

**IMPLEMENTING AN INTENTIONAL TEACHING MODEL TO INVESTIGATE
GRADE 9 LEARNERS' WAYS OF WORKING WITH RATIONAL ALGEBRAIC
FRACTIONS**

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A thesis submitted in fulfilment of the degree M.Ed. (Mathematics Education) in the School
of Science and Mathematics Education (SSME), University of the Western Cape.



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DECLARATION

I declare that **Implementing an intentional teaching model to investigate grade 9 learners' ways of working with rational algebraic fractions** is my own work and that it has not been submitted before for any degree or examination in any other university. All the sources that I have used or quoted have been indicated and acknowledged by complete referencing.

Nwabisa Vivian Maphini

Date: June 2018

Signed _____



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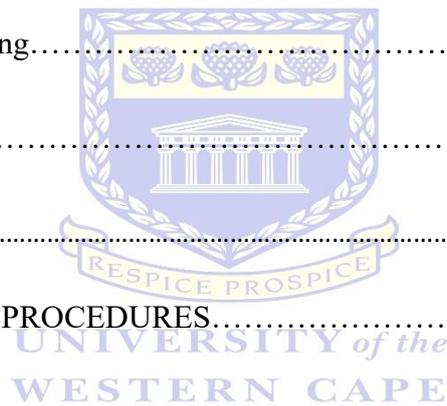
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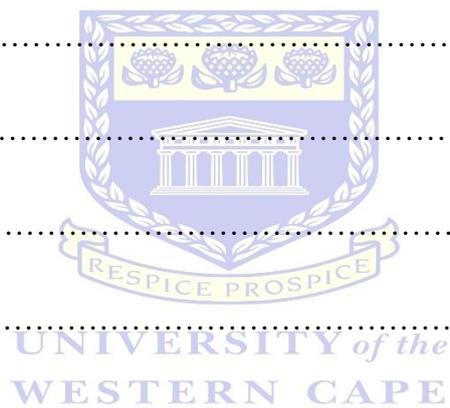


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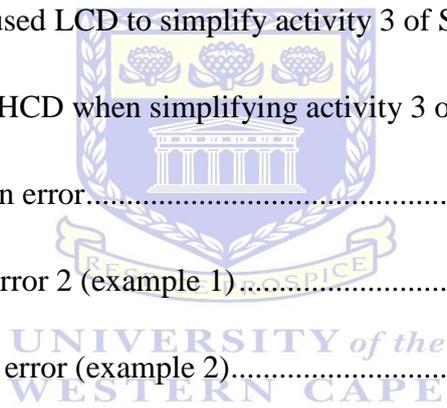
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ABSTRACT

In South Africa it is widely known that most learners struggle with mathematics. The results for mathematics are poor. The department of basic education offers a number of intervention programmes to assist learners in mathematics but the problem still persists. Algebra is the most basic and important topic in mathematics as it becomes an element in almost all the other topics in mathematics curriculum. Algebraic fractions in particular are a challenge for most learners. Research shows that learners commit a number of errors when they work with algebraic fractions.

The study investigated the implementation of an intentional teaching model into grade 9 mathematics learners' ways of working with rational algebraic fractions. An intentional teaching model is a teaching strategy which emphasizes teaching intentions or teaching objectives are brought to the fore during a lesson, the model emphasizes the use of spiral revision and assessment for learning. Ways of working in this study refers to the way in which learners deal with algebraic fractions when they simplify them including the errors they commit from the misconceptions they have about aspects of working with fractions. The study was conducted in a group of grade 9 mathematics learners at Gugulethu High school, which is located in Gugulethu, a township in the Western Cape Province of South Africa.

The study is premised on a qualitative research paradigm which focuses on studying situations in their natural settings and applying an interpretive perspective. Data was collected by means of observation and video recording of lessons while learners were engaged in working with algebraic fractions. Learners' written work was analysed as part of the data collection. The results of the study show that learners commit a number of errors when they manipulate algebraic fractions. Among other errors are: (i) Cancellation errors which had the highest frequency of occurrence (ii) Defractionalisation (iii) No recognition of

the common factor and (iv) Exponential laws error. It was found that the learners' ways of working with algebraic fractions are mostly characterised by their misunderstanding of exponential laws and difficulty in working with fractions needing the use of factorisation to simplify and find the lowest or highest common denominator during addition or subtraction. The results of the study also reveal that learners struggle to articulate extensively or in detail what they are actually doing as they simplify rational algebraic fraction.



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KEY WORDS

Rational algebraic fractions

Ways of working

Intentional teaching model

Design-based research

Errors and misconceptions



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ACRONYMS

OBE- Outcomes based Education

NCS- National Curriculum Statement

CAPS- Curriculum and Assessment Policy statement

LEDIMTALI – local Evidence Driven Improvement in Mathematics teaching & Learning Initiative

AfL- Assessment for Learning



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CHAPTER 1

INTRODUCTION, BACKGROUND, RATIONALE AND RESEARCH QUESTION

1.1 Introduction

Fractions and algebra are critically important components of mathematics education (Brown & Quinn, 2007). The understanding of rational algebraic fractions is fundamental to the successful study of advanced mathematical concepts required in science, technology, engineering and mathematics. This, therefore, implies that learners who intend to choose careers in fields of engineering, science, technology and mathematics need an understanding of algebraic fractions.

Algebraic fractions are defined as fractions whose numerators and or denominators are algebraic expressions. An algebraic fraction whose numerator is a polynomial of lower degree than the polynomial in the denominator is referred to as proper algebraic fraction and an algebraic fraction whose numerator is the polynomial of higher degree than that of the denominator is referred to as an improper fraction (McDonald & Dalla, 2004).

There are different types of algebraic fractions such as rational algebraic fractions, irrational algebraic fractions and complex algebraic fractions. A rational algebraic fraction is a fraction whose numerator and denominator are both rational polynomials. A function $f(x)$ is a rational polynomial function if it is the quotient of two polynomials $p(x)$ and $q(x)$ i.e. $f(x) = \frac{P(x)}{Q(x)}$

: $q(x) \neq 0$. For example $f(x) = \frac{x^2 - 6x + 5}{x + 1}$ is a rational algebraic fraction. Irrational algebraic

fractions contain a variable under a fractional exponent for example $\frac{z^3 - \frac{1}{3}a}{z^2 - z^3}$. Complex algebraic

fractions have a numerator and/ or denominator which are fractions. For example $\frac{\frac{6}{x+1} - \frac{4}{x+2}}{\frac{x+2}{x+2} - \frac{x+1}{x+1}}$ is a

complex fraction.

All algebraic fractions are subject to the same laws of arithmetic fractions when they are simplified. It was not the aim of this thesis to investigate different types of algebraic fractions. Therefore, algebraic fractions which have been investigated are rational algebraic fractions.

Research results indicate clearly that improving students understanding of fractions occurs through more precise instruction rather than through significant increases in time spent learning about fractions (Watanabe, 2012).

This therefore implies that teachers need some strategies that work successfully in helping learners understand fractions than spending a lot of time using the same methods that have failed. The Ontario Ministry of Education (2016) encourages teachers to be thoughtful and intentional in the selection of resources and learning opportunities to maximise learning.

This study reports on learners' ways of working with algebraic fractions in an intentional teaching environment. Ways of working in this study refers to the way in which learners dealt with algebraic fractions when simplifying algebraic fraction including the errors they committed and the misconceptions they have about aspects of working with fractions. An intentional teaching model was implemented in a class of grade 9 mathematics learners in one of the public high schools in Gugulethu, a residential area located in the City of Cape Town in the Western Cape Province of South Africa.

1.2 Research question

This research sought to answer the following question:

How do grade 9 mathematics learners engage with algebraic fractions when an intentional teaching model is implemented?

1.3 Rationale for the study

In South Africa, it is widely known that most learners struggle with mathematics and consequently perform poorly in the subject. The Department of Basic Education offers a number of intervention programmes to assist the learners in mathematics but the problem still persists. Consequently, the Department of Basic Education amended the progression criteria for the grade 7-9 by allowing learners who obtain a minimum of 20% in mathematics to be condoned so that they can be able to progress to the next grade. This was done because most learners could not get the 40% pass requirement for mathematics in grade 7-9 and they had to repeat the grade because mathematics was one of the compulsory subject to pass in order to progress (Department of Basic education; National Assessment Circular No. 3 of 2016).

Algebra is the most basic and important topic in mathematics as it becomes an element in almost all the other topics in mathematics curriculum. Algebraic fractions in particular are a challenge for most learners. According to Wu (2001, p. 1) cited in Brown & Quin (2007) there are at least two major bottlenecks in mathematics education, the teaching of fractions and the introduction of algebra” This statement emphasize that fractions are a challenging topic in mathematics that affects the understanding of mathematics in learners. Aldrich (2015) argues that the students who are able to understand and perform operations on fractions (with or without a calculator) do significantly better in algebra courses than those who lack this ability. This means that if learners can improve their understanding of algebraic fractions, their performance in mathematics can improve.

Algebraic fractions are used in many areas of mathematics. For example in sequences or series, proficiency in working with fraction is required when working with sequences such

as: $\frac{1}{k}$; $\frac{3}{k}$; $\frac{5}{k}$ + $\frac{k-1}{k}$. Whether the learner needs to identify the type of the sequences,

find the sum of the terms or find the value of k, the knowledge of working with fractions will be required. In geometric sequences such as this: $k + 1$; $k - 1$; $2k - 5$ learners can be required to find the value of k, with the wrong conception of how to work with fractions they can struggle to solve the problem. Fractions can also be used in trigonometry, for example

simplifying: $\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta}$ need the understanding of adding fractions where one needs to

make the denominators the same. In differentiation or limits finding the limit of a function

like $\lim_{x \rightarrow 1} \frac{x^2-1}{x-1}$ before substituting with $x = 1$ learners need to simplify the fraction to avoid

having zero in the denominator. There are many other topics in mathematics where the

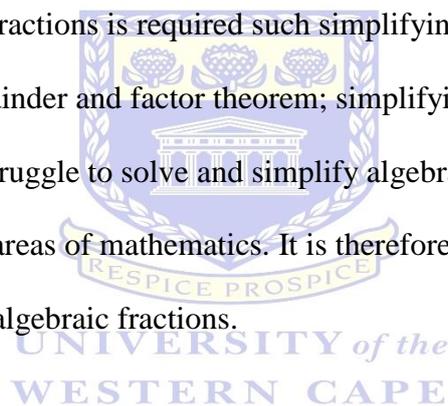
knowledge of working with fractions is required such simplifying surds with fractions;

dividing polynomials in remainder and factor theorem; simplifying exponents; simplifying

logarithms etc. If learners struggle to solve and simplify algebraic fractions, then they will

struggle to progress in other areas of mathematics. It is therefore important for learners to

acquire skills of simplifying algebraic fractions.



In my teaching experience, I have observed many errors that learners commit when they solve algebraic fractions. Research also exposes a number of errors that learners commit when simplifying algebraic fractions and hence have also motivated this study. Makonye & Khanyile (2015) found eight categories of errors that learners commit when simplifying algebraic fractions. The categories of errors they identified were: (i) Cancellation error (ii) factors error (iii) trinomial factorisation error (iv) lowest common denominator error (unable to identify the lowest common denominator) (v) careless error (vi) obtaining correct answer using incorrect mathematical procedures.

Working with algebraic fractions involves many operations such as factorizing, adding, subtracting, multiplying, dividing fractions and simplifying. For example when simplifying the following fraction: $\frac{x^2+5xy-4y^2}{x^2-16y^2} - \frac{2xy}{2x^2+8xy}$, factorisation of the denominators, identifying the common denominator so that one can subtract the two fractions, adding and subtracting like terms and cancelling common factors are some of the procedures which are required to simplify this fraction.

1.4 Definition of terms

Rational algebraic fractions: A rational algebraic fraction is a fraction whose numerator and denominator are both rational polynomials.

Ways of working: Ways of working in this study refers to the way in which learners dealt with algebraic fractions when simplifying algebraic fraction including the errors they committed and the misconceptions they have about aspects of working with fractions⁵

Intentional teaching model: Teaching models represent teaching strategy or methodology. An intentional teaching model is a teaching strategy which emphasizes teaching intentions or teaching objectives are brought to the fore during a lesson, the model emphasizes the use of spiral revision; assessment for learning and prioritization of legitimate mathematical knowledge.

Design-based research : According to Brown & Collins (1992), cited in The design-based research collective (2002), design based research is an emerging paradigm for the study of learning in context through the systematic design and study of instructional strategies and tools.

Errors and misconceptions : A mathematical error refers to a mistake or condition of being wrong in the process of solving a mathematical problem (Hurrel, 2013 ; Godden, Mbekwa &

Julie, 2013). A misconception is the implicit belief held by a learner which governs the errors that a pupil makes(Bell, 1984 cited in Hurrell , 2013).

1.5 Outline of the study

This study is divided into five chapters.

Chapter 1: This introduces the study on algebraic fractions and also highlights the research questions for the research and the rationale for the study. The chapter also gives the definitions of the terms used in the study.

Chapter 2: This chapter discuss the literature review for the study. Definition for algebraic fractions is outlined. The chapter gives a detail discussion on the errors and misconceptions found in the literature on algebraic fractions. The intentional teaching, model implemented in this study is explained in detail.

Chapter 3: This chapter deals with the research design and methodology used in the study. A detailed discussion of design based research is covered in this chapter. The chapter also gives indication of the research sample, data collection methods, issues of reliability and validity and ethical consideration are also outlined.

Chapter 4: The chapter deals with matters relating to how the data was analysed.

Chapter 5: This chapter gives reflections on results, and conclusions in reference to the research question. The chapter also concludes the thesis and give recommendations for future researchers on the topic.

1.6 Conclusion

The chapter gave a brief introduction of the topic for the research, research question and the rationale for the study. The terms used in this study have been defined. The following chapter will give the literature review for the study.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter outlines a detailed explanation of what algebraic fractions are, what simplifying algebraic fractions entails, what are the research findings on the errors which learners commit and possible reasons behind learners' errors. Finally, a detailed account of the intentional teaching model used in this study, intentional teaching in general and how it applies to this study is given.

Educational research shows that the learning of algebraic fractions has become one of the most difficult and challenging content areas of mathematics for learners. Mhakure, Jacobs, & Julie (2014) argue that the learning of algebraic fractions is the most problematic area in school mathematics.

The teaching methods used by teachers may have a positive or negative impact on how well learners will do in fractions. Naiser, Wright & Caprano, (2013) assert that in order to improve instruction on fractions teachers need to use different methods of presenting their lessons to learners. If teachers can reflect on their teaching methods to see what is working and what does not they can be in a better position to improve their teaching strategies and

hence help their learners understand mathematics, in this case, with particular reference to algebraic fractions.

2.2 Algebraic fractions

Algebraic fractions were defined in chapter 1. It was highlighted that there are different types of algebraic fractions namely, rational algebraic fractions; irrational algebraic fractions and complex algebraic fractions.

This research will only investigate rational algebraic fractions.

2.3 Simplifying algebraic fractions

Simplifying in mathematics is the process of reducing an expression to a briefer, lowest terms or one easier form to work with.

As it has already been explained in chapter 1 that a rational algebraic fraction is a fraction whose numerator and denominator are both rational polynomials. It therefore indicates that a rational algebraic fraction can have a number of terms or factors which will need to be simplified into lowest terms.

To simplify rational fractions means writing a fraction such that there are no common factors other than 1 or -1. Simplifying algebraic fractions can be done by multiplying or dividing the numerator and denominator by the same non-zero number or expression, which is to cancel any common numerical or variable factors. To do this it is necessary to factorize the numerator and denominator to see if there are any common factors.”(Constanta, 2012,p.3-4)

According to grade 9 mathematics syllabus in South Africa learners must be taught to Simplify algebraic expressions involving the following operations: addition of like and unlike terms; division of monomials, binomials and trinomials by monomials or integers. They should simplify algebraic fractions using factorization. (Department of Basic Education Curriculum and Assessment Policy Statement, 2013,)

This, therefore means that simplifying algebraic fractions in grade 9 will be limited to adding and subtracting fractions; simplifying algebraic fractions by factorising and adding or subtracting like terms.



2.4 Errors and misconceptions

As it has already been stated in chapter 1, a mathematical error refers to a mistake or condition of being wrong in the process of solving a mathematical problem (Hurrell 2013; Godden, Mbekwa & Julie, 2013). A misconception is the implicit belief held by a learner which governs the errors that a pupil makes (Bell, 1984 cited in Hurrell, 2013). Errors often reveal the misconceptions that learners have about the mathematical ideas and procedures. Student errors reflect their understanding of concepts, problems or procedures (Roselizawati, Sawradi & Shahrill 2004).

Misconceptions are referred to by many researchers as systematic errors and mistakes are referred to as unsystematic errors (Steinle, 2004 cited in Hurrell 2013); Riccomini, 2005 cited in Luneta & Makonye 2010). Mistakes can be easily corrected by learners themselves but misconceptions are being held with the thinking that what is done is correct (Erlina, 2011). A systematic error is usually a consequence of misconception which can emanate from the learners' failure to make connections between new knowledge and previous knowledge. Misconceptions are more resistant to change and hence difficult to eradicate because they are usually embedded in the learners' conceptual map or schema (Godden, Mbekwa & Julie, 2013). Hurrell (2013) argues that misconceptions are problematic because they interfere with subsequent understandings if the student attempts to use them as the basis for further learning. Misconceptions have been actively constructed by the student and therefore have emotional and intellectual attachment for that student, and consequently are only relinquished by the student with great reluctance.

