PURCHASING POWER PARITY BETWEEN BOTSWANA AND SOUTH AFRICA: A COINTEGRATION ANALYSIS

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REFERENCE
PURCHASING POWER PARITY BETWEEN BOTSWANA AND SOUTH AFRICA: A COINTEGRATION ANALYSIS

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Abstract

This paper tests the Purchasing Power Parity (PPP) hypothesis for Botswana and South Africa using cointegration analysis. The data used are the spot exchange rate between the Botswana and South Africa (Rand and Pula) and their Consumer Price Indices. The cointegration test indicated that, for Botswana and South Africa, the Purchasing Power Parity hypothesis fails to hold in the long run. This means that the exchange rate between the two countries do not appreciate/depreciate to equalize the ratio of the two countries’ price levels. An error-correction model could not be constructed because the variables were not cointegrated showing that there is no stable long-term relationship between them.
KEYWORDS

Purchasing Power Parity
Law of one price
Exchange rates
Consumer price index
Cointegration
Stationarity
Error-correction
Mean reversion
Deviation
Temporal aggregation
DECLARATION

I declare that *Purchasing Power Parity between Botswana and South Africa: A cointegration Analysis* is my own work, that it has not been submitted for any degree or examination in any other university, and that all the sources I have used or quoted have been indicated and acknowledged by complete reference.

KATSO TSHIPINARE

Signed.................................. Date....................................
CHAPTER ONE

1. Introduction

The theory of Purchasing Power Parity (PPP) states that exchange rates between currencies are in equilibrium when their purchasing power is the same in each of the two countries (Antweiler, 2005). This means that the exchange rate between countries should equal the ratio of the countries’ price levels of a fixed basket of goods and services. When a country’s domestic price level is increasing more rapidly than its major trading partners (i.e., a country experiencing inflation), that country’s exchange rate must depreciate in order to return to purchasing power parity.

Broadly, Purchasing Power Parity is a theory of exchange rate determination and a way to compare the average costs of goods and services between countries. The theory further assumes that the actions of importers and exporters, motivated by cross country price differences induce changes in the spot exchange rate. In another vein, purchasing power parity suggests that transactions on a country’s current account affect the value of the exchange on the foreign exchange market (Antweiler, 2005). This contrasts with the interest rate parity theory, which assumes that the actions of investors, whose transactions are recorded on the capital account (now called the financial account), induce changes in the exchange rate.

Purchasing power parity is based on the extension and variation of the law of one price as applied to the aggregate economy. To explain the theory it is best, first, to review the idea behind the law of one price. The law of one price states that, in the absence of transportation and other transaction costs, competitive markets will equalize the price of an identical good in two countries when the prices are expressed in the same currency. If it makes sense from the law of one price that identical goods should sell for identical prices in different markets, then the law ought to hold for all identical goods sold in both markets. However, there is a slight twist added to the law of one price to convert it to the purchasing power parity theory. In the law of one price, goods arbitrage in a particular
product is expected to affect the prices of the goods in the two markets. The twist that is included in the purchasing power parity theory is that arbitrage, occurring across a range of goods and services in the market basket, will affect the exchange rate rather than the market prices (goods prices), (Antweiler, 2005).

The Purchasing Power Parity relationship becomes a theory of exchange rate determination by introducing assumptions about the behaviour of importers and exporters in response to changes in the relative costs of national market baskets. Recall, in the story of the law of one price, when the price of a good differed between two country's markets, there is an incentive for profit-seeking individuals to buy the good in the low price market and resell it in the high price market. Similarly, if a market basket, containing many different goods and services, costs more in one market than another, it is expected, likewise that profit-seeking individuals will buy the relatively cheaper goods in the low cost market and resell them in the higher priced market. If the law of one price leads to the equalization of the prices of a good between two markets, then it seems reasonable to conclude that purchasing power parity, describing the equality of market baskets across countries, should also hold, (Suranovic, 1999).

There are two versions of purchasing power parity theory – absolute and relative purchasing power parity. Absolute purchasing power parity theory states that a basket of goods costs the same domestically and abroad if the goods prices are converted into a common currency (Suranovic, 1999). In other words, absolute purchasing power parity theory postulates that the purchasing power of money should be equal between countries.

By contrast, relative purchasing power parity theory does not compare domestic and foreign levels of purchasing power, but rather focuses on changes in this purchasing power. Relative purchasing power parity theory, therefore, states that the inflation rate differentials between two countries or regions are offset through inverse changes in the nominal exchange rate so that the purchasing power ratio between the two remains constant. It, therefore, follows that the validity of absolute purchasing power parity
implies the validity of relative purchasing power parity theory, but not vice versa (Bundesbank Monthly Report, June 2004).

1.1 Statement of the Problem

Although simple in theory, the real world is characterised by a number of complications such as differentiated products, tastes and wide range of costs, which created considerable problems for economists testing the theory empirically in the post-Bretton Woods era. With the move to flexible exchange rates in the early 1970s, it was generally assumed that the exchange rate would quickly adjust to changes in relative price levels. In determining the validity of purchasing power parity, the results from several empirical studies have been mixed. Few, if any, studies have found evidence for the theory in the short run; while the results on Purchasing Power Parity in the long run have been more varied.

