Search for Low Spin Collective Structures in $^{158}\text{Er}$ and $^{159}\text{Er}$

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Level scheme
Declaration

I, the undersigned, hereby declare that the work contained in this thesis is my own original work and that I have not previously in its entirety or in part submitted it at any university for a degree.

Tshepo Samuel Dinoko

Signed: .............................. Date: ..............................
Abstract

Lying in a rapidly-changing transitional region between nuclei that may behave as spherical vibrators \( N \leq 88 \) or good rotors \( N \geq 92 \), the \( N = 90 \) isotones present a unique testing ground where new nuclear collective phenomena may be uncovered.

As part of a general investigation of collective structures and the coupling of single-particle states to these structures, the \( ^{150}\text{Sm}(^{12}\text{C},4n\gamma\gamma) \) and \( ^{150}\text{Sm}(^{13}\text{C},4n\gamma\gamma) \) reactions at a beam energy of \( E_{\text{lab}} = 65 \) MeV were used to study the low and medium spin structure of \( ^{158,159}\text{Er} \) nuclei below spin \( 20\hbar \).

The \( \gamma-\gamma \) coincidence events were detected in the nine escape-suppressed HPGe Clover detectors using the AFRODITE \( \gamma \)-ray spectrometer at the iThemba Laboratory for Accelerator Based Sciences. DCO ratios and \( \gamma \)-ray polarization measurements were used to establish the spins and parities of newly observed and confirm previously established rotational bands. The observed bands will be discussed in terms of both traditional quadrupole rotational-vibration models and recent predictions of octupole correlations in the \( N = 90 \) isotones and neighbouring nuclei. The data obtained will be compared with the spectroscopic systematics of neighbouring nuclei.
# Acronym

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tr>
<td>PACE</td>
<td>Projected Angular momentum Coupled Evaporation</td>
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<tr>
<td>AFRODITE</td>
<td>AFRican Omnipurpose Detector for Innovative Techniques-and Experiments</td>
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<tr>
<td>DCO</td>
<td>Directional Correlations from Oriented states</td>
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<tr>
<td>HIFE</td>
<td>Heavy Ion Fusion Evaporation</td>
</tr>
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<td>SM</td>
<td>Shell Model</td>
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To

My late father, Nkoetsile Justice Dinoko. This one is for you. Robala ka kagiso Mokgatla.

My family, Mankwana (mother), Israel, Eric, Kagiso, Boitumelo le Mpho.

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- Onalerona-Ofile
- Larona

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sa ga Rre, ke a leboga Dinoko.
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Introduction

1.1 Introduction

The way in which the atomic nucleus is excited as angular momentum increases represents a delicate interplay between single-particle and collective degrees of freedom. The addition of valence nucleons to spherical, closed-core nuclei breaks the spatial symmetry, allowing rotation of a now deformed system to compete energetically with single-particle and vibrational modes of excitation [Ree11]. The Er nuclei, lie in a ‘transitional’ region where nuclear collectivity rapidly changes from apparently vibrational to rotational motion [Cas81]. This transition is reflected in a sharp change in the experimental $R_4 \equiv E(4^+_1)/E(2^+_1)$ energy ratios between $N = 88$ and $N = 92$ nuclei for isotopes near $Z = 64$, as shown in Figure 1.1. $E(4^+_1)/E(2^+_1)$ ratios of 2.00 and 3.33 are expected for pure vibrational and rotational behavior, respectively. For the Er isotopes ($Z = 68$), $^{154}\text{Er}$ has an $E(4^+_1)/E(2^+_1)$ ratio that lies close to the vibrational limit, while $^{160}\text{Er}$, with only six more valence neutrons, already lies close to the rotational limit. The intermediate $^{158}\text{Er}$ has an $E(4^+_1)/E(2^+_1)$ ratio approaching 2.70, the value expected for a $\gamma$-soft nucleus [Cas85], where vibrational modes of excitation couple to rotations [Fae62, Ree11].

The nuclei $^{158,159}\text{Er}$ are classic examples of rich nuclear-structure phenomena at high-spin [Wan13]. The yrast states, shown in Figure 1.2, have first a neutron alignment ($\text{back-bend}$), then a proton alignment ($\text{up-bend}$), then the band termination at spin $46\hbar$. Above band termination, there is a shower of weak $\gamma$-rays feeding the high-spin states, and finally a triaxially superdeformed band has been discovered [Pau07] at very
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Figure 1.1: The ratio \( R_4 = E(4^+) / E(2^+) \) for selected even-even nuclei with \( 144 \leq A \leq 168 \). Isotopes are joined by solid lines and isotones by dashed lines. The rotational limit, \( R_4 = 3.333 \) is indicated. The plot is based on data taken from Nuclear Data Sheets [nndc13].

Although the multi-quasiparticle structures of \(^{158,159}\)Er are well established [Sim87, Sim94], there is scarce spectroscopic information on the vibrational structures in \(^{158}\)Er or how these couple to the odd neutron in \(^{159}\)Er. The relative lack of information at low spins is probably due to a combination of the low spins of the ground states of the nuclei \(^{158,159}\)Tm, 2\(^-\) and 5\(^-\), respectively, that \( \beta^+ \) or electron capture decay to \(^{158,159}\)Er. The known bands in \(^{158}\)Er are presented in [Pip06] showing a partial level scheme where the reaction \(^{114}\)Cd\((^{48}\)Ca,4n\(\gamma\))\(^{158}\)Er at a beam energy of \( E_{lab} = 215 \) MeV was used to populate \(^{158}\)Er at high spins. Figure 1.4 shows the current knowledge of the lowest \( k = 2 \) (\( \gamma \) band) and \( k = 0 \) (\( \beta \)) bands in \(^{158}\)Er [Hel04].

It can be seen from both parts of Figure 1.4, that the \( \beta \)-band is known only up to the 4\(^+\) level and the \( \gamma \)-band up to 10\(^+\) in the even spins and to the 7\(^+\) in the odd spins. In the lighter neighbouring \( N = 90 \) isotones the \( \gamma \)-band is known up to 32\(^+\) in \(^{156}\)Dy [Maj11] and up to 17\(^+\) in \(^{154}\)Gd [Mad08]. In both these neighbouring nuclei, the first excited 0\(^2\)\(^+\) state is about 125 keV lower in energy than in \(^{158}\)Er, where it is at 806.4

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Figure 1.2: Spectrum of the yrast $\gamma$-rays of $^{158}$Er. TESSA data [Sha88].

keV. In $^{154}$Gd and $^{156}$Dy these $\beta$-bands backbend early and become yrast above spin $16^+$. $^{158}$Er is less deformed than the lighter $^{154}$Gd and $^{156}$Dy nuclei and the strong backbend is seen in the ground-state band. In the lighter $N = 90$ isotones this $0^+_2$ state has been suggested to be not a $\beta$-vibration, but a two-particle, two-hole seniority zero neutron state lowered into the pairing gap by the configuration dependent pairing interaction and the low density of the oblate states near the Fermi surface. This is the ‘pairing isomer’ concept of Ragnarsson and Broglia [Rag76] [Sha11]. The Nilsson orbital responsible for producing $| 0^+_2 \rangle$ is the high-K [505] $\frac{11}{2}^-$ orbital [Sha10].
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Figure 1.3: Pedagogic picture of the major Yrast phenomena in the nucleus $^{158}\text{Er}$ [Pau07].
Figure 1.4: The current knowledge of the $k = 0$ ($\beta$) and $k = 2$ ($\gamma$) bands of $^{158}$Er [Hel04].
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In $^{154}$Gd and $^{156}$Dy the $\gamma$-bands track the ground state band \( |0^+_1\rangle \) and there is evidence that the $\gamma$-band belonging to the ‘second vacuum’ \( |0^+_2\rangle \) tracks this band through its backbend. Clearly, there is great interest in investigating the lower-spin yrare states of $^{158}$Er. The odd neutron nuclei with \( N = 91 \) are particularly interesting. A suggested proof that the $0^+_2$ states in the \( N = 90 \) nuclei are not $\beta$-vibrational but are 2p-2h states is that this $0^+_2$ state never couples to the $[505] 11^2_-$ high-K orbital, Figure 1.5 and [Sha10].

![Figure 1.5: Schematic showing the rotational band heads arising from the coupling of the second vacuum \( |0^+_2\rangle \) and the first $\gamma$-vibration, at 681 keV and 996 keV, respectively, in $^{154}$Gd to the Nilsson single-particle neutron orbital in $^{155}$Gd with K = $\Omega$. The data for the K $\leq \frac{5}{2}$ orbitals are taken from [Sch86] and the data for the $[505] 11^2_-$ orbital are taken from [Mad08, Sha10].](image)

The currently known decay scheme for $^{159}$Er is outlined by Mustafa et al. [Mus11]. The core excitations in $^{158}$Er are very close together Figure 1.4 at 806 keV for \( |0^+_2\rangle \) and 820 keV for the $\gamma$-band. Coupling of the $[505] 11^2_-$ neutron to \( |0^+_2\rangle \) would produce a K = $11^2_-$ band, if it is not blocked, and coupling to the $\gamma$-vibration K = $15^2_-$ and K = $7^2_-$ bands, depending on the relative orientation of $K_{\text{core}}$ and $K_{\text{neutron}}$. In $^{159}$Er the
\[ \frac{11}{2}^- \] band back-bends at the AB crossing mimicking the yrast band in \(^{158}\text{Er}\). It is very interesting to discover if the \( K = \frac{15}{2}^- \) coupling to the \( \gamma \)-vibration also backbends in this manner.

Following the introduction in Chapter 1, Chapter 2 outlines various theoretical models, which may be applied to describe these nuclei. The experimental methods on \( \gamma \)-ray spectroscopy are discussed in Chapter 3. The experimental details, results, and discussion of \(^{158}\text{Er}\) and \(^{159}\text{Er}\) are presented in Chapter 4. Finally, conclusions and an outlook are presented in Chapter 5.
Nuclear Structure Theory

2.1 Introduction

No complete theory exists which fully describes the structure and behaviour of complex nuclei based solely on a knowledge of the force acting between nucleons. However, great progress has been made with the aid of simple models. A model embodies certain aspects of our knowledge and, almost invariably, incorporates simplifying assumptions which enable calculations to be made. A successful model should be able to give a reasonable account of the properties it was designed to address and also make predictions of other properties which can be checked by experiment [Lil01].

One of the simplest nuclear models is one in which the nucleus is regarded as a collection of neutrons and protons forming a droplet of incompressible fluid which behaves in some ways like a classical liquid drop. In the 1930s, this liquid-drop model was shown to account very well for the systematic behaviour of the nucleon binding energy with mass number, and confidence in its validity grew. However, as more information about nuclei became available, certain discrepancies with the liquid-drop model were noted and efforts to understand them led to the development of the nuclear shell model. The basis of this model is that there is an ordered structure within the nucleus in which the neutrons and protons are arranged in stable quantum states in a potential well that is common to all of them. Indeed, many nuclei behave as if most of the the nucleons form an inert core and low-energy excited states are determined by a few nucleons 'outside' the core. The picture is similar to that of an atom in which electrons are arranged
in shells and any chemical activity is determined by the most weakly bound, valence
electrons [Lil01].

The nuclear shell model has proven to be very successful in accounting for the ground-
state properties and low-lying excited states of very many nuclei. However, there are
certain modes of excitation, found in most nuclei, which are better described as collec-
tive vibrations and rotations of the nucleus represented as a liquid drop. Conceptually,
the liquid-drop and shell models appear to be very different and the idea of nucleons
moving in stable orbits and interacting weakly is not obviously consistent with that of
a collection of nucleons forming a drop of nuclear fluid. Reconciling these two appar-
ently contradictory pictures of nuclear behaviour has been a central goal of scientists
developing unified theories of nuclear structure [Lil01].

2.2 The Liquid Drop Model

There were many early attempts to develop models that could explain what was seen
in experimental studies of the nucleus. One phenomenon was the saturation of binding
energy per nucleon \( E_B/A \), see Figure 2.1. This provided, at least in part, the impetus
to develop the Liquid Drop Model (LDM). In this model the nucleus is likened to a
drop of liquid and should have analogous interactions. This idea was the basis for a
proposal by C. F. Weizäcker in 1936 which came in the form of the semi empirical mass
formula given by Equation 2.1.

\[
E_B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z - 1)}{A^{1/3}} - a_A \frac{(A - 2Z)^2}{A} + \delta(A, Z) \tag{2.1}
\]

The term \( E_B \) is the binding energy of the nucleus. The first term, \( a_v A \), is the volume
term arising from the strong nuclear force. This force has a very limited range and
nucleons only interact with their nearest neighbors. The number of particles that
interact is roughly proportional to \( A \), hence it is called the volume term. The second
term, \( a_s A^{2/3} \), is the surface term also arising from the strong nuclear force but differing
from the volume term due to the proximity of other nucleons and is the analogue to
the surface tension of a liquid droplet. The third term, \( a_c \frac{Z(Z - 1)}{A^{1/3}} \), is the Coulomb term
and represents the electrostatic repulsion of the protons of the nucleus with each other.
It is proportional to the number of proton pairs \( Z^2 \) and inversely proportional to the
radius. However, since a nucleus must have more than one proton for electrostatic repulsion to occur, the $Z^2$ term becomes $Z(Z - 1)$. The fourth term, $a_A \frac{(A-2Z)^2}{A}$, is the asymmetry term which arises from the Pauli exclusion principle and describes the energy cost to the nucleus due to an unmatched number of protons and neutrons. The last term, $\delta(A, Z)$, is the pairing term and is indicative of unpaired nucleons of either type.

![Figure 2.1](image)

**Figure 2.1:** A plot of experimental values of binding energy per nucleon ($B/A$) versus mass number $A$.

A figure derived from the semi-empirical mass formula is shown in Figure 2.1. In this figure, it is seen that there is good agreement for mass $A \geq 20$ where the dotted line represents the semi-empirical mass formula and the dots represent the average binding energy per nucleon ($E_B/A$).

While the LDM successfully explained other collective effects, it also had certain draw-
backs, one of which is that it failed to account for the quantum nature of the nucleus. Too many nuclear details were lost due to the averaging or macroscopic nature of the model. To account for the microscopic nature of the nucleons in the nucleus, a new approach was implemented using the Shell Model.

2.3 The Shell Model

To discuss the Shell Model we must point out what seems to be obvious [Chu08]. The nuclear force is strong and attractive. The evidence for the existence of this force is that the nucleons of the nucleus are confined in a potential well in the presence of a Coulomb repulsion in nuclei with proton number above $Z=1$. Furthermore, the force must be brief as the available binding energy per nucleon ($E_B/A$) does not scale proportionally to the volume of the nucleus. Additionally, the nucleons are fermions and obey the Pauli Exclusion Principle. Therefore the nucleus has a finite dimension.

If one examines the neutron and the proton separation energies with increasing nucleon number and compares them to the ionization energy patterns observed in atomic systems one may be tempted to describe nuclear structure in terms of shells.

Although there are some fundamental differences between the two systems, we assume that, like the electrons in the atomic system, the valence nucleons orbit in the presence of a central potential provided by a spherical nuclear core. We also assume as a reasonable first guess that the potential is that of the simple harmonic oscillator (SHO) given by Equation 2.2. The classical form of the Hamiltonian is then described by Equation 2.3.

$$V(r) = \frac{1}{2}m\omega^2r^2 + V_0$$  \hspace{1cm} (2.2)

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2r^2 + V_0$$  \hspace{1cm} (2.3)

remembering that this is a quantum system and the selection of this potential permits us to separate the wave function into its radial and angular components, shown by Equation 2.4, then Equation 2.5 is used to solve for the energy eigenvalues given by Equation 2.6.
2.3 The Shell Model

Figure 2.2: Above: Two-proton separation energies for a series of isotones. For continuity, the lower Z is noted. Below: Two-neutron separation energies for a series of isotopes. Adapted from [Kra88] with the data coming from [Wap77].
2. NUCLEAR STRUCTURE THEORY

\[
\psi_{nlm}(\mathbf{r}) = \psi_{nlm}(r, \theta, \phi) = \frac{R(\theta, \phi)}{r}
\]  \hspace{1cm} (2.4)

\[
H = \left[ \frac{p^2}{2m} + V(r) \right] \psi_{nlm}(\mathbf{r}) = E_{nlm} \psi_{nlm}(\mathbf{r})
\]  \hspace{1cm} (2.5)

where \( H \) is the Hamiltonian, \( E \) are the energy eigenvalues corresponding to the major oscillator quantum number \( N \) and \( n, l, m \) are the radial, orbital momentum and magnetic quantum numbers, respectively. Solving Equation 2.5, gives the eigenvalues of the Hamiltonian, as equally spaced energy levels according to:

\[
E = \left[ N + \frac{3}{2} \right] \hbar \omega + V_0
\]  \hspace{1cm} (2.6)

such that,

\[ N = 2(n - 1) + l, \ n = 1, 2, 3, ... \text{ and } l = 0, 1, 2, ... \]

Unfortunately, the energy gaps occurring at 2, 8, 20, 40, 70 and 112 presented in the left-hand column of Figure 2.3 do not agree with what has been observed experimentally. To address this problem, two extensions were made. An \( l^2 \) term was added to the SHO potential. Although the energy gaps were still incorrect, this had the effect of splitting the energy levels where \( l > 0 \) and thus reduced their degeneracy. The other effect that this modification had was to flatten out the potential inside the nuclear well as this term is attractive and becomes larger with increasing orbital angular momentum. The level energies are reduced, as is seen in the center column of Figure 2.3.

The second modification that was made to the SHO potential was spin orbit coupling, as proposed in 1948 [Hax49, May50-I, May50-II]. This term introduces a difference in energy of the total angular momentum vector \( \mathbf{j} \) which arises due to the alignment or anti alignment of the intrinsic spin of the nucleon. This spin-orbit interaction serves to further split the energy levels for orbital angular momentum \( l > 0 \) and correctly reproduces the experimentally observed ‘magic numbers’ (complete shell occupation) 2, 8, 20, 28, 50, 82 and 126, which have large energy gaps between them and are shown in the third column of Figure 2.3. The energy of the higher-spin state is depressed, while the energy for the lower-spin state is elevated.
Figure 2.3: The left column of energies represent those of the simple harmonic oscillator (SHO). These levels do not reproduce what is seen experimentally. The center column is a modification to the SHO, which flattens out the potential in the nuclear well. In the right-hand column are energy levels after the addition of the spin-orbit interaction. With this term, proposed by [Hax49, May50-I, May50-II], the shell model successfully reproduces the shell gap energies and magic numbers for spherical nuclei.
2.4 The Deformed Shell Model

The spherical Shell Model met with great success in predicting energy and angular momentum states for spherical nuclei. However, many nuclei having proton number (Z) and neutron number (N) far from shell closure are deformed. Therefore, a means of modifying the theory was needed. This addition came in the form of the Nilsson model which was proposed by [Nil55]. There are two assumptions of the Nilsson model that one should note. One is that there are many orbits made by the valence nucleons for every rotation of the nuclear core, thus the two motions may be decoupled. The other assumption in the Nilsson model is that the single or valence particles orbit a nuclear core and that through their interaction make the nucleus symmetrically deformed. The Hamiltonian then requires a cylindrically symmetric potential instead of the previously spherical potential and may be written as:

\[ H = \frac{p^2}{2m} + \frac{1}{2} m [\omega_x^2 (x^2 + y^2) + \omega_y^2 z^2] + Cl \cdot s + Dl^2 \]  

(2.7)

which reproduces the spherical shell model Hamiltonian when \( \omega_z = \omega_x \). The kinetic energy of the particle is given by the first term while the second term represents the deformed potential.

In the deformed potential term, \( \omega_x, \omega_y \) and \( \omega_z \) are the orthogonal oscillator frequencies in three dimensional space. The third term is a consequence of the spin-orbit interaction. The last term \( (l^2) \) is a correction added to the Nilsson Hamiltonian to bring it in line with what is seen experimentally [Eis70]. Although there are more realistic single particle potentials, this form of the single particle Hamiltonian correctly reproduces the previously mentioned magic numbers for \( \beta_2 = 0 \), where \( \beta_2 \) is the quadrupole deformation along the axis of symmetry. While the single particle energies are well described by Equation 2.7 where \( \beta_2 = 0 \), there is a more useful form of the equation that emerges when we change our system to cylindrical coordinates. To this end we make the following transformations,

\[ \omega_x^2 = \omega_y^2 = \omega_0^2 (1 + \frac{2}{3} \epsilon_2) \]
\[ \omega_z^2 = \omega_0^2 (1 + \frac{4}{3} \epsilon_2) \]  

(2.8)

where \( \omega \) is the oscillator frequency in the spherical potential described by

\[ h\omega_0 = 41A^{-\frac{1}{3}} \]  

(2.9)
and $\varepsilon_2(0.95\beta)$ is the adjusted quadrupole deformation parameter. We then make the assumption that the nuclear volume is a constant as a function of $\omega_0$ and, therefore, $\omega_x\omega_y\omega_z$ is a constant and thus leads to the relationship:

$$\omega_0 = \left(1 - \frac{4}{3}\varepsilon_2^2 - \frac{16}{27}\varepsilon_2^3\right)^{-\frac{1}{2}} = \text{constant} \quad (2.10)$$

Substituting Equations 2.8, 2.9 and 2.10 into 2.7 yields the more natural equation:

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega_0^2r^2 - m\omega_0^2\varepsilon_2^2\frac{4}{5}\sqrt{\frac{\pi}{5}}Y_{20}(\theta,\phi) + Cl \cdot s + Dl^2 \quad (2.11)$$

At this point it should be stressed that this equation is dependent on the nucleon type for the single particle interaction. This statement is manifest by the differences in the Nilsson plots of proton and neutron single particle energies. These differences in the single particle energy levels result from the magnetic fields that arise from the orbital motion and intrinsic spin of the proton and neutron, which must vary, as the proton has a single positive charge, whereas the neutron is neutral. This Coulomb effect appears in the Hamiltonian in the form of the Nilsson parameters $\mu$ and $\kappa$. The Nilsson parameters are related to the coefficients $D$ and $C$ in the Nilsson Hamiltonian by Equations 2.12 and 2.13. These effects are reflected in the single particle energies for deformed and spherical nuclei alike.

$$\mu = \frac{2D}{C} \quad (2.12)$$

$$\kappa = \frac{C}{2\hbar\omega_0} \quad (2.13)$$

For spherical nuclei, the orbit of the valence nucleon has no bearing on its potential energy state, however, for deformed nuclei the orbital path does make a difference. Nucleons orbiting in the plane that contains the axis of symmetry ($z$) are on average, closer to the nucleus than those that orbit the axis of symmetry creating prolate shapes. In this case the nucleon then has a lower potential energy and thus the single particle energy in the Nilsson Hamiltonian tends to decrease with increasing deformation. Those nucleons with orbits more about the axis of symmetry tend to increase the energy of the Hamiltonian. The upshot of this is that the difference in orbit gives rise to the phenomenon of $K$-splitting and establishes the projection of total angular momentum
(\(K\)) on the symmetry axis as one of the good quantum numbers of the nuclear system. The permissible values of \(K\) are governed by quantum mechanics with \(j+1/2\) possible values of suborbitals, where \(j\) is the total angular contribution from the single particle. Each of these suborbitals is doubly degenerate, each with a full suborbital having two like nucleons with anti-aligned spin in order to conform to the Pauli exclusion principle. The configuration for each suborbital is expressed by

\[
K^\pi[Nn_z\Lambda] \tag{2.14}
\]

This configuration is unique for each suborbital, where \(K\) is the projection of the single-particle total angular momentum on the symmetry axis, \(\pi\) is the parity of the state, \(N\) is the principal quantum number, \(n_z\) is the number of nodes along the axis of symmetry and \(\Lambda\) is the projection of single-particle orbital angular momentum on the symmetry axis.

### 2.5 Pairing Correlations

Pairing in nuclei is the coupling of nucleons in pairs to states of zero angular momentum. The phenomenon is most evident in nuclei with two neutrons or two protons outside of doubly-closed shells. However, it occurs to some extent in all nuclei and is responsible for the fact that all even-even nuclei have zero angular momentum \((J = 0)\) ground states [Row10]. The tendency of spin 1/2 particles to form \(J = 0\) coupled pairs is well known in a variety of many-fermion systems. For example, it is believed to be responsible for superconductivity [Coo56, Bar57, Bog58, Bog59] in macroscopic condensed systems. In this case, the fermions are electrons and the pairs are commonly called ‘Cooper pairs’. The effects of pairing in a superconductor are dramatic because they cause a many-fermion system to exhibit properties, like persistent current flows, more usually associated with a (superfluid) many-boson system [Row10].