Misconceptions have various terminologies that are used across the literature for example (Confrey (1990) cited in Steinle, 1999) identified varied terms from his review of literature on misconceptions in the field of science, mathematics and programming. He

referred to errors as alternative conceptions; pre-conceptions; conceptual primitives; critical barriers to learning; naive theories etc. Confrey (1990) characterizes misconceptions as surprising, pervasive and resilient.

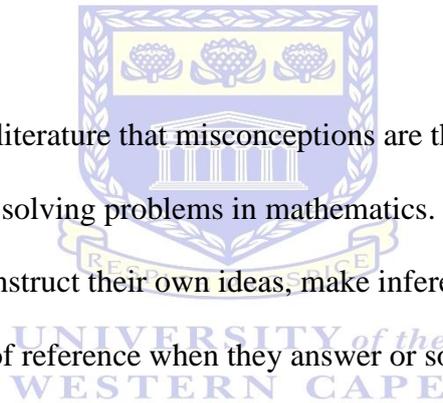
Graeber & Johnson (1991) cited in Steinle (1999) characterize misconceptions as being.

(i) **Self evident:** This means that learners do not feel the need to prove them because they believe they are correct.

(ii) **Coercive:** This means that one is compelled to use them in an initial response.

(iii) **Widespread:** This is because they are common among both naïve and academically able students.

It is evident from the literature that misconceptions are the main reason behind errors which learners commit when solving problems in mathematics. When learners receive new information, they actively construct their own ideas, make inferences and interpretations which becomes their source of reference when they answer or solve problems.



There are various ways in which errors are categorized. For example, Godden; Mbekwa & Julie (2013) categorized errors into four namely: careless errors, procedural errors, application errors and calculation errors. Careless errors are just learners' mistakes not caused by any cognitive or conceptual challenge. Procedural errors are wrong application of mathematical procedures; calculation errors occur in the application of four basic operations, namely: addition, subtraction, multiplication and division and lastly application errors occurs when a learner knows the procedure to apply but applies it wrongly for example factorizing correctly but have wrong signs for factors.

There are many categories or types of errors that we find in the literature on rational algebraic fractions. We find these errors in studies such as Otten, Males & Figueras (2008); Mhakure, Jacobs & Julie (2014); Makonye & Khanyile, (2015) ; Ruhl, Balatti, & Belward, (2011) and others. This research focuses more on those types of errors as they relate to students' work with fractions.

2.5 Learner errors on algebraic fractions

Otten, Males & Figueras (2008) identified seven categories of errors that learners commit when they simplify algebraic fractions. Their study examined how algebra students simplify rational expressions. A survey was administered to grade 7, 10, 11 & 12 learners who had been taught rational algebraic expressions before the survey. The errors that were identified from learners work were categorised as:

(i) **Cancellation error:** This refers to cancelling of either the constant term, variable or coefficient that is common both in the numerator and denominator. For example: $\frac{4x+20}{12+4x} = \frac{20}{12}$

(ii) **Partial division:** This refers to using a denominator of a rational expression to divide only some of the terms in the numerator not dividing the whole numerator for example

$\frac{2x+4}{2} = 2x + 2$ in this case 4 has been divided by 2 to get 2 but 2x was not divided by 2.

(iii) **Like-term error 1:** This error refers to learners using another operation instead of division. Usually learners would subtract a denominator from one of the terms in the

numerator. For example : $\frac{5x+15}{2x+6} = 3x + 9$

(iv) Like term error (2): Learners commit this error by inappropriately performing a

inappropriate operation in the term of the denominator or numerator .For example: $\frac{7x+x}{7x+3x} =$

$$\frac{8x^2}{10x^2}$$

(v) Defractionalisation: Transformation of a fraction with a numerator of 1 to a non-fraction for example $\frac{1}{x} = x$

(vi) Equationisation: This refers to transforming a rational expression into a linear expression. An example was $\frac{10+4x}{2x} = 10 + 2x$.

(vii) Linearization: Breaking up a rational expression with a compound into two separate rational expressions $\frac{4}{8+4x} = \frac{4}{8} + \frac{4}{4x}$



The most common error that learners committed was cancellation error. This therefore attests to what Grossman (1924) cited in Rulh; Balatti & Bellward(2011) states:

Every teacher of experience knows that a great number of his algebra pupils all the way from the first year in high school up to college continue with almost comical regularity to make strange mistakes in the subject of —cancellation in fractions— mistakes that show clearly that the essence of the matter has escaped them. (p.104)

Learners have a misconception of cancellation in algebraic fractions.

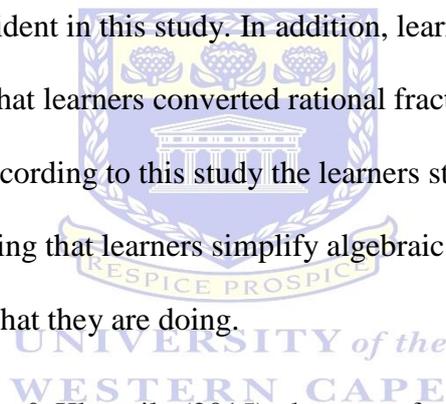
Most learners fail to factorise when they manipulate and they just cancel whatever they find common in the numerator and denominator either terms or co-efficient. Some do not fail to factorise but cancel like terms wrongly for example:

$\frac{b^3+6b}{3b} = \frac{b(b^2+6)}{3b} = \frac{b^2+6}{2b}$; in this case (b) has been subtracted from (3b) to get (2b) and

another case of wrong cancelation is the cancelation of the co-efficient in the terms for

example: $\frac{b^3+6b}{3b} = \frac{b^3+2b}{b}$ / (Rulh; Balatti & Bellward, 2011)

Mhakure, Jacobs & Julie (2014) conducted a study on rational algebraic fractions. The study investigated the proficiencies of grade 10 mathematics learners in simplifying rational algebraic fractions in a high-stakes end of year examination. The ultimate aim of the study was to investigate how proficient are grade 10 mathematics learners in the simplifying rational algebraic expressions. The results of the study showed that learners struggle to solve algebraic fractions and they commit many errors. The errors found by Otten, Males & Figueras (2008) were also evident in this study. In addition, learners also committed algebraic equation error which means that learners converted rational fractions to algebraic equations and then tried to simplify. According to this study the learners struggle with rational algebraic fractions is conceptual, meaning that learners simplify algebraic fractions without really understanding or justifying what they are doing.



In a study by Makonye & Khanyile (2015), the type of errors that learners commit when simplifying algebraic fractions were determined. The following discussion is based on those errors.

ERROR CATEGORY

EXAMPLES

1. Cancellation error:

$$\frac{\cancel{a^2b} - \cancel{ab}}{a^3 + a^2} \times \frac{a^3 - a}{\cancel{a^2b} - 2ab + b}$$

Learners who committed this error could not factorise before they simplify but only cancel the like terms. Learners seem to have a misconception on cancellation of common factors when two rational fractions are being multiplied. a^2b in the numerator has been cancelled with a^2b in the denominator of the other fraction without considering that they are

terms in the expressions not factors. The same procedure was applied to a^3 and ab . Like terms act as a strong visual cues to learners which leads to cancelling incorrectly.

2. Confusing factors :

$$\frac{4x^2+16}{x^2+4} = \frac{4(x^2+4)}{(x+2)(x+2)}$$

$$= \frac{4\cancel{(x+2)}(x+2)}{\cancel{(x+2)}(x+2)}$$

Learners committing this error could not recognise common factors after factorising the numerator. The sum of two squares is confused with the difference of two squares.

3. No recognition of common factor

$$\frac{a^2b-ab}{a^3+a^2} \times \frac{a^3-a}{a^2b-2ab+b}$$

Learner committing this error failed to recognise the common factors in the expressions, could see common terms and treated them as common factors.

4. Unable to factorise trinomial

$$\frac{3}{x^2+6x+9} - \frac{2}{x^2-9} - \frac{1}{x^2-6x+9}$$

$$\frac{3}{(x+3)(x+3)} - \frac{2}{(x+3)(x-3)} - \frac{1}{(x-3)(x+3)}$$

Learners committing this error had a challenge with signs when factorising a trinomial. x^2-6x+9 could not be correctly factorised. Learners tend to focus getting two numbers that can give the last term of the trinomial when multiplied ($3 \times 3 = 9$) and give the middle term when added ($3+3=6$) but they fail to check if the signs will allow them to get what they seeking to find

5. Lowest Common denominator error

$$\frac{3}{(x+3)(x+3)} - \frac{2}{(x+3)(x-3)} - \frac{1}{(x-3)(x-3)}$$

LCD $(x+3)(x-3)$

$$= \frac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}(x+3)} \frac{3}{\cancel{(x+3)}(x+3)} - \frac{(x+3)\cancel{(x-3)}}{(x+3)\cancel{(x-3)}} \frac{2}{\cancel{(x+3)}(\cancel{x-3})} - \frac{(x+3)\cancel{(x-3)}}{(x+3)\cancel{(x-3)}} \frac{1}{\cancel{(x-3)}(x-3)}$$

Learners committing this error were able to correctly factorise the trinomials but lack conceptual understanding of what is the Lowest common denominator. $(x-3)(x+3)$ was used as the LCD because learners realised that each denominator had either $x-3$ or $x+3$ and for them that made their LCD.

6. Careless error or random error $\frac{x}{x} + \frac{1}{x} = \frac{1}{x}$

Learners committing this error could not write the solution for the quotient where the dividend is equal to the divisor. $(x \div x)$ but just ignored it or considered it to be zero.

7. correct answer obtained by using $\frac{4x^2+16}{x^2+4}$

incorrect mathematical rule
$$= \frac{4(x^2+4)}{(x+2)(x+2)}$$

$$= \frac{4(x+2)(x+2)}{(x+2)(x+2)} = 4$$

Learners committing this error were found to have obtained the correct answer but used the wrong mathematical rule. x^2+4 was wrongly factorised but the final answer was the correct answer.

8. Dropping the denominator $\frac{2}{(x+3)} - \frac{1}{(x+3)^2}$

LCD= $x+3$

$$= \frac{2}{(x+3)} - \frac{1}{(x+3)(x+3)}$$

$$= (x+3)(2) - (x+3)(1)$$

$$= (2x+6) - 3x+3$$

$$= -x+3$$

This error was identified in learners who, after finding the lowest common denominator, used it to multiply each value in the numerator, dropped the denominator and the simplified. (Makonye & Khanyile, 2015, p. 62)

Makonye & Khanyile (2015) assert that the errors that learners tend to commit are systematic and persistent. Learners' instrumental understanding led to misconceptions and consequently resulted to a number of errors learners committed. Instrumental understanding means the ability to execute mathematical rules and procedures or possession of a rule and ability to use it. It is rules without reasons. Instrumental understanding involves multiplicity of rules rather than fewer principles of more general application (Skemp, 1978). This is evident in the Lowest common denominator error identified by (Makonye & Khanyile ,2015)

$$\frac{3}{(x+3)(x+3)} - \frac{2}{(x+3)(x-3)} - \frac{1}{(x-3)(x-3)}$$



 LCD (x+3)(x-3)

$$= \frac{3}{\cancel{(x+3)}(x-3)} - \frac{2}{\cancel{(x+3)}(x-3)} - \frac{1}{(x+3)\cancel{(x-3)}}$$

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This learner knows the rule to be followed when subtracting rational algebraic fractions which is finding the lowest common denominator and is able to apply it by multiplying in each fraction then cancel. But she/he does not understand the reason why do you need to multiply with the common denominator in each fraction as a result the LCD used to multiply was wrong.

Another error that learners commit which was identified by Judah, Makonye, & Nzima, (2016) is conjoining.

Conjoining is defined as the senseless combination of unlike terms by multiplication when adding or subtracting algebraic expressions. For example a learner simplifying $\frac{b^3+6b}{3b} = \frac{6b^4}{3b}$, in this case the learner multiplied b^3 and b to get b^4 .

2.6 Possible reasons for learner's errors

Research shows a number of possible reasons behind errors that learners commit when simplifying rational algebraic fractions. I will mention a few.

Reliance on visual cues which is in line with the use of instrumental understanding leads to many errors. Instrumental understanding is defined as the ability to execute mathematical rules and procedures or possession of a rule and ability to use it (Skemp 1976). Learners follow procedures without fully understanding them (Otten, Males & Figueras, 2008; Judah, Makonye & Nzima, 2016). Learners depend on their instrumental understanding which require their procedural knowledge and prior-knowledge of on simplifying common fractions. They draw from this incomplete or flawed knowledge and end up making many errors. For example the learners' committing a cancellation error usually cancels the common terms on rational algebraic expressions as if they cancel common factor

3. Apply your knowledge of factorising to simplify the following fractions if it is possible to c

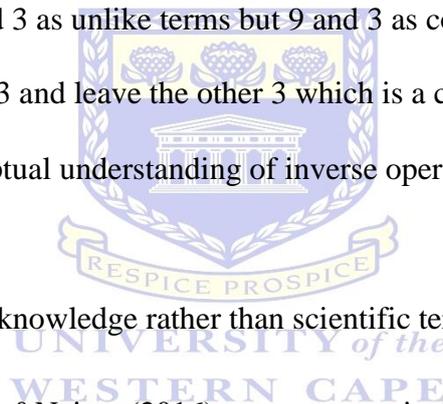
$$(i) \frac{x^2 + 3x + 2}{x + 2} \quad \frac{x^2 + 3x + \cancel{2}}{x + \cancel{2}} = \frac{x^2 + 3x}{x} = x + 3$$

Figure 2.1 Example of cancelation error.

In the above example seeing 2 in the numerator and in the denominator became a strong visual cue which made the learner to cancel the terms of the expression. The learner used the incomplete understanding of simplifying common fractions

According to Otten, Males & Figueras (2008) other possible reasons are:

- Lack of a solid comprehension of rational numbers which makes it more difficult for them to work with something that looks like a fraction with unknowns (x)
- Fixation and overemphasis of like terms: learners give over attention to like terms and perform operations only among like terms ignoring other terms in the rational expression. This is evident in the cancellation error $\frac{4x+20}{12+4x} = \frac{20}{12}$ and partial division, $\frac{3x+9}{3} = 3x+3$. In the case of cancellation error, they could see 4x in the numerator and denominator as like terms cancelled as if it were factors. In the case of partial division 3x was not divided by 3 but only 9 was divided. This could have resulted from the learners seeing 3x and 3 as unlike terms but 9 and 3 as constants which became the reason to divide 9 by 3 and leave the other 3 which is a co-efficient of x.
- An inadequate conceptual understanding of inverse operations leads to wrong cancellation.
- Reliance on informal knowledge rather than scientific terminology.



According to Judah Makonye & Nzima (2016) some reasons include

- Belief that an answer cannot be an algebraic expression but must be a numerical answer.

This is evident in a study by (Hall, 2002) where a learner simplified the rational algebraic fraction $\frac{x^2+3x-10}{x^2+2x-8} = \frac{(x+5)(x-2)}{(x+4)(x-2)} = \frac{(x+5)}{(x+4)} = \frac{5}{4}$. The learner simplified the fraction correctly but could not leave the answer with unknowns, therefore the unknowns were discarded and final solution was left with only numerical.

Learners perceive algebraic expressions only as processes and fail to accept them as objects. Otten, Males & Figueras (2008) argue that it is difficult for learners to transition

from arithmetic to algebra. Learners struggle to perceive polynomials as objects (structurally) rather than process (operationally). For example, a learner looking at $3x+9$ can perceive as a number of processes to follow, which are multiplying 3 by x and the adding 9. In the cancelation error that learners usually commit, it is assumed that these learners may be viewing algebraic fractions as processes rather than objects, cancellation of similar things in the fraction which can either be factors or terms is perceived as one of the operations to be done in the process.

Mhakure, Jacobs & Julie (2014) argue that learners with insufficient knowledge of fractional sub-constructs (part-whole, ratio, operator, measure and quotient) in the senior primary phase usually struggle with simplifying rational algebraic fractions in grade 10.

- The presence of visual cues in problems involving rational algebraic fractions act as a distractive stimulus for learners.

Rational algebraic fractions contain all the difficulties of ordinary fractions, but in addition they include those difficulties that are usually associated with algebra generally. These include addition and subtraction involving like and unlike terms, multiplication and division. These operations take place inside each algebraic fraction and thus mirror some of the sub-constructs of ordinary fractions. Further, other difficulties included in the addition of algebraic fractions, are the use of a lowest common denominator and the operations involved in adding these terms, in much the same way that ordinary fractions are added.

Sometimes learners simplifying $\frac{x^2-9}{x-3}$ divide the x^2 by x and divide 9 by 3 to get $(x-3)$.

These learner fails to understand the part-whole relationship. x^2-9 is not understood as a whole that must be divided into parts by $(x-3)$. If the learner would understand x^2-9 as a whole for example ($25-9 =16$) divided into equal parts by $(5-3=2)$ then there would a small chance of wrong cancelation.

The ratio construct talks to the natural way of showcasing the procedures associated with finding the equivalent fractions. In the case of algebraic fractions there are unknown values in a fraction which represent a certain number. Learners who understand the ratio construct when simplifying $\frac{(x+5)}{(x+4)}$ would not just cancel the x's and come up with $\frac{5}{4}$ as the learner did in the study by Hall (2002) because for any random value of x substituted in $\frac{(x+5)}{(x+4)}$ the ratio will not be equal to 5:4.

2.7 Models of teaching

This study is implementing a teaching model, it therefore becomes imperative to give a brief view and definition of what a teaching model is in general.

Teaching is not an easy job. A teacher is faced with a group of learners who come from different backgrounds with different learning abilities. He /she has to convey the subject matter or content to all of them in way that will help them conceptually understand it and be able to apply it. It is therefore important for the teacher to have the knowledge of the subject content, strategies to pass that knowledge and design some materials that will be relevant and helpful in the process of transmitting knowledge. Models of teaching help teachers to accomplish their teaching goals.

