EXPLORING GRADE 9 MATHEMATICS LEARNERS LEARNING OF CONGRUENCY BASED PROOFS IN GEOMETRY VIA A WEB-BASED LEARNING SYSTEM

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Declaration

I declare that *Exploring grade 9 mathematics learners learning of congruency based proofs in geometry via a web-based learning system*, is my own work, that it has not been submitted before for any degree or examination in any other university, and that all the sources I have used or quoted have been indicated and acknowledge by complete references.

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Taariq Chetty December 2022

Dedication

This humble piece of work is in memory of my beloved parents; Fatima Chetty and Adeel Chetty and my sisters Tahseen Chetty and Kishmia Chetty for their love and support which surpasses time and space. To my parents for their prayers, support and the sacrifices that they made for me. I also thank my supervisor Professor Rajendran Govender for his encouragement, patience understanding and diligent supervision of my work.

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I thank them for helping me get through all my difficult moments which would have stopped me from completing this degree and for always encouraging me to complete this degree.

Abstract

Globally, research and evaluation reports show that students are not learning congruency-based proofs, as part of the Geometry section of mathematics, efficiently. One identifier of student understanding related to geometry is the teacher's method of instruction. In order to attain success in mathematics the understanding of proofs and writing of proofs are of utmost importance. In this regard, web-based learning could be used in school mathematics to enhance activities involving "proof". Proofs are the heart of mathematics and digital resources may be used to teach learners effectively, starting from primary school level. The success of this process does not only depend on how effectively teachers use resources to teach proofs, but also on students' perceptions on using web- based learning to understand congruency- based proofs.

This study involved exploring how grade 9 learners learn congruency- based proofs via a web-based learning system. In this sense, the study was oriented by the research question: To what extent do grade 9 learners discover, apply and provide proofs for congruency theorems?

Firstly, an investigation task was issued to learners to complete in a web-based learning environment, secondly students were interviewed individually and thirdly an observation schedule was conducted and completed by the researcher. There were 22 grade 9 students who voluntarily participated in this study. After participating in the study each student was interviewed about what they thought about the investigation task on congruency-based proofs and also their experiences of using web-based learning.

The findings of the study reveal that the use of web- based learning does enhance learners understanding of congruency-based proofs. Also, the success of teaching congruency- based proofs lies in the quality of the lesson taught. Another result of this study concerns students' perceptions of using web-based learning to understand congruency-based proofs. Here, students agreed that it helped them understand congruency-based proofs better.

This study revealed that the use of web-based learning can lead to success in learners' proving of congruency-based proofs. What underlies this is that web-based learning creates and environment that is conducive for learners to understand and write congruency based- proofs. Another possible reason is that learners today are more

technologically inclined and are, thus, able to respond to web-based learning better when compared to traditional teaching methods.

Keywords: Congruency, Geometry, Proof, Web based learning

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Key Words

Congruency: Equal in size and shape. Two objects are congruent if they have the same dimensions and shape.

Geometry: Is a branch of mathematics concerned with questions of shape, size, relative position of figures, and the properties of space.

Proof: A proof can be described as the process of justifying by explaining, arguing and verifying a particular statement**.**

Web based learning: Often called online learning or e learning because it includes online course content

CHAPTER 1

INTRODUCTION, BACKGROUND, RATIONALE AND RESEARCH QUESTION

1.1. Introduction

Congruency constitutes a key topic in school geometry. The reasons for this are two-fold. First, the four conditions for triangle congruency (side-angle-side, SAS; side-side-side, SSS; side, angle, angle, SAA; right angle, hypotenuse, side, $90°HS$) can be used to prove many more propositions, as in the books of Euclid. Second, triangle congruency links to the even more powerful mathematical topic of similarity (via the idea of similar triangles). Despite congruence in general, and congruent triangles, holding a key position in the geometry curriculum, there is, as far as I have been able to ascertain, little research on the teaching of congruency; at least, little published in English.

Problem solving in geometry becomes enhanced, when learners actively look for the relationships between geometric figures. According to Driscoll et al (2010) "This may be congruence, similarity, equality, supplementary, parallelism, and other relationships Often, the relationships can be exploited to arrive at exact solutions to the problem, and without relationships, solvers are usually reduced to finding approximate solutions, at best." Furthermore, generalizing geometric ideas, which is a habit of wanting to understand and describe the 'always' and the 'every' related to geometric phenomena, involves establishing what remains invariant among the attributes of a geometric figure through experimentation in most instances.

This study focuses on the geometry section of mathematics. It deals with congruency, mathematical experimentation, discovering proofs, inductive reasoning, conjecturing, deductive reasoning, justifying, solving of geometrical problems and proof competencies, as well as proof competency strategies. Discovery and re-discovery through investigation, using an invariant as the focal point, making conjectures with inductive or deductive reasoning is a core aspect of the study. This lends itself to proving the statements by writing proofs. Proof writing is the act of logically explaining a statement or phenomena in mathematics. What is required would be understanding, mathematical skills and the necessary competencies in order to make a justification. Strategies used by the learners involved in this study will be scrutinised and assessed.

This study provided an opportunity for grade 9 learners at a school in the southern suburbs in Cape Town, South Africa, to experiment with congruency problems within a classroom setting and discover attributes which are invariant. Subsequently, it explored their ability to apply what is taught when solving congruency problems and then logically explain (prove) the validity of their invariants, which would have been captured as conjectures. It also investigated their ability to prove or solve congruency problems invoking the proven congruency theorems. The theorems were investigated and analysed using six proof strategies. The review of research for this study included experimentation within a classroom context, inductive and deductive reasoning, conjecturing, generalizing, proof and proving.

1.2. Background and Context

In South Africa (SA) the mathematics curriculum has changed over the past twenty years and more vastly over the past fifty years (Jojo, 2019). The change has led to a lack of consistency and proper planning when it comes to the teaching and learning of geometry in classrooms. There is a shared view amongst researchers that teacher's inability to give learners the opportunity to apply themselves and discover the connections between proofs, has led to misconceptions with regards to the topic and creates a gap in knowledge when learners progress to the next grade. The respective diagnostic reports Trends in International Mathematics and Science Study (TIMSS) on the ANA grade 9 Mathematics examinations, have consistently reported that learners struggle with doing congruency proofs. For example, TIMSS (2015, p.74) states that given a question based on space and shape (geometry):

"Only 3.5% of South African learners answered this correctly. Learners could not identify the similar triangles and hence could not calculate the ratio between the given sides of the triangles. Learners struggle with the topics similarity and congruency in grades 8 and 9."

So it is quite clear that there is a major gap in knowledge when it comes to application and proofs in congruency theorems with respect to grade 9 learners. This motivates for improved teaching and learning methods with regards to this specific topic. According to TIMSS (2015, p.74) "Learners should be encouraged to apply various maths concepts to solve different challenging questions. Teachers must constantly present problems that deepen cognitive thinking and ability." There was a shared view that the DBE (2017 and 2018) promotes the idea of learners applying themselves with regards to congruency and teachers have a responsibility for facilitating this.

Given the above mentioned, it is evident that Congruency is not a well answered section in the grade 9 examinations and there are many aspects that need work. It is the inability of learners to write proofs, which shows that there is a lack of understanding of congruency theorems.

Researchers agree that software programs or any technology in the classroom enhances learning and allows more time for discovery (Hölzl, 1996; Arzarello, Otivero, Paola, & Robutti, 2002; Geiger, Faragher, Redmond & Lowe, 2008). In addition, there is a shared view that teachers should structure their lessons in a way which allows learners to make connections when it comes to geometry. However, the teachers have stressed that the demands of the curriculum prevents them from giving learners this opportunity. These demands are due to the constraints that the curriculum comes with and time management is an issue. Regardless, congruency proof writing remains an issue for most grade 9 learners across our South African classrooms, which leads to adverse effects as they progress.

1.3. Rational and Motivation

Congruency theorems, particularly the use of the discovery learning method to analyse and enhance how learners apply their thinking and abilities in proof writing has not been well studied. Research regarding application and proofs when it comes to congruency theorems explains the various types of strategies which are used. However, there is limited studies analysing learner's process through discovery, resulting in reasoning with understanding and ultimately their proof writing abilities in South Africa. As an educator, it is notable that learners lack the necessary skills to develop proofs and apply what is taught. I have been strongly motivated to not only investigate the capability of the learner's proof writing competency skills, but also to determine whether learners identify the relationship between application and proofs.

My research into mathematics education is based on observations of poor math performance in South African schools. Using technology to teach mathematics in the classroom is something I am passionate about, as I believe it enhances students' mathematical thinking and can help improve their performance. Furthermore, learners should see that mathematical investigations are essential when it comes to solving congruency theorems.

The correct use of the discovery learning method could lead to an improvement in the teaching and learning of mathematics. Mathematical investigations are not being used enough in schools. Due to time constraints and demands from the curriculum itself, many teachers are not able to allow learners to investigate for themselves. I believe that the use of discovery learning method could solve this crucial problem and thus the importance of mathematical investigations will not be neglected in schools.

As a high school student, I was taught geometry through traditional instructional approaches. When it came to the topic of congruency theorems, I simply memorized the theorems written on the board by my mathematics teacher. I encountered an issue with this approach because there was no scaffolded guidance, where we could apply what was taught properly and develop a proof for the relevant congruency theorems. If the problems tested, were not like those taught in class, my classmates and I would struggle to make connections and solve the congruency-based proof problem. However, during my studies in completion of my B.Ed. Honours degree at the University of Western Cape, I came across the discovery learning method, which allows learners to take charge of their own learning through scaffolded worksheets. This ultimately helped me to see the importance in teachers creating an opportunity for learners to apply themselves, make connections and ultimately write proofs in the context of geometry. For example, I could work with complex diagrams, identify relationships, illustrate reasoning and create arguments consisting of several connected statements. Hence, as a teacher now in high school, I would like to respond to the crisis highlighted by the DBE (2017 and 2018) and support the learning trajectory which I had encountered in my B.Ed. Honours classes, to my grade 9 learners.

1.4. Problem statement

Taking into consideration the mentioned contextual aspects prevailing in the teaching of congruency across our classrooms in high schools, as well as findings articulated in the DBE (2017 and 2018) diagnostic reports it is quite evident that very minimal is done to create an environment and create opportunities for learners to discover and explore the relevance between application and proofs in congruency. Furthermore, Diagnostic reports and evaluation reports confirm that Grade 9 learners tended to struggle with reasoning proving of congruent triangles and making related mathematical deductions (DBE, 2017 and DBE, 2018). Understanding what a proof is and writing proofs are essential for success in mathematics. Thus, school mathematics at grade 9 should include proving activities. Proofs are the heart of mathematics and proving is complex; teachers should help their leaners develop these processes in the early grades (like grades 8 and 9). The success of this depends on the kind of pedagogical moves that a teacher incorporates in engaging leaners in proving kind of activities in geometry. Moreover, findings show that learners struggle to formulate proofs for congruency theorems due to their inability to apply what is taught to them (DBE, 2017 and DBE, 2018) . Hence, it is necessary to explore grade 9 learners' learning of congruency-based proofs in geometry via a web-based learning system.

1.5. The aim of the study

The aim of this study was to explore grade 9 learners experience of learning congruency-based proofs in geometry in a web-based learning system especially with regard to their proof writing ability and making mathematical deductions.

1.6. Research objectives

To achieve the aim of the study the following objectives were set out:

(i) To explore the proof writing competencies that grade 9 learners demonstrate when attempting to prove triangles congruent in a web-based learning environment.

(ii) To ascertain the extent to which grade 9 learners can use their constructed congruency proofs to make mathematical deductions in a web-based learning environment.

1.7. Research questions

(i) What proof writing competencies do grade 9 mathematics learners demonstrate when attempting to prove triangles congruent in a web-based learning environment?

(ii) To what extent can grade 9 learners use the congruency- based proofs to make mathematical deductions in a web-based learning environment?

1.8. Significance of the study

The results from this study could be very beneficial to educators in South African schools, especially those who have access to computers, but do not utilise them for mathematics teaching. It should increase the knowledge about the importance of the relationship between application and proofs when it comes to congruency theorems and how students can use them to their benefit.

It should help understand different ways to improve congruency theorems and the use of exploratory methods to creating proof. Through teaching using application and proofs, the study can help educators have a deeper understanding into learners reasoning skills and proof competency. The poor performance of South African learners in geometry is an indication that it is important to find solutions to enhance student performance not only in Congruency Theorems but also in different strands of geometry. Ultimately, the study hopes to provide a positive contribution to the teaching of mathematics to all South African schools.

1.9. Chapter Outline

This thesis comprising five chapters, outlining the study in the following way:

Chapter 1: Introduction, Background, Rationale and Research Question

In this chapter, an introduction to the study was given along with reporting on the background, the problem statement, purpose, research question, and its significance. Furthermore, the research design and methodology, study limitations are discussed.

Chapter 2: Literature Review and Theoretical Framework

This chapter started with discussing congruency-based proofs and web-based learning. It argued the need for using web-based learning in order to teach congruency-based proofs. The theoretical framework characterised by Mcrone and Martin (2011, p.8) proof writing competency model provided a lens to explore the topic under investigation.

Chapter 3: Research Methodology

In this chapter, the study's research design and methodology are presented and explained. First, the research paradigm resonating the study purpose is highlighted, then the research approach, population and sampling techniques, and method and procedure of collecting the data are described and discussed. These are followed by strategies used in analysing the data to gain an understanding of Grade 9 learners' proof writing competencies demonstrated through solving congruency-based proofs in a Western Cape school. Issues of trustworthiness of the results, along with ethical considerations, are dealt with, to ensure a reliable reflection for the community of researchers and practitioners to use and consider in further studies.

Chapter 4: Data Analysis

In this chapter, the analysed data results was reported and a detailed discussion provided for the findings according to the literature review and theoretical framework presented in Chapter two.

Chapter 5: Discussion of results, recommendations and conclusion

Chapter five comprises a summary of the findings, which assisted in answering the research question, the discussion of these findings' implications and recommendations. These are followed by suggestions for future research, acknowledgement of the study limitations, and ultimately the study conclusion.

1.10. Conclusion

This chapter gave a brief introduction to the topic of this research, research questions and the rationale for the study. The terms used in this study has been correctly defined. The following chapter will provide the literature review and theoretical framework for the study.

CHAPTER 2

LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2. Literature review.

The historical development of congruency and the relevance of applications and proofs has been well documented. According to Dokai (2014, p.27) “For over 2000 years the SAS theorem was proved by the method of superposition to establish the congruence of two triangles by superimposing one triangle on the other.” So it took quite a while for an established proof to be developed for SAS theorem. According to Dokai (2014, p.27) “The search for an analytical proof involved digging deep into past literature on the beginnings of geometry including the masterpiece, Euclid’s Elements.”

Despite Euclid using superposition to prove congruency in triangles, it was well documented that he was not satisfied with this method. According to Hann (2019, p.1) “he was not happy with the proof, as he avoided similar proofs in other situations. The way he proved it, is to move one triangle until it is superimposed on the other triangle.”

Presently, superposition is not considered to be legal now, since it involves some complicated assumptions. So as a result of this, SAS is given as an assumption or postulate, it is called the Side-Angle-Side Postulate. A shift away from using assumptions was came about using a proof for each congruency case. According to Hann (2019, p.1) “These two are now assumed as postulates, it is easy to construct a triangle given the lengths of the three sides, or two sides and the included angle, or two angles and the included side.” Most authors have found it strange that the corresponding congruence theorems cannot be proven, except by a slightly shady means called superposition. Furthermore, according to Haan (2019, p.1) “The situation SSA (or ASS) is not a theorem or postulate, as it is often ambiguous there are often two different non-congruent triangles with two congruent sides and a congruent angle at the end of one of the sides.” So this indicates that authors have questioned the validity of the SSA theorem, which sparked a shift away from the older methods to an analytic method of proof.

Today, the Two Column proof is now taught and used within mathematics classrooms. According to Hann (2019, p.1) the two-column proof is “a kind of proof in which the statements or conclusions are listed in one column, and the reasons for each statements truth are listed in another column. Identical in content, but different in form, from a paragraph proof. These proofs are based on set conditions or facts which have been proved to be true about triangles which are congruent. The two column proofs highlight a shift towards mathematical reasoning when it comes to solving problems. According to Hann (2019, p.1) “there are six facts for every set of congruent triangles. Fortunately, when we need to prove that triangles are congruent, we do not need to show all six facts are true.” So the historical development shows a shift away from using assumptions, to actually use well developed proofs for congruency.

A considerable amount of research has focussed on the importance of Application and Proofs in high school geometry. According to Goldberger (2002, p.1) “a proof of a theorem is a finite sequence of claims, each claim being derived logically (i.e. by substituting in some tautology) from the previous claims, as well as theorems whose truth has been already established. The last claim in the sequence is the statement of the theorem, or a statement that clearly implies the theorem.”

The importance of proofs in mathematics teaching and learning has been stressed by many researchers. According to Goldberger (2002, p.1) “It is difficult to overestimate the importance of proofs in mathematics. If you have a conjecture, the only way that you can safely be sure that it is true, is by presenting a valid mathematical proof.” Another importance of mathematical proof is the insight that it may offer. Being able to write down a valid proof may indicate that you have a thorough understanding of the problem

There has been a great deal research when it comes to peoples understanding of proofs, according to Goldberger (2002, p.3) “part of the difficulties of people in understanding the notion of proofs stem from the fact that people do not have the right picture of what mathematics is.” According to Goldberger (2002, p.3) “mathematics is about understanding the laws behind numbers, algebra and geometry. It is about finding new and non-routine ways to look at these systems and explain strange phenomena that we may encounter.” One can conclude that if learners struggle to provide proofs for congruency theorems, they have limited knowledge on the mathematics behind topic.

In addition, there are many authors which emphasize the importance of proofs when it comes to learners linking mathematics to real life situations. According to Cooper (undated, p.3) “written proofs are a record of your understanding, and a way to communicate mathematical ideas with others. Doing mathematics is all about finding proof and real life has a lot to do with mathematics, even if it does not look that way very often.” So for learners to best understand congruency theorems, it is important for teachers to design lessons which allow learners to construct logical mathematical proofs. If this is not achieved, it could lead to learners unable to see the relevance of mathematics in their daily lives.

Design-based methods have been extensively researched, specifically in Mathematics Education in developing countries; it is a method which allows for a structured lesson. Furthermore, through studying the enactments of the activities designed and compared to the results of the desired or undesired outcomes of the study, the research is acquired (Cooper; Stein & Lane, 1996; Barab & Squire, 2004). Sandoval and Bell (2004) state that designed lessons in the classroom allow for good clinical trials. Since this research methodology involves designing investigations, which allows learners to make deductions and conjectures, it is best suited for this study.

There is a shared view that improved reasoning, problem solving and learner performance can be achieved when lessons include a number of approaches to find solutions and encourage learners to provide explanations for their reasoning and ultimately make conjectures (Tarr, 2008; Stand & Lane, 1996). So in essence lessons of this nature promote learner application and providing proofs to further enhance their understanding of congruency theorems.

Based on the above-mentioned information, learning theories which will be utilised were decided as it best suits this study. The basis of which learning theories have been chosen were based on the studies that determine how a learner gains information and ultimately learns. This study will use the constructivist approach to learning in mathematics, which is a theory that is underpinned by guided learning.

Therefore, this study aimed to determine, firstly, to what extent grade 9 learners discover, apply and provide proofs for congruency theorems. Secondly, the study will have an analysis of learner responses and how the learners justify conjectures based on their explanations and findings. Lastly, to determine if learners can apply congruency theorems when it comes to more complex questions in geometry. This study will focus on theorems in Congruency that involves triangles.

It is important to discuss the four cases of congruency within the context of this research, because one must first have sound knowledge and an adequate understanding of the topic under discussion, before exploring congruency-based proofs and learners proof writing competencies. If one does not understand the four cases of congruency thoroughly it could lead to misunderstanding of learner’s ability to solve congruency-based proof problems.

2.1. The Four cases of Congruency.

There are four cases of triangles congruency, Side Angle Side (SAS), Angle Angle Side (AAS), Side Side Side (SSS), Angle Side Angle (ASA). SAS is a rule used to prove whether a given set of triangles are congruent. In this case, two triangles are congruent if two sides and one included angle in a given triangle are equal to the corresponding two sides and one included angle in another triangles. It is important to remember for this case of congruency, that the included angle must be formed by the two sides for the triangles to be congruent.

AAS states that two triangles are congruent if their corresponding two angles and one non included side are equal. The SSS rule states that two triangles are congruent if their corresponding three side lengths are equal. The ASA rule states that two triangles are congruent their corresponding two angles and one included side are equal.

For this research it’s important to understand the four cases of congruency, if one does not understand it, it becomes difficult to understand congruency-based proofs. The four cases of congruency was the basis of the proofs in this study.

2.2. How do learners construct a triangle given three sides?

There was much research on learners construct a triangle given three sides. There are ten steps that learners can use in order to sketch this triangle given the information. Before learners construct a triangle given three sides it is important for the learner to know some important information. According to Byjus (2021) “ SSS Congruence rule: If three sides of one triangle are equal to the corresponding sides of another triangle, then the triangles are congruent. Constructing triangles using SSS congruence criteria is possible when all the three sides are known to us. The requirements for construction are: A ruler and a compass. Side-Side-Side is one of the properties of congruent triangles.” So learners usually use a ruler, compass and protractor in order to sketch a triangle, these are important tools that learners must have. Furthermore according to Byjus (2021) “By SSS rule, construction of a triangle is possible with three given side measures. For the construction of a triangle, first identify the longest measure among the three side measures. Draw the longest side measure as the base of the triangle, then take other measurements using a ruler to mark the arcs by taking the endpoints of the base as vertices. Finally, join the intersection of arcs with the endpoints of the base to get the required triangle.”

Constructing SSS Triangles

Let us consider a triangle ABC, having the measurement of sides equal: AB = 7 cm, BC = 4 cm and CA = 6 cm. The steps for construction of triangle are:

- **Step 1:** Mark a point A
- **Step 2:** Measure the length of 7 cm using compass and scale
- **Step 3:** With the help of Compass mark an arc placing pointer at point A
- **Step 4:** Mark a point B on the arc
- **Step 5:** Now measure the length of 6 cm
- **Step 6:** Again using compass mark an arc above point B using the same point (A)
- **Step 7:** Measure the length of 4 cm
- **Step 8:** Using the compass placed at point B cut an arc such that it crosses the previous arc.
- **Step 9:** Name the point as C where the two arcs cross each other
- **Step 10:** At the end, join the points A, B and C with the help of a ruler to give the required triangle.

Thus, the obtained triangle is the required triangle ABC with the given measurements. The above-mentioned steps are fundamental to this study as it provides a step by step procedure that could prove useful to learners sketching triangles correctly and thus allowing them to compare which triangles are congruent correctly.

2.3. What are the challenges that learners have when it comes to constructing triangles?

There are many challenges that learners have when it comes to constructing triangles. According to Leon & Timon (2017) "A very particular aspect of the study of geometry is the construction of triangles. The triangle is the simplest flat figure. It comprises, with only three sides, three angles and three vertices, a defined part of the plane." Furthermore according to Leon & Timon (2017) "Its simplicity makes it a basic tool for decomposing more complex figures and making measurements, and is therefore considered of the basic mosaic of geometry."

According to Da Silva & Santos (2019) "In the process of constructing triangles, the abstraction of the concept of angle and of measurement are essential, since some of the errors produced by the students in the resolution of tasks that involve the construction of triangles are related to the construction of angles given a certain amplitude and with the measurement of line segments."

Furthermore, according to Da Silva & Santos (2019) "When constructing triangles with ruler, compass and protractor, various concepts and properties should be used. The previously studied concepts and properties must be related and applied at the time of construction." So it is of utmost importance that learners apply all of these concepts, otherwise errors will occur, when constructing triangles.

There is a link between constructions and proofs of congruent triangles, if learners are unable to sketch a triangle it could lead to a lack of understanding when comes to solving congruency-based proof problems. It is important that learners accurately construct triangles in order to gain the necessary knowledge when it comes to identifying which sides and angles are the same between two triangles, which helps with proving that the two triangles are congruent. Often we assume that learners know the difference between an angle and a side at grade 9 level, but constructions can expose and bring out learner misconceptions of the difference between an angle and a side. It is this misconception that could lead to errors when learners attempt to prove that two triangles are congruent.

There are many reasons as to why learners struggle with constructing triangles, according to Zamgni & Crespo (2016) "In the various stages involved in the construction of the triangle, different difficulties can arise if the properties of the triangles are not taken into account in their construction." Also, according to Van de Walle (2009) "The student's difficulties in solving tasks involving the construction of two-dimensional representation of triangles are sometimes also due to the lack of experience and skills in the use of the didactic materials needed for this construction." Furthermore according to Da Silva & Santos (2019) "These difficulties are also due to the appeal to their properties and the fact that visualization plays an important role in the decoding and transformation of mathematical information."

There is a shared view that in order to understand the difficulties that learners have with constructing triangles, it is important to understand the concept of measurement and to understand the concept of angle. According to Da Silva & Santos (2019) "The difficulty in understanding the construction of triangles begins first by understanding the concept of angle and the concept of measurement, then goes through the use of ruler, compass and transferor, and by understanding its properties."

There exists extensive research on the topic of measurement. According to Battista (2007) "The concept of measurement is built along the construction of the geometric conceptualization, reasoning and application." Furthermore, according to da Silva & Santos (2019) "Based on other authors, Battista (2007) considers that despite the importance of geometric measurement, the performance of some students in measurement tasks is low. Furthermore according to Da Silva & Santos (2019) "That is, they do not adequately establish the connection between numerical measurements and the iteration process of unitary measures." So it is clear that learners struggle to make connections between these concepts, which is critical to learner success in terms of constructing triangles. According to Da Silva & Santos (2019) "For example, students who incorrectly measure an object's length when one of its edges is not zero aligned in a ruler do not conceptualize clearly how the numerical ruler marks indicate the iteration of the lengths of the unit."