An excess of experimental features exists, which cannot be explained within the framework of the Shell Models discussed so far. Examples of these nuclear phenomena include the following:

* The ground states of all even-even nuclei have spin and parity of \(I^\pi = 0^+\), \((a \text{ force exists that acts to couple nucleons pairwise, such that their angular momenta cancel})\).
Figure 2.4: Nilsson diagram for protons, $50 \leq Z \leq 82$ ($\epsilon_4 = \epsilon_2/6$) [Mot55, Nil55]. The convention for labeling each state is $K^\pi = [N, n_z, \Lambda]$ where $K$ is the projection of the angular momentum along the axis of symmetry, $\pi$ is the parity of the wave function of that state, $N$ is the primary quantum number from the harmonic oscillator, $n_z$ is the number of nodes in the wave function in the $z$ direction and $\Lambda$ is the component of the total orbital angular momentum along the symmetry axis $\Lambda = K \pm \frac{1}{2}$. The broken lines represent odd parity states, while the unbroken lines represent even parity states.
2. NUCLEAR STRUCTURE THEORY

Figure 2.5: Nilsson diagram for neutrons, $50 \leq N \leq 82$ ($\epsilon_4 = \epsilon_2/6$) [Mot55, Nil55]. The convention for labeling each state is $K^\pi = [N, n_z, \Lambda]$ where $K$ is the projection of the angular momentum along the axis of symmetry, $\pi$ is the parity of the wave function of that state, $N$ is the primary quantum number from the harmonic oscillator, $n_z$ is the number of nodes in the wave function in the $z$ direction and $\Lambda$ is the component of the angular momentum along the symmetry axis $\Lambda = K \pm \frac{1}{2}$. The broken lines represent odd parity states, while the unbroken lines represent even parity states.
In even-even deformed nuclei, there is a large gap $\sim 1-2$ MeV between the ground state and the first excited intrinsic configuration, which is absent in neighboring odd-$A$ nuclei, (there exists a ‘pairing gap’).

The ground state spin of odd-$A$ nuclei is determined by the spin of the last nucleon, (because it is unpaired).

It is found that even-even nuclei are more tightly bound than odd-even nuclei, which are in turn more bound than odd-odd nuclei, (attractive force).

The moment of inertia for deformed nuclei is $\approx 30-50\%$ of the expected rigid-body values at low spin (superfluidity).

In order to explain the above nuclear mysteries, and more, there was a need to account for the tendency of fermions (i.e., protons and neutrons) to pair with similar fermions. These, so called, ‘pairing correlations’ are the consequence of the short range, attractive component of the nucleon-nucleon force, and push the nucleons into states with momentum equal in magnitude or / and opposite in sign, while obeying the Pauli Exclusion Principle (PEP). In the extremely short range limit, this attractive nuclear interaction may be described by a delta function, $\delta(r_1 - r_2)$, which only affects particles that occupy the same position in space, and is weak otherwise. Of course, completely overlapping orbitals are forbidden, as mentioned above, and thus the next possible case scenario, is for the pair of nucleons to travel in ‘time reversed orbits’. This means that the two nucleons may have the same quantum numbers, except for the magnetic quantum number, which is taken to be the same magnitude but of opposite sign.

The time reversed orbits result in a coupling of the nucleons to zero angular momentum pairs in the nuclear ground state. This coupling of particles to spin $I = 0$ is called the monopole pair interaction. This way, the pair may achieve lower energies. The qualitative description is of two like-nucleons moving in the same $j_1$ shell, but in opposite directions $K_1$ and $-K_1$. At this point, there is high probability that a collision may occur. If it happens, the two nucleons will be scattered into another equal and opposite orbit, depicted as $(j_2, \pm K_2)$, in Figure 2.6(a). Far below the Fermi surface the Pauli principle forbids such scattering to occur, since there are no free orbits available for the pairs to scatter into. The energy levels are all filled up to a maximum energy called the Fermi energy. All the levels below this Fermi surface are occupied, while the levels above it are empty. Therefore, the Fermi surface, represents the energy of the upper-most occupied state, that is filled in the absence of the pairing force. However, at the
Figure 2.6: (a) The scattering of a pair of nucleons from one time reversed orbital \((j_1, K_1)\), \((j_1, -K_1)\) to another \((j_2, K_2)\), \((j_2, -K_2)\) due to the pairing force in a deformed nucleus. (b) The occupation for the diffuse Fermi surface of a pair correlated state compared with that of a sharp Fermi surface for an uncorrelated state [Pip06].

The scattering, mentioned above, leads to a mixture of states and a partial occupancy of levels, as opposed to strict particle and hole states. The picture therefore, of a particle
occupying a particular state, has to be changed into combinations of a particle and a hole known as a ‘quasiparticle’ [Bar57, Boh58]. In other words, this can be considered as a linear combination of particle and hole wavefunctions. At large distances below the Fermi surface the quasiparticle is a hole state, whereas far above the surface, it is a particle state. It is in the region around the Fermi surface that the quasiparticle is ‘part hole and part particle’. Occupation probabilities now need to describe the nuclear state. The probability that a state $i$ is occupied by a hole or a particle is given by the expression [Pip06, Cas00]:

$$U_i^2 = \frac{1}{2} \left[ 1 + \frac{(\epsilon_i - \lambda)}{\sqrt{(\epsilon_i - \lambda)^2 + \Delta^2}} \right], \quad V_i^2 = \frac{1}{2} \left[ 1 + \frac{(\epsilon_i - \lambda)}{\sqrt{(\epsilon_i - \lambda)^2 + \Delta^2}} \right],$$

(2.15)

and the normalization requires:

$$U_i^2 + V_i^2 = 1.$$

(2.16)

where $U_i^2$ is the probability that the orbit $i$ is occupied by a hole, $V_i^2$ is the probability the orbit $i$ is occupied by a particle, $\epsilon_i$ is the single-particle energy of the state $i$ and $\lambda$ is the Fermi energy. Clearly, far below the Fermi energy, ($\epsilon_i \ll \lambda$) $V_i^2 = 1$, and far above the Fermi energy ($\epsilon_i \gg \lambda$) $U_i^2 = 1$. Close to the Fermi surface the occupation probabilities are mixed. In Equation 2.15, $\Delta$ is known as a pair gap parameter [Pip06, Cas00]:

$$\Delta = G \sum_{i,j} U_i V_j.$$  

(2.17)

The pairing gap is the difference between the ground state and the excited band. In Equation 2.17 $i, j$ refer to different orbits and $G$ is the strength of the pairing force, which decreases with $A$ in heavier nuclei; here the outer nucleons are generally further apart and so spatial overlaps are likely to be less. $G$ may also be different for protons and neutrons, being lower for the former because of the Coulomb repulsion. Commonly used prescriptions are [Pip06, Cas00]:

$$G_p = \frac{17}{A}, \quad G_n = \frac{23}{A} MeV$$

(2.18)
Note that if there was no pairing, the Fermi surface \( \lambda \) would coincide with the last orbit being filled, and \( (\epsilon_i - \lambda) \) would be the excitation energy required to excite one of the nucleons, in this last orbit, to one of the highest orbits \( \epsilon_i \). In the presence of pairing, however, this single-particle excitation energy \( (\epsilon_i - \lambda) \) is replaced by the quasiparticle energy \( E_i \) given by [Pip06, Cas00]:

\[
E_i = \sqrt{(\epsilon_i - \lambda)^2 + \Delta^2}
\]  

(2.19)

The behavior of \( E_i \) is interesting and has important consequences. In even-even nuclei, the ground state is a quasiparticle vacuum or zero quasiparticle state \( 0\text{-qp} \), and the simplest excitation consists of breaking one pair and raising a particle to the next higher orbit. Without pairing, this is a particle-hole excitation. In the presence of pairing, it appears as a two quasiparticle excitation \( 2\text{-qp} \), one quasiparticle \( 1\text{-qp} \) being the hole left behind and the other being the particle excitation newly created. Thus, the excitation energy is equal to the sum of the individual \( (i \text{ and } j) \) quasiparticle energies [Pip06, Cas00]:

\[
E_{xij} = \sqrt{(\epsilon_i - \lambda)^2 + \Delta^2} + \sqrt{(\epsilon_j - \lambda)^2 + \Delta^2}
\]  

(2.20)

Clearly, the minimum energy required for this \( 2\text{-qp} \) excitation is \( 2\Delta \), giving the famous 'pairing gap' that is a nearly universal feature of even-even nuclei. In fact, most \( 2\text{-qp} \) structures occur at \( \sim 1.5\text{-}2 \text{ MeV} \) and typical values of \( \Delta \) for even-even nuclei range from 0.7-1 MeV. On the other hand, in an odd-\( A \) nucleus the ground state is a \( 1\text{-qp} \) state, since it is formed by one unpaired particle outside an even-even core. The first excited state can be created by merely changing the orbital of the odd particle. Basically, all it takes is to promote the unpaired quasiparticle, rather than to create one, as in the even-even case. Therefore, the excitation energy is now only the difference in energy between the states [Pip06, Cas00]:

\[
E_{xi} = E_i - E_0 = \sqrt{(\epsilon_i - \lambda)^2 + \Delta^2} + \sqrt{(\epsilon_0 - \lambda)^2 + \Delta^2}
\]  

(2.21)

where \( E_0(\epsilon_0) \) is the quasiparticle single particle energy of that orbit nearest the Fermi energy \( \Delta \). In this case, however, there is no minimum energy required for an intrinsic excited state. The effect of pairing here is to actually decrease the excitation energies of the low lying states, compressing the excitation energy spectrum, for orbitals near
Fermi surface. At higher energies, where $\epsilon_i - \Delta \gg \Delta$, the effect is to lower all states by an amount $\sim \Delta$ [Pip06].

2.6 Nuclear Rotation

So far we have considered theoretical models, which do not take into account the effects of rotation on the nuclear system. However, in order to describe experimental spectra from deformed nuclei, a combination of single particle and collective models are necessary. Hence, an appropriate place to begin this discussion would be the different modes of generating nuclear angular momentum, but before continuing with such a description, it is necessary to clarify the concept of nuclear rotation.

For spherical systems, due to the symmetry involved, all three axes in space ($x$, $y$, and $z$) are quantum mechanically indistinguishable, and therefore, collective rotation around any of these axes of symmetry is forbidden. Because of the axial symmetry, the energy of the nuclear wavefunction, rotated by an arbitrary angle about a symmetry axis, is the same as its unrotated value. This is justified as the effect of rotation is only to introduce a phase change, so there is no energy associated with such rotation. Put in another way, there is no way of measuring a rotation of the whole nucleus about a symmetry axis, and hence, there is no associated collective rotational energy. However, in the case of axially deformed nucleus, there is a set of axes of rotation ($x$, $y$), perpendicular to the symmetry axis ($z$). This implies that the rotational operator describing rotations about the $z$ axis, $\hat{R}_z$, must have zero effect (\textit{It is not that the operator is zero, but that its action only produces a phase change, which is not observable}), and thus, any rotational angular momentum $\vec{R}$ must be perpendicular to the symmetry axis; for instance we may choose $x$ as our axis of rotation see subsection 2.6.3 [Jel90].

2.6.1 Non-Collective Single-Particle Excitation

In the proximity of closed shells, spherical or near spherical nuclei reside. These nuclei can only generate angular momentum ‘spin’ non collectively, by the rearrangement of the valence nucleons; for example they can align the spins of individual nucleons all in the same direction, as shown in Figure 2.7(b). The total angular momentum is, then, calculated as the sum of the valence nucleons that are not coupled to spin zero. The bulk of the nucleons from the rest of the nuclear matter (the core) make no contribution
due to pair correlations. This mechanism by which spin is built is based on the single
particle motion, which represents an out of phase motion and leads to shell structure
and single nucleon states. For instance, the ‘Spherical Shell Model’ describes those
particular nuclear properties in which only the nucleons in the vicinity of the Fermi
surfaces are involved. Figure 2.7(b) also illustrates how complex and irregular the
energy level spectrum is when single particle excitation is involved [Eva04, Pau03].

2.6.2 Collective Rotation

For axially deformed nuclei, angular momentum can be generated by increasing their
collective rotational velocity. As stated on subsection 2.6, the rotation is perpendicular
to the symmetry axis, with the bulk of the nucleons making a coherent contribution to
the angular momentum. This collective motion, represents an ‘in phase’ or coherent
motion of all the nucleons and leads to deformation, vibrational and rotational degrees
of freedom. The ‘Liquid Drop Model’ belongs in this category, and describes the bulk
nuclear properties, which depend on the nucleon number in a smooth way. The char-
acteristic signature of this mode of excitation, is the observation of smooth rotational
band sequences in the nuclear level scheme, and Figure 2.7(a) clearly displays such
a scenario. Nuclear structure physics involves the delicate interplay between the two
fundamental modes of excitation, collective and non collective [Pau03].

2.6.3 Nuclear Angular Momentum

To describe the interplay between the motion of particles and the collective rotation,
[Boh53, Boh55] proposed to take into account only a few valence particles, which move
more or less independently in the deformed well of the core, and to couple them to a
collective rotating core the ‘rotor’, which stands for the rest of the particles. Consider
an axially deformed nucleus rotating about the x-axis, with an orbiting valence nucleon
as shown in Figure 2.8. The total angular momentum of this system, $\vec{T}$, is composed
of two parts. The first, $\vec{R}$, is the angular momentum generated by the collective rotation
of many nucleons about the x-axis (core), and the second, $\vec{J}$, that generated by the
intrinsic motion of the valence nucleons:

\[ \vec{T} = \vec{R} + \vec{J} \quad (2.22) \]
Figure 2.7: Two different mechanisms of generating angular momentum (spin I), namely collective rotation (a) and (b), non collective (single particle excitation). In the first case, (a) corresponds to a deformed nucleus, such as $^{158}\text{Er}$, and the result is a smooth rotational band displayed by a regular level scheme. However, (b) corresponds to a nearly spherical nucleus, such as $^{147}\text{Gd}$, in which case the level scheme is very complex [Pip06].

The angular momentum of the valence nucleons, $\vec{J}$, is the sum of the angular momentum of the individual valence nucleons:

$$\vec{J} = \sum_{i=1}^{\text{valence}} \vec{j}_i \quad (2.23)$$

For one valence nucleon ($i=1$), Equation 2.23 reduces to $\vec{J} = \vec{j}$. The projection of the total angular momentum $\vec{I}$, onto the symmetry axis, is $K$, and is the same as the
projection of $\vec{J}$, whereas the projection of $\vec{I}$ is denoted by $I_x$:

$$I_x = \left[ \sqrt{I(I+1) - K^2} \right] \hbar, \quad (2.24)$$

and is known as the aligned angular momentum. The projection of a valence nucleon's total angular momentum ($\vec{J}$), onto the symmetry axis, is $\Omega$.

How do we distinguish the different mechanisms for the generation of angular momentum (single particle alignment and collective rotation) experimentally? We mentioned earlier in subsection 2.6.1 and subsection 2.6.2 that the level scheme of a deformed nucleus is smooth, as opposed to a spherical nucleus. We may start probing this statement by considering the simple case of an even-even deformed nucleus. In the band built on the ground state (ground state rotational band) of such a nucleus, the valence particles are paired off, such that $\vec{J} = 0$. Because at low spin ($\sim 10\hbar$) the pairing force dominates, $\vec{I} = \vec{R}$. Hence, if the nucleus, in the simplest approximation, is regarded as a classical rigid body, then its rotational energy $E$ is given by [Eji89, Kra88]:

$$E = \frac{1}{2} I \omega^2, \quad (2.25)$$

where $I$ is the (static) moment of inertia (sometimes the static moment of inertia is represented by the symbol $I^{(0)}$), assumed to be constant, and $\omega$ is the rotational frequency. We can modify this expression in terms of the rotational angular momentum $|\vec{R}| = R$:

$$R = I = \Omega \omega \quad (2.26)$$

so that the energy now becomes:

$$E = \frac{R^2}{2 \Omega} \quad (2.27)$$

The real nucleus, however, is a quantum object and the variable $R = I$ is not continuous. Taking the quantum mechanical value of $I$:

$$I^2 = I(I+1)\hbar^2, \quad (2.28)$$

results in:
2.6 Nuclear Rotation

\[ E = \frac{\hbar^2}{2\mathcal{I}} I(I + 1) = A I(I + 1), \text{with} \ A = \frac{\hbar^2}{2\mathcal{I}} = \text{const.} \quad (2.29) \]

Equation 2.29 defines a series of levels, which have energies proportional to \( I(I + 1) \); the \( \gamma \)-ray energies increase smoothly, see subsection 2.6.2. It is a simple expression for the energies of a collectively rotating nucleus, when there are no single particle excitations (\( K = 0 \)). Increasing spin \( I \) corresponds to adding rotational energy to the nucleus, and the nuclear excited states should cascade down toward the ground state, through a sequence of \( E2 \gamma \) transitions, known as a rotational band. One can calculate from Equation 2.29 that a pure rigid rotor assuming constant \( \mathcal{I} \) would have \( E(I = 4^+)/E(I = 2^+) = 3.33 \), This simple formula is one of the most famous results of the rotational model and still remains one of the best signatures for rotational motion and deformation \[\text{[Cas00]}\]. The energy of a transition in the rotational band, between two successive levels \( (I \rightarrow I - 2) \) is given by (from Equation 2.29):

\[ E_\gamma = E(I) - E(I - 2) = \frac{\hbar^2}{2\mathcal{I}} (4I - 2) \quad (2.30) \]

Thus, the transition energy \( E_\gamma \) is linear in \( I \). From this it follows that successive transition energy differences, \( dE_\gamma/dI \), should be constant at a value of \( 2\hbar^2/\mathcal{I} \). A spectrum of \( \gamma \)-ray transitions, representing a \( \Delta I = 2 \) cascade down a rotational band, should therefore show a set of uniformly spaced peaks. However, with increasing rotation (angular velocity), the Coriolis force causes nucleons to align their spin along the rotation axis. This occurs for pairs of nuclei in specific orbits, at certain rotational frequencies; the \( \gamma \)-rays bunch together at such points (not equally spaced anymore). The observed rotational energy levels begin to deviate more and more from the \( I(I + 1) \) rule.

The agreement between Equation 2.29 and experiment, for the calculated energy levels, is not quite exact, but is good enough to give us confidence that we have at least a rough idea of the origin of the excited levels. The reason is that the rotational behavior of the nucleus is intermediate between a rigid object, in which the particles are tightly bonded together, and a fluid, in which the particles are only weakly bonded. Let us consider two extreme cases, first that of a solid ellipsoid of revolution of mass \( M \) with rigid moment of inertia \( \mathcal{I}_{\text{rigid}} \), and second, that of a fluid inside a rotating ellipsoidal vessel, with fluid moment of inertia \( \mathcal{I}_{\text{fluid}} \). By comparing the corresponding values for the rotational energy-constant, \( \hbar^2/2\mathcal{I} \), with that of a typical nucleus, say in the \( A \approx 150 \)
rare earths deformed region where the constant is roughly 15 keV, we conclude that [Kra88]:

\[ \Im_{\text{rigid}} > \Im_{\text{nucleus}} > \Im_{\text{fluid}} \] (2.31)

The nucleus is not a rigid body and measured moments of inertia are somewhat less of the order of 30% to 50%, than rigid body values at low spins [Bar57]. This is due to the effects of the pairing interaction, which makes the nucleus behave like a superfluid. As the nucleus rotates, it is found that the moment of inertia changes as a function of spin.

Figure 2.8: A schematic illustrating an angular momentum coupling scheme for an axially deformed core, rotating about an axis (x) perpendicular to the symmetry axis (z), with angular velocity (rotational frequency) \( \omega \). In the figure, \( I \) is the total angular momentum, composed of two parts: the angular momentum of the valence nucleon \( j(=J) \) and the angular momentum \( R \) due to collective rotation. \( K(=\Omega) \) is the projection of \( I \) (and \( j \)) onto the symmetry axis and \( I_x \) is the projection of \( I \) onto the rotating axis, modified from [Pip06].
2.6 Nuclear Rotation

2.6.4 Moments of Inertia

We begin this discussion by relating the rotational frequency $\omega$ to the experimentally observed transitions, $E_\gamma$. Classically, the rotational frequency $\omega$ can be defined from (Equation 2.25 and Equation 2.26):

$$\omega = \frac{dE}{dI}$$  \hspace{1cm} (2.32)

In a quantum mechanical treatment of Equation 2.32, one should use:

$$\omega = \frac{dE}{dI_x},$$  \hspace{1cm} (2.33)

where $I_x$ is given by Equation 2.24. We can simplify the expression for $\omega$, by concentrating on the $K = 0$ case $I_x = I\hbar$. Also, for $I \gg 1$, using Equation 2.29 and Equation 2.30 [Eji89]:

$$\hbar \omega \approx \frac{E_\gamma}{2}$$  \hspace{1cm} (2.34)

The above equation is also a good approximation for a simple reason; all experimental data come in the form of discrete energy transitions, and thus, Equation 2.33 can be written as:

$$\hbar \omega = \frac{\Delta E}{\Delta I}$$  \hspace{1cm} (2.35)

Since, the numerator equals just $E_\gamma$ and $\Delta I = 2$, for stretched quadrupole transitions within a rotational band, we can easily derive Equation 2.34. According to this equation, each transition within a band is assumed to have some frequency $\omega$, that is the average of the rotational frequency values of the two levels connected by the transition.

The moment of inertia of a rotating deformed nucleus is expected to be strongly affected by changes in the pairing correlations (the Coriolis anti-pairing effect). Other dynamic variables, which may influence the moment of inertia, include deformation changes, shell effects and the spin alignment of quasiparticles. It is, therefore, very difficult to associate a certain observed change in the moment of inertia with one specific aspect of nuclear dynamics. To overcome this obstacle, [Boh75] introduced two possible definitions for the moment of inertia reflecting two different classes of nuclear dynamical...
properties. The (kinematic moment of inertia, \( \mathcal{I}^{(1)} \)), governing an ideal collective rotation is defined as a quantity proportional to the inverse of the slope (first derivative) of the \( E \) versus \( I \) curve. The name of \( \mathcal{I}^{(1)} \) is appropriate, since it emphasizes its relation with the kinematics (motion) of the nucleus expressed in the \( I/\omega \) ratio, and contains information on nuclear properties like deformation and pairing. Using Equation 2.24 and Equation 2.29 for \( K = 0 \) and \( I \gg 1 \):

\[
\mathcal{I}^{(1)} = \hbar^2 I \left( \frac{dE}{dI} \right)^{-1} = \hbar \frac{I}{\omega}
\]

(2.36)

On the other hand, the dynamic moment of inertia, \( \mathcal{I}^{(2)} \), expresses the response of the nucleus to a force; a dynamic effect like backbending will cause abrupt changes in \( \mathcal{I}^{(2)} \). Using the same equations as above, the dynamic moment of inertia is related to the curvature (second derivative) of the \( E \) versus \( I \) curve:

\[
\mathcal{I}^{(2)} = \hbar^2 \left( \frac{d^2E}{dI^2} \right)^{-1} = \hbar \frac{dI}{d\omega}
\]

(2.37)

Such definitions are analogous to the two possible definitions for the effective mass of an electron in a crystal lattice. Experimentally, the two moments of inertia, using Equations 2.30, 2.34 and, 2.40 are defined as:

\[
\mathcal{I}^{(1)} = \hbar^2 \frac{2I}{E_\gamma},
\]

(2.38)

and

\[
\mathcal{I}^{(2)} = \frac{2\hbar^2}{\Delta E_\gamma}
\]

(2.39)

The dynamic moment of inertia is not a function of the spin of the levels in the rotational band, only in the transition energy difference of consecutive \( \gamma \)-rays, and its measurement is very sensitive to the structural changes taking place within the band. If \( \mathcal{I}^{(2)} \) were constant, \( \Delta E_\gamma \) would be the same for all values of spin. Often this is not true and \( \mathcal{I}^{(2)} \) is found to increase with spin. The two moments of inertia are related via their definitions:

\[
\mathcal{I}^{(2)} = \hbar \frac{dI}{d\omega} = \hbar \frac{d[\omega \mathcal{I}^{(1)}]}{d\omega} = \hbar \left[ \mathcal{I}^{(1)} + \omega \frac{d\mathcal{I}^{(1)}}{d\omega} \right],
\]

(2.40)

and in the limit of rigid rotation, \( \mathcal{I}^{(2)} \approx \mathcal{I}^{(1)} \).
2.6 Nuclear Rotation

2.6.5 The Coriolis Anti-Pairing Effect

The Coriolis name in this discussion is used for forces created by a body, which is moving with velocity $\vec{v}$, on a rotating system animated by an angular velocity $\vec{\omega}$. The effect of the Coriolis force is an apparent deflection of the path of this object, which does not actually deviate from its path, but it rather appears to do so because of the motion of the coordinate system. Since we live on a rotating planet, the Coriolis effect is very common. The resulting ocean currents, weather patterns and the vortices formed during draining of our tubs are accepted consequences of these forces. Nevertheless, the forces are generally weak, and they do not affect our human activities. This is probably the reason why only advanced studied in Physics deal with this phenomenon [Pip06].