Model of teaching can be defined in various ways by many researchers but what is common in all definitions is that teaching models represent teaching strategy or methodology, for example according to Pateliya (2013) "Model of teaching can be defined as instructional design which describes the process of specifying and producing particular environmental situations which cause the students to interact in such a way that a specific change occurs in their behaviour." p.125. Bruce Joyce and Marsha Weil's (1980) cited in Ellis (1979) define model of teaching as a plan or pattern that can be used to shape curricula (long-term courses

of studies), to design instructional materials and to guide instruction in the classroom and other settings.

Ellis (1979) defines models of teaching as strategies that are based on theories (and often the research) of educators, psychologists, philosophers and others who question how individuals often learn.

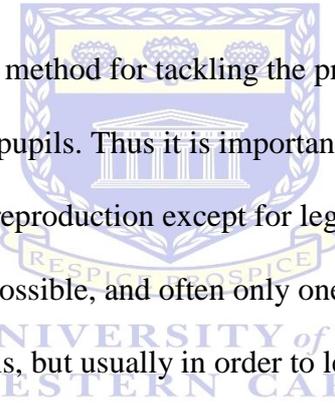
Models of teaching deal with the way, in which instructional experiences can be constructed, sequenced or delivered (Wilson, 2016). According to Pateliya (2013) there are types of modern teaching models namely: Information processing models, Social interaction models, Personal models and Behaviour modification models.

The research shows that each teaching model is designed to fulfil a specific purpose, for example (Joyce, 1985) indicates that some models are designed for cognitive and conceptual development of learners. According to (Ellis,1979) some models are designed to help learners grow in self-awareness or creativity. Whilst others foster the development of self-discipline, others stimulate inductive reasoning or theory building and others provide for the mastery of the subject matter (Wilson 2016). Pateliya (2013) argues that models of teaching are helpful in a number of ways including guiding the teacher to select appropriate techniques, strategies and methods for effective utilization of the teaching situation and material for realizing the objectives. Models bring about positive changes in learner behaviours; teacher –learner interaction and favourable environmental situation for carrying out teaching process.

In mathematics instruction, especially in high school, the traditional models of teaching mathematics are still dominant (Wachira, Pourdavood & Skitzki, 2013) . Teachers usually use the methods of teaching that they observed from their mathematics teachers which are more teacher centred. But there is a need for the models of teaching mathematics

that will focus also on the learner, which create an environment for learners to engage in mathematical enquiry, reason and the construction of knowledge.

Research shows that in mathematics teaching and learning there is a model that is commonly used in the mathematics classrooms. Wigley (1992) refers to this model as the path-smoothing model. The features and methodology of this model are meant to make things easy for learners. In outlining its features and methodology Wigley (1992) explains that the teacher or text states the kind of problem on which the class will be working. The teacher or text attempts to classify the subject matter into a limited number of categories and to present them one at a time. There is an implicit assumption that, from the exposition, pupils will recognize and identify with the nature of the problem being posed.

The logo of the University of the Western Cape is centered in the background of this paragraph. It features a shield with a building facade, flanked by two trees, and a banner below with the motto 'RESPICERE PROSPICERE'. Below the shield, the text 'UNIVERSITY of the WESTERN CAPE' is written in a stylized font.

Learners are led through a method for tackling the problems. The key principle is to establish secure pathways for the pupils. Thus it is important to present ways of solving problems in a series of steps. No reproduction except for legitimate academic purposes, for permissions which is as short as possible, and often only one approach is considered seriously. Teachers question pupils, but usually in order to lead them in a particular direction and to check that they are following.

Learners work on exercises to practice the methods given aimed at involving learners more actively. These are usually classified by the teacher or text writer and are graded for difficulty. Pupils repeat the taught processes until they can do so with the minimum of error.

Hence revision is done by returning to the same or similar subject matter throughout the course. Although this model emphasizes repetitive rather than insightful activities, almost all teachers who use it as their basic approach will also consciously offer some insightful experiences (p.5).

This methodology is also evident in a study by Julie (2013). Teachers using this model focus on helping learners succeed in the public examination which does not prove to work. The disadvantage of the model is that learners do not understand concepts and they struggle when they are faced with unfamiliar problems. It is, therefore, vital that we explore different models of teaching mathematics so that we can establish successful ways of teaching to help learners to succeed and also understand mathematical concepts, their relationships and how they apply them in the process of problem solving.

2.8 Intentional teaching

Intentional teaching is a teaching strategy that is used most commonly in the early childhood education. It is one of the pedagogical practices in early childhood learning framework. Though this concept is used in the early childhood education, the model of intentional teaching developed and which is discussed in this thesis was developed for high school mathematics teaching. Intentional teaching therefore will be discussed with reference to mathematics teaching in high school.



Intentional teaching is a term used to describe teaching that is purposeful, thoughtful and deliberate. Intentional teaching is not an accidental act but it is an act of thoughtfully applying knowledge to act on a situation and teach a lesson in the moment (Tompkins, 2013). It means systematically introducing content, in all domains, using developmentally based methods and respecting children's modes of learning (Epstein, 2007).

Intentional teaching is the opposite of teaching by rote or continuing with traditions simply because things have always been done that way (Australian Department of Education Employment and Work Relations, 2009, p. 15).

Epstein (2007) explains that being intentional is purposeful and is done with a goal in mind and a plan of achieving that goal. This act emanates from careful thought and is

accompanied by consideration of its potential effects. Pianta (2003) cited in Epstein (2007, p.5) defines intentionality as “directed, designed interactions between children and teachers in which teachers purposefully challenge, scaffold, and extend children’s skills”. An intentional teacher therefore is someone whose actions

...originate from careful thought and are accompanied by careful consideration of their potential effects. Thus an “intentional” teacher aims at clearly defined learning objectives for children, employs instructional strategies likely to help children achieve the objectives, and continually assesses progress and adjusts the strategies based on that assessment (Epstein, 2007, p.4)

Slavin (2000, p.7) asserts that although “there is no formula for good teaching, the one attribute that seems to be characteristic of outstanding teaching is *intentionality*, doing things on purpose”. Educators have an important role to play in facilitating children’s learning and development. Barnes (2012), cited in Slavin (2000, p.8) states that “Teachers who get better each year are the ones who are open to new ideas and who look at their own teaching critically”. Intentional teachers maintain a “working knowledge of relevant research, are purposeful and think about *why* they do what they do ... and combine knowledge of research with professional common sense” (Slavin 2000, p. 17).

Intentional teachers establish the habit of informed reflection in their teaching. Intentional teachers act purposefully, have defined learning objectives, employ instructional strategies that are likely to help challenge every learner, continuously assess the progress and adjust the strategies based on the results of their assessment. Epstein (2007) argues that intentional teaching requires teachers to have a vast knowledge of how learners learn. They need to apply different strategies to help learners learn. Teachers need to know which strategy to apply, when to apply it and how to apply it so that they can accommodate

individual differences of learners. When using intentional teaching there is no one recommended approach to teaching, we cannot say a teacher approach or learner centred approach is better.

According to Epstein, (2007) sometimes child guided experiences seem to help learners learn certain content and sometimes adult guided experiences help learners learn certain content. Child guided experiences is, where learners acquire knowledge and skills mainly through their own exploration and experience, including through interactions with peers and adult guided experiences is, in planned situations in which their teachers introduce information, model skills, and the like.

Teachers identify the strategies that are effective for each content, if for a particular content, child guided experiences work better then teacher plays a vital role in creating an environment suitable to optimize learning in that mode. Teachers know that each of these experiences is important and there is no one that is better than the other. They use each of them according to the needs of their content and they can use both of them in some cases.

Slavin (2000,p.11) suggests five questions that intentional teachers should consider as they plan, teach, reflect on, and revise their practices:

What am I trying to accomplish?

What are my students' relevant experiences and needs?

What approaches and materials are available to help me challenge every student?

How will I know whether and when to change my strategy or modify my instruction?

What information will I accept as evidence that my students and I are experiencing success?

Slavin (2000) argues that as an intentional teacher, besides understanding one's subject one needs to understand the developmental levels and needs of one's learners. Teachers need to understand how learning, memory, problem-solving skills and creativity can be acquired and the strategies that can be used to promote their acquisition. Intentional teachers need to set objectives, design activities that will help learners attain the set objectives, and then assess the learners' progress towards attaining objectives.

According to Berliner (1992) cited in (Epstein, 2007) the characteristics of a good intentional teacher are:

High expectations: Teacher has high expectations from their learners. They assume that learners are capable of attaining meaningful educational goals. They transfer these expectations to learners and help them to believe in themselves.

Planning and management: Teachers carefully plan for their teaching. They have clear objectives to achieve and can manage both individual and the group in a climate that captures the child's interest to learning.

Learning –orientated classroom: Both the teacher and learners value their classroom as the place where learning takes place.

Engaging activities: Teachers understand how their learners learn so they plan activities that connect with the child's experiences and they are careful of giving activities that will demotivate learners and make them believe it's difficult to do.

Thoughtful questioning: Teachers pose questions to learners to get to the learners thinking and they try to stimulate the learners thinking process.

Feedback: Good intentional teachers understand the importance of supportive and evaluative feedback. Their evaluation and feedback focuses on learning rather than judgment. (P. 6)

Intentional teaching is a good approach to teaching that has a child's interest at its core. All teachers want to help their learners understand their subjects the best way they can and be able to apply what they have learned in different situations. Implementing the concept of being intentional in teaching can help teachers achieve their teaching goals.

2.8.1 Intentional teaching environment

As indicated above the purpose of this study is to investigate the learners' ways of working with algebraic fractions in an intentional teaching environment. An intentional learning environment is one where the educator's role is to mentor, coach and guide. The learner's role is to question, connect, reflect and apply knowledge, to create, act and achieve (American Colleges and Universities (AAC&U), 2002). According to Radich (2010) in an intentional teaching environment relationships of mutual respect and trust are established, children and adults collaborate to solve problems, children's ideas and opinions count and adults contribute knowledge from their own experience to enrich, challenge and extend children's learning.



2.8.2 Essence of intentional teaching

We are coming from a time where the main focus of the school was the transmission of content and mastery of facts. Teachers were to transmit the content to the learners, who were considered as empty vessels to which one has to fill in the knowledge prescribed to what was called the core syllabus. The intentional teaching model assumes that knowledge should not be regarded as something outside of the learner which needs to be transmitted into the learner's mind. Although OBE was seen as an approach that would bring about the holistic development of the learner, it did not prove to be working.

OBE was revised and a new approach developed which was National Curriculum Statement (NCS). Complaints about the implementation of NCS, teachers overloaded with

admin work, the different interpretations of the curriculum requirements and the learner's performance under NCS approach became the reasons for the amendment of the NCS to Curriculum and Policy Statement (CAPS). CAPS is not a new curriculum but an amendment of NCS. In all these changes in the curriculum and methods of teaching, the learner performance in mathematics continues to be the challenge in our country.

From 2009 to 2013 the number of grade 12 learners passing mathematics with a mark above 50% has not improved substantially. Trends in the NCS mathematics since 2008 to 2013 show that the percentage of learners taking mathematics has gradually decreased from 56.1% in 2008 to 42.96% in 2013. The pass percentage slightly increased from 45.7% in 2008 to 59.1% in 2013. The proportion of all matriculates passing mathematics decreased from 25.6% to 25.38% in 2013 (Education for all, 2014). After the implementation of CAPS there was no positive impact in terms of the matric results in mathematics. The results decreased from 59 % in 2013 to 53,5% in 2014 and 49.1% in 2015.

There are a number of factors contributing to the learner performance in mathematics. One cannot only change the curriculum (what we teach) in trying to bring change but it is also important to focus on the method (how we teach the curriculum). It is therefore imperative that we explore different methods of teaching the curriculum so that at the end of the day we can identify the methods that work better in improving the learners understanding of mathematics and their performance.

The intentional teaching model brings about the departure from traditional ways of teaching to a curriculum which has children's interests at its centre, while at the same time introducing outcomes to frame teaching practice (Leggett & Ford, 2007). Intentional teaching model involves a constructivist approach to teaching where learning is constructed, active, reflective, inquiry based, collaborative & evolving.

Constructivist approaches to teaching typically make extensive use of cooperative learning which put the emphasis on the social nature of learning and the use of groups of peers...[and] discovery learning, in which students are encouraged to learn largely on their own through active involvement with concepts and principles... (Slavin, 2000, p. 259).

In direct instruction, on the other hand, "the teacher transmits information directly to the students; lessons are goal-oriented and structured by the teacher" (Slavin ,2000, p. 220). It is therefore essential to investigate the model of intentional teaching in order to find out whether it will bring about the change.

Chapter one has already highlighted that learners struggle with simplifying rational algebraic fractions. Otten, Males &Figueras (2008) argue that even students who are adept at simplifying a variety of polynomial expressions often face a challenge when confronted with rational expressions. Watanabe (2012) assert that improving students understanding of fractions occurs through more precise instruction rather than through significant increases in time spent learning about fractions. In this study intentional teaching is implemented in this study as a teaching method that is assumed to be a method that can improve the learning of rational expressions.

2.9 The intentional teaching model

This study implemented an intentional teaching model to investigate grade 9 learners' ways of working with rational algebraic fractions. An intentional teaching model is a teaching strategy which emphasizes that teaching intentions or teaching objectives are brought to the

fore during a lesson. The study proposes that when an intentional teaching model is employed, learners will be less prone to commit errors and will gain proficiency in the simplification of fractions by having the appropriate conceptual understanding and by following the correct procedures of working with fractions. The model to be implemented in this study is eloquently expounded by (Julie, 2013).

Julie was inspired to develop this model during his involvement in the LEDIMTALI (local Evidence Driven Improvement in Mathematics teaching & Learning Initiative) project of the University of the Western Cape, a higher education institution in the Western Cape Province of South Africa. The LEDIMTALI is a project where mathematics educators, mathematicians and mathematics curriculum advisors work collectively and collaboratively to develop quality teaching of mathematics. Professor Julie observed 40 mathematics teachers demonstrating a detailed explanation of the steps to be followed when solving mathematics problems. The explanation is written on the chalkboard and in the process the teacher asks questions from learners, and answers some of the questions by him/herself. After the demonstration of few examples the teacher gives learners similar problem to solve, he monitors their progress and help those who need assistance. Towards the end of the lesson learners would be given homework which will be checked the next day and corrections be made. Julie sought means of making good use of this existing model to bring about a more learner centred teaching. The need to design a model was intensified by part of the project where a common end of year examination for grade 10 was designed for the participating schools. After the analysis of learner performance teachers were asked to prepare strategies that they would use to help learners on their difficulties with question. The strategies that teachers came with were just descriptions of solutions procedures. Observing the teacher's

presentation emphasized the importance of developing a model of teaching that will be based on intentional teaching and some principles of assessment for learning (AFL).

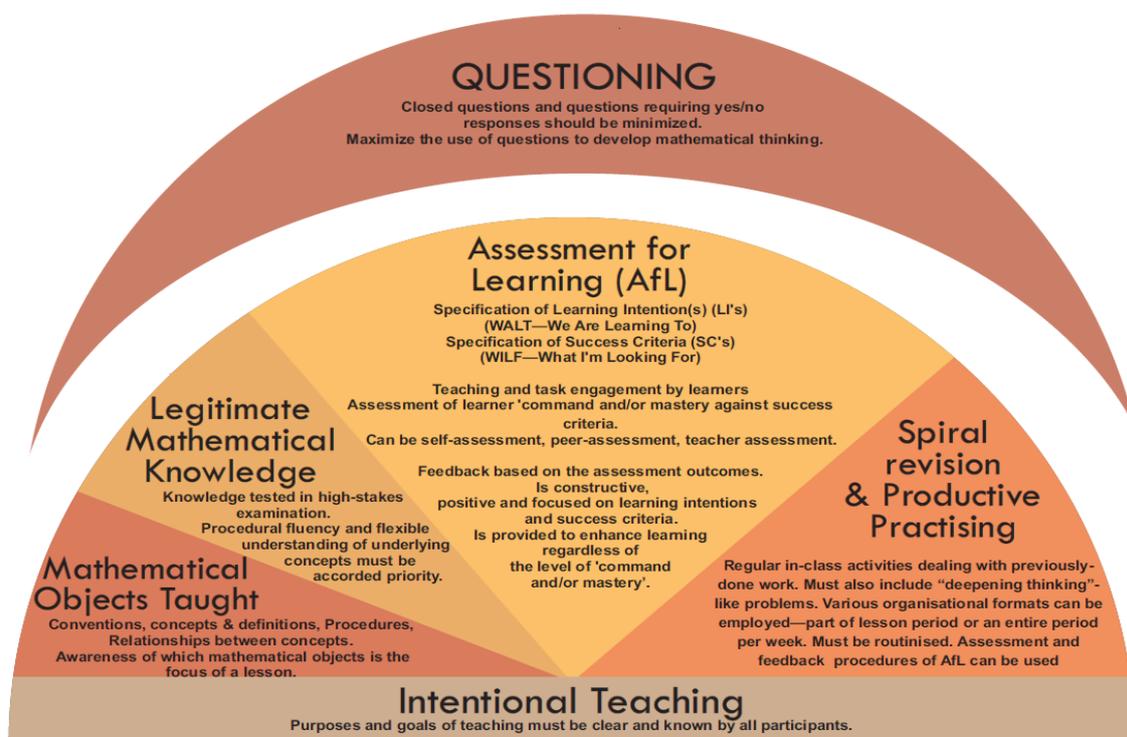


Figure 2.2 Intentional teaching model (Julie .2013)

The intentional teaching model emphasizes that the purposes and goals of learning must be clear to all participants i.e. teacher and learners. Both the teacher and the learner must be aware of which of the objects of mathematical knowledge is the focus of teaching. According to this model learning intention need to be specific. Learners need to be clear of what they are learning to do, for example learners must know they are learning to simply algebraic fractions by using some operations such as factorising, dividing, subtracting,

multiplying algebraic expressions. Together with understanding of learning intention, learners need to be clear about the success criterion which determines whether the learning intentions have been achieved. Success criteria means knowing what they are looking for. When teaching the lesson, the teacher should state clear procedure on how to get what we are looking for, for example they need to know that if for example they are adding algebraic fractions, they need to find the common denominator and they can do so by first factorising, if the after factorising a common denominator can be identified then they can add numerators of fractions with similar denominators then simplify. The success criterion is also used for assessment and after the assessment a positive feedback is given based on the learning intentions and the success criteria specified. Both the teacher and the learners must articulate the learning intentions and success criteria.