In the wake of these studies, this paper conducts a cointegration analysis of purchasing power parity for Botswana and South Africa to determine the validity of the theory in the long run. The paper also intends to move beyond the purely statistical issues of whether the real exchange rates contains a unit root, to focus on the purchasing power parity puzzle summarized by Rogoff (1996), that involves the reconciliation of slow speeds of convergence to purchasing power parity and the high short-term volatility of real exchange rates.

The empirical inconclusiveness of purchasing power parity, which is well-known in the literature, to an extent that it has been regarded as the purchasing power parity puzzle by Rogoff (1996), has motivated many economists to return to the examination of the assumptions underlying the purchasing power parity theory. The main problem, in this study therefore, is how to reconcile the excessively high volatility of real exchange rates in the short term with the seemingly “long” half-life of deviation from purchasing power parity.
1.2 Research Questions

This study aims to answer the following key questions;

- What would the empirical test of the purchasing power parity theory for Botswana Pula and South African Rand show?
- What impact does frequency of data have on the speed of adjustment of exchange rates?

1.3 The Purpose of the Study

The purpose of this paper is to empirically test the purchasing power parity hypothesis for Botswana and South Africa using cointegration and error-correction modelling. Specific attention will be given to the problems of data pollution and sources of deviation from purchasing power parity.

1.4 Organisation of the Study

The remainder of the paper is organised as follows; chapter two gives theoretical and empirical literature that relate to this study. Chapter three outlines the methodology adopted and specifies the model to be used. The empirical analysis and discussion of the results will be provided in chapter four before concluding in chapter five.
CHAPTER TWO

2. Literature Review

The revival of interest in purchasing power parity has been documented in a number of comprehensive and influential surveys. This section aims to provide an overview of academic debate on the deviations from parity, the speed of adjustment, the empirical evidence and potential pitfalls.

2.1 The academic debate on the deviations from the Purchasing Power Parity

Over the last one and half decades, much academic research has examined the empirical validity of the purchasing power parity theory. These academic researchers used various techniques to empirically test this theory and different results were obtained depending on the methodology applied. These mixed results suggested that it is not easy to prove the purchasing power parity theory (Sarno and Taylor, 2002). The Balassa-Samuelson effect, which has gained considerable empirical support and continues to be highly influential, is one possible reason why the theory is comparatively difficult to prove (Bundesbank Monthly Report, June 2004). According to this hypothesis, the purchasing power parity theory applies only to some goods that are internationally tradable. If the productivity level in a country’s tradables sector increases, according to the Balassa-Samuelson hypothesis, the prices of these goods will not fall as they are determined by the conditions of competition on the world market. Put differently, as the cost of resources used in producing non-tradables rise, the relative prices of non-tradables rise, thus inducing an appreciation of the real exchange rate.

Besides the Balassa-Samuelson hypothesis, there are other hypotheses identified, which seek to explain deviations from purchasing power parity and whether those deviations are transitory or permanent. One of the factors identified is the role of news and, in particular, the response of exchange rates to new developments (Darius and Williams, 1999). The reaction of exchange rates to news formed the basis of the Dornbusch (1976)
model of overshooting exchange rates. The basis of the theory is that news is processed faster in exchange rates than in goods prices. The rationale has been that prices are determined in the commodity markets, where signals tend to be digested very slowly. This is compared to exchange rates, which are determined in the auction markets where news is quickly assimilated.

Based on the above hypothesis, deviations from purchasing power parity are largely the results of price stickiness. Hence, it is logical to conclude that such deviations should disappear overtime as prices adjust to a new equilibrium given nominal disturbances. In the case where a real disturbance occurs and the price indices contain different goods and weights in various countries, the purchasing power parity deviations may decline, but might not disappear altogether (Suranovic, 1999). The role of news as a source of deviation from purchasing power parity is likely to be relevant in the developed countries. This is related to the fact that in developed countries, where financial markets systems are sophisticated, exchange rates movements are usually influenced by developments in the asset markets.

Another explanation for deviations from the purchasing power parity, which is gaining increasing popularity, is the idea of partial pass-through of exchange rates. This hypothesis has been analyzed and developed by a number of researchers, including Froot and Rogoff (1995) and Freenstra and Kendall (1997). The basic tenet of the proposition is that under conditions of imperfect competition, firms involved in the export of goods and services may adjust prices by less than the complete change in the exchange rate. A firm, which wants to maintain market share, may decrease profit margins in order to absorb some of the price increases associated with a currency appreciation. Hence, only a certain percentage of the price increase associated with the currency change is passed through to the importer price. The above discussion mainly highlights the explanation for short term deviations from PPP, and in the long run such deviations should diminish significantly. In contrast, most empirical studies indicate that in a number of cases these deviations persist in the long run.
The other reason why purchasing power parity fails to hold is transport costs and trade restrictions (Suranovic, 1999). Since the purchasing power parity theory is derived from the law of one price, the same assumptions are needed for both theories. The law of one price assumed that there are no transportation costs and no differential taxes applied between the two markets. This means that there can be no tariffs on imports or other types of restrictions on trade. However, in the real world transport costs and trade restrictions do exist and this would tend to drive prices for similar goods apart. Transport costs should make goods cheaper in the exporting market and more expensive in the importing market. Similarly, an import tariff would drive a wedge between the prices of an identical good in two trading countries’ markets, raising it in the import market relative to the export market price. Thus, the greater are transportation costs and trade restrictions between countries, the less likely for the costs of market baskets to be equalized.