![Figure 2.9: The effect of the Coriolis force, $F_{\text{cor}}$, for two nucleons (1 and 2) moving in time reversed orbitals. The Coriolis force acts in opposite directions for each nucleon, and thereby decreases the pairing correlations with respect to the non-rotating case. The rotational axis points out of the page [Pip06].](image)

It turns out that nuclear rotation generates inertial forces, which simulate the effects of electric and magnetic fields. As in the case of any rotating system, the inertial force can be separated into centrifugal and Coriolis forces. The former act radially and result in a stretching of the nucleus. The effect is analogous to the stretching of a polar molecule.
by the application of an electric field. The Coriolis force takes the form:

\[ F_{\text{cor.}} = -2m \mathbf{\omega} \times \mathbf{v}, \]  

(2.41)

where \( m \) is the mass \( (i.e.\) nucleon in our case) and \( \mathbf{v} \) the velocity of the moving object (nucleon). This expression inescapably reminds us of the force \( e \mathbf{v} \times \mathbf{B} \) on an electric charge \( e \), moving with velocity \( \mathbf{v} \) in a magnetic field \( \mathbf{B} \). Thus, the effects of these forces are similar to those induced by electric and magnetic fields \([\text{Gel92}]\) At low spins \( (\sim 10\hbar) \) the nucleus displays well established superfluid properties with nucleons teaming up in time reversed orbits, or ‘Cooper pairs’. For two nucleons paired together in time reversed orbitals, an increase in rotational frequency \( \omega \) will increase the Coriolis force which becomes greater than the pairing energy for the two nucleons. At a certain critical angular momentum \( I_{\text{crit.}} \) (or rotational frequency \( \omega_{\text{crit.}} \)), the pairing correlations will be quenched completely. This process is the analogue of the quenching of superconductivity by a sufficiently high magnetic field (Meissner Effect), and it is called the Coriolis Anti-Pairing (CAP) effect, first discussed by \([\text{Mot60}]\). In fact, in this interpretation all nucleons correspond to the force coherently, and a phase transition, from a super-fluid \( (\text{paired}) \) to a normal fluid \( (\text{unpaired}) \) state, is involved.

In 1972 \([\text{Ste72}]\) refined the idea of the CAP effect, by showing that the Coriolis force affects certain orbitals more than others see Equation 2.42. At low rotational frequencies the story essentially remains the same, where with a sufficiently deformed nuclear field the nucleonic motion is coupled to the deformation of the core. The angular momentum of the valence nucleon will subsequently be aligned with symmetry \( z \)-axis. This extreme coupling limit is known as the strong or deformed aligned (DAL) scheme. As the rotation increases, at a critical frequency \( \omega_{\text{crit.}} \), the two paired particles split and begin to travel around the rotating nuclear core in different orbitals; at this point, each nucleon will align its angular momentum \( \mathbf{j} \) along the rotational axis \( x \). The nucleons’ motion is now coupled to the rotation of the core and this coupling represents the weakly deformed aligned or strongly rotation-aligned (RAL) scheme. The angular momentum, generated by these two rotationally aligned nucleons, is given by the sum of their angular momentum projections on the rotational axis \( x \). This contributes to the total nuclear angular momentum \( \mathbf{I} \) and the nucleus can essentially reduce its angular frequency of rotation \( \omega \) \( (i.e.\) ‘slow down’), by increasing its moment of inertia \( \mathcal{I} \), whilst preserving the same total angular momentum. Note that the moment of inertia
2.6 Nuclear Rotation

which is a measure of mass distribution changes, since the effect of the Coriolis force results in the transfer of the nucleons into different orbits. Although large centrifugal stretching would also lead to an increase in the moment of inertia, centrifugal forces are not thought to play a major role until higher angular momentum [Cul91]. An estimate of the maximum Coriolis energy \( V_{\text{Cor.}}^{\text{max}} \), associated with a nucleon, in a rotating nucleus can be made by using [Cas00]:

\[
V_{\text{Cor.}}^{\text{max}} = \frac{\hbar^2}{2\mathcal{I}} I j, \tag{2.42}
\]

where \( I \) is the total angular momentum, \( j \) is the single particle angular momentum, and \( \mathcal{I} \) is the moment of inertia. Evidently from Equation 2.42, the Coriolis force is not only dependent on how fast the nucleus is rotating, but also which orbital a nucleon occupies. Consequently, high-\( j \) orbitals are predicted to be aligned first, as they are especially sensitive to the Coriolis force. In the \( A \approx 150 \) mass region, the proton \( h_{11/2} \) and neutron \( i_{13/2} \) orbitals, have proven to be greatly affected by the Coriolis force at high spin [Pip06]. Typically the \( i_{13/2} \) neutron orbital is the last filled neutron single-particle orbital, in the rare-earth nuclei, and the \( h_{11/2} \) is the one for protons. So, for \( I = j = 13/2 \) and a typical inertial parameter \( \hbar^2/2\mathcal{I} = 15 \text{ keV} \), for rare-earth nuclei, Equation 2.42 yields \( V_{\text{Cor.}}^{\text{max}} \approx 600 \text{ keV} \). This indicates that the Coriolis force is not necessarily a minor perturbation, and it thus cannot be neglected.

2.6.6 Band Crossing and Backbending

In the region between 10 and 20 units of angular momentum (\( \hbar \)), an anomaly is observed in the transition energies of the ground state rotational bands of many deformed nuclei. The nuclear excitation energy is observed to deviate from the expected rigid rotor \( I(I+1) \) dependence. In fact, in 1971 [Joh71] found, for the first time, a sudden change in the ground state rotational bands of \( ^{162}\text{Er} \) and \( ^{158,160}\text{Dy} \). The phenomenon seen by [Joh71], has come to be known as ‘backbending’ and denotes a change in the intrinsic structure of the nucleus. Backbending occurs when the Coriolis force, becomes greater than the pairing energy of two nucleons moving in time-reversed orbits. When that happens, at an energy corresponding to a critical rotational frequency (\( \omega_{\text{crit}} \)), the unpaired nucleons go into different orbits and increase significantly the nuclear moment of inertia, while the rotational frequency decreases. This phenomena, in the \( A \sim 160 \)
mass region, is a consequence of breaking a pair of \( i_{13/2} \) neutrons, which then align their angular momentum with the rotational axis. The situation then remains stable until another pair (\( h_{11/2} \) protons) is broken and another change in the moment of inertia occurs. This second backbending is usually more challenging to observe experimentally, because it happens at higher rotational frequencies. However, it has been seen in various bands of \( ^{157,158,159} \)Dy and \( ^{158} \)Er, which have been observed by [Pip06].

Backbending can be most easily demonstrated if one plots the moment of inertia as a function of \( \hbar \omega \). The term backbending arose from the ‘S’ shape of the curve seen in the plot, since the moment of inertia bends back and up. Figure 2.10(a) is an example of this phenomena for the nucleus \( ^{158} \)Er, up to \( \hbar \omega \approx 0.4 \) MeV. The moment of inertia gradually increases with rotational frequency up to about 0.27 MeV, at which point a radical change occurs; the moment of inertia increases dramatically, while the nucleus reduces its collective rotation. An alternative backbending plot, that reproduces this S-shape, is a spin-projection or ‘alignment \( i \)’ plot, shown in Figure 2.10(b). In addition, this type of a plot allows us to extract the gain in the angular momentum, due to the alignment of the quasiparticles, and thus compare it with theoretical values.

**Figure 2.10:** (a) Kinematical moment of inertia as a function of rotational frequency \( \hbar \omega \) for the yrast sequence of \( ^{158} \)Er. The ‘S’ shape is usually referred to as a backbending and it occurs at \( \hbar \omega \approx 0.27 \) MeV, due to the alignment of a pair of \( i_{13/2} \) neutrons. Below this critical frequency value the yrast sequence is composed of the ground state band, but after the backbend the S-band becomes yrast. (b) A similar plot of (a) displaying the same backbend phenomenon. However, this alignment \( i \) plot contains additional information about the gain in angular momentum, during the backbend, because of the \( i_{13/2} \) neutrons. Modified from [Pip06].
2.6 Nuclear Rotation

Backbending can be described by one set of rotational state crossing another one, as shown in Figure 2.10(a) and Figure 2.10(b). Both figures display a maximum range of $I \approx 25\hbar$, for the ground state $0-qp$ and ‘S’ 2-$qp$ bands of $^{158}$Er. In even-even nuclei like $^{158}$Er, the band built upon the first pair of aligned quasiparticles is known as the ‘Stockholm’ ‘S’-band. For $I \leq 12\hbar$ the yrast sequence is the ground-state band, in which all the nucleons are paired off and the angular momentum is generated solely by the collective rotation of the nucleus. At $I \approx 12 \sim 14\hbar$, the two bands cross and the first backbend occurs. At this point it is equivalent to say that the nucleus changes its intrinsic structure. This corresponds to a transition from one band to another. For $I > 12 \sim 14\hbar$, the most energetically favorable band yrast becomes the S-band. The reason that one usually follows the yrast band is that these levels are experimentally the most accessible. It can be immediately seen that the angular frequency of rotation, which is related to the slope in the curve of $E_x$ versus $I$ must decrease at the crossing point, whereas $I$ increases. This means physically that a part of angular momentum is transferred into the intrinsic degrees of freedom, thus leading to a slowing down of the collective rotation [Voi83].

![Figure 2.11](image)

**Figure 2.11**: (a) A graph of excitation energy vs spin for the ground state and $S$ bands of $^{158}$Er, up to $I = 25\hbar$. The two bands cross at $I \approx 12\hbar$, at which point the S-band becomes yrast. (b) A rigid rotor plot for the $^{158}$Er data, with rigid rotor reference of $\hbar^2/2\Omega_{rig.} [I(I+1)] = 0.007I(I+1)$ subtracted from the excitation energies. Modified from [Pip06].

Figure 2.11(a) and Figure 2.11(b) are similar plots. The former one is simply based on direct experimental observables, like the excitation energies $E_x$ and the spin $I$ of the
nuclear states. The latter one, is commonly known as the ‘rigid rotor’ plot, where the excitation energy is plotted minus the standard rigid-rotor reference defined in Equation 2.29 for $\mathcal{S}_{\text{rig}}$. The rigid-body moment of inertia is normalized to $^{158}\text{Er}$ [Afa99]:

$$\frac{\hbar^2}{2I} = 0.007 \frac{158^{5/3}}{A} \text{MeV}$$

Equation (2.43)

The subtraction of the rigid-rotor reference has the effect of magnifying the differences between the ground-state and the ‘S’ bands, which are not visible in Figure 2.11(a), and turns out to be a very important kind of plot when it comes to identifying other changes in structure.
Experimental Methods

3.1 Introduction

The experimental determination of electromagnetic transition matrix elements often begins with the detection of $\gamma$-rays from transitions in the nucleus. Precision $\gamma$-ray spectroscopy with large detector arrays is an important part of the research programs at virtually all nuclear physics accelerator laboratories [Sve03]. These spectrometers permit the determination of the energies of $\gamma$-rays and their intensities, which are important to an understanding of the decaying nuclear excited states from which they are emitted [Sch09].

3.2 Heavy-Ion Fusion Evaporation (HIFE)

In studies of low-spin states one obviously searches for reactions which transfer to the nucleus the lowest possible angular momentum and the largest possible cross section. Heavy-ion fusion-evaporation reactions provide the most efficient way to achieve high spins. Fusion reactions also produce several nuclei or reaction channels, depending on how many particles ($p, n, \alpha$) are evaporated.

This mechanism involves the fusion of an incident projectile nucleus with a target nucleus, resulting in formation of a compound system [Boc80]. In order for fusion to occur, the projectile kinetic energy must be greater than the Coulomb repulsion energy, due to protons, or the projectile will be scattered away from the target nucleus [Pip06]. The Coulomb barrier that needs to be overcome is given approximately by [Kra88].
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\[ E_C(MeV) = \frac{1.44Z_1Z_2}{1.16(A_1^{1/3} + A_2^{1/3} + 2)} \]  

where \( Z_1 \) and \( Z_2 \) denote the beam and target atomic numbers, respectively, \( A_1 \) and \( A_2 \) refer to the beam and target mass number. One speaks about the formation of a compound nucleus only if the lifetime of the composite system is long enough (\( \geq 10^{-20} s \)) for thermodynamic equilibrium to be reached before it decays. According to Niels Bohr, the decay probability depends only on the total energy given to the system and is independent of the formation processes. The compound nucleus resembles a hot, charged, rotating liquid drop, and is highly unstable. After a relatively short period of time (\( \sim 10^{-19} s \)) it decays rapidly, evaporating particles to carry away energy (heat). A wide variety of particles may be evaporated, the relative binding energies dictating which ones are favored. Although, protons, neutrons and alpha particles evaporation are all possible reaction channels, the Coulomb barrier is sufficiently strong in the (\( A \sim 160 \)) region such that charged particles are greatly inhibited from emission. Therefore, neutron evaporation dominates the early decay process of the compound nucleus. An emitted neutron lowers considerably, the excitation energy, by its separation energy \( \sim 8-10 \text{ MeV} \) plus its kinetic energy \( \sim 1-2 \text{ MeV} \), but carries off little angular momentum on average \( \sim 1\hbar \). This occurs because the particles face a centrifugal barrier, created by the rapid rotation. Thus, particle evaporation leaves the nucleus cooled but still spinning with very high rotational frequency. Particle evaporation continues until the excitation energy remaining in the system is within one neutron separation energy or less, above the yrast line.

As particle emission is no longer possible at this point, the compound nucleus de-excitation may now proceed by electromagnetic radiation emission. The nucleus begins to cool off, first, through the emission of statistical \( \gamma \) rays, \( i.e. \), \( \gamma \)'s whose strengths are dictated by random overlaps between the initial state wavefunction and states lying below them in energy and close to them in spin see Figure 3.1. These are predominantly electric dipole (\( E1 \)) \( \gamma \)-ray transitions which remove excitation energy but little angular momentum. These transitions produce a tail in the high energy region of the \( \gamma \)-ray spectra. In fact, in this region the level density is extremely high.
3.2 Heavy-Ion Fusion Evaporation (HIFE)

Figure 3.1: Schematic illustrating a heavy-ion fusion-evaporation reaction; statistical transitions ‘cooling’ the nucleus, collective rotational cascades decelerating the angular rotation, and the collective discrete transitions between low-spin, near yrast states that are resolvable with existing detection systems. Modified from [Kha11].

and the γ-rays are too weak and closely spaced in energy, to be individually resolved. The result is a so-called γ-ray continuum. As the energy of the system approaches the yrast line (within ∼2-3 MeV of the yrast) the level density decreases sufficiently, so that stretched quadrupole ($E2$) transitions dominate and de-excite the nucleus (if it is deformed) through many collective bands; energy is dissipated in the process and, also, angular momentum. As the γ-ray emission proceeds, the cascade becomes concentrated into fewer near-yrast sequences. Figure 3.1 illustrates this. The nucleus begins to approach its ground state approximately $10^{-9}$ seconds after it is formed. By the time the nucleus reaches its ground state it has rotated $10^{11}$ times, i.e., only one order of magnitude less than the number of rotations of the earth since its creation [Eji89]. In the end, the cascades become so concentrated, that we are actually able to detect discrete transitions, using unique techniques.
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Figure 3.2: The schematic diagram of the excitation energy, \( E_{\text{exc}} \), versus the nuclear spin \( I \). The diagram illustrates typical \( \gamma \)-ray de-excitation paths, following the production of a very hot compound nucleus in a heavy ion fusion evaporation reaction. The yrast line represents the minimum excitation energy of a level at a given spin. The entry line (dashed curve) marks the start of the \( \gamma \)-ray cascades following the particle evaporation.

3.3 Detection of Gamma Radiation

In their basic principles of operation, most detectors of nuclear radiations follow similar characteristics. The incoming photon during an interaction produces an energetic electron. The interaction time is very small and can be considered as instantaneous [Del92, Kno00]. It is these secondary electrons which finally loose their energy in the medium and produce electron-hole or electron-ion pairs which constitute an electric charge \( Q \). In semi-conductor detectors these charges are collected by the application of an electric field and form the basic signal carrying the information about the detected
3.3 Detection of Gamma Radiation

The time required for collection of charges depends on the type of detector, for example a semiconductor detector has a collection time of about a few nanoseconds while for a gas detector it is a few milliseconds. The amplitude of the signal pulse is proportional to the energy of radiation deposited [Tai80, Neg11].

3.3.1 Semiconductor Detectors

Solid semiconducting materials germanium (Ge) and silicon (Si) are frequently used as γ-ray detectors. Both germanium and silicon form solid crystals in which four valence electrons form four covalent bonds with neighboring atoms. All valence electrons thus participate in covalent bonds, and the band structure shows a filled valence band and an empty conduction band. The difference between an insulator (i.e., NaI) and a semiconductor is in the size of the energy gap, which is perhaps on average 5 eV in an insulator (∼ 7 eV for NaI) and 1 eV (∼ 1.1 eV for silicon and ∼ 0.7 eV for germanium) in a semiconductor. Even at room temperature, a small number of electrons (perhaps 1 in 10⁹) is thermally excited across the gap and into the conduction band, leaving a valence band vacancy known as a “hole”. While the electron from a neighboring atom fills the hole (creating in a process another hole) the hole appears to migrate through the crystal (but of course the positively charged atoms do not move) [Kra88].

3.3.2 High-Purity Germanium (HPGe) Detectors

Germanium detectors belong to a class of semiconductor detectors known for their superior energy resolution as compared to scintillation detectors. This good energy resolution is possible because a comparatively low energy, ∼ 3 eV, is needed to create a charge pair (in this case an electron-hole pair). This translates into a higher number of charge pairs created for a given energy of incident radiation and, therefore, a large number of charge carriers and relatively low statistical fluctuation. The energy needed to create a charge pair is slightly less in germanium than in silicon at both room temperature and at 77 K. Another important parameter is the active volume of the detector which is determined by the thickness $d$ of the depletion region. The thickness $d$ depends on the bias voltage $V$ applied and the concentration of impurity $N$ as:

$$d = \sqrt{\frac{2eV}{eN}}$$  \hspace{1cm} (3.2)
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Figure 3.3: A diagram of band structure difference between metals, semiconductors and insulators [Wik2].

where $\epsilon$ is the dielectric constant of the medium. From Equation 3.2 it is clear that for increasing the thickness of the depletion region the parameter which can be varied is the concentration of impurity $N$. To minimize the impurity concentration, there are two approaches that can be implemented. The first approach is to add an impurity of opposite type after the crystal formation, which would compensate for the impurity already present. This approach is performed for both silicon (Si) and germanium (Ge) by a process called lithium (Li) ion drifting. The germanium detectors fabricated by this process are called Ge(Li) or ‘jelly’ detectors. The disadvantage these detectors have is that they have always to be maintained at liquid nitrogen temperature to prevent the Li from migrating. In addition, they are highly prone to neutron damage and, therefore, not preferred for in-beam spectroscopy. The second approach to get a low value of $N$ is to make a hyper pure crystal by a technique called zone refining. By this technique the impurity concentration can be made as low as $10^9$ atoms/cm$^3$. However, this technique can only be applied to germanium, which has a lower melting point ($959^\circ$C) as compared to silicon ($1410^\circ$C). The detectors made with this method are called Hyper-Pure Germanium (HPGe) detectors. Unlike Ge(Li) these detectors need not always be maintained at liquid nitrogen temperature and can be annealed at
3.3 Detection of Gamma Radiation

elevated temperature to nullify the effect of neutron damage. Another advantage of germanium over silicon is its high atomic number $Z$, which enhances the absorption of $\gamma$ radiation via the photoelectric effect.

The major disadvantage of germanium detectors in comparison with scintillator detectors are their poor timing properties. The major factors which limit the timing resolution of a germanium detector (which may be few tens of nanoseconds) are the charge collection time and the variable shape of pulse rise from event to event. In spite of these deficits they are now the most important tool for $\gamma$-ray spectroscopy due to their superior energy resolution, which compensates for these deficits.

3.3.3 pn Junction

Though exceedingly pure germanium crystals are desired, perfectly pure, or intrinsic, crystals are currently unobtainable. Residual impurities fall roughly into two categories based on the number of their valence electrons. Germanium is tetra-valent, or contains four electrons in its valence shell, and covalently bonds with its four nearest neighbors. Impurities with fewer than four valence electrons, such as boron, form fewer bonds. The result is a void, or a hole, where an electron can be trapped. This type of impurity is labeled as p-type. Arsenic, on the other hand, is an n-type impurity. This is because it possesses five valence electrons, contributing an extra negative charge carrier. When n-type material comes in contact with p-type material, a depleted region forms adjacent to the junction. In this region, the mobile electrons and holes have canceled each other until the electric field established by the interstitial impurities prevents further migration, see Figure 3.4.

This volume serves as the active portion of the detector. The application of a reverse biased voltage, positive to n-type and negative to p-type, facilitates detection in two ways. First, the depletion region increases in size as remaining charge carriers migrate away from the junction toward the appropriate contact. This improves the detector’s efficiency by increasing its active volume. Second, the voltage allows collection of the subsequent charge carriers freed by detected radiation.

There are many possible configurations for the germanium crystal in a $\gamma$-ray detector. The availability of options allows a detector to be tailor made with its final application in mind. The n-type-coaxial configuration is illustrated by in Figure 3.5.
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Figure 3.4: Left: p-type material, with an excess of positive charge and n-type material with an excess of negative charge. Middle: After contact, a depletion zone forms where some of the excess charge cancels. Right: When a reverse biased is applied, the depletion zone grows, forming the active volume of the detector.

Figure 3.5: Illustration of a typical coaxial n-type HPGe detector and its crystal.
This detector geometry consists of a cylindrical crystal of n-type germanium, with roughly half the core removed. Additionally, the outside edge of the forward face is beveled. The outer contact has a thin region of highly concentrated p-type impurities. Created with a technique called doping, this small volume is called p+ contact. A complementary n+ contact is located on the inner surface, and formed through lithium implantation.

The n-type coaxial HPGe detectors have many advantages when employed in γ-ray spectroscopy studies. Coaxial detectors in general have relatively large volumes, providing good efficiency over a wide energy range, 100 keV to 4 MeV. Additional benefits such as maximizing the active detection volume, arise from the use of n-type material. Creating contacts on the crystal’s surface renders the doped material ineffective for γ-ray detection. While the n+ contact is roughly 700 µm deep, radially, the p+ contact requires a depth of about 0.3 µm. By placing the thinner region on the outer surface, more material remains capable of detecting incoming radiation. The n-type crystals are also more efficient over a large range and are less susceptible to radiation damage than their p-type counterparts [Cam04].
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3.3.4 Clover Detectors

Clover detectors were the first composite detectors used in the EUROGAM2 setup [Lei97, Mos91, Nol94]. A Clover detector consists of four HPGe crystals placed in four quadrants of a square and housed in a cryostat, see Figure 3.6. The length and diameter of each crystal is about 70 mm and 50 mm respectively and have a 36 mm tapering length and a front width of 41 mm. Each of the crystals can be used as an independent detector. This increases granularity of the Clover detector compared with a conventional HPGe detector. Since the crystals are closely packed with a small gap of 0.2 mm, there is a high probability of detecting a Compton-scattered $\gamma$ transition scattered from the neighbouring crystal. If the crystals are used as independent detectors the $\gamma$ transitions corresponding to these events are added to the Compton background.

Specifications of the Clover detector [Lei97, New98];

- Distance from the crystal surface to the target centre: $D_{tc} = 196$ mm
- Distance from the detector end-cap to the crystal surface: $D_{ec} = 20$ mm
- Total opening angle: $\Theta = 23.2^\circ$
- Solid angle per detector: $\Omega_{Ge} = 1.34\%$ of the $4\pi$ (for a 0.2 mm distance between crystals)
- Photo-peak efficiency for 1.33 MeV: $\epsilon_{ph,\Omega_{Ge}} = 17.8 \times 10^{-4}$
- Add-back factor for 1.33 MeV: 1.56
- Peak-to-total ratio for 1.33 MeV: $(P/T)_{Ge} = 0.30$
Figure 3.6: Top: A picture of Clover detector that can fit into a BGO shield with its cylindrical liquid nitrogen dewar. Bottom: The crystal arrangement of a 4 segmented Clover detector [Ash07].
3. EXPERIMENTAL METHODS

3.3.5 Compton Suppression

The goal of Compton suppression is to improve the quality of the data by reducing the number of partial-energy events in the spectrum. This is accomplished by surrounding the Ge crystal with a secondary detector. If a $\gamma$-ray is detected in both the Ge detector and this secondary detector within a specific time frame, then it is interpreted as a scattering event and not written as data. Due to the requirement of good timing properties and cost considerations, the secondary detector is typically a scintillation detector. The lower energy resolution of this type of detector is not a factor because we just need to know if a $\gamma$-ray is there or not. The most common scintillation material for this application is bismuth germanate ($\text{Bi}_4\text{Ge}_3\text{O}_{12}$), abbreviated as BGO. This material works well due to its high $Z$ which increases the likelihood of a $\gamma$-ray interacting with the detector. See Figure 3.5.

