AN INVESTIGATION INTO THE METHODOLOGIES OF VALUE-AT-RISK AND A SIMULATION PROCESS OF A PORTFOLIO OF FINANCIAL INSTRUMENTS.

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A thesis submitted in partial fulfillment of the requirements for the degree of Magister Scientiae in the Department of Statistics, University of the Western Cape.

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KEYWORDS

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ABSTRACT

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Financial companies such as investment and commercial banks as well as insurance companies, mutual and pension funds hold assets in the form of financial instruments in portfolios. Nowadays, financial instruments have proliferated so much that there are so many forms of them namely: derivatives, common stock, corporate and government bonds, foreign exchange and contracts. With so many financial instruments, companies can have very large and diversified portfolios for which they must quantify the risk.

With high profile calamities that have rocked the financial world lately, the need for better risk management has never been so in demand as before. Value-at-Risk (VaR) is the latest addition in the investor's toolkit as far as measurements of risk is concerned. This new measure of risk complements well the existing risk measures that exist. Unfortunately, VaR is not unanimous and it has attracted a lot of critics over the years.

This research thesis is threefold: to introduce the reader to the VaR concept; to discuss the different methods that exist to calculate VaR; and, finally, to simulate the VaR of a portfolio of government bonds. The first part of this research is to introduce the reader to the general idea of risk forms and its management, the role that the existing risk measures have played so far and the coming up of the new technique, which is VaR. The pros and cons that accompany a new technique are discussed as well as the history of VaR.

The second part is about the different methods that exist to compute the VaR of a portfolio. Usually, VaR methodologies fall into three categories namely: Parametric; Historical; and Monte Carlo. In this research, the advantages and disadvantages of these three methods are discussed together with a step-wise method on how to proceed to calculate the VaR of a portfolio using any of the three methods.

The practical side of this thesis deals about the VaR simulation of a portfolio of financial instruments. The chosen financial instruments are four South African government bonds with different characteristics. VaR for this particular portfolio will then be simulated by the three main methods. Eleven different simulations are run and they are compared against a Control Simulation (Benchmark Portfolio) to see how factors influencing VaR measure cope under different conditions. The main idea here was to check how VaR measures can change under different portfolio characteristics and to interpret these changes. Moreover, the VaR estimates under the three different methods will be compared.

Finally, the reliability of the research, when the practical side is compared to existing theory of VaR, as well as the limitations of the topic is discussed. VaR is a very useful risk measurement but if on one hand it does provide useful information to investors, over-dependence can be misleading. As a result, VaR must be handled with care and maybe it must always be complemented with other existing risk measures.

November 2004.

DECLARATION

I declare that *An investigation into the methodologies of Value-at-Risk and a simulation process of a portfolio of financial instruments* is my own work, that it has not been submitted before for any degree or examination in any other university, and that all sources I have used or quoted have been indicated and acknowledged as complete references.

Gamal Abdel Hussein Ballam November 2004

Signed:.........................

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This list will not be complete without me mentioning a very important person in my life who unfortunately is not among us anymore but whose contributions in my achievements have been enormous. I would like to dedicate this thesis to my late beloved uncle, Abdool Rajack Ballam.

November 2004

List of Acronyms

BCBS	Basle Committee on Banking Supervision
CAD	European Capital Adequacy Derivatives
CAPM	Capital Asset Pricing Model
EWMA	Exponential Weighted Moving Average
EWRM	Enterprise-Wide Risk Management
FSB	Financial Services Board
RAPM	Risk-Adjusted Performance Measurement
RAROC	Risk-Adjusted On Capital
SEC	Securities and Exchange Commission
UNCR	Uniform Net Capital Rule
ZAR	South African Rand

CONTENTS

Chapter 1

Introduction

1.1 Background to Value-at-Risk (VaR)

Everyday, in every aspect of life and in any line of business, people or organizations are pursuing new avenues and new ventures in the hope of achieving higher returns. These ventures come in different forms and are uncertain. People have different aims and accordingly they allocate capital and resources to achieve them. In doing so, they face uncertainty, i.e. they face risk. Gugi et al. (1999) define risk as the:

"Danger of not achieving certain return".

The face of risk management has changed drastically over the past couple of years. This is mainly because of the rapid changes in the fields of investment, portfolio and fund management. These changes have brought along more complications into a system, which was already complex.

Dowd (1999a) claims that the benefits a risk management system will bring to an organization can be:

- Probabilities of the organization getting into financial distress - organizations are in distress when they are in financial turmoil;
- Reduction of cash flow volatility.

Risks come in all forms. Some can be measured or quantified while others cannot. Market risk, which is the risk of loss sustained as a result of adverse changes in the values of market prices of traded instruments, falls into the category of risk that can be measured. Dowd (1999a) reports that quantitative approaches to risk management are based on three basic essential steps, namely:

(1) **The formulation of a risk management policy:** this will indicate to the organizations what risks they want to bear, what risks they are prepared to offset and what methods they will use to manage the risk exposures.

(2) **The quantification of the risk exposure(s):** this involves the measurement of the relevant risk exposures by appropriate methods which include:

- Duration, duration-convexity and gap analyses to measure interest rate risk;
- Portfolio methods that focus on certain types of risk (i.e., equity risk) which offset one another in a portfolio;
- Regression analyses, which estimate exposure to interest-rate, equity, foreign exchange, commodity and other related risks.
- Scenario analyses, which estimate the gain or loss under, specified situations; and
- Zero-arbitrage methods, which calculate risk exposures of derivatives positions.

(3) **Response to exposure:** with the management policy in mind, organizations can then decide what risks they would want to manage and what instruments will better serve their purpose.

Each of these methods mentioned in (2) above has its own particulars and advantages in risk management, but they are also limited. Dowd (1999a) reports that, since the late 1970s, financial institutions have realized these limitations and were accordingly working on their own models to measure risks. Progress has been steady but slow, both from academics and industry. Perhaps the most notable private sector initiative towards better risk management is that of J P Morgan, which unveiled its RiskMetrics™ in October 1994 (Jorion, 2004).

The introduction of the RiskMetrics™ methodology in risk management triggered the industry into more developments for measurements of risks and this gave rise to the modern era Value-at-Risk (VaR). History, however, traces its origins as early as the 1950s when VaR was developed from basic mathematics (Holton, 2002a).

Value-at-Risk is a method to measure market risk and is defined as the maximum amount (in relevant currency) that a portfolio can lose with a certain probability over a period of time. VaR collapses an entire profit and loss distribution of portfolio returns into a single number, which summarizes exposure to market risk as well as the probability of an adverse move (Jorion, 2004).

In J P Morgan's RiskMetrics^{TM} , VaR is computed from a system based on standard portfolio theory. In fact, the system is closely related to Modern Portfolio Theory, using estimates of the standard deviations and correlations between the returns of different trading instruments. However, the system relies on too many over-simplifications and the implementation requires a huge amount of work such as measurement methods, constructing data and computing systems to carry out the estimations.

During the same period, i.e. early nineties, that J P Morgan was busy developing its RiskMetrics^{TM}, other financial institutions and also banks were working on their own VaR systems, as they viewed it as a chance to find a common measure of risk across multiple financial products (McGin, 1998). The principles of their VaR were also based on portfolio theory though some major differences could be picked up from the assumptions and procedures. With rapid changes in the field of information technology and the advent of more powerful computers, some other VaR systems were being developed at the same time. These include a historical approach, which estimated VaR from a histogram of past profit and loss data (i.e. using past information) for the portfolio as a whole and a Monte Carlo simulation approach, which was based on a random number generator to obtain the hypothetical distribution.

Since that period of sustained information technology advancement, VaR has come a long way and has spread rapidly among financial institutions including securities houses. Its use is being encouraged by the Bank for International Settlements, the Federal Reserve Bank and the Securities and Exchange Commission for just about every derivatives user (Falkenstein, 1997). With the promise that it holds of combining all quantifiable risks across the business lines of an institution, yielding one firm measure of risk (Simons, 1996), VaR has also attracted the attention of regulators such as the Basel Committee in Banking Supervision (BCBS) and regulators in the European Union like Britain's Financial Supervisory Authority (Jorion, 2004), Financial Services Board (FSB) South Africa.

High profile financial disasters- Orange County, Sumitomo, Barings, Daiwa and others, which have rocked the financial world in the past, all highlighted the importance of a better risk management. Regulators are now more concerned about the development of a set of accepted risk management guidelines (Financial Risk Management, Contingency Analysis, 1996). The important one is the BCBS report from which the main recommendations (BCBS, Contingency Analysis, 1996) are:

- Need for senior management to understand the risk of their business and the importance of them overseeing the risks that lower level managers will take;
- Separation of trading and administrative offices to help detect fraud;
- Need for an independent risk management link that reports directly to top management;
- Need for full and complete audit and control;
- Importance of good and safe information systems; and
- Use of value-at-risk and stress testing to measure financial risks across the business.

With these recommendations in mind as well as for practical limitations risk managers were led to develop alternative methods to implement VaR and further add on to the existing ones (Ganief, 2001).

1.2 Definition of Research Problem

The financial world experienced some rough times in the late 1980s with the 1987 stock market crash and the 1990s series of collapses. It all started with the distress in the bond market in 1994 followed by market crises in Mexico in 1995, in Asia in 1997 and in Russia in 1998 (Barone-Adesi et al, 2000). These events have been the key issues in finance and henceforth in risk management as well as for international regulatory bodies.

Risk management has witnessed much development. Simons (1996) reports that *"risk management"* has become a popular buzzword- the phrase appearing in the American Banker *"72 times in 1990 and 325 times in 1995"*. Risk management has grown in sophistication and usage for the last few years (Winterton, 2003). At the center of all this interest, is a new approach to risk management called Value-at-Risk (VaR).

According to Schachter (1997), the standard deviation is all what is needed to:

1. Encapsulate all the information about risk which is relevant; and
2. Construct risk-based rules to optimal risk management decisions.

However, managers think of risk in terms of relevant currency of loss and not in terms of deviations as defined by the standard deviation.

An alternative measure of risk was therefore required, which led to a new interest in an objective way of gauging the adequacy capital (Simons, 1996). In their search, financial institutions turned partly to analytical tools and VaR emerged as the favoured method for measuring risks (Simons, 1996). VaR expresses, in relevant currency terms, the major concern of risk management, which is the loss to portfolio value.

VaR can be calculated across financial instruments. However, various methods can be used to calculate VaR, each resulting in a different answer. There are methods that handle only linear instruments, while others can handle any type of instruments. Jorion (2004) reports on a couple of commercial vendors offering risk management systems that compute VaR. According to McGin (1998), VaR is still a phenomenon in the area of risk management which is advancing beyond the existing less sophisticated ways to measure risk and is evolving since it is the measure regulatory bodies look to for domestic and international portfolios.

When attempts, to apply the theory from literatures to the practical world of risk management, are made, a few daunting questions arise. More importantly if "given two VaR measures, how can the risk manager pick the best one" and "given a VaR measure, how does the risk manager know it is specified according to the portfolio"?

Despite its popularity for measuring market risk, no common platform has yet been reached as to the best implementation of VaR approach. This absence of a consensus particularly based on the implementation of the methods currently in use has some significant drawbacks (Ganief, 2001).

1.3 Specific Aims of Study

The aims of this research are threefold:

1. To introduce the reader to the VaR concept;
2. To present the different methods to calculate VaR; and
3. To simulate VaR for a portfolio of financial instruments.

In recent years, VaR has become a popular measure of market risk. It is widely used by financial institutions and non-financial corporates to control the market risk in a portfolio of financial instruments (Hull and White, 1998). The reason for this acquired interest could be traced to the advantages that VaR holds over traditional risk measures. The first objective of this research is to provide the reader with some background relationship between VaR and its counterparts while also discussing the advantages and disadvantages with respect to each other. The evolution of VaR through the years will also be investigated.

As mentioned earlier, the computation of VaR can be a daunting process. The power of the concept lies in its generality, but the challenge of calculating a VaR measure also crops up from its generality (Measuring VaR, Contingency Analysis, 1996). The most important step in computing a VaR metric is to find the return (profit and loss) distribution of the portfolio. The VaR approach is still evolving and experiments and research on the topic are continuing.

The most common VaR methodologies are:

1. Parametric Approach.
2. Historical Simulation.
3. Monte Carlo Simulation.

The parametric approach to calculate VaR is closely related to the theory of Modern Portfolio Theory whereby the VaR is expressed as a product of the standard deviation of the portfolio returns. The historical simulation is somewhat different as it assumes that the future cannot be that different from the past. As such, it simulates a hypothetical histogram of returns for a portfolio using past market prices and comparing with current prices. The VaR statistic is then read from the histogram. The Monte Carlo simulation also computes a histogram of hypothetical returns, which are obtained by selecting at random (usually a number generator) from a given distribution of price changes estimated with past data.

All three of these techniques have their advantages and disadvantages. The second aim of this research will be to take a closer look at these approaches and also elaborating on their strengths and weaknesses. The conditions under which the different techniques perform best will also be investigated and a stepwise description of how to apply the different techniques on a multiple instruments portfolio will also be discussed. This knowledge will certainly assist investors in selecting the most appropriate approach considering their scenario.

The third and final aim of this research is the simulation process of a selected portfolio. A portfolio of financial instruments consisting of four South African government bonds will be chosen and the VaR measures will be computed by three main methods: Historical Simulation; Variance-Covariance Method; Monte Carlo Simulation. The simulation procedures will be run on the SAS Risk Dimension software and the results will be analyzed. The research findings and interpretations can be used to model similar or linear portfolios. The reason to simulate VaR of such a portfolio is to measure how VaR estimates vary when changes in the dependent factors take place. Accordingly, eleven different simulation procedures will be run and their VaR estimates will be computed and compared. The VaR measures under the three different methods will also be analyzed and attempts to answer a question like: why the VaR estimates differ or are the same under different methods when the portfolio is the same, will be made.

In chapter two, the history and evolution of Value-at-Risk over the years will be analyzed as well as the contributions and critics that VaR has attracted since its concept. Chapter three will deal with the existing measures of risk as well as the theory of VaR. The three known techniques to compute VaR will also be discussed together with their advantages and disadvantages. In chapter four, the results and the findings of the eleven different simulation runs will be analyzed and, finally, chapter five will be about the conclusion of this research.

Chapter 2

Literature Review

2.1 Risk Management: The History of Value-at-Risk

Risk management was considered a novelty in 1990, but the term "risk management" is not new. It has existed for long and its use can be traced back as far as the 1960s and 1970s when firms were looking for alternatives to insurance (Holton, 2002a). The new "risk management" that has evolved during the 1990s is quite different from any of its earlier forms and it is then that the term "*value-at-risk*" entered the financial lexicon, although VaR origins also go back a long time. The history of VaR will be split up into two parts namely:

1. The early days - which will include the years 1920-1980; and
2. The modern era - including years 1990 to now.

2.1.1 The Early Days of Value-at-Risk

Value-at-Risk (VaR) has its roots both in Capital Requirement and Portfolio Theory. Its first appearance can be traced back as early as the year 1922, when the New York Stock Exchange applied an informal test on the United States (US) Securities firms for capital requirement (Holton, 2002b).

During the 1950s, portfolio theorists had started to develop basic mathematics for VaR measures. Academic papers from James Tobin, Jack Treynor, William Sharpe and Jan Mossin (Holton, 2002a) were contributing to the development of VaR. The kind of VaR measures they employed at the time were best applicable to equity portfolios (Holton, 2002a). In fact in the same period, Harry Markowitz and Arthur Roy independently published VaR measures that were supporting portfolio optimization (as reported by Holton, 2002b).

The 1970s saw an outburst of VaR. First, Schrock and Dusak (Holton, 2002b) came up with simple VaR measures for futures portfolios and then Lietaer (Holton, 2002b) described a practical VaR measure for foreign exchange risk positions. In 1975, the US Securities Exchange Commission (SEC) established a uniform Net Capital Rule, which included a system of "haircuts", applied to firms' capital as safeguard against market losses and they were based upon statistical analyses of past data. At the same time US regulators were prompting securities firms to come up with procedures for aggregating data to support capital computations that were reported in their "FOCUS" reports (Holton, 2002b). From the late 1970s, a number of major financial firms had started to work on internal models to compute and aggregate risks across the organization as a whole (Dowd, 1999a). At the start of 1980, the "haircuts" from the US SEC were made to reflect a 0.95 confidence interval of the amount of money a firm might stand to lose over a month liquidation period. Whatever crude the "haircuts" could have been, they were a VaR measure (Holton, 2002b).

The early days of VaR were much ruled by regulators. It was only by 1980 that organizations saw a real need to develop more advanced VaR measures, but these remained as practical tools known only to the professionals within the organizations (Holton, 2002a). Tracing the historical development of institutional VaR is quite tedious for two main reasons. First of all, VaR was used for internal purposes only by the firms and secondly, the VaR were not published and very rarely mentioned in literatures (Holton, 2002b).

One interesting piece of document, however, was a letter from Stephen C. Francis of Fischer, Francis, Trees and Watts to the Federal Reserve Bank of New York, to indicate that their VaR measure was similar to SEC's Uniform Net Capital Rule (UNCR) but only that they had employed more asset categories - namely 27 of them (Holton, 2002b).

Round about the 1980s, while working at the Bankers Trust, Kenneth Garbade described advanced VaR measures for fixed income markets (Holton, 2002b). They were believed to have been influenced, but certainly different from an internal VaR measure Bankers Trusts had themselves implemented earlier for use with its Risk-adjusted On Capital

(RAROC) system of capital allocation (Holton, 2002a). Bankers Trusts threw in even more efforts to improve existing VaR measures following the 1987 stock market crash (Holton, 2002b).

In the late 1980s, VaR was certainly not a household name but a lot of organizations were starting to get interested and involved. Chase Manhattan bank developed, during that period, a Monte Carlo based VaR measure for its use with its return on RAROC international capital allocation system. At the same time Citibank had implemented another VaR measure for capital allocation (Holton, 2002b).

2.1.2 Modern Era Value-at-Risk

The largest share of advancement and development on the topic of VaR came in the 1990s when the concept of VaR really took off (Holton, 2002b; Winterton, 2003) with Linsmeir and Pearson (1996) also believing that the concept and the use of VaR is recent. Though Linsmeir and Pearson (1996) reported that VaR was really being used in the late 1980s to measure the risks of active portfolios, they are adamant that the use of VaR really exploded in the 1990s.

The reasons for the upcoming of VaR during that time could be attributed to the proliferation of derivative instruments and the publicized losses that have spurred the world of finance and the field of risk management (Holton, 2002b). By 1993, a fair number of financial organizations were employing VaR measures to assess market risk, allocate capital or monitor risk limits (Holton, 2002a). In the same year, a study by the Group of Thirty entitled "Derivatives: Practices and Principles" strongly recommended

VaR analysis. The study's recommendations were largely accepted by the industry as the standard of "best practices" (Simons, 1996). Still, in 1993, the Group of Thirty requested a survey, which was conducted by Price Waterhouse. One of the main findings of the survey was that: among 80 responding derivatives dealers, 30% were using VaR to support risk limits with another 10% planning to do so (Holton, 2002a).

Linsmeir and Pearson (1996) reported that:

"Currently VaR is finding more importance and most major financial firms are using it".

In 1994, a follow-up to the survey of the Group of Thirty's global derivatives project reported that 43% of dealers were using some kind of VaR with 37% indicating at the time that they planned to use VaR by the end of 1995 (Linsmeir and Pearson, 1996).

The biggest breakthrough on the concept of VaR came from J P Morgan when they released their RiskMetrics™ in 1994 (Dowd, 1999a). The RiskMetrics™ system is said to have originated when J P Morgan's chairman at the time, Dennis Weatherstone requested from his staff a daily one-page report indicating the risk and potential losses over the next day across the organization's entire active portfolio (Dowd, 1999a). To meet up with the chairman's demand, the staff of J P Morgan had to develop a system to measure risk across different trading positions over the whole of the organization and then aggregate these risks into a single number. The measure used was VaR or the "most likely loss over the next trading day" (Dowd, 1999a; Holton, 2002b; Linsmeir and Pearson, 1996).

It could be said that RiskMetrics™ triggered quite a revolution among the financial institutions. VaR became increasingly important and was also being used by smaller financial firms, non-financial corporations and institutional investors (Linsmeir and Pearson, 1996). In 1995, a Wharton/CIBC Wood Grundy Survey on derivatives usage among US non-financial firms reported that 29% of respondents were using VaR to evaluate the risks of derivatives transactions (Linsmeir and Pearson, 1996). In the same year, 1995, a related survey by the Institutional Investor that time, showed 32% of firms using VaR as a measure of market risk (Linsmeir and Pearson, 1996). Moreover, a 1995 survey by the New York University Stern School of Business reported that 60% of firms managing pension funds were using VaR (Linsmeir and Pearson, 1996).

In the modern era of VaR, regulators also played an important role just like in the early days. They did not remain insensible to the VaR revolution. In 1995, the Basle Commission on Banking Supervision (BCBS) proposed allowing banks to calculate their capital requirement for market risk with their own VaR models, but using certain parameters imposed by the committee (Holton, 2002b). In June 1995, the US Federal Reserve proposed a "precommitment" approach to allow banks to use their own VaR models to compute market risks with fines to be imposed in the event that losses exceed capital requirement (Holton, 2002b). In December of the same year, the US SEC had released for comment a proposed rule for corporate risk disclosure, which listed VaR as:

"One of the three possible market risk disclosure measures".

The European Union's Capital Adequacy Directive (CAD) also jumped on the bandwagon in 1996 when they allowed VaR models to be used in capital requirements for foreign exchange positions and they were moving towards a decision to allow VaR to calculate capital requirements for other market risks (Holton, 2002b).

The rapid development of VaR and its acceptance was something inevitable, as already the Bank of New York, considered to be the world's largest custodian with $6.8 trillion under custody publicly announced its interests in VaR and the release of its "RiskManager"- a sophisticated tool to compute VaR (Bank of New York Press Release, 1999) a move that did not leave other leaders in VaR technology without any reactions. In 2003, as financial firms were preparing themselves for Basel II (Marlin, 2003), J P Morgan Chase and SunGard had joined forces to come up with sophisticated offerings to aid analyze risk (Marlin, 2003).

2.2 Perception of Value-at-Risk by Investors

Value-at-Risk (VaR) has hit the financial world at a time when a new tool to measure risk was desperately needed. It has attracted a lot of praises from both academics and practitioners, but VaR has not escaped its detractors who firmly believed that it is more of a cult than what the market needed. This has brought up the VaR debate and has split the financial world into two about the subject. This section analyses the pros and cons of VaR that has been documented. The debate, however, is still heating up.

2.2.1 The Contributions of Value-at-Risk

The advent of Value-at-Risk (VaR) has been welcomed by a large portion of the community of investors and praises are still drooling over its benefits. Simons (1996) was found to be full of praises about the arrival of VaR on the scene when she said:

"In the last two years an approach to risk management called VaR has been accepted by both practitioners and regulators as the right way to measure risks becoming a de facto industry standard".

Schachter (1997) believes that the idea behind the development of VaR was to provide a single number, one that could encapsulate all information about a portfolio's risk, one number that could be computed quickly and one that could be communicated to non-technical senior managers. It was a statement repeated in another article (Measuring Value-at-Risk, Contingency Analysis, 1996) where it could be read:

"VaR is a powerful tool for assessing market risk; being applicable to all liquid assets and encompassing, at least in theory, all sources of market risk, VaR is an all-encompassing measure of market risk".

According to McGinn (1998), VaR is still a new phenomenon in the field of risk management, which is advancing beyond the less sophisticated methods to measure risks and which is finding a lot of takers and certainly continuous usage in financial organizations as a risk management tool- a feeling completely shared by numerous people. One of them, Winterton (2003) quoted:

"Any risk manager would be interested in measuring risk and thus VaR do have some use in risk management".

Perhaps leading the pack as far as praises for VaR are concerned must be Falkenstein (1997) who reported that VaR has become an *"indispensable tool"* for monitoring risks and an *"integral part of methodologies that allocate capital to various lines of business".* This may sound a little like an over-statement but Jorgensen (1998) backed this argument by saying that VaR has emerged as a major tool to measure market risks and that it is being used as a regulatory tool *"for ensuring the soundness of the financial system".*

Amman and Reich (2001) went one step further and reported:

"Most widely used tool to measure, gear and control market risk is VaR";

and was found to say that various financial firms and *"interest groups"* have recommended VaR as a portfolio risk measurement tool. Even the Bank of New York has jumped on the bandwagon of VaR praises when, in a press release in 1999, the bank described VaR as a *"critical tool in the risk management process"* and is backing its development by investing into RiskManager - software to compute VaR (Bank of New York Press release, 1999).

Regulatory bodies have been omnipresent in the development of VaR and it is without surprise that they think highly of the concept. Hull and White (1998) reported that central bank regulators have adopted VaR as:

"The major determinant of the capital banks are required to keep to cover potential losses arising from market risks they are bearing".

This is placing a lot of faith in a fairly new concept. However, McGinn (1998) backed this argument by saying that:

"VaR is still the measure regulators are asking for; there is still a demand to produce a VaR".

Dowd (1999a) has investigated the contributions of VaR from another angle and was found saying that VaR definitely brings a plus to the risk management process. Dowd (1999a) quoted that it gives top management:

"A much better handle on risks, thus leading to more informed and better risk management".

Dowd (1999a) went one step further by saying that VaR:

"Leads to a robust new control system that makes it much harder for fraud and human error to go undetected".

VaR also helps to discourage excessive risk taking (Dowd, 1999a) and before its advent, shareholders were not in a position to access the total trading risks financial organizations were assuming (Jorion, 2002).

It has to be left to Jorion (2002), considered to be one of the pioneers of VaR to wrap up the list of VaR contributions. Jorion (2002) agreed with the earlier authors:

"VaR has become a standard benchmark for measuring risks".

Jorion (2002) backed his statement with an extract from the Group of Thirty's report on derivatives which stated that *"market risk is best measured as VaR".* Overall, it seems that VaR is an:

"Indispensable tool for navigating through financial markets". (Jorion, 2004).

2.2.2 The Critics on Value-at-Risk

Like any new concept that wants to establish itself, Value-at-Risk (VaR) has not escaped the critics and over times VaR has attracted a fair share of criticism. According to Holton (2002a), criticisms of VaR tend to follow three themes:

1. Different implementations of VaR produced inconsistent results;
2. As a measure of market risk, VaR is conceptually flawed; and
3. Widespread application of VaR entails systemic risks.

Critics in the first camp include Beder (Holton, 2002a) who performed an experiment using Monte Carlo and Historical approaches to compute sixteen different VaR measurements for each of three portfolios; all the results tended to be inconsistent, which led the experimenter to describe "*VaR as seductive but dangerous*". Simons (1996) also backed up this inconsistency when she wrote:

"*There is no generally accepted way to calculate it and various methods can yield widely different results*".

Marshall and Siegel (Holton, 2002a) also carried out a little test on their own when they approached eleven software vendors and provided each one of them with several portfolios, such that each vendor would be calculating VaR for the same portfolios; the vendors should have got the same results, but they did not. Winterton (2003) on his side believes that one problem could be in the methodology assumption:

"The future cannot differ very much from the past, and in some cases, only relatively recent past is taken; intuitively, new developments can occur".

McGinn's (1998) remark also fall in this category of criticism when he said that:

"VaR offers a snapshot; it is not comprehensive. With all the data requirements that exist, to do a good VaR, you may have to run several iterations".

The second line of criticism attacked VaR on its conceptuality and believed it is flawed. Leading the pack, here, is Taleb (Holton, 2002a) who was found to say:

"The condensation of complex factors naturally does not just affect the accuracy of the measure. Critics of VaR argue that simplifications could result in such distortions as to nullify the value of the measurement".

Perhaps this could have been a critic that could be overlooked. However, first Hoppe (1999) reported that:

"The powerful industry consensus behind VaR cannot hide the fact that the measure rests on statistical assumptions that do not correspond to the real world. The results of VaR calculations are thus literally nonsensical",

and second, Dowd (1999b) was found to say that:

"A major problem facing VaR practitioners is that VaR is an extreme quantile on a return distribution and yet we have relatively few extreme observations with which to estimate it. Our VaR estimates are therefore imprecise".

However, the above problem could be resolved by Extreme Value Theory (Ganief, 2001).

The founder of the Algorithmics software, Dembo (Holton, 2002a) also goes along with this criticism and believes that the concept of VaR is a pretty good idea, but the way that it is being calculated nowadays is bad news as the calculation errors could be huge. He said:

"Often the number that is being computed is almost meaningless".

Winterton (2003) could not have been more direct when he talked about the different "guises" of VaR and believed VaR is compromised by too many unrealistic assumptions. Simons (1996) thinks that VaR is only one of the many tools to manage risk and according to her also VaR is based on a number of unrealistic assumptions. Wallace (1997) went even further with his article on non-financial corporations and reported:

"As a state-of-the-art risk management tool, VaR has been remarkably unsuccessful in catching on with non-financial corporations. This is part due to its relative statistical complexity".

These critics are based on a practical note, but underlying them are certainly philosophical issues identified by Harry Markowitz and reported by Holton (2002a):

"If probabilities are subjective, it makes no sense to speak of the 'accuracy' of a VaR measure or of a 'forecast' of the correlation matrix".

The third line of criticism suggests that, if numerous market players use VaR for capital allocation or maintain risk limits, they will tend to simultaneously liquidate positions during market turmoil periods (Holton, 2002a). This is also a feeling shared by Falkenstein (1997) who wrote:

"One major problem in equating VaR to risk capital, however, is that it is contradicted by how actual firms have historically 'used up' their capital (i.e. defaulted) from losses due to position taking".

These days, VaR is being adopted for just every need: risk reporting, regulatory capital, and internal allocation of capital and performance measurement. However, the question is:

"Is VaR the answer to all risk management challenges?" (Schachter, 1997)

Dowd (1999b) believes, on the other hand, that there is no theory that exists to prove that VaR is the adequate measure to rely for optimal decision rules.

2.2.3 The Verdict

With the two camps well anchored on their positions and both backing their arguments, it might not ever be possible to close the curtains on the VaR debate. Schachter (1997) reported that: maybe it is the holy scale after all, but others are well present to contradict especially when it seems that VaR is not unanimous especially that according to Winterton (2003) it has failed the corporate world.

Schachter (1997) goes one extra yard and poses the question of whether VaR is:

"A tool or a rule"?

about which there seems very much of confusion as Schachter (1997) believes that VaR is getting used for simply anything and that is where investors are committing the mistake.

Value-at-Risk is a powerful tool to measure market risk and that is because it captures the risk of an asset or a portfolio of financial instruments across different positions of risks. However, care must be applied about its applications. It is better to look at VaR as a tool but certainly not a rule. Specialists must investigate whether its use in an organization is going to be a value-added and whether it is applicable to the business of the firm. It is recommended that VaR is not used alone as a market risk measure and it is better if it is complemented by other existing auxiliary methods.

Chapter 3

Value-at-Risk

3.1 Risk

In any line of business and in every aspect of life, people venture in new avenues in the pursuit of high returns. Accordingly they allocate resources and capital. In return for their investments they expect rewards in the form of money. However, these investments are not sure or guaranteed of achieving the rewards that might be expected. They are exposed to an element of uncertainty. According to Contingency Analysis (1996), risk is made up of two components namely:

1. Exposure.
2. Uncertainty.

Another word for uncertainty is ignorance. The reason why investors face market risk is simply because of their ignorance of the future behaviour of the markets they are trading in. They can assess and even make predictions about the market behaviours but to know exactly what is going to happen is impossible. To be able to make predictions or even take decisions, investors or risk managers must quantify this exposure to uncertainty, i.e. this risk.

Organizations accept willingly or not, the assumption, management and pricing of risk. In a broader picture, risk entails different forms. These usually include:

- Market Risk;
- Credit Risk;
- Liquidity Risk; and
- Operational Risk.

"Market risk is exposure to the uncertain market value of a portfolio", (Holton, 2002a).

Credit risk is the cropping up from failure of counterparty to meet its legal obligation. Liquidity risk, on the other hand, is the risk of loss arising from the inability to settle payments or inability to re-finance financial obligations. Finally, operational risk is the exposure to a wide range of risks namely: processing failure (operational), legal, regulatory and technological.

Knowing the existence of all these risks is one thing, but managing them is certainly a different perspective. The process of managing risks, as it is understood today is called "Risk Management". According to the article Financial Risk Management, Contingency Analysis (1996) and Linsmeir and Pearson (1996), Risk Management has become more pronounced lately because of some events that have certainly marked the financial world namely:

- Increase in risk profile of organizations;
- Volatility of markets;
- Proliferation of derivative instruments;
- High-profile disasters;
- Crashes of financial markets; and
- Regulatory requirements for a better management of risk.

The emphasis of this research is on market risk. Trading and market risk management encompass the overall risk profile of a firm. Strategic planning accesses the identified risk factors. It is important to understand and manage market risk before making investments decisions.

3.2 Market Risk and its Management

The earlier definition of market risk is too broad. Perhaps, to arrive to a more precise definition of market risk, it is better to analyze the factors composing market risk. The four most common market risk factors are:

1. Interest rates.
2. Foreign exchange rates.
3. Equity prices.
4. Commodity prices.

Interest rate risk is the unpredictable changes in interest rates that may adversely affect the value of a financial instrument or the valuation of a portfolio or the condition of the firm as a whole. Foreign-exchange risk is the uncertain movements in exchange rates that may affect the value of an organization's holdings and thus its financial position. Equity-price risk is the potential for adverse changes in the value of an organization's equity-related investments. Finally, commodity-price risk is the potential for adverse changes in the value of an organization's commodity-related holdings. With all the factors making up market risk, a more precise definition of market risk can be formed. Market risk can be defined as the risk to an entity of losses arising from potential adverse changes in the asset prices they are exposed to, including changes in interest rates, foreign-exchange rates, equity prices and commodity prices.

The exposure to market risk can be measured by the loss in capital invested. Such a measurement is important to the management of market risk. Sound market risk management will require that each market risk exposure is identified and compared to a firm's tolerance (riskpsychology.net, 2003). A nominal exposure report is one such method of reporting market risk. The key to the management of market risk is to decide whether or not to hedge the risky assets. Hedging of risky assets or simply the offsetting of risky investments is achieved through derivative instruments, more precisely through derivative contracts.

The method to measure market risk is quite straightforward. A portfolio is decomposed into its underlying risk factors according to the presence of the different types of financial instruments. The risk decomposition process will entail a further breakdown of each financial instrument into its pure risk components.

The decomposed portfolio will be then processed in two separate ways, namely:

1. **Risk Measurement:** This is the projected rates and prices used to estimate the risk of the portfolio.
2. **Valuation or Pricing:** This is revaluating the portfolio using current prices and rates for the relevant risk factors to estimate the earnings of the portfolio.

The valuation or pricing process of the portfolio of financial instruments implies the marking of the portfolio to market using current prices and rates. The mark-to-market will establish the value of the portfolio on a liquidation basis. This will provide valuable information on the success or failure of the transaction entered into, the earnings of the portfolio and the liquidation value of the portfolio.

Market Risk Management is helped through sophisticated market risk measurement techniques (riskpsychology.net, 2003) designed to estimate potential adverse changes in the market prices and rates and the quantification of the impact of these changes on the portfolio's value.

3.3 Traditional Market Risk Measures

There are numerous Market Risk measures that exist to compute such kind of risks. This has been termed traditional measures simply because they are measures that investors or risk managers have been using all this time. It is not to say that they are of no use nowadays. On the contrary, they are still here and are used to complement the sophisticated techniques. The traditional market risk measures that will be developed in this section are:

1. Volatility;
2. Beta;
3. The Greeks;
4. Duration and convexity; and
5. The other Market Risk measures.

3.3.1 Volatility

The volatility of financial variables is their degree of variability. A variable that fluctuates widely over time is said to have high volatility and one, which is stable, has low volatility. In finance the standard definition for volatility is:

"*The volatility of a random variable is the standard deviation of its returns*" (Volatility, Contingency Analysis, 1996).

In practice, volatilities are computed for variables such as: market value of a portfolio, interest rates, stock prices, exchange rates and so on. Two methods exist to estimate volatility and they are namely:

1. Historical volatility: This approach to estimating volatility is based on the application of time series techniques to historical data for a variable whose volatility must be estimated. This method is usually based on daily data.
2. Implied volatility: This method of estimating volatility is derived from option prices. Options pricing require volatility estimates as inputs. Options prices are volatile and the volatility models for the option can be used for the underlier.

A common question arising as a result of those two existing methods is: which one offers a better indication of market risk? No adequate answer exists as each has its strengths and definitely its limitations.

According to the article 'Volatility' (Contingency Analysis, 1996) implied volatilities are an

"Indication of risk that combines the insights of many market participants".

However, since implied volatilities are mainly prices, they can be biased. Historical volatilities, on the other hand, are highly flexible and reflect actual market fluctuations. They are applicable to any financial instrument or portfolio for which market data exist. The limitation of historical volatility is precisely about its data dependence. It may be that the data upon which historical volatility is based is stale, i.e. encompassing a period, which is not reflective of current market conditions. As a result, this estimate can measure a false or not adequate measure of risk. Also, for many instruments, historical volatility will say nothing about the riskiness, e.g. if this technique is applied to a call option, which was out-of-the-money but now is in-the-money, the historical volatility will be misleading.

3.3.2 Beta

Beta (β) is a market risk measure, which is employed mostly in the equity markets. Since it is related to equity markets, it will have equity-related risks (Beta, Contingency Analysis, 1996). According to the Capital Asset Pricing Model (CAPM), equity-related risk has two components:

1. Systematic Risk: This is the risk of holding the market portfolio.
2. Specific Risk: This is the risk arising from causes unique to individual stocks.

Specific risk can be diversified. Diversification is the reduction in market risk by investing in unrelated financial instruments. If a portfolio is largely diversified, an investor may find himself left with a portfolio which is close to the market portfolio. Such a portfolio has no specific risk. Since, technically, the composure of such a portfolio is the same as the market portfolio, it will bear only systematic risk. However, systematic risk is a risk, which cannot be diversified. Beta is such a market risk measure that calculates an instrument's (a share's) or a portfolio's systematic risk.

Mathematically, beta is equal to (Beta, Contingency Analysis, 1996):

$$\text{Beta}(\beta) = \frac{Cov(x_p, x_m)}{\sigma_m^2}.$$

Where: $Cov(x_p, x_m)$ is the covariance between a portfolio's (instrument) return and market return and

σ_m^2 is the squared market volatility.

Beta is more often used as a measure for a portfolio's risk. For a largely diversified portfolio, it can be informative as systematic risk will be the primary source of risk for such portfolios. However, for lesser diversified portfolios, specific risk together with systematic risk will be present. As such, beta for these portfolios will be misleading.

3.3.3 The Greeks

Derivative instruments as well as options tend to create a lot of risk exposures which are quite unpredictable but confinable. When trying to hedge a financial instrument or a portfolio, it is important to understand specific exposures to all sources of risks.

The Greeks are a set of factor sensitivities, which are extensively used by investors to calculate the exposures of portfolios that contain options and derivatives (Greeks, Contingency Analysis, 1996). Each one of the measures will calculate how the portfolio's market value will respond to changes in some variable, namely an underlier, implied volatility, interest rate or time. An underlier is the value from which a derivative derives its value. There are five Greeks namely (Greeks, Contingency Analysis, 1996):

1. Delta;

2. Gamma;
3. Theta;
4. Rho; and
5. Vega.

Delta (δ): The changes in values of an underlier are more often the primary source of risk in a portfolio containing derivatives instruments. Delta represents a first-order measure of sensitivity to an underlier (Delta, Contingency Analysis, 1996).

Assume that a portfolio, P, responds to changes of some underlier, x, with a current market value. Then there exists a relationship P = g (x) between the value of the portfolio and the price of the underlier, assuming other market variables to be constant. Accordingly, the value of the portfolio increases if the price of the underlier increases and the value of the portfolio will decrease if the price of the underlier decreases. This is the kind of information that delta conveys, along with the magnitude of such sensitivity (Delta, Contingency Analysis, 1996).

If a tangent line is fitted to the curve at the underlier current market value, the gradient or slope of that line will capture the magnitude and direction of the portfolio's sensitivity to the underlier. In fact, the value of the gradient of that tangent line is equal to the value of delta.

Analogously, in calculus, this is simply calculating the slope of a tangent line and it can be achieved by using differentiation:

If P = g(x), then $g'(x) = \frac{\Delta P}{\Delta x}$.

Since δ is the slope of the tangent line, then $\delta = \frac{\Delta P}{\Delta x}$.

As a result, an approximation for the behaviour of a portfolio can be obtained as:

$\Delta P \approx delta \times \Delta x$.

This is called the delta approximation and for any small change in the current value of the underlier, the portfolio will experience a corresponding small change (Delta, Contingency Analysis, 1996).