The concept of information asymmetry is also not trivial. The law of one price assumes that individuals have good, even perfect, information about the prices of goods in the other markets (Suranovic, 1999). Only with this knowledge will profit-seekers begin to export goods to the high priced market and import goods from the low priced market. Consider a case in which there is imperfect information or only a small group of traders know about a price discrepancy and that group is unable to achieve the scale of trade needed to equalize the prices for that product. In either case, traders without information about price differences will not respond to the profit opportunities and thus prices will not be equalized. Thus, the law of one price may not hold for some products, which would imply that purchasing power parity would not hold either.

In the purchasing power parity theory, it is the behaviour of profit-seeking importers and exporter that forces the exchange rate to adjust to the purchasing power parity level (Suranovic, 1999). These activities would be recorded on the current account of a country’s balance of payments. Thus, it is reasonable to say that the purchasing power parity theory is based on current account transactions. This contrasts with the Interest Rate Parity (IRP) theory in which the behaviour of investors seeking the highest rates of
return on investment motivates adjustment in the exchange rate. Since investors are trading assets, these transactions would appear on a country’s capital account of its balance of payments. Thus, the interest rate parity is based on capital account transactions. For the law of one price and, subsequently, purchasing power parity to hold, both current and capital accounts transactions must be driving the exchange rate to the same equilibrium level.

2.2 Speed of Adjustment

As Rogoff (1996) points out, every reasonable theoretical model suggests that there should be at least some temporary component to purchasing power parity deviations. In other words, even if purchasing power parity deviations are persistent, relative purchasing power parity should hold in the long run, hence the real exchange rate should display mean reversion.

Other than the stochastic deviation from traditional purchasing power parity, an alternative measure of speed of adjustment is the “half-life” of a process, a concept derived originally from physics. This concept indicates how long it takes for the impact of a unit shock to dissipate by half. Empirically, the half-life of purchasing power parity can be estimated not only from an autoregressive data generating process of the real exchange rate, but also from variance ratios. The time required for a shock to diminish to half of its initial size is an alternative estimator of the half-life of real exchange rate innovations. (See Table 2.1).
Table 2.1: Estimates of PPP half-lives

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Half life (Years)</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frankel (1990)</td>
<td>4.6</td>
<td>Dollar – pound</td>
</tr>
<tr>
<td>Manzur (1990, 1993)</td>
<td>5</td>
<td>Seven industrial countries</td>
</tr>
<tr>
<td>Fung and Lo (1992)</td>
<td>6.5</td>
<td>Six industrial countries</td>
</tr>
<tr>
<td>Wei and Parsley (1995)</td>
<td>4.25</td>
<td>European Monetary System (EMS)</td>
</tr>
<tr>
<td></td>
<td>4.75</td>
<td>Non-EMS</td>
</tr>
<tr>
<td>Frankel and Rose (1996)</td>
<td>4</td>
<td>150 countries</td>
</tr>
<tr>
<td>Lothian and Taylor (1996)</td>
<td>2.8</td>
<td>Franc – pound</td>
</tr>
<tr>
<td></td>
<td>5.9</td>
<td>Dollar – pound</td>
</tr>
<tr>
<td>Papell (1997)</td>
<td>1.9</td>
<td>European Community</td>
</tr>
<tr>
<td></td>
<td>2.8</td>
<td>EMS</td>
</tr>
<tr>
<td>Higgins and Zakrajsek</td>
<td>5</td>
<td>Europe, CPI</td>
</tr>
<tr>
<td>(1999)</td>
<td>3</td>
<td>Europe, WPI</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>OECD, WPI</td>
</tr>
<tr>
<td></td>
<td>11.5</td>
<td>Open economies, CPI</td>
</tr>
<tr>
<td>Cheung and Lai (2000)</td>
<td>2.5</td>
<td>Industrial countries</td>
</tr>
<tr>
<td></td>
<td>Under 3</td>
<td>Developing countries</td>
</tr>
<tr>
<td>MacDonald and Ricci (2002)</td>
<td>1.2</td>
<td>Industrial countries</td>
</tr>
<tr>
<td>Apte, Sercu and Uppal</td>
<td>1</td>
<td>Industrial countries</td>
</tr>
<tr>
<td>(2003)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pittis, N. et al. (2005)</td>
<td>2.5</td>
<td>Pound – US Dollar</td>
</tr>
</tbody>
</table>

Source: Different publications

Table 2.1 summarizes estimates of half-lives from various studies in the purchasing power parity literature. It can be seen that most of the estimates lie between 3 and 5 years. The median and mean of half-lives are 3.65 and 4 years, respectively. These figures are in broad agreement with the length of the long run insofar as purchasing power parity is concerned as reported in the survey of Froot and Rogoff (1995) (usually
four to five years). The only exception is from the study by Murray and Papell (2005) (not included in table 1 as it inflates the median and mean, hence making it difficult to compare with other studies), which argued that the true half-life lies between 3.72 and 34.31 years.

2.3 Recent Empirical Findings

To review the empirical evidence, studies were classified into three kinds of samples: (1) Industrial countries; (2) mixed samples of both developed and developing countries; and (3) developing countries.