To determine the effectiveness of Compton suppression we use a $\gamma$ source and compare the number of counts in the well known photopeaks versus the total counts collected. This is known as the peak-to total ratio (P/T ratio). Traditionally a $^{60}\text{Co}$ source is used for this task. When $^{60}\text{Co}$ $\beta^-$ decays, a $^{60}\text{Ni}$ daughter nucleus is formed that then decays to its ground state through the emission of two $\gamma$-rays of energies 1173 and 1333 keV. An unsuppressed germanium detector has a P/T ratio of about 20%. This means that 80% of the events collected are background.

A Compton suppressed Ge detector typically has a peak-to-total ratio of about 60%. Spectra showing a $^{60}\text{Co}$ source for both suppressed and unsuppressed Ge detector are illustrated in Figure 3.7. The technique of Compton suppression allows us to observe weaker and more exotic $\gamma$-rays that would have been otherwise unobservable.
Figure 3.7: Spectra from $^{60}$Co source demonstrating the dramatic difference in the P/T ratio with the employment of Compton suppression. The Compton background is reduced, the 1173 keV and 1332 keV photopeaks height is emphasized with respect to the background.
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3.3.6 AFRODITE Array

The AFRODITE Ge detector array at iThemba LABS [New98] Figure 3.8 is mounted in a rhombicuboctahedron frame with 16 detector positions. The array normally consists of nine Clover detectors, four positioned at 135°, five positioned at 90° and seven low-energy photon spectrometers. For the purpose of the experiment described in this thesis only the Clover detectors were used. They are similar to those of EUROBALL III [Duc99]. Each Clover consists of four 50 x 70 mm HPGe crystals and the nine Clovers subtend a solid angle of 11% of $4\pi$. These detectors are very suitable for measuring high-energy $\gamma$-rays due to their large volume. The total active Ge volume is $\approx 470$ cm$^3$. Moreover, the crystals are closely packed without any material between them, thus enabling good energy resolution for signals added from more than one crystal. The distance between the front face of the detector and the center of the AFRODITE array is 17 cm [Lip06].

Figure 3.9 is the Schematic diagram showing the orientation of the Clover detectors in the AFRODITE array.

![AFRODITE Array](image)

**Figure 3.8:** A picture of the AFRODITE array spectrometer.
Figure 3.9: Schematic diagram of the detector orientation in the AFRODITE array. Modified from [Law13].
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3.3.7 AFRODITE Data Acquisition System and Electronics

The block diagram of the AFRODITE electronics is shown in Figure 3.10. In this work, nine Clover detectors were used and no LEPS detectors were used, even though they are discussed in this section. The Clover detectors provide four channels corresponding to each crystal. The signals from the LEPS detectors preamplifiers are amplified by the CAEN N568 amplifiers. These amplify and shape the LEPS detector signal in a form compatible with both linear and fast signals. The fast signals produced by the amplifier were fed into a constant fraction discriminator (CFD), where they were transformed from analog into logic signals. These were then fed into a FAN IN circuit connected to a gate and delay generator (G&DG). The FAN IN unit was operated in an OR mode for the input signals from the four elements of a LEPS detector.

The RIS modules perform the signal processing for the Clover detectors. As shown in Figure 3.11 and Figure 3.12, respectively, the RIS modules include the standard fast-slow processing circuits as well as a circuit to create a veto signal, if a Ge and a BGO detector fire, to suppress Compton-scattering events and to produce a ‘clean Ge’ signal. The ‘clean Ge’ signal was fed into the trigger logic consisting of two multiplicity units, the first of which was set to two for this experiment, i.e., for an event to be counted the trigger demanded that a suppressed Clover detector had fired. The second unit also accepted logic signals from the LEPS detector. The output from both multiplicity units can be ‘AND’ed in the 365 AL coincidence unit. In this way, more complex trigger conditions can be demanded. Once the trigger signal is generated, it is fanned out to the various ADC’s, TDC’s and RIS modules. In one branch, Silena 4418/V ADC modules were gated to digitize the energies of the LEPS detectors. Another branch enables the digitization of energy and time in the RIS module by routing the clean Ge signal to the RIS module back, into its ‘trigger’ input, which initiates the module to commence conversion.

A further branch of the trigger logic was used to create a radio-frequency (RF) gated trigger signal, being used both to gate the various TDC modules and to provide a time reference. The timing diagram is shown in Figure 3.13. The cyclotron RF signal provides the time reference. The pulse selection was 70.34 ns for $^{158,159}$Er measurements. The arrival of two clean Ge signals at the coincidence unit, Ge1 and Ge2, generates
3.3 Detection of Gamma Radiation

Figure 3.10: Diagram of the AFRODITE array electronics.

a trigger signal, which is true during the time when both logic signals overlap. The width of the clean Ge signals was set to almost twice the time between beam bursts, in this case about 180 ns. This defines the coincidence window. In this example, Ge2 is delayed with respect to Ge1, and a delayed trigger signal is generated at C, as shown in Figure 3.10. The trigger signal is stretched by a gate and delay generator at D and gated with the RF signal to produce the RF-gated trigger signal, Figure 3.10. This signal was fanned out to the TDC modules, 4418/T of the LEPS detectors, where it serves as a ‘common start’ for the time measurement in the TDC. The individual timing signal ‘stops’ the TDC allowing the time to be measured. The RIS modules also have the internal TDC, which operates in a common stop mode. The RF gated trigger is
3. EXPERIMENTAL METHODS

**Figure 3.11**: Part of the electronics which processes signals from the Clover detectors in the RIS modules.

**Figure 3.12**: The Compton suppression electronics of the RIS module using the BGO detector signals.

used as the common stop, by applying it to the trigger 2 input. The RIS and 4418/V ADC modules are read out via a FERA BUS by a VME module called F2VB and the data are sent to a Linux workstation. This workstation allows the RIS modules to be controlled and to write the data to tape.
Figure 3.13: The RF time signal in coincidence with Ge detector time signal for the experiments $^{158,159}$Er.
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3.4 Data Analysis Methods

In this section, methods for extracting information from the raw data collected in an experiment are explained. Once the data are gathered and copied from tape into hard disk, the analysis begins. A typical AFRODITE experiment (Analogue system) contains \( \approx 10^8 \) coincidence events with at least two prompt coincident \( \gamma \)-rays.

3.4.1 Energy Calibration

When data are digitized and stored in the hard disk in a sequence of channels, the correspondence between the channels and the energies is not the same for any two crystals and is not known. This correspondence between the energies and the channels for each crystal is made with the help of data from a source of known \( \gamma \) energies, in this case a \(^{152}\)Eu radioactive source. Once the corresponding channels where these \( \gamma \)-ray peaks are present are noted, the calibration for the complete range of channels is carried out. The calibration is either a linear fit or a quadratic fit or a higher order polynomial fit, with the general equation for fit given by:

\[
E = a_0 + a_1 x + a_2 x^2
\]

(3.3)

where \( x \) is a channel number and the constants; \( a_0 \) is the intercept on the energy axis and gives an idea about the amount of shift present in the data channel, \( a_1 \) gives an idea about the rate of change of energy with the change in channel number while \( a_2 \) gives an idea about the curvature or non linearity present.

The \(^{152}\)Eu source was run for \( \approx 1 \) hour, to accumulate sufficient counting statistics. Thereafter, centroids of the photopeaks from saved source spectra were determined using the SFIT program [Rad95]. This program finds the centroids of the \(^{152}\)Eu automatically and those that fail to fit are determined manually by using the GF3 program [Rad95]. The centroids were then fed to the SCAL program which fit an energy calibration to each detector using Equation 3.3. Thereafter, the program DOP_COR was used to determine the gain matching coefficients, discussed in subsection 3.4.3, which map the channels to calibration of \( E = 0.5 \xi \) keV for Clover detectors, where \( \xi \) are new channels as shown in Figure 3.14. The DOP_COR program gives the option of including the Doppler shift corrections. For this work no Doppler corrections were done [Nts11].
3.4 Data Analysis Methods

![Energy Calibration Plot](image)

**Figure 3.14:** Plot of the energy calibration, the arrows indicate mapping direction from one equation to the other with $x'$ on the $x$ axis, which shows the new channel ultimately used in the analysis in these measurements. Modified from [Nts11].

### 3.4.2 Time Calibration

The aspiration to make all Clover detectors time peaks to be at the same position requires time calibration. Therefore, the time was calibrated by finding the prompt peak position and the separation of the beam pulse using GF3 [Rad95] program. Knowing the RF (Radio frequency of the SSC) the separation in terms of seconds can be known. Then the separation of the successive beam pulses in terms of channels and time can be used to calculate the slope ($m$) in units of channels/seconds. The separation between the beam pulse was about 70.34 ns for $^{150}\text{Sm}(^{12}\text{C},4n\gamma\gamma)^{158}\text{Er}$ and 67.39 ns for $^{150}\text{Sm}(^{13}\text{C},4n\gamma\gamma)^{159}\text{Er}$. These pulses have a smaller amplitude than the prompt peak since they are caused by $\gamma$-rays being detected outside of the coincidence window. Figure 3.15 shows how time gain matching was performed. The slope ($m$) was calculated using:

$$m = \frac{Y \times \text{number of pulses}}{X_i - X_f}, \text{ where } Y = 70.34 \text{ or } 67.39$$

(3.4)

where $X_i$ and $X_f$ are the centroids of the initial and final time peaks. The prompt peak at channel $X_0$ was assumed to occur at time $t_0 = 1000$. 

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3. EXPERIMENTAL METHODS

One has to map the time \( t = mx + C \) onto \( t = x' \) where \( x' \) is the new channel number and \( C \) is a constant. In order to determine \( x' \) the two equations can be equated since the time intervals are equal:

\[
x' = t = mx = C \tag{3.5}
\]

\[
t_0 = 1000 = mx_0 + C \tag{3.6}
\]

from these equations \( C \) becomes;

\[
C = 1000 - mx_0 \tag{3.7}
\]

By substituting Equation 3.7 into Equation 3.5, we obtain:

\[
x' = mx + 1000 - mx_0 \tag{3.8}
\]

and finally:

\[
x' = m(x - x_0) + 1000 \tag{3.9}
\]

Using this method, all the prompt peaks can be calibrated in such a way that they are positioned at channel 1000 [Nts11].

![Figure 3.15: The time gain matching plot.](image)

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3.4 Data Analysis Methods

3.4.3 Gain Matching

In general, the gain of all ADC’s are different, the correspondence between the energy and the channels is not the same for all the ADC’s or in other words the calibration constants are different for different ADC’s. Before the data from such ADC’s are combined for singles data analysis or used for matrix formation, all the ADC’s have to be gain matched. The reason behind this is the fact that all the data sorting operations are operated on the channels and the reference to energy is made only in the form of the calibration equation [Neg11].

3.4.4 Gain Drift Correction

The in-beam data were sorted in order to trace possible gain drifts that might have occurred in the duration of the experiment. Investigation lead to the conclusion that there were some gain drifts. Figure 3.16 shows how the gain changed with time. It was established that temperature differences in the AFRODITE vault might have an effect on the amplifiers and the rest of the electronics such that drifts occur in the output signal. The corrections were done by using a correctly calibrated spectrum obtained from the first runs of the experiment, referred to as in-beam reference spectrum. Well known peaks like the 511 keV, tungsten X-rays were used as a hint to choose the in-beam reference spectrum and to correct the drift. All date runs were collected in a GF3 [Rad95] file and were compared.

3.4.5 Add-back

A great advantage of the composite detectors is the enhancement of the peak-to-total ratio P/T by applying an add-back procedure. Therefore, γ-rays can be fully absorbed in a single crystal or can be detected first in one crystal and Compton scatter to another crystal. The energy deposited in the first crystal can be added to the energy deposited in the second crystal to sum-up the total energy of the γ-ray. There is no definite rule for the add-back procedure, and several scenarios may be used based on probability concepts. The following criteria have been used in our AFRODITE experiment Figure 3.17:
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![Plot showing the gain drift in the 4-elements of Clover 4 as a result of temperature variation](Nts11).

- Signals from two neighbouring segments (horizontally or vertically) are added, *e.g.* AB or AC.
- Hits in two diagonal segments, *e.g.* AD, are discarded, because they probably do not originate from the same $\gamma$-ray.
- The angle $\theta$ of the $\gamma$-ray is assumed to be the average of the two angles of the two added $\gamma$-rays.

3.4.6 Efficiency Calibration

The efficiency calibration measurement of the AFRODITE Clover detectors was performed at the end of the experiment using a $^{152}$Eu radioactive source, where the source was placed exactly at the target position. The data were recorded event-by-event and later sorted offline, in order to obtain the addback of the energies deposited from different individual crystal elements. The relative efficiency curve for various AFRODITE settings shown in Figure 3.18 was obtained. The efficiency was fitted to the Equation 3.10:

$$In(\text{eff}) = [(A + B + Cx^2)^{-G} + (D + E_y + Fy^2)^{-G}]^{1/\delta}$$  \hspace{1cm} (3.10)
3.4 Data Analysis Methods

Figure 3.17: Schematic of a Clover detector. The four crystals are labelled A, B, C and D to explain the add-back procedure.

where A, B and C describes the efficiency at low energies and D, E and F describes the efficiency at high energies. The parameters x and y are given by Equation 3.11 and Equation 3.12 respectively. G is an interaction parameter between the low and the high energy region, and it determines the efficiency in the turn over region.

\[ x = \log \frac{E_\gamma}{100} \]  
\[ y = \log \frac{E_\gamma}{1000} \]  

The relative efficiency curve was obtained using the Radware EFFIT program [Rad95] illustrated in Figure 3.18. The maximum detection efficiency for Clovers occurs at 150 keV. It decreases monotonically with increase in energy as shown in Figure 3.18. This decrease in efficiency is caused by the decrease in cross section of both the photoelectric effect and Compton scattering as a function of γ-ray energy [Nts11].
Figure 3.18: Relative efficiency curve for AFRODITE’s 9 Clover detectors measured with $^{152}\text{Eu}$ radioactive source for the current work.
3.4 Data Analysis Methods

3.4.7 Transition Intensities

The measured $\gamma$-ray intensities are given by;

$$I_{\gamma} = \frac{S}{\epsilon}(1 + \epsilon)$$  \hspace{1cm} (3.13)

where $S$ is the area under the $\gamma$-ray peak and $\epsilon$ is the efficiency of the detector for a $\gamma$-ray discussed in subsection 3.4.6. The total transition intensity, $I_t$ is given by;

$$I_t - I_{\gamma}(1 + \alpha) = \frac{S}{\epsilon}(1 + \epsilon)$$  \hspace{1cm} (3.14)

where $\alpha$ is the electron conversion coefficient, which gives the probability of electron emission relative to $\gamma$-ray emission \[Kra88]\ defined as:

$$\alpha = \frac{I_e}{I_{\gamma}}$$  \hspace{1cm} (3.15)

3.4.8 Coincidence Matrices

A large amount of data is collected during the fusion-evaporation reaction experiments. After these data have been stored in a hard disk, they are sorted and converted into matrices compatible with the RADWARE software. A matrix is a two-dimensional $E_{\gamma 1}, E_{\gamma 2}$ energy histogram with the energy of one photon on one axis and the energy of a coincident photon on the other axis. An experimental technique, particularly useful for the study of the $\gamma$-ray energy correlations, consists of registering coincidences between the $\gamma$ quanta with the help of at least two detectors. A convenient means of representing the results of such a coincidence experiment is to plot the number of coincidences vs $E_{\gamma 1}$, and simultaneously vs $E_{\gamma 2}$, the energies of the coincident quanta registered by the first and the second detector, respectively. Such a two dimensional plot is sometimes referred to as a correlation matrix or two-dimensional matrix \[Pip06]\ Figure 3.19. In principle, when a gate is set on a particular photopeak using channel numbers from the total projection or ungated spectrum, a coincidence spectrum comprised of all the $\gamma$-rays in coincidence with it is observed. The time and energy calibrations were used in the MTSort program \[Cre06]\ which matched the energies and the times and updated the matrix. The total projection of the matrix is formed by the projection of all the number of counts onto one energy axis resulting in a one dimensional $\gamma$-ray spectrum. After
the matrix is created, it is necessary to convert it into a (.m4b) RADWARE format [Rad95, Rad00] which, can be analyzed in a GF3 program. The eg2rad program is used to do this conversion. The converted matrix with dimensions of 4095 X 4095 channels has a dispersion of 0.5 keV for Clover detectors. These \( \gamma \gamma \) matrices are used to construct a nuclear level scheme [Nts11].

\[ E_{\gamma_1} \]

\[ E_{\gamma_2} \]

**Figure 3.19**: Contour plot of the \( \gamma \gamma \) correlation matrix of the nucleus \(^{158}\)Er. The orange colors correspond to the higher-intensity \( \gamma \)-rays, with respect to the cyan color, which correspond to the lower intensity \( \gamma \)-rays.

### 3.4.9 The \( \gamma-\gamma \) Coincidence

In \( \gamma \)-ray spectroscopy, a \( \gamma-\gamma \) coincidence implies the simultaneous detection of two \( \gamma \)-rays by the detection system within a certain time interval \((\Delta t)\), called the coincidence window. For an array, it translates to the simultaneous detection of \( \gamma \)-rays by at least two of the detectors of the array within the time \((\Delta t)\). This technique makes it possible to ascertain the origin of the two radiations from the same source. If the origin of the \( \gamma \) rays is from the same source, in this case the nucleus, such events are referred to as ‘true events’ otherwise they are called ‘random events’. The probability of random events increases with an increase in the width of the coincidence window. On the other hand, decreasing the width of the window reduces the statistics of true or genuine events.
Therefore, for a meaningful spectroscopic experiment a compromise of these opposing scenarios is made.

### 3.5 Correlations of Gamma Transitions

The multipolarity of $\gamma$-transition and the relative spins and parities of the levels can be determined by the measurements of angular distributions, angular correlations ($R_{DCO}$), and polarizations of $\gamma$-rays in Heavy-Ion Fusion-Evaporation (HIFE) reactions.

#### 3.5.1 Angular Distribution

The angular distributions $W(\theta)$ of $\gamma$-rays, is a measurement of the intensity of $\gamma$-rays as a function of the angle $\theta$ with respect to the beam direction. In order to have an angular distribution, oriented nuclear states are required. Oriented means that the population of the $m$ substates depends only on the magnitude of $m$, and not on its sign. To give a non-isotropic $\gamma$-ray angular distribution, relative population parameters $P(m)$ of the angular momentum substates have to be unequal, thus $P(m) \neq P(m')$, where $m$ is the magnetic quantum number given by $m = -I, ..., +I$. $P(m)$ denotes the fraction of the nuclei that occupy this state. The angular distribution of $\gamma$-rays emitted from such states has the form:

$$W(\theta) = \sum_k a_k P_k(cos\theta) = a_0 P_0(cos\theta) + a_1 P_1(cos\theta) + a_4 P_4(cos\theta) + ... \quad (3.16)$$

where $a_k$ are the coefficients of the corresponding Legendre Polynomials $P_k(cos\theta)$ that depend upon the spins of the initial and final states from which the $\gamma$-ray transition originates and upon the multipolarity of the $\gamma$-ray transition. The quantity $k$ takes even integer values only from conservation of parity, thus $k = 0, 2, ... 2I$.

In HIFE reactions, a Gaussian distribution of the substates is formed with the population parameters $P(m)$ given by:

$$P(m) = \frac{e^{-m^2/2\sigma^2}}{\sum_{m'} e^{-m'^2/2\sigma^2}} \quad (3.17)$$
3. EXPERIMENTAL METHODS

where σ represents the width of the substate distribution.

3.5.2 Angular Correlations

Angular correlation measurements involve the determination of the coincident intensities of two γ-rays in cascade detected at angles θ₁ and θ₂. Consider an event where two γ-rays are in cascade from an initial state \(I_i\) to a final state \(I_f\) via an intermediate state, \(I\) as shown in Figure 3.20 [Bvu08]

\[
W(\theta_1, \theta_2, \phi) = \sum_{\lambda_1, \lambda_2} B_{\lambda_1}(I_1) A^{\lambda_2 \lambda_1} (X_1) A_{\lambda_2}(X_2) H_{\lambda_1 \lambda_2}(\theta_1, \theta_2, \phi)
\] (3.18)

The parameters in the relation given in Equation 3.18 are well defined and explained in [Kra73]. These angular correlations are the basis of a key technique, called the Directional Correlations of gamma rays from Oriented states of nuclei (DCO), and will be discussed in subsection 3.5.3.
3.5 Correlations of Gamma Transitions

Figure 3.21: The angles in a directional correlation of two successive radiations emitted from an axially oriented state (DCO) [Kra73].

3.5.3 Directional Correlations of Gamma Rays from Oriented States (DCO)

The definition of DCO follows from two angular correlation functions $W(\theta_1, \theta_2, \phi)$ and $W(\theta_2, \theta_1, \phi)$. DCO measurements determine $\lambda$, the $\lambda$ is used to determine $I_i$ if $I_f$ is known, Figure 3.20. Therefore, the DCO ratio is defined as the ratio of angular correlation functions [Bvu08]:

$$R_{DCO} = \frac{W(\theta_1, \theta_2, \phi)}{W(\theta_2, \theta_1, \phi)}$$  \hspace{1cm} (3.19)

where $W(\theta_1, \theta_2, \phi)$ is the angular correlation of $\gamma_1$ detected at $\theta_1$ in coincidence with $\gamma_2$ detected at $\theta_2$, and $W(\theta_2, \theta_1, \phi)$ is the angular correlation of $\gamma_1$ detected at $\theta_2$ in coincidence with $\gamma_2$ detected at $\theta_1$, see Figure 3.22. The experimental $R_{DCO}$ is given by:

$$R_{DCO} = \frac{I_{\theta_2}^{\gamma_1}(Gated_{\theta_1}^{\gamma_2})}{I_{\theta_1}^{\gamma_1}(Gated_{\theta_2}^{\gamma_2})}$$ \hspace{1cm} (3.20)
where $I_{\theta_2}^{\gamma_1}(Gated_{\theta_1}^{\gamma_2})$ is the intensity of $\gamma_1$ detected at $\theta_2$ and $I_{\theta_1}^{\gamma_1}(Gated_{\theta_2}^{\gamma_2})$ is the intensity of $\gamma_1$ detected at $\theta_2$ gated on $\gamma_2$ detected at $\theta_1$. In the case of the AFRODITE $\gamma$-ray spectrometer the angles $\theta_1$ and $\theta_2$ are $90^\circ$ and $135^\circ$. For this work the angle $\phi$ is $\approx 15^\circ$ on average.