Gamma (γ): If delta summarizes the most significant information about a portfolio's sensitivity to an underlier, gamma captures the second most order significant piece of information. While delta captures the sloping effect of graph P = g(x), gamma will capture its curvature effect. Since gamma is a second-order measure it comes as no surprise that it will be obtained by the second derivative, i.e. g"(x) (Gamma, Contingency Analysis, 1996).

An approximation of gamma can be obtained by best-fitting a parabola to P = g(x) at its current market value. Generally, the best-fitting parabola has the form:

$$\text{Best-fit parabola} = ax^2 + bx + c$$

where a, b, c are constants which can be determined to achieve the best fit.

Gamma is equal to twice the coefficient of x^2, i.e. 2a. Moreover, the best-fit parabola also gives the portfolio's delta and that is equal to the constant b. Gamma not only provides information about the magnitude of the curvature, but about its directions as well. Positive gamma implies an open-upward curvature while a negative gamma corresponds to an open-downward curvature (Gamma, Contingency Analysis, 1996).

Formally, gamma is defined as follows, using the techniques of calculus:

$$\gamma = g''(x) = \frac{\Delta^2 P}{\Delta x^2}.$$

As a result, the portfolio's value can be calculated in response to small changes in the underlier as such:

$$\Delta P \approx \frac{\gamma}{2}\Delta x^2 + \delta \Delta x$$

This is the delta-gamma approximation (Gamma, Contingency Analysis, 1996).

Rho (ρ): It is one of the Greek factor sensitivities which are used by investors to measure exposures in portfolios containing derivatives instruments. In fact, rho calculates the linear exposure to the changes in the risk-free interest rate of a portfolio. The risk-free interest rate is considered to be a theoretical interest rate at which an investment may earn interest without incurring any risk. In practice, the risk-free rate is often assumed to be a short term Treasury rate (Rho, Contingency Analysis, 1996).

If P is the current value for the portfolio and underlier, then P = g (r) represents the relationship between the portfolio and the interest rate, r. Since rho is a first-order sensitivity measure and rho represents the gradient to the tangent line of the above function, then by using the techniques of calculus rho can be formally derived:

If $P = g(r)$

then $\rho = g'(r) = \frac{\partial P}{\partial r} = \frac{\Delta P}{\Delta r}$.

Thus, rho can be approximated as: $\Delta P \approx \rho \times \Delta r$

where Δr is the small change in the risk free rate and ΔP is the corresponding change in portfolio's value (Rho, Contingency Analysis, 1996).

For most portfolios, sensitivity to risk free rate is minor compared to other possible sensitivities. Thus rho is less significant but certainly not unimportant.

Theta (θ): This is a factor sensitivity applied by investors to calculate exposures to a portfolio containing derivatives. It is the only one amongst the Greeks that measures a portfolio's linear exposure with respect to time. Accordingly, theta gives an indication of the evolution of a portfolio when time changes, assuming all other market variables remain constant (Theta, Contingency Analysis, 1996).

If T denotes time and P_T denotes a portfolio's value at time T, then

$$\theta = \frac{\partial P_T}{\partial T} = \frac{\Delta P_T}{\Delta T}$$

with the derivative evaluated at time = 0. Analogously, in calculus, this is simply the rate of change of the value of the portfolio with time.

Theta can be approximated as follows:

$$\Delta P_T \approx \theta \times \Delta T$$

where ΔT is the small interval change in time and ΔP_T is the corresponding portfolio's change in value (Theta, Contingency Analysis, 1996).

Vega: Also referred to as kappa, this is the fifth factor sensitivity used by investors to measure exposures in a portfolio containing derivatives. Vega is mostly informative to portfolios that contain options which are either direct or imbedded. These portfolios are sensitive to the implied volatility of the underliers (Vega, Contingency Analysis, 1996). Generally, it can be seen that a long option position benefits from rising implied volatilities and will suffer from declining of such effects. On the other hand, a short position will show opposite behaviour.

Mathematically, vega is defined much the same like the delta and as the delta, it is also a first-order linear approximation of the price sensitivity of the portfolio. The only difference from those two measures is that delta calculates sensitivity to the underlier while vega calculates the sensitivity to its implied volatility. Formally, vega is defined as follows (Vega, Contingency Analysis, 1996):

If portfolio P is a function of implied volatilities, then

$$P = g(\sigma).$$

A tangent line is to be fitted to the curve at the current volatility. The slope of that line is the instrument's vega. Fitting of a tangent line and calculating the gradient are analogous to differentiation in calculus. Thus,

$$\text{Vega} = g'(\sigma) = \frac{\partial P}{\partial \sigma} = \frac{\Delta P}{\Delta \sigma}$$

As a result, vega can be approximated as such:

$$\Delta P \approx vega \times \Delta\sigma$$

where $\Delta\sigma$ is a small change in implied volatility from its current value and ΔP is the corresponding change in the portfolio's value as a result of the change in implied volatility. If a portfolio holds options on different underliers, it will have a different vega for each of the implied volatilities.

3.3.4 Duration and Convexity

Duration and convexity are factor sensitivities describing exposures to parallel shifts in the spot curve. A spot curve is yield in the interest rate curve. They are both applicable to fixed income instruments and a portfolio containing fixed income. Fixed income instruments are particularly sensitive to the changes in interest rates (Duration and Convexity, Contingency Analysis, 1996).

The fractional change in a portfolio containing fixed income instruments is a function of parallel shifts in the spot curve, i.e. a parallel shift in interest rates. It is important to note that the curve describing the relationship $\frac{\Delta P}{P} = g(\Delta r)$ captures the important information that a portfolio value will decrease if interest rate increases and the value will rise if interest rate falls. If a tangent line is fitted to the curve of price of fixed income against parallel shift in the interest rate, that tangent line will capture the magnitude and direction of the portfolio's sensitivity to interest rates. Duration is defined as that tangent line multiplied by negative one (Duration and Convexity, Contingency Analysis, 1996).

Mathematically, duration can be described as follows:

Let ΔP be the change in a portfolio's value,

Δr be a parallel shift in spot curve, measured in percentage,

$\frac{\Delta P}{P}$ is the percentage change in portfolio's value.

Accordingly, $\frac{\Delta P}{P} = g(\Delta r)$ is a relationship describing a portfolio's sensitivity to shifts in spot curve. From calculus, the tangent line is obtained as such:

$$\frac{\partial P}{p\partial r} = \frac{\Delta P}{p\Delta r} = \frac{\Delta P}{p} \times \frac{1}{\Delta r}$$

Duration is obtained by multiplying by -1 thus:

$$\text{Duration} = -1 \times \frac{\partial P}{p\partial r} = -1 \times \frac{\Delta P}{p\Delta r} = -1 \times \left(\frac{\Delta P}{p} \times \frac{1}{\Delta r}\right).$$

This leads to the approximation:

$$\frac{\Delta P}{P} \approx -duration \times \Delta r.$$

The unit of duration is normally years. Duration captures a fixed income instrument or a portfolio containing such instruments with a single number.

While duration investigates the downward sloping nature of the relationship $\frac{\Delta P}{P} = g(\Delta r)$, it says nothing about its upward curvature. Convexity is the measurement describing curvature (Duration and Convexity, Contingency Analysis, 1996).

To approximate convexity, a parabola is best fitted to the relationship of the function.

$$\text{Best-fit parabola} = \quad U(\Delta r)^2 + V(\Delta r)$$

where U and V are constants.

Hence, convexity = 2U.

Convexity also investigates direction on top of magnitude. Positive convexity is curvature bending upward while negative convexity is curvature bending downward.

Duration and convexity are considered good means to measure market risks in different situations. However, their limitations come from the fact that they only consider exposure to parallel shift in the spot curves.

3.3.5 Other Measures

According to Dowd (1999a), the following are also used as market risk measures:

- Regression analyses that estimate the exposures to interest rates, foreign exchange, equity, commodity and other market risks based on estimated regression relationships;
- Scenario (or 'what if') analyses estimating what is expected to be gained or lost under specific situations/scenarios;
- Zero arbitrage methods which use stochastic models to estimate risk exposures of portfolios containing derivatives; and
- Portfolio analyses focusing on the ways in which certain types of risks offset each other in a portfolio.

All these above measures have their particular uses but they are also flanked with limitations. Again according to Dowd (1999a), these above measures are limited as such:

- Regression analyses rely on the stability of assumed regression relationships and can be inexact if there are changes in these relationships;
- Scenario analyses can be difficult to carry out;
- Models are difficult to use and implement and they have their own limitations; and
- Portfolio analyses require too much data and run into problems if assumptions are violated.

While each of these measures has their own limitations, they also share common ones like the difficulty to compare and encompass risk across different financial instruments in a portfolio.

3.4 Value-at-Risk (VaR)

Value-at-risk (VaR) is a statistical risk measure that captures the market risk exposure of an asset or a portfolio. A technical definition is given by (Value-at-Risk, Contingency Analysis, 1996):

"VaR is an amount of money such that the portfolio will lose less than that amount over a specified period with a specific probability".

Jorion (2004) carries on on this path and further provides a technical definition of VaR:

"VaR summarizes the predicted maximum loss (or worst loss) over the target horizon with a given confidence interval".

Both those definitions are broad and technical and a simpler definition has to be found to explain VaR.

In simpler words, VaR is a risk measure enabling investors and risk managers to determine how much the value of an asset or a portfolio could decline over a given time horizon with a defined probability as a result of adverse changes in market conditions (Gugi et al, 1999). All these definitions show that VaR is based on two factors namely:

1. Time horizon: The period over which the asset in the portfolio will be held, also called holding period. For active portfolios with liquid assets, the typical holding period is 1-trading day, although regulators like the European Capital Adequacy Derivatives (CAD) require 10 days. Ideally the time horizon should correspond to the largest period required for orderly portfolio liquidation.
2. Probability: This is the confidence interval or significance level at which the estimate will be made. Choices about the confidence interval depend on its use. Risk aversion or high costs will imply that a larger amount of cash should cover possible losses, thus implying a higher confidence interval. Popular choices of confidence intervals are 95% and 99%.

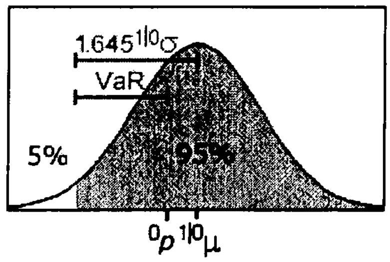

Diagram1: A 1 - day 95% VaR (Value-at-Risk, Contingency Analysis, 1996)

Basically, VaR expresses in relevant currency units the expected worst loss that may be incurred over a defined time horizon and with a specific significance level (Gugi et al, 1999). For example, if the daily VaR of an organization's trading portfolio is ZAR 50 million at the 99% confidence interval then it means that there is only 1 chance out of a 100 assuming normal market conditions for a loss greater than ZAR 50 million to be incurred.

According to Jorion (2004), VaR is a simple number that captures the exposure of a portfolio to market risk together with the probability of an adverse market move. VaR measures risk in the same units, i.e. the relevant country currency. This is the main difference from the other market risk measures. It is then left to the investors or risk managers to decide whether they are comfortable with this level of risk.

While VaR measures how much could be lost on the value of an asset or portfolio, it also gives an idea of how much cash that should be put aside as cushion for days when losses will unexpectedly be large. As a result, VaR is not only a market risk tool to quantify risk but it also aids risk management (Value-at-risk, Contingency Analysis, 1996). While Jorion (2002) believes that VaR captures the effects of leverage, diversification and probability of adverse market movements into a single relevant currency amount which is easy to communicate to management, Schachter (1997) reports that VaR was designed to produce a single number that would encapsulate information about a portfolio's risk.

Formally, Value-at-Risk (VaR) is a type of risk measures that describes the market risk of a portfolio probabilistically. In itself, VaR is a powerful tool but it is also quite a challenge. The main force of VaR is its generality and its applicability to all liquid assets. Since VaR encompasses, at least in theory, all the sources of market risk it is therefore an all-encompassing measure of market risk (Value-at-Risk, Contingency Analysis, 1996).

The challenge posed by VaR also comes from its generality, as with its force. To be able to measure market risk in a trading portfolio using VaR, some means have to be found for determining the probability distribution of the portfolio's returns. The two following concepts must be distinguished:

1. A VaR measure
2. A VaR metric.

3.4.1 Developing a Value-at-risk Measure

A VaR measure is a series of operations that are performed to calculate VaR of a trading portfolio. In order to apply a VaR measure, it has to be implemented in some manner. In this section, a VaR measurement derivation will be elaborated.

Before deriving a VaR measure, the following notations must be defined (Value-at-Risk, Contingency Analysis, 1996):

- Time is measured in trading days;
- Current time is 0;
- Portfolio current market value is P^0;
- Market value in trading day 1 is P^1 and is unknown; and
- $x^{1|0}$ indicates a parameter for the portfolio in time 1 conditional on information available in time 0.

The task is about finding a probability distribution for P^1. One way that exists to achieve this is to assume a standard statistical distribution. Specifically, the Normal distribution is adopted, as its parameters (μ and σ^2) are fully described. Now, if P^1 follows a Normal distribution, then all that is needed to compute VaR is to estimate the $\mu^{1|0}$ and the $\sigma^{1|0}$ of that characterized distribution. Assuming a 95% confidence interval, VaR can be obtained as follows:

$$\text{VaR} = 1.645\ \sigma^{1|0} + (P^0 - \mu^{1|0}) \qquad \text{(A)}$$

where the 5% quantile in Normal distribution is 1.645. In practice, however, $\mu^{1|0}$ is often close to P^{0}. Thus, the above equation A reduces to:

$$\text{VaR} = 1.645\ \sigma^{1|0} \ldots\ldots\ldots\ldots\ldots\ldots \text{(B)}$$

If $\sigma^{1|0}$ can be estimated, then equation B will be able to give a value of VaR.

Estimating $\sigma^{1|0}$ of a portfolio's market value is analogous to the task of estimating σ of a portfolio of returns as in Modern Portfolio Theory, only that VaR deals with market values and not returns.

To obtain $\sigma^{1|0}$, the following is derived:

Suppose $X_1 \ldots X_m$ are random variables with standard deviations σ_i and correlations $\rho_{i,j}$.

Let Y, a random variable, be defined as a linear polynomial of the X_i such that

$$Y = a + b_1 X_1 + \ldots + b_m X_m \ldots\ldots\ldots\ldots\ldots\ldots \text{(C)}$$

Then, σ_Y will be given by

$$\sigma_Y = \sqrt{\sum_i (b_i,\sigma_i)^2 + 2\sum_{j\geq 1} (b_i,\sigma_i)(b_j,\sigma_j)\rho_{i,j}} \ldots\ldots\ldots\ldots \text{(D)}$$

Expression D can now be used to estimate $\sigma^{1|0}$ of the portfolio's market value.

For that to happen, however, let the portfolio's holdings (elements) be υ_i instruments in m assets. The accumulated market values of the m assets at time 1 are random variables, which will be denoted by S_i^1. As a result, the portfolio's value at time T = 1 will be:

$$P^1 = \upsilon_1 S_1^1 + \ldots + \upsilon_m S_m^1 \ldots\ldots\ldots\ldots\ldots\ldots \text{(E)}$$

Based on expression (E) and applying expression (D), $\sigma^{1|0}$ can be obtained. All that are needed are simply information about σ_i and $\rho_{i,j}$ of S_i^1. This can be a daunting process and a manageable solution could be to model the portfolio's behaviour, not in terms of its assets but rather in terms of the relevant risk factors that are specific to the assets in the portfolio.

The n modeled risk factors are termed key factors and their values are denoted at time T = 1 as $\mathbf{R}^1$, where

$$\mathbf{R}^1 = \begin{pmatrix} R_1^1 \\ . \\ . \\ . \\ R_n^1 \end{pmatrix}.$$

A valuation formula λ_i must be defined for each asset such that:

$$S_i^1 = \lambda_i(\mathbf{R}^1) \text{.. (F)}$$

P^1 is a linear polynomial of the assets values S_i^1 (Expression E), thus:

$$P^1 = \sum_{i=1}^{m} \upsilon_i S_i^1 = \sum_{i=1}^{m} \upsilon_i \lambda_i(\mathbf{R}^1) \text{.. (G)}$$

As a result,

$$P^1 = \theta(\mathbf{R}^1) \text{.. (H)}$$

where $\theta = \lambda_i \upsilon_i$.

Expression (H) is called a portfolio mapping where θ is the portfolio mapping function.

The portfolio mapping function θ, will map the n-dimensional space of the key factors to the 1-dimensional space of the portfolio's market value. Knowing one realization of $\mathbf{R}^1$, θ will yield the corresponding value of P^1. This process, however, does not give the entire distribution of P^1, which is needed to be able to estimate $\sigma^{1|0}$. This is because $\mathbf{R}^1$ is independent of the composition of the portfolio and will therefore not be able to tell how risky the portfolio is.

The problem is how to achieve the entire distribution of P^1?

One way to achieve this is to assume the linearity of the portfolio, but what if the portfolio is not linear?

Since the concept of VaR is being generalized here, it is preferable that the above problem is being solved in a general way. To be able to succeed here, the general problem facing the calculations of VaR must be formulated.

To calculate VaR, the distribution of P^1 must be characterized conditional on information at time T = 0. The problem is two-fold (Linsmeir and Pearson, 1996; Value-at-Risk, Contingency Analysis, 1996):

1. The first part is about the key factors, R_i^1. Since they are observable financial variables, data about their past (historical data) must be available for them. Basing on these data, the joint distribution of $\mathbf{R}^1$ can be characterized using the information of $\sigma_i^{1|0}$ and $\rho_{i,j}^{1|0}$ for R_i^1. The distribution of $\mathbf{R}^1$ must be converted into a characterization for $\mathbf{R}^1$. On its own, however, the characterization of the distribution of $\mathbf{R}^1$ cannot achieve this procedure, as it is independent of the composition of the portfolio. As such, the distribution of $\mathbf{R}^1$ alone cannot tell how risky the portfolio is.

2. The second part is about the mapping that relates P^1 to $\mathbf{R}^1$. This is a formula that will change constantly to reflect the portfolio's evolving composition. The expression (H) contributes to the analysis what the characterization of the distribution of $\mathbf{R}^1$ (part 1) does not. It will reflect the composition of the portfolio. However, on its own, the mapping cannot also give an indication of the riskiness of the portfolio, as it does not contain any information about market factors.

To obtain an estimate for $\sigma^{1|0}$, it is therefore necessary to merge the two parts of the problem. What has to be done is simply to somehow filter the market information contained in the characterization of the distribution of $\mathbf{R}^1$ (part 1) into the portfolio information contained in the portfolio mapping function (part 2).

Every VaR measure will address this two-folded problem. All of the VaR measures share common components for solving this issue. All of them must somehow specify a portfolio mapping function; all must also characterize the distribution of $\mathbf{R}^1$; and all VaR measures must combine those two pieces to draw the distribution of P^1.

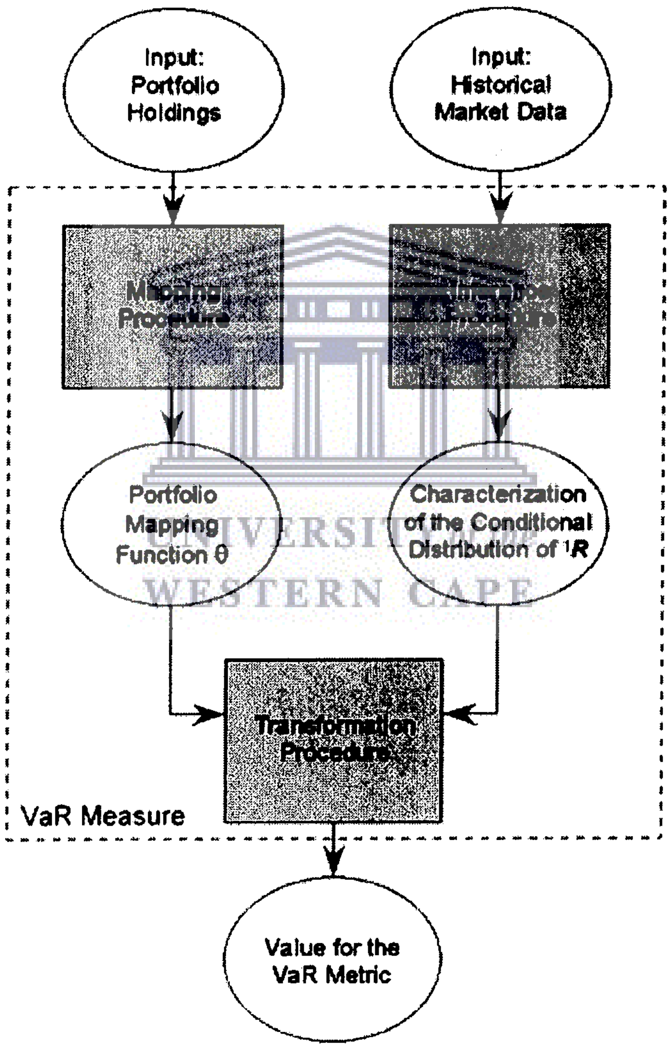

Diagram 2: Procedural Steps of VaR Measure (Value-at-Risk, Contingency Analysis, 1996)

Any practical VaR measure as shown by the above flowchart includes three basic procedures namely:

1. The Mapping Procedure;
2. The Inference Procedure; and
3. The Transformation Procedure.

By specifying a portfolio mapping function, a mapping procedure will describe the portfolio's exposures. By characterizing the joint distribution of $\mathbf{R}^1$, an inference procedure will on its turn describe the portfolio's uncertainty. Now, both exposure and uncertainty are the two components of risk and the transformation procedure will then combine these two components of risk to describe the distribution of P^1 that will summarize the value in the form of a VaR metric. As a result, a transformation procedure will describe risk (Value-at-Risk, Contingency Analysis, 1996).

A mapping procedure will accept a portfolio's composition as its input. The output will be a portfolio's mapping function θ that defines P^1 as a function of $\mathbf{R}^1$, i.e. $P^1 = \theta(\mathbf{R}^1)$. To specify θ, the portfolio mapping function, is a task belonging more to the field of financial engineering (Value-at-Risk, Contingency Analysis, 1996). This issue will not be addressed here as it falls outside the scope of this research.

An inference procedure consists of characterizing the joint distribution of the key vector $\mathbf{R}^1$ conditional on the availability on information at the moment time T = 0. Generally, it will accept past information (historical data) as input and will apply time series analysis techniques to characterize the joint distribution conditional on past information available. The most common technique applied is the Exponential Weighted Moving Average (EWMA).

A transformation procedure will combine those two outputs from the relevant two described procedures, i.e. the mapping and the inference procedures, and will use them to characterize the distribution of P^1, conditional on past information at time T = 0. Based on that characterization and maybe the portfolio's current value P^0, the transformation procedure determines the value of the desired VaR metric. The result is a VaR measurement.

Three basic forms of transformations exist (Value-at-Risk, Contingency Analysis, 1996) and they are namely:

1. Linear;
2. Historical Simulation; and
3. Monte Carlo Simulation.

All three forms have different ways to actually characterize the distribution of P^1 and thus obtaining the value of $\sigma^{1|0}$, from which the VaR can be computed. Traditionally, VaR measures have always been categorized according to the different transformation procedures they employ as enumerated above. They are namely:

1. A linear VaR measure, also called Parametric VaR;.
2. Historical VaR measure; and
3. Monte Carlo VaR measure.

Those three above categories actually characterize the three methodologies that exist to compute VaR (extensively discussed in section 3.5).

3.4.2 Interpretation of a VaR Metric

A measure is simply an operation for assigning a number to something. A metric is defined as the interpretation of the number assigned. And finally, a measurement is the outcome of applying a measure and obtaining a number (VaR Metric, Contingency Analysis, 1996). There are several risk metrics like: volatility, the Greeks, duration and convexity, beta and so on. Value-at-Risk (VaR) is also a risk metric since it measures risk.

The same variables definition used in the previous section is applicable here and will thus not be redefined. Formally, a VaR metric is just a real-valued function of namely:

1. The portfolio's current value P^0; and
2. The distribution of P^1, conditional on past information available at time T = 0.

A couple of VaR metrics exist and they will be investigated here (VaR Metric, Contingency Analysis, 1996):

- Standard Deviation of portfolio simple return Z^1, conditional on past information is a VaR metric:

$$std(Z^1) = std\left(\frac{P^1 - P^0}{P^0}\right) = \frac{1}{P^0} std(P^1)$$

- Quantiles of portfolio's loss: $L^1 = P^0 - P^1$.
- Expected tail loss (ETL), also called expected shortfall, is also a good VaR metric. This is the mean loss of the portfolio assuming that loss exceeds some quantiles of loss.

Formally, to specify a VaR metric the three following things must be distinguished:

1. Time period: this is the VaR horizon, e.g. 1 day, 1 month, 1 year and so on;
2. The base currency: this is the currency in which P^0 and P^1 are denominated; and
3. The function of P^0 and the conditional distribution of P^1.

Reporting of VaR metrics follow some kind of convention so as to make them standard and meaningful for different countries. The following is adopted as some kind of convention for naming VaR metrics (VaR Metric, Contingency Analysis, 1996):

- The metric's name is given in the order: horizon, function and currency followed by "VaR";
- If horizon is expressed in days but without further qualification, it is understood that that they are actually trading days; and

- If function is a quantile of loss, it will be indicated as a percentage.

The following are examples of VaR metrics which are quoted for portfolios:

- 1-day standard deviation of simple return ZAR VaR;
- 1-week 90% JPY VaR; and
- 2-week 95% ETL USD VaR.

3.4.3 Advantages of Value-at-Risk

Value-at-risk (VaR) is known as the maximum likely loss on a portfolio, which is predicted on a level of likelihood on a time horizon. A VaR has important characteristics and consequently advantages (Dowd, 1999a):

- A VaR figure or digit (i.e. a VaR metric) provides a common consistent quantification of risk across different positions and risk factors. VaR eases the comparisons of risks across different portfolios and certainly across different assets or financial instruments;
- It enables risk managers and investors to aggregate risk across different positions and risk factors, so that such risks can be added, like adding fixed-income risk to equity risk; and
- VaR takes account of the correlations between distinct risk factors, e.g. if two risk factors offset each other, VaR will allow this offset while informing that the overall risk is quite low.

3.4.4 Limitations of Value-at-Risk (VaR)

Value-at-Risk is an important tool and a useful tool in the management of risk but it is certainly not a panacea (Simons, 1996). For investors and traders, VaR is just another measurement item in their toolkit. The risk managers will be looking at the different traditional measures of risk which means that they will go beyond VaR. Limitations of Value-at-risk include (Simons, 1996):

- VaR focuses on a single arbitrary point on the profit and loss distributions, while it would be preferable to be looking at the whole distribution;
- VaR provides little information on how risks are to be measured in conditions of extreme market; and
- VaR computations are difficult during times of market crises when correlations between financial products break down, liquidity vanishes and price data might be unavailable. To model risk in such conditions would be quite a daunting task as a lot of information would be withheld due to competition.

3.5 Value-at-Risk Methodologies

As seen in Section 3.4, VaR methodologies are actually categorized by the way they process the transformation procedure. There are three important transformation procedures and accordingly they characterize the three different VaR methodologies that will be discussed in this section. The three methods are namely:

1. Parametric Method;
2. Historical Simulation Method; and
3. Monte Carlo Simulation Method.

Each method will be discussed separately followed by a five-step procedure to apply each method for a multiple instrument portfolio just like the one understudy in this research. The advantages and disadvantages of each method will also be discussed and finally, in a subsection, the three methods will be put on the same platform and compared accordingly.

3.5.1 The Parametric Method

When Value-at-risk was first developed, the parametric approach was the standard as it was computationally efficient. Its efficiency depends on its analytical approach that directly calculates a solution (Capital Market Risk Advisors, 2001). The parametric approach is most of the time termed the Variance-Covariance Method, precisely because of its analytical approach.

The Variance-Covariance approach is based on the assumption that the underlying market factors follow a multivariate Normal distribution. With this assumption in mind, it is possible to find the distribution of mark-to-market portfolio profit and loss, which will also be Normal (Linsmeir and Pearson, 1996). Once the distribution of Profit and Loss has been obtained, and because it follows a Normal distribution, its properties can be applied to determine the loss that will be equaled or exceeded x % of the time, i.e. the VaR. The reason why the properties are determinants for the loss is because the Normal distribution is fully defined with its two parameters.

While this approach seems more like a "black box" because it depends on a handful of statistical formulas, it does capture the determinants of VaR (Linsmeir and Pearson, 1996). The Variance-Covariance method identifies the notions of variability and the co-movements with concepts from Statistics like standard deviation and correlation. These two Statistical concepts will determine the Variance-Covariance matrix of the assumed Normal distribution of changes in the market factors (Linsmeir and Pearson, 1996).

A key step about this approach is known as the "risk mapping", which was discussed earlier. This involves taking the actual instruments and "maps" them on a set of simpler, standardized instruments or positions. Each of these positions is actually associated with a single market factor. This approach remains an excellent one especially for a portfolio containing minimal optionality (or option instruments) and holdings in highly efficient markets when they can be expected to follow a Normal distribution (Capital Market Risk Advisors, 2001).

Variance-Covariance Methodology Application for a Multiple Instrument Portfolio (Linsmeir and Pearson, 1996)

Step 1: Investigate the different market factors and the following standardized positions, which are directly related to these market factors. Map these instruments onto the standardized positions.

Step 2: Assume a multivariate Normal Distribution for the percentage changes in the market factors. Further estimate the parameters of that distribution (σ_i and $\rho_{i,j}$).

Step 3: Using the standard deviations and correlations of the market factors, determine the σ_i's and $\rho_{i,j}$'s of changes in the values of the standardized positions. The σ_i's in the values of the standardized positions can be determined by multiplying the standard deviations of the market factors and the sensitivities of the standardized positions to the changes in the market factors. The correlations between changes in the values of the standardized positions and the correlations between the market factors are equal, except that the sign of the correlation will change if the value of one of the standardized positions changes inversely with changes in the market factors.

Step 4: With the standard deviations and correlations between changes in the value of the standard positions now known (from step 3), the variance and accordingly the standard deviation of the portfolio can be computed using the properties of the sum of Normal random variables. The distribution of the portfolio profit and loss can be obtained.

Step 5: One of the properties of the Normal distribution is that outcomes less or equal to 1.65 standard deviations below the mean will occur only 5% of times. If a probability of 5% is therefore used to determine VaR, then VaR will be equaled to 1.65 times the portfolio standard deviation.

Advantages of the Variance-Covariance Method

- Easy to implement;
- Calculations are quickly performed;
- Easy to investigate alternative assumptions about correlations/standard deviations;
- Based on well-known applications in Modern Portfolio Theory and widely disseminated by J P Morgan RiskMetrics^{TM}; and
- Easy to compute in an excel spreadsheet if the input values are known.

Disadvantages of the Variance-Covariance Method

- Inability to capture the risks of portfolios including options;
- Difficult to report to top management;
- Produces Misleading VaR estimates when the past is atypical;
- The assumption of Normality is not always true; and
- The linearization of the prices is quite a problem when the portfolio contains a fair share of options.

3.5.2 The Historical Simulation Approach

The historical simulation methodology repeatedly values the financial instruments of a portfolio according to the market conditions that have existed over a specific period of time. This method is therefore quite intuitive (Capital Market Risk Advisors, 2001). Historical simulation is a plain, atheoretical method which requires relatively few assumptions about the statistical distribution of the market factors. In essence the method is about using past information (historical data) in market rates and prices to determine a distribution of potential future profit and loss of a portfolio and then reading off the VaR as simply the loss that exceeds only x% of the time, with x% to be decided on (Linsmeir and Pearson, 1996).

Generally, the historical simulation works as follows:

A distribution for the profit and loss distribution is characterized by taking the current portfolio and subjecting it to the actual changes in the market factors that have been experienced for some N periods of time, in this case over trading days.

In a simpler language, N sets of hypothetical market factors are being constructed using the current values and the changes experienced over the past N periods. With these hypothetical values of market factors obtained, N hypothetical mark-to-market portfolio values are calculated. This process will yield N hypothetical mark-to-market profit and loss of the portfolio when compared to the current mark-to-market portfolio values. Once the hypothetical mark-to-market profit and loss values for each of the past N periods have been computed, the distribution can be characterized and the value-at-risk can be obtained (Linsmeir and Pearson, 1996).

Historical Simulation Application to a Multiple Instrument Portfolio (Linsmeir and Pearson, 1996)

Step 1: Investigate the different market factors and try to obtain formulas expressing mark-to-market values of the instruments in terms of the market factors, risk factors.

Step 2: The past information (historical values) of all the market factors have to be obtained for the last N periods. The daily changes in these rates will be used to construct hypothetical values of the market factors which are used in the computation of the hypothetical profit and loss (step 3) since the daily VaR metric is a measure of the portfolio loss caused by changes over a daily holding period.

Step 3: This is the crucial step. It is important that the mark-to-market profit and loss on each instrument in the portfolio be calculated and then added together for every day.

Step 4: Rank the mark-to-market profit and loss from biggest profit to the smallest loss.

Step 5: Choose the loss which is equaled or exceeded x% of the time, where x has to be predetermined. Using a probability of x%, this is the value-at-risk.

Advantages of Historical Simulation (Linsmeir and Pearson, 1996)

- Ability to capture the risks of portfolios that contain options;
- Ease of implementation;
- Computations are performed rather quickly; and
- Fairly easy to communicate to top management.

Disadvantages of Historical Simulation (Linsmeir and Pearson, 1996)

- Will produce quite misleading VaR measures when the recent past is atypical;

- Difficult to perform the analysis for examining the effects of alternative assumptions; and
- The recent past might not be reflective of the changes for the period understudy.

3.5.3 The Monte Carlo Simulation

Monte Carlo simulation is seen as a hybrid between the parametric approach and the historical approach. The Monte Carlo technique will use the variance-covariance matrix as the parametric approach to compute an analytical solution, which will drive the simulation (Capital Market Risk Advisors, 2001).

The Monte Carlo simulation technique has numerous similarities to historical simulation. The main difference noticed is the that instead of driving the simulation using observed changes in the market factors over the past N periods to characterize N hypothetical portfolio profit and loss values, the Monte Carlo technique selects a statistical distribution that is thought to adequately capture the potential changes in the market factors.

A pseudo-random number generator is used afterwards to generate thousands or even many more hypothetical changes in the market factors. These are further used to characterize thousands of hypothetical portfolio profit and loss values based on the current portfolio, and the distribution of potential portfolio profit and loss. Finally, the VaR is computed from this distribution (Linsmeir and Pearson, 1996).

Monte Carlo Simulation Application to a Multiple Instrument Portfolio (Linsmeir and Pearson, 1996)

Step 1: Investigate the different market factors and try to obtain pricing formulas expressing the mark-to-market values of the different financial instruments in terms of the market factors.

Step 2: Determine or assume a joint distribution of potential changes in the values of all of the market factors that are present. The ability to pick up on the distribution is the main feature of Monte Carlo simulation. Once the distribution has been specified, estimate the values of σ_i and $\rho_{i,j}$.

Step 3: Once the distribution has been chosen, a pseudo-random number generator is used to generate N hypothetical values of the changes in the market factors. Then calculate the mark-to-market profit and loss on every instrument present in the portfolio and add together for each day.

Step 4: Rank the mark-to-market profit and loss from largest profit to smallest loss.

Step 5: Choose the loss that is equaled or exceeded x% of the time, where x% has to be predetermined as usual. Using a probability of x%, this is the value-at-risk.

Advantages of the Monte Carlo Simulation (Linsmeir and Pearson, 1996)

- The ability to capture the risks of portfolios containing options; and

- Easy to perform analyses for examining effect of alternative assumptions.

Disadvantages of the Monte Carlo Simulation (Linsmeir and Pearson, 1996)

- Computations take long;
- Not easy to explain to top management; and
- Will produce misleading VaR when recent past is atypical.

3.5.4 Comparison between the Methodologies

Schachter (1999) talks about the accuracy of the VaR methodologies by saying:

"Accuracy is in the eye of the beholder".

For Linsmeir and Pearson (1996), the question is simple; from the three existing methodologies of VaR:

"Which method of calculating Value-at-Risk is best?"

Unfortunately, there is no simple answer to what looks to be an easy question, as it will depend on the nature of the portfolio and certainly on the data that will be used in the estimation of VaR (Schachter, 1997). The three methods differ in different aspects as discussed in the advantages and disadvantages in the previous section, and summarized here (Linsmeir and Pearson, 1996):

- Ability to capture the risks of options and option-like financial instruments;

- The ease of the implementation of the methodologies;
- The ease of communicating and explaining to top management;
- The reliability of the results obtained; and
- The flexibility of analyses of the effects of potential changes in the assumptions.

3.6 Applications of Value-at-Risk

Financial organizations can either be dealers or simply investment firms. They both are exposed to risk the same way as a result of their trading activities and their investment positions. Value-at-risk can measure the risks of these different types of institutions, although a few differences will exist in the application of the VaR. However, the principal elements in the use of VaR are similar. According to Dowd (1999a), VaR figures have numerous uses. Therefore, VaR measurements attract a lot of users. According to Dowd (1999a) the main applications of Value-at-risk are namely:

- Performance Evaluation;
- Capital Allocation;
- Trading Decision; and
- Enterprise-wide Risk Management (EWRM).

Value-at-risk can also be the basis for communicating market risk to the other players in the financial world in the form of incorporating VaR values in end-of-year reports of companies, or even disclosing them to shareholders (Jorion, 2002).

3.6.1 Performance Evaluation

VaR is used in performance evaluation to assess decisions taken by decentralized fund managers and also traders (Dowd, 1999a). The information contained in VaR does help risk mangers to compare the Risk-adjusted Performance Measurement (RAPM) across distinct portfolios. As an example RAPM will help to assess the trading revenues of distinct traders in similar markets and will compare them with respect to the following ratios (Dowd, 1999a):

- Sharpe Ratio: Profit and Loss/ Volatility;
- Risk Ratio: Profit and Loss/ VaR; and
- Efficiency Ratio: VaR/ volatility.

So far, performance of positions takers and traders has been assessed basing on returns only. The RAPM certainly brings another perspective of assessment and is definitely more meaningful for the purposes of comparison (Dowd, 1999a).

3.6.2 Capital Allocation

VaR is being used to determine an organization's capital requirement. VaR is also helpful when it comes to the allocation of capital across an organization's different business units (Dowd, 1999a).

The existence of RAPM system helps evaluating both organizations and their products in terms of Risk-adjusted returns. This involves the evaluation of returns from single activities and then comparing them with the organization's cost of capital. If the return turns out to be lower than the cost of capital, the activity should therefore be discontinued to avoid a potential loss of value.

3.6.3 Trading Decision

The ability of a VaR measure to encapsulate and consolidate risk across different positions in a portfolio or simply across different assets classes and communicate the total risk on an overall basis in the form of a single digit representing the relevant currency (e.g. Rand) is one of the most important applications of VaR.

With the VaR information, risk managers can take better-informed decisions about their trading or investment strategies. The taking of position should be directed towards the maximization of returns given a level of risk tolerance. If a risk manager calculates the incremental rise in the value of the VaR of any investment, better decisions for optimal performance of active trading portfolios can be taken (Dowd, 1999a)

3.6.4 Enterprise-Wide Risk Management (EWRM)

On top of the above applications, VaR also opens up the chance of a radical approach to enterprise-wide risk management. These radically new approaches go beyond the existing risk management and definitely require a major transformation in the existing way that organizations position and govern themselves. It certainly improves on traditional management techniques and is summarized below (Dowd, 1999a):

- It provides top management with a much better grip on risks, thus leading to a more informed risk management;
- It leads to a robust new control system, which renders it more difficult for fraud and errors to go undetected;
- It helps firms respond more appropriately to regulators in particular to the capital adequacy regulations that firms face, thus helping them on how to deal with the burden of such regulations; and
- Systems based on VaR methods are useful to quantify other risks like liquidity, credit, cash flow as well as other forms of market risks leading to a more integrated approach to the different forms of risks.

3.7 Conclusion on Value-at-Risk

The concept of Value-at-risk (VaR) does bring a plus in the computation and management of risk. However, VaR also has its limitations and some critics even go as far as saying that it should be buried. To already bury VaR is a little premature, as VaR has shown some promise ever since its first implementations. Extensions are now being applied by a wider audience.

To use or not to use VaR as a measure of market risk is therefore the biggest question. Objectively, the use of VaR will return better results when it is complemented by other market risk measures and existing techniques to quantify risk. After all, VaR is considered just like another tool in the investor's toolkit. The techniques that complement VaR measures well are:

- Stress testing: This is a measure of potential losses as a result of possible events in an abnormal market environment. There are two common types of stress testing. The first is based on economic scenarios. Pretend that a portfolio experiences the 1987 stock market crash again. The second is "matrix" based. Change some assumptions about variances and correlations and see what happens. Neither is statistical in nature, in contrast to VaR, i.e. the probability of the scenario is unknown (Schachter, 1997).
- Back testing: This is a statistical procedure to validate the accuracy of VaR models. Banking regulators require back testing for banks that use VaR for capital allocation. It involves a comparison between the frequency a VaR model under-predicts the subsequent day's loss, against the number of time such an under-prediction is expected. If losses exceeding VaR have a 1 in 100 chance of occurring, then 2 or 3 of those are expected in a year (Schachter, 1997).