Furthermore according to Da Silva & Santos (2019) "many traditional curricula prematurely teach numerical procedures for geometric measurements, students have little opportunity to think about the adequacy of the numerical procedures they apply and do not have sufficient opportunities to develop skills with spatially structuring units of measurement." So it is clear that there is a view that the curriculum restricts learners from learning construction of triangles effectively.

According to Joram, Subrahmanyam & Gelman (1998) cited in Da Silva & Santos (2019) argue that although younger students can learn some simple measurement skills, real physical measurement, such as determining lengths, for example with physical objects serving as a unit of measure, is a challenge for many, which may mean that student's knowledge of linear measures may be more superficial than it seems." Furthermore according to Battista (2007) "The research of several authors suggests that students construct a meaningful understanding of the measure of length as they abstract and reflect on the process of iteration of units of length."

The next concept that needs to be understood would be an angle. According to Da Silva & Santos (2019) "Students have great difficulty in learning the concept of angle. Angle is considered to be a multifaceted concept (Douek, 1998), since there is a great variety of definitions presented in school textbooks and textbooks for teaching and training (Mitchelmore & White, 2000)." According to Da Silva & Santos et al (2019) "Some authors consider that the angle can be defined based on three different aspects, as a quantity of rotation between two lines meet at a point (turn), as a union of two rays with a common extreme point (ray) and as the intersection of two half planes."

This concept is constructed slowly and progressively (Da Silva & Santos, 2019). According to White & Mitchelmore (2003) "it is clear from the research literature that students have great difficulty in co-ordinating the various facets of the angle concept. In addition, several authors have observed that students also have different angle conceptions." Furthermore according to Mitchelmore (1998) "The process of constructing the concept of angle crosses several obstacles, and the difficulties that students encounter in learning angles and the mistakes made were observed in several experimental studies."

There are many studies that have brought forward learner misconceptions. According to Berthelot & Salim (1996) "report, based on a Close Study (1982), that when students respond that the larger angle is the one that has one or both sides longer, without taking into consideration the space between them." Furthermore, according to Da Silva & Santos (2019) "These studies reveal other misconceptions of some students because they consider that one side of the angle must always be horizontal and the direction of the angular aperture must always be counter-clockwise, or that the angle is a sector of a circle."

There are many difficulties that learners face with regards to angles and triangle constructions. According to Da Silva & Santos (2019) "Students face some difficulties in using a protractor, which have been emphasized the literature. Sometimes students align the base of the protractors body along one side of the angle, rather than the protractors own reference line, or they do not place the origin of the protractor on the vertex of the angle to be measured. According to Tanguay (2012) "They can also read the angle measure clockwise when they should read counter-clockwise or vice versa." This is a common mistake that learners do make when it comes to the construction of triangles. Furthermore according to Mitchelmore & White (2000) " the difficulty students have in learning to use a protractor may result from the fact that in a protractor several lines may be chosen for the initial side of an angle but the terminal side must be imagined. On the other hand, the cause of the students' difficulties seems to be the absence of structural angular components, which leads to failure to establish adequate structural mappings." So it is clear from the literature that there are many challenges that learners face when it comes to constructing triangles and it is up to the teacher remove these misconceptions.

2.4. Geometrical reasoning.

In a global context, there exists lots ofliterature on geometry. According to Gunhan (2014, p.3) "Geometry is an important branch of mathematics. It allows for people to understand the world by comparing shapes, objects and their connections." The phenomenon of geometrical reasoning has been researched quite extensively. According to Arcavi (2013) & Battista (2007) "geometric reasoning refers to the act of inventing and using formal conceptual systems to investigate shape and space." Furthermore, research has been done when it comes to the issues encountered by learners understanding of geometry. According to Duval (1998) "in order to determine the difficulties encountered by students in geometry, it is necessary to identify the cognitive processes that underlie geometric processes."

The following are some of the examples of geometrical reasoning skills displayed by students. According to Trends in International Mathematics and Science Study (Acat, Sisman, Aypay & Karadug, 2011) "a student with reasoning skills must able to perform the following: identify and use interrelations between variables in mathematical situations, dissociate geometric shapes in order to facilitate the resolution of a geometrical problem, draw the expansion of an object; visualize the transformation of three dimensional objects and deduce valid results based on the information provided." Geometrical reasoning provides students with an opportunity to discover the relationship between Congruency theorem applications and proofs.

Within the context of this research, geometrical reasoning was important for this study. If one does not understand the principles of geometrical reasoning it becomes a challenge to understand congruency-based proofs, since geometrical reasoning is a critical part of proving.

2.5. Reasoning.

Reasoning encompasses all activities of thinking, this includes sense-making and conjecturing which are created by judgements and inferences (Govender, 2013). For high school geometry, inductive reasoning and deductive reasoning are most prevalent (Goel, Gold, Kapur, & Houle, 1997).

For the purpose of this study the following description of reasoning is accepted: According to Goel, Gold, Kapur, & Houle (1997, p.1305). "Reasoning in the activity of evaluating arguments.

All arguments involve the claim that one or more propositions (the premise) provide some grounds for accepting another proposition (the conclusion)"

In this study reasoning is of vital importance when it comes to congruency-based proofs and the various proof writing competencies which was discussed in the latter. It is important to understand the concept of reasoning properly within the context of this research

2.5.1. Inductive Reasoning.

Inductive reasoning is a way of coming up with general conclusions from specific observations, these observations may support the conclusion but they do not ensure it (Grigoridou, 2012, p.4). This is because there might exist counter examples which we haven't observed yet and which would make our conclusion actually wrong.

According to Grigoridou (2012, p.4) "Thus although inductive reasoning might serve as a tool for coming up with interesting conjectures and it is closer to our intention, it certainly gives less valid outcomes to formal mathematics than deductive reasoning." So inductive reasoning is clearly not the way if one strives to find truth.

2.5.2. Deductive Reasoning.

Deductive reasoning, also known as proving, requires using known axioms and theorems in order to prove a theorem or conclusion based on logical rules (Goel, Gold,Kapur & Houle, 1997). According to Johnson-Laird (1999,p.4) "Deduction is a specific kind of reasoning concerned with the yielding of true conclusions from true given premises, or else the yielding of valid conclusions." In other words, deductive reasoning is about drawing specific conclusions from the given premises by following certain rules of logic. According to Clements & Battista (1992, p.437) "Deductive reasoning is indeed the only way for mathematicians to establish truth, empirical and intuitive methods, like induction, are necessary and helpful." This observation is seen to be very important in mathematics education.

2.6. What is an argument in Geometry?

Krummheur (2002) and Conner (2007) define argumentation "as a social phenomenon, when cooperating individuals tried to adjust their intentions and interpretations by verbally presenting the rationale for their actions."

In Toulmins model of an argument consists of three elements (Toulmin, 1974):

C (claim): the statement of the speaker

D (data): data justifying claim C

W (warrant): the inference rule, which allows data to be connected to the claim.

According to Boero (2010, p.3) “in any argument, the first step is expressed by a viewpoint. In Toulmin’s terminology the standpoint is called the claim.” The second step is made up of providing data to support the claim. The warrant provides the justification for using the data as a support for the claim. The warrant which can be expressed as a principle a rule, acts a bridge between the data and the claim (Boero, 2010, p.3). Three other elements that describe an argument can be taken into account: B (Backing) the support of the rule; Q (Qualifer) the strength of the argument; Re (Rebuttal) the exception to the rule. Overall Toulmins model of argumentation contains six related elements.

2.7. Deductive arguments in Geometry.

As a phenomenon, deductive arguments have been researched widely. With reference to geometrical proofs, it is seen as an important part when solving a proof. Before one discusses a deductive argument, it important to understand what an argument is. For the purpose of this research an argument consists of one or more statements set out as support for some other statement. According to Engel (2020) “deductive arguments are arguments in which the conclusion is presented as following from the premises with necessity.” Furthermore according to Besnard & Hunter (2013, p.1) “A deductive argument is a pair where the first item is a set of premises, the second item is a claim and the premises entail claim.” So it is clear that a deductive arguments consists of two parts. According to Besnard & Hunter (2013, p.1) “This can be formalized by assuming a logical language for the premises and the claim, and logical entailment (or congruence relation) for showing that the claim follows from the premises.” It is important for a deductive argument to be concise and logical. Furthermore according to IEP staff (2020) “A deductive argument is an argument that is intended by the arguer to be deductively valid, that is, to provide a guarantee of the truth of the conclusion provided that arguments premises are true.” So it is clear that a deductive argument is well structured. According to IEP staff (2020) “This point can be expressed also by saying that, in a deductive argument, the premises are intended to provide such strong support for the conclusion that, if the premises are true, then it would be impossible for the conclusion to be false.”

2.8. Proof.

In history proof was used mainly as a tool to verify accuracy of a mathematical statement (Zaslavsky, Nickerson, Stylianides, Kidron, Landman, 2010). This means that a proof was seen as an argument to eradicate doubt of a particular mathematical conjecture (Govender, 2013; De Villiers, 1990).

2.8.1. What is proof?

A considerable amount of research has focussed on the importance of Application and Proofs in high school geometry. According to Conner (undated, p.2) "I define a proof as logically correct deductive argument built up from given conditions, definitions, and theorems within an axiom system." This definition of proof suggests that a proof is specific kind of argument and this highlights the relationship between argumentation and proof, as previously discussed. According to Goldberger (2002, p.1) "a proof of a theorem is a finite sequence of claims, each claim being derived logically (i.e. by substituting in some tautology) from the previous claims, as well as theorems whose truth has been already established. The last claim in the sequence is the statement of the theorem, or a statement that clearly implies the theorem."

Proofs are described hypotheses results generated by arguments that consist of logically strict deductions (NCTM, 2000). Similarly, a proof can be described as the process of justifying by explaining, arguing and verifying a particular statement. Logical arguments of justification are commonly called proof (Govender, 2013). Mata-Pereira and da Ponte, (2017), describe a proof as follows. "A proof is a connected sequence of assertions that includes a set of accepted statements, forms of reasoning and modes of representing arguments". Therefore, proofs are inclusive of building arguments through using theorems and definitions as stipulated in the Department of Basic Education CAPS curriculum.

2.8.2. Functions of proof.

There are several aspects of proof that have been referred to as the functions of proof in the research literature of mathematical proof (Bell, 1976; de Villiers, 1999; Hanna & Jehnke, 1993; Schoenfeld, 1992). According to Oren (2007, p.8) "Traditionally, proof has been seen almost exclusively in terms of the verification of the correctness of mathematical statements. According to this aspect, proof is used mainly to remove either personal doubt and/or those of skeptics."

Several researchers have supported this idea of proof (Bell, 1976; Hanna, 1989; Rav, 1999; Volmink, 1990).

According to Hanna (1989, p.20) “A proof is an argument needed to validate a statement, an argument that may assume several different forms as long as it is convincing.” Similarly Bell (1976, p.24) declared that “The mathematical meaning of proof carries three senses. The first is verification or justification, concerned with the truth of a proposition.”

Besides verification some researchers emphasized other functions of proof. According to Bell (1978, p.48) “not simply with the formal presentation of arguments, but with the student’s own activity of arriving at conviction, of making verification, and of communicating convictions about results to others”. According to Oren (2007, p.8) “Hersh (1993) has claimed that proof has one purpose in the world of mathematical research: that of providing conviction.” Furthermore, according to Hersh (1993, p.389) “In mathematical research, the purpose of proof is to convince. The test of whether something is a proof is whether it convinces qualified judges…..a proof is just a convincing argument, as judged by competent judges….In mathematical practice, in the real life of living mathematicians, proof is convincing argument, as judged by qualified judges.”

According to Oren (2007, p.9) “Hersh (1993) strongly believes that whereas the primary role of proof in the mathematics community is to convince, in schools and at undergraduate level its role is to explain.”

Furthermore Hanna (1995, p.47) elaborates this idea as followings “While in mathematical practice the main function of proof is justification and verification, its main function in mathematics education is surely that of explanation.”

Hanna (1998) has called for using proofs to create a meaningful experience; that is, as a means to help students understand why results are true. Hersh (1993) and Hanna (1995) assert that the main function of proof in the classroom should be to promote understanding by explaining.

According to Oren (2007, p.9) “de Villiers (1990, 1999) points out that proofs have multiple functions that go beyond mere verification and that can also be developed in computer environments: such as explanation (providing insight into why it is true), discovery (the discovery or invention of new results), communication (the negotiation of meaning), intellectual challenge (the self-realization/fulfilment derived from constructing a proof), systematization (the organization of various results into a deductive system of axioms, concepts and theorems).”

Recently, Hanna (2000) has provided a comprehensive list of the various purposes of mathematical proof:

1. Verification (concerned with the truth of a statement)

2. Explanation (providing insight into why it is true);

3. Systematization (the organization of various results into a deductive system of axioms, major concepts and theorems);

4. Discovery (the discovery or invention of new results);

5. Communication (the transmission of mathematical knowledge);

6. Construction of an empirical theory;

7. Exploration of the meaning of a definition or the consequences of an assumption;

8. Incorporation of a well-known fact into a new framework and thus viewing it from a fresh perspective. (p. 9)

A learner who attempts a proof, will acquire a deep understanding to the functions above. Not only it provides the verification of the mathematical statement or conjecture, it also provides the function of discovery whereby the individual discovers new results and it discusses the realization or self-fulfilment that arises from proving (Govender, 2013). Teachers must recognize the different proof strategies that a learner can utilize, in order for them to understand the ways in which a learner would attempt a proof or justify a geometrical statement.

2.9. Functions of Proof and their Role in Instructional Exchanges.

There are seven roles that proofs play in the instructional exchanges of mathematics. The first one is that the work of proving may count as verification of or conviction about the truth of a statement. According to Herbst, Miyakawa & Chazan (2009) "Mathematical statement don't go without saying in the discipline of mathematics as they might, for example in revealed or mystical knowledge. They are claimed to be truth and their truth can be verified or refuted." With this in mind according to Herbst, Miyakawa & Chazan (2009) "Thus in the classroom the work of proving could be accounted for as accomplishing the verification of a statement." Furthermore, according to Herbst, Miyakawa & Chazan (2009) "The management work of the teacher, effecting a transaction of the work done for the claim that the truth of a statement is known, includes not only attesting that the claim has been verified but also that students are convinced of the truth of the statement." So the role that the teacher plays when it comes to proving is critical. According to Bass (2009, p.630) "This one function of proof is to show that a proposition is demonstrably true in mathematics. The truth of the statement being what matters, it might be just as good to know one proof as to know another one, or perhaps to know only that a proof exists."

The second role that proof plays is that the work of proving may count as explanation or understanding of a statement. According to Herbst, Miyakawa & Chazan (2009) "Mathematical statements are connected with other statements by way of the concepts they predicate about." Furthermore, according to Herbst, Miyakawa & Chazan (2009). "A second set of stakes of the work of proving is associated with the contingencies of on the one hand explaining mathematically why a statement is true and on the other hand, of attesting to students understanding of what the statement means." According to Herbst, Miyakawa & Chazan (2009) "A proof can have the function of explaining why a theorem is a reasonable thing to say about a known concept by showing how the statement of a theorem coheres and connects with the key properties of the concepts involved in the proof." Furthermore according to Herbst, Miyawaka & Chazan (2009) "This explanatory function of proof has also been given a subjective interpretation in the notion that proofs can help students understand the meaning of mathematical ideas." Many authors have supported this function of proof, according to Herbst, Miyawaka & Chazan (2009) "Thus Hanna (2000) has expressed support for this function of proof in preferring proofs that explain over proofs that merely prove and in noting that the most important role of proof is to promote students understanding of mathematics." Another author Knuth (2002) "has echoed it arguing to teachers that proofs are valuable because they can help students understand mathematics."

The third role that proofs plays in mathematics, is that the work of proving may count as discovery of a reasonable statement. According to Herbst, Miyakawa & Chazan (2009) " A third set of stakes of the work of proving is associated with the contingencies of on the one hand representing a rational discovery of a mathematical statement and on the other hand, of attesting to students perception that the statement is plausible or reasonable." Furthermore, according to Herbst, Miyakawa & Chazan (2009) "Proof is not merely a process done after the formulation of statements but actually a process that enables the production, the shaping of plausible statements." With this in mind Hanna & Jahnke (1996) "the exploration function of proof is also captured by this discovery function in particular as proof plays the role of exploring consequences of a definition or assumption." This role also plays is important when it comes to instructional exchanges, according to Ball & Bass (2003) "In terms of theory of instructional exchanges this encourages us to look at the work the teacher does or the students do to come up with or introduce a new piece of mathematical knowledge, a definition or a theorem and to place special value to this work when it represents mathematical knowledge as reasonable." Also according to Herbst, Miyakawa & Chazan (2009) "This discovery function of proof stresses how the work of proving can be the source of mathematical propositions."

The fourth role of proof is that the work of proving may count us negotiation or demonstration of standards for communication. According to Herbst, Miyakawa & Chazan (2009) "A fourth exchange value on the work of proving is the claim that a mathematical argument has been communicated." Furthermore according to Herbst Miyakawa & Chazan (2009) "This requires the teacher to manage an exchange that involves not only whether the argument has been communicated but also whether the argument has been communicated but also whether students know how to communicate it." Communication in proofs is play an important role, according to Herbst, Miyakawa & Chazan (2009) "Most of the time the work of showing that a proof exists, for example in communicating as a result through a journal publication, includes some negotiation of how much about the argument is needed in order to communicate the result as one for which a proof exists." With this in mind according to Herbst, Miyakawa & Chazan (2009) "Any proof done in class can therefore have as an exchange value not just that it lays claim on the truth or on the explanation of the statement proved but also that it instantiates the nature of what counts as proof."

The fifth role that proof plays, is that the work of proving may count as systematizing mathematical knowledge. According to Herbst, Miyakawa & Chazan (2009) "A fifth exchange value on the work of proving is the claim that a mathematical statement has been incorporated into a theory or mathematical system of postulates, definitions, and other theorems." Furthermore according to Herbst, Miyawaka & Chazan (2009) "This has been referred to as the systematizing function of proof by De Villiers (1990) as well as by Bell (1976) and also alluded to by Hanna & Jahnkes (1996) "incorporation" function of proof, whereby a proof may incorporate a known proposition into a different theory, thus enabling the representation of that proposition in a new light." With this in mind according to Herbst, Miyawaka & Chazan (2009) "the work of producing a proof of a statement may be accounted for as showing that a statement is deducible or derivable from some other statements." Many authors have written about this function of proof, according to Herbst, Miyawaka & Chazan (2009) "The systematization function of proof can help the teacher manage allocating value when the work done shows that the student knows how the probability of a statement depends on the truth of other statements. According to Herbst, Miyawaka & Chazan (2009) "The doing of a particular proof in class may have its exchange value the representation of the deductibility of a given proposition from other propositions."

A sixth role that proof plays in instructional exchanges is, the work of proving may count as containing or slowing a mathematical technique. According to Herbst, Miyakawa & Chazan (2009) "A sixth exchange value for the work of proving is the claim that a mathematical technique has been represented and or learnt." This function of proof is that proofs are bearers of mathematical knowledge. According to Herbst, Miyawaka & Chazan (2009) "This is one of the reasons why mathematicians are keen to attend to the proofs rather than only to the theorem statements when they read scholarly articles." Furthermore according to Rav (1999) "the entire mathematical know how is embedded in the collection of proofs. With this in mind according to Hanna & Barbeau (2008) "Further show how relevant this function of proof is in mathematics education by exemplifying how school level proofs are bearers of mathematical knowledge. By participating in the proof of a statement, techniques become part of the body of mathematical knowledge."

The seventh role that proof plays in instructional exchanges is, the work of proving may count as establishing the theoretical predictability of an empirical fact. According to Herbst, Miyawaka & Chazan (2009). "A seventh exchange value for the work of proving is the claim that a mathematical theory can predict an empirical fact." Furthermore, according to Hanna & Jahnke (1996) "the third function of proof is the construction of an empirical theory, proofs are crucial elements for constructing an empirical theory." With this in mind according to Herbst, Miyawaka & Chazan (2009) "An empirical theory can be seen as a system of propositions, each of which asserts empirical statements of fact, connected by deductive relationships that are established by proofs." In conclusion according to Herbst, Miyawaka & Chazan (2009) "This function of proof is particularly important as it helps relate theoretical knowledge of mathematics to some empirical aspects of students mathematical activity including drawing geometric drawings, sketching graphs, estimating calculations, and so on." It is quite clear that there are many roles that a proof plays in instructional exchanges and it is important for teachers understand all of these roles clearly.

2.10. Proof writing competencies.

Students need the necessary levels of competency in order to display sufficient proof writing skills. Proof writing competencies involve the learners abilities to communicate and justify through problem solving and reasoning (Stepelman, 2006).

Proof writing competencies will be assessed using a rubric adapted from Posamentier, Smith and Stepelman's book (2006), titled *Teaching mathematics: Techniques and enrichment*. The rubric provides a criterion to assess the work of learners based on their competency level. There are four levels, namely; unsatisfactory, approaching satisfactory, satisfactory, and superior. The criteria on which these levels are assessed, addresses the learners conceptual understanding, mathematical problem solving and reasoning, and the learners' ability to communicate through explanation (Posamentier, Smith &Stepelman, 2006).

2.11. Six Geometrical proof strategies.

For the purpose of learning proof and proving in high school mathematics classes, various strategies have been proposed, (Craine, 2009). Jager, Fitton, and Blake, (2004), put forth the following six kinds of geometrical proof strategies that can be used when proving or solving geometry mathematics.

1. The congruency approach (solving using congruency that two item are equal)

2. Direct application of theorem(s)

3. The algebraic approach

4. Use of other branches of mathematics

5. Reductio ad absurdum (Proof by Contradiction)

6. Analytic approach

2.12. The importance of proofs in school.

Before one understands the importance of proofs in school, it is important to understand the fundamental underpinnings of proofs which was discussed previously. The importance of teaching mathematical proofs in schools has been highlighted by many researchers. According to Styliandes (2017, p.237) "The concept of proof has received attention from mathematics education researchers for many decades (since at least Fawcett's work in the 1930's; Fawcett, 1938), but more explicitly so in the past few decades." There are many advantages when it comes to the teaching of proofs in school, according to Styliandes (2017, p.237) "For example, proof can allow even young children to explore or debate the truth of mathematical assertions based on the logical structure of the mathematical system rather than on the authority of the teacher or the textbook." So it is clear that proofs empowers learners to engage with the mathematics they are taught. According to TESS-India (2014, p.1) "There are many discussions around the world about whether mathematical proofs should be part of the school curriculum." Furthermore, according to TESS-India (2014, p.1) "Researchers suggest working on mathematical learning opportunities."

Learners are offered the following learning opportunities by mathematical proofs; according to TESS India (2014,p.1) " Hanna (2000) summarized these as verification of the truth of a statement; explanation by providing insight why it is true; systematisation by organising various results into a deductive system of axioms, major concepts and theorems, discovery or invention of new results; communication in order to transmit mathematical knowledge; construction of an empirical theory; exploration of the meaning of a definition or the consequences of an assumption and the incorporation of a well-known fact into a new framework and thus viewing it from a fresh perspective." As a result of the above summary, it is clearly obvious that mathematical proofs are of utmost importance in schools and must be taught properly to mathematics learners. Furthermore, according to Styliandes (2017, p.237) "Proof can also serve a broad range of functions, including the following: verification or falsification, explanation, discovery, communication, illustration of new methods of deductions and justification of the use of a definition or an axiom system." So it clearly evident that the teaching of mathematical proofs is critical to deepening learner understanding of mathematical concepts.

2.13. The importance of congruency in school mathematics.

The importance of congruency in schools cannot be undermined, since it serves as the foundation for FET geometry in schools. If learners have a proper foundation set at GET level, it makes the work of FET educators much easier when it comes to the teaching of geometry. According to Huber (2016) "The congruence criteria allow you to decide, based on a pretty small amount of information, that two triangles are congruent, there is quite a bit of information that you can copy over from the triangle you know about to the other triangle." It is clear that it allows learners to make connections between shapes. According to Huber (2016) "For example, if you know that two triangles have two of the same angles and the side between those angles is the same, then you know that the third angle is the same, the other two sides are the same, the areas are the same." Furthermore, according to Otálora (2016, p.251) "Congruence is an important mathematical idea for humans to understand the structure of their environment. Congruence is embedded is young children's everyday experiences that allow them to develop intuitive senses of this geometric relationship." So it is quite clear that congruence is relevant to everyday life, which is an important part of mathematics. Furthermore, according to Huang & Witz (2011) & Wu (2003) "Understanding the concept of congruence provides strong foundations for learning more advanced mathematical processes such as area and volume measurement." Having discussed previously the importance of proofs, it is important that learners understand congruency- based proofs as it further enhances their understanding of congruency.

There is a view that congruency needs to be taught because of its relevance to real life problems. According to Passy (2013) "Congruent triangles are an important part of our everyday world, especially reinforcing many structures." If learners can see how relevant congruent triangles are, it could generate interest and encourage learners to study it further. According to Passy (2013) "in the real-world congruent triangles are used in construction when we need to reinforce structures so that they are strong and stable, and do not bend or buckle in strong winds or when under load." This puts emphasis on the importance of teaching congruency to learners. It might even lead to an increase in attracting learners to the world of construction. This is what mathematics promotes, critical thinking and making connections.

2.14. The challenges that learners have with congruency proofs and theorems.

There have been many challenges experienced by learners and teachers when it comes to the topic of congruency-based proofs and theorems. According to Wu (2005) "middle school students have difficulties in understanding the precise mathematical definition of congruence and fail to grasp how it underlays other mathematical processes." It is clear that there are misconceptions when it comes to congruency. According to Clements & Samara (2014) "the natural development of congruence also represents critical challenges for young children because they tend to analyse only parts of the shape, but not the relationships between these parts and privilege aspects of the shapes that are salient perceptually rather than aspects that are mathematically relevant." Furthermore, according to Otálora (2016, p.251) "Thus young children fail when one of the two figures is rotated or flipped or when the figures are unusual for them." So it clear that there gaps in knowledge that grade 9 learners have before learning congruency. According to Otálora (2016, p.251) "The author suggests that traditional teaching of geometry in early grades is implemented in rigid ways, which that children are exposed to only prototypical shapes and have little experience with non-examples or variant shapes." So it is clear that prior knowledge that grade 9 learners have is negatively affected due to poor teaching of concept in earlier grades. According to Clements & Sarama (2014) "Students difficulties can endure until adolescence if not well addressed educationally, limiting their access to formal mathematics in higher grades." With this in mind it is of utmost importance for teachers to address these gaps in knowledge which learners have on the topic of congruency.