Figure 3.22: Setup for the correlation measurements of $\gamma$-rays.
3.5.4 Linear Polarization Measurements

Linear polarization measurements allow the electric \( (E) \) or the magnetic \( (M) \) nature of \( \gamma \)-rays to be determined. Linear polarization measurements depend on Compton scattering of \( \gamma \)-rays, because the intensity of the scattered radiation depends on the direction of the electric field vector \( \vec{E} \) \cite{Bvu08}, see Figure 3.23. When \( \gamma \)-rays are linearly polarized, the angular distribution function of scattered \( \gamma \)-rays does not only depend on their outgoing direction \( \theta \) with respect to the beam axis, but also on their electric field direction with respect to the reaction plane defined by an outgoing \( \gamma \)-ray and the beam axis. Since the linear polarization of \( \gamma \)-rays emitted from oriented states of nuclei has a close relation to their angular distribution, the linear polarization of these \( \gamma \)-rays in terms of their angular distribution functions is given by \cite{Lee02}:

\[
P(\theta) = \frac{W(\theta, \zeta = 0^\circ) - W(\theta, \zeta = 90^\circ)}{W(\theta, \zeta = 0^\circ) + W(\theta, \zeta = 90^\circ)}
\]  
(3.21)

Linear polarization of \( \gamma \)-rays is defined as the difference between the angular distribution function when their electric field is in or parallel to the reaction plane \( W(\theta, \zeta = 0^\circ) \), and where it is perpendicular to the reaction plane \( W(\theta, \zeta = 90^\circ) \), see Figure 3.23. Here \( \theta \) is the angle of the outgoing \( \gamma \)-rays with respect to the beam axis. Clover detectors can be used to perform linear polarization measurements \cite{Bvu08}. These detectors can be used as both a scatterer to scatter incident \( \gamma \)-rays and an analyzer or a counter to detect \( \gamma \)-rays. This will help in measuring the number of \( \gamma \)-rays scattered perpendicular \( (N_\perp) \) and parallel \( (N_\parallel) \) to the scattering plane. The difference between \( (N_\perp) \) and parallel \( (N_\parallel) \) with respect to the beam direction divided by their sum defines the polarization anisotropy as:

\[
A_p = \frac{aN_\perp - N_\parallel}{aN_\perp + N_\parallel}
\]  
(3.22)

The normalization constant \( a \) can be found by taking the ratio of \( N_\perp \) to \( N_\parallel \) for unpolarized \( \gamma \)-rays emitted from a standard calibration source and it also corrects for any instrumental effect between horizontal and vertical planes. The relation between \( P(\theta) \) and \( A_p \) is given by:
$A_p = Q_k P(\theta)$ (3.23)

where $Q_k$ is the polarization sensitivity. In general, the polarization anisotropy has positive sign for stretched electric transitions and negative sign for stretched magnetic transitions, and the opposite for unstretched transitions. Table 3.1 shows the signs of linear polarization of $\gamma$-rays for different radiation types.

<table>
<thead>
<tr>
<th>Radiation Type</th>
<th>$a_2$</th>
<th>$a_4$</th>
<th>$P(\theta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stretched M1</td>
<td>-</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>Unstretched M1</td>
<td>+</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>Stretched E1</td>
<td>-</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>Unstretched E1</td>
<td>+</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>Stretched M2</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Stretched E2</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>
Experimental Details and Results

4.1 History of Erbium

Erbium is a chemical rare-earth element in the periodic table that has the symbol ‘Er’ and atomic number 68. Erbium was discovered by Carl Gustaf Mosander in 1843. Mosander separated ‘yttria’ from the mineral gadolinite into three fractions which he called yttria, erbia and terbia. He named the new element after the town Ytterby in Sweden, where large concentrations of yttria and erbium are located. Erbia and terbia, however, were confused at this time. After 1860 terbia was renamed erbia, and after 1877 what had been known as erbia was renamed terbia [Wik1].

4.2 Experimental Details

The nuclei $^{158,159}\text{Er}$ were studied at iThemba Laboratory for Accelerator Based Sciences using the $^{150}\text{Sm}(^{12}\text{C},4n\gamma \gamma)^{158}\text{Er}$ and $^{150}\text{Sm}(^{13}\text{C},4n\gamma \gamma)^{159}\text{Er}$ reactions at a bombarding energy of $E_{lab} = 65\text{MeV}$. The $\gamma$ decays, which resulted from the reaction products, have been detected using the AFRODITE $\gamma$-ray spectrometer equipped with nine escape-suppressed Clover detectors; five positioned at $90^\circ$ and four positioned at $135^\circ$. The $^{12,13}\text{C}$ beams were delivered by the K = 200 Separated Sector Cyclotron (SSC) and used to bombard a 1 mg/cm$^2$ $^{150}\text{Sm}$ target, backed on 7.3 mg/cm$^2$ Au foil. A time window of 110 ns between two $\gamma$-rays being detected in the AFRODITE array characterized $\gamma$-$\gamma$ coincidence events. An average beam current of 15 enA was used and a total of about $4.2 \times 10^8$ events were accumulated for each reaction during approximately fifty hours of beam time.
4. EXPERIMENTAL DETAILS AND RESULTS

In the offline analysis, $\gamma$-$\gamma$ coincident events were unfolded from the raw data and replayed into Radware-format [Rad95] for subsequent analysis. In addition, there were open reaction channels from 5n and 3n evaporations, the break-up of the carbon beam and from reactions with the Au backing. The choice of beam energy was made from the PACE2 [Gov80] calculations, (see Figure 4.1).
Figure 4.1: PACE2 simulation of $^{150}\text{Sm}(^{13}\text{C},4\text{n}\gamma\gamma)$ reaction cross sections. In this work a beam of $E_{\text{lab}} = 65$ MeV was used to obtain cross-section (a) and partial cross-section (b) at low angular momentum to study erbium nuclei.
4. EXPERIMENTAL DETAILS AND RESULTS

4.3 Data Analysis and Results

The experiment was conducted at iThemba LABS' AFRODITE γ-ray spectrometer and was primarily focused on the search for low spin collective structures in $^{158,159}$Er. The $4n$ channel permitted the exploration of $^{158,159}$Er to 65 MeV of excitation energy. New transitions were observed, spins and parities were properly assigned by using DCO ratios and linear polarization measurements.

4.3.1 DCO Ratios Analysis

In order to deduce information about the multipolarities of transitions observed in the $^{158,159}$Er level schemes, two DCO coincidence asymmetric matrices for each nucleus were constructed. The first matrix contained γ-rays detected at the 135° Clover detectors (y-axis) in coincidence with the γ-rays detected at the 90° Clover detectors (x-axis). The second matrix was the transpose of the first matrix. The total projection of the matrix is obtained by projection of all counts onto one energy axis, which results in a one dimensional γ-ray spectrum.

The spins (I) and parities (π) associated with the excited levels in the level scheme offer valuable information into the structure and behavior of a nucleus. As the observed γ-rays correspond to transitions between these states, determining the characteristics of the γ-ray allows classification of the corresponding initial and final nuclear states. Spin values were obtained from many transitions in the data using the DCO procedure [Eks92, Kra89].

Electromagnetic radiation, including γ-rays, can be described in terms of a multipole expansion. As a result, γ-rays can be considered as being composed of differing degrees of constituent terms. As the expansion converges rapidly, only the first couple of terms are considered relevant. The index L indicates the order of the multipole expansion; for L = 1, the transition is a dipole while quadrupole transitions are L = 2. The total projection spectrum of the matrix used for the analysis of DCO ratios of $^{158}$Er is shown in Figure 4.2(a). The $R_{DCO}$ ratio for the γ-ray transitions in the $^{158,159}$Er decay schemes using the AFRODITE array is defined by:

$$R_{DCO} = \frac{I(135°,90°)}{I(90°,135°)}$$ (4.1)
where the numerator $I(135^\circ, 90^\circ)$ is the intensity of $\gamma_1$ detected by the 135° Clover detectors in coincidence with $\gamma_2$ detected by the 90° Clover detectors, and the denominator $I(90^\circ, 135^\circ)$ is the intensity of $\gamma_1$ detected in the 90° Clover detectors in coincidence with $\gamma_2$ detected in the 135° Clover detectors. Quadrupole transitions exhibit a value of $R_{DCO} \approx 1$ while dipole transitions have $R_{DCO} \approx 0.5$. Transitions that are mixed in nature can display intermediate values. Additionally, transitions can be classified by whether they are primarily magnetic ($M$) or electric ($E$) in nature. While rotational bands are typically composed of numerous stretched $E2$ transitions, dipoles and transitions of a mixed nature, $M1$ and $E2$, are frequently observed for transitions between bands. The change in parity between the initial and final states is indicated by the following relationship:

\begin{equation}
\pi(ML) = (-1)^{L+1}
\end{equation}

\begin{equation}
\pi(EL) = (-1)^L
\end{equation}

The uncertainties were calculated as follows:

\begin{equation}
\Delta R_{DCO} = R_{DCO} \sqrt{\left( \frac{\Delta I(135^\circ, 90^\circ)}{I(135^\circ, 90^\circ)} \right)^2 + \left( \frac{\Delta I(90^\circ, 135^\circ)}{I(90^\circ, 135^\circ)} \right)^2}
\end{equation}

where $\Delta I(135^\circ, 90^\circ)$ and $\Delta I(90^\circ, 135^\circ)$ are the uncertainties in the numerator and denominator.
4. EXPERIMENTAL DETAILS AND RESULTS

4.3.2 Linear Polarization Measurements

The linear polarization anisotropy measurements were performed, using the Clovers as Compton polarimeters [Jon95] in the AFRODITE array, as described by Starosta et al. [Sta99]. The polarization anisotropy $P$, is defined as:

$$P = \frac{A_p}{Q},$$

(4.5)

on the basis of the experimental asymmetry between vertically and horizontally scattered $\gamma$-rays,

$$A_p = \frac{aN_V - N_H}{aN_V + N_H}$$

(4.6)

where $N_H$ is the number of $\gamma$-rays that scattered parallel to the plane of the beam and the initial gamma ray that was emitted. The relative efficiencies of $N_H$ and $N_V$, the quantity $a(E_\gamma)$, was measured using unpolarized $\gamma$-rays from the many contaminant $\beta$-decay transitions. It is defined as: $a = N_H/N_V$ and the measured value of $a = 1.015$ was found to have essentially no energy dependence, and $N_V$ is the number of $\gamma$-rays that scattered perpendicular to the beam direction, for this is where the polarization sensitivity $Q$, and hence $A_p$, are maximal. The polarization sensitivity for Clover detectors has been measured by [Sta99] to be:

$$Q = Q_0[0.31(2) + 7(2) \times 10^{-5}E_\gamma],$$

(4.7)

where $E_\gamma$ is in keV. $Q_0$ is the polarization sensitivity for an ideal polarimeter [Rou12],

$$Q_0 = \frac{(1 + \alpha)}{(1 + \alpha + \alpha^2)},$$

(4.8)

where $\alpha = E_\gamma[\text{keV}]/511$. The uncertainty in the $A_p$ measured was calculated by using Equation 4.9:

$$\Delta A_p = \frac{2}{(N_V + N_H)^2} \sqrt{(N_H\sigma_V)^2 + (N_V\sigma_H)^2}$$

(4.9)

where $\sigma_V$ and $\sigma_H$ are the uncertainties in the number of $\gamma$-rays scattered vertically at 90° and horizontally at 90° Clover detectors respectively. Hence, in order to measure $A_p$, two coincidence matrices are constructed. These are basically ‘add-back’ spectra. One matrix is ‘add-back’ for $\gamma$-rays scattered in a plane including the beam line versus
4.3 Data Analysis and Results

any other $\gamma$-ray in any other detector to give $N_H$. The other matrix is ‘add-back’ for $\gamma$-rays scattered in a plane perpendicular to the beam line to give $N_V$.

In the case where there is little or no mixing, the polarization anisotropy is known to have a positive sign for stretched electric transitions and a negative sign for stretched magnetic transitions. For mixed, $(\Delta I = 1, E2/M1)$ transitions the linear polarization $P(\theta)$ also depends on the mixing ratio. A single $P(\theta)$ value may correspond to more than one value of $\sigma$ [Twi73]. Figure 4.2(b) is the total projection of the linear polarization anisotropy measurements in $^{158}\text{Er}$. 
Figure 4.2: Total projection spectra used to calculate (a) the DCO ratio and (b) linear polarization anisotropy in $^{158}$Er.
4.4 The $^{158}$Er Experimental Results and Level Scheme

The level scheme for $^{158}$Er deduced from the present work is shown in Figure 4.17. Tentative transitions and levels are indicated by dashed lines, and tentative spins and parities are within parentheses, wherever applicable. New transitions are highlighted in red, transitions that were previously placed in different bands are highlighted in blue and previously published ones are in black. Transitions were positioned in the level scheme based on coincidence relationships and intensity arguments. Spins and parity assignments are either adopted from [Pip06, Sim84], wherever applicable, or were determined based on the linear polarization anisotropy and DCO ratio measurements, see Table 6.1 in Appendix A6. Overall thirty six new transitions including one new band were added to the level scheme of $^{158}$Er. Figure 4.3 is the total projection of the $\gamma - \gamma$ coincidence spectrum used to build the level scheme of $^{158}$Er.

Tables showing DCO ratios, linear polarization anisotropy and multipolarity assigned for the $\gamma$-ray transitions in different levels of the $^{158}$Er decay scheme are given in Appendix A6. The discussion for spin and parity assignments is outlined in this chapter.
4. EXPERIMENTAL DETAILS AND RESULTS

Figure 4.3: Background-subtracted total projection of the $\gamma - \gamma$ coincidence matrix used to build the level scheme of $^{158}$Er. All labelled transitions belong to $^{158}$Er. Asterisks are used to denote transitions from other reaction channels.
4.4 The $^{158}$Er Experimental Results and Level Scheme

4.4.1 $^{158}$Er Bands and Spectra

About nine bands, including one new band and ten spectra of $^{158}$Er, are included and discussed in this section. Bands 1 - 5 are presented in the magnified partial level scheme Figure 4.10, while, the partial level scheme for Bands 6 to 9 is shown in Figure 4.14. The measured DCO ratios and linear polarization anisotropy ($A_p$) plots are displayed in Figures 4.15(a-b) and 4.16, respectively.

4.4.2 Band 1 (The Ground State Band)

Band 1 is the ground state band, which continues above the S-band crossing near spin $12^+$. Previous work [Pip06] from the reaction $^{114}$Cd($^{48}$Ca,4n$\gamma\gamma$) at $E_{lab} = 215$ MeV observed this sequence up to $40^+$. In this study spins up to $20^+$ were populated from the reaction $^{150}$Sm($^{12}$C,4n$\gamma\gamma$) at $E_{lab} = 65$ MeV. The presence of the triplet 649, 651 and 653 keV was confirmed by the new 184 and 835 keV $\gamma$-rays, which pins down the $16^+$ and $18^+$ levels, respectively. The ordering of the 651 and 649 keV $\gamma$-rays still remains uncertain [Ril84]. The $16^+$ 835-keV $\gamma$-ray, which decays into the $14^+$ level in Band 2 was assigned $E2$ character, with the measured values of DCO and $A_p$ of 0.70(0.30) and 0.07(0.01), respectively, (see Table 6.1). DCO and $A_P$ measurements for the $14^+$ 184-keV $\gamma$-ray that decays into the $14^+$/level of the S-band (Band 2) could not be performed because of low statistics. For all the measured transitions in Band 1, $R_{DCO} \sim 1$ and $A_p >0$ are consistent with stretched $E2$ character, (see Figures 4.15(a) , 4.16 and Appendix A6). A magnified portion of the $^{158}$Er level scheme, showing Band 1 with clarity, is given in Figure 4.10, and a representative spectrum for this cascade is presented in Figure 4.4. This spectrum was obtain by gating on the 694-keV peak.
Figure 4.4: The 694 keV gated spectrum for Band 1 in $^{158}$Er. Asterisks are used to denote contaminants from other reaction channels and / or other bands of $^{158}$Er, not associated with this cascade.
4.4.3 Band 2 (The S Band)

Band 2 is the yrast $(\pi,\alpha) = (+,0)$ $S$ band, a sequence of 8 rotational states up to the $26^+_1$ level at 7278 keV. In the middle, spin $12^+$ and $14^+$ states of Band 2 cross Band 1 (ground-state band), and for spin $14^+$ and higher, Band 2 is yrast, (see Figure 4.10). Previously [Pip06] this band was observed up to spin $38^+$. In this work we observed up to spin $24^+$. DCO ratios and polarization measurements have confirmed the spins and parities of this band, with the in-band being assigned to $E2$ character and 510-keV transition from decay to the ground state band ($14^+ \rightarrow 12^+$) has $E2$ character. The 201-keV decay into the ground state band ($12^+ \rightarrow 12^+$) with $R_{DCO} = 0.48(0.02)$ and $A_p = -0.03(0.06)$ has been assigned as a mixed $E2/M1$ transition. The 394 keV $\gamma$-ray decays into $0^+_2$ ($12^+ \rightarrow 10^+$) via an $E2$ transition. (See, Figures 4.15(a) , 4.16 and Appendix A6). Figure 4.5 shows the spectrum obtained in the present work by making a single gate transition depopulating levels between $16^+h$ and $22^+h$ in the yrast band. This spectrum clearly indicates the decay scheme previously proposed by [Pip06] up to this level is correct.
Figure 4.5: The 809-keV gated spectrum for Band 2 in $^{158}\text{Er}$. Asterisks are used to denote contaminants from other reaction channels or / and other bands of $^{158}\text{Er}$, not associated with this cascade.
4.4 The $^{158}$Er Experimental Results and Level Scheme

4.4.4 Band 3 (The $|0^+_2\rangle$ Band)

Band 3 is the band built on the $0^+_2$ excitation, which was previously known up to spin $4^+_2$ [Agu75], and has been extended in this work to $16^+$. The confirmation of in-band transitions for the $0^+_2$ band is supported by the two $\gamma - \gamma$ coincidence spectra shown in Figure 4.6, where gates set on the 430- and 469-keV $\gamma$-rays allow an arrangement of the band. The ordering of the $\gamma$-rays is also supported by the arrangement of the in-band and out-of-band decays. The $\gamma$-ray transitions were previously observed but placed in different bands [Pip06, Sim80, Hel04, nndc13]. The 405-keV $\gamma$-ray decays from the $6^+$ state into the $4^+$ state of the $\gamma$-band, Band 4. It has its 382 keV counterpart from $6^+_\gamma \rightarrow 4^+_{|0^+_2\rangle}$, because the $\gamma$ and the $|0^+_2\rangle$ bands cross. The $6^+$ level was given as $(4^+)$ in the Data Tables [Hel04, nndc13] but our data plus the connections with the $\gamma$-band and the 394 keV connection to the established $12^+$ state at the bottom of Band 2 (S-band), rule out anything except $6^+$. The $12^+ \rightarrow 16^+$ in Band 3 also helps to establish the spin sequence of the levels. The 183 keV could not be observed in this work, but it was confirmed from P. I. Mashita’s data [Mas13], where $^{158}$Yb $\beta^+$ decays to $^{158}$Tm which decays to $^{158}$Er, see Figure 4.7.

The spins and parities of the states above the $4^+_2$ level in Band 3 have been firmly assigned in this work and the transitions linking states between $16^+_2$ and $10^+_2$ are new. See Table 6.1, for the measured DCO and $A_p$ values. Most deformed nuclei in the $N = 90$ range have bands based on the low lying first excited $K^\pi = 0^+$ state. This band has usually been interpreted as a $\beta$-vibrational band [Bes63]. Band 3 is such a band in $^{158}$Er. The assignment of $\beta$-vibrational band to the lowest excited $K^\pi = 0^+_2$ state has been questioned previously by [Cha79, She80]. It was recently suggested by [Sha06] that the $K^\pi = 0^+$ structure of Band 3 may not be a $\beta$-vibrational band but is a two-particle, two-hole seniority zero neutron state lowered into the pairing gap by the configuration dependent pairing interaction and the low density of the oblate states near the Fermi surface. Gamma-ray spectra gated on the 430 keV $8^+_2 \rightarrow 6^+_2$ transition depopulating the $0^+_2$ band present similar intensities at $\theta_{lab} = 90^\circ$ and $135^\circ$ Clover detection angles and suggest a predominant $E2$ character for in-band transitions. This band is shown in the magnified partial level scheme in Figure 4.10.
Figure 4.6: Background-subtracted coincidence γ-ray spectra gated by the 469 keV (top) and 430 keV (bottom) γ-rays. Transitions depopulating the $0^+_2$ band are detailed in the insets (Band 3 Figure 4.10) in $^{158}$Er.
Figure 4.7: Partial level scheme of $^{158}\text{Er}$, from the $\beta^+$ decay of $^{158}\text{Tm}$, populated in the $\beta$-decay of $^{158}\text{Yb}$. The ground state, $0^+_2$, even-spin gamma and odd-spin gamma bands are labelled by A, B, C and D respectively [Mas13].
4. EXPERIMENTAL DETAILS AND RESULTS

4.4.5 Bands 4 and 5 (Gamma Bands)

Band 4 is the $K^\pi = 2^+$ even $\gamma$ band, and the band has been previously observed up to $4^+$ [Agu75]. In this work this band has been extended to spin $12^+$ with three new in-band transitions (500, 546 and 588 keV) assigned as $E2$ in character. Four new out of band transitions were observed in this band with the 382 keV $\gamma$-ray decaying into the $0_2^+$ band via an $E2$ transition and the 646, 669, 1113 and 1192 keV $\gamma$-rays decaying into the ground-state band. A spectrum displaying the photopeak energies of this band is shown in Figure 4.8. For the DCO and $A_p$ values, see Table 6.1.

Band 5 is the odd-spin members of the $\gamma$ band, [Sim80, Pip06, Hel04]. This band was previously observed up to tentative spin $5^+$. We extended this band to tentative spin $15^+$ with a total of six new transitions being added. The measured DCO and $A_p$ values for the 542 and 592-keV $\gamma$-rays are; 1.10(0.02), 0.23(0.07) and 1.20(0.05), 0.38(0.03) respectively, which indicate the $E2$ character. DCO and $A_p$ values for the tentative $15^+$ level and the 1043-keV $\gamma$-ray could not be measured because of limited statistics. The 1021, 962 and 956-keV $\gamma$-rays that decay into Band 1 are mixed transitions $E2/M1$. Band 5 is displayed in a magnified partial level scheme in Figure 4.10 and the spectrum showing transitions in Band 5 is in Figure 4.9.
4.4 The $^{158}$Er Experimental Results and Level Scheme

Figure 4.8: Background subtracted spectrum for Band 4 in $^{158}$Er, gated by the 546 keV (blue) and 1113 keV (red). Asterisks are used to denote contaminants from other reaction channels and / or other bands of $^{158}$Er, not associated with this cascade.
4. EXPERIMENTAL DETAILS AND RESULTS

Figure 4.9: The 592 keV gated background subtracted spectrum for Band 5 in $^{158}$Er, all labelled transitions belong to $^{158}$Er. Asterisks are used to denote contaminants from other reaction channels and / or other bands of $^{158}$Er, not associated with this cascade.
Figure 4.10: Positive parity bands, Bands 1 - 5 in $^{158}$Er. Tentative transitions and levels are indicated by dashed lines, and tentative spins and parities are within parentheses, wherever applicable. New transitions are highlighted in red, transitions that were previously placed in different bands are highlighted in blue and previously published ones are in black. The width of the arrows are proportional to the transition intensities.
4.4.6 Band 6

Band 6 is the (parity $\pi$, signature $\alpha$) ($-$, 1) sequence previously observed by [Pip06], beyond band termination based on a high-energy $\gamma$-ray decaying from spin $49^-$.

In this work this band is observed up to spin $23^-$, (see Figure 4.14).

Similar to Band 2, this band demonstrates an abrupt change from a regular rotational cascade at low spin.

Two new transitions of 215 keV, the 7$^-$ and 629 keV (see Figure 4.17), an $E1$, that decays to the 6$^+$ level in Band 3, have been added. Our DCO and $A_p$ measurements (see, Figures 4.15(b), 4.16 and Table 6.1) confirm previous assignments.

![Figure 4.11: The 635 keV gated spectrum for Band 6 in $^{158}$Er. Asterisks are used to denote contaminants from other reaction channels or / and other bands of $^{158}$Er, not associated with this cascade.](image_url)
4.4 The $^{158}$Er Experimental Results and Level Scheme

4.4.7 Band 7

Band 7 is the ($\gamma$,0) cascade previously observed [Sim80] up to the termination state of $48^{-}$. In this work we observed levels up to $22^{-}$. One new $\gamma$-ray, of 547-keV was added to the tentative level ($6^{-}$), this $\gamma$-ray decays into the $6^{+}$ level in Band 4, see Figure 4.17. The in-band $\gamma$-rays have been assigned $E2$ character based on the measured DCO and $A_p$ values. The decays into the ground-state band, Band 1, via $E1$ transitions have been measured, see Table 6.1. Figure 4.12 shows the spectrum of $\gamma$-rays for Band 7, and the band is shown in a magnified Figure 4.14.

![Figure 4.12](image_url)

**Figure 4.12:** The 520 keV gated spectrum for Band 7 in $^{158}$Er. Asterisks are used to denote contaminants from other reaction channels or / and other bands of $^{158}$Er not associated with this cascade.
4. EXPERIMENTAL DETAILS AND RESULTS

4.4.8 Band 8

Band 8 is an odd negative parity band, and it was previously observed up to the tentative $21^{-}$ state [Pip06]. Spins up to $17^{-}$ were observed for this band. New transitions 420, 326, 1000 and 1150-keV in the low spin states were added. DCO ratios and polarization measurements confirmed the spins and parities of these transitions except for the 326-keV and tentative 1150-keV $\gamma$-rays. The DCO and $A_p$ values for the 420-keV and 1000-keV $\gamma$-rays are 1.20(0.06), 0.40(0.02) and 0.39(0.02), -0.23(0.03), respectively. A tentative $(3^{-})$ level was also added at 1342-keV. (See, Figures 4.15(b) and 4.16).