The next chapter is about the simulation procedures of this research. A portfolio of four South African bonds will be exposed to the three main methodologies described in this chapter and the different VaR estimates will be computed and compared. Eleven different simulations will be undertaken and the underlying factors of VaR will be measured.

Chapter 4

Research Findings and Analysis

4.1 The Portfolio of Financial Instruments

The previous chapter (Chapter 3) dealt with the theory, the methodologies and the applications of VaR. The next step now, is to apply the techniques of VaR to a portfolio of financial instruments and to analyze how the dependent factors of VaR react. For this research, a portfolio of financial instruments consisting of four South African government bonds was selected.

The four government bonds included in the portfolio to be analyzed are:

1. R153
2. R157
3. E168
4. DV07

Each of the four bonds has different characteristics as far as the maturity dates, book-closed and coupon payments are concerned. Table 1 (Appendix III) summarizes the features of the different bonds.

4.2 The factors to be measured

In Chapter 3, (Section 3.4), it was found that VaR is based on two main factors namely:

1. Time horizon: The period over which the asset in the portfolio will be held, also called holding period. For active portfolios with liquid assets, typical holding period is 1-trading day, although regulators like the European Capital Adequacy Derivatives (CAD) require 10 days. Ideally, time horizon should correspond to the largest period required for orderly portfolio liquidation.
2. Probability: This is the confidence interval or significance level at which the estimate will be made. Choices about confidence interval depend on its use. Risk aversion or high costs will imply that a larger amount of cash should cover possible losses, thus implying a higher confidence interval. Popular choices of confidence intervals are 95% and 99%.

The two above-mentioned factors will then be measured and analyzed. The SAS Risk Dimension software will conduct the analysis. It was found that Risk Dimension is also sensitive to the dates (day/month/year) on which the simulation of the portfolio takes place. Thus, the third factor of measurement will be "effective date" and depending on when the simulation is run, the different effective dates will be termed:

1. The Past - simulation run on effective date 25 Sep 2002;

2. The Present - simulation run on effective date 25 Sep 2003; and
3. The Future - simulation run on effective date 14 May 2004.

The reason why this factor is worth measuring is simply because it is interesting to see how the VaR of a portfolio evolves through time, i.e. it is quite informative to observe what VaR a portfolio has now, what it was a year back and what it will be in a year's time, with other conditions being fixed. With this kind of information on hand, decision-making is rendered easier for an investor as far as a time frame is concerned. This piece of information gives the investor the freedom to decide on the best time to act on the portfolio. Knowing how the VaR of a portfolio evolved over the past year and by how much it will change over the coming year is quite helpful to the investor and accordingly measures can be taken to limit the changes to a minimum, i.e. actions can be taken to minimize the overall risk.

Throughout this research, it was found that market risk of a portfolio is dependent on the composition of the particular portfolio, i.e. the individual risks of the instruments have an effect on the overall portfolio risk. As a result, changing the composition of the portfolio will actually change the portfolio overall risk and will have an effect on the value of the VaR. The composition of this particular portfolio comprises of four government bonds. However, for this research, the techniques of VaR will be carried out with the assumption that each government bond has an equal weightage, i.e. with the assumption that an equal amount of money is invested in the four different government bonds of this portfolio.

The objective in this research is to try minimizing VaR value for this particular portfolio and to identify which government bond causes more influence on the VaR estimates and explain why. In real-life situations, this is common practice, since a fund manager or an investor will definitely look at minimizing the overall risk of a portfolio. One of the features of a VaR analysis is that it allows the user to vary the composition or weightage of a portfolio to bring the risk down. However, it is very difficult to incorporate 'portfolio composition' in SAS Risk Dimension and accordingly, it will be assumed that the different bonds have an equal weightage.

4.3 The Processes

The portfolio will be subjected to the main methodologies of VaR described in this research. In Chapter 3, all three methodologies were elaborated. Now, the technical process on how these methodologies act upon a portfolio and especially how SAS Risk Dimension handles these different methodologies, will be described. The three main processes are:

(1) Historical VaR;

(2) Variance-Covariance VaR; and

(3) Monte Carlo VaR.

4.3.1 Historical VaR

The data available for the four bonds range from Dec 1999 to 14 May 2004. To calculate a VaR estimate, the basic market prices and rates that affect a portfolio must be known (Ganief, 2001). In fact, these can be considered as the risk factors for simple portfolios. In Chapter 3(Section 3.4.1), a general approach was taken for the development of a VaR estimate.

Accordingly, in that section, risk decomposition of the portfolio was discussed in terms of risk factors, but it should be noted that indeed two situations of portfolio decomposition exist to calculate the VaR of a portfolio (Ganief, 2001). They are:

1. The Fully Aggregated Position – this situation is mostly applicable for portfolios with few instruments and in stable conditions.
2. The Market Position – this situation is suitable for complex portfolios with many different instruments and a time changing composition. This is also the situation which selects the risk factors of the different instruments to decompose the portfolio and whose process is outlined in section 3.4.1.

Due to the fact that the portfolio contains few financial instruments, all of the same type, it is not necessary to decompose the risk by the general approach outlined in section 3.4.1, i.e. by the risk factors (The Market Position).

The process indeed will be based on the financial instruments themselves. Historical VaR requires little information about the statistical distribution of market risk factors and as a result can be applied directly on the financial instruments in the portfolio, which is the way SAS Risk Dimension will handle the Historical Simulation for this portfolio.

Historical VaR re-evaluates the current portfolio using historical rates and prices to obtain the risk of the portfolios. Applications of this process require the following procedures, (Ganief, 2001):

1. The portfolio is defined in terms of risk factors, either in a fully aggregated situation or a market situation. Under the fully aggregated situation, a set of historical data for the different instruments is needed while under a market situation, each instrument will have to be decomposed into defined risk factors and the market values for these risk factors will have to be known. Usually, complex portfolios will use the market situation to compute Historical VaR, while simple ones will apply the fully aggregated situation to arrive to a Historical VaR.
2. A historical set of data is needed. Usually a period between 90 to 500 days will be sufficient. This data consists of market rates and prices, which have been recorded.
3. The historical set of data is changed to the current valuation date of the VaR estimate.
4. The portfolio is revalued by utilizing pricing models based on the historical set of data to obtain the changes in the portfolio values.

5. The VaR estimate is obtained from the set of value changes computed using a percentile ranking. This requires the matching of the value of the confidence level with the ranked profit and loss histogram.

For this research, the portfolio is composed of only one type of financial instrument and there are only four of them. As a result, instead of decomposing the portfolio by the risk factors (outlined in section 3.4.1), also known as the Market Situation, the Fully Aggregated situation will decompose the portfolio. This means that the Historical VaR will be obtained by applying the method directly on the financial instruments.

4.3.2 Variance-Covariance VaR

This process arises from the Variance-Covariance Method or Analytical VaR although very often, it is also called the Delta-Normal method as in SAS Risk Dimension. This elegant way of calculating VaR was developed by J P Morgan through its RiskMetrics™ (Jorion, 2004). Analytical VaR is commonly applicable to portfolios and it is about using historical correlations and volatilities to derive the portfolio's market risk (Ganief, 2001).

VaR of a portfolio is the measurement of risk over a given horizon. Usually, VaR requires the construction of a return distribution for the portfolio. However, for this method the distribution is assumed to be Normal and accordingly the problem of the assumed distribution is eliminated as the Normal distribution is fully defined by its two parameters namely μ and σ (Ganief, 2001).

With the distribution assumed to be Normal and defined by the above parameters, then all that are required to compute a VaR value is to estimate the two parameters μ and σ. These two estimates will provide the necessary information needed to compute the VaR statistics. Usually, VaR is a measure calculated as the maximum loss that can occur at a confidence level of 95% (Ganief, 2001). As such, under this process, the VaR statistic will simply be:

$$1.645\,\sigma - \mu \quad \text{.....................................Equation 1}$$

To find an estimate for μ is quite simple. However, since VaR is usually calculated over short horizons, μ is typically set as zero. As a result, Equation 1 is reduced to the following:

$$1.645\,\sigma \quad \text{...Equation 2}$$

Accordingly, estimating VaR through this process resembles a simple task of calculating the standard deviation, σ, of the return distribution of the portfolio. Generally, there exist three ways to compute such a standard deviation (Ganief, 2001):

1. Estimation from the historical data;
2. The Fully Aggregated Approach; and
3. Decomposition of the portfolio into identified risk factors.

In this research, the computation of the standard deviation for the Variance-Covariance VaR is estimated from the historical data in terms of the volatilities and correlations between the financial instruments, in the form of a matrix supported by SAS Risk Dimension. Once the standard deviation is estimated, a VaR value can be calculated by using equation 4.3.3. For different confidence levels, the Normal table is used to read off the quantile values.

4.3.3 Monte Carlo VaR

Another way to compute VaR is by the Monte Carlo Simulation. Monte Carlo Simulation is made up of three important factors (Risk Dimension Documentation, 2001), namely:

1. Simulation of the world's future states;
2. State variable transformation; and
3. Pricing of the portfolio.

Because of the fact the world's future states are not known, models of state variables are used to forecast the future states. Historical data of the state variables can be used to develop future states though a lot of emphasis is being put on model application. SAS Risk Dimension supports Monte Carlo Simulation through both the model specification and the use of historical data in the form of a matrix. For this research, the Monte Carlo Simulation performed was supported by a matrix of statistical information rather than a defined model. As a result, only this type of Monte Carlo Simulation will be described here.

Such a Monte Carlo Simulation randomly generates scenarios based on some assumed joint probability distribution of the risk factors (Ganief, 2001). Historical data can be used to obtain the statistics needed to form the matrix. The statistics composing the matrix are volatilities and correlations and they will define the distribution. Once the assumed distribution is set up, a random number generator, which has to be chosen, will produce a selection of scenarios (Ganief, 2001). As such, the selected scenarios will be inferred from the assumed distribution and they will reflect the statistical characteristics drawn from the available historical data.

4.4 The Analysis from SAS Risk Dimension

Section 4.2 investigated the different factors that will be analyzed for a VaR value for this portfolio of government bonds under the three different methodologies described in this thesis. The portfolio will be subjected to different conditions and accordingly the VaR value will be measured. The different conditions or factors to be measured will be:

1. Time horizon;
2. Probability; and
3. Effective date.

The aim is to observe how the portfolio reacts to these conditions under the three different techniques outlined earlier. An attempt to find an optimal environment for this particular portfolio will be made, i.e. under what level of conditions will this portfolio be exposed to the lowest risk possible. SAS Risk Dimension features VaR values in two different forms under a Historical Simulation namely:

1. Profit and Loss;
2. Exposure, also called exposure-at-risk: This is the relevance to non-performance at some point in the remaining life of the portfolio.

Different simulations based on different conditions will be run for this particular portfolio. The Control Simulation or Benchmark Portfolio will bear the following conditions in mind:

- Effective date: 25 September 2003, "The Present";
- Confidence level: Probability: 95%;
- Holding Period: 1-trading day for each government bond; and
- Composition/Weightage: Each instrument is assumed to have equal weightage.

The Control Simulation in this situation will be the reference point and the benchmark against which all the other simulations will be analyzed. The conditions selected for the control simulation are as such because 95% probability and 1-trading day are most common levels for calculating VaR. The present (corresponding to the purchase dates of the bonds) value of the VaR is what is required and an equal weightage for the different bonds is the assumption.

4.5 The Different Simulations

For this research, the following simulations will be run at various levels for the different factors under measurement. The different simulations run will bear the following conditions:

1. The **Control Simulation** (As defined in section 4.3), which is also **'The Present'**.
2. Equal Weightage, **1**-trading day holding period at **99%** probability run on effective date **25 September 2003**.

3. Equal Weightage, **1**-trading day holding period at **90%** probability run on effective date **25 September 2003**.
4. Equal Weightage, **1**-trading day holding period at **92.5%** probability run on effective date **25 September 2003**.
5. Equal Weightage, **1**-trading day holding period at **97.5%** probability run on effective date **25 September 2003**.
6. Equal Weightage, **1**-trading day holding period at **95%** probability run on effective date **25 September 2002**, which is also **'The Past'**.
7. Equal Weightage, **1**-trading day holding period at **95%** probability run on effective date **14 May 2004**, which is also **'The Future'**.
8. Equal Weightage, **bond R153 with a 10**-trading day holding period at **95%** run on effective date **25 September 2003**.
9. Equal Weightage, **bond R157 with a 10**-trading day holding period at **95%** run on effective date **25 September 2003**.
10. Equal Weightage, **bond E168 with a 10**-trading day holding period at **95%** run on effective date **25 September 2003**.
11. Equal Weightage, **bond DV07 with a 10**-trading day holding period at **95%** run on effective date **25 September 2003**.

Simulations 1 to 5 will be used to measure how VaR estimates react to changes in probability level while simulation 1 together with simulations 6 and 7 will be used to measure the factor effective date on VaR values. Finally, simulation 1 together with simulations 8 to 11 will be used to check upon the effects of holding period on the VaR values. It must be noted that the measurements of the factors will be for the three different methodologies.

4.5.1 The Control Simulation

The Control Simulation or Benchmark Portfolio is also termed "The Present". It is the simulation which is run on the effective date of 25 September 2003 at a confidence level of 95%. The different government bonds are assumed to have an equal weightage and their holding period is fixed at 1-trading day. The reason why the effective date, 25 September 2003, is termed "The Present" is simply because the instruments in the portfolio all have a purchase date (Table 1, Appendix III) corresponding to the above effective date. The three techniques of computing VaR are applied on that portfolio with the above conditions and run in SAS Risk Dimension.

Section 1 (Appendix I) shows the different tables for the Control Simulation under the three different techniques together with the VaR distributions figures, both in absolute values and percentage. Table A1.1 (Appendix I) summarizes the statistics for this Control Simulation under Historical Simulation whilst Table A1.2 (Appendix I) shows the statistics for the exposure-at-risk (EaR) under Historical Simulation.

It can be seen that under Historical Simulation process, the VaR estimate for the Control Simulation is 255,748.47 whilst the EaR is 46,514,815.66. This means that 95 times out a 100, the Control Simulation or Benchmark Portfolio will stand to lose at most ZAR 255,748.47. Table B1.1 on the other hand summarizes the Variance-Covariance VaR estimates for the Control Simulation. It is seen that the VaR value under this methodology is 43,155.45. Finally table C1.3 gives the statistics for this Benchmark portfolio under the Monte Carlo Simulation and it is found that the VaR value this time is 29,117.16.

4.5.2 Equal Weightage, 1-trading day holding period at 90% probability run on effective date 25 September 2003.

This second simulation run has the above characteristics. Compared to the Control Simulation, there is only one difference – instead of the simulation being run at the 95% probability, it is being run at a lesser level, i.e. the 90% probability level with the other conditions being the same as the Control Simulation.

Tables A2.1 to A2.2 (Appendix I) summarize the statistics and the results for this simulation run. Under Historical Simulation, it is found that the VaR estimate is 174,730.57 while the exposure-at-risk is 46,444,548.25. This means that 90 times out of 100, this portfolio with the above characteristics will lose at most ZAR 174,730.57. When the portfolio is subjected to the Variance-Covariance Method, it is found that the VaR value is 33,623.62 while under the Monte Carlo Simulation, the VaR estimate is 20,951.89. As a result, 90 times out of 100, this particular portfolio will lose at most ZAR 33,623.62 under the Variance-Covariance Method and ZAR 33,623.62 under the Monte Carlo Simulation.

4.5.3 Equal Weightage, 1-trading day holding period at 99% probability run on effective date 25 September 2003.

This simulation run bears the above conditions. Compared to the Control Simulation, there is only one change – instead of a 95% probability, the simulation is now run at 99% with the other factors being kept the same as the Control Simulation.

Tables A3.1 and A3.4 (Appendix I) summarize the results for this simulation. It is found that for this simulation run, the VaR is 567,792.30 and the exposure-at-risk to be 46,713,354.12. This is now under the Historical Simulation and it implies that 99 times out of 100, this portfolio will lose at most ZAR 567,792.30. Under Variance-Covariance, the VaR value was found to be 61,035.58 and the VaR estimate is 54,034.87. As a result, the portfolio will lose at most ZAR 61,035.58 under the Variance-Covariance Method and at most ZAR 54,034.87 under the Monte Carlo Simulation and that 99 times out of 100.

4.5.4 Equal Weightage, 1-trading day holding period at 92.5% probability run on effective date 25 September 2003.

This fourth simulation run has the above characteristics. Compared to the Control Simulation, there is only one difference – instead of the simulation being run at the 95% probability, it is being run at a lesser level, i.e. the 92.5% probability level with the other conditions being the same as the Control Simulation.

Tables A4.1 and A4.2 (Appendix I) summarize the statistics and figures A4.1 and A4.2 show the distributions for this simulation run. Under Historical Simulation, it is found that the VaR estimate is 201,749.24 while the exposure-at-risk is 46,474,519.85. This means that at a confidence level of 92.5%, this portfolio with the above characteristics will lose at most ZAR 201,749.24. When the portfolio is subjected to the Variance-Covariance Method, it is found that the VaR estimate is 37,768.49 while under the Monte Carlo Simulation, the VaR value is 24,762.08. As a result, at a confidence level of 92.5%, this particular portfolio will lose at most ZAR 37,768.49 under the Variance-Covariance Method and ZAR 24,762.08 under the Monte Carlo Simulation. Figure C4.1 displays the Monte Carlo Simulation distribution for this simulation run.

4.5.5 Equal Weightage, 1-trading day holding period at 97.5% probability run on effective date 25 September 2003.

This simulation run bears the above conditions. Compared to the Control Simulation, there is only one change – instead of a 95% probability, the simulation is now run at 97.5% with the other factors being kept the same as the Benchmark Portfolio.

Tables A5.1 and A5.2 (Appendix I) summarize the results for this simulation. It is found that for this simulation run, the VaR is 336,032.02 and the exposure-at-risk to be 46,604,229.75. This is now under the Historical Simulation and it implies that at a 97.5% confidence level, this portfolio will lose at most ZAR 336,032.02. Figures A5.1 and A5.2 display the distribution of the VaR and EaR under the Historical Simulation. Under Variance-Covariance, the VaR value was found to be 51,422.89 and the VaR estimate is

52,609.27, under the Monte Carlo Simulation. As a result, the portfolio will lose at most ZAR 51,422.89 under the Variance-Covariance Method and at most ZAR 52,609.27 under the Monte Carlo Simulation at 97.5% confidence level.

4.5.6 Equal Weightage, 1-trading day holding period at 95% probability run on effective date 25 September 2002 – 'The Past'.

This simulation run is termed 'The Past' as its effective date is 25 September 2002, a year ago from the time of the purchase of the instruments in the portfolio (purchase date: 25 September 2003, Table 1, Appendix III). Compared to the Control Simulation, the only difference is about the effective date with the other conditions being the same. The reason why this simulation is important is because it is informative to know how this particular portfolio was performing in the past compared to the present.

Tables A6.1 to A6.2 (Appendix I) summarize the statistics and figures A6.1 and A6.2 display the results for this simulation run. Under Historical Simulation, it is found that the VaR estimate is 281,357.6 while the exposure-at-risk is 46,963,303.69. This means that 95 times out of 100, this portfolio with the above conditions will lose at most ZAR 281,357.68. When the portfolio is subjected to Variance-Covariance Method, it is found that the VaR value is 47,363.37 while under the Monte Carlo Simulation the VaR estimate is found to be 32,948.70. As a result, when the portfolio is subjected to the Variance-Covariance Method, 95 times out a 100, it will lose at most ZAR 47, 363.37

and at most ZAR 32,948.70 under the Monte Carlo Simulation. Figure C6.1 (Appendix I) display the Monte Carlo VaR analysis for this simulation run.

4.5.7 Equal Weightage, 1-trading day holding period at 95% probability run on effective date 14 May 2004 – 'The Future'.

This simulation is termed 'The Future' as it ran on effective date 14 May 2004. Compared to the control simulation, once again there is only one difference and that is about the effective date. If running a simulation in the past is informative, then running one in the future in no less informative. It does give an edge to know how a portfolio with the same characteristics will perform in a future not so distant. Due to the fact that the historical data ranges until 14 May 2004, the simulation run "The Future" cannot take place exactly a year in time, i.e. on 25 September 2004. If this is performed, then there will be a case of missing values.

Tables A7.1 to A7.2 (Appendix I) summarize the statistics and figures A7.1 to A7.2 display the results for this simulation run. Under Historical Simulation, it is found that the VaR estimate is 215,244.60 while the exposure-at-risk is 45,445,069.80. This means that 95 times out of 100, this portfolio with the above conditions will lose at most ZAR 215,244.60 under the technique of Historical Simulation. When the portfolio is subjected to Variance-Covariance Method, it is found that the VaR value is 41,019.38 while under the Monte Carlo Simulation the VaR estimate is found to be 424.08. As a result, when the portfolio is subjected to the Variance-Covariance method, it will lose ZAR 41,019.38

and ZAR 424.08 under Monte Carlo Simulation and that 95 times out a 100. Figure C7.1 (Appendix I) displays the Monte Carlo VaR analysis for this simulation run.

4.5.8 Equal Weightage, bond R153 with a 10-day holding period at 95% run on effective date 25 September 2003.

This simulation run has the above conditions. Compared to the Control Simulation, the only change is the holding period of one bond in the portfolio and that is bond R153 with a 10-trading day holding period. The three remaining bonds in the portfolio still have the same holding period, i.e. 1-trading day each.

Tables A8.1 and A8.2 (Appendix I) summarize the statistics for this simulation run. It is found that the VaR estimate is 793,167.88 and the exposure-at-risk to be 151,108,949.72 under Historical Simulation. This implies that 95 times out of 100, this portfolio will lose at most ZAR 793,167.88. Under the methodology of Variance-Covariance, the VaR estimate is found to be 129,526.43 and the Monte Carlo VaR is 100,244.78. Figures A8.3 and C8.2 display the contribution of the instruments towards risk under the conditions. These figures are very informative, though it is quite logical that the more an investor hold to an instrument, the more risk that instrument is exposed to.

4.5.9 Equal Weightage, bond R157 with a 10-day holding period at 95% run on effective date 25 September 2003.

This simulation run bears the above characteristics. Compared to the Control Simulation, there is one single difference and that is bond R157 has a 10-trading day holding period compared to a holding period of 1-trading day in the Control Simulation. The remaining three bonds in this particular portfolio still bear a 1-trading day holding period.

Tables A9.1 and A9.2 (Appendix I) summarize the statistics of this simulation. It is found that the VaR estimate of this portfolio is 1,157,801.80 while the exposure-at-risk is 156,981,146.64 under Historical Simulation. This implies 95 times out of 100, under Historical Simulation, this particular portfolio stands to lose at most ZAR 1,157,801.80. Under Variance-Covariance the VaR estimate is found to be 183,774.89 and the Monte Carlo VaR estimate is 127,957.55. This means that under the Variance-Covariance Method, this portfolio will lose at most ZAR 183,774.89 and ZAR 127,957.55 under Monte Carlo and that 95 times out of a 100.

4.5.10 Equal Weightage, bond E168 with a 10-day holding period at 95% run on effective date 25 September 2003.

This simulation run has the above conditions. Compared to the Control Simulation, there is again one change – bond E168 in the portfolio has a 10-trading day holding period compared to the 1-trading day holding period. The remaining four instruments still have their original holding period of 1-trading day.

Tables A10.1 and A10.4 (Appendix I) summarize the simulation statistics. Under Historical Simulation, the VaR estimate is found to be 650.358.82 while the exposure-at-risk is 136,403,977.59. This implies that under Historical Simulation, 95 times out of 100, this particular portfolio will lose at most ZAR 650.358.82. Under Variance-Covariance, the VaR estimate is equal to 110,282.71 while under the Monte Carlo Simulation, the VaR value is found to be 57,549.41. As a result, 95 times out of a 100, this portfolio will lose at most ZAR 110,282.71 and ZAR 57,549.41 under the Variance-Covariance and Monte Carlo Simulation respectively.

4.5.11 Equal Weightage, bond DV07 with a 10-day holding period at 95% run on effective date 25 September 2003.

This simulation run bears the above characteristics. Compared to the Control Simulation, there is one change – this time it is bond DV07 that has a 10-trading day holding period with the remaining four bonds still having the same 1-trading day holding period.

Tables A11.1 and A11.2 (Appendix I) summarize the statistics for this simulation run. It is found that the VaR estimate is 790,159.07 and the exposure-at-risk to be 160,387,408.21 under Historical Simulation. This implies that 95 times out of 100, this portfolio will lose at most ZAR 790,159.07. Figures A11.1 to A11.3 display the simulation under this technique and also the instrument risk contribution for this portfolio. Under the Variance-Covariance Methodology, the VaR estimate is found to be 149,999.62 and the Monte Carlo VaR is equal to 111,700.66.

4.6 The effects on VaR due to changes in the factors

The simulations that have been run were for a purpose and that was to see how the dependent factors react to changing conditions. The factors under measurement here were:

1. Probability Level;
2. Effective Date; and
3. Holding Period.

Accordingly, different simulations were described that would indeed measure the effects on these factors.

4.6.1 Effect of a changing "confidence interval" on VaR

The Control Simulation as well as simulations run 4.5.2 to 4.5.5 are the portfolios that have been set up accordingly to measure the effects of a changing confidence interval. The following table summarizes the different VaR values under the different level of confidence at that for all three different methodologies and the exposure.

Method	90%	92.5%	95% (Control Simulation)	97.5%	99%
Historical	174,730.57	201,749.24	255,748.47	336,032.02	567,792.30
Exposure	46,444,548.25	46,474,519.85	46,514,815.66	46,604,229.75	46,713,354.12
Variance-Covariance	33,623.62	37,768.49	43,155.45	51,422.89	61,035.58
Monte-Carlo	20,951.89	24,762.08	29,117.16	52,609.27	54,034.87

Table 4.6.1: Summary of VaR values for different Confidence Levels

It can be seen from table 4.6.1 that when the confidence level increases, the VaR estimate also increases and that is true for any of the three methodologies as well as the exposure. The lesser the probability level, the less reliable the VaR estimate will be. It is therefore, misleading to try to bring the VaR estimate down, i.e. the risk, by decreasing the probability level, as by no means is it informative. An investor would want to be able to rely on the figures of VaR and not make the VaR estimates look reasonable. Increasing the level of the probability is a case of risk aversion, as higher cost will be needed to minimize possible losses. Therefore, the VaR values being higher as the probability level increases, just confirm the theory of VaR for this factor.

Figures 1.1 to 1.4 (Appendix II) depict scatter-plots of VaR estimates according to the methodologies with confidence levels. All four plots show a positive slope, which implies that the value of VaR will increase with increase in probability level. The regression line is also fitted in all four cases and the regression equations given in each situation. This is the functional form of VaR where it gives VaR as a function of the confidence level, i.e. VaR = f(probability level) and this equation is given for all three methodologies as well as the exposure-at-risk in Appendix II.

4.6.2 Effect of a changing "effective date" on VaR

To investigate the effects of changing the effective dates on the value of VaR, the following simulations were run and they were:

1. The Control Simulation;
2. The simulation run, "The Past"; and
3. The simulation run, "The Future".

The following table summarizes the results for the VaR estimates for the different methodologies as well as the exposure.

Method	"The Past"	Control Simulation	"The Future"
Historical Simulation	281,357.60	255,748.47	244,119.82
Exposure-at-risk	46,963,303.69	46,514,815.66	46,030,534.61
Variance-Covariance	47,363.37	43,155.45	41,679.86
Monte-Carlo	32,948.70	29,117.16	25,887.17

Table 4.6.2: Summary of VaR values for different effective dates

From table 4.6.2, it can be noted that as the portfolio is moved backward in time, the value of the VaR estimates increase – "The Past" simulation having a larger value than the Control Simulation and finally "The Future" simulation. This is notable for all three methodologies as well as the exposure-at-risk.

To explain the rise in the VaR estimates, the characteristics of the government bonds in the portfolio must be analyzed. One reason why 'The Past' portfolio bears a higher VaR value could be because the maturity dates of the different bonds are still far away. If this is the case, then the closer to the maturity dates this portfolio is valued, the VaR estimate must decrease.

To show this trend, the functional form of VaR against effective date has been found. Figures 2.1 to 2.4 (Appendix III) depict scatter plots of VaR estimates against numerous dates for this portfolio. The range of the dates is from 25 March 2000 to 25 March 2004 (historical data permitting). To plot the two variables, the date values were given ranks with 25 March given a rank '1' up to the last date available. The regression line is also fitted for the four situations and the equations also provided (Section 2, Appendix III). It can be seen that in all four situations, the regression line has a decreasing slope which means that as the portfolio is moved forward in time, VaR decreases. For a portfolio consisting of government bonds, this can be attributed to moving closer to the different maturity dates of the bonds.

It was not possible to run "The Future" simulation exactly one year ahead in time. This is because the historical data available ends on 14 May 2004. However, if there was a need to actually know the VaR value exactly one year ahead, a prediction can be made from the regression analysis. For the Historical Simulation, the regression equation is

VaR = 345702 – 10883(effective date)...........Equation 1

To obtain an estimate for the VaR under this method for the effective date, 25 September 2004, which is exactly a year ahead in time from the purchase of the instruments, the following can be done:

The rank of the effective date 25 September 2004 is equal to 10. Substituting in equation 1, the VaR estimate will be equal to 236872. This value is less than the simulated value of "The Future" simulation whose effective date is 25 March 2004. It shows that the forecasted value is indeed lesser for an effective date even further forward in time.

4.6.3 Effect of a changing "holding period" on VaR

The Control Simulation as well as simulations runs 4.5.8 to 4.5.11 were set up to measure the changes on VaR values due to a changing holding period of the bonds in the portfolio.

The following table summarizes the VaR results of these simulations:

Method	Control Simulation	R153 with a 10-day holding period (4.5.8)	R157 with a 10-day holding period (4.5.9)	E168 with a 10-day holding period (4.5.10)	DV07 with a 10-day holding period (4.5.11)
Historical Simulation	255,748.47	793,167.88	1,157,801.80	650,358.82	790,159.07
Exposure	46,514,815.66	151,108,949.72	156,981,146.64	136,403,977.59	160,387,408.21
Variance-Covariance	43,155.45	129.526.43	183,744.89	110,282.71	146,999.62
Monte-Carlo	29,117.96	100,244.78	127,957.55	57,549.41	111,700.66

Table 4.6.3: Summary of VaR estimates for the different bonds at a 10-day holding period.

The only difference of the above simulations with the Control Simulation is that each time, one of the bonds is having a holding period of 10-trading days with the rest kept at 1-trading day. The Control Simulation has all the bonds kept at a holding period of 1-trading day.

From Table 4.6.3, a few trends can be noted. It can be seen that keeping bond R157 at a 10-day holding period will cause the highest value of the VaR estimate. This is because the maturity date of bond R157 is 15 September 2015 (Table 1, Appendix III). Holding on to more days to a bond whose maturity date is latest in the portfolio will automatically have more risk. As a result, under all three methodologies, the VaR values are high.

On the other hand, holding on to 10-trading days on to bond E168 causes the lowest VaR value compared to the other simulations. This comes as no surprise as already maturity dates have been identified as the component on which risk in a portfolio of bonds depends. The maturity date of bond E168 is 01 June 2008 (Table 1, Appendix III). Compared to the other bonds in the portfolio, E168 will reach maturity first. As a result, its VaR value is lowest compared to the other simulation runs.

One notable fact when comparing these simulations runs to measure holding period is between bonds R153 and DV07. If maturity date is responsible for the risk of a portfolio of bonds, then bond R153 with a maturity date of 08 August 2010 (Table 1, Appendix III) must have a smaller VaR value than bond DV07 whose maturity date is 30 September 2010. However, this is not the case for all the methodologies. If indeed this is true for the Variance-Covariance method, the Monte Carlo Simulation and the exposure-at-risk, the historical simulation shows a bigger VaR value for bond R153 compared to bond DV07. This can be explained by the coupon dates of the two bonds. Under the Historical Simulation, coupon dates are important when holding period is being measured. Though bond DV07 has a later maturity date than R153, its first coupon date, CD1 is earlier than that of R153 (CD1 of DV07 is 31 March 2004 whilst CD1 of R153 is 31 August 2004, Table 1, Appendix III).

The factor holding period shows that holding on to risky instruments will cause larger VaR values. In this portfolio holding on to bonds with the latest maturity dates will cause the VaR values to increase. However, if the holding period of the whole portfolio is changed and not only one bond at a time, the VaR values increase proportionally. It means that if the holding period of all bonds is changed to 10-trading days the VaR values will be 10 times the VaR value when all bonds are kept at 1-trading day, i.e. 10 times the VaR estimates of the Control Simulation.

As a result, based on the assumption that change in holding period occurs throughout the portfolio, the functional form VaR = f(holding period) is a proportional increasing function as described in section 3(Appendix III) by the scatter plot and the regression analysis. This trend is true for all three methodologies.

4.7 Comparison of the VaR between the different Methodologies

It will be noted that the three different methodologies have been giving very different VaR values for the same portfolio. It must be mentioned that first of all the portfolio is composed of the same type of financial instruments. As a result, it would have been expected that the VaR values, though not equal to, but would rather be close to one another. However, the three methodologies have yielded contrasting values but have kept up with all the patterns.

If reference is made to Tables 4.6.1 to 4.6.3, this notable effect will be seen. It can be noted that the Historical Simulation gives very big VaR values compared to the Variance-Covariance Method and the Monte Carlo Simulation. For the latter two, though the VaR values are not quite the same, they do not show high differences. It would be thought that for such a simple portfolio containing the same type of instruments, the VaR values will rather coincide in one way or the other. For an investor this situation will be quite confusing and if care is not taken, problems are bound to arise. The main question here will be: with three very different VaR estimates available for the same portfolio, which one does the investor chooses?

The answer is definitely not to choose the lowest value of the VaR estimate. This can be quite misleading. It will depend on how much faith the investor will want to put in these VaR values. The differences in the VaR values in this case can be attributed to the way the different methodologies handle bonds and also to the data. Moreover, SAS Risk

Dimension must also be looked at and that is why the three technical processes were described in Section 4.3. The way that SAS Risk Dimension handles these three processes was also mentioned for that matter. It is known that the Monte Carlo Simulation handle financial instruments that are difficult to be priced like derivatives. This in no means can justify the difference of the VaR values for a portfolio of bonds. The Monte Carlo simulation was run in SAS Risk Dimension based on the covariance matrix but not on a defined model. SAS Risk Dimension has the option of running a Monte Carlo Simulation based on either a covariance matrix or a defined model. In fact, it is the same covariance matrix that was defined to be used by the Variance-Covariance Method. It could be one reason why the two VaR values are closer but yet very different from the Historical Simulation. Moreover, the Variance-Covariance Method assumes that the distribution is Normal. It could be that in this case, the distribution cannot be assumed to be Normal. All these suppositions, however, will not help the investor on which VaR estimate to choose.

Chapter 5

Conclusion and Recommendations

5.1 Reliability of Results

The previous chapter (Chapter 4) is about the practical part of this research. In that chapter, the portfolio is presented as well as the financial instruments. The software that is used is also introduced. The chapter also deals with the factors under measurement in this research as well as providing details about the different simulation runs under the three main methods of VaR. Finally, the results are presented and analyzed as well as interpreted.

The next step is to discuss the reliability of the different results (Appendix I). Chapter 3 deals with the theory and methodologies of VaR and already then, an idea on how the results are supposed to turn out, were made. It happens that the different set of results for the distinct simulation runs fit the theory of VaR accordingly.

The first simulation run was termed the Control Simulation (4.5.1) and against which the other ten (4.5.2 - 4.5.11) were compared. Simulation runs 4.5.2 to 4.5.5 measured the influence of the factor probability level on VaR estimates. With the probability level set at 95% in the Control Simulation, the simulation run of 4.5.2 to 4.5.5 had theirs set at 90%, 99%, 92.5% and 97.5% respectively. The results are in accordance to what the theory of VaR suggests about an increase in confidence level from 90% to 95% and then to 99%.

The results for the three simulations above are summarized (Appendix I) and they show that, when the probability level increases from 90% to 99% (with other factors staying unchanged in the portfolio), the VaR estimates increase as predicted by the theory of VaR (Section 3.4.2).

Simulation runs 4.5.6 and 4.5.7 were set to measure the effect of the factor "effective date" on VaR estimates. It is important to note here that "effective date" is not a factor that is discussed by the theory of VaR. It is a practical factor which advantageously is featured by SAS Risk Dimension. The simulation run 4.5.6 was termed "The Past" while the simulation run 4.5.7 was called "The Future". The Control Simulation was also termed "The Present".

The chosen "effective date" is about the date the simulation is run. For this research, the chosen "effective date" is 25 September 2003. The choice of this particular date is appropriate since it will represent the purchase dates of the different bonds in the portfolio and it will match the "effective date" of the Control Simulation.

Analyzing the results (Appendix I) of "The Past" and "The Future" and comparing both with the Control Simulation ("The Present"), a simple logical and sensible pattern could be detected. Compared to the Control Simulation, "The Past" had a larger VaR estimate while "The Future" had a lesser VaR estimate. The reason why "The Future" have a lesser VaR estimate than the Control Simulation could be that the more the portfolio is valued in time the closer it gets to the different maturity dates of the government bonds and accordingly less risk will be taken in by the portfolio. Another plausible reason could be that a couple of coupons are already paid for the different government bonds. Accordingly, the portfolio has already cashed in and the VaR estimates decrease logically.

"The Past" had a higher VaR estimate compared to both the Control Simulation and "The Future". "The Past" arose when the same portfolio is measured for VaR back in time, i.e. when the Control Simulation is run a year ago. The fact of "The Past" bearing a higher VaR value than "The Present" can be explained by the same arguments as above, only that this time, it will be that the maturity dates are too far away and also no coupon payments have been made yet.

On a more positive note, however, effective date provides useful insight of how a very same portfolio performed back in time and will perform in the future. What is also interesting, is that the portfolio can be run as far away as possible both in the past and in the future, obviously to the limit of plausibility; like for this particular portfolio it will be insensible to run the portfolio after the latest maturity date of the government bonds.

Unfortunately, it could be that not all computer software available to estimate the VaR of a portfolio bears this feature.

Simulation runs 4.5.8 to 4.5.11 measure the influence of holding period on the estimation of the VaR for a portfolio. Holding period is the second factor that is discussed by the theory of VaR and which, when varied, should produce different values of VaR estimates. In real life, investors do change holding period of financial instruments to try and bring overall risk of a portfolio down.

One of the requirements of Banking Regulation is that financial instruments must bear a 10-day holding period for capital (Chapter 3). In this research, the Control Simulation had all of its five government bonds with 1-day holding period. Simulation runs 4.5.8 to 4.5.11 had each time one government bond with a 10-day holding period with the other bonds with a 1-day holding period. The different results (Appendix I) are very informative about the factor "holding period" and depending on which bond had its holding period increased, the VaR estimates changed accordingly. The reason why some VaR estimates were higher than others for the simulation runs 4.5.8 to 4.5.11 was explained in Chapter 4.

5.2 Problems Encountered

This research explores the factors influencing the measurement of VaR estimates. If the SAS Risk environment was successfully set up as well as the different simulations run, it was definitely not without hassles. A few problems were encountered in the build-up of the Risk Environment and afterwards not everything that was desirable was implemented.

The problem encountered during this research was to accommodate the factor "weightage" in the measurement of VaR estimates. It is common knowledge that, in practice, when a financial instrument in a portfolio has been analyzed to bear less risk over a period of time, more money is invested in that instrument. In other words, the ratio of capital invested switches in the portfolio. This indeed has an impact on the value of a VaR estimate for a portfolio. Unfortunately, it has not been possible to measure the factor weightage for this particular portfolio and it was assumed that an equal amount of money was invested in each of the different government bonds. The problem arose from SAS Risk Dimension itself. It was discovered at a later stage during this research that SAS Risk Dimension does not support dynamic portfolios yet. The problem has already been taken up by the software vendor and it looks like it is to be remedied in the future.

5.3 Limitations of the Research

In section 5.1, the reliability of the results for this research was discussed. Although, it could be found that the results are reliable and in accordance with theory of VaR, it cannot go without mentioning that some over-simplifications have taken place. On top of those over-simplifications, it must be said that if care is not taken in the setting up of the Risk Environment, then all the results can become misleading.

First of all, one limitation of this research is about the chosen portfolio of financial instruments. The particular portfolio contains only one type of financial instrument and that is the government bond. All four of them have different characteristics but they are all still of the same type. As a result, one can argue aspects about the reliability of these results though one must accept that the results fit the theory of VaR pretty well. The main question here would be: if the portfolio had different types of financial instruments, would the result still be fitting the theory of VaR?