2.15. Congruency- based proofs teaching methods and its successes.

There has been many successes and mathematical breakthroughs when it comes to the effective teaching of congruency-based proofs. These successful teaching methodologies varies amongst each other. There is a shared view that the use of educational technology can be used in order to teach congruency effectively. According to Clements & Sarama (2014) et al "Researchers have stressed that utilizing digital interactive technologies in early childhood education can promote new ways of mathematical thinking in young learners." The Geometers Sketchpad is an example of a successful educational tool that can be used to teach congruency effectively. According to Otálora (2016, p.251) "The use of dynamic geometry software such as Geometers Sketchpad could support young children's reasoning on properties of two-dimensional shapes and facilitate their access to more complex." It is understood that using the Geometers Sketchpad with the use of discovery learners enhances learners understanding of congruency. According to Hegedus (2013) "The addition of multitouch devices could foster direct interaction with the mathematical configurations and collaborative behaviours that, in turn, could support the co-construction of shared mathematical meanings."

Furthermore according to Otálora (2016, p.252) "This study posits that combining Sketchpad with the iPad through application Sketchpad ® Explorer, could enhance young children's learning experiences of congruence by helping them grasp in a dynamic way what means "same shape and same size", so that they can link these properties to continuous geometric motions and to a variety of triangles." If the Geometers Sketchpad is used collaboratively the results will be positive. According to Otálora (2016, p.252) "Moreover, children could work in small groups manipulating the dynamic shapes directly and simultaneously on the iPad, to also benefit from gestural expressivity and social interaction. Such an environment could help students overcome some of learners challenges stated above." So it is quite clear that using the above mentioned technological educational tools will lead to more successes in the teaching and learning of congruency.

2.16. Preservice classroom teachers proof schemes in Geometry.

The role that teachers play in children learning of proofs is critical. According to Oflaz, Balut & Akcakin (2016, p.134) “One of the determinants influencing students learning of geometry is teachers’ knowledge structures, which play a fundamental role in student learning. When considered from this point of view, classroom teachers geometric understanding and thinking becomes crucial.” Furthermore, according to Oflaz, Bulut & Akcakin (2016, p.135) “As geometry is not taught through reasoning and is not proof based primary and secondary schools, this situation is far from beneficial to student thought processes and is instead restrictive.” The proving process has been classified, according to Bell (1976) “researchers have classified the proving process in terms of different dimensions. Of these Harel & Sowder (1998) put forward what is called a proof scheme, according to Oflaz, Bulut & Akcakin (2016) “A proof scheme is a collective cognitive characteristic of the proofs one produces. The taxonomy consists of three classes: external conviction, empirical, and deductive, each group comprising subclasses.”

It is important for teachers to understand what a proof scheme is and how it can influence a learner. According to Flores (2006) “In the externally based proof scheme, what convinces the student and what the student offers to convince others comes from an outside source. The outside source may be an authority such as a teacher or a book, symbol manipulation having no coherent system of references in the eyes of the student. Deductive proofs schemes are also important to understand, according to Mathematicians Flores (2006) “deductive proof schemes are appropriate types of justifications in mathematics. When a student is at a deductive proof scheme, he or she will be able to make a generality, an operational thought, and a logical inference.”

The importance of proofs in geometry has been highlighted by many authors. According to Oflaz, Bulut & Akcakin (2016, p.136) “Geometry teaching should not attach importance only to geometric proofs. Proof processes in terms of proof schemes have a key position in order to explain the meaning of learner’s configuration of geometric knowledge.” Furthermore, according to Oflaz, Bulut &Akcakin (2016, p.136) “In order to teach geometry efficiently, integration of proof into geometry curriculum comes into prominence. It is obvious that a proof free geometry curriculum does not teach geometry.” So it is clear that there is a shared view that geometry is nothing without proofs and proofs must be taught properly at all costs. According to Senk (1989) “Understanding what proving means and being able to write proofs are essential for success in mathematics.” There is also a view on how geometry should be taught, according to Oflaz, Bulut & Akcakin (2016, p.146) “Teaching geometry should focus on geometric understanding instead of rote learning.” So teachers should avoid rote learning and rather focus more on geometrical understanding of learners. According to Harel (2008) “Proofs are explained as ways of understanding associated with the mental act of proving. Likewise, proof schemes are ways of thinking that represent the collective mental characteristics of one’s proofs.”

There is a shared view that the role of the teacher is once again critical to the success of learners in geometrical proofs. According to Oflaz, Bulut & Ackakin (2016, p. 146) “Since proofs are the heart of mathematics and proving is complex, teachers should help their students to develop these processes in the early grades.” Furthermore, according to Hanna et al (2009) “The success of this process depends on the teachers views about the essence and forms of proofs.” So it is clear that the opinions and approaches that teachers have towards proofs has a net effect on the success of learner’s geometrical understanding. There is a shared view that are many reasons for the failure of learners when it comes to proofs. According to Oflaz, Bulut & Ackakin (2016, p.146) “Another reason for students’ failure in proving may be the attitude of teachers about proofs. When investigated, it is shown that school curricula is not sufficient in terms of developing proving ability (Tall, 1995).” In turn, teachers play a pivotal role in learner success of proving, it is important for teachers to have the right skills and attitude needed to impart knowledge and skills of proving to their learners.

According to Oflaz, Bulut & Ackakin (2016, p.146) “However, according to the results of this study and other studies in the field, it is understood that preservice teachers are not able to prove even a simple geometry theorem.” This is a major concern and needs to address tertiary institutions in order to upskill preservice teachers on proving in geometry. What underlies this is thought to be insufficient knowledge in the students about definitions of geometric concepts, as well as the misconceptions they have.” This is definitely a major concern, if pre-service teachers can prove a simple geometry theorem, it will definitely lead to the failure of learners understanding of geometrical proofs. According to Oflaz, Bulut & Ackakin (2016, p.146) “Furthermore, another reason for this can be that they did not experience any proving processes in their former education. Hence, students should realize how valuable proving is and acquire knowledge through the counselling of a teacher.” So, it is imperative that pre- service teachers seek help to fill these major gaps in knowledge.

According to Martin & Harel (1989) "What a teacher thinks about the role of a proof in mathematical thinking is important in in-class training." Also according to Hersh (1993) stated "due to restricted time, the subject being difficult for a students' level, and the insipidity of the proving process, the expected importance of a proof is not extended in the classroom." So it is clear that there is a number factors that hinders teachers from raising the importance of proofs. In conclusion the role of the teacher is important when it comes to proof schemes and when it comes to learners understanding of geometrical proofs.

2.17. Reasoning and proof in geometry: Effects of a learning environment based on heuristic worked out examples.

There is a shared view that there must be classroom setting that is conducive for effective teaching of proofs. According to Reisse, Heinze, Renkl & Gros (2008), "In order to design an effective learning environment for mathematical proof ie. Classroom based learning framework for mathematical proof which offers well organized learning opportunities to the students, it is essential to characterize not only the expected result, but also the process of getting to this result." Furthermore, according to Reisse et al (2008) "The process of proving a theorem can take a long time and may include a sudden progress as well as unexpected setbacks." There are challenges for mathematics learners when it comes to proving a theorem and it can lead to an undesired outcome.

With this mind according to Reisse et al (2008) "Being aware of the iterative character of performing a proof, mathematics educators generally argue that the teaching and learning of proof should not be restricted to the presentation of a correct result but should emphasize the procedural aspects of proving." Many mathematicians have similar views towards proofs, according to van der Waerden (1954) et al "It is well known through a number of reports from mathematicians how this process might work." According to Polya (1957) "In particular, mathematicians stress that proving is a process in which not only deductive reasoning but also exploration plays a dominant role."

The use of worked out examples for proofs has become ever popular and is important for success in mathematics. According to Reisse et al (2008) "Worked out examples consist of a problem and its detailed solution of a problem. Research results how that in the beginning of the learning process on a topic, worked out examples lead to higher learning gains than other forms of instruction, particularly in well-structured domains such as mathematics (Atkinson, 2000)."

Worked out examples can be of benefit to students when it comes to mathematics learning. According to Reisse et al (2008, p.458) "There are some characteristics of worked out examples which enable students to learn successfully." Also, according to Atkinson et al (2000) "the structure of the single worked out example or of the set of worked out examples may influence to what extent learners can profit from this learning environment." According to Reisse et al (2008, p.458) "Thus, learning by self-explaining examples seem to be most promising even without a school context and worked out examples have been positively evaluated for mathematics learning." Worked out examples helps learners to understand concepts better.

According to Reisse et al (2008, p.458) "Worked out examples, which are supposed to foster the ability to perform mathematical proof should accordingly offer process-oriented learning opportunities. Thus, they may lead to a deeper understanding of the heuristics used during the solution process." Furthermore, according to Reisse et al (2008, p.458) "Schoenfeld (1983) investigated experts thinking processes during problem solving and found out that they used various heuristic methods."

There is a shared view that the use of heuristic worked out examples can lead to success in learners understanding of mathematical proofs. According to Reisse et al (2008) "Heuristic worked out examples include characteristics of traditional worked out examples and aspects of the heuristics that are important for the solution process." These aspects are scaffolding and independent learning of learner's own activity. So heuristic worked out examples is a combination of two aspects and has been proven to be a powerful tool in mathematics learning. According to Renkl & Atkinson (2003) "Research suggests that example-based learning can be a primarily recommended in the beginning of a skill acquisition in the beginning of a skill acquisition process, when students still lack a basic understanding and are not able to work on problems on their own.

Heuristic worked out examples can help weak students understand better. According to Reisse et al (2008, p.458) "Accordingly, we assume that an intervention with heuristic worked out examples might primarily be helpful for weaker students. The heuristic examples that have to be self- explained provide them with a model how to solve proof problems." Furthermore, according to Reisse et al (2008, p.458) "More advanced students have already a basic understanding how to deal with proof problems. Therefore, self -explaining with the help of an example how to proceed in proving may be redundant, or the provided heuristic may even interfere with the students own strategies."

Many other researchers have spoken about the positive effects that heuristic worked out examples has on learners. According to Reisse, Heinze, Renkl & Gros (2008, p.463) " the data suggest, that the positive effect of self-explaining heuristic worked out examples might be independent of the specific teacher." The study further says, according to Reisse et al (2008, p.458) "There was a gain of competency in all classrooms participating in the experiment. Probably the effects of autonomous and self-regulated learning particularly in a well-structured learning environment are still underestimated in the mathematics classrooms.". Furthermore, according to Reisse et al (2008, p.458) "The results indicate that self-explaining heuristic worked out examples are qualified instrument for improving students achievement on reasoning and proof in the mathematics classroom." So it is clear that heuristic worked out examples can lead to success in mathematics.

There are many reasons as to heuristic worked examples can lead to success. According to Reisse et al (2008, p.463) "Evidently the learning environment has influenced the students abilities to argue in a mathematical setting." Also, according to Reisse et al (2008, p.463) "Probably the scaffold that the heuristic worked out examples provided might have enabled the students to better understand what a mathematical proof respectively a deductive argument is." With this in mind according to Reisse et al (2008, p.463) "Theoretical arguments from both, educational psychology and mathematics education, suggested that heuristic worked out examples might be helpful for learning mathematical proof." So, it is clear that it is beneficial. According to Reisse et al (2008, p,463) "The data revealed that in particular students with low proving competencies were able to benefit from working with examples that emphasized the heuristic nature of proving and encouraged them to explicate the process of proving." So it is clear that heuristic worked out examples can help students who are weak when it comes to mathematical proofs and teachers should expose learners to these examples.

2.18. Students reasons for introducing auxiliary lines in proving situations.

There are many reasons for introducing auxiliary lines in mathematical proving situations. According to Polya (1957, p.46) "an element that we introduce in the hope that it will further the solution is called an auxiliary element." Furthermore, according to Senk (1985, p.455) "the introduction of auxiliary lines is a critical part in the solution of geometric proof problems. She concluded that the difficulty with auxiliary lines exemplifies the need to teach students how, why and when they can transform a diagram in a proof." So it clear that the use of auxiliary lines is of utmost importance.

Despite its importance, there are also challenges that learners have with auxiliary lines. According to Palatnik & Dreyfus (2018, p.2) "Ding and Jones (2006) also identified the introduction of an auxiliary line for use in proof as a source of great difficulty for students. Furthermore according to Hsu (2007) "this difficulty is caused by students need to perceive the diagrams dynamically and apply transformational observation to visualize a solution that can be generated with the help of auxiliary lines." These difficulties have also been recognized extensively, according to Palatnik & Dreyfus (2018, p.2) "Students difficulty with the introduction of auxiliary lines when proving a geometric statement can partially be explained by a form of instruction, which does not encourage students to add new lines to the given diagram."

There is also the belief that students' views on auxiliary lines create these difficulties. According to Herbst & Brach (2006) "Students expect to play a passive role in interaction with diagrams when proving. They believe that they are usually not expected to introduce auxiliary lines, and if so, they expect to receive a hint to do so from the teacher." This dependency that learners have on teachers is the root of the problem. In order to overcome this problem, according to Hsieh, Horng & Shy (2012) "the incorporation of exploration in proving helps students to reveal new information which makes the introduction of auxiliary lines more transparent."

Despite the challenges that auxiliary lines has when it comes to proofs, many authors still support it use and recognizes its importance. According to Palatnik & Dreyfus (2018, p.3) "To summarize, the introduction of auxiliary lines is a critical decision for students in proving situations." Also according to Hsu & Silver (2014), Senk (1985) "It contributes to the complexity of proving and proof comprehension." Furthermore, according to Palatkin & Dreyfus (2018, p.13) "Based on our findings we suggest that the introduction of auxiliary lines in proving situations can be expected to provide a rich context for development of geometrical thinking."

There are also suggestions on what teachers should do when incorporating auxiliary lines. According to Palatkin & Dreyfus (2018, p.13) "Teacher should encourage proactive and reflective approaches to the introduction of auxiliary elements. It is impossible to develop mastery related to the introduction of auxiliary lines without creating experiences of drawing, imagining or dynamically modifying auxiliary lines." Furthermore, according to Palatkin & Dreyfus (2018, p.13) "Namely, teachers should encourage students to draw various auxiliary lines and inspect alternative situations in which auxiliary lines are subject to the scrutiny of deductive reasoning." In conclusion, if teachers introduce auxiliary lines in a way that contributes towards the understanding of a proof, it will lead to successes in their learning and create a positive effect. The introduction of auxiliary lines can be aided with the use of web-based learning which is discussed next.

2.19. What is web-based learning?

For the purpose of this research, it is important to understand what Web-Based Learning (WBL) is and the nature of it as well. According to Poon, Low & Yong (2004, p.374) "Web-based learning (WBL), also known as electronic learning (e-learning), refers to Internet technologies used to deliver a broad array of solutions that enhance the instructional process."

In order for web-based learning to perform its function properly, there needs to be an environment suitable for web-based learning. According to Poon, Low & Yong (2004, p.374) "The WBL environment is an interactive network system consisting of a variety of functions to support a virtual classroom to enhance the quality of teaching and learning activities."

2.20. How different authors define web-based learning.

There are many authors, who have come forth in defining WBL even further. According to Relan & Gillian (1997) "have defined web-based learning as a cognitive learning strategy application in a constructive and collaborative learning environment using web-based facilities."

According to Hashim (2016, p.8) "E-learning is often confused with definitions of other terms like online learning, blended learning, distributed learning, mobile learning, internet-based learning, technology-based learning, computer-based learning, web-based learning and virtual learning."

According to Yusup (2012) "There are those who equate e-learning definition with Learning Management Systems (LMS) like Blackboard, WebCt, Moodle, My Guru, MyLM's and others."

Furthermore, according to McKimm, Jollie & Cantillon (2003) "Web based learning is often called online learning or e-learning because it includes online course content. Discussion forums via email, video conferencing, and live lectures (video streaming) are all possible through the web."

It is important that one understands the definitions of web-based learning from many different authors in order to develop a more refined definition of WBL. For the purpose of this study the definition from Relan & Gillian (1997) is used, which defined web- based learning as a cognitive learning strategy application in a constructive and collaborative learning environment using web based facilities.

2.21. Nature and form of web based learning and how it takes place.

The nature of web-based learning has been described by many authors and researchers have broken up web-based learning into two contexts being technology and pedagogy. According to Hashim (2016, p.8) “E-learning actually involves technology and pedagogy. The technology refers to computers, CD-ROMS, electronic devices and the internet.” Furthermore, according to Hashim (2016, p.8) “Meanwhile pedagogy covers online learning, blended learning, distance learning, face to face learning, web-based learning, computer aided instruction, individual learning, network learning and interactive learning.” The features of a typical web-based course is fundamental to the effectiveness of teaching and learning via a web based learning system. According to McKimm, Jollie & Cantillon (2003) these features include “course information, notice board, timetable, curriculum map, teaching materials such as slides, handouts, articles, communication via email, discussion boards, formative and summative assessments, student management tools (records, statistics, student tracking) and links to useful internal and external websites- for example, library, online databases and journals.” So it quite evident that a web based learning system can consist of a variety of resources, and it can be seen as a main source of information for teachers and learners as well.

2.22. Benefits, advantages and disadvantages of web based learning.

There are many advantages when it comes to using WBL, according to Khan (1999) “Web-based environment has the potential to offer more advantages to learners to improve the learning process compared to the traditional learning package.” Web based learning has many advantages such as distance learning, according to Kassim & Razaq (2009) “This method enables individuals to participate in open and independent learning sessions at their own pace without following timetables or attending designated classes.” Web based learning is a shift away from a teacher centred approach to more of a student-centred approach, according to Jonassen (1999) “WBL is able to support learning based on behaviourism to change to constructivism.” So its offers a constructivist approach which is suitable for mathematics teaching and learning, according to Hashim (2016, p,12) “constructivism learning involves the students to construct their own knowledge and apply a variety of skills and make logical synthesis.” Thus making it more learner centred.

According to Hill (1997) “WBL gives the chance for students to acquire higher thinking strategies that encourage divergent thinking with the existence of guides from different perspectives.” In essence WBL instils higher order thinking skills of learners. According to Bonk & dan Reynolds (1997) “ A constructivist WBL, where students are encouraged to state their individual opinions and ideas, conducts reflections, explore diverse knowledge and identify the strengths and inclinations of arguments make the core to nurture higher order thinking skills.”

WBL also makes it more convenient for students and it saves a lot of time if used properly. According to Hashim (2016, p.12) “students can also use the web to find information and the latest development regarding their studies quickly with great ease.” With this in mind examples are also given as to how WBL is more convenient, according to Hashim (2016, p.12) “For example, UTHM andragogy approach requires students to work in groups to find information using Web Based learning. This is because the aim of adult education is broad involving their determination to develop skills, enriching knowledge as well as improving their qualification and professionalism.”

Another advantage of Web based learning is cost reduction, according to Hashim “some of the courses offered in the field of training and skills incur high cost. Therefore, technology assistance can help reduce the cost of acquiring the needed skills.” Furthermore, according to Harun (2003) “WBL is useful in replacing real situation T&L that require high cost and difficult to manage. Using web-based learning are cost reduction, for example in medicine, manufacturing, technical and vocational education and military.”

An important advantage of Web based learning is the increase in motivation and access that it offers, according to Tennyson & Nielsen (1998) “aspects that include motivation and emotion can affect executive control in increasing attention and also to arrange cognitive strategies, facilitating planning and use of knowledge.” According to Che Wan (2007) “Teachers experiences in applying the web in teaching has developed a sense of excitement and interest among students toward teaching in technical and vocational education.” So it is quite evident that web based learning has a lot of advantages if used correctly. Furthermore, according to Ahmad & Tamuri (2020) “Through the application of information technology, the T&L environment has become more interesting and has a high potential in increasing T & L quality.”

There are also a number of disadvantages that Web Based learning has. The first of these would be technical obstacles. According to Hashim (2016, p.13) “Access to computer and technology equipment and also internet might be a problem to students, especially those who live in rural areas." Hashim (2016, p.13) adds that "Weak internet access has stopped students from accessing every display in a website."

The next disadvantage would be a lack of moral value implementation, According to Hashim (2016, p.13) "Most websites that have been built or software being developed lack moral value implementation." Furthermore, according to Jasmy et.al (2003) "Without having a teacher who acts as a facilitator, students who uses WBL have failed to master effective learning methods like simple note taking." So it is clearly evident that teachers have to be technologically inclined in order to make use of WBL effectively. According to Hashim (2016, p.13) "Most software being developed in the market do not have the equilibrium between entertainment and education." So there is a need for balance when it comes to use of WBL. According to Majid (2012) "This has led to problems in character, behaviour, performance and achievement among students nowadays." So it is clearly evident that despite having its disadvantages, there are many more advantages which WBL offers, which is why it is best to use when it comes to mathematics teaching and learning, because of technological advances which have been made.

2.23. Studies on web-based learning.

There has been extensive research into the effective use of web-based learning, when it comes to teaching, learning and assessing Mathematics. It is important for teachers to design lesson plans that are suitable for the effective use of web- based learning. If lessons are planned correctly, learners will benefit from the advantages which web- based learning offers. According to McKimm, Jollie& Cantillon (2003), "Course designers need to remember that younger students are more likely to be familiar with using the internet than older learners, who may feel less comfortable with a web- based course." So it's important for teachers to realize the audience and their capabilities. According to McKimm, Jollie & Cantillion (2003) "to get the best out of their learning experience, learners need basic computer skills, support and guidance." So the role of the teacher is crucial when it comes to equipping learners with basic skills before teaching via a web based learning system. Furthermore, according to McKimm, Jollie & Cantillon "teachers must design their courses to encourage effective web based learning rather than aimless "surfing"." So it's important that teachers have web based lessons that are goal driven, in order for their lessons to be effective. According to Mckimm, Jollie & Cantillon (2003) "programme design should therefore filter out poor information as well as signpost key information sources." There is also a need for teachers to be encouraged via training and support in order to use the web and other information technology systems in their teaching. Furthermore, according to

McKimm, Jollie & Cantillon (2003) "They need examples and awareness of good practice, and standards should be set in relation to how teachers present information and manage the learning environment." So it clear that teachers need to be well equipped when teaching via a web based learning system and learners need to be equipped with basic computer skills before learning via a web based learning system.

When comes to the assessment of learning with the use of web- based learning many researchers have researched it extensively and have also highlighted the advantages and disadvantages of assessment via web-based learning. According to McKimm, Jollie & Cantillon (2003) "With all types of learning, including web-based learning, it is useful for students to receive constructive, timely, and relevant feedback on their progress." According to McKimm, Jollie & Cantillon "Online assessment is sometimes constrained by the medium in which it is operating. Computer marked assessments alone are not appropriate for marking or giving feedback on assignments such as essays or projects that require more than mere reproduction of knowledge." Furthermore, according to McKimm, Jollie & Cantillon (2003) "when planning online assessment it is important to determine what is to be assessed. If knowledge reproduction is being tested objective questions (such as multiple choice or "true or false" questions) with instant or model answers can provide excellent feedback." However online assessment also comes with its challenges, according to McKimm, Jollie & Cantillon (2003) "assessment of higher cognitive functions, such as analysis and synthesis, will require more complex tests. Automated marking may be difficult for such assessments, and the teacher is likely to have to do a substantial amount of work before he can add his or her comments to the student's record." So, it is quite evident that some much need planning has to be done before using online assessments.

2.24. What is web-based learning pedagogical approaches?

The pedagogical approaches to web-based learning has been researched quite extensively by many researchers. There is a shared belief that constructivism is the embodiment of web-based learning. According to Chiriac (2019, p.182) "Constructivism highlights the value of the active self-involvement of students, thus managing and encouraging personalized learning activities via collaborative learning within social contexts." Since collaborative learning is an important part of WBL, it clear that constructivism is the learning theory of which WBL stems from. According to Kahiigi et al (2008) "specified this type of learning facilitates critical thinking and problem solving, helps build new ideas using previous knowledge and experience attained and implies students to take on the responsibility of learning by actively participating in the learning process.” So in essence this is the characteristic which makes WBL a success in terms of mathematics teaching and learning.

Furthermore according to Chiriac (2019,p.182) “Knowledge construction within WBL refers to the process of developing ideas, attitudes and belief as a way by which a learner produces and links its new knowledge understanding with existing ones.” So it is clear that WBL puts emphasis on the importance of prior knowledge, which is fundamental in mathematics teaching and learning. According to Chiriac (2019, p.182) “In terms of knowledge building and educational technology, WBL needs to imply the development of learning patterns, the combination of e-learning strategies, collaborative learning techniques and pedagogical insights.” So it is clear that collaborative learning is once again an important pedagogical principle in WBL.

2.25. The scaffolding of web based learning into lessons.

The importance of scaffolding of web based learning into lessons cannot be emphasized more clearly. Scaffolding is seen as the determinant as to whether web based learning is successful or not in the teaching of mathematical concepts. According to Shin, Brush & Glazewski (2017), “Many researchers have argued that scaffolding provides the framework for assisting students with these challenges. Scaffolds are tools, strategies, and guides that help individual learners to accomplish tasks that are beyond their ability to complete alone.” It is quite clear that an example of a scaffolding tool could be an instructional YouTube video that could be used in a web-based learning environment. Furthermore according to Shin, Brush & Glazewski (2017) “Scaffolding can appear in multiple forms depending on the various types of support provided to engage students in an inquiry-based learning activity.”

According to Lee & Calandra (2004) & Walker & Zeidler (2007) “Some researchers have reported that web-based scaffolding tools are effective in promoting students scientific reasoning skills.” So it is clearly evident that scaffolding is effective when it comes to mathematical reasoning. According to Lee & Calandra (2004) “reported that annotations embedded in web based resources encouraged students to access prior knowledge which is essential in understanding contextual information and generating their own explanations during problem solving.” So scaffolding puts emphasis on the importance of prior knowledge when learners are faced with new concepts. It has been reported that there is a need for more research into scaffolding, according to Kin & Hannafin (2011) “Scaffolding research typically has focused on certain features and affordances of technology in various settings rather than on the holistic use of scaffolds to support the overall learning experience."