4.4.9 Band 9

Band 9 is the newly constructed band in this work with three tentative in-band transitions of 412, 417 and 434-keV, the arrangement of these transitions is uncertain. The DCO ratio for the 412-keV $\gamma$-ray was measured to be 1.10(0.10) with 0.87(0.10) for the 417-keV transition. The $A_p$ measurements for the three $\gamma$ rays were not possible, because the gated spectrum to measure these transitions did not have enough statistics. A total of six new tentative out-of-band transitions of 500, 607, 800, 968, 1058 and 1174-keV were observed in this band. The band decays mainly into Band 1, except for the 500-keV $\gamma$-ray that decays into Band 8 ($((6^{-}) \rightarrow (5^{-}))$). Contaminants in these data were a contributing factor because most of the transitions in this band have the same energies as in some neighbouring nuclei. This band is shown in the level scheme (Figure 4.14) and the spectrum showing the $\gamma$-rays in this band is Figure 4.13.
Figure 4.13: The spectrum for Band 9 in $^{158}$Er. Asterisks are used to denote contaminants from other reaction channels and/or other bands of $^{158}$Er, not associated with this cascade.
Figure 4.14: The ground state band with negative parity bands, Bands 1, 6, 7, 8 and 9 in $^{158}$Er. Tentative transitions and levels are indicated by dashed lines, and tentative spins and parities are within parentheses, wherever applicable. New transitions are highlighted in red, and previously published ones are in black. The width of the arrows are proportional to the transitions intensities.
Figure 4.15: Measured DCO ratios of (a) positive parity bands and (b) negative parity bands in $^{158}$Er. A ratio near 0.5 is consistent with a stretched dipole transition while a value near 1.0 corresponds to stretched quadrupole transition. All measurements are gated by $E2$ transitions.
Figure 4.16: Plot of results of linear polarization anisotropy measurements in $^{158}$Er.
Figure 4.17: Partial level scheme for $^{158}$Er deduced in this work. Tentative transitions and levels are indicated by dashed lines, and tentative spins and parities are within parentheses wherever applicable. New transitions are highlighted in red, transitions that were previously placed in different bands are highlighted in blue and previously published ones are in black. The width of the arrows are proportional to the transitions intensities.
4. EXPERIMENTAL DETAILS AND RESULTS

4.5 The $^{159}\text{Er}$ Experimental Results and Level Scheme

The partial level scheme for $^{159}\text{Er}$ deduced from the present work is shown in Figure 4.33. We observe levels in the yrast band up to the 6176-keV ($53/2^+$) level, Band 1 up to the 4911-keV ($43/2^+$) level, Band 2 up to the 5593-keV ($49/2^-$) level, Band 3 up to the 835-keV ($15/2^-$) level, Band 4 up to the 4360-keV ($41/2^-$) level, Band 5 up to the 5245-keV ($43/2^-$) level, Band 6 up to the 4993-keV ($45/2^+$) level, Band 7 up to the 4562-keV ($41/2^-$) level, Band 8 up to the 4234-keV ($39/2^-$) level, Band 9 up to the 3726-keV ($31/2^-$) level, Band 10 up to the 3414-keV ($29/2^-$) level, Band 11 up to the 4240-keV ($39/2^+$) level, Band 12 up to the 6036-keV ($51/2^-$) level, Band (a) up to the 4350-keV ($39/2^-$) level, Band (b) up to the 2284-keV ($17/2^-$) level, Band (c) up to the 4186-keV ($33/2^+$) level, Band (d) up to the 3162-keV ($35/2^+$) level, Band (e) up to the 4209-keV ($41/2^+$) level and Band (f) up to the 4281-keV ($39/2^+$) level.

This work also permitted the establishment of low spin states $21/2^-$ in Band 4, $25/2^-$ in Band 5, new Band 11, and $23/2^-$ in Band 12. The DCO ratio and polarization anisotropy measurements are given in Table 6.2 (see Appendix A6) and are plotted in Figures 4.31 and 4.32.

Previously published bands/transitions are presented in black, whereas, the new ones from this work are presented in red. Transitions were positioned in the level scheme based on coincidence relationships and intensity arguments. The discussion of spin and parity assignments is included in this chapter. The total projection spectrum of the matrix of $\gamma - \gamma$ coincidences from the $^{150}\text{Sm}(^{13}\text{C},4\text{n})^{159}\text{Er}$ reaction is presented in Figure 4.18.
Figure 4.18: Background-subtracted total projection of the $\gamma - \gamma$ coincidence matrix used to build the level scheme of $^{159}$Er. All labelled transitions belong to $^{159}$Er. Asterisks are used to denote transitions from other reaction channels.
4. EXPERIMENTAL DETAILS AND RESULTS

4.5.1 $^{159}$Er Bands and Spectra

About thirteen bands and twelve spectra are constructed to represent the $^{159}$Er data in this work. The yrast band to Band 5 are presented in the magnified partial level scheme Figure 4.23, while, the partial level scheme for Bands 6 to 12 is represented in Figure 4.29.

4.5.2 Yrast Band and Band 1

In the positive parity yrast band of the $^{159}$Er transitions have been previously reported by [Mus11] up to tentative excited state of spin and parity (105/2$^+$), whereas in this work spin up to 53/2$^+$ was populated. No new $\gamma$-rays or placements in this band were observed in this work. A coincident $\gamma$-ray spectrum showing the photopeak energies correspond to all of the transitions associated with the Yrast band is presented in Figure 4.19. The $\gamma$-ray spectrum in Figure 4.19 was produced by placing a single gate at 710 keV. The DCO ratios and $A_\rho$ values were measured and found to be of stretched quadrupole character, see Table 6.2, establishing positive parity and signature ($\pi, \alpha$) = (+, +1/2) for the connected levels.

The band based on the tentative 183 keV level, Band 1, is the unfavored signature partner of the lowest-lying $i_{13/2}$ [651 3/2] structure with (+, -1/2) initially established by [Sim84] is populated weakly, because it becomes increasingly non-yrast with increasing spin. (See Figure 4.23). This band has previously been observed up to (91/2$^+$) [Mus11]. The present work this band is observed up to (43/2$^+$) and no new transitions were added. Figure 4.20 is the 487 keV gated spectrum used to illustrate $\gamma$-rays of Band 1.
Figure 4.19: Background subtracted spectrum, showing $\gamma$-ray transitions for the Yrast Band gated by the 710 keV, the $41/2^+ \rightarrow 37/2^+$ transition. Asterisks are used to denote contaminants from other reaction channels or other bands of $^{159}$Er.
Figure 4.20: The 487 keV gated spectrum illustrating $\gamma$-ray energies of Band 1. Asterisks are used to denote contaminants from other reaction channels or other bands of $^{159}$Er.
4.5.3 Bands 2 and 3

Band 2 has previously been established up to (105/2−) [Mus11], and in this work up to spin 49/2− was observed and no new transitions were added. Figure 4.21 shows transitions in coincidence with the 490-keV γ-ray. The DCO ratios are consistent with the in-band transitions up to 49/2− level being stretched $E2$s.

Band 3 (Figure 4.23) is the band based on the ground state. The present work has not extended this sequence beyond the known 835 keV 15/2− level. Band 3 has previously been observed to (91/2−) [Mus11]. Simpson et al. [Sim84] observed these γ-rays from (3/2−) to (15/2−) and from (27/2−) to (83/2−) but as two separate bands. In this work no link whatsoever was observed between the two bands, which we label Band 3 and Band 12 as indicated by [Mus11]. Transitions linking Band 2 and 3 have DCO ratio and $A_p ∼ 0.5$ and -0.04 respectively which suggested they are $E2/M1$ and indicates the $M1$ is favored, see Table 6.2.

![Figure 4.21: Spectrum gated by the 490 keV γ-ray showing Bands 2 and 3 with M1 transitions that link the two bands. Asterisks are used to denote contaminants from other reaction channels or other bands of 159Er.](image-url)
4. EXPERIMENTAL DETAILS AND RESULTS

4.5.4 Bands 4 and 5

Band 4 has the same signature as Band 2, the ground state band, and has been previously studied up to a tentative spin \((85/2^-)\) by [Mus11]. In the present work this band has been observed up to spin \(37/2^-\). Three new transitions; 355-keV \((25/2^- \rightarrow 21/2^-)\), 773-keV (Band 4 \(\rightarrow\) Band 1) and 839-keV (Band 4 \(\rightarrow\) Yrast band) have been placed in this band. The DCO and \(A_p\) values for the three transitions support the \(E2\) character for the 355-keV and \(E1\) character for 773 and 839-keV transitions, (see Table 6.2). The gated spectrum illustrating Band 4 is presented in Figure 4.22.

Band 5 has a sequence of four \(\gamma\)-rays with energies 571, 619, 664 and 720-keV decaying into the \(25/2^-\) level of Band 2 by a 656-keV \(\gamma\)-ray [Del87] and the new 749-keV \(\gamma\)-ray that decays into \(21/2^-\) of Band 2. Band 5 has the same parity and signature as the Band 2, the ground-state band, and is based on these four transitions. The DCO ratio value of 0.89(0.04) and \(A_p\) value of 0.01(0.04) indicate that this transition is probably a stretched \(E2\).
Figure 4.22: Background subtracted spectrum of Band 4 with the 658 keV gate. Asterisks are used to denote contaminants from other reaction channels or other bands of $^{159}$Er.
Figure 4.23: Partial level scheme for $^{159}$Er representing the Yrast band and Bands 1 - 5 seen in the current work. Tentative transitions and levels are indicated by dashed lines, and tentative spins and parities are within parentheses, wherever applicable. New transitions are highlighted in red and previously published ones are in black. The width of the arrows are proportional to the transitions intensities.
4.5.5 Band 6

Band 6 was first observed by Mustafa et al. [Mus11] and has been interpreted as a $\gamma$-vibration based on the yrast configuration. In this work only six $\gamma$-rays were observed up to spin $45/2^+$. This band is connected to the yrast band though a series of high-energy $\gamma$ decays. The DCO and $A_p$ values of the six observed in-band transitions were measured and found to be consistent with being of $E2$ character. The DCO and $A_p$ values of $\gamma$-rays decaying into the yrast band could not be measured because these transitions were weak. The 351 and 550-keV gated spectra of this band is presented in Figure 4.24.

![Figure 4.24: Background subtracted spectrum of Band 6, with the 550 keV gate in blue and the 351-keV gate in red. Asterisks are used to denote contaminants from other reaction channels or other bands of $^{159}$Er.](image-url)
4. EXPERIMENTAL DETAILS AND RESULTS

4.5.6 Strongly Coupled Bands, Bands 7, 8, 9 and 10

Bands 7 and 8 have previously been observed by [Del87, Sim98]. Mustafa et al. [Mus11] extended Band 8 with two tentative transitions at the bottom of the bands and established a new sequence of transitions in parallel with Band 7 at high spins. Figure 4.25 shows the photopeak energies of Bands 7 and 8, this figure confirms the linking transitions that connect Band 7 and 8 to the yrast band and Bands 1 and 2. The DCO and $A_p$ values confirm that the in-band transitions have $E2$ character, and the linking transitions have $E1$ character. (See Table 6.2). A new 1434-keV $\gamma$-ray was observed in Band 8, decaying from the $27/2^-$ state to the $25/2^+$ member of the yrast Band. In this work we saw no signs of the levels below the $25/2^-$ and $23/2^-$ seen by [Mus11].

The rotational band structure in deformed $^{159}$Er, based on the [505]11/2$^-$ neutron in Bands 9 and 10 is well established. But the excitation energy and the half life remain undetermined [Sha13]. Also, collective structures built on this band head have not been found. These are expected to be the $K^\pi = 2^+$ core $\gamma$-vibration excitation coupling to this neutron orbit but not to the alleged $\beta$-vibration [Sha06]. A 600±60 ns isomer was reported in $^{159}$Er in 1970 [Lei70]. It was assumed to be the head of a $K^\pi = 11/2^-$ band based on the [505]11/2$^-$ Nilsson orbital, the long life-time being due to its low excitation energy and the inhibition of any dipole or quadrupole transitions to lower states by the $\Delta K = 3$ change required. This isomer was also observed [Str75] in the $\beta^+/EC$ decay of $^{159}$Tm, and the life-time was reported as 550±150 ns. These two ‘measurements’ account for the 0.59±0.06 $\mu$s given in the Nuclear Data Sheets [nndc13]. Both of the experiments identify the isomer as having an energy of 428 keV. But this 11/2$^-$ state belongs to the [321]3/2$^-$ ground-state band and cannot possibly have this life-time [Sim84]. It is rather unlikely that there is no isomer with a $\sim$0.6 $\mu$s lifetime, but the question arises is this actually the band head of the expected [505]11/2$^-$ band? [Sha13]

The existence of the [505]11/2$^-$ band is well established [Sim98] and the $B(M1)/B(M2)$ ratios show that indeed this is the correct Nilsson assignment for this band. However the excitation energy decay modes and lifetime are not known at all. Table 4.1 gives the excitation energies, the lifetimes and the states they decay to, of the known [nndc13]
Figure 4.25: Background subtracted spectrum of Bands 7 and 8, with the 293 keV gate in blue and the 163 keV gate in red. Asterisks are used to denote contaminants from other reaction channels or other bands of $^{159}$Er.
neutron \([505]11/2^–\) bands in the odd-neutron nuclei neighbouring \(^{159}\text{Er}\). The systematics of the half lives and energies in Table 4.1 suggest that the \([505]11/2^–\) band head in \(^{159}\text{Er}\) will have an excitation energy of less than 400 keV and a life-time between 10 \(\mu\)s and 20 ms.

Table 4.1: The known \([505]11/2^–\) bands neighbouring \(^{159}\text{Er}\).

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>(E_x) (keV)</th>
<th>(T_{1/2}) (isomer half-life)</th>
<th>Decay Spin/Parity, (intensity)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{151}\text{Sm}_{89})</td>
<td>216</td>
<td>1.4 (\mu)s</td>
<td>(9/2^–(7), 13/2^+(14))</td>
</tr>
<tr>
<td>(^{153}\text{Gd}_{89})</td>
<td>171</td>
<td>76 (\mu)s</td>
<td>(9/2^+(100), 7/2^–(64))</td>
</tr>
<tr>
<td>(^{155}\text{Dy}_{89})</td>
<td>234</td>
<td>6 (\mu)s</td>
<td>(13/2^+(47), 9/2^+(100))</td>
</tr>
<tr>
<td>(^{153}\text{Sm}_{91})</td>
<td>98</td>
<td>10.6 ms</td>
<td>(9/2^+(100))</td>
</tr>
<tr>
<td>(^{153}\text{Gd}_{91})</td>
<td>121</td>
<td>32 ms</td>
<td>(9/2^+(100))</td>
</tr>
<tr>
<td>(^{157}\text{Dy}_{91})</td>
<td>299</td>
<td>21.6 ms</td>
<td>(9/2^+(100), 7/2^–(7))</td>
</tr>
<tr>
<td>(^{159}\text{Er}_{91})</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>(^{157}\text{Gd}_{93})</td>
<td>427</td>
<td>11.5 (\mu)s</td>
<td>(9/2^+(49), 9/2^–(100))</td>
</tr>
<tr>
<td>(^{159}\text{Dy}_{93})</td>
<td>353</td>
<td>122 (\mu)s</td>
<td>(9/2^+(78), 9/2^–(100))</td>
</tr>
<tr>
<td>(^{161}\text{Dy}_{93})</td>
<td>396</td>
<td>7.5 (\mu)s(^\text{a})</td>
<td>(11/2^+(49), 13/2^+(61))</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(9/2^+(100), 9/2^+(49))</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(7/2^–(21))</td>
</tr>
</tbody>
</table>

Performing \(^{150}\text{Sm}(^{13}\text{C,4n})^{159}\text{Er}\) and \(^{124}\text{Sn}(^{40}\text{Ar,5n})^{159}\text{Er}\) reactions at beam energies of 70 MeV and 165 MeV respectively, to search for the \([505]11/2^–\) isomer in \(^{159}\text{Er}\) have been proposed at the iThemba LABS PAC. The DCO and \(A_p\) values confirmed the in band transitions of the two bands to be of the \(E2\) character and the \(\gamma\)-rays decaying between the two bands as \(M1/E2’\)s. New Band b has been established in this work with tentative a 184-keV \(\gamma\)-ray and two tentative \(\gamma\)-rays of 672-keV and 896-keV that decay into the 15/2\(^–\) state in Band 9 and 13/2\(^–\) state in Band 10 respectively.
Figure 4.26: Background subtracted spectrum of Bands 9 and 10 gated at the 225 keV $\gamma$-ray.
4. EXPERIMENTAL DETAILS AND RESULTS

4.5.7 Band 11

Band 11 is a newly constructed band in this work, (see Figures 4.29 and 4.27). It consists of four transitions; 520-keV (27/2\(^+\) → 23/2\(^+\)), 610-keV (31/2\(^+\) → 27/2\(^+\)), 679-keV (35/2\(^+\) → 31/2\(^+\)) and the 714-keV (39/2\(^+\) → 35/2\(^+\)) level and is connected to the yrast band via the high-energy 930-keV (23/2\(^+\) → 21/2\(^+\)), 984-keV (27/2\(^+\) → 25/2\(^+\)), 1037-keV (31/2\(^+\) → 29/2\(^+\)), 1089-keV (35/2\(^+\) → 33/2\(^+\)) and 1126-keV (39/2\(^+\) → 37/2\(^+\)), \(E2/M1\) transitions. The DCO and \(A_p\) values of the four observed transitions were found to be consistent with their being of \(E2\) character, which means this band is of positive parity with signature (+1/2). The DCO and \(A_p\) values of some \(\gamma\)-rays decaying into the yrast band could not be measured because they were weak.

![Figure 4.27: Background subtracted spectrum of Band 11, with the 351-keV gate in blue and the 466-keV gate in red. Asterisks are used to denote contaminants from other reaction channels or other bands of \(^{159}\)Er.](image-url)
4.5.8 Band 12

Band 12 has previously been observed from $\left(27/2^- \rightarrow 83/2^-\right)$ by Simpson et al. [Sim84]. In this work a tentative new level $\left(23/2^-\right)$ was added with two new $\gamma$-rays, 599-keV and 1065-keV decaying into the $25/2^+$ and $21/2^+$ of the yrast band, respectively. The 351-keV gated spectrum in Figure 4.28, shows the energies of the in-band transitions in blue and $\gamma$-rays decaying into the yrast band in red. DCO and $A_p$ measurements have firmly confirmed the multipolarity of the 599-keV transition as $E1$. The 1065-keV transition could not be assigned because of low statistics.
Figure 4.28: Background subtracted spectrum of Band 12, with the 856 keV gate in blue and the high-energy part in red, gated at the 688 keV transition. Asterisks are used to denote contaminants from other reaction channels or other bands of $^{159}$Er.
4.5 The Experimental Results and Level Scheme

Figure 4.29: Partial level scheme for $^{159}$Er representing the yrast band and Bands 6 - 12 from the current work. Tentative transitions and levels are indicated by dashed lines, and tentative spins and parities are within parentheses, wherever applicable. New transitions are highlighted in red and previously published ones are in black. The width of the arrows are proportional to the transitions intensities.
4.5.9 Band a-f

New tentative levels, labeled Bands a to f in Figure 4.30, have been observed in this work. They consist of 31 transitions decaying into the Yrast band through a series of high energy $\gamma$-rays. The multipolarities of these transitions could not be measured because high-energy levels were weakly populated in this experiment.
Figure 4.30: Partial level scheme for $^{159}$Er, for levels not belonging to obvious bands.
4. EXPERIMENTAL DETAILS AND RESULTS

**Figure 4.31:** Measured DCO ratios in $^{159}$Er. A ratio near 0.5 is consistent with a stretched dipole transition while a value near 1.0 corresponds to stretched quadrupole transition.

**Figure 4.32:** Plot of linear polarization measurements in $^{159}$Er.
Figure 4.33: Partial level scheme for $^{159}$Er deduced in this work. Tentative transitions and levels are indicated by dashed lines, and tentative spins and parities are within parentheses, wherever applicable. New transitions are highlighted in red and previously published ones are in black. The width of the arrows is proportional to the transition intensities.
5

Discussion

The implication of the experimental results described in Chapter 4 leads to useful insights into the structure of these erbium nuclei, which will be explored in this chapter.

Experimental aligned angular momenta, $i_x$, Routhians, $e'$ [Naz85], and excitation energies, $E_x$, have been extracted from the present data for observed bands in $^{158,159}$Er. Harris parameters [Har65, Sim84]$J_0 = 15.5\text{ MeV}^{-1}\hbar^2$ and $J_1 = 90\text{ MeV}^{-3}\hbar^4$ for $^{158}$Er and $J_0 = 19.1\text{ MeV}^{-1}\hbar^2$ and $J_1 = 90\text{ MeV}^{-3}\hbar^4$ for $^{159}$Er have been used to define the contribution of the core so as to reveal quasiparticle characteristics.

5.0.10 Interpretation of $^{158}$Er

The low-frequency part of the ground state band in $^{158}$Er is interpreted as the vacuum state in which all the levels below the Fermi surface are filled and all the levels above are empty [Sim84]. The $|0^+_2\rangle$ states (Band 3) in the rare-earth transitional nuclei with $N = 88$ and 90, that lie at low energies within the pairing gap, are interpreted to be two-particle, two-hole seniority-zero neutron states lowered into the pairing gap by the configuration dependent pairing interaction and the low density of the oblate states near the Fermi surface. Figure 5.1 shows the energy systematics of available $0^+_1$ and $0^+_2$ bands in $N = 90$ isotones. The $0^+_2$ excitation in $^{158}$Er lies at 806.4 keV, about 125 keV higher than in the lighter $N = 90$ isotones. The $E(J + 2)/E(J)$ energy ratios for the $0^+_2$ bands in the $N = 90$ isotones are very similar, except for a slightly smaller $E(4^+_2)/E(2^+_2)$ ratio in $^{158}$Er and $^{160}$Yb [Bar13]. This systematic trend may suggest a
similar structure for the $0^+_2$ bands related to a two-particle ($2p$) two-hole ($2h$) neutron configuration [Bje66, Sha80], [Bvu08].

![Figure 5.1: Plot of the excitation energy as a function of angular momentum for the members of the $0^+_1$ and $0^+_2$ bands in the N = 90 isotones [nndc13].](image)

In $^{152}\text{Sm}$, $^{154}\text{Gd}$ and $^{156}\text{Dy}$, the similarity in energy spacing between the ground and $0^+_2$ bands, together with the large $E0$ strengths between the two $K = 0$ bands, has been associated with strong mixing of coexisting bands with different deformations [Hey11]. In contrast, as shown in Figure 5.1, the similarity in energy spacing does not continue in the heavier isotones, which indicates a reduction in deformation of the ground-state bands of $^{158}\text{Er}$ and $^{160}\text{Yb}$ [Bar13], which is not mirrored in the $0^+_2$ bands. This would support a shape coexistence picture with the displacement upwards of the oblate 505 configuration [Woo13]. A plot of alignment $i_x$ and Routhian $e'$ as a function of rotational frequency $\omega$ of the $0^+_2$ band in comparison with the ground-state (Band 1) and $S$ (Band 2) bands is shown in Figure 5.2. As identified by Simpson et al. [Sim84], the crossing frequency between Bands 1 and 2 occurs at $\hbar\omega \approx 0.275$ MeV, which is
the normal crossing frequency for the alignment of \( i_{13/2} \) neutrons in even-even and odd proton nuclei [Gar82]. The \( 0^+ \) band is crossed by the S-band, Band 2, at a smaller frequency \( \hbar \omega \approx 0.2 \) MeV. This crossing is the same type of frequency that is observed in the isotones \(^{154}\text{Gd}\) [War73] and \(^{156}\text{Dy}\) [And74]. It is just a normal band crossing and not due to the alignment of any nucleons by the Coriolis force. Figure 5.2 shows that the \( 0^+ \) band is steadily aligning towards a value of \( i_x \) achieved by the S-band. The alignment of the ground state \( 0^+_1 \) band is much sharper giving rise to the well known spectacular ‘back bend’. The irregularity in Band 3, Figures 5.3(a) and (b) is due to band mixing at spins \( 12^+ \) and \( 14^+ \) with Band 2. When these two bands mix energy is partly not purely rotational [Woo13].