Secondly, in theory government bonds are considered to be risk-free financial instruments. Their presence in a portfolio is usually to try to bring down the overall risk. VaR of a portfolio is the amount of money that a portfolio can expect to lose at most over a period of time. Now, if government bonds are risk free and this portfolio is entirely made up of government bonds, how can that particular portfolio be at risk? It would make sense to calculate the VaR of a portfolio composed of a mixture of financial instruments or even for a portfolio which is made up of only one type of financial instrument but which certainly are not risk-free. But still, a portfolio of government bonds was selected for this research and VaR estimates could be obtained from that portfolio and under different conditions.

The one limitation of this research concerns the results directly. The factor "effective date" that was measured in this research is not a theoretical factor of VaR but more a practical factor of SAS Risk Dimension that certainly is important and informative. In section 5.1, its reliability is discussed but the factor "effective date" has to be handled with care. If on one hand it does provide valuable information to an investor, it can be misleading if attention is not paid to the particulars of the portfolio. It simply means that the dates of running the simulation have to be decided and known. Accordingly, if this is overlooked, then the VaR estimate will be very misleading.

One other limitation that was noticed during this research and which is very important is about the VaR values under the different methodologies. It will definitely not be helpful for an investor to have three very different VaR values for a very same portfolio and none of them coincide, though the VaR estimates from the Variance-Covariance Method and Monte Carlo Simulation are close. The reason for that was provided in Chapter 4. Which one must the investor choose and why, will be a very difficult question to answer and it will depend a lot on his experience and his knowledge of VaR. However, even so though he can land into trouble.

5.4 Value-at-Risk - The Final Verdict

This research is about the theory and methodologies of Value-at-Risk (VaR). The application of the theory to a physical portfolio of financial instruments is also explored (Chapter 4). The factors that can influence a VaR estimate for a portfolio are under scrutiny in this research. The three main methodologies described in this research are applied on the physical portfolio and the VaR values they produced are discussed. The pros and cons of VaR are weighted. In all, this research covers most of the areas on the topic of VaR.

Now, the inevitable question can be asked about VaR: Should VaR be adopted or must it be rejected? Based on this research alone, a stand cannot be taken. It must be said, that many positive things were discovered about VaR during the research exercise. The simulations were very informative. On the other hand, VaR is definitely not the answer to muster financial risk. The literature review of this thesis together with the practical problems illustrated are proofs that VaR is far away from being the "holy scale" to measure risk.

VaR is certainly a very powerful measure of risk but it is definitely not the only one. The important thing here is that VaR must be used together with other existing measures of risk and then VaR estimates become very helpful. If an investor relies solely on VaR measures, he has to be extremely careful and it will all depend on how much faith he wants to put in these VaR estimates. His problem gets worse when he has three different VaR estimates for the very same portfolio. One author has said and reported in this research that: "*VaR is seductive but dangerous*" while another one makes mention about VaR being: "*the answer to investors' prayers*".

A true and fair reflection about the VaR hysteria will be: while VaR is very informative it can also be misleading. Does it boil down to say that after all VaR is just like the existing risk measures? The answer to this question is definitely no. VaR is certainly more powerful than the existing risk measures as it gives more information and can definitely look into different aspects of a portfolio. However, VaR must be used in parallel with the existing risk measures and before putting a lot of faith in VaR estimates, it is important to check on the reliability of the results.

5.5 Prospect for new research

The topic Value-at-Risk still remains very mysterious. It must be agreed that over the years and with technology progressing, VaR has gained more and more importance. However, not all the problems of VaR have been solved. If many financial firms have worked on the technological development of computer software to more easily compute VaR, even more people want to look into the theoretical aspect of the topic and discuss its reliability.

To find the minimum value of a VaR estimate for a particular portfolio (if this exists), an investor has to do it by trial and error. Obviously, he will not change the probability level to reduce the VaR estimate of his portfolio. He will try to change the weightage of investment of the different financial instruments and also change the holding period of the instruments in the portfolio, once he has identified which financial instrument is more at risk than the others.

To look at a portfolio and know which financial instrument is the most at risk is an impossible task and even an experienced investor will be unable to detect this easily. VaR estimates can detect the above and accordingly measures can be taken to minimize the risk of the portfolio.

Having already investigated a portfolio of government bonds under the three main methodologies, this analysis should be taken a step further in future research. It will be very interesting to analyze the VaR results of a portfolio of diversified financial instruments. There are a few other portfolio permutations that are interesting to research under different VaR methodologies. Historical Simulation's ability to cope with a portfolio of derivatives, swaps or contracts should be studied in future projects.

All the above can be investigated at a higher level for further research. A few important factors must be considered for research of this nature. First of all, it must be made certain that financial data relative to the different financial instruments, to be included in the different portfolios, are available and over a long enough period of time. Secondly, more training on the SAS Risk Dimension software will be required. Finally, no important research can be achieved or made possible without the availability of the necessary funds.

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APPENDIX I

(1) EQUAL WEIGHTAGE, 95% PROBABILITY, 1–DAY HOLDING PERIOD, RUN ON 25 SEPTEMBER 2003, "THE PRESENT", CONTROL SIMULATION

(A) HISTORICAL VaR

	Statistic	Estimate
1	At-Risk Value (ZAR)	255,748.47
2	Lower Tolerance Limit of At-Risk Value (ZAR)	222,658.44
3	Upper Tolerance Limit of At-Risk Value (ZAR)	277,547.52
4	At-Risk Value as percent of Base Value	0.55
5	Lower Tol Limit of VaR as percent of Base	0.48
6	Upper Tol Limit of VaR as percent of Base	0.60
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	1092
9	Number of Replications Actually Used	1092
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	16,515.16
13	Standard Deviation of Profit/Loss	216,887.28
14	Skewness of Profit/Loss	-1.98977
15	Kurtosis of Profit/Loss	42.27753
16	Minimum Profit/Loss over Simulations (ZAR)	-2,997,430.29
17	Median Profit/Loss (ZAR)	14,801.38
18	Maximum Profit/Loss over Simulations (ZAR)	1,875,322.80

Table A1.1: VaR statistics by Historical Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	46,514,815.66
2	Lower Tolerance Limit of At-Risk Value (ZAR)	46,492,933.94
3	Upper Tolerance Limit of At-Risk Value (ZAR)	46,555,832.25
4	At-Risk Value as percent of Base Value	100.62
5	Lower Tol Limit of VaR as percent of Base	100.57
6	Upper Tol Limit of VaR as percent of Base	100.71
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	1092
9	Number of Replications Actually Used	1092
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	46,244,615.47
13	Standard Deviation of Profit/Loss	216,887.28
14	Skewness of Profit/Loss	-1.98977
15	Kurtosis of Profit/Loss	42.27753
16	Minimum Profit/Loss over Simulations (ZAR)	43,230,670.02
17	Median Profit/Loss (ZAR)	46,242,901.69
18	Maximum Profit/Loss over Simulations (ZAR)	48,103,423.11

Table A1.2: EaR statistics by Historical Simulation

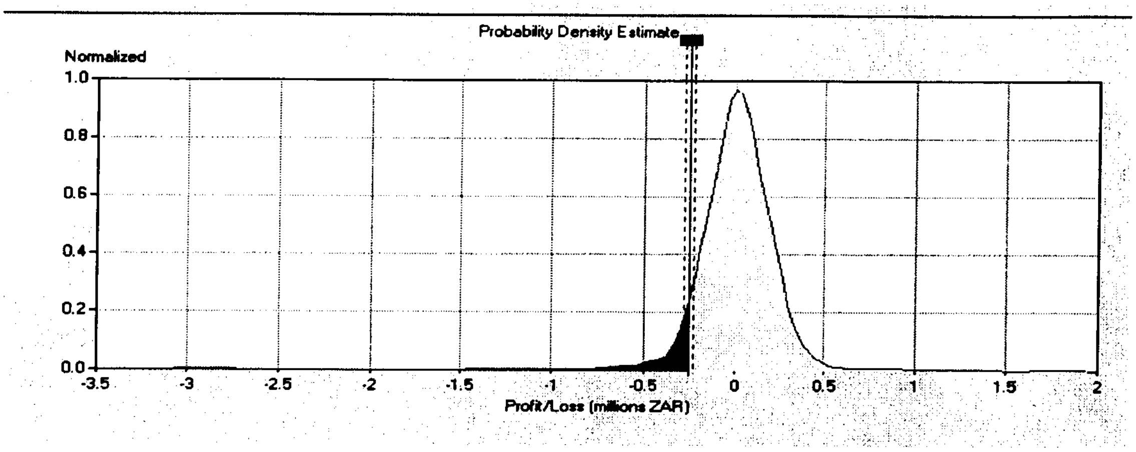

Figure A1.1: VaR distribution by Historical Simulation (Absolute Values)

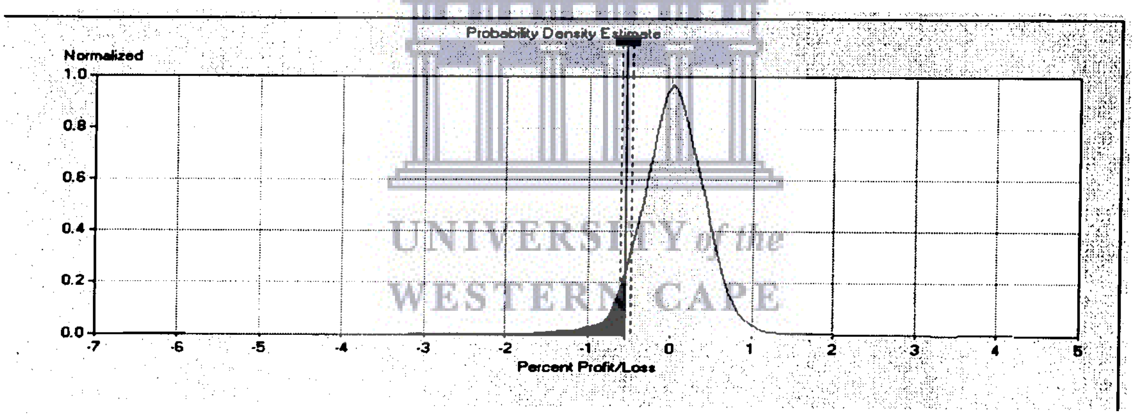

Figure A1.2: VaR Distribution by Historical Simulation (Percentage)

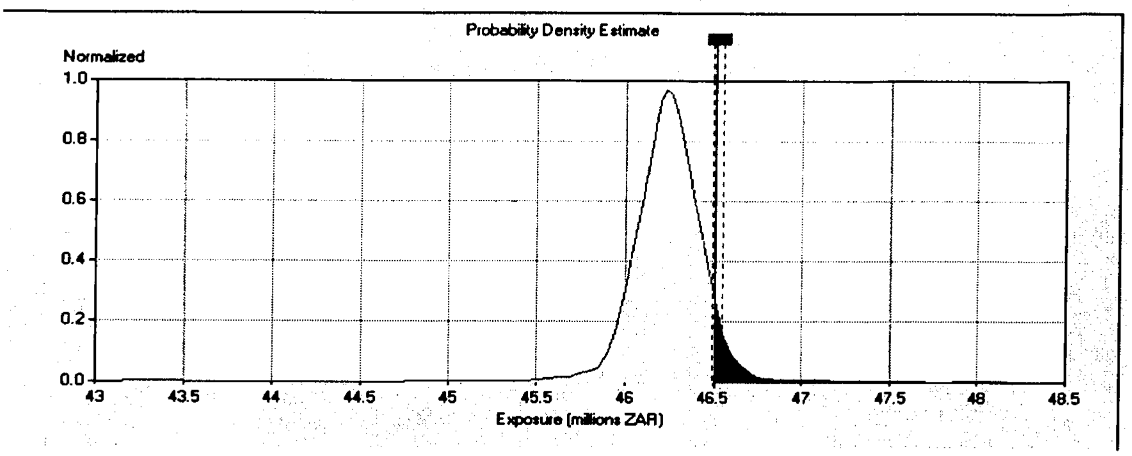

Figure A1.3: EaR Distribution by Historical Simulation (Absolute Values)

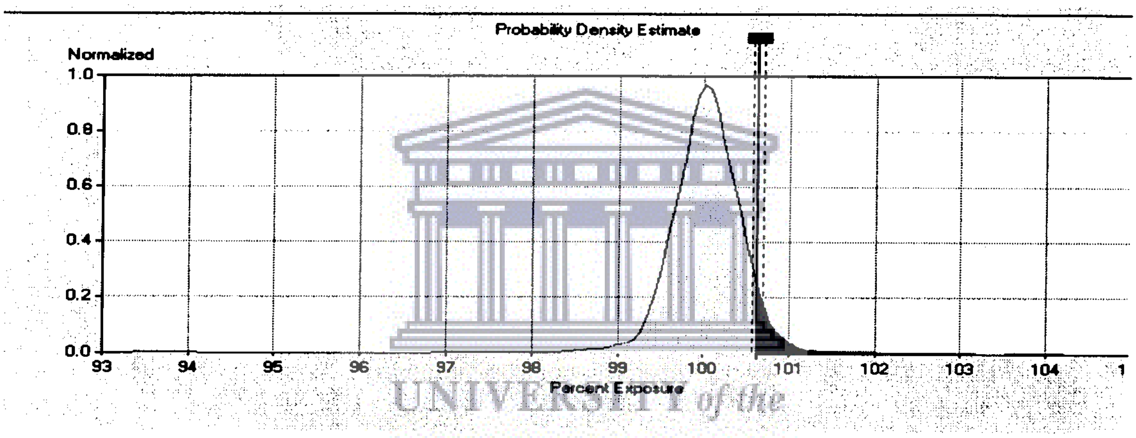

Figure A1.4: EaR Distribution by Historical Simulation (Percentage)

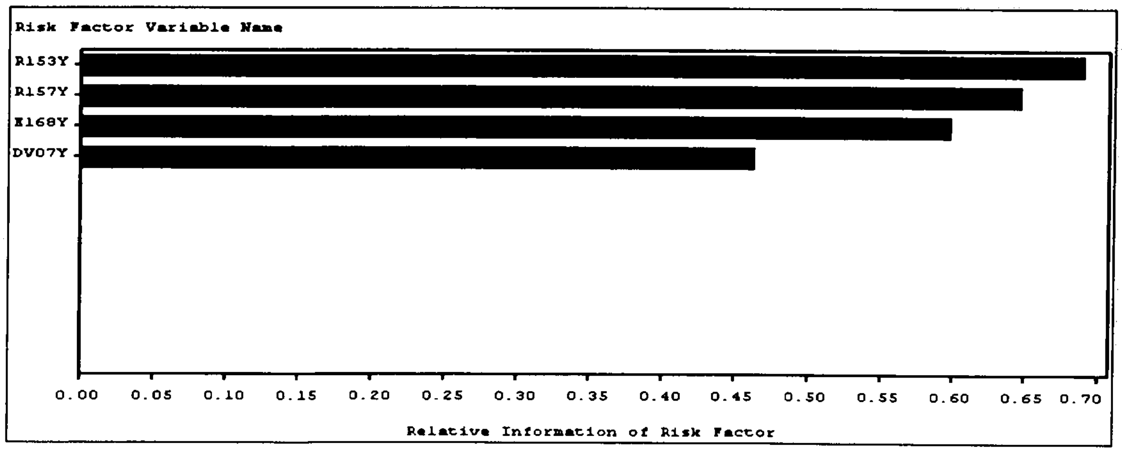

Figure A1.5: Historical Simulation Information Measures

(B) Variance-Covariance Method

ANLSYS	1. DeltaNormal
NInst	4
NMissing	0
AnalysisNumber	3
BaseDate	25SEP2003
date	26SEP2003
MtM	46,228,100.31
VaR	43,155.45
VaRPct	0.09
DV07Y	-5799363
E168Y	-3871764
R153Y	-5035836
R157Y	-8076768

Table B1.1: VaR Statistics by Variance-Covariance Method

(C) Monte Carlo Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	29,117.96
2	Lower Tolerance Limit of At-Risk Value (ZAR)	20,951.89
3	Upper Tolerance Limit of At-Risk Value (ZAR)	53,823.20
4	At-Risk Value as percent of Base Value	0.06
5	Lower Tol Limit of VaR as percent of Base	0.05
6	Upper Tol Limit of VaR as percent of Base	0.12
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	100
9	Number of Replications Actually Used	100
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	11,408.39
13	Standard Deviation of Profit/Loss	26,504.40
14	Skewness of Profit/Loss	-0.00944
15	Kurtosis of Profit/Loss	0.00455
16	Minimum Profit/Loss over Simulations (ZAR)	-54,034.87
17	Median Profit/Loss (ZAR)	8,475.84
18	Maximum Profit/Loss over Simulations (ZAR)	84,198.48

Table C1.1: VaR Statistics by Monte Carlo Simulation

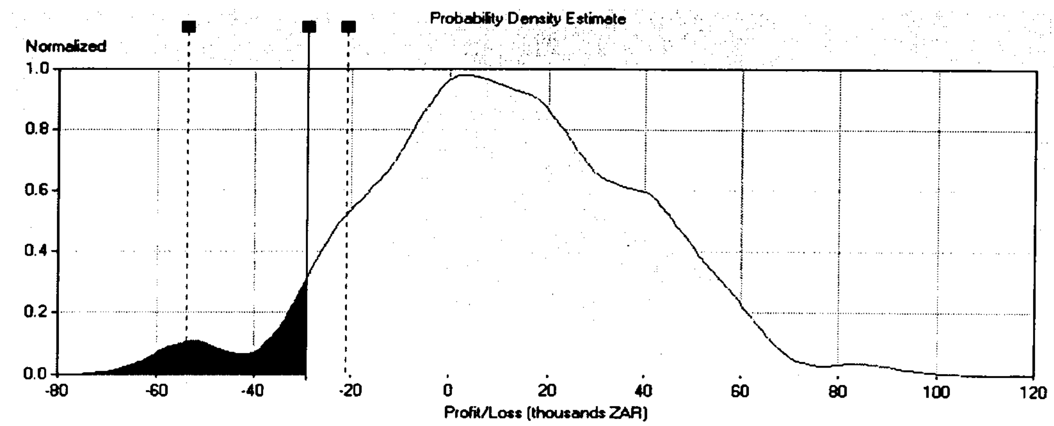

Figure C1.1: VaR Distribution by Monte Carlo Simulation (Absolute Values)

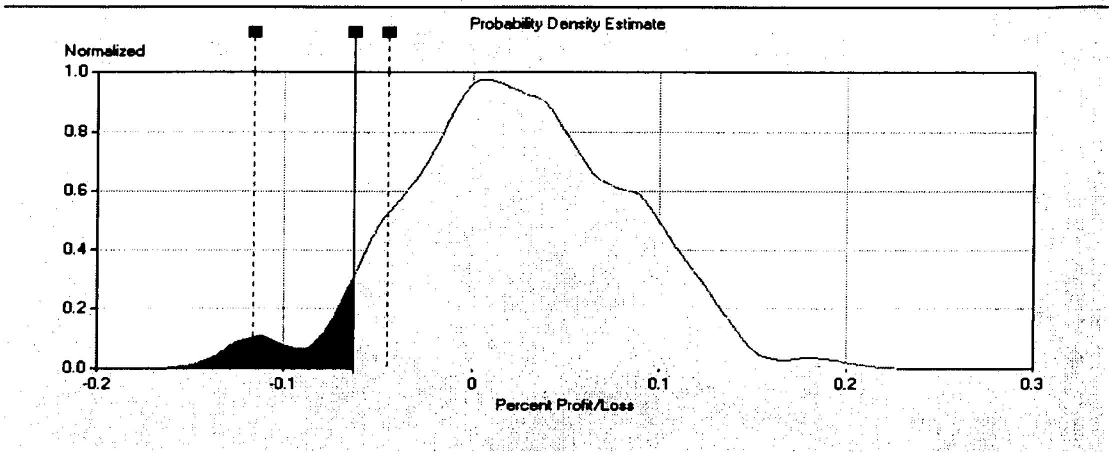

Figure C1.2: VaR Distribution by Monte Carlo Simulation (Percentage)

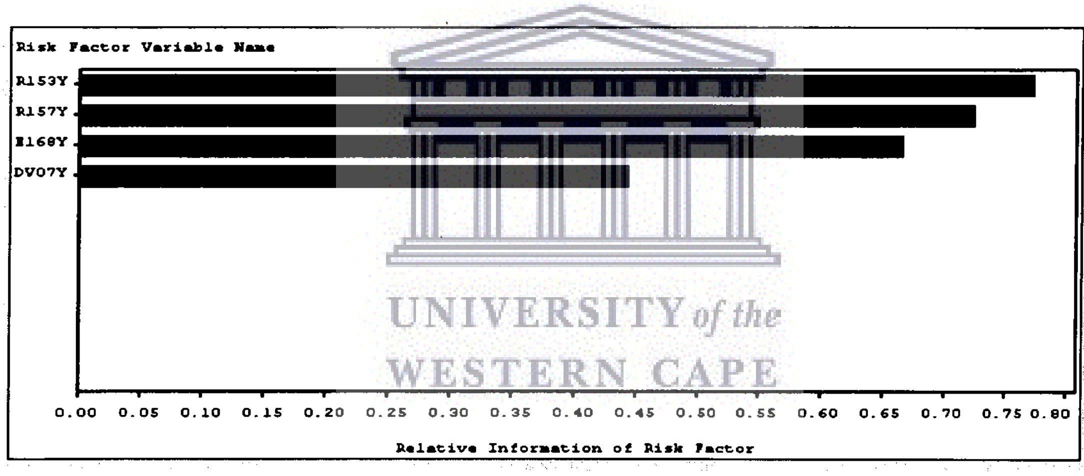

Figure C1.3: Monte Carlo Simulation Information Measures

(2) EQUAL WEIGHTAGE, 90% PROBABILITY, 1-HOLDING DAY, RUN ON 25 SEPTEMBER 2003.

(A) Historical Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	174,730.57
2	Lower Tolerance Limit of At-Risk Value (ZAR)	160,348.50
3	Upper Tolerance Limit of At-Risk Value (ZAR)	188,476.08
4	At-Risk Value as percent of Base Value	0.38
5	Lower Tol Limit of VaR as percent of Base	0.35
6	Upper Tol Limit of VaR as percent of Base	0.41
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	1092
9	Number of Replications Actually Used	1092
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	16,515.16
13	Standard Deviation of Profit/Loss	216,887.28
14	Skewness of Profit/Loss	-1.98977
15	Kurtosis of Profit/Loss	42.27753
16	Minimum Profit/Loss over Simulations (ZAR)	-2,997,430.29
17	Median Profit/Loss (ZAR)	14,801.38
18	Maximum Profit/Loss over Simulations (ZAR)	1,875,322.80

Table A2.1: VaR statistics by Historical Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	46,444,548.25
2	Lower Tolerance Limit of At-Risk Value (ZAR)	46,427,471.09
3	Upper Tolerance Limit of At-Risk Value (ZAR)	46,461,078.59
4	At-Risk Value as percent of Base Value	100.47
5	Lower Tol Limit of VaR as percent of Base	100.43
6	Upper Tol Limit of VaR as percent of Base	100.50
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	1092
9	Number of Replications Actually Used	1092
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	46,244,615.47
13	Standard Deviation of Profit/Loss	216,887.28
14	Skewness of Profit/Loss	-1.98977
15	Kurtosis of Profit/Loss	42.27753
16	Minimum Profit/Loss over Simulations (ZAR)	43,230,670.02
17	Median Profit/Loss (ZAR)	46,242,901.69
18	Maximum Profit/Loss over Simulations (ZAR)	48,103,423.11

Table A2.2: EaR statistics by Historical Simulation

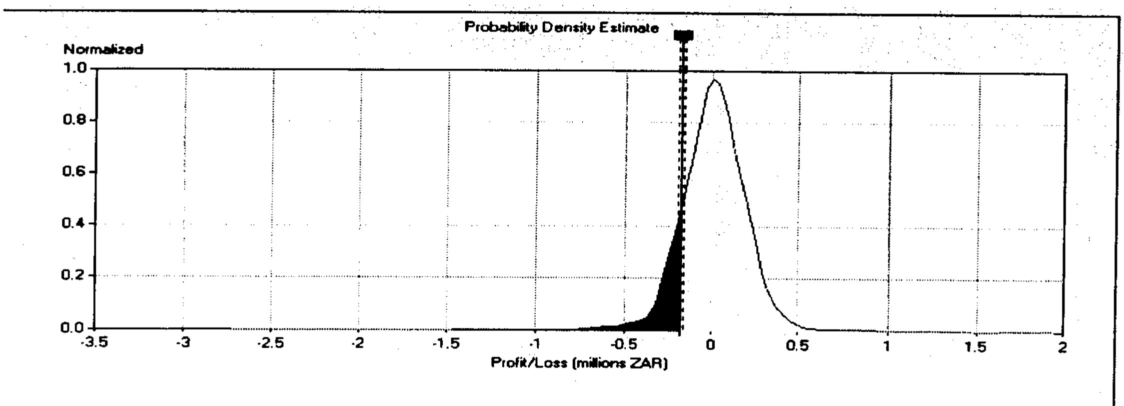

Figure A2.1: VaR Distribution by Historical Simulation

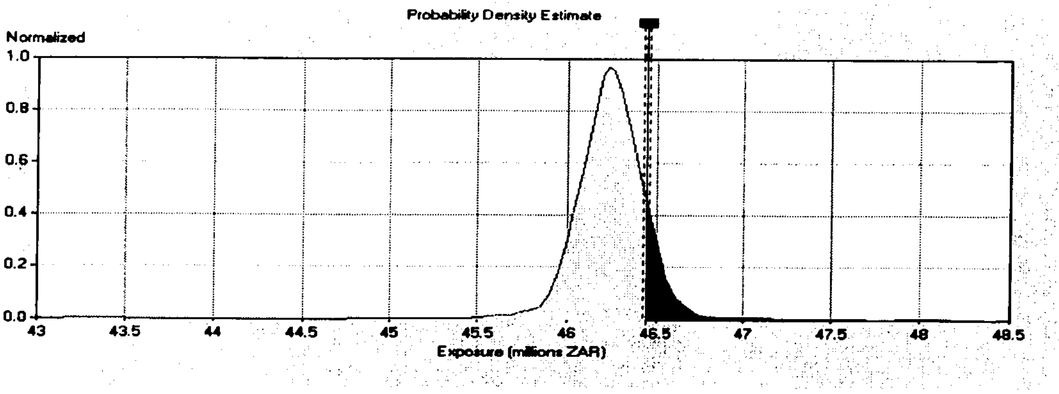

Figure A2.2: EaR Distribution by Historical Simulation

(B) Variance-Covariance Method

ANLSYS	1. DeltaNormal
NInst	4
NMissing	0
AnalysisNumber	3
BaseDate	25SEP2003
date	26SEP2003
MtM	46,228,100.31
VaR	33,623.62
VaRPct	0.07
DV07Y	-5799363
E168Y	-3871764
R153Y	-5035836
R157Y	-8076768

Table B2.1: VaR by Variance-Covariance Method

(C) Monte Carlo Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	20,951.89
2	Lower Tolerance Limit of At-Risk Value (ZAR)	14,999.33
3	Upper Tolerance Limit of At-Risk Value (ZAR)	26,738.85
4	At-Risk Value as percent of Base Value	0.05
5	Lower Tol Limit of VaR as percent of Base	0.03
6	Upper Tol Limit of VaR as percent of Base	0.06
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	100
9	Number of Replications Actually Used	100
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	11,408.39
13	Standard Deviation of Profit/Loss	26,504.40
14	Skewness of Profit/Loss	-0.00944
15	Kurtosis of Profit/Loss	0.00455
16	Minimum Profit/Loss over Simulations (ZAR)	-54,034.87
17	Median Profit/Loss (ZAR)	8,475.84
18	Maximum Profit/Loss over Simulations (ZAR)	84,198.48

Table C2.1: VaR Statistics by Monte Carlo Simulation

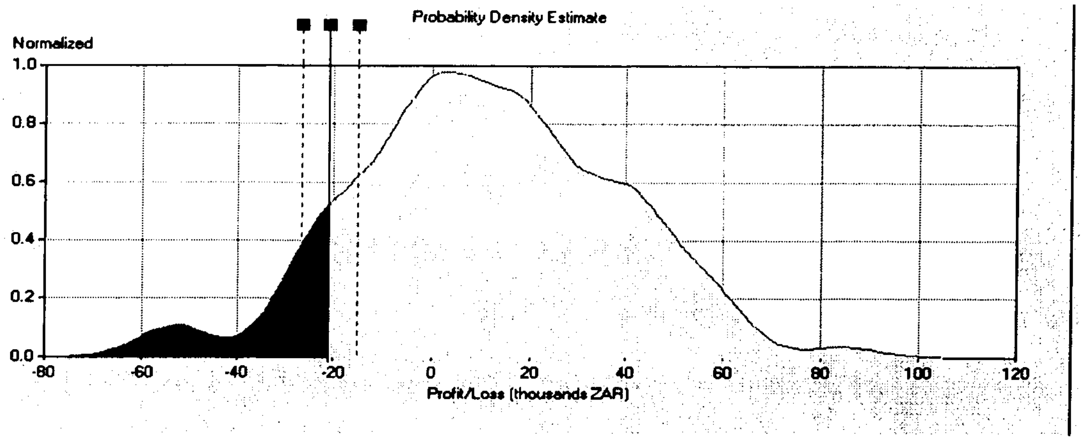

Figure C2.1: VaR Distribution by Monte Carlo Simulation

(3) EQUAL WEIGHTAGE, 99% PROBABILITY, 1-HOLDING DAY, RUN ON 25 SEPTEMBER 2003.

(A) Historical Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	567,792.30
2	Lower Tolerance Limit of At-Risk Value (ZAR)	477,794.91
3	Upper Tolerance Limit of At-Risk Value (ZAR)	716,338.43
4	At-Risk Value as percent of Base Value	1.23
5	Lower Tol Limit of VaR as percent of Base	1.03
6	Upper Tol Limit of VaR as percent of Base	1.55
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	1092
9	Number of Replications Actually Used	1092
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	16,515.16
13	Standard Deviation of Profit/Loss	216,887.28
14	Skewness of Profit/Loss	-1.98977
15	Kurtosis of Profit/Loss	42.27753
16	Minimum Profit/Loss over Simulations (ZAR)	-2,997,430.29
17	Median Profit/Loss (ZAR)	14,801.38
18	Maximum Profit/Loss over Simulations (ZAR)	1,875,322.80

Table A3.1: VaR by Historical Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	46,713,354.12
2	Lower Tolerance Limit of At-Risk Value (ZAR)	46,634,085.46
3	Upper Tolerance Limit of At-Risk Value (ZAR)	46,963,898.60
4	At-Risk Value as percent of Base Value	101.05
5	Lower Tol Limit of VaR as percent of Base	100.88
6	Upper Tol Limit of VaR as percent of Base	101.59
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	1092
9	Number of Replications Actually Used	1092
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	46,244,615.47
13	Standard Deviation of Profit/Loss	216,887.28
14	Skewness of Profit/Loss	-1.98977
15	Kurtosis of Profit/Loss	42.27753
16	Minimum Profit/Loss over Simulations (ZAR)	43,230,670.02
17	Median Profit/Loss (ZAR)	46,242,901.69
18	Maximum Profit/Loss over Simulations (ZAR)	48,103,423.11

Table A3.2: EaR by Historical Simulation

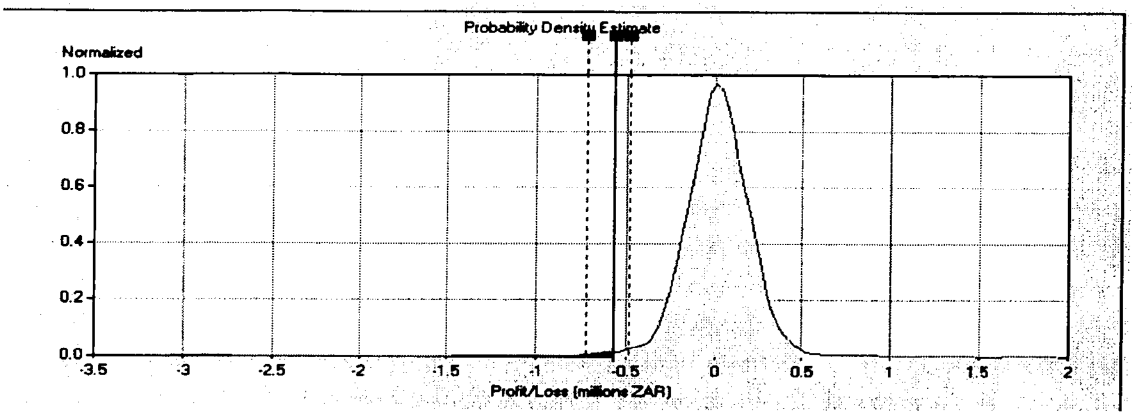

Figure A3.1: VaR Distribution by Historical simulation

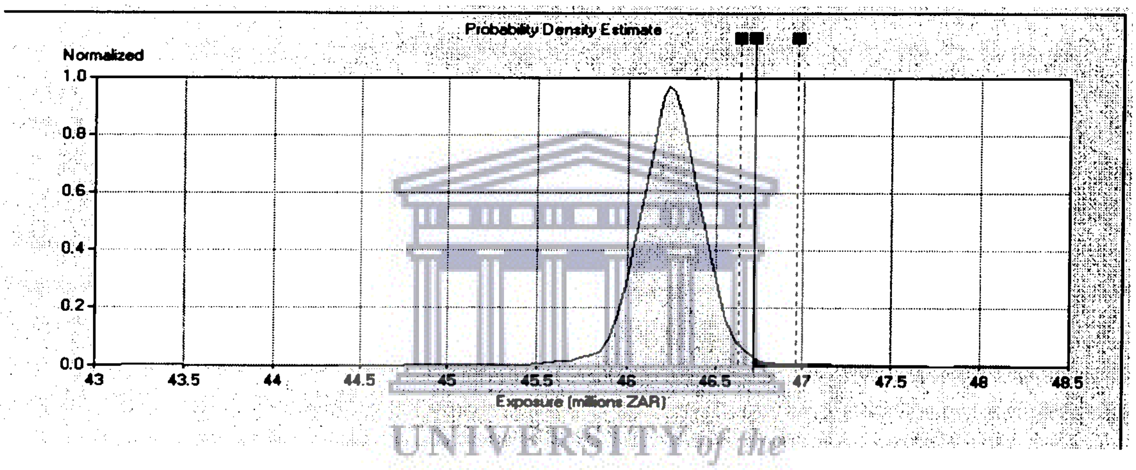

Figure A3.2: EaR Distribution by Historical Simulation

(B) Variance-Covariance Method

ANLSYS	1. DeltaNormal
NInst	4
NMissing	0
AnalysisNumber	3
BaseDate	25SEP2003
date	26SEP2003
MtM	46,228,100.31
VaR	61,035.58
VaRPct	0.13
DV07Y	-5799363
E168Y	-3871764
R153Y	-5035836
R157Y	-8076768

Table B3.1: VaR by Variance-Covariance Method

(C) Monte Carlo Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	54,034.87
2	Lower Tolerance Limit of At-Risk Value (ZAR)	
3	Upper Tolerance Limit of At-Risk Value (ZAR)	
4	At-Risk Value as percent of Base Value	0.12
5	Lower Tol Limit of VaR as percent of Base	
6	Upper Tol Limit of VaR as percent of Base	
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	100
9	Number of Replications Actually Used	100
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	11,408.39
13	Standard Deviation of Profit/Loss	26,504.40
14	Skewness of Profit/Loss	-0.00944
15	Kurtosis of Profit/Loss	0.00455
16	Minimum Profit/Loss over Simulations (ZAR)	-54,034.87
17	Median Profit/Loss (ZAR)	8,475.84
18	Maximum Profit/Loss over Simulations (ZAR)	84,198.48

Table C3.1: VaR statistics by Monte Carlo Method

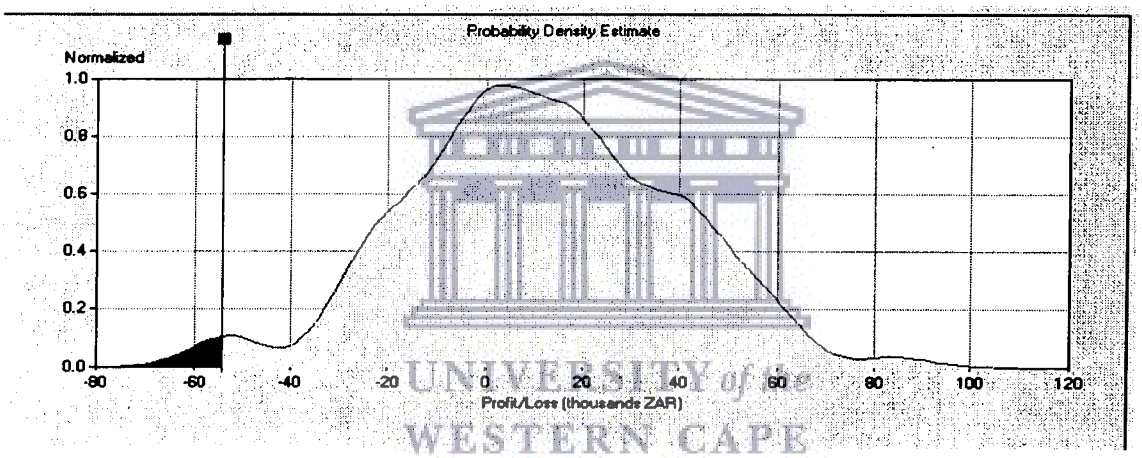

Figure C3.1: VaR Distribution by Monte Carlo Simulation

(4) EQUAL WEIGHTAGE, 92.5% PROBABILITY, 1-HOLDING DAY, RUN ON 25 SEPTEMBER 2003.