2.9.1 Spiral revision

The intentional teaching model advocates for “*spiral revision*” and productive practising. Spiral revision or repeated revision is the strategy devised to address the aspect of practising of skills and processes in class. Spiral revision is a construct by Julie which means the repeated practising of work previously covered (Julie, 2013). The work that has been done need to be revised with learners and the questions that the teacher asks must probe mathematical thinking.

When new content is introduced, learners usually do well in exercises while the content is still fresh in their minds but when time examinations or test comes they cannot even remember what they did to get it write. Spiral revision therefore aims at helping learners practice mathematics procedural skills so that they can become part of their long term memory and they can use their existing knowledge to solve problems or make connections when learning new content.

Spiral revision is coupled with productive practising which is a strategy that exposes learners to deepening thinking problems. For this case learners must be exposed to different type of asking algebraic fraction only by asking learners to simplify or add the fractions but to the type of questions that require them to reason and explain in detail what they are doing in each step as they work with fractions.

2.9.2 Prioritization of legitimate knowledge.

The model also advocates for the prioritization of the knowledge that is tested in high stakes examinations. Teachers need to understand the development of the knowledge that will be asked in the examinations. An example would be, when teaching algebraic fractions in grade 10, you must be aware which part of this topic is mostly asked in the exams and how can you help your learners to develop their knowledge of the topic in such a way that they will be competent in the topic up to their grade 12 examinations. Fractions of the type $\frac{x^2-9}{x-3}$ give learners difficulty to solve when asked in trigonometric identities such as $\frac{\sin^2\theta-\cos^2\theta}{\sin\theta+\cos\theta}$. In grade 12 examinations learners are required to simplify or prove trigonometric identities and they struggle to apply procedures of simplifying algebraic fractions where necessary. Also grade 12 mathematics examinations involve functions and graphs. In this topic learners need skills of simplifying algebraic fractions in order to solve some of the problems. An example is found in the 2012 mathematics examination paper 1. Question 6.1.2 required learners to show that $\frac{x-2}{x+1}$ can be written as $\frac{-3}{x+1} + 1$. According to the national report learners struggled to solve the question and it was suggested that learners should be exposed to fractions of this nature from grade 10 (Department of Basic Education, 2013). Intentional teaching model therefore advocates for the emphasis of the type of rational fractions that will help learners solve problems in their grade 12 examinations.

2.9.3 Assessment for learning

The assessment for learning is defined by Black, Harrison, Lee, Mashall & William (2004) quoted in Julie (2013) as any assessment for which the first priority in its design and practice is to serve the purpose of promoting students' learning. It thus differs from assessment designed primarily to serve the purposes of accountability, or of ranking, or of certifying competence... it provides information that teachers and their students can use as feedback in assessing themselves and one another and in modifying the teaching and learning activities in which they are engaged.p.10.

Assessment for learning is not just a tool to measure how much learners managed to grasp or understand but rather describes the ways in which teachers observe and try to understand student learning, and then use that information to further future learning (Drummond, 2003).

The main aim of AfL is to ensure that assessment is used in a positive way to both improve learner performance and to empower students (Black & Wiliam, 1998) cited in (Lauf &Dole, 2010). The purpose of assessment for learning would have been achieved if students reach a point where they are motivated to learn; own their learning process; are able to evaluate their work and make judgements about where they are in their understanding.

The focus of AfL process is how students respond to questions or activities given for the assessment. It is therefore vital that the type of questions given to learners are designed carefully to ensure that they open up an opportunity for learners to demonstrate their thinking and justify their solutions. AfL practises involve self; peer and teacher assessment and positive feedback is also one of the important aspects of assessment for learning.

According to Julie (2013) there are some areas of overlap between AfL and intentional teaching hence the model incorporates principles of AfL and intentional teaching. Assessment for learning put emphasis on specification of learning intentions, understanding of the success criteria, teaching & task engagement by learners and finally assessment and feedback. This research focusses more on this aspect of intentional teaching and use it to evaluate learners' ways of working with rational algebraic fractions.

Intentional teaching forms the base of this model; therefore, teachers need to do all things with intention in mind. Being clear of the learning objectives they want to achieve and then plan the activities that will help them achieve the set objectives.

2.10 Conclusion

This chapter has outlined what algebraic fractions are and the effect that misconceptions have in the errors that learners commit when they simplify algebraic fractions. It has been highlighted that misconceptions are difficult to eradicate because they are strongly embedded in the learners thinking system. Teachers need to explore methods of teaching that can help them eradicate misconceptions that learners have and one of the methods advocated by the intentional teaching model is spiral revision.

CHAPTER 3

RESEARCH DESIGN AND PROCEDURES

3.1 Introduction

This chapter discusses the research paradigm followed in this study and the research method used. A motivation for the choice of research paradigm and method is outlined. Issues of sampling, data collection, how validity and reliability have been addressed and finally the ethical considerations are outlined in this chapter.

3.2 Research design

Each research project has its specific objectives and goals and therefore different researchers use different research methods because they have different research objectives. Each research method is relevant and appropriate for a specific task. Research method use should also be relevant to other aspects of research such as how data will be collected, treated, analysed and interpreted.



The study is premised on a qualitative research paradigm which focuses on studying situations in their natural settings and applying an interpretive perspective. The study has followed a design-based research methodology.

3.3 Qualitative research

Qualitative research is defined by many researchers in different ways but all definitions seek to define the same essence of qualitative research. Denzin & Lincoln, (2005, pp. 3) state that-

Qualitative research is a situated activity that locates the observer in the world. It consists of a set of interpretive, material practices that makes the world visible. These practices transform the world. They turn the world into a series of representations,

including field notes, interviews, conversations, photographs, recordings, and memos to the self. At this level, qualitative research involves an interpretive, naturalistic approach to the world. This means that qualitative researchers study things in their natural settings, try to make sense of, or to interpret phenomena in terms of the meanings people bring to them.

This definition corresponds with the five features of a qualitative research mentioned by Miles & Huberman(1994) cited in Mbekwa (2002).

- Qualitative research is conducted through an intense and/or prolonged contact with a field or life situation. These situations are typically “banal” or normal ones, reflective of the everyday life of individuals or groups, societies and organisations.
- The researcher’s role is to gain a holistic (systematic, encompassing, integrated) overview of the context under study: Its logic, its arrangements, and its explicit and implicit rules.
- The main task is to explicate the ways in particular settings some to understand, account for, take action and otherwise manage their day-to-day situations.
- Many interpretations of this material are possible, but some are more compelling for theoretical reasons or on grounds of internal consistency.
- Relatively little standardised instrumentation is used at the outset. The researcher is the main “measuring device “in the study.
- Most analysis is done with words. The words can be assembled, sub clustered, broken into semiotics segments. They can be organised to permit the researcher to contrast, compare, analyse and bestow patterns upon them. (p. 65)

From the quotation it is evident that qualitative research is more relevant when one wants to understand or analyse a certain behaviour, phenomena or situation that happens in a

particular context or environment because one works closely with the participants in their own environment. When giving account of what has been observed during the research process the researcher mostly uses words to explain the findings rather than numerical interpretation. In qualitative research the researcher doesn't necessarily need statistical data to support his propositions that lead to the conclusions.

The qualitative research approach is naturalistic in a sense that it does not seek to manipulate the objects of the study but it focuses on the natural behaviour of the objects and provide the description of their behaviour. Its analysis aims at describing the environment as it is perceived by participants. It seeks to illuminate the ways individuals interact to sustain or change social settings (Boadua, 2005). The researcher interacts closely with the participants in order to get a full view of how they interact either with each other or the research instrument.



3.4 Motivation for selecting qualitative approach

In this study the researcher has implemented a teaching methodology which is an intentional teaching model. The aim was to investigate how grade 9 mathematics learners engage with algebraic fractions when an intentional teaching model is implemented. In order to observe how learners, engage with algebraic fractions, the researcher needs to interact closely with, and study how the learners interact with materials and respond to the model in a classroom environment. The researcher has to gather data by means of observing, audio recording, video recording, learners' written work and field notes from the class while learners are interacting with each other simplifying algebraic fractions. The qualitative research approach gives the researcher the opportunity to collect data by observing and interpreting even the non-verbal communications from the participants by interacting with

them in their own settings. In a qualitative research the analysis is mostly done with words, as (Atkinson,1991 & 1992) cited in Boadua (2005) argues that the words chosen for description are usually constructed by the field worker on the basis of observation and participation. When analysing data, the researcher writes a detailed description of what was observed during the class. A qualitative approach is more relevant for the objectives of this study.

3. 5 Design based research

The research methods that have continuously been used in educational research have received criticism from many researchers in that they are disconnected from the actual challenges faced by teachers in teaching and learning environments and also the challenges faced by policy makers and other stakeholders in the process of learning and teaching. Therefore, there has been a need for a research method that will address the real issues faced by practitioners in real contexts, a type of research that will connect learning environments and learning theories. Hence design based research has been employed as the type of research method to be used in this study.

According to Anderson & Shattuck, (2012) design based research evolved near the beginning of the 21st century and was heralded as a practical research methodology that could effectively bridge the chasm between research and practice in formal education.

Design based research is defined as systematic but flexible methodology. Its aim is to improve educational practices through iterative analysis, design, development and implementation, based on collaboration among researchers and practitioners in real-world settings and to leading contextually –sensitive principles and theories, (Samson,2006). Brown & Collins (1992) cited in The design-based research collective, (2002) define design based

research as an emerging paradigm for the study of learning in context through the systematic design and study of instructional strategies and tools.

Anderson & Shattuck (2012, p.2-4) argue that a quality design based research study is one that is defined by:

Being situated in a real educational context.

Focussing on the design and testing of a significant intervention.

Using mixed methods.

Involving multiple iterations.

Involving collaborative partnership between researcher and practitioners (*educator is both a researcher and teacher*)

Evolution of design principles.

Comparison to action research.

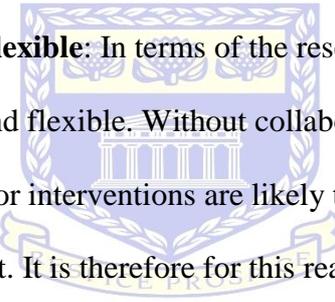
Practical impact on practice.



Wang & Hannafin (2005, p.8) summarise these qualities of design based research nicely into five basic characteristics viz. (a) pragmatic ; (b) grounded ; (c) interactive , iterative and flexible ; (d) intergrative and (e) contextual.

Pragmatic : Design based research is pragmatic in the sense that it refines both theory and practice (Wang & Hannafin , 2005). The development of the theory is inseparable from practice. The aim is to refine theory and practice and provide new possibilities either by improving practice or theory. The theory can only be commended if its principles and concepts inform and improve practice.

Grounded: design based research is grounded on theory, relevant research and real-world contexts. Before conducting research, a theory about teaching and learning is identified or selected. Using that selected theory, a thorough research is done to examine literature and identify gaps or areas that need to be improved or eliminated. The aim is to revise and refine theory and practice. Design based research is also based on real world contexts where participants interact socially with one another and within a design setting rather than a laboratory setting isolated from everyday practice (Brown & Campione, 1996; Collins, 1999 cited in Wang & Hannafin, 2005). Practitioners and researchers work together to produce meaningful change in contexts of practice such as classroom, after school programmes etc. (Design Based research collective, 2003).



Interactive, iterative & flexible: In terms of the research process design based research is interactive, iterative and flexible. Without collaboration of practitioners the proposed means of improvement or interventions are likely to have no effect or will not be implemented in real world context. It is therefore for this reason that design based research stresses collaboration among participants and researchers. The theories and interventions are continuously developed and refined throughout the process in an iterative cycle. The design process also allows flexibility for the designers to make changes where necessary.

Integrative: Design based research allow the researcher to integrate methods of data collection. Researchers mix a variety of data collection methods to maximise the credibility of the ongoing research. This is helpful because data collection from a number of sources increase objectivity, validity and applicability of the research (Wang & hannafin , 2005).

Contextual: The research results are connected with the design process and the setting from which the results were generated. Design based research process, findings and changes from the initial design are well documented to allow interested researchers or

designers to trace where or how innovations emerged and whether they can be applicable to their context. Thus guidance for applying generated principles is provided and the generalizability of the design should to be validated.

The design-based research collective (2002) argues that design based research is central in efforts to foster learning and create usable knowledge, and advance theories of learning and teaching in complex settings. It may also contribute to the growth of human capacity for subsequent educational reform. Design based research goes beyond merely designing and testing particular interventions but its interventions embody specific theoretical claims about teaching and learning, and reflect a commitment to understanding the relationships among theory, designed artefacts, and practice. At the same time, research on specific interventions can contribute to theories of learning and teaching.

The study implements an intentional teaching model as an intervention and will therefore study its impact on learning. Design based research allows this to be fulfilled because its basic principle is to test theories in real world contexts.

3.6 Participants

The learners that participated in the study were grade 9 learners who were doing mathematics in one of the high schools at Gugulethu. Gugulethu is a residential area located in the Western Cape Province of South Africa. The residential area has both formal and informal settlements as homes from where learners come. Majority of learners are black South Africans and three learners who were international learners from other African countries

Data was collected from a class of 38 grade 9 mathematics learners who were doing the rational algebraic fractions as one of the topics in grade 9 mathematics syllabus for 2016.

These learners were between the ages of 13 and 16 years. The selection of the group was not based on how learners perform in mathematics but has been selected as the grade 9

mathematics class which the researcher teaches. The sampling is convenient because intentional teaching needs more interaction between the learners and the researcher, this will be possible because the researcher teaches the class.

Sampling in qualitative research is defined as the selection of specific data sources from which data are collected to address the research objectives (Gentles, Charles, Ploeg, & Mckibbon, 2015). The type of sampling that is used in this research is purposive sampling and the purposive sampling strategy that is followed is criterion sampling. It is a criterion sampling because the sample selected is selected on basis of the research purpose.

3.7 Data collection

Data collection took place during the fourth term of 2016 academic year in South Africa. Learners had already done algebraic fractions in the previous term. Hence in data collection algebraic fractions were not taught in class or introduced as new topic but as a revision and consolidation exercise. The intentional teaching model was implemented in a grade 9 mathematics class with a specific emphasis on assessment for learning. A brief introduction of the principles of intentional teaching was given to learners with emphasis on learning intentions and goals of learning. The research approach and perspective was in the form of AfL and spiral revision because the topic had already been taught in the previous terms and learners were assessed on the topic in a form of test and mid-year examinations.

Three sets of activities were designed to probe learners' thinking. Learners were not only required to produce answers for problems through the application of methods taught in class but were required to give their reasoning behind their solutions. The main focus was to help learners avoid or correct the errors they committed when simplifying algebraic expressions. The data collection took place over three days. Each day had its own activity

designed to probe learners' thinking and to deal with some misconceptions and errors that learners would possibly display. On the first day learners were seated in six groups of five to six learners to cater for the limited number of recording devices and also to allow learners to discuss about the activities given to them, but each learner had his / her own piece of answer sheet to write on.

The activities used in this study were taken from a set of questions by LEDIMTALI (Local Evidence Driven Improvement in Mathematics Teaching and Learning Initiative) project at University of the Western Cape, a higher education institution in the Western Cape province of South Africa. The LEDIMTALI is a project where mathematics educators, mathematicians and mathematics curriculum advisors work collectively and collaboratively to develop quality teaching of mathematics.

A set of activities as shown below was given to grade 9 learners to engage and answer on the given sheets with questions. Questions for SET A activities were designed to deal with the most common error that learners display when they simplify algebraic fractions which is cancellation of same variables. Also activity 3 on SET A was designed to get learners' understanding of the elementary simplification of algebraic fractions.

SET A

ACTIVITY 1

Which of the following fractions will give an answer of 2? tick (✓) *Yes or No*

		Yes	No
1.1	$\frac{2x}{x}$		
1.2	$\frac{6xy}{3xy}$		

1.3	$\frac{6a^2b}{3ba^2}$		
1.4	$\frac{8m^4n}{4m^2n}$		
1.5	$\frac{5x^3}{10x^3}$		

Jenny said that $\frac{p^2q^4}{q^2p^2} = 2$ because the “p’s” all cancel and for the “q’s” the $4 \div 2 = 2$. Do you agree with Jenny? Give reasons for your answer.

Zolile simplified $\frac{5(x-1)}{6} + \frac{x-3}{3} - \frac{x-1}{2}$ as follows:

$$\begin{aligned} & \frac{5(x-1)}{6} + \frac{x-3}{3} - \frac{x-1}{2} \\ &= \frac{30(x-1)}{36} + \frac{12(x-3)}{36} - \frac{18(x-1)}{36} \\ &= \frac{30x-30+12x-36-18x+18}{36} \\ &= \frac{24x-48}{36} \\ &= \frac{2x-4}{3} \end{aligned}$$



Explain with reasons why you agree or disagree with the way Zolile simplified the expression. Can you find another way to simplify the expression?