Panel A of table 2 shows that recent evidence from industrial countries is generally supportive of long run purchasing power parity, and interestingly, such favourable results are obtained through a diverse variety of econometric techniques. Mixed evidence is found in studies whose samples include both industrial and developing countries; see Panel B of table 2. Due to limited data availability and quality, there are only a handful of studies that examine the validity of long run purchasing power parity for developing economies. Panel C of table 2 lists such papers.
<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Nature of data</th>
<th>Sample period</th>
<th>Price index used</th>
<th>Approach</th>
<th>Does PPP hold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kawai and Ohara (1997)</td>
<td>Time-series</td>
<td>A. 1973-96</td>
<td>Industrial CPI and WPI</td>
<td>Unit root test</td>
<td>No</td>
</tr>
<tr>
<td>Cheng (1999)</td>
<td>Time-series</td>
<td>1954-94</td>
<td>CPI</td>
<td>Unit root and cointegration</td>
<td>Yes</td>
</tr>
<tr>
<td>May and Rothman (1999)</td>
<td>Time-series</td>
<td>1960-97</td>
<td>CPI</td>
<td>Unit root test</td>
<td>Yes</td>
</tr>
<tr>
<td>Choong et al. (2003)</td>
<td>Time-series</td>
<td>1973-97</td>
<td>CPI and WPI</td>
<td>Nonlinear unit root test</td>
<td>Yes</td>
</tr>
<tr>
<td>Bec, Salem and Rahbek (2004)</td>
<td>Time-series</td>
<td>-</td>
<td>-</td>
<td>TVECM</td>
<td>Yes</td>
</tr>
<tr>
<td>Pittis, Christou, Hassapis and Kalyvitis (2005)</td>
<td>Time-series</td>
<td>-</td>
<td>WPI</td>
<td>Near-to-unit roots test</td>
<td>Yes</td>
</tr>
<tr>
<td>Papell and Time-series</td>
<td>-</td>
<td>CPI</td>
<td>Unit root test</td>
<td></td>
<td>Yes</td>
</tr>
<tr>
<td>Source</td>
<td>Type</td>
<td>Industrial</td>
<td>Country</td>
<td>Time-Range</td>
<td>And</td>
</tr>
<tr>
<td>--------------------------------</td>
<td>------------</td>
<td>------------</td>
<td>--------------------------</td>
<td>------------</td>
<td>-----</td>
</tr>
<tr>
<td>Prodan (2005)</td>
<td>Series</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wu and Chen (1999)</td>
<td>Panel</td>
<td>1980-96</td>
<td>Panel unit test</td>
<td></td>
<td>CPI</td>
</tr>
<tr>
<td>Lee (1999)</td>
<td>Time-series</td>
<td>1957-54</td>
<td>Generalized error tests</td>
<td></td>
<td>CPI</td>
</tr>
<tr>
<td>Higgins and Zakrajsek (1999)</td>
<td>Panel</td>
<td>1973-97</td>
<td>Panel unit error tests</td>
<td></td>
<td>CPI</td>
</tr>
<tr>
<td>Harris, Leyborne McCabe (2004)</td>
<td>Panel</td>
<td>1974-98</td>
<td>Panel unit root test</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Salehizadeh and Taylor (1999)</td>
<td>Time-series</td>
<td>1975-97</td>
<td>Cointegration</td>
<td></td>
<td>CPI</td>
</tr>
<tr>
<td>Doganlar (1999)</td>
<td>Time-series</td>
<td>1980-95</td>
<td>Cointegration</td>
<td></td>
<td>CPI</td>
</tr>
<tr>
<td>Guimaraes-Filho (1999)</td>
<td>Time-series</td>
<td>1855-90</td>
<td>Robust rank test</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2.4 Purchasing Power Parity Puzzle and Potential Pitfall

The Purchasing Power Parity puzzle is based on empirical evidence that international price differences for individual goods (in the case of the law of one price) or baskets of goods (in the case of PPP) appear highly persistent or even non-stationary (Taylor, 2000). The present consensus is that these price differences have a half-life that is of the order of five years at best and infinity at worst. This seems unreasonable in a world where transportation and transaction costs appear so low as to encourage arbitrage and the convergence of price gaps over much shorter horizons, typically days or weeks. However, current empirical analyses of such gaps rely on a particular choice of methodology, involving relatively low-frequency monthly, quarterly, or annual data.

2.5 Pitfall: Temporal Aggregation

The problem of temporal aggregation has been appreciated in the econometric literature ever since the seminal contribution of Holbrook Working – 1960 (Taylor, 2000). But it is notable that Working’s approach has left virtually no imprint on the PPP and Law of One Price (LOOP) literature, especially given his original focus on price adjustment and the data problems endemic to this field. In the last decade, when literally hundreds of papers have appeared on purchasing power parity and law of one price, only a handful cite Working’s approach and the temporal aggregation problem; and just two of those appeared in the last ten years. The most recent paper, by Taylor (2000), is an important contribution that, like the present paper, notes how temporal aggregation can affect unit root tests.