(A) Historical Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	201,749.24
2	Lower Tolerance Limit of At-Risk Value (ZAR)	185,493.87
3	Upper Tolerance Limit of At-Risk Value (ZAR)	222,658.44
4	At-Risk Value as percent of Base Value	0.44
5	Lower Tol Limit of VaR as percent of Base	0.40
6	Upper Tol Limit of VaR as percent of Base	0.48
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	1092
9	Number of Replications Actually Used	1092
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	16,515.16
13	Standard Deviation of Profit/Loss	216,887.28
14	Skewness of Profit/Loss	-1.98977
15	Kurtosis of Profit/Loss	42.27753
16	Minimum Profit/Loss over Simulations (ZAR)	-2,997,430.29
17	Median Profit/Loss (ZAR)	14,801.38
18	Maximum Profit/Loss over Simulations (ZAR)	1,875,322.80

Table A4.1: VaR statistics by Historical Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	46,474,519.85
2	Lower Tolerance Limit of At-Risk Value (ZAR)	46,454,454.09
3	Upper Tolerance Limit of At-Risk Value (ZAR)	46,492,933.94
4	At-Risk Value as percent of Base Value	100.53
5	Lower Tol Limit of VaR as percent of Base	100.49
6	Upper Tol Limit of VaR as percent of Base	100.57
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	1092
9	Number of Replications Actually Used	1092
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	46,244,615.47
13	Standard Deviation of Profit/Loss	216,887.28
14	Skewness of Profit/Loss	-1.98977
15	Kurtosis of Profit/Loss	42.27753
16	Minimum Profit/Loss over Simulations (ZAR)	43,230,670.02
17	Median Profit/Loss (ZAR)	46,242,901.69
18	Maximum Profit/Loss over Simulations (ZAR)	48,103,423.11

Table A4.2: EaR statistics by Historical Simulation

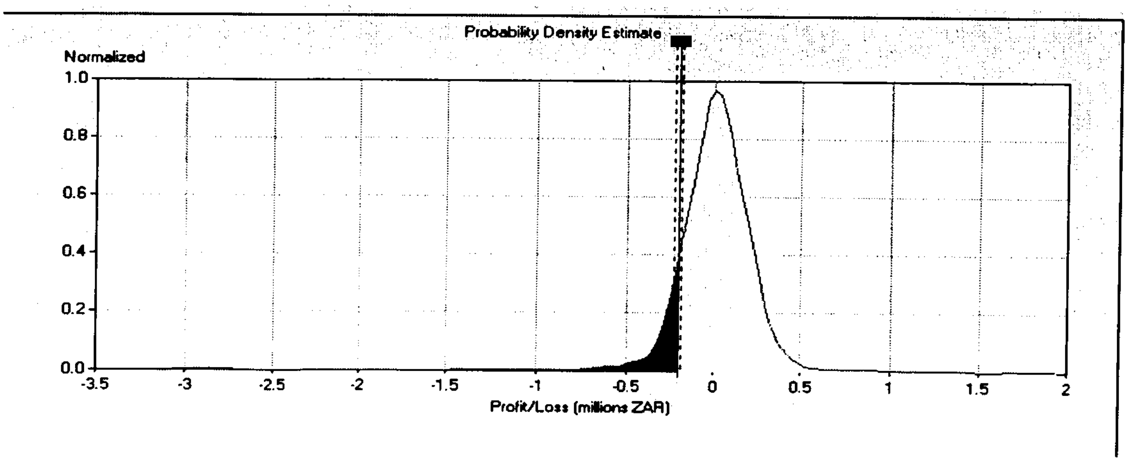

Figure A4.1: VaR Distribution by Historical Simulation

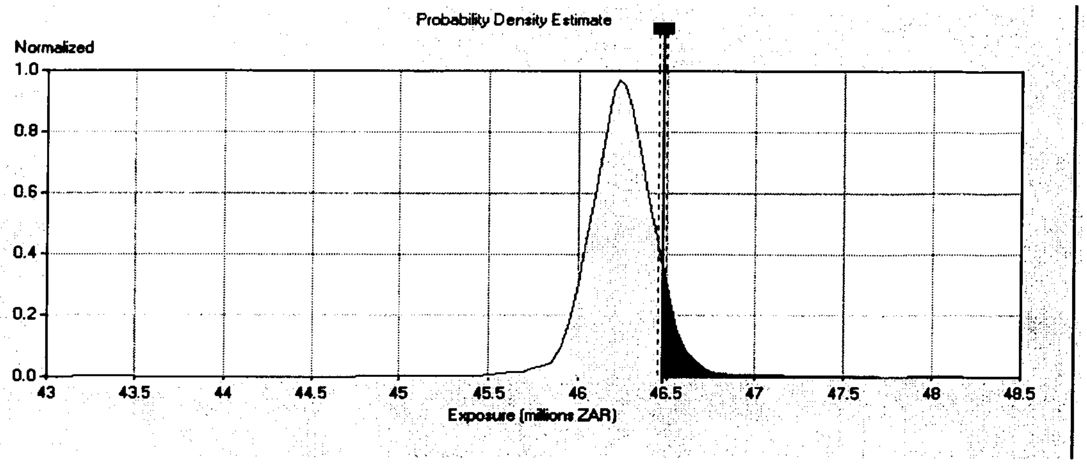

Figure A4.2: EaR Distribution by Historical Simulation

(B) Variance-Covariance Method

ANLSYS	1. DeltaNormal
NInst	4
NMissing	0
AnalysisNumber	3
BaseDate	25SEP2003
date	26SEP2003
MtM	46,228,100.31
VaR	37,768.49
VaRPct	0.08
DV07Y	-5799363
E168Y	-3871764
R153Y	-5035836
R157Y	-8076768

Table B4.1: VaR statistics by Variance-Covariance Method

(C) Monte Carlo Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	24,762.08
2	Lower Tolerance Limit of At-Risk Value (ZAR)	18,668.47
3	Upper Tolerance Limit of At-Risk Value (ZAR)	30,269.10
4	At-Risk Value as percent of Base Value	0.05
5	Lower Tol Limit of VaR as percent of Base	0.04
6	Upper Tol Limit of VaR as percent of Base	0.07
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	100
9	Number of Replications Actually Used	100
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	11,408.39
13	Standard Deviation of Profit/Loss	26,504.40
14	Skewness of Profit/Loss	-0.00944
15	Kurtosis of Profit/Loss	0.00455
16	Minimum Profit/Loss over Simulations (ZAR)	-54,034.87
17	Median Profit/Loss (ZAR)	8,475.84
18	Maximum Profit/Loss over Simulations (ZAR)	84,198.48

Table C4.1: VaR statistics by Monte Carlo Simulation

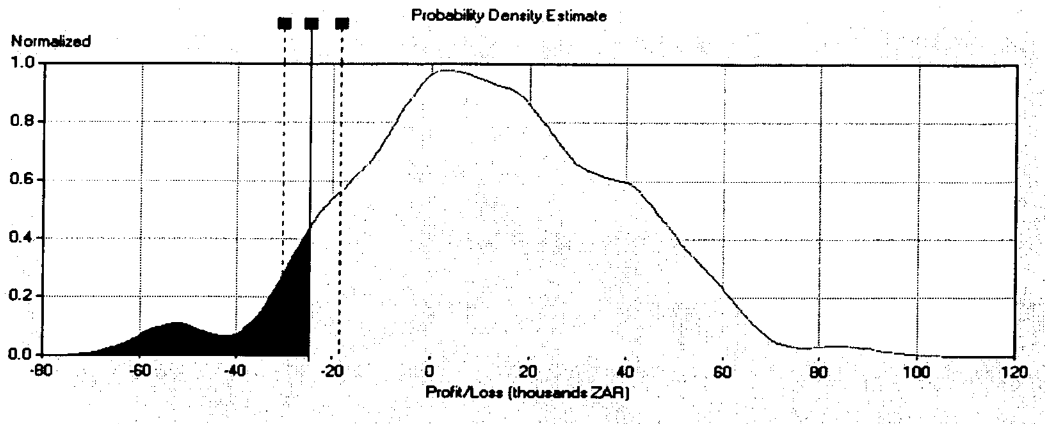

Figure C4.1: VaR Distribution by Monte Carlo Simulation

(5) EQUAL WEIGHTAGE, 97.5% PROBABILITY, 1-HOLDING DAY, RUN ON 25 SEPTEMBER 2003.

(A) Historical Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	336,032.02
2	Lower Tolerance Limit of At-Risk Value (ZAR)	300,876.05
3	Upper Tolerance Limit of At-Risk Value (ZAR)	449,389.85
4	At-Risk Value as percent of Base Value	0.73
5	Lower Tol Limit of VaR as percent of Base	0.65
6	Upper Tol Limit of VaR as percent of Base	0.97
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	1092
9	Number of Replications Actually Used	1092
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	16,515.16
13	Standard Deviation of Profit/Loss	216,887.28
14	Skewness of Profit/Loss	-1.98977
15	Kurtosis of Profit/Loss	42.27753
16	Minimum Profit/Loss over Simulations (ZAR)	-2,997,430.29
17	Median Profit/Loss (ZAR)	14,801.38
18	Maximum Profit/Loss over Simulations (ZAR)	1,875,322.80

Figure A5.1: VaR statistics by Historical Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	46,604,229.75
2	Lower Tolerance Limit of At-Risk Value (ZAR)	46,571,006.93
3	Upper Tolerance Limit of At-Risk Value (ZAR)	46,628,678.37
4	At-Risk Value as percent of Base Value	100.81
5	Lower Tol Limit of VaR as percent of Base	100.74
6	Upper Tol Limit of VaR as percent of Base	100.87
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	1092
9	Number of Replications Actually Used	1092
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	46,244,615.47
13	Standard Deviation of Profit/Loss	216,887.28
14	Skewness of Profit/Loss	-1.98977
15	Kurtosis of Profit/Loss	42.27753
16	Minimum Profit/Loss over Simulations (ZAR)	43,230,670.02
17	Median Profit/Loss (ZAR)	46,242,901.69
18	Maximum Profit/Loss over Simulations (ZAR)	48,103,423.11

Figure A5.2: EaR statistics by Historical Simulation

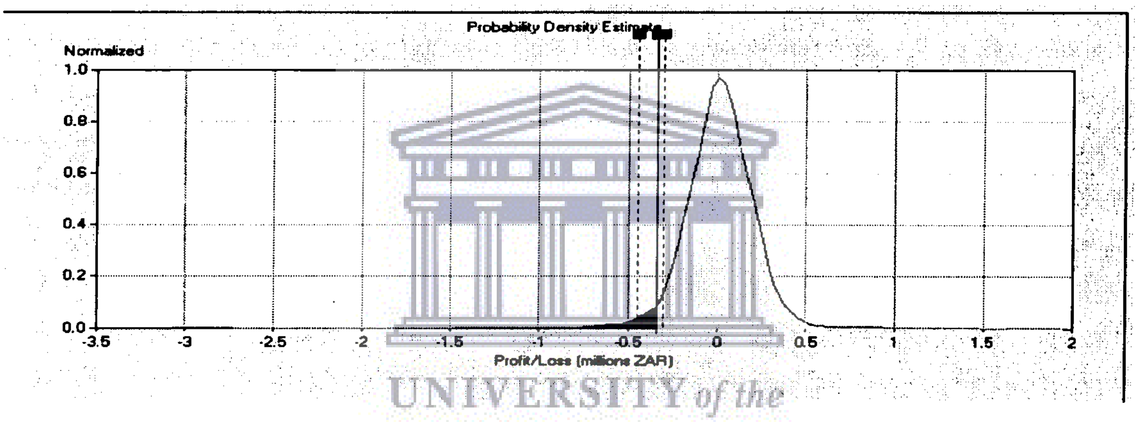

Figure A5.1: VaR Distribution by Historical Simulation

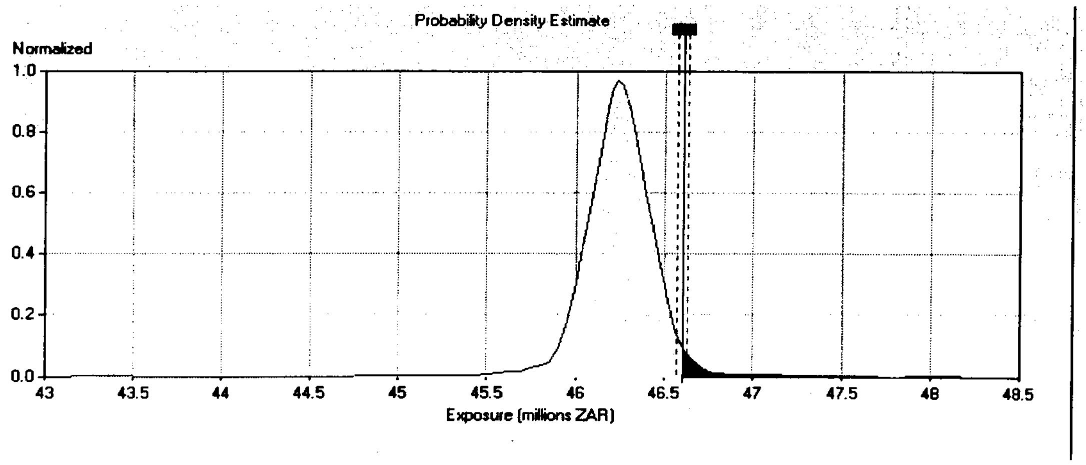

Figure A5.2: EaR Distribution by Historical Simulation

(B) Variance-Covariance Method

ANLSYS	1. DeltaNormal
NInst	4
NMissing	0
AnalysisNumber	3
BaseDate	25SEP2003
date	26SEP2003
MtM	46,228,100.31
VaR	51,422.89
VaRPct	0.11
DV07Y	-5799363
E168Y	-3871764
R153Y	-5035836
R157Y	-8076768

Figure B5.1: VaR statistics by Variance-Covariance Method

(C) Monte Carlo Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	52,609.27
2	Lower Tolerance Limit of At-Risk Value (ZAR)	25,915.53
3	Upper Tolerance Limit of At-Risk Value (ZAR)	54,034.87
4	At-Risk Value as percent of Base Value	0.11
5	Lower Tol Limit of VaR as percent of Base	0.06
6	Upper Tol Limit of VaR as percent of Base	0.12
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	100
9	Number of Replications Actually Used	100
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	11,408.39
13	Standard Deviation of Profit/Loss	26,504.40
14	Skewness of Profit/Loss	-0.00944
15	Kurtosis of Profit/Loss	0.00455
16	Minimum Profit/Loss over Simulations (ZAR)	-54,034.87
17	Median Profit/Loss (ZAR)	8,475.84
18	Maximum Profit/Loss over Simulations (ZAR)	84,198.48

Table C5.1: VaR statistics by Monte Carlo Simulation

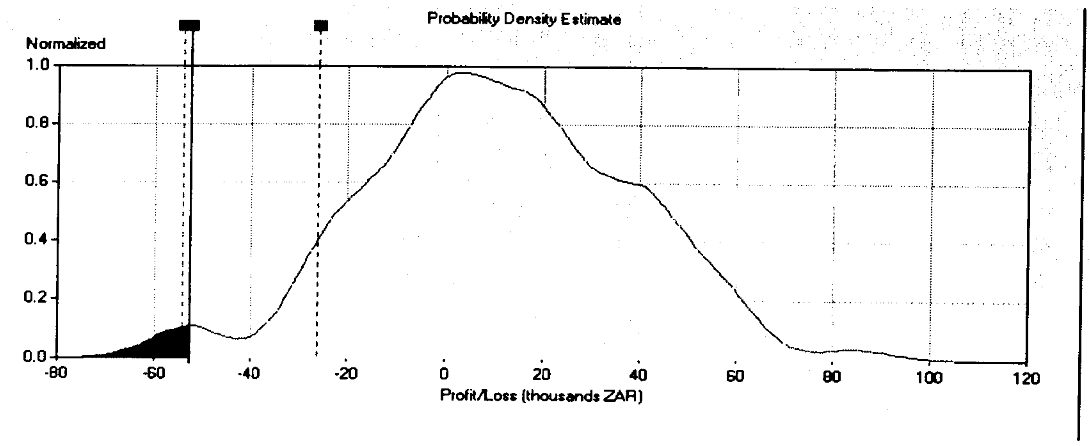

Figure C5.1: VaR Distribution by Monte Carlo Simulation

(6) EQUAL WEIGHTAGE, 95% PROBABILITY, 1-HOLDING DAY, RUN ON 25 SEPTEMBER 2002 – The Past.

(A) Historical Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	281,357.68
2	Lower Tolerance Limit of At-Risk Value (ZAR)	246,672.94
3	Upper Tolerance Limit of At-Risk Value (ZAR)	305,319.28
4	At-Risk Value as percent of Base Value	0.60
5	Lower Tol Limit of VaR as percent of Base	0.53
6	Upper Tol Limit of VaR as percent of Base	0.65
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	1092
9	Number of Replications Actually Used	1092
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	17,252.33
13	Standard Deviation of Profit/Loss	237,826.96
14	Skewness of Profit/Loss	-1.95652
15	Kurtosis of Profit/Loss	41.91976
16	Minimum Profit/Loss over Simulations (ZAR)	-3,277,133.84
17	Median Profit/Loss (ZAR)	14,993.44
18	Maximum Profit/Loss over Simulations (ZAR)	2,061,801.65

Table A6.1: VaR statistics by Historical Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	46,963,303.69
2	Lower Tolerance Limit of At-Risk Value (ZAR)	46,939,238.91
3	Upper Tolerance Limit of At-Risk Value (ZAR)	47,009,659.59
4	At-Risk Value as percent of Base Value	100.67
5	Lower Tol Limit of VaR as percent of Base	100.62
6	Upper Tol Limit of VaR as percent of Base	100.77
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	1092
9	Number of Replications Actually Used	1092
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	46,667,824.02
13	Standard Deviation of Profit/Loss	237,826.96
14	Skewness of Profit/Loss	-1.95652
15	Kurtosis of Profit/Loss	41.91976
16	Minimum Profit/Loss over Simulations (ZAR)	43,373,437.85
17	Median Profit/Loss (ZAR)	46,665,565.13
18	Maximum Profit/Loss over Simulations (ZAR)	48,712,373.34

Table A6.2: EaR statistics by Historical Simulation

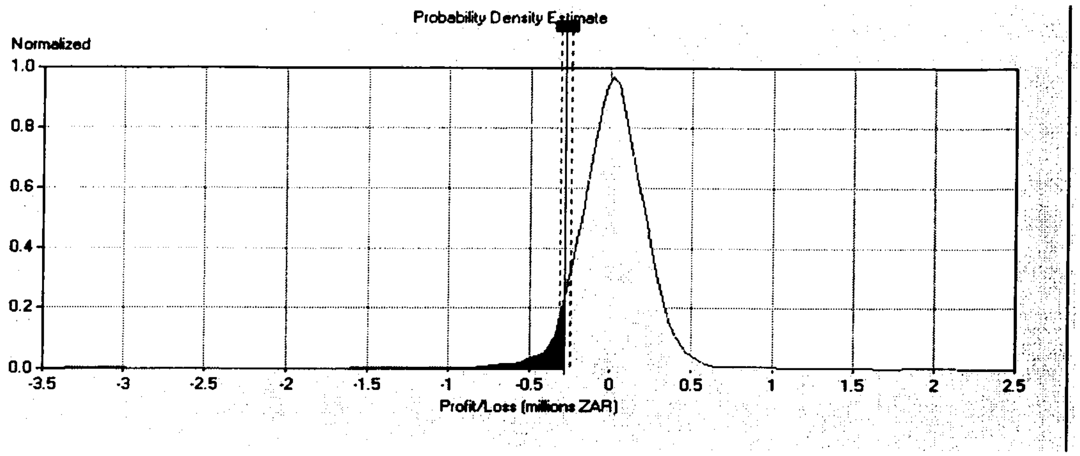

Figure A6.1: VaR Distribution by Historical Simulation

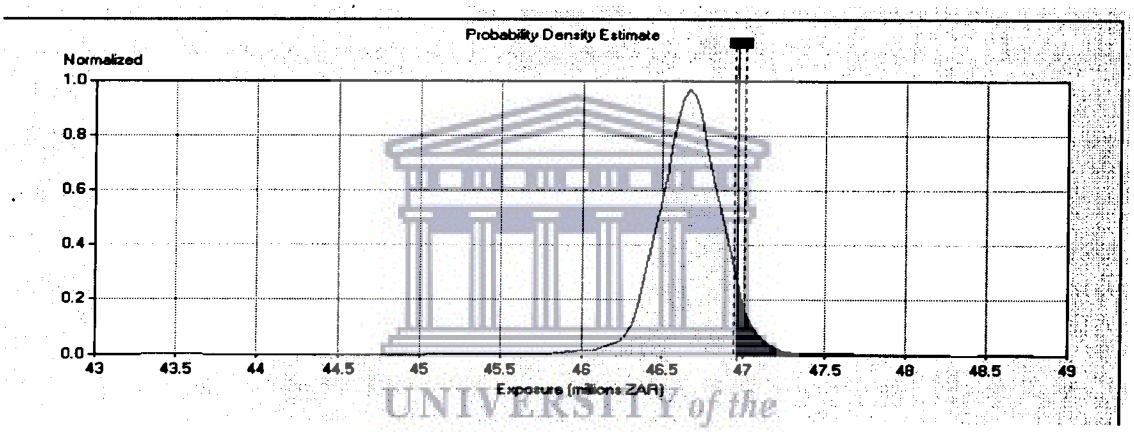

Figure A6.2: EaR Distribution by Historical Simulation

(B) Variance-Covariance Method

ANLSYS	1. DeltaNormal
NInst	4
NMissing	0
AnalysisNumber	3
BaseDate	25SEP2002
date	26SEP2002
MtM	46,650,571.69
VaR	47,363.37
VaRPct	0.10
DV07Y	-6410617
E168Y	-4488992
R153Y	-5637046
R157Y	-8472279

Table B6.1: VaR statistics by Variance-Covariance Method

(C) Monte Carlo Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	32,948.70
2	Lower Tolerance Limit of At-Risk Value (ZAR)	23,962.08
3	Upper Tolerance Limit of At-Risk Value (ZAR)	60,106.61
4	At-Risk Value as percent of Base Value	0.07
5	Lower Tol Limit of VaR as percent of Base	0.05
6	Upper Tol Limit of VaR as percent of Base	0.13
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	100
9	Number of Replications Actually Used	100
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	11,567.12
13	Standard Deviation of Profit/Loss	29,086.94
14	Skewness of Profit/Loss	-0.00893
15	Kurtosis of Profit/Loss	0.00672
16	Minimum Profit/Loss over Simulations (ZAR)	-60,140.28
17	Median Profit/Loss (ZAR)	8,383.25
18	Maximum Profit/Loss over Simulations (ZAR)	91,625.57

Table C6.1: VaR statistics by Monte Carlo Simulation

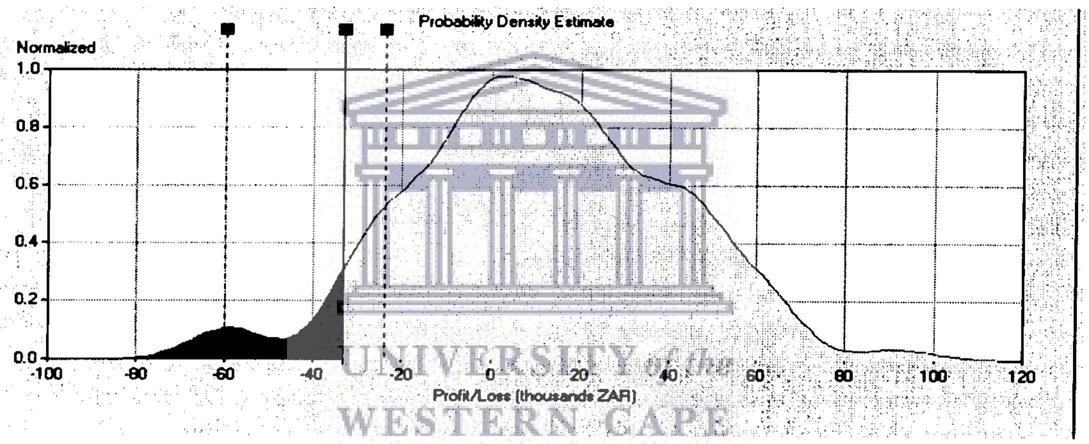

Figure C6.1: VaR Distribution by Monte Carlo Simulation

(7) EQUAL WEIGHTAGE, 95% PROBABILITY, 1-HOLDING DAY, RUN ON 14 May 2004 – The Future.

(A) Historical Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	244,119.82
2	Lower Tolerance Limit of At-Risk Value (ZAR)	211,280.84
3	Upper Tolerance Limit of At-Risk Value (ZAR)	266,687.62
4	At-Risk Value as percent of Base Value	0.53
5	Lower Tol Limit of VaR as percent of Base	0.46
6	Upper Tol Limit of VaR as percent of Base	0.58
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	1092
9	Number of Replications Actually Used	1092
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	18,209.94
13	Standard Deviation of Profit/Loss	209,629.33
14	Skewness of Profit/Loss	-2.00188
15	Kurtosis of Profit/Loss	42.38182
16	Minimum Profit/Loss over Simulations (ZAR)	-2,897,883.56
17	Median Profit/Loss (ZAR)	16,667.47
18	Maximum Profit/Loss over Simulations (ZAR)	1,812,312.15

Table A7.1: VaR statistics by Historical Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	46,030,534.61
2	Lower Tolerance Limit of At-Risk Value (ZAR)	46,008,811.52
3	Upper Tolerance Limit of At-Risk Value (ZAR)	46,069,506.86
4	At-Risk Value as percent of Base Value	100.61
5	Lower Tol Limit of VaR as percent of Base	100.56
6	Upper Tol Limit of VaR as percent of Base	100.70
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	1092
9	Number of Replications Actually Used	1092
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	45,768,637.02
13	Standard Deviation of Profit/Loss	209,629.33
14	Skewness of Profit/Loss	-2.00188
15	Kurtosis of Profit/Loss	42.38182
16	Minimum Profit/Loss over Simulations (ZAR)	42,852,543.51
17	Median Profit/Loss (ZAR)	45,767,094.55
18	Maximum Profit/Loss over Simulations (ZAR)	47,562,739.23

Table A7.2: EaR statistics by Historical Simulation

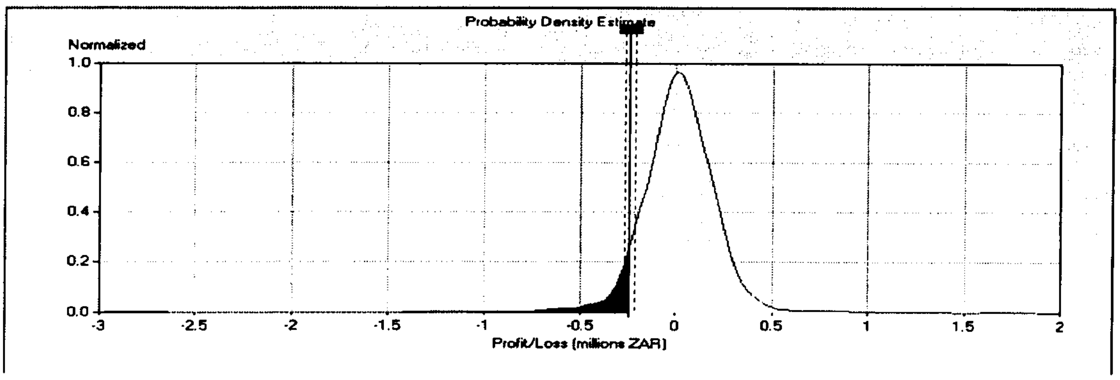

Figure A7.1: VaR Distribution by Historical Simulation

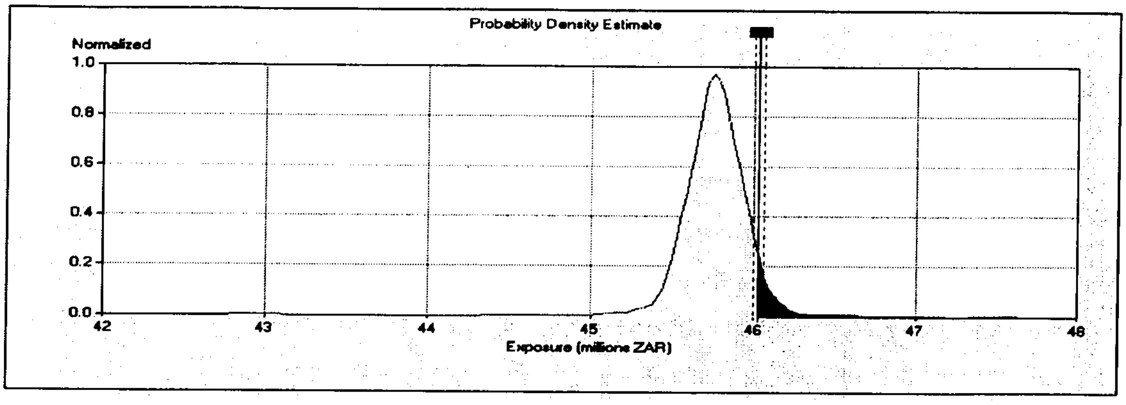

Figure A7.2: EaR Distribution by Historical Simulation

(B) Variance-Covariance Method

ANLSYS	1. DeltaNormal
NInst	4
NMissing	0
AnalysisNumber	3
BaseDate	25MAR2004
date	26MAR2004
MtM	45,750,427.07
VaR	41,679.86
VaRPct	0.09
DV07Y	-5476839
E168Y	-3550986
R153Y	-5108467
R157Y	-7866949

Table B7.1: VaR statistics by Variance-Covariance Method

(C) Monte Carlo Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	25,887.17
2	Lower Tolerance Limit of At-Risk Value (ZAR)	17,832.40
3	Upper Tolerance Limit of At-Risk Value (ZAR)	49,696.93
4	At-Risk Value as percent of Base Value	0.06
5	Lower Tol Limit of VaR as percent of Base	0.04
6	Upper Tol Limit of VaR as percent of Base	0.11
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	100
9	Number of Replications Actually Used	100
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	13,301.49
13	Standard Deviation of Profit/Loss	25,594.73
14	Skewness of Profit/Loss	-0.01011
15	Kurtosis of Profit/Loss	0.00419
16	Minimum Profit/Loss over Simulations (ZAR)	-50,041.16
17	Median Profit/Loss (ZAR)	10,408.00
18	Maximum Profit/Loss over Simulations (ZAR)	83,514.94

Table C7.1: VaR statistics by Monte Carlo Simulation

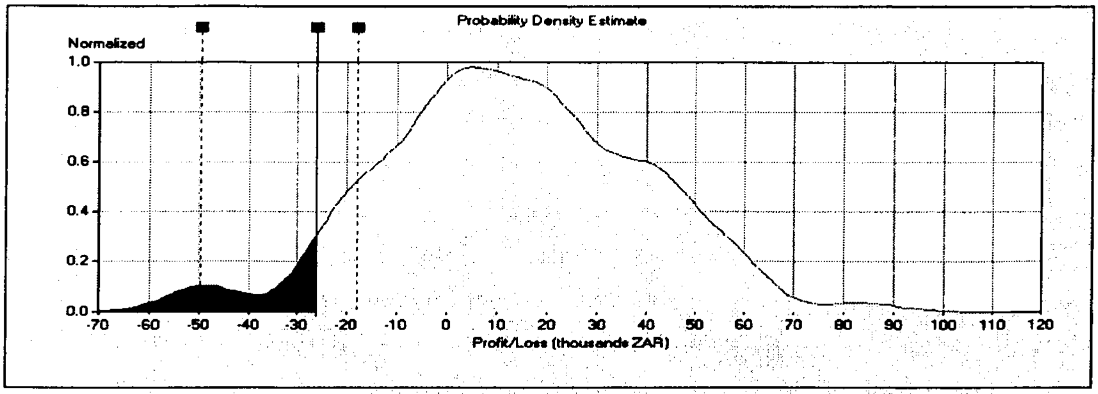

Figure C7.1: VaR Distribution by Monte Carlo Simulation

(8) EQUAL WEIGHTAGE, 95% PROBABILITY, 10-HOLDING DAYS FOR R153, RUN ON 25 SEPTEMBER 2003.

(A) Historical Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	793,167.88
2	Lower Tolerance Limit of At-Risk Value (ZAR)	707,423.50
3	Upper Tolerance Limit of At-Risk Value (ZAR)	911,655.38
4	At-Risk Value as percent of Base Value	0.53
5	Lower Tol Limit of VaR as percent of Base	0.47
6	Upper Tol Limit of VaR as percent of Base	0.61
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	1092
9	Number of Replications Actually Used	1092
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	37,352.96
13	Standard Deviation of Profit/Loss	662,086.29
14	Skewness of Profit/Loss	-2.03240
15	Kurtosis of Profit/Loss	41.43560
16	Minimum Profit/Loss over Simulations (ZAR)	-9,143,878.35
17	Median Profit/Loss (ZAR)	30,618.16
18	Maximum Profit/Loss over Simulations (ZAR)	5,603,083.41

Table A8.1: VaR statistics by Historical Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	151,108,949.72
2	Lower Tolerance Limit of At-Risk Value (ZAR)	151,008,584.99
3	Upper Tolerance Limit of At-Risk Value (ZAR)	151,203,430.50
4	At-Risk Value as percent of Base Value	100.60
5	Lower Tol Limit of VaR as percent of Base	100.54
6	Upper Tol Limit of VaR as percent of Base	100.67
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	1092
9	Number of Replications Actually Used	1092
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	150,239,992.65
13	Standard Deviation of Profit/Loss	662,086.29
14	Skewness of Profit/Loss	-2.03240
15	Kurtosis of Profit/Loss	41.43560
16	Minimum Profit/Loss over Simulations (ZAR)	141,058,761.34
17	Median Profit/Loss (ZAR)	150,233,257.86
18	Maximum Profit/Loss over Simulations (ZAR)	155,805,723.10

Table A8.2: EaR statistics by Historical Simulation

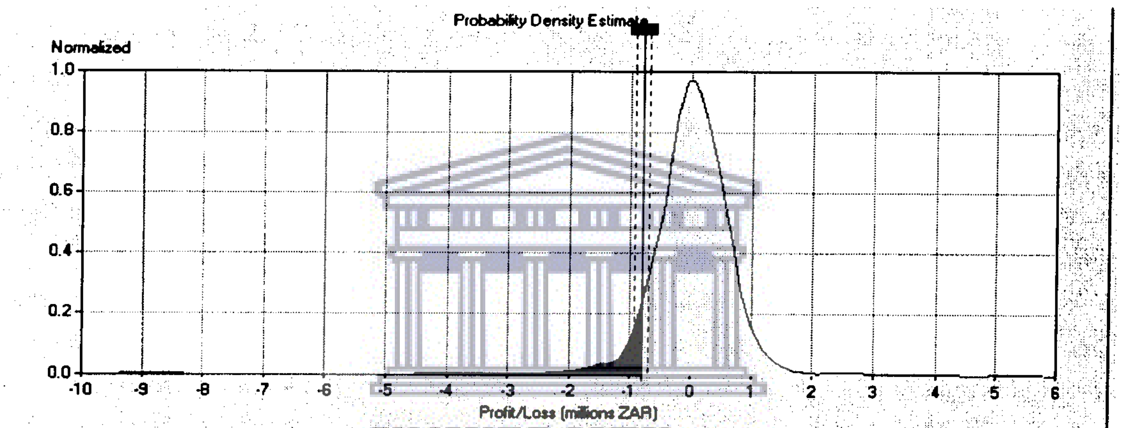

Figure A8.1: VaR Distribution by Historical Simulation

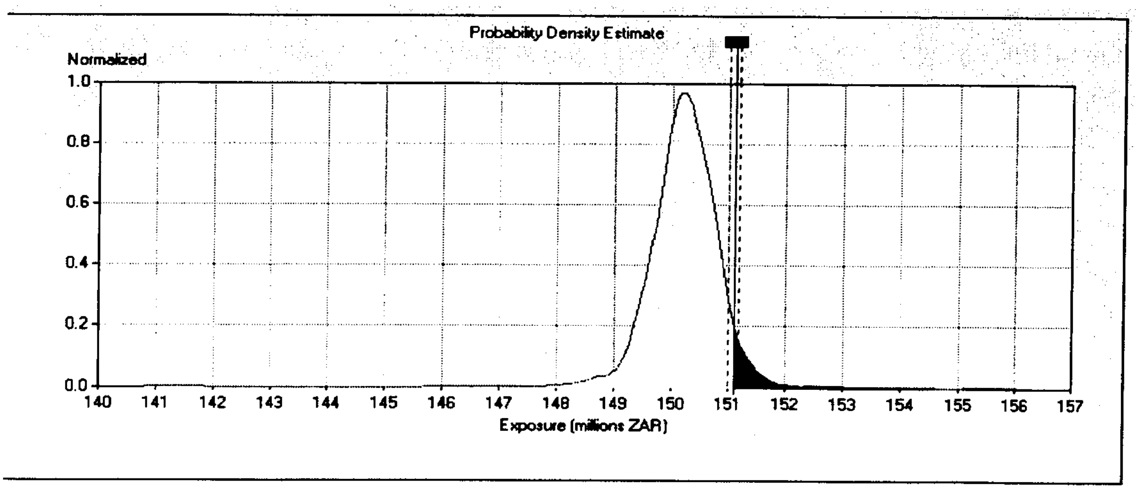

Figure A8.2: EaR Distribution by Historical Simulation

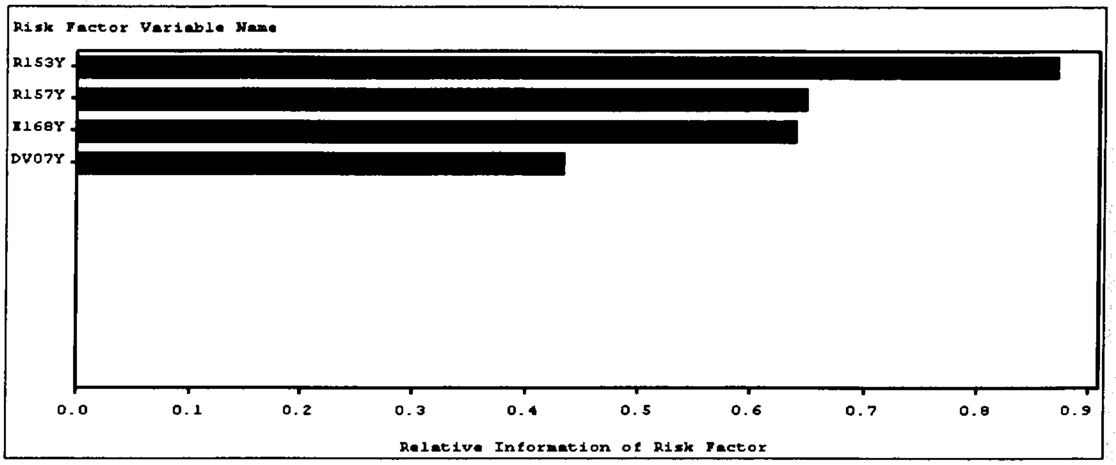

Figure A8.3: Information Measures by Historical Simulation

(B) **Variance-Covariance Method**

ANLSYS	1. DeltaNormal
NInst	4
NMissing	0
AnalysisNumber	1
BaseDate	25SEP2003
date	26SEP2003
MtM	150,202,639.69
VaR	129,526.43
VaRPct	0.09
DV07Y	-5799363
E168Y	-3871764
R153Y	-50358364
R157Y	-8076768

Table B8.1: VaR statistics by Variance-Covariance Method

(C) Monte Carlo Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	100,244.78
2	Lower Tolerance Limit of At-Risk Value (ZAR)	79,476.04
3	Upper Tolerance Limit of At-Risk Value (ZAR)	177,910.69
4	At-Risk Value as percent of Base Value	0.07
5	Lower Tol Limit of VaR as percent of Base	0.05
6	Upper Tol Limit of VaR as percent of Base	0.12
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	100
9	Number of Replications Actually Used	100
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	21,930.36
13	Standard Deviation of Profit/Loss	79,174.01
14	Skewness of Profit/Loss	-0.02644
15	Kurtosis of Profit/Loss	0.04851
16	Minimum Profit/Loss over Simulations (ZAR)	-182,799.52
17	Median Profit/Loss (ZAR)	17,521.02
18	Maximum Profit/Loss over Simulations (ZAR)	239,592.78

Table C8.1: VaR statistics by Monte Carlo Simulation

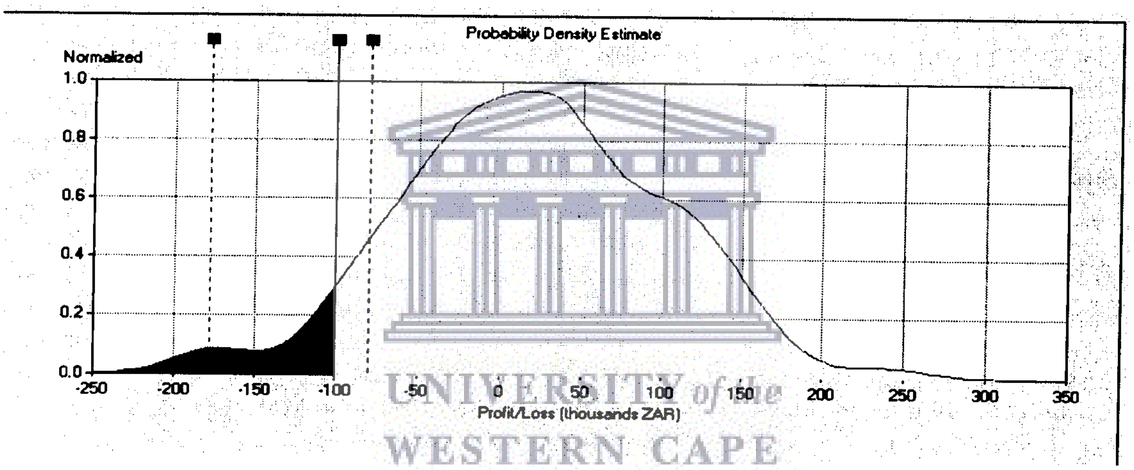

Figure C8.1: VaR Distribution by Monte Carlo Simulation

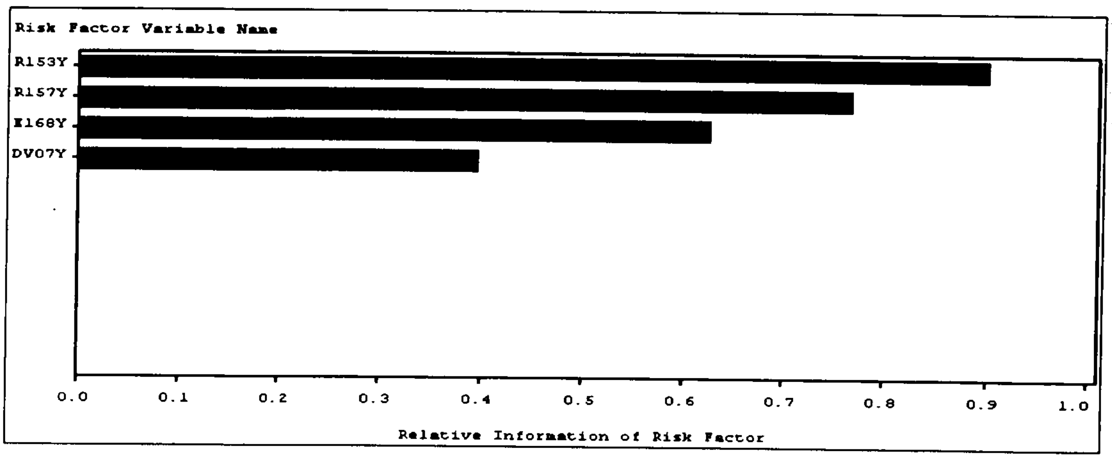

Figure C8.2: Information Measures by Monte Carlo Simulation

(9) EQUAL WEIGHTAGE, 95% PROBABILITY, 10-HOLDING DAYS FOR R157, RUN ON 25 SEPTEMBER 2003.

