Comparing South African Financial Markets Behaviour to the Geometric Brownian Motion Process

by

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DECLARATION

I declare that **Comparing South African markets behaviour to the Geometric Brownian motion process** is my own work, that it has not been copied from anywhere, and that all sources I have used have been indicated and acknowledged by complete references.

KARANGWA Innocent   Date: 14 November 2008

Signed …………………………….

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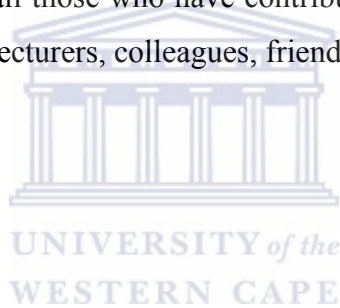
ABSTRACT

This study examines the behaviour of the South African financial markets with regards to the Geometric Brownian motion process. It uses the daily, weekly, and monthly stock returns time series of some major securities trading in the South African financial market, more specifically the US dollar/Euro, JSE ALSI Total Returns Index, South African All Bond Index, Anglo American Corporation, Standard Bank, Sasol, US dollar Gold Price , Brent spot oil price, and South African white maize near future.

 The assumptions underlying the Geometric Brownian motion in finance, namely the stationarity, the normality and the independence of stock returns, are tested using both graphical (histograms and normal plots) and statistical test (Kolmogorov-Simirnov test, Box-Ljung statistic and Augmented Dickey-Fuller test) methods to check whether or not the Brownian motion as a model for South African financial markets holds. The Hurst exponent or independence index is also applied to support the results from the previous test. Theoretically, the independent or Geometric Brownian motion time series should be characterised by the Hurst exponent of ½. A value of a Hurst exponent different from that would indicate the presence of long memory or fractional Brownian motion in a time series. The study shows that at least one assumption is violated when the Geometric Brownian motion process is examined assumption by assumption.

It also reveals the presence of both long memory and random walk or Geometric Brownian motion in the South African financial markets returns when the Hurst index analysis is used and finds that the Currency market is the most efficient of the South African financial markets. The study concludes that although some assumptions underlying the process are violated, the Brownian motion as a model in South African financial markets can not be rejected. It can be accepted in some instances if some parameters such as the Hurst exponent are added.

Key-words:

Acronyms

GBM: Geometric Brownian motion

FBM: Fractional Brownian motion

SPSS: Statistical Package for Social Sciences

EURUSD: US dollar/Euro

J2O3T: JSE All Share Index Total Returns Index

ALBI: South African All Bond Index

FCRB: CRB Commodity Price Index

AGL: Anglo American Corporation

SBK: Standard Bank

SOL: Sasol

DGLDS: US dollar Gold Price

BRSPOT: Brent spot oil price

WMAZN: South African white maize near future

Table of contents

List of Figures

List of tables

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CHAPTER ONE: INTRODUCTION

1.1 Introduction

This chapter discusses the background information covering the Brownian motion, and defines the main concepts used in this research. It also presents the research problem, the main assumptions underlying the Brownian motion in the financial stock markets and states the aims, objectives, importance, limitations of the study and the report structure.

1.2 Background

Models and algorithms, such as the Brownian motion, are formulated to help explain unpredictable movements and predict uncertainties.

The Brownian theory is named after Robert Brown, a Scottish botanist who discovered the motion [56, 57, 58, 59, 60, 61]. Brown observed the chaotic behavior of pollen grains suspended in a fluid under a microscope and reasoned that their motion was due to large numbers of random molecular forces that affected the grains.

In the 20th century, Guoy [58], revised the Brown theory and concluded that the Brownian motion was a clear demonstration of the existence of molecules in continuous motion.

All the 19th century research remained at a qualitative level. At that stage, the characteristic of the Brownian motion was the completely irregular and unceasing motion of the particles that was not attributed to external causes, or related to nature, but only on the particle size. It was only in 1905 that Einstein [3, 58, 59], in his research on mathematical laws that govern the movements of particles based on principals of kinetic molecular theory and heat, made the research quantitative. He demonstrated that by using a formula, the path described by a molecule on the average is not proportional to time but proportional to the square root of time [18].

Thereafter, the Brownian motion theory has since been applied in many fields including physics, astronomy, medicine (medical imaging), robotics, and stock markets (which is our research area of interest), amongst others.

1. 3. Research problem

Similar to the Brownian motion theory, uncertainties are always found in economic relationships. Therefore, people require ways that can help in making better informed decisions or choose more optimal business strategies.

Formulated workable hypotheses that take into consideration uncertainty and randomness of the process help form the basis of their decisions.

Using the same reasoning as Robert Brown, Louis Bachelier [1, 59, 62, 63], in his doctoral thesis "La théorie de la spéculation" from the University of Paris, proposed for the first time the Brownian motion as a model for market prices. He stated that the latter follows a random walk or a Brownian motion. That is, the previous change in the value of variable is unrelated to the future or past changes.

The shortcoming of the Bachelier findings was that it allowed the price to be negative. Samuelson [64] solved the problem by introducing the Geometric Brownian Motion (GBM), which assumed that the logarithm of the share prices rather than the prices themselves follows a Brownian motion.

As in other financial markets, the South African financial markets components, namely the share or equity market, the foreign currency market, the bond market and the commodity market [88], are all exposed to uncertainties.

In our project, we used the Samuelson reasoning to study the behaviour of the major securities trading in the South African financial markets, namely, Euro / US dollar, JSE ALSI Total Returns Index, South African All Bond Index, Anglo American corporation, Standard Bank, Sasol, CRB Commodity Price Index, US dollar Gold Price, Brent spot oil price, and South African white maize near future.

1.4.   Assumptions

The critical assumptions that underlie the Brownian motion model for stock price as discussed in [11] are:

1. Statistical independence of price changes (price changes or increments are uncorrelated or follow a random walk). This means that the current change of a price is not influenced by the past changes and does not have any influence on the future changes. This assumption seems to be relevant, at least on a long enough term. From time to time (second to second, hour to hour, day to day, month to month, etc), price changes are probably independent. This assumption has been documented by several studies [1, 70, 71, 72] and constitutes the essence of what economists generally call the Efficient Market Hypothesis, or the Random Walk Hypothesis [63, 64, 73, 75]. This hypothesis states that if the price changes are random and therefore unpredictable, it is because investors are properly doing their jobs. In this case, all arbitrage opportunities are exploited as much as possible.

2. Normality of price changes (meaning that changes follow a bell shaped curve) provides a distribution function characterized by only the mean and the volatility and implies a certain contained behaviour of the changes. This assumption seems reasonable for stock price fluctuations but does no take into consideration the fact that negative stock prices could result from large negative changes. This problem is solved by using the log normal distribution from the geometric Brownian motion.

3. The price-change indexes or statistics do not vary with time, meaning that the mean and the standard deviation do not change with time. The assumption that the variance remains unchanged on different intervals of the same lengths is not correct since the variance of stock price changes does not need to be proportional to the length of time.

The above three assumptions constitute the definition of white noise, equivalent to the changes of a Brownian motion.

1.5. The aim of the study

The aim of this study was to test if the above assumptions apply in the South African financial markets. Thereafter, the study aimed at determining the efficiency of the African financial markets and validates the assumptions in financial analysis, including the pricing of options and calculation of Value-at-Risk.

1.6. Research objectives

1. In order to investigate the above aim, the Box- Ljung statistic for autocorrelation in the series together with the Hurst exponent or index was used for the first assumption, the Kolmogorov-simirinov test was used for the second assumption, and the Dickey-Fuller test was utilized for the last assumption. All the above tests were supported by graphical methods.

2. This study has helped to assess whether or not the Geometric Brownian motion may be used as a model in the South African financial markets on the basis of the results obtained in 1.

3. Lastly, the study aimed to lay a foundation for future research about the South African financial markets behaviour.

1.7 Importance of this Research

The results obtained from this study could help in understanding the behaviour of the South African financial markets and improve market assessments as they vary over time. The findings could also be more useful to investors who make investment decisions based on risk analysis, optimized portfolios, and derivatives structures. They could as well have profound input on improving the practices involving the use of Value- at-Risk and option pricing models.

1.8 Research Design and Analysis.

The general concepts used for the different approaches were discussed and an analysis of quantitative data was carried out. The time series data on of the South African Financial major securities were used. The test for each assumption of the Geometric Brownian motion using both graphical and statistical test methods was done. The Hurst exponent significance values were used to support our findings in the test above.

The SPSS, Eviews, and Microsoft Excel spreadsheet were used for analysis.

1.9. Study limitations

Among our study limitations, were a limited sample of index numbers and a lot of missing data. The problem of missing data was handled by removing the dates for which values were missing.

1.10 Report structure

This report is structured into five chapters as follows: The first chapter gives an overall introduction of the study; the second chapter discusses in brief the major concepts used in the study and what has been published so far about this particular topic. The third and fourth chapters give a description and analysis of the data. The fifth and last chapter provides a conclusion on what has been discussed in the whole study.

CHAPTER TWO: REVIEW OF THE LITERATURE

2.1 Introduction

In this chapter, the general concepts used in this study are defined and discussed, and a brief overview of the application of the Brownian motion in stock markets is given.

2.2 Definition of concepts

2.2.1 Brownian motion

A Brownian motion is discussed by [59] as a stochastic process $W(t)$, for $t \geq 0$, with the following properties:

1. Every increment $W(t)-W(s)$ over an interval of length $t-s$ is normally distributed with mean 0 and variance $t-s$.
2. For every pair of disjoint time intervals $[t_1,t_2]$ and $[t_3,t_4]$, with $t_1 < t_2 < t_3 < t_4$, the increments $W(t_4) - W(t_3)$ and $W(t_2) - W(t_1)$ are independent random variables with distributions given as in part 1, and similarly for n disjoint time intervals, with n being an arbitrary integer.
3. $W(0) = 0$
4. For all t, $W(t)$ must be continuous.

2.2.2 Geometric Brownian motion

The Geometric Brownian motion (GBM), also called the exponential Brownian motion, is a continuous time stochastic process in which the logarithm of the randomly varying quantity follows a Brownian motion or Wiener process [44].

It is used to model financial markets data, especially in option pricing [87] because it accommodates positive values, and only fractional changes of the random variate are significant.

As an illustration, let S_t be a stochastic process. Then S_t is said to follow a GBM if the following stochastic differential equation is satisfied:

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$ --- eq.1

where W_t is a Wiener process or a Brownian motion and the percentage drift μ and the volatility σ are constant.

The analytic solution of the equation is as follows:

$$S_t = S_0 \exp(\mu - \frac{\sigma^2}{2})t + \sigma W_t$$ -- eq.2

where S_0 is an initial value which is taken arbitrary.

The random variable $\log(\frac{S_t}{S_0})$ is normally distributed with mean $(\mu - \frac{\sigma^2}{2})$ t and variance $\sigma^2 t$.

This reflects that the increments of Geometric Brownian Motion are normal relative to the current price (when dealing with prices), and this is the reason why the process is named "geometric".
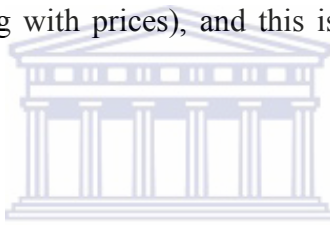
2.2.3 Hurst exponent

When dealing with financial time series data, it is always crucial to check whether or not they are predictable before attempting to model them and forecast their development. The Hurst exponent or independence index as a numerical estimate of predictability is always applicable.

The Hurst exponent is referred to as the relative tendency of a time series to either regress to a longer term mean value or "cluster" in a direction [43] and helps to classify time series in terms of predictability.

It is used in addition to test for the independence of time series and to inform on the presence of long memory or long range correlations in time series. Several studies in the past have used the Hurst exponent [76, 77, 78, 79, 80], ARFIMA (Autoregressive Fractional Integration Moving Average) [81], and FIGARCH (Fractionally Integrated Generalized Autoregressive Conditional Heteroscedasticity) [82] to quantify the long-term memory property in time series data.

There are many methods that are used to calculate the Hurst exponent. These include the classical rescaled range analysis [34], the generalized Hurst exponent [77], the modified Rescale Range (R/S) analysis [84], the GPH method [85], and the Detrended Fluctuation Analysis (DFA) [86].

In our study, we used the Beran approach [69] to calculate the Hurst exponent (H) as follows:

$$H = \tfrac{1}{2}\{[\ln(\rho +1)/\ln(2)] + 1\}$$ --- eq.3

where H denotes the Hurst exponent or Hurst index, and ρ, the first order autocorrelation of the time series data.

The values of the Hurst exponent range from zero to one and are interpreted as follows:

- H= ½ or close to that value indicate a random walk or a Brownian motion. In this case no correlation is present between any past, current, and future elements. In other words, there is no independence behaviour in the series. Such series are not easy to predict.
- H < ½ indicates the presence of anti-persistence, meaning that if there is an increase, the decrease will automatically follow and vice versa. This behaviour is also called the mean reversion in the sense that the future values will always tend to return to a longer term mean value.
- H > ½ indicates the presence of the persistence behaviour, meaning that the time series is trending. It may be a decreasing or increasing trend.

 The Hurst exponent different from a half also means that the series is not independent.

Series with higher H values are easier to predict than series with lower H values.

The Hurst exponent is also discussed in [35] where some other meanings of its values are cited:

- If H = ½, then the process is Brownian motion.

- If H > ½, then the increments of the process are positively correlated and the process exhibits long range dependence. The Hurst parameter above a half also means that there is persistence.
- If H < ½, then the increments of the process are negatively correlated, and it *also* means that there is an anti-persistence

2.2.4 Fractional Geometric Brownian motion

The concept of Fractional Geometric Brownian motion process is described by [21] as follows:

In the Black & Scholes pricing model, the randomness of a stock price S is due to the Brownian motion W such that:

$$dS_t = S_t(\mu dt + dW_t), S_0 > 0$$ --- eq.4

According to this model, the logarithmic returns are supposed to be independent normal variables.

The independence issue was studied using the rescale range analysis, a technique developed by Hurst [34] and characterized by the Hurst exponent described above.

The Hurst exponent of a half indicates that the returns are independent. However, some studies have revealed a Hurst parameter or index which is different from ½ [23, 24, 25, 26, 27, 28, 29, 30, 31, 32].

To overcome this problem, the Brownian motion W must be replaced by the fractional Brownian motion (fBm).

If X is a fBm, then it is called a continuous and centered Gaussian process having stationary increments and variance $E[X_t^2] = t^{2H}$, where H denotes the Hurst index and t denotes the time increment of the process.

By replacing W with X in the Black and Scholes model, a Geometric fractional Brownian motion (GfBm) is generated as follows:

$$dS_t = S_t \, t \, (\mu dt + dX_t)$$ --- eq.5

The properties of the fBm assuming for instance B_H is a Brownian motion that includes [46]:

- $B_H(0) = 0$, almost surely.
- B_H has strictly stationary increments, that is the random function $M_h(t) = B_H(t+h) - B_H(t)$, $h => 0$, is strictly stationary.
- B_H is self-similar of order H denoted H – ss.

Mathematically speaking, an object is said to be self–similar, if it is exactly or approximately similar to part of itself.

- Finite dimensional distributions of B_H are Gaussian with E $B_H(0) = 0$
- B_H is almost surely continuous.
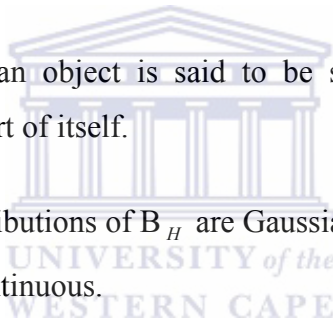
2.2.5 Return

The term return, as discussed in [20], refers to any number of metrics of the change in asset's or portfolios' accumulated value over some period of time. In investment management, there are two types of returns that need to be distinguished, namely the total returns and the net returns.

Net returns refer to the returns obtained from accumulated values that reflect only price appreciation and income from dividends or interest.

Net returns are obtained from accumulated values that reflect items such as management fees, transaction costs, taxes, etc.

Two standard metrics of returns, the simple returns and log returns are always calculated based on the total or net return.

If P_t is a portfolio's or asset's accumulated value at time t, then these returns can be mathematically written as follows:

$$\text{Simple return} = (P_t - P_{t-1})/P_{t-1}$$

$$\text{Log returns} = \ln (P_t / P_{t-1})$$ where the term ln denotes the natural logarithm.

The simple return, known as arithmetic or discrete rate of return, is defined as the capital gain plus any interim payment such as a dividend or a coupon:

$$R_t = (P_t + D_t - P_{t-1})/P_{t-1}$$ --- eq.6

where P denotes the capital gain, and D_t, an interim payment.

R_t is also given as:

$$R_t = (P_t - P_{t-1})/P_{t-1}$$ --- eq.7

when there is no interim payment as seen previously.

Simple returns are close to logarithmic returns.

Returns are generally calculated over one year or less but always reported on an annual basis. This is referred to as annualized returns or rate of returns.

When one has to focus on long-horizon returns, the log returns also known as geometric rate of returns are preferred. They are defined in terms of the logarithm of the ratio:

$$R_t = \ln (P_t + D_t / P_{t-1})$$ --- eq.8

where the term D_t denotes the interim payment when there is one, or $R_t = \ln(P_t / P_{t-1})$ when there is none [20].

The use of these geometric returns has two advantages [9].

Firstly they may be more economically meaningful than arithmetic returns. If geometric returns are normally distributed, then the distribution can lead to a price that is negative.

The second advantage of using geometric returns is that they easily allow extensions into multiple periods.

For example, considering the return over a 2- month period the geometric return can be decomposed as:

$$R_{t,2} = \ln(P_t / P_{t-2}) = R_{t,2} = \ln(P_t / P_{t-1}) + \ln(P_{t-1} / P_{t-2}) = R_t + R_{t-1} \quad \text{...eq.9}$$

This is particularly convenient since the two-month geometric return is simply the sum of the two monthly returns. However, the decomposition is not possible with discrete returns.
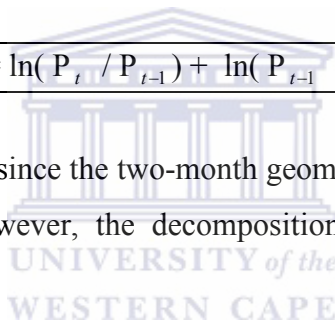
2.2.6 Normality

Most of statistical methods require data to be normally distributed. When decisions about processes are to be made, the normality assumption for the error rates to accept is needed.

Invalid assumptions on a specified distribution also lead to incorrect conclusions. Therefore, random errors need to follow a normal distribution for the test results to be reliable.

The null hypothesis for the test for normality states that the actual distribution of the variable is equal to the expected distribution or briefly, the variable is normally distributed.

There are graphical and statistical tests methods that are used to test for normality. Graphical methods inform on the shape of the distribution but do not give us a guarantee that the distribution is normal and do not test whether the difference between the normal distribution and the sample distribution is significant. With graphical methods, small sample sizes tend to pass all tests for normality whereas large sample sizes do not [66]. That is, there are always small deviations from normality that may lead to confusion when making a decision.

It is therefore advisable to always use both graphical and statistical test methods when testing for normality.

The concept of normality and some approaches that are used to test for normality are discussed by [66] and [68] in the following sections:

2.2.6.1 Graphical methods

2.2.6.1.1 Histogram

The histogram is obtained by splitting the range of data into equal-sized bins which are also referred to as classes, then number of points from data set falling into each class is counted.

The histogram graphically summarizes the distribution of a data set and shows its location, its spread, the presence of outliers, and the multiple modes in the data [39]. Those features explain the proper distributional model for the data.

 Figure 2.1 shows an example of a bell-shaped and symmetrical histogram with data points equally distributed around the middle.
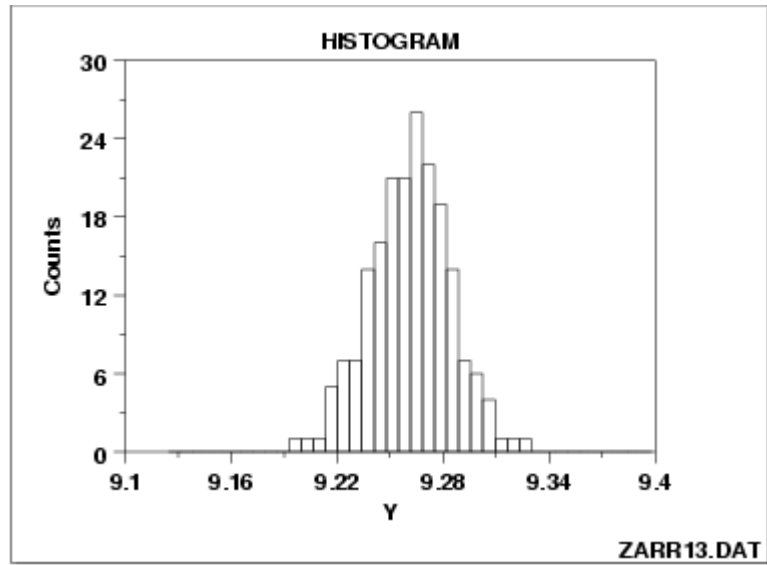
Figure 2.1: A bell-shaped and symmetrical histogram. Source, [36].

2.2.6.1.2 Stem and leaf plot

The stem and leaf plot combines the features of a graphic and a table in that the original data values are explicitly shown in the display as a stem and a leaf for each value. The stems determine a set of bins in which leaves are sorted, and the resulting list of leaves for each stem looks like a bar in a histogram. An example of a stem and leaf plot of monthly returns of South African bonds share prices (ALBI) from 31/01/1999 to 31/07/2008 is shown in Figure 2.2. The distribution is normal.

```
Frequency    Stem & Leaf

    3.00 Extremes    (=<-.037)
    1.00       -2.  5
    2.00       -2.  00
    4.00       -1.  5777
    4.00       -1.  0112
    9.00       -0.  555667888
    6.00       -0.  012244
   11.00        0.  00000111123
   15.00        0.  666678888999999
   13.00        1.  0111223333344
   12.00        1.  555567788999
   15.00        2.  000112222233344
    6.00        2.  666678
    6.00        3.  001344
     .00        3.
    3.00        4.  011
    2.00        4.  67
    1.00        5.  3
    2.00 Extremes    (>=.060)


 Stem width:   .0100000
 Each leaf:       1 case(s)
```
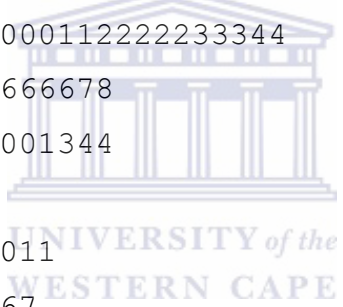
Figure 2.2 ALBI (CL) Stem-and-Leaf Plot

2.2.6.1.3 Box plot

The box plot provides a summary of many aspects of the distribution. It is based on a 5-number summary (minimum, first quartile, median, third quartile, maximum) of the data. It helps to detect and illustrate the location and variation of changes between different

groups of data. An example of a box plot is given by Figure 2.3 in which the distribution is not normal.
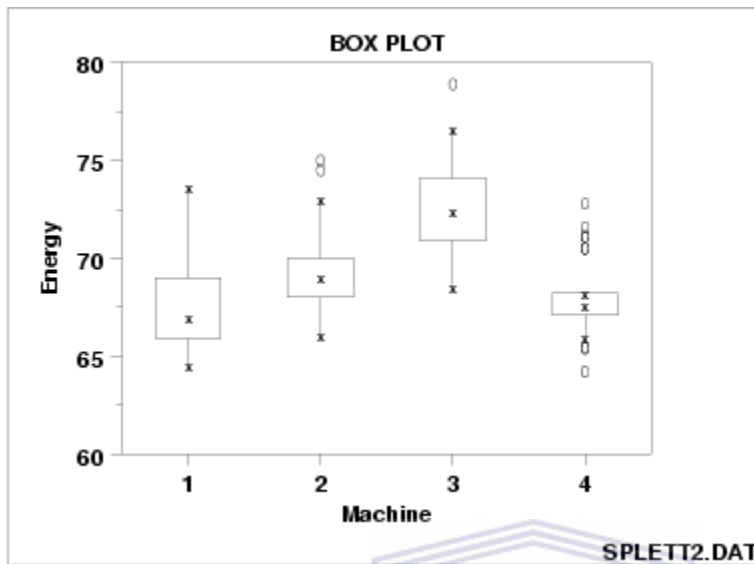
Figure 2.3: The box plot of a non-normal distribution. Source [38]

2.2.6.1.4 Normal quantile plot (Q-Q Plot)

This method inspects how a population distribution appears to differ from the normal distribution. Normal Q-Q plot shows the quantiles of a variable distribution against the quantiles of the normal distribution.

For values sampled from normal distribution, the points of a Normal Q-Q plot appear on or near the straight line drawn through the middle half of the point. The scattered points that fall away from the line are suspected outliers that may cause the sample to fail a normality test. An example of a normal quantile plot is shown in Figure 2.4. The data points do not follow a straight line and therefore the data is not normally distributed.
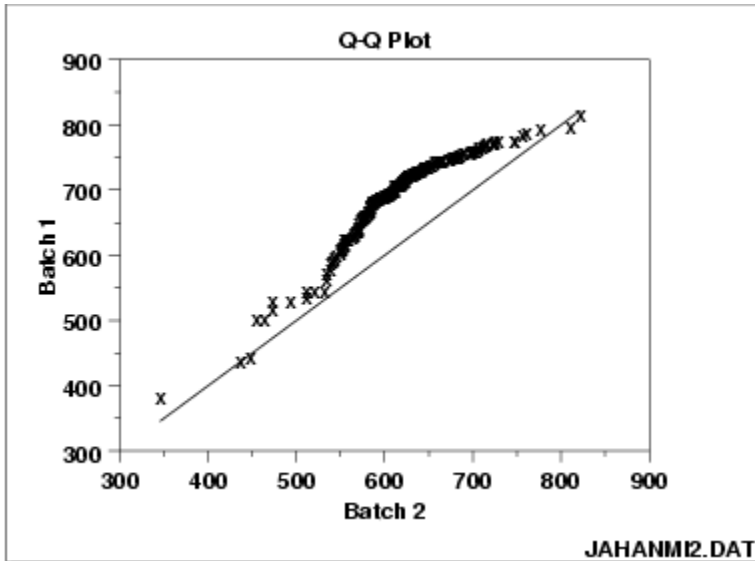
Figure 2.4: A normal quantile from a non-normal distribution. Source [42]

2.2.6.1.5 Normal probability plot (P-P Plot)

The normal probability plot is a graphical method that is used to assess whether data follow a given distribution [39].

The data is plotted against the theoretical distribution and the result should approximately be a straight line for the assumption for the given distribution to hold. The departure from the obtained straight line is a sign of the departure for the given distribution [40].

The method also plots observed cumulative probabilities of occurrence of the standardized residuals on the Y axis and of the expected normal probabilities of occurrence on the X axis. This results in a 45-degree line appearing when the observed errors conforms to the normally expected errors and the assumptions of normality distributed errors are met .Figure 2.5 shows an example of a normal probability plot that shows that the data comes from a normal distribution.

29
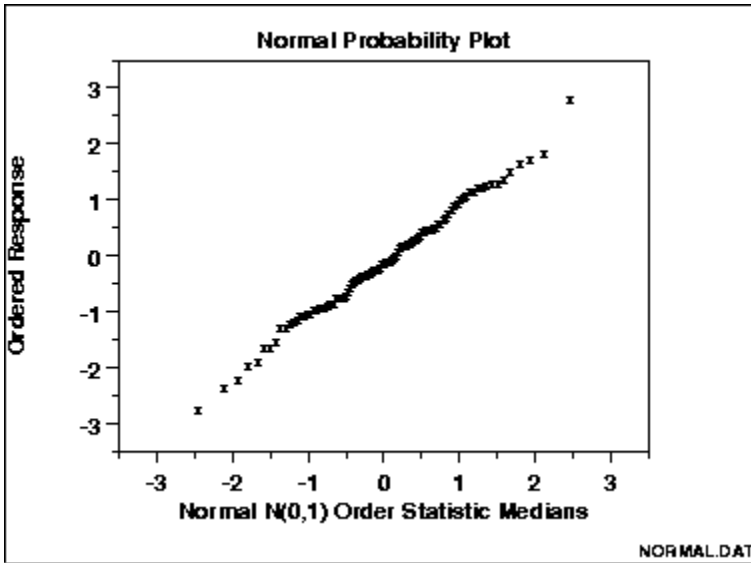
Figure 2.5: The normal probability plots of data coming from a normal distribution. Source [37].

2.2.6.2 Test methods

2.2.6.2.1 Kolmorgorov-Simirinov statistic

This test is used to check if the sample originates from a hypothesized distribution and is based on the empirical cumulative distribution function (ECDF).

Assuming that $x_1, \ldots, x_n$ is a random sample from some continuous distribution with CDF F(x). The empirical CDF is denoted by

$$F_n(x) = \frac{1}{n} *[\text{number of observation} \leq x]$$ ----------------------------eq.10

The Kolmorgorov-Simirinov statistic (D) is based on the largest vertical difference between $F_x(x)$ and $F_n(x)$ and is defined as

$$D_n = \sup_x |F_n(x) - F_x(x)|$$ --eq.11

The hypotheses to test are:

H_0 : The data follows the specified distribution (normal distribution in our case)

H_a : The data do not follow the specified distribution

H_0 is rejected at a chosen significance level (α) if the test statistic is greater than the critical value obtained from the table.

2.2.6.2.2 Lillierfors corrected Kolmogorov-Simirnov statistic

The Lillierfors corrected Kolmogorov-Simirnov statistic is used to make a comparison between the cumulative distribution of the data and the expected cumulative normal distribution. Unlike the Kolmogorov-Simirnov (K-S) test, because unknown population parameters can be estimated, while the test statistic is the same. Their test statistics differ, and therefore, the decisions to be taken must also differ.

2.2.6.2.3 Shapiro-Wilk test

The Shapiro-Wilk test depends on the correlation between data that are given and their corresponding normal scores. If the test statistic W is significant, the assumption that the distribution is normal is rejected.

The test statistic is as follows:

$$W = \frac{\{\sum a_i x_{(i)}\}^2}{\sum (x_i - \bar{x})^2}$$ --eq.12

where x_i is the ith largest order statistic, $\bar{x}$ the sample mean and n the number of observations.

2.2.6.2.4 D'Agostino-Pearson (DAP) Omnibus test

The D'Agostino-Pearson Omnibus test first analyzes the skewness and kurtosis, calculates how each of these values differs from the values expected in a normal distribution, and computes a single p-value from the sum of squares of these

discrepancies. DAP is a combination of the D'Agostino skewness test and Anscombe-Glynn kurtosis test.

The test statistic s given by:

$$K^2 = Z^2(\sqrt{b_1}) + Z^2(\sqrt{b_2})$$ ---eq.13

where $Z^2(\sqrt{b_1})$ and $Z^2(\sqrt{b_2})$ are the standard normal deviates equivalent to observing

$\sqrt{b_1}$ (Skewness) and $\sqrt{b_2}$ (kurtosis).

The K^2 statistic has approximately the chi-squared distribution with 2 degrees of freedom when the population is normally distributed.

2.2.6.2.5 Jarqua-Bera (JB) test

This test depends on the skewness and kurtosis statistics.

The test statistics is given by:

$$T = n\left(\frac{(\sqrt{b_1})^2}{6} + \frac{(b_2 - 3)^2}{24}\right)$$ ---eq.14

The test statistic has approximately a chi-square distribution with 2 degrees of freedom.

If the test statistic equals zero, then it means that the distribution has zero skewness and 3 kurtosis, and therefore the conclusion that the assumption of normality holds becomes valid.

2.2.6.2.6 Anderson-Darling test

The Anderson-Darling test was named after Theodore Wilbur Anderson [90] and Donald A. Darling [68, 91]. It is used with a small sample size $n \leq 8$, because large samples may reject the assumption of normality with slight imperfections.

The test assesses whether the sample comes from a specified distribution.

The formula for the test statistic A to assess if data $\{Y_1 < Y_2 < \ldots < Y_N\}$ (the data must be ordered first) originates from a distribution with cumulative distribution function (CDF) F is

$$A^2 = -N - S$$ ---eq.15

where $S = \sum_{k=1}^{N} \frac{2k-1}{N}\left[\ln F(Y_k) + \ln(1 - F(Y_{N+1-k}))\right]$ ---------------------------------eq.16

The test statistic can be compared to the critical values of the theoretical distribution dependent on what F is used, to determine the P-values. The Anderson-Darling test for normality is a distance or Empirical Distribution function (EDF) test. It is based upon the concept that when given a hypothesized underlying distribution, the data can be transformed to a uniform distribution. The transformed sample data can be then tested for uniformity with a distance test.
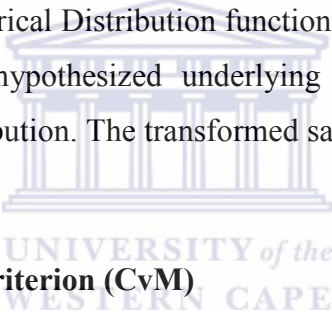
2.2.6.2.7 Cramer-von-Mises criterion (CvM)

CvM tests the goodness of fit of a probability distribution F^* that is compared to a given distribution F.

It is given by the formula:

$$nW^2 = n\int_{-\infty}^{\infty} \left[F(x) - F^*(x)\right]^2 dF(x)$$ -------------------------------------eq.17

This test has two applications; on either one sample or on two samples.

For the one sample, the observed values x_1, …, x_n increase in order. It can then be demonstrated that:

$$T = nW^2 = \frac{1}{12n} + \sum_{i=1}^{n} \left[\frac{2i-1}{2n} - F(x_i) \right]^2 \qquad \text{-------------------------eq.18}$$

If this value exceeds the tabulated value, then the hypothesis that the data comes from the distribution F (.) is rejected.

For the two samples, consider the observed values $x_1, \ldots, x_n$ and $y_1, \ldots, y_m$

that are increasing in order in the first and the second sample respectively.

Suppose also that $r_1, \ldots, r_n$ are the ranks of the x's in the combined sample and $s_1, \ldots, s_m$ are the ranks of the y's in the combined sample. Then, it can be proved that

$$T = nW^2 = \frac{U}{nm(n+m)} - \frac{4mn-1}{6(m+n)} \qquad \text{-----------------------------------eq.19}$$

where U is given by:

$$U = n \sum_{i=1}^{n} (r_i - i)^2 + m \sum_{j=1}^{m} (s_j - j)^2 \qquad \text{-----------------------------eq.20}$$

If the value of t exceeds the tabulated value, then the hypothesis that the two samples originate from the same distribution is rejected.

2.2.6.2.8 Pearson's chi-square test

This test is one of the several chi-square tests- statistical procedures that use a chi-square distribution to reference its results. It is used to test the null hypothesis that the frequency distribution of certain events observed in a sample is consistent with a particular theoretical distribution. The events that are taken into consideration have to be mutually exclusive and have the probabilities that sum up to one. The Pearson's chi-square test is also used to test the goodness of fit (whether or not the frequency distribution differs from the theoretical distribution), and the independence to assess whether paired

observations on two variables, expressed in a contingency table are independent of each other.