2.26. The use of Videos as a Teaching aid.

Globally, the use of videos as a teaching aid has been widely researched. According to Bates (2016) "The influence of digital videos on our everyday culture is undeniable. Online video sharing sites such as YouTube, Vimeo, and Metacafe boast monthly audience numbers in the millions." So it is clear that the use of videos is very popular in the global market, the Youtube videos used in this study would boast huge popularity amongst spectators.

According to Bevan (2019) "With digital videos continuing to gain popularity, it seems only natural that this familiar and widespread platform should extend into the education system." So it is evident that videos has gained popularity in the education system. Students today are using educational videos as a tool for learning everything: from basic skills - like changing a tire - to the latest dance craze. According to Bevan (2019) "Remarkably, millennials make up 92% of the digital video viewing audience." So, it is clear that videos are common for learners to watch, which makes it good for teachers to use. Furthermore, according to Bevan (2019) "Abstract topics that once seemed difficult to teach and learn are now more accessible and understandable thanks to the availability of effective educational video platforms for online learning." So it clear that videos makes things easier for both the teacher and learners when comes to understanding concepts.

According to Bevan (2019) "Studies have shown that the use of short video clips allows for more efficient processing and memory recall." So short videos help with memory learning, which is important particularly for the subject of mathematics. Furthermore according to Bevan (2019) "The visual and auditory nature of videos appeals to a wide audience and allows each user to process information in a way that's natural to them. In a nutshell, videos are good teachers." So it is clearly evident that videos would be a good teaching aid for teachers and also appeal to different types of students, with different learning styles.

According to Bevan (2019) "The use of videos in teaching and learning serves to not only benefit students, but also teachers, their affiliated institutions, and the entire school system." So it is evident that videos benefits all parties. According to Bevan (2019) "A 2015 study conducted by software company Kaltura concluded that 93% of teachers believe that the use of educational videos improves the learning experience." So it is clear that most teachers are for the use of videos in the classroom, which is encouraging particularly because of the shift from traditional practices to more modern day practices. Furthermore, according to Bevan (2019) "They also serve to break down barriers, such as student and campus location, which were once insurmountable." So videos breaks down learning barriers which makes an effective teaching aid.

Despite the many benefits of using video there is a demand for it, which does make it difficult for teachers. According to Bevan (2019) "As a result, educational institutions are faced with the task of meeting the rising demand for quality learning videos, online course offerings, and campus accessibility." This rising demand for quality videos can prove to be a challenge for teachers, which is why teachers prefer making their own videos. According to Bevan (2019) "Indeed, many are choosing to create their own educational video learning materials. While this idea may seem daunting, it can be a positive and enjoyable experience if you contract a full-service video production company that can tailor your videos to your institution's individual needs." So if teachers makes their own videos it can improve the quality that teachers need from videos and teachers can thus easily design tasks based on the videos they have created.

2.27. The use of Videos for facilitating teaching and learning in Mathematics.

There are five ways in which videos can have a powerful impact on the teaching and learning of mathematics.

- The first way is, Engagement**,** According to EdSurge (2020) "Studies have shown that video learning has positive outcomes on multiple levels, including increased motivation and deeper learning, and can specifically impact students' ability to facilitate discussions and identify problems." So engagement allows learners to identify problems which a critical part of mathematics.
- The second impact is Effectiveness, according to EdSurge (2020) "Video learning is effective on both sides of the classroom; educators can use it to create time and space for active learning." So its effectiveness allows learners to actively engage in the mathematics. Furthermore according to EdSurge (2020) "Once a video is created, it can be reused and updated as needed, leaving more time in the classroom for live discussions and engagement with students." Once again it offers a critical thing which is important in mathematics which is engagement.
- Thirdly, videos offer Authenticity**,** according to EdSurge (2020) "Video engages both the student and educator in a one-on-one relationship without ever being in the same room."

So this is the advantage that videos offers which is one on one learning, this is an important part on the teaching of mathematics. Teach can now actively focus on improving learners' mathematical skills individually. Furthermore, according to EdSurge (2020) "A compelling 2016 study by the Online Learning Consortium found that video helped educators build and foster authentic relationships with students." So videos help facilitate the teaching and learning of mathematics by strengthening the relationship between teachers and learners.

- The fourth impact is Inspired Thinking, according to EdSurge (2020) "Visual cues combined with audio play a huge role in the comprehension and retention of new material." So learner can benefit from videos through retaining information, which is important in mathematics. Furthermore, according to EdSurge (2020) "Forrester Research analyst James McQuivey claims one minute of video equals approximately 1.8 million written words. Thus, when video is used in the classroom, students are are forced to think critically when introduced to complex content." So videos basically stimulates critical thinking, which is an important skill that mathematics aims to equip learners with.
- Fifthly its impact is, Video for All, according to EdSurge (2020) "Video can help address this gap in training by giving both general and special education teachers the opportunity to teach students at their own pace." So, it helps mathematics with the pace of teaching, which is beneficial in the teaching of mathematics. Furthermore, according to EdSurge (2020) "Students can rewatch a video multiple times in order to gain and retain learning material. And captions, for example, enable deaf students to read the video." So the use of video also breaks learning barriers and allows a mathematics teacher to teach disabled learners.

2.28. The use of Videos in the teaching of Geometry.

For a number of years videos have been used for the teaching of Geometry. On video sites such as YouTube, there exists a wide variety of quality videos for learners to view. This wide variety of videos covers the syllabus with regards to geometry. Learners can easily view videos on their cell phones and laptops as well, this ease of access makes videos popular when it comes to learning geometry. The important attributes that videos have which makes geometry easy to teach is that it prevents information overload and visually stimulates learners with regards to geometrical sketches.

The demand for quality geometry video lessons has risen increasingly because the topic of geometry is an important one in the field of mathematics. Geometry can be challenging for learners and videos provide an extra resource for teachers and learners to make the teaching and learning of geometry easier. The fact that learners can easily memorize geometrical sketches and proofs via videos makes it convenient for learners to watch. Geometry is a discipline that requires learners to repeatedly learn proofs and theorems and video provides a platform for learners to do just that. Learners can easily watch videos repeatedly if they did not understand, whilst traditional practices do not allow teachers enough time to repeatedly teach lessons. Videos helps teachers with the timing of lessons, since geometry can be a time-consuming topic to teach.

2.29. The benefits, advantages and disadvantages of using Videos in a lesson.

There are several benefits that videos have to offer not only to learners but also to teachers. The benefits to learners are as follows, Videos create a more engaging sensory experience than using print materials alone. Visual learning is of utmost importance when it comes to congruency and videos offers a platform for visual learning to take place. According to Das (2019) “The vast population of learners has a variety of learning styles. Auditory learners learn the best through listening. Kinaesthetic learners learn the best when they do an actual activity, while visual learners are comfortable retaining information when ideas, words, and concepts are associated with images and visuals. Videos have something for everyone. Auditory learners can enjoy the audio narration, visual learners can enjoy the rich animation and text/images, while kinaesthetic learners can enjoy the demonstration of activities. Videos, as a single platform, cater to all categories of learners.”

According to Bevan (2019) “Learners actually get to see and hear the concept being taught, and they can process it in the same way they process their everyday interactions.” So videos stimulate learning by allows learners to use various sensory experiences. The second benefit is that, they provide a go-to resource that can be watched from anywhere with an internet connection. According to Bevan (2019) “Videos are accessible on a multitude of devices including laptops, tablets, and smartphones. This allows for viewing at the student’s convenience and from wherever they are.” So videos are easily accessible which is important when it comes to the learning experience of learners, ease of access is critical. Thirdly, Videos increase knowledge retention, according to Bevan (2019) “since they can be stopped and replayed as many times as needed. They can also be reviewed long after the initial lesson was taught.” So its supports the teaching skill of repetition which is important for learners to experience.

Fourthly, they greatly assist in the learning of all subjects, according to Bevan (2019) "but particularly those topics that are complex and/or highly visual, such as step-by-step procedures, problem-solving, or science and math formulas." So not only does it assist learners with mathematics, it also assists with other subjects. The last benefit for learners that videos offer is, they increase proficiency in digital literacy and communication, which are important 21st-century skills (Bevan, 2019). This is fundamental in any facet of the learners learning experience.

There are also many benefits for Teachers, the first benefit is that videos increase student engagement, and according to Bevan (2019) "which in turn helps boost achievement, if students are interested in the material, they will process and remember it better." Videos basically arouses interest of learners, which makes it the job of a teacher so much easier. Secondly, they offer the flexibility to pause, rewind, or skip throughout the video to have class discussions or review particular areas (Bevan, 2019). This function is one of the ground-breaking things which videos offer, teaching of lessons becomes so much easier, since learners can watch lessons repeatedly.

The third benefit for teachers is that videos can create a blended learning environment, which is the ideal teaching approach for teachers of the 21st century. According to Bevan (2019) "They enable teachers to create a flipped classroom or "blended" learning environment. However, videos are also beneficial to teachers who teach in traditional classroom settings." Blended learning is the future of teaching mathematics in the classroom, since it utilizes a variety of resources to teach lessons, both teachers and learners will have these resources readily available.

Fourthly, according to Bevan (2019) "Digital videos facilitate remote learning opportunities so that teachers can reach students from all over the world." This basically enables teachers to reach a broader scope of learners and provides teachers with the opportunity to grow. The fifth benefit is that, many videos now contain analytics features that enable teachers to track student engagement and attendance while viewing. According to Bevan (2019) "Companies like NextThought even provide platforms that can track how long a video was viewed for and what percentage of the video was watched. This allows the facilitator to gauge the effectiveness of the video." Videos basically provides teachers the opportunity to monitor accurately the work rate of learners. When it comes to the subject of mathematics this is critical, since the depth of knowledge is important.

Sixthly, videos provide opportunities for student feedback and assistance through video. According to Bevan (2019) "This is helpful for students who are unable to attend classes, or who need tutoring or review sessions." Feedback is of utmost importance in teaching and learning and videos provides a platform for effective feedback. Lastly, according to Bevan (2019) “Videos seek to change the roles of teachers from lecturers to facilitators. It’s important to note that videos are meant to enhance course materials and lectures — not replace them.” With this in mind, it is clearly evident that videos allow teachers to be facilitators, which is of utmost importance for mathematics teachers, since they must facilitate the learning of the mathematics from the beginning to the end.

Although videos clearly have many benefits, there are also many disadvantages. Firstly, according to Bates (2016) “unless your video is captioned with subtitles it can be difficult for the hearing impaired to access.” So, it is important that videos have subtitles in order to avoid disadvantaging different types of learners. The second disadvantage is as follows, according to Bates (2016) “many faculty have no knowledge or experience in using video other than for recording lectures.” This lack of experience can really disadvantage learners, so it is important that teachers get experience with working with videos. Thirdly, according to Bates (2016) “there is currently a very limited amount of high-quality educational video free for downloading because the cost of developing high quality educational video that exploits the unique characteristics of the medium is still relatively high.” So it is clear that the cost of the video is quite high and it will disadvantage teachers and learners whom are poor or have no funds. The fourth disadvantage of using videos in lessons is that according to Bates (2019) “creating original material that exploits the unique characteristics of video is time consuming and still relatively expensive, because it usually needs professional video production.” So, it is time consuming for educators to create their own videos and they must pay attention to detail.

Fifthly, according to Bates (2016) “to get the most out of educational video, students need specially designed activities that sit outside the video itself.” In essence, if teachers do not set up specially designed activities that is relevant to the video, it will defeat the purpose of the lesson, which is a major disadvantage, a lot of planning must go into this. The last disadvantage of using videos is that, according to Bates (2019) “students often reject videos that require them to do analysis or interpretation; they often prefer direct instruction that focuses primarily on comprehension. Such students need to be trained to use video differently, which requires time to be devoted to developing such skills.” So, it is clear that if teachers do not teach learners how to interpret videos they will be disadvantaged.

2.30. Why videos should be used in a lesson.

Videos arouses interest amongst learners and keeps them captivated from start of the lesson to the end. According to Rajadell & Garriga- Grazon (2017) “With the evolution of digital technology and fast access streaming video channels via the Internet, video has moved from being an important element to being considered as teaching methodology, with the increase in the number of settings which use dynamic image as a captivating element for students attention, to the point that some authors argue that it is more efficient than other methodologies based on books based on books or text material.” So, it is clear that authors feel that videos should be used in lessons.

An example of a popular data base for educational videos would be the Khan Academy, according to Thompson (2011) “In addition to these videos, the website offers software that generates practice problems and rewards good performance with videogame-like badges-for answering a streak of questions correctly, say, or mastering a series of algebra levels.” So it clear that Khan Academy videos is suitable for learners solving mathematical problems. Videos also visually stimulates learners and therefore helps with the various learning styles that learners have such as audio and visual learning. It is also a shift from the traditional practices to more modern day practices and this is important for the growth of teaching and explores new avenues of teaching and learning methods.

In the subject of mathematics, the topic of geometry requires learners to memorize sketches and become familiar with the properties of a geometrical sketch. Videos allows learners to memorize sketches which is critical in geometry. Learners can easily pause and rewind parts of video, which makes it easier learning concepts repeatedly. Videos also helps learners to concentrate more which is important in the subject of mathematics, learners often lose focus in mathematics lesson which leads to major gaps in knowledge and videos prevents this from happening.

The reason as to why videos should be used is that it is revolutionizes the classroom. According to Khan (2011) “By removing the one-size-fits-all lecture from the classroom, and letting students have self-paced lecture at home, then when you go to the classroom, letting them do the work, having the teacher walk around, having peers actually be able to interact with each other, these teachers have used technology to humanize the classroom.” So it is clear that videos should be used during lesson because it has a positive impact on any given classroom setting.

A reason as to why videos should be used in a lesson is because videos are popular amongst youth, since they watch videos such as YouTube videos quite regularly, so it’s not as if they are not accustomed to videos. Videos also prevents lessons from becoming boring and dull, which most learners encounter, so it pleases learners and boosts morale amongst learners which is important for teachers to be able to do.

2.31. How learners respond to the use of Videos in a lesson.

The perceptions of learners on videos are of utmost importance. A study that gathered responses from learners based on their feelings towards videos, provided the following views.

- According to Kosterelioglu (2016) "Use of videos allow us to move away from classical and boring narrations and let us participate more actively and hang on to every word during the class". So it is clear that this student feels that videos arouses interest and prevents lessons from being boring. Furthermore, according to Kosterelioglu (2016) "Videos help me gather back my thoughts just when I am about to lose all my concentration. I believe the lesson is more effective and better. We are not losing our concentration**".** So this basically tells us that videos helps learners concentrate better.
- According to Kosterelioglu(2016) "I can get easily bored from classes with educational content, and lessons such as history. But then the lesson is supported with video both the lesson becomes livelier and the students are not bored anymore". So once again backs up another students view that video prevents lessons from being boring. A video must be used to arouse interest from learners.
- According to Kosterelioglu (2016) "Use of videos really affects me positively. I cannot participate in classes since I cannot speak much in class and I shy away from talking in public. Therefore I get bored in class. You're (turning on) videos during educational psychology classes increases my interest towards the lesson. Although I get bored in many of the other classes, I don't get bored in this one". So it is clear that videos has a positive impact on learners with learning barriers or even personal problems.
- According to Kosterelioglu (2016) "I found the videos I watch very beneficial. The topic does not feel unrelated anymore and we comprehend it. It also prevents monotony in the classroom". So videos basically avoids lessons from becoming disinteresting and lacking variety. Similarly, according to Kosterelioglu (2016) "Peppering the lesson with the videos keeps our interest alive and prevents the class from being monotonous". So once again another student's opinion is that videos prevents lessons from being disinteresting.
- There is a shared view that videos are very effective in arousing learner interest, according to Kosterelioglu (2016) "Use of videos during class is an important tool by

concretizing what is taught and presenting it to the student, I believe videos are supporting materials to increase student interest".

- Furthermore, according to Kosterelioglu (2016) "I adapt better to the lesson since I find videos interesting. Visual content in the videos help us to place materials in our minds better". So, videos basically visually stimulates learners in the classroom and this feature is ground breaking. There is also a view that videos remove gaps in learning that exists amongst learners. According to Kosterelioglu (2016) "One second loss of concentration while the topic is being taught makes you lose the train of thought. But in my opinion, when you watch a related video immediately after helps us fill the gap.
- A certain interest is created when you say we will watch a video. That plays an effective role in learning". So, students can even see how videos removes gaps in learning. There a shared view amongst students that if videos are not played regularly it might hamper learner concentration, this is the only negative recorded from the study on students perceptions. According to Kosterelioglu (2016) "After each topic, there is a continuing expectation whether there is a video to watch. When there is none, we fell like off!. That's the only negative aspect I can mention. Videos should definitely be included in classes". So it clear that learners firmly believe that videos should be used in lessons.

In conclusion, this study on the perceptions of students towards using videos in lessons, is that it should be used. There is a shared view that videos arouses interest, helps with learner concentration and prevents lessons from becoming monotonous. Since videos are popular amongst learners, teachers should use it as a teaching aid to maximize learning.

2.32. How do Videos support learner's conceptual understanding of mathematics?

The conceptual understanding of mathematics from learners, is critical when it comes the subject of mathematics. There are many ways in which videos support the conceptual understand of mathematics. The first way is belief, videos strengthen learner's belief in understanding mathematics by making mathematical concepts easier to understand. The second way is sense making, videos allow learners to come to a conclusion on their own. Since learners making conclusions when it comes to proofs and theorems in mathematics is important. Videos shifts teaching methodologies from rote memorization to conceptual understanding According to Alcala (2016) " I found that by developing justification as a norm, I could understand exactly what students believed, why they believed it, and respond with more effective questions. Moreover, in order to have those conversations, the task they were responding to had to be a rich, inquiry-based task". So the use of videos, encourages teachers to develop inquiry based tasks, which is important for problem solving in mathematics.

Another way in which videos support learner's conceptual understanding is through scaffolding. Learners can easily scaffold their video lessons in order to support learner's different levels of thinking, higher order questions are asked at important stages of the videos. Time is also a factor in which videos strengthen learners conceptual understanding, according to Alcala (2016) "conceptual ideas are not built in one day or even two. They are developed after repeated exposure to a particular mathematical idea in various contexts. Students have to struggle and resolve that struggle to internalize of concept." So videos allow for repeated exposure to mathematical ideas, through the pausing and rewinding features, this is important to boost learner conceptual understanding of mathematics.

The last factor that videos offer to boost learner conceptual understanding of mathematics is multiple representations. Videos are able to provide multiple representations for mathematical ideas such as dynamic diagrams and mathematical notation. According to Alcala (2016) "When developing conceptual understanding, it's imperative to give students freedom of choice in how they might potentially respond." Videos provides this freedom for learners to think mathematically, which is important for conceptual understanding of mathematics. Videos are examples of web-based learning and is one of the most popular and easily accessible forms of web-based learning. Learners find it easier to learn via videos as a web-based learning tool.

2.33. How is feedback important in web-based learning?

The role of feedback in web-based learning is of crucial importance. It is important that learners give feedback and teachers are reflective when it comes to web-based learning. Constructive feedback leads to effective teaching and learning. According to Bangwert-Drowns et al(1995) "Research generally supports the notion that learners tend to provide feedback resulting in more effective learning." Furthermore, according to Clark & Dwyer (1998) "Research suggests that feedback is one of the most significant sources of information helping individual students to correct misconceptions, reconstruct knowledge, support metacognitive processes, improve academic achievement, and enhance motivation." With this in mind, it is important to note that this study will provide learners with an opportunity to not only provide feedback but receive feedback whilst engaging in web- based learning.

According to Johnson & Johnson (1993) "Although feedback sources include self, technological devices, and other people Johnson & Johnson identify receiving feedback from other people as the most powerful." The importance of feedback is also emphasized further, according to Dempsey et al (1993) "Researchers suggest that feedback can impact learner's motivation and self-esteem. In particular feedback signifies progress." So it is evident that feedback plays an important role in learner performance. According to Dintrich & Schunk (2002) "Feedback that emphasizes mastery, self-improvement, and achievement should therefore have positive effects on learner's self-efficacy."

A popular example of web-based learning would be dynamic geometry software. DGS is being used in high school mathematics and universities in order to further enhance the teaching and learning of mathematics and is viewed by many as a significant step forward in the teaching of congruency-based proofs and geometry as a whole.

2.34. Students Conceptions of Congruency through the use of Dynamic Geometry Software.

Globally, the use of Dynamic Geometry Software tools is an issue that has not been extensively researched. The purpose of this literature review is to summarize the article named, Students Conceptions of Congruency Through the Use of Dynamic Geometry Software. Dynamic Geometry Software also known as DGS is a tool used to teach mathematical concepts such as congruency. This paper argues that the use of DGS provides a new and innovative way of teaching the concept of congruency and should be implemented in the mathematics classroom in order to extend learners knowledge of any given mathematical concept. The difference between dynamic and static diagrams is also highlighted. Dynamic diagrams allow learners to use dragging and measuring tools in order to discover and make mathematical conjectures.

DGS such as GeoGebra, Cabri and the Geometers Sketchpad are examples of new and innovative tools that can be used to teach geometry in the classroom. These platforms provide learners with the opportunity to use dynamic diagrams which allows learners to drag component objects of a shape. Whilst static diagrams do not offer learners the opportunity and thus restricts learners of using innovative tools and rather using more orthodox tools such as compass and straightedge. By using DGS in lessons it enables learners to a measure preserving conception in geometry. According to Gonzalez & Herbst (2009, p.155) "the measure preserving conception of congruency describes the sphere of practice in which a student establishes that two objects are congruent by way of checking that they have the same measure." How the measure preserving conception is used via DGS is when a student uses the measure tool and dragging one extreme.

The possibilities of DGS tools for interacting with diagrams are endless. According to Gonzalez & Herbst (2009, p.158) "By dragging objects students can trace the locus of points, collect data about measurements, and perceive relationships between elements of a configuration." So it is clear that dragging provides learners with an innovative way to explore mathematical concepts. When students use DGS tools to do constructions they can align their understanding of relationships between geometric objects with theoretical notions in geometry, move towards abstracts and generalizations, and get experience with geometric objects and their connections that can enable them to justify relationships between geometric objects and eventually produce proofs of theorems (Gonzalez & Herbst, 2009, p.160). So, it is clear that the dragging tool provides learners with the necessary skills needed in order to understand a mathematical concept, it is beneficial in geometry since it allows learners to produce proofs of theorems, which is a critical part of geometry.

The paper also provides episodes where students used dragging and measuring in order to investigate the relationship between quadrilaterals. According to Gonzalez & Herbst (2009, p.162) "We argue that the combination of dragging and measuring enabled students to investigate relationships between a quadrilateral and the quadrilateral resulting by connecting the midpoints of the original one." The results of the episodes showed that tools within dynamic geometry provided new affordances for students to make discoveries. According to Gonzalez & Herbst (2009, p.177) "In the parallelogram and the square episodes, the dragging and the measuring tool opened the possibility of investigating many drawings instead of one and then seeing what things changed and which remained constant." So, it is clear that DGS tools provides learners with the opportunity to not only investigate but also to make many discoveries which is a fundamental part of mathematics in classrooms.

Furthermore, according to Gonzalez & Herbst (2009, p.177) "More importantly, students could only see what remained constant because they knew what to look at by acts they made on diagrams such as choosing what to measure." Thus through the use of measuring and dragging, students discoveries were supported by a measuring conception of congruency where they took as congruent objects that had equal measure (Gonzalez & Herbst, 2009, p.177).There were also more conclusions derived from the parallelogram and square episodes of the study, according to Gonzalez & Herbst (2009, p.179) "The analyses of the parallelogram and square episodes show how the use of technological artifacts could bring about new conceptions of mathematical ideas in students work. By incorporating new actions, dragging and measuring, the teacher and students in this study developed new knowledge, with the use of new tools."

The above mentioned provides evidence that if teachers wish for learners to discover new mathematical ideas such as conjectures, they have to implement new teaching and learning tools in the classroom such as the dragging and measuring tool which DGS offers. There is a shared view that DGS tools needs to be used in all mathematics classrooms in order to improve mathematics teaching and learning, and to also enhance learners understanding of mathematical concepts. DGS tools are the future of mathematics teaching and learning since it allows learners to make new discoveries, which is fundamental in the field of mathematics. An example of DGS would be GeoGebra, has gained popularity worldwide in the community of mathematics teachers

2.35. Use of GeoGebra in explorative, illustrative and demonstrative moments.

Globally, the use of GeoGebra (GGB) has gained popularity when it comes to the teaching of geometry. According to Lasa & Wilhelmi (2010) "In recent years, GeoGebra (GGB) has displaced Cabri ll Plus at Spanish Universities, in Primary School Teacher Grades and Secondary School Teacher Masters." So it is clear that GeoGebra is more favourable compared to other dynamic geometry software tools. According to Lasa & Wilhelmi (2010) "GGB presents tools for integrated development of notions, processes and meanings on Geometry, Algebra and Functions Theory that highlights the essentially relational aspect of mathematics."

Furthermore, according to Lasa & Wilhelmi (2013) "In addition, since version 4.0, a statistical package contributes to its versatility and the 3D version is on the roadmap for software developers. All this justifies why its use is gradually spreading at Primary and Secondary schools. With this in mind, according to Lasa & Wilhelmi (2013) "Despite these advantages, the widespread use of the program at Secondary schools is far from being a done deal. Therefore, centres for teacher assistance carry out concrete activities in order to increase their digital competence in this area."

So, it is clear that there is still work to be done when it comes to the integration of GGB. According to Lasa & Wilhelmi (2013, p.53) "These activities are reinforced by these organized by the various GGB institutes in Spain: discussion forums, attendance seminars, training courses for teachers and classroom activities for students." Furthermore, there is a shared view that GGB must be used in university education, according to Lasa & Wilhelmi (2013, p.53) "Therefore in this context, it's pertinent to include GGB at University Education. The use of GGB has been adapted to new Primary School Teacher Grades and Secondary School Teacher Masters (SSTM)."