The missing connection is strange given that many economists are aware of these issues, and cite Working’s approach and the temporal aggregation problem in other kinds of economic studies. An extensive literature notes the role of temporal aggregation as it affects models of consumption, the permanent income hypothesis and various other business-cycle phenomena. The same aggregation issues have been raised in a number of other areas, ranging from stock market and exchange rate volatility the asset-pricing
models (Taylor, 2000). Since so much of the testing of purchasing power parity and law of one price has focused on testing stationarity, we should also note that the unit root literature has also paid some attention to the Working’s approach effect, and has concluded that problems of temporal aggregation can certainly affect the way we make inferences from standard unit roots tests by biasing coefficients and having lower power (Taylor, 2000). Still, these findings have not yet made an impact on the mainstream of the purchasing power parity and law of one price literature, an oversight hoped to be addressed in this paper.

In order to consider the temporal aggregation problem as it affects the testing of purchasing power parity and law of one price, it is better to see, first, when the problem might arise. Taylor (2000) used the following example to show the origin of the problem. Suppose that prices are contracted daily in the relevant markets, and that arbitrage happens each day after prices are set, but that we only observe data that is a weekly average of the market prices. In that case, the true process for the price gap is the daily \( x_t \), where the index \( t = 0, \ldots, T-1 \) runs over each successive day. Instead, the limitations of the data force us to use a moving average of the data, in the form:

\[
x^*_s = 1/p (x_{ps} + \ldots + x_{ps+p-1})
\]

(1)

where \( s = 0, \ldots, s-1 \).

Here \( p \) denotes the period over which averaging takes places. In this example with a daily process and weekly sampling \( p = 7 \). In another example, with a weekly process and annual sampling we would have \( p = 52 \).

The dynamics of the sampled data \( x^*_s \) will not correspond to those of the process \( x_t \), even adjusting of a rescaling in the time dimension. In particular, it can be shown that estimates of the convergence speed of the process \( x^*_s \) will always be slower than the process \( x_t \), and that this bias increases as the degree of temporal aggregation \( P \) increases. Given the half-life formula as;
Where

\[ x^*_s = \rho^*_s x^*_{s-1} + \varepsilon^*_s \],

a regression equation used to estimate the basic model on temporally aggregated data, it is instructive to examine the implications of temporal aggregation using some numerical examples by way of illustration. Table 3.1 shows the values of estimated half-life \( H^* \) using the above formula, and the bias factor \( H^*/H \), for various values of the true half-life \( H \) and the temporal aggregation parameter \( P \). What is shown here is just an example of daily data, with true half-lives and sampling frequencies varying between one day, two days, one week, two weeks, one month, two months, one quarter, six months, one year and two years.

The results show that temporal aggregation biases are not small. For example, look at column four of table 2.3: a process with a true half-life of 7 days (one week) in daily data would be estimated to have a half-life of 13.8 (about 14 days – two weeks) with monthly data \( (P = 30) \), 22.6 days (over three weeks) with quarterly data \( (P = 90) \) and 59.5 days (over eight weeks) with annual data \( (P = 365) \). In the penultimate column, a process with a half-life of one year in daily data would appear to have a half-life of almost two years with annual data.

Aside from noting the size of these biases, some more systematic variation is apparent. First, we note that the bias is always upwards: the estimated half-life always exceeds the true half-life. Second, reading across the table, we note that the estimated half-life appears to be an increasing function of the true half-life. Third, reading down the table, it appears that the size of the bias increase as the degree of temporal aggregation increases (that is for given \( H \), \( H^* \) is an increasing function of \( P \)). Fourth, reading across the lower table, it seems that for a given \( P \), the bias factor is a decreasing function of \( H \) and
converges monotonically to a limit for large $H$. Fifth, it appears that the extent of the bias potentially increases without limit, as might be seen from reading down the first column. Sixth, we may note that above and to the right of the diagonal in the lower panel, the bias factor is bounded; it reaches its highest value on the diagonal; and, as we move down the diagonal, this bias factor appears to reach an upper limit of about 1.59 or 59%.
### Table 2.3: Temporal Aggregation Bias for Half-life Estimates

<table>
<thead>
<tr>
<th>Temporal Aggregation (P)</th>
<th>True half-life (H*, days)</th>
<th>Estimated half-life (H*, days)</th>
<th>Bias Factor (H*/H)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>1.4</td>
<td>2.7</td>
<td>9.4</td>
</tr>
<tr>
<td>7</td>
<td>2.2</td>
<td>3.7</td>
<td>11.0</td>
</tr>
<tr>
<td>14</td>
<td>3.3</td>
<td>4.7</td>
<td>11.8</td>
</tr>
<tr>
<td>30</td>
<td>5.5</td>
<td>7.0</td>
<td>13.8</td>
</tr>
<tr>
<td>90</td>
<td>12.8</td>
<td>15.1</td>
<td>22.6</td>
</tr>
<tr>
<td>180</td>
<td>22.3</td>
<td>25.8</td>
<td>35.5</td>
</tr>
<tr>
<td>365</td>
<td>40.1</td>
<td>45.6</td>
<td>59.5</td>
</tr>
<tr>
<td>730</td>
<td>72.3</td>
<td>81.1</td>
<td>102.0</td>
</tr>
</tbody>
</table>

Notes: Given P and H, H* is calculated from (2).