The chi-square statistic is given by:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$ --eq.21

Where χ^2 stands for chi-square;

    O_i = observed frequency;

    E_i = expected (theoretical) frequency that is assessed by the null hypothesis.

    n = number of possible outcomes for each event.

2.2.7 Stationarity

Stationarity and time-varying volatility are the most crucial characteristic of financial time series data. Therefore, these two properties have to be taken into consideration whenever time series data are being analyzed.

This section discusses in brief the concept of stationarity and nonstationarity and some approaches that are used to test them [10, 16, 44].

A time series is said to be stationary if its mean and variance are constant over time and the value of the covariance between the two time periods depends only on the distance or gap or lag between the two time periods and not the actual time at which the covariance is computed. Such series is in general called weakly stationary or covariance stationary.

As an illustration, consider a time series Y_t with mean $\mu = E[Y_t]$,

variance $\sigma^2 = Var(Y_t) = E[Y_t - \mu]^2$, and covariance $\gamma_k = E[(Y_t - \mu)(Y_{t+k} - \mu)]$, where γ_k is the covariance or autocovariance at lag k between Y_t and Y_{t+k}.

35

In the case k = 0,

$$\gamma_0 = E\left[(Y_t - \mu)(Y_t - \mu)\right] = E\left[Y_t - \mu\right]^2 = \sigma^2$$

In case k=1,

γ_1 becomes the covariance between two adjacent values of Y.

If Y_t is shifted to Y_{t+n}, then for Y_t to be stationary, its mean, variance, and covariance have to be the same as those of Y_{t+n}, that is, they have to be constant over all lags. The type of time series that tends to its mean is called mean reversion and fluctuations about it will always have constant amplitude.

There is a special type of time series called purely random or white noise process, which has a zero mean, a constant variance and no serial correlation (or autocorrelation).

A time series that is not stationary is referred to as a nonstationary time series. Its mean or its variance changes with time, or both the mean and the standard deviation vary with time. Nonstationarity also means that a variable has a tendency to return to a constant value or linear trend [44]

The Random Walk model is an example of a nonstationary time series. Also nonstationary is synonymous to random walk.

There are two types of random walks: a random walk without drift and a random walk with drift.

(1) A random walk without drift (no constant or intercept term).

Its form is as follows:

$$\boxed{Y_t = Y_{t-1} + \mu_t}$$ --eq.22

The equation means that the value of Y at time t equals its time value at time t-1 plus a random shock μ_t.

By decomposition,

$$Y_1 = Y_0 + \mu_1$$

$$Y_2 = Y_1 + \mu_2 = Y_0 + \mu_1 + \mu_2$$

.

.

$$Y_t = = Y_0 + \sum \mu_t$$

The mean and the variance in this case will be respectively:

$$E\,[Y_t] = Y_0,$$

$$Var\,(Y_t) = t\sigma^2$$

That is, the mean of Y equals its starting or initial value Y_0 which is a constant, but its variance grows with time, thus violating the stationarity condition. Y_0 is always set to zero, which makes the mean to be zero.

(2) A random walk with drift

Its form is as follows:

$$Y_t = \delta + Y_{t-1} + \mu_t \quad \text{--eq.23}$$

where δ is a drift parameter.

The name drift originates from the fact that $\Delta Y_t = Y_t - Y_{t-1} = \delta + \mu_t$, which means that Y_t can drift upward or downward depending on the value of δ which can be negative or positive.

The mean and the standard deviation will be respectively:

$$E[Y_t] = Y_0 + t * \delta,$$

$$Var(Y_t) = t\sigma^2.$$

Thus, both the mean and the variance vary with time, which also violates the stationarity condition.

The random walk is also an example of a unit root process.

As an illustration, consider a random walk model Y_t such that

$$Y_t = \rho Y_{t-1} + \mu_t, \ -1 <= \rho <= 1$$

For $\rho = 1$, $Y_t = Y_{t-1} + \mu_t$, is a random walk with drift.

This case is called the unit root problem, meaning also the situation of nonstationarity.

Therefore, the terms nonstationarity, unit root, and random walk mean the same thing.

In case $|\rho| < 1$, Y_t becomes stationary.

2.2.7.1 Some approaches to test for stationarity

2.2.7.1.1 The graphical analysis

In this case, time series data is first plotted. If it shows for example an upward trend, then it is a sign that the mean is changing, and therefore it is not stationary [10]. If it shows a time mean reverting, then it is an indication of the same uncertainty in the price

a day in the future and in a month in the future. It can also be a realization of a random walk [41].

An example of time series prices plots is given in Figure 2.6.

The first part of the figure gives the idea of a mean reverting, the second the random walk and the last the trend.
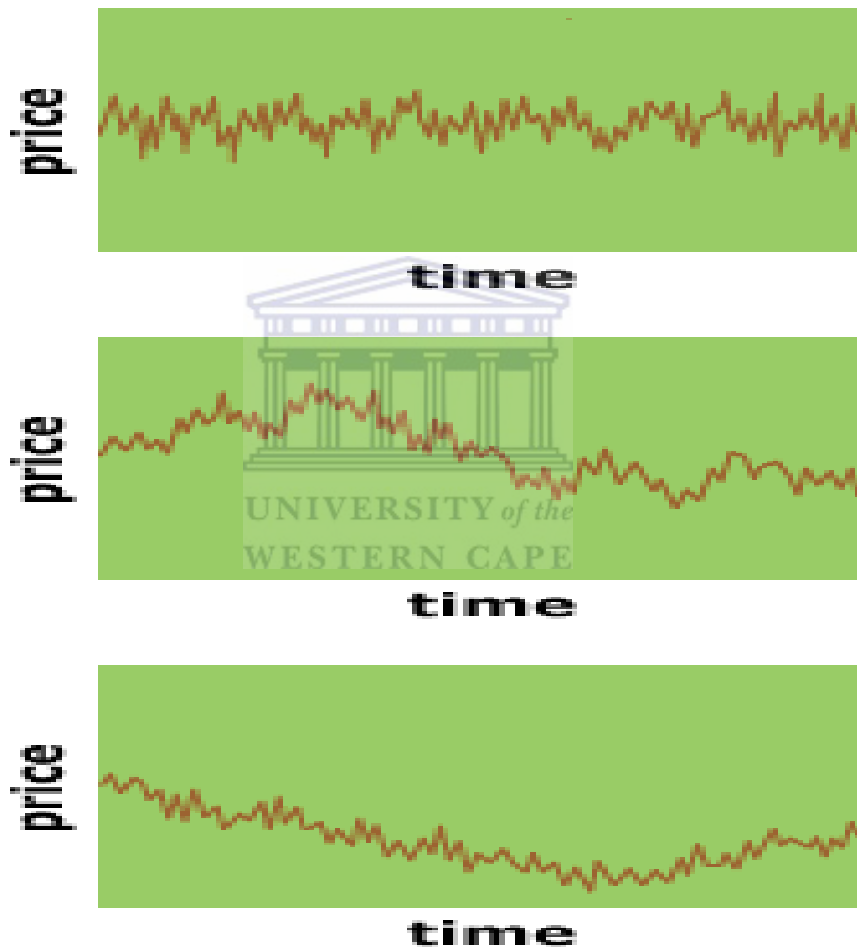

Figure 2.6: Time series prices plots. The mean reverting, the random walk and the trend. Source [41]

2.2.7.1.2 Autocorrelation function (ACF) and correlogram

The ACF is defined as:

$$\rho_k = \frac{\gamma_k}{\gamma_0} = \text{cov at lag k / variance}$$ --eq.24

where the covariance and variance are as previously defined.

If k = 0, then $\rho_0 = 1$.

The graph of ρ_k against k (lags) is referred to as a population correlogram.

The sample autocorrelation function is also computed as:

$$\hat{\rho}_k = \frac{\hat{\gamma}_k}{\hat{\gamma}_0}$$ ---eq.25

where $\hat{\gamma}_k$ is the covariance given by

$$\hat{\gamma}_k = \frac{\sum (Y_t - \bar{Y})(Y_{t+k} - \bar{Y})}{n};$$

$$\hat{\gamma}_0 = \frac{\sum (Y_t - \bar{Y})^2}{n};$$

$\bar{Y}$ is the mean, and n is the sample size.

A graph of $\hat{\rho}_k$ versus k is called the sample correlogram.

If the autocorrelation coefficient starts at a higher value at lag 1 and declines slowly, it is an indication that the time series is nonstationary, indicating a change in mean or variance, or in both.

If the autocorrelations at various lags prowl about zero on a diagram, then it is an indication of stationarity.

The significance of the autocorrelation coefficients is discussed by Box and Ljung. According to them, the joint hypothesis test that all the correlations (ρ_j) up to certain lags are simultaneously equal to zero is done, instead of testing the statistical significance of any individual autocorrelation. The test statistic is defined as:

$$Q_{LB} = n\,(n+2)\sum_{j=1}^{h}\frac{\rho^2(j)}{n-j}$$ ---eq.26

Where n is the sample size, $\rho(j)$ is the autocorrelation at lag j, and h is the lag length.

If the computed Q_{LB} exceeds the Q value from the chi-square distribution at the chosen level of significance, the null hypothesis that all (ρ_j) are zero is rejected; at least some of them must be nonzero.
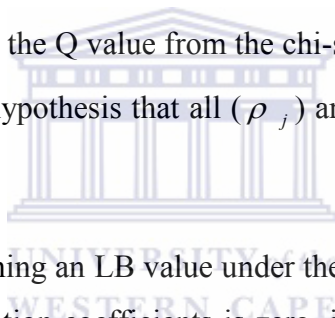
Also if the probability of obtaining an LB value under the null hypothesis that the sum of j squared estimated autocorrelation coefficients is zero, is practically zero; then one can conclude that the given time series is nonstationary.

Nonstationarity can also be tested by a correlogram of a time series, which is a graph of autocorrelation at various lags. The correlogram dies or diminishes gradually for nonstationary time series [10].

2.2.7.1.3 The unit root test

Another powerful suggested method to test for stationarity is the unit root test [10, 44, 83].

Before looking at the test itself, we first explain the concept.

If $Y_t = \rho\, Y_{t-1} + \mu_t$, $-1 <= \rho <= 1$, where μ_t is a white noise error term, for k = 1, this corresponds to a unit root case or random walk without drift, which is nonstationary.

By taking the first difference, we have:

$$\Delta Y_t = Y_t - Y_{t-1} = \rho\, Y_t - Y_{t-1} + \mu_t$$

$$= (\rho - 1)\, Y_{t-1} + \mu_t$$

$$= \delta\, Y_{t-1} + \mu_t \text{ , where } \delta = \rho - 1 \text{, and } \Delta \text{ is the first difference.}$$

The null hypothesis to be tested is $H_0 : \delta = 0$.

If $\rho = 1$, then we have a unit root, meaning that the time series is nonstationary.

In that case, we have $\Delta Y_t = |Y_t - Y_{t-1}| = \mu_t$, which is a stationary white noise.

One of the approaches to test for unit rule is the Dickey -Fuller (DF) and the Augmented Dickey- Fuller tests [10, 47, 48, 49, 50]. The assumptions underlying these approaches are as follows: the error term is uncorrelated and the error term is correlated.
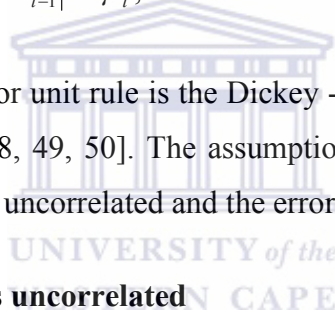
(1)Assuming the error term is uncorrelated

The approach is discussed under three different hypotheses:

a) The case where Y_t is a random walk. That is,

$$\Delta Y_t = \delta - Y_{t-1} = \delta\, Y_t - Y_{t-1} + \mu_t \text{ as previously seen.}$$

b) The case where Y_t is a random walk with drift. That is,

$$\Delta Y_t = Y_t - Y_{t-1} = \beta_1 + \delta\, Y_t - Y_{t-1} + \mu_t$$

c) The Case where Y_t is a random walk with drift around a stochastic trend:

That is,

$$\Delta Y_t = Y_t - Y_{t-1} = \beta_1 + \beta_2 * t + \delta Y_t - Y_{t-1} + \mu_t$$, where t denotes a

time or trend variable.

The hypotheses to test are:

H_0: $\delta = 0$, that is, there is a unit root, or the time series is nonstationary.

H_1: $\delta < 0$, that is, the time series stationary.

The value of the test statistic:

$$DF_r = \frac{\hat{\gamma}}{SE(\hat{\gamma})}$$ --eq.27

 is computed and compared to value of the critical for the test (DF or ADF) If the test statistic is greater (in absolute value) than the critical value, then the null hypothesis is rejected and no unit root is present in the series.
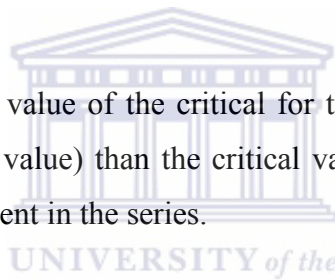
If the null hypothesis H_0 is rejected, then the time series is stationary with the following characteristics:

Mean $\mu = 0$ in case of a)

Mean $\mu = \dfrac{\beta_1}{1-\rho}$ for the case of b) above.

Y_t is a stationary around a deterministic trend, or predictable trend.

(2)Assuming the error term is correlated

With his approach, lagged values of the dependent variable ΔY_t are added to equations in a), b), and c) above.

As an example, if we consider the equation in c), the Augmented Dickey-Fuller test (ADF) has a regression form such as:

$$\Delta Y_t = \beta_1 + \beta_2 * t + \delta Y_{t-1} + \sum_{i=1}^{m} \beta_i \Delta Y_{t-i} + \varepsilon_t$$

where ε_t is a pure white noise error term.

Also

$$\Delta Y_{t-1} = Y_{t-1} - Y_{t-2}$$

$$\Delta Y_{t-2} = Y_{t-2} - Y_{t-3}, \text{ etc.}$$

The hypotheses to test for DF and ADF statistics are the same, as well as the critical values that they both use.

2.2.8 Volatility

The concept of volatility is discussed in [9, 10, 19, 20, 41]

Volatility is referred to as a measure of frequency and size of fluctuation in the price of a share. It is an annualized standard deviation of daily percentage changes in stock prices. Volatility plays a major role in the value of the options. Nowadays, investors tend to make decisions on the basis of how stock prices move.

If they believe that the stock prices will go up, they buy the stock. If they believe that the price will fall, they sell the stock or avoid buying it. Their success always depends on the decisions they make depending on how the stock prices behave [13]

In mathematical term, the volatility can be expressed as:

$$\boxed{\text{Volatility} = \text{std} \left(\log \left(Q_t / Q_{t-1} \right) \right)} \text{ ---eq.28}$$

where Q_t Q_{t-1},…, are stochastic processes, which in terms may represent prices, accumulated values, exchange rates, interest rates and so on, and std denotes the standard deviation of the time t return, and log stands for a natural logarithm. Simple returns may also be used, especially in the context of portfolio theory.

The above definition (expression) is more precise if one assumes that returns are conditionally heteroscedastic, that is, if the volatility is stationary. In this case returns are conditionally heteroscedastic, the volatility represents the standard deviation of the time t log return conditional n the information available at time t-1.

The expression may be changed to:

$$\boxed{\text{Volatility} = \text{std}_{t-1} \ (\log \ (Q_t / Q_{t-1}))}$$ --eq.29

When fluctuations in a stochastic process from one time to another are correlated, there is no relationship between, say daily volatility and weekly volatility, monthly volatility, etc.

However, if those fluctuations are independent; the so-called square root of time rule is introduced. This concept says that the volatility grows with the square root of unit time.

Prices that follow a Brownian motion, a random walk, and a geometric Brownian motion satisfy the independence condition, and therefore, their volatilities increase with the square root of time.

This rule is precise if volatilities are calculated on the log returns basis.

As previously seen, the geometric returns can be decomposed into multiple periods.

If for instance we consider two-month period returns, then we will have the following:

$$\boxed{R_{t,2} = \ln \ (P_t / P_{t-2} = \ln \ (P_t / P_{t-1}) + \ln \ (P_{t-1} / P_{t-2}) = R_{t-1} + R_t}$$ --------------------eq.30

It is also known that

$$E[X_1 + X_2 \ ] = E[X_1] + E[X_2]$$

and

$$\text{Var } (X_1 + X_2) = \text{Var}(X_1) + \text{Var}(X_2) + 2\text{Cov}(X_1, X_2)$$

If price follows a random walk (uncorrelated) as it is commonly believed, then

$$\text{Cov } (X_1, X_2) = 0.$$

Assuming the returns are identically distributed over time,

i.e.,

$$E[R_{t-1}] = E[R_t] = E[R]$$

and

$$\text{Var } (R_{t-1}) = \text{Var}(R_t) = \text{Var } (R),$$

we have

$$E[R_{t,2}] = E[R_{t-1}] + E[R_t] = 2E[R]$$

and

$$\text{Var}(R_{t,2}) = \text{Var}(R_{t-1}) + \text{Var}(R_t) = 2\text{Var}(R)$$

Thus, we see that the expected return over two days is twice the expected return. So is the estimated variance. Therefore, it is generalized that the variance and the expected return have a linear growth over time. In contrast, the volatility grows with the square root of time. As a summary, if one has to go from annual to daily, monthly or quarterly data, we will have the following:

$$\mu = \mu_{annual} T$$

$$\sigma = \sigma_{annual} \sqrt{T}$$

where T stands for the number of years

It can be seen for instance that 1/12 is used for monthly data because we have 12 months in a year, 1/252 if it is daily data assuming there are 252 trading days in a year.

The table 2.1 illustrates the change in means and volatility over various horizons, assuming the volatility (growing with the square root of time) is 13.5% and the mean (growing with time) is 12.2%.

HORIZON	YEARS (T)	MEAN (μ)	VOLATILITY (σ)
Annual	1	12.2%	13.5%
Quarterly	0.25000	3.0500%	6.75000%
Monthly	0.08333	1.0166%	3.89704%
Weekly	0.01918	0.2340%	1.86964%
Daily	0.00397	0.0484%	0.85061%
Hourly	0.00050	0.0061%	0.301869%

Table 2.1: Mean and volatility over various horizons. Source, [9]

The volatility between two time periods is also expressed as follows:

$$\sigma_{X+Y} = \sqrt{\left(\sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y\right)} \text{ for x \# y}$$

$$= \sigma_x \sqrt{2(1+p)}, \text{ for x = y and } \rho \# 0$$

$$= \sigma_x \sqrt{2}, \text{ for } \rho = 0.$$

The above approach is considered when returns are not correlated across periods.

Consider the case where they are correlated:

Suppose that X_t follows a first order autoregression with shocks or error terms in returns related to shocks in previous time period.

Then $X_t = \rho X_{t-1} + \mu_t$, where μ_t is innovations, which are assumed to have the same variance as previously seen.

If the volatility changes with time, then we will have the following:

the variance over a 2-day return = Var $(X_t + X_{t-1}) = \sigma^2 + \sigma^2 + 2\rho\sigma^2$

$$= \sigma^2 (2 + 2\rho)$$

The volatility in this case will be:

$$\sigma_x = \sigma_x \sqrt{2(1+\rho)} \quad \text{for} \quad \rho \neq 0$$

If $\rho > 0$, then the value of this volatility will be higher than the one of the independent identically distributed returns, which is the case of trending markets.

Generally, if we have M periods, the variance will be expressed as:

$$\boxed{\text{Var} \left(\sum_{i=1}^{M} X_{t+i} \right) = \sigma^2 \left[M + 2(M\text{-}1)\rho + 2(M\text{-}2)\rho(2) + \ldots + 2(1) \right] \rho(M\text{-}1)} \ldots\text{eq.31}$$

where $\rho(2)$, ..., $\rho(2)$ denotes the second order autocorrelation, $\rho(M\text{-}1)$ stands for the (M-1) order autocorrelation, etc.
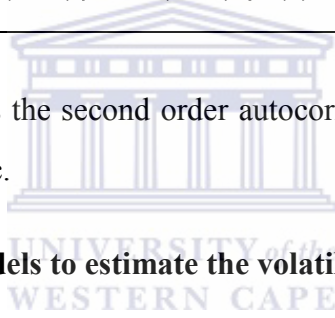
2.2.8.1 Some appropriate models to estimate the volatility

There are appropriate models to estimate the volatility. Some of them are the moving average, the Expected Weighted Moving Average (EWMA), the autoregressive conditional heteroscedacity (ARCH) and the generalized autoregressive conditional heteroscedacity (GARCH) models [5, 9, 10, 12, 89].

2.2.8.1.1 The moving average approach

Suppose that we observe returns r_t over N days. Then the volatility which is constructed from the moving average is as follows:

$$\boxed{\sigma_t^2 = (1/N) \sum_{i=1}^{N} r_{t-i}^2} \text{---eq.32}$$

In this case, the focus is on raw returns instead of returns around the mean.

The forecast is updated daily by adding information from the previous day and dropping the information from (N+1) days ago. The weights on past returns are all equal to 1/N.

2.2.8.1.2 The Expected Weighted Moving Average (EWMA) approach

Variances and means are modelled using an exponentially weighted moving average (EMW) forecast. The forecast for time t is a weighted average of the previous forecast, using weight λ, and the latest squared innovation, using weight $1 - \lambda$:

$$\boxed{h_t = \lambda\, h_{t-1} + (1 - \lambda)\, r_{t-1}^2}$$ --eq.33

The parameter λ is called the decay factor and it is supposed to be less than unity.

This model places geometrically declining weights on past observations, thus assigning greater importance to recent observations.

2.2.8.1.3 The Generalized Autoregressive Heteroscedastic (GARCH) approach

GARCH stands for the Generalized Autoregressive Heteroscedastic model. This is proposed by Engle [5] and Bollerslev [89], which assumes that the variance of returns follows a predictable process as described by [9].

The conditional variance depends not only on the latest innovation but also on the previous conditional variance.

Consider the conditional variance h_t using information up to time t-1, and r_{t-1} as the previous day's return.

The simplest such model is the GARCH (1, 1) process:

$$\boxed{H_t = \alpha_0 + \alpha_1\, r_{t-1}^2 + \beta\, h_{t-1}}$$ --eq.34

This means that the conditional variance of r at time t depends not only on the squared error term in the previous period, but also on its conditional variance in the previous time period.

This model can be generalized to GARCH (p, q) model with p lagged terms of the squared error term and q terms of the lagged conditional variances.

The average, unconditional variance is found by setting $E(r_{t-1}^2) = h_t = h_{t-1} = h$. Solving for h, we find the following:

$$h = \alpha_0 / (1 - \alpha_1 - \beta)$$ ---eq.35

This model is stationary if $\alpha_1 + \beta < 1$, otherwise it is nonstationary.

2.3 Overview of the application of the Brownian motion in stock markets.

In the past years, the application of the Brownian motion process to analyze financial time series has been under the scrutiny of empirical research. This process, as suggested earlier on, was originally used by Louis Bachelier [1] and later on, was reviewed in a quite number of pieces of research including:

- Osborne [7] who applied the Brownian motion in stocks markets and managed to show that the logs of common stock prices and the value of the money can be regarded as an ensemble of decisions in a statistical equilibrium. He found that this ensemble of logs of prices, each changing with time, is similar to the ensemble of coordinates of a large number of molecules in the Brownian motion theory.
- Black and Scholes [87] for pricing options, which was based on the statistical properties of the Brownian motions.
- Smith [8] who applied the Brownian motion theory to investigate the price controls. He analyzed the effects of price stabilization schemes on investment when the demand is uncertain, by using the method of regulated Brownian motion. The methods and conclusions he came up with in his research are applicable to any economic situation that involves smooth costs of adjustment of stocks when there is uncertainty of prices, but subject to government control.
- Etc.

CHAPTER 3: DATA DESCRIPTION

3.1 Introduction

Our data was sourced from I-Net Bridge [54]. They are daily, weekly and monthly stock prices of some of the major securities trading in the South African financial market, more specially the Rand/ US$, Rand/Euro, JSE ALSI Total Returns Index, South African All Bond Index, Anglo American Corporation, Standard Bank, Sasol, Gold Price US$, Brent Spot oil price, and South African White Maize Near Future.

The returns were calculated for each one as follows:

$$R_t = \ln (P_t + D_t / P_{t-1})$$ --eq.36

 where the R_t denotes the stock returns, P_t stands for the closing stock price on day, week or month t, and D_t denotes the dividends on day, week or month t.

3.2 EURO/US DOLLAR

The data set for the logarithmic daily returns of EURUSD was composed of 369 observations from 17 January 2007 to 23 July 2008.

The weekly data set included 399 observations from 17 December 2000 to 27 July 2008

Lastly, the monthly data set was composed of 399 observations from 31 May 1975 to 31 July 2008.

3.3 OTHER SECURITIES

Similarly, the composition and time periods of data sets for other securities returns for each frequency are found in the table 3.1.

	DAILY		WEEKLY		MONTHLY	
	No. of observations	Time period	No. of observations	Time period	No. of observations	Time period
BRSPOT	369	17/01/2007 to 23/07/2008	399	17/12/2000 to 27/07/2008	329	30/09/1975 to 31/07/2008
J2O3T	369	17/01/2007 to 23/07/2008	399	17/12/2000 to 27/07/2008	157	31/07/1995 to 31/07/2007
ALBI	369	17/01/2007 to 23/07/2008	399	17/12/2000 to 27/07/2008	115	31/01/1999 to 31/07/2008
FCRB	369	17/01/2007 to 23/07/2008	399	17/12/2000 to 27/07/2008	399	31/05/1975 to 31/07/2008
AGL	369	17/01/2007 to 23/07/2008	399	17/12/2000 to 27/07/2008	399	31/05/1975 to 31/07/2008
SBK	369	17/01/2007 to 23/07/2008	399	17/12/2000 to 27/07/2008	399	31/05/1975 to 31/07/2008
SOL	369	17/01/2007 to 23/07/2008	399	17/12/2000 to 27/07/2008	344	31/02/1979 to 31/07/2008
DGLDS	369	17/01/2007 to 23/07/2008	399	17/12/2000 to 27/07/2008	182	30/06/1993 to 31/07/2008
WMAZN	369	17/01/2007 to 23/07/2008	399	17/12/2000 to 27/07/2008	137	31/03/1997 to 31/07/2008

Table 3.1: Compositions and time periods of data sets of major securities other than the EURUSD, trading in the South African financial market.

CHAPTER 4: DATA ANALYSIS

4.1 Introduction

To assess whether each security follows a Geometric Brownian motion or not, we studied the assumptions underlying this process, namely the sationarity, the normality and the independence.

We first tested the stationarity through the sequence plots together with the Dickey-Fuller test. Secondly, we studied the normality through the graphical methods (histograms and normal quantile plots) and test method (Kolmogorov- Simirnov statistic) provided in the literature. Lastly, we studied the independence behaviour through the Box-Ljung statistic significance of autocorrelations at the significance level of a half for the first order autocorrelations only. We used the Hurst exponent in order to support our analysis when making the final decision (in assessing the Geometric Brownian motion as a model for South African financial markets). The results are provided in appendix (Sequence plots, histograms together with normal quantile plots and the Augmented Dickey-Fuller test Eviews outputs results for the three frequencies (daily, weekly and monthly), in tables 4.4, 4.5, 4.6 for Kolmogorov-Simirnov, Box-Ljung and Hurst exponent statistics respectively).

4.2 EURO/US DOLLAR

The sequence plots of the EURUSD returns reveal a kind of periodic/cyclic time series [4], with the volatility varying over time: large changes (upwards or downwards) are often being followed by large fluctuations, and small changes are tending to be followed by small fluctuations; thus visually violating the stationarity assumptions for a Geometric Brownian motion process.

The Augmented Dickey-Fuller test Eviews outputs results for the three frequencies (daily, weekly and monthly) are also shown in table 4.1 below.

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-20.14382	0.0000
Test critical values: 1% level	-3.447914	
5% level	-2.869176	
10% level	-2.570905	

*MacKinnon (1996) one-sided p-values.

Table 4.1: Eviews output results for the Augmented Dickey-Fuller test for stationarity in the daily data.

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-20.51208	0.0000
Test critical values: 1% level	-3.446567	
5% level	-2.868583	
10% level	-2.570588	

*MacKinnon (1996) one-sided p-values.

Table 4.2: Eviews output results for the Augmented Dickey-Fuller test for stationarity in the weekly data.

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-9.107412	0.0000
Test critical values: 1% level	-3.482879	
5% level	-2.884477	
10% level	-2.579080	

*MacKinnon (1996) one-sided p-values.

Table 4.3: Eviews output results for the Augmented Dickey-Fuller test for stationarity in the monthly data.

The results show that the test statistics from all the frequencies are less than the relevant critical values. Therefore, we fail to reject the null hypothesis of a unit root in the EURUSD daily, weekly and monthly returns series at any of significance levels, and conclude that there are unit roots or the EURUSD time series returns are nonstationary.

We then studied the normality assumptions through the histogram and normality plot together with the Kolmogorov-Simirnov statistic.

The graphical methods, as mentioned in the literature [66], show the shape of the distributions but do not guarantee that the data originates from a specified distribution. . That is why; we need to support them with the test methods.

In the case of the EURUSD daily time series data, the histogram shows a kind of a symmetrical distribution but the normal Q-Q plot shows some dots that are flying away from the straight line which makes us doubt about the distribution of the data.

Similarly the EURUSD weekly and monthly data plots can not allow us as well to decide on the distribution of the data since the shape of their histograms are not fully bell-shapes and their normal Q-Q plot does not as well convince us that all dots will be on or close to the straight lines.

We supported our results by the Kolmogorov-Simirnov test. Normally, the hypothesis test for the normality tests the null hypothesis that the variable is normal [66]. That is, the actual distribution of the variable fits the pattern that we would expect if it is normal. Failure to reject the null hypothesis leads to conclude that the null hypothesis is normal. In Kolmogorov-Simirnov test for normality, the null hypothesis says that the actual distribution of the variable equals the expected distribution, that is, the variable is normally distributed. The distribution of EURUSD is associated with the low (<0.05) significance value of 0.035 in the weekly data which leads us to reject the null hypothesis and conclude that the weekly EURUSD logarithmic returns are not normally distributed.

Again the distribution of EURUSD logarithmic returns are associated with the high (>0.05) significance values of 0.080 (for daily returns), and 0.200 (for weekly returns)

that lead us not to reject the null hypothesis and conclude that the normality assumption holds or daily and monthly logarithmic returns of the EURUSD are normally distributed.

Security	Daily		Weekly		Monthly	
	Statistic	Significance	Statistic	Significance	Statistic	Significance
EURUSD	0.044*	0.080	0.047	0.035	0.027*	0.200
J203T	0.049	0.031	0.64	0.001	0.071*	0.053
ALBI	0.056	0.008	0.084	0.000	0.062*	0.200
FCRB	0.051	0.021	0.055	0.005	0.034*	0.200
AGL	0.039*	0.200	0.041*	0.115	0.043*	0.079
SBK	0.062	0.002	0.044*	0.062	0.053	0.009
SOL	0.035*	0.200	0.049	0.021	0.046*	0.083
DGLDS	0.059	0.003	0.064	0.001	0.070	0.028
BRSPOT	0.057	0.006	0.064	0.000	0.076	0.000
WMAZN	0.049	0.033	0.056	0.004	0.046*	0.200

* Not significant at the level of $\alpha = 0.05$

Table 4.4: Kolmogorov- Simirnov test for normality results for daily, weekly and monthly data: SPSS output results.

To identify the independence behaviour, we studied the first order autocorrelations at each frequency (daily, weekly and monthly basis) using the Box-Ljung statistics for the significance of autocorrelation at the level of 0.05, as discussed in the literature.

The results are found in Table 4.5.

SECURITY	DAILY	WEEKLY	MONTHLY
EURUSD	-0.054*	-0.031*	0.115
J203T	-0.031*	-0.000*	0.-0.024*
ALBI	0.191	0.113	0.094*
FCRB	-0.032*	-0.009*	0.004*
AGL	0.003*	-0.067*	0.032*
SBK	-0.099*	-0.112	0.070*
SOL	0.011*	-0.112	0.028*
DGLDS	-0.034*	-0.094*	-0.035*
BRSPOT	-0.100	0.005*	0.041*
WMAZN	0.050*	0.117	0.190

 Not significant (the probability is substantially greater than 0.05)

Table 4.5: First order autocorrelations for daily, weekly, and monthly data. Results obtained using SPSS.

Our findings show that the first order autocorrelations of the EURUSD daily and weekly logarithmic returns are not statistically significant (the probability is greater than 0.05) whereas the autocorrelations for monthly returns are significant (the probability is less than 0.05). This means that for the first case, the autocorrelation is zero (series are independent) between the returns at time t and the returns at time t-1, otherwise the autocorrelation is present in the series (there is a serial correlation). Therefore, we conclude that the EURUSD daily and weekly returns are independent whereas the EURUSD monthly returns are dependent (serially correlated).

Putting everything together, we find that we can not assess whether or not the EURUSD daily and weekly and monthly logarithmic returns follow a random walk or a Geometric Brownian motion on the basis of the study of the assumptions underlying the process, since at least one of the three assumptions underlying the process is violated in the study.

To solve our problem, we studied the independence of the returns or simply the Geometric Brownian motion through the significance of the Hurst exponent values. Table

4.6 contains the results calculated manually using the Hurst exponent formula provided in the literature and the first order autocorrelations in Table 4.5.

SECURITY	DAILY	WEEKLY	MONTHLY
EURUSD	0.46	0.48	0.58
J203T	0.48	0.50	0.49
ALBI	0.63	0.58	0.56
FCRB	0.48	0.49	0.50
AGL	0.50	0.45	0.52
SBK	0.42	0.41	0.55
SOL	0.51	0.41	0.52
DGLDS	0.48	0.43	0.47
BRSPOT	0.42	0.50	0.53
WMAZN	0.54	0.58	0.58

Table 4.6 Hurst exponent values for daily, weekly, and monthly data.

Hurst exponent values of 0.46, 0.48 and 0.58 were found in the daily, weekly and monthly logarithmic returns respectively.

The first two values are close to 0.5, which is consistent with what was found other currency studies such as [23], and because no autocorrelations are present, the series are random walks or Geometric Brownian, or simply there is a short memory (non-long memory). This also means that there is independence behaviour in the returns of the EURUSD security and therefore, the data can be modeled using the Geometric Brownian motion.

The last value is far greater than 0.5 which indicates that that there is a long memory or long range correlations in the logarithmic returns of the EURUSD logarithmic returns series. In this case the Fractional Geometric Brownian motion is appropriate to model the data.

Therefore, based on Hurst exponent significance values, our analysis shows that the Geometric Brownian motion can not be rejected as a model for daily, weekly and monthly returns of the EURUSD.

