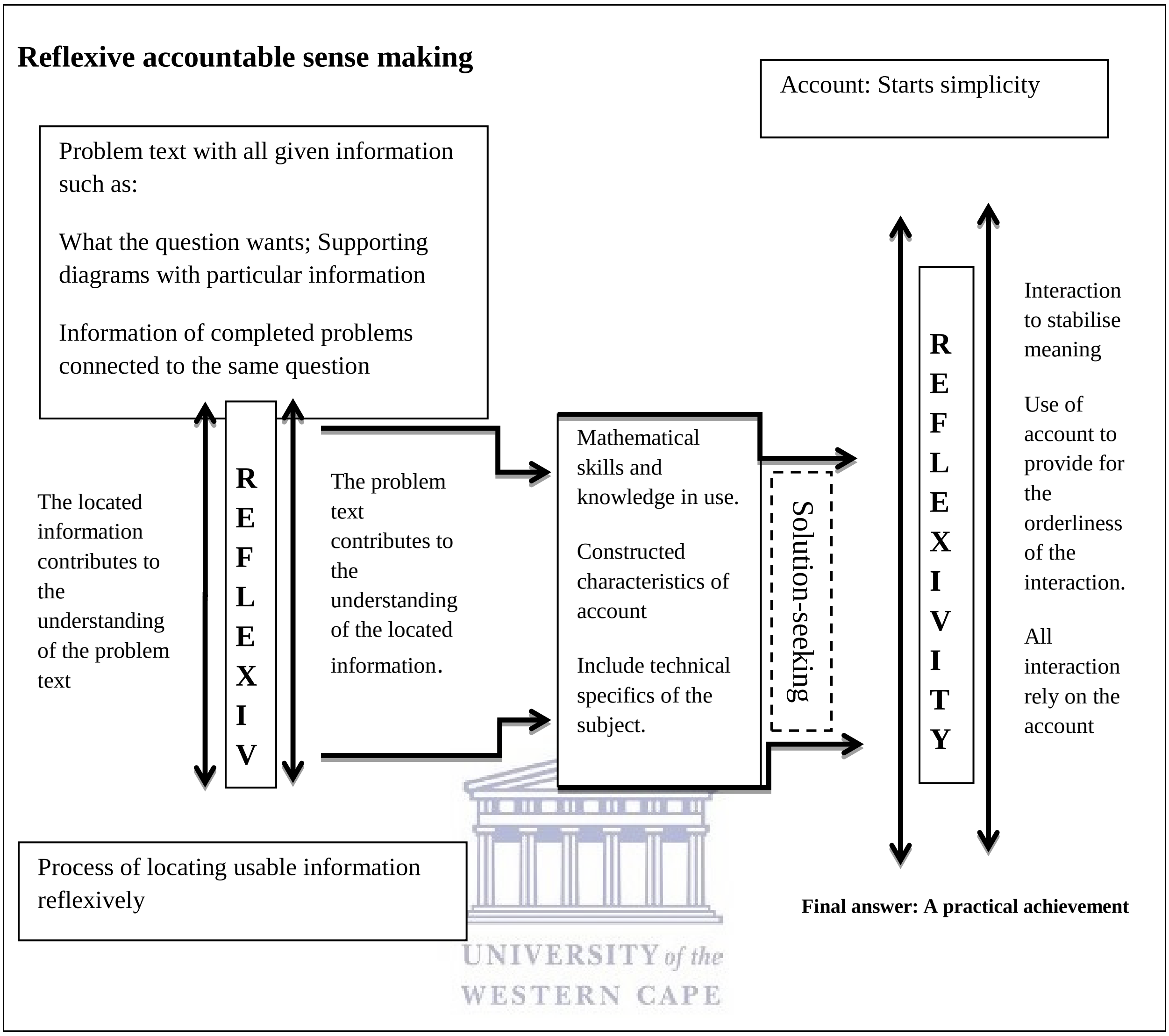


Figure 3.1: Reflexivity, a constitutive part of all practical sense making

However it is important to note that practical sense-making is free from judging the rationality of account building in terms of whether it is correct or incorrect. It treats all produced accounts as practical accomplishments. This feature of ethnomethodological indifference forms a crucial element in the analysis of the examinees responses is research.

3.2.2 Ethnomethodological indifference

Ethnomethodological indifference is an instructive way of working as it involves deliberately ignoring the judgmental attention of the established corpus of social science. It is an ethnomethodological policy whereby decisions of what phenomena consist of is neither made in advance nor is it based upon prior formal analytical studies (Garfinkel 2002, p. 171). Thus Ethnomethodological indifference to policies, methods and the corpus of formal analytical investigations makes no use of the privilege of transcendental analysis. This includes the position of universal observer, which in this regard satisfies the demands of empirical claims. In this instance and relevant to this research the central idea of the use of ethnomethodological studies is to provide an alternative procedural description of phenomena of order and method. These phenomena are things that exhibit the endogenous recognisable production of the mathematics produced by examinees. As Erickson (1986) points out, "Answering the question, "What is happening?" with a general answer often is not very useful" (p. 121).

3.3 Ethnomethodological analysis

Ethnomethodological analysis aims to understand the ordinary methods that people use in a specific social setting to realise their ordinary actions (Garfinkel, 1967). The scientific approach to ethnomethodology is to analyse the methods or procedures members of society use for conducting their affairs in their ordinary daily actions. Thus one can say that ethnomethodology may be defined as the science of procedures that constitute what Garfinkel (1967) has called "practical sociological action and reasoning". Similarly, it can be said that ethnomethodology is a special approach to doing and understanding qualitative social research. Ethnomethodology seeks to study and understand the finer features of how members go about producing social order. According to Have (2004, p.17) ethnomethodology focuses on local accountability as well as on the local time-bound features of certain phenomena. Livingston (1987) is of the opinion that ethnomethodological research and discovery advocate the way in which a production cohort produces and manages a particular, situated, materially specific social objective.

The research question of this study is closely aligned to its approach and finds support and verification by means of an ethnomethodological analysis of examinees' ways of working in the high-stakes mathematics examination setting. Ethnomethodology provides insight into the production of the subject matter and places the phenomenon of learners' ways of working in context. The intention was to seek an understanding of the production of mathematics in such a setting. The guiding perspective taken was that the phenomenon (ways of working within a high-stakes examination setting) exists in an intimate relationship with interpretation (the analysis of the ways of working in a high-stakes examination) such that object and action mutually elaborate upon one another in a "hermeneutic spiral of meaning" (Mehan and Wood (1975).

According to Mehan and Wood (1975), the "hermeneutic spiral of meaning" formulates a core understanding of the sociological approach called Ethnomethodology in which meaning is seen to exist in a documentary relationship with itself. Harold Garfinkel (1967) proposed that meaning was not absolute in some sense, but a continual practical matter of making do. The documents and underlying structure rely upon one another for their elucidation-knowledge, understanding and even truth, which are all the products of this documentary method. The use of the documentary method in qualitative research is further discussed in Chapter Four, which is concerned with the methodological approach of this study.

Ethnomethodology is also an approach, which brings about an understanding of how practice is interlinked with or embedded in ordinary abilities. According to Have (2004) ethnomethodology's objective is to show that there is no essential difference between scientific practice and another kinds of daily-life or real-time practice. These practices are unique because they must always be part of a social orderliness. According to Livingston (1987), ethnomethodologists investigate the problem of the production of social order through the construction and examination of that order's material detail, not through the construction and exploitation of definitions in order to theorize about it.

Livingston (1986) demonstrates how a proof is developed through the interaction of the on-going actions of the 'prover', his/her written recordings on a chalkboard or on paper of the progress being made, and the final rendition of the finished product. Of importance for this research study is Livingston's (1986) differentiation between the "lived work" and "the account" of a proof. The account of the proof is what is recorded in textbooks or research papers whilst the "lived work" is that which is characterised as rough work. This differentiation becomes clear as a phenomenon of ways of working in a high-stakes examination setting, in the data analysis process.

In addition, research into the ways of working in Mathematics from the stance of the ethnomethodological and social study of science has also been reported on by Pickering (1995), Merz and Knorr-Cetina (1997), Julie (2003), Greiffenhagen (2008) and Roth (2012). Their studies cover a diverse range of contexts within which mathematical work is done. A discussion of the work of these authors follows in the framework suggested later in this chapter.

It is important to note that ethnomethodology is not a substitute for formal analytical knowledge or its application, but rather as an addition, supplementation or completion thereof (Have, 2004). Thus, the ethnomethodological constructs noted earlier are what will help to explain and interpret the problem of practical objectivity and practical observability in what ethnomethodology sets out to analyse. Learners' solutions to mathematical problems are only part of the account of their understanding of the mathematics in terms of the totality of the natural occurrence in situ.

Social structures are processed to be understood through the social interactions that reflexively constitute them (Garfinkel 1967). The ethnomethodological concern with the production of meaning demands a narrowly-focused analytical context. It is especially important to note that Garfinkel (1967) intends ethnomethodology to be used to analyse naturally occurring events, which later on include interactions or data such as newspaper articles, television dramas, interviews and historical documents. While some scholars argue that this data might not display the analytical strengths of Garfinkel's ethnomethodology, it is also true that today scholars use ethnomethodology in various ways and settings, and with a range of data, which Garfinkel had not anticipated. That notwithstanding, Watson & Seiler, (1992) comment that many other scholars have found ethnomethodology to be a suitable framework for analysing a wide range of data including interviews, cultural texts and ethnographies.

Researchers who analyse the dialectic production of mathematics as an account-situated social order are concerned with responses that indicate how examinees go about doing the work and interpreting the order in situ, how claims are accepted or discredited and the shared assumptions, or tacit knowledge underlying the accomplishment. This last point may be regarded as a defining feature of ethnomethodological analysis. Heritage (1984, p. 181) is of the opinion that analysing accounts should not only be for what was said, but should also point towards “a mass of unstated assumptions”. In this study the analytical focus relies on Garfinkel’s (1967, p. 78) documentary method that is treated by researchers as documents pointing to or standing in place of a presupposed underlying pattern that is reflexive in its appearance and underlying textures. According to Pascale (2013), documentary analysis seeks to examine the tacit knowledge underlying the textures of participants’ ways of working when doing mathematics. The ways of working are what I want to elaborate on by taking a closer look at how the act of making visible processes in mathematics that go largely unobserved, manifests itself. This I want to discuss through what Pickering (1995) calls the dance of agencies.

3.4 Human and Disciplinary Agencies

Pickering's (1995) efforts to understand the practice and concept of the "mangle" capture a muddled relationship between human and material agencies. Collins and Yearley (1992a, 1992b) defend the humanist positioning of the traditional sociology of scientific knowledge which gives preference to the human subject through an unbalanced distribution of agency to humans and none to the material world. Callon and Latour (1992) think that we should see non-human agency neither as scientists present it nor as how humanist sociologists perceive it in terms of scientific knowledge, but rather that it should be thought of in terms of semiotics. Like other practice theorists, Pickering (1995, p. 5) defines entanglement between human and material agencies as a concept of practice which involves cultural and historical activity. He sees this relationship as the "the work of cultural extension and transformation in time".

However, Pickering (1995, p. 29) proposes that the ongoing social routines of human agency are located in disciplinary agency consisting of resistance within conceptual practices. Human agency accompanies conceptual structures in conceptual practice as does material agency in material practice. Disciplinary agency is thus the counterpart to material agency in mathematical practice. It consists of historically established, routinized and structured operational techniques that are applied and used in a mathematical domain. Performing such techniques is not dependent on the goals and intentions of the human practitioners. During these times, the creativity of the scientist is passive and the disciplinary agency in an active mode.

In mathematics one can argue that the tools or non-human agencies and the role these play in the production of mathematics are not easily recognisable in the final answer of any mathematics problem. However, it can be accepted that the final answer is when humans give up control to the routine ways of reacting to the world or some mathematical problem text. Thus, this study recognises the composite relationship of human and disciplinary agency and draws further insight from what Holland, Lachicotte, Skinner & Cain (2003, p. 279) state regarding agency: that it is not just individual, but rather that "agency lies in the improvisation that people create in response to particular situations". These responses people create can only be observed and reacted upon. The product of this entanglement constitutes the answer in texts produced by examinees, which represent disciplinary agency whether or not the practical action and manoeuvring reacts upon the dialectic action of resistance and accommodation.

To sum up the relevance of agency to this study, one can define agency as the capacity to do things, or in this case to produce mathematics. The examinees have agency, which they use to do mathematics. The disciplinary agency has the capacity to bring forth the unexpected, which in turn exerts agency that the examinee can observe and react to. This notion of acting and reacting is what Pickering (1995) refers to as the dialectic of resistance and accommodation.

3.4.1 Resistance and Accommodation

Pickering defines the "occurrence of a block on the path to some goal" as resistance (Pickering, 1993 p. 569). This resistance is accommodated by different interactions. Pickering (1993) sees accommodation as some tentative human approach that circumvents the obstacles. This dialectic mechanism captures how sense-making emerges over time in a social order.

If mathematics production is to be fully understood, the tools used in mathematical activity are not to be reduced to avoidable phenomena. In the instance of this research, the situated environment has many elements that are part of the existing social order. It is important to note that these tools do not operate independently, but instead require a skilled person to channel their agency in the desired direction. This implies that the skilled operator, in this case the examinee, and the non-human agency come together as a single unit in the production of mathematics in the situated environment. Furthermore, in elaborating on the co-existence of human and disciplinary agencies by referring to them as scientific practices, Pickering (1995) views science as practice and culture. Thus, what exists in this social order is the sociological engagement of the human agency with the disciplinary agency coupled to the scientific dialectic of resistance and accommodation.

3.5 Framework

The role of frameworks in Mathematics Education Research (MER) is very important and is central to the field of enquiry. According to the online Encarta World Dictionary cited by Lester (2005), a framework is defined as "a set of ideas, principles, agreements or rules that provides the basis or outline for something that is more fully developed at a later stage". Furthermore, Lester (2005) describes a framework as a scaffold, which builders use to reach inaccessible heights or portions of a building. Thus, a framework is a basic structure of the ideas that underpins the phenomenon that is being investigated.

This structure of ideas alludes to the abstractions and assumed interrelationships of the features relevant to the investigation of the phenomena as determined by the adopted research perspective. Hence, the abstractions and interrelationships are to be used as a basis and justification in all aspects of the research. Eisenhart (1991) refers to three types of frameworks that are common in mathematics education research. These are the theoretical framework, practical framework and conceptual framework. In a theoretical framework the researcher uses accepted conventions of argumentation and experimentation associated with the theory and research. The practical framework suggested by Eisenhart (1991) is not informed by theory but by practice, knowledge, findings of other research and in some cases, viewpoints of the public. Eisenhart (1991) points out that a practical framework tends to ignore how those directly involved make sense of their situation.

Another framework described by Eisenhart (1991) is the conceptual framework. Eisenhart (1991, p. 459) describes a conceptual framework as "a skeletal structure of justification, rather than a skeletal structure of explanation". Furthermore, a conceptual framework consists of concepts chosen specifically for the problem that is being investigated. Like theoretical frameworks, conceptual frameworks are based on previous research, but the latter are also constructed from an array of current and possibly far-ranging sources. The conceptual framework may be based on different theories and various aspects of practitioner knowledge, depending on what the researcher can argue will be relevant to and important in addressing the research problem. Thus, as changes occur in the state-of-knowledge, of available empirical evidence, the conceptual framework may be reassembled.

With reference to Levi-Strauss (1966) and Gravemeijer (1994), Cobb (2007, p. 29) suggests a bricolage approach to frameworks. It bears some similarities to a conceptual framework. A bricolage is used to provide a firm structure for the domain of a study. Cobb (2007) sees a bricoleur as a skilful repair person who uses any tools available in any given situation to complete a job. In this type of framework the problem drives the research and assists in identifying the theoretical constructs used to build and develop the research.

To best guide the research question and support the structure of the domain of this study, I used a multi-faceted framework with different theories of diverse orientations. Niss (2007) is of the opinion that the most important purpose of theory is to provide a structured set of lenses through which to approach, observe, study, analyse and interpret phenomena. Lave (1993) acknowledges this by stating that, without theory it is impossible to analyse activities. Theories of situated activities do not separate action, thought, feeling and values from their collective cultural and historical forms of located conflict and meaningful activity. According to Denzin and Lincoln (2000), knowledge production and making sense of the world is a complex process. As an approach to research they therefore suggest that multiple theories would provide richness and depth to a study. Hence, the bricolage approach seemed to be the most suitable and appropriate framework for this study, where coexistence of theories is of importance.

The combination and coexistence of theories coincides with the principles of ethnomethodology and its important constructs. All these constructs have a subtle and unavoidable role to play in the analysis processes. Based on the foregoing discussion, I conceptualise the use of a bricolage framework for the purpose of this study as one which entails the underlying features of ethnomethodology and its relevant constructs as well as the sociological influences underpinning the in situ-ness of the context. This conceptualisation exemplifies the ethnomethodological constructs emanating from the data as well as the scientific temporality in the ways of working within the dialectic of resistance and accommodation.

It is Pickering (1995, p. 22) who states that: "It becomes possible to imagine that science is not just about representation. However, there is another way of thinking about science. One can start from the idea that the world is filled not, with facts and observations, but with agency." Pickering (1995, p. 22) analyses the existence of a series of resistances and accommodations as instances of mangling, which he metaphorically called the "the dance of agency". The following are instances of how Pickering's theory has been used by researchers.

Brown and Redmond (2008, p. 107) interpret Pickering's theory in terms of teachers' approaches in the classroom. When teachers teach by focussing on the algorithms and the way of doing the mathematical problems, they work with disciplinary agency. If they use open-ended questions and encourage learners to discuss new ideas, they encourage human agency. The learners' movements from the disciplinary to the human agency are what constitute the dance of agency. The agency in this instance would be the teacher's way of "being, seeing and responding" to learners.

Boaler (2003) observes different approaches to teaching in the Mathematics classrooms and finds that in classes where students were given open-ended questions and were guided through the problem-solving process, students learnt to mingle the standard algorithms with their own thoughts when engaging in solving mathematical problems. Hence, they were actively involved in the dance of agency, as Pickering (1995) would have observed.

When comparing the difference between the ways of working done by research mathematicians and mathematics students, research mathematicians as Grootenboer and Jorgensen (2009) observe, are more collaborative while students are focused on the procedures and algorithms of the discipline. They assert that the learning of Mathematics should be mathematician-like as shown in the research. When students shift from one way of working to the other, the dance of agency takes place.

Livingston (1986) conducted an ethnomethodological study on proofs using his own data to open the lived work on proving in mathematics (Livingstone 1986, 2006). He differentiated between the lived work and the account of the work. For proving, he asserts that:

> What is written and said is not really the whole proof. It is a proof-account as one intelligible social object, consisting of the proof-account /the lived-work or proving to

which the proof-account is essentially and fundamentally linked. The irremediable pairing of the proof-account /the lived-work or proving as one integral object, not as two distinct parts circumstantially joined. The proof-account /the lived-work or proving is the proof in and as the details of its own accomplishment (1986, p. 111).

Pickering (1995) renders a description of Hamiltin's construction of quaternions which is different to Livingston's study. He proposes disciplinary agency as "the agency of discipline relating to elementary algebra, for example that leads us through a series of manipulations within an established conceptual system" (p. 115). According to the pursuance of solution-seeking in mathematics there are periods when the problem-solvers are passive and the discipline exerts agency. In this sense mathematical work is characterised as a dance of agency where human and disciplinary agency intertwine during the solution-seeking process. The way goals mutate in practice is a theme in Pickering's description of the construction of Hamilton's quaternions. He describes his interpretation as a pattern of mathematical work where:

- the human agency makes some extension of what is known
- the human agency yields to disciplinary agency to see where that takes you
- when you run into resistance – something is not working – you start thinking of making a new extension
- you yield to disciplinary agency again, and consider where it takes you then
- you repeat this process until you are satisfied with the outcome.

He refers to this intertwinement as "the dialectic of resistance and accommodation. This is where the resistance signifies failure to recognise and attain an intended capture of agency in practice and on the other hand accommodation (of) the active human strategy of response to resistance" (p. 22) Resistance occurs when an action does not produce an expected outcome.

Pickering demonstrates how Hamilton in his construction of quaternions encountered resistance when the algebraic and geometric approaches rendered contradictory results. Accommodation is the path embarked upon to resolve resistance.

Similarly, Merz and Knorr-Cetina (1997) analysed "the work and accomplishments of theoretical physicists" (p.73). Their work is situated within the application of procedures of the new sociology of scientific practices on mathematics. Their work emulates to a certain extent the work of Livingston and Pickering to render an analytic account of the practical work of theoretical physicists. Theoretical physics is "like mathematics … a thinking science, where work is done at the desk, instruments are concentrated around the pencil and the computer, and processing is realized through writing" (p. 74). Merz and Knorr-Cetina (1997) used the emails, which the physicists had used to communicate their ongoing work, and they used the data they got from conducting interviews with the physicists. They emphasize that theoretical physicists achieve resolution to problems through a first-order demonstration, and, if this is hard to achieve, through a second round of diversions and tricks. However, in a third course undertaken by theorists' practical work of dealing with computational objects, they used strategies such as variation, "doing examples", and model objects (p. 95).

Another way of working identified by Knorr-Cetina (1999) was in her work on the production of scientific knowledge; she used the term 'blind variation' to describe the process by which problems with anomalous data are dealt with in small laboratory settings. According to her study, when experiments do not work it is not common for scientists to go to the main cause of the problem: "Instead, they will try several variations in the belief that these will result in workable evidence" (p. 91). The way of working was labelled by Knorr-Cetina (1999) as follows:

Variation: This way of working concerns a problem of interpretation or when different variations are used in the belief that they will ultimately result in workable evidence.

Work done by Julie (2003) involved practising teachers who worked in groups. He analysed how with no prior experience of modelling they went about constructing a mathematical model. He showed how a particular table constructed by the teachers drove the mathematical model-building activity at a given time. This table played a crucial mediating and guidance-giving role. The performative actions they employed were identified as shedding, compensation and variation. Julie (2003, p.120) observed the following types of patterns in the mathematical work:

Compensation refers to a pattern of work where it compensates for missing data. Julie (2003) differentiates between two types of compensation:

- **Assumption-driven calculation:** when calculations are made by using numbers, which were derived from assumed information
- **Assumptive-driven calculation:** when other given or known information is adequate for replacing missing data.