The last observation is encouraging, since it suggests that we might be able to limit temporal aggregation biases by appropriate choices of sampling frequency. The most important lesson that is becoming clear from all these observations is that in order to get fairly accurate estimates of half-life we need to ensure that the temporal aggregation in the data is of an order of magnitude not greater than the size of the half-life itself.
However, this, in turn, will require some prior estimation about what the half-life might be, so as to design a decent experiment.
CHAPTER THREE

3. Methodology

The study intends to use the Ordinary Least Square technique (OLS). There are two main reasons for using the OLS method of estimation. First, it allows proper comparison of the results from this study with other studies that used the same technique. Second, it allows stationarity tests, as it is important to determine whether the regressions in question are spurious.

3.1 Model Specification

The Purchasing Power Parity theory is used to determine prices and the exchange rate. If one assumes zero transactions cost or other impediments to trade, given that all goods are tradable, effective arbitrage would result in the strongest version of purchasing power parity, namely absolute purchasing power parity, which is stated as follows:

\[ E = \frac{P}{P^*} \]  

Where \( E \) is the exchange rate and represents the number of units of domestic currency required for purchasing one unit of foreign currency. \( P \) and \( P^* \) are the domestic and the foreign price indices, respectively. This version of the purchasing power parity is premised on the law of one price.

In reality, the equilibrium price of a good may not be the same when converted into a common currency. The reasons for this include the wedge created because of transport costs, quotas, tariffs and information asymmetry, which reduces the effectiveness of arbitrageurs. In addition, the presence of non-traded goods can prevent arbitrageurs from responding to profitable investment opportunities. Furthermore, the non-neutrality of money\(^1\) in the short run can generate price differences in similar goods across countries.

\(^1\) The possibility for the monetary authorities to control the rate of output
To account for the shortcoming of the absolute version of purchasing power parity, an alternative referred to as relative purchasing power parity is often specified. According to this version the relative or percentage change in the exchange rate is equivalent to the difference in inflation rates and is given by the following:

\[ \Delta E = \Delta P - \Delta P^* \]  

(4)

Where \( \Delta E \) is the percentage change in the exchange rate, while \( \Delta P \) and \( \Delta P^* \) represents the rate of change of the domestic and foreign price level. The above equation states that the rate of change of the exchange rate approximates the domestic rate of inflation minus the foreign rate of inflation.

However, the model used for this study is the following purchasing power parity model in standard logarithmic form:

\[ \text{Ln}S_t = \beta_0 + \beta_1 \ln \left( \frac{P_t^*}{P_t} \right) + \varepsilon_t \]  

(5)

Where \( \text{Ln}S_t \) is the logarithmic of the actual exchange rate (foreign currency to domestic currency – Botswana Pula being the domestic currency and the South Africa Rand is the foreign currency), and \( P_t^* \) and \( P_t \) are the foreign and domestic CPIs, respectively. For the purchasing power parity relationship to hold the coefficient \( \beta_1 \) should be equals to one (\( \beta_1 = 1 \)).

3.2 Data

The data required to test the purchasing power parity theory are obtained online from of the South African Reserve Bank and the Bank of Botswana databases. The three time series variables that this study intends to use to test the purchasing power parity theory, just like other studies, are the spot exchange rate and the Botswana and South African
Consumer Price Indices (CPI)\textsuperscript{2}. The countries chosen to test purchasing power parity are South Africa and Botswana. The data are in a high-frequency monthly format and span a period of twenty years from January 1985 to January 2005 for both countries. Because of geography, history and their joint membership in Southern African Customs Union (SACU), Botswana and South Africa exhibit a greater degree of economic and financial integration along with fewer commercial and financial restrictions. This offers a better platform for analysing and testing the validity of the purchasing power parity theory between these two countries.

3.2.1 Price Indices

A price index represents the cost, over time, for a representative basket of goods relative to some arbitrary base year (Rothman and May, 1999). Upward changes in the price indices indicate inflation (i.e., an increase in the relative cost of the same basket of goods over time). Consequently, price indices are an important component in evaluating the merits of the purchasing power parity theory. Despite their importance in this respect, the use of price indices as a measure of inflation is not void of controversy. In addition to different price indices that are available, supposedly similar indices are often calculated and comprised of differing baskets of goods.

The Consumer Price Index (CPI) and Producer Price Index (PPI) represent two of the more common indices of price. The Consumer Price Index is meant to be a representative sample of goods and services consumed, while the Producer Price Index tracks prices received by firms for goods and services. Since international firms and tradable goods are key actors in the foreign exchange market, the use of the Producer Price Index in an evaluation of purchasing power parity would be a sound exercise. However, this study uses CPI data because testing the relative purchasing power parity utilizes the percentage change in inflation to predict movements in exchange rates, so any discrepancy between

\textsuperscript{2} The are problems associated with using price indices to explain exchange rates changes. The inclusion of non-traded goods such as housing in constructing these indices is the source of the problem (Ramirez and Khan, 1999). Changes in the prices of these goods do not translate into changes in international trade flows, therefore, they do not affect exchange rates. However, using these indices can be rationalized by arguing that changes in the prices of non-traded goods affect the prices of traded goods indirectly through their impact on wage demands and the cost of living.
the Consumer Price Index and Producer Price Index derived data is likely to be negligible. Also the CPI data can easily be obtained from the two countries being studied.