4.3 Other securities

The same analysis was also done for other securities .The sequence plots, histograms (together with normality plots) and Dickey- Fuller unit root test results of the logarithmic returns of the major securities trading in the South African financial market are also found in Appendix, whereas the test results (Kolmogorov-Simirnov, Box-Ljung and Hurst exponent statistics) are contained in Tables 4.4, 4.5 and 4.6 respectively. Table 4.7 provides a summary of our findings.

SECURITY	DAILY	WEEKLY	MONTHLY
EURUSD	• Independent • Nonstationary • Normally distributed	• Independent • Nonstationary • Not normally distributed	• Not independent • Nonstationary • Normally distributed
J203T	• Independent • Nonstationary • Not normally distributed	• Independent • Nonstationary • Not normally distributed	• Independent • Nonstationary • Normally distributed
ALBI	• Not independent • Nonstationary • Not normally distributed	• Not independent • Nonstationary • Not normally distributed	• Independent • Nonstationary • Normally distributed
FCRB	• Independent • Nonstationary • Not normally distributed	• Independent • Nonstationary • Not normally distributed	• Independent • Nonstationary • Normally distributed
AGL	• Independent	• Independent	• Independent

	• Nonstationary	• Nonstationary	• Nonstationary
	• Normally distributed	• Normally distributed	• Normally distributed
SBK	• Independent	• Not independent	• Independent
	• Nonstationary	• Nonstationary	• Nonstationary
	• Not normally distributed	• Normally distributed	• Not normally distributed
SOL	• Independent	• Not independent	• Independent
	• Nonstationary	• Nonstationary	• Nonstationary
	• Normally distributed	• Not normally distributed	• Normally distributed
DGLDS	• Independent	• Independent	• Independent
	• Nonstationary	• Nonstationary	• Nonstationary
	• Not normally distributed	• Not normally distributed	• Not normally distributed
BRSPOT	• Independent	• Not independent	• Independent
	• Nonstationary	• Nonstationary	• Nonstationary
	• Not normally distributed	• Not normally distributed	• Not normally distributed
WMAZN	• Independent	• Not independent	• Independent
	• Nonstationary	• Nonstationary	• Nonstationary
	• Not normally distributed	• Not normally distributed	• Normally distributed

Table 4.7: Overall summary of findings about the assumptions underlying the Brownian motion process in the major securities trading in the South African financial markets.

From the results in the table above, we see that high frequency financial market returns tend to be more normally distributed than the lower ones, thus making it easy to make inferences about them.

South African financial markets returns were found to be nonstationary. This is consistent with what was mentioned by some other studies in other financial markets which state that financial market returns are nonstationary [2, 10, 33]. We also realize that most of the South African financial markets returns are independent, which is also consistent with what was mentioned by the same studies [2, 10, and 33].

In Table 4.6, the Hurst exponent values of 0.48, 0.50, and 0.49 were found in the daily, weekly and monthly of the J203T logarithmic returns respectively. These values are equal or close to 0.5, but because there are no autocorrelations in the series, this indicates that the series are independent or random walk or follow a Geometric Brownian motion. The Hurst exponent values of 0.63, 0.58 and 0.56 were found in the daily, weekly, and monthly ALBI returns series respectively. These values are bigger than the 0.50 and because of the presence of autocorrelations in the series; this indicates the presence of long memory or long range correlations in ALBI returns series. In this case, the fractional Brownian motion can be used to model these series. Hurst exponent values of 0.48, 0.49, and 0.50 were found in the daily, weekly and monthly of the FCRB returns respectively. These values are equal or close to 0.5, but because of the absence of autocorrelations in the series, these are independent or follow the Geometric Brownian motion. The Hurst exponent values of 0.50, 0.45, and 0.52 were found in the daily, weekly and monthly of the AGL logarithmic returns respectively. These values are equal or close to 0.5, and because no autocorrelations are found in the series, the series are independent or behave according the Geometric Brownian motion. The Hurst exponent values of 0.42 and 0.41 with negative autocorrelations were found in the daily and weekly SBK logarithmic returns respectively. These values are less than 0.5, indicating the presence of long memory (antiperisistence behaviour) in the series. Such series are modeled using the fractional Geometric Brownian motion.

Hurst exponent values of 0.51 and 0.52 were found in the daily and monthly SOL logarithmic returns respectively. These values are equal or close to 0.5 and because the series are not serially correlated, this indicates that they are random walk or follow the Geometric Brownian motion. The Hurst exponent value of 0.41 (less than 0.5) associated with the presence of negative autocorrelation was also found in the weekly Sol returns

series, which indicates the presence of long memory in the series, more specifically the antiperisistence behaviour or fractional Brownian Motion process.

Hurst exponent values of 0.48 and 0.47 which are close to 0.50 were found in the daily and weekly DGLDS logarithmic returns respectively. Because the series are not independent, the process is a random walk or Geometric Brownian motion.

The Hurst exponent value of 0.43 (< 0.50) associated with the negative autocorrelation was also found in the weekly DGLDS returns series, indicating the presence of long memory or antiperisistence behaviour. Therefore, the process follows a fractional Brownian motion.

Hurst exponent value of 0.42 (<0.50) was found in the daily BRSPOT logarithmic returns, indicating the presence of long memory or Fractional Brownian motion process.

The Hurst exponent values of 0.50, and 0.53 were found as well in the weekly and monthly BRSPOT logarithmic returns respectively. But because the series are not independent, the process is a random walk or Geometric Brownian motion.

The Hurst exponent values of 0.54, 0.58 and 0.58 (all greater than 0.50) were found in the daily, weekly, and monthly WMAZN returns series respectively. Because of the presence of autocorrelations in the series; this indicates that there is a long memory in the WMAZN returns series or the process is a fractional Brownian motion.

Therefore, based on the Hurst exponent significance values analysis, we cannot deny the presence of the long memory in the South African financial market returns, and therefore the assumption of long memory should be added to the other assumptions underlying price variations in the markets.

More generally, we find that studying the South African financial markets behaviour through the assumptions underlying the process is not an advisable method that can lead to a good conclusion in assessing whether or not the financial market follows a Geometric Brownian motion because at least one assumption among the three is violated.

This study must be supported by any other method such as the Hurst exponent significance values to strengthen the conclusion.

Generally, we find that although some assumptions underlying the Geometric Brownian motion are violated in the EURUSD logarithmic returns at some frequencies, the Geometric Brownian motion process cannot be rejected as a model in South African financial markets.

CHAPTER 5: CONCLUSION

While examining the behaviour of the South African financial markets with regard to the Geometric Brownian motion, we analysed the securities indices of the South African financial markets. We looked at the US dollar/Euro, JSE ALSI Total Returns Index, South African All Bond Index, Anglo American Corporation, Standard Bank, Sasol, Gold Price US$, Brent Spot oil price, and South African White Maize Near Future, to study the assumptions underlying the process namely the stationarity, the normality, as well as the independence.

We used both graphical and statistical methods which are appropriate for each assumption, specifically the Dickey-Fuller test (for stationarity), the Kolmogorov-Simirnov test (for normality) and the Box-Ljung statistic test (for independence).

The results have not allowed us to verify the applicability of the Geometric Brownian motion as a model of the South African financial markets, since at least one assumption among the three assumptions underlying the process was violated.

Therefore, we concluded that studying the behaviour of the South African financial markets through the assumptions underlying the Geometric Brownian process is not a proper method. An attempt must be made to extend or refine the model.

To solve our problem, we utilised the Hurst exponent, which is a tool used to test the memory in time series, and therefore helps to determine the behaviour and efficiency of the markets.

A Hurst exponent who is equal to 0.50 indicates independence behaviour of the series or a Geometric Brownian motion, whereas the Hurst exponent values different from ½ show the presence of long memory or long range dependence which is characterised by the fractional Brownian motion model.

Our findings have revealed the presence of both short and long memory in the South African financial market time series when the Hurst exponent analysis is used. They have also allowed us to classify the South African Financial markets behaviour in three categories, that is, those which follow a random walk or Geometric Brownian motion process (GBM), those which are inconclusive or mixed (both random walk and long memory are present in their series) and those which have long memory or follow a fractional Geometric Brownian motion process. The table 5.1 below gives a brief summary of our findings by showing the behaviour of each financial security grouped in its respective market category.

MARKET	BEHAVIOUR		
	Random Walk or GBM	**Inconclusive or Mixed**	**Long memory or fGBM**
Currency	-	EURUSD	-
Equity	J203T AGL	SBK SOL	-
Bond	-	-	ALBI
Commodity	FCRB	BRSPOT DGLDS	WMAZN

Table 5.1: Summary of our findings of each financial security grouped in its respective market category.

The key-findings as shown in Table 5.1 are:
- The currency market is inconclusive or mixed; that is, it is characterised by both long memory (at higher frequency) and random walk or Geometric Brownian motion (at lower frequency).
- Among the components of the equity market, the J203T and the AGL follow a random walk or a Geometric Brownian motion and therefore they are more

efficient than the other components (SBK and SOL) which are inconclusive or mixed.

- The bond market is characterised by a long memory and therefore it is inefficient.
- In commodity market, the FCRB follows a random walk or a Geometric Brownian motion and therefore, it is more efficient than other commodity market components, more specifically the BRSPOT and DGLDS which are inconclusive (both long memory and random walk are found in their logarithmic returns) and the WMAZN which is characterised by the long memory and therefore becoming inefficient.

In view of the findings in Table 5.1, we have found that the currency market is the most efficient of the South African financial markets.

In summary, the findings highlighted above led us to conclude that we cannot reject the hypothesis that both the Geometric Brownian motion and the fractional Brownian motion processes as models of the South African financial markets prices fluctuations holds. We also suggest that long memory assumption should be added to the assumptions underlying price variations in the South African financial markets in particular and in quantitative finance in general. In fact, the presence of memory in financial market does not imply an inefficient market. Indeed, it was shown that some of the most efficient markets (such as the currency market) contain some memory. This should be expected since it would be unrealistic to expect market participants, who are real life people after all, to be without memory. In fact, much of the trading in financial and commodity markets are not driven by forward looking fundamental analysis, but backward looking technical analysis which is wholly dependent on historical prices and volumes.

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APPENDIX

This section contains the sequence plots, histograms together with normal quantile plots and the Augmented Dickey-Fuller unit root test results of daily, weekly and monthly logarithmic returns of the major securities trading in the South African financial market; namely the EURO/US dollar, JSE ALSI Total Returns Index, South African All Bond Index, Anglo American Corporation, Standard Bank, Sasol, US dollar Gold Price, CRB Commodity Price, Index Brent spot oil price, and South African white maize near future. The sequence plots of these securities show a kind of periodic/cyclic time series [4] with a volatility changing with time and thus violating the stationarity assumption of a Geometric Brownian motion process.

The Augmented Dickey-Fuller unit test results reveal that the test statistics at all the frequencies (daily, weekly and monthly data) are lower than the relevant critical values, which allows us to conclude that there are unit roots in the logarithmic returns of these securities or the log returns of these securities are nonstationary.

Histograms of most of the securities show a substantial violation of normality caused generally by some extremely large values and outliers and their normal quantile plots contains dots that do not fit (on or closer) the straight line. Only the histograms and normal plots of the EURO/US dollar (for daily and monthly data), Anglo American Corporation (for daily, weekly and monthly data), Sasol (for weekly and monthly data), All Bonds Index, All, CRB Commodity Price Index, and South African white maize near future (for monthly data only) fulfill the normality assumption.

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1 DAILY DATA

1.1 EURO/US DOLLAR (EURUSD)

1.1.1 Sequence plot

1.2 Histogram and normal quantile plot of the data

Histogram

Mean =5.25E-4%
Std. Dev. =0.
0048089150%
N =369

Normal Q-Q Plot of EURUSD (CL)
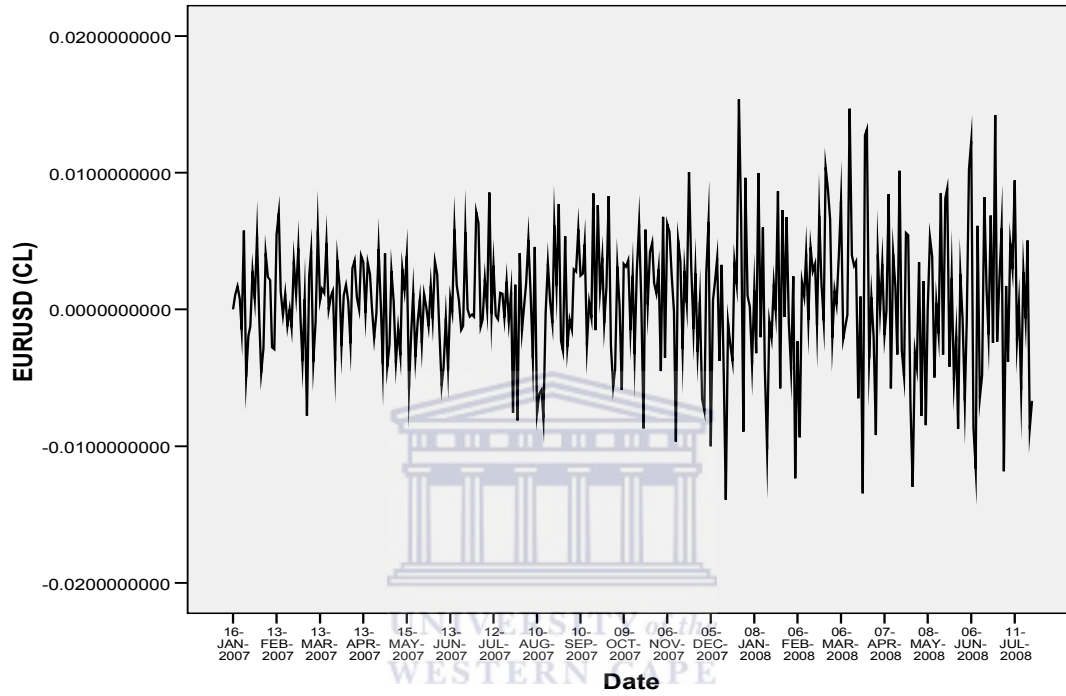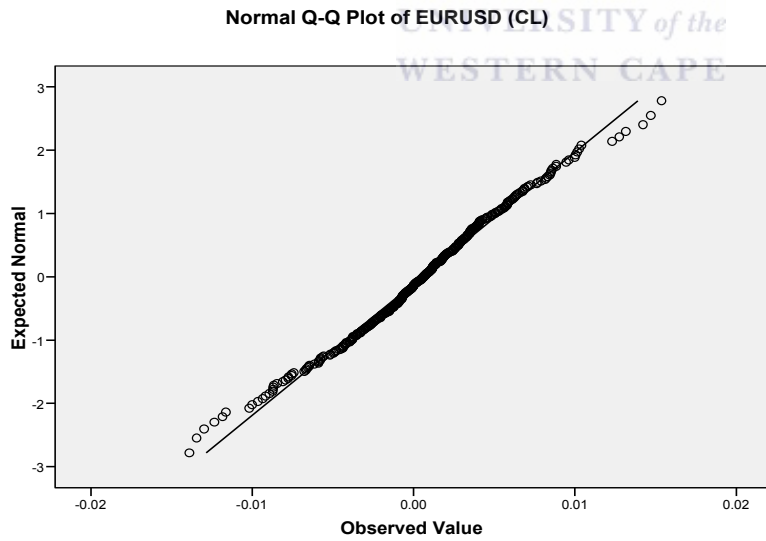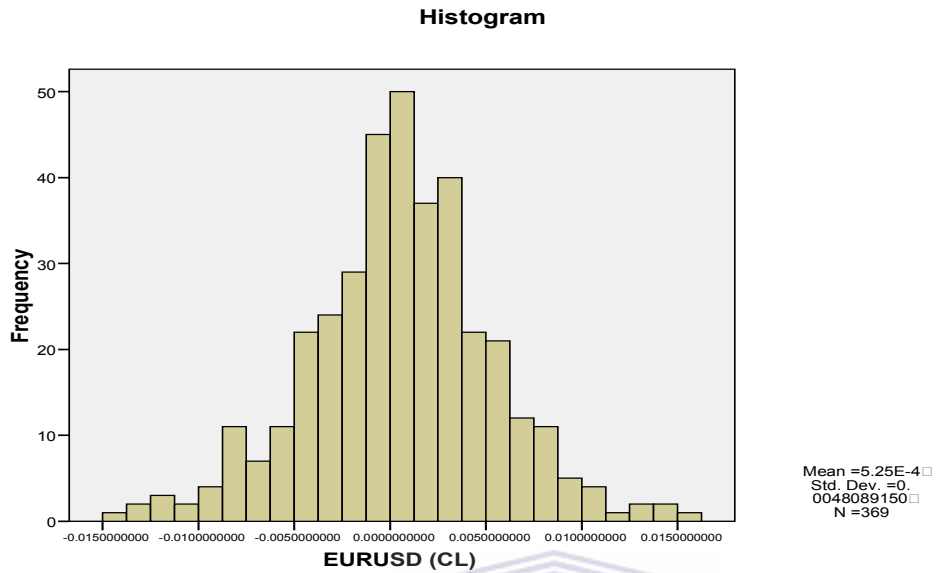
1.1.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-20.14382	0.0000
Test critical values:	1% level	-3.447914	
	5% level	-2.869176	
	10% level	-2.570905	

*MacKinnon (1996) one-sided p-values.

1.2 JSE ALL SHARE TOTAL RETURNS INDEX (J203T)
1.2.1 Sequence plot
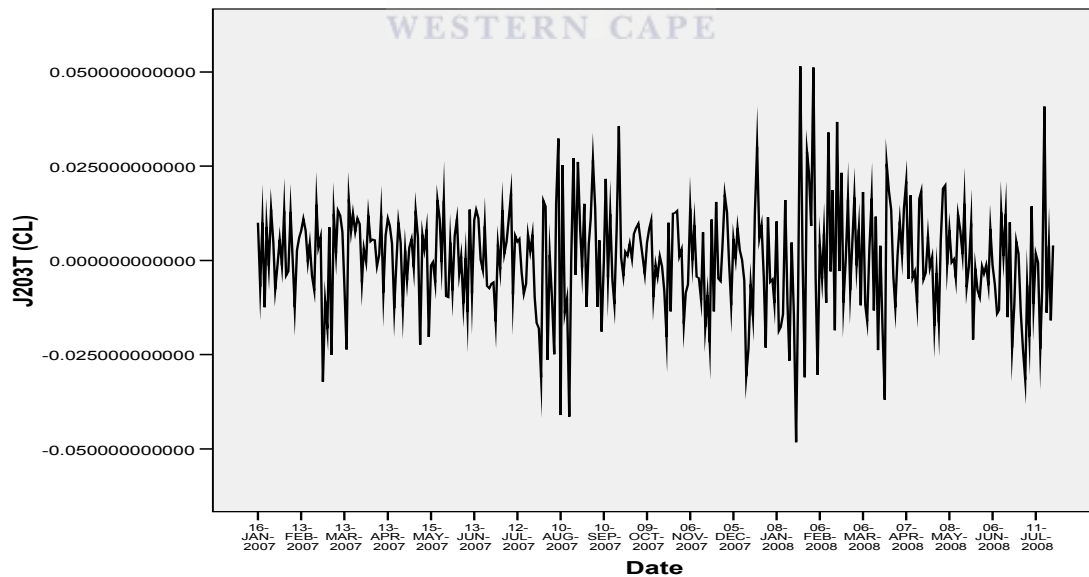
1.2.2 Histogram and normal quantile plots of the data

Histogram

Mean =3.61E-4%
Std. Dev. =0.
013844844890%
N =369
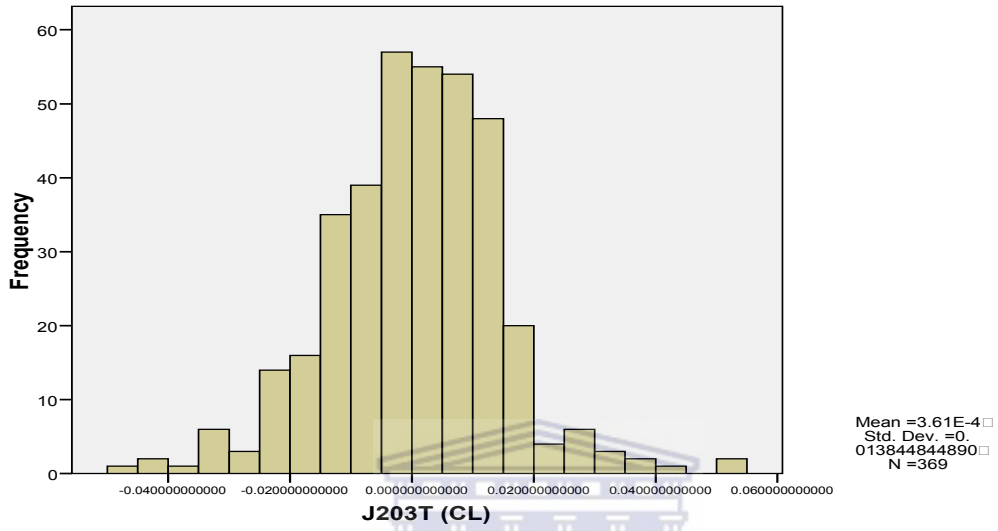
Normal Q-Q Plot of J203T (CL)
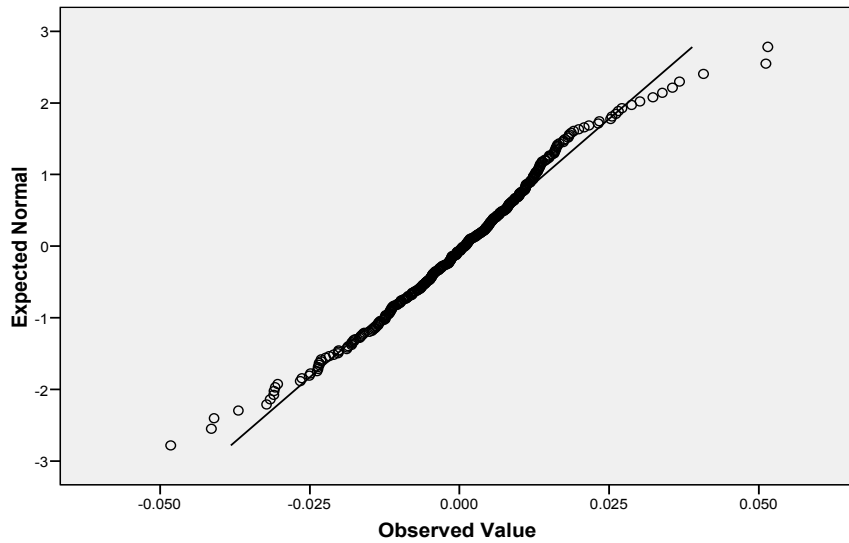
1.2.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-19.74172	0.0000
Test critical values:	1% level	-3.447914	
	5% level	-2.869176	
	10% level	-2.570905	

*MacKinnon (1996) one-sided p-values.

1.3 SOUTH AFRICAN ALL BOND INDEX (ALBI)

1.3.1 Sequence plot

1.3.2 Histogram and normal quantile plot of the data

Histogram

Mean =4.88E-5%
Std. Dev. =0.
002938703301%
N =369
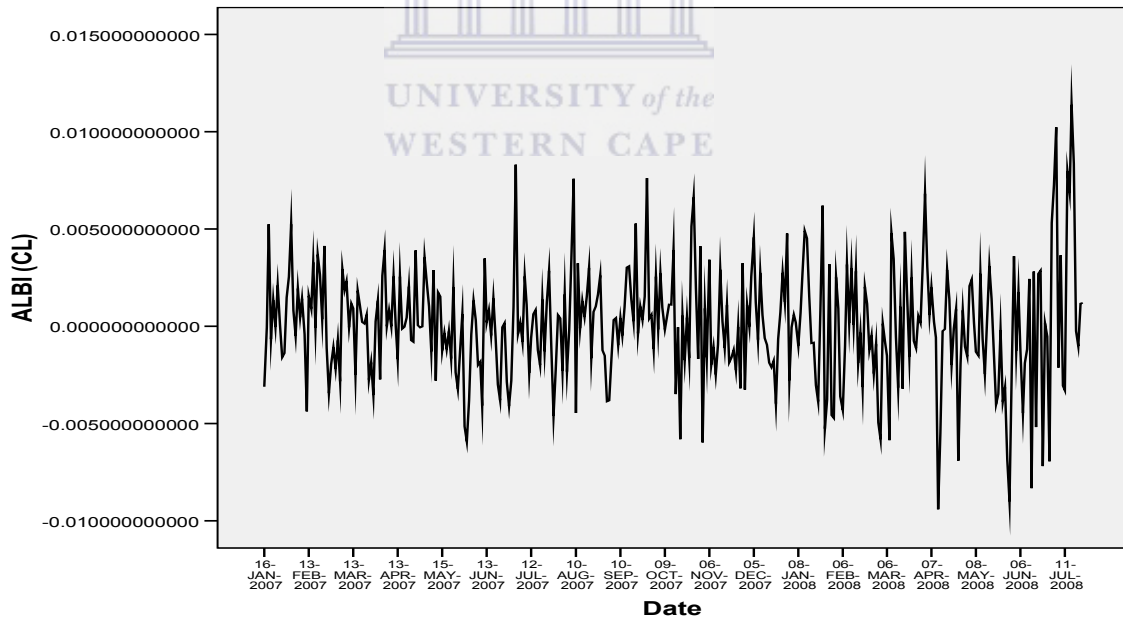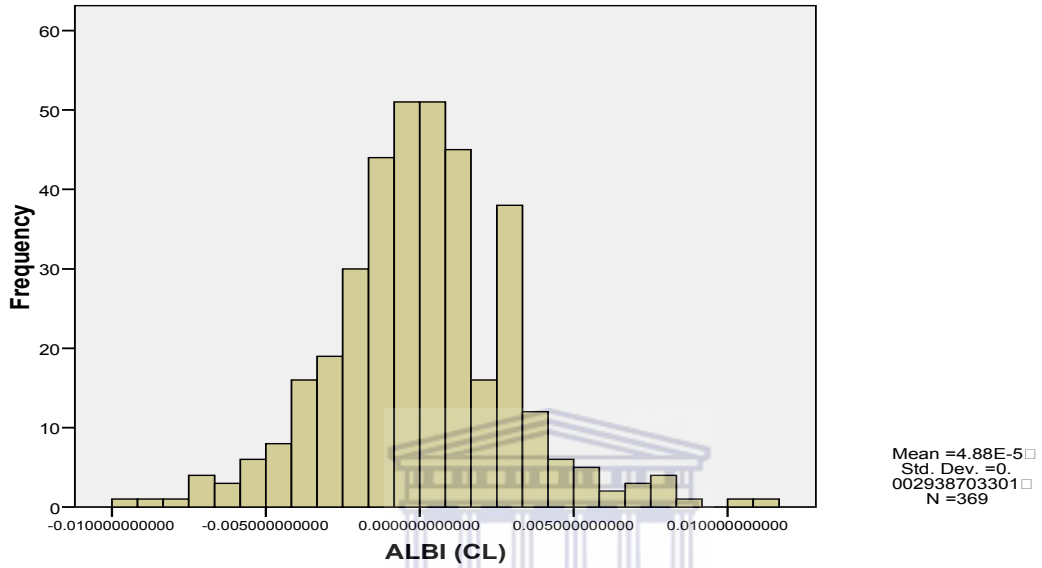
Normal Q-Q Plot of ALBI (CL)
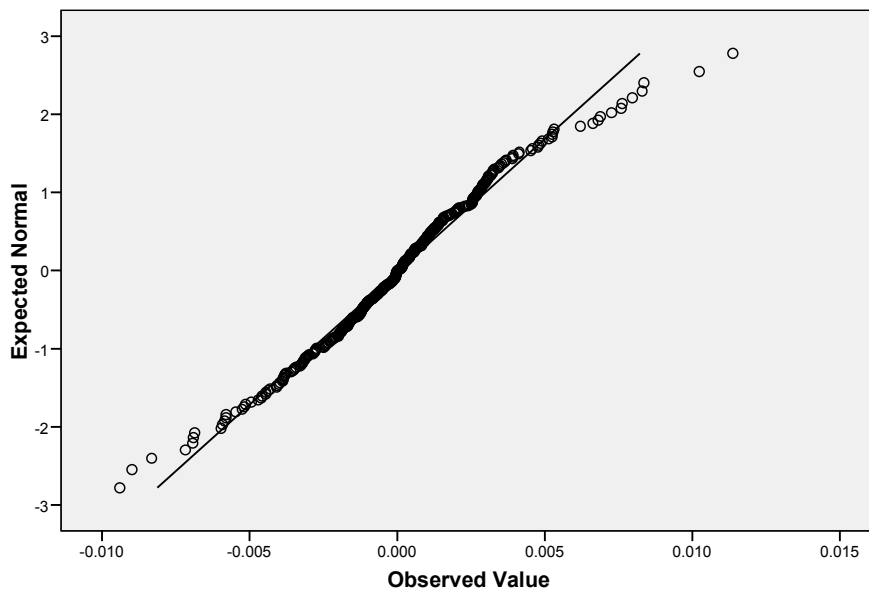
1.3.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-15.78104	0.0000
Test critical values:	1% level	-3.447914	
	5% level	-2.869176	
	10% level	-2.570905	

*MacKinnon (1996) one-sided p-values.

1.4 CRB COMMODITY PRICE INDEX (FCRB)

1.4.1 Sequence plot

1.4.2 Histogram and normal quantile plot of the data

Histogram

Mean =9.60E−4%
Std. Dev. =0.
010996231865%
N =369

Normal Q-Q Plot of FCRB (CL)
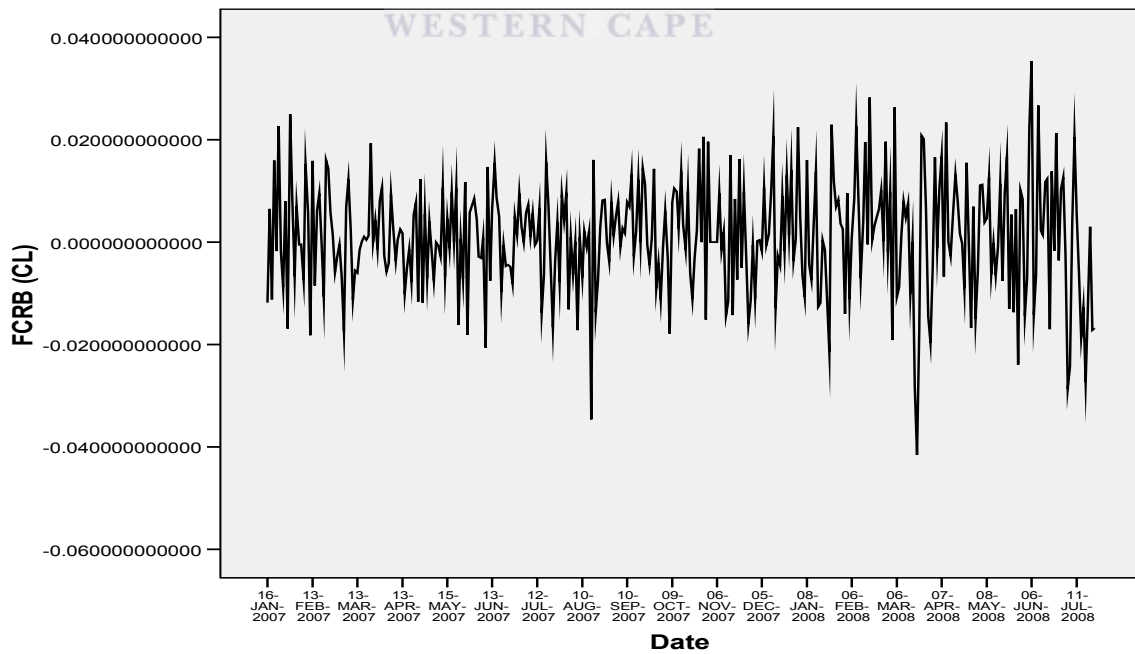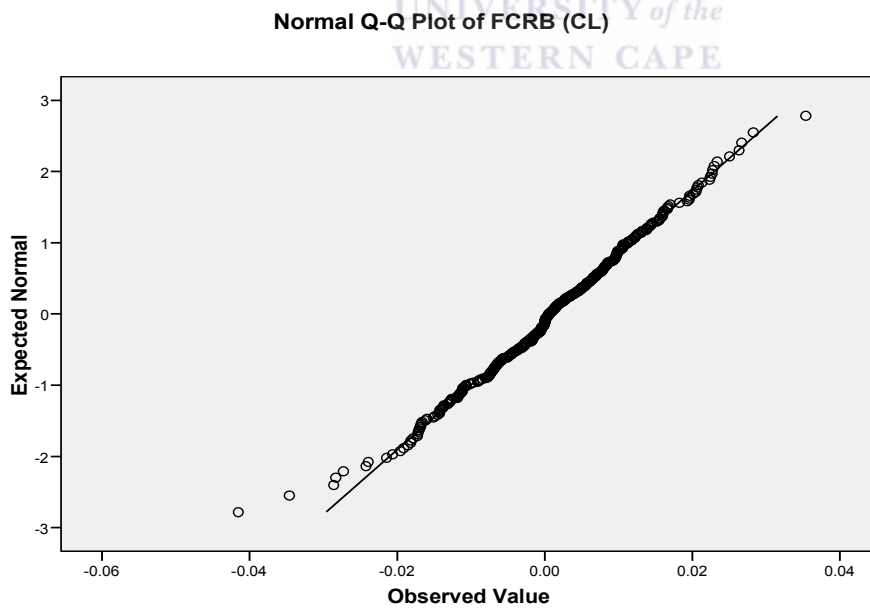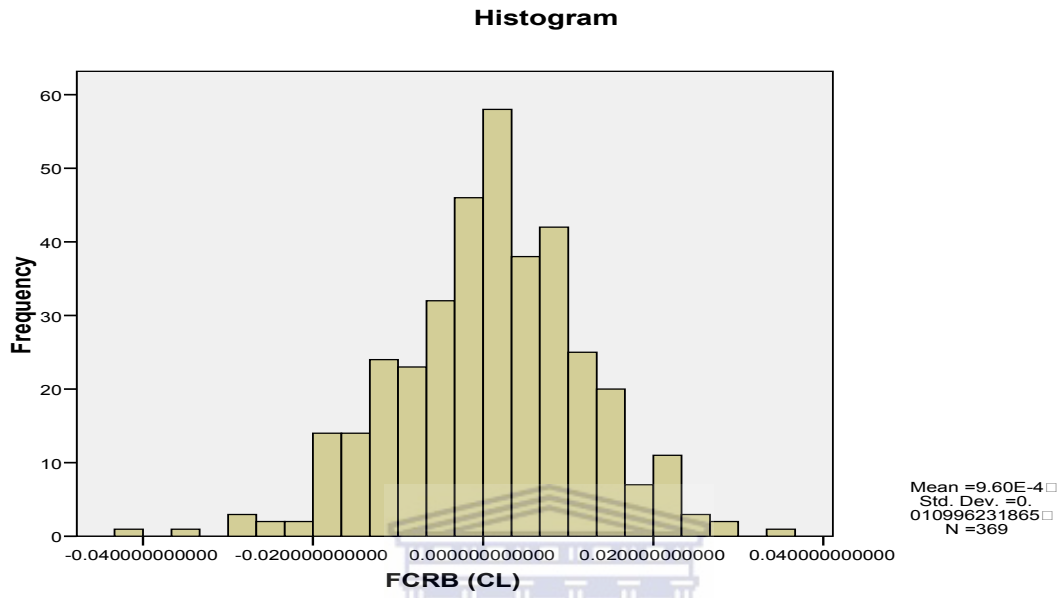
1.4.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-19.71570	0.0000
Test critical values:	1% level	-3.447914	
	5% level	-2.869176	
	10% level	-2.570905	

*MacKinnon (1996) one-sided p-values.