(A) Historical Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	1,157,801.80
2	Lower Tolerance Limit of At-Risk Value (ZAR)	1,033,653.30
3	Upper Tolerance Limit of At-Risk Value (ZAR)	1,314,119.32
4	At-Risk Value as percent of Base Value	0.74
5	Lower Tol Limit of VaR as percent of Base	0.66
6	Upper Tol Limit of VaR as percent of Base	0.84
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	1092
9	Number of Replications Actually Used	1092
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	64,835.89
13	Standard Deviation of Profit/Loss	940,111.93
14	Skewness of Profit/Loss	-1.72517
15	Kurtosis of Profit/Loss	36.34546
16	Minimum Profit/Loss over Simulations (ZAR)	-12,466,759.85
17	Median Profit/Loss (ZAR)	67,371.55
18	Maximum Profit/Loss over Simulations (ZAR)	8,020,816.05

Table A9.1: VaR statistics by Historical simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	156,981,146.64
2	Lower Tolerance Limit of At-Risk Value (ZAR)	156,897,820.02
3	Upper Tolerance Limit of At-Risk Value (ZAR)	157,125,946.96
4	At-Risk Value as percent of Base Value	100.83
5	Lower Tol Limit of VaR as percent of Base	100.78
6	Upper Tol Limit of VaR as percent of Base	100.92
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	1092
9	Number of Replications Actually Used	1092
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	155,753,566.22
13	Standard Deviation of Profit/Loss	940,111.93
14	Skewness of Profit/Loss	-1.72517
15	Kurtosis of Profit/Loss	36.34546
16	Minimum Profit/Loss over Simulations (ZAR)	143,221,970.49
17	Median Profit/Loss (ZAR)	155,756,101.88
18	Maximum Profit/Loss over Simulations (ZAR)	163,709,546.38

Table A9.2: EaR statistics by Historical Simulation

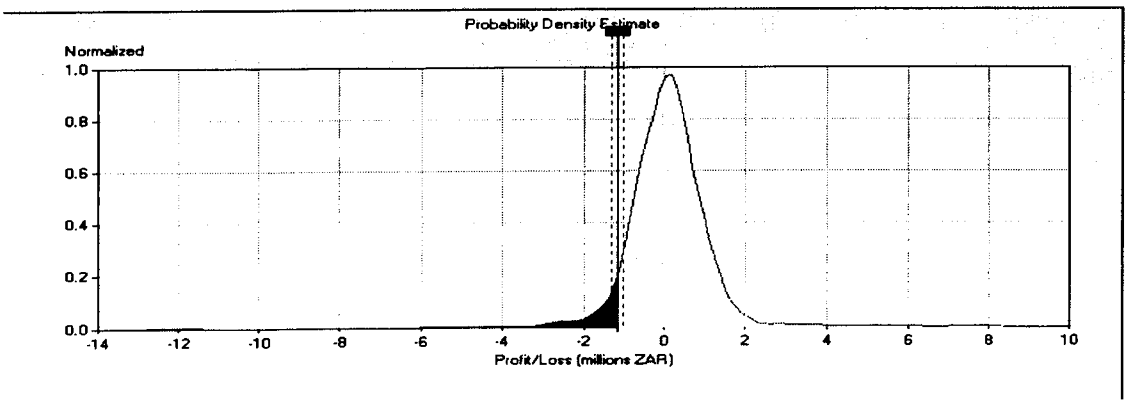

Figure A9.1: VaR Distribution by Historical Simulation

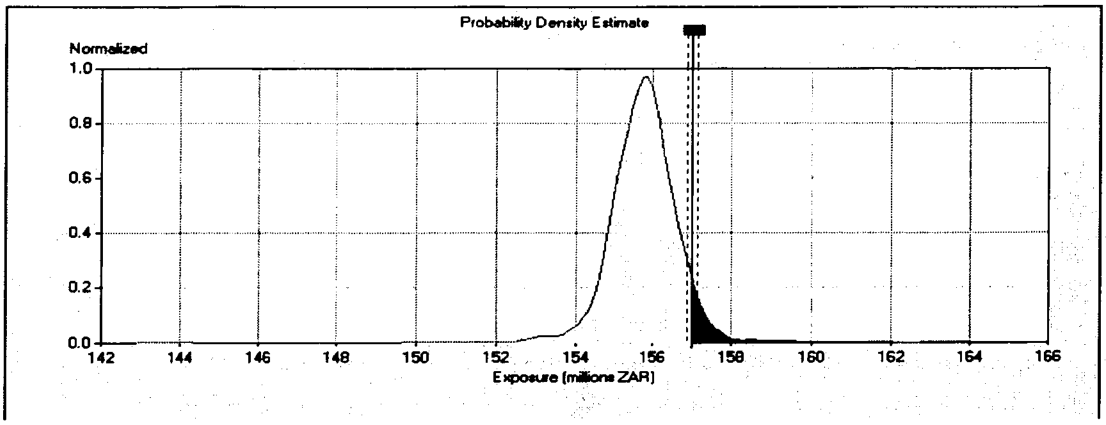

Figure A9.2: EaR Distribution by Historical Simulation

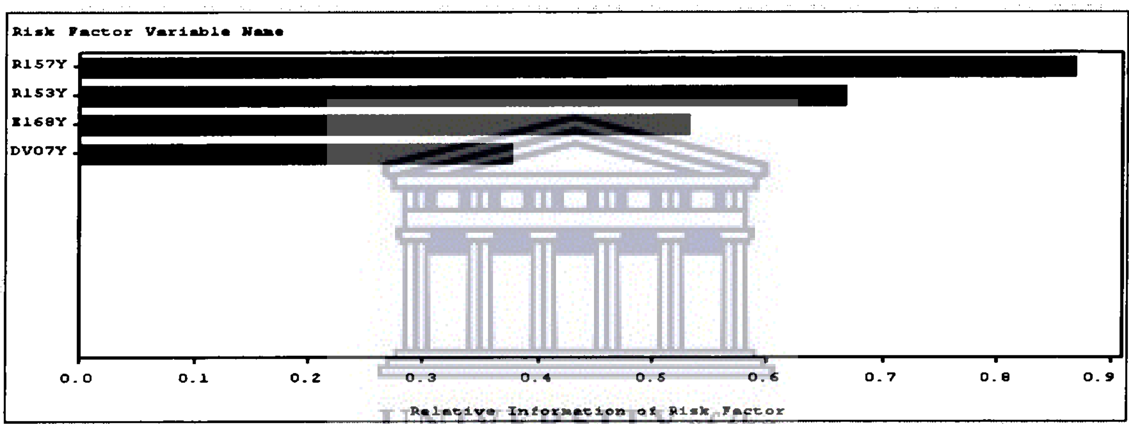

Figure A9.3: Information Measures by Historical Simulation

(B) Variance-Covariance Method

ANLSYS	1. DeltaNormal
NInst	4
NMissing	0
AnalysisNumber	3
BaseDate	25SEP2003
date	26SEP2003
MtM	155,688,730.34
VaR	183,774.89
VaRPct	0.12
DV07Y	-5799363
E168Y	-3871764
R153Y	-5035836
R157Y	-80767679

Table B9.1: VaR statistics by Variance-Covariance Method

(C) Monte Carlo Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	127,957.55
2	Lower Tolerance Limit of At-Risk Value (ZAR)	88,210.05
3	Upper Tolerance Limit of At-Risk Value (ZAR)	229,754.72
4	At-Risk Value as percent of Base Value	0.08
5	Lower Tol Limit of VaR as percent of Base	0.06
6	Upper Tol Limit of VaR as percent of Base	0.15
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	100
9	Number of Replications Actually Used	100
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	43,029.22
13	Standard Deviation of Profit/Loss	113,069.72
14	Skewness of Profit/Loss	-0.03728
15	Kurtosis of Profit/Loss	-0.03063
16	Minimum Profit/Loss over Simulations (ZAR)	-256,363.48
17	Median Profit/Loss (ZAR)	42,754.59
18	Maximum Profit/Loss over Simulations (ZAR)	333,636.81

Table C9.1: VaR statistics by Monte Carlo Simulation

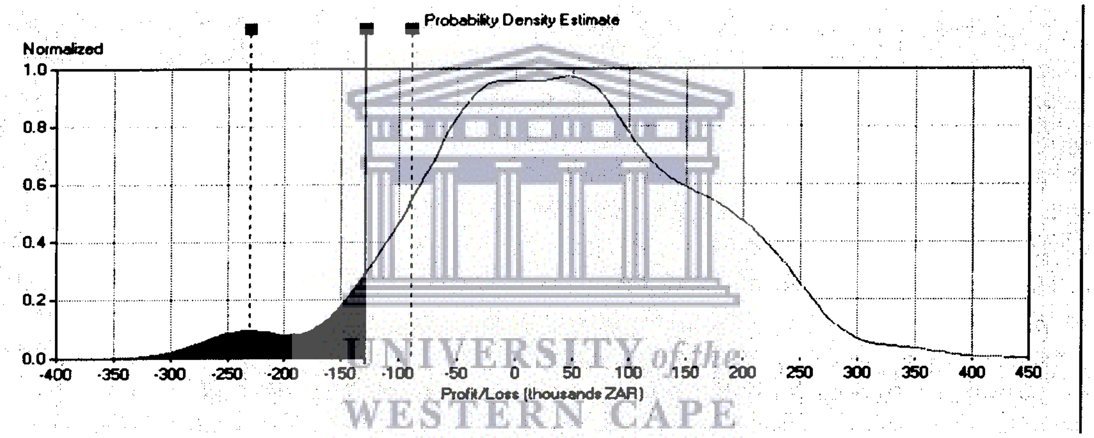

Figure C9.1: VaR Distribution by Monte Carlo Simulation

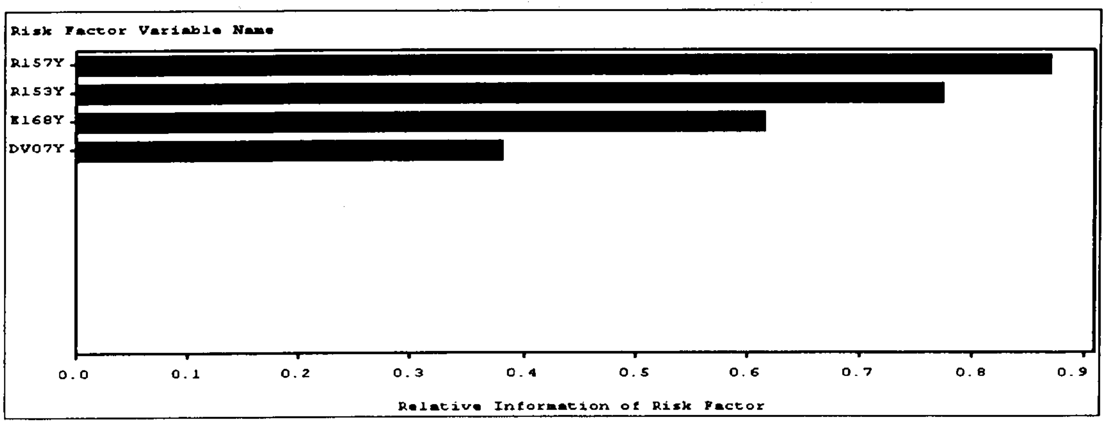

Figure C9.2: Information Measures by Monte Carlo Simulation

(10) EQUAL WEIGHTAGE, 95% PROBABILITY, 10-HOLDING DAYS FOR E168, RUN ON 25 SEPTEMBER 2003.

(A) Historical Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	650,358.82
2	Lower Tolerance Limit of At-Risk Value (ZAR)	608,279.36
3	Upper Tolerance Limit of At-Risk Value (ZAR)	743,146.60
4	At-Risk Value as percent of Base Value	0.48
5	Lower Tol Limit of VaR as percent of Base	0.45
6	Upper Tol Limit of VaR as percent of Base	0.55
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	1092
9	Number of Replications Actually Used	1092
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	50,027.83
13	Standard Deviation of Profit/Loss	557,747.66
14	Skewness of Profit/Loss	-1.98619
15	Kurtosis of Profit/Loss	39.90732
16	Minimum Profit/Loss over Simulations (ZAR)	-7,610,425.40
17	Median Profit/Loss (ZAR)	49,024.88
18	Maximum Profit/Loss over Simulations (ZAR)	4,682,746.23

Table A10.1: VaR statistics by Historical Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	136,403,977.59
2	Lower Tolerance Limit of At-Risk Value (ZAR)	136,322,107.65
3	Upper Tolerance Limit of At-Risk Value (ZAR)	136,499,784.58
4	At-Risk Value as percent of Base Value	100.57
5	Lower Tol Limit of VaR as percent of Base	100.51
6	Upper Tol Limit of VaR as percent of Base	100.64
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	1092
9	Number of Replications Actually Used	1092
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	135,683,812.17
13	Standard Deviation of Profit/Loss	557,747.66
14	Skewness of Profit/Loss	-1.98619
15	Kurtosis of Profit/Loss	39.90732
16	Minimum Profit/Loss over Simulations (ZAR)	128,023,358.93
17	Median Profit/Loss (ZAR)	135,682,809.21
18	Maximum Profit/Loss over Simulations (ZAR)	140,316,530.57

Table A10.2: EaR statistics by Historical Simulation

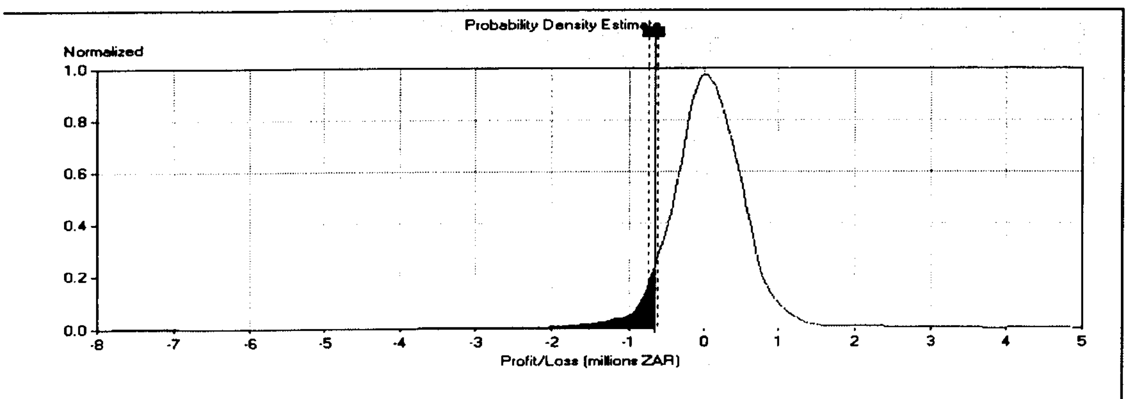

Figure A10.1: VaR Distribution by Historical Simulation

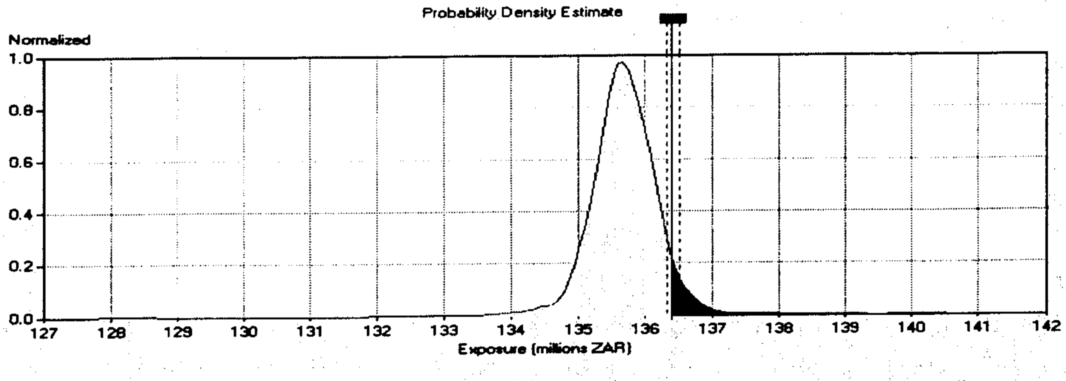

Figure A10.2: EaR Distribution by Historical Simulation

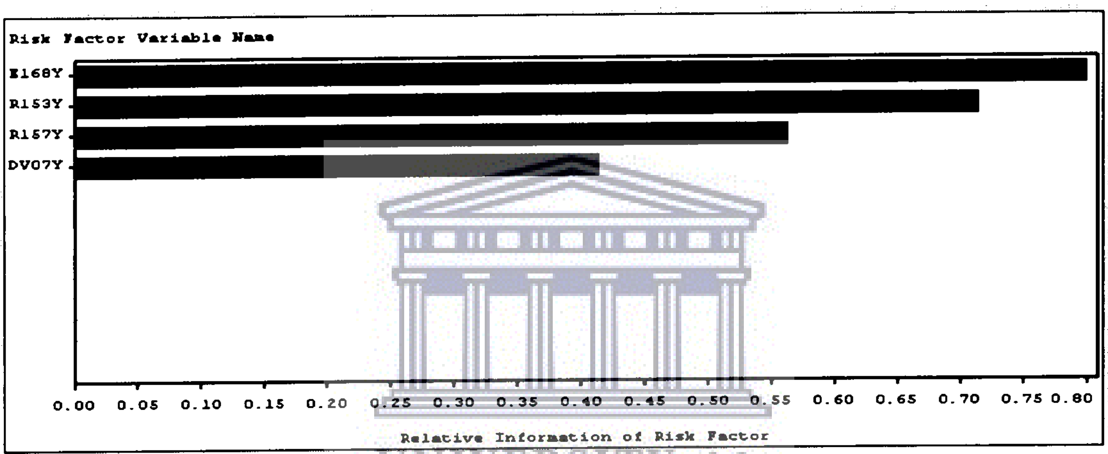

Figure A10.3: Information Measures by Historical Simulation

(B) Variance-Covariance Method

ANLSYS	1. DeltaNormal
NInst	4
NMissing	0
AnalysisNumber	1
BaseDate	25SEP2003
date	26SEP2003
MtM	135,633,784.34
VaR	110,282.71
VaRPct	0.08
DV07Y	-5799363
E168Y	-38717640
R153Y	-5035836
R157Y	-8076768

Table B10.1: VaR statistics by Variance-Covariance Method

(C) Monte Carlo Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	57,549.41
2	Lower Tolerance Limit of At-Risk Value (ZAR)	49,606.13
3	Upper Tolerance Limit of At-Risk Value (ZAR)	131,379.29
4	At-Risk Value as percent of Base Value	0.04
5	Lower Tol Limit of VaR as percent of Base	0.04
6	Upper Tol Limit of VaR as percent of Base	0.10
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	100
9	Number of Replications Actually Used	100
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	37,337.94
13	Standard Deviation of Profit/Loss	67,613.68
14	Skewness of Profit/Loss	-0.02368
15	Kurtosis of Profit/Loss	0.09393
16	Minimum Profit/Loss over Simulations (ZAR)	-138,154.02
17	Median Profit/Loss (ZAR)	36,584.66
18	Maximum Profit/Loss over Simulations (ZAR)	231,042.75

Table C10.1: VaR statistics by Monte Carlo Simulation

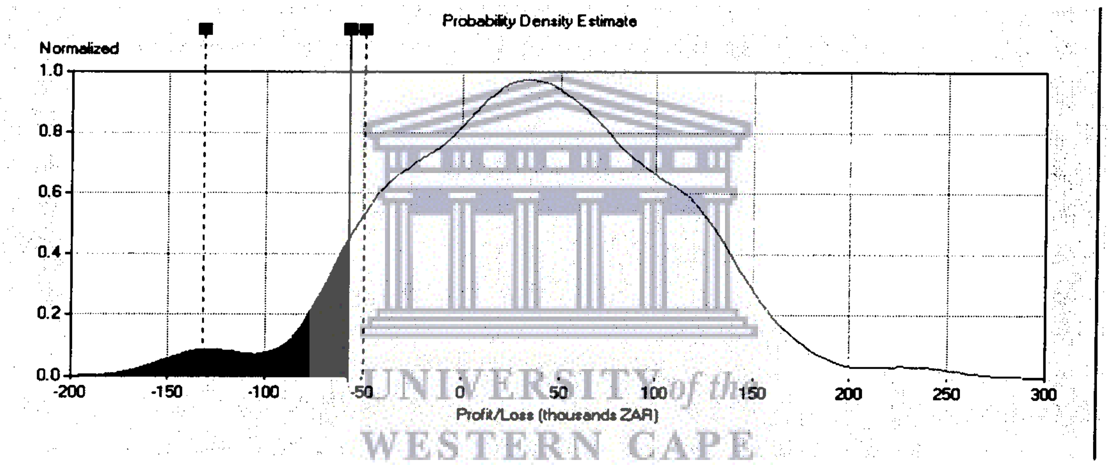

Figure C10.1: VaR Distribution by Monte Carlo Simulation

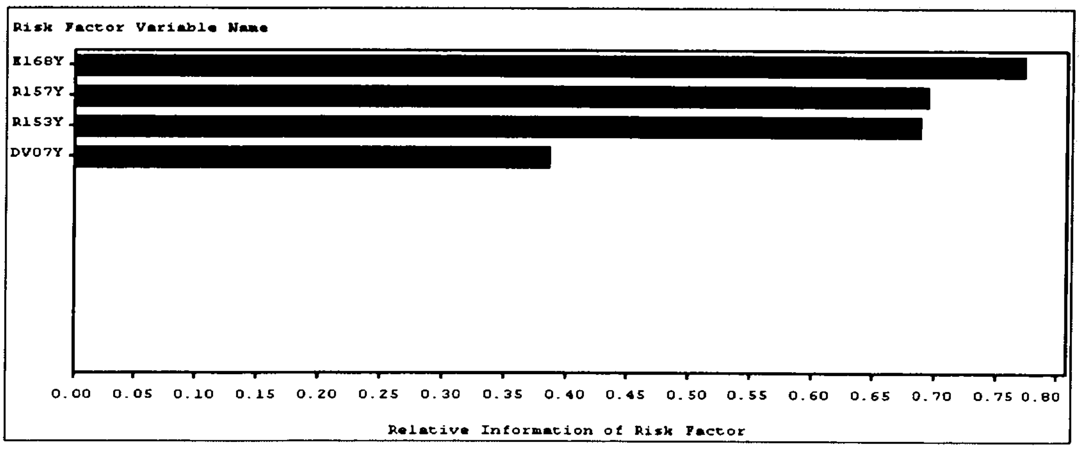

Figure C10.2: Information Measures by Monte Carlo Simulation

(11) EQUAL WEIGHTAGE, 95% PROBABILITY, 10-HOLDING DAYS FOR DV07, RUN ON 25 SEPTEMBER 2003.

(A) Historical Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	790,159.07
2	Lower Tolerance Limit of At-Risk Value (ZAR)	713,007.95
3	Upper Tolerance Limit of At-Risk Value (ZAR)	895,033.28
4	At-Risk Value as percent of Base Value	0.50
5	Lower Tol Limit of VaR as percent of Base	0.45
6	Upper Tol Limit of VaR as percent of Base	0.56
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	1092
9	Number of Replications Actually Used	1092
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	62,480.41
13	Standard Deviation of Profit/Loss	716,647.26
14	Skewness of Profit/Loss	-1.93056
15	Kurtosis of Profit/Loss	40.39238
16	Minimum Profit/Loss over Simulations (ZAR)	-9,745,530.15
17	Median Profit/Loss (ZAR)	49,859.23
18	Maximum Profit/Loss over Simulations (ZAR)	6,072,550.68

Table A11.1: VaR statistics by Historical Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	160,387,408.21
2	Lower Tolerance Limit of At-Risk Value (ZAR)	160,308,276.66
3	Upper Tolerance Limit of At-Risk Value (ZAR)	160,527,685.67
4	At-Risk Value as percent of Base Value	100.59
5	Lower Tol Limit of VaR as percent of Base	100.54
6	Upper Tol Limit of VaR as percent of Base	100.68
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	1092
9	Number of Replications Actually Used	1092
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	159,502,630.09
13	Standard Deviation of Profit/Loss	716,647.26
14	Skewness of Profit/Loss	-1.93056
15	Kurtosis of Profit/Loss	40.39238
16	Minimum Profit/Loss over Simulations (ZAR)	149,694,619.53
17	Median Profit/Loss (ZAR)	159,490,008.91
18	Maximum Profit/Loss over Simulations (ZAR)	165,512,700.36

Table A11.2: EaR statistics by Historical Simulation

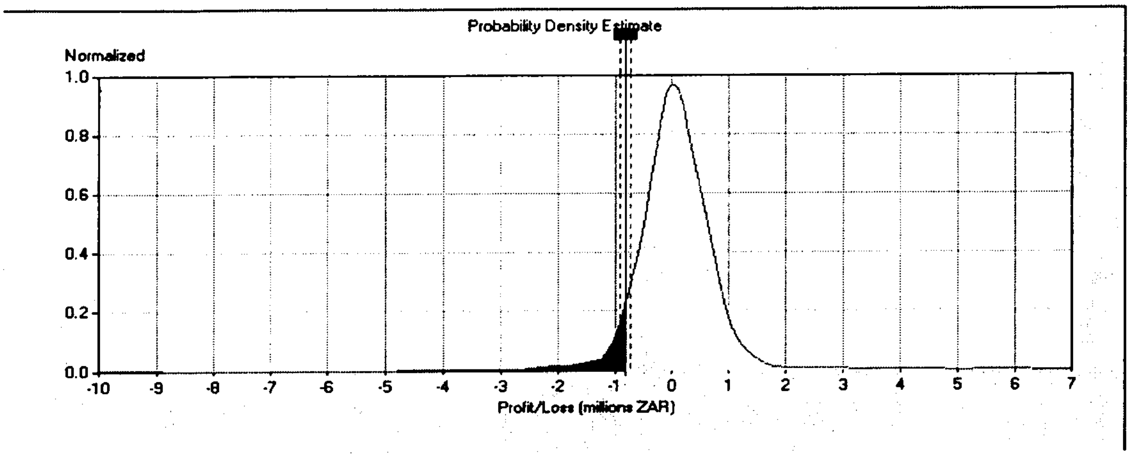

Figure A11.1: VaR Distribution by Historical Simulation

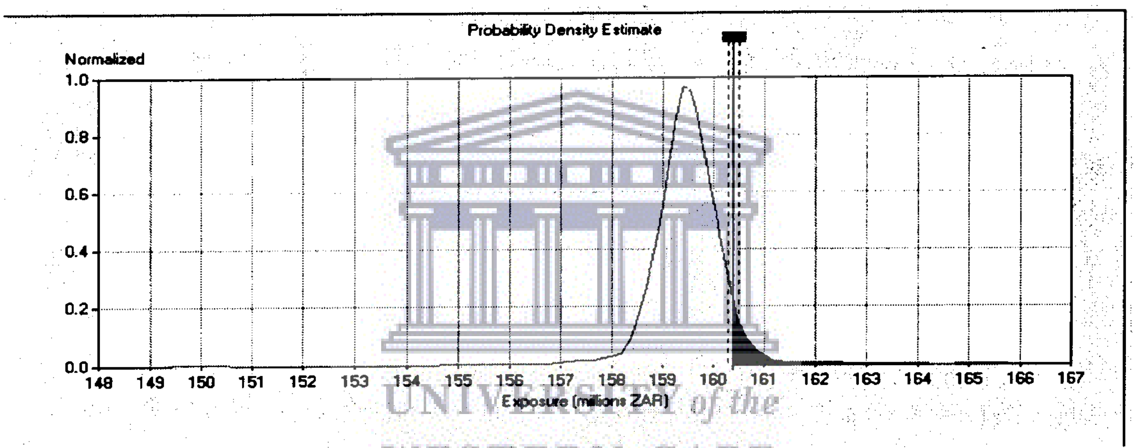

Figure A11.2: EaR Distribution by Historical Simulation

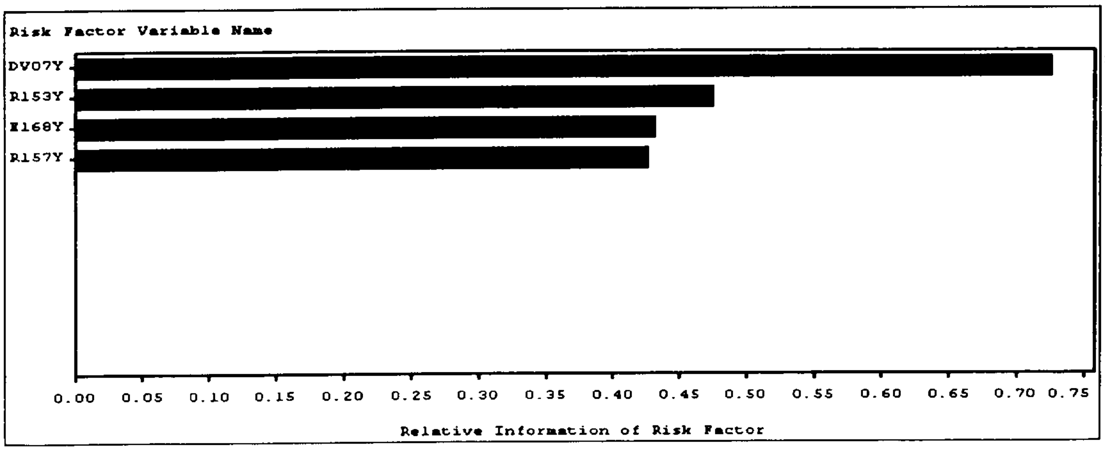

Figure A11.3: Information Measures by Historical Simulation

(B) Variance-Covariance Method

ANLSYS	1. DeltaNormal
NInst	4
NMissing	0
AnalysisNumber	3
BaseDate	25SEP2003
date	26SEP2003
MtM	159,440,149.68
VaR	146,999.62
VaRPct	0.09
DV07Y	-57993630
E168Y	-3871764
R153Y	-5035836
R157Y	-8076768

Table B11.1: VaR statistics by Variance-Covariance Method

(C) Monte Carlo Simulation

	Statistic	Estimate
1	At-Risk Value (ZAR)	111,700.66
2	Lower Tolerance Limit of At-Risk Value (ZAR)	76,500.42
3	Upper Tolerance Limit of At-Risk Value (ZAR)	153,882.15
4	At-Risk Value as percent of Base Value	0.07
5	Lower Tol Limit of VaR as percent of Base	0.05
6	Upper Tol Limit of VaR as percent of Base	0.10
7	Number of Instruments in Portfolio	4
8	Total Number of Simulation Replications	100
9	Number of Replications Actually Used	100
10	Number of Unadjusted Missing Replications	0
11	Number of Missing Adjusted Replications	0
12	Mean Profit/Loss over Simulations (ZAR)	46,011.54
13	Standard Deviation of Profit/Loss	90,452.20
14	Skewness of Profit/Loss	0.02604
15	Kurtosis of Profit/Loss	-0.05460
16	Minimum Profit/Loss over Simulations (ZAR)	-188,299.13
17	Median Profit/Loss (ZAR)	44,758.01
18	Maximum Profit/Loss over Simulations (ZAR)	290,307.85

Table C11.1: VaR statistics by Monte Carlo Simulation

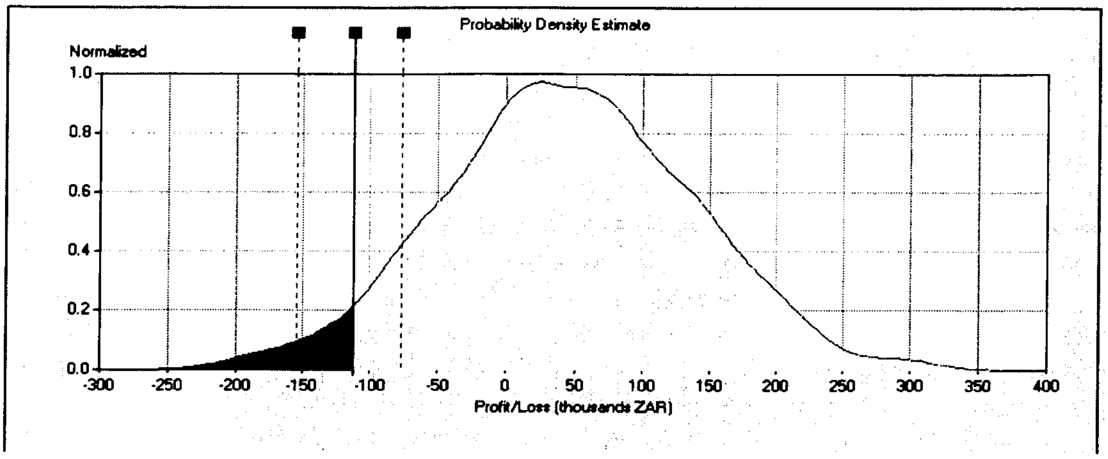

Figure C11.1: VaR Distribution by Monte Carlo Simulation

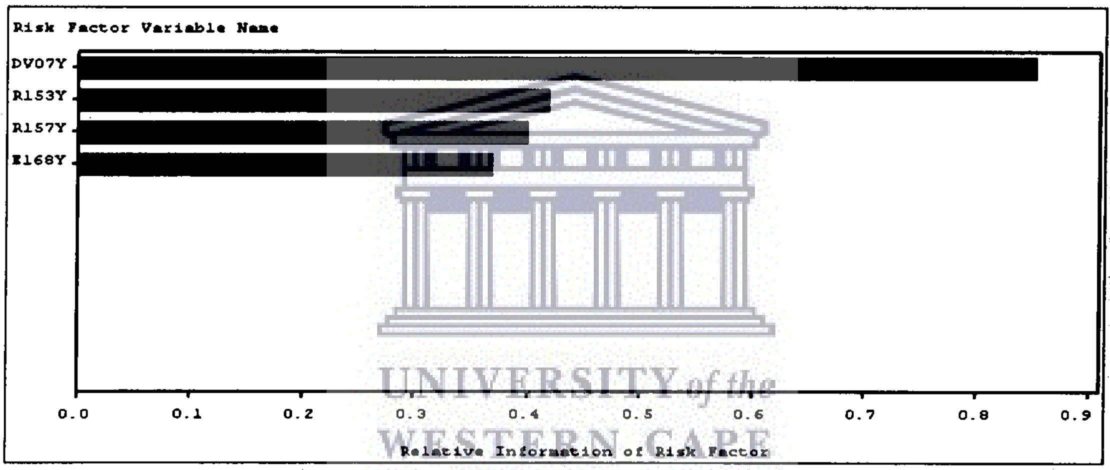

Figure C11.2: Information Measures by Monte Carlo Simulation

APPENDIX II

(1) **Functional Form: VaR = f(probability level)**

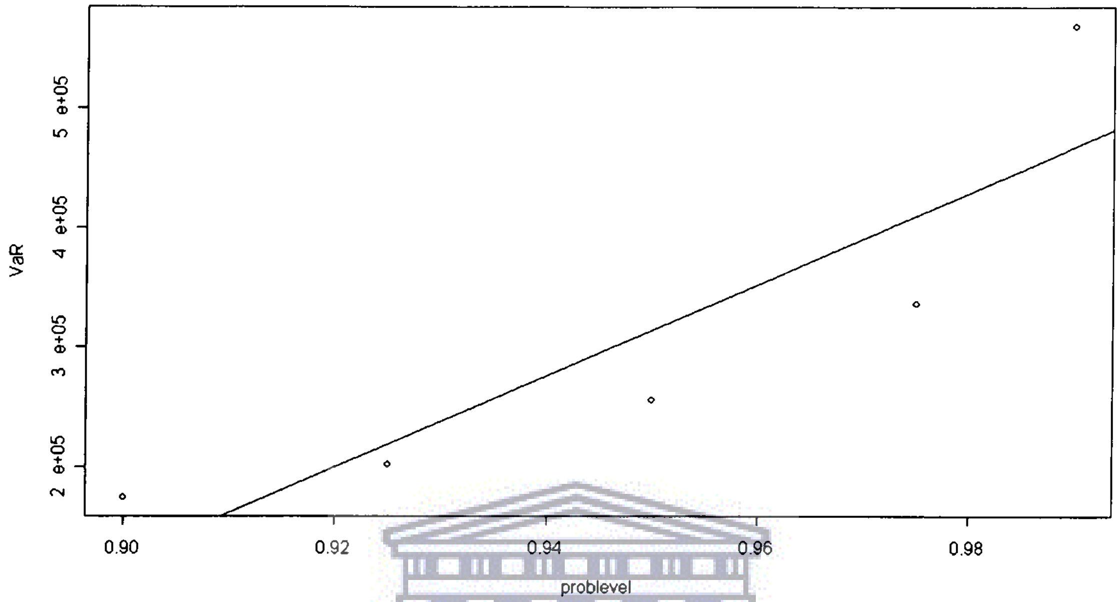

Figure 1.1: Linear Regression of VaR (Historical Simulation) against Probability Level

Coefficients:
(Intercept) problevel
-3321929 3828206

Regression line (Historical Simulation):

VaR = -3321929 + 3828206(problevel)

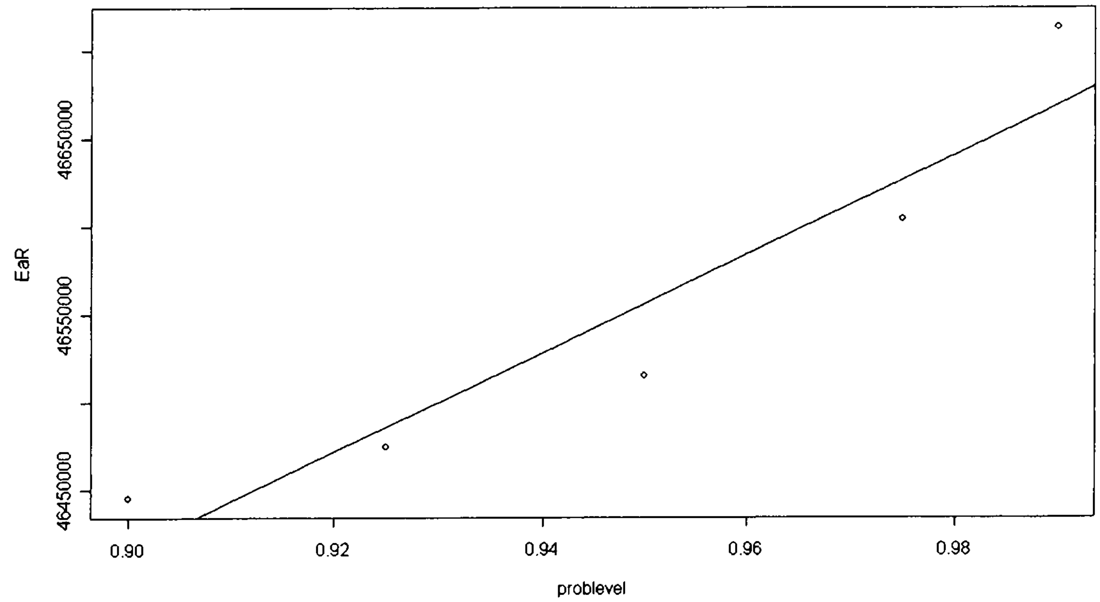

Figure 1.2: Linear Regression of EaR against Probability

Coefficients:
(Intercept) problevel
43873050 2824097

Regression line:

EaR = 43873050 + 2824097(problevel)

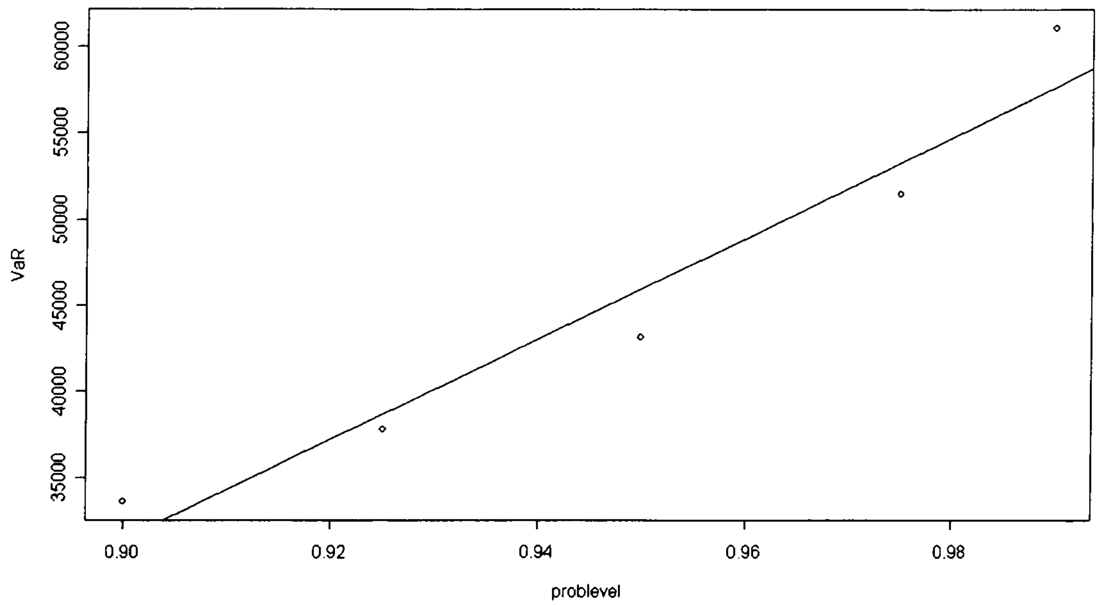

Figure 1.3: Linear Regression of VaR (Variance-Covariance) against probability level