There are three moments on mathematical, where the use of GGB is pertinent, these moments are exploration, illustration and demonstration of a property." The first is exploration, according to Lasa & Wilhelmi (2013, p.54) "Dynamic Geometry software allows the construction of explorative models for solving exercises and problems." Furthermore, according to Lasa & Wilhelmi (2013, p.54) "These models serve to the purpose of inferring properties from a geometric figure or construction." With this mind according to Lasa & Wilhelmi (2013, p.54) "The goal is to design a construction that satisfies the restrictions of a proposition or the initial conditions of a problem. After manipulating the construction, students deduce its properties."

The second moment is the illustration of a property. According to Lasa & Wilhelmi (2013, p.55) "As mentioned before, the widespread use of dynamic geometry software- in particular GGB-consists in giving examples of properties by means of concrete cases selected adhoc." With this in mind, according to Lasa &Wilhelmi (2013, p.55) "A construction is presented which shows the veracity of a given property. This construction serves as a manipulative model and its use may be complemented by a digital whiteboard." Furthermore, according to Lasa & Wilhelmi (2013, p.55) "Thus, for example, to study properties of a triangle, the dynamic software can generate multitude of triangles, instead of only a few of them as in the ordinary state." So it is clear that GGB exposes learners to an array of geometrical problems which ideal for the learning of proofs.

According to Lasa & Wilhelmi (2013, p.55) "This widespread use should motivate new examples that would improve students confidence in the formulated conjectures." Furthermore according to Holz (2001) as cited by Burke & Kennedy (2011), "states that in an environment of dynamic geometry software, students who observe the truth of a conjecture has the urge to know the reason for the claim. After all the illustration of a property is just a "picture" of it."

The third moment is the demonstration of a property. According to Lasa & Wilhelmi (2013, p.55) "Traditionally, the step by step formal proof of a geometrical property is carried out on a blackboard." With this in mind, according to Lasa & Wilhelmi (2013, p.55) "However, since the ordinary blackboards are substituted by digital whiteboards and dynamic geometry software, these formal proofs are left out, illustrative constructions are not designed considering elements of the formal proof, and sometimes, computing steps differ from pure logical reasoning." So it is clear that the transition from blackboards to digital whiteboards has caused a change in the instructional exchange of proofs. According to Lasa &Wilhelmi (2013, p.55) "Its teachers' job to select situations which permit to join both reasoning, inductive reasoning due to dynamic geometry software, and deductive reasoning, traditionally linked to formal proofs-pencil and paper proofs." So the role of the teacher when it comes to the use of GGB is critical.

There is a shared view that the use of GGB stimulates an environment that is conducive for the learning of proofs. According to Lasa & Wilhelmi (2013, p.56) "in addition, dynamic software such as GGB or Sketchpad present an environment where it is easy to find counter examples, therefore, the notion of an axiom is extended and few propositions require formal proof, if you fail to find a counter example with the dynamic model, they're believed to be true (De Villiers, 2004)." With this mind according to Lasa & Wilhelmi (2013, p.56) "In this context, different authors present a number of classifications for roles demonstration may play, such as explanation, verbal argument or formal proof." Furthermore, more research is being done when it comes to the use of GGB, according to Lasa & Wilhelmi (2013, p.56) "Since its clear from the literature that GGB serves to the purposes of illustration, we would like to make a step forward and discuss which characteristics should a GGB construction have in order to aid a formal argument." So it is clear that the use GGB is in imperative when it comes to the teaching of geometrical proofs and can lead to success in mathematics.

2.36. Theoretical Framework

A theoretical framework is made up of one or several interrelated theories or concepts relevant to the intended study (Driver, 1989). The theoretical framework plays a pivotal role in guiding the study by giving it some form of direction. It prevents the researcher from getting confused in the process of the study since there are many things worth including. It acts as a lens through which the researcher views the whole study. This study was situated within a socio-constructivist theoretical framework, which in the latter gave rise to the discovery learning theory, through the use of technology which was web-based learning. The framework for this study is an investigation of grade 9 mathematics learner's ability to do congruency-based proofs within a web-based learning environment. The rise of this constructivist learning approach led to a group discussion in mathematics classrooms. According to Driver et al (2000, p.298) "the literature on constructivist teaching continues to be an important source of information about appropriate strategies for promoting discussion and argument in order to develop students conceptual understanding."

2.36.1. What is constructivism?

The Constructivist approach promotes the incorporation between teaching strategies and learner responses. Furthermore, it also provides the platform for students to analyse, interpret and predict information. Learning theories help explain how learning takes place, especially mathematics learning (Jaworski, 2006). Yet, many theories are not reflective of the social environments that learners find themselves in. Learning should be a way of being, it should be habit and inscribed in the attitude of the learner. (Vaill, 1996).

Cicconi (2014) explains that technology supports high cognitive functions that are necessary when learning mathematics such as, evaluating and analysing. This then leads to exploratory investigations and creating. Ulrich, Tillema, Hackenberg and Norton (2014) expresses the fact that constructivism emphasizes exploratory investigation through the model of teaching. This involves creative learner interactions, in order to guide the learners from their existing framework of knowledge by exploring what they already know. Constructivism encourages teaching strategies that utilise cognitive functions which include analysing, interpreting, and predicting information (Ulrich et al, 2014). It also promotes problem solving and incorporates these skills in the learning process. So, learners actively invent their own learning, making it more meaningful.

2.36.2. Social constructivism

A social constructivist approach influenced by Vygotsky's work, emphasizes the social contexts of learning and that knowledge is mutually built and constructed. According to Kalpana (2014, p.27) "By interacting with other students get the opportunity to share their views and thus generate a shared understanding related to the concept."

From Piaget to Vygotsky the conceptual shift is from individual to collaboration or assisted performance, social interaction and socio-cultural activity. According to Kalpana (2014, p.28) "Two important assumptions in social constructivists approaches are situated cognition which refers to the idea that thinking is located in social and physical contexts not within individuals mind." Which ultimately means that knowledge is tied to the situation in which they are harmed, and it is difficult to apply in other situations. In essence, learning situations should be as close to real life situations as possible. The second assumption would be Zone of proximal development which according to Kalpana (2014, p.28) refers to "the range of tasks that are too difficult for children to accomplish independently" but can attain mastery if they are provided assistance and guidance by the adults or more able peers.

2.36.3. Proof Writing Competency

The proof writing competency model identifies seven dimensions that needs to be prevalent in student's actions towards the development and writing of proofs in geometry (Mcrone & Martin ,2011). The proof writing competency model is important for this study because it served as the theoretical framework for the study in order to analyse the data, especially the congruency-based proof problems in both task-based activities. Each of the 7 dimensions of the model was used in order to analyse student actions and whether each learner solved the congruency-based proof problems correctly or not. So, it was significant to the study as it basically provided a criteria to be satisfied for each problem to be solved by every learner.

Table 2.1: Student actions that contribute to the development and writing of geometry proofs (adapted from Mcrone and Martin (2011, p.8)

Dimension	**Student Actions**
Make Conjectures	Make claims or ask questions about relationships.
Justification	All conjectures must be justified or refuted
Role of Proof	Proof is used to: • Establish validity of statements • Explain why conjectures are true
Provide Warrants	Justify claims or supply reasons for others' claims. • Use geometry property or relationship • Appeal to logic structure
Build Chain of Reasoning	Create argument consisting of several connected statements.
Standards for Reasoning	Reasons must meet certain standards in order to be valid
Use a diagram	Use a diagram to identify relationships and illustrate reasoning.

It is important to understand each of the 7 dimensions before one uses the proof competency model in order to analyse the data, as this is a critical part of the data analysis. For the purpose of this study, below is detailed explanations of each of these dimensions which were important for this study.

What is a conjecture?

In mathematics, a conjecture is a mathematical statement which appears to be true but has not been formally proven. A conjecture can be thought of as the mathematician's way of saying "I believe that this is true, but I have no proof yet." A conjecture is a good guess or an idea about a pattern. Furthermore, a conjecture is a conclusion or a proposition that is proffered on a tentative basis without proof.

What is justification?

In mathematics teachers should encourage students to focus on more than just the right answer, students need to understand the process and underlying concepts to derive the right answer (Johnson and Watson, 2011). In other words students need to find and justify their solutions. To justify a solution, students will need to be able to use appropriate mathematical language to give reasons for the particular approach used to solve a problem, that solution needs to be justified. That is, the students need to explain how they know that their "solution" is correct. Justification of a solution can also arise in the context of a class discussion of mathematics, where students will need to explain their solutions orally.

What is the role of proof?

A proof is a formal demonstration of a result, a sequence of logical arguments that allows establishing the veracity of a mathematical property. According to Bleiler-Baxter & Pair (2017) "for a mathematician, a proof serves to convince or justify that a certain statement is true. But it also helps to increase the understanding of the result and the related concepts. That is why a proof has the role of explanation." So it is clear that proof serves more than one role in mathematics and is very useful when it comes to explaining a solution to a specific geometric problem.

What are warrants?

Warrants in mathematics is that which secures knowledge and its aim is for truth in an argument. According to Toulmins (1958) "The warrant (W) justifies the connection between data and conclusion; warrants include appealing to a definition, a rule, an example, or an analogy." The sequence of reasoning for each statement can be the warrant.

What is chain of reasoning?

A chain of reasoning is constructed for supporting multi-step and dynamic reasoning on changed relations and objects. In detail, iteratively, the relational reasoning operations form new relations between objects, and the object refining operations generate new compound objects from relations. A chain of reasoning is basically a structured argument which is used to prove whether a statement is true or false.

Thus, with the above mentioned in mind, the study utilized the literature to determine the thinking processes of the learners using the recognition of the 7 dimensions of proof writing competencies. The literature has focused on many misconceptions and studies conducted with regard to congruency-based proofs however, this study proposes research be done on with regard to learner understanding of congruency-based proofs in a web-based learning environment, in South Africa, more specifically, in the Western Cape in a small case study. Having conducted a study on Congruency based proofs within a Web-Based learning environment, with a group of grade 9 learners, hence addressing the 'gap' in research in developing countries.

2.37. Conclusion

Geometrical reasoning connects the learning and teaching of congruency-based proofs in grade 9 to FET Geometry in high school. Web based learning was chosen as the platform because according to Hill (1997) "WBL gives the chance for students to acquire higher thinking strategies that encourage divergent thinking with the existence of guides from different perspectives." Generalisation is the principle or rule that can be used as the basis for geometrical reasoning. Therefore, students have the ability to develop generalisations through drawings and proofs. In this research, I wish to direct learner's attention to the relationship between congruency theorems and proofs, so that they can move from geometrical thinking to geometrical reasoning. The next chapter discusses the research methodology for this study.

CHAPTER 3

RESEARCH METHODOLOGY

3.1. Research design

A research design dictates the manner in which the researcher will conduct their research, Jacobs (2005) suggests that it refers to the plan according to which the study is executed. According to Dlamini (2017) "It refers to all the planning involved regarding the study as well as all the decisions that the researcher had to make in order to answer the research question as effectively and efficiently as possible." Simply put, research design describes the major procedure to be followed in carrying out research. It is a specification of the operations to be performed (Dlamini, 2017).

This study took the form of a case study and used a mix method approach by using both qualitative and quantitative methods (and techniques). The quantitative approach involves the use of numerical data to analyse data while the qualitative approach focuses on the web or meanings of how people make sense of their worlds and observations (Hofstede, Neuijen, Ohayv, & Sanders, 1990). This study consists mainly of a collection of qualitative data, which is supplemented with quantitative data analysed using elementary descriptive statistics including tables.

The Case study method enables a researcher to closely examine the data within a specific context. According to Zainal (2007, p.1) "In most cases, a case study method selects a small geographical area or a very limited number of individuals as the subjects of study." Furthermore, according to Zainal (2007, p.1) "case studies, in their true essence, explore and investigate contemporary real-life phenomenon through detailed contextual analysis of a limited number of events or conditions, and their relationships." Also, Yin (1984, p.23) defines the case study research method "as an empirical inquiry that investigates a contemporary phenomenon within its real-life context when boundaries between phenomenon and context are not clearly evident, and which sources of evidence are used."

As a result of using the case study research method, data can be obtained from interviews, direct observations, participant observations, and documentation and physical artefacts. Substantial data collected through such a combination will allow the researcher to gain an in-depth insight of grade 9 learner's discovery of congruency invariants, conjecture formulation and justification, and their proof writing competency. According to Yin (2012), case study designs can be exploratory, explanatory and descriptive. In this study, the researcher will use a descriptive case study design which will allow the researcher to describe the unobstructed phenomena emerging from the range of data collected. Roller and Lavarakas (2015) acknowledge that a case study design may include single or multiple case studies, and that prior to commencing the study the researcher must determine the unit of analysis. This study had taken the form of a single case study, where the unit of analysis was grade 9 mathematical learners at a high school located in the Southern Suburbs in Western Cape.

3.2. Research Paradigm

Research paradigm is the set of common beliefs and agreements shared between scientists about how problems should be understood and addressed (Kuhn, 1970). Research paradigms can be characterized by the way scientists respond to three basic questions: ontological, epistemological and methodological questions" (Guba, 1990). Social scientists can ground their inquiries in any number of paradigms. None is right or wrong, merely more or less useful in particular situation. These paradigms shape the kind of theory created for general understanding (Babbie, 1998) According to Kuhn (1970) paradigm contains "universally recognized scientific achievements that, for a time, provide model problems and solutions for a community of researchers". Data triangulation is when data is collected at different times and source and combined, compared to increase confidence. Investigator triangulation is when data is gathered by different investigators, independently and compared/combined to increase confidence. Lastly methodological triangulation is using both qualitative and quantitative methods to increase confidence.

3.3. Population and sampling

According to Bless and Higson-Smith (2000, p.85) a population is "the set of elements that the research focuses on and to which the obtained results should be generalized". Given the focus of this description, the population that will be used in this study will be 22 grade 9 learners doing mathematics at a high school located in the Southern Suburbs in Western Cape. It is important to note that this sample was reduced into a smaller sample in relation to the subjects that were taught by the teacher to gather information for this study, due to not all participants whom gave consent to be involved in the study.

A convenient sampling procedure will be used in this study. It means that learners that are chosen for this study are only chosen because they belong to a specific class which is available for the study. In addition, learners will be self-selected, meaning that they will decide for themselves if they wish to be a part of the study (Laerd, 2012).

3.4. Data collection methods and instruments

A mixed methods study employed the following data collection methods: task-based activities, observations, discourse and discussions and focus group interviews. The data collection in this study was done according to the data collection plan indicated in Table 1. It was envisaged that the triangulation of the various instruments and data sets yielded a clearer picture of grade 9 learner's capabilities of discovering methods of application and writing proofs within the context of congruency theorems.

In this study, two worksheets, observation and semi-structured interviews were used to collect the relevant data needed to provide answers to the research question. The worksheet (see Appendix 1 and 2) in the context of this study was deemed to be two the task-based activities, the first of which consisted of sketching of triangles mainly and the second was based mainly on congruency-based proofs, which learner's had to solve in double mathematics period of 80 minutes. The purpose of the worksheet sheet was to determine learners' proof writing competencies, ascertain how they solved problems pivoted around congruency-based proofs, and identify challenges and difficulties learner's experienced in solving such problems. As learners were working through each problem individually, the researcher observed learners' moves in attempting to solve the congruency-based proof problems using a structured observation schedule (see Appendix 3) According to McLeod (2015), "a structured observation schedule is used in controlled observations, and it is the duty of the researcher to brief research participants of the study aims so to let them know why they are being observed". In controlled observations, the researcher usually avoids any direct contact with the group, also known as non-participation. Hence, in this research study, the researcher acted as participant observer, the researchers role was to play the YouTube videos on congruency based proofs and also guide learners with the task based activities, ensuring that learners do their own work. The researcher used the observational checklist was used to systematically record results of the behaviour of learners as they worked through each of the word problems. The intention of the observation was to supplement the information provided by the participants to get credible information with regard to proof writing competencies in solving congruency-based proof problems. Semi-structured interviews was a third strategy that was planned to be used to collect data The interviews with the learners were scheduled to be completed after school hours for about 45 minutes for 5 days over a week. The reason for conducting the interviews after school hours was due to the fact the researcher did not want to interrupt the teaching and learning process. The interview with each learner was to be about 10 minutes. The researcher designed an interview schedule (see Appendix 4) to ensure focus without imposing too much pressure on participants.

3.4.1. How data was collected from participants using the Task-based activity

Day 1:

On the first day of data collection, the learners or research participants watched a video via YouTube, based on the sketching of triangles. This video showed learners how to sketch triangles properly and will also discuss the properties of triangles. After watching the video, learners asked questions based on the sketching of triangles. Once all questions was answered by teacher, the teacher or researcher did question 1.1 of the congruency task with them, so that learners are properly guided in terms of sketching triangles. Learners then continued to sketch triangle 1.2 to 1.6. in task-based activity 1. If learners wanted to watch the YouTube video again, the teacher was readily available to play the video, in order to remove any misconceptions.

Day 2:

After learners were done sketching all triangles, the teacher played a YouTube video based on congruent triangles. This video explained the properties for triangles to be congruent. The video also provided an example for the different cases for congruency. Learners then completed question 1.7 which asked learners to state which of the previous six triangles were congruent to each other and which triangles are not congruent. Learners then completed question 2 which was a table based on then conditions which made triangles congruent or not, learners would have to answer yes, or no. Learners were allowed to watch the video based on the properties of congruent triangles, in order to remove any misconceptions. After learners were done with question 2, the teacher explained question 3 clearly to the learners and learners then watched a video based on the differences between, congruent, similar and parallel lines. Learners then completed question 3 and were allowed to watch the YouTube video again.

Day 3:

Learners made sure that Question 3.1 to 3.3 was completed. The teacher then played a YouTube video based on congruency-based proofs, which explained clearly on how to complete congruency proof related problems, learners were then allowed to ask questions based on the statement and reasoning parts to a proof, or any other questions based on congruency-based proofs. The teacher allowed learners to complete question 3.4 based on a proof problem. After all questions were completed by learners, the teacher had an open-ended discussion based on the congruency- based task and made a note of learner responses.

Day 4:

The teacher played a different YouTube video based on congruency-based proofs, which explained clearly on how to complete congruency proof related problems, learners were then allowed to ask questions based on the statement and reasoning parts to a proof, or any other questions based on congruency-based proofs. The teacher allowed learners to complete question 1.a-1.d based on a variety of congruency based proof problems. Once again making the YouTube video readily available for learners to watch. After all questions was completed by learners, the teacher had an open ended discussion based on the congruency based task and made note of learner responses.

Day 5:

At the beginning of the lesson, the teacher played a YouTube video based on congruency-based proofs, which explained clearly on how to complete congruency proof related problems, learners were then allowed to ask questions based on the statement and reasoning parts to a proof, or any other questions based on congruency-based proofs. The teacher allowed learners to complete question 2 to question 5 based on a congruency-based proof problem. Once again making the YouTube video readily available for learners to watch. After all questions are completed by learners, the teacher will have an open-ended discussion based on the congruency-based task and make note of learner responses.

3.4.2. Data collection plan

The Data collection plan shows the summary of the following methodological framework in table 1 below and an outline thereof. These data are typically analysed quantitatively. Task based activity and observations are attached as Appendices 1 and 2. This process is explained in the four steps below and outline of the data collection plan.

Table 3.1 Data collection plan

Research Question	Data collection Method	Instrument
1. What invariant(s) learners discover when experimenting with congruency theorems?	1. Task based activity 2. Observations	1. Task Based Activity 1: Congruency invariants 2. Observation Schedule
2. How do learners formulate each of their discovered congruency as conjectures?	1. Task based activity 2. Discourse and Discussions	1. Task Based Activity 1: Congruence invariants 2. Audio recording during task-based activity discussions.
3. How do grade 9 learners attempt to prove (i.e. logically explain) each of their congruency conjectures?	1. Task-based activity 2. Observations	1. Task Based Activity 1: Congruency invariants 2. Observation Schedule
4. What proof writing competencies learners demonstrate when proving or solving congruency theorems?	1. Task-based activity 2. Focus group interviews	1. Task Based Activity 2: Solving and Proving Congruency theorems 2. Focused Group-Interview schedule

3.4.3. Task-based activity

A tasked based activity in the form an Investigation Task based on congruency theorems was used in this study, according to Pokhrel (2016, p.26) " Activity-based learning as the name suggests is a process whereby learners are actively engaged in learning process, rather than "passively" absorbing lectures." It is based on the core premise that learning should be based on doing some hands-on experiments and activities rather than just listening to lessons only. According to Pokhrel (2016, p.26) "Actively-based learning involves reading, writing, discussion, practical activities, engagement in solving problems, analysis, synthesis, and education." There are several strategies to be carried by teachers in the classroom in activity based. Some of the activities as mentioned by Festus (2013) are discovery approach of teaching aids, co-operative learning or small group learners, discussion in class. With regards to this study, the tasked based activity or investigation task was applied in such a manner that it was the source of physical or tangible data needed for the research. Each participant received the same tasked based activity and was allocated the same amount of time to complete it. The task-based activity was facilitated by me the researcher, and it was set under strict examination conditions, to avoid any irregularities.

3.4.4. Focus group interviews

According to Freitas et al. (1998) "Focus group is a type of in depth-interview accomplished in a group, whose meetings present characteristics defined with respect to the proposal, size, comparison, and interview procedures." The focus or object of analysis is the interaction inside the group. The participants influence each other through their answers to the ideas and contributions during the discussion. According to Freitas et al (1998) "Focus group can contribute to a project built around the individual interview especially in the planning phase of the interview route." In this case, the idea was to use a small number of exploratory groups, in an initial stage of the research, to guide the construction of the topics of the interview. In this study, focus group interviews was conducted by the researcher in such a manner, that the researcher divided learners in groups based on their competency levels. Well performing, average and underperforming learners will be separated and interviewed.

3.4.5. Observations

According to Kawulich (2012, p.2) "Observation used in the social sciences as a method for collecting data about people, processes and cultures." Simply put, Observation is also a tool used regularly to collect data by teacher researcher in their classrooms, by social workers in community settings, and by psychologists reducing human behaviour. Furthermore, according to Kawulich (2012, p.2) "Observation, particularly participation observation, has been the hallmark of much of the research conducted in anthropological and sociological studies and is a typical methodological approach of ethnography." Furthermore, according to Marshall & Rossman (1989, p.79) "Observation is the systematic description of events, behaviours, and artefacts of a social setting." In this study, observations were used by the teacher researcher; in such a manner that the research writes down in detail clearly on what was observed during the task based activity or any stage of the classroom interactions. The interactions between the participants taking part in the research as well the researchers own experiences was recorded clearly as a result of the observation process.

3.4.6. Discourse and Discussions

Classroom discussions should help students learn but getting students to actively participate can sometimes be a difficult task. Silberman (1996) asserts that to stimulate class discussion, you first have to build interest. How, then, can we make that happen? A number of strategies are presented here which can change the once ―quiet classroom into one that has lively and meaningful discussion. According to Barton, Heilker, and Rutkowski (n.d.) "our students should be attentive and involved and engaged to help them construct their own learning and engage in discussion." Burton et al. also point out that effective classroom discussion occurs when students talk with other students and not just the instructor (para. 7). Dialogue among classroom peers can be monopolized by a few talkative students while other students sit back and passively observe. Helping to break the habit of rote, two-way responses between the instructor and the student while the rest of the class remains uninvolved can be achieved by implementing some of the strategies presented here. The classroom discussion tool was mediated by the researcher throughout the duration of it. The researcher ensured that the participants engaged with each in a respectable manner and be allowed to express their feelings and experiences openly with regards to their interactions with the Congruency task-based activity. Learners would also provide clearly their conclusions with regards to the relationship they have discovered between application and proofs in Congruency.

3.5. Validity and Reliability of the instruments

Validity is the extent to which an instrument measures what it is supposed to measure and performs as it is designed to perform. Reliability can be thought of as consistency, it questions whether the instrument consistently measures what it is intended to measure. This means that the methods and underlying research design was of a high quality in to obtain data that is both valid and reliable. I envisaged that by using multi-methods strategies I was able to triangulate data from variety of sources and obtain a more vigorous data set than would otherwise be the case. In addition, a peer review system was used to validate the instruments, in order to ensure that the questions are unbiased, trustworthy and consistent with the research questions being investigated.

3.6. Data analysis

The quantitative and qualitative methods of data analysis was used in this study. Quantitative methods will be restricted to simple statistical analysis of frequencies and measures of central tendency. To display patterns and shifts, tables and graphs will be used. Qualitative methods will be analysed using methods of discourse and content analysis (Morrison, Cohen and Manion, 2007). Mcrone and Martin's (2011) Proof Competency Framework, Posamentier et al's (2010) Proof Performance Geometric Rubric, and Jager et al's (2004) Categories of Rider Strategies will be used to track the participant's progress on the respective task based activities. The Proof Competency Framework, Proof Performance Geometric Rubric, and Categories of Congruency theorems will also be used as units of analysis to interpret the findings.

3.7. Limitations of the study

Since the researchers' class had learners who were extremely weak or extremely strong in their mathematical abilities, there was discrepancies in terms of the accuracy of the data. The instrument used had to accommodate for learners of different competency levels. This could be a time consuming activity, given the amount of work grade 9 learners must cover in the curriculum.

3.8. Ethical Issues and Considerations

Research which involves human subjects is clearly known to be sensitive because of potential abuse of subjects' rights or issues of security and avoidance of physical, emotional or material harm. Taking this into consideration, all participants were informed, and their consent was sought, and the purpose of the study was explained fully to them. The participants were assured anonymity, confidentiality and nondisclosure of all information, and their right to withdraw from the study if they wish to at any stage. In line with the university's ethics practice, the ethical requirements of the University of the Western Cape were strictly adhered to. In accordance with research procedures in place, the ethical clearance of the Western Cape Education Department was also obtained.

3.9. Conclusion

This chapter explained the methodology of the study, which in turn described the sample and how data was collected. Qualitative research provided a platform for the analysis from being gained from the natural settings of where learners were completing their task based on congruency-based proofs. The documents that were analysed is from a Grade 9 class of 2022. The issues of validity, reliability and ethical considerations were completely covered in this study. The following chapter looked at the findings of the research.