1.5 ANGLO AMERICAN CORPORATION (AGL)

1.5.1 Sequence plot
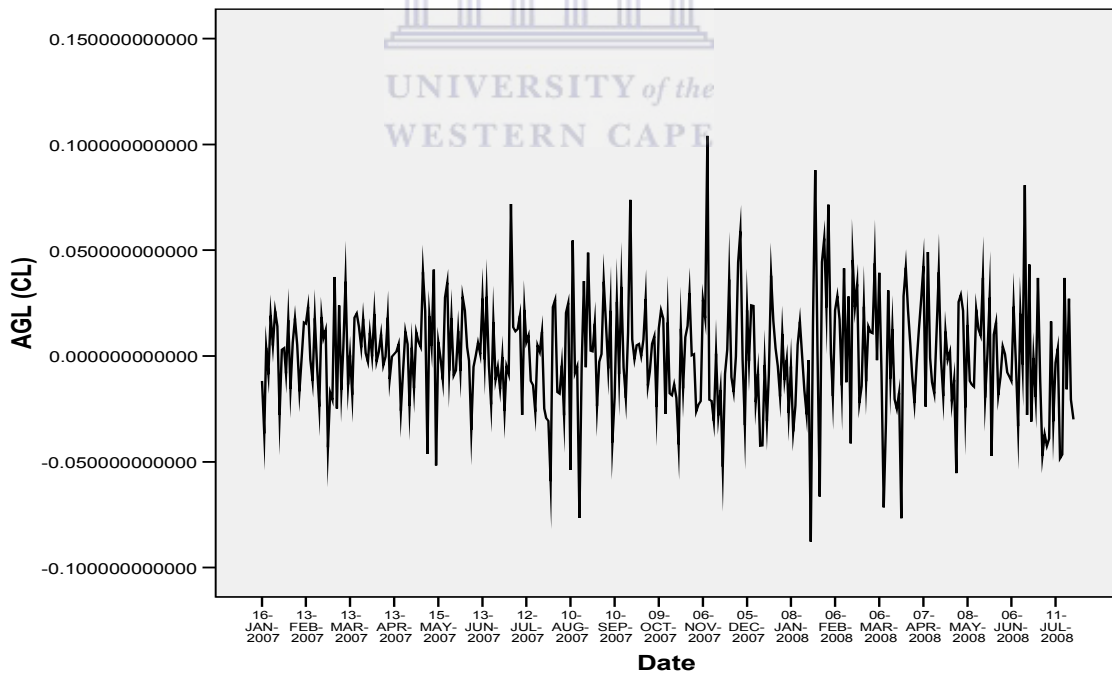
1.5.2 Histogram and normal quantile plot of the data

Histogram
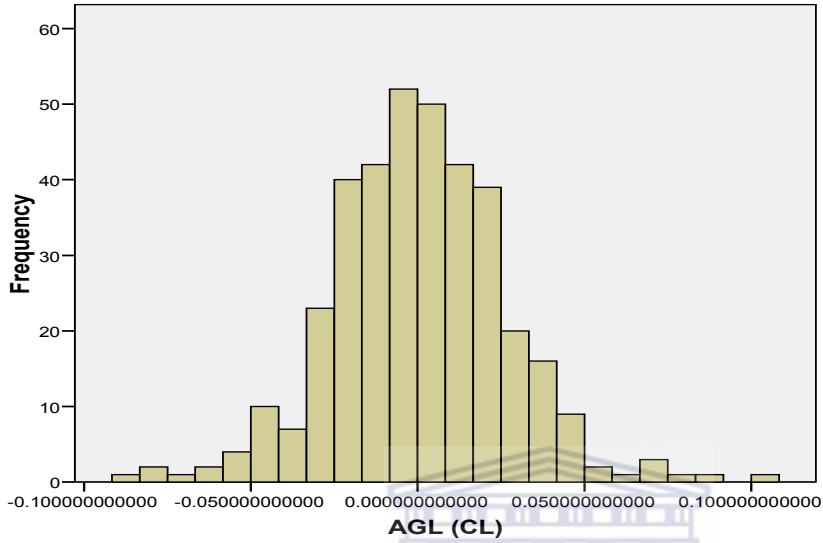
Mean =4.77E-4%
Std. Dev. =0.
025971514480%
N =369

Normal Q-Q Plot of AGL (CL)
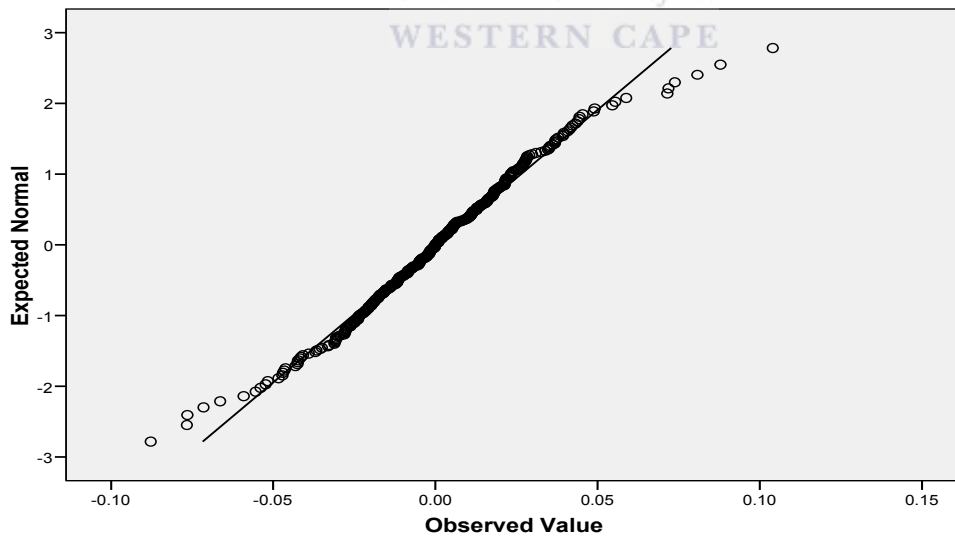
1.5.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-19.04376	0.0000
Test critical values:	1% level	-3.447914	
	5% level	-2.869176	
	10% level	-2.570905	

*MacKinnon (1996) one-sided p-values.

1.6 US DOLLAR GOLD PRICE (DGLDS)

1.6.1 Sequence plot

1.6.2 Histogram and normal quantile plot of the data

Histogram

Mean =0.001038627825%
Std. Dev. =0.
012373797399%
N =369
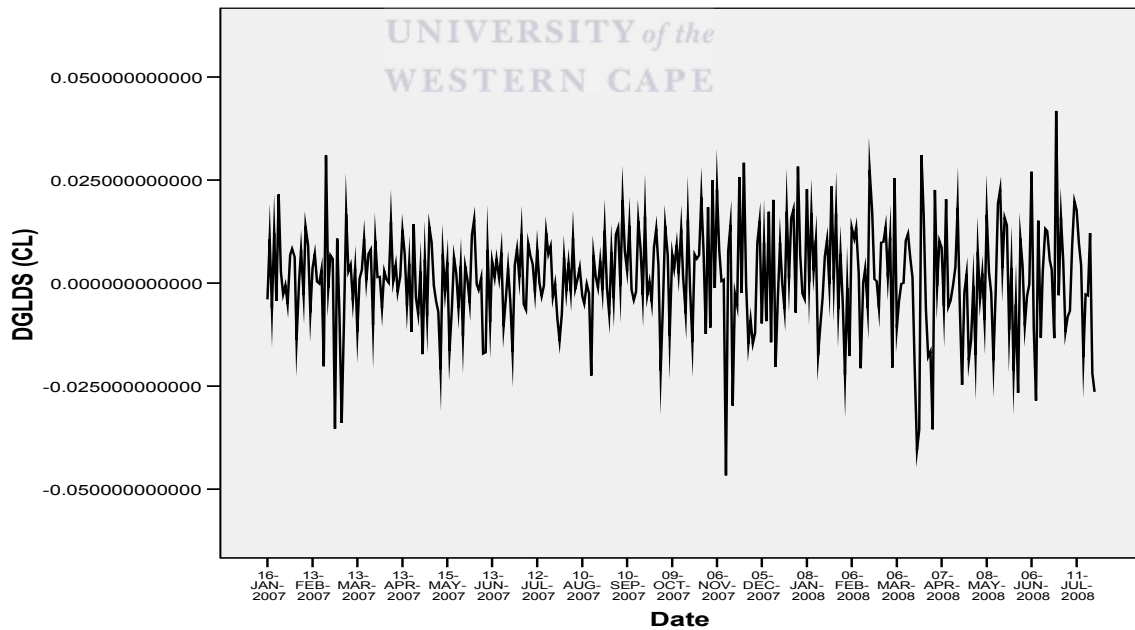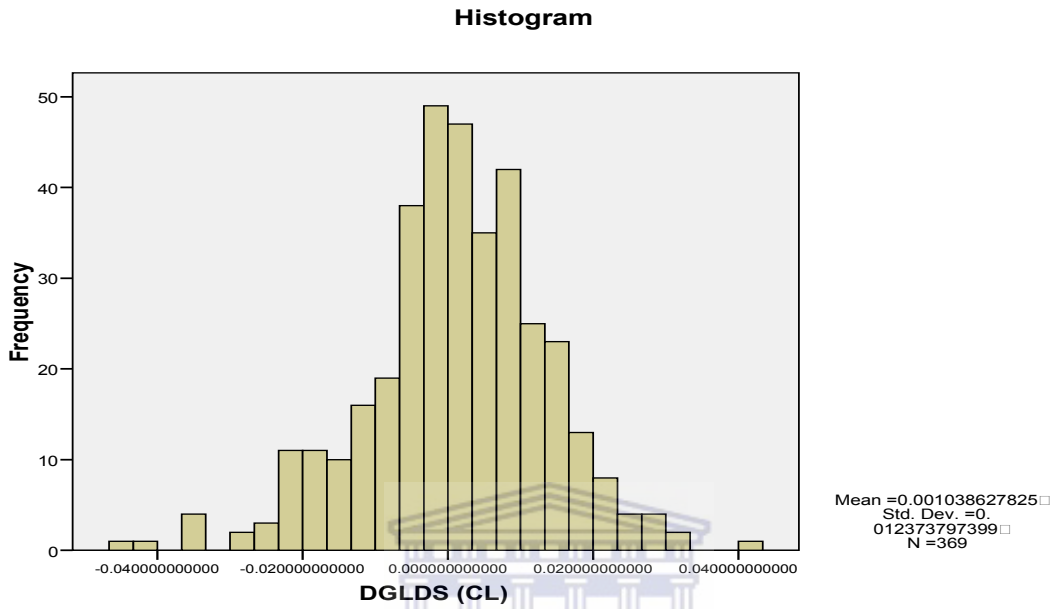
Normal Q-Q Plot of DGLDS (CL)
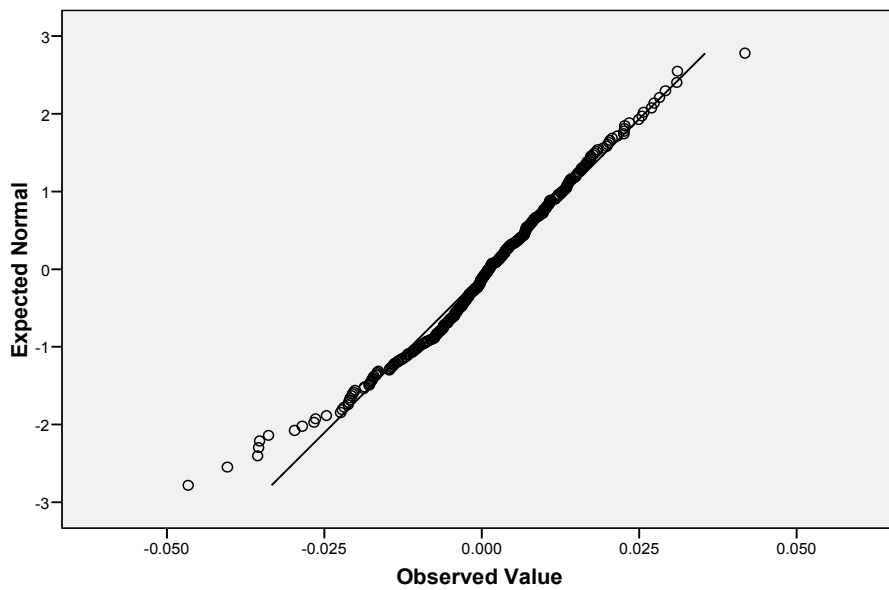
1.6.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-19.67798	0.0000
Test critical values:	1% level	-3.447914	
	5% level	-2.869176	
	10% level	-2.570905	

*MacKinnon (1996) one-sided p-values.

1.7 BRENT SPOT OIL PRICE (BRSPOT)

1.7.1 Sequence plot

1.7.2 Histogram and normal quantile plot of the data

Histogram

Mean =0.002393959096%
Std. Dev. =0.
018813008228%
N =369
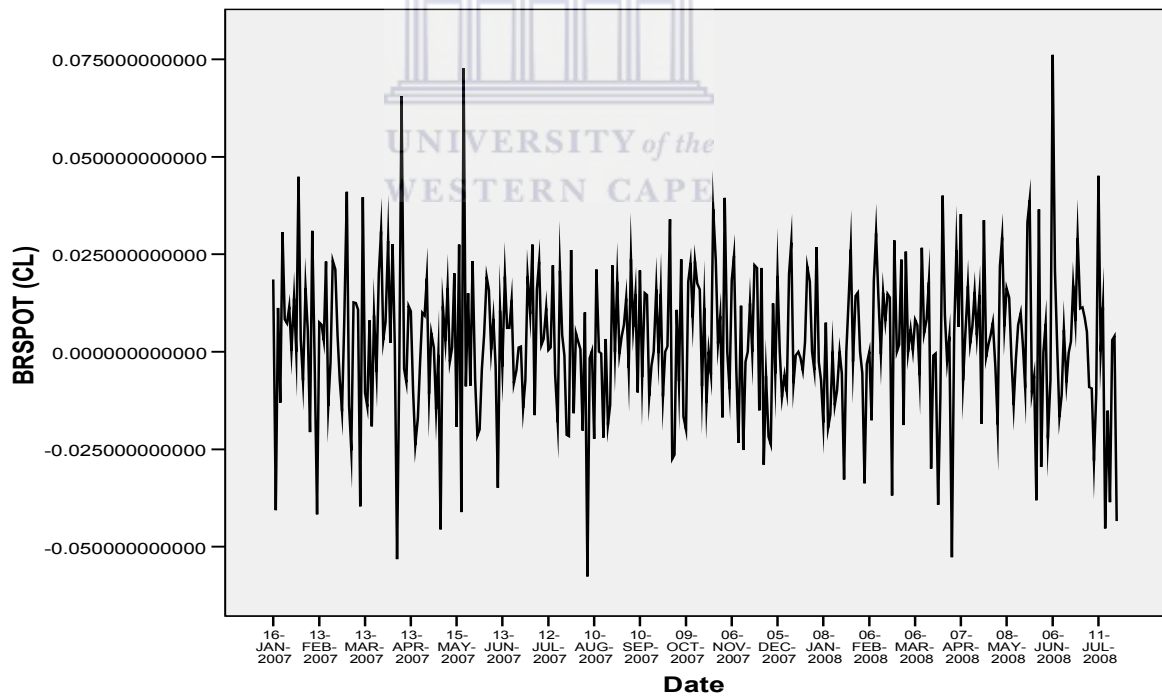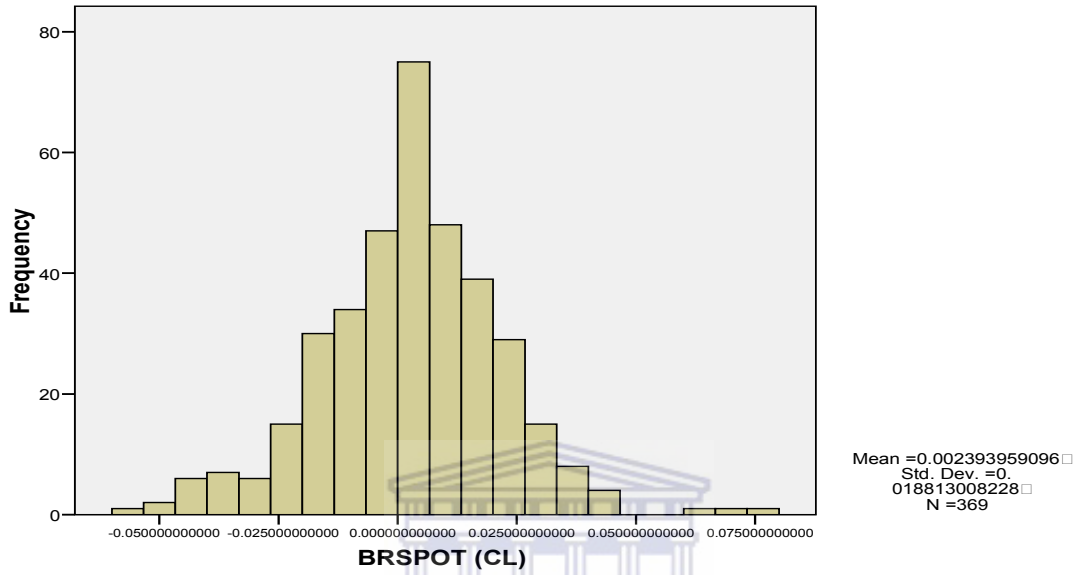
BRSPOT (CL)

Normal Q-Q Plot of BRSPOT (CL)
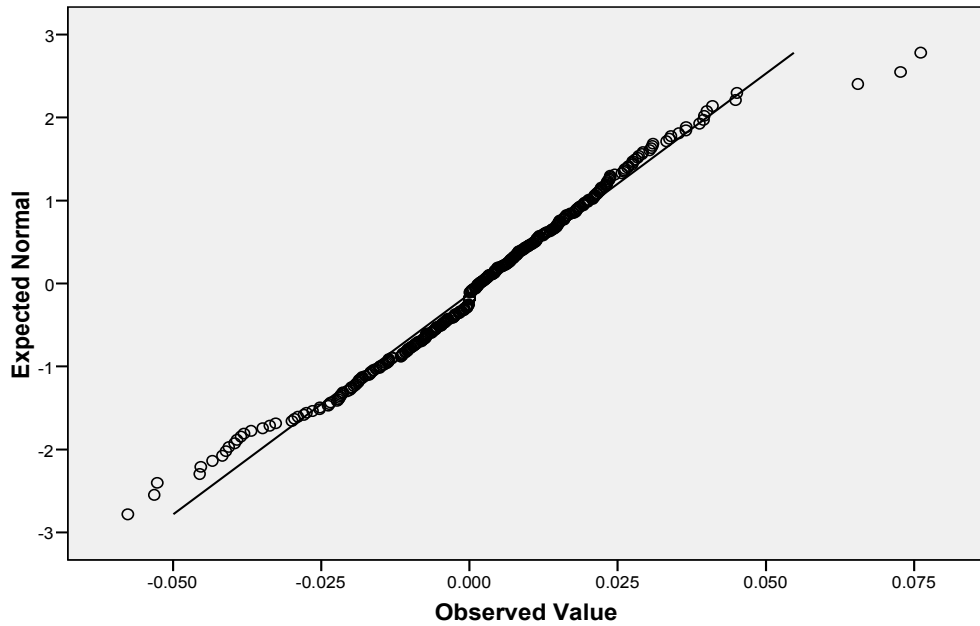
1.7.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-21.03959	0.0000
Test critical values:	1% level	-3.447914	
	5% level	-2.869176	
	10% level	-2.570905	

*MacKinnon (1996) one-sided p-values.
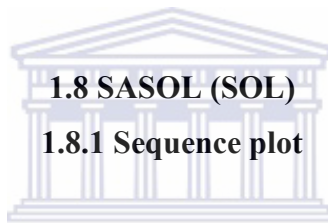
1.8 SASOL (SOL)
1.8.1 Sequence plot

1.8.2 Histogram and normal quantile plot of the data

Histogram

Mean =0.001407798546%
Std. Dev. =0.
021723258421%
N =369
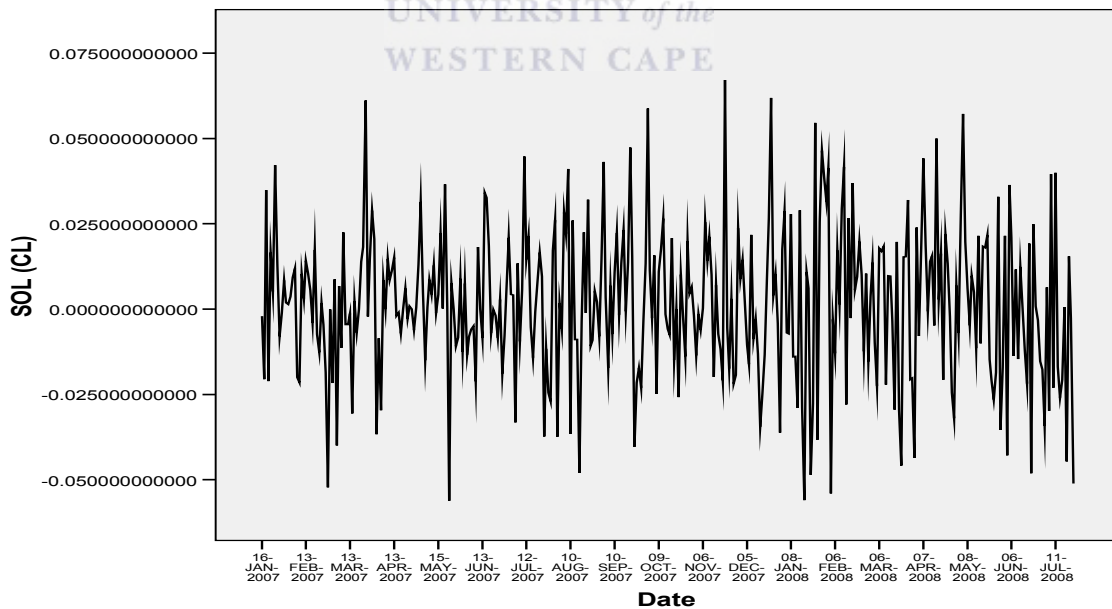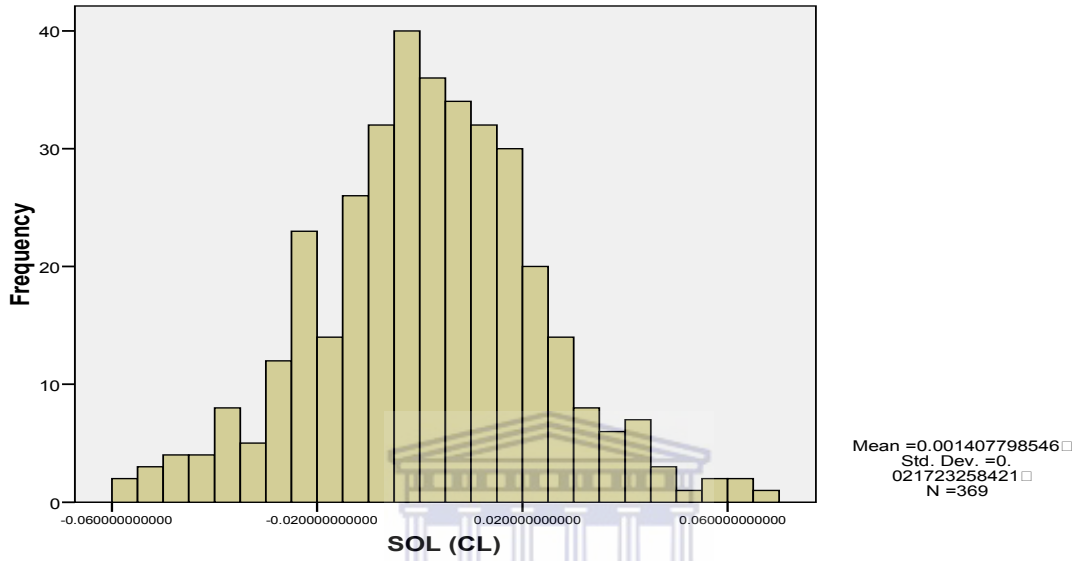
Normal Q-Q Plot of SOL (CL)
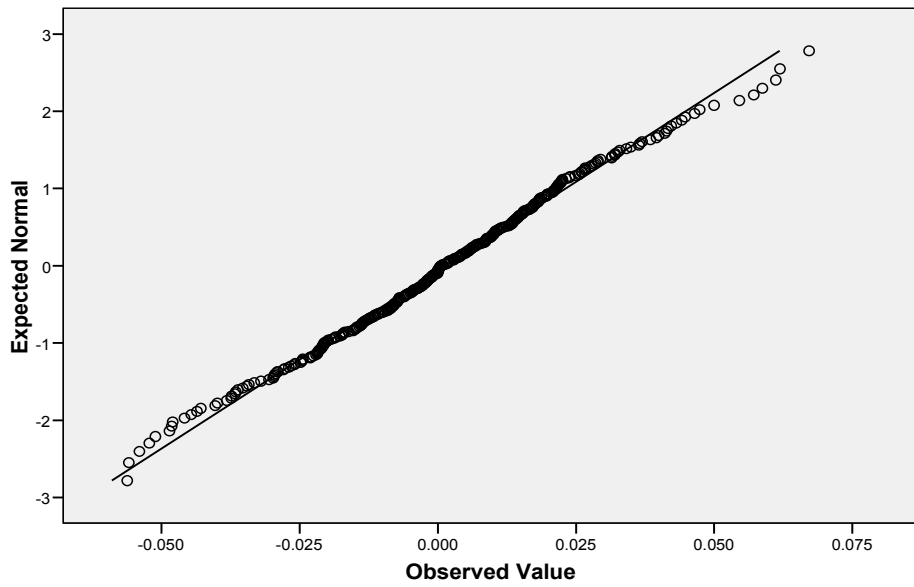
1.8.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-18.77467	0.0000
Test critical values:	1% level	-3.447914	
	5% level	-2.869176	
	10% level	-2.570905	

*MacKinnon (1996) one-sided p-values.

1.9 SOUTH AFRICAN WHITE MAIZE NEAR FUTURE (WMAZN)
1.9.1 Sequence plot

1.9.2 Histogram and normal quantile plot of the data

Histogram

Mean =8.74E-4%
Std. Dev. =0.
020106394191%
N =369
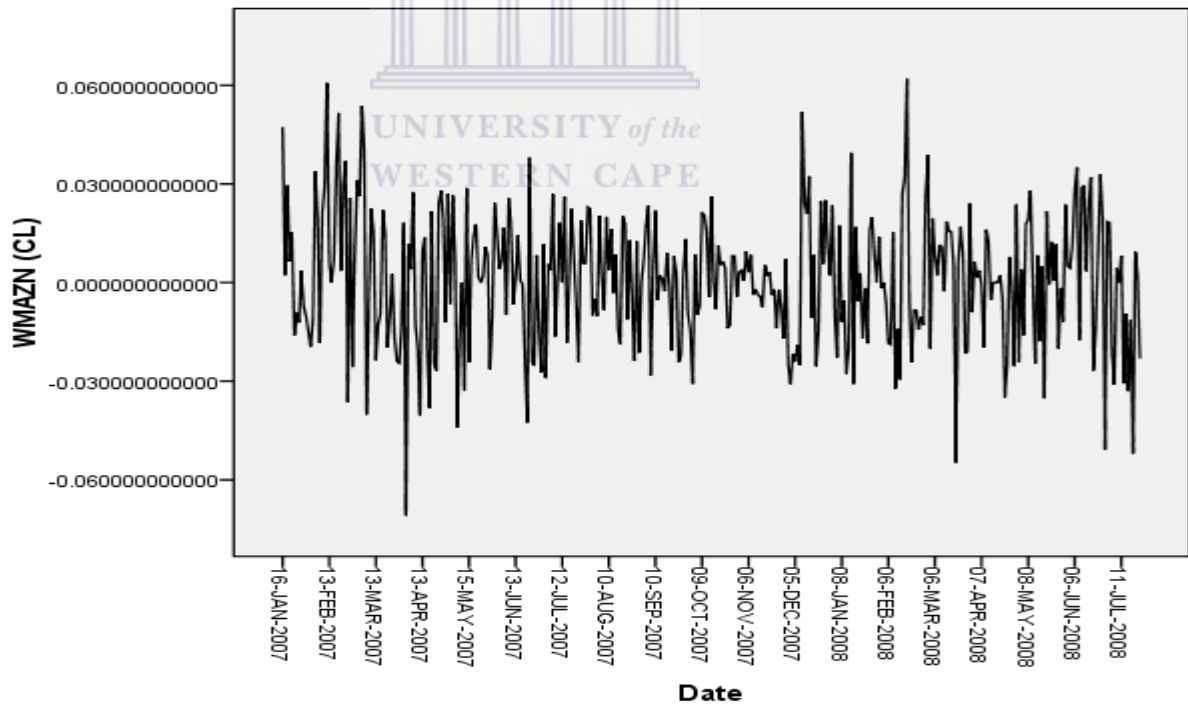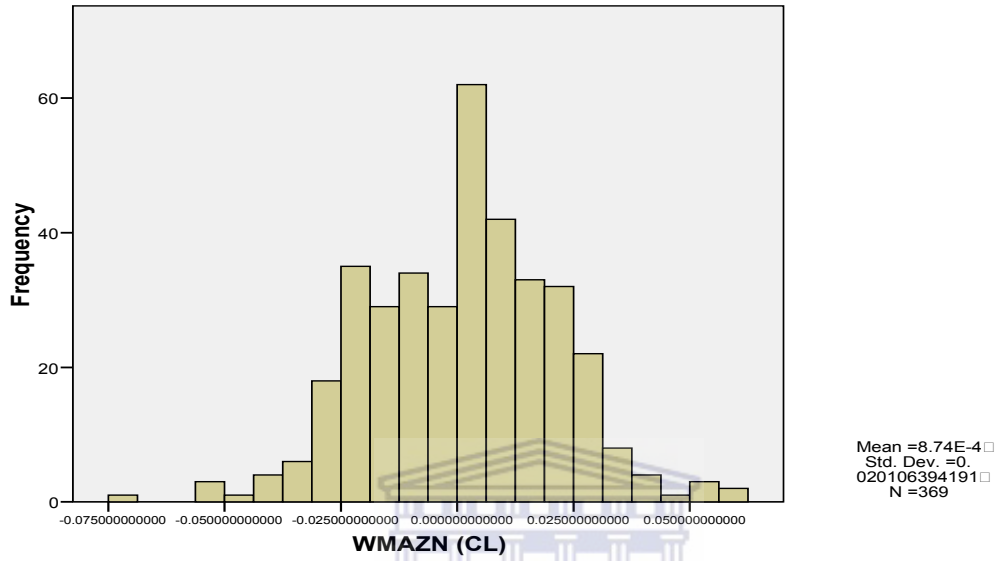
WMAZN (CL)

Normal Q-Q Plot of WMAZN (CL)
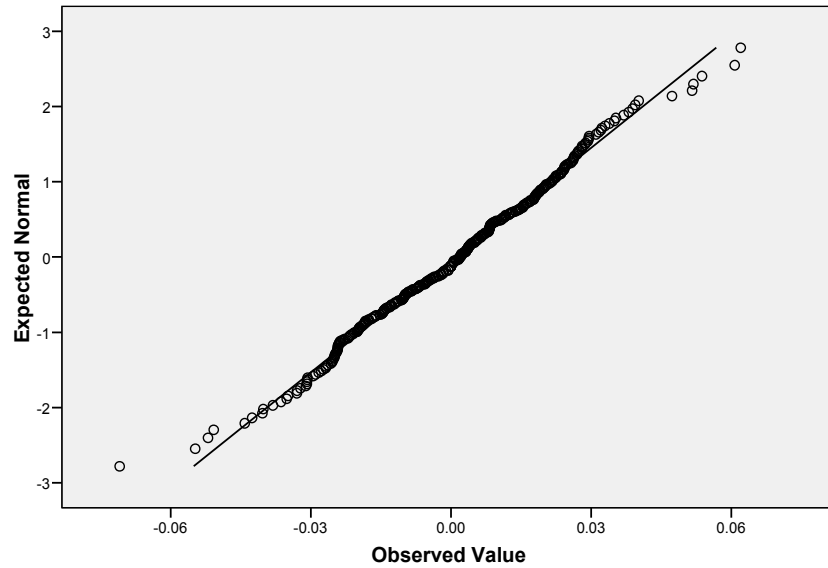
1.9.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-18.29253	0.0000
Test critical values:	1% level	-3.447914	
	5% level	-2.869176	
	10% level	-2.570905	

*MacKinnon (1996) one-sided p-values.