Learners were given time to engage these activities in their groups and later after the scripts with their answers had been collected, a discussion was opened to the whole class as a form of giving feedback to learners. The discussion was co-ordinated by the teacher who probed learners' thinking by asking follow up questions on the responses that were given by

learners in answering the questions on activities. It was very interesting to see some learners being so confident of the wrong answers they gave for the activity and they were so convinced that they are right.

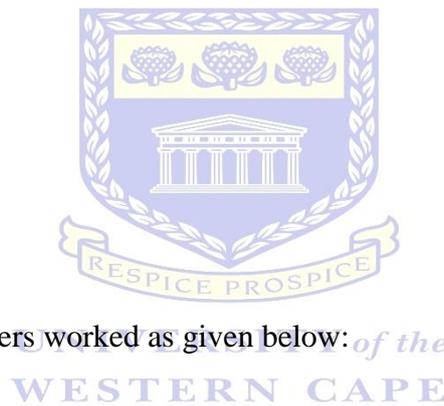
On the second day of data collection, the class setting was still the same as on the first day. Learners engaged with activities on SET B as shown below. These activities were designed to help learners sort out confusions and have a productive practising to deepen their thinking as they manipulated the fractions.

SET B

Activity 1.

1.1

To simplify $\frac{3a+3b}{a+b}$ two learners worked as given below:



Zodwa:

$$\frac{3a+3b}{a+b} = \frac{3a+3b}{a+b} = 3 + 3 = 6$$

Zizi:

$$\frac{3a+3b}{a+b} = \frac{3(a+b)}{a+b} = 3$$

Which learner performed the calculation correctly? Write down reasons for your choice.

1.2. To simplify $\frac{a^2-b^2}{a+b}$ two learners worked as given below:

Patrick:

$$\frac{a^2-b^2}{a+b} = \frac{a^2-b^2}{a+b} = a - b$$

Andiswa:

$$\frac{a^2-b^2}{a+b} = \frac{(a-b)(a+b)}{(a+b)} = a - b$$

With whose method do you agree? Write down reasons for your answer.

When simplifying fractions learners have a tendency of cancelling either the constant term, variable or co-efficient that is common both in the numerator and denominator. The activities for SET B were designed to address cancellation of terms rather than cancelling factors when simplifying algebraic fractions.

On the third day learners engaged with SET C activities which were based on “explaining & unscrambling”. These were an interesting set of activities as they gave learners an opportunity to reflect on their methods of simplifying algebraic fractions and see if they did really understand the procedures they apply or they just follow them as formulae for solving problems.

SET C

Activity 1

A way to simplify $\frac{3}{a^2-4} + \frac{2}{a-2}$ is given below. Write down what was done in the steps numbered A, B, C & D.

$$\frac{3}{a^2-4} + \frac{2}{a-2}$$

$$= \frac{3}{(a+2)(a-2)} + \frac{2}{a-2} \quad \text{Step A: } \dots\dots\dots$$

$$= \frac{3+2(a+2)}{(a+2)(a-2)} \quad \text{Step B: } \dots\dots\dots$$

$$= \frac{3+2a+4}{(a+2)(a-2)} \quad \text{Step C: } \dots\dots\dots$$

$$= \frac{7+2a}{(a+2)(a-2)} \quad \text{Step D: } \dots\dots\dots$$

Activity 2

The expressions given below can be arranged to give steps to arrive to the answer of

“simplify $\frac{3x+2}{2} + \frac{3+x}{2} - \frac{7}{6}$ ”. Put the steps in correct order to get the answer.

1. $\frac{(9x+6)+(9+3x)-7}{6}$

2. $\frac{12x+8}{6}$

3. $\frac{3(3x+2)}{6} + \frac{3(3+x)}{6} - \frac{7 \times 1}{6}$

4. $\frac{6x+4}{3}$

5. $\frac{(9x+3x)+(6+9-7)}{6}$



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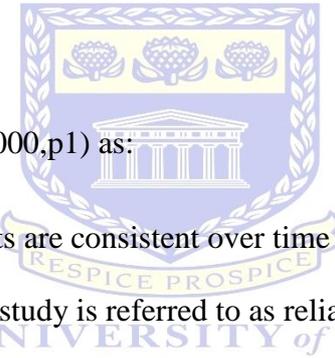
3.8 Data collection procedures

This study used four methods of data collection, observation, student’ written work, audio recordings and video recordings. The researcher observed learner’s behaviours as they dealt with activities, a draft of notes was taken during the session and later the researcher

wrote comprehensive notes of what transpired during the session. Also the learners' written work which shows how they answered questions for each set of activities was collected and kept safely for data analysis. During the three sessions with learners, the recording was done using a tablet. The groups were recorded on alternating basis, one group would be recorded for a particular period of time and then the recorder would be moved to another group. The sound recorder was left running on that particular group to avoid intimidating learners while they have their conversations about the activities. Lastly video recording of learners was taken by a colleague who is not a maths teacher to avoid bias. A maths teacher is in a position to identify whether the group is doing right or wrong and may be tempted to record mostly the group that is giving correct answers.

3.9 Validity and Reliability

Reliability is defined by Joppe (2000,p1) as:



The extent to which results are consistent over time and an accurate representation of the total population under study is referred to as reliability and if the results of a study can be reproduced under a similar methodology, then the research instrument is considered to be reliable.

Joppe (2000, p.1) again defines the validity as:

a form of determining whether the research truly measures that which it was intended to measure or how truthful the research results are. In other words, does the research instrument allow one to hit "the bull's eye" of your research object? Researchers generally determine validity by asking a series of questions, and will often look for the answers in the research of others.

Qualitative researchers argue that the concepts of validity and reliability are important for qualitative researchers to consider when designing the study, analysing results and

judging the quality of research (Patton, 2002). To be applicable in qualitative research these concepts need to be redefined.

Qualitative researchers use different terms to describe reliability and validity. For example Lincoln & Guba, (1985) make use of the term ‘trustworthiness’ of the study instead of reliability and validity. Guba and Lincoln (1989) proposed four criteria for judging trustworthiness or the soundness of qualitative research. The four terms that can be used to answer the question of validity and reliability in qualitative research which are: credibility (in a place of internal validity), transferability (in place of external validity), dependability (in place of reliability) and conformability (in place of objectivity) (Guba & Lincoln, 1985).

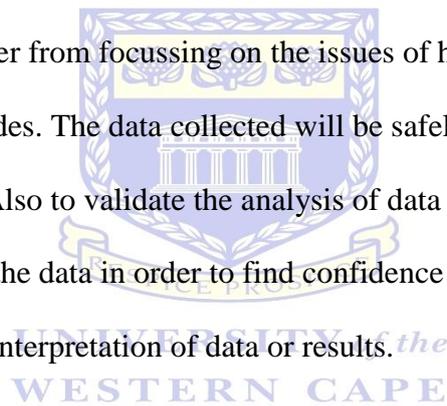
Credibility answers the question of how congruent are the results with reality i.e. a true picture of the phenomena under scrutiny is being presented, (Shenton, 2004). The implementation of the credibility criterion is twofold. Firstly the researcher has to carry out the inquiry in such a way that the probability that findings will be found to be credible is enhanced and secondly the researcher has to demonstrate the credibility of the findings by having them approved by constructors of the multiple realities being studied. Shenton, (2004) argue that to increase the probability of high credibility in the field one should consider methods such as firstly adopting research methods that are well established in studies of the same nature as his/her study. Methods of collecting data and the analysis should be derived from those that have been used successfully in previous comparable projects. In this research video recording of lessons, audio recording and learners’ written work were used as methods of collecting data. These methods have been used by many researchers successfully in projects of the same nature. Secondly random sampling is recommended in ensuring the credibility of research. This is to ensure that those selected are a representative of the larger sample. In this research a purposive sampling was used and hence no random sample was used.

Another form of ensuring credibility is triangulation which is the application of multiple data collection methods. According to (Guba, 1989) the use of different methods in concert compensate for their individual limitations and exploit their individual benefits. If one method has a limitation, then the other method can cover for the limitation while the researcher is not losing the advantage that the former had. In this research a number of methods of data collection have been used as mentioned earlier. Transferability is used by some researchers in preference to external validity or generalizability which are concerned with the extent to which results of the study can be applied to other situations. In case of transferability, the concern is on demonstrating that the research findings can be applied to a wider population. For a qualitative research it is impossible to specify that the results and conclusions are applicable to other situations or larger population. Therefore, to address transferability of the research findings a thick description can be provided which is necessary to enable someone interested in making a transfer to reach conclusion about whether transfer can be contemplated as a possibility. Therefore, the responsibility of the researcher is to provide a data base that makes transferability judgements possible on the part of the potential appliers. Purposive sampling can be used to address transferability. In this research a detailed explanation of the setting from data was collected, how data was collected and analysed is provided so that a reader or researcher can be able to make decisions about the transferability considering the contextual factors.

Dependability: In quantitative research it is possible to employ techniques to ensure the possibility of the consistency or repeatability of measures. This is not so easy in a case of qualitative research considering the changing nature of phenomena scrutinised by qualitative researchers. In order to address the dependability issue more directly, the processes within the study should be reported in detail, thereby enabling a future researcher to repeat the work,

if not necessarily to gain the same results. Thus, the research design may be viewed as a “prototype model”. (Shenton, 2004, p. 72).

Confirmability: The concept of confirmability is the qualitative investigator’s comparable concern to objectivity. Here steps must be taken to help ensure as far as possible that the work’s findings are the result of the experiences and ideas of the informants, rather than the characteristics and preferences of the researcher. It must be clear that the findings emerge from the data and not their predispositions, (Shenton, 2004). The researcher needs to have measures in place in order to avoid researcher bias (Sheton, 2004). To address bias the researcher asked a colleague who is not a mathematics teacher to do the recording so as to avoid selective recording. Also for data analysis, a colleague was asked to act as a sounding board to prevent the researcher from focussing on the issues of her interest but give a detail explanation of the data provides. The data collected will be safely kept for evidence or verification of the findings. Also to validate the analysis of data a peer researcher was involved in interpretation of the data in order to find confidence and also an expert was involved in the analysis and interpretation of data or results.

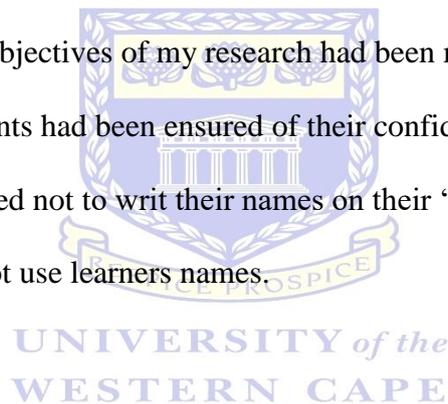


3.10 Ethical considerations

“Ethical issues are the concerns and dilemmas that arise over the proper way to execute research, more specifically not to create harmful conditions for the subjects of inquiry, humans, in the research process” (Schurink, 2005, p. 43). The researcher was informed of the importance of being respectful and sensitive to the research participants and taking it into consideration the Ethical code of the University of the Western Cape was followed and respected. The researcher applied for the permission to conduct research at a public school from the Western Cape Education Department of South Africa and also applied

to the school management of the school to which research was conducted. The permission was therefore granted for research to be conducted in grade 9 mathematics class.

MacMillan & Schumacher,(2010) argue that privacy, anonymity and confidentiality are elements of ethics in research. Those who participate in research have a right to privacy and protection and there must be no connection between the data and the participants. If possible data must be collected such that the researcher cannot identify the participants from the collected data. In respect of participants privacy , anonymity and confidentiality: (i) the participants in the study had been informed that their participation in the research is voluntary and if at any time of the research one wants to quit, he/she can do so. (ii) Consent forms had been given for the participants and their parents to sign before they begin to participate in my research. (iii) The aims and objectives of my research had been made clear to all the participants..(v) All participants had been ensured of their confidentiality and anonymity. (vi)The learners were informed not to writ their names on their ‘work and during data analysis the researcher did not use learners names.



3.11 Conclusion

This chapter has addressed the research design for the study and the motivation for the selection of the research approach applied in this study. A discussion of the design based research as a methodology followed in this study has been given. The chapter also touched the issues pertaining to data collection, how reliability and validity have been addressed in the study and finally how the researcher addressed the ethical considerations. The next chapter will discuss data analysis.

CHAPTER 4

DATA ANALYSIS

4.1 Introduction

This chapter outlines how the collected data was analyzed. The data analysis process followed is explained and the summaries of results are tabulated. The chapter also discusses some errors that learners committed as they were simplifying algebraic fractions and give a brief summary in a table form. Finally, the level of deeper explanation on learners' responses to questions is discussed. It should be noted that in this chapter some remarks from learners' scripts were quoted as they are written on their scripts with no correction in language. The quoted extracts are all indicated by inverted commas.

4.2 Analysis

"Data analysis is the process of bringing order, structure and meaning to the mass of collected data. It is a messy, ambiguous, time-consuming, creative, and fascinating process. It does not proceed in a linear fashion; it is not neat. Qualitative data analysis is a search for general statements about relationships among categories of data." (Marshall and Rossman, 1990:111).

According to the quotation above, collected data is just a mass of information which does not have order or meaning until a process which brings order and meaning of the information is applied. There is no one form or formula for analyzing data but at the end of the process one can make sense of the data collected and finally come to a conclusion as to what the study reveals. Miles & Huberman (1994) suggest three sub processes of data analysis namely: data reduction; data display, drawing and verifying conclusions.

From the set of 3 activities that were given to learners in three different days, for SET A activity there was a batch of 38 scripts which were all used for analysis. For SET B there were 38 learners and all their scripts were used for analysis of question 1.1 of Set B activity but for question 1.2 eight learners did not answer the question and hence there were 30 scripts that were used for the analysis. The last set of activities viz. SET C, only 16 learners attempted to answer the question and the other 22 learners did not answer, that left only 16 scripts to be analyzed.

The data was first summarized in tables that detailed the learner's responses to each activity. The scripts were then later coded according to the types of errors identified which were the Defractionalisation error, Cancellation error and Non-recognition of common factors, Dropping the denominator and the Exponential law error.

Data was categorized according to the errors that learners committed when simplifying algebraic fractions. The errors that were mentioned in the literature review chapter were used as a conceptual framework. Other errors that were not mentioned in the literature review were given names and hence coded. The following are the codes for errors:

(DF) DE fractionalization error which is the transformation of a fraction with a unity numerator to a non-fraction e.g. $\frac{1}{2} = 2$ or $\frac{1}{x} = x$ (Otten, Males & Figueras ;2008). (CE)

cancelation error & (NRCF) no recognition of common factor. The cancelation error is characterized by cancelation of a common term in the numerator and denominator as if it's a common factor. The error can also be as a result of failure to recognize a common factor in the numerator and or denominator. Another error is Dropping the Denominator(DD).

;Makonye & Khanyile (2015). (ELE₁) exponential law error one, this is characterized by learners subtracting the exponents of the terms in the denominator from the exponent of the similar term in the numerator. E.g. $\frac{x^2-y^2}{x-y} = x^{2-1} - y^{2-1} = x - y$. (ELE₂) exponential law

error two, in this type of error learners hold a misconception about a variable with exponent of zero. For some learners if the exponent is not visibly written as a superscript then it means it is zero and for others the variable with zero exponent is equal to zero. E.g. $\frac{a^2b^3}{a^2b} =$

$$a^{2-2}b^{3-2} = ab. \quad \text{Or} \quad \frac{a^2}{a^2} = a^{2-2} = 0$$

Table 4.1 summarizes learners' responses to the questions on SET A activities and TABLE 4.2 summarize learner's responses to questions on SET C activities.

4.3 Analysis of learner's work on algebraic fractions

The activities that were given to learners were targeted at sorting out the confusions, misconceptions or errors that are usually persistent in learners when they simplify algebraic fractions through productive practice and constructive feedback. The main method for data analysis in this was document analysis of the learners' scripts in which learners did their calculations. The audio and video recordings were used to remind and to keep a fresh memory of what was happening in class as learners were simplifying algebraic fractions. The recordings were also used to find learners reasoning and comments as they were working. The first set of activities was a batch of 38 scripts.

The first activity was based on scratching out the same letters on a monomial divided by another monomial. The table below analyses the learners' responses on monomial division.