Coefficients:

(Intercept)	problevel
-231282	291860

Regression line:

VaR = -231282 + 291860(problevel)

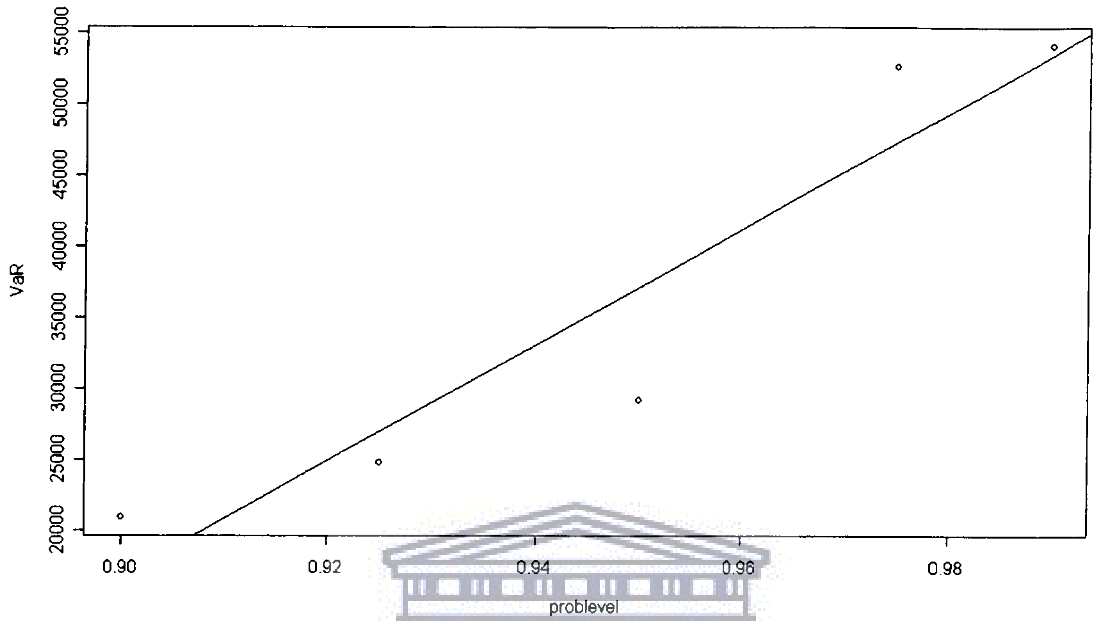

Figure 1.4: Linear Regression of VaR (Monte Carlo) against Probability level

Coefficients:

(Intercept)	problevel
-350155	407649

Regression Line:

VaR = -350155 + 407649(problevel)

(2) **Functional form VaR = f(effective date)**

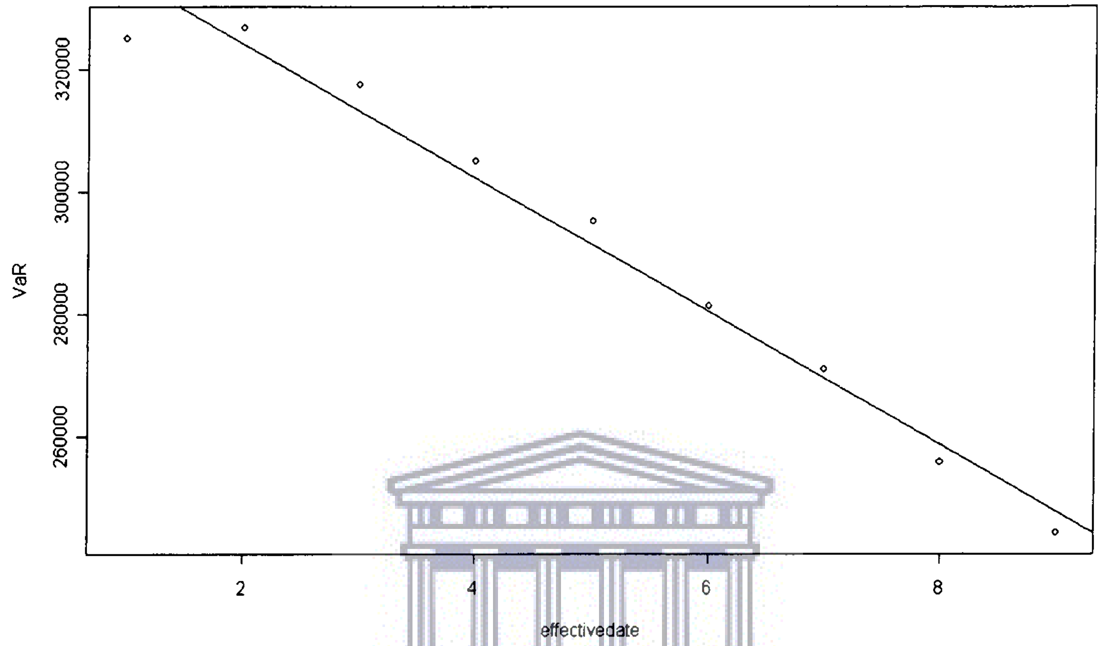

Figure 2.1: Linear Regression of VaR (Historical Simulation) against effective date

Coefficients:

(Intercept)	effectivedate
345702	-10883

Regression line:

VaR = 345702 – 10883(effectivedate)

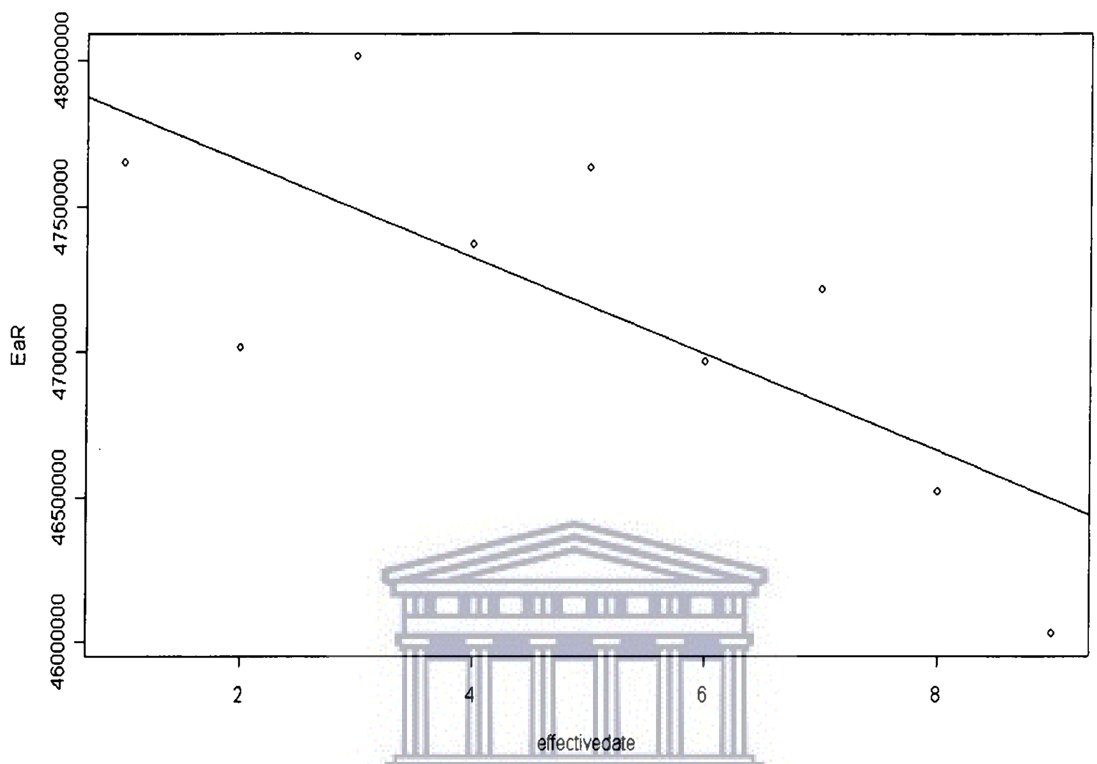

Figure 2.1: Linear Regression of EaR (Historical Simulation) against effective date

Coefficients:
(Intercept) effectivedate
47986675 -166299

Regression line:

EaR = 47986675 – 166299(effectivedate)

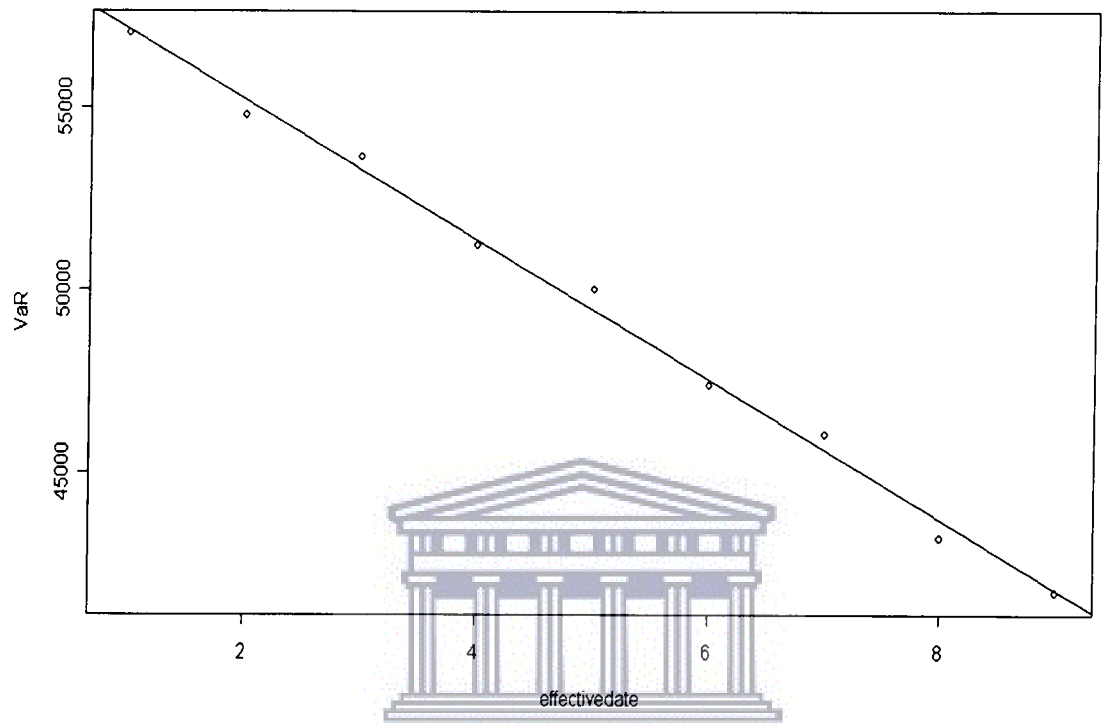

Figure 2.3: Linear Regression of VaR (Variance-Covariance) against effective date

Coefficients:

(Intercept)	effectivedate
59065	-1925

Regression Line:

VaR = 59065 – 1925(effectivedate)

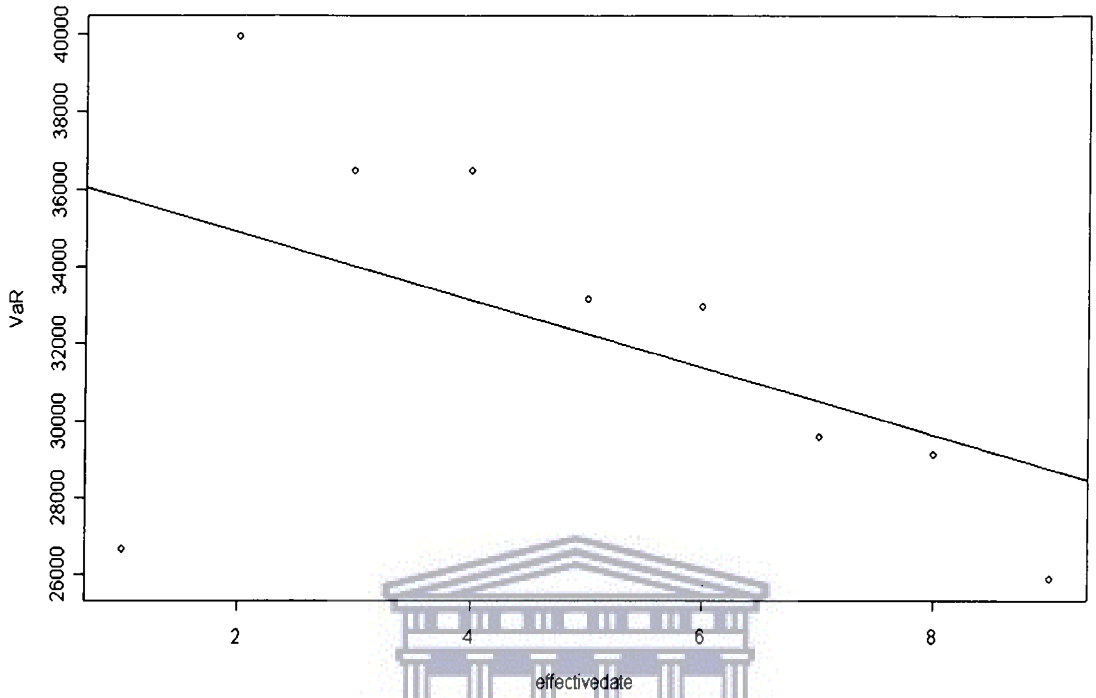

Figure 2.4: Linear Regression of VaR (Monte Carlo) against effective date

Coefficients:

(Intercept)	effectivedate
36650.7	-881.5

Regression Line:

VaR = 36650.7 – 881.5(effectivedate)

(3) Functional form VaR = f(holding period)

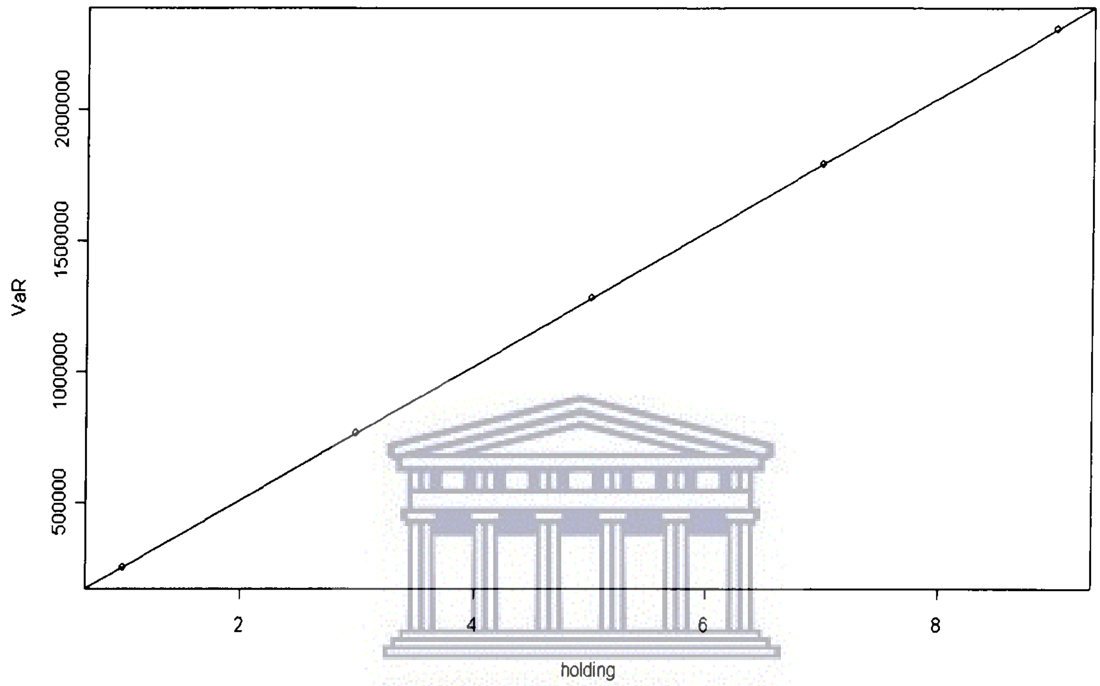

Figure 3.1: Linear Regression of VaR (All Methodologies) against holding period

Coefficients:
(Intercept) holding
-8.603e-11 2.557e+05

Regression Line:

VaR = -8.603e-11 + 2.557e+05(holding)

APPENDIX III

InstrID	Yield	Coupon	PurchaseDate	Maturities	CD1	CD2	BC1	BC2	Holding
R153	R153Y	13	25-Sep-03	8-Aug-10	31-Aug-04	28-Feb-05	31-Jul-04	31-Jan-05	1
R157	R157Y	13.5	25-Sep-03	15-Sep-15	15-Sep-04	15-Mar-05	15-Aug-04	15-Feb-05	1
E168	E168Y	11	25-Sep-03	1-Jun-08	1-Jun-04	1-Dec-04	1-May-04	1-Nov-04	1
DV07	DV07Y	14.5	25-Sep-03	30-Sep-10	31-Mar-04	30-Sep-04	29-Feb-04	31-Aug-04	1

Table 1: Portfolio Characteristics for Control Simulation

InstrID	Yield	Coupon	PurchaseDate	Maturities	CD1	CD2	BC1	BC2	Holding
R153	R153Y	13	25-Sep-03	8-Aug-10	31-Aug-04	28-Feb-05	31-Jul-04	31-Jan-05	10
R157	R157Y	13.5	25-Sep-03	15-Sep-15	15-Sep-04	15-Mar-05	15-Aug-04	15-Feb-05	1
E168	E168Y	11	25-Sep-03	1-Jun-08	1-Jun-04	1-Dec-04	1-May-04	1-Nov-04	1
DV07	DV07Y	14.5	25-Sep-03	30-Sep-10	31-Mar-04	30-Sep-04	29-Feb-04	31-Aug-04	1

Table 2: Portfolio Characteristics for Simulation Run 4.5.8

InstrID	Yield	Coupon	PurchaseDate	Maturities	CD1	CD2	BC1	BC2	Holding
R153	R153Y	13	25-Sep-03	8-Aug-10	31-Aug-04	28-Feb-05	31-Jul-04	31-Jan-05	1
R157	R157Y	13.5	25-Sep-03	15-Sep-15	15-Sep-04	15-Mar-05	15-Aug-04	15-Feb-05	10
E168	E168Y	11	25-Sep-03	1-Jun-08	1-Jun-04	1-Dec-04	1-May-04	1-Nov-04	1
DV07	DV07Y	14.5	25-Sep-03	30-Sep-10	31-Mar-04	30-Sep-04	29-Feb-04	31-Aug-04	1

Table 3: Portfolio Characteristics for Simulation Run 4.5.9

InstrID	Yield	Coupon	PurchaseDate	Maturities	CD1	CD2	BC1	BC2	Holding
R153	R153Y	13	25-Sep-03	8-Aug-10	31-Aug-04	28-Feb-05	31-Jul-04	31-Jan-05	1
R157	R157Y	13.5	25-Sep-03	15-Sep-15	15-Sep-04	15-Mar-05	15-Aug-04	15-Feb-05	1
E168	E168Y	11	25-Sep-03	1-Jun-08	1-Jun-04	1-Dec-04	1-May-04	1-Nov-04	10
DV07	DV07Y	14.5	25-Sep-03	30-Sep-10	31-Mar-04	30-Sep-04	29-Feb-04	31-Aug-04	1

Table 4: Portfolio Characteristics for Simulation Run 4.5.10

InstrID	Yield	Coupon	PurchaseDate	Maturities	CD1	CD2	BC1	BC2	Holding
R153	R153Y	13	25-Sep-03	8-Aug-10	31-Aug-04	28-Feb-05	31-Jul-04	31-Jan-05	1
R157	R157Y	13.5	25-Sep-03	15-Sep-15	15-Sep-04	15-Mar-05	15-Aug-04	15-Feb-05	1
E168	E168Y	11	25-Sep-03	1-Jun-08	1-Jun-04	1-Dec-04	1-May-04	1-Nov-04	1
DV07	DV07Y	14.5	25-Sep-03	30-Sep-10	31-Mar-04	30-Sep-04	29-Feb-04	31-Aug-04	10

Table 5: Portfolio Characteristics for Simulation Run 4.5.11

Date	R153Y	R157Y	E168Y	DV07Y
14-May-04	0.1014	0.1036	0.10645	0.1064

Table 6: Current Market Information for Pricing of Portfolio

The Data

DATE	R153Y	E168Y	R157Y	DV07Y
14607	0.13670	0.14135	0.13655	0.14490
14613	0.13755	0.14230	0.13730	0.14560
14614	0.13925	0.14385	0.13905	0.14740
14615	0.13915	0.14375	0.13890	0.14720
14616	0.13665	0.14115	0.13635	0.14470
14619	0.13485	0.13960	0.13460	0.14290
14620	0.13550	0.14015	0.13530	0.14350
14621	0.13480	0.13955	0.13465	0.14290
14622	0.13315	0.13805	0.13300	0.14120
14623	0.13205	0.13720	0.13185	0.14040
14626	0.13130	0.13615	0.13110	0.13950
14627	0.13215	0.13695	0.13195	0.14030
14628	0.13430	0.13695	0.13410	0.14250
14629	0.13270	0.13740	0.13260	0.14100
14630	0.13310	0.13785	0.13300	0.14130
14633	0.13320	0.13795	0.13315	0.14160
14634	0.13420	0.13865	0.13400	0.14160
14635	0.13325	0.13790	0.13325	0.14160
14636	0.13455	0.13900	0.13460	0.14270
14637	0.13790	0.14245	0.13800	0.14610
14638	0.13725	0.14185	0.13745	0.14560
14641	0.13590	0.14045	0.13610	0.14390
14642	0.13525	0.14015	0.13540	0.14330
14643	0.13340	0.13800	0.13355	0.14140
14644	0.13330	0.13785	0.13340	0.14120
14647	0.13335	0.13785	0.13350	0.14160
14648	0.13210	0.13685	0.13220	0.14030
14649	0.13255	0.13710	0.13270	0.14070
14650	0.13380	0.13835	0.13400	0.14200
14651	0.13515	0.13935	0.13530	0.14320
14654	0.13480	0.13905	0.13495	0.14290
14655	0.13565	0.13985	0.13585	0.14370
14656	0.13645	0.14065	0.13665	0.14450
14657	0.13630	0.14055	0.13660	0.14450
14658	0.13645	0.14055	0.13670	0.14460
14661	0.13660	0.14110	0.13685	0.14490
14662	0.13640	0.14085	0.13670	0.14450
14663	0.13460	0.13895	0.13490	0.14260
14664	0.13510	0.13945	0.13545	0.14310
14665	0.13415	0.13845	0.13450	0.14210
14668	0.13630	0.14085	0.13665	0.14430

14669	0.13655	0.14105	0.13690	0.14460
14670	0.13625	0.14075	0.13665	0.14430
14671	0.13730	0.14175	0.13775	0.14540
14672	0.13870	0.14325	0.13890	0.14680
14675	0.13945	0.14405	0.13965	0.14720
14676	0.14005	0.14495	0.14020	0.14790
14677	0.13870	0.14345	0.13885	0.14640
14678	0.13840	0.14305	0.13855	0.14620
14679	0.13855	0.14300	0.13860	0.14640
14682	0.14070	0.14515	0.14080	0.14850
14683	0.13925	0.14385	0.13935	0.14690
14684	0.14025	0.14495	0.14025	0.14800
14685	0.14040	0.14505	0.14040	0.14800
14686	0.14000	0.14460	0.13995	0.14770
14689	0.13900	0.14365	0.13895	0.14670
14691	0.13820	0.14275	0.13815	0.14580
14692	0.13810	0.14275	0.13800	0.14590
14693	0.13800	0.14255	0.13790	0.14570
14696	0.13830	0.14285	0.13820	0.14590
14697	0.13850	0.14305	0.13855	0.14620
14698	0.13980	0.14405	0.13985	0.14750
14699	0.14135	0.14575	0.14135	0.14910
14700	0.14090	0.14540	0.14085	0.14840
14703	0.14225	0.14665	0.14220	0.14980
14704	0.14175	0.14645	0.14170	0.14940
14705	0.14375	0.14825	0.14370	0.15140
14706	0.14300	0.14775	0.14290	0.15080
14707	0.14250	0.14730	0.14240	0.15030
14710	0.14210	0.14645	0.14200	0.14970
14711	0.14280	0.14735	0.14265	0.15060
14712	0.14095	0.14555	0.14075	0.14860
14713	0.14035	0.14485	0.14010	0.14800
14714	0.14080	0.14525	0.14060	0.14850
14717	0.14300	0.14765	0.14270	0.15070
14718	0.14325	0.14770	0.14305	0.15080
14719	0.14345	0.14795	0.14310	0.15110
14720	0.14515	0.14960	0.14485	0.15280
14725	0.14475	0.14940	0.14445	0.15250
14726	0.14515	0.14970	0.14480	0.15260
14728	0.14470	0.14905	0.14430	0.15200
14732	0.14465	0.14890	0.14425	0.15210
14733	0.14550	0.14980	0.14515	0.15300
14734	0.14595	0.15005	0.14565	0.15350
14735	0.14775	0.15175	0.14750	0.15550
14738	0.14890	0.15285	0.14865	0.15650
14739	0.15165	0.15560	0.15135	0.15920

14740	0.15340	0.15730	0.15310	0.16110
14741	0.14965	0.15365	0.14915	0.15710
14742	0.14930	0.15350	0.14870	0.15690
14745	0.14870	0.15285	0.14820	0.16620
14746	0.14910	0.15315	0.14850	0.15650
14747	0.14935	0.15335	0.14880	0.15660
14748	0.14820	0.15260	0.14725	0.15540
14749	0.14865	0.15305	0.14795	0.15590
14752	0.14640	0.15085	0.14580	0.15370
14753	0.14580	0.15015	0.14530	0.15280
14754	0.14640	0.15095	0.14600	0.15330
14755	0.14640	0.15085	0.14600	0.15340
14756	0.14650	0.15105	0.14615	0.15350
14759	0.14595	0.15025	0.14570	0.15300
14760	0.14460	0.14885	0.14420	0.15190
14761	0.14405	0.14855	0.14365	0.15130
14762	0.14410	0.14855	0.14370	0.15130
14763	0.14265	0.14710	0.14225	0.14990
14766	0.14250	0.14665	0.14195	0.14950
14767	0.14295	0.14735	0.14255	0.15010
14768	0.14410	0.14835	0.14365	0.15120
14769	0.14540	0.14965	0.14480	0.15250
14770	0.14450	0.14890	0.14395	0.15160
14773	0.14590	0.15025	0.14535	0.15300
14774	0.14590	0.15025	0.14535	0.15300
14775	0.14495	0.14930	0.14435	0.15170
14776	0.14470	0.14925	0.14400	0.15160
14780	0.14365	0.14795	0.14305	0.15040
14781	0.14300	0.14745	0.14240	0.14980
14782	0.14340	0.14755	0.14270	0.15010
14783	0.14270	0.14670	0.14190	0.14930
14784	0.14325	0.14715	0.14260	0.14980
14787	0.14260	0.14665	0.14190	0.14920
14788	0.14215	0.14595	0.14140	0.14870
14789	0.14140	0.14535	0.14075	0.14750
14790	0.14185	0.14580	0.14115	0.14780
14791	0.14180	0.14575	0.14110	0.14790
14792	0.14180	0.14575	0.14110	0.14790
14794	0.14240	0.14630	0.14170	0.14880
14795	0.14125	0.14530	0.14050	0.14760
14796	0.13990	0.14385	0.13890	0.14620
14797	0.14065	0.14450	0.13955	0.14700
14798	0.13860	0.14250	0.13765	0.14500
14801	0.13750	0.14150	0.13655	0.14380
14802	0.13790	0.14195	0.13705	0.14430
14803	0.13850	0.14250	0.13780	0.14490

14804	0.13890	0.14285	0.13810	0.14520
14805	0.13845	0.14250	0.13775	0.14480
14808	0.13845	0.14245	0.13775	0.14480
14809	0.13920	0.14310	0.13860	0.14550
14810	0.14005	0.14400	0.13945	0.14630
14811	0.13950	0.14350	0.13895	0.14560
14812	0.13830	0.14240	0.13780	0.14460
14815	0.13870	0.14250	0.13815	0.14490
14816	0.13885	0.14255	0.13830	0.14490
14817	0.13785	0.14170	0.13730	0.14400
14818	0.13755	0.14145	0.13680	0.14390
14819	0.13755	0.14140	0.13685	0.14360
14822	0.13715	0.14090	0.13640	0.14320
14823	0.13645	0.14020	0.13565	0.14250
14824	0.13685	0.14060	0.13600	0.14290
14825	0.13670	0.14050	0.13590	0.14270
14826	0.13580	0.13950	0.13500	0.14160
14829	0.13510	0.13870	0.13430	0.14100
14830	0.13500	0.13865	0.13425	0.14080
14832	0.13410	0.13785	0.13330	0.13980
14833	0.13370	0.13740	0.13295	0.13940
14836	0.13370	0.13750	0.13285	0.13950
14837	0.13460	0.13830	0.13375	0.14030
14838	0.13520	0.13890	0.13435	0.14080
14839	0.13630	0.13995	0.13540	0.14190
14840	0.13620	0.13980	0.13540	0.14180
14843	0.13665	0.14030	0.13590	0.14220
14844	0.13600	0.13960	0.13530	0.14140
14845	0.13525	0.13905	0.13455	0.14080
14846	0.13435	0.13810	0.13350	0.13980
14847	0.13400	0.13765	0.13315	0.13890
14850	0.13370	0.13720	0.13280	0.13880
14851	0.13380	0.13735	0.13290	0.13900
14852	0.13420	0.13760	0.13325	0.13920
14853	0.13540	0.13885	0.13440	0.14040
14854	0.13565	0.13920	0.13475	0.14080
14857	0.13660	0.14015	0.13570	0.14170
14858	0.13765	0.14090	0.13675	0.14270
14859	0.13750	0.14085	0.13660	0.14260
14860	0.13690	0.14010	0.13605	0.14190
14861	0.13635	0.13975	0.13545	0.14130
14864	0.13695	0.14015	0.13605	0.14200
14865	0.13770	0.14080	0.13680	0.14270
14866	0.13750	0.14055	0.13665	0.14250
14867	0.13700	0.14010	0.13615	0.14200
14868	0.13740	0.14040	0.13660	0.14230

14871	0.13800	0.14090	0.13720	0.14300
14872	0.13870	0.14150	0.13795	0.14380
14873	0.13825	0.14120	0.13745	0.14320
14874	0.13810	0.14110	0.13725	0.14310
14875	0.13695	0.14010	0.13615	0.14200
14879	0.13595	0.13890	0.13515	0.14090
14880	0.13655	0.13960	0.13575	0.14170
14881	0.13605	0.13915	0.13525	0.14110
14882	0.13545	0.13840	0.13465	0.14040
14885	0.13465	0.13775	0.13385	0.13970
14886	0.13460	0.13765	0.13380	0.13970
14887	0.13505	0.13810	0.13425	0.14010
14888	0.13550	0.13840	0.13480	0.14040
14889	0.13515	0.13830	0.13450	0.14020
14892	0.13600	0.13910	0.13545	0.14100
14893	0.13690	0.13990	0.13645	0.14200
14894	0.13760	0.14060	0.13710	0.14300
14895	0.13810	0.14130	0.13760	0.14340
14896	0.13780	0.14085	0.13730	0.14300
14899	0.13825	0.14120	0.13780	0.14360
14900	0.13825	0.14130	0.13775	0.14360
14901	0.13975	0.14290	0.13925	0.14530
14902	0.13770	0.14060	0.13720	0.14320
14903	0.13720	0.14020	0.13670	0.14270
14906	0.13665	0.13970	0.13615	0.14220
14907	0.13735	0.14035	0.13685	0.14280
14908	0.13790	0.14095	0.13740	0.14360
14909	0.13855	0.14165	0.13810	0.14410
14910	0.13820	0.14130	0.13780	0.14360
14913	0.13855	0.14170	0.13820	0.14410
14914	0.13745	0.14050	0.13720	0.14290
14915	0.13660	0.13965	0.13635	0.14200
14916	0.13520	0.13860	0.13485	0.14090
14917	0.13430	0.13775	0.13390	0.14010
14920	0.13210	0.13600	0.13170	0.13790
14921	0.13310	0.13685	0.13270	0.13890
14922	0.13375	0.13765	0.13345	0.13960
14923	0.13495	0.13885	0.13470	0.14080
14924	0.13380	0.13760	0.13360	0.13970
14927	0.13480	0.13860	0.13455	0.14060
14928	0.13490	0.13880	0.13480	0.14080
14929	0.13455	0.13835	0.13430	0.14040
14930	0.13440	0.13820	0.13420	0.14030
14931	0.13335	0.13695	0.13310	0.13890
14934	0.13350	0.13720	0.13320	0.13930
14935	0.13280	0.13640	0.13260	0.13860

14936	0.13345	0.13705	0.13315	0.13900
14937	0.13335	0.13695	0.13305	0.13900
14938	0.13320	0.13690	0.13290	0.13890
14941	0.13300	0.13660	0.13270	0.13850
14942	0.13385	0.13735	0.13365	0.13910
14943	0.13390	0.13740	0.13375	0.13940
14944	0.13245	0.13595	0.13225	0.13800
14945	0.12990	0.13340	0.12955	0.13590
14948	0.12900	0.13250	0.12855	0.13450
14950	0.12975	0.13335	0.12935	0.13510
14951	0.12925	0.13275	0.12890	0.13440
14952	0.13000	0.13350	0.12965	0.13520
14955	0.13030	0.13380	0.13000	0.13560
14956	0.13010	0.13360	0.12970	0.13530
14957	0.12950	0.13300	0.12925	0.13460
14958	0.12830	0.13180	0.12805	0.13350
14959	0.12810	0.13155	0.12780	0.13330
14962	0.12720	0.13065	0.12685	0.13230
14963	0.12740	0.13080	0.12715	0.13250
14964	0.12645	0.12990	0.12615	0.13150
14965	0.12580	0.12925	0.12550	0.13100
14966	0.12625	0.12965	0.12595	0.13140
14971	0.12630	0.12970	0.12600	0.13140
14972	0.12670	0.13000	0.12645	0.13185
14973	0.12720	0.13050	0.12690	0.13250
14977	0.12720	0.13050	0.12690	0.13240
14978	0.12575	0.12915	0.12545	0.13100
14979	0.12630	0.12970	0.12605	0.13140
14980	0.12585	0.12935	0.12595	0.13100
14983	0.12670	0.13020	0.12695	0.13190
14984	0.12695	0.13045	0.12705	0.13220
14985	0.12795	0.13145	0.12805	0.13290
14986	0.12735	0.13045	0.12750	0.13220
14987	0.12675	0.13015	0.12690	0.13170
14990	0.12605	0.12955	0.12615	0.13100
14991	0.12480	0.12830	0.12485	0.12970
14992	0.12575	0.12925	0.12590	0.13070
14993	0.12610	0.12960	0.12620	0.13100
14994	0.12585	0.12935	0.12595	0.13080
14997	0.12650	0.13000	0.12660	0.13140
14998	0.12660	0.13010	0.12680	0.13160
14999	0.12655	0.13005	0.12670	0.13160
15000	0.12570	0.12920	0.12580	0.13060
15001	0.12490	0.12840	0.12500	0.12980
15004	0.12405	0.12755	0.12410	0.12890
15005	0.12385	0.12735	0.12390	0.12870

15006	0.12360	0.12710	0.12365	0.12840
15007	0.12185	0.12505	0.12200	0.12690
15008	0.12220	0.12560	0.12235	0.12690
15011	0.12180	0.12520	0.12200	0.12650
15012	0.12035	0.12365	0.12045	0.12500
15013	0.12035	0.12375	0.12050	0.12500
15014	0.12080	0.12420	0.12100	0.12540
15015	0.11970	0.12315	0.11995	0.12470
15018	0.11880	0.12210	0.11890	0.12370
15019	0.11935	0.12265	0.11945	0.12430
15020	0.11975	0.12295	0.11985	0.12470
15021	0.11950	0.12270	0.11960	0.12430
15022	0.11895	0.12215	0.11900	0.12370
15025	0.11820	0.12140	0.11820	0.12310
15026	0.11870	0.12190	0.11870	0.12350
15027	0.11780	0.12100	0.11780	0.12260
15028	0.11705	0.12035	0.11700	0.12180
15029	0.11735	0.12055	0.11735	0.12200
15032	0.11775	0.12095	0.11775	0.12250
15033	0.11745	0.12065	0.11740	0.12220
15034	0.11645	0.11965	0.11640	0.12120
15035	0.11680	0.11995	0.11675	0.12150
15036	0.11665	0.11985	0.11665	0.12130
15039	0.11555	0.11875	0.11555	0.12020
15040	0.11560	0.11880	0.11560	0.12030
15041	0.11540	0.11860	0.11540	0.12010
15042	0.11530	0.11850	0.11535	0.12000
15043	0.11485	0.11805	0.11500	0.11960
15046	0.11500	0.11820	0.11510	0.11970
15047	0.11605	0.11925	0.11625	0.12070
15048	0.11745	0.12065	0.11765	0.12210
15049	0.11820	0.12140	0.11840	0.12290
15050	0.11715	0.12045	0.11735	0.12180
15053	0.11800	0.12140	0.11820	0.12280
15054	0.11970	0.12310	0.11990	0.12440
15056	0.11985	0.12305	0.12010	0.12450
15057	0.11985	0.12305	0.12010	0.12450
15060	0.12010	0.12350	0.12035	0.12490
15061	0.12110	0.12440	0.12140	0.12590
15062	0.12175	0.12530	0.12205	0.12650
15063	0.12185	0.12525	0.12215	0.12660
15064	0.12200	0.12550	0.12230	0.12680
15067	0.12345	0.12695	0.12375	0.12830
15068	0.12380	0.12730	0.12410	0.12860
15069	0.12180	0.12520	0.12210	0.12660
15070	0.12030	0.12370	0.12060	0.12510

15071	0.11950	0.12300	0.11980	0.12430
15074	0.11880	0.12225	0.11910	0.12360
15075	0.11960	0.12315	0.11990	0.12440
15076	0.11970	0.12325	0.12000	0.12450
15077	0.11950	0.12305	0.11985	0.12430
15082	0.12065	0.12420	0.12095	0.12540
15083	0.12070	0.12425	0.12100	0.12550
15084	0.12120	0.12470	0.12155	0.12610
15085	0.12230	0.12580	0.12255	0.12720
15088	0.12360	0.12710	0.12400	0.12850
15089	0.12325	0.12665	0.12360	0.12820
15090	0.12335	0.12680	0.12375	0.12830
15091	0.12140	0.12480	0.12185	0.12630
15095	0.12040	0.12380	0.12085	0.12530
15097	0.12050	0.12380	0.12090	0.12540
15098	0.12155	0.12485	0.12215	0.12640
15099	0.12020	0.12350	0.12060	0.12500
15102	0.11920	0.12250	0.11960	0.12400
15103	0.12010	0.12340	0.12040	0.12470
15104	0.11980	0.12310	0.12010	0.12440
15105	0.11970	0.12290	0.12000	0.12430
15106	0.11995	0.12315	0.12025	0.12450
15109	0.11930	0.12250	0.11960	0.12390
15110	0.11875	0.12195	0.11920	0.12340
15111	0.11815	0.12135	0.11860	0.12260
15112	0.11775	0.12095	0.11815	0.12220
15113	0.11735	0.12055	0.11770	0.12190
15116	0.11775	0.12095	0.11810	0.12230
15117	0.11780	0.12100	0.11810	0.12240
15118	0.11770	0.12090	0.11800	0.12220
15119	0.11760	0.12080	0.11790	0.12210
15120	0.11715	0.12035	0.11755	0.12160
15123	0.11730	0.12060	0.11770	0.12180
15124	0.11750	0.12080	0.11790	0.12200
15125	0.11760	0.12090	0.11790	0.12210
15126	0.11815	0.12145	0.11845	0.12260
15127	0.11770	0.12105	0.11800	0.12210
15130	0.11715	0.12050	0.11745	0.12150
15131	0.11625	0.11955	0.11655	0.12030
15132	0.11620	0.11945	0.11650	0.12020
15133	0.11595	0.11900	0.11620	0.11990
15134	0.11590	0.11910	0.11620	0.11990
15137	0.11610	0.11930	0.11635	0.12010
15138	0.11520	0.11835	0.11545	0.11910
15139	0.11490	0.11790	0.11510	0.11880
15140	0.11305	0.11620	0.11335	0.11690