1.10 STANDARD BANK (SBK)

1.10.1 Sequence plot

1.10.2 Histogram and normal quantile plot of the data

Histogram

Mean =-3.58E-4%
Std. Dev. =0.
022161524770%
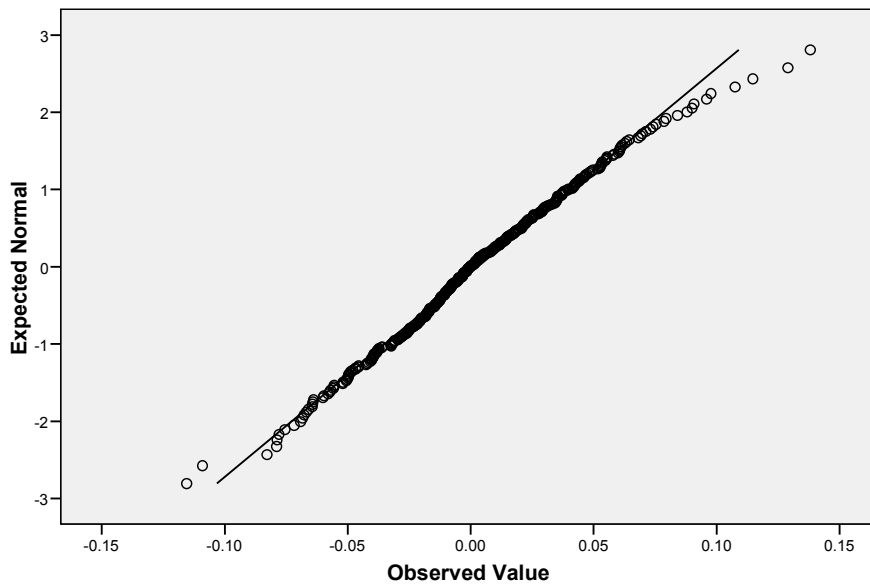N =369
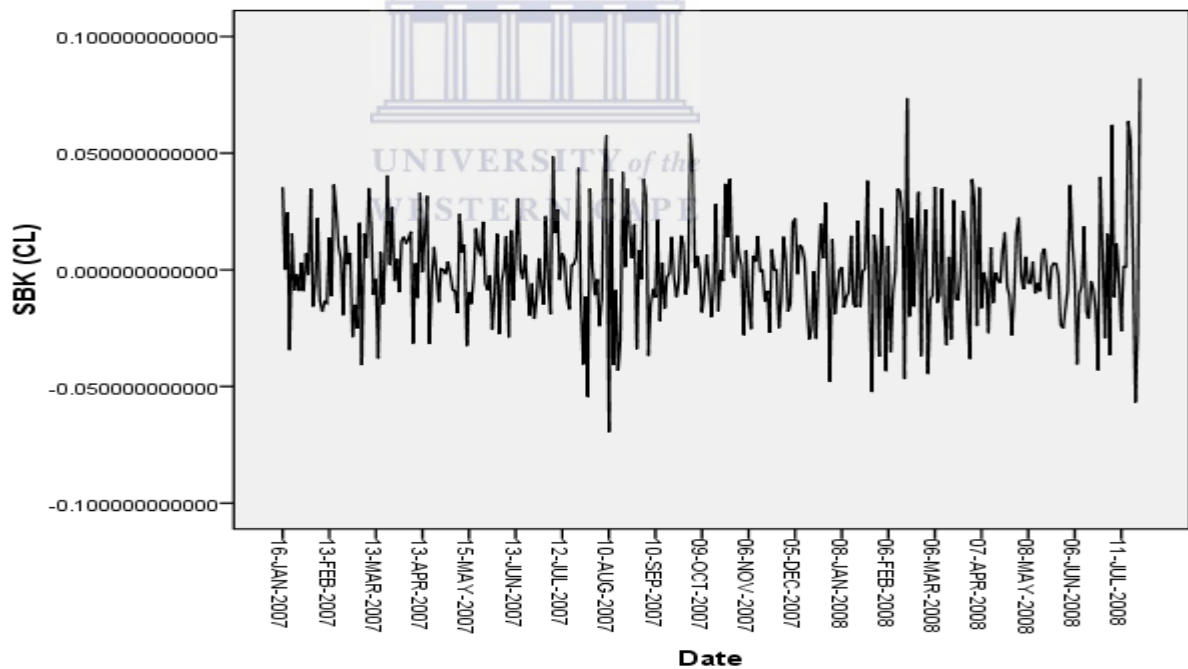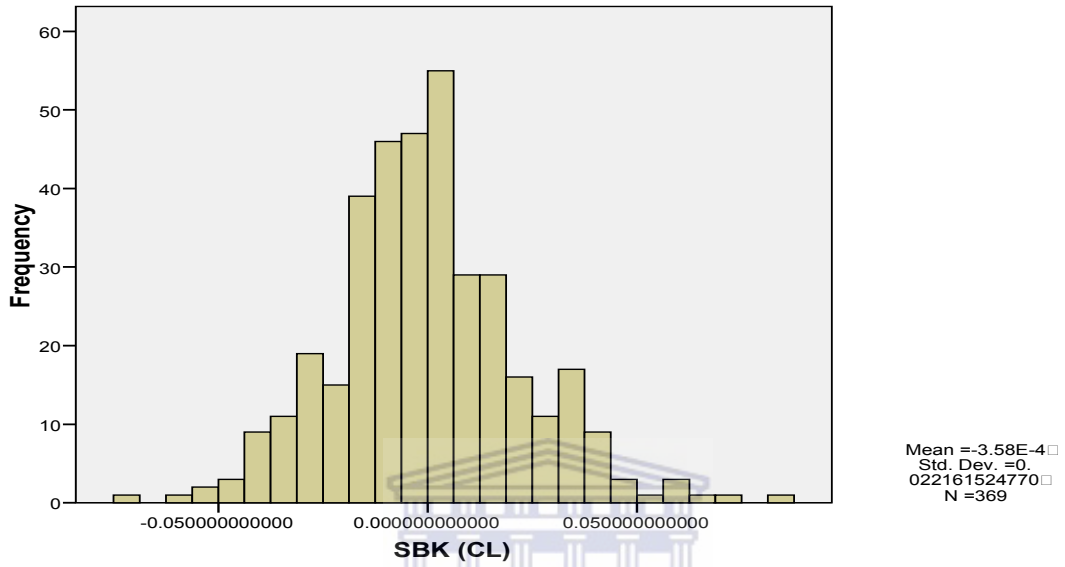
Normal Q-Q Plot of SBK (CL)
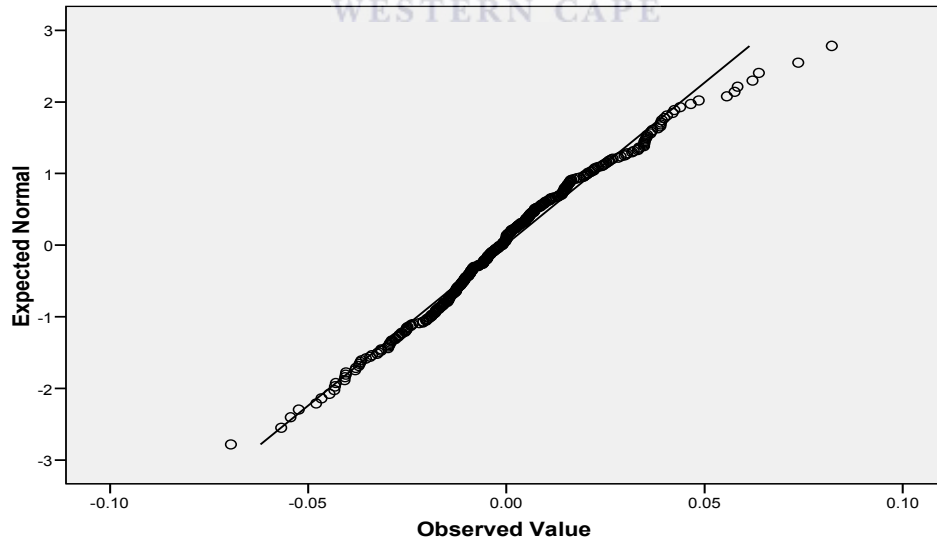
1.10.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-20.88485	0.0000
Test critical values:	1% level	-3.447914	
	5% level	-2.869176	
	10% level	-2.570905	

*MacKinnon (1996) one-sided p-values.

2 WEEKLY DATA
2.1 EURO/USD DOLLAR (EURUSD)
2.1.1 Sequence plot
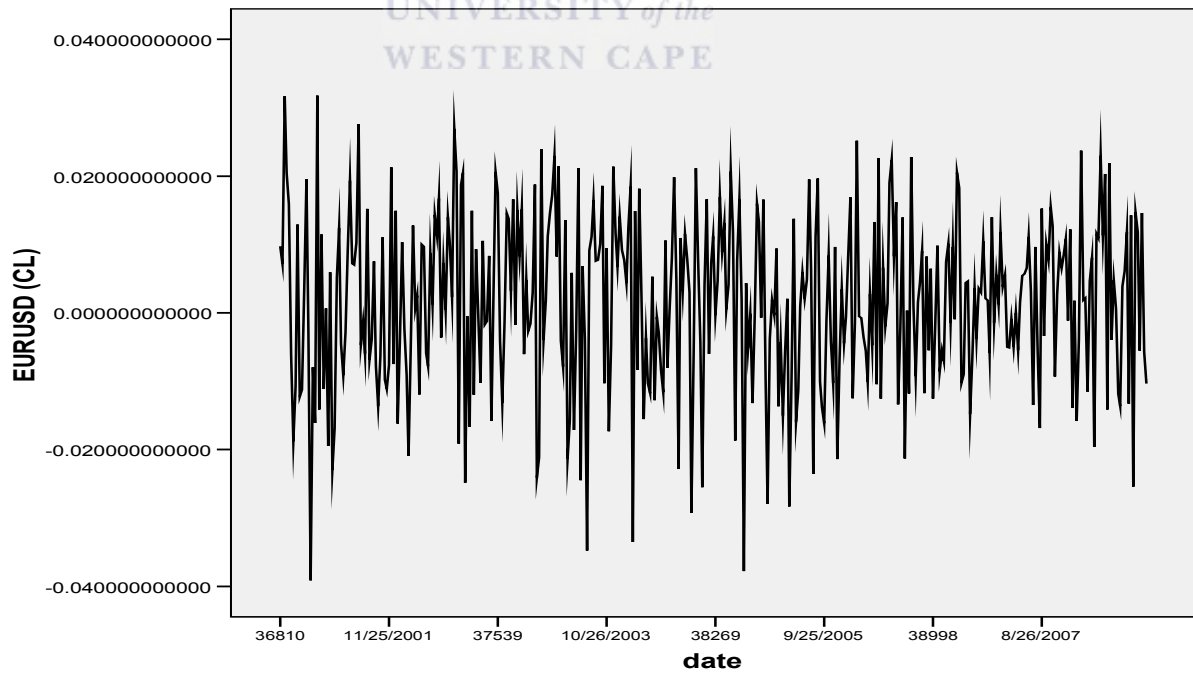
2.1.2 Histogram and normal quantile plot of the data

Histogram

Mean =0.001450178879%
Std. Dev. =0.
012557478216%
N =399

EURUSD (CL)

Normal Q-Q Plot of EURUSD (CL)

Observed Value
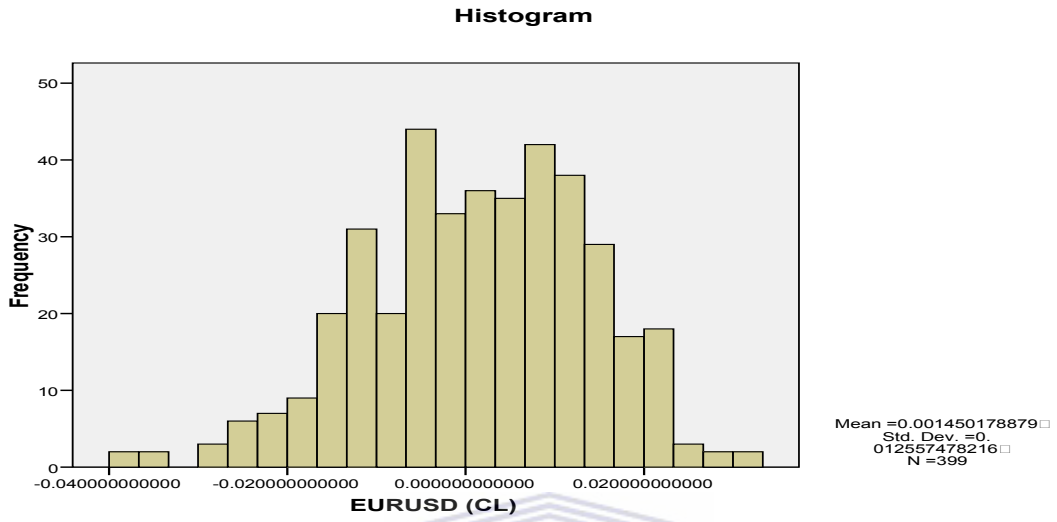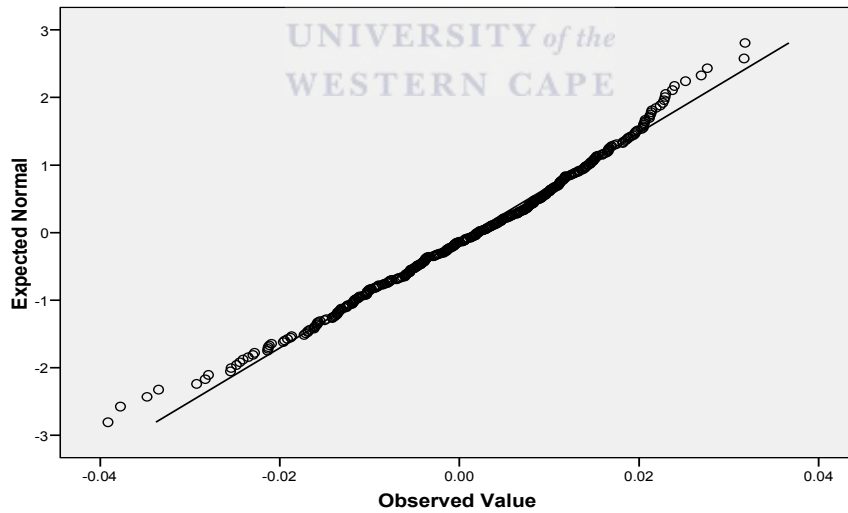
2.1.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-20.51208	0.0000
Test critical values:	1% level	-3.446567	
	5% level	-2.868583	
	10% level	-2.570588	

*MacKinnon (1996) one-sided p-values.

2.2 JSE ALL SHARE TOTAL RETURNS INDEX (J203T)

2.2.2 Histogram and normal quantile plot of the data

Histogram

Mean =0.003763270603%
Std. Dev. =0.
026877853123%
N =399

Frequency (y-axis)
J203T (CL) (x-axis)

Normal Q-Q Plot of J203T (CL)

Expected Normal (y-axis)
Observed Value (x-axis)
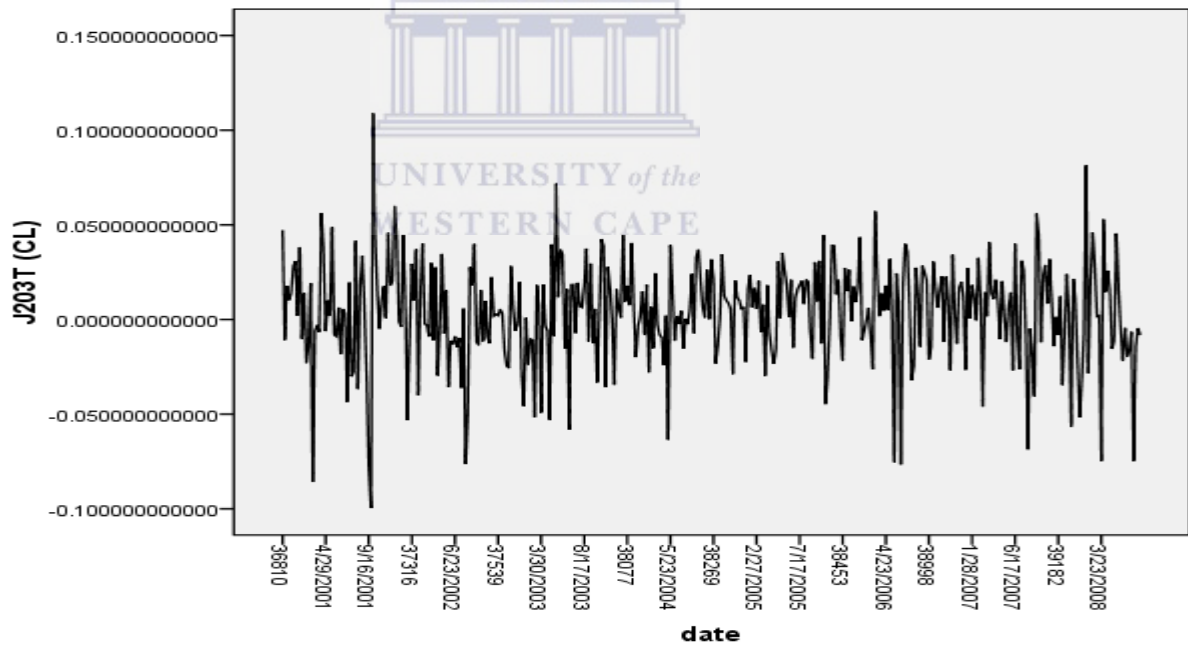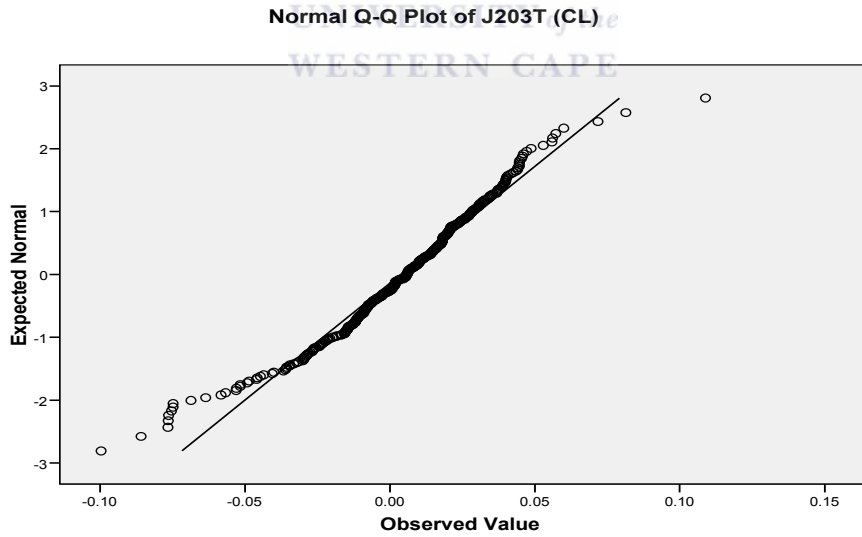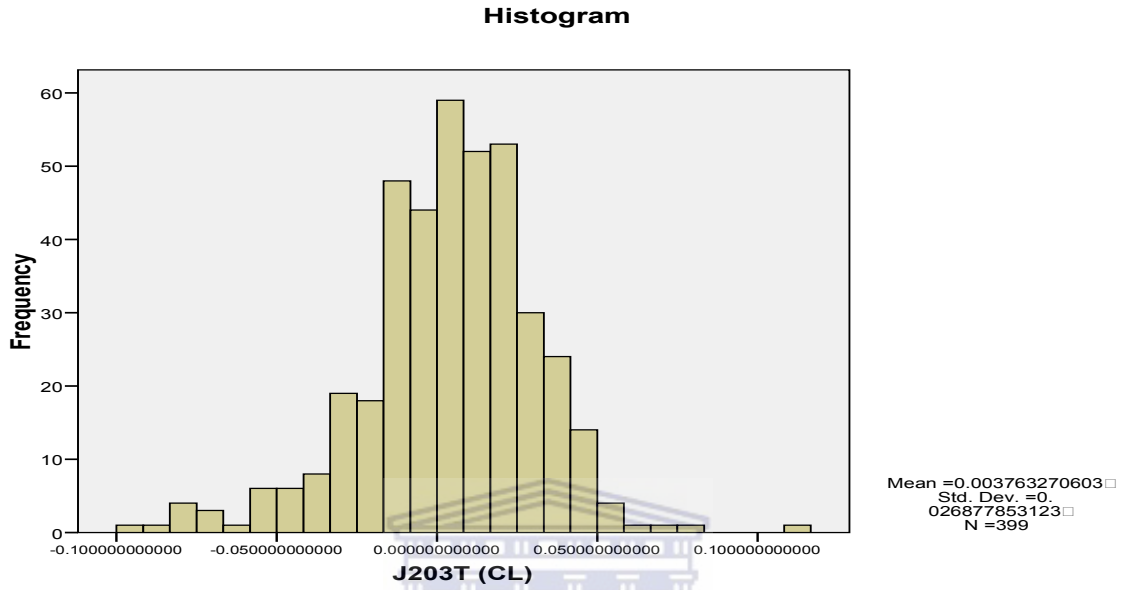
2.2.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-19.95405	0.0000
Test critical values:	1% level	-3.446567	
	5% level	-2.868583	
	10% level	-2.570588	

*MacKinnon (1996) one-sided p-values.

2.3 SOUTH AFRICAN ALL BOND INDEX (ALBI)
2.3.1 Sequence plot

2.3.2 Histogram and normal quantile plot of the data

Histogram

Mean =0.00204812306%
Std. Dev. =0.
00972913539%
N =399
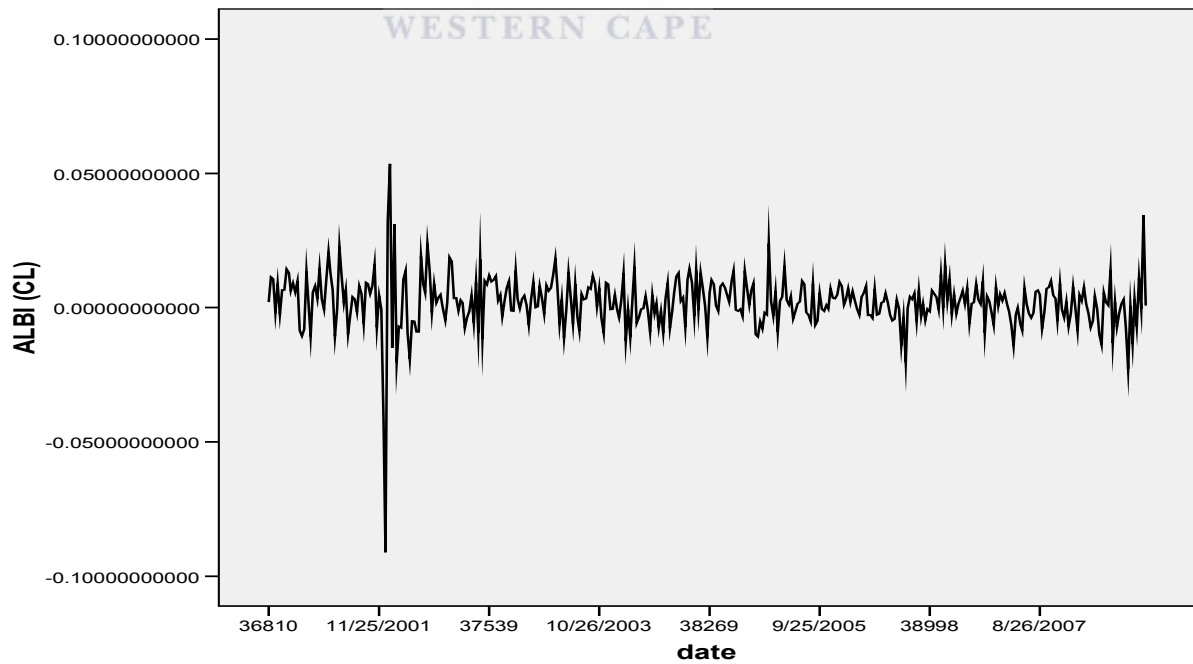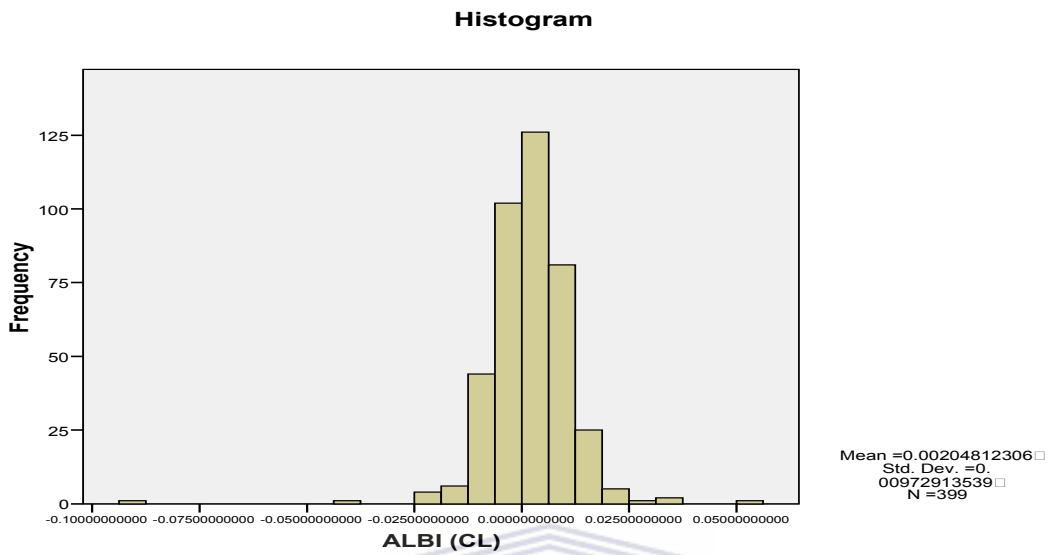
Normal Q-Q Plot of ALBI (CL)
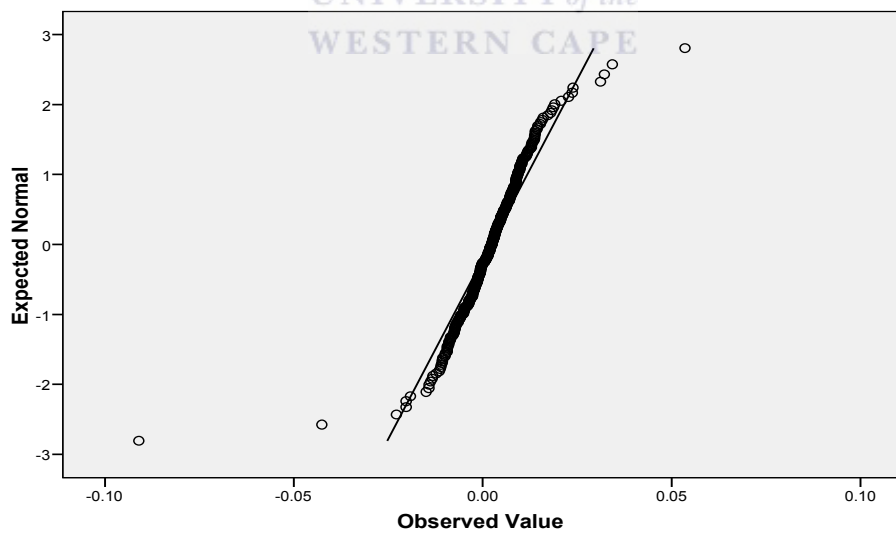
2.3.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-17.76120	0.0000
Test critical values:	1% level	-3.446567	
	5% level	-2.868583	
	10% level	-2.570588	

*MacKinnon (1996) one-sided p-values.

2.4 CRB COMMODITY PRICE INDEX (FCRB)

2.4.2 Sequence plot

2.4.2 Histogram and normal quantile plot of the data

Histogram

Mean =0.00
Std. Dev. =0.019
N =399

FCRB (CL)

Normal Q-Q Plot of FCRB (CL)
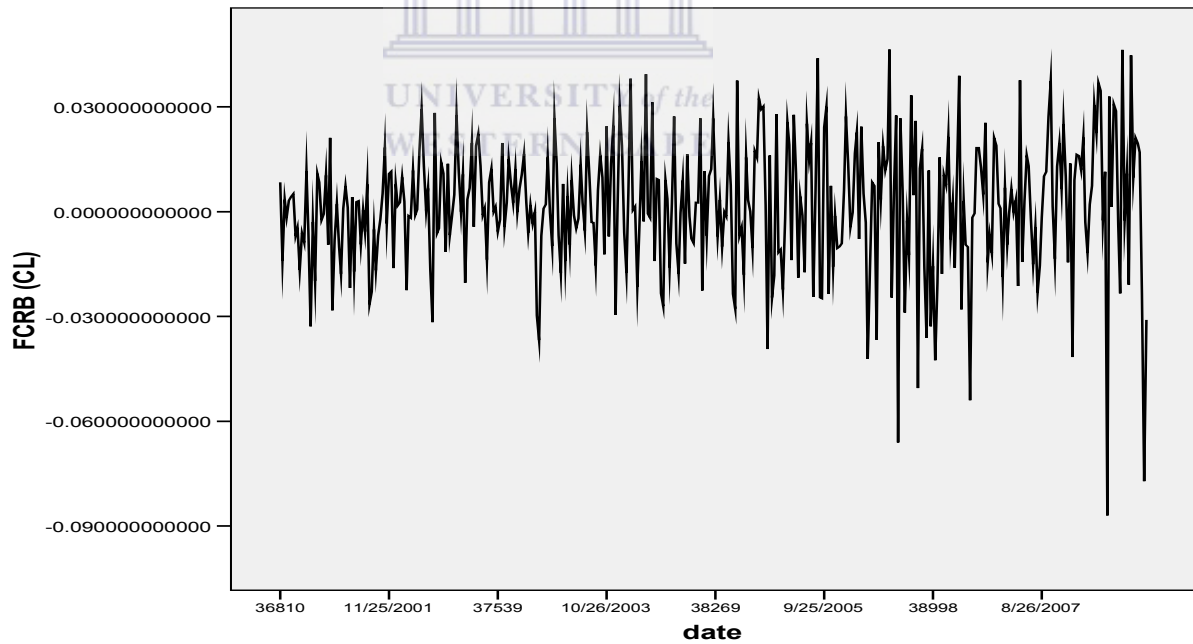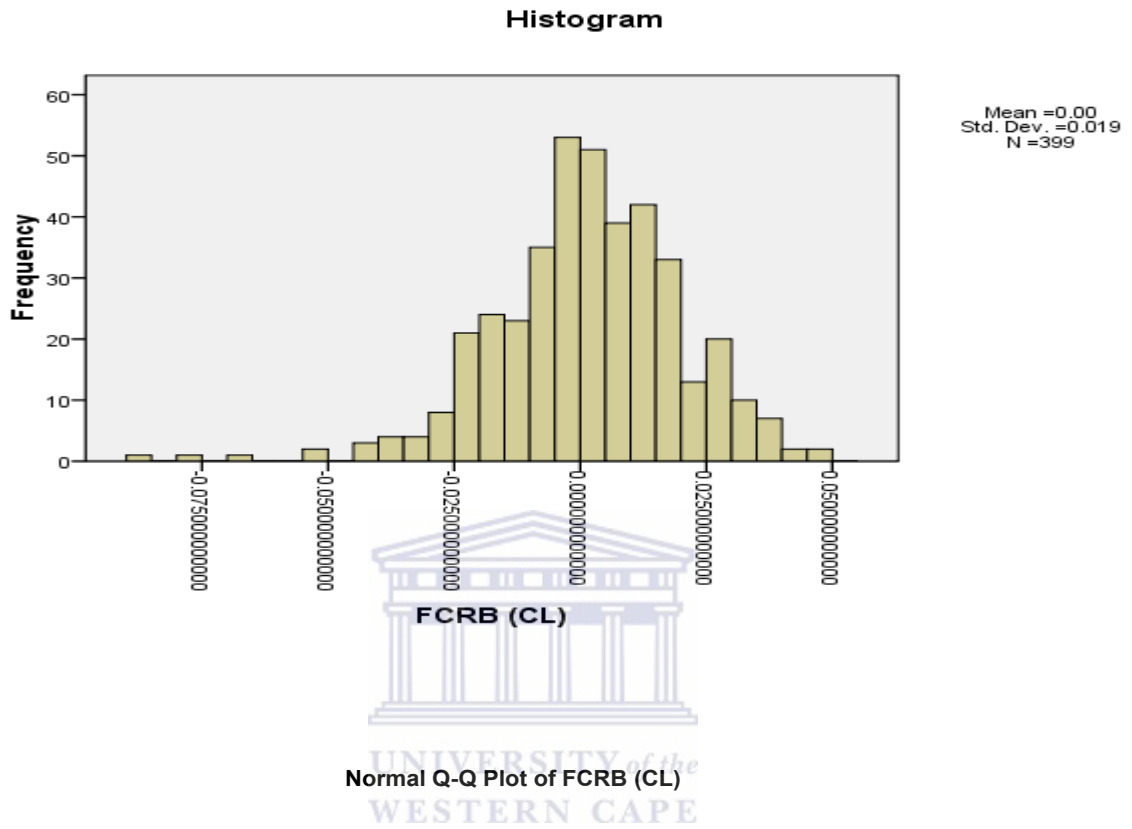
2.4.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-20.00923	0.0000
Test critical values:	1% level	-3.446567	
	5% level	-2.868583	
	10% level	-2.570588	

*MacKinnon (1996) one-sided p-values.