Shedding is referred to by Mason (1999) as mathematical work that points to the phenomenon of being stuck.

Julie (2015) did a study using the notion of ethnomethodology and the sociological study of work in science. The research conducted concerned the ways of working with modelling and application-like problems in a time-restricted examination. The actual scripts of examinees were analysed to tease out the ways of working with such problems taken from a high-stakes examination context. The analysis was anchored by various agencies present in the high-stakes context.

The ways of working that emanated from this study included the "U-turn", which was categorized by three distinct features, which were as follows:

- Go back to the start; restart with a different interpretation of some of the given strategy exhibited
- Go back to the start; restart with a different strategy
- Go to the start; restart because the result obtained does not comply with the dictates of the extra-mathematical context (pp. 478-485).

Other ways of working identified by Julie (2003) using Pickering's Dance of Agency and ethnomethodological analysis involve:

- **Reversal:** referred to by Julie (2003), in which a calculated resistance was created and the learner removed the produced work by drawing a line through it, immediately starting again from a certain point
- **Convenience:** referred to by Julie (2003), in which the learner creates a situation using a faulty method by which to simplify the problem to get to a suitable objective.

Greiffenhagen (2008), by contrast, collected data by using videos of actual lectures to graduate mathematics students, as well as interactions between mathematics doctoral students and their supervisors, to illuminate such lived work in mathematics. He found that mathematical work at its site of production is "both retrospective and prospective, i.e. the lecturer is summarizing what he will do next" (p.17).

Roth (2012) also focused on the difference between the lived work and the account of such work in mathematics. Using the proof of the theorem, "the sum of the internal angles of a triangle is 180", he demonstrates how the lived work of the proof emerges through drawing, seeing and concluding. He is of the opinion that "the lived (subjective) work has to be enacted each and every time the person is actually doing or following (observing) the proof" (Roth 2012, pp. 239-240).

In this study, examinees' ways of working were analysed while observations were made of the dialectic instances of resistances and how these resistances were overcome by accommodation, as well as the instances of non-firing of resistance. The non-firing of resistance refers to the ways of working, which exemplify resistance in the solution-seeking path, while the work continues uninterrupted. The dialectic relationship as described by Pickering (1995) becomes visible in the analysis through the textures of creating order and ultimately visible practical accomplishments within the pursuit of solutions.

3.6 Conclusion

After establishing the gap in the way feedback is given to all role players the study suggests a more comprehensive way of giving feedback. In this case, an ethnomethodological analysis is embraced. It takes into account the natural setting and the reflexive notion of accounts that are being created. The visible textures of examinees' ways of working when producing mathematics are the source of investigation. This notion of ethnomethodological analysis and scientific knowledge brings into play the interaction between the dialectic features of resistance and accommodation and ethnomethodology. The chapter that follows gives a detailed outline of the research methods that were used in this study.

CHAPTER FOUR

RESEARCH DESIGN

4.1 Introduction

The previous chapter provided a discussion of the theoretical issues relevant to this study. It gave a review of the literature consulted to conceptualize a thinking framework of what this study tried to achieve. In this chapter I provide a structured outline of specific detailed features of this study by describing the design and methodological procedures used. I explain the sampling procedures and describe the sample chosen for this study. Descriptions of the data collection organization as well as the analysis of it are discussed. The data is also presented in its preliminary labelled stage. The reliability and validity concerns together with the relevant strategies taken are also highlighted. The chapter concludes with a discussion and explanation of the ethical issues that were dealt with during the course of the research study.

4.2 Research design

A qualitative design was undertaken in this study because examinees' ways of working in producing their responses to questions in a high-stakes examination were used to describe the phenomena under investigation. The research question posed in this study could best be answered by means of a qualitative approach which seeks to appreciate and understand phenomena in a context such as "real world setting where the researcher does not attempt to manipulate the phenomenon of interest" (Patton, 2001, p. 39). Qualitative research approaches seek clarification, understanding, and extrapolation of findings (Hoepfl, 1997). According to Jacob (1987), a qualitative research methodology attempts to present the data from the perspective of learners. However, there is a range of different ways to make sense of the world to discover the meaning as seen by those who are being researched and to understand their view of the world rather than that of the researchers (Denzin & Lincoln 1994). This is a very important statement for this study, because how the data was analysed and interpreted was strengthened by the notion of ethnomethodology. According to Patton (2002) Seale, 1999, Seale 2007, there is no single way to carry out qualitative research. The approach to the study researchers choose depends upon factors such as the researchers' belief of the social world, the nature of the knowledge and how it can be acquired as well as the purpose and goal of the study. Also, the research participants (examinees), the audience at whom the study is aimed and the environment of the researchers themselves, become important factors.

Morse, (2001) argues that maintaining consistency in the methods and approaches used by researchers can result in better quality work being produced. Denzin and Lincoln (2011, p. 3) describe qualitative research as:

> a set of interpretive, material practices that makes the world visible. These practices transform the world. They turn the world into a series of representations, including field notes, interviews, conversations, photographs, recordings and memos. Qualitative researchers study things in their natural setting, attempting to make sense of or interpret phenomena in terms of the meanings people bring to them.

This description by Denzin and Lincoln (2011, p. 3) is further strengthened by the interpretive, hermeneutics and semiotics paradigms which are common in qualitative research. Paradigms are viewed as sets of beliefs that deal with ultimates of first principle. A paradigm represents a view that defines, for its holder, the nature of the world, the individual's place in it and the range of possible relationships to that world and its parts (Kaplan and Maxwell, 1994).

4.3 Theoretical considerations

4.3.1 Ethnomethodology

The NSC high-stakes examination setting is limited to those who are obligated to take part in it. Ethnomethodology is a way to bring out the everyday taken-for-granted aspects of social life. As the theoretical discussion that follows will show, the significance of this approach in my study involves the observation of examinees in the natural setting a process that is not permitted in a high-stakes examination setting.

Garfinkel, in his pioneering work, paid careful attention to practical actions of laboratory scientists and mathematicians. These studies which focused on what people were doing when they were doing their jobs, involved close examination of the details of work practice. Under the banner of the Sociology of Scientific Knowledge (SSK), work is done to recover the specifics of some scientific and mathematical work.

From the work of Pickering (1995), Livingston (1986) and other ethnomethodologists one can also study the produced artefacts to trace the "ethnomethods" used by others in their efforts to construct an artefact. This research is ethnomethodological as it focuses on the detailed practices in how examinees engage with their solution-seeking pursuance while solving trigonometry problems in the context of the time-restricted high-stakes NSC mathematics examination. It employed an ethnomethodological analytical perspective using the constructs of ethnomethodology such as reflexivity, as well as the documentary method of analysis to makes sense of examinees' ways of working.

Other important features related to how the data was perceived and interpreted were:

- that the accounts and practical action are reflexive or indicative of the occasion of their use

- that all accounts and actions are practical accomplishments Garfinkel (1967).

These features serve as resources in ethnomethodological investigations (Garfinkel 1967, p. 4). Garfinkel (1967, p. 37) identified a number of interpretive resources for professional sociologists. The first resource is the use of background expectancies, which points to those which can be seen but go largely unnoticed. These are everyday features that people actively gravitate towards. In making this statement the researcher perceives the textures of examinees' ways of working as the same phenomena, especially because in the marking process these textures are treated as unseen. Furthermore, ethnomethodologists identify background features by distancing themselves from the familiar routine nature of things. However, in situated interactions they start with the familiar scenes to disrupt the taken-for-granted assumptions to make sense of them. Hence, the analysis of data does not go without the causal implications of social facts on the examinees' ways of working. Nor does it ignore the historical connections in the building up of solution-seeking pursuance to achieve a practical accomplishment.

4.3.2 Dialectic Resistance and Accommodation

The ways of working that represent how examinees navigated their solution-seeking process is exemplified by a "stop and continue" or a "stop and start over" movement which is grounded in the notion of dialectic resistance and accommodation. The framework used in this study to analyse procedures in mathematics production is based on Pickering's (1995) distinction of agencies. Pickering (1995) classifies three types of agency: human, disciplinary and material. While one would not usually think of materials or disciplines as having agency' Pickering (1995, p. 52) describes the human engagement with each of these agencies as a "dialectic of resistance and accommodation". The way in which the use of agency is adopted in the analysis of examinees' ways of working includes only the human and disciplinary agency. Both Pickering (1995) and Boaler (2002, p. 42-47) also do not refer to material agency in mathematics. Pickering offers material agency as only being evident in scientific advancements. Pickering's (1995, p. 115) view is that mathematics is a product of human activity and therefore human agency plays a major role in any conceptual advancement. The analysis of examinees' ways of working included the examinees' written responses and the action taken by them when there was some dissatisfaction with the work produced. In Pickering's (1995, p. 115) argument for disciplinary agency he describes how a conceptual system can carry human conceptual practices along independently of individual wishes and intents. Therefore, although humans exercise their agency in their intentions and actions, they often meet with resistance or obstacles. This resistance is put into existence by the agency of the concept produced by the examinees. However when examinees produce responses to mathematical questions, they actively do the work. The finished product is a representation of disciplinary agency. Pickering (1995, p. 29) proposes that in this instance the resistance in conceptual practice within the mangle is now located in the disciplinary agency. The dance of agency is then enacted by having the examinee accommodate their actions to appropriate the resistance. This dialectical interaction is the framework from which the production of mathematics in pursuit of a solution was analysed. Thus, the collaboration between ethnomethodology and the dialectic resistance and accommodation is what steered the way in which the examinees' responses within a high-stakes examination setting was analysed.

4.4 Research setting

4.4.1 High-stakes examination setting

The high-stakes examination setting was the social context in which the examinees produced their responses to questions on trigonometry. The examinees oriented themselves towards the situated-ness of this setting. This study interpreted the high-stakes examination setting as a normal, natural practice of organisation which reflects the connectedness of social facts. This made the context of the high-stakes examination an important central and referential part of all mathematics produced in it.

4.4.2 The examination room (local setting)

According to the Regulations pertaining to the conduct, administration and management of the National Senior Certificate Examination (2014), the chief invigilator is the principal or a senior educator at the school. He/she must ensure the readiness of the examination room. The DBE prescribes the regulations which must be followed thus:

> Candidates must be seated at least one metre apart and are not allowed to sit two to a desk. It is expected that the chief invigilator draws up a seating plan, indicating the desk arrangements and the examination numbers of candidates. Such seating plan must be submitted for each examination question paper written. All subject matter, such as drawings, must be removed from the walls and chalkboards must be cleared of any writing, formulae, or drawings. Examination rooms must have sufficient ventilation and illumination (Department of Basic Education, 2014).

Upon entering the examination room, the examinee must produce an admission letter as well as proof of identification. Examinees must enter the examination session thirty minutes before the start of the examination. During this period the chief invigilator will go through all the printed pages in the question paper to make sure there are no printing errors. After the completion of the checking of the examination paper and other administration issues and announcements, the examinees get ten minutes reading time where they are allowed to read through the question paper. Each session must commence and be terminated according to the time specified on the examination timetable. The time allotted to complete the examination is indicated on the examination paper. All examination rooms must have a visible clock to indicate the starting and finishing time to the examinees. The mathematics Paper 2 used for this study was a 3-hour paper (Department of Basic Education, 2014).

Examinees were not allowed to have a book, memorandum, notes, maps, photos or other documents or papers (including unused paper), or any other material which may be of help to them when writing the examination. Examinees were not allowed in with any loose paper or additional answer books for "rough work". All work including "rough work", had to be done on the examination answer script according to the Regulation Notice No. 371 in Government Gazette No. 37651 dated16 May 2014 (Department of Basic Education 2014).

4.5 Documentary method of interpretation in a qualitative research approach

Document analysis in qualitative research is a systematic procedure for reviewing or evaluating documents printed or written or electronic. Bailey (1194, p.194) differentiates between two types of documents in qualitative research, the primary documents and the secondary documents. Those not present at a particular event present secondary documents. The documents employed in this study are considered primary documents because of the participants' involvement in the event of production.

Like other analytical methods in qualitative research, documentary analysis requires that data be examined and interpreted in order to elicit meaning, to gain understanding and to develop empirical knowledge (Corbin and Strauss, 2008). Although documentary analysis has served mostly as a complement to other research methods, it has also been used as a stand-alone method (Bowen, 2009). Furthermore, Bowen (2009) explains that documents can serve a variety of purposes as part of a research undertaking. He specifies five uses of documents. Firstly, documents can provide data on the context within which research participants operate. Such information and insight can help researchers understand the historical roots of specific issues and can indicate the conditions that impinge upon the phenomena under investigation. Secondly, information in documents can suggest some questions which require asking and situations that need to be observed as part of the research. Thirdly, documents can provide primary or supplementary research data. The information and insight derived from documents can become valuable additions to a knowledge base. Fourthly, documents provide a means of tracking changes and development. Fifthly, documents can be analysed as a way to verify findings or corroborate evidence from other sources.

However, literature is very clear on the advantages and disadvantages of such a qualitative method of interpretation and analysis. Yin (1994, p. 80) cited by Bowen (2009), outlines the advantages of using the qualitative document analysis research method as follows: it is less time consuming; it does not require data selection but rather data collection; it can be reviewed repeatedly; it is cost effective and documents are unaffected by the research process.

However, this study draws its use of documentary method of analysis from the viewpoint of ethnomethodology. According to Garfinkel (1967, p. 95) the documentary method of interpretation is common to both qualitative and quantitative research as when investigators frequently use certain observed features of the thing they are referring to as 'characterising indicators of the intended matter'. Garfinkel (1967) gives a re-specification of this after completing a study of patients' pathways through a clinic using their medical records. At the start of the study it was found that the medical records were incomplete. Instead of abandoning or supplementing the data Garfinkel (1967) concluded that the documentary method of interpretation could clearly be understood as a constitutive phenomenon (p. 16). This was the background in which the answer scripts of the examinees were analysed, interpreted and discussed. These answer scripts were seen as documents that were constitutively linked to a time-restricted high-stakes examination unique to the South African education context. The constitutive connectedness Garfinkel (1967, p.16) demonstrates through his study of a clinic's medical records as noted in Chapter 2 is the same connectedness ascribed to the answer scripts of the examinees in this study. Garfinkel (1967) suggests the documentary method of interpretation involves complex issues such as organizationally distributed and temporally articulated interactions amongst various parties. He defines the documentary method of interpretation in the following way:

> The method consists of treating an actual appearance as the "document of", as "pointing to", as "standing on behalf of" a presupposed underlying pattern. Not only is the underlying pattern derived from its individual documentary evidence, but the individual documentary evidence (is in turn) interpreted on the basis of "what is known" about the underlying pattern. Each is used to elaborate the other. (Garfinkel 1967a, p.78)

This reflects the complexities of the social structure in which examinees' responses were produced. These complexities include the rules and regulations that guide the organization of the NSC high-stakes mathematics examination as well as the need to establish time-based order in the ways of working of examinees.

Another important factor regarding documents being used for qualitative research is the quality of the documents. Quality refers to whether the credibility, authenticity and the meaning of these scripts is evaluated for its trustworthiness.

4.6 Trustworthiness of data

4.6.1 Credibility

Credibility refers to whether the evidence is free from error and distortion. According to Scott (1990) the question of credibility should concern the extent to which the researcher is sincere in his/her opinion and in their attempt to record an accurate account from that chosen standpoint. On the question of credibility, I can safely say that the examination scripts were free from distortion. I can safely say that none of the scripts was produced for my benefit. I, therefore believe that they were truthful and could not have been altered either for the benefit of this study or to have misled the researcher in any way. The question of credibility also applies to the examinees. As explained in Chapter Two, the NSC examination is the key to various future endeavours for examinees and it therefore is believed that the responses the examinees expressed were honest and justify the situation.

4.6.2 Authenticity

The evidence for analysis was a fundamental part of the research so the examination scripts consulted were therefore genuine and original in their entirety. The Regulation Notice No. 371 in Government Gazette No. 37651 dated16 May 2014 detailed the processes to establish the authenticity of the examination papers and answer books.

The documents used in this study showed all the official printing and bar-coding of the DBE. According to Platt (1981) and in the case of this study all examination papers (documents) were scrutinised in terms of printing errors and ambiguities. As for the examinees, the identity of those who responded to the questions in the NSC mathematics examination was not of interest. However the examination regulations are very strict in the case of identification when entering the examination room, as explained earlier in this chapter. The examination answer scripts presented an examination number and not the name of the examinee (Department of Basic Education 2014).

6.6.3 Meaning

According to Scott (1990, p.28) meaning refers to whether the evidence is clear and comprehensible. The ultimate purpose of examining documents is to understand the meaning and significance of the content of the document. However documents contain literal and interpretative meanings. According to Scott (1990), the literal meaning gives the face value meaning from which its real significance must be reconstructed, whereas an interpretative understanding relates the literal meaning to the contexts in which the documents are produced in order to assess the meaning of the text as a whole. In this study the latter aspect is particularly significant as it concerns the contexts of its production. Social texts do not merely reflect objects, events and categories which exist in the social world, but actively construct a version of those objects, events, and categories (Potter and Wetherell, 1987).

4.7 Reliability and Validity

Qualitative research uses a naturalistic approach that seeks to understand phenomena in context-specific settings such as real world settings (Patton 2001, p.39). Qualitative analysis emanates from the philosophical nature of various paradigms. For qualitative output or accounts of analyses to be of substance, they must draw the proof of reliability and validity. The rough aspect of validity refers to the extent to which a test measures what we actually wish to measure (Blumberg, 2005). If research is not valid, reliability hardly matters. Research needs to be valid to answer the research question. However, Blumberg (2005) posits that the optimal situation is to conduct research that is both valid and reliable. According to Eisner (1991, p. 58) a good qualitative study can help us to understand a situation. This relates to the concept of good quality research when reliability is a concept of quality evaluation. For Stenbacka (2001), without the concept of reliability as a criterion, the study is no good. However, Patton (2001) states that validity and reliability are what every qualitative study should be concerned with in its design, with analyses of the results and judgment of the quality of the study. Furthermore, the inquirer has to persuade the audience the findings of the research are worth paying attention to (Lincoln & Guba 1985, p. 290).

Validity is a kind of qualifying measure. Gresswell & Muller (2000) suggest validity is affected by the researcher's perception of validity and choice of paradigm assumption. Thus Mishler's (2000) statement that the concept of reliability and validity brings a sense of trustworthiness and confidence to the findings, is supported by Lincoln & Guba (1985) and Johnson (1997, p. 282).

In my study, firstly the categories were validated by in-depth discussion and debate with my supervisor. Secondly, the initial results were presented at a seminar to mathematics educators at a neighbouring university. Thirdly, the results of two categories were presented at a national conference, the Association for Mathematics Education of South Africa (AMESA). These initial results were presented as a long paper to researchers in the field of mathematics education, lecturers from tertiary institutions and high school mathematics educators (AMESA 2015, p. 456).

Such forms of validation strengthen the reliability of the results. Blumberg (2005) sees reliability as a measurement that supplies consistent results. Defining a valid measure as one that "really measures what we wish to measure", ethnomethodologists' observations are probably valid simply because they tend not to follow the traditional practice of proposing an indirect measurement for some concepts. Ethnomethodology searches for observable regularities and then labels them and this process has almost perfect validity Bailey (1994) since the practice is the operational definition of the concept. Hence the answer scripts in which the examinees produced their responses constitute the concept to be measured in terms of the ways of working in the NSC high-stakes examination.

Joppe (2000) defines reliability as:

> …The extent to which results are consistent over time and an accurate representation of the total population under study is referred to as reliability and if the results of a study can be reproduced under a similar methodology, then the research instrument is considered reliable (p. 147).

4.8 Interpretive paradigm

An interpretive paradigm was an important feature of this qualitative study because of the meaning assigned to the text. Interpretive research focuses on the full complexity of human sense-making as the situation emerges (Kaplan and Maxwell, 1994). The interpretive approach was used to look into examinees' ways of working in the high-stakes examination as it aimed at understanding the textures that were produced. The interpretations of textures were linked to the context in which it was produced (Walsham 1993, p. 4-5). Gephart (1999) argues that interpretive assumes that knowledge and meaning are acts of interpretation, hence there is no objective knowledge which is independent of thinking reasoning humans.

The thinking and reasoning was made visible through the textures produced by the ways of working and was a phenomenon this study attempted to understand through the meanings that were assigned to it by examinees (Deetz, 1996). However as noted earlier, the setting was of equal importance. The interpretive paradigm is concerned with understanding the world as it is from the subjective experiences of individuals as noted by Reeves and Hedberg (2003, p. 32); the "interpretivist" paradigm stresses the need to put analysis in context.

Another relevant feature of the interpretive paradigm is the role it plays in enhancing theories. The interest of interpretivists according to Walsham (1995) is to judge or evaluate and refine interpretive theories. Walsham (1995b) presents three different uses of theory in interpretive case studies. Firstly, theory guides the design and collection of data. Secondly, theory is used as an iterative process of data collection and analysis, and lastly, theory is used as an outcome in research. The theoretical constructs of importance in this iterative process emanate from ethnomethodology and the social study of scientific work. Ethnomethodological analyses of interpretation are concerned with what Garfinkel (1967, p. 1) calls "practical sociological action and reasoning" with the objective of extracting and understanding the influence of social facts concerning practical social action.

Furthermore, this notion of meaning-making of the mathematical discoveries of examinees connects with the hermeneutics paradigm, which asks the question, "What is the meaning of text?" (Radnitzky 1970, p. 20). Hermeneutics is primarily concerned with the meaning-making of text or text-analogue. The text referred to is in this instance the written responses of examinees. The study attempts to form an understanding of the written mathematical text as a whole as well as the interpretation of its parts by explaining it as a constitutive phenomenon (Gadamer 1976, p. 117). The written responses of the examinees and the textures that were noticeable in the solution-seeking pursuance were treated as items that carried meaning.