CHAPTER 4

DATA ANALYSIS

4.1. Introduction

The previous chapter involved a discussion of the research design, which includes the research method and the manner in which the data was collected and analysed. This chapter discusses the findings from the study.

4.2. Analyses of Worksheets

The research was conducted in the researcher's class. The researcher was the implementer of the task, he conducted the lessons and stayed for the entire duration for the whole process of data collection. He used a web-based learning system as his teaching method, so that learners could extract information via it, in order to complete the Congruency based task. Worksheets were given to them, and they worked on them for about two days. The researcher went through the task with them, and the researcher observed the process as well. These documents were collected to be analysed.

These documents were used in order to identify the similarities and differences in how learners answered the questions. The analysis is presented in a discussion form, based on each question.

The researcher observed the learners work individually with the tasks and group discussions they had afterward. Learners discussed question explaining to one another how they got to a particular answer.

The following tasks as embedded in Appendix 1 and Appendix 2 were given to the learners to do after the initial exemplary exercises.

The learners first worked individually and afterwards had discussions. The participants were 22 learners from the researchers' class. The following is based on the learners' individual work. When it came to the group discussions some only gave the answers, but from the submitted scripts one could see what was done when looking at their congruency-based proofs.

Question 1 was relatively easy for learners to do. The learners used different methods to complete the tasks. About ten learners could not complete the question pertaining to showing which triangles were congruent or not in Question 1. Most learners out of the group of 22 learners completed the Question 3 based on congruency-based proofs in task-based activity 1.

4.2.1. Analysis of constructions: Question 1.1 on investigation task

Question 1.1 in the investigation task reads as follows:

If three sides are given: side, side, side (S, S, S): (4)

ΔDEF with DE = 7 cm, DF = 6 cm and EF = 5 cm.

An expected solution for Question 1.1 is as follows:

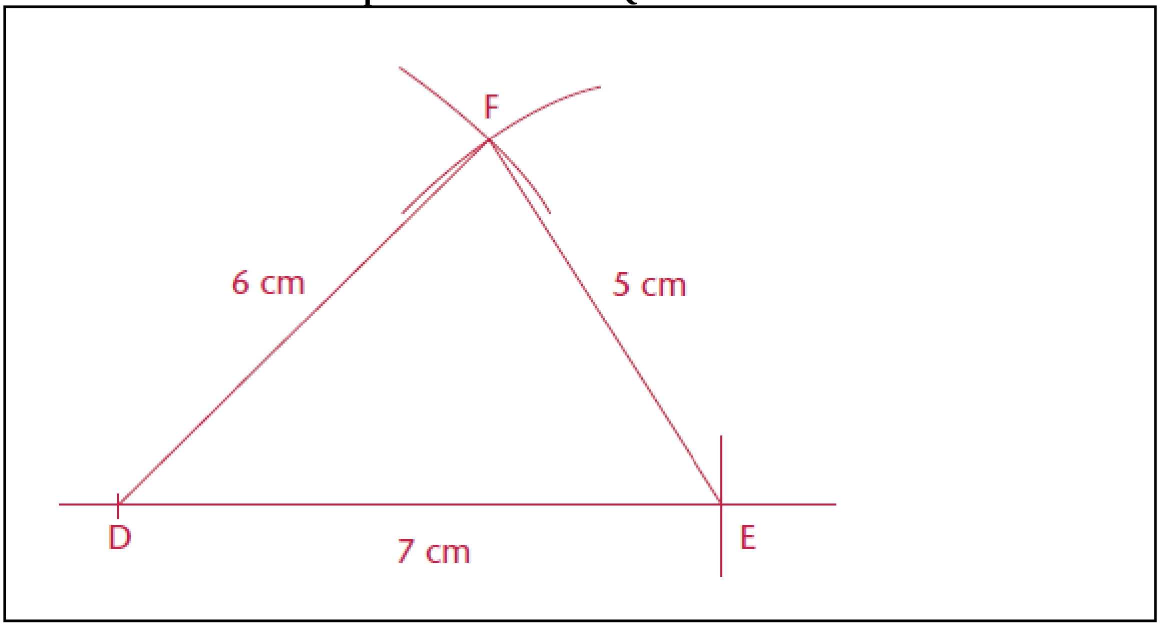


Figure 4.1: An expected solution for Question 1.1

As illustrated in Table 4.1, analysis of learners' responses to Question 1.1 showed that 14 learners attempted the problem correctly, 8 learners attempted the problem but made errors, and none of the learners did not attempt Question 1.1.

Table 4.1: Analysis of Question 1.1

Learner response	Attempted correctly	Attempted but made errors	Not attempted
Number of learners	14	8	0
Learner number	2,3,4,7,8,9,10,14,15,16,17,18,20 21,	1,5,6,11,12,13,19 22	None

As illustrated in the table above it is clear that learners performed well in this question. The reason why learners performed well in this question could be due to the fact learners found the video to be very helpful when sketching the triangle. The video clearly explained on how to go about sketching this specific type of triangle.

4.2.2. Exemplification of learners who attempted Question 1.1 correctly.

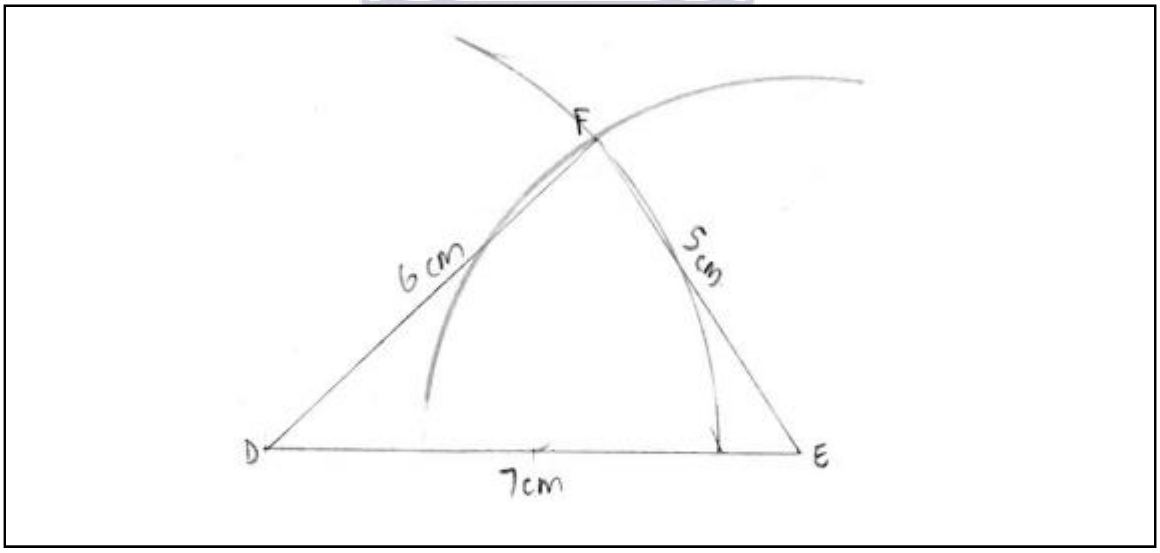


Figure 4.2: Learner 21 response to Question 1.1

Learner 21 (L21) showed a clear understanding on how to sketch the triangle given three sides. The learner labelled the triangle in the correct order, labelling it in an anticlockwise direction as instructed by the teacher, this was an instruction that was not followed by all learners thus leading to errors. The learner also had an accurate measurement of each side and thus sketches the triangle successfully. The learner left the arc marks behind which showed evidence of proper use of the compass. Unfortunately, there was a trend amongst most learners that they do not leave the arc marks behind and instead they erase it. The learner displayed evidence of the necessary competence to sketch a triangle given three sides.

4.2.3. Exemplification of learners who attempted Question 1.1 but made errors.

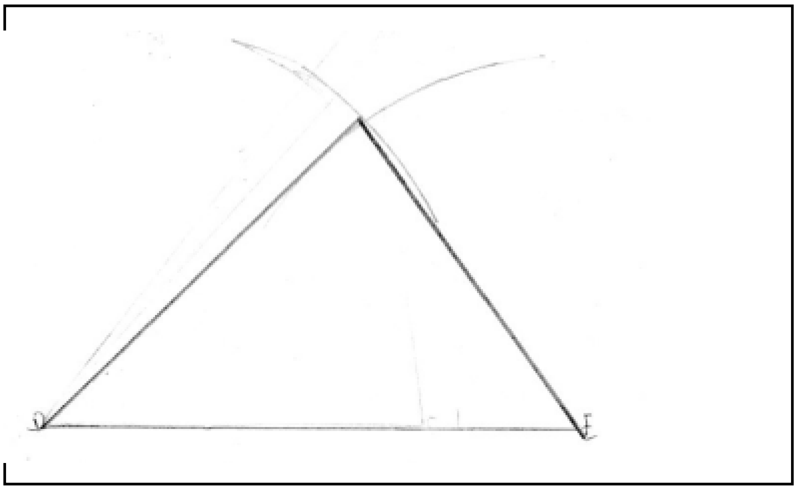


Figure 4.3: Learner 5 response to Question 1.1

L5 did not sketch the triangle properly and showed a clear lack of understanding on how to sketch a triangle given three sides. There are multiple errors here, firstly, the learner did not sketch the triangles with accurately with the compass and secondly the learner did not label the triangle in the anticlockwise direction and thus did not follow instruction properly, this was a common trend amongst most learners leading to them being penalised. This learner showed clear misconceptions when it comes to sketching a triangle. This implied that the learner did not have the necessary competence to sketch the triangle. It is also possible that learners did not have enough practice to use mathematical instruments in order to sketch the triangle. Which ultimately led to errors being made by the learner.

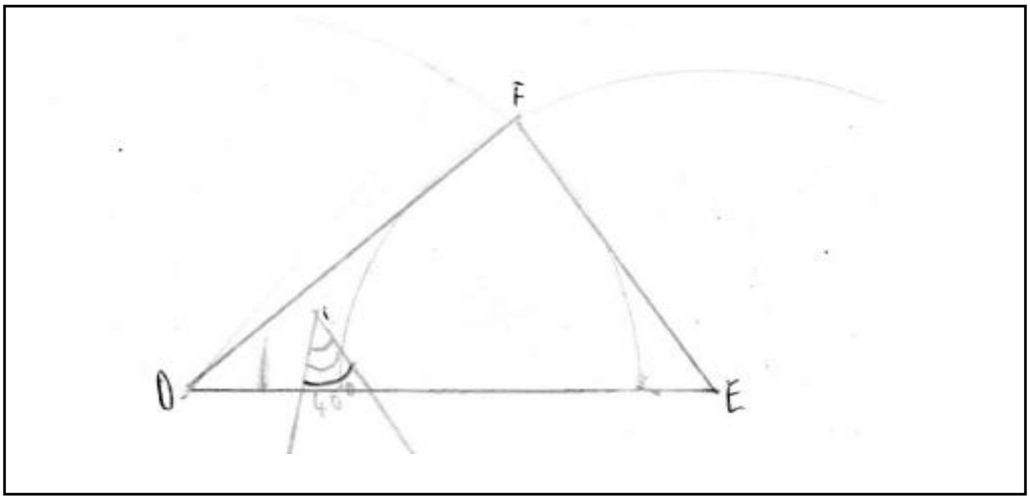


Figure 4.4: Learner 13 response to Question 1.1

L13 displayed the wrong approach to sketching the triangle, the learner sketched the triangle correctly but sketches triangle 1.2 onto triangle 1.1 and this led to many errors. This was a common trend amongst most learners who attempted the problem but made errors. The learner did not indicate the measurements as well which makes it a challenge for learners to distinguish between the sides and its specific measurements. This made it difficult for the learner to compare which triangles are congruent or not. The learners showed many misconceptions when it comes to sketching a triangle correctly. So the learner does not have the necessary competence to sketch the triangle.

4.2.4. Analysis of constructions: Question 1.2 on investigation task

Question 1.2 in the investigation task reads as follows:

If three angles are given: angle, angle, angle (∠,∠,∠): (4)

ΔABC with $\angle A = 80°$, $\angle B = 60°$ and $\angle C = 40°$.

An expected solution for Question 1.2 is as follows:

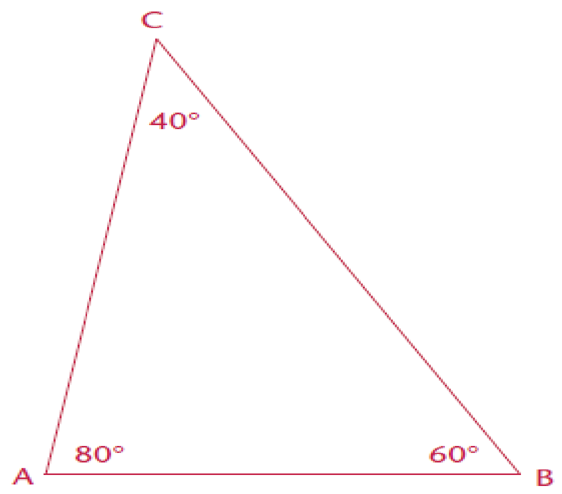


Figure 4.5: An expected solution for Question 1.2

As illustrated in Table 4.2, analysis of learners' responses to Question 1.2 showed that 1 learner attempted the problem correctly, 19 learners attempted the problem but made errors, and 1 learner did not attempt Question 1.2.

Table 4.2: Analysis of Question 1.2

Learner response	Attempted correctly	Attempted but made errors	Not attempted
Number of learners	2	19	1
Learner number	2,16	1,3,4,5,6,7,8,9,10,11,12,13,14,15,17,18,19 20,21	22

As illustrated in the table above it is clear that learners performed poorly in this question. The reason why learners performed so poorly in this question could be due to the fact learners still do not know how to measure angles correctly and thus could not sketch the triangle properly. This clearly needs to be addressed by the teacher because it is critical for learners to understand how to measure angles with the use of a protractor. So, it is clear there are major misconceptions with regards to this question. The reason there are these misconceptions was learners did not know how use their mathematical drawing tools in order to sketch the triangles, this exposed the fact that learners were not taught properly in previous grades on how to sketch triangles.

4.2.5. Exemplification of learners who attempted Question 1.2 correctly.

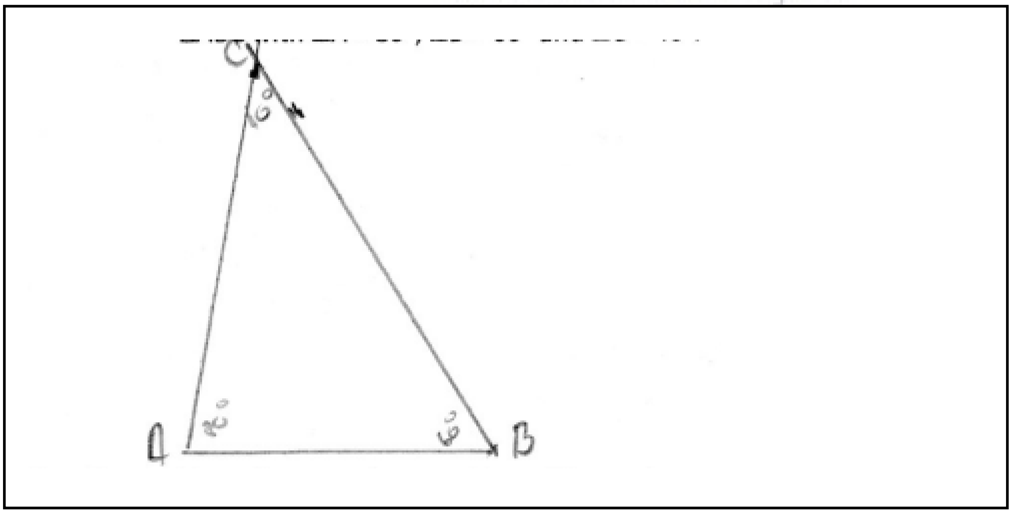


Figure 4.6: Learner 16 response to Question 1.2

L16 showed a clear understanding of how to measure angles correctly with the use of a protractor, many learners struggled with using a protractor to measure angles. The learner also labelled the triangle correctly and thus sketches the triangle successfully. The learner showed no evidence of misconceptions when it comes to sketching a triangle given three angles. The learner showed evidence of having the necessary competence to sketch a triangle given three angles.

4.2.6. Exemplification of learners who attempted Question 1.2 but made errors.

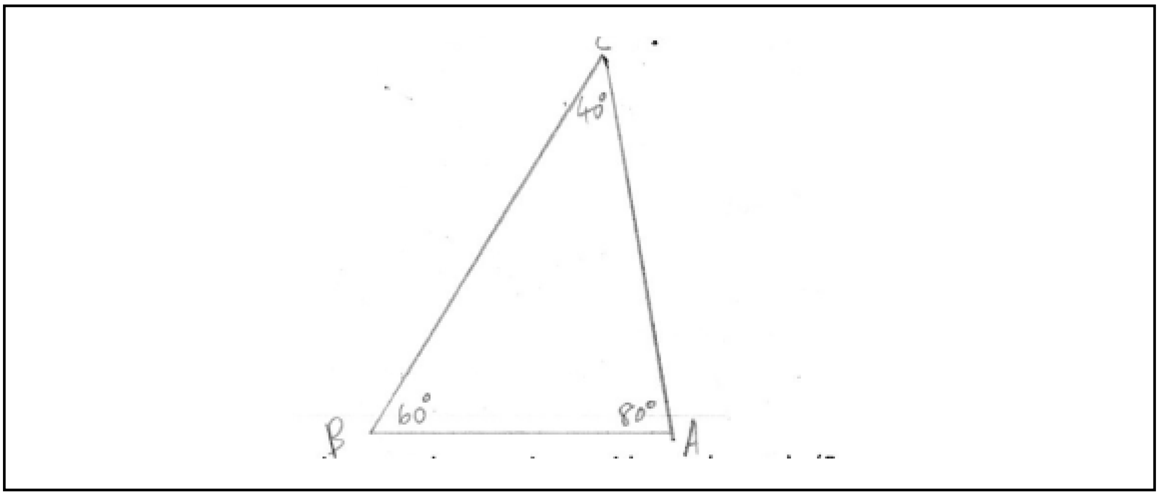


Figure 4.7: Learner 17 response to Question 1.2

L17 does not measure the angles in the correct order and sketches the triangle incorrectly, the learner also does not label the triangle at all. This learner clearly does not understand the significance of sketching a triangle properly. The learner did not follow the instructional video properly on how to sketch a triangle given three angles. The learner does not have the necessary competence to sketch a triangle given three angles.

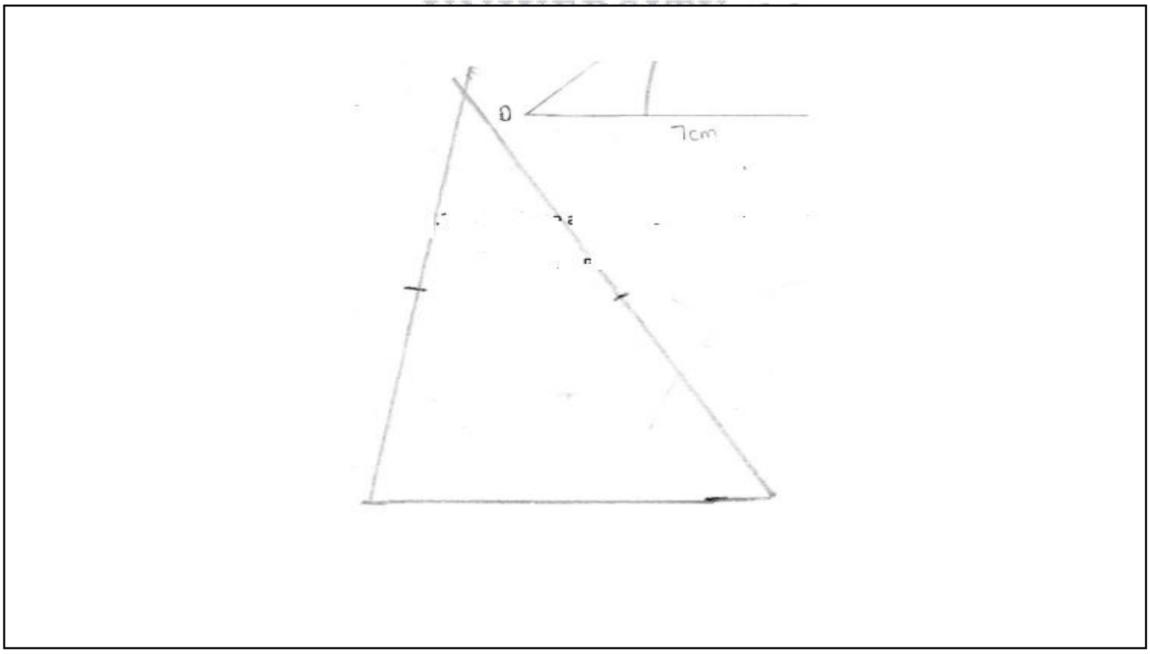


Figure 4.8: Learner 3 response to Question 1.2

L3 did not sketch the triangle properly, used a completely wrong approach and thus displayed major gaps in knowledge when it comes to sketching triangles. A similar sketch was picked up with another participant. The learner had a lack of understanding on where to start when sketching a triangle given three angles. The learner clearly needs to be re taught on how to sketch this triangle. This is a major concern because if the learner has this major gap in knowledge, it will be difficult for the learner to sketch more complex triangles and to even compare which triangles are congruent or not. This learner clearly does not have the competence to sketch a triangle given three angles properly and needs to work harder to sketch such a triangle

4.2.7. Analysis of constructions: Question 1.3 on investigation task

Question 1.3 in the investigation task reads as follows:

If one side and two angles are given: side, angle, angle (S, ∠,∠): (4)

ΔGHI with GH = 8 cm, ∠G = 60° and ∠H = 30°.

An expected solution for Question 1.3 is as follows:

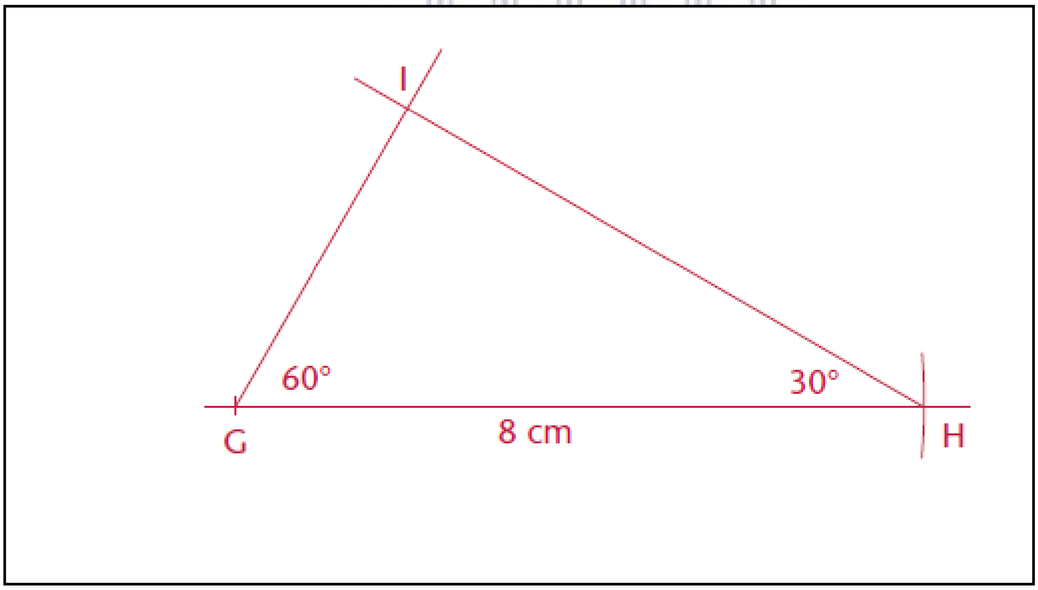


Figure 4.9: An expected solution for Question 1.3

As illustrated in Table 4.3, analysis of learners' responses to Question 1.3 showed that 9 learners attempted the problem correctly, 12 learners attempted the problem but made errors, and 1 learner did not attempt Question 1.3.

Table 4.3: Analysis of Question 1.3

Learner response	Attempted correctly	Attempted but made errors	Not attempted
Number of learners	9	12	1
Learner number	1,2,7,8,9,11,16,18 21,	3,4,5,6,10,12,13,14,15,17 19,20	22

As illustrated in the table above it was clear that learners performed poorly in this question. The reason why learners performed so poorly in this question could be due to the fact learners still do not know how to measure angles correctly and thus could not sketch the triangle properly. This clearly needs to be addressed by the teacher because it is critical for learners to understand how to measure angles with the use of a protractor. So it is clear there are major misconceptions with regards to this question and these misconceptions could lead to learners making errors in the more challenging questions.

4.2.8. Exemplification of learners who attempted Question 1.3 correctly.

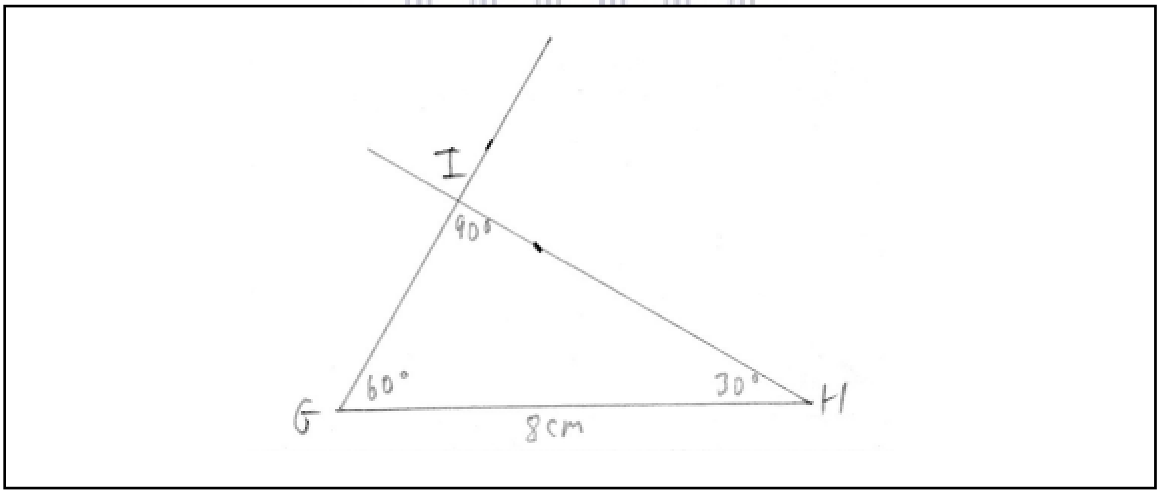


Figure 4.10: Learner 16 response to Question 1.3

L16 sketches the triangle correctly and shows a clear understanding on how to sketch a triangle given two angles and a side. The learner has an accurate measure of the angles and the side and also labels the triangle correctly. The only flaw once again is that the learner shows know evidence of the use of a compass which was illustrated by the teacher. The learner does have the necessary competence to sketch a triangle given two angles and a side, only needs to show the arc marks in future.