2.5 ANGLO AMERICAN CORPORATION (AGL)

2.5.1 Sequence plot
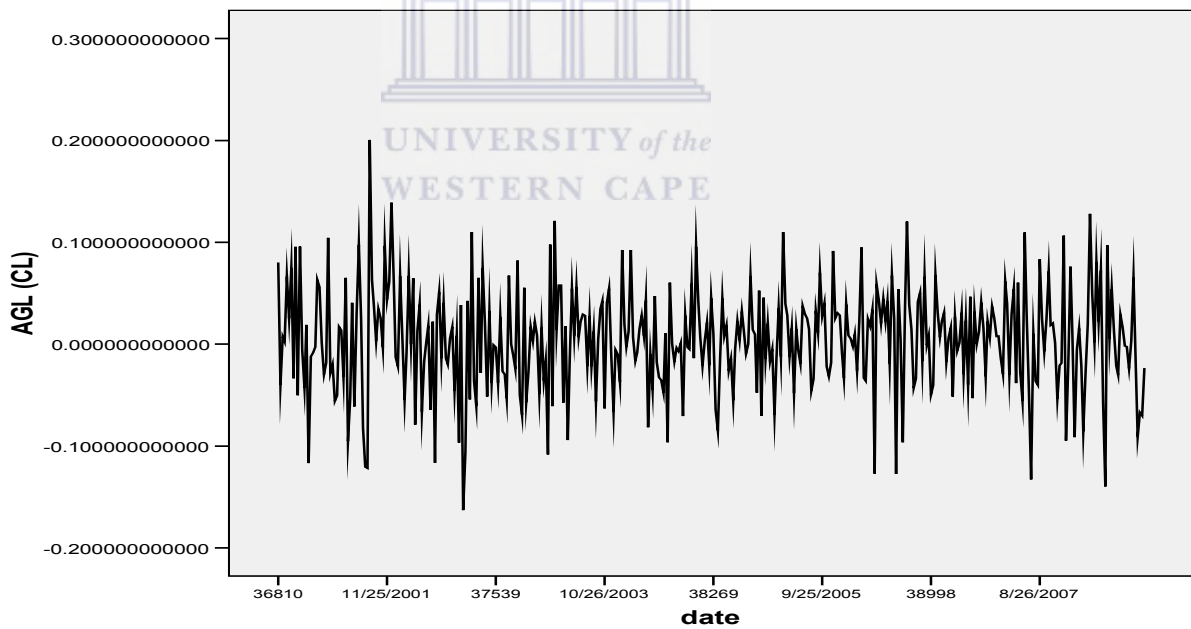
2.5.2 Histogram and normal quantile plot of the data

Histogram

Frequency

AGL (CL)

Mean =0.003609505642%
Std. Dev. =0.
050509034360%
N =399

Normal Q-Q Plot of AGL (CL)

Expected Normal

Observed Value
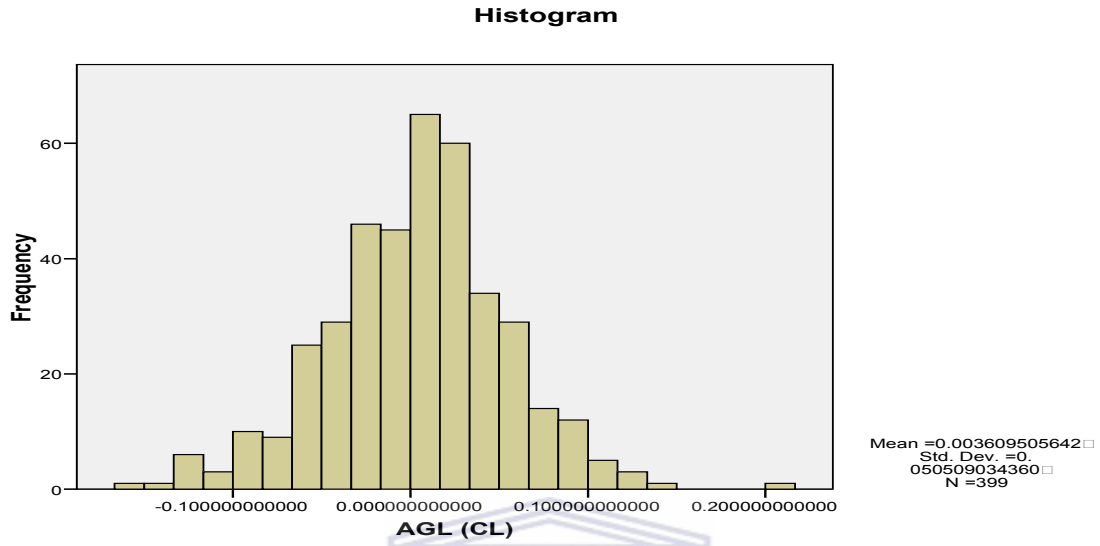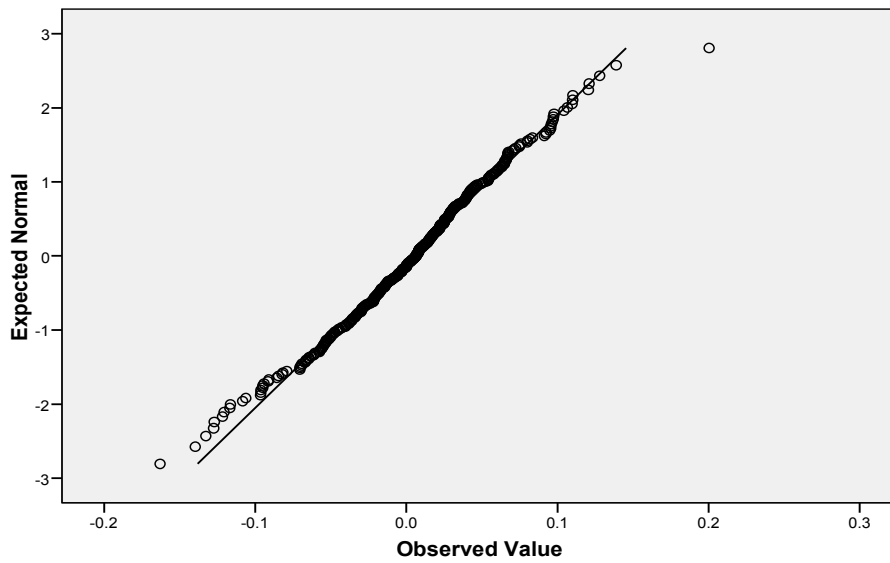
2.5.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-21.32824	0.0000
Test critical values:	1% level	-3.446567	
	5% level	-2.868583	
	10% level	-2.570588	

*MacKinnon (1996) one-sided p-values.

2.6 STANDARD BANK (SBK)

2.6.1 Sequence plot

2.6.2 Histogram and normal quantile plot of the data

Histogram

Mean =0.002897535797%
Std. Dev. =0.
037802863343%
N =399

Normal Q-Q Plot of SBK (CL)
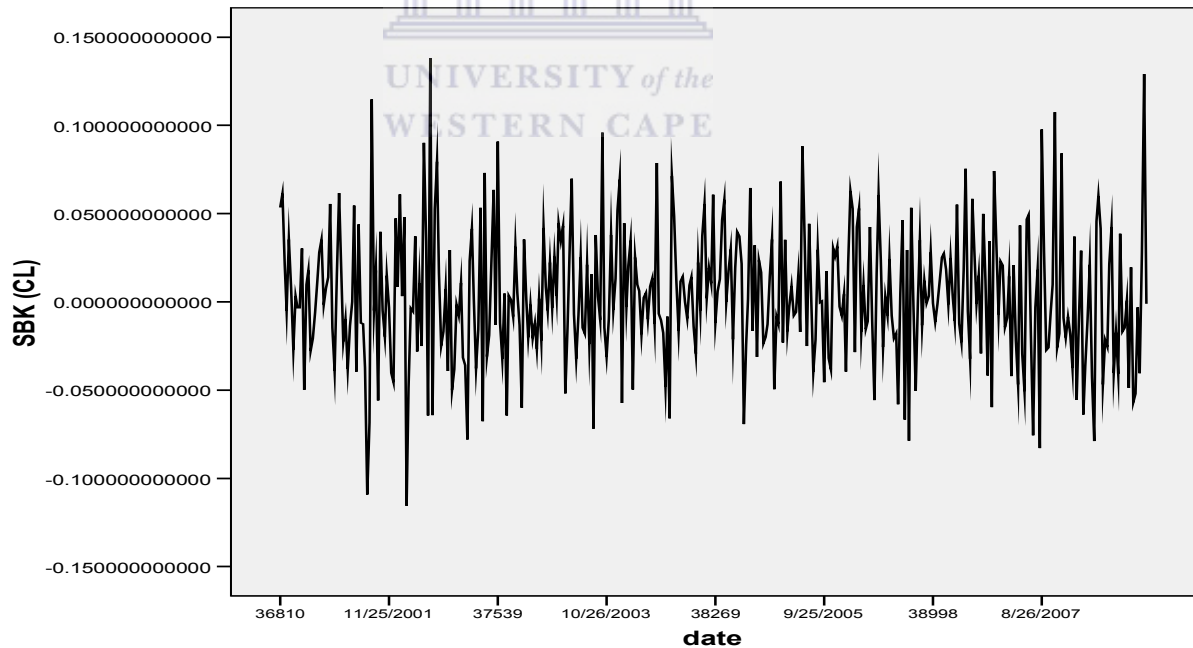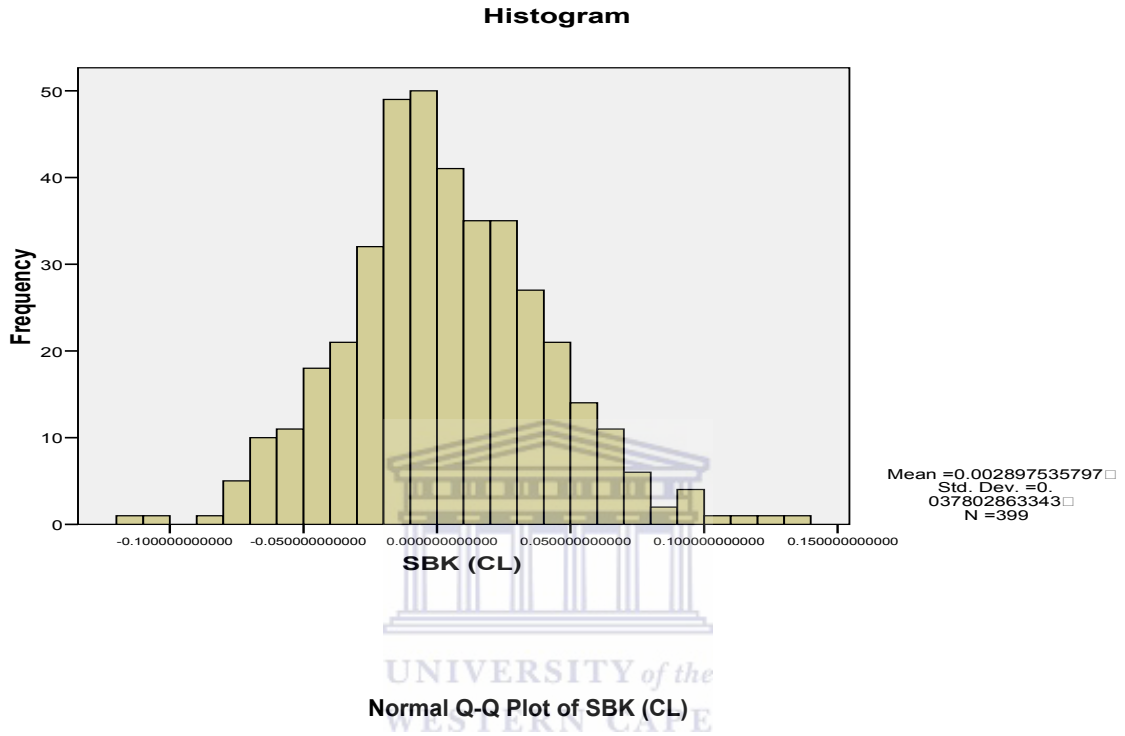
2.6.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-22.32941	0.0000
Test critical values:	1% level	-3.446567	
	5% level	-2.868583	
	10% level	-2.570588	

*MacKinnon (1996) one-sided p-values.

2.7 US DOLLAR GOLD PRICE (DGLDS)

2.7.1 Sequence plot

2.7.2 Histogram and normal quantile plot of the data

Histogram

Mean =0.003080178675%
Std. Dev. =0.
023820872045%
N =399
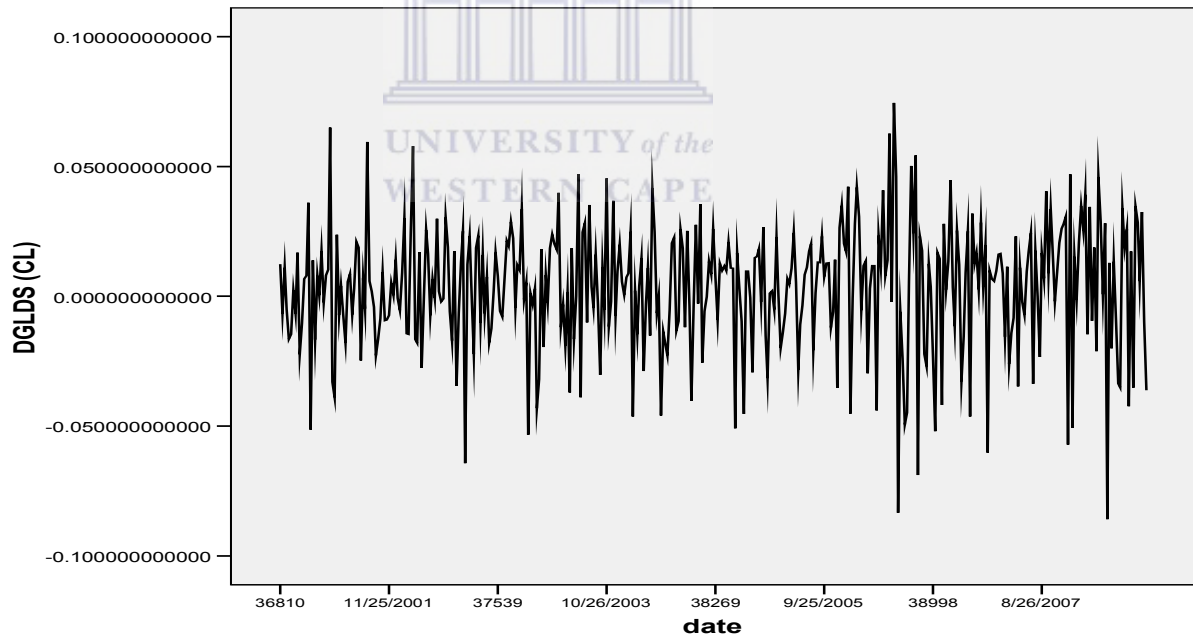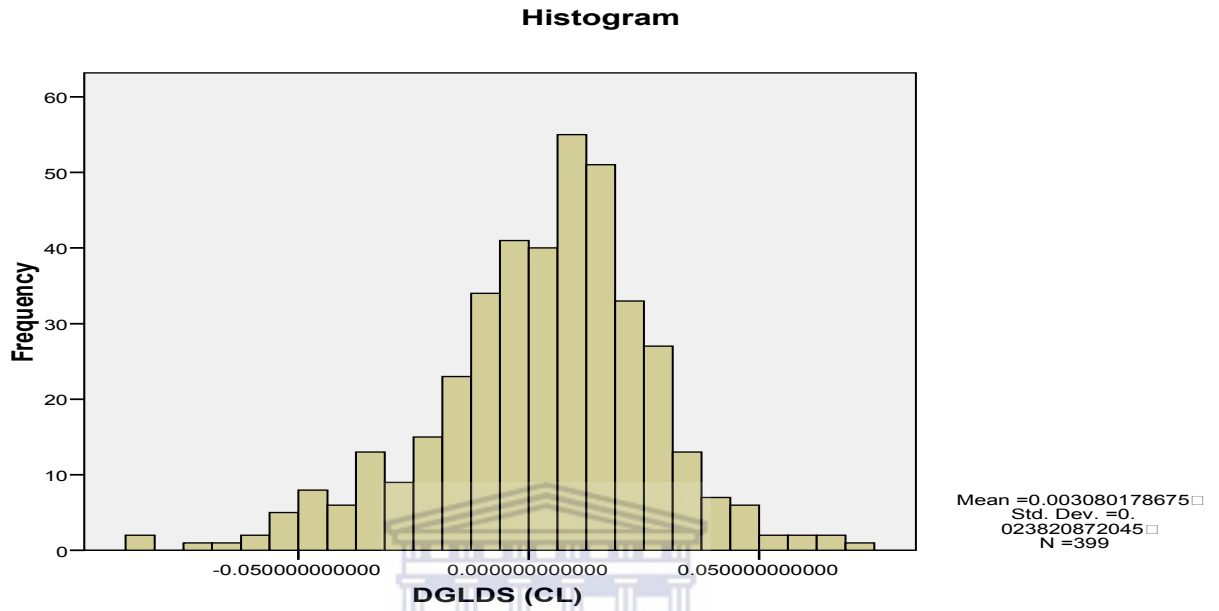
Normal Q-Q Plot of DGLDS (CL)
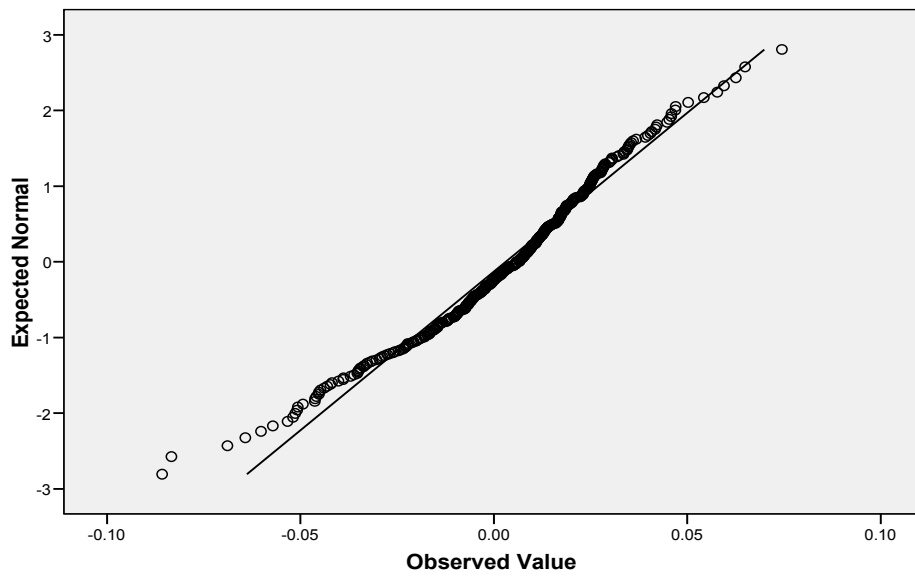
2.7.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-21.81857	0.0000
Test critical values:	1% level	-3.446567	
	5% level	-2.868583	
	10% level	-2.570588	

*MacKinnon (1996) one-sided p-values.

2.8 BRENT SPOT OIL PRICE (BRSPOT)

2.8.1 Sequence plot

2.8.2 Histogram and normal quantile plot of the data

Histogram

Mean =0.003399882133%
Std. Dev. =0.
044114465271%
N =399

Normal Q-Q Plot of BRSPOT (CL)
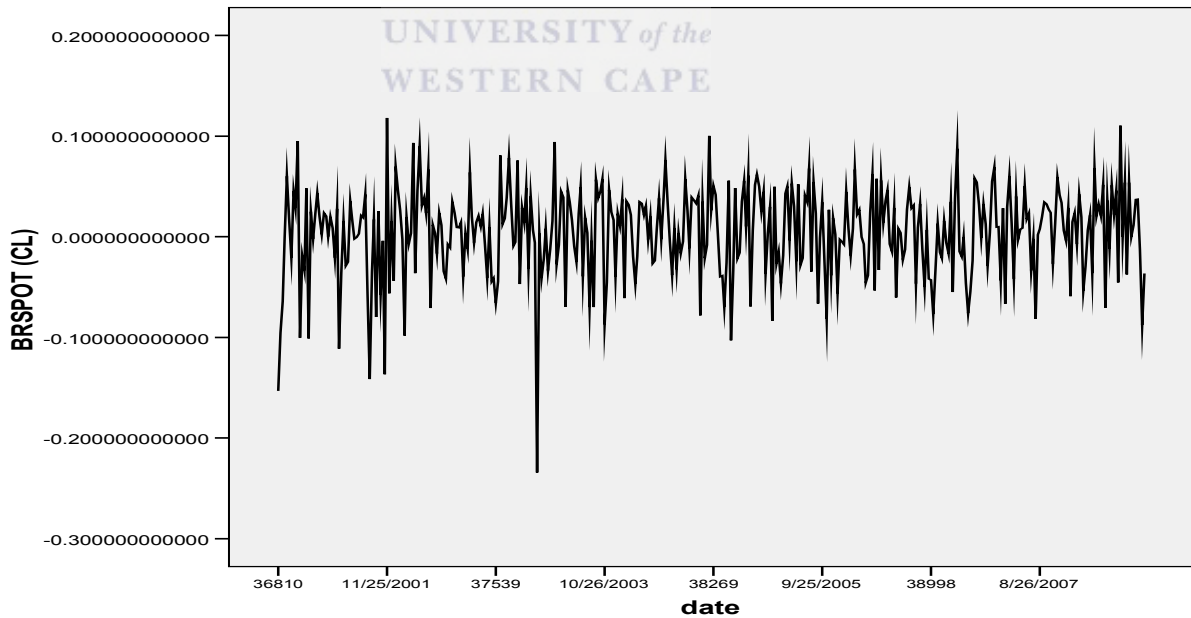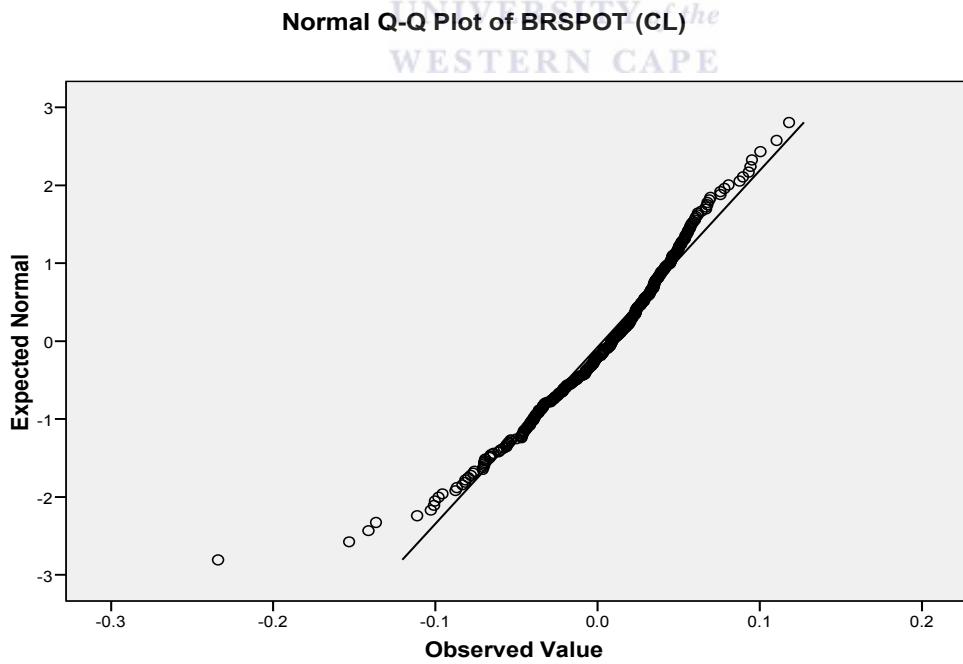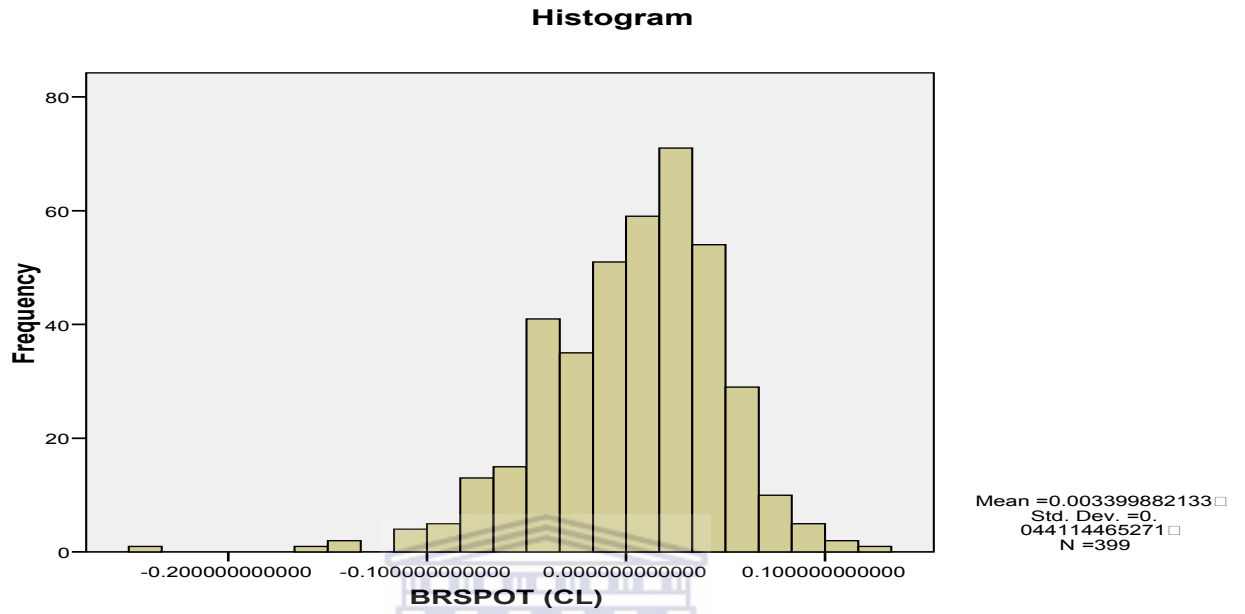
2.8.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-20.10967	0.0000
Test critical values:	1% level	-3.446567	
	5% level	-2.868583	
	10% level	-2.570588	

*MacKinnon (1996) one-sided p-values.

2.9 SOUTH AFRICAN WHITE MAIZE NEAR FUTURE (WMAZN)
2.9.1 Sequence plot

113

2.9.2 Histogram and normal quantile plot of the data

Histogram

Frequency

WMAZN (CL)

Mean =0.00248488880%
Std. Dev. =0.
04983846164%
N =399

Normal Q-Q Plot of WMAZN (CL)

Expected Normal

Observed Value
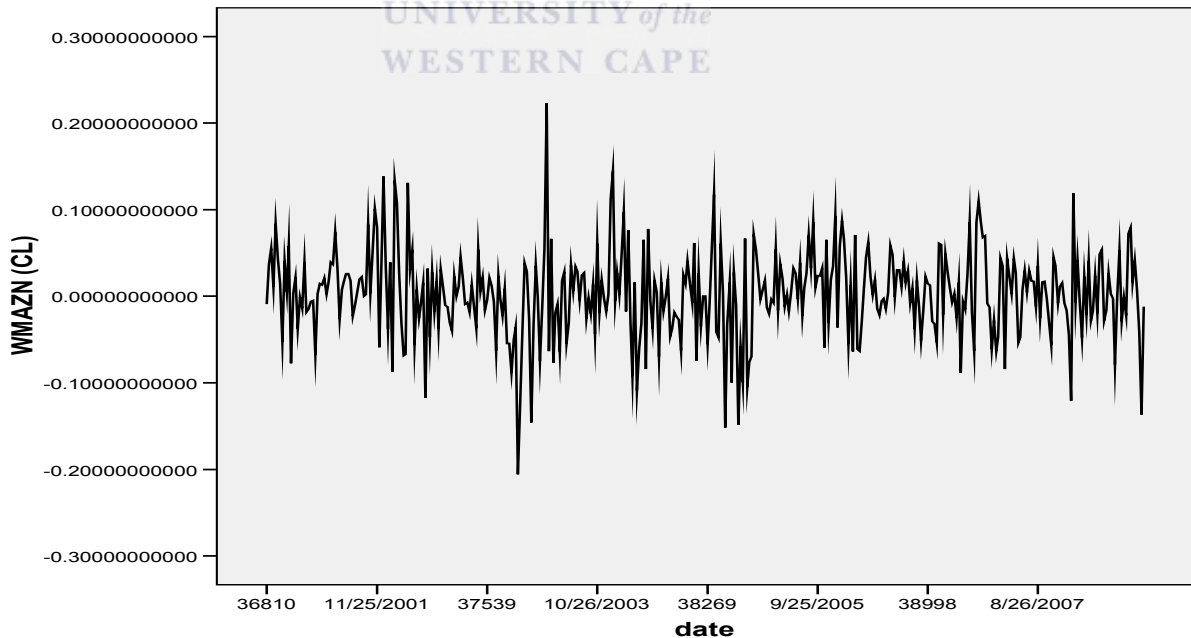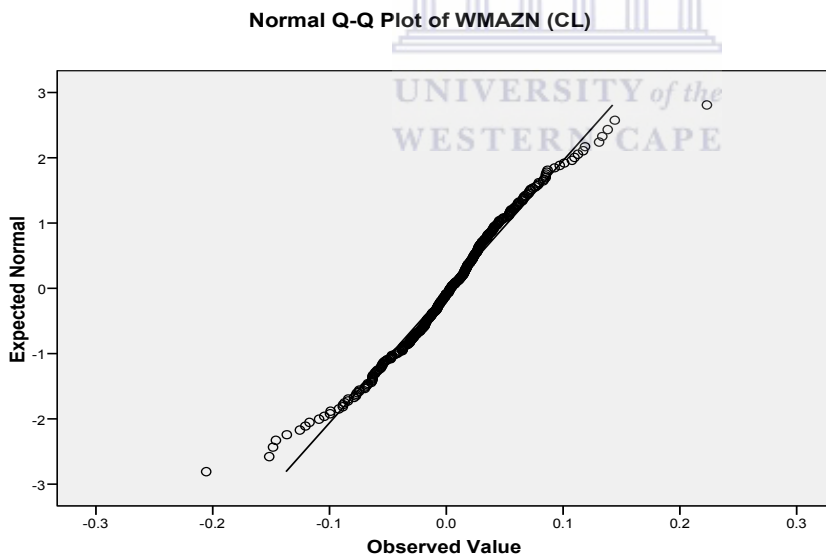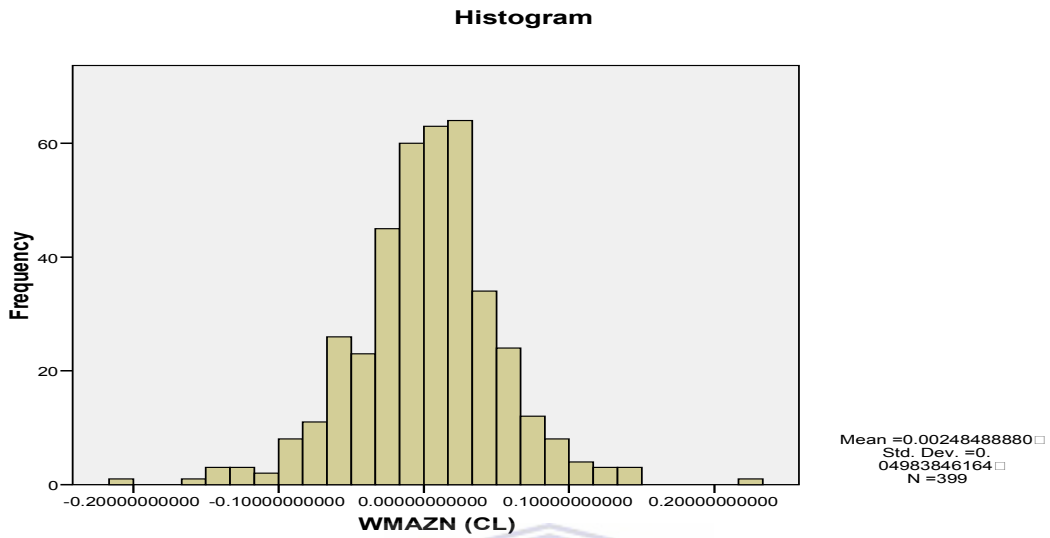
2.9.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-17.69808	0.0000
Test critical values:	1% level	-3.446567	
	5% level	-2.868583	
	10% level	-2.570588	

*MacKinnon (1996) one-sided p-values.

2.10 SASOL (SOL)

2.10.1 Sequence plot
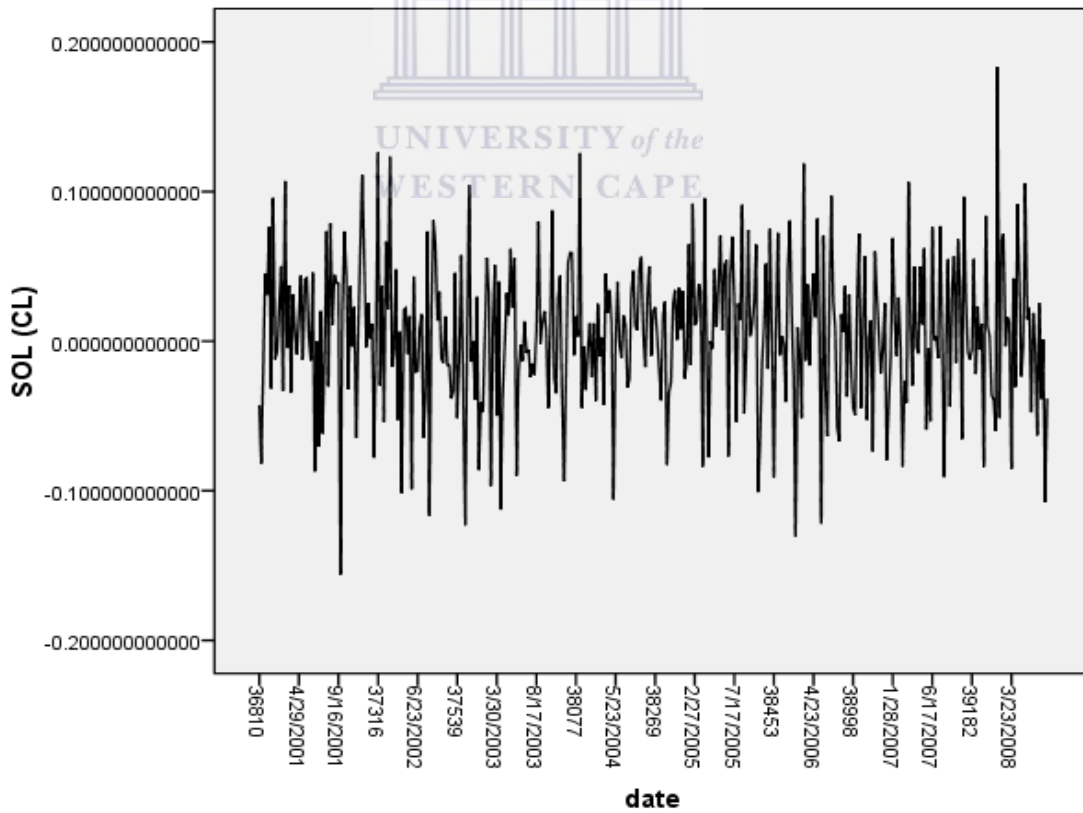
2.10.2 Histogram and normal quantile plot of the data

Histogram

Mean =0.004921563625%
Std. Dev. =0.
049223918640%
N =399
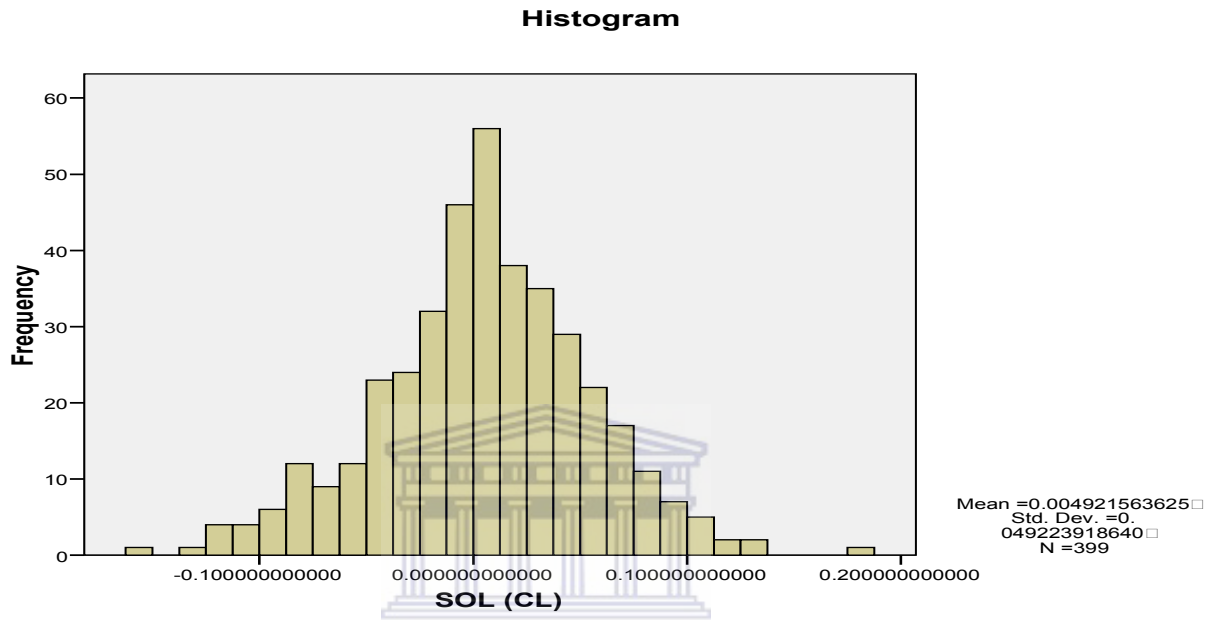
Normal Q-Q Plot of SOL (CL)
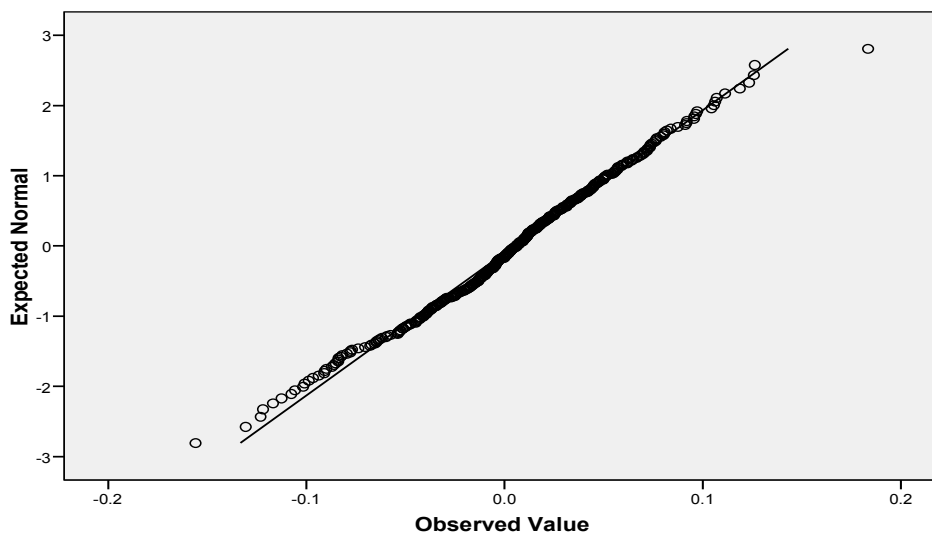
2.10.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-22.28731	0.0000
Test critical values:	1% level	-3.446567	
	5% level	-2.868583	
	10% level	-2.570588	

*MacKinnon (1996) one-sided p-values.