**Figure 5.2:** Experimental alignments \( i_x \) (top panel) and Routhians (bottom panel) as a function of rotational frequency for the \( 0^+_1, 0^+_2 \) and S bands in \(^{158}\text{Er}\).
5. DISCUSSION

Figure 5.3: Alignment $i$ as a function of rotational frequency $\hbar \omega$ (a) and $E_\gamma$ as a function of spin (b) [Woo13], for Band 1 (ground state band), Band 2 ($S$-band), Band 3 ($|0^+_2\rangle$ band), Band 4 (even-$\gamma$-band) and Band 5 (odd-$\gamma$-band) in $^{158}$Er.
A notable feature of $\gamma$-bands (Bands 4 and 5) is that they track the intrinsic configuration, usually the ground state, that they are based on, Figure 5.5. The $\gamma$-band tracks the ground state band configuration up to spin $12^+\hbar$. The aligned band (Band 2), which causes a back bend in the band based on the $0_2^+$ state [War73] shows no sign of interaction with the $\gamma$-bands. Figure 5.5 shows that Band 1 has a very weak crossing with Band 2. The $\gamma$-band tracks Band 1 at lower spins and then starts to align to follow the aligned yrast states. The alignment in the $\gamma$-band comes at a slightly lower spin and hence a slightly lower rotational frequency than for the yrast states. The question then arises, are there $\gamma$-vibrations based on the aligned configurations? Indeed such a crossing has been observed in $^{164}$Er [Joh78, Yat80]. The experimental data showing this crossing is shown in Figure 5.4 together with a calculation based on the rotational-alignment model. The data shows that the ground state band is crossed by the usual aligned- or $S$-band. The $\gamma$-band tracks the ground state band at lower spins and then aligns to follow the aligned yrast states. It can be seen from Figure 5.4 that the alignment in the $\gamma$-band will come at a slightly lower spin and hence a slightly lower rotational frequency than for the yrast states. This is because the band crossing is just from two different configurations; the $\gamma$-band built on the ground state $|0_1^+\rangle$ is crossed by a band which is the $\gamma$-band built on the aligned band $|0_1^+ + (i_{13}/2)^2\rangle$ [Sha11].

Some algebraic models including the extended version of the interacting boson (sdg-IBM) [Ari75, Yos86] and pseudosymplectic models [Cas87] have also been employed to study the the $\gamma$-excitation modes and these predict high collectivity for the double-$\gamma$ vibration [Gar97]. A considerable effort has been devoted in understanding the $\gamma$-excitation mechanism by using the random phase approximation (RPA) approach [Mar70, Egi80, She08]. Recently, the triaxial projected shell model (TPSM) has been employed to describe $\gamma$ bands [She99, Sun00]. This model uses the shell model diagonalization approach and, in this sense, it is similar to the conventional shell model approach except that the basis states in the TPSM are triaxially deformed rather than spherical. In the present version of the model, the intrinsic deformed basis is constructed from the triaxial Nilsson potential [She08]. The good angular momentum states are then obtained through an exact three-dimensional angular momentum projection technique. In the final stage, the configuration mixing is performed by diagonalizing the pairing plus quadrupole-quadrupole Hamiltonian in the projection basis [Har95, Har91].
5. DISCUSSION

Figure 5.4: Plot of the level excitation energies for positive parity bands in $^{164}$Er against spin on an I(I + 1) scale [Yat80]; (a) experiment and (b) the rotational-aligned model. The solid circles correspond to even-spin states and the open circles to odd-spin states [Sha11].
advantage of the TPSM is that it describes the deformed single-particle states micro-
scopically as in the quasiparticle-phonon nuclear model (QPNM), multi-phonon method
(MPM), and the dynamic deformation model (DDM), but its total many-body wave
functions are exact eigenstates of the angular momentum operator [She08]. Correc-
tions beyond the mean field are introduced by mixing the projected configurations. An
intrinsic triaxial state in the TPSM is a rich superposition of different $K$ states. The
triaxial deformed vacuum state is composed of $K = 0, 2, 4, \ldots$ configurations. The
projected band from these $K = 0, 2, 4$ intrinsic states are the dominant components of
the ground, $\gamma$, and $2\gamma$ bands, respectively [Sun00]. In Figure 5.6, the projected bands
associated with the qp configuration are shown for $K = 0, 2, 4$, namely the $(0,0)$,
$(2,0)$ and $(4,0)$ bands. These bands are referred to as ground-state, $\gamma$-, and $2\gamma$-bands.
The ground-state band has $K = 0$ and is, therefore, comprised of only even-$K$ values
[She08]. It is evident from Figure 5.6 that the $(2,0)$ band for $^{158}$Er lies very close to
the $(0,0)$ band. This means that $\gamma$-vibration has low excitation energy in this nucleus.
For high-spin states, it is further noted that the $(0,0)$ and $(2,0)$ band energies become
almost degenerate, and, in fact, for $I = 16$ and above, the energy of even-spin states in
the $(2,0)$ band is slightly lower than that of the $(0,0)$ band.
It is well-known fact that that the $\gamma$-bands become lower in energy with increasing
triaxiality and they become favored with increasing angular momentum. As can be
seen from Figure 5.6, the $(2,0)$ band in $^{158}$Er has a degree of signature spitting with
the splitting amplitude increasing with spin. The $(4,0)$ band lies at a slightly higher
excitation energy. The $(4,0)$ band is also noted to have signature splitting for higher
angular momentum, and the splitting amplitude is nearly the same for the $(2,0)$ and
$(4,0)$ bands. In Figure 5.6, several representative multi-qp bands, namely projected 2-
and 4-qp configurations, are also plotted. Although the $K = 1$ 2qp neutron (1,2$n$)
and 2-qp proton (1,2$p$) bands are close in energy for low spins, with increasing spin
the 2$n$–qp bands are lower in energy than the 2$p$-qp bands due to larger rotational
alignment [She08]. It is noted that neutrons are occupying the $1i_{13/2}$ and protons are
occupying $1h_{11/2}$ intruder sub-shells. For each of the $(1,2n)$ and $(1,2p)$ bands, the
projected energies are also shown for the corresponding $\gamma$-bands with configurations $(3,2n)$
and $(3,2p)$. The $(1,2n)$ band is noted to cross the $(2,0)$ and the $(0,0)$ bands at $I = 12$.
It is also seen that the $(3,2n)$ band crosses the $(0,0)$ band at slightly higher spin value
of $I = 14$. It is interesting to note that after the band crossing, the lowest even-spin
states originate from the \((1,2n)\) band, whereas the odd-spin members are the projected states from the \((3,2n)\) configuration. Finally, the 4-qp \((4,4)\) configuration lies at high excitation energies and does not become yrast, at-least up to spin values shown in the Figure 5.6.

**Figure 5.5:** Excitation energy, minus a rigid-rotor reference, plotted as a function of spin for the Bands 1 to 5 in \(^{158}\text{Er}\).
Figure 5.6: Theoretical results: Band diagrams for $^{158}$Er. The labels (0,0), (2,0), (4,0), (1,2n), (3,2n), (1,2p), (3,2p), (2,4) and (4,4) correspond to ground, $\gamma$, 2$\gamma$ and 2n-aligned $\gamma$ band on this 2n-aligned state, 2p-aligned $\gamma$ band on this proton-aligned state, (2n + 2p)-aligned band, and $\gamma$ band built on this 4-quasiparticle state. [She08].
The experimental results for the Bands 1, 3, and 4 (ground-state band, first excited $0^+$ band and the $\gamma$-band) in our data were compared with the theoretical predictions of density functional theory (DFT), a model which is well-known for its numerous applications in the description of nuclear ground and excited states. In particular, the covariant version of DFT takes the Lorentz symmetry into account in a self consistent way. In practical applications, there are two widely used models in the covariant density functional theory (CDFT) framework: the relativistic Hartree (RH) and relativistic Hartree-Fock (RHF) models. The former one is usually known as the relativistic mean field (RMF) model \( ^1 \). For comparison with our experimental data a newly developed point-coupling parametrization PC-PK1, which particularly improves the description for isospin dependence of the binding energy \([\text{Zha12, Zha10}]\) was used. The systematics of low-lying states in the nuclei with \( Z = 62 \sim 70, \) and \( N = 88 \sim 100 \) \([\text{Son13}]\) calculations were done in collaboration with the Southwest University in China, \([\text{Li13}]\) and a review paper will be published \( ^2 \). Figure 5.7 shows the agreement between our experimental data and the theoretical calculations and Figure 5.8 shows the \( N = 90 \) isotones theoretical PES plots. These potential energy surfaces were drawn for the positive parity and even-spin states \((0^+_1, 0^+_2 \) and ground-state bands) of \(^{152}\text{Sm}, ^{154}\text{Gd}, ^{156}\text{Dy} \) and \(^{158}\text{Er}\). The plots show that the oblate minima are really saddle points associated with the deeper prolate minima due to $\gamma$ softness \([\text{Li1-09}]\), see Figures 5.7 and 5.8. These results will be fully discussed in \([\text{Li13}]\).

\( ^1 \) Hartree and Hartree-Fock theories are not density functional theories (DFTh’s). Though the equations look similar. Essentially, DFTh minimizes an energy functional which is the sum of a kinetic energy that depends on the gradients of orbitals (summed over all particles) and a potential energy that depends on the particle density. Hartree and Hartree-Fock theory minimize the quantum mechanical expectation value of the sum of the kinetic energy operator (summed over all particles) and the sum over all particles of a two-body force.

\( ^2 \) The actual procedure from DFTh to calculating nuclear collective properties is not as direct as implied in this statement. Two key steps are not mentioned. First, the calculation of a set of parameters, on a discrete grid of shape coordinates, for a five-dimensional Bohr collective Hamiltonian; second, the solution of the Schrodinger equation for this Hamiltonian by numerical integration. The potential energy mentioned, emerge after the first of these steps \([\text{Woo13}]\).
PC-PK1 parametrization

The parametrization of PC-PK1 is determined by a multiple parameter fitting to both the binding energies for 60 selected spherical nuclei, and 17 charge radii, with the Levenberg-Marquardt method. Meanwhile, the empirical pairing gaps for 6 nuclei obtained with the five-point formula are also employed to constrain the pairing strengths. With the experimental observables and the calculated values, the ensemble of parameters can be obtained by minimizing the square deviation. More details for the fitting procedure and coupling constants of the parametrization PC-PK1 can be found in [Zha10, Zha12].

Our experimental data shows a small signature splitting in Bands 4 and 5 which, is not in the calculation. This could be that the even spins mix with other even spin bands whereas the odd spins have no similar spin nearby to mix with.
5. DISCUSSION

Figure 5.7: The experimental data compared to the PC-PK1 relativistic density functional model for the $0_1^+$, $0_2^+$ and $\gamma$ bands, and the potential energy surface in $^{158}$Er.
Figure 5.8: The excitation spectrum of the $N = 90$ isotones calculated with the PC-PK1 relativistic density functional and their potential energy surface [Son13]. $0_1^+$; (blue), $0_2^+$; (green) and $\gamma$; (maroon) bands.
Figure 5.9, shows results of a systematic calculation for the first excited $0^+_2$ state in Gd, Dy, and Er isotopes, and compares them with the experimental data. The energy levels $2^+_1$, $4^+_1$, and $6^+_1$ of the ground state rotational band are also shown [Che13]. It is found that with a single set of parameters in the Hamiltonian, the characteristic behavior of the shell evolution is well produced. It is seen that starting from the neutron number $N = 90$ on the Left hand side of Figure 5.9 and going to heavier isotopes the ground state band becomes more and more compressed, eventually following the rotational rule $E \sim I(I + 1)$ at $N = 98$. On the other hand the $0^+_2$ state is found low for isotopes with neutron number 90. Without any adjustable parameter, the increase of the energy of the $0^+_2$ with increasing neutron number is correctly produced for all three isotopic chains. The case with the largest discrepancy between calculation and data is $^{158}\text{Er}$ [Che13].
Figure 5.9: Comparison of calculated $0^+_2$ state with experimental data for Gd, Dy, and Er isotopes. The energy $2^+_1$, $4^+_1$, and $6^+_1$ of ground-state rotational band are also shown. Calculated results (open circles) are compared with data (filled squares) [Che13].
5. DISCUSSION

5.0.11 Negative Parity States

Bands 6 and 7 were observed to upbend at $\hbar \omega_c \sim 0.35$ MeV. This band crossing involves configuration change in the two bands. Band 1 is crossed for the second time at $\hbar \omega_c \sim 0.33$ MeV as shown in Figure 5.10(a), Figure 5.10(b) shows clearly the alignment of Bands 6, 7 and 8 compared to Band 1. At spin above $12^+$, Bands 6 and 7 align to Band 1 and below this spin the bands deviate and cross Bands 8 and 9, (see Figure 5.10(c)).

Band 8 and its members decay predominantly to the yrast band and cross Band 7 at $\hbar \omega_c \sim 0.33$ MeV in Figure 5.10(a). This band indicates a gain in alignment. (Figure 5.10(b) [Woo13] and Figure 5.10(c)) and it connects a $K^\pi = 0^-$ octupole band with a $2qn$ band as similar crossing as observed in $^{160}$Yb [Sim84, Rie80]. At low spins Band 8 tracks the yrast structure. Band 9 is a new band and its nature is uncertain. The Routhian and the energy function for Band 9 could not be plotted because the spins and parity of this band are not yet confirmed, but Figure 5.10(c) indicates the alignment of this band to Bands 1 and 8. Bands 6 and 7 cross Band 8 at $\sim \text{spin } 6$. 
Figure 5.10: Plots for negative parity bands in $^{158}$Er with Band 1 as a reference band. (a) The alignment versus rotational frequency, (b) $\gamma$-energy versus spin [Woo13] and (c) the energy minus rigid rotor versus spin.
5. DISCUSSION

5.0.12 Interpretation of $^{159}$Er

Experimentally, the nuclear structure properties of bands can be identified from the alignment, $\gamma$-ray energies as a function of spin, and the excitation energy relative to a rotating liquid drop rigid rotor reference based on the Lublin Strasbourg model [Pom03, Car06], as a function of spin. $I_x$ is the aligned angular momentum, $I_x = \sqrt{I(I+1) - K^2}$, at a given spin and the rotational frequency can be related to the $\gamma$-ray transition energy $\omega \simeq E_\gamma/\Delta I_x \simeq E_\gamma/2$, whereas the rotational frequency and the rigid rotor references are given by:

\[ I_{x,\text{ref}}(\omega) - (J_0 + J_1)\omega - i_0 \]  \hspace{1cm} (5.1)

\[ E_{RLD} = \frac{\hbar^2}{23_{\text{rig}}} I(I+1) \]  \hspace{1cm} (5.2)

Harris parameters [Har65, Sim84] $J_0 = 19.1 \text{ MeV}^{-1}\hbar^2$ and $J_1 = 90 \text{ MeV}^{-3}\hbar^4$ have been used to define the contribution of the core so as to reveal quasiparticle character only. The only values are chosen from the mean values for two even-even neighboring isotopes with an offset $i_0 = 0$ to ensure that the alignment for ground state bands of $^{158,160}$Er at low rotational frequencies is approximately zero. These values provide constant alignment for the quasineutron configuration after the backbend.
5.0.13 Positive Parity Bands: The Yrast Band, Bands 1, 6 and 11

The yrast Band is based on the odd neutron occupying the favored signature, (+,+1/2), of the $i_{13/2}$ Nilsson orbital $[651]3/2^+$. This was previously assigned by [Sim84]. Transitions in the yrast band were observed up to spin $53/2^+$. The observation of $\gamma$-ray transitions and the establishment of the multipolarity of transitions between states of the yrast band helped to provide a complete description of the structure of this band. The experimental energy alignment plot of the yrast band is presented in Figure 5.11, in which the yrast band has an initial alignment of $\approx 6.3 \hbar$. This reflects its one particle configuration at low spin, with the odd neutron in the $[651]3/2^+$ orbital.

Band 1 is the unfavored signature partner of the yrast Band (+,-1/2) [Sim84]. It lies at excitation energy of 137 keV above the yrast band [Sim84, Mus11].

Band 6 has been interpreted as a $\gamma$-vibrational excitation coupled to the occupied $i_{13/2}$ neutron in the $[651]3/2^-$ Nilsson orbital of the yrast configuration. This band has the same signature and parity as the yrast Band (+,+1/2). This idea is supported by the angular intensity ratio measurement for most of the decay-out $\gamma$-rays to the yrast Band. Figure 5.11 shows the alignment plot of Band 6. In this plot the nature of this band appears to be more complex than the other bands [Mus11].

Band 11 is the new band observed in this work. This band is at an excitation energy of 1490-keV higher than that of the yrast band and is connected via a high-energy transitions to the yrast band. It is labelled as Band 11 in Figures 4.29 and 4.33. It has the parity signature (+,-1/2). At spin $27/2^+$ this band tracks the yrast band and Band 1, see Figure 5.11.
Figure 5.11: Plots for positive parity bands in $^{159}$Er with the Yrast as a reference band. (a) The alignment versus rotational frequency, (b) $\gamma$-energy versus spin [Woo13] and (c) the energy minus rigid rotor versus spin.
5.0.14 Negative Parity Bands: Bands 2, 3, 4 and 5

In this section the negative parity bands in $^{159}$Er are interpreted. The extensions of the previously known band, Band 4 and new transitions added to Bands 5 and 12 in this work are shown in Figure 4.33 and will be discussed.

Band 2 has previously been assigned to the occupying of the $h_{11/2}$ Nilsson [521]3/2$^-$ orbital by the odd neutron [Sim84, Del87]. This band has the favored signature ($-$, +1/2) and is the ground-state band in $^{159}$Er. The energy alignment plot (Figure 5.12, [Woo13]), shows that at low spins this band exhibits collective rotational motion. There is a gain in alignment of $\sim 8\hbar$, see Figure 5.12. This gain in alignment is consistent with the initial aligned angular momentum of the quasi-neutrons in the signature-partner, positive-parity bands.

Band 3 could not be extended in this work. A sequence of three transitions, $E_\gamma = 145$, 285- and 404-keV, which cascade to the ground-state band was observed [Sim84]. The level at 145-keV was observed by Strusny et al. [Str75] and interpreted as the 7/2$^-$ state. The 114- and 85-keV $\gamma$-rays which connect this level with low-spin members of Band 2 were found to be $M1$ and $E2/M1$, respectively (Strusny et al. [Str75]). The 171-, 144- and 157-keV $\gamma$-rays between this band and Band 2 are measured to be $M1$ transitions.

Band 12 as observed by [Mus11] could not be linked to Band 3 in the manner that Mustafa et al. did. The energy-alignment plot in Figure 5.12 shows the alignment of $\sim 10\hbar$ of this band to Band 2.
Figure 5.12: Plots for negative parity bands in $^{159}\text{Er}$. (a) The alignment versus rotational frequency, (b) $\gamma$-energy versus spin [Woo13] and (c) the energy minus rigid rotor versus spin.
Band 4 has the same signature and parity as Band 2, and depopulates through the 641-keV $\gamma$-ray to Band 1 at the $23/2^+$ level and by an 839-keV $\gamma$-ray to the yrast $25/2^+$. A new $21/2^-$ level, fed by a 355-keV $E2$ transition from the $25/2^-$ level, decays by a 773-keV $\gamma$-ray to the $19/2^-$ of Band 1. This band has previously been assigned to be the next higher-lying negative-parity positive-signature quasineutron configuration originating from the $h_{9/2}[523]5/2^-$ Nilsson orbital [Del87]. The initial alignment of $\sim 8.5\hbar$ can be seen in Figure 5.13. The gain in alignment of this band increases gradually with increasing rotational frequency.

Band 5 was previously observed by [Del87], (see Figure 4.33). In this work a $25/2^-$ level and an out-of-band transition, the 749-keV $\gamma$-ray decaying to Band 2, was observed. The alignment characteristics of Band 5 lend weight to this assignment, with the initial alignment of $\sim 8\hbar$, see Figure 5.13, [Woo13].
Figure 5.13: Plots for negative parity bands in $^{159}$Er, Bands 2, 4 and 5. (a) The alignment versus rotational frequency, (b) in-band $E2\gamma$-energy versus spin [Woo13] and (c) the energy minus rigid rotor versus spin.
5.0.15 Strongly Coupled Bands: Bands 7, 8, 9 and 10

The structure of the strongly-coupled signature partner Bands (7 and 8) and (9 and 10) observed in this study, has previously been discussed [Mus11, Sim98]. The proposed configuration of the coupled structures was based on several experimental observations; absence of the occurrence of the quasiparticle configuration $AB \left( h_{11/2} \right)^2$ proton crossing in the coupled structures, similarity in the intensity of different multiple character $\gamma$-ray transitions at the bottom of the bands, the dipole nature of the assigned 808-, 1272- and 1434-keV decay out transitions from Bands 7 and 8 to the low-K yrast structures. No decays out of the $(11/2^-)$ and $13/2^-$ states in Bands 9 and 10, respectively, have been observed, suggesting that the band head is the [505]$11/2^-$ neutron configuration. In neighboring odd-neutron nuclei this band head is isomeric with half-lives in the $\mu$s to ms region. Figure 5.14 shows the alignment of Bands 7 to 10 and the yrast band is the reference band.
5. DISCUSSION

Figure 5.14: Plots for high-K negative parity bands in $^{159}$Er with Yrast band as a reference. (a) The alignment versus rotational frequency, (b) in-band $E2\gamma$-energy versus spin [Woo13] and (c) the energy minus rigid rotor versus spin.
Conclusions

The nuclei $^{158,159}$Er are textbook examples of the behavior of nuclei at high spins, but surprisingly little is known on their low spin structures. The detailed spectroscopic study at low and medium spins in nuclei $^{158,159}$Er have been carried out using the $^{150}$Sm($^{12}$C,4nγγ)$^{158}$Er and $^{150}$Sm($^{13}$C,4nγγ)$^{159}$Er reactions at beam energy of $E_{\text{lab}} = 65$ MeV using AFRODITE γ-ray spectrometer at iThemba LABS. The current work has revealed new information on the yrare states below spin $20\hbar$ of these nuclei. New rotational bands (Band 9 in $^{158}$Er, Band 11 and levels, Figure 4.30, not belonging to obvious bands in $^{159}$Er), and extensions at low spins to fill the gaps between levels in $^{158,159}$Er were discovered in this work. The spins and parities of the band built on the first excited $0^+_2$ state in $^{158}$Er have been assigned and the band extended to $16^+_1$ from the analysis of coincidence relationships, intensity arguments, the assignment of DCO ratios and $A_p$ values. Furthermore, transitions which were previously placed in different bands [Sim84, Pip06, Hel04, nndc13] have been placed accordingly, with the 405-keV γ-ray decaying to the $4^+_1$ state and its 382-keV counterpart from the $6^+_\gamma \rightarrow 4^+_0[0^+_2>$, because Band 3 and Band 4 cross. The $0^+_2$ and γ-band in $^{158}$Er is complicated when they cross. This has not been noticed in previous experiments but accounted for the fact that these positive parity bands had not been properly observed before. Additionally, the $|0^+_2>$ and the γ-bands in this work are in good agreement with the theoretical calculations. The $K^{\pi} = 2^+$ bands that we associate with the γ-vibrations are truly ‘collective’ structures. They invariably track their intrinsic structures, both in even and in odd nuclei. It is not clear if the ‘γ-vibrational’ bands are just a $K^{\pi} = 2^+$ projection of the zero point motion on the symmetry axis, or if they are more of a
6. CONCLUSIONS

traditional boson or phonon, but unlike the ‘β-vibrations’, γ-vibrations exhibit real collective motion. The existence of the [505]11/2− band in $^{159}$Er is well established, however, the excitation energy, decay modes, and lifetime still remain unknown. Analyzing the data was complex as, in order to populate states at the lower spins, $^{12,13}$C beams had to be used and the only available target was backed with Au. This lead to many other reaction channels being open and contamination from strong γ-rays from reactions on the Au target. Overall, new transitions, including one band in $^{158}$Er and one band in $^{159}$Er, were placed as a result of the work in this thesis. These results help in understanding the behavior of the $N = 88 – 90$ rare-earth isotones at low spin.
APPENDIX A

The tables include experimentally determined properties for the nuclei $^{158,159}$Er, including:

- $E_\gamma \rightarrow \gamma$-ray energy. The energies of the $\gamma$-rays are estimated to be correct to 0.3 keV for strong transitions rising to 0.6 keV for weaker transitions.
- $J_i^\pi \rightarrow J_f^\pi \rightarrow \text{Spin / parity assignment}$
- $R_{DCO} \rightarrow \text{Spin and parity measurements}$
- $A_p \rightarrow \text{Polarization measurements}$