4.2.9 Exemplification of learners who attempted Question 1.3 but made errors.

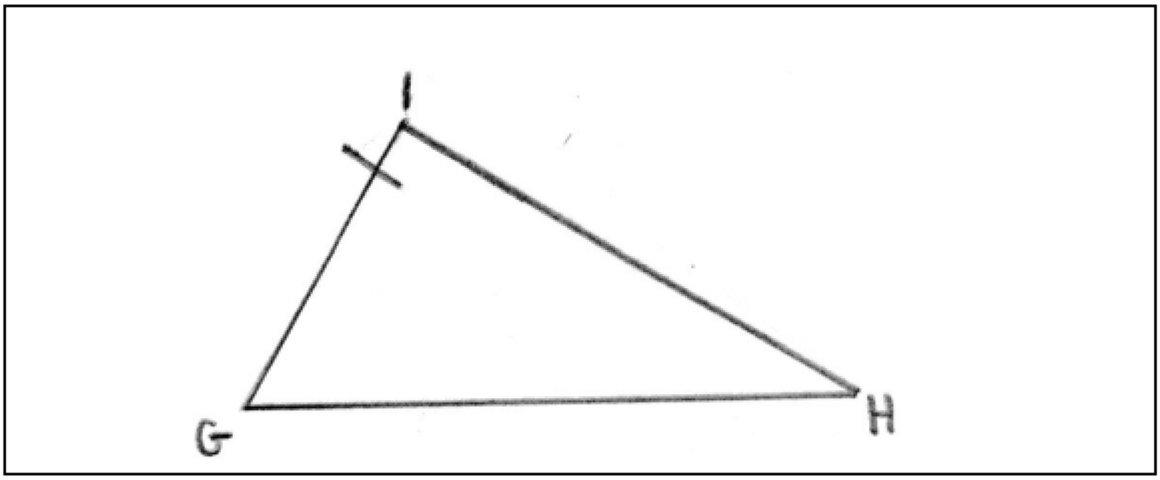


Figure 4.11: Learner 3 response to Question 1.3

L3 shows major gaps in knowledge when comes to sketching this triangle. Firstly, the learner does not start from the base and sketch, secondly the learner does not measure the angles properly with a protractor thus leading to major discrepancies. The learner also does not label the triangle correctly. This learner clearly needs to be retaught on how to sketch a triangle to remove these misconceptions. The learner shows evidence of not having the necessary competence to sketch a triangle given two angles and a side.

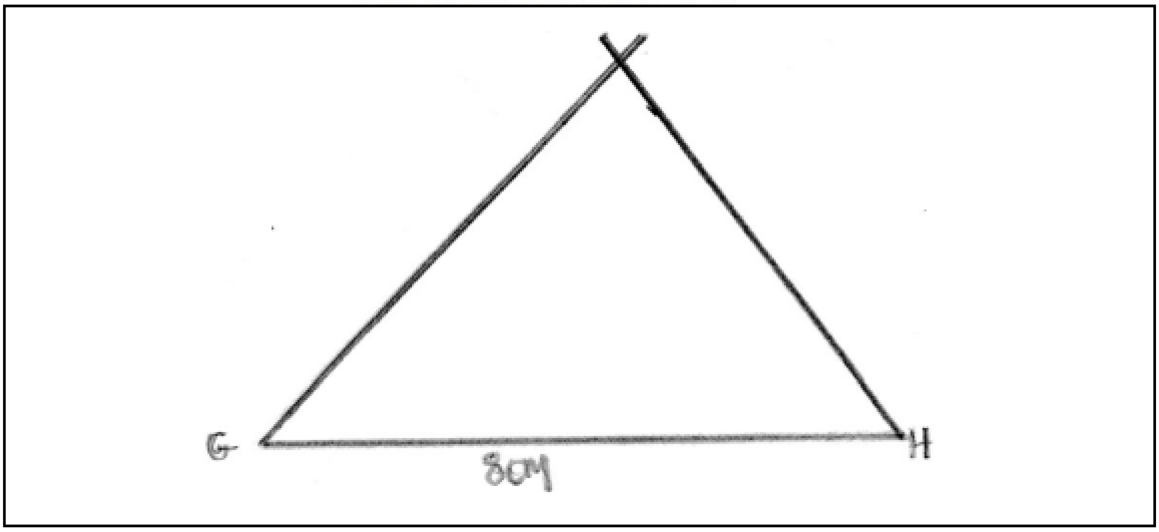


Figure 4.12: Learner 10 response to Question 1.3

L10 did not sketch the triangle to scale and supposed to measure the angles accurately. The learner clearly did not read the question properly and did not interpret the information given, which ultimately led to the learner sketching the triangle incorrectly. The learners need to be taught on how to read a problem with understanding and apply the information that is given properly. The learner does not have the necessary competence to sketch such a triangle.

4.2.10 Analysis of constructions: Question 1.4 on investigation task

Question 1.4 in the investigation task reads as follows:

If two sides and an including angle are given: side, angle, side (S,∠,S):

ΔJKL with JK = 9 cm, ∠K = 130° and KL = 7 cm. (4)

An expected solution for Question 1.4 is as follows:

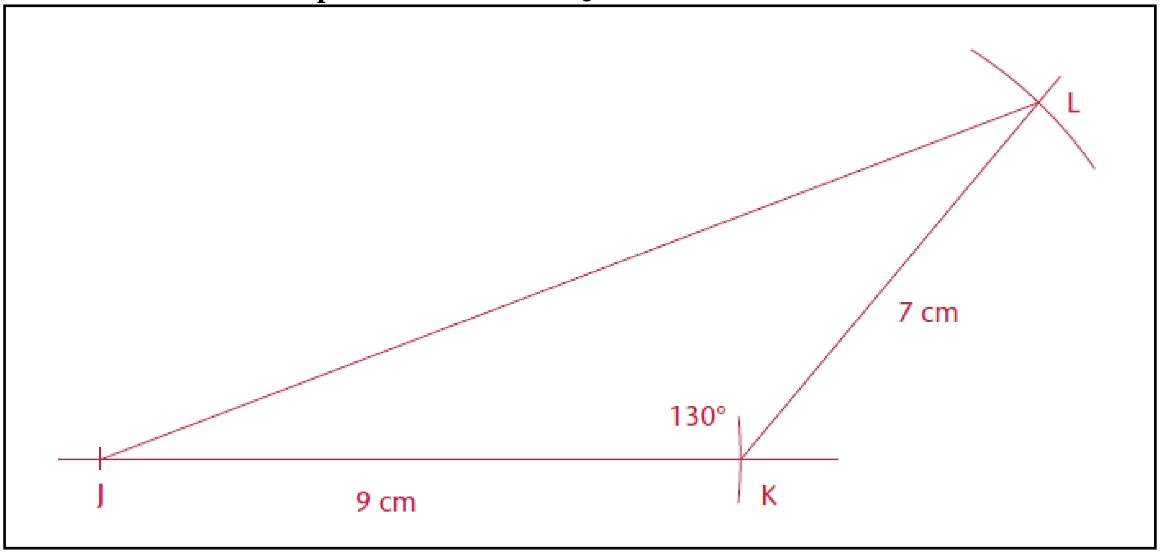


Figure 4.13: An expected solution for Question 1.4

As illustrated in Table 4.4, analysis of learners' responses to Question 1.4 showed that 9 learners attempted the problem correctly, 12 learners attempted the problem but made errors, and 1 learner did not attempt Question 1.4.

Table 4.4: Analysis of Question 1.4

Learner response	Attempted correctly	Attempted but made errors	Not attempted
Number of learners	9	12	1
Learner number	1,2,9,10,12,14,17 19,21	3,4,5,6,7,8,11,13,15,16,18,20	22

As illustrated in the table above it is clear that learners performed poorly in this question. The reason why learners performed so poorly in this question could be due to the fact learners still do not know how to measure angles correctly and thus could not sketch the triangle properly. This clearly needs to be addressed by the teacher because it is critical for learners to understand how to measure angles with the use of a protractor. So it is clear there are major misconceptions with regards to this question.

4.2.11. Exemplification of learners who attempted Question 1.4 correctly

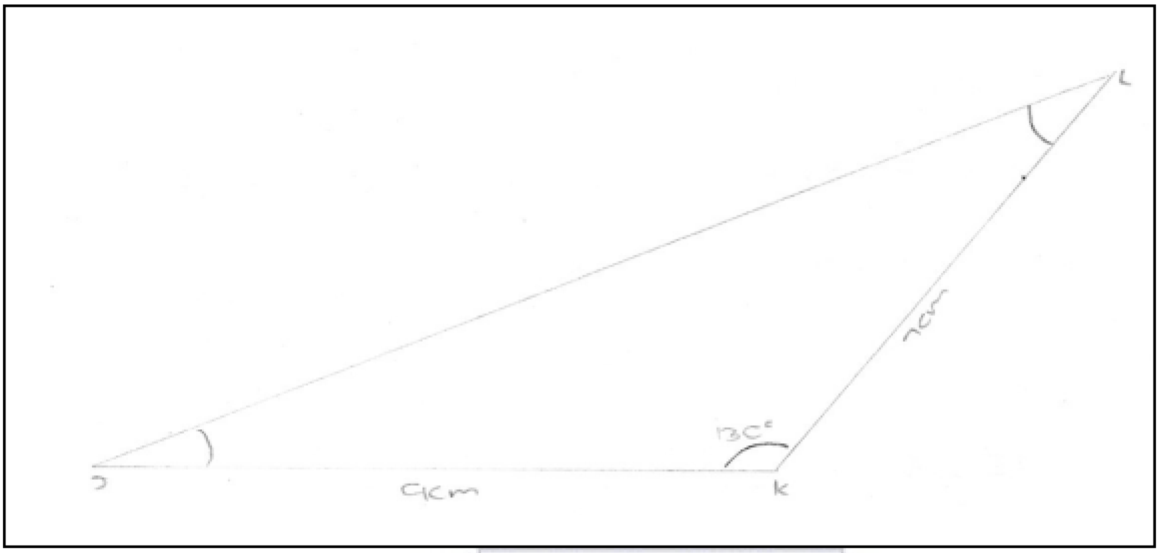


Figure 4.14: Learner 12 response to Question 1.4

L12 sketches the triangle correctly and shows a clear understanding on how to sketch a triangle given two sides and an included angle. The learner has a accurate measure of the angles and the side and also labelled the triangle correctly. The only flaw once again is that the learner shows know evidence of the use of a compass which was illustrated by the teacher. The learner does have the necessary competence to sketch a triangle given two sides and an included angle, the learner only needs to show the arc marks in future.

4.2.12. Exemplification of learners who attempted Question 1.4 but made errors.

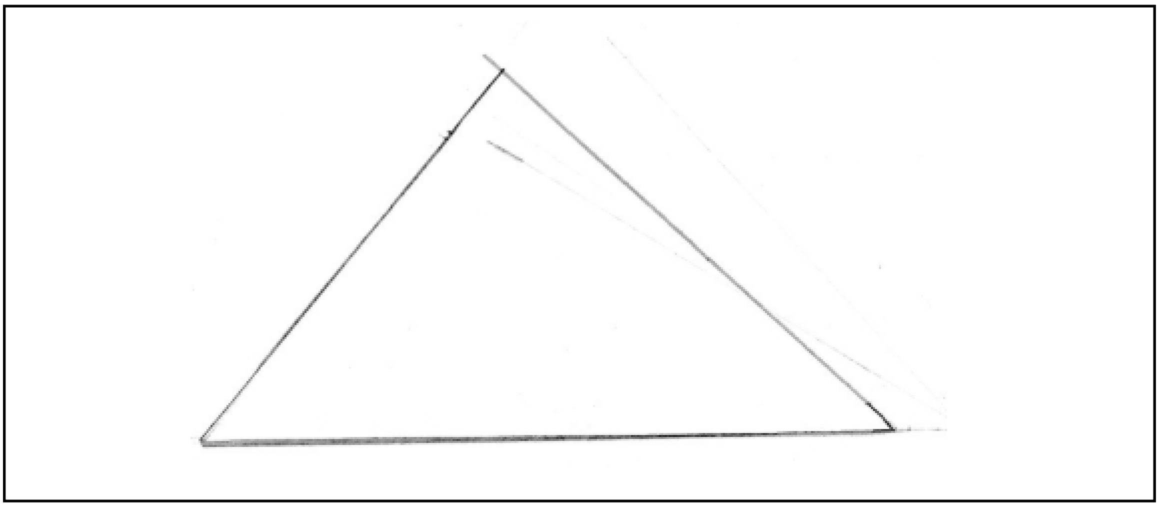


Figure 4.15: Learner 5 response to Question 1.4

L5 showed a clear gap in knowledge when it comes to measuring angle K and sketching it properly. This is a major concern because the learner is clearly confused on what angle K is and does not understand the concept of an included angle in a triangle and that it lies between two given sides. The learner clearly does not have the necessary competence to sketch a triangle given two sides and an included angle.

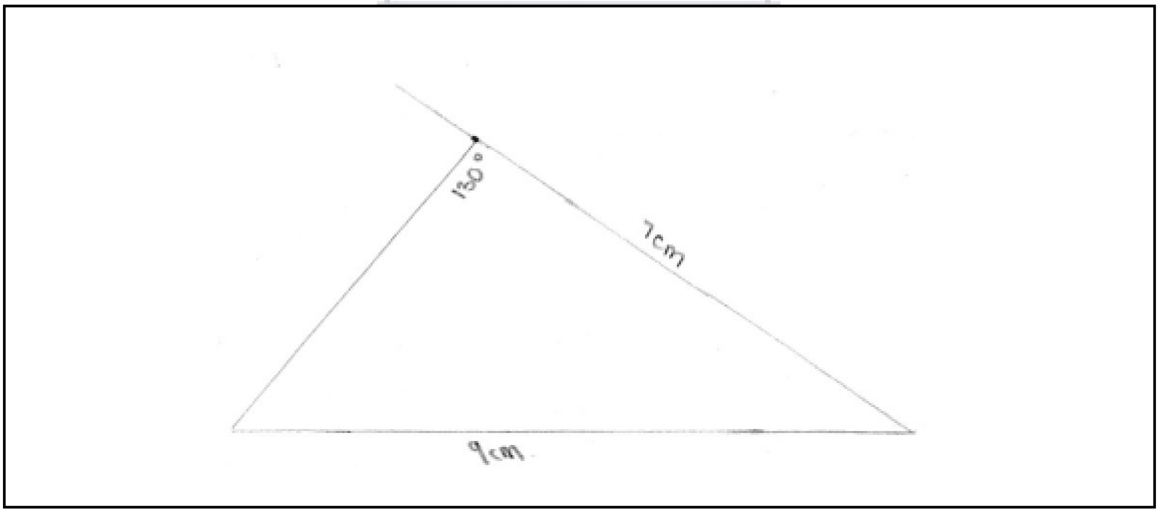


Figure 4.16: Learner 20 response to Question 1.4

L20 showed major gaps in knowledge when it comes to sketching this triangle. The learner clearly did not read the question properly, the learner also labels the triangle incorrectly which is a trend amongst most learners. L20 similar to L5, does not have the necessary competence to sketch this type of triangle.

4.2.13. Analysis of constructions: Question 1.5 on investigation task

Question 1.5 in the investigation task reads as follows:

If two sides and an angle which are not included are given: side, side, and angle

(S, S, ∠): ΔMNP with MN = 10 cm, ∠M = 50° and PN = 8 cm. (4)

An expected solution for Question 1.5 is as follows:

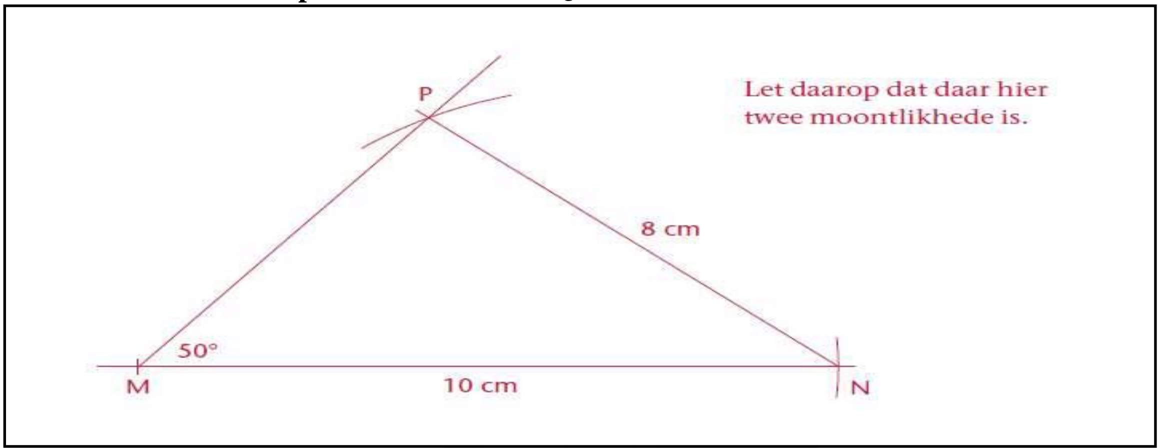


Figure 4.17: An expected solution for Question 1.5

As illustrated in Table 4.5, analysis of learners' responses to Question 1.5 showed that 16 learners attempted the problem correctly, 4 learners attempted the problem but made errors, and 2 learners did not attempt Question 1.5.

Table 4.5: Analysis of Question 1.5

Learner response	Attempted correctly	Attempted but made errors	Not attempted
Number of learners	16	4	2
Learner number	1,2,3,4,6,7,8,9,10,11,12,13 14,17,19,20	5,16,18,21	15,22

As illustrated in the table above it is clear that learners performed well in this question. The reason why learners performed well in this question could be due to the fact learners found the video to be very helpful when sketching the triangle. The video clearly explained on how to go about sketching this specific type of triangle.

4.2.14. Exemplification of learners who attempted Question 1.5 correctly.

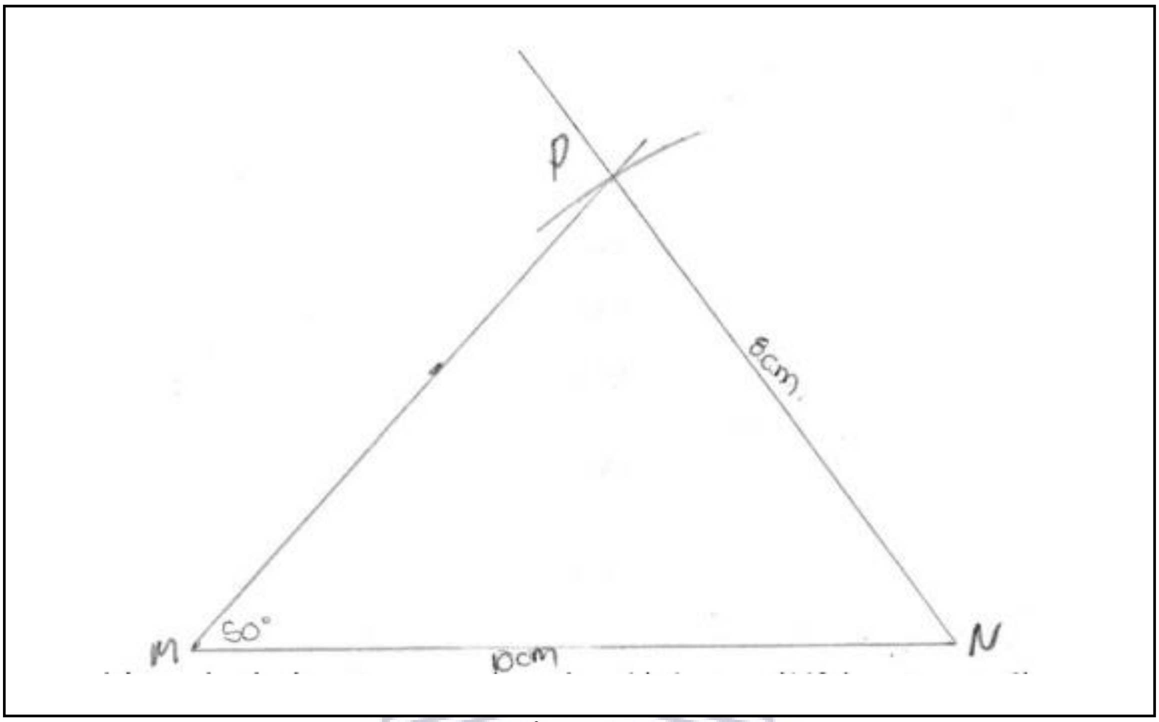


Figure 4.18: Learner 20 response to Question 1.5

L20 sketches the triangle with relative ease, diagram was drawn to scale. Angle was measured correctly and sides were measured and labelled correctly too. The learner showed evidence for the use of a compass to draw the arcs, which is important for this sketch. The learner does show evidence of having the necessary competence to sketch a triangle, given two sides and a non-included angle.

4.2.15. Exemplification of learners who attempted Question 1.5 but made errors.

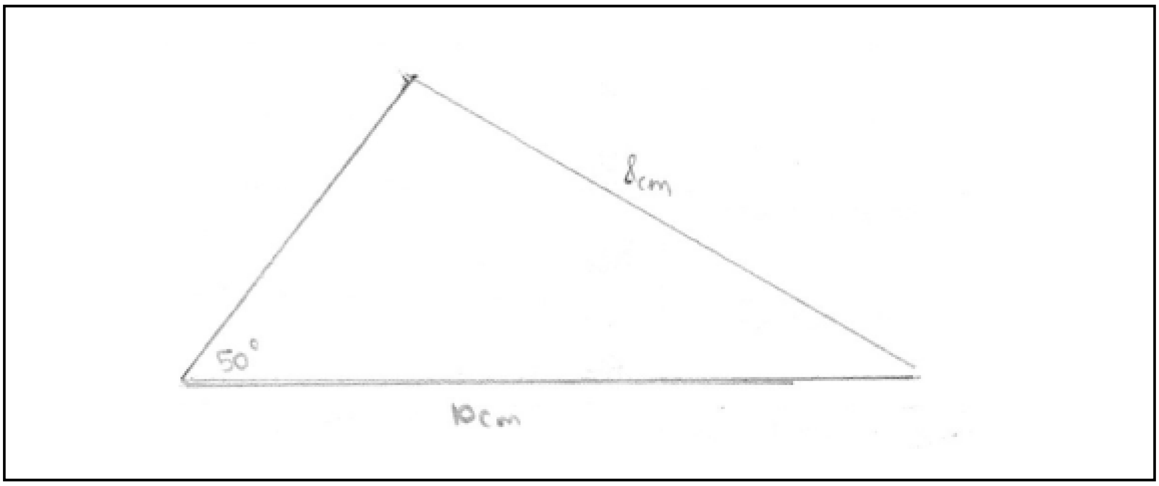


Figure 4.19: Learner 18 response to Question 1.5

L18 did not sketch the triangle properly, uses a completely wrong approach and thus displays major gaps in knowledge when it comes to sketching triangles. There is no evidence of accurate use of compass and the triangle is not labelled properly. The learner clearly needs to be re taught on how to sketch this triangle, in order to remove the misconception, the learner has with regards to constructing triangles. The learner clearly does not have the necessary competence to sketch a triangle given two sides and a non-included angle. This learner should be a major concern.

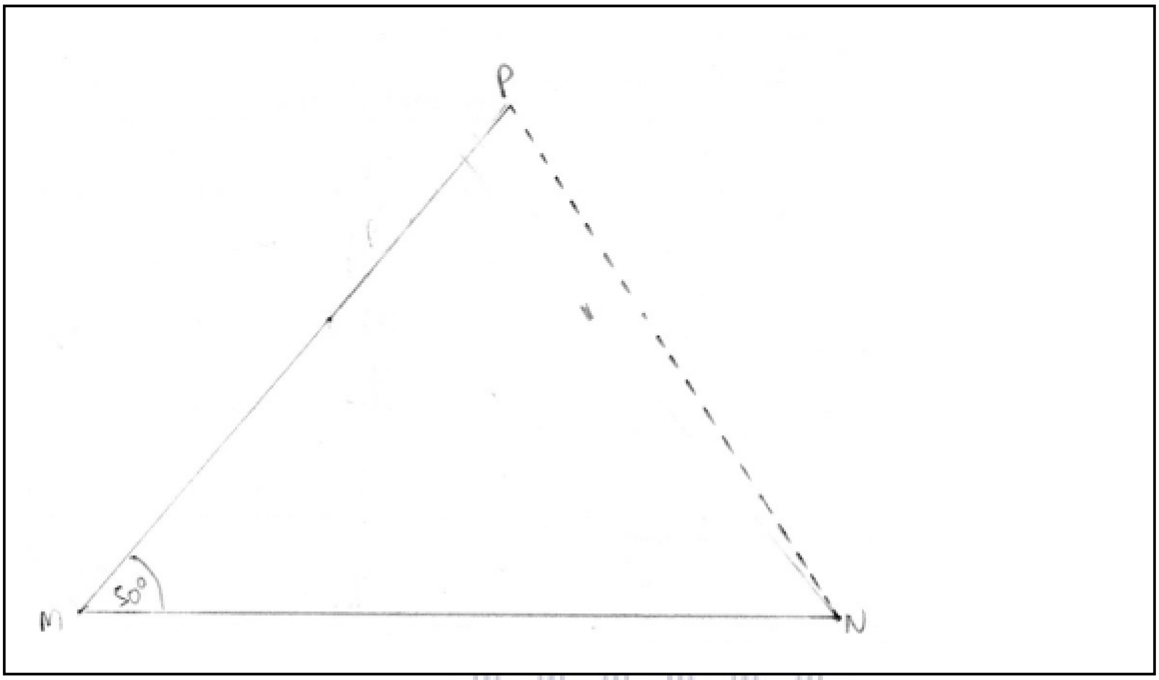


Figure 4.20: Learner 21 response to Question 1.5

L21 clearly did not read the question properly and thus used the information needed to sketch the triangle incorrectly. The learner did not measure the sides correctly and mixed up the measurements, leading to an incorrect sketch of the triangle. The learner clearly needs help with how to read information given properly and what steps to follow to apply what is given. This is also common with a few other participants. The learner does not have the basic competence to sketch such a triangle, the learner is careless and neglectful which is not good qualities to have for such a topic.