3 MONTHLY DATA

3.1 EURO/US DOLLAR (EURUSD)

3.1.1 Sequence plot

3.1.2 Histogram and normal quantile plot of the data

Histogram

Mean =0.001612974885%
Std. Dev. =0.
029654721250%
N =399

Normal Q-Q Plot of EURUSD (CL)
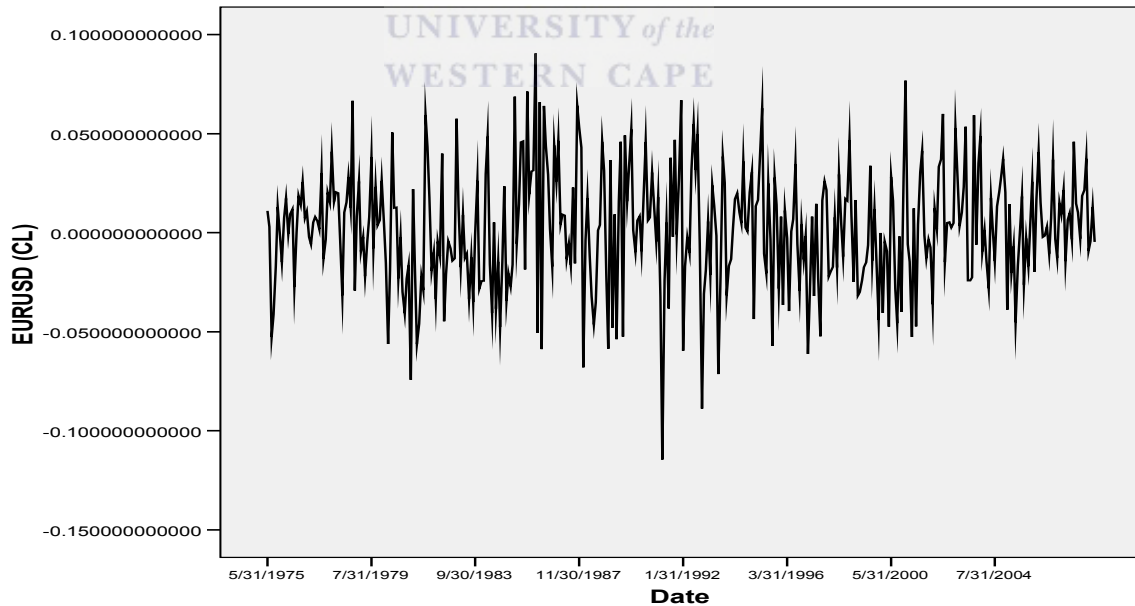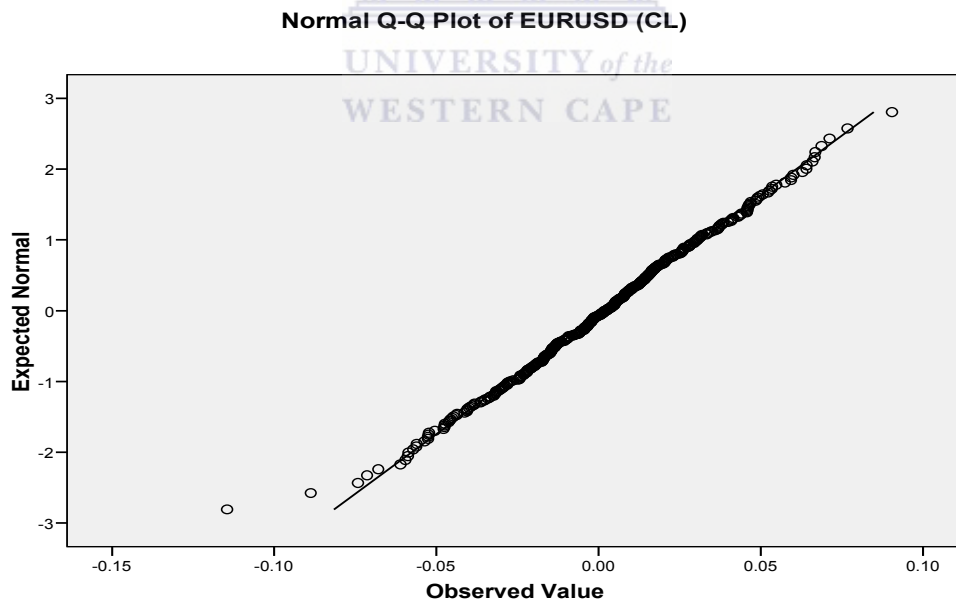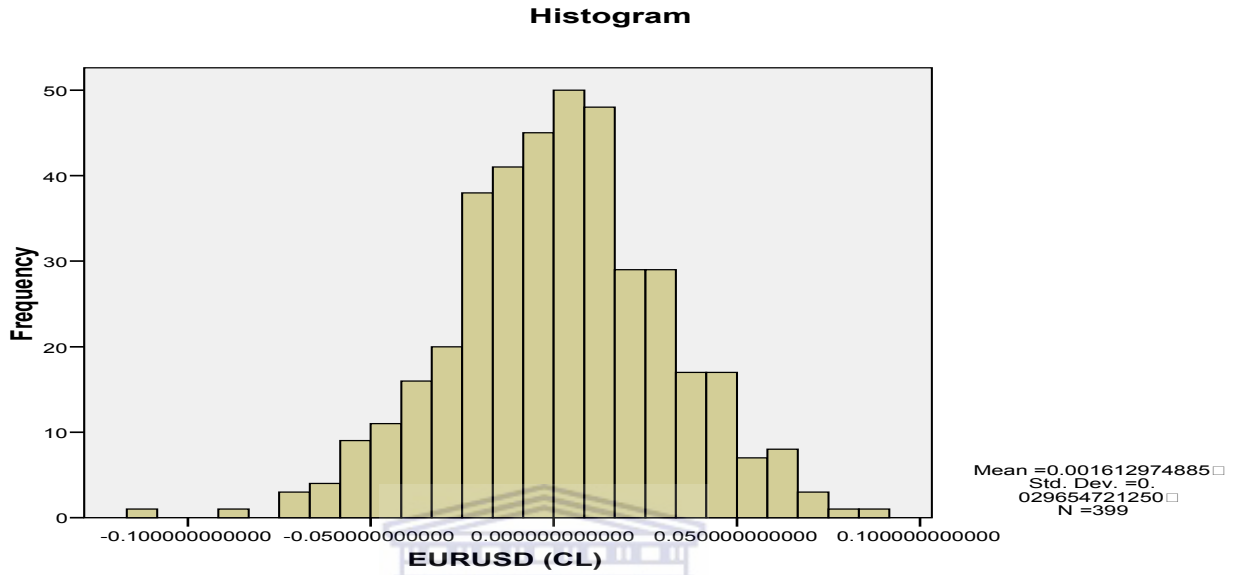
3.1.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-9.107412	0.0000
Test critical values:	1% level	-3.482879	
	5% level	-2.884477	
	10% level	-2.579080	

*MacKinnon (1996) one-sided p-values.

3.2 JSE ALL SHARE TOTAL RETURNS INDEX (J203T)

3.2.1 Sequence plot

119

3.2.2 Histogram and normal quantile plot of the data

Histogram

Mean =0.013262444019%
Std. Dev. =0.
060516864437%
N =157
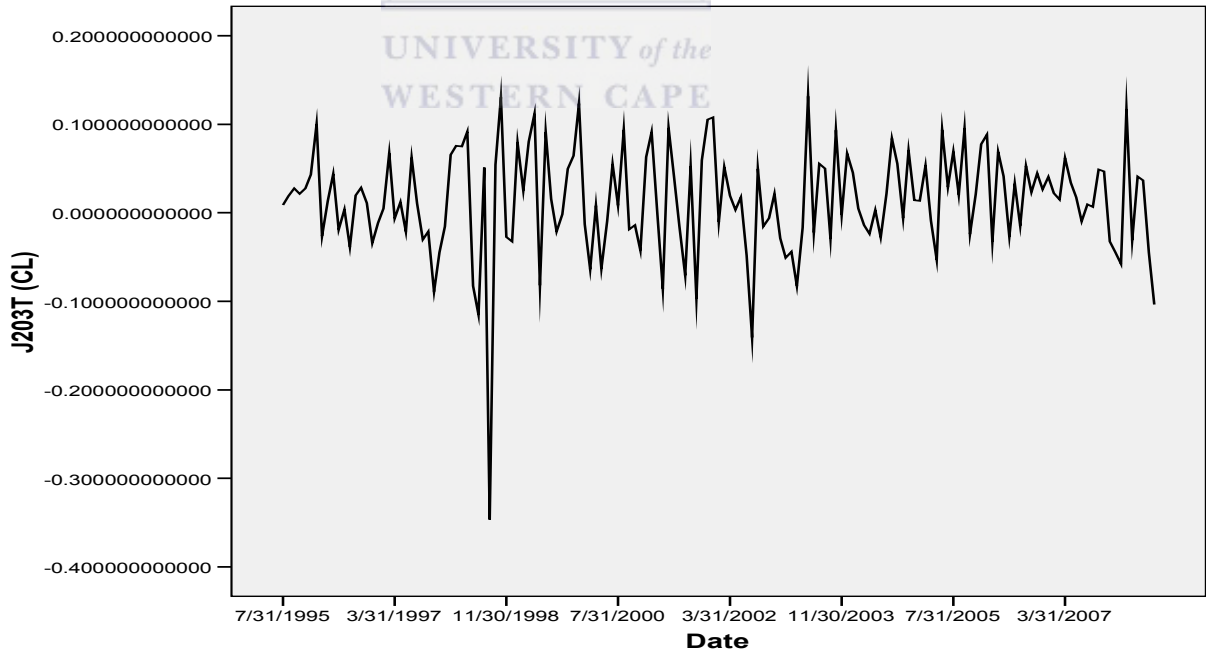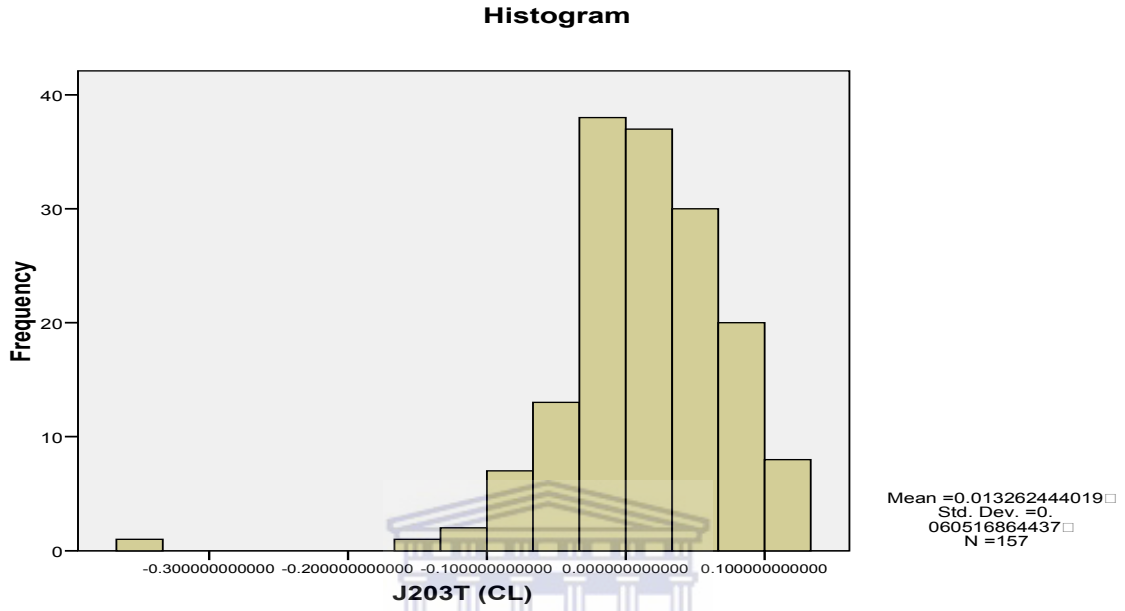
J203T (CL)

Normal Q-Q Plot of J203T (CL)
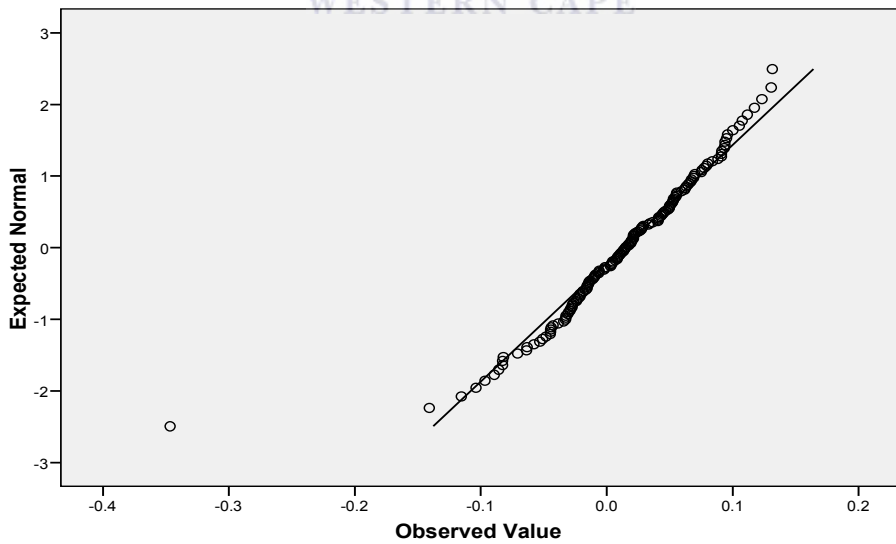
3.2.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-12.56450	0.0000
Test critical values:	1% level	-3.472534	
	5% level	-2.879966	
	10% level	-2.576674	

*MacKinnon (1996) one-sided p-values.

3.3 SOUTH AFRICAN ALL BOND INDEX (ALBI)

3.3.1 Sequence plot

3.3.2 Histogram and normal quantile plot of the data

Histogram

Mean =0.010652961015%
Std. Dev. =0.
019233415352%
N =115

Normal Q-Q Plot of ALBI (CL)
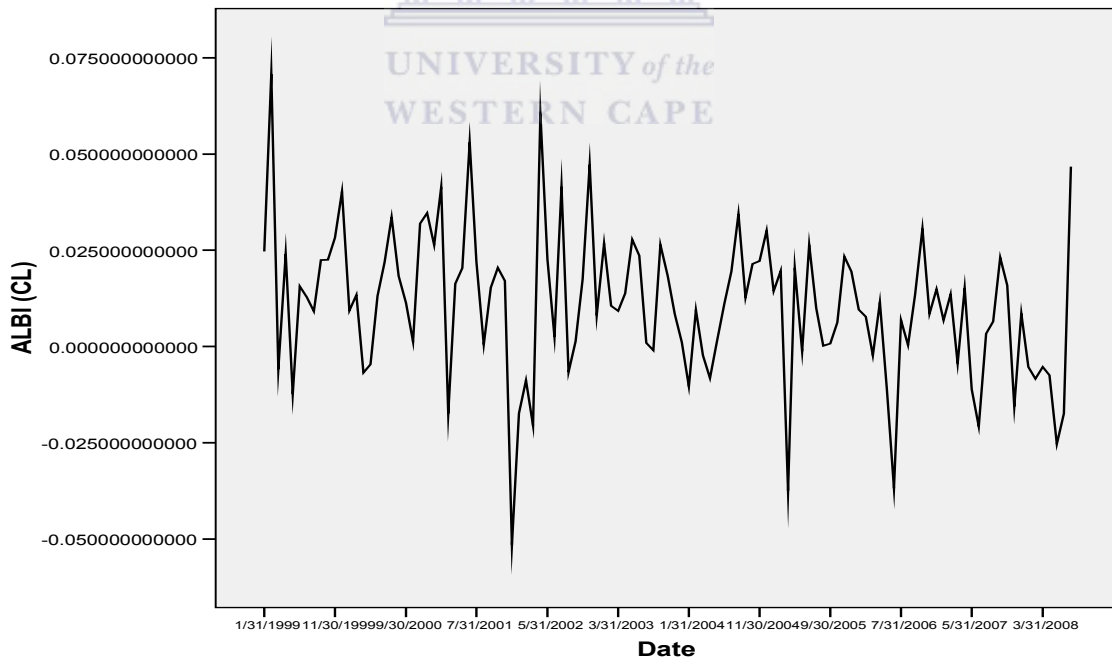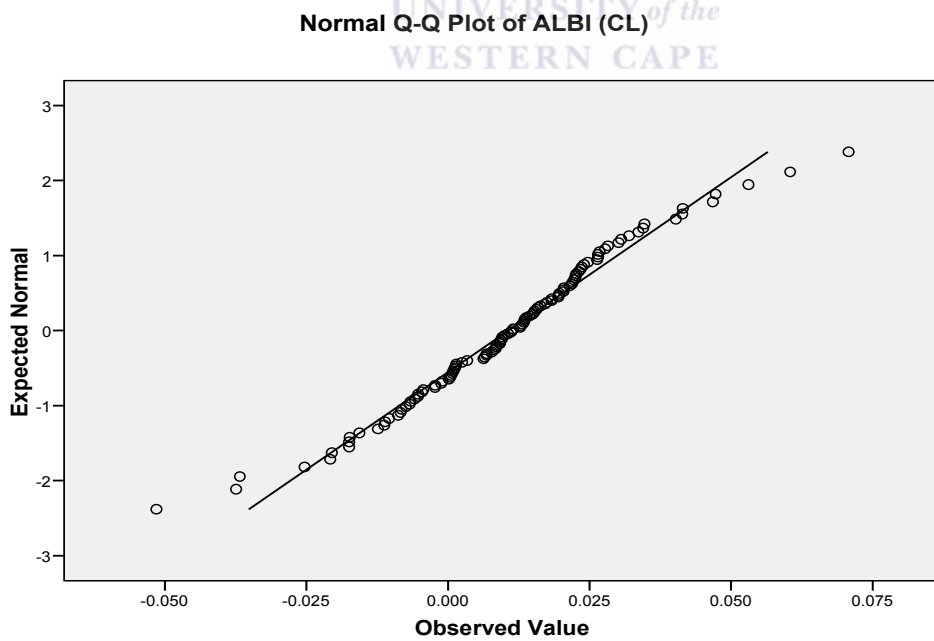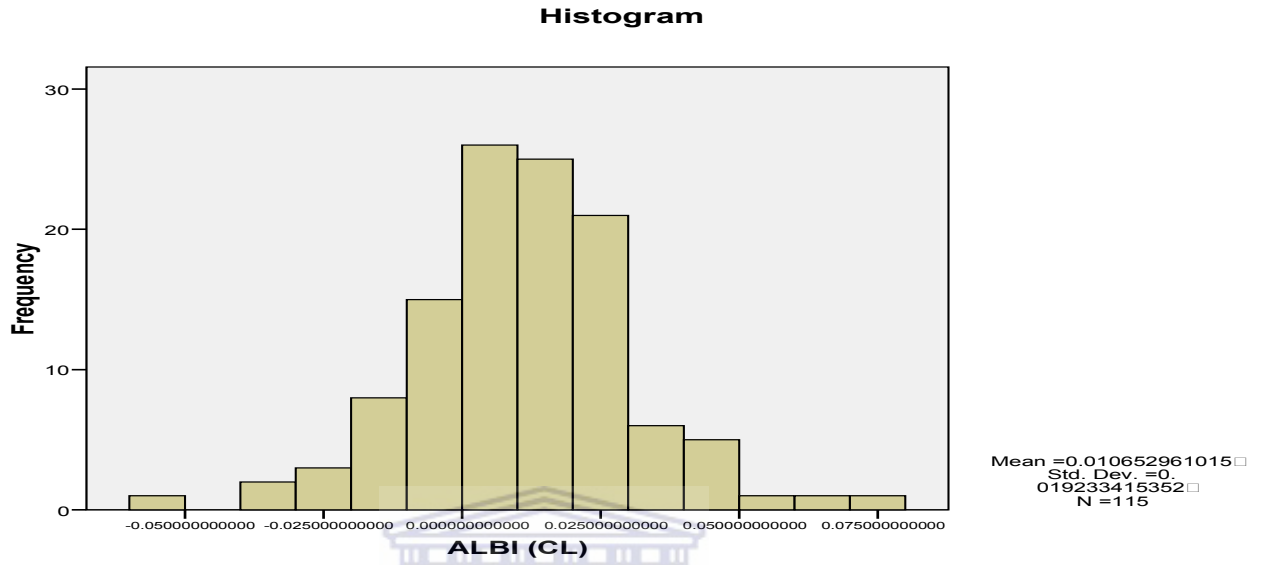
3.3.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-8.864867	0.0000
Test critical values:	1% level	-3.488585	
	5% level	-2.886959	
	10% level	-2.580402	

*MacKinnon (1996) one-sided p-values.

3.4 ANGLO AMERICAN CORPORATION (AGL)

3.4.1 Sequence plot

3.4.2 Histogram and normal quantile plot of the data

Histogram

Mean =0.01477483282%
Std. Dev. =0.
09131503031%
N =399

Normal Q-Q Plot of AGL (CL)
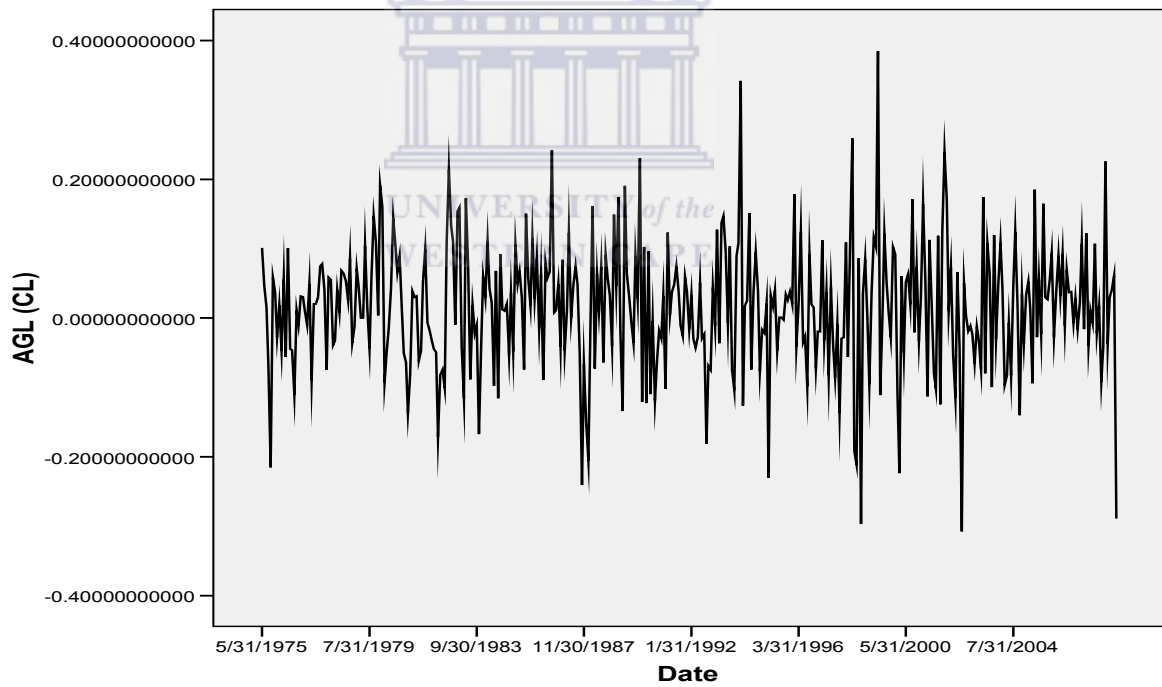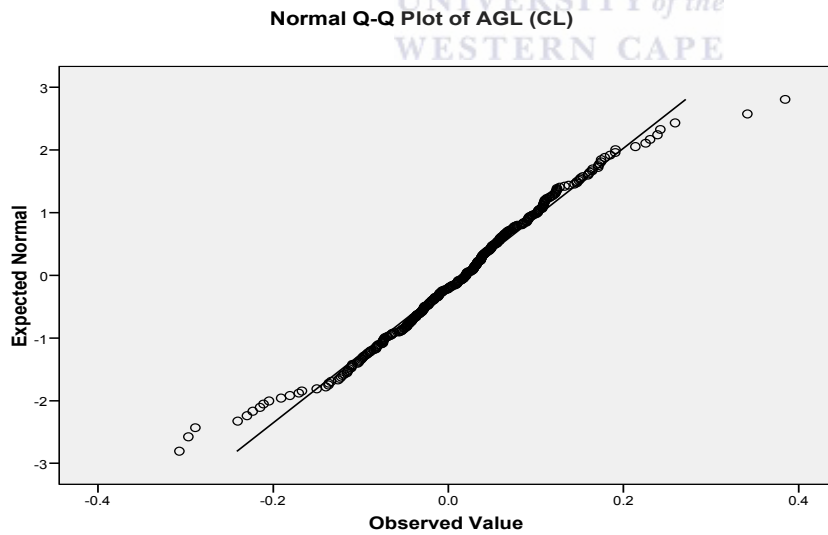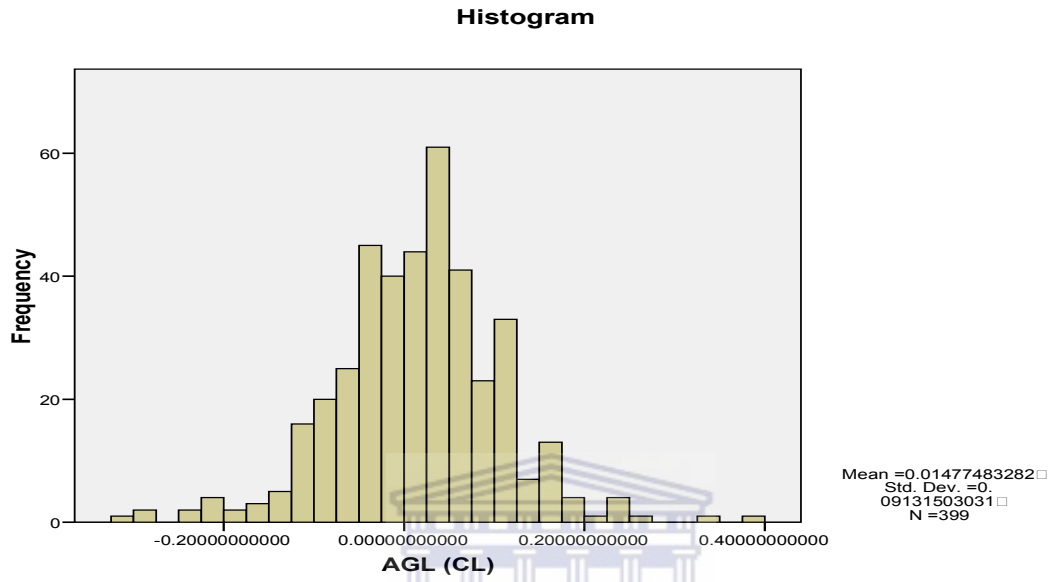
3.4.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-8.838207	0.0000
Test critical values:	1% level	-3.482879	
	5% level	-2.884477	
	10% level	-2.579080	

*MacKinnon (1996) one-sided p-values.

3.5 US DOLLAR GOLD PRICE (DGLDS)

3.5.1 Sequence plot

3.5.2 Histogram and normal quantile plot of the data

Histogram

Mean =0.00489972772%
Std. Dev. =0.
03829396257%
N =182

Normal Q-Q Plot of DGLDS (CL)
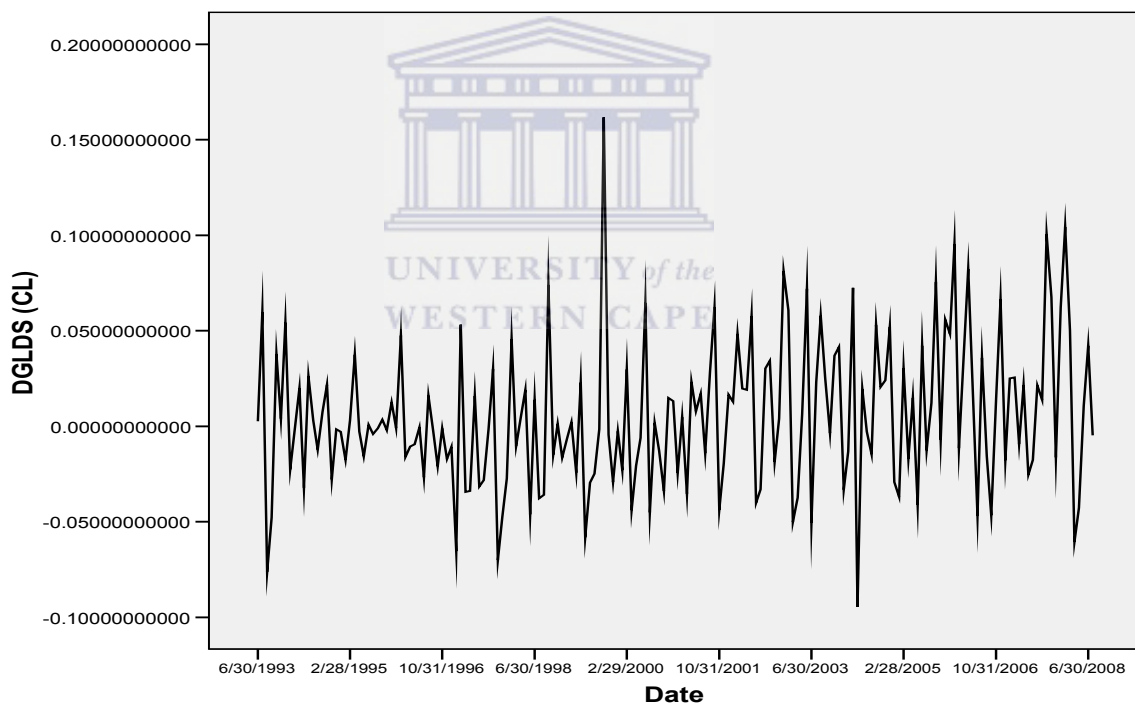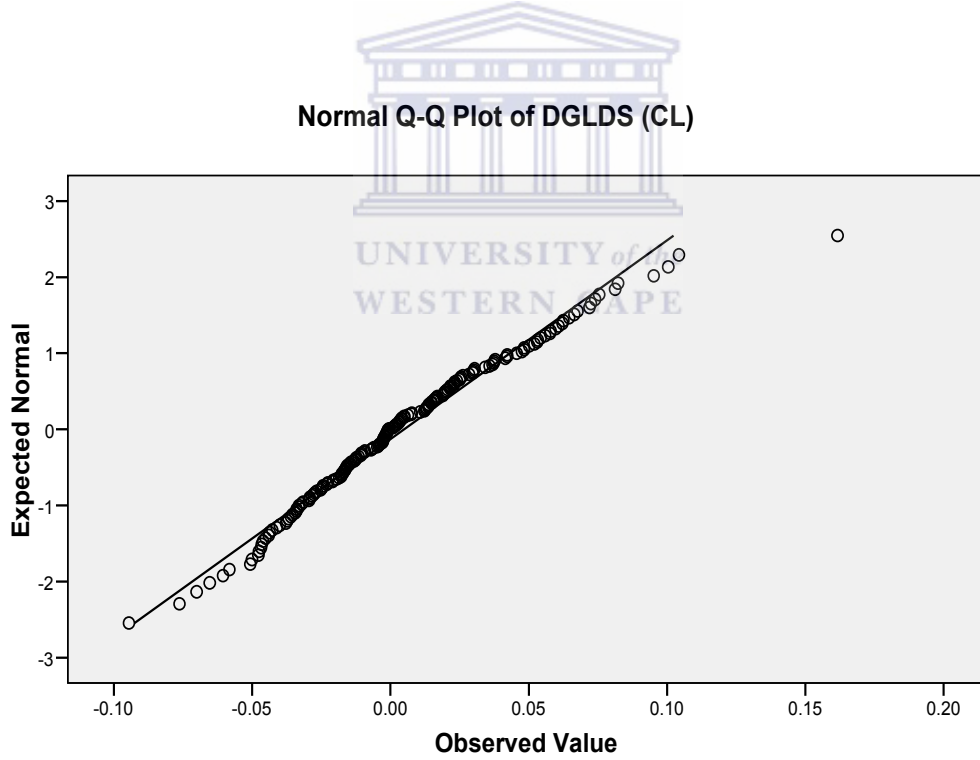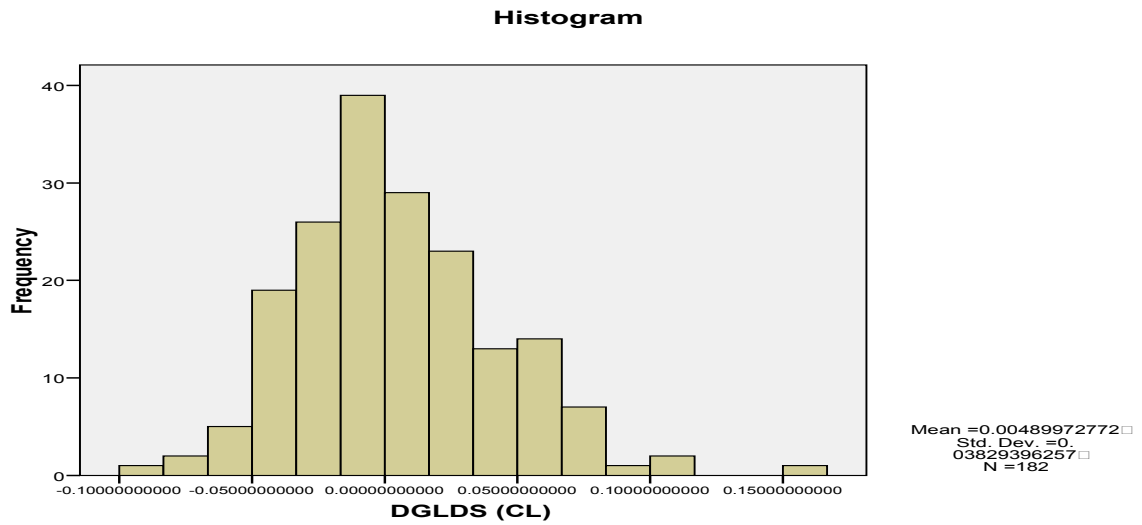
3.5.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-13.84980	0.0000
Test critical values:	1% level	-3.466580	
	5% level	-2.877363	
	10% level	-2.575284	

*MacKinnon (1996) one-sided p-values.