4.9 Research Sample

Sampling in qualitative research can occur at several stages, both while collecting data and while interpreting and reporting on it (Yorkshire and Humber, 2009). Marshall (2006, p.523) presents three broad approaches to selecting a sample which are: convenience sampling, judgmental sampling and theoretical sampling.

Convenience sampling is the least rigorous technique, involving the selection of subjects. It is not expensive, does not take up much time and not much effort goes into it (Marshall, 2006, p.523). The researchers select the available subjects who are likely to participate for a specific period. The second type of sampling strategy Marshall (1996) describes as judgemental and as the most common sampling technique. This type of sampling technique is actively selected by the researcher and is the most productive sample in terms of extracting from examinees' ways of working the visible textures that are produced. The last sampling strategy identified by Marshall (1996) is the theoretical sample. This type of sampling constructs interpretive theories from the emergence of data and selects a new sample to examine and elaborate upon.

4.9.1 The convenience sample

Data were organised by means of a convenience sampling strategy. This sampling strategy fits into the already selected sample called Local Evidence-Driven Improvement of Mathematics Teaching and Learning Initiative (LEDIMTALI) (Julie 2011). LEDIMTALI is a project where educators, mathematics educators, mathematicians, and curriculum advisors collectively and collaboratively work to ultimately induce the learners' achievement at their highest potential in mathematics (Julie 2012).

The sample of schools in the LEDIMTALI was chosen according to the following input agreed upon by all relevant role-players:

- ten schools were part of the initiative
- schools were all functional
- schools were from the same district
- the school had at least one class for mathematics at Grade 12 level

The schools are located in a 25km radius of the University of the Western Cape (UWC). Schools are situated in areas where clusters could form easily. The total number of mathematics question papers (Paper 2) of all ten schools amounted to 380.

4.9.2 Data selection

The foregoing conditions constitute a broad outline of the sample that was used. However, a more specific database chosen from these schools consists of the examination scripts. There were a total of 380 Mathematics Paper 2 scripts from the 10 schools. Furthermore, only questions on trigonometry were selected for analysis. Therefore as noted earlier, only the questions pertaining trigonometry in Paper 2 (Questions 8.9.10.11, and 13) were selected for analysis. All these questions were scrutinized and questions which showed visible textures in the ways of working were selected. 94 scripts were found to have some form of texture. Here the textures are a product of the ways of working of examinees. In other words,

- the textures represent some type of abandonment of a pursuit in a solution-seeking path
- the ways of working are responses to the problem text in pursuit of a suitable solution.

4.9.3 Snowball sampling

The categories were obtained through the notion of snowball sampling which refers to a type of sampling in sociology and statistics research. This is a non-probability sampling technique where an existing phenomenon recruits similar subjects that show an association with the existing phenomena. According to Handcock and Gile (2011, p. 1) the definition of snowball sampling reflects a phenomenon in the sociology of science that multi-disciplinary fields tend to produce which features a plethora of inconsistent terminology. In the case of this study, the phenomena that drove this snowball idea were the textures of ways of working. The problems that were selected for this study were preliminarily labelled and described as follows:

Problem 1: Partly removed

This category refers to abandonment at the earlier part of the work and then re-attempted from scratch afterwards:

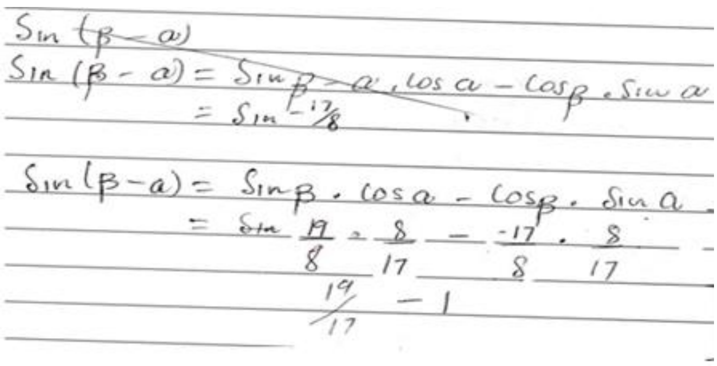


Figure 4.1: Partly removed

Problem 2: Cancelled

This category refers to lines drawn through the work and the word cancelled written on the solution. The examinee starts a new pursuit:

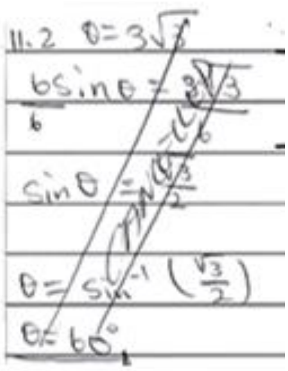


Figure 4.2: Cancelled

Problem 3: Disorderly texture

This category refers to unsystematic lines drawn (in a disorderly fashion) through part of the produced work. A continuation of the production follows the original production path:

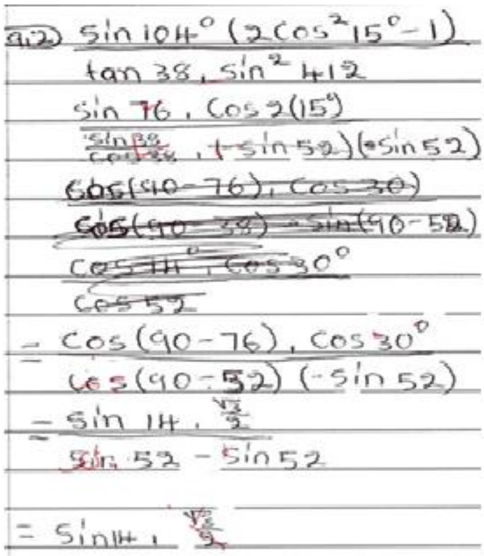


Figure 4.3: Disorderly texture

Problem 5: Rough work

This category shows the examinee indicating produced work as rough work by drawing a line through it and indicating it as rough work. A new pursuit starts:

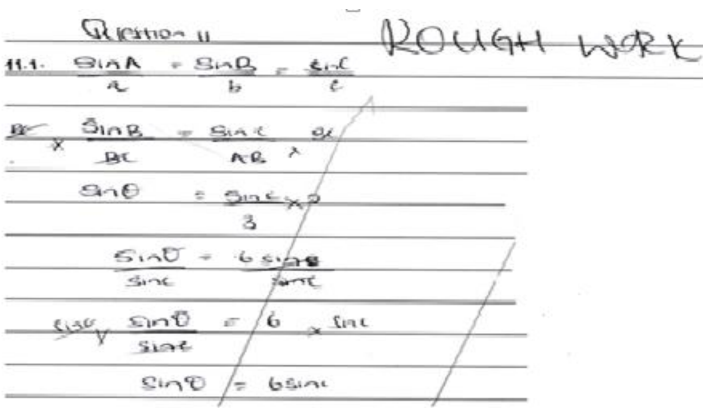


Figure 4.4: Rough work

Problem 6: Crossing out

This texture refers to work that was crossed out by the examinee. This is followed by a new pursuit:

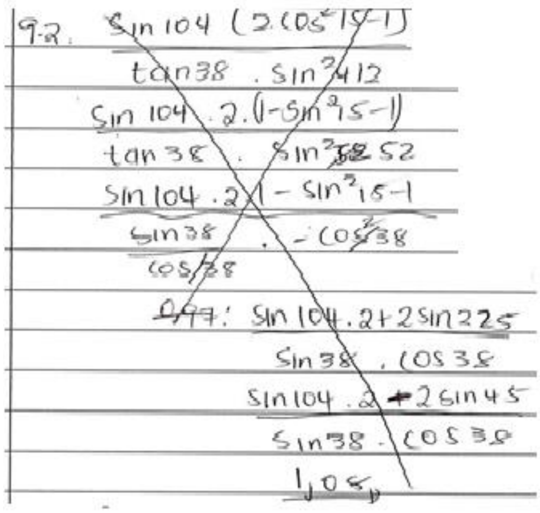


Figure 4.5: Crossing out

Problem 7: Making a U-turn

This texture shows a line drawn through the last part of the work. A new way of working starts at some point of the pursuit:

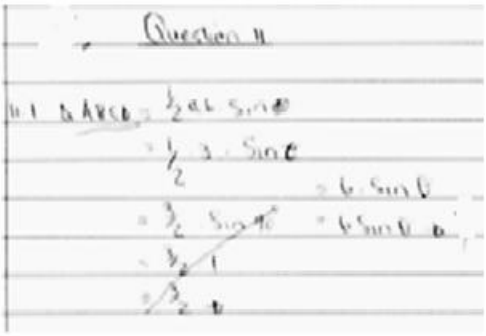


Figure 4.6: Making a U-turn

Problem 8: Cancelled and error

This refers to textures that show abandonment consisting of lines and the words “cancelled and error”:

Figure 3.7: Cancelled and error

4.10 Ethical Considerations

Ethical considerations present a set of moral principles, rules, or standards governing a person, profession or institution. This research study was part of the LEDIMTALI. Permission and approval for this project were obtained from the Western Cape Education Department (WCED) and the University of the Western Cape (UWC). Individuals (Head of the Mathematics Department and educators) and institutions (schools) participating in this project gave their written consent for their inclusion. Schools and educators were duly informed about the details of the project and were under no circumstances coerced into participating. Data was analysed in a manner that avoids mis-statements, misinterpretation, or fraudulent analysis.

The data was kept anonymous so that participating learners and schools might not be identified (McMillan & Schumacher, 2010). More specifically, copies of tables or excerpts of examinees' mathematical work that were included in the study showed no labels that could reveal information about participants. No unauthorised access was allowed regarding electronic information. All electronic information was stored with a secure password in accordance with Adler & Lerman (2013, p. 29) who posit that education research should "take sufficient care of those being researched".

The notion of confidentiality is considered a very important issue in this study because the data presented responses of examinees taken from the NSC high-stakes examination. According to McMillan & Schumacher (2010, p. 22), the right to confidentiality means access to the individual data or names of participants is kept in strictest confidence and should be available only to the researcher. Furthermore, marking evidence was deleted from the responses and no evidence of the nature of the actual marking of the scripts was divulged or used in presentations and conversations about this research.

The examination scripts were collected from the WCED and were securely stored by the LEDIMTALI and the researcher had access to these scripts only under the supervision of the project director. No unauthorised access to these scripts was allowed.

4.11 Concluding remarks

This chapter presented the research design and methodology. It gave a description of the rationale of research question and the qualitative approach employed as a research method. This chapter described the type of sample used and the sampling process which the researcher undertook to streamline the study in a logical and focussed way. Furthermore, the chapter presented the data in its preliminary labelled stage, which includes the definitions for each label. Other important issues discussed in this chapter are those of validity and reliability in qualitative research as well as how it connects with the ethnomethodological analytical perspective. I have shown how the theory of ethnomethodology and the dialectics resistance and accommodation collaborated in the analysis of data and how conclusions were drawn from it. Finally, I outlined how the ethical considerations were dealt with to safeguard the ethical credibility of the study.

The next chapter contains an analysis of the data, which will examine and guarantee the quality of the methodological design.

CHAPTER FIVE

DATA ANALYSIS

5.1 Introduction

In this chapter the results of the analysis of the data are presented. The data was collected and then processed in response to the research question posed in Chapter One. A fundamental aim drove the collection of the data and the subsequent analysis of it. The aim was to investigate the examinees' ways of working with questions dealing with trigonometry in the Grade 12 high-stakes NSC examination written in November 2012. As mentioned in the methodology chapter, a convenient sample was used comprising the examination scripts of the examinees attending the 10 LEDIMTALI schools. All the scripts were investigated and assessed to identify those which showed visible textures, such as deletions and other features indicating that the examinees did not work in some linear fashion to reach responses to questions pertaining to trigonometry. This data selection process rendered 98 scripts for in-depth analysis. The textures are described in terms of their relevance to the research question. They were labelled according to the researcher's interpretation. Eight ways of working were found. The category identification process was driven by two fundamental theoretical perspectives. The first of these is ethnomethodology (Garfinkel 1967) and its constructs, and the second is the mangle of practice (Pickering 1995) with an emphasis on identifying the dialectic of resistance and accommodation.

Through the dialectic resistance and accommodation and the ethnomethodological constructs the textures of the examinees' ways of working in their pursuit of an objective or solution became noticeable. According to De Vos (1998, p. 203) data analysis entails the analyst's differentiation of data into constituent parts to obtain answers to the research question. The research data alone does not provide the answers to the research question. However, De Vos

(1998, p. 203) argues that the purpose of interpreting the data is to reduce it to an intelligible and interpretable form. In so doing the relation in terms of patterns or trends between concepts, constructs or variables can be identified or isolated. As with established themes in the data the research problems can be studied and tested and conclusions drawn. The following sections present the textures produced from the ways of working that were identified from the examinees' responses to questions on trigonometry in the 2012 NSC Mathematics Examination.

5.2 Abandonment after a nearly complete attempt

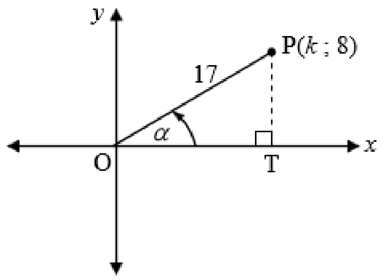
This way of working refers to the commencement of a pursuit to find a solution to a mathematical problem. The attempt shows some form of working right through nearly the final answer, and then early abandonment of the path of pursuit and a recommencement of the solution-seeking process through the making of adaptations to the previous path of pursuit.

The problem in Figure 5.1 below appeared in the examination paper. It is presented in its entirety since the category under discussion focuses on 8.1.4, but other parts of the question play a part in the production of the response to 8.1.4.

QUESTION 8

Answer this question WITHOUT using a calculator.

8.1 The point P(k ; 8) lies in the first quadrant such that OP = 17 units and $T\hat{O}P = \alpha$ as shown in the diagram alongside.



8.1.1	Determine the value of k.	(2)
8.1.2	Write down the value of $\cos\alpha$.	(1)
8.1.3	If it is further given that $\alpha + \beta = 180°$, determine $\cos\beta$.	(2)
8.1.4	Hence, determine the value of $\sin(\beta - \alpha)$.	(4)

Figure 5.1: November 2012 NSC Mathematics Paper 2, question 8

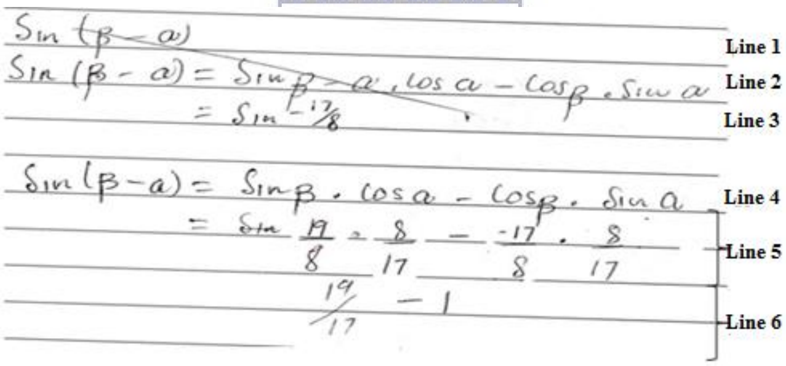


Figure 5.2: Abandonment after a nearly-complete attempt

Figure 5.2 is a representative response to Question 8.1.4. It is an example of the category under discussion and presents the texture, which is the product of the way of working. In line 1 of Figure 5.2 the examinee first writes down the question to which a response is needed. In line 2 of Figure 5.2 this "what must be determined" is written down again in the expansion of $sin(\beta - \alpha) = sin\beta - \alpha . cos\alpha - cos\beta . sin\alpha$ using the compound angle formula. A continuation of the pursuit in line 3 of Figure 5.2 shows that the notation when you do trigonometry had not been adhered to by the examinee when he/she writes $sin\frac{-17}{8}$. Thus having to evaluate the first function in the expansion formula the pursuit is abandoned as indicated by the line drawn through the work.

The examinee starts again by transferring and applying the compound angle formula correctly from the information sheet and in the same process reconstructs the formula to fit the conditions of the question. From this point, the examinee engages in a simplification process by using the values of the sides in a trigonometric relationship between sides and angles of a right-angled triangle. This ultimately renders the expression $\frac{19}{17}$ – 1 in line 6 of Figure 5.2.

5.3 Abandonment of a solution

This way of working refers to the complete working through of a mathematical question followed by an abandonment of the solution. Two kinds of abandonment of solutions emerged and these are discussed in the sections that follow.

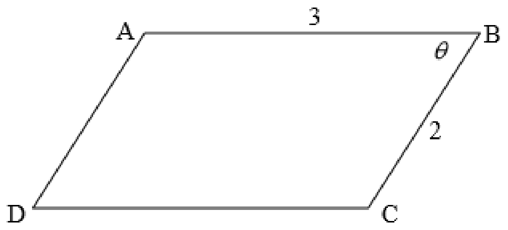
5.3.1 Abandonment and the construction of a new problem by "proving the given"

This way of working refers to the completion of a pursuit showing a solution to a mathematical problem and then the abandonment of all the work, the construction of a different new problem and the pursuit of the solution of this newly-constructed problem.

The problem in Figure 5.3 which follows appeared in the examination paper. It presents Questions 11.1 and 11.2 despite the fact that the category under discussion focuses on 11.2; however, other parts of the question play a part in the production of the response to 11.2:

QUESTION 11

ABCD is a parallelogram with AB = 3 units, BC = 2 units and $A\hat{B}C = \theta$ for $0° < \theta \leq 90°$.



11.1 Prove that the area of parallelogram ABCD is $6 \sin \theta$.

11.2 Calculate the value of θ for which the area of the parallelogram is $3\sqrt{3}$ square units.

Figure 5.3: November 2012 NSC Mathematics Paper 2, Question 11

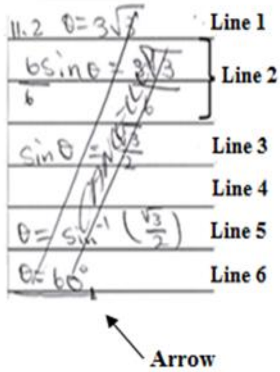


Figure 5.4: Abandonment of the first solution

In Question 11.2 the examinee needed to calculate the value of θ for which the area of the parallelogram is $3\sqrt{3}$ square units. In line 1 of Figure 5.4 the examinee wrote down the equation $\theta = 3\sqrt{3}$ in an attempt to calculate the size of the angle θ. Line 2 of Figure 5.4 shows the introduction of $6sin\theta$, which in this instance is linked to the area of the parallelogram, which is the link to what was given. This is to simplify the pursuit to get θ as the subject of the equation thus rendering $\sin \theta = \frac{\sqrt{3}}{2}$ in line 3 of Figure 5.4. In line 4 of Figure 5.4 the examinee deduces from line 3 that θ is equal to $sin^{-1}(\frac{\sqrt{3}}{2})$ to get closer to the objective, which is to determine the value of the angle θ. The examinee calculates $sin^{-1}(\frac{\sqrt{3}}{2})$ and obtains $\theta = 60°$. The examinee abandons the entire pursuit by drawing two lines through it and writes the word “CANCELLED” in between the two lines.

Figure 5.5 is a representation of the newly-constructed problem where the examinee’s objective was to show that the area of the parallelogram ABCD is $3\sqrt{3}$ square units, which was given in the problem text.

11.2 Area of ABC = AB . BC sin θ **Line 1**

= 3×2 sin 60° **Line 2**

= $3\sqrt{3}$ units **Line 3**

Figure 5.5: Newly-constructed problem

The new pursuit shows the examinee’s proof of what is given in the problem text in Figure 4.3 Question 11.2.

4.3.2 Abandonment and reconstruction of the same problem “proving the procedural objective”

This way of working shows that the examinee abandoned the solution and starts over. The new solution-seeking path shows the reconstruction of the same problem with some adaptation and rerouting to “without the calculator”.

The problem in Figure 5.6 that follows appeared in the examination paper. It presents only Question 9.2, which is the focus of discussion:

9.2 Simplify without the use of a calculator: $\dfrac{\sin 104°(2\cos^2 15° - 1)}{\tan 38°.\sin^2 412°}$

Figure 5.6: November 2012 NSC Mathematics Paper 2, Question 9.2

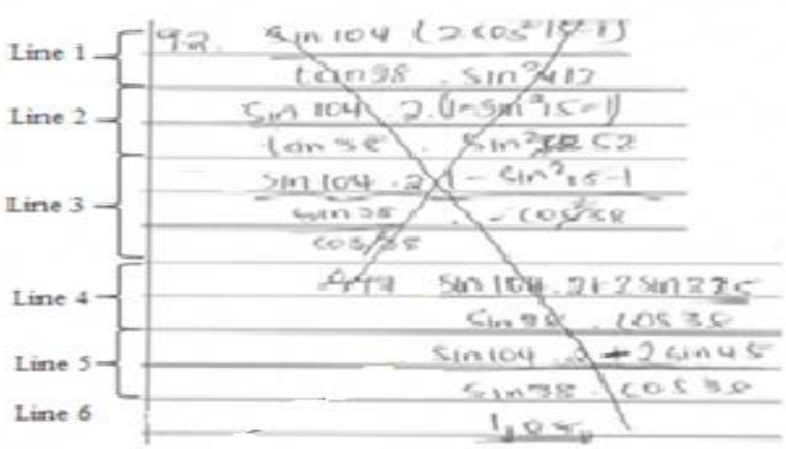


Figure 5.7: Abandonment of first solution-seeking path

Figure 5.7 is a representative response to Question 9.2 of Figure 5.6. It is an example of the pursuit under discussion and presents the texture which is the product of the way of working. The expression to be simplified "without the use of a calculator" is copied from the problem text in Figure 5.6 onto the answer script as shown in line 1 of Figure 5.7. Line 2 of Figure 5.7 shows no change to the expression $sin104°\ and\ \tan 38°$. The examinee simplifies the expression $(2cos^2 15° - 1)$ by substituting $cos^2 15°$ with $1 - sin^2 15°$, making use of the trigonometric identity relation of $cos^2 x + sin^2 x = 1$ to render $2.(1 - sin^2 15° - 1)$. The continuous navigation to work towards a solution sees the examinee substitute $sin^2 412°$ with $sin^2 52°$ using $\sin(412° - 360°)$. Line 3 of Figure 5.7 sees a replication of $sin104°$ in the first term and the removal of the brackets of the expression $2.(1 - sin^2 15° - 1)$ in the second term in the numerator to render $2.1 - sin^2 15 - 1$. The next step in this pursuit shows the examinee evaluate $2.1 - sin^2 15 - 1$ to obtain 2 + $2sin225°$ in line 4 of Figure 5.7. This way of working later contributes towards the abandonment:

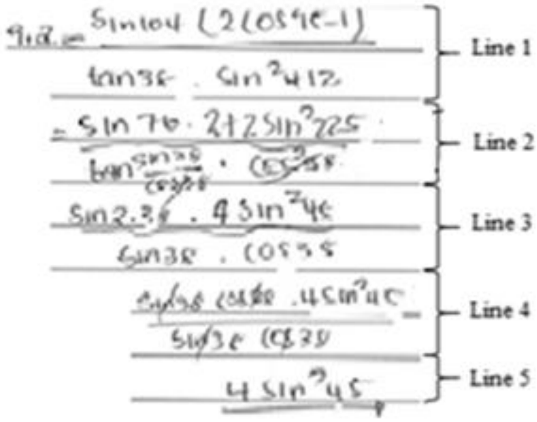


Figure 5.8: New extension

The new extension shows that the examinee continuing with the same line of thinking, repeating some of the work produced in the abandoned pursuit. In lines 2 of Figure 5.8, in the numerator, the examinee simplifies $sin104°$ by using the supplementary angle relationship $\sin(180° - 104°) = sin76°$. In the numerator of line 3 of Figure 5.8, the examinee writes $sin76°$ as a double angle, $\sin(2 \times 38°)$. The way of working shows the examinee produced the expression $\frac{sin38°\ cos38°.4sin^2 45°}{sin38°\ cos38°}$ rendered in line 4 of Figure 5.8. After doing cancellation of function the examinee left $4sin^2 45°$ as the final result.