SET A

ACTIVITY 1

Question 1: which of the following fractions will give an answer of 2? Tick yes or no

Total number of scripts: 38

TABLE 4.1: Analysis of activity 1, Set A

Problem	Answered yes	Answered no	Comments
1.1 $\frac{2x}{x}$	100%	0	<p>The aim of the questions was to check if learners know that the numerator and denominator are divided by the same common factors present in the numerator and the denominator. All learners answered the question correctly and they had no debates about the answer. For 1.2 the expected error was a learner may divide $\frac{6}{3} = 3$ but no learner committed this type of error.</p>
1.2 $\frac{6xy}{3xy}$	100%	0	
1.3 $\frac{6a^2b}{3ba^2}$	67%	33%	<p>This activity was based on commutative property. Most learner's way of simplifying this question showed no application of the commutative property. Learners first rearranged the variables so that they appear in pattern where the order in the numerator is the same as in the denominator then they cancelled.</p> <p>Learners showed understanding of cancelling based on subtraction of exponents. Some learners wrote that $b^0 = 1$ and $a^0 = 1$ and they had their final answer equal to 2.</p>

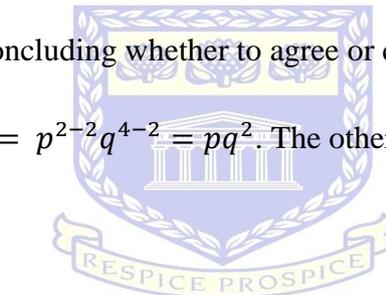
			<p>4 learners first ticked on the NO and later scratched the answer and selected YES. It can be assumed that they changed their answers after they realized their group members had different answers or they were not sure about their answers.</p> <p>A misconception of a variable power with 1 as its exponent such as b. learners held a misconception that b has no exponent therefore $\frac{b}{b} \neq 1$ but can only be zero. Some learners only recognize an exponent if it is visibly written as a superscript of a number or variable.</p>
1.4 $\frac{8m^4n}{4m^2n}$	13%	87%	<p>Most learners answered correctly by saying no, but 3 learners answered yes the answer will be 2.</p> <p>2 learners first answered YES and later scratched their answers and selected NO.</p> <p>One learner who answered No had a final answer of $2m^2n$ which was obtained by subtracting the exponents but for $n^{1-1} = n$.</p>
1.5 $\frac{5x^3}{10x^3}$	14%	86%	<p>Most learners answered correctly but 4 learners answered YES which was the wrong option.</p> <p>6 learners seemed not to be sure about the answer because they first chose YES and later scratched and selected NO.</p>

			The learners who answered Yes argued that 5 goes two times to 10 and the x^3 's cancel then the final answer would be 2.
Total % for correct answers		85%	

Question 2: Jenny said that $\frac{p^2q^4}{q^2p^2} = 2$ because “p’s “ all cancel and for “q’s” the $4 \div 2 = 2$.

Do you agree with Jenny? Give reasons for your answer.

Only 5 % of the learners agreed or supported the way Jenny simplified the fraction and 95% of the learners disagreed with Jenny’s answer and the steps she followed. Of the learners who showed calculations before concluding whether to agree or disagree five learners simplified this fraction as follows: $\frac{p^2q^4}{q^2p^2} = p^{2-2}q^{4-2} = pq^2$. The other 9 learners had their final answer equal to q^2 .



One reason that most learners gave for why they disagree was that instead of dividing the exponents Jenny should have subtracted the exponents. Some learners who argued that Jenny’s way of simplifying the fraction is wrong because she should have subtracted instead of dividing fractions, they said but the final answer will still be the same. “*No instead of dividing he should subtract but his answer is correct*” quoted from a learner’s script. These learners argue strongly that the answer will be 2. Because the exponents for p will subtract and leave no exponent for p and $4-2 = 2$.

Question 3:

Zolile simplified $\frac{5(x-1)}{6} + \frac{(x-3)}{3} - \frac{x-1}{2}$ as follows:

$$\frac{5(x-1)}{6} + \frac{(x-3)}{3} - \frac{x-1}{2}$$

$$\begin{aligned}
&= \frac{30(x-5)}{36} + \frac{12(x-3)}{36} - \frac{18(x-1)}{36} \\
&= \frac{30x-30+12x-36-18x-18}{36} \\
&= \frac{24x-48}{36} \\
&= \frac{2x-4}{3}
\end{aligned}$$

Explain with reasons why you agree or not with the way Zolile simplified the expression. Can you find another way to simplify the expression?

Out of 38 learners only 20 attempted to answer the question; which means 47% of the learners could not attempt the question. Of the 20 learners who gave answers none of them stated that they agree with the way a fraction was simplified 5 tried to simplify the problem but could not get the correct answer and they did not write if they agree or disagree. One learner simplified the expression correctly but did not answer if Zolile's method is right or wrong. The other 7 learners only wrote that they disagree with how Zolile simplified but could not give reasons or show their own way of simplifying expression. The other 7 learners wrote that they disagree and then simplified the fractions but only one of them managed to simplify correctly.

Learners consulted their note books when they got to this question and others just looked at the question and made no attempt of finding a solution. In the overall for this question only 5% of the learners could simplify the fraction correctly, 13 % of the group simplified correctly but made a few mistakes in their final answer. Of those 13 learners who attempted to solve the problem most had a lowest common denominator of 6 except for two learners who multiplied $6 \times 2 \times 3 = 36$ and wrote that HCD = 36.

Figure 4.1 below shows an example of a learner who used the common denominator of 36 and added the fractions correctly.

$$\begin{aligned}
 & \frac{5(x-1)}{6} + \frac{x-3}{3} - \frac{x-1}{2} \\
 & \frac{5x-5}{6} + \frac{x-3}{3} - \frac{x-1}{2} \\
 & = \frac{6(5x-5)}{36} + \frac{12(x-3)}{36} - \frac{18(x-1)}{36} \\
 & = \frac{30x-30 + 12x-36 - 18x + 18}{36} \\
 & = \frac{24x-48}{36} = 12 \text{ (All)} \\
 & = \frac{2x-4}{3}
 \end{aligned}$$

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Figure 4.1: learner used common denominator correctly to simplify activity 3 of SET A

Figure 4.1 shows one of the learner’s solutions for activity 3 of SET A. The learner started by simplifying the numerator of the first fraction $5(x-1)$ and opened the brackets. The learner found the common denominator by multiplying all denominators ($6 \times 3 \times 2 = 36$). The denominator of each fraction was then divided by 36 and the answer multiplied to the numerator. This learner managed to completely simplify the fraction using a denominator of 36. The indication of the ticks (\checkmark) and the cross (\times) shows the learners method of identifying the like terms before adding them. After getting an answer $\frac{24x-48}{36}$ the learner continued to simplify by dividing each term by 12 or identified a common factor which is 12.

Figure 4.2 below shows an example of learners' work, who used the HCD 6.

$$\begin{aligned}
 & \frac{\cancel{5}(x-1)}{6} + \frac{\cancel{2}(x-3)}{3} - \frac{x-1}{2} \\
 & = \frac{5x-5 + x-3 - x+1}{6} \\
 & = \frac{30(x-5) + 6(x-3) - 6(x+1)}{6} \\
 & = \frac{30x-150 + 6x-18 - 6x-6}{6} \\
 & = \frac{30x-174}{6} \\
 & = \frac{30x-174}{6}
 \end{aligned}$$

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Figure 4.2: learner who wrongly used LCD to simplify activity 3 of SET A

The last groups are those who used the lowest common denominator of 6. From this group only 5 learners managed to simplify the fraction correctly but 7 learners failed to simplify the fraction correctly.

Figure 4.2 shows the work of a learner who used 6 as the LCD for the fractions. The learner's way of working shows a misconception of how the common denominator is used when adding fractions. This learner multiplied all the numerators with the lowest common denominator 6(LCD). The next steps are worked out correctly but because the LCD was wrongly used the learner got a wrong answer.

Figure 4.3 below shows an example of learner who used the LCD = 6 and added fractions correctly.

$$\begin{aligned}
 & \cancel{5(x-1)} + \frac{5(x-1)}{6} + \frac{x-3}{3} - \frac{x-1}{2} \\
 &= \frac{5(x-1)}{6} + \frac{x-3}{3} - \frac{x-1}{2} \\
 &= \frac{5(x-1)}{6} + \frac{2(x-3)}{6} - \frac{3(x-1)}{6} \\
 &= \frac{5x-5 + 2x-6 - 3x-3}{6} \\
 &= \frac{4x-14}{6} = \frac{2x-7}{3}
 \end{aligned}$$

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Figure 4.3: learner correctly used LCD to simplify activity 3 of SET A

Figure 4.3 above shows the work of the learner who managed to correctly use the LCD to simplify the algebraic fractions but could not get a correct final answer. It can be assumed that the learner had a careless error because in the third step of the solution the negative sign was left and when multiplying with 3, the learner multiplied by positive 3 rather than negative 3. This led the final answer being wrong.

Figure 4.4 below is an example of learner's way of working when simplifying the algebraic fraction given in activity 3 of SET A.

$$\frac{5(x-1)}{6} + \frac{x-3}{3} - \frac{x-1}{2} \quad H \neq D$$

$$6 \times 3 \times 2 = 36$$

$$= 36(5+x-1) + 12(x-3) - 6(x-1)$$

Figure 4.4: learner who used common denominator 36 when simplifying activity 3 of set A

The learner in **figure 4.4** found the highest common denominator which is 36 but could not correctly use it in simplifying the fraction. The learner eliminated the denominators for each fraction by multiplying with each denominator by 6 and then multiplied the answer to the numerator. Hence there was no correct use of the denominator and evidence that shows understanding how the common denominator is used.

SET B

Activity 1

Question 1.1

On activity 1 of set B, learners were required to look at the answers of two learners who simplified the fraction $\frac{3a+3b}{a+b}$, state who simplified the fraction correctly and give reasons for their choices. The fraction was simplified as follows

Zodwa:

$$\frac{3a+3b}{a+b} = \frac{3\cancel{a}+3\cancel{b}}{\cancel{a}+\cancel{b}} = 3 + 3 = 6$$

Zizi:

$$\frac{3a+3b}{a+b} = \frac{3(a+b)}{a+b} = 3$$

38 learners did the activity, 23 learners chose Zodwa's simplification as a correct answer and 15 learners chose Zizi's answer as a correct answer. The reasons learners gave for choosing Zodwa include the following

"Zodwa, like terms cancel each other, then its 3+3 and the answer is 6"

"Zodwa because she managed to cancel the variables and added constants"

"I think its Zodwa because when you divide its algebraic fraction you need to cancel all the alphabet that is in the numerator so that you can have the numbers and add them"

"Zodwa because when you divide the same bases they cancel each other."

From the learners' reasoning it is evident that learners have a number of misconceptions about cancelling. Learners hold a misconception that you can cancel any variable in a numerator with a like variable in the denominator. Understanding of cancelling same bases is not based on the correct operation of simplifying by cancelling common factors. Learners do cancel like terms.

The learners who chose Zizi's answer as a correct answer gave the following reasons for their choice:

“Zizi is the one who perform the correct calculation because she first take out the common factor”

“Zizi is the one who calculated correctly because she first factorized before coming with an answer”

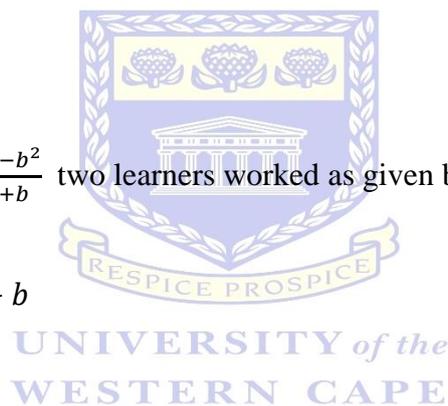
“Zizi is the one who performed the calculation correct because Zodwa just cancelled alphabets before simplifying”

“Zizi is correct because you can't cancel variables”

These learners gave a correct answer but their reasons do not reveal a clear understanding of algorithms that are applied in simplifying the fraction.

Question 1.2: To simplify $\frac{a^2-b^2}{a+b}$ two learners worked as given below:

Patric: $\frac{a^2-b^2}{a+a} = \frac{a^{\cancel{2}}-b^{\cancel{2}}}{\cancel{a}+\cancel{a}} = a - b$



Andiswa: $\frac{a^2-b^2}{a+a} = \frac{(a-b)(a+b)}{a+b} = a - b$

This question was based on factorizing the difference of two squares and it had strong visual cues for learners to be tempted to cancel the a 's and b 's. Also different ways of simplifying the fraction came to the same answer. So a correct answer was obtained from wrong mathematical methods.

21% learners did not attempt this question, 24% learners chose Patric's answer as the correct method and 55% learners chose Andiswa's method as the correct method of simplifying the fraction.

All learners that chose Patric's answer argued that it was correct because when you divide the same bases you subtract exponents.

Those who chose Andiswa's method as correct argued that Andiswa was correct because:

“ she first factorized answer simplified the equation”

“ she showed all steps and came up right answer”

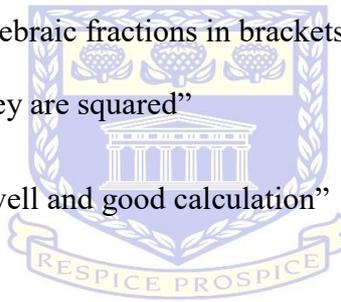
“ Simplified variables first then cancelled like terms”

“She factorized without cancelling”

“ she didn't cancel exponents”

“ Because when you multiply algebraic fractions in brackets you must subtract or divide them with the denominators if they are squared”

“ because she performed a very well and good calculation”



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SET C

This set of activities was based on explaining and unscrambling. The purpose was to help learners to understand the algorithms behind what they do as they simplify algebraic fractions and develop their algebraic thinking in mathematics.

Activity 1:

In this activity learners were given a simplified algebraic fraction in steps and were required to explain what was done in each step. The fraction was $\frac{3}{a^2-4} + \frac{2}{a-2}$

1st step: $\frac{3}{(a+2)(a-2)} + \frac{2}{a-2}$

$$2^{\text{nd}} \text{ step: } \frac{3+2(a+2)}{(a+2)(a-2)}$$

$$3^{\text{rd}} \text{ step: } \frac{3+2a+4}{(a+2)(a-2)}$$

$$4^{\text{th}} \text{ step: } \frac{7+2a}{(a+2)(a-2)}$$

There were Only 16 learners in class who attempted this question. Most of the learners in class did not understand what they were required to do, so they kept asking and seeking clarity on the question.

TABLE 4.2: learners' responses for activity 1 of SET C

steps	Learner response	Number of learners
1 st step: $\frac{3}{(a+2)(a-2)} + \frac{2}{a-2}$	Factorized	6
	Simplified the denominator	9
	Looking for common denominator	1
2 nd step: $\frac{3+2(a+2)}{(a+2)(a-2)}$	Take out the common denominator	3
	Right hand side added to the left hand side	2
	She did multiplication inverse	2
3 rd step: $\frac{3+2a+4}{(a+2)(a-2)}$	Solving (brackets) by multiplication	6
	Opened brackets in the numerator	2

	Multiplied the number that is added to three to the numbers in the bracket.	2
4 th step: $\frac{7+2a}{(a+2)(a-2)}$	Add like terms	8
	Get the answer	1

Activity 2

In this activity learners were given steps of simplified algebraic fractions, the steps were not in correct order and learners were asked to arrange the steps to give an answer to the following question: simplify $\frac{3x+2}{2} + \frac{3+x}{2} - \frac{7}{6}$. Steps according to the question were as follows:

Step 1 : $\frac{(9x+6)+(9+3x)-7}{6}$

Step 2: $\frac{12x+8}{6}$

Step 3: $\frac{3(3x+2)}{6} + \frac{3(3+x)}{6} - \frac{7 \times 1}{6}$

Step 4: $\frac{6x+4}{3}$

Step 5 : $\frac{(9x+3x)+(6+9-7)}{6}$



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TABLE 4.3: learner's responses for activity 2 of SET C

Steps arrangement	1 st	2 nd	3 rd	4 th	5 th
% of learners with right answer	100%	92%	96%	96%	96%

Comments	All learners found LCD	2 learners collected like terms before simplifying brackets	Two learners who missed 2 nd step	One learner could not answer	One learner could not answer
----------	------------------------	---	--	------------------------------	------------------------------

For this question 100% of the learners who answered the question could identify the first step of the solution where the common denominator was made. 8.3 % of the learners could not arrange correctly the second and the third step, some learners started by collecting like terms in the numerator before simplifying.



4.4 Errors Identified

It has already been stated in chapter two that learners commit errors as a result of the misconceptions they have in mathematics. I will reveal and discuss the errors learners' committed and their misconceptions. Learners were found to have committed the following errors:

4.4.1 Defractionalisation (DE)

According to Otten, Males & Figueras (2008) the term defractionalisation refers to a transformation of a fraction with a unitary numerator to a non-fraction.

When dividing the fraction $\frac{5x^3}{10x^3}$ in activity 1.5 of SET A some learners ignored the factor in

the numerator and wrote the fraction $\frac{1}{2}$ as 2. When simplifying $\frac{5x^3}{10x^3}$, learners cancelled x^3

and x^3 , then they divided 5 by 10 to get 2. Then their final answer was 2 instead of $\frac{1}{2}$. When listening to them on audio recordings learners argued strongly that x^3 and x^3 cancel; 5 goes to 10 two times therefore the final answer is 2. This showed the lack of understanding that we cancel the factors. If learners would understand the fraction in terms of its factors $\frac{5.1.x.x.x}{5.2.x.x.x}$ then it would be helpful for them to get to the correct final answer. This also shows the lack of adequate conceptual understanding of inverse operations. It can also be assumed that learner's conceptualization of fractional sub-constructs mentioned by Charalambous & Pitta-Pantazi (2007) is the reason for learners problem in simplifying this fraction. After cancelling x^3 and x^3 , the understanding of a fraction as part-whole relationship would have helped them to simplify $\frac{5}{10}$ correctly. Charalambous & Pitta-Pantazi (2007) states that when considering a fraction as representing a part-whole relationship, the numerator represents the count and the denominator represents the unit. When working with a fraction as a quotient the numerator is a quantity (dividend) which is being divided by the denominator (divisor).

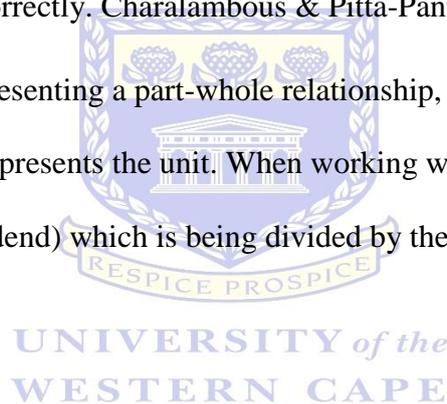


Figure 4.5 below shows an example of a Defractionalisation error from learner's scripts.