3.2.2 Exchange Rates

The exchange rates can be expressed in real or nominal terms. Real exchange rates are a function of the nominal exchange rate and the relative price level between two countries. The nominal exchange rates provide some measure of the value of one currency in terms of another. Price levels provide relevant information pertaining to the cost of a representative basket of goods for any given country. Multiplying the nominal exchange rate by the ratio of two countries’ price levels thus provides a value for the cost of one country’s goods relative to another country (Ramirez and Khan, 1999).

It is important to graphically examine the graphs of the exchange rates and price indices for the countries in question to determine whether the variables exhibit any time trend. The existence of a time trend might be a hint for nonstationarity of data, which can create serious problems\(^3\) in Ordinary Least Squares (OLS) estimation of purchasing power parity.

\(^3\) Spurious regressions
Figure 1: Rate of Inflation - Botswana
Figure 2: Consumer Price Index – South Africa

![Chart of Consumer Price Index (SA_CPI) from 1986 to 2004.](chart)

Figure 3: Botswana and South Africa’s exchange rate (Rand per Pula)

![Chart of exchange rate (EXCHANGERATE) from 1986 to 2004.](chart)
The graphical examination of the countries consumer price indices and the bilateral spot exchanges rates shows that there is a time trend involved in those variables, respectively. Though the trend appear not to be obvious in the case of Botswana’s consumer price index, the South African consumer price index shows a clear negative time trend whereas the exchange rates between two countries portrays a positive time trend. The presence of time trend provides indirect evidence that the variables for these countries are nonstationary. However, more precise tests will be conducted to determine the stationarity of the variables.
CHAPTER FOUR

4. Empirical Analysis

The results from the regression for equation (5) are shown in Table 4.

Table 4.1: PPP Estimates Using Monthly Data – January 1985 through January 2005

<table>
<thead>
<tr>
<th>Dependent variable: $S_t$</th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>F-statistic</th>
<th>Durbin-Watson statistics</th>
<th>Adjusted $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rand/Pula</td>
<td>0.2865 (56.4846)</td>
<td>-0.0375 (-5.2768)</td>
<td>27.8442</td>
<td>0.0377</td>
<td>0.10</td>
</tr>
<tr>
<td>N = 241</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The numbers in parentheses are t-statistics. The Consumer Price Index, for each country, was used as the price index for the independent variable, $\ln(P^*/P)$.

From Table 4.1, the estimates from regression equation 5 do not fulfil the requirement that $\beta_1 = 1$ and the estimated coefficient has wrong sign. Although t-statistics, Durbin-Watson and F-statistics are included in Table 4.1, they should not be used to determine the significance of the coefficients, since they may be misleading, as the variables were not tested for stationarity. But judging from the values of the coefficients and ignoring their significance for now, the results indicate that the purchasing power parity failed to hold in the long run for South Africa and Botswana, using data from 1985 to 2005.

Having established that purchasing power parity does not hold for South Africa and Botswana using the OLS method of estimation, it must be examined whether these results are spurious. Results from the regression models will not be meaningful if the exchange rate or price indices are non-stationary, that is, if they possess a time trend. According to Granger and Newbold (1974) and Phillips and Perron (1988), spurious regressions will tend to have inflated statistics, such as high t-statistics, $R^2$ values and F-statistics. The
results reported in Table 4.1 exhibit this pattern. The t-statistics and the F-statistics are relatively high considering the fact that their coefficients have the wrong signs and the purchasing power parity relationship does not hold.

4.1 Testing for Stationarity

The OLS regression (specifically inflated statistics) from the previous section shows that the variables may not stationary. It is important to check whether the time series variables; exchange rates and CPIs, are stationary. A stationary time-series variable possesses a constant mean and variance over time and an autocorrelation function that depends solely on the length of the expressed lag (Rothman and May, 1999). By definition, purchasing power parity suggests that real exchange rate series should exhibit these stationary qualities, thus any percentage change in the price level between the two countries would be offset by an equal appreciation/depreciation of the nominal exchange rate. If it can be shown that the real exchange rate exhibits stationarity, this would provide much needed support for the purchasing power parity theory. Conversely, if the real exchange rates exhibit non-stationarity (i.e., if deviations from PPP are permanent), then purchasing power parity theory will be rejected.

A formal test of the null hypothesis of non-stationarity will be conducted via the Augmented Dickey-Fuller (ADF) test. The principle behind the Augmented Dickey-Fuller equation is to test for the presence of a unit root in the coefficient of lagged variables. If the value of the coefficient of a lagged variable is one (1), then the regression exhibits the properties of a non-stationary random walk process. The null hypothesis of a unit root is rejected if the computed ADF test statistic is greater in absolute value than the critical values (MacKinnon critical value at given percentage level), (Ramirez and Khan, 1999).
Table 4.2: Augmented Dickey-Fuller (ADF) Unit Root Tests – 5 % level

<table>
<thead>
<tr>
<th>Variables</th>
<th>ADF</th>
<th>Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>At levels</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spot exchange rate</td>
<td>-2.4625</td>
<td>-2.8739</td>
</tr>
<tr>
<td>SA_CPI</td>
<td>-1.7970</td>
<td>-2.8739</td>
</tr>
<tr>
<td>BOTS_CPI</td>
<td>-2.4805</td>
<td>-2.8739</td>
</tr>
<tr>
<td><strong>First difference</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spot exchange rate</td>
<td>-6.5548</td>
<td>-2.8739</td>
</tr>
<tr>
<td>SA_CPI</td>
<td>-6.5863</td>
<td>-2.8739</td>
</tr>
<tr>
<td>BOTS_CPI</td>
<td>-6.7634</td>
<td>-2.8739</td>
</tr>
</tbody>
</table>

The results in Table 4.2 are consistent with those of prior studies that have found the exchange rate and the price indices to be non-stationary for most of the countries in question. However, the null hypothesis of a unit root in the first difference of the entire variables can be rejected at the 5 percent significance level. This implies that the variables are stationary after the first difference.