5.4 Reversal

"Reversal is where a calculated resistance was created and the learner removes the produced work by drawing a line through it and starts over immediately from a certain point" (Julie 2003 p. 121).

The re-start is on the same solution-seeking path, with a different strategy until a final result is reached.

Figure 5.9 is the response to question 9.2 (see Figure 5.9 before):

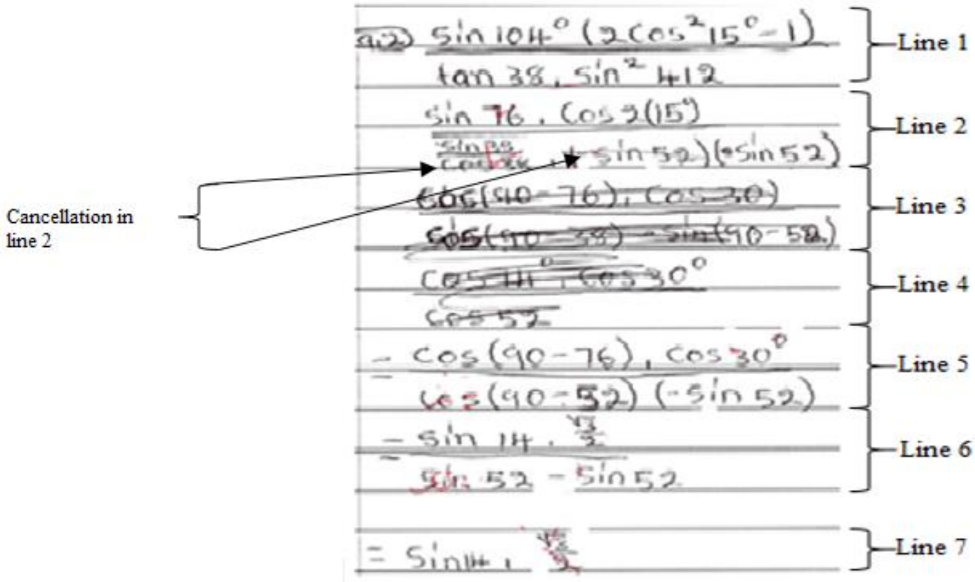


Figure 5.9: Reversal

As with the solution-paths illustrated in Figure 5.9, the start is made by writing down the symbolic objective in $line$ 1 which is part of the question. Line 2 of Figure 5.9 shows the examinee replacing $sin104°$ by the acute angle for the relationship $\sin(180° - 104°)$ to $sin76°$. The examinee then uses the trigonometric relationship of double angles to transform $(2cos^2 15° - 1)$ to $cos2(15°)$. The further pursuit shows the examinee using the identity relationship to substitute $tan38$ with $\frac{sin38°}{cos38°}$ in line 2 of Figure 5.9. The reduction formula of $\sin(360° + 52°) = sin52°$ is also used to express $sin^2 412°$ as $(-sin52°).(-sin52°)$. The continued pursuit sees the produced work in lines 3 and 4 abandoned, scratched out, upon obtaining the $cos14°$ and $cos52°$ in line 4. The re-starting point, "reversal" is where the examinee copies the abandoned work $\cos(90° - 76°)$ and $cos30°$ in the exact form – the numerator in line 3 to the numerator in line 5 – of Figure 5.9. The examinee moves back and follows a different strategy to fit the mathematical context. In the same instance, the examinee adapts $\cos(90 - 38)$ to $\cos(90 - 52)$ and then evaluates it as $-sin52$. Evidence in line 5 of Figure 5.9 shows the examinee replaces $cos14°$ with $sin14°$ and $cos52°$ with $sin52°$. The pursuit proceeds in line 6 of Figure 5.9, showing the simplification of the work in lines 5 of Figure 5.9 giving the expression $\frac{sin14.\frac{\sqrt{3}}{2}}{sin52°-sin52°}$. The last part of the pursuit sees the examinee simplifying $cos30°$ with its angle-side ratio of $\frac{\sqrt{3}}{2}$, and cancelling $sin52° - sin52°$ to obtain $sin14°.\frac{\sqrt{3}}{2}$ as a final result.

5.5 Convenience

"Convenience is where the examinee creates a situation using a faulty method to simplify a mathematical problem to get to a suitable objective" (Julie 2003, p. 121). This way of working shows the abandonment of a completed pursuit and the examinee's recommencement with minor adjustments to their first attempt to reach the objective given in the problem text.

Figure 5.10 is a response to Question 11.1 (see Figure 5.3):

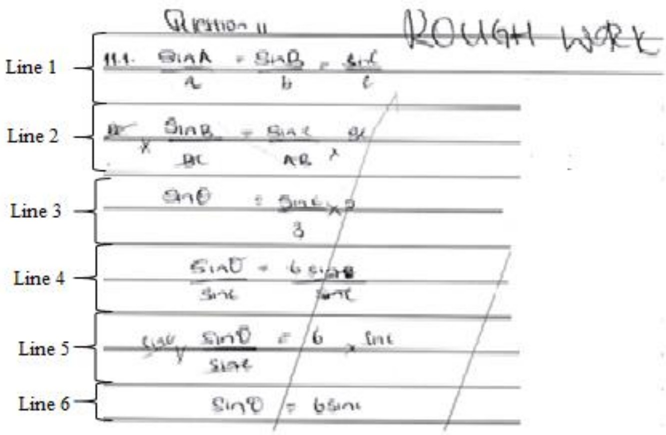


Figure 5.10: Convenience: Abandonment labelled as rough work

In line 1 of Figure 5.10, the examinee copies the formula in the inverse form as $\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$ from the information sheet. Line 2 of Figure 5.10 shows the values of a replaced by BC, b replaced by AB and c unchanged. In line 3 of Figure, 5.10 BC is substituted by 2 and AB by 3. In line 4 of Figure 5.10 the examinee produces $6sinC$ as a solution for $\frac{sinC}{3} \times \frac{2}{1}$. Both sides of the equation in line 4 are then divided by $sinC$, which renders $\frac{sin\theta}{sinC} = 6sinc.$ Further work in lines 5 Figure 5.10 shows the examinee multiplying both sides of the equation with $\frac{sinc}{1}$ to obtain a solution of $sin\theta = 6sinC$ in line 6. This way of working leads to the complete abandonment of the response and is labelled as rough work:

Figure 5.11: Convenience: New pursuit with minor adjustments

The new pursuit sees the examinee persist with the same formula. Line 1 of Figure 5.11 shows the examinee copying $\frac{sinA}{a} = \frac{sinB}{b}$ from line 2 of the previous response with an adjustment to the formula by excluding the $\frac{sinC}{c}$. The examinee continues in lines 3 of Figure 5.11 to multiply both sides of the equation with ‘b’. Further work line 4 of Figure 5.11 sees the examinee substitute 2 in the place of ‘a’ and 3 in place of ‘b’. Line 4 of Figure 5.11 shows $\frac{sinB}{b}$ replaced by $sin\theta$. The examinee leaves $6 = sin\theta$ as the final result.

5.6 Targeting

This way of working refers to the targeting of an objective in a solution-seeking pursuit to render a solution to what the problem text set. Multiple abandonment strategies indicate that agency was exerted on what was produced to get to the objective. The new pursuit showed similarities in terms of the said target and was left as the final result.

The problem in Figure 5.12 appeared in the examination paper. It presents Questions 8.2 and 8.2.1 which are the focus of discussion in the foregoing category:

8.2	Consider the expression: $\dfrac{1-\cos 2x-\sin x}{\sin 2x-\cos x}$	
	8.2.1 Prove that: $\dfrac{1-\cos 2x-\sin x}{\sin 2x-\cos x}=\tan x$	(4)

Figure 5.12: November 2012 NSC Mathematics Paper 2, Question 8.2

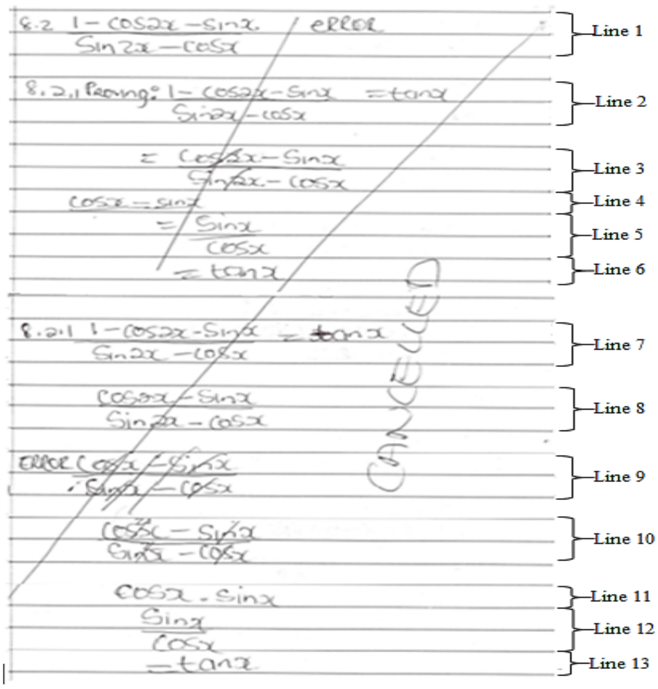


Figure 5.13: Abandonment showing multiple textures

In line, 1 of Figure 5.13 the examinee copied the expression which needs to be considered from the problem statement. Line 2 of Figure 5.13 shows that the examinee copied the problem text 8.2.1 onto the answer script. A scan of the entire script of Figure 5.13 and 5.14 indicates that the examinee has set the target to reach $\frac{\sin x}{\cos x}$ through simplification of the left-hand side.

Figure 5.14: The final Attempt

The new path followed in Figure 5.14 starts the same way as the two previous attempts by first recording the question. $1 - cos2x$ is replaced by cos^2x and $sin2x$ by sin^2x in line 2 of Figure 5.14. The quest to obtain $\frac{\sin x}{\cos x}$ is still exerting agency and cos^2x and sin^2x are cancelled as seen in line 2, because their cancellation will render the target which is sought.

5.7 U-turn

This way of working refers to the commencement of a pursuit to find a solution to a mathematical problem. The "U-turn shows that after obtaining the final answer the examinee abandoned some work, (went up) to a certain point in the pursuit, restarted because the result obtained did not comply with the dictates of the mathematical context" (Julie 2015, p. 483).

In category 5.7, the examinee works right through to the final answer and then abandons some of the work. Category 5.2 shows an early abandonment before the final result was reached.

Figure 5.15 is a response to Question 11.1. See Figure 5.3.

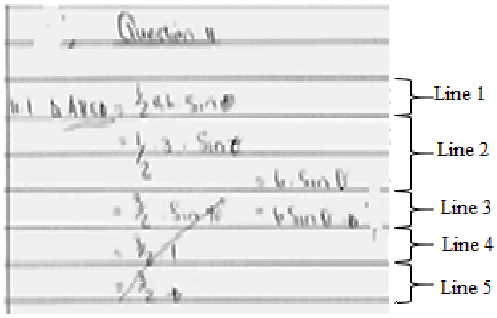


Figure 5.15: Examinee making a "U-turn" in solution-seeing pursuit

It is observable in line 1 of Figure 5.15 that the examinee first copies the formula for the area of a triangle from the formula sheet that forms part of the question paper. In line 4 ($\frac{3}{2}$ sin 90°), 90° was substituted for θ with 90° most probably drawn from the restriction $0° < \theta \leq 90°$ given in the problem text. In lines 4 and 5 of Figure 5.15, the simplifications are carried out to reach $\frac{3}{2}$ as the final answer indicated by the arrow (→), a popular indicator used in school mathematics to signify "this is my final answer". The work is abandoned by the drawing of a line through lines 6, 5 and 4. The work is restarted in line 3 and 6 sin θ is left as the final answer.

5.8 Conclusion

The analysis of the qualitative data that was collected was presented in this chapter. The analysis demonstrated the ways of working which occurred during the resolution pursuance in a time-restricted high-stakes examination setting. The analysis rendered 6 different categories. These categories as described are firstly, (1) Abandonment after a nearly complete attempt; secondly, (2) Abandonment of a solution, consisting of two sub-categories which are (2a) abandonment and the construction of a new problem by "proving the given," and (2b)

Abandonment and reconstruction of the same problem “proving the procedural objective”. The other categories are (3) Reversal, (4) Convenience, (5) Targeting and (6) U-turn. The next chapter will discuss the results of the analysis in terms of the research question.

CHAPTER SIX

DISCUSSION OF FINDINGS

6.1 Introduction

The purpose of the study was to investigate examinees' ways of working in the high-stakes NSC Mathematics Examination. It focussed on an ethnomethodological analysis to see whether it is possible to meaningfully analyse the production of the responses of examinees in the NSC mathematics examinations. The analysis was primarily also driven by the dialectic of resistance and accommodation which is triggered by the exertions of agencies referred to by Pickering (1995) as the dance of agencies. Perspectives arising from the analyses made visible the textures of examinees' mathematical work. From an ethnomethodological perspective the study provides insight into these textures produced by examinees' ways of working in relation to trigonometry. It shows the exploration of these textures, exposing how examinees navigate their need for solution seeking and discovery in pursuit of what was set out for them to accomplish in the examination.

The examination problem text was the NSC Mathematics Paper 2 that was written in 2012. The data was collected from a convenient sample from the 10 schools that participated in the LEDIMTALI based at the University of the Western Cape. The data collected from Paper 2 appeared in questions on trigonometry. The focus was to extract only those problems which consisted of visible textures of some sort of abandoned work. These textures were organised for the purpose of analysis to offer responses to the research question:

"What are the textures of ways of working exhibited by learners' work in Trigonometry in the Grade 12 high-stake examinations?"

This chapter discusses the results given in the previous chapter and explains these results by using the relevant constructs from ethnomethodology. In the first instance, the textures were identified and labelled based on the framework discussed in Chapter Three. The study revealed texture that was not part of the initial framework. The textures were initially grouped into categories. Table 6.1 gives a summary of the textures and the total percentages found in the examination scripts from Question 8 to 13. The textures in the data summary are represented as they were labelled in the data analysis chapter. From the 133 textures observed in 94 examination papers, Table 6.1 shows that the Reversal texture was most prominent in the examinees' ways of working, followed by the "Abandonment after a nearly completed attempt". "Targeting" and "Convenience" constituted the least number of textures in ways of working exhibited by examinees, amounting to only 0.8% and 1.5% respectively. "Abandonment of a solution" was divided into two sub-categories. These two categories are, Abandonment and constructing a new problem by "proving the given," and Abandonment and constructing the same problem "proving the procedural objective". The following table presents one total for both categories, thus showing it as Abandonment of a solution.

DATA SUMMARY FOR THE NUMBER OF TEXTURES FOR Q8 – Q13						
	CATEGORIES					
Description of textures	Abandonment after a nearly completed attempt	Abandonment of a solution: by construction of a new problem by "proving the given" / "constructing the same problem "proving the procedural objective"	Reversal	Convenience	U-turn	Targeting
Number of question papers selected	94	94	94	94	94	94
Total number of textures observed	133	133	133	133	133	133
Total number of textures per category	31	23	37	2	24	1
% per category	23.3	24.8	27.8	1.5	18	0.8

Table 6.1: The data summary of textures

The "Abandonment and the construction of a new problem by "proving the given" and "Abandonment and reconstruction of the same problem by "proving the procedural objective" are the two categories that were distinguished in terms of what the examinees tried to prove according to the mathematical context of the solution seeking pursuance. However, the number of categories found of the kind, "Abandonment of a solution" amounts to 24.8%.

The teasing out of the examinees' responses to the items (trigonometry problems) of high-stakes mathematics examinations rendered the textures. The textures show the way in which the examinees directed their pursuance to reach what they considered defensible answers to the examination questions. The ways of working detectable in the analysis showed a variety of abandonments, which were made visible through different kinds of textures. These textures of ways of working are illustrated in the analysis. What follows is a discussion of the results found.

6.2 Textures of ways of working

6.2.1 Abandonment after nearly completed attempt

This way of working showed some form of working almost to the point of the final answer, and then an early abandoning of the path of pursuit and a recommencement of the solution-seeking process having made adaptations to the previous path of pursuit.

The commencement of solution-seeking showed that the objective to be pursued was written down. This served as a kind of reminder of "what must be determined". The expansion of the compound angle formula showed a transcription error. The information sheet gave the compound angle formula in the form of $\sin(\alpha-\beta) = sin\alpha.cos\beta - cos\alpha.sin\beta$ whereas for the question $\sin(\beta-\alpha)$ was used. The angles used in the problem statement Figure 5.1, (8.1.4) were in different positions to those given in the information sheet. The examinee wrote $\sin\beta - \alpha$ in the compound angle formula such that $sin\beta - \alpha$ appeared on the right-hand side of the equation as illustrated here $[sin(\beta - \alpha) = sin\beta - \alpha.cos\alpha - cos\beta.sin\alpha]$ instead of only writing $sin\beta$. This was probably due to time pressure, which resulted in the examinee hastening to finish in time. Furthermore, the general notation is to write the "sin of an angle", where the value of angle "β" or "α" is to be seen as the size of an angle in degrees. In this case the examinee meant for $\frac{-17}{8}$ to be the value of $sin\beta$ in which case one does not rewrite the sin as was done in Figure 5.2 line 3. In terms of the negative in $\frac{-17}{8}$, the probable explanation was a faulty interpretation of the reduction formula where the examinee interpreted $\sin(180° - \beta)$ as a negative ratio. This was seen in line 3 of Figure 5.2 where the examinee wrote $sin\frac{-17}{8}$ for $\sin\beta$. In addition, there was no triggering that $-1 \leq sin\theta \leq 1$ since $\frac{-17}{8} < -1$. Thus having to evaluate the first function in the expansion formula triggered a resistance, line 3 of Figure 5.2 showed that the examinee did not continue with the substitution of values for the other ratios. It thus appeared that there was a realisation that "something must be wrong" and the solution pursuit was abandoned as indicated by the line drawn through the work.

Figure 6.1 represented the examinee's response to Question 8.1.1 and Figure 6.2 the one to Question 8.1.2. These figures are presented here because of the relevance to the final answers in Figure 5.2.

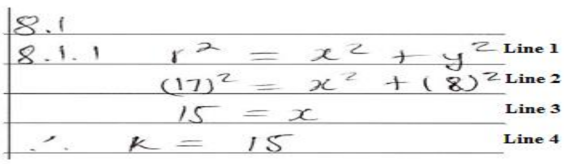


Figure 6.1: Question 8.1.1. Examinee determining the value of "k"

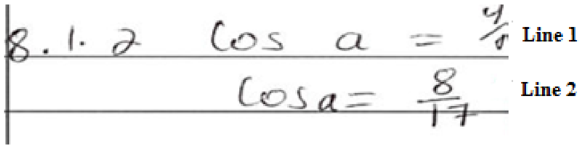


Figure 6.2: Question 8.1.2. Examinee capturing the value for $cos\alpha$

In line 4 in Figure 6.1 for the Question 8.1.1, the examinee had to determine the value of "k" $k = 15$. The 15 represented the $x - value$, which indicated the length of line OT. In the first instance, the examinee wrote $\frac{19}{8}$ indicated by line 5 of Figure 5.2 for $sin\beta$. However, the examinee first wrote sin in front of $\frac{19}{8}$ in line 5 of Figure 5.2 and then deleted it. This demonstrated the notion of reflexivity, the rationality of "this was tried before and it did not work". The hypotenuse was given as 17 and one can assume that the examinee made a mistake in the process of transferring the value for the hypotenuse from the question paper to the script. Furthermore, Question 8.1.3 stated, "If it is further given that $\alpha + \beta = 180°$, determine $cos\beta$". The examinee calculated $cos\beta\ to\ be\ \frac{-17}{8}$ which showed the examinee reversed the hypotenuse and the opposite side. One can deduce from the ways of working in line 2 of Figure 6.2 from the response-production that the ratios for $cos\beta\ and\ cos\ \alpha$ were triggered by $\alpha + \beta = 180°$. This can be viewed in the same way by the examinee as $sinA = \cos(90° - A)$. It can also be that by deducing $cos\beta$ in line 2 of Figure 5.2 in the second quadrant the examinee pursued the consequences of sine and cosine as "opposites". This was triggered in the sense of "hypotenuse and adjacent" sides requiring to be inverted for the $\beta = 180° - \alpha$ which was deduced from the Question 8.1.3 Figure 5.1, where it is given as $\alpha + \beta = 180°$. Although the way of working showed visible resistance such as the evaluation of the relationship ratio of side and angle of a right-angled triangle, there was no resistance observed and the examinee continued and did not resort to any sort of accommodation. For example, in line 5 of Figure 5.2 the examinee's work showed the following: [$sin\beta = \frac{19}{8}$ instead of $\frac{8}{17}$; $cos\alpha = \frac{8}{17}$ instead of $\frac{15}{17}$; and $\cos\beta = \frac{-17}{8}$ instead of + $\frac{15}{17}$]. Although the examinee determined the value for "k" in line 4 of Figure 5.3 correctly, it was never used when the functions of the formula in the side angle relation as illustrated in line 5 of Figure 5.2 were substituted. The examinee continued without any attempt to reconstruct the ways of working to accommodate the resistance. The process of writing an angle in relation to the side's ratio resulted in problems with the simplification of fractions consisting of two terms such as $\frac{19}{8} \cdot \frac{8}{17} - \frac{-17}{8} \cdot \frac{8}{17}$. In the first term in line 2 of Figure 5.2, the examinee dealt with $\frac{19}{8} . \frac{8}{17}$ as derived from the previous step. The 8's were cancelled to render $\frac{19}{17}$. In the second term in line 2 of Figure 5.2, the examinee overlooked the notion of multiplying with a negative. This forms part of the historicized mathematical knowledge; referring to a negative multiplied with a negative equals a positive, hence "-1" was obtained by solving the fraction $-\frac{-17}{8} . \frac{8}{17}$. Thus the subtraction sign was not taken into consideration and no resistance was triggered. This is probably a result of the pressure of time to get an answer, and that the context inhibited the firing of resistance.