Table 6.1: Table of observed $\gamma$ rays in $^{158}$Er

<table>
<thead>
<tr>
<th>$E_\gamma$(keV)</th>
<th>$J_i^\pi \rightarrow J_f^\pi$</th>
<th>$R_{DCO}$</th>
<th>Band 1</th>
<th>Multip.</th>
<th>Band</th>
</tr>
</thead>
<tbody>
<tr>
<td>184</td>
<td>$14^+ \rightarrow 14^+$</td>
<td>$^1$</td>
<td>$^2$</td>
<td>$^3$</td>
<td>$1 \rightarrow 2$</td>
</tr>
<tr>
<td>192</td>
<td>$2^+ \rightarrow 0^+$</td>
<td>1.11(0.03)</td>
<td>0.40(0.02)</td>
<td>E2</td>
<td>1</td>
</tr>
<tr>
<td>335</td>
<td>$4^+ \rightarrow 2^+$</td>
<td>1.05(0.03)</td>
<td>0.37(0.01)</td>
<td>E2</td>
<td>1</td>
</tr>
<tr>
<td>443</td>
<td>$6^+ \rightarrow 4^+$</td>
<td>1.15(0.05)</td>
<td>0.32(0.02)</td>
<td>E2</td>
<td>1</td>
</tr>
<tr>
<td>524</td>
<td>$8^+ \rightarrow 6^+$</td>
<td>1.01(0.04)</td>
<td>0.24(0.02)</td>
<td>E2</td>
<td>1</td>
</tr>
<tr>
<td>580</td>
<td>$10^+ \rightarrow 8^+$</td>
<td>1.02(0.05)</td>
<td>0.19(0.02)</td>
<td>E2</td>
<td>1</td>
</tr>
<tr>
<td>609</td>
<td>$12^+ \rightarrow 10^+$</td>
<td>1.04(0.07)</td>
<td>0.14(0.04)</td>
<td>E2</td>
<td>1</td>
</tr>
<tr>
<td>694</td>
<td>$14^+ \rightarrow 12^+$</td>
<td>1.05(0.01)</td>
<td>0.04(0.02)</td>
<td>E2</td>
<td>1</td>
</tr>
<tr>
<td>651</td>
<td>$16^+ \rightarrow 14^+$</td>
<td>1.09(0.04)</td>
<td>0.06(0.02)</td>
<td>E2</td>
<td>1</td>
</tr>
<tr>
<td>653</td>
<td>$18^+ \rightarrow 16^+$</td>
<td>1.30(0.10)</td>
<td>$^2$</td>
<td>$^*$E2</td>
<td>1</td>
</tr>
<tr>
<td>649</td>
<td>$20^+ \rightarrow 18^+$</td>
<td>1.02(0.20)</td>
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<th>$E_\gamma$(keV)</th>
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<th>$R_{DCO}$</th>
<th>$A_p$</th>
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<td><strong>Band 4</strong></td>
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<td>0.19(0.07)</td>
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Table 6.1 – continued from previous page

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<th>$J_\gamma^\pi \rightarrow J_\gamma^\pi$</th>
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<th>$A_p$</th>
<th>Multip.</th>
<th>Band</th>
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<td>1.20(0.20)</td>
<td>0.17(0.04)</td>
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<tr>
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<td>0.11(0.09)</td>
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<td>821</td>
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<td>0.06(0.04)</td>
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<td>M1/E2</td>
<td>4$\rightarrow$1</td>
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<td>-0.35(0.02)</td>
<td>M1/E2</td>
<td>4$\rightarrow$1</td>
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<tr>
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<td>-0.04(0.03)</td>
<td>M1/E2</td>
<td>4$\rightarrow$1</td>
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<tr>
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<td>-0.05(0.01)</td>
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<td>4$\rightarrow$1</td>
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<tr>
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<td>4$\rightarrow$3</td>
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<tr>
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<td>$^3$</td>
<td>4$\rightarrow$1</td>
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<tr>
<td>1192</td>
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<td>$^1$</td>
<td>$^2$</td>
<td>$^3$</td>
<td>4$\rightarrow$1</td>
</tr>
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</table>

Band 5

| 395            | $5^+ \rightarrow 3^+$            | 1.20(0.03) | 0.41(0.02) | E2 | 5 |
| 475            | $7^+ \rightarrow 5^+$            | 0.87(0.02) | 0.32(0.04) | E2 | 5 |
| 542            | $9^+ \rightarrow 7^+$            | 1.10(0.02) | 0.23(0.07) | E2 | 5 |
| 592            | $11^+ \rightarrow 9^+$           | 1.20(0.05) | 0.38(0.03) | E2 | 5 |
| 628            | $13^+ \rightarrow 11^+$          | 0.90(0.10) | 0.35(0.03) | E2 | 5 |
| 852            | $3^+ \rightarrow 2^+$            | 0.31(0.20) | -0.04(0.01) | M1/E2 | 5$\rightarrow$1 |
| 912            | $5^+ \rightarrow 4^+$            | 0.44(0.06) | -0.03(0.04) | M1/E2 | 5$\rightarrow$1 |
| 943            | $7^+ \rightarrow 6^+$            | 0.38(0.06) | -0.05(0.02) | M1/E2 | 5$\rightarrow$1 |
| 962            | $9^+ \rightarrow 8^+$            | 1.10(0.30) | -0.02(0.01) | M1/E2 | 5$\rightarrow$1 |
| 956            | $11^+ \rightarrow 10^+$          | 1.40(0.20) | -0.03(0.03) | M1/E2 | 5$\rightarrow$1 |
| 1021           | $13^+ \rightarrow 12^+$          | $^1$ | $^2$ | $^3$ | 5$\rightarrow$1 |
| 1043           | $15^+ \rightarrow 14^+$          | $^1$ | $^2$ | $^3$ | 5$\rightarrow$1 |

Band 6

| 215            | $7^- \rightarrow 9^-$            | 0.99(0.03) | 0.48(0.07) | E2 | 6$\rightarrow$8 |
| 300            | $11^- \rightarrow 9^-$           | 1.00(0.01) | 0.44(0.01) | E2 | 6 |
| 424            | $13^- \rightarrow 11^-$          | 1.04(0.02) | 0.34(0.01) | E2 | 6 |
| 541            | $15^- \rightarrow 13^-$          | 1.20(0.02) | 0.30(0.08) | E2 | 6 |
| 635            | $17^- \rightarrow 15^-$          | 0.94(0.07) | 0.30(0.02) | E2 | 6 |
| 693            | $19^- \rightarrow 17^-$          | 1.10(0.02) | 0.39(0.02) | E2 | 6 |
| 939            | $9^- \rightarrow 8^-$            | 0.50(0.01) | 0.25(0.03) | E2 | 6$\rightarrow$1 |

Continued on next page
Table 6.1 – continued from previous page

<table>
<thead>
<tr>
<th>$E_{\gamma}$(keV)</th>
<th>$J_i^x \rightarrow J_f^x$</th>
<th>$R_{DCO}$</th>
<th>$A_p$</th>
<th>Multip.</th>
<th>Band</th>
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<td>$7^- \rightarrow 6^+$</td>
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<td>0.06(0.04)</td>
<td>E1</td>
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<td>$11^- \rightarrow 10^+$</td>
<td>0.60(0.03)</td>
<td>0.24(0.01)</td>
<td>E1</td>
<td>6 → 1</td>
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<tr>
<td>414</td>
<td>$9^- \rightarrow 8^+$</td>
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<td>0.33(0.01)</td>
<td>E1</td>
<td>6 → 3</td>
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<tr>
<td>475</td>
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<td>0.05(0.01)</td>
<td>E1</td>
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<td>159</td>
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<td>578</td>
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<td>0.44(0.03)</td>
<td>E2</td>
<td>6 → 8</td>
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Band 7

| 237              | $10^- \rightarrow 8^-$   | 1.20(0.03)| 0.41(0.05)| E2      | 7     |
| 386              | $12^- \rightarrow 10^-$  | 1.09(0.01)| 0.14(0.02)| E2      | 7     |
| 520              | $14^- \rightarrow 12^-$  | 1.20(0.02)| 0.28(0.02)| E2      | 7     |
| 630              | $16^- \rightarrow 14^-$  | 1.20(0.04)| 0.09(0.01)| E2      | 7     |
| 710              | $18^- \rightarrow 16^-$  | 1.10(0.07)| 0.20(0.09)| E2      | 7     |
| 727              | $20^- \rightarrow 18^-$  | 1.30(0.10)|       | $^3$    | 7 → 8 |
| 683              | $22^- \rightarrow 20^-$  | $\rangle^1$| $\rangle^2$| $^3$    | 7 → 8 |
| 299              | $10^- \rightarrow 9^-$   | 0.30(0.10)|       | $^3$    | 7 → 8 |
| 307              | $8^- \rightarrow 6^-$    | 1.20(0.10)| 0.06(0.03)| E2      | 7 → 9 |
| 481              | $8^- \rightarrow 7^-$    | 0.40(0.08)| $\rangle^2$| $^3$    | 7 → 8 |
| 547              | $6^- \rightarrow 6^+$    | $\rangle^1$| $\rangle^2$| $^3$    | 7 → 4 |
| 841              | $8^- \rightarrow 8^+$    | 0.80(0.02)| -0.22(0.02)| E1      | 7 → 1 |
| 421              | $8^- \rightarrow 7^+$    | 0.50(0.02)| -0.15(0.05)| E2      | 7 → 5 |
| 498              | $10^- \rightarrow 10^+$  | $\rangle^1$| $\rangle^2$| $^3$    | 7 → 8 |
| 274              | $12^- \rightarrow 12^+$  | $\rangle^1$| $\rangle^2$| $^3$    | 7 → 1 |
| 284              | $14^- \rightarrow 14^+$  | $\rangle^1$| $\rangle^2$| $^3$    | 7 → 2 |

Band 8

| 326              | $7^- \rightarrow 5^-$    | $\rangle^1$| $\rangle^2$| $^3$    | 8     |
| 420              | $9^- \rightarrow 7^-$    | 1.20(0.06)| 0.40(0.02)| E2      | 8     |
| 488              | $11^- \rightarrow 9^-$   | 0.90(0.07)| 0.37(0.01)| E2      | 8     |
| 544              | $13^- \rightarrow 11^-$  | 1.12(0.04)| $\rangle^2$| $^3$    | 8     |
| 602              | $15^- \rightarrow 13^-$  | 1.05(0.10)| 0.19(0.07)| E2      | 8     |
| 664              | $17^- \rightarrow 15^-$  | 0.85(0.20)| 0.41(0.02)| E2      | 8     |
| 1150             | $3^- \rightarrow 2^+$    | $\rangle^1$| $\rangle^2$| $^3$    | 8 → 1 |

Continued on next page
### Table 6.1 – continued from previous page

<table>
<thead>
<tr>
<th>$E_\gamma$(keV)</th>
<th>$J_i^\pi \rightarrow J_f^\pi$</th>
<th>$R_{DCO}$</th>
<th>$A_p$</th>
<th>Multip.</th>
<th>Band</th>
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<td>5$^-$ → 4$^+$</td>
<td>0.39(0.20)</td>
<td>-0.23(0.03)</td>
<td>E1</td>
<td>8 → 1</td>
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<tr>
<td>883</td>
<td>7$^-$ → 6$^+$</td>
<td>1.20(0.08)</td>
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<tr>
<td>360</td>
<td>7$^-$ → 8$^+$</td>
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<td>$^2$E2</td>
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<td>780</td>
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<td>-0.44(0.01)</td>
<td>E1</td>
<td>8 → 1</td>
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<td>688</td>
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<td>0.72(0.06)</td>
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<tr>
<td>624</td>
<td>13$^-$ → 12$^+$</td>
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<table>
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<td>433</td>
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<td>$^1$</td>
<td>$^2$</td>
<td>$^3$</td>
<td>9</td>
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<tr>
<td>412</td>
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<td>416</td>
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<td>$^3$</td>
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<td>$^2$</td>
<td>$^3$</td>
<td>9 → 1</td>
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<td>1058</td>
<td>6$^-$ → 6$^+$</td>
<td>1.05(0.20)</td>
<td>$^2$</td>
<td>$^3$</td>
<td>9 → 1</td>
</tr>
<tr>
<td>967</td>
<td>8$^-$ → 8$^+$</td>
<td>0.90(0.10)</td>
<td>$^2$</td>
<td>$^3$</td>
<td>9 → 1</td>
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<tr>
<td>501</td>
<td>6$^-$ → 5$^-$</td>
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<td>607</td>
<td>8$^-$ → 7$^-$</td>
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<td>0.80(0.10)</td>
<td>$^2$</td>
<td>$^3$</td>
<td>9 → 1</td>
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</tbody>
</table>

$^1$ Insufficient statistics to measure DCO ratios
$^2$ Insufficient statistics to measure linear polarization anisotropy
$^3$ Multipolarity could not be obtained
$^*$ Previously measured and confirmed transition
## Table 6.2: Table of observed $\gamma$ rays in $^{150}\text{Er}$

<table>
<thead>
<tr>
<th>$E_\gamma$(keV)</th>
<th>$J^+_i \rightarrow J^+_f$</th>
<th>$R_{DCO}$</th>
<th>$A_p$</th>
<th>Multip.</th>
<th>Band</th>
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</tr>
<tr>
<td>210</td>
<td>$17/2^+ \rightarrow 13/2^+$</td>
<td>0.82(0.04)</td>
<td>0.03(0.03)</td>
<td>E2</td>
<td>Yrast</td>
</tr>
<tr>
<td>351</td>
<td>$21/2^+ \rightarrow 17/2^+$</td>
<td>0.97(0.05)</td>
<td>0.05(0.04)</td>
<td>E2</td>
<td>Yrast</td>
</tr>
<tr>
<td>466</td>
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<td>1 → Yrast</td>
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<td>-0.03(0.03)</td>
<td>M1/E2</td>
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<td><strong>Band 2</strong></td>
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<td>200</td>
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<td>0.03(0.02)</td>
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<tr>
<th>E(_g) (keV)</th>
<th>(J_i^\pi \rightarrow J_f^\pi)</th>
<th>(R_{DCO})</th>
<th>(A_p)</th>
<th>Multip.</th>
<th>Band</th>
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<td>1.04(0.02)</td>
<td>0.04(0.04)</td>
<td>E2</td>
<td>2</td>
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<td>438</td>
<td>33/2(^-) \rightarrow 29/2(^-)</td>
<td>0.89(0.06)</td>
<td>0.06(0.05)</td>
<td>E2</td>
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<td>528</td>
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<td>0.96(0.05)</td>
<td>0.04(0.05)</td>
<td>E2</td>
<td>2</td>
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<tr>
<td>625</td>
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<td>0.02(0.04)</td>
<td>E2</td>
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<td>2</td>
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<tr>
<td>388</td>
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<td>0.88(0.05)</td>
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<td>E2</td>
<td>2 \rightarrow 4</td>
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<tr>
<td>450</td>
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<td>0.79(0.08)</td>
<td>-0.06(0.04)</td>
<td>E1</td>
<td>2 \rightarrow 1</td>
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<tr>
<td>114</td>
<td>9/2(^-) \rightarrow 7/2(^-)</td>
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<td>2 \rightarrow 3</td>
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<tr>
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<td>M1/E2</td>
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<td>157</td>
<td>17/2(^-) \rightarrow 15/2(^-)</td>
<td>0.63(0.03)</td>
<td>-0.05(0.03)</td>
<td>M1/E2</td>
<td>2 \rightarrow 3</td>
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**Band 3**

| 285         | 11/2\(^-\) \rightarrow 7/2\(^-\) | 1.01(0.03) | 0.02(0.03) | E2     | 3    |
| 405         | 15/2\(^-\) \rightarrow 11/2\(^-\) | 0.89(0.05) | 0.05(0.05) | E2     | 3    |
| 84          | 7/2\(^-\) \rightarrow 5/2\(^-\) | )¹²        | )³         | M1/E2  | 3 \rightarrow 2 |
| 171         | 11/2\(^-\) \rightarrow 9/2\(^-\) | 0.51(0.01) | -0.02(0.01) | M1/E2  | 3 \rightarrow 2 |

**Band 4**

| 464         | 29/2\(^-\) \rightarrow 25/2\(^-\) | 1.03(0.02) | 0.08(0.02) | E2     | 4    |
| 548         | 33/2\(^-\) \rightarrow 29/2\(^-\) | 0.88(0.01) | 0.06(0.01) | E2     | 4    |
| 597         | 37/2\(^-\) \rightarrow 33/2\(^-\) | 1.01(0.04) | 0.04(0.04) | E2     | 4    |
| 658         | 41/2\(^-\) \rightarrow 37/2\(^-\) | 1.01(0.02) | 0.03(0.02) | E2     | 4    |
| 641         | 25/2\(^-\) \rightarrow 23/2\(^+\) | 0.59(0.01) | 0.01(0.01) | E1     | 4 \rightarrow 1 |
| 746         | 29/2\(^-\) \rightarrow 29/2\(^+\) | 1.04(0.02) | 0.04(0.02) | E1     | 4 \rightarrow Yrast |
| 667         | 33/2\(^-\) \rightarrow 33/2\(^+\) | 0.99(0.04) | 0.01(0.04) | E1     | 4 \rightarrow Yrast |
| 541         | 29/2\(^-\) \rightarrow 25/2\(^-\) | 0.87(0.03) | 0.02(0.03) | E2     | 4 \rightarrow 2 |
| 355         | 25/2\(^-\) \rightarrow 21/2\(^-\) | 1.01(0.03) | 0.02(0.02) | E2     | 4 \rightarrow 4 |
| 733         | 21/2\(^-\) \rightarrow 19/2\(^+\) | 1.04(0.02) | 0.04(0.04) | E1     | 4 \rightarrow 1 |
| 839         | 25/2\(^-\) \rightarrow 25/2\(^+\) | 0.77(0.06) | 0.02(0.03) | E1     | 4 \rightarrow Yrast |

**Band 5**

| 571         | 33/2\(^-\) \rightarrow 29/2\(^-\) | 1.03(0.03) | 0.04(0.03) | E2     | 5    |
| 619         | 37/2\(^-\) \rightarrow 33/2\(^-\) | 0.99(0.04) | 0.01(0.04) | E2     | 5    |

Continued on next page
Table 6.2 – continued from previous page

<table>
<thead>
<tr>
<th>$E_r$(keV)</th>
<th>$J_i^\pm \rightarrow J_f^\pm$</th>
<th>$R_{DCO}$</th>
<th>$A_p$</th>
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<th>Band</th>
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<tbody>
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**Band 6**

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$41/2^+$

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**Band 7**

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<td>567</td>
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<td>-0.09(0.05)</td>
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**Band 8**

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<th>$A_p$</th>
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<td>0.08(0.04)</td>
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<tr>
<td>417</td>
<td>$31/2^- \rightarrow 27/2^-$</td>
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<tr>
<td>273</td>
<td>$35/2^- \rightarrow 33/2^-$</td>
<td>0.55(0.04)</td>
<td>-0.04(0.04)</td>
<td>M1/E2</td>
<td>8 \rightarrow 7</td>
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<tr>
<td>223</td>
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<td>8 \rightarrow 7</td>
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<tr>
<td>163</td>
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<td>0.41(0.05)</td>
<td>-0.05(0.05)</td>
<td>M1/E2</td>
<td>8 \rightarrow 7</td>
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<td>428</td>
<td>$15/2^- \rightarrow 11/2^-$</td>
<td>0.99(0.04)</td>
<td>0.03(0.04)</td>
<td>E2</td>
<td>9</td>
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<tr>
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<td>$19/2^- \rightarrow 15/2^-$</td>
<td>1.02(0.01)</td>
<td>0.01(0.01)</td>
<td>E2</td>
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<td>$27/2^- \rightarrow 23/2^-$</td>
<td>0.87(0.05)</td>
<td>0.01(0.05)</td>
<td>E2</td>
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</tr>
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<td>$31/2^- \rightarrow 29/2^-$</td>
<td>0.58(0.0)</td>
<td>-0.02(0.04)</td>
<td>M1/E2</td>
<td>9 \rightarrow 10</td>
</tr>
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<td>-0.04(0.06)</td>
<td>M1/E2</td>
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<td>M1/E2</td>
<td>9 \rightarrow 10</td>
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<td>$19/2^- \rightarrow 17/2^-$</td>
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<td>0.03(0.03)</td>
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<td>0.01(0.02)</td>
<td>E2</td>
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<tr>
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<td>1.03(0.05)</td>
<td>0.01(0.05)</td>
<td>E2</td>
<td>10</td>
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<td>620</td>
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<td>0.02(0.06)</td>
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<td>-0.01(0.05)</td>
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<td>M1/E2</td>
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<td>-0.03(0.01)</td>
<td>M1/E2</td>
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<td>244</td>
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<td>0.62(0.04)</td>
<td>-0.04(0.04)</td>
<td>M1/E2</td>
<td>10 \rightarrow 9</td>
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<td>$13/2^- \rightarrow 11/2^-$</td>
<td>0.55(0.06)</td>
<td>-0.01(0.06)</td>
<td>M1/E2</td>
<td>10 \rightarrow 9</td>
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<td>$25/2^+ \rightarrow 21/2^+$</td>
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<td>E2</td>
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<td>0.05(0.02)</td>
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<td>11</td>
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<td>0.04(0.04)</td>
<td>E2</td>
<td>11</td>
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<td>930</td>
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<td>M1/E2</td>
<td>11 \rightarrow Yrast</td>
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<td>984</td>
<td>$25/2^+ \rightarrow 25/2^+$</td>
<td>0.78(0.08)</td>
<td>-0.04(0.08)</td>
<td>M1/E2</td>
<td>11 \rightarrow Yrast</td>
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Continued on next page
Table 6.2 – continued from previous page

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<th>$E_{\gamma}$ (keV)</th>
<th>$J_i^\pi \rightarrow J_f^\pi$</th>
<th>$R_{DCO}$</th>
<th>$A_p$</th>
<th>Multip.</th>
<th>Band</th>
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<td>1037</td>
<td>29/2$^+$ → 29/2$^+$</td>
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<td>-0.05(0.05)</td>
<td>M1/E2</td>
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<td>1089</td>
<td>33/2$^+$ → 33/2$^+$</td>
<td>$^1$</td>
<td>$^2$</td>
<td>$^3$</td>
<td>11 → Yrast</td>
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<td>1126</td>
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<td>$^1$</td>
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Band 12

<table>
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<th>$R_{DCO}$</th>
<th>$A_p$</th>
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<th>Band</th>
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<tbody>
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<td>1.06(0.04)</td>
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<td>39/2$^-$ → 35/2$^-$</td>
<td>1.01(0.02)</td>
<td>0.01(0.02)</td>
<td>E2</td>
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<td>688</td>
<td>43/2$^-$ → 39/2$^-$</td>
<td>0.94(0.05)</td>
<td>0.02(0.05)</td>
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<td>1.08(0.08)</td>
<td>0.05(0.08)</td>
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<td>715</td>
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<td>0.44(0.04)</td>
<td>0.01(0.04)</td>
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<td>0.57(0.07)</td>
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<td>12 → Yrast</td>
</tr>
<tr>
<td>1011</td>
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<td>E1</td>
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<td>0.02(0.08)</td>
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<td>12 → Yrast</td>
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<tr>
<td>1065</td>
<td>23/2$^-$ → 21/2$^+$</td>
<td>0.52(0.05)</td>
<td>$^2$</td>
<td>$^3$</td>
<td>12 → Yrast</td>
</tr>
</tbody>
</table>

$^1$ Insufficient statistics to measure DCO ratios
$^2$ Insufficient statistics to measure linear polarization anisotropy
$^3$ Multipolarity could not be obtained
$^*$ Previously measured and confirmed transition
APPENDIX B

Contaminants from other reaction channels in the $^{158,159}$Er data were observed and some of them are presented in this section. Various reasons contribute to the population of other neighbouring nuclei to the one of interest, i.e. the break down of the $^{12}$C beam into two or three alphas, the reaction of these beam fragments with the target and the reaction between the beam and the target backing, which in our case was gold.

This is because in HIFE reactions the difference between the number and type of particles emitted in the initial cooling process produces different species of residual nuclei. These nuclei can contaminate the nuclei of interest and make it challenging to construct the level schemes, because some of the contaminants have the same energies as the ones that are investigated. Some of the contaminants observed in this work include: $^{32}$S, $^{152}$Gd, $^{155,156,158,159}$Dy, $^{156,157,160}$Er, $^{205,206}$Po, $^{205}$Bi and $^{205}$At.
Figure 6.1: Partial level scheme of $^{156}$Dy
Figure 6.2: Partial level scheme of $^{158}$Dy
Figure 6.3: Partial level scheme of $^{206}\text{Po}$
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‘The way to happiness: keep your heart free from hate, your mind from worry. Live simply, expect little, give much. Fill your your life with love. Scatter sunshine. Forget self, think of others. Do as you would be done by’

‘You are in a position, where you can only fall up’ JFSS