3.6 CRB COMMODITY PRICE INDEX (FCRB)

3.6.1 Sequence plot

3.6.2 Histogram and normal quantile plot of the data

Histogram

Mean =0.002004872339%
Std. Dev. =0.
032086346910%
N =399
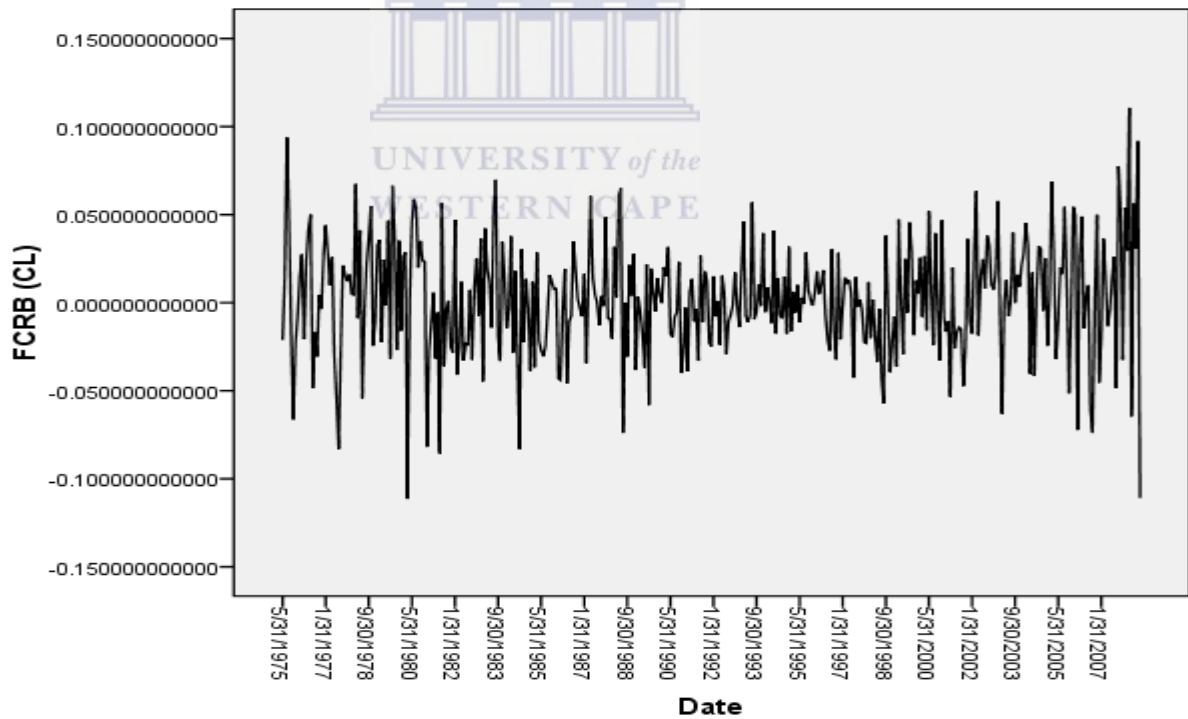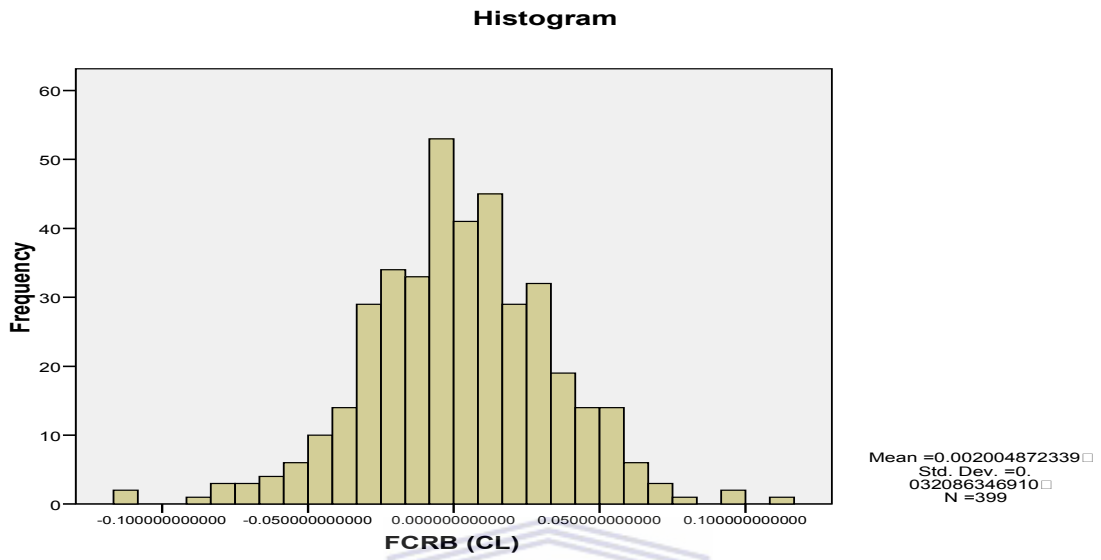
FCRB (CL)

Normal Q-Q Plot of FCRB (CL)
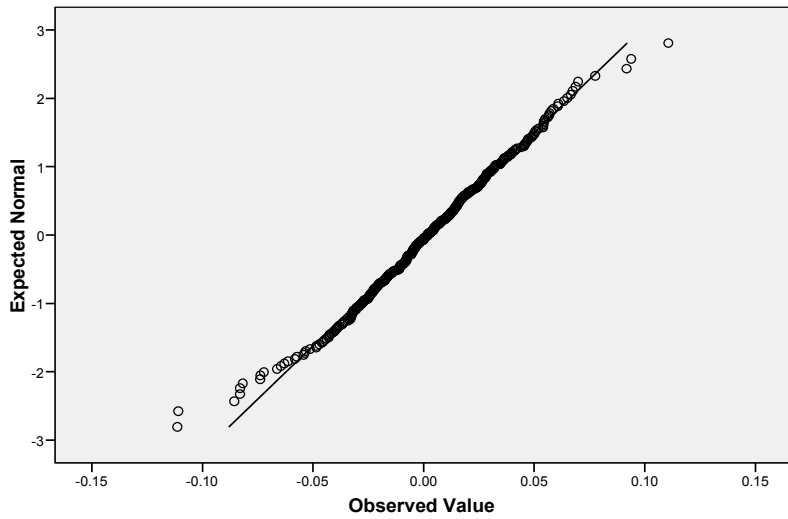
3.6.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-10.59615	0.0000
Test critical values:	1% level	-3.482879	
	5% level	-2.884477	
	10% level	-2.579080	

*MacKinnon (1996) one-sided p-values.

3.7 STANDARD BANK (SBK)

3.7.1 Sequence plot
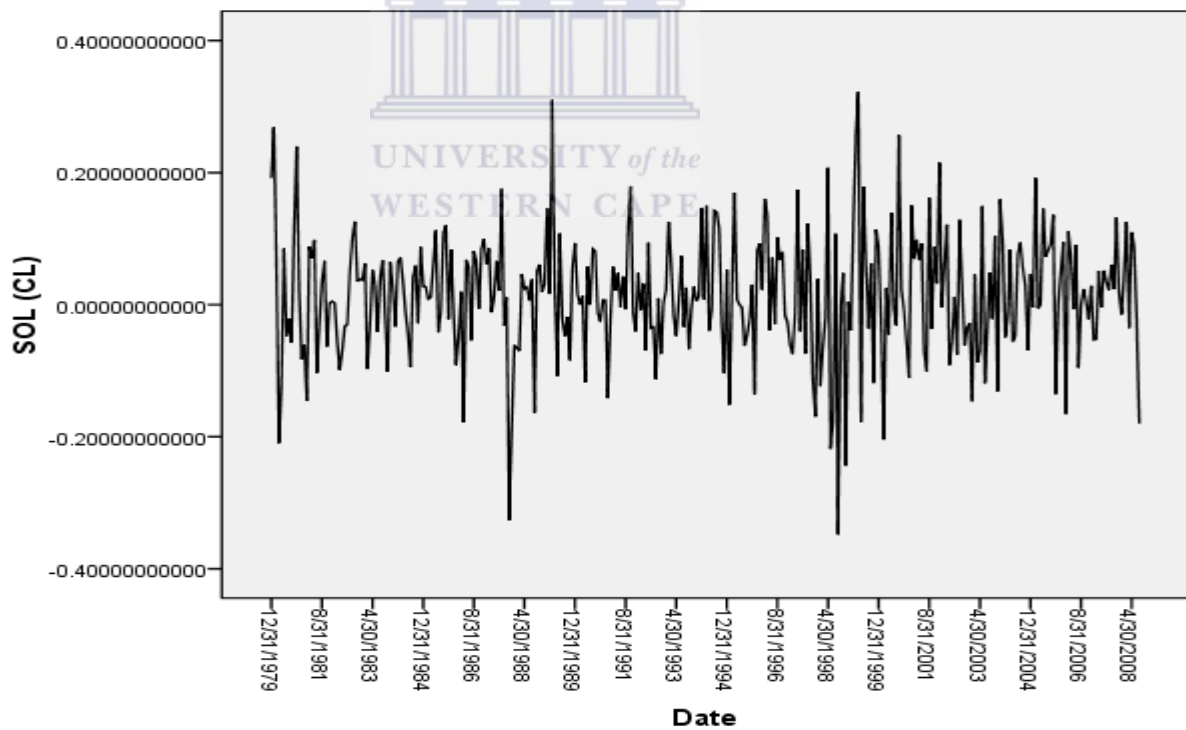
3.7.2 Histogram and normal quantile plot of the data

Histogram

Mean =0.014711197023%
Std. Dev. =0.
080970286393%
N =399

Normal Q-Q Plot of SBK (CL)
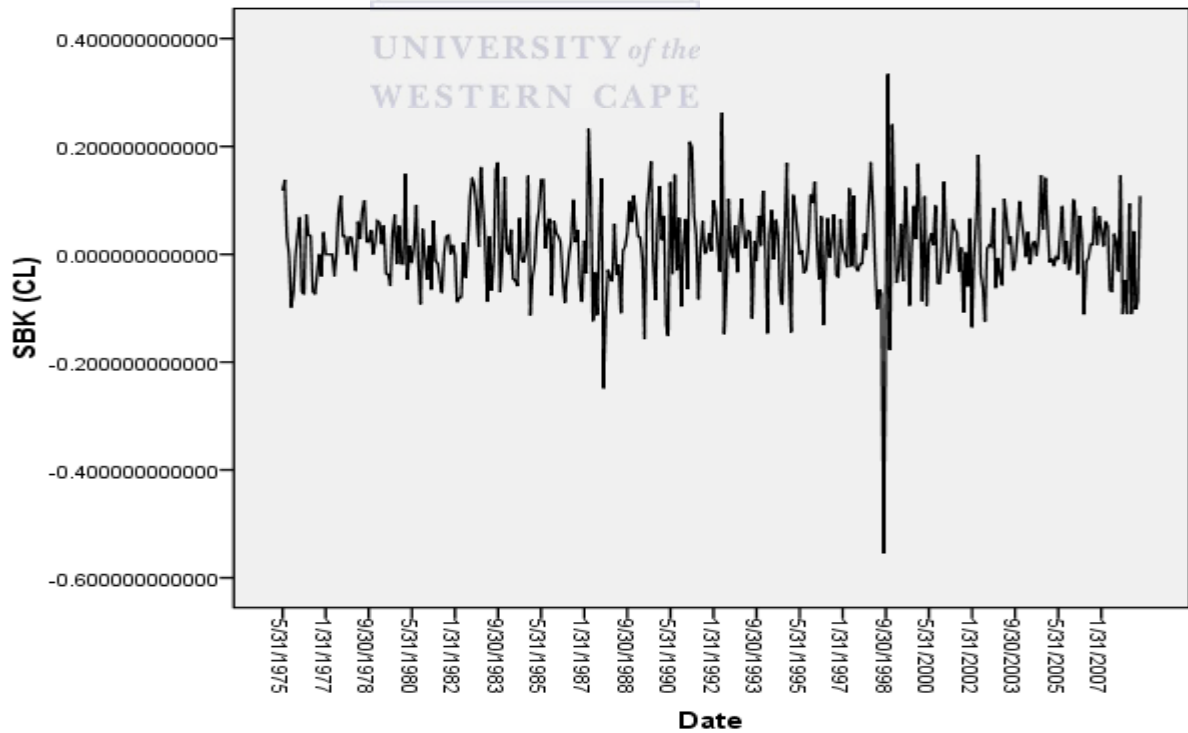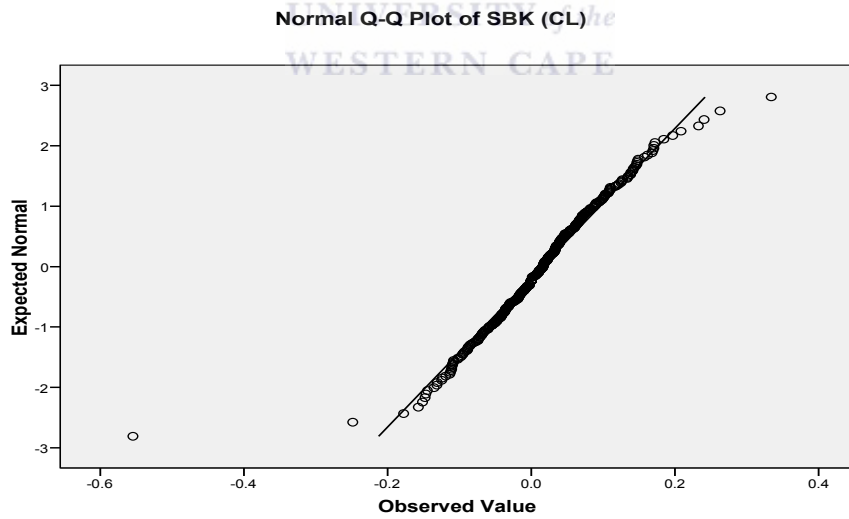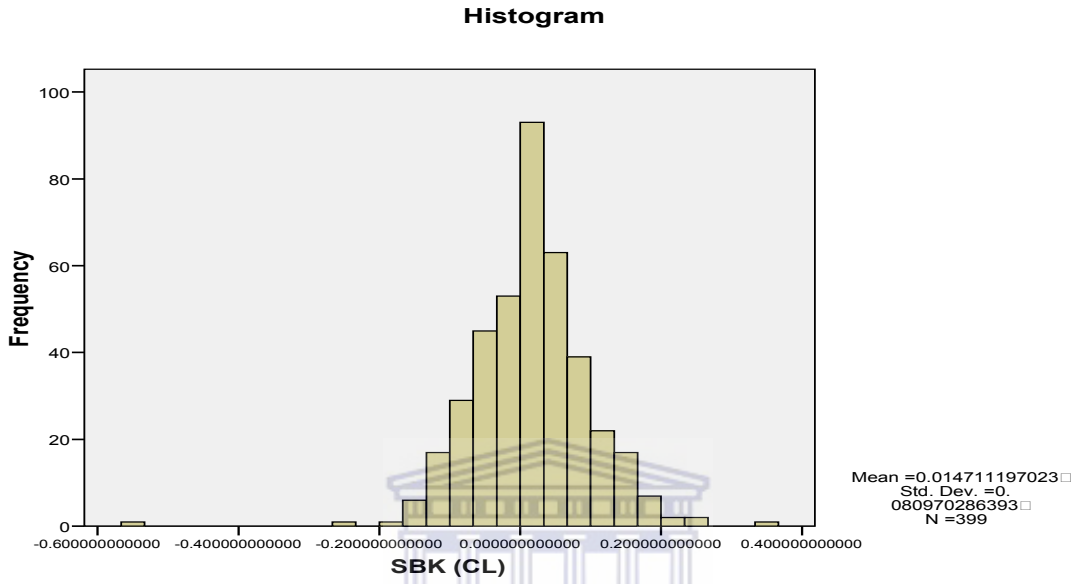
3.7.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-8.816134	0.0000
Test critical values:	1% level	-3.482879	
	5% level	-2.884477	
	10% level	-2.579080	

*MacKinnon (1996) one-sided p-values.

3.8 SASOL (SOL)

3.8.1 Sequence plot

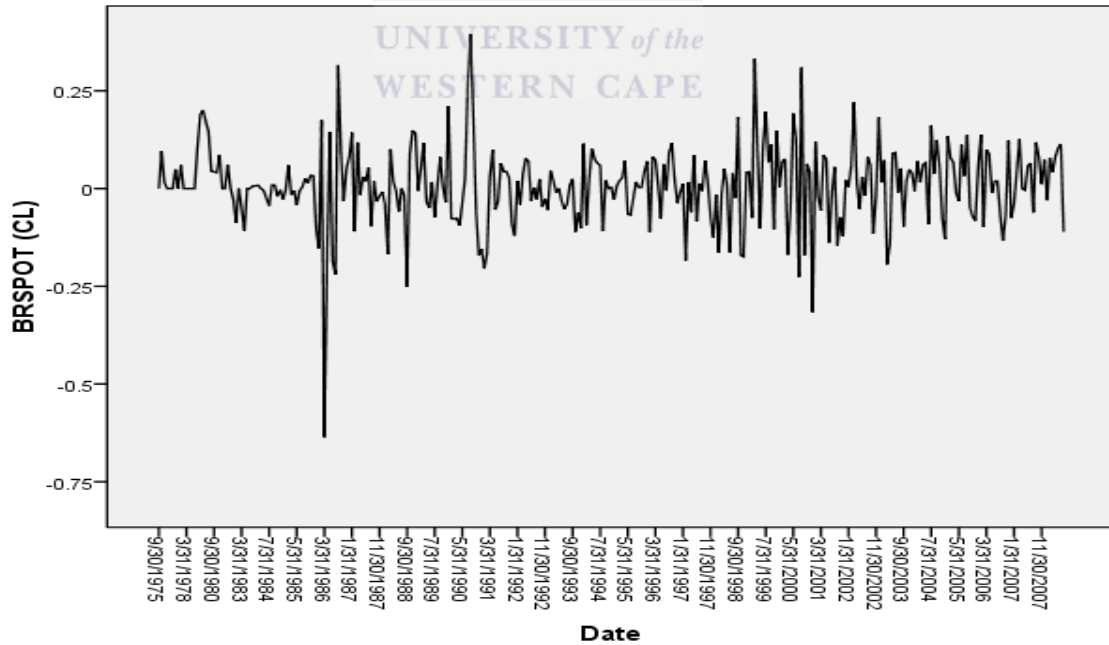**3.8.2 Histogram and normal quantile plot of the data**

Histogram

Mean =0.01452829038%
Std. Dev. =0.
09298029794%
N =344
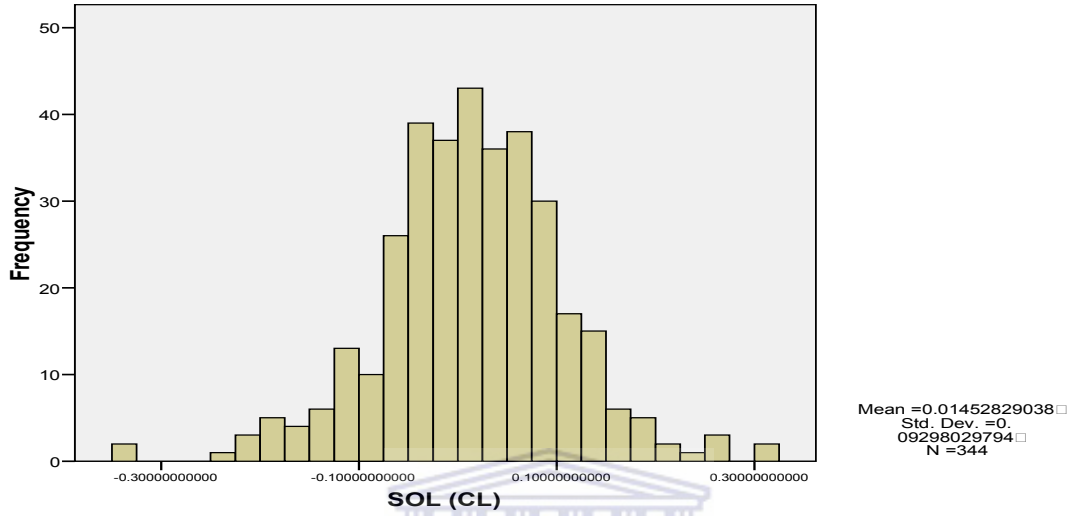
Normal Q-Q Plot of SOL (CL)
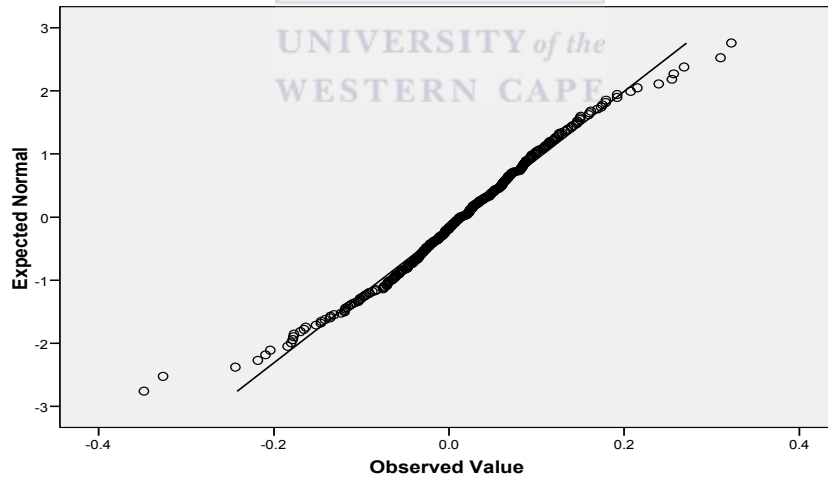
3.8.3 Augmented Dickey- Fuller unit root test Eviews output results

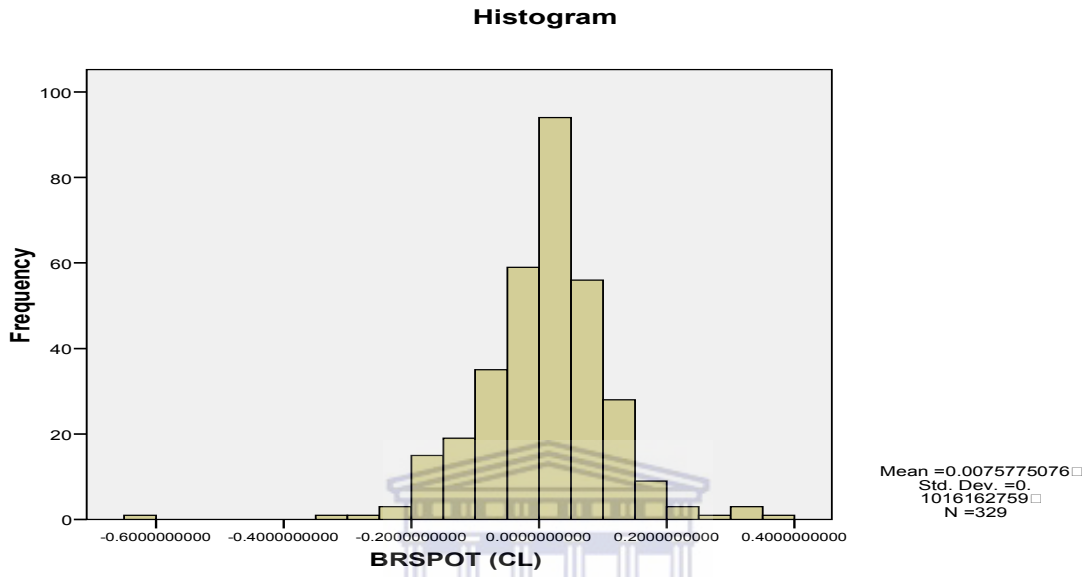|		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-17.92814	0.0000
Test critical values:	1% level	-3.449220	
	5% level	-2.869750	
	10% level	-2.571213	

*MacKinnon (1996) one-sided p-values.

3.9 BRENT SPOT OIL PRICE (BRSPOT)
3.9.1 Sequence plot
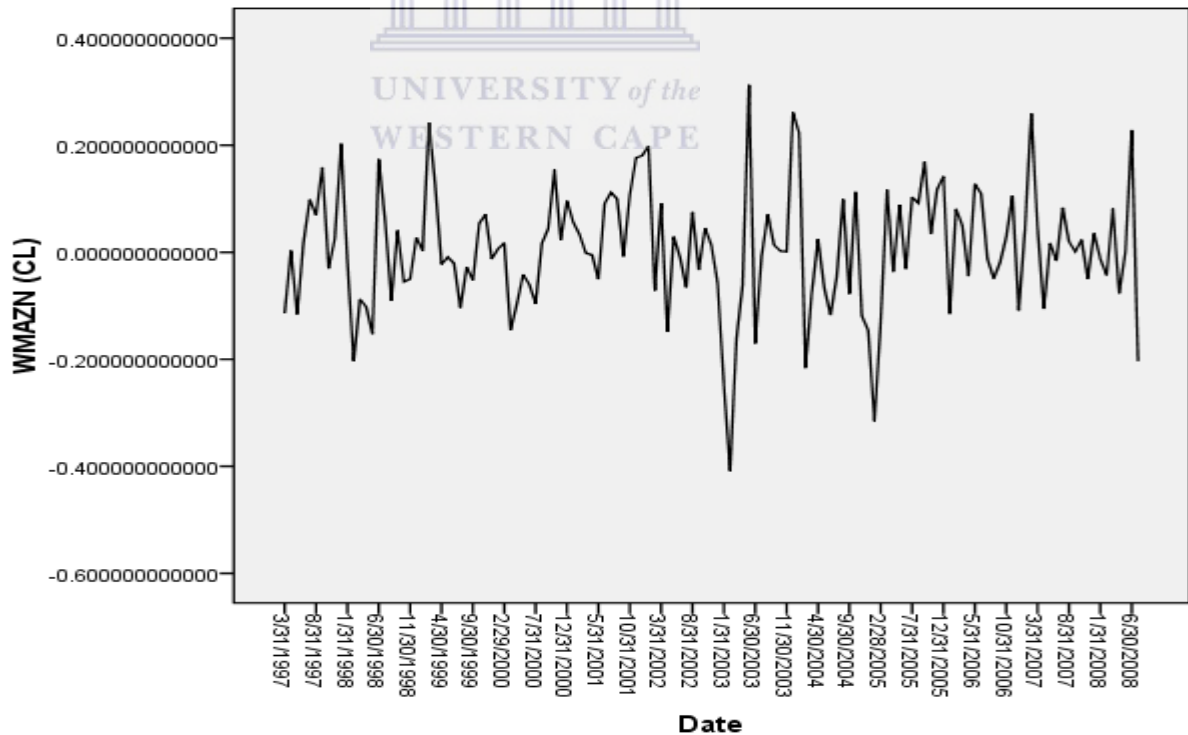
3.9.2 Histogram and normal quantile plot of the data

Histogram

Mean =0.0075775076%
Std. Dev. =0.
1016162759%
N =329

Normal Q-Q Plot of BRSPOT (CL)
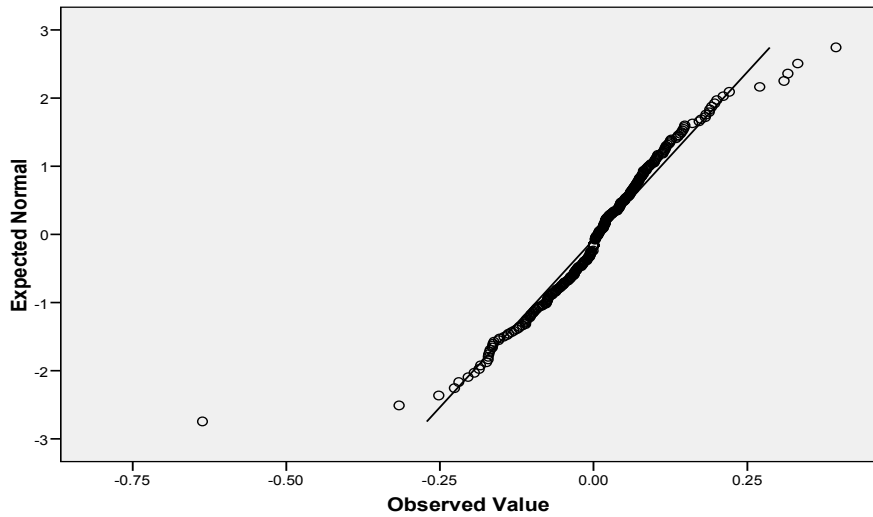
3.9.3 Augmented Dickey- Fuller unit root test Eviews output results

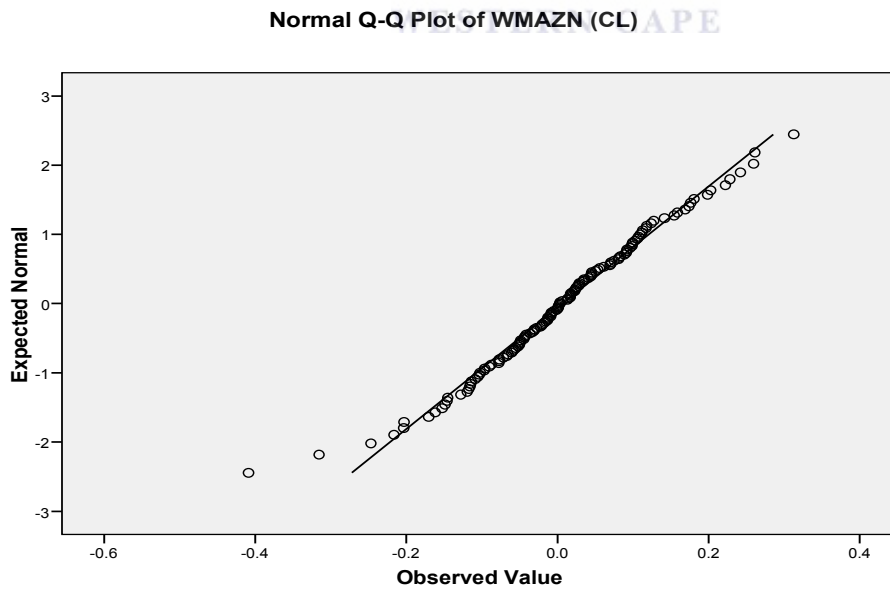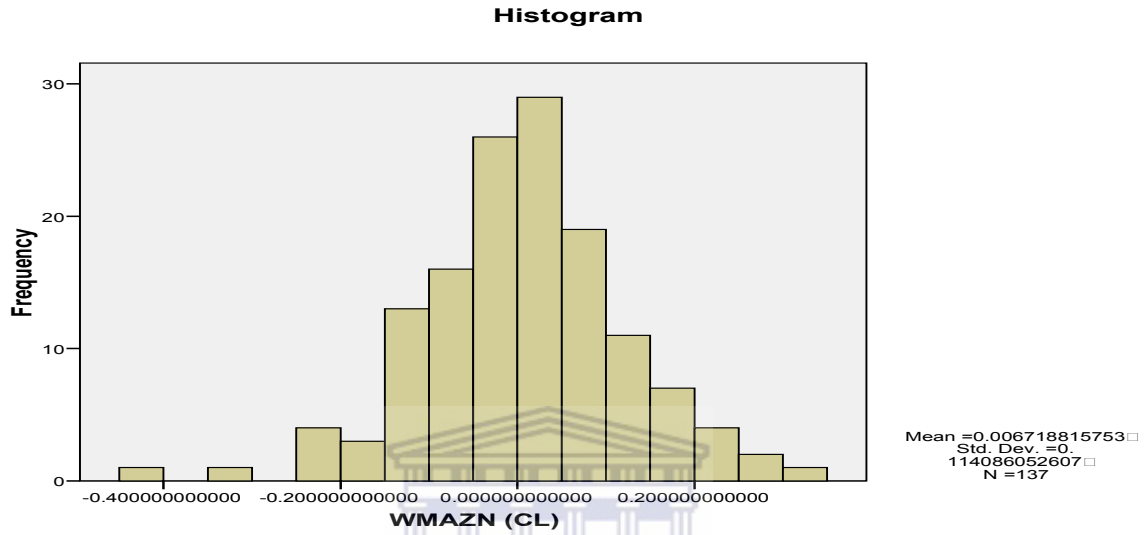|		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-17.29689	0.0000
Test critical values:	1% level	-3.450099	
	5% level	-2.870137	
	10% level	-2.571420	

*MacKinnon (1996) one-sided p-values.

3.10 SOUTH AFRICAN WHITE MAIZE NEAR FUTURE (WMAZN)

3.10. Sequence plot

3.10.2 Histogram and normal quantile plot of the data

Histogram

Frequency vs WMAZN (CL)

Mean =0.006718815753%
Std. Dev. =0.114086052607%
N =137

Normal Q-Q Plot of WMAZN (CL)

3.10.3 Augmented Dickey- Fuller unit root test Eviews output results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-9.422120	0.0000
Test critical values:	1% level	-3.478911	
	5% level	-2.882748	
	10% level	-2.578158	

*MacKinnon (1996) one-sided p-values.