1. Which of the following fractions will give an answer of 2 ? tick(✓) Yes or No.

		Yes	No
1.1	$\frac{2x}{x}$	✓	
1.2	$\frac{6xy}{3xy}$	✓	
1.3	$\frac{6a^2b}{3ba^2}$		✓
1.4	$\frac{8m^4n}{4m^2n}$	✓	✓
1.5	$\frac{5x^3}{10x^3}$	✓	

Figure 4.5: Defractionalisation error



4.4.2 Exponential law error two (MEL₂)

For some learners if the exponent of a variable is 1 then they regard it as if there is no exponent or the exponent is zero. In simplifying $\frac{6a^2b}{3ba^2}$ learners argued strongly that when dividing same bases you subtract exponents, therefore $a^{2-2} = 0$ and b and b cancel each other.

“Jonga u b no b baya khansilishana then xa udiviyida exponents yazi subtracta so 2-2 ngu 0 so kosala u $2a$ ”

Tranlation: “ look b and b cancel each other ,then when you divide you subtract exponents so $2-2$ is zero , $2a$ will remain(as answer)”.

This group of learners did not use their knowledge of exponents correctly, they only applied some laws and neglected others hence for them a variable with zero exponent was the same to the variable with the exponent of one.

1. Which of the following fractions will give an answer of 2 ? tick(✓) Yes or No.

		Yes	No
1.1	$\frac{2x}{x}$	✓	
1.2	$\frac{6xy}{3xy}$	✓	
1.3	$\frac{6a^2b}{3ba^2}$		✓
1.4	$\frac{8m^4n}{4m^2n}$		✓
1.5	$\frac{5x^3}{10x^3}$		✓

Figure 4.6: Exponential law error 2 (example 1)

Learners have another misconception of a variable with the exponent zero and exponent of one. This was shown also when simplifying number 2 of SET A activity. The learners show an understanding of subtracting exponents when dividing but if the final answer for the exponent is zero, they cannot apply the exponents rule that a number or variable with a zero exponent is equal to one.

2. Jenny said that $\frac{p^2q^4}{q^2p^2} = 2$ because the "p's" all cancel and for the "q's" the $4 \div 2 = 2$

Do you agree with Jenny? Give reasons for your answer.

I disagree because $\frac{p^2 q^4}{q^2 p^2}$
 $= p^{2-2} q^{4-2}$
 $= pq^2$

Figure 4.7: exponential law2 error (example 2)

The same misconception of laws of exponents lead to errors when learners were

simplifying $\frac{8m^4n}{4m^2n}$. There are learners who simplified $\frac{8m^4n}{4m^2n}$ and got $2m^2n$, for these learners

when subtracting exponents for n , $n^{1-1} = n^0 = n$. if there is no exponent written visibly for the

variable then they assume the exponent to be zero.

1.1 $\frac{2x}{x} = 2$

1.2 $\frac{6xy}{3xy} = 2$

1.3 $\frac{6a^2b}{3ba^2} =$

1.4 $\frac{8m^4n}{4m^2n} = 2m^2n$

1.5

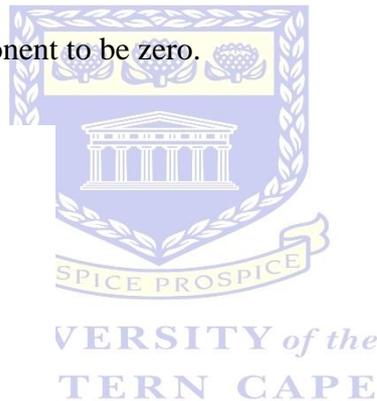


Figure 4.8: exponential law error 2 (example 3)

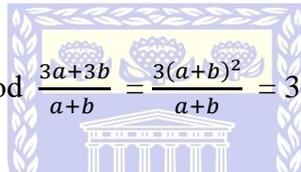
4.4.3 Cancellation error (CE) & No recognition of common factor.

Also the cancellation of terms comes from the same misconception of dividing same bases. On Set B activity 1, some learners said Zodwa is right when simplifying $\frac{3a+3b}{a+b}$ by cancelling a 's and b's. They argue that she is correct because when you divide same bases you cancel them. Makonye & Khanyile (2015) refer to this error as no recognition of the common factor, where a learner cancel terms instead of factorizing and cancel factors.

This learner was drawn to cancellation by a strong distracting stimulus of having the same variables in the numerator and denominator. That led to cancelling the terms rather than factors. One learner in answering activity 1.1 simplified the fraction $\frac{3a+3b}{a+b}$ in two ways.

Firstly following Zodwa's method : $\frac{3a+3b}{a+b} = \frac{3a+3b}{a+b} = 3+3 = 6$

Secondly following Zizi's method $\frac{3a+3b}{a+b} = \frac{3(a+b)^2}{a+b} = 3^2 = 6$.



1.1

To simplify $\frac{3a+3b}{a+b}$ two learners worked as given below:

Zodwa:

$$\frac{3a+3b}{a+b} = \frac{3a+3b}{a+b} = 3+3 = 6$$

$$\frac{3a+3b}{a+b} = \frac{3a+3b}{a+b} = 3+3 = 6$$

Zizi:

$$\frac{3a+3b}{a+b} = \frac{3(a+b)}{a+b} = 3$$

$$\frac{3a+b}{a+b} = \frac{3(a+b)^2}{a+b} = 3^2 = 6$$

Which learner performed the calculation correct. Write down reasons for for your choice.

.....

Figure 4.9 Cancellation and non- recognition of common factor error

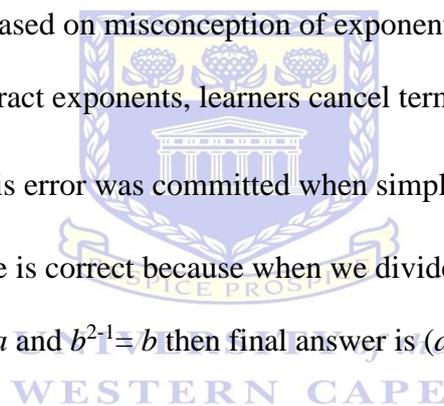
In following Zodwa's method the learner could not recognize the common factor of 3 and cancelled the variables in the terms of the numerator with the variable in the terms of the

denominator. In this case the learner followed exactly the same way of simplifying as Zodwa.

In the second case the presence of brackets for the learner was a stimulus which made the learner to think of squaring the bracket so that factors can be cancelled. Although the learner did not explain what he/she was trying to do it can be assumed after cancelling (a+b) then the exponent was transferred to 3 because it was not used in the process. This learner does not understand the cancellation of common factors because the expected answer from what the learner has done could have been $3(a+b)$.

4.4.4 Exponential law error (1)

This type of error is based on misconception of exponential law which says that when dividing same bases, we subtract exponents, learners cancel terms because they subtract exponents of those terms. This error was committed when simplifying $\frac{a^2-b^2}{a+b}$ by cancelling a and b. Learners argue that she is correct because when we divide powers with the same base, exponents subtract, so $a^{2-1} = a$ and $b^{2-1} = b$ then final answer is (a-b).



1.2

To simplify $\frac{a^2-b^2}{a+b}$ two learners worked as given below:

Patrick:

$$\frac{a^2-b^2}{a+b} = \frac{a^2-b^2}{a+b} = a-b$$

Andishwa:

$$\frac{a^2-b^2}{a+b} = \frac{(a-b)(a+b)}{(a+b)} = a-b$$

With whose method do you agree? Write down reasons for your answer.

~~Andishwa's~~ method. Patrick's method because when you divide the bars you subtract the exponents.



Figure 4.10: Exponential law error1

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The table below gives an indication of the frequency of errors committed by learners in percentages.

Table 4.4 frequency of errors committed

Type of error	Number of errors	% for error type
Defractionalisation error (DF)	$\frac{4}{38}$	11%
Cancellation error (CE)	$\frac{23}{38}$	61%

No recognition of common factor(NRCF)	$\frac{19}{33}$	58%
Exponential law (1)	$\frac{4}{38}$	11%
Exponential law (2)	$\frac{5}{38}$	13%

4.5 Deeper level of explanations

In most activities that were given to learners to work with, learners were supposed to give reasons and some form of explanation for the answers they gave. From this research it was evident that learners struggle to explain the reasoning behind their solutions to rational algebraic fractions. For Set A activity, most learners gave the reasons why they agree with the solution of $\frac{p^2q^4}{q^2p^2} = 2$. But only 26 % of learners attempted to give reasons why they agreed or disagreed with the way an algebraic fraction was simplified by Zolile in question number 3 of activity 1, SET A. From the 26% that attempted some gave reasons clearly explaining what was mathematically incorrect in the solution. The figure below gives an example of the learner who gave the reason for the answer.

Lack of deeper explanation (example 1)

3. Zolile simplified $\frac{5(x-1)}{6} + \frac{x-3}{3} - \frac{x-1}{2}$ as follows:

$$\begin{aligned} & \frac{5(x-1)}{6} + \frac{x-3}{3} - \frac{x-1}{2} \\ &= \frac{30(x-1)}{36} + \frac{12(x-3)}{36} - \frac{18(x-1)}{36} \\ &= \frac{30x-30+12x-36-18x+18}{36} \\ &= \frac{24x-48}{36} \\ &= \frac{2x-4}{3} \end{aligned}$$

Explain with reasons why you agree or not with the way Zolile simplified the expression. Can you find another way to simplify the expression?

NO. I do not agree because he made a mistake in step 3, we used the LCD 6.

Figure 4.11. Lack of deeper explanation. (Example 1)

The learner in the figure above did not explain clearly what was wrong and how it could be corrected. The learner used 6 as an LCD to simplify the fractions and did manage to simplify it correctly. In the activities of set B only 34% of learners gave the reasons or explanation for their solutions. Most learners only gave the answers without reasons.

1.1

To simplify $\frac{3a+3b}{a+b}$ two learners worked as given below:

Zodwa:

$$\frac{3a+3b}{a+b} = \frac{3a+3b}{a+b} = 3 + 3 = 6$$

Zizi:

$$\frac{3a+3b}{a+b} = \frac{3(a+b)}{a+b} = 3$$

Which learner performed the calculation correct. Write down reasons for for your choice.

Zodwa. This because.....
.....
.....

1.2

To simplify $\frac{a^2-b^2}{a+b}$ two learners worked as given below:

Patrick:

$$\frac{a^2-b^2}{a+b} = \frac{a^2-b^2}{a+b} = a - b$$

Andishwa:

$$\frac{a^2-b^2}{a+b} = \frac{(a-b)(a+b)}{(a+b)} = a - b$$

With whose method do you agree? Write down reasons for your answer.

~~Andishwa~~ Patrick method.....
.....

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Figure 4.12: Lack of deeper explanation (example 2)

Most learners managed to arrange the steps of a simplified algebraic fraction in activity 2 of Set C but could not correctly answer activity 1 of the same Set C which required them to explain what was done in each step as the fraction was simplified. This shows that they have no deeper understanding of what they are doing as they simplify fractions and hence cannot give a deeper explanation of what they are doing. The questions that required explanations were not answered by most learners; those who answered only gave short explanations with no detail to support their choices. Learners usually use only one method of

simplifying a fraction and do apply their thinking into other ways that can be used to work out problems, this results in learners committing an error when simplifying a fraction in one question using a particular method but same learner is able to solve another that needed the application that he did apply in the previous question. For example in figure 4.9 the learner who incorrectly simplified $\frac{8m^4n}{4m^2n} = 2m^2n$; used the laws of exponents incorrectly because the learner believes $n^{1-1} = n$. But the same learner simplified $\frac{2x}{x}$ correctly by cancelling x . If the learner applied his/her mind and thought about cancelling; could have seen that the n 's can cancel therefore it cannot be correct to have n in the final answer. Only 36% of learners attempted to answer activity 1 of Set C, 16% only explained one step of the solution and 46% did not attempt to answer the question



Activity 1

1. A way to simplify $\frac{3}{a^2-4} + \frac{2}{a-2}$ is given below. Write down what was done in the steps numbered A, B, C & D.

$$\begin{aligned} & \frac{3}{a^2-4} + \frac{2}{a-2} \\ &= \frac{3}{(a+2)(a-2)} + \frac{2}{a-2} \quad \text{Step A: } \underline{\text{They simplify "a^2" and "-4" .}} \\ &= \frac{3+2(a+2)}{(a+2)(a-2)} \quad \text{Step B: } \dots\dots\dots \\ &= \frac{3+2a+4}{(a+2)(a-2)} \quad \text{Step C: } \dots\dots\dots \\ &= \frac{7+2a}{(a+2)(a-2)} \quad \text{Step D: } \dots\dots\dots \end{aligned}$$

Activity 2

The expressions given below can be arranged to give steps to arrive the answer of "simplify $\frac{3x+2}{2} + \frac{3+x}{2} - \frac{7}{6}$ ". Put the steps in correct order to get the answer.

$$\begin{aligned} 1. & \frac{(9x+6)+(9+3x)-7}{6} &= & \frac{3(3x+2)}{6} + \frac{3(3+x)}{6} - \frac{7 \times 1}{6} \\ 2. & \frac{12x+8}{6} &= & \frac{(9x+6) + (9+3x) - 7}{6} \\ 3. & \frac{3(3x+2)}{6} + \frac{3(3+x)}{6} - \frac{7 \times 1}{6} &= & \frac{(9x+3x) + (6+9-7)}{6} \\ 4. & \frac{6x+4}{3} &= & \frac{12x+8}{6} \\ 5. & \frac{(9x+3x)+(6+9-7)}{6} &= & \frac{6x+4}{3} \end{aligned}$$

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Figure 4.13: Lack of deeper explanation example 3

The learner's way of working in the figure above shows that he/she struggles to explain what is being done at each step of simplifying an algebraic fraction. The same learner who cannot explain the steps in a mathematical language but is able to re-arrange the steps correctly. It can be assumed that the child can simplify the algebraic fractions but cannot express what he / she is doing at each step as she/he simplifies. This shows that learners lack conceptual understanding of rational algebraic fractions which means that they cannot really justify why they do what they are doing. It can therefore be argued that learners use their instrumental understanding in working with algebraic fractions. Instrumental understanding is defined by Skemp (1978) as the ability to apply a rule to the solution of a problem without

understanding how it works. This application of rules without understanding how they work has been evident in most learner's work. Learners were also found to apply exponential laws wrongly. Skemp (1978) also describes relational or conceptual understanding as the ability to deduce specific rules and procedures from more general mathematical relations. Learners need to be helped to reach a level of conceptual understanding by using methods of intentional teaching which emphasizes that the learning intentions be made clear, and the success criteria be known. As learners articulate their learning intentions or success criteria for each activity they will challenge their minds to find reasons for what they do and in so doing they can develop their conceptual understanding.

SET C

Activity 1

1. A way to simplify $\frac{3}{a^2-4} + \frac{2}{a-2}$ is given below. Write down what was done in the steps numbered A, B, C & D.

$\frac{3}{a^2-4} + \frac{2}{a-2}$ $= \frac{3}{(a+2)(a-2)} + \frac{2}{a-2}$	Step A: The equation was factorised
$= \frac{3+2(a+2)}{(a+2)(a-2)}$	Step B: The right hand side was added to the left hand side.
$= \frac{3+2a+4}{(a+2)(a-2)}$	Step C: Then we multiplied the number that is added to three to the numbers in the bracket
$= \frac{7+2a}{(a+2)(a-2)}$	Step D:

Figure 4.14: lack of deeper explanation example 4

In the figure above the learner attempted to answer the question by writing what was done in each step as the fraction was simplified. But it seems that the learners struggle to explain mathematically the algorithms followed in simplifying the fraction.

4.6 Conclusion

This chapter has discussed data collection in the form of algebraic problems given to learners to work through and an analysis of how the learners dealt with the problems. It has furthermore discussed the different kinds of errors committed by learners. In addition, learners failed to give extensive and detailed explanations of their answers. The next chapter will provide a summary and discussion of the findings and recommendations for further study.



CHAPTER 5

DISCUSSION OF THE RESULTS, RECOMMENDATIONS AND CONCLUSION

5.1 Introduction

The previous chapter gave a detailed analysis of the data collected and discussed how learners engaged with the algebraic fractions they were required to simplify or the responses to problems whose simplifications were given. This chapter will discuss the findings of the study according to the purpose of the study and the research question. Recommendations will then be given for future researchers and other educational practitioners who will read the findings of this study.

The study investigated grade 9 mathematics learners' ways of working with rational algebraic fractions when an intentional teaching model is implemented. Ways of working refers to the way in which learners engage with rational algebraic fractions when they simplify them including the errors they commit from the misconceptions they have about aspects of working with fractions.

The intentional teaching model implemented emphasizes that the purpose and goals of learning be stated clear to all participants. It also advocates for spiral revision; assessment for learning and fostering of mathematical thinking through appropriate questioning strategies.

The specific question for the research was:

How do grade 9 learners engage with algebraic fractions when an intentional teaching model is implemented?

Learner's scripts with solutions for the given activities focused on rational algebraic fractions were analysed and the questions are outlined in chapter 4. The errors identified were analysed and this chapter will discuss the findings and conclude the thesis.

5.2 Discussion of the results

5.2.1 What the learners' ways of working with rational algebraic fractions in an intentional teaching environment?

It has been a common practice for teachers to present lessons and later give learners activities as means of assessing if learners understood and can be able to implement the knowledge and concepts that have been taught. After the completion of the activities and corrections for a particular topic they then move to the next topic because of time and rushing for syllabus coverage. Contrary to this approach intentional teaching advocates for assessment for learning and spiral revision. In this research the topic that learners dealt with was taught already in the previous term, hence the approach was in the form of revision. From this revision it was clear that learners did not remember most of the concepts they learned before because they consulted their books most of the time before attempting to answer the questions.