4.2 Cointegration Tests

To perform a cointegration test, it is necessary that the order of integration of all the variables in the long-run relationship be the same (Ramirez and Khan, 1999). The order of integration can be defined as the number of times a time series variable must be differenced before it become stationary.

The ADF tests rule out directly OLS estimation for testing purchasing power parity in the long run. However, given that the order of integration for all the countries’ variables is I(1), cointegration analysis is warranted. The advantage of using this procedure is that it

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4 The bivariate case requires that the variables be of the same order of integration.
5 Generally, cointegration attempts to find a linear combination of the nonstationary variables that is of an order less than the highest ordered variables. In this case, since the highest order of the variables is I(1), an order less than this would be I(0), which implies stationarity at levels.
can determine the existence of a stable long-run (equilibrium) relationship among the non-stationary time series variables. It also ignores the short-run dynamics that might cause the relationship not to hold in the short run.

There are two alternative techniques for running cointegration tests; the Engle and Granger (1987) one or two-step test and the maximum likelihood method developed by Johansen (1988) and Johansen and Juselius (1990). The latter test is preferred when there are more than two time series variables involved, because it can determine the number of cointegrating vectors. In this study, Engle-Granger two-step test is used since there are only two variables involved.

Table 4.2 shows that at levels, the variables are not stationary, but after being differenced once, they become stationary. Again, all the variables are integrated of the same order; hence, equation (5) can be estimated at levels and the residuals from it should be tested for stationarity. As with univariate unit root tests, the null hypothesis of a unit root and thus no cointegration is based on a t-test with a non-normal distribution. However, unless the estimate of the long-run model is already known, not estimated using the static model, it is not possible to use the standard Dickey-Fuller table of critical values (Harris, D. et al. (2004). Thus, different critical values are needed as the number of observations changes. Fortunately, MacKinnon (1991) has linked the critical values for particular tests to a set of parameters of an equation of the response surface estimates. That is, with the table of the response and the relation below, it is possible to obtain the appropriate critical values for any test involving the residuals from an OLS equation where the number of regressors (excluding the constant and trend) lies between 1 and 6 (1< n < 6).

\[
C(p) = \phi_{\infty} + \phi_1/T + \phi_2/T^2
\]

(6)

Where \( C(p) \) is the \( p \) percent critical value and \( T \) is the number of observation.

The estimated 5 percent critical value for 236 observations when \( n = 3 \) is -4.4767. Since the ADF test statistic is less negative than -4.4767 (it is -2.8064), the decision rule is not
to reject the null hypothesis of no cointegration at the 5 percent significant level. Therefore, the conclusion is that the variables are not cointegrated.

From this result it can be concluded that there is no stable long-run relationship between the Rand/Pula exchange rate and price levels between Botswana and South African CPI ratios. This means that the time series variables may diverge from each other in the short run and will stay away from each other (drift apart) in the long run.

According to the Granger representation theorem, the existence of a stable long-run relationship between the exchange rate and price levels enables the analyst to estimate at least one error-correction model (Ramirez and Khan, 1999). However, this study fails to find any stable long-run relationship between the exchange rates and price levels; hence no error-correction model can be estimated.
5. Conclusion

This study tested the theory of purchasing power parity for Botswana and South Africa using cointegration modelling. An error-correction model could not be constructed because the variables were not cointegrated – do not have any stable long term relationship. Hence, this study does not find any evidence to support purchasing power parity in the long run between the Botswana and South African CPIs and exchange rate.

The pollution of data by temporal aggregation creates a serious pitfall on the way to an understanding of the purchasing power parity puzzle. In an ideal world, high frequency data could solve our problems, but Researchers, it would seem, are between a rock and a hard place. They are forced to use low frequency data for most purchasing power parity and law of one price testing of contemporary and historical data, simply because that is all they have had available. As a result, the estimated half-lives might still be way off target. If a complete reworking of the research agenda were contemplated, a case can be made that researchers should adopt a new empirical strategy of developing shorter spans of high-frequency data to get more powerful tests of stationarity and more reliable estimates of half-lives.

This study failed to support the purchasing power parity hypothesis and, hence it could not estimate the speed of convergence through an error-correction mechanism. But the implications for calculations of convergence speeds in price processes need to be considered and the future research papers should aim taking that direction. Furthermore, if measurements of half-lives can be found to be out by, say, a factor of ten or more, then one of the following should be done. First, researchers should search for higher-frequency data with which to pursue testing, looking for frequencies that match the underlying arbitrage process. Second, researchers should consider the implications of non-linear models that might deliver more rapid adjustments.
REFERENCE