4.2.16. Analysis of constructions: Question 1.6 on investigation task

Question 1.6 in the investigation task reads as follows:

If a right angle, the hypotenuse and another side is given: (90°, hypotenuse, S):

ΔTRS with TR ⊥ RS, RS = 7 cm and TS = 8 cm. (4)

An expected solution for Question 1.6 is as follows:

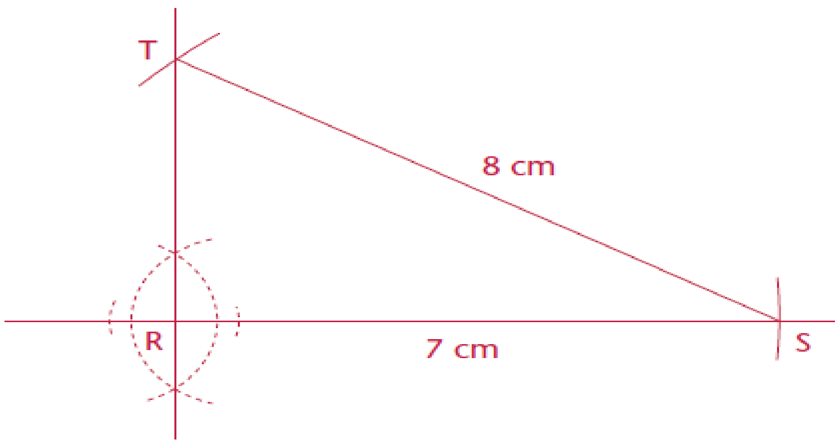


Figure 4.21: An expected solution for Question 1.6

As illustrated in Table 4.6, analysis of learners' responses to Question 1.6 showed that 7 learners attempted the problem correctly, 12 learners attempted the problem but made errors, and 3 learners did not attempt Question 1.6.

Table 4.6: Analysis of Question 1.6

Learner response	**Attempted correctly**	**Attempted but made errors**	**Not attempted**
Number of learners	7	12	3
Learner number	6,9,10,11,12,13,20	1,2,3,5,7,8,14,16,17,18,19,21	4,15,22

As illustrated in the table above it is clear that learners performed poorly in this question. The reason why learners performed so poorly in this question could be due to the fact learners still do not know how to measure angles correctly and thus could not sketch the triangle properly. This clearly needs to be addressed by the teacher because it is critical for learners to understand how to measure angles with the use of a protractor. So, it is clear there are major misconceptions with regards to this question.

4.2.17. Exemplification of learners who attempted Question 1.6 correctly.

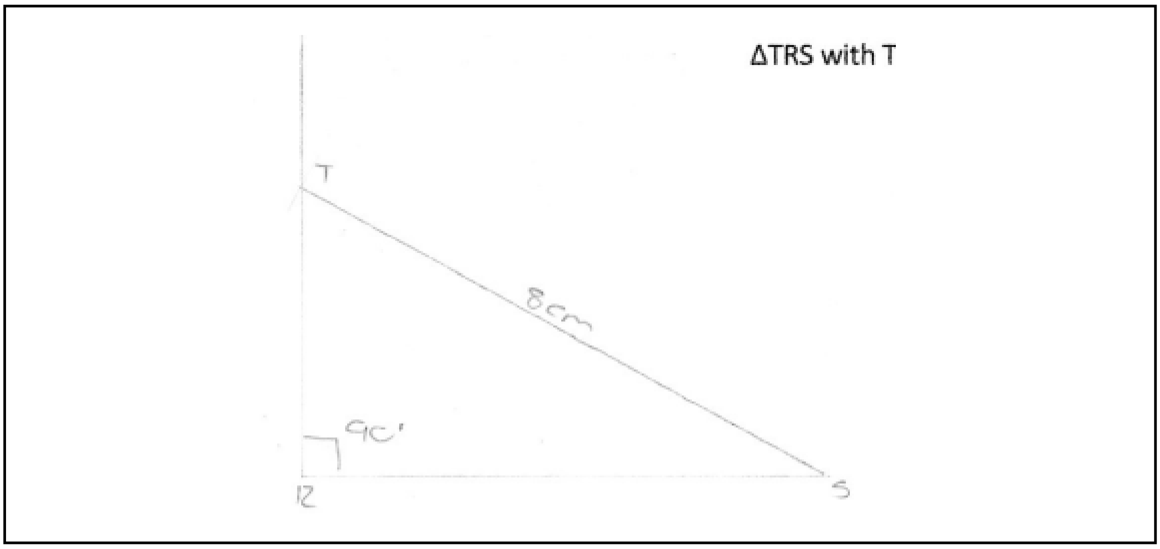


Figure 4.22: Learner 12 response to Question 1.6

L12 sketched the triangle correctly, all sides have the correct measurements and labels, the right angle is also drawn in the correct place. However, the learner shows no evidence of an arc which is critically important to sketch this type of triangle for accuracy, nonetheless the learner still manages to be accurate with the measurements. The learner probably did use arcs to construct the triangle, but then erased it which is a common trend amongst all participants. The learner does have the necessary competence to sketch a triangle given a right angle, hypotenuse and a side.

4.2.18. Exemplification of learners who attempted Question 1.6 but made errors.

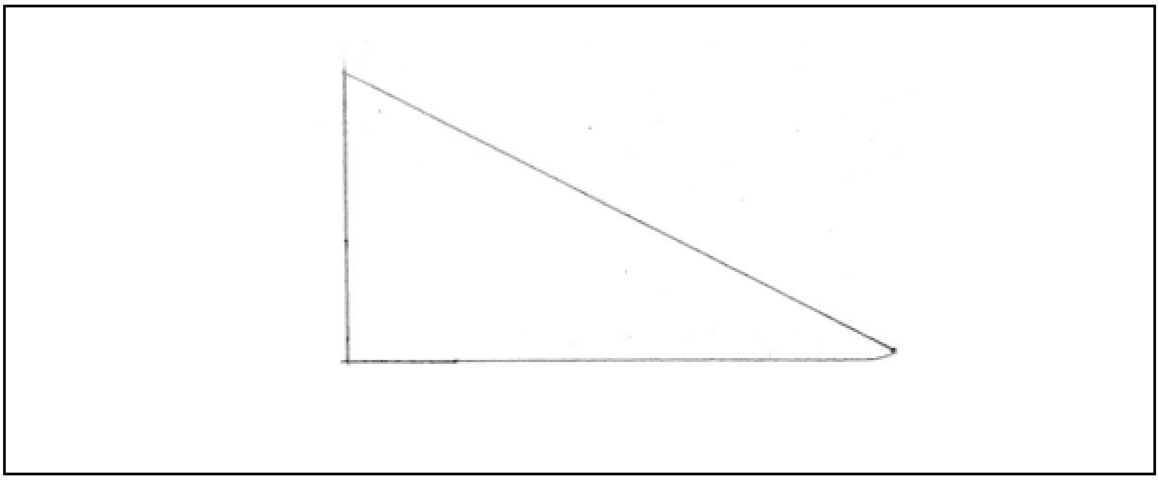


Figure 4.23: Learner 5 response to Question 1.6

L5 drew a right-angled triangle, did not label the triangle. This was also picked up with another learner. The learner clearly shows a lack of effort in terms of sketching the triangle correctly and did not follow instructions properly. This is a major concern as the learner clearly lacks motivation to sketch the triangle properly and inevitably would struggle to compare which triangles are congruent or not. The learner clearly does not have the necessary competence to sketch such a triangle accurately and rather prefers to draw any right-angled triangle.

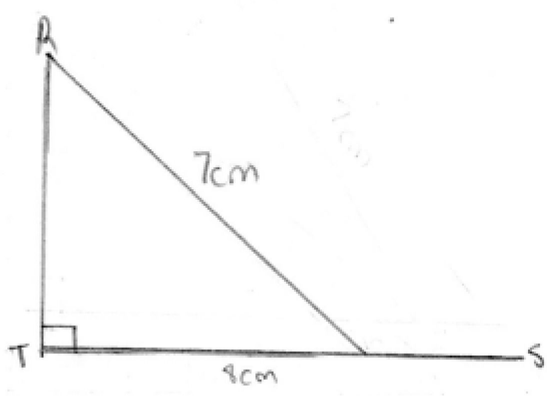


Figure 4.24: Learner 16 response to Question 1.6

L16 does not even draw a right-angled triangle properly and thus shows major misconceptions when it comes to this question, no measurements were indicated. The learner did not label the triangle properly and clearly did not understand the question. The learner clearly has major misconceptions and needs to be retaught on how to sketch a triangle given a right angle, hypotenuse and a side. This is a major concern for the teacher as it shows that some learners do not know their properties of triangles. The learner does not have the necessary competence to sketch a right-angled triangle and neglects the information that was given.

4.2.19. Analysis of Question 1.7 on investigation task

Question 1.7 in the investigation task reads as follows:

Compare your triangles with the triangles of three of your classmates. Which of your triangles are:

1.7.1 congruent to theirs?	(2)
--	
1.7.2 not congruent?	
--	(2)

An expected solution for Question 1.7 is as follows:

1.7.1) Congruent triangles are:
1.1/ 1.3/ 1.4 and 1.6

1.7.2) Triangles that are not congruent are:
1.2 and 1.5

Figure 4.25: An expected solution for Question 1.7

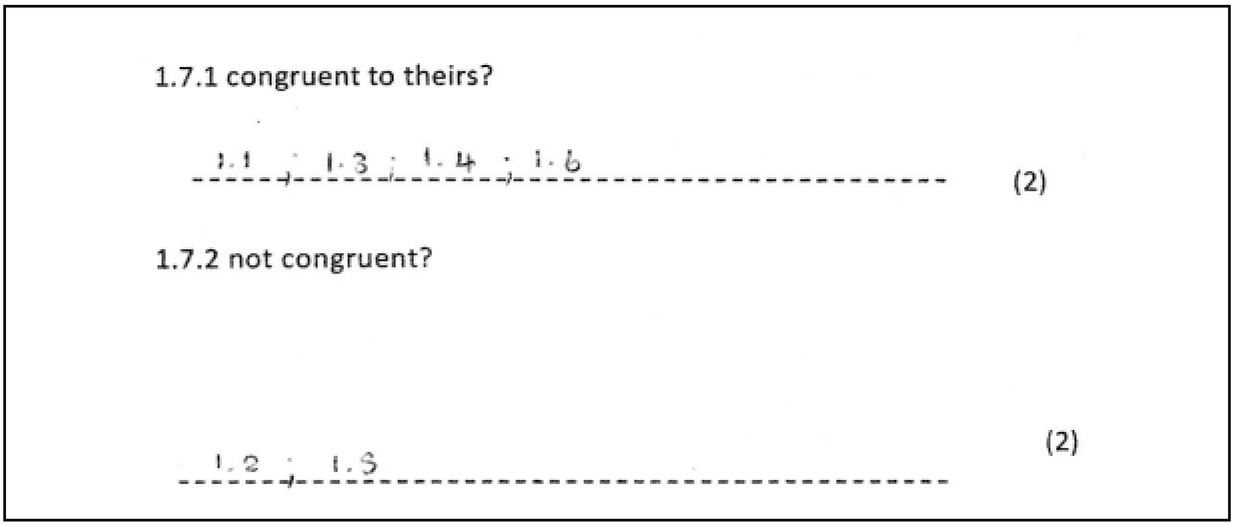
As illustrated in Table 4.7, analysis of learners' responses to Question 1.7 showed that 21 learners attempted the problem correctly, 1 learner attempted the problem but made errors, and none of the learners did not attempt Question 1.7.

Table 4.7: Analysis of Question 1.7

Learner response	**Attempted correctly**	**Attempted but made errors**	**Not attempted**
Number of learners	21	1	0
Learner number	1,2,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18 19,20,21,22	3	None

As illustrated in the table above it is clear that learners performed well in this question. The reason why learners performed well in this question could be due to the fact learners found the video to be very helpful when answering this question. The video clearly explained on how to go about comparing which triangles were congruent to each other or not.

4.2.20. Exemplification of learners who attempted Question 1.7 correctly


1.7.1 congruent to theirs?

1.1 ; 1.3 ; 1.4 ; 1.6 (2)

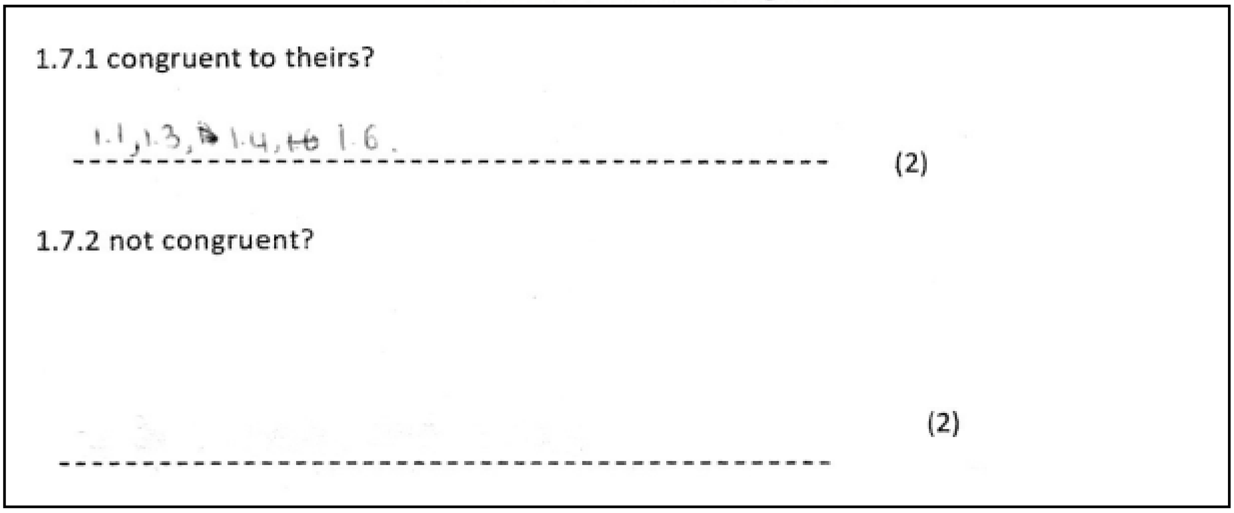
1.7.2 not congruent?

1.2 ; 1.5 (2)

Figure 4.26: Learner 7 response to Question 1.7

L7 for question 1.7.1 on which triangles are congruent and for question 1.7.2 on which triangles are not congruent, answers the questions with relative ease. The learner shows a great understanding when it comes to comparing which triangles are congruent and which are not congruent. The learner does have the necessary competence levels to answer such a question.

4.2.21. Exemplification of learners who attempted Question 1.7 correctly but made errors.


1.7.1 congruent to theirs?

1.1, 1.3, 1.4, 1.6 . (2)

1.7.2 not congruent?

(2)

Figure 4.27: Learner 3 response to Question 1.7

L3 for question 1.7.1 on which triangles are congruent, got it correct but clearly showed signs of being indecisive, thus for question 1.7.2 on which triangles are not congruent, does not answer the question. The learner showed clear gaps in knowledge when it comes to comparing which triangles not congruent. The learner does not have the necessary competence levels to answer such a question and probably got most of the sketches incorrect which is why the learner failed to match which triangles was congruent or not congruent with their fellow classmates.

4.2.22. Analysis of Question 2 on investigation task

Question 2 in the investigation task reads as follows:

Complete the table below. Write down if congruent triangles can be constructed if the following conditions are given:

Conditions	Congruent (Yes/No)
3 sides (SSS)	
2 sides (SS)	
3 angles (∠,∠,∠)	
2 angles and a side (∠,∠, S)	
2 sides and an angle that is not situated between the sides (S, S, ∠)	
2 sides and an angle that is situated between the two sides (S, ∠, S)	
A right angle with a hypotenuse and another side (90°HS) or (90°, hypotenuse, side)	

[7]

An expected solution for Question 2 is as follows:

Conditions	Congruent (Yes/No)
3 sides (SSS)	Yes
2 sides (SS)	No
3 angles (∠,∠,∠)	No
2 angles and a side (∠,∠, S)	Yes
2 sides and an angle that is not situated between the sides (S, S, ∠)	No
2 sides and an angle that is situated between the two sides (S, ∠, S)	Yes

A right angle with a hypotenuse and another side (90°HS) or (90°, hypotenuse, side)	Yes

Figure 4.28: An expected solution for Question 2 **(7)**

As illustrated in Table 4.8, analysis of learners' responses to Question 2 showed that 22 learners attempted the problem correctly, none of the learners attempted the problem but made errors, and none of the learners did not attempt Question 2.

Table 4.8: Analysis of Question 2

Learner response	**Attempted correctly**	**Attempted but made errors**	**Not attempted**
Number of learners	22	0	0
Learner number	1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17 18,19,20,21,22	None	None

As illustrated in the table above it is clear that learners performed well in this question. The reason why learners performed well in this question could be due to the fact learners found the video to be very helpful when answering this question. The video clearly explained on how to go about stating what are the conditions for congruency. Learners could then play what was taught to them via the video.

4.2.23. Exemplification of learners who attempted Question 2 correctly.

Conditions	Congruent (Yes/ No)
3 sides (SSS)	Yes
2 sides (SS)	No
3 angles (∠,∠,∠)	No
2 angles and a side (∠,∠, S)	Yes
2 sides and an angle that is not situated between the sides (S, S, ∠)	No
2 sides and an angle that is situated between the two sides (S, ∠, S)	Yes
A right angle with a hypotenuse and another side (90°HS) or (90°, hypotenuse, side)	Yes.

Figure 4.29: Learner 11 response to Question 2

L11 got the entire table correct and answers the question with relative ease. The learner clearly understood the four cases of congruency, which made it easy for the learner to answer this question. This learner clearly did not struggle with Question 3.1-3.3.3 as the learner has sound knowledge of congruency and similarity. The learner shows evidence of a great level of competence when it comes to knowing the conditions for congruency and similarity. This question was the best answered question, which clearly showed that the videos helped the learners understand the difference between congruency and similarity clearly, which is critical.

Conditions	Congruent (Yes/ No)
3 sides (SSS)	Yes
2 sides (SS)	No
3 angles (∠,∠,∠)	No
2 angles and a side (∠,∠, S)	Yes
2 sides and an angle that is not situated between the sides (S, S, ∠)	No
2 sides and an angle that is situated between the two sides (S, ∠, S)	Yes
A right angle with a hypotenuse and another side (90°HS) or (90°, hypotenuse, side)	Yes

Figure 4.30: Learner 5 response to Question 2

L5 got the entire table correct and answers the question with relative ease. The learner clearly understood the four cases of congruency, which made it easy for the learner to answer this question. This learner, similarly, to L11 did not struggle with Question 3.1-3.3.3 as the learner has sound knowledge of congruency and similarity.

4.2.24. Analysis of constructions: Question 3.1-3.3.3 on investigation task

Question 3.1-3.3.3 in the investigation task reads as follows:

Question 3

3.1) Name the 4 cases of congruence in triangles:

-- [4]

3.2) Complete: If 2 triangles have equal angles, we may say that the two triangles are:

-- [1]

3. 3) Write down in words the meaning of:

3.3.1) $\Delta ABC \equiv \Delta XYZ$

-- (2)

3.3.2) $\Delta ABC /// \Delta XYZ$

--- (2)

3.3.3) $RM \parallel EH$

--- (2) [6]

An expected solution for Question 3.1-3.3.3 is as follows:

3.1) Name the 4 cases of congruence in triangles:

- S,S,S
- S, ∠,S
- ∠, ∠,S
- 90°, Hypotenuse, S

-- [4]

3.2) Complete: If 2 triangles have equal angles, we may say that the two triangles are:

-------Similar, but not necessarily congruent. --------------------------------------- [1]

3. 3) Write down in words the meaning of:

3.3.1) $\Delta ABC \equiv \Delta XYZ$

---------Triangle ABC is congruent to triangle XYZ--- (2)

3.3.2) $\Delta ABC /// \Delta XYZ$

----------Triangle ABC is similar to triangle XYZ---(2)

3.3.3) $RM \parallel EH$

-------------Line segment/ Line RM is parallel to line segment/ line EH------------(2) [6]

Figure 4.31: An expected solution for Question 3.1-3.3.3

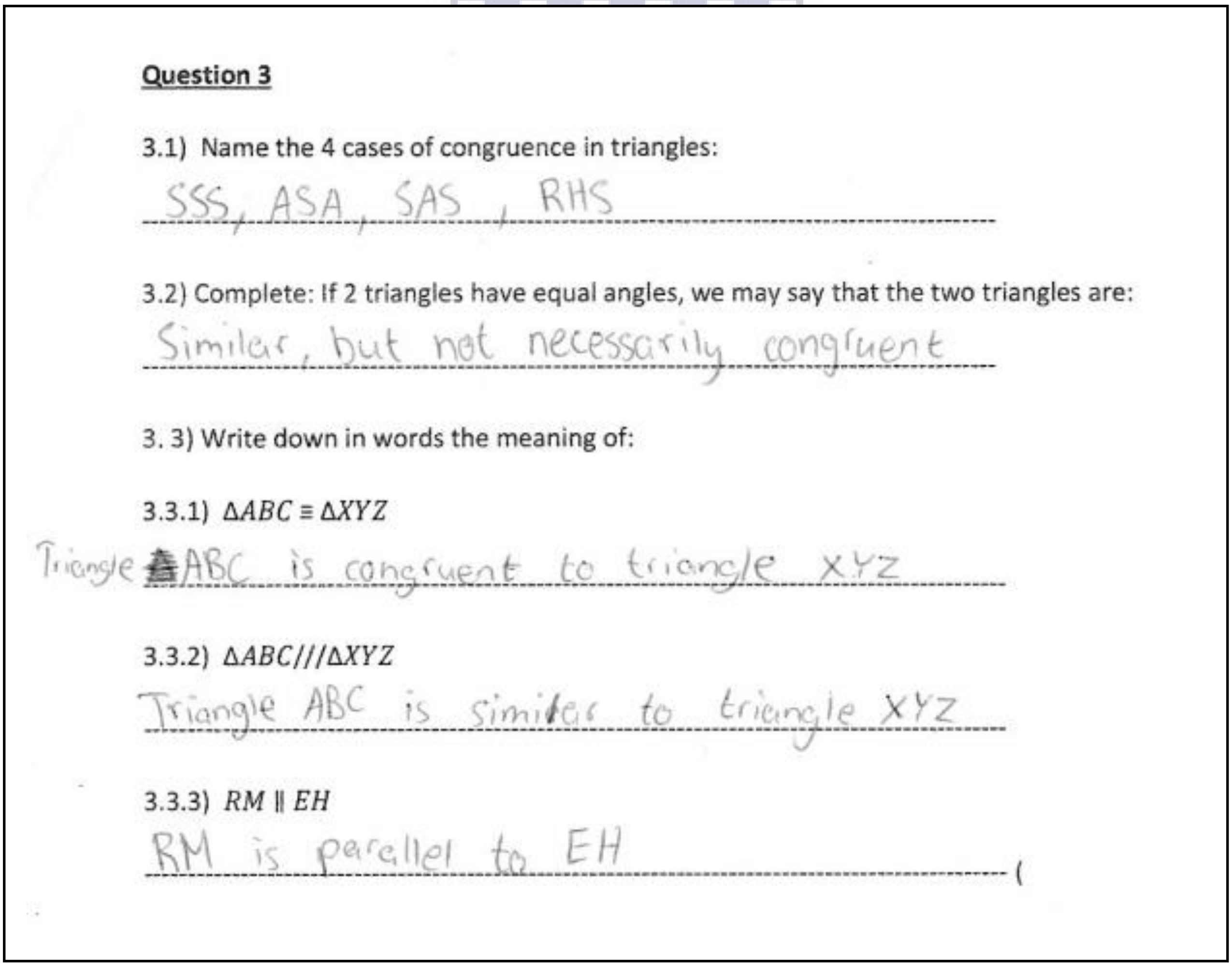
As illustrated in Table 4.9, analysis of learners' responses to Question 3.1-3.3.3 showed that 22 learners attempted the problem correctly, none of the learners attempted the problem but made errors, and none of the learners did not attempt Question 3.1-3.3.3.

Table 4.9: Analysis of Question 3.1-3.3.3

Learner response	Attempted correctly	Attempted but made errors	Not attempted
Number of learners	22	0	0
Learner number	1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17 18,19,20,21,22	None	None

As illustrated in the table above it is clear that learners performed well in this question. The reason why learners performed well in this question could be due to the fact learners found the video to be very helpful when answering this question. The video clearly explained on how to go about differentiating between congruency, similarity, and parallel lines.

4.2.25. Exemplification of learners who attempted Question 3.1-3.3.3 correctly.


Question 3

3.1) Name the 4 cases of congruence in triangles:

SSS, ASA, SAS, RHS

3.2) Complete: If 2 triangles have equal angles, we may say that the two triangles are:

Similar, but not necessarily congruent

3. 3) Write down in words the meaning of:

3.3.1) $\Delta ABC \equiv \Delta XYZ