As learners were working with algebraic fractions, errors relating to the misconception of exponential laws were identified. In the grade 9 mathematics curriculum in South Africa, exponents are dealt with at the beginning of the first term of the year and algebraic fractions are introduced later towards the end of the term and completed in the second term.

The learners way of simplifying algebraic fractions seemed to be more influenced by their knowledge of exponential laws than other sub-constructs are that are used when dealing with algebraic fractions such as factorizing (understanding of lowest common factor and highest common factor; factorising difference of two squares), ratio (which leads to the understanding of equivalence for example a learner simplifying $\frac{5x^3}{10x^3}$ after cancelling the x^3 would have known that $\frac{5}{10}$ is not equivalent to 2 which can be written as $\frac{2}{1}$. It can be argued that learners are prone to use exponential laws in simplifying algebraic fractions because they did not engage into productive spiral revision that foster their mathematical thinking. The type of activities they work with do not probe their mathematical reasoning behind the correct and wrong answers they give as solutions to the activities. It can also be assumed that, the over emphasis on exponents topic since its done at the beginning of the year could have resulted to the learners ways of simplifying algebraic fractions.

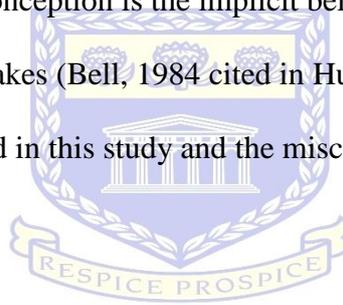
Intentional teaching incorporates principles of assessment for learning (Afl). In chapter 3 assessment for learning was defined as a form of assessment whose priority in design and practice is to serve the purpose of promoting student learning. It does not serve as means of accountability of measure of competence. But it helps learners to evaluate their learning and improve. In this research the questions were designed to assess for the purpose of learning.

The activities helped learners to realise their misconceptions on the topic that was already covered. In the process of working with activities learners enjoyed the different form of questioning and they engaged with each other in groups as they were trying to solve problems. Combining assessment for learning and spiral revision, learners can be helped to understand topics better.

From this research it cannot be concluded that learners were less prone to commit errors when an intentional teaching model is implemented because there was no post data collection after the learners worked with algebraic expressions that would have worked as basis to assess the impact. But it can be stated that the learners reasoning behind their ways of working with algebraic fractions were identified and learners showed positive response to the feedback given after each Activity.

5.2.2 Common Errors and Misconceptions

As it was stated in chapter 2, a mathematical error refers to a mistake or condition of being wrong in the process of solving a mathematical problem (Hurrell, 2013); Godden, Mbekwa & Julie, 2013). A misconception is the implicit belief held by a learner which governs the errors that a pupil makes (Bell, 1984 cited in Hurrell, 2013). Following below are the errors that learners committed in this study and the misconceptions that lead to those errors.



5.2.2.1 Defractionalisation error.

The results indicate that 11% of learners committed this error as they were working with algebraic fractions and this was a lower percentage of errors as compare to other errors that were committed. This type of error has been found in the literature to have a lower frequency in the number of errors that learners commit when simplifying algebraic fractions. Otten, Males & Figueras (2008) found that 5 % of learners committed this error on their research on simplification of algebraic fractions. Learners lack of sub- fractional constructs and their misconception of the inverse operations lead to type of error.

5.2.2.2 Exponential law error₂

Learners' ways of simplifying algebraic fractions used mostly the laws of exponents but learners had some misconceptions about the laws of exponents. Learners understand that when dividing same bases, we subtract exponents. The misconception is that they do not apply all the laws of exponents. When subtracting the value for the exponent such that the resulting exponent is zero, learners still write the variable. This error contributed 11% percent of the errors committed by learners. This error is in two folds; some learners hold a misconception that a variable with zero exponent is zero and others hold the misconception that a variable with zero exponent is equal to that variable as seen in Figure 2.8 and 2.9 in chapter 4.

5.2.2.3 Exponential law error₁

This type of error contributed 13% of the errors that learners committed when working with rational algebraic fractions. The error is made by subtracting exponents of the variables (terms) in the denominator from those of the terms in the numerator. Learners do not use factorisation but rather apply laws of exponents incorrectly. Productive practising can help learners reduce this type of error. Also the assessment for learning can assist learners, if fractions that involve only numbers and then fractions with variables are given to learners to simplify and give explanations for their answers, such can help learners learn to apply their thinking broadly when simplifying fractions.

5.2.2.4 Cancellation error

This type error had 61% of learners who committed it. This is in line with what is found in literature. The research shows that cancellation is most common error when learners work with algebraic fractions. Otten, Males &Figueras (2008) found the cancelation to be the most common error on learner work.

Mhakure, Jacobs & Julie (2005) found the cancellation error to be the most frequent when analysing learners work for sub-fractional constructs. When learners see like terms in the numerator and denominator and denominator they cancel term without factorising. Learners hold a misconception that when you have same variable in the numerator and denominator, you must cancel them for example when learners simplified Activity 1 of SET 8, most learners argued that Zodwa is correct “ *I think it’s Zodwa because when you divide algebraic fraction you need to cancel all the alphabet that is in the numerator so that you can have the numbers and add them*”.

The learners’ argument is that variables must be cancelled so that one can only have numbers. The learner’s way of simplifying rational fractions is that variables cancel, they do not have a correct understanding of why cancellation is done and under which conditions variables can be cancelled. Exercises on fractions where the numerator or denominator is a sum or difference of two numbers with no variables can be another way of alleviating this error. E.g $\frac{2+3}{2}$. Learners can easily realise that if they cancel the 2’s the final answer will be 3 but $\frac{5}{2} \neq 3$. Learners need activities that will help them realise that a variable can represent a certain numerical values.

It can also be assumed that learners fail to recognise the common factor or learners’ ways of working do involve factorisation. An emphasis on the questions that require factorization than on the ones that apply laws of exponents can help learners overcome this error.

5.3 Lack of fluency

Learners struggle to express themselves in mathematical language when they need to explain what they are doing as they simplify algebraic fractional. In answering the questions in activities for Set A and Set B, most learners only gave their answers to whether they agree or disagree without detailed reasoning for their answers. Some only performed the calculations that show how the rational algebraic fractions can be simplified. Set C activity required learners to explain what was done in each step of a simplified rational algebraic fraction but only 36% of learners attempted to answer the question and even from the 36% that answered most explanations were not correctly explaining the algorithms in mathematical language. This is line with the findings of the research by (Ruhl & Balatti , 2011) that showed that when learners are required to write their thinking of what they do when working with fractions, only a few provide a rationale for the procedures they have used to get to the solutions or conclusions.

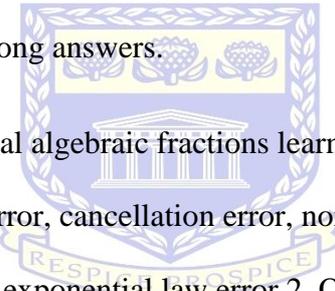
This therefore suggests that educators need to expose learners to more questions that require them to explain what they are doing so that they develop the skill of understanding and using the correct mathematical language to explain the algorithms applied when simplifying rational algebraic fractions.

5.4 Recommendations

The results of this study indicates that learners rely more on exponential laws when working with rational algebraic fractions. The use of factorization and finding the lowest common denominator has been found to be challenging for learners as a result most learners struggle to add or subtract algebraic fractions. Educators need to apply assessment for learning. As much as it is difficult to give much attention to each topic because of time and

pressure to complete syllabus, educators need to give assessments that will combine activities from the topics previously done and the topic that learners are currently doing as part of spiral revision.

These type of assessments can help learners overcome their misconceptions and build a good link between each topic. They would be able to correctly apply knowledge previously learned in another topic in the topic that they are doing. For example in this research learners were incorrectly applying their knowledge of simplifying exponents in simplifying algebraic fractions. If these learners did engage in some form of spiral revision where they do activities that will help them realise how exponents apply in simplifying algebraic fractions and where they do not apply. Assessments should not focus on the right or wrong answers but on the reasoning behind the right and wrong answers.



When working with rational algebraic fractions learners were found to have committed DE-fractionalization error, cancellation error, non-recognition of common factor error, exponential law error 1 and exponential law error 2. Of the errors that learners committed the cancelation error was found to have a higher frequency. In committing this error, it was found that learners cancel because they believe any like terms (which are variables) in the numerator and denominator must be cancelled; another reason for learner's cancelation error was that they subtract exponents of the same bases when they divide but they apply this law in terms without factorising first.

According to Piaget (1985), new information is shaped using the learners' existing knowledge and the existing knowledge is then modified to accommodate the new information. This process happens in three major concepts namely: assimilation; accommodation and equilibration. In the assimilation stage the learner perceives new information in terms of the existing cognitive structures. In this particular case when learners

see division of the same variables they interpret the information or problem in terms the existing knowledge of laws of exponents? Accommodation is when a learner's previous knowledge has been modified to fit in the new knowledge. The last concept equilibrium encompasses both assimilation and accommodation.

The results of this study show that learners do not accommodate new information. They only assimilate knowledge and do not get a point of modifying their previous knowledge to accommodate new knowledge hence they commit cancellation error because of incorrect use of exponential laws. Their incorrect use of exponential laws when working with algebraic fractions shows that they did not accommodate the new information and modified their previous knowledge of exponential laws to fit into the simplification of rational algebraic fractions. The learners need challenging questioning that will help them complete puzzle of all the content they receive and not build it as separate pieces that do not complement each other.

From the study we also realise that learners forget the content previously taught, because in simplifying algebraic fractions learners kept referring on their note books. This shows that a form of spiral revision is needed to keep the memory of learned content fresh on learners' minds.

I propose that teachers use the type of questions where learners are required to explain what they are doing in each step as continue working with algebraic fractions. This can help them to reflect and realise they are doing the wrong thing when they get to a point where they cannot give reason for what they are doing. It can also help teachers to identify learner's misconceptions which lead to a number of errors.

Marshall & Drummond, (2006) cited in Moreland & Cowie (2013) argue that "the spirit of AfL is evoked when teachers have a pedagogical mind-set that foregrounds the sharing of responsibility with students as the norm, and when they provide students with opportunities,

and the means, to exercise responsibility for their learning and learning progress. This implies that teachers should use assessment positively with the intention of creating a classroom atmosphere that is conducive for learners to own their learning process and be able to self-evaluate their learning and that will reduce the teacher's responsibility because it is shared.

5.5 Recommendations for further research

I recommend that further research be conducted on the implementation of intentional teaching over a longer period, where the assessment for learning with spiral revision can be implemented over a longer period. Learners need to be exposed to the type of questioning that requires them to give in detail the rationale behind their manipulations as they work with fractions. The researcher can do a pre-assessment and the post assessment of learners' ways of working fractions so that an impact of revision can be determined.

5.6 Conclusion

The aim of the research was to investigate the learner's ways of working with algebraic fractions including the errors they commit when they simplify fractions. The results of the study indicate that learners rely on their understanding of laws of simplifying exponents when they simplify algebraic fractions and they struggle to add and subtract fractions. With fractions that need factorization learners have difficulties in simplifying them. The study also reveals that learners use their instrumental understanding when they work with rational algebraic fractions, they lack adequate conceptual or relational understanding of what they do in each step as they work with rational algebraic fractions.

The type of errors committed by learners included cancellation error; no recognition of common factor and exponential laws errors. Of the errors committed, cancellation error had a higher frequency. It was also identified from this study that learners struggle to give a detailed explanation of what they do as they simplify rational algebraic fractions. From the

conceptual framework we expected to see these types of errors because they are common in the literature for algebraic fractions but the exponential law errors came as a surprise.

Misconception of exponential laws appeared to be common to most learners and it is the result of wrong cancellation at times.

Learners have a number of misconceptions about the topic because it encompasses other topics such as laws of exponents; factorization; addition and subtraction of ordinary fractions and other algebraic operations such as addition of like and unlike terms. When working with rational algebraic fractions they bring all the misconceptions they had from each topic which makes them commit a number of errors.



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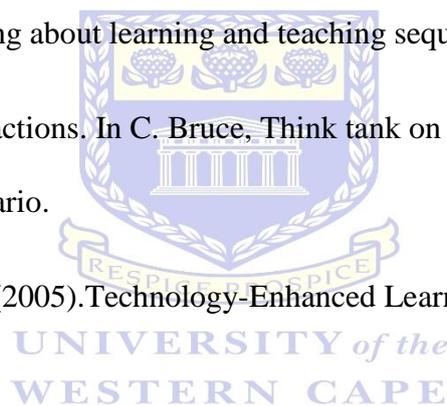
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APPENDICES

Appendix 1a

Dear Participant

Project details and information

I am currently busy with my masters at the University of the Western Cape, Faculty of Education and would be conducting research in your school. I will collect the data in mathematics classes that I request you to participate in.

My topic is: Implementing an intentional teaching model to investigate grade 10 learners' ways of working with rational algebraic fractions.

The purpose of the study is: To investigate the learners' ways of working with algebraic fractions in an intentional teaching environment.

Consent

Should you agree to take part in this research, you will be asked to sign this letter of consent.

Two copies are required, one for our records and one for your records.

You will be aware that data collected during this research might result in research which may be published, but your name will **not** be used and any information you disclose will be kept confidential.



You may also refuse to answer any questions that you are not comfortable with.

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You may withdraw from this study at any time.

Date: 21 january 2016.....

Learner's Name:

Learner's Signature:

University Official's Name: Prof M. MBEKWA.....

University Official's Signature.....

Researcher Name: N.V MAPHINI

FACULTY OF EDUCATION

Researcher Signature:

Private Bag X17,
Belville, 7535
South Africa
Tel: +27 (0) 21 959 2861
Fax: +27 (0) 21 959 3358
Email: cjulie@uwc.ac.za

If you have any questions concerning this research, free to contact call Ms N.V MAPHINI

at(0769416323) email address:3406582 @myuwc.ac.za or my supervisor, Prof M. Mbekwa

at (021) 959 2957, email address: mmbekwa@uwc.ac.za



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Appendix 1b

Letter of Consent

Dear Parent

Project details and information

I am currently busy with my master's research at the University of the Western Cape, Faculty of Education.

My topic is: Implementing an intentional teaching model to investigate grade 10 learners' ways of working with algebraic fractions.

The aim of the study is: To investigate the learners' ways of working with algebraic fractions in an intentional teaching environment.

Consent

This consent asks for your permission to allow your child to participate in this project.

Should you agree to let her/him to take part in this research, you will be asked to sign this letter of consent. Two copies are required, one for our records and one for your records.

You will be aware that data collected during this research might result in research which may be published, but your name will **not** be used.

Your child may also refuse to answer any questions that he/she is not comfortable with.

The child may withdraw from this study at any time.

Date: 21 january 2016
.....

Parent's Name:

Parents's Signature:

University Official's Name.....

University Official's Signature.....

Researcher Name: ...NWABISA VIVIAN MAPHINI.....

Researcher Signature:

If you have any questions concerning this research, free to contact call Ms N.V MAPHINI

(0769416323) at email address: 3406582 @myuwc.ac.za or my supervisor, Prof M.

Mbekwa at (021) 959 2957, email address: mmbekwa@uwc.ac.za



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Appendix 1c

Letter of Consent

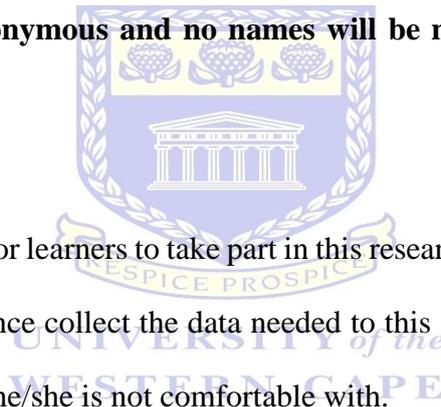
DEAR PRINCIPAL, SGB & GRADE 10 HEAD

I am currently busy with my masters at the University of the Western Cape, Department of Science and Mathematics Education.

My topic is: Implementing an intentional teaching model to investigate grade 10 learners' ways of working with rational algebraic fractions

The consent form herewith asks for your permission to conduct research in your grade 10 mathematics class. I would like collect data from learners through written notes, audio records, and learners written work. This will support me with the data collection process for the study.

All information will be anonymous and no names will be mentioned in any reports or discussion documents.



Should you give permission for learners to take part in this research, I will have the opportunity to work with learners and hence collect the data needed to this study. The learner may refuse to answer any questions that he/she is not comfortable with.

You will be aware that the data collected might result in research which may be published, but no name of the learner or school will be used.

Date: 21 JANUARY 2016.....

Principal Name: MR NGCENGE.....

Principal Signature:

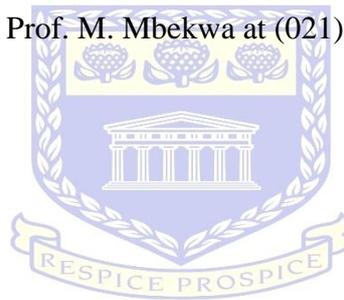
SGB Member Name..... Signature.....

SGB Member Name.....Signature.....

Researcher/Interviewer Name: NWABISA MAPHINI

Researcher/ Interviewer Signature:

If you have any questions concerning this research, free to call Ms.N.V MAPHINI at
(0769416323), or my supervisor, Prof. M. Mbekwa at (021) 9592957



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