6.2.2 Abandonment of a solution

This way of working refers to a complete attempt at solving a mathematical question and then the abandonment of that solution. Two kinds of abandonment of solutions emerged and these are discussed in the sub-section that follows.

6.2.2.1 Abandonment and the construction of a new problem by "proving the given"

This way of working showed a solution to a mathematical problem and then the abandonment of all the work, the construction of a new problem and the pursuit of a solution to this newly constructed problem.

Figure 5.3 represents a response to Question 11.1 as indicated in the problem text in Figure 4.3. The examinee had to prove that the area of the parallelogram ABCD is $6sin\theta$. In so doing the examinee had to use the information given in the problem text. In this case the objective is stated clearly and the examinee presents a proof in line 3 of Figure 4.4 showing the final answer by drawing an arrow (→) underneath it as indicated in Figure 4.4.

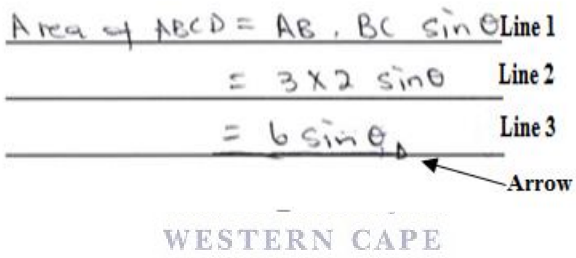


Figure 6.3: Response to Question 11.1

The pursuit commenced where the examinee put $3\sqrt{3} = \theta$. We can reasonably infer that this was merely writing down the "unknown- θ" and the "given-$3\sqrt{3}$". This is a common teaching sequence used by teachers. They normally ask learners to start writing down the unknown and what is given from the problem text. Learners normally write down "little reminders" from the problem text to organise their solution-seeking pursuit. This way of working can be linked to the mathematically historicized-self particularly with respect to instructions given to learners when solving problems such as "write down the unknown and what is given". Another probable reason is the examinee's understanding of the (=) sign, thus putting one representation of the area as equal to another representation of that same area. The $6sin\theta$ was a response to Question 11.1 from the previous pursuit in line 3 of Figure 5.4, as indicated in the problem text in Figure 5.3. Both sides are related to the area of the parallelogram. The $3\sqrt{3}$ represents the numerical square units of the area of the parallelogram ABCD and $6sin\theta$ is the trigonometric representation of the area of the parallelogram ABCD. Question 11.1 in Figure 5.3 asks the examinee to show that the area is $6sin\theta$. Both these representations were given as part of the problem text in Questions 11.1 and 11.2 in Figure 5.3. Line 2 of Figure 5.4 showed the examinee divided by 6 on both sides of the equation. The examinee had to make the connection that the value refers to the size of the angle in degrees. Rendering the $60°$ as the final answer is an example of the way learners are normally instructed to find an angle with the use of a calculator. This way of instruction is of the kind "press the sin^{-1} button". This way of working sees the mathematically historicized-self exerting agency in the path of getting to $60°$ as indicated by line 6 of Figure 5.4 and showing it as the final answer for that pursuit by drawing an arrow underneath it. This indicates that s/he did not accept $60°$ as the answer. The examinee then proceeded to construct a new problem in the form of "proving the given".

The newly-constructed problem shows a pattern of work which compensates for missing data. The way of working shows calculations derived from given information. The formula used by the examinee in line 1 of Figure 5.5 was given on the information sheet; the ways of working showed the examinee copied the formula omitting the $\frac{1}{2}$. The same ways of working can be seen in Figure 6.3, which shows a case of reflexivity. This is probably because the examinee took ABC as a parallelogram consisting of two congruent triangles without specifically stating it. This is a "normal way" of working under examination conditions, where incomplete things are written down while working with the complete thing. This way of working was in a sense enforced by the context of limited time for completion under which the examination was written.

Part of mathematics examination preparation for questions with multiple parts is that learners are instructed to use what is given as "true" in one part of the question to solve other parts. This is part of the examinees' mathematical historicized-self and forms part of the armoury with which the examinee enters the examination situation. This way of working is what Julie (2003) refers to as compensation, where a pattern of work emerges that compensates for missing data, which in this instance was a value for θ. The type of compensation relevant to this pursuit is assumptive-driven calculation, which Julie (2003) described as a way of working to replace missing data when other given or known information is adequate. This applies because the value of 60° in the abandoned attempt seemed adequate for replacing the missing data.

6.2.2.2 Abandonment and reconstruction of the same problem "proving the procedural objective"

This refers to the abandonment of a solution and starting over. The new solution-seeking path showed the reconstruction of the same problem with some adaptation and re-routing to do it "without the calculator".

This pursuit started where the examinee wrote down that which had to be determined without the use of a calculator. In line 2 of Figure 5.7 the examinee wrote down the "2." and in order to retain the intact state of the reformulated $cos^2 15°$, an opening bracket, "(", is written. The closing bracket, ")", is taken as the given closing after "-1". This exerted agency may be linked to the examinee's use of structural sense which enables them to make better use of previously learned techniques. The presence of the brackets helped examinees to "see" structure. A feature of using structural sense is "looking" before "doing" something that teachers commonly emphasize when reviewing previously learned work. The non-recognition of structural change in terms of the missing bracket might be caused by the exertion of “time to complete” and continuing to work without checking.

The navigation approach indicated that the examinee focused on the simplification of the “difficult” part “$2.(.(1 - sin^2 15° - 1)$” thus keeping $sin 104°$ intact. The pursuit in Figure 4.7 showed the examinee continued with $sin 104°$ to nearly the end of the pursuit before the final answer in line 6.

In line 4 of Figure 5.7 the ways of working showed the examinee obtained the number 225. The examinee probably got 225 from taking 15 squared. The $+sin^2 15°$ was possibly obtained by multiplying $-sin^2 15°$ with -1. The examinee then multiplied the expression 1+$sin^2 225°$ by 2, to obtain 2 + $2sin 225°$. This way of working saw the examinee use common algorithms to evaluate the expression $2.1 - sin^2 15 - 1$. This non-firing of resistance can be linked to the linear way examinees proceed due to time pressure, which did not allow time to “look back” when a solution path to follow had been decided. Rather, advice from teachers is that they have to “look over their work”. This means that examinees need to allow time for checking their answers for accuracy and completeness after they have finished the test. This is an activity they engage in after having found a solution. The way of working showed that the examinee continued with this line of reasoning. Line 3 of Figure 5.7 showed the substitution of the function $tan 38°$ in the denominator by its trigonometric identity, which was shown as $\frac{sin 38°}{cos 38°}$.

The examinee then used the complementary angle relationship $sinx = \cos(90° - x)$ to express $sin^2 52°$ as $-cos^2 38°$ which again exerted agency when the negative sign was put in front of cos. This is probable because the examinee used the sign of the co-ratio. Equivalently, in the clockwise direction, this angle has a $(-)$ measure. Further investigation saw that the examinee simplified the expression as shown in line 3 of Figure 5.7 by cancelling $-cos^2 38°\ and\ cos 38°$ in the numerator. The ratios that were left from the previous expression in line 3 of Figure 5.7 were presumably calculator-generated to obtain the 0,97 as indicated by line 4 of Figure 5.7. The examinee drew a line through the final result. The work continued to result in a calculator-generated final solution in line 6. The cancellation of the entire solution path was ostensibly triggered at this point by the problem texts "without the calculator". This is similar to Pickering's (1995) reasoning of Hamilton's construction of quaternions, finding resistance between an algebraic and geometric solution upon reaching the end of his pursuit. In Hamilton's pursuit, the resistance was that the algebraic and geometric solutions rendered different results and Hamilton's expectation was that the results would be the same. Hamilton knew the rules for algebra, so he started manipulating equations to see if the result would correspond with any sensible extrapolation of multiplying two vectors. This mathematical work took him through a series of manipulations within a conceptual system. The resistance was encountered when these manipulations did not produce an expected result (Pickering, 1995). When encountering resistance, meaning something is not working, you start thinking of "making a new extension" (Marick, 2004)

Pickering's (1995) interpretation of Hamilton's work sees accommodation as the path embarked on to resolve a resistance. Evidence of this is the examinee's way of dealing with $sin 104°$. The pursuance of solution-seeking shows a form of reflexivity, combining the "where it will take me now" and as put by Coulon (1995), the "rationality of what we doing", in building up meaning (p. 23) "without the calculator". The intention showed in the way of working is to produce a result "without the calculator". It is almost as if the mathematical work in the abandoned pursuit and the objective set by the problem text with its underlying conceptual condition, pre-determined the accounts of the new extension. This is shown in the handling of $sin104°$ to render $sin38°$. Line 5 of Figure 5.7 sees the $104°$ being accommodated for by $38°$ in line 4 of Figure 5.8 as a more suitable replacement, which was ultimately cancelled later in the pursuit, thus showing the extension to be a continuation of the conceptual conditions flowing from the abandoned work. This shows that, although the previous attempt was abandoned, some of the calculations were used without showing the steps; instead, they were copied into the new extension. The working towards cancellation is fuelled by the procedural instruction in the problem text, "without the calculator". The examinee cancelled $sin38\ and\ cos38$ in the numerator and denominator to obtain a more suitable outcome of $4sin^2 45°$ as indicated in line 5 of Figure 5.8, rendering $45°$ as a special angle which need not to be solved with the use of a calculator. This was shown when the examinee left $4sin^2 45°$ as the final result.

6.2.3 Reversal

"Reversal is where a calculated resistance was created and the learner removed the produced work by drawing a line through it and started over immediately from a certain point" (Julie 2003, p.121). The re-start was on the same solution-seeking path with a different strategy until a final result was reached.

After writing down the symbolic objective in line 1 of Figure 5.9, the examinee continued the solution-seeking pursuit. After making use of various trigonometric relationships such as side angle relationship of right-angled triangles, acute angle relationship and double angles, the examinee's pursuit rendered $(-sin52°).(-sin52°)$ in line 2 of Figure 5.9. However, the expression $(-sin52°).(-sin52°)$ exerted agency upon rendering the negative, $" - "$ signs of the expression in the brackets. The exertion of agency was where the examinee 'saw' sin as negative in the fourth quadrant. The $360°$ obtained in the evaluation process contributed to this because the normal course of teaching a relationship with $360°$ is associated with the fourth quadrant. In line 2 of Figure 5.9, $\cos 38°$ and $(-sin52°)$ was cancelled as indicated by the arrows pointing to cancelled parts of the fraction in line 2. It was clear from the deleted parts $cos38$ which appeared in the numerator of the expression $\frac{sin38}{cos38}$ and $(-sin52°)$ that the objective was to obtain the same ratio in terms of cos. This way of working was a case of reflexivity where things done before were referred to whilst working influence the future direction taken.

In line 4 of Figure 5.9 the pursuit rendered $cos14°$ and $cos52°$. Although the $cos14°$ and $cos52°$ flow from the previously produced account it did not comply with the chosen mathematical context to achieve an objective “without the calculator” this abandonment indicated the calculated resistance caused by the mathematical context in terms of the direction taken in the solution-seeking pursuit. Further work after the abandonment showed the examinee copied the $\cos(90° - 76°)$ and $cos30°$ in the exact form from the numerator in line 3 to the numerator in line 5 of Figure 5.9. The examinee moved back and followed a different strategy to fit the mathematical context. In the same instance, the examinee adapted $\cos(90 - 38)$ to $\cos(90 - 52)$ and then evaluated it as $-sin52$. This could be observed in line 5 of Figure 5.9, where it showed the examinee replaced $cos14°$ with $sin14°$ and $cos52°$ with $sin52°$.

The pursuance of solution-seeking shows the rationality of work from the abandoned pursuit that might still be appropriate in its exact form or that might need some reworking and can still lead to a desired result “without the calculator”.

The ways of working up to this point show the continued non-firing of resistance in terms of the retention of $\cos(90 - 76)$ rendering of $sin14°$. The evaluation of $\cos(90 - 76)$ in terms of the acute angle relation would have taken the pursuit back to line 2 ending up with $sin76$.

Although, $sin14°$ exerted agency in terms of the use of the co-function the examinee persisted with it up to the final result.

6.2.4 Convenience

"Convenience is where the examinee created a situation using a faulty method to simplify a mathematical problem to get to a suitable objective" (Julie 2003, p. 121). The way of working showed the abandonment of a completed pursuit and the examinee started from the beginning with minor adjustments to attempt to reach the objective given in the problem text.

The texture of ways of working emanating from the analysis started with the writing down of the formula to prove the area of the parallelogram equal to $6sin\theta$. The examinee's interpretation of the area of the parallelogram was triggered by the formula given in the formula sheet, which showed the ΔABC: $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$. The selection of this formula exerted agency in the form of a "convenience", which indicated the use of a faulty method in pursuance of a suitable solution proving the objective given in the problem text. The examinee 'saw' the diagram in Figure 5.3, which represented parallelogram ABCD, consisting of two triangles, which allowed the mathematical argument to prove its area, ABCD = $6sin\theta$. The concept of congruent and similar triangles made up the mathematical argument in relation to the information given in the text and the diagram in Figure 5.3, as well is formulae provided in the information sheet.

The work in line 2 of Figure 5.10 showed the examinee cross-multiplied BC with $\frac{sinB}{b}$ and $\frac{sinC}{c}$ $\left[\frac{BC}{1} \times \frac{sinA}{a} = \frac{sinB}{b} \times \frac{BC}{1}\right]$ and omitted $\frac{sinA}{a}$ probably because there are only two values given in the diagram ABCD in Figure 5.3. The examinee's way of working is towards a formulation where given values could be used. The examinee replaced $sinB$ with $sin\theta$ on the right side of the equation in line 3 of Figure 5.10. This is probably due to the position of θ in parallelogram ABCD in Figure 5.3. The diagram in Figure 5.3 shows θ as angle B, which is noticeable in the given problem text "$A\hat{B}C = \theta$". The work produced up to the final answer was abandoned and a new pursuit was started. The rough work was a completed solution pursuit. This was not in the form of helpful notes, scribblings or short reminders of what were important planned calculations. It was not of a kind common in the way of working in mathematics when learners do rough work in the margin of the page or in a space away from the original pursuit. In this instance, contributing to this form of labelling was the instruction given to examinees in the NSC examination, to do all rough work on the examination script.

Plausibly the examinee checked the final result in line 6 of Figure 5.10, $sin\theta = 6sinc$ against the problem text to "prove the area of parallelogram ABCD $= 6sin\theta$". The examinee found the final result not suitable in terms of the objective given in the problem text. The checking is part of the examinee's historicized mathematical knowledge. A common instruction given by teachers to learners and by invigilators to examinees is "make sure of your answer". It was this type of checking which triggered the resistance and caused the examinee to label the work as rough work and draw lines through it.

In the new pursuit the work showed some adjustments; however the examinee still created a situation using a method which does not serve the purpose of obtaining a suitable solution. You will perhaps have to make some statement in your framework of this construct so that you do not focus on 'wrong or correct' ways/ methods to solve the same mathematical problem, as indicated in Figure 5.11. The continuation with this way of working showed the non-firing of resistance in terms of "convenience". This could probably be due to the representation of sin in the sine rule formula as shown in the information sheet and the sin in $6sin\theta$ given in the problem text. The exclusion of $\frac{sinC}{c}$ was to accommodate for final result in line 6 of Figure 5.10 in the abandoned pursuit. The adjustment of the formula probably exerted the expectation that the pursuit would render something closer to the objective. The examinee's way of working – of replicating and adjusting work (see from Figure 5.10 as used in Figure 5.11) – shows reflexivity. Although the work in Figure 5.10 was labelled as rough work, the examinee saw this work as part of the evidence of a solution-seeking pursuit to solve a mathematical problem. However, it also gives an indication that the examinee saw this work as useful and as a relevant attempt to do the actual work that would produce a suitable solution.

The examinee's ways of working in line 6 of Figure 5.11 start out with the $" \therefore "$, which means "therefore" showing that what follows is derived from the previous work. This notion of making $" \therefore "$ is a pursuance which shows procedural fluency, giving reason and meaning to the path that was travelled so far. However the pursuit re-routed to a totally different path by taking the given and constructing a final answer. The rerouted path shows the examinee writes $AB \times BC = sin\theta$ and substitutes $AB \times BC$ with 3 and 2 respectively. The examinee multiplies "3×2" and leaves the outcome as $6 = sin\theta$ in line 7 of Figure 5.11. In this instance, the problem text probably exerted agency, which brought the examinee's work closer to the given objective stated in the problem text.

6.2.5 Targeting

This way of working refers to the targeting of an objective in a solution-seeking pursuit to render a solution to what the problem text set. Multiple abandonment strategies indicate that agency was exerted on what was produced to get to the objective. The new pursuit showed similarities in terms of the said target, and was left as the final result.

The pursuit commencing with the quest led to something akin to "I must get rid of the 1 in the numerator" of line 2 of Figure 5.13. This "get rid of" is a common colloquial form used in school mathematics. I infer that the mathematically historicized-self in terms of this common colloquial form exerted its agency which rendered the line 3 of Figure 5.13 to be written with the "1" being omitted. The cancellations done in line 3 of Figure 5.13 are a familiar misconception of cancelling by regarding the numerators and denominators of an algebraic fraction as monomials when they are not. This 'method' is applies to the 2's of sin 2*x* and cos 2*x*. By now declaring cos *x* = sin *x* and ostensibly by substitution the examinee arrives at $\frac{\sin x}{\cos x}$ of the intended target showing in line 5 of Figure 5.13. For some reason, to which outside readers have no access, this is deemed not correct, the work is scratched out and "error" added, indicating that the way of pursuit is abandoned.

The next attempt starts at line 7 of Figure 5.13 where the problem, 8.2.1 is written again. In line 8 of Figure 5.13 the "get rid of 1" is applied again and in line 9 the same cancellation in line 10 of Figure 5.13 as was done before is executed, rendering $\frac{\cos \mathrm{x} - \sin x}{\sin \mathrm{x} - \cos x}$. This is deleted and "error" written next to it. This is a case of reflexivity of which it is said, "In the course of our ordinary activities… we are building up…the meaning, the order, and the rationality of what we are doing" (Coulon, 1995, p. 23). The "rationality" here is of the form "this road has been travelled before and did not lead to the desired target". This spurs the selection of a different path of pursuit in the replacement of $\cos 2x$ by $cos^2 x$ in line 11 of Figure 5.13. Part of the context is the formula sheet that is part of the question paper and the formulae given for double angles for the cosine function given as $\cos 2\alpha = \begin{cases} cos^2\alpha - sin^2\alpha \\ 1 - 2sin^2\alpha \\ 2\,cos^2\alpha - 1 \end{cases}$.

I infer that this object of the context was used but the second part, $sin^2\alpha$, was omitted due to the attempt to obtain the set objective to "work towards the target $\frac{\sin x}{\cos x}$". In the same vein and using the supplied formulae, $sin2x$ is replaced by $sin^2 x$, the other term of $cos2\alpha = cos^2\alpha - sin^2\alpha$. The same erroneous cancellations are performed in line 10 of Figure 5.13

and a further attempt is made to obtain $\frac{\sin x}{\cos x}$. It appears that there is a search for a relationship between cos x and sin x by recording cos x - sin x. The entire effort is abandoned by scratching out all the work as seen in Figure 5.13 where the word "cancelled" appears.

6.2.6 U-turn

This way of working refers to the abandonment after a completed attempt. The "U-turn shows that, after obtaining the final answer the examinee abandons some work, (goes) to a certain point in the pursuit, restarts because the result obtained does not comply with the dictates of the mathematical context" (Julie, 2015, p. 483).

In terms of the mathematically historicized-self the experience of the examinee with area and trigonometry is linked to the area of the triangle. The figure re-inforces this because it closely resembles the figure used when the area of a triangle is being dealt with in textbooks and during teaching. In line 2 of Figure 5.15 the data for the sides is substituted. Lacking access to any further information, the best plausible explanation is that 2 were multiplied by 3 and the division by 2 was carried out. However, the $\frac{1}{2}$ was retained probably due to the rush of the moment. There are a total of 13 questions and the linear fashion with which most examinees navigate examinations meant that this question was attempted close to the moment of expired time.

What is noticeable is that a line with an arrowhead is drawn across the last 3 lines from the lower left side to the right side of lines 2 and 3 as indicated by Figure 5.15. It is plausible that the examinee worked solely with the answer text after line 3 of Figure 5.15 and upon the declaration of the answer $\frac{3}{2}$ there was a return to the problem text, which indicates that the target had to be 6 sin θ. This meant that what Pickering (1995) defines as a resistance was encountered. This causes the examinee to make a reversal defined by Julie (2015) as working a problem through to its conclusion and when encountering resistance reverting to some prior point and making a fresh attempt from that point onwards. The new attempt starts at line 2 of Figure 5.15. A probable explanation is that the examinee realized that the figure consists of two congruent triangles, a property of parallelograms which forms part of examinee's entire corpus of mathematical experiences (the mathematically historicized examinee). With this U-turn the examinee arrives at the correct answer although nowhere on the answer text is there any indication of the congruency.

6.3 Solution-seeking pursuance in a high-stakes examination

The textures of ways of working highlighted above were produced through the dialectic resistance and accommodation. In most cases resistance occurred when the examinee made use of trigonometric relationships and a selection of formulae from the information sheet.

The pattern of work that emerged from the analysis is of a kind in which agency is exerted upon the examinees' path of pursuance of a solution to a mathematical problem. "Pattern" has to be understood as something that is accountable, that is, reportable-observable-describable, something that refers to a meaning and therefore to a process of interpretation (Signorini, 1985). The foregoing discussion indicates that the ways of working and the exertions of agency led to a resistance in the solution-seeking path which was then accommodated for. In some cases agency was exerted and the non-firing of resistance occurred.

6.4 Non-firing of resistance

The dialectic of resistance and accommodation explained by Pickering (1995) where the resistance denotes the failure to achieve an intended outcome is captured by the agency in practice. Accommodation is an active human strategy of response to a resistance, which can include concepts like adaptations, inclusions, exclusions, etc. This happens when there is a “looking back or revision of goals and intentions. When the resistance goes unnoticed then this is a case of non-firing of resistance.

Roth, (2007) understands agency in a dialectic relationship with structure and as a dynamic competence of human activity, in this instance examinees who act independently to make choices. Sometimes the choices are conscious, however sometimes we act as agents unaware of our options. This appeared to be the case from the results of the analysis. Agency is not just individual; it is exercised within social practices (Cohen, 1994) making the high-stakes examination one such structure mentioned by Roth (2007), and this determines the examinees' relationship as social beings acting in situ (Garfinkel 1967). According to Durkheim (1952), every social structure consists of its own unique social facts. Garfinkel's (1967) idea of social facts is something that is local and indigenous. Garfinkel (1967) further explains these social facts as reflexive accountable ongoing practical achievements (p.11). This phenomenon is noticeable in the examinees' ways of working when there is no action taken upon the exertion of agency. Hence, the non-firing of resistance caused by the social facts, which exerts agency, inhibits these processes, which in most cases leads to not rendering a suitable practical achievement.

However, we can ask the question, “What are these social facts?” and “In which way (do they) cause the examinee not to act on the exertion of agency?” Garfinkel (1991) sees the notion of social facts as a coordinated assembly of people self-organizing and self-discovering in a local natural setting. Through this objective ethnomethodology tries to catch the defining features, the “just what-ness” of mundane activities that make them what they are; that is, what methods, means and procedures examinees use in the activity they each perform (Francis and Hester, 2004). From a sociological perspective the argument of non-firing of resistance impacted by social facts may be extended. The question asked by traditional sociology of knowledge, is “how” and “to what extent” do the social factors that constitute your environment influence the product of the mind (Steven 1995, p. 289). In the same instance Steven (1995, p. 289) argues that the sociology of scientific knowledge seeks to show that knowledge is constitutively social. Durkheim (1952) argues that this (pressure) commented upon in the discussion may be identified as social facts driven by the causal nature of the social structure (high-stakes examination). It can therefore be deduced that the non-firing of resistance is caused by it. Research conducted by Schroeder (2006) in America accords with these views. According to the study it was found that the underlying pressure on teachers and students to perform well affected classroom behaviours and social interaction between students and teachers. The study showed that it had a particularly negative impact on children from disadvantaged groups and children with disabilities.

These social facts are restrictions to human actions that transcend time and space according to Pickering (1995). The social facts exerting upon individuals are not material but reside in the prestige of power. They are mental in nature and are a representation of the set of rules that determine behaviour. In other words social reality constituted by institutionalized ways of acting and thinking according to Glasër and Laubel (2004) are forms of social currents. This links up with the Durkheim's (1982) argument in terms of the identification of social facts through establishing whether or not they are sanctioned. If the context within which individuals (examinees) act acknowledges whether or not individuals behave in established ways according to institutional injunctions, and if it rewards or punishes according to individual compliance, then we can be sure that we have identified a social fact.

The NSC examination is the most externally orientated examination written by Grade 12 learners in South Africa. The NSC examination is set externally, moderated externally, and marked externally (Department of Basic Education, 2008) which increases the pressure on examinees. The examinees use this examination to gain entrance to tertiary education opportunities to further their careers. A pass in this examination serves as a benchmark for finding a job to sustain an income. The context of the examination setting expects of examinees to behave in a particular way to be in compliance with the rules and regulations set by the DBE. In Chapter Three these rules and regulations are noted. This is in line with the notion of institutional injunction referred to by Durkheim (1952). Furthermore, if examinees do not adhere to these rules and regulations set out for them, if there is an irregularity in the process they encounter, they may lose the marks for that particular subject and ultimately fail the examination (Government Gazette, Vol. 522, No.31680 dated 12 December 2008). This invokes two very important phrases used by Garfinkel (1967) when he explains social fact as "the objective reality of social facts are "in that" and "just how", where the "in that" refers to the situation or setting (high-stakes examination) and the "just how" refers to the ways of working by which examinees make sense of their world. Watson and Mason (2006, p. 91-111) warn that there are no guarantees no matter how carefully an exercise is structured because many other relevant factors influence the situation.

The important feature of the high-stakes NSC examination setting is the prominence of time. The time given to complete the examination paper is indicated on the front page of the question paper. Examinees are allowed 10 minutes reading time which they use to acquaint themselves with the content of the paper. During these 10 minutes, they are not allowed to write anything. The clock in the examination room will indicate to them when to start writing as well as when to stop. According to the rules examinees are not allowed to write outside of the given time indicated on the question paper. This visual time keeping in the examination room is part of the evaluation requirements which are checked when district officials do their evaluation rounds. In the process of writing the examinees are always aware of the time because of the visual emphasis it is given in the examination room. All these injunctions are what the examinees deal with when approaching and writing the NSC high-stakes examination hence the "pressure" that is exerted on them. Regarding time strictures, it can be deduced from the analysis that the way of working has a lot to do with finishing the examination paper in the given time. The pressure of finishing in time and completing all or most of the questions is a challenge confronting the examinees and exerts pressure on them.

Earlier I mentioned how important it is for the examinees to perform well in the high-stakes NSC examination. Stiggins (1999) speaks of the pressure which testing puts on students to do well which, when not accompanied by adequate support, can lead to anxiety and a sense of futility. The negative impact of the pressure on individual students to perform well in the high-stakes-testing regime according to Senate References Committee on Education, Employment & Workplace Relations (2010) is a worrying factor. Durkheim (1982) is of the opinion that social facts are identifiable through the power of external coercion which is capable of exerting agency upon the individual. The presence of this power as Durkheim (1982, p. 56) called it, is recognisable because of the existence of some pre-determined sanction or through the resistance that the fact imposes on any individual. Garfinkel (1967) sees this as an ongoing practical achievement with no time-outs.

From all the issues relating to social facts and social structures and their influence on the individual acting in situ, one can infer that the reality of the non-firing of resistance is a causal factor of social structures from which social facts exert agency on the individual participating in a time-restricted high-stakes examination such as the NSC examination.

Learners learn the properties of quadrilaterals from Grade 6, dealing with elementary outlined features of the two-dimensional figure and moving to the senior phase in Grade 9 where they deal with more advanced properties of two and three-dimensional quadrilaterals. Because the Grade 12 curriculum does not allow teachers to venture far back in their teaching this is accepted as existing knowledge. In their common approach to teaching teachers may frequently be heard to comment, “This you did in grade...” reminding learners’ of what they are supposed to know.

The examinee chose the formula for similarity triangles to prove the statement in the problem text. This was more a case of understanding the concepts of similarity and congruency. Both formulae that dealt with triangles showed the representation of the function sin. Hence, the examinee saw the parallelogram consisting of two similar triangles and chose to persist with it. Another possible explanation for this is the fact that similarity triangles are part of the Grade 12 geometry syllabuses and congruency was taught in Grade 9. Therefore, the examinee acts upon the most ‘recent’ knowledge of Grade 12 and uses that as more appropriate for solving the problem.

Furthermore, from the analysis, it could be seen that the examinees were exposed to trigonometric relationships, but was more a case of not understanding the concepts. From this I infer that learning occurred by rote. It was clear that the examinees could not make the link with previous knowledge, and as a result, the knowledge became isolated and was difficult to recall. Such learning (by rote) is the cause of many exertions of agency as the examinees recall only partially-remembered and distorted rules (Oliver 1989). In building the mathematical context, the analysis showed the triggering of hinds, short reminders and distorted knowledge or rules as put by Oliver (1989), which led to a new strategy in pursuit of a solution. The time-restrictedness of the high-stakes examination made it difficult for the examinees to recall previous work or work that has been prepared (studied) for the purpose of the examination. However, examinees reacted on these hinds and abandoned the produced work. In these instances, it was the mathematical context which exerted agency on the examinee.

These ways of working influenced the time it took the examinees to complete a problem. All the textures required of the examinee to redo parts or to complete the problem thus leaving less time to spend on other problems. Table 5.2 gives a summary of the questions that were not attempted, referred to as blanks. There are many reasons for these phenomenona. They include not having enough time to complete the examination (abandonment and restart); having not known the answer to a question, or it could be speculated that the examinee left the question blank and decided to come back to it but ran out of time, which could also point to time spent on one problem. The fact is that one cannot argue otherwise when one looks at the textures of ways of working. Working through a problem to the final answer, abandoning the approach or attempt, then restarting is time-consuming, hence this puts more pressure on the examinee, which ultimately leads to a case of not completing the paper.

BLANKS PER QUESTION SUMMARY																				
	Q8						**Q9**		**Q10**				Q11			**Q12**			**Q13**	
Sub-question (Trigo-nometry)	8.1.1	8.1.2	8.1.3	8.1.4	8.2.1	8.1.2	9.1	9.2	10.1	10.2	10.3	10.4	11.1	11.2	11.3	12.1	12.2	12.3	13.1	13.2
No. of question papers	94	94	94	94	94	94	94	94	94	94	94	94	94	94	94	94	94	94	94	94
No. of blanks	6	3	10	14	2	14	2	1	6	16	17	26	12	28	44	24	39	29	21	50

Table 6.2: Blanks per question

The graph in Figure 5.4 below gives an indication of the increase of blanks from Question 8 to Question 13.

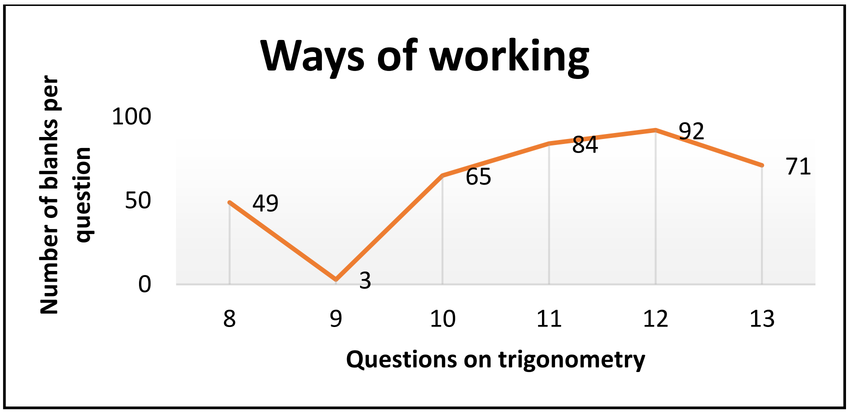


Figure 6.4: Graph illustrating the ways of working in terms of blanks

The strategies and tactics are highly driven by the time-restricted context within which the high-stakes examination is conducted. Hence the tactics took on various characteristics and did not show exact copies of textures although the way of using it in the pursuit rendered similarities in the purpose of its use.

6.5 Reflexivity

The analysis showed how the examinees decided on the way forward by recognising work produced that did not comply with the solution-seeking pursuance and from a chosen objective. This way of working was shown in the continued pursuit of work; in lines 3 and 4 of Figure 5.13 work was abandoned (scratched out) upon the examinee having obtained the $cos14°$ and $cos52°$ in line 4. The reflexive way of working in this instance was controlled by the mathematical context which was to find a solution "without the calculator". Reflexivity in this instance is inherent in the ways of working which showed how examinees employ the mathematical procedures and concepts. Pollner (1991) sees reflexivity as endogenous, meaning how things are done, in this instance by the examinees – what they do in, to, and about the social reality. The analysis shows the examinee moved back and followed a different strategy to fit the mathematical context and in the process abandoned the produced work to choose a direction to fit the solution-seeking pursuit.

The relevance of the abandoned work and the reflexive account of pursuance in a time-restricted high-stakes examination such as the NSC examination, emphasizes the notion of the "lived" work and proof-account, which according to Livingston (1986, p.111) is irremediably tied. Reflexivity incorporates the realisation of mutual circularity of account processes and the social reality they construe (Garfinkel 1967, p.8). It can be argued that the examinees conformed to customary patterns of interaction such as practical rationality. The reflexive accounts produced by examinees are not only a way to achieve a solution to a mathematical problem but also a way of adapting to the time-restricted context of the examination. Ramsden, (1998, p. 48) sees that the idea that such action be recognized as intentional will rest upon those presuppositions said to constitute "shared social knowledge". According to Goodwin (2000) reflexivity within this social context (e.g. time-restricted high-stakes mathematics examination) means the inseparability of a "theory" of representation from the social contexts in which mathematical representations are composed and used. The way in which examinees experience the high-stakes examination setting renders the setting a constituent feature of its organization, which may be observed through the textures produced from it.

The notion of reflexivity and rationality in actions brings to the fore an understanding of social experience of the world or the local setting. In this instance and relevant to the context of mathematics production is the experience of school mathematics. Many forms of reflexivity were displayed in the examinees' ways of working in the build-up to the final result. In this instance reflexivity is used as a practice which describes and constitutes a social framework of the high-stakes time-restricted examination. Reflexivity used as a feature of social action presumes the conditions of production in the solution-seeking pursuance and at the same time makes the action observable through textures of ways of working. From this I infer that social action is reflexive and it emphasises the fact that the accountability of an activity such as writing a high-stakes examination has constitutive elements. Furthermore, from the discussion and as it is put by Coulon (1995), “the rationality of what we doing” shows that social action is rational. Thus to say social action is rational emphasises the fact that it is systematically produced in situ. Activities such as these are intelligible, and can be described and evaluated. The features of reflexivity that became observable from the examinees’ ways of working are highlighted here:

- Case of reflexivity 1: refers to looking back at the abandoned pursuits and re-use of the work in the exact form
- Case of reflexivity 2: refers to the use of work from abandoned pursuits and a reconceptualization of it in the new pursuit
- Case of reflexivity 3: refers to the ways of working by selecting certain accounts of the procedural work from what was abandoned, skipping the intermediate steps
- Case of reflexivity 4: refers to the rationality of produced work that was tried before, which did not suit the chosen mathematical context; these accounts of mathematical work are not used in the repeated attempt
- Case of reflexivity 5: refers to adapting work from the abandoned pursuit to use it in the new pursuit.

Other than the reflexive notion of building up meaning, this has shown that the examinee enters the examination room with the accumulated mathematical experience from years of schooling. The reflection on past mathematical classroom experience is what is referred to as the mathematically historicized-self.

6.6 Mathematically historicized-self

A mathematically historicized-self retains all school mathematics that the examinee was exposed to in a school context in all forms. The examinees take into account that which is known in terms of ways of doing mathematics in a certain routine way, a set of skills which were developed over the years of schooling in the subject. From Grade 4, the examinee was exposed to at least two high-stakes examinations: a mid-year exam as well as the final examination at the end of each year. The mid-year examination acts as a measure for determining the progress that has been made in terms of attainment of the content that was taught, whereas the final examination is purely a promotion examination, to secure or to halt promotion to the next grade or phase. However, these examinations are known as school-based assessments and are somewhat different from the NSC examination. It is thus understandable that the examinee enters the examination room not only with the knowledge that he/she will be tested on, in this case trigonometry, but rather with all school mathematical knowledge known to him/her.

The analysis has shown the mathematically historicized-self comes into play in the form of little reminders which examinees use to build up their understanding of the mathematical problems put to them to solve. These little reminders present in the analysis are captured in the list that follows:

- checking of work – a common instruction given to examinees by invigilators in a high-stakes examination, of the kind, “make sure you check your answers”; see line 6 of Figure 5.14
- a common colloquial notion: I must “get rid of” – which can be seen in line 2 of Figure 5.21

- mathematical experience concerning properties of quadrilaterals and congruency triangles, which is acquired in the early grades as part of geometry; see line 2 of Figure 5.21
- a common classroom instruction given to learners when solving mathematical problems is to "write down the unknown and what is given"; an indication of this in the analysis can be seen in line 1 of Figure 5.7
- making use of common algorithms by multiplying integers of the kind "$(-) \times (-) = (+)$"; evidence of this can be found in the analysis in line 2 of Figure 5.2.

The capacity to discern in any situation is dependent on a disposition to engage with mathematics, prior experience of mathematical practices, current conceptualizations, social and affective aspects of the situation, and much more (Watson and Mason 2006, p. 91-111). Because of the intensity of the NSC examination being set, moderated and marked externally it is common practice for learners to go through a demanding study program in preparation for the NSC examination, which can be seen as the order of things in the run-up to the final NSC examination. This preparation as discussed in Chapter Two is concentrated on past NSC question papers, examination guidelines put forward by the DBE, revision classes and tutoring sessions as well as other study initiatives. This is almost like a closed process which allows learners to take in as much information as they can about the specific subject matter. In the process of preparing for the trigonometry examination, the mathematically historicized-self exists and is unavoidably part of the preparation but goes unnoticed. However when confronted with mathematical problems in the high-stakes examination context the examinees start making the prepared knowledge explicit as they do with the mathematically historicized-self.

Schoenfeld et. al. (1993) proffers the following definition of a mathematical problem:

> For any student a mathematical problem is a task (a) in which the learner is interested and engaged and for which he wished to obtain a resolution, and (b) for which the learner does not have a readily accessible mathematical means by which to achieve that resolution (quoted in Carlsen , 2008, p. 28; Bjuland, 2002, p. 9).

Taken from this perspective it can be inferred that it is in such situations where the mathematically historicized-self comes into existence and is just looking for a gap in which to assert or insert him-/herself. In this regard, the mathematically historicized-self exerts agency and is acted upon by human agency, the examinee. This is merely triggered by the mathematical problem the examinees are confronted with. Holland, Lachicotte, Skinner, & Cain (2003, p. 279) are of the opinion that “agency lies in the improvisations that people create in response to particular situations”.

6.7 Conclusion

This chapter presented a discussion of the results. The discussion was done within the framework of ethnomethodology. The enthomethodological procedures and constructs underpinning the discussion were used to answer the research question. These underpinnings included the following: (1) that the objective structures are rooted in social practices; (2) that social conduct is responsive to background activities or rules; (3) that social activities are practical accomplishments and (4) that social fact influences social activities. From the discussion the examinees’ way of working have shown rationality in the production of mathematics. This rationality is an observable feature of the solution-seeking pursuance. The discussion explains this through the reflexive way in which examinees navigate their solution-seeking pursuance. This makes the textures of ways of working in a high-stakes examination an activity of rational action. The discussion showed that the actions taken by examinees were controlled by the exertion of agencies, which were or were not acted upon.

One such feature of agency exertion was the examinees' historicized mathematical knowledge. The discussion also concluded that if no action was taken upon the exertion of agency then the non-firing of resistance occurred. One causal feature which brought on the non-firing of resistance was the time pressure under which examinees wrote the examination. The next chapter will show how practices such as those displayed in the foregoing discussion have implications for the teaching and learning of mathematics. Suggestions for further research are offered.

CHAPTER SEVEN

CONCLUSION

7.1 Introduction

In this chapter the main findings with regard to the research question are summarized and general conclusions based on the findings of the study are described. Furthermore, the strengths and limitations of the thesis are considered and suggestions for further research into examinees' ways of working in a time-restricted high-stakes examination are presented. This chapter concludes with recommendations relating to the teaching and learning of mathematics, particularly as these processes pertain to high-stakes examinations.

7.2 Summary of findings

The research question posed in Chapter One was as follows: "What are the textures of ways of working exhibited by learners' work in trigonometry in the Grade 12 high-stakes examinations?"

The aims of the research were to explore the textures of the learners' ways of working with mathematics in a high-stakes examination. Exploring these aims revealed how learners grapple with the mathematical problems set out for them to solve in the NSC Mathematics Examination. Thus pursuing the objective of interpreting and analysing the procedures examinees employ to produce responses in a high-stakes mathematics examination setting as presented in Chapter Four, brought to the fore the textures of abandonment. Within these textures of abandonment other tactics and strategies such as U-turn, reversal, convenience and targeting was exhibited in the ways of working adopted by examinees. The exploration of these textures of abandonment displayed by examinees' ways of working with trigonometry brought into focus the research question posed in Chapter One. It was found that the strategies and tactics used by examinees are highly driven by the context within which the high-stakes examination is situated. Furthermore, making visible ways of working is a common practice in a community of mathematicians. It is not strange to find that mathematicians discuss with one another their ways of working in their endeavours to solve problems they are engaged in. They will discuss their false starts, their blockages, strategies they have used to get out of dead-ends and hints they got from others that advanced their thinking to resolve dilemmas they face (Julie 2015). The important issue arising from this research study was that the ways of working of the examinees emulate the ways or strategies mathematicians use to make sense of mathematical problems they confront. According to Floyd (1981, p.1) mathematicians "will go up many a blind alley, following up hunches". She asserts that it is unlikely that mathematicians will proceed "unerringly" to any sort of solution without a single digression. It is my view that these types of mathematical discoveries and such flawed pursuance to solve mathematical problems are what the textures of ways of working by examinees in a time-restricted high-stakes examination represent and thus illustrate that examinees exhibit the same kinds of ways of working that may be found in the practice of mature mathematicians.

These practices remind us of things we know and things we normally do not notice. Investigating social phenomena from an ethnomethodological perspective enhances our understanding of everyday situations, which includes writing and responding to questions in a high-stakes examination (Silverman 1998, p. 24-31). However, this investigation was not without certain limitations. What follows is a discussion of the limitations which delineated the parameters under which the study was done as well as the validity and reliability of the methodology that was used.

7.3 Limitations

Limitations are those issues over which the researcher has no control and which limit the breadth of the study (Simon & Goes 2013). According to Wierma (2000, p. 211) limitations in qualitative research tell us more about the reliability and validity of the study, the size of the sample and the data collection method. This study presented some limitations insofar as these aspects are concerned, such as the use of a convenient sample which included only the schools that formed part of the LEDIMTALI, as stated in Chapter Three. The socio-economic contexts of the schools were more or less similar. These schools are all in a 25km radius of the higher education institution where the project was housed, which was one of the criteria stipulated by the funders of the project. Therefore, the adequacy of the sample is not representative of all schools in the South African context. Readers should therefore approach the results and findings with caution and not regard them as generalizable. According to Rosenthal and Rosnow (1991) an understanding of results from non-random and cross-sectional data should be restricted to the groups studied at the time of the research. The limited heterogeneity in the schools' demographic characteristics affected the nature and extent of the results and findings. The responses that were investigated dealt only with the Grade 12 NSC Mathematics Examination Paper 2 written in 2012. The analysis of ways of working was applied to questions on trigonometry only. The findings might differ in other topics of the Grade 12 mathematics curriculum. With regard to the foundations of this study and according to the research problem stated in Chapter One, not much literature was found to adequately conceptualise the scope of the research topic.

The limitations of applying ethnomethodology to a text revolve around the lack of observation by the researcher to the lived experiences of the social activities. The examination scripts are a product of the mathematics produced in the NSC high-stakes examination.

Without observing the social dynamics of such inquiry, the researcher's depiction of social practices may be inconsistent with the lived experiences. However, the documentary method of interpretation played a pivotal role in the textual explanation. The evidence was completed inductively linking it to a presupposed underlying pattern.

It is also the case that the data collected for this study was constitutively artificial compared to naturally occurring data. However Silverman (1985, p.156) is of the opinion that neither of the two kinds of data can be better than the other; each is reliant on the method of analysis. From an ethnomethodological perspective the setting is uniquely linked to practical accomplishments, which in this instance was the time-restricted high-stakes NSC examination, thus making the textures of ways of working, an inseparable part of the social setting.

7.4 Methodological reflections

Concerning the relationship between theory and practice, the relevant considerations to the field of mathematics education and classroom practice have not been explicated in the study. The methodological reflections form a critical part of the research conducted and are vital to the evaluation of the investigation and to the field of mathematics education research. However this research presented the reflections in an implicit way. It was therefore important to necessitate mathematics education research to relate the complexity of the research objective. This study provided an opportunity to expand the practice of ethnomethodology as applied to mathematics education research, particularly assessment and the type of feedback it produces. This was evident in the discussion in Chapters Four and Five, whereas other comparative studies such as the diagnostic reports of the Department of Basic Education 2012-2015 have been linked to errors and misconceptions only. Furthermore, this study reflected on the interplay between the written responses of examinees' (ways of working) and the setting in which these were produced.

Another important aspect of the methodological reflection entailed the process by which the research was to relate to its purpose. The purpose was to make explicit the textures of ways of working in a time-restricted high-stakes examination. This may be deduced from the sociological and ethnomethodological reflections on the mathematical responses as well as conditions under which these took place. The use of the documentary method of analysis and the notion of the "lived work" and the proof account of the work (Livingstone, 1986), gave the investigation empirical meaning. This formed a critical part of the research and in a big way the purpose has been fulfilled through the concept of ethnomethodological constructs such as reflexivity, social facts, mathematically historicized-self and ethnomethodological indifference. The empirical feature of ethnomethodology establishes the text in situ, giving it a different approach to the responses of examinees' work. Critical theorists have reacted upon the notion of the susceptibility of readers to naturalize texts by finding common-sense naturalistic frameworks with which to interpret them, which according to these theorists do not relate to the traditional norms of realism (Culler, 1975). However, Garfinkel's (1967) early work on the reading of clinical notes in order to understand the clinical routine of patients demonstrates that clinical personnel could find evidence of clinical work in the notes. This was done by taking the clinical notes as they were presented. An ethnomethodological understanding of social facts sees the responses as a reflexive accountable feature of the time-restricted high-stakes examination thus, emphasizing the importance of the causal influences of the context on the mathematics that is produced. It further allows the researcher to bring out that which normally goes unnoticed (textures) during examinations.

The intention of the research was that it have some practical value for the teaching and learning of mathematics while providing teachers with thinking tools rather than with a guideline for teaching practice. From my experience as a mathematics teacher, statements such as, “He knows nothing about functions” or “She knows systems of equations very well” are common to teacher-talk. These statements imply that a learner may or may not know some of the work (Even, 1993). When a learner gets 80% in a test, does that mean that the learner knows 80% of the test material or if a learner gets 100% of the answers correct, does that mean that the learner knows everything perfectly? In the case of a learner failing a test, does that mean that the learner does not know the work? In this regard this research showed that “knowing” is not easy to pinpoint. However, the use of ethnomethodological analysis provides important information regarding the solution-seeking processes examinees’ employ to navigate their pursuit of a suitable objective. It also presented concrete illustrations of what it might mean for examinees to construct and develop their own understanding and ideas about the mathematics they learn, and how these ideas are not necessarily identical to the structure of the discipline or to what was intended by the instruction.

Furthermore, ethnomethodological indifference provides a broader insight into examinees’ understanding of the mathematics in the analysis of examinees’ ways of working. It takes the focus away from recognizing errors and misconceptions and exemplifies the examinees’ understanding of the use of the mathematical constructs in a particular mathematical context. In Chapter One I referred to an alternative type of feedback to the usual errors, misconceptions and suggestions for improvement. This research offers a narrative for classroom instruction in terms of getting to know what learners understand and how their ways of working make that explicit. The notion of ethnomethodological indifference has rendered observable the rationality of examinees’ pursuance in the context of the time-restricted high-stakes examination, hence making the textures of ways of working an understandable and intelligible feature of the solution-seeking process.

The examinees used every account (including abandoned work) to build new accounts reflexively. This highlighted the strategies used by examinees to make sense of the problem text as well as the mathematical context which underpinned the objective to be pursued. Not only did it show the examinees' understanding of the mathematical concepts, but it also showed the tactics used by examinees to reflexively build up their understanding to an acceptable level of practical accomplishment. This notion of reflexivity can be seen as a way of working which is not only relevant to a high-stakes examination setting but to any setting in which learners are challenged to solve a mathematical problem. An ethnomethodological analysis of responses to the trigonometry question from the NSC high-stakes examination has resulted in capturing much information while refraining from being critical of the participant. More importantly, the methodological approach used in this research has shown the necessity for maintaining an understanding of previously learned mathematics (mathematically historicized-self) or mathematical techniques. It is my hope that teachers employ the theoretical concepts presented in this study to reflect on and regulate their classroom practice.

7.5 Recommendations for teaching and learning

It is my contention that making visible the ways of working of examinees in a high-stakes examination can contribute towards enhancing future examinees' coping strategies in high-stakes examination questions within the examination setting. One way to achieve this is to provide these future examinees with the produced work of past examinees and for them to analyse this in a manner similar to what was done in this study. This will have to be accompanied by exemplification of such analyses. For example, learners can be provided with one of the examples used here with the analysis provided in learner-comprehensible language. They can then be provided with a near-similar example drawn from the 'real' examination scripts, asked to do a similar analysis and then asked to find the given solution. Activities such as this from the "errors and misconceptions" paradigm have found their way into textbooks and other learning resources. Mason (2000, p. 10) gives an example of such a situation where learners are asked to discuss and evaluate whether $\frac{(5a)^{-2}}{5a^{-2}}$ is simplified correctly as follows: $\frac{(5a)^{-2}}{5a^{-2}} = \frac{(5a)^{-2}.a^{2}}{5} = \frac{5a}{5} = a$. Mason (2000, p. 10) asserts that exposing learners to "confusion [committing common errors], with the expectation that [learners] have been awakened to [them]…will make them more alert in the future". According to the findings of this study the doing of mathematics in a high-stakes examination resembles a mathematician-like way of working. As suggested by Schoenfield (1987) it should therefore be the aim of mathematics educators to introduce learners to such approaches to doing mathematics. Schoenfield (1987) suggests that learners should be exposed to such mathematics practices. Julie (1992, p. 19) goes further and asserts that learners should get the opportunity to practise this mathematician-like way of doing mathematics at their level. In discussions with one of my colleagues this notion of exposing learners to confusion was not positively accepted because of the rigid focus on the correct thing. I thus acknowledge that mathematician-like practice in the mathematics classroom is something to get used to and it should be seen as an attempt to move the thinking of educators away from focusing on right and wrong. It is my recommendation that learners be provided with problems to do or situations where they are able to develop their own problems instead of the educator just explaining how to get to the answer (Julie (1992, p. 19).

Furthermore, we need to allow ourselves to see examples such as the abandoned work presented in the analysis as a useful object of teaching and learning. When we allow learners to make sense of it, it will bring out a greater understanding of their own ways of ways of working in a time-restricted examination. According to Lampert (1990, p. 17) the content of mathematical lessons should show mathematical contexts which expose learners to strategies that support or reject solutions instead of simply a search for the answer. This way of doing mathematics should encourage interaction between learners whereby they discuss their solution-seeking strategies. The feedback gained from such mathematical practice will be learner-centered and in a language which will be comfortable for them to understand. In these instances teachers must refrain from giving the results but rather allow the class to seek their own resolution Julie (1992, p. 19). Teachers should facilitate such practice. It is the "this" which a teacher might refer to by saying, "look at this" and in which case the learner might reply "I am looking at this" or even "I am thinking about this". Exposing learners to such activities will force learners to label the "this" as the object, which belongs to the confines of a mathematical problem. The meaning-making in such ways of working will give learners a greater understanding of their own ways of working. This process of getting learners to analyze, explain, and interpret textures of abandonment will provide learners with a more direct approach to feedback and the use of that feedback to improve their understanding of their own work in a specific mathematical context.

Watson and Watson & Mason (2006, p. 91-111) are of the opinion that the use of such mathematical objects (textures of abandonment) even in highly organized situations in which learners reach for the same answers or practical accomplishments using the exact same data, may offer learners experiences that vary. In terms of teacher planning, it would allow teachers to plan their teaching from the learners' perspectives. This would include analysis of constructs in the conventional way that one anticipates learners will encounter. This approach to planning would allow teachers to identify regularities in conventional examples of those concepts that could assist learners' to (re)construct generalities related to concept. It is my view that ethnomethodological principles and the dialectic of resistance and accommodation will influence the underlying techniques unobtrusively. Garfinkel (1967, p.8) notes that when engaged in everyday activities, people are not concerned (generally) with discussing practical actions in a self-reflexive fashion:

> They recognize, demonstrate, and make observable for each other the rational character of their actual, and that means their occasional, practices while respecting that reflexivity as an unalterable and unavoidable condition of their inquiries. (Garfinkel, 1967, p. 8)

The use of textures presented in the analysis as an object for teaching and learning would present something different and unique. Wertheimer (1945) further advocates such uniqueness when he presents the notion of productive thinking in the process of teaching and learning. According to Wertheimer (1945) productive thinking is a way to show what he calls “blind drill” and its consequences. He describes the responses he obtained from students when he asked them to work out problems such as $\frac{357+357+357}{3}$ =?. He concluded that some “bright students” saw through such problems, observing that the division “undoes” the addition, rendering the original number. However, many students who had obtained high marks in school were blind to the structure of the problem, and insisted on working through it mechanically. These are examples of how the mathematical historicised-self can influence our ways of working. The way students are taught in arithmetic, the representation of problems in textbooks, the specific psychology books on which their methods were based places the emphasis on mechanical drill from which learners develop blind habits. Wertheimer (1945) argues that although repetition is useful, the continuous use of mechanical repetition also has harmful effects. It is dangerous because it easily induces habits of sheer mechanized action, blindness, tendencies to perform slavishly instead of thinking, instead of facing a problem freely (Wertheimer, 1945, p. 130–131).

7.6 Suggestions for further research

Future research may involve the practice of learners in collaborative group work to achieve an indication of the strategies learners employ when analyzing and interpreting responses taken from school-based assessment, such as class tests and examinations as well responses from high-stakes examinations. The researcher can employ the same theoretical approach as in the case of this study. They can do a qualitative study with the collaboration of ethnomethodology and the dance of agencies, focusing on the dialectic resistance and accommodation put forward by Pickering (1995).

Data can comprise observations, video and tape recordings. Interviews will elicit greater information regarding participants' knowledge and attitudes. The assemblage of a variety of qualitative data will strengthen the researchers' understanding of the participants' thoughts and opinions. The sample selection should represent a diverse group of learners from different socio-economic backgrounds, geographical locations of schools and various performance levels in mathematics. A future research question may read as follows:

> What knowledge and beliefs do learners have about examinees' ways of working? An ethnomethodological analysis of the strategies learners employ when analyzing and interpreting mathematical responses from a high-stakes examination

7.7 Concluding remarks

This study has contributed towards exposing the practices learners are engaged in when solving mathematical problems in high-stakes examinations. It has shown how examinees engaged the problem text, which resulted in various textures of ways of working in the solution-seeking paths.

This study adds value to the type of feedback generated from within the NSC mathematics high-stakes examination. It illustrates an alternative type of feedback, which can be extracted from the learners' ways of working with trigonometry. Furthermore, this study has contributed to the drive of the DBE to improve mathematics results in our schools.

Preliminary theories of ethnomethodological research in mathematics education have been generated together with a contribution which it is hoped will have enriched the field mathematics education empirically and methodologically in terms of ethnomethodological analysis. Finally, this study strengthens and adds to the notion of the sociology of scientific knowledge. This however was done in a limited and restricted environment.

Mehan (1979) concludes that the significance of questions is not the same for everybody. The testing adult and the testing student do not share the same meaning. What are considered wrong answers very often represent a different interpretation of the conceptual material and neither a lack of knowledge nor an inability to reason correctly. Treating the test results as objective facts hides the processes used by examinees by which to construct their answers. However, Mehan (1979) sees this construction as something that should be judged as fundamental by educators. Reports from the DBE on a year-to-year basis show that our schools are not performing well in mathematics. As indicated in this study various ways to improve results in mathematics have been suggested by the DBE. However, the suggestions of the DBE reveal that what is proposed is no different from the normal. These suggestions and interventions are things such as increasing the number of drills, of standard manipulations to counter errors and misconceptions and various intervention programmes such as telematics, or spring and winter school programmes. Whether these routine strategies will get us out of the dilemma of poor performance in mathematics is still to be seen Julie (1992, p. 20). In this study I refer to textures of abandonment and other tactics and strategies used by learners to solve mathematical problems. I recommend that the doing of mathematics in terms of classroom practice should become more mathematician-like and that this way of working becomes an accepted method by which to enhance the teaching and learning of mathematics.

8. References

Adler, J., & Lerman, S. (2003). Getting the description right and making it count: Ethical practice in mathematics education research. In *Second international handbook of mathematics education* (pp. 441-470). Springer Netherlands.

Anderson, G., Benjamin, D., & Fuss, M. A. (1994). The determinants of success in university introductory economics courses. *The Journal of Economic Education*, *25*(2), 99-119.

Andersson, A., & Norén, E. (2011, February). Agency in mathematics education. In *Proceedings from 7th conference for European research in mathematics education* (pp. 1389-1398).

Antippa, A. F. (2003). The combinatorial structure of trigonometry. *International Journal of Mathematics and Mathematical Sciences*, *2003*(8), 475-500.

Bailey, K., 1994, Methods of Social Research, Fourth Edition, New York: The Free Press.

Barksdale-Ladd, M. A., & Thomas, K. F. (2000). What's at stake in high-stakes testing teachers and parents speak out. *Journal of Teacher Education*, *51*(5), 384-397.

Bjuland, R. (2002). Problem-solving in geometry. Reasoning processes of learner teachers working in small groups: A dialogical approach. Unpublished doctoral dissertation.

Bergen, Norway: University of Bergen. Black, P., & Wiliam, D. (1998). Assessment and classroom learning. *Assessment in education*, *5*(1), 7-74.

Boaler, J. (2002). The development of disciplinary relationships: Knowledge, practice and identity in mathematics classrooms. *For the learning of mathematics*, *22*(1), 42-47.

Boaler, J. (2003). Studying and Capturing the Complexity of Practice – The Case of the. *International Group for the Psychology of Mathematics Education*, *1*, 3-16.

Bowen, G. A. (2009). Document analysis as a qualitative research method. *Qualitative research journal*, *9*(2), 27-40.

Brodie, K. (2005). Using cognitive and situative perspectives to understand teacher interactions with learner errors. *International Group for the Psychology of Mathematics Education*, 177.

Brodie, K. (2010). *Teaching mathematical reasoning in secondary school classrooms*. London: Springer.

Brown, P., Lauder, H., & Ashton, D. (2008). Education, globalisation and the future of the knowledge economy. *European Educational Research Journal*, *7*(2), 131-156.

Brown, P., Lauder, H., & Ashton, D. (2008). Education, globalisation and the future of the knowledge economy. *European Education Research Journal*, 7(2), 131-156.

Brown, R., & Redmond, T. (2008). Reconceptualising agency through teachers talking about a sociocultural approach to teaching mathematics in the classroom. *Navigating currents and charting directions*, 101-108.

Brown, S. A. (2005). *The trigonometric connection: Students' understanding of sine and cosine*. ProQuest.

Brown, S. A. (2006, July). The trigonometric connection: students' understanding of sine and cosine. In *Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, p. 228).

Callon, M., & Latour, B. (1992). Don't throw the baby out with the bath school! A reply to Collins and Yearley. *Science as practice and culture*, *343*, 368.

Carlsen, M. (2008). Appropriating mathematical tools through problem-solving in collaborative small group settings. University of Agder.

Chaiklin, S., & Lave, J. (Eds.). (1993). *Understanding practice: Perspectives on activity and context* (Vol. 78). Cambridge: Cambridge University Press.

Challenger, M. (2009). *From triangles to a concept: a phenomenographic study of A-level students' development of the concept of trigonometry. Unpublished PhD thesis*. Coventry: University of Warwick.

Chauke, W. (2013).A report on analysis of grade 12 students' performance in mathematics paper II examination of 2012. Gauteng Department of Education.

Chee-Cheong, C. (1999). Public examinations in Hong Kong. *Assessment in Education: Principles, Policy & Practice*, *6*(3), 405-417.

Cobb, P. (2007). Putting philosophy to work. *Second handbook of research on mathematics teaching and learning: a project of the National Council of Teachers of Mathematics*, *1*(1).

Cohen, D. (1994). *Law, sexuality, and society: The enforcement of morals in classical Athens*. Cambridge University Press.

Collins, Harry M. 1992. Changing Order: Replication and Induction in Scientific Practice, 2d ed. Chicago: University of Chicago Press.

Collins, Harry M., and Steven Yearley. 1992a. "Epistemological Chicken." Pp. 301- 26 in Science as Practice and Culture, edited by A. Pickering. Chicago: University of Chicago Press. . 1992b. "Journey into Space." p. 369-89 in Science as Practice and Culture, edited by A. Pickering. Chicago: University of Chicago Press.

Collins and Yearley. Science as practice and culture, 343,368.

Corbin, J., & Strauss, A. (2008). Basics of qualitative research 3e.

Cotton, T., & Hardy, T. (2004). Problematising culture and discourse for mathematics education research. In *Researching the socio-political dimensions of mathematics education* (pp. 85-103). Springer US.

Coulon, A. (1995). *Ethnomethodology* (Vol. 36). Sage.

Culler, J. (1975). Defining narrative units. *Style and structure in literature*, 123-42.

Davis, R. (1984). Diagnostic reasoning based on structure and behavior. *Artificial intelligence*, *24*(1), 347-410.

DBE, RSA. (2010). Curriculum Assessment Policy Statement (CAPS).

De Villiers, M. & Jugmohan, J. (2012). Learners' conceptualisation of the sine function during an introductory activity using sketchpad at grade 10 level. Educ. Matem. Pesq., São Paulo, 14(1), 9-30.

De Vos, A. S., Strydom, H., Fouche, C. B., & Delport, C. (1998). *Research at grass roots.* Pretoria: Van Schaik.

De Vos, A.S.(ed.) 1998. Research at Grass Roots. A primer for the caring professions. Pretoria J.L. van Schaik Publishers.

Deetz, S. (1996). Crossroads-describing differences in approaches to organization science: Rethinking Burrell and Morgan and their legacy.*Organization science*, *7*(2), 191-207.

Delice, A. (2002). Recognizing, recalling and doing in the 'simplification' of trigonometric expressions. The 26th Annual Conference of the International Group for the

Psychology of Mathematics Education (PME26), the School of Education and Professional Development at the University of East Anglia, Norwich: England, 1, 247

Delice, A., & Roper, T. (2006). Implications of a comparative study for mathematics education in the English education system. *Teaching Mathematics and its Applications*, *25*(2), 64-72.

Demir, O. 2012. Students' concept development and understanding of sine and cosine functions.(Thesis). Universiteit van Amsterdam.

Denzin, N. K., & Lincoln, Y. S. (2000). Strategies of inquiry. *Handbook of qualitative research*, 367- 378.

Denzin, N. K., & Lincoln, Y. S. (1994). *Handbook of qualitative research*. Sage Publications, Inc.

Denzin, N. K., & Lincoln, Y. S. (2011). *The SAGE handbook of qualitative research*. Sage.

Department of Basic Education (DoBE). (2011). *Curriculum and assessment policy statement/ Grades 10–12*. Pretoria: Mathematics.

Department of Basic Education. (2014). 2013 National Senior Certificate Examination. *National diagnostic report*.

Devadoss, S., & Foltz, J. (1996). Evaluation of factors influencing student class attendance and performance. *American Journal of Agricultural Economics*, *78*(3), 499-507.

Dietel, R. J., Herman, J. L., & Knuth, R. A. (1991). What does research say about assessment. *NCREL, Oak Brook*.

Grand, D. (2012). WCED programmes to help improve matric performance. Western Cape Department of Education

Dourish, P., & Button, G. (1998). On" technomethodology": Foundational relationships between ethnomethodology and system design. *Human-computer interaction*, *13*(4), 395-432.

Drews, D. (2005). Children's errors and misconceptions in mathematics.*Understanding common misconcepfions in primary mathemafics*, 14-22.

Durkheim, E. (1938). The Rules of Sociological Method, trans. *George EG Catlin.(Original Work published 1895)*.

Durkheim, E. (1982). What is a social fact?. In *The rules of sociological method* (pp. 50-59). Macmillan Education UK.

Durkheim, E., & Suicide, A. (1952). *A study in sociology*. Routledge & K. Paul.

Eisenhart, M. A. (1991, October). Conceptual frameworks for research circa 1991: Ideas from a cultural anthropologist; implications for mathematics education researchers. In *Proceedings of the 13th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 202-219).

Eisner, E. W. (1991). The enlightened eye. Stenbacka, C. (2001). Qualitative research requires quality concepts of its own. *Management decision*, *39*(7), 551-556.

Erickson, F. (1986). Qualitative methods in research on teaching (pp. 119-161). *Handbook of research on teaching*.

Even, R. (1993). Subject-matter knowledge and pedagogical content knowledge: Prospective secondary teachers and the function concept. *Journal for research in mathematics education*, 94-116.

Filmer, P. (1972). On Harold Garfinkel's ethnomethodology. In Paul Filmer et al. New Directions in Sociological Theory. 203-234. Cambridge:MIT Pres.

Floyd, A. (Ed.). (1981). *Developing Mathematical Thinking: A Reader*. Addison-Wesley.

Foster, D. (2007). Pedagogical content coaching. In *Professional Continuum Conference (National Science Foundation-Educational Development Corporation)"Instructional Coaching in Mathematics: Researchers and Practitioners Learning Together." Boston, MA. Retrieved March*, (Vol. 16, p. 2009).

Fox, N., Hunn, A., & Mathers, N. (2007). Sampling and sample size calculation. *The NIHR RDS for the East Midlands/Yorkshire & the Humber*

Francis, D., & Hester, S. (2004). *An invitation to ethnomethodology: Language, society and interaction*. Sage.

Gadamer, H. G. (1976). On the scope and function of hermeneutical reflection. *Philosophical hermeneutics*, 18-43.

Garfinkel, H. (1959). Parsons Primer. *Unpublished manuscript of Sociology*, *251*.

Garfinkel, H. (1967). Studies in ethnomethodology.

Garfinkel, H. (1991). Respecification: Evidence for locally produced, naturally accountable phenomena of order, logic, reason, meaning, method, etc. in and as of the essential haecceity of immortal ordinary society (I)—an announcement of studies. *Ethnomethodology and the human sciences*, 10-19.

Garfinkel, H. (2002). Ethnomethodology's program: Working out Durkheim's aphorism. *Lanham: Rowman & Littlefield Publishers*.

Gephart, R. (1999, January). Paradigms and research methods. In *Research methods forum,* (Vol. 4, No. 1, p. 11).

Goodwin, C. (2000). Practices of seeing visual analysis: An ethnomethodological approach. *Handbook of visual analysis*, 157-182.

Gravemeijer, K. (1994). Educational development and developmental research in mathematics education. *Journal for research in Mathematics Education*, 443-471.

Greiffenhagen, C. (2008, September). Video analysis of mathematical practice? Different attempts to" open up" mathematics for sociological investigation. In *Forum Qualitative Sozialforschung/Forum: Qualitative Social Research* (Vol. 9, No. 3).

Griffin, B. W., & Heidorn, M. H. (1996). An examination of the relationship between minimum competency test performance and dropping out of high school. *Educational Evaluation and Policy Analysis*, *18*(3), 243-252.

Grootenboer, P., & Jorgensen, R. (2009). Towards a theory of identity and agency in coming to learn mathematics. *Eurasia Journal of Mathematics Science and Technology Education*, *5*(3), 255-266.

Gur, H. (2009). Trigonometry Learning. *New Horizons in Education*, *57*(1), 67-80.

Hall, D.T. (2004). The protean career: A quarter-century journey. Journal of Vocational Behavior, 16, 1-13.

Hancock B., Windridge K., and Ockleford E. An Introduction to Qualitative

Research. The NIHR RDS EM / YH, 2007

Handcock, M. S., & Gile, K. J. (2011). Comment: On the concept of snowball sampling. *Sociological Methodology*, *41*(1), 367-371.

Harlen, W., & Deakin Crick, R. (2002). A systematic review of the impact of summative assessment and tests on students' motivation for learning (EPPI-Centre Review, version 1.1). *Research evidence in education library*, *1*.

Hatano, G. (1996). A conception of knowledge acquisition and its implications for mathematics education. In P Steffe, P Nesher, P Cobb, G Goldin, & B Greer (Eds.), *Theories of mathematical learning* (pp. 197–217). New Jersey: Lawrence Erlbaum.

Hattie, J., & Timperley, H. (2007). The power of feedback. *Review of educational research*, *77*(1), 81-112.

Hoepfl, M. C. (1997). Choosing qualitative research: A primer for technology education researchers.

Holland, D. L. W., Skinner, D. and Cain, C. (2003). *Identity and agency in cultural worlds*.

Holland, D., Lachicotte Jr, W., Skinner, D. & Cain, C. (2003). Identity and Agency in Cultural Worlds. London, Cambridge: Harvard University Press.

Jacob, E. (1987). Qualitative research traditions: A review. *Review of educational research*, *57*(1), 1-50.

Jacobs, M., Mhakure, D., Fray, R. L., Holtman, L., & Julie, C. (2014). Item difficulty analysis of a high-stakes mathematics examination using Rasch analysis: The case of sequences and series. *Pythagoras*, *35*(1), 7-pages.

Johnson, B. R. (1997). Examining the validity structure of qualitative research. Education, 118(3), 282-292.

Joppe, M. (2003). The research process. The Research Process. Retrieved April 20, 2016, From http://www.ryerson.ca/~mjoppe/rp.htm

Julie, C. 1992. Doing Mathematics--What does it mean?. Unpublished keynote address presented at *The Second Annual Convention of the Mathematics Association of Transkei*. Transkei Inservice College: Mthatha.

Julie, C. (1998a). The production of artifacts as goal for school mathematics. In A. Olivier and K. Newstead (Eds.), Proceedings of the 22nd international conference for the psychology of mathematics education, Volume 1. (pp. 49-65).

Julie, C. (2002). The Activity System of School-Teaching Mathematics and Mathematical Modelling. For the Learning of Mathematics, 22(3), 29 – 37.

Julie, C. (2003). Work moments in mathematical modelling by practising mathematics teachers with no prior experience of mathematical modelling and applications. New Zealand Journal of Mathematics, 32 (Supplementary Issue), 117–124.

Julie, C. (2012). The primacy of teaching procedures in school mathematics. In Nieuwoudt, S., Laubscher, D. & Dreyer, H. (Eds.), Proceedings of the 18th Annual National Congress of the Association for Mathematics Association of South Africa. Potchefstroom: North-West University.

Julie, C. (2015). Learners' Dealing with a Financial Application-Like Problem in High-Stakes Examination. *Mathemathematics Modelling in Education Research and practice.* Cultural, Social and Cognitive Influences. *International Perspectives on the Teaching* *and Learning of Mathematical Modelling.* Springer Switzerland , 477-486

Kaplan, B., & Maxwell, J. A. (1994). Evaluating health care information systems: Methods and applications. *Qualitative Research Methods for Evaluating Computer Information Systems. JG Anderson, CE Ayden and SJ Jay. Thousand Oaks, Sage*.

Kellaghan, T., & Madaus, G. (2003). External (public) examinations. In *International handbook of educational evaluation* (pp. 577-600). Springer Netherlands.

Kendal, M., & Stacey, K. 1997. Teaching Trigonometri. Viculum, 34 (1), 4-8.

Kennedy, M. (1997). "The connection between research and practice", Educational Researcher, 26(7), 4 – 7

Khalloufi-Mouha, F., & Smida, H. (2012). Constructing mathematical meaning of a trigonometric function through the use of an artefact. *African Journal of Research in Mathematics, Science and Technology Education*, *16*(2), 207-224.

Klein, A. (2002). Audit committee, board of director characteristics, and earnings management. *Journal of accounting and economics*, *33*(3), 375-400.

Kluger, A. N., & DeNisi, A. (1996). The effects of feedback interventions on performance: a historical review, a meta-analysis, and a preliminary feedback intervention theory. *Psychological bulletin*, *119*(2), 254.

Knorr-Cetina, K. (1999). Epistemic cultures: How scientists make sense.*Chicago, Indiana*.

Kraak, A. (2013). State failure in dealing with the NEET problem in South Africa: which way forward?. *Research in Post-Compulsory Education*, *18*(1-2), 77-97.

Kress, G. (1989). Linguistic processes in sociocultural practice. (2nd ed.). Oxford: Oxford

University Press.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American educational research journal*, *27*(1), 29-63.

Lave, J. (1988). *Cognition in practice: Mind, mathematics and culture in everyday life*. Cambridge University Press.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge university press.

Lave, J. (1993). Introduction: The practice of learning. In J. L. Seth Chaiklin (Eds), Understanding Practice: Perspectives on Activity and Content. Cambridge: Cambridge University Press.

Lee, Y. J., & Roth, W. M. (2007). The individual| collective dialectic in the learning organization. *The Learning Organization*, *14*(2), 92-107.

Lerman, S. (2000). The social turn in mathematics education research. *Multiple perspectives on mathematics teaching and learning*, (pp. 19-44). Westport, CT, USA: Ablex

Lester, F. K. (2005). On the theoretical, conceptual, and philosophical foundations for research in mathematics education. *ZDM*, *37*(6), 457-467.

Levi-Strauss, C. (1966). *The savage mind*. University of Chicago Press.

Li, S., & Seale, C. (2007). Learning to do qualitative data analysis: An observational study of doctoral work. *Qualitative Health Research*, *17*(10), 1442-1452.

Liberman, K. (2012). Semantic Drift in Conversations. *Human Studies*, *35*(2), 263-277.

Lincoln, Y. S., & Guba, E. G. (1985). Naturalistic inquiry. Beverly Hills, CA: Sage.

Livingston, E. (1986). The ethnomethodological foundations of mathematics.

Livingston, E. (1987). *Making sense of ethnomethodology*. Taylor & Francis.

Lochead, J. & Mestre, J. (1988). From words to algebra: mending misconceptions. In A. Coxford & A. Shulte (Eds.), The Ideas of Algebra, K- 12 (1988 Yearbook of the National Council of Teachers of Mathematics, pp. 127-135). Reston, VA: National Council of Teachers of Mathematics.

Luneta, K, & Makonye, PJ. (2010). Learner errors and misconceptions in elementary analysis: a case study of a grade 12 class in South Africa. *Acta Didactica Napocensia, 3*(3), 35–46.

Lynch, M. (1997). *Scientific practice and ordinary action: Ethnomethodology and social studies of science*. Cambridge University Press.

Madaus, G. F. (1991). The effects of important tests on students: Implications for a national examination system. *The Phi Delta Kappan*, *73*(3), 226-231

Maharaj, A. (2008). Some insights from research literature for teaching and learning mathematics. *South African Journal of Education*, *28*(3), 401-414.

Makgato, M. (2007). Factors associated with poor performance of learners in mathematics and physical science in secondary schools in Soshanguve, South Africa. *Africa Education Review*, *4*(1), 89-103.

Marick, B. (2004). Agile methods and agile testing. *STQE Magazine*, *3*(5).

Marshall, M. N. (1996). Sampling for qualitative research. *Family practice*,*13*(6), 522-526.

Martínez-Sierra, G. (2008). From the analysis of the articulation of the trigonometric functions to the corpus of eulerian analysis to the interpretation of the conceptual breaks present in its scholar structure. *Proceedings of the HPM*.

Mason, C. (1999). The TRIAD approach: A consensus for science teaching and learning. In *Examining pedagogical content knowledge* (pp. 277-292). Springer Netherlands.

Mason, J. (2000). Asking mathematical questions mathematically. *International journal of mathematical Education in Science and Technology*,*31*(1), 97-111.

Mcmillan, H., & Schumacher, S. (2010). Researcher in Education.

Mehan, H. (1979). *Learning lessons*. Cambridge, MA: Harvard University Press.

Mehan, H., & Wood, H. (1975). *The reality of ethnomethodology*. New York: Wiley.

Meier, C. (2011). The Foundations for Learning Campaign: helping hand or hurdle?. *South African Journal of Education*, *31*(4), 549-573.

Merz, M., & Cetina, K. K. (1997). Deconstruction in a thinking science: Theoretical physicists at work. *Social Studies of Science*, *27*(1), 73-111.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook*. Sage.

Mishler, E. G. (2000). Validation in inquiry-guided research: The role of exemplars in narrative studies. *Acts of inquiry in qualitative research*, 119-146.

Morgan, C. (1999). Assessment in mathematics education: A critical social research perspective.

Morgan, R. K. (1999). *Environmental impact assessment: a methodological approach*. Springer Science & Business Media.

Morse, J.M., Kuzel, A.J. and Swanson, J.M. (eds) (2001) The Nature of Qualitative Evidence, Thousand Oaks, CA: Sage

Murphy, R., Nuttall, D. L., & Broadfoot, P. (1995). *Effective assessment and the improvement of education: A tribute to Desmond Nuttall*. Taylor & Francis.

Nagy, P. (2000). The three roles of assessment: Gatekeeping, accountability, and instructional diagnosis. *Canadian Journal of Education/Revue canadienne de l'éducation*, 262-279.

Nesher, P. (1987). Towards an instructional theory: the role of student's misconceptions. *For Learning of Mathematics, 7*(3), 33–40.

Niss, M. (1999). Aspects of the nature and state of research in mathematics education. *Educational studies in mathematics*, *40*(1), 1-24.

Niss, M. A. (2007). Reflections on the state of and trends in research on mathematics teaching and learning. In *Second handbook of research on mathematics teaching and learning*. Information Age Publishing, incorporated.

Olivier, A. (1989). Handling pupils' misconceptions.

Orhun, N (2002). Solution of verbal problems using concept of least common multiplier (LCM) and greatest common divisor (GCD) in Primary School Mathematics and Misconceptions. Retrieved 6th September 2006 from http://math.unipa.it/~grim/SiOrhun.PDF

Orhun, N. (2010). The gap between real numbers and trigonometric relations.*Quaderni di Ricerca in Didattica*, *20*, 175-184.

Orton, A. (1983a). Students' understanding of differentiation. *Educational Studies in Mathematics*, *14*(3), 235–250.

Orton, A. (1983b). Students' understanding of integration. *Educational Studies in Mathematics, 14*, 1–18.

Pascale, C. M. (2013). *Making sense of race, class, and gender: Commonsense, power, and privilege in the United States*. Routledge.

Patton, M. Q. (2002). Qualitative interviewing. *Qualitative research and evaluation methods, 3*, 344-347.

Pickering, A. (1995). The mangle of practice: Time, agency. *& science. Chicago: University of Chicago*.

Polesel, J., Dulfer, N., & Turnbull, M. (2012). The experience of education: The impacts of high stakes testing on school students and their families. *Literature Review prepared for the Whitlam Institute, Melbourne Graduate School of Education, and the Foundation for Young Australians. Available online at: http://www. whitlam. org/data/assets/pdf_file/0008/276191/High_Stakes_Testing_Literature_Review.pdf (accessed 20 september 2015).*

Pollner, M. (1991). Left of ethnomethodology: The rise and decline of radical reflexivity. *American Sociological Review*, 370-380.

Potter, J., & Wetherell, M. (1987). *Discourse and social psychology: Beyond attitudes and behaviour*. Sage.

Presmeg, N. C. (2006). A semiotic view of the role of imagery and inscriptions in mathematics teaching and learning. In *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 19-34).

Radnitzky, G. (1970). [The relation to practice in research. Early studies for the theoretical basis of science policy. *Studium generale; Zeitschrift fur die Einheit der*

Wissenschaften im Zusammenhang ihrer Begriffsbildungen und Forschungsmethoden, *23*(9), 817-855.

Raj, M., & Nega, M. (2011). History of Trigonometry. With a Classroom Application. Presented at AMATYC 37, Austin Texas. 11 November 2011. Georgia Perimeter College.

Ramsden, P. (1998) Learning to Lead in Higher Education, 2nd ed.London, UK: Routledge.

Ramsden, P. (1998). Managing the effective university. *Higher education research & development*, *17*(3), 347-370.

Reeves, T. C., & Hedberg, J. G. (2003). Evaluating interactive learning systems. *Athens, GA: University of Georgia, College of Education*.

Republic of South Africa Department of Basic Education. (2010). Curriculum Assessment Policy Statement (CAPS).

Rey, O. (2010). The use of external assessments and the impact on education systems. *Beyond Lisbon 2010: Perspectives from Research and Development for Education Policy in Europe*, 137-157.

Rogers, M. F. (1983). *Sociology, ethnomethodology and experience*. CUP Archive.

Rosenthal, R., & Rosnow, R. L. (1991). *Essentials of behavioral research: Methods and data analysis*. McGraw-Hill Humanities Social.

Roth, W. M. (2007). Theorizing passivity. *Cultural Studies of Science Education*, *2*(1), 1-8.

Roth, W. M. (2012). *Authentic school science: Knowing and learning in open-inquiry science laboratories* (Vol. 1). Springer Science & Business Media.

Roth, W. M. (2013). Ethnomethodology in/for Science Education. In *What More in/for*

Science Education (pp. 3-24). Sense Publishers.

Ryan, J. T. & Williams, J. S. (2000). Mathematical discussions with children: exploring methods and misconceptions as a teaching strategy. Manchester: University of Manchester

Schmidt, N. B., Lerew, D. R., & Jackson, R. J. (1999). Prospective evaluation of anxiety sensitivity in the pathogenesis of panic: replication and extension. *Journal of abnormal psychology*, *108*(3), 532.

Schoenfeld, A. H. (1987). *Cognitive science and mathematics education*. Psychology Press.

Schoenfeld, A. H., Smith, J. P., & Arcavi, A. (1993). Learning: The microgenetic analysis of one student's evolving understanding of a complex subject matter domain. *Advances in instructional psychology*, *4*, 55-175.

Schroeder, L 2006, 'What high-stakes testing means for the emotional wellbeing of student and teachers', unpublished PhD thesis, Graduate University, Claremont.

Scott, J., 1990, A Matter of Record, Documentary Sources in Social Research, Cambridge: Polity Press.

Seale, C. (1999). Quality in qualitative research. *Qualitative inquiry*, *5*(4), 465-478.

Shapin, S. (1995). Here and everywhere: sociology of scientific knowledge. *Annual review of sociology*, 289-321.

Siegfried, J. J., & Fels, R. (1979). Research on teaching college economics: A survey. *Journal of Economic Literature*, *17*(3), 923-969.

Silverman, D. (1985). Qualitative methodology and sociology: describing the social world.

Silverman, D. (1998). *Harvey Sacks: Social science and conversation analysis*. Oxford

University Press on Demand.

Simon, M. K., & Goes, J. (2013). *Scope, limitations, and delimitations.* Dissertation and Scholary Rsearch Recipes for Success. Seattle, WA: Dissertation Success LLC

Smith, JP, DiSessa, AA, & Rosehelle, J. (1993). Misconceptions reconceived: a constructivist analysis of knowledge in transition. *The Journal of the Learning Science, 3*(2), 115–163.

Stenbacka, C. (2001). Qualitative research requires quality concepts of its own. *Management decision*, *39*(7), 551-556.

Stiggins, R. J. (1999). Assessment, student confidence, and school success. *The Phi Delta Kappan*, *81*(3), 191-198.

Ten Have, P. (2004). *Understanding qualitative research and ethnomethodology*. Sage.

Thompson, P. W. (2008). Conceptual analysis of mathematical ideas: Some spadework at the foundations of mathematics education. In *Proceedings of the annual meeting of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 45-64). PME Morelia, Mexico.

Umalusi (2004) Report on the investigation into the Senior Certificate Examination. Pretoria: Umalusi.

Walsham, G. (1995). Interpretive case studies in IS research: nature and method. *European Journal of information systems*, *4*(2), 74-81.

Watson, A., & Mason, J. (2006). Seeing an exercise as a single mathematical object: Using variation to structure sense-making. *Mathematical thinking and learning*, *8*(2), 91-111

Watson, G., & Seiler, R. M. (1992). Text in context. *Contributions to ethnomethodology. Newbury Park etc*.

Webb, N., & Coxford, A. F. (1993). *Assessment in the mathematics classroom* (Vol. 55). National Council of Teachers of Mathematics.

Weber, K. (2005). Students' understanding of trigonometric functions. *Mathematics Education Research Journal*, *17*(3), 91-112.

Weber, K. (2008). Teaching trigonometry functions: lessons learned from research. *Mathematics teacher*, *102*(2), 144 –150.

Wenger, E. (1998). Communities of practice: Learning as a social system. *Systems thinker*, *9*(5), 2-3.

Wertheimer, M. (1945). Productive thinking: Harper, New York 1945.

Wheelock, A., Bebell, D., & Haney, W. (2000). What can student drawings tell us about high-stakes testing in Massachusetts. *Teachers College Record*, *2*.

Wieder, D. L. (1974). *Language and social reality: The case of telling the convict code* (Vol. 10). The Hague: Mouton.

Wiersma, W. 2000. *Research methods in education*: An introduction. 7th ed. Boston: Allyn & Bacon.

Yin, R. (1994). *Case study research*: Design and methods. Beverly Hills.