15141	0.11220	0.11540	0.11250	0.11610
15144	0.11270	0.11570	0.11295	0.11660
15145	0.11195	0.11515	0.11215	0.11590
15146	0.11100	0.11420	0.11130	0.11490
15147	0.10905	0.11230	0.10930	0.11290
15148	0.10970	0.11300	0.10995	0.11410
15151	0.11030	0.11300	0.11055	0.11460
15152	0.10970	0.11295	0.11000	0.11410
15153	0.10980	0.11315	0.11005	0.11420
15154	0.10900	0.11240	0.10920	0.11340
15155	0.10860	0.11200	0.10880	0.11320
15158	0.10835	0.11175	0.10855	0.11270
15159	0.10900	0.11240	0.10920	0.11340
15160	0.11070	0.11400	0.11090	0.11510
15161	0.11150	0.11490	0.11170	0.11590
15162	0.11140	0.11470	0.11160	0.11600
15165	0.11080	0.11430	0.11100	0.11530
15166	0.11025	0.11375	0.11050	0.11480
15167	0.11250	0.11600	0.11280	0.11710
15168	0.11260	0.11605	0.11290	0.11720
15169	0.11210	0.11555	0.11240	0.11670
15172	0.11050	0.11400	0.11070	0.11500
15173	0.10870	0.11220	0.10890	0.11320
15174	0.10790	0.11140	0.10815	0.11240
15175	0.10795	0.11140	0.10815	0.11250
15176	0.10780	0.11125	0.10805	0.11230
15179	0.10740	0.11070	0.10765	0.11170
15180	0.10640	0.10990	0.10665	0.11070
15181	0.10675	0.11015	0.10700	0.11090
15182	0.10635	0.10975	0.10660	0.11040
15183	0.10590	0.10925	0.10620	0.11000
15186	0.10530	0.10850	0.10555	0.10940
15187	0.10610	0.10930	0.10635	0.11020
15188	0.10600	0.10920	0.10620	0.11010
15189	0.10540	0.10860	0.10565	0.10950
15190	0.10570	0.10895	0.10600	0.10980
15193	0.10590	0.10915	0.10620	0.10990
15194	0.10615	0.10935	0.10645	0.11020
15195	0.10510	0.10830	0.10540	0.10910
15197	0.10490	0.10810	0.10530	0.10890
15200	0.10490	0.10815	0.10530	0.10900
15201	0.10530	0.10855	0.10570	0.10950
15202	0.10590	0.10910	0.10635	0.11010
15203	0.10740	0.11055	0.10790	0.11160
15204	0.10740	0.11050	0.10780	0.11170
15207	0.10785	0.11095	0.10830	0.11200

15208	0.10840	0.11140	0.10880	0.11250
15209	0.10930	0.11230	0.10980	0.11340
15210	0.10870	0.11170	0.10920	0.11280
15211	0.10830	0.11130	0.10890	0.11240
15214	0.10835	0.11135	0.10890	0.11250
15215	0.10780	0.11070	0.10850	0.11190
15216	0.10690	0.10975	0.10750	0.11100
15217	0.10750	0.11035	0.10820	0.11170
15218	0.10790	0.11075	0.10860	0.11210
15221	0.10840	0.11125	0.10910	0.11260
15222	0.10870	0.11155	0.10940	0.11290
15223	0.10850	0.11135	0.10910	0.11270
15224	0.10850	0.11130	0.10910	0.11260
15225	0.10780	0.11060	0.10835	0.11200
15228	0.10765	0.11045	0.10820	0.11180
15229	0.11020	0.11300	0.11070	0.11440
15230	0.10935	0.11210	0.10975	0.11360
15231	0.10890	0.11170	0.10930	0.11310
15232	0.10870	0.11150	0.10910	0.11290
15235	0.10890	0.11170	0.10940	0.11320
15236	0.10735	0.11010	0.10780	0.11160
15237	0.10550	0.10825	0.10610	0.10970
15238	0.10535	0.10820	0.10610	0.10940
15239	0.10765	0.11040	0.10835	0.11190
15243	0.10690	0.10970	0.10770	0.11100
15244	0.10690	0.10980	0.10770	0.11110
15245	0.10660	0.10950	0.10750	0.11080
15246	0.10680	0.10970	0.10740	0.11100
15249	0.10725	0.11015	0.10790	0.11140
15250	0.10790	0.11080	0.10860	0.11210
15251	0.10770	0.11050	0.10850	0.11190
15252	0.10800	0.11080	0.10870	0.11220
15253	0.10820	0.11100	0.10890	0.11240
15256	0.10875	0.11155	0.10960	0.11290
15257	0.10790	0.11070	0.10880	0.11200
15258	0.10640	0.10930	0.10720	0.11050
15259	0.10610	0.10900	0.10690	0.11020
15260	0.10665	0.10955	0.10750	0.11080
15263	0.10490	0.10780	0.10590	0.10900
15264	0.10620	0.10905	0.10720	0.11030
15265	0.10600	0.10885	0.10690	0.11010
15266	0.10565	0.10850	0.10655	0.10980
15267	0.10535	0.10820	0.10625	0.10940
15270	0.10650	0.10935	0.10740	0.11060
15271	0.10530	0.10815	0.10615	0.10940
15272	0.10530	0.10815	0.10610	0.10940

15273	0.10455	0.10735	0.10535	0.10870
15274	0.10470	0.10735	0.10550	0.10880
15277	0.10460	0.10725	0.10540	0.10870
15278	0.10460	0.10725	0.10550	0.10890
15279	0.10420	0.10690	0.10520	0.10880
15280	0.10320	0.10595	0.10410	0.10750
15281	0.10340	0.10620	0.10420	0.10770
15284	0.10260	0.10540	0.10360	0.10680
15285	0.10210	0.10490	0.10310	0.10630
15286	0.10180	0.10460	0.10280	0.10600
15287	0.10115	0.10395	0.10210	0.10530
15288	0.10070	0.10350	0.10160	0.10490
15291	0.10085	0.10370	0.10175	0.10500
15292	0.10090	0.10365	0.10180	0.10510
15293	0.10135	0.10410	0.10225	0.10540
15294	0.10200	0.10475	0.10295	0.10620
15295	0.10255	0.10530	0.10345	0.10680
15298	0.10200	0.10475	0.10290	0.10620
15299	0.10190	0.10465	0.10280	0.10600
15300	0.10160	0.10435	0.10250	0.10570
15301	0.10210	0.10485	0.10285	0.10620
15302	0.10210	0.10485	0.10290	0.10620
15305	0.10100	0.10375	0.10170	0.10510
15306	0.10030	0.10300	0.10105	0.10440
15307	0.10000	0.10270	0.10070	0.10410
15308	0.10015	0.10285	0.10080	0.10420
15309	0.10240	0.10525	0.10315	0.10650
15312	0.10510	0.10795	0.10585	0.10940
15313	0.10955	0.11240	0.11030	0.11370
15314	0.11250	0.11540	0.11310	0.11670
15315	0.11510	0.11795	0.11560	0.11930
15316	0.11165	0.11450	0.11215	0.11580
15319	0.11250	0.11535	0.11285	0.11670
15320	0.11200	0.11485	0.11240	0.11610
15321	0.11270	0.11555	0.11315	0.11680
15322	0.11520	0.11805	0.11565	0.11930
15323	0.13150	0.13430	0.13180	0.13550
15327	0.12580	0.12870	0.12605	0.12980
15328	0.12930	0.13230	0.12950	0.13340
15329	0.13600	0.13910	0.13600	0.14010
15330	0.12510	0.12820	0.12500	0.12920
15333	0.12030	0.12340	0.12040	0.12440
15337	0.11815	0.12125	0.11835	0.12230
15338	0.11435	0.11750	0.11475	0.11830
15340	0.11520	0.11840	0.11560	0.11920
15342	0.11860	0.12180	0.11890	0.12260

15343	0.12250	0.12560	0.12280	0.12650
15344	0.11795	0.12105	0.11825	0.12200
15347	0.11150	0.11450	0.11170	0.11550
15348	0.11020	0.11320	0.11050	0.11420
15349	0.11300	0.11600	0.11340	0.11710
15350	0.11190	0.11490	0.11210	0.11590
15351	0.11200	0.11500	0.11230	0.11610
15354	0.11130	0.11430	0.11165	0.11530
15355	0.11760	0.12060	0.11790	0.12160
15356	0.11810	0.12110	0.11830	0.12200
15357	0.11690	0.11990	0.11710	0.12090
15358	0.11640	0.11940	0.11655	0.12040
15361	0.11760	0.12060	0.11780	0.12150
15362	0.11930	0.12230	0.11950	0.12330
15363	0.11860	0.12160	0.11890	0.12260
15364	0.11910	0.12210	0.11950	0.12310
15365	0.11825	0.12125	0.11865	0.12240
15368	0.11930	0.12230	0.11980	0.12330
15369	0.11950	0.12250	0.12000	0.12350
15370	0.12210	0.12510	0.12270	0.12620
15371	0.12090	0.12400	0.12150	0.12490
15372	0.12030	0.12340	0.12090	0.12430
15375	0.12125	0.12435	0.12205	0.12530
15376	0.11890	0.12200	0.11960	0.12300
15377	0.11785	0.12095	0.11860	0.12190
15378	0.11900	0.12210	0.11975	0.12320
15379	0.11880	0.12190	0.11950	0.12300
15382	0.11840	0.12150	0.11910	0.12260
15383	0.11760	0.12070	0.11830	0.12180
15384	0.11730	0.12040	0.11790	0.12150
15385	0.11700	0.12010	0.11750	0.12120
15386	0.11630	0.11940	0.11690	0.12050
15389	0.11650	0.11960	0.11710	0.12070
15390	0.11620	0.11925	0.11650	0.12040
15391	0.11720	0.12020	0.11740	0.12140
15392	0.11800	0.12090	0.11810	0.12220
15393	0.11900	0.12190	0.11915	0.12310
15396	0.11890	0.12180	0.11870	0.12310
15397	0.11890	0.12180	0.11870	0.12310
15398	0.12370	0.12660	0.12350	0.12790
15399	0.12510	0.12800	0.12500	0.12930
15400	0.12360	0.12655	0.12350	0.12780
15403	0.12180	0.12475	0.12165	0.12600
15404	0.12300	0.12595	0.12290	0.12720
15405	0.12290	0.12585	0.12280	0.12710
15406	0.12480	0.12770	0.12460	0.12900

15407	0.12530	0.12820	0.12500	0.12940
15410	0.12480	0.12770	0.12450	0.12900
15411	0.12580	0.12870	0.12550	0.13000
15412	0.12670	0.12960	0.12630	0.13090
15413	0.12600	0.12890	0.12560	0.13020
15414	0.12690	0.12985	0.12650	0.13110
15417	0.12720	0.13010	0.12670	0.13140
15418	0.12790	0.13080	0.12740	0.13210
15419	0.12890	0.13180	0.12870	0.13310
15421	0.12940	0.13230	0.12920	0.13340
15424	0.13000	0.13290	0.12980	0.13450
15425	0.13150	0.13440	0.13130	0.13600
15426	0.13320	0.13605	0.13300	0.13770
15427	0.13200	0.13495	0.13170	0.13650
15432	0.13220	0.13505	0.13200	0.13670
15433	0.13080	0.13365	0.13055	0.13520
15434	0.12970	0.13255	0.12890	0.13400
15435	0.12820	0.13110	0.12730	0.13250
15438	0.12890	0.13180	0.12730	0.13320
15439	0.12930	0.13220	0.12800	0.13360
15440	0.12810	0.13100	0.12660	0.13210
15441	0.12770	0.13065	0.12635	0.13180
15442	0.12690	0.12990	0.12540	0.13100
15445	0.12590	0.12885	0.12440	0.13010
15446	0.12550	0.12845	0.12400	0.12970
15447	0.12460	0.12755	0.12310	0.12870
15448	0.12530	0.12825	0.12390	0.12940
15449	0.12620	0.12915	0.12460	0.13030
15452	0.12630	0.12920	0.12480	0.13060
15453	0.12640	0.12930	0.12490	0.13050
15454	0.12330	0.12630	0.12150	0.12750
15455	0.12180	0.12480	0.11960	0.12590
15456	0.12160	0.12460	0.11870	0.12570
15459	0.12050	0.12350	0.11760	0.12460
15460	0.12090	0.12390	0.11800	0.12500
15462	0.11890	0.12190	0.11590	0.12300
15463	0.11880	0.12180	0.11580	0.12290
15466	0.11960	0.12260	0.11700	0.12370
15467	0.11910	0.12210	0.11680	0.12320
15468	0.11880	0.12180	0.11660	0.12290
15469	0.12030	0.12330	0.11860	0.12440
15470	0.11960	0.12260	0.11770	0.12370
15473	0.11770	0.12070	0.11570	0.12180
15474	0.11750	0.12050	0.11530	0.12160
15475	0.11760	0.12060	0.11530	0.12170
15476	0.11820	0.12120	0.11610	0.12230

15477	0.11800	0.12100	0.11610	0.12220
15480	0.11810	0.12110	0.11620	0.12220
15481	0.11770	0.12085	0.11550	0.12180
15482	0.11780	0.12100	0.11520	0.12180
15483	0.11890	0.12210	0.11670	0.12300
15484	0.11800	0.12120	0.11570	0.12200
15487	0.11870	0.12190	0.11670	0.12270
15488	0.11880	0.12210	0.11680	0.12280
15489	0.11910	0.12240	0.11730	0.12310
15490	0.11710	0.12040	0.11540	0.12110
15491	0.11770	0.12100	0.11600	0.12170
15494	0.11730	0.12060	0.11530	0.12130
15495	0.11580	0.11910	0.11380	0.11980
15496	0.11690	0.12020	0.11460	0.12090
15497	0.11690	0.12020	0.11470	0.12090
15498	0.11740	0.12070	0.11510	0.12150
15501	0.11730	0.12060	0.11500	0.12140
15502	0.11750	0.12080	0.11550	0.12160
15503	0.11850	0.12180	0.11650	0.12260
15504	0.11800	0.12130	0.11600	0.12210
15505	0.11820	0.12150	0.11590	0.12230
15509	0.11780	0.12110	0.11520	0.12190
15510	0.11830	0.12160	0.11580	0.12240
15511	0.11900	0.12230	0.11660	0.12310
15512	0.11970	0.12310	0.11720	0.12380
15515	0.11970	0.12310	0.11720	0.12380
15516	0.11960	0.12300	0.11710	0.12370
15517	0.11910	0.12260	0.11670	0.12320
15518	0.11940	0.12305	0.11690	0.12350
15519	0.11990	0.12355	0.11720	0.12400
15522	0.11950	0.12305	0.11670	0.12390
15523	0.11710	0.12065	0.11410	0.12150
15524	0.11680	0.12035	0.11360	0.12120
15525	0.11680	0.12035	0.11350	0.12120
15526	0.11620	0.11975	0.11280	0.12060
15529	0.11570	0.11925	0.11230	0.12010
15530	0.11390	0.11745	0.11050	0.11820
15531	0.11330	0.11680	0.11000	0.11770
15532	0.11320	0.11670	0.11010	0.11750
15533	0.11270	0.11625	0.10940	0.11700
15536	0.11250	0.11605	0.10910	0.11680
15537	0.11340	0.11715	0.11010	0.11780
15538	0.11440	0.11815	0.11140	0.11880
15539	0.11350	0.11725	0.11040	0.11790
15540	0.11230	0.11605	0.10930	0.11660
15543	0.11220	0.11595	0.10930	0.11650

15544	0.11350	0.11725	0.11070	0.11790
15545	0.11300	0.11675	0.11030	0.11720
15546	0.11190	0.11565	0.10940	0.11610
15547	0.11190	0.11565	0.10930	0.11610
15550	0.11290	0.11665	0.11020	0.11710
15551	0.11390	0.11765	0.11130	0.11810
15552	0.11270	0.11660	0.10990	0.11680
15553	0.11180	0.11570	0.10880	0.11600
15554	0.11270	0.11660	0.10990	0.11690
15557	0.11320	0.11710	0.11030	0.11690
15558	0.11355	0.11745	0.11090	0.11770
15559	0.11320	0.11710	0.11050	0.11740
15560	0.11270	0.11660	0.10990	0.11690
15564	0.11220	0.11610	0.10940	0.11640
15565	0.11300	0.11700	0.11020	0.11720
15566	0.11270	0.11670	0.11000	0.11720
15567	0.11340	0.11740	0.11060	0.11760
15568	0.11270	0.11670	0.11000	0.11690
15571	0.11300	0.11700	0.11025	0.11720
15572	0.11380	0.11770	0.11100	0.11800
15573	0.11470	0.11860	0.11200	0.11890
15574	0.11510	0.11900	0.11230	0.11930
15575	0.11470	0.11860	0.11180	0.11930
15578	0.11430	0.11820	0.11140	0.11850
15579	0.11380	0.11770	0.11080	0.11800
15580	0.11580	0.11970	0.11260	0.12010
15581	0.11580	0.11970	0.11260	0.12000
15582	0.11610	0.12040	0.11280	0.12030
15585	0.11670	0.12070	0.11330	0.12080
15586	0.11830	0.12230	0.11480	0.12240
15587	0.11710	0.12110	0.11370	0.12130
15588	0.11750	0.12150	0.11400	0.12170
15589	0.11710	0.12110	0.11350	0.12130
15592	0.11680	0.12080	0.11300	0.12100
15593	0.11690	0.12090	0.11330	0.12110
15594	0.11700	0.12100	0.11330	0.12120
15595	0.11660	0.12070	0.11300	0.12060
15598	0.11670	0.12080	0.11270	0.12070
15599	0.11780	0.12250	0.11390	0.12180
15600	0.11790	0.12240	0.11410	0.12190
15601	0.11750	0.12210	0.11370	0.12150
15602	0.11750	0.12210	0.11370	0.12150
15603	0.11810	0.12270	0.11420	0.12210
15606	0.11890	0.12330	0.11480	0.12290
15608	0.11890	0.12380	0.11420	0.12290
15609	0.11810	0.12310	0.11290	0.12200

15610	0.11740	0.12260	0.11150	0.12140
15613	0.11880	0.12460	0.11260	0.12280
15614	0.11890	0.12510	0.11220	0.12290
15615	0.11960	0.12580	0.11330	0.12360
15616	0.11940	0.12530	0.11330	0.12320
15617	0.11990	0.12570	0.11390	0.12390
15620	0.11950	0.12560	0.11340	0.12350
15621	0.11760	0.12270	0.11090	0.12160
15622	0.11660	0.12210	0.11000	0.12070
15623	0.11580	0.12130	0.10940	0.11980
15624	0.11670	0.12210	0.11030	0.12080
15627	0.11640	0.12190	0.11030	0.12040
15628	0.11725	0.12260	0.11140	0.12130
15629	0.11930	0.12460	0.11420	0.12330
15630	0.11890	0.12390	0.11390	0.12290
15631	0.11870	0.12370	0.11360	0.12260
15634	0.11910	0.12410	0.11420	0.12300
15635	0.11970	0.12410	0.11550	0.12360
15636	0.11790	0.12300	0.11340	0.12180
15637	0.11780	0.12240	0.11310	0.12160
15638	0.11720	0.12195	0.11220	0.12110
15641	0.11690	0.12160	0.11180	0.12080
15642	0.11590	0.12070	0.11120	0.11990
15643	0.11640	0.12080	0.11230	0.12030
15644	0.11610	0.12020	0.11210	0.12000
15645	0.11540	0.11960	0.11140	0.11930
15648	0.11540	0.11950	0.11190	0.11930
15649	0.11630	0.12040	0.11340	0.12020
15650	0.11550	0.11970	0.11230	0.11940
15651	0.11380	0.11800	0.11080	0.11770
15652	0.11270	0.11700	0.10970	0.11660
15655	0.11300	0.11765	0.11000	0.11690
15656	0.11320	0.11805	0.11000	0.11710
15657	0.11190	0.11695	0.10850	0.11580
15658	0.11140	0.11665	0.10800	0.11530
15659	0.11120	0.11640	0.10770	0.11510
15662	0.11160	0.11660	0.10780	0.11540
15663	0.10950	0.11510	0.10590	0.11340
15664	0.11000	0.11560	0.10640	0.11340
15665	0.11030	0.11590	0.10670	0.11420
15666	0.10930	0.11470	0.10560	0.11320
15669	0.10840	0.11370	0.10460	0.11230
15670	0.10770	0.11280	0.10400	0.11160
15671	0.10670	0.11160	0.10370	0.11060
15672	0.10740	0.11230	0.10460	0.11130
15673	0.10740	0.11220	0.10460	0.11130

15676	0.10830	0.11310	0.10560	0.11260
15677	0.10860	0.11310	0.10600	0.11250
15678	0.10730	0.11190	0.10460	0.11130
15679	0.10740	0.11220	0.10460	0.11130
15680	0.10700	0.11200	0.10410	0.11090
15683	0.10670	0.11180	0.10370	0.11060
15684	0.10590	0.11100	0.10310	0.10980
15685	0.10610	0.11110	0.10340	0.11000
15686	0.10650	0.11140	0.10390	0.11040
15687	0.10630	0.11110	0.10370	0.11020
15691	0.10680	0.11130	0.10420	0.11080
15692	0.10700	0.11190	0.10430	0.11090
15693	0.10750	0.11220	0.10470	0.11140
15694	0.10750	0.11230	0.10470	0.11140
15697	0.10750	0.11230	0.10460	0.11140
15698	0.10750	0.11230	0.10450	0.11130
15701	0.10750	0.11230	0.10450	0.11140
15704	0.10750	0.11220	0.10450	0.11140
15705	0.10740	0.11220	0.10440	0.11140
15707	0.10680	0.11170	0.10390	0.11070
15708	0.10630	0.11120	0.10340	0.11020
15711	0.10400	0.10900	0.10100	0.10790
15712	0.10410	0.10890	0.10120	0.10800
15713	0.10460	0.10950	0.10160	0.10850
15714	0.10470	0.10950	0.10170	0.10860
15715	0.10470	0.10960	0.10180	0.10860
15718	0.10480	0.10960	0.10180	0.10870
15719	0.10570	0.11040	0.10270	0.10960
15720	0.10580	0.11050	0.10280	0.10970
15721	0.10490	0.10990	0.10160	0.10880
15722	0.10530	0.11060	0.10210	0.10920
15725	0.10630	0.11170	0.10310	0.11020
15726	0.10700	0.11230	0.10360	0.11090
15727	0.10700	0.11250	0.10360	0.11090
15728	0.10610	0.11160	0.10240	0.11000
15729	0.10620	0.11160	0.10250	0.11010
15732	0.10590	0.11150	0.10200	0.10980
15733	0.10540	0.11090	0.10070	0.10930
15734	0.10410	0.10930	0.10020	0.10800
15735	0.10450	0.10970	0.09950	0.10840
15736	0.10380	0.10900	0.09920	0.10770
15739	0.10380	0.10910	0.09930	0.10770
15740	0.10330	0.10880	0.09890	0.10720
15741	0.10380	0.10930	0.09970	0.10770
15742	0.10350	0.10910	0.09940	0.10740
15743	0.10300	0.10860	0.09910	0.10690

15746	0.10310	0.10870	0.09930	0.10700
15747	0.10410	0.10920	0.10030	0.10800
15748	0.10400	0.10910	0.09990	0.10790
15749	0.10390	0.10880	0.09980	0.10780
15750	0.10370	0.10860	0.09980	0.10760
15753	0.10420	0.10910	0.10020	0.10810
15754	0.10380	0.10890	0.09930	0.10770
15755	0.10410	0.10970	0.09990	0.10800
15756	0.10390	0.10940	0.09980	0.10800
15757	0.10330	0.10890	0.09910	0.10740
15760	0.10340	0.10910	0.09930	0.10750
15761	0.10290	0.10870	0.09890	0.10690
15762	0.10210	0.10760	0.09830	0.10610
15763	0.10200	0.10760	0.09840	0.10650
15764	0.10230	0.10790	0.09870	0.10630
15767	0.10210	0.10760	0.09850	0.10630
15768	0.10240	0.10770	0.09880	0.10660
15769	0.10240	0.10770	0.09880	0.10660
15770	0.10270	0.10820	0.09920	0.10690
15771	0.10240	0.10790	0.09900	0.10660
15774	0.10240	0.10760	0.09910	0.10660
15775	0.10250	0.10780	0.09910	0.10670
15776	0.10305	0.10805	0.09975	0.10725
15777	0.10400	0.10890	0.10080	0.10820
15778	0.10410	0.10920	0.10110	0.10830
15781	0.10380	0.10890	0.10080	0.10800
15782	0.10420	0.10940	0.10140	0.10840
15783	0.10420	0.10970	0.10150	0.10840
15784	0.10380	0.10925	0.10070	0.10800
15788	0.10390	0.10950	0.10100	0.10810
15789	0.10330	0.10920	0.10050	0.10750
15790	0.10230	0.10810	0.09920	0.10650
15791	0.10250	0.10830	0.09950	0.10670
15792	0.10210	0.10790	0.09910	0.10630
15795	0.10160	0.10750	0.09880	0.10580
15796	0.10190	0.10770	0.09960	0.10610
15797	0.10230	0.10820	0.09990	0.10650
15798	0.10240	0.10830	0.09980	0.10660
15799	0.10210	0.10790	0.09970	0.10680
15802	0.10260	0.10840	0.10040	0.10730
15803	0.10290	0.10870	0.10090	0.10760
15804	0.10260	0.10850	0.10060	0.10730
15805	0.10260	0.10840	0.10050	0.10730
15806	0.10230	0.10820	0.10010	0.10700
15809	0.10240	0.10820	0.10020	0.10710
15810	0.10260	0.10820	0.10030	0.10700

15811	0.10180	0.10750	0.09950	0.10610
15812	0.10110	0.10670	0.09830	0.10540
15817	0.10100	0.10630	0.09830	0.10530
15818	0.10110	0.10615	0.09870	0.10540
15819	0.10050	0.10565	0.09840	0.10480
15820	0.10050	0.10555	0.09840	0.10490
15824	0.10000	0.10475	0.09820	0.10440
15825	0.10000	0.10485	0.09790	0.10440
15827	0.10140	0.10625	0.09940	0.10540
15830	0.10080	0.10560	0.09880	0.10480
15831	0.10040	0.10510	0.09860	0.10440
15832	0.10010	0.10450	0.09840	0.10410
15833	0.09990	0.10410	0.09860	0.10390
15834	0.09990	0.10375	0.09860	0.10390
15837	0.09920	0.10335	0.09800	0.10320
15838	0.09870	0.10265	0.09720	0.10250
15839	0.09920	0.10315	0.09790	0.10300
15840	0.09970	0.10375	0.09860	0.10350
15841	0.09890	0.10285	0.09780	0.10270
15844	0.09890	0.10285	0.09800	0.10270
15845	0.09790	0.10135	0.09730	0.10170
15846	0.09750	0.10105	0.09730	0.10130
15847	0.09740	0.10135	0.09700	0.10120
15848	0.09730	0.10125	0.09700	0.10110
15851	0.09700	0.10105	0.09670	0.10080
15852	0.09670	0.10070	0.09640	0.10050
15853	0.09730	0.10140	0.09680	0.10110
15854	0.09700	0.10100	0.09620	0.10080
15855	0.09540	0.09915	0.09470	0.09920
15858	0.09230	0.09570	0.09180	0.09610
15859	0.09250	0.09610	0.09260	0.09630
15860	0.09290	0.09670	0.09380	0.09670
15861	0.09150	0.09540	0.09200	0.09530
15862	0.09100	0.09490	0.09180	0.09480
15863	0.09110	0.09480	0.09185	0.09490
15866	0.09160	0.09520	0.09250	0.09540
15867	0.09175	0.09530	0.09270	0.09555
15868	0.08990	0.09285	0.09140	0.09370
15869	0.08970	0.09270	0.09095	0.09350
15873	0.08910	0.09165	0.09060	0.09500
15874	0.09050	0.09325	0.09220	0.09500
15875	0.09090	0.09360	0.09270	0.09550
15876	0.09120	0.09375	0.09350	0.09475
15879	0.09170	0.09425	0.09385	0.09440
15880	0.09095	0.09345	0.09330	0.09440
15881	0.08910	0.09105	0.09260	0.09440

15882	0.09000	0.09195	0.09340	0.09450
15883	0.09060	0.09190	0.09410	0.09345
15886	0.09070	0.09190	0.09415	0.09710
15887	0.08965	0.09110	0.09295	0.09710
15888	0.09085	0.09275	0.09460	0.09710
15889	0.09285	0.09515	0.09715	0.09745
15890	0.09330	0.09575	0.09680	0.09875
15893	0.09365	0.09615	0.09665	0.09845
15894	0.09495	0.09760	0.09735	0.09845
15895	0.09450	0.09750	0.09645	0.09845
15896	0.09380	0.09620	0.09605	0.09815
15897	0.09465	0.09650	0.09685	0.09895
15900	0.09415	0.09610	0.09635	0.09705
15901	0.09475	0.09680	0.09695	0.09705
15902	0.09530	0.09700	0.09720	0.09705
15903	0.09410	0.09585	0.09565	0.09700
15904	0.09265	0.09445	0.09430	0.09690
15907	0.09260	0.09450	0.09425	0.09620
15908	0.09250	0.09455	0.09425	0.09620
15909	0.09210	0.09380	0.09430	0.09620
15910	0.09220	0.09375	0.09485	0.09645
15911	0.09180	0.09330	0.09450	0.09665
15914	0.09205	0.09330	0.09460	0.09835
15915	0.09225	0.09350	0.09480	0.09835
15916	0.09330	0.09500	0.09540	0.09835
15917	0.09320	0.09470	0.09525	0.09910
15918	0.09395	0.09605	0.09585	0.09895
15921	0.09470	0.09675	0.09625	0.09760
15922	0.09455	0.09630	0.09605	0.09760
15923	0.09475	0.09650	0.09630	0.09760
15924	0.09340	0.09480	0.09485	0.09785
15925	0.09260	0.09410	0.09400	0.09790
15928	0.09285	0.09425	0.09450	0.09880
15929	0.09290	0.09440	0.09470	0.09880
15930	0.09305	0.09490	0.09485	0.09880
15931	0.09390	0.09635	0.09530	0.09885
15932	0.09380	0.09640	0.09470	0.10035
15935	0.09385	0.09670	0.09460	0.10100
15936	0.09535	0.09825	0.09620	0.10100
15937	0.09530	0.09830	0.09640	0.10100
15938	0.09560	0.09850	0.09670	0.10130
15939	0.09600	0.09890	0.09715	0.10245
15942	0.09630	0.09915	0.09745	0.10040
15943	0.09745	0.10035	0.09805	0.10040
15944	0.09650	0.10000	0.09690	0.10040
15945	0.09620	0.09975	0.09665	0.10055

15948	0.09540	0.09835	0.09625	0.10105
15949	0.09555	0.09800	0.09645	0.10025
15950	0.09605	0.09855	0.09730	0.10025
15951	0.09735	0.09985	0.09865	0.10025
15952	0.09665	0.09950	0.09790	0.09920
15953	0.09525	0.09675	0.09665	0.09950
15956	0.09420	0.09625	0.09595	0.09955
15957	0.09450	0.09670	0.09650	0.09880
15958	0.09475	0.09720	0.09675	0.09860
15959	0.09540	0.09805	0.09715	0.09880
15960	0.09455	0.09690	0.09655	0.09805
15963	0.09380	0.09620	0.09610	0.09830
15964	0.09360	0.09610	0.09600	0.09750
15965	0.09380	0.09610	0.09620	0.09750
15966	0.09305	0.09550	0.09535	0.09750
15967	0.09330	0.09560	0.09540	0.09730
15970	0.09250	0.09490	0.09460	0.09730
15971	0.09250	0.09475	0.09465	0.09775
15972	0.09250	0.09475	0.09465	0.09650
15973	0.09230	0.09400	0.09450	0.09630
15974	0.09230	0.09400	0.09450	0.09540
15977	0.09275	0.09450	0.09495	0.09540
15978	0.09150	0.09325	0.09335	0.09510
15979	0.09130	0.09315	0.09305	0.09545
15980	0.09040	0.09220	0.09210	0.09480
15981	0.09020	0.09200	0.09235	0.09375
15984	0.09040	0.09230	0.09270	0.09385
15985	0.09010	0.09180	0.09255	0.09390
15986	0.09045	0.09205	0.09290	0.09390
15987	0.08980	0.09125	0.09225	0.09305
15988	0.08875	0.09060	0.09090	0.09420
15991	0.08885	0.09090	0.09105	0.09455
15992	0.08890	0.09110	0.09095	0.09440
15993	0.08890	0.09115	0.09105	0.09385
15994	0.08805	0.08955	0.09020	0.09360
15995	0.08920	0.09070	0.09135	0.09300
15998	0.08955	0.09170	0.09165	0.09300
15999	0.08940	0.09150	0.09155	0.09285
16000	0.08885	0.09050	0.09080	0.09230
16001	0.08860	0.08985	0.09055	0.09290
16002	0.08800	0.08885	0.09000	0.09380
16005	0.08800	0.08875	0.09040	0.09400
16006	0.08785	0.08890	0.09045	0.09440
16007	0.08730	0.08785	0.09000	0.09450
16008	0.08790	0.08860	0.09055	0.09550
16009	0.08880	0.08950	0.09180	0.09625

16012	0.08900	0.08990	0.09230	0.09660
16013	0.08940	0.09050	0.09280	0.09555
16014	0.08950	0.09055	0.09305	0.09500
16015	0.09125	0.09165	0.09365	0.09500
16016	0.09160	0.09330	0.09395	0.09440
16019	0.09055	0.09215	0.09275	0.09370
16020	0.09000	0.09150	0.09235	0.09420
16021	0.09000	0.09125	0.09295	0.09360
16022	0.08995	0.09095	0.09375	0.09325
16023	0.08940	0.09005	0.09345	0.09290
16026	0.08870	0.08950	0.09255	0.09285
16027	0.08920	0.09020	0.09295	0.09370
16028	0.08860	0.08920	0.09275	0.09300
16029	0.08825	0.08890	0.09205	0.09340
16030	0.08790	0.08835	0.09165	0.09335
16033	0.08785	0.08840	0.09175	0.09370
16034	0.08870	0.08860	0.09315	0.09360
16035	0.08800	0.08770	0.09215	0.09380
16036	0.08840	0.08825	0.09240	0.09380
16037	0.08835	0.08845	0.09210	0.09400
16040	0.08870	0.08895	0.09250	0.09440
16041	0.08860	0.08880	0.09235	0.09500
16042	0.08880	0.08860	0.09275	0.09340
16043	0.08880	0.08870	0.09255	0.09340
16044	0.08900	0.08955	0.09215	0.09430
16047	0.08940	0.09010	0.09235	0.09340
16048	0.09000	0.09070	0.09325	0.09340
16049	0.08840	0.08830	0.09165	0.09370
16050	0.08840	0.09060	0.09105	0.09420
16051	0.08930	0.09140	0.09030	0.09440
16054	0.08840	0.09080	0.08925	0.09660
16055	0.08840	0.09080	0.08925	0.09590
16056	0.08870	0.09090	0.08965	0.09550
16057	0.08920	0.09180	0.09020	0.09550
16058	0.08940	0.09225	0.09095	0.09550
16061	0.09160	0.09580	0.09300	0.09530
16062	0.09090	0.09480	0.09185	0.09490
16063	0.09050	0.09465	0.09150	0.09540
16068	0.09030	0.09430	0.09130	0.09420
16069	0.08990	0.09370	0.09075	0.09380
16070	0.09040	0.09405	0.09125	0.09350
16072	0.09030	0.09400	0.09115	0.09290
16075	0.08920	0.09230	0.09030	0.09260
16076	0.08880	0.09190	0.08990	0.09430
16077	0.08850	0.09115	0.08960	0.09570
16078	0.08830	0.09070	0.08940	0.09540

16079	0.08790	0.09040	0.08880	0.09620
16082	0.08760	0.09050	0.08855	0.09660
16083	0.08930	0.09245	0.09010	0.09680
16084	0.09070	0.09365	0.09150	0.09630
16085	0.09040	0.09370	0.09110	0.09600
16086	0.09120	0.09550	0.09140	0.09650
16089	0.09160	0.09570	0.09180	0.09730
16090	0.09180	0.09530	0.09210	0.09930
16091	0.09130	0.09440	0.09230	0.09760
16092	0.09100	0.09420	0.09200	0.09940
16093	0.09150	0.09495	0.09245	0.09910
16096	0.09230	0.09575	0.09330	0.09900
16097	0.09430	0.09780	0.09620	0.09760
16098	0.09260	0.09630	0.09480	0.09680
16099	0.09440	0.09810	0.09665	0.09700
16100	0.09410	0.09810	0.09590	0.09730
16103	0.09400	0.09795	0.09605	0.09850
16104	0.09260	0.09660	0.09485	0.09780
16105	0.09180	0.09595	0.09430	0.09750
16106	0.09230	0.09655	0.09470	0.09770
16107	0.09350	0.09780	0.09525	0.09650
16110	0.09280	0.09715	0.09490	0.09590
16111	0.09250	0.09725	0.09445	0.09680
16112	0.09270	0.09700	0.09455	0.09680
16113	0.09150	0.09575	0.09345	0.09620
16114	0.09090	0.09500	0.09225	0.09780
16115	0.09180	0.09590	0.09310	0.09760
16118	0.09190	0.09625	0.09275	0.09750
16119	0.09120	0.09510	0.09225	0.09720
16120	0.09280	0.09615	0.09395	0.09760
16121	0.09260	0.09575	0.09400	0.09790
16124	0.09250	0.09590	0.09390	0.09860
16125	0.09220	0.09555	0.09350	0.09900
16126	0.09260	0.09635	0.09365	0.09930
16127	0.09290	0.09715	0.09390	0.10060
16128	0.09360	0.09760	0.09475	0.10040
16131	0.09400	0.09775	0.09535	0.09870
16132	0.09430	0.09785	0.09625	0.09800
16133	0.09560	0.09945	0.09770	0.09820
16134	0.09540	0.09950	0.09740	0.09880
16135	0.09370	0.09800	0.09545	0.09860
16138	0.09300	0.09740	0.09455	0.09930
16139	0.09320	0.09710	0.09470	0.09920
16140	0.09380	0.09770	0.09555	0.09900
16141	0.09360	0.09770	0.09535	0.09920
16142	0.09430	0.09835	0.09585	0.09910

16145	0.09420	0.09830	0.09580	0.09860
16146	0.09400	0.09815	0.09560	0.09860
16147	0.09420	0.09835	0.09585	0.09850
16148	0.09410	0.09815	0.09580	0.09900
16149	0.09360	0.09765	0.09545	0.09920
16153	0.09350	0.09740	0.09545	0.09930
16154	0.09400	0.09810	0.09580	0.09930
16155	0.09420	0.09845	0.09590	0.10020
16156	0.09430	0.09840	0.09600	0.10040
16159	0.09430	0.09845	0.09605	0.10130
16160	0.09430	0.09840	0.09630	0.10140
16161	0.09520	0.09920	0.09720	0.10130
16162	0.09540	0.09930	0.09755	0.10110
16163	0.09630	0.10030	0.09840	0.10080
16166	0.09640	0.10065	0.09805	0.10080
16167	0.09630	0.10065	0.09795	0.10080
16168	0.09610	0.10040	0.09795	0.10120
16169	0.09580	0.10015	0.09770	0.10120
16174	0.09620	0.10050	0.09815	0.10170
16176	0.09690	0.10130	0.09860	0.10230
16177	0.09670	0.10125	0.09855	0.10190
16180	0.09670	0.10125	0.09860	0.10200
16181	0.09700	0.10140	0.09880	0.10220
16182	0.09730	0.10175	0.09900	0.10220
16183	0.09690	0.10145	0.09860	0.10260
16184	0.09700	0.10150	0.09860	0.10290
16187	0.09720	0.10175	0.09880	0.10360
16189	0.09760	0.10195	0.09990	0.10320
16190	0.09790	0.10260	0.10000	0.10350
16191	0.09860	0.10340	0.10065	0.10310
16194	0.09820	0.10285	0.10025	0.10440
16195	0.09820	0.10280	0.10030	0.10440
16196	0.09850	0.10270	0.10045	0.10600
16197	0.09840	0.10260	0.10040	0.10590
16198	0.09940	0.10360	0.10135	0.10620
16201	0.10100	0.10580	0.10310	0.10640
16202	0.10090	0.10560	0.10310	0.10640
16203	0.10120	0.10590	0.10330	0.10660
16204	0.10240	0.10730	0.10465	0.10740
16205	0.10140	0.10645	0.10360	0.10640

Table 8: Historical Data of the Four Government Bonds used in the Historical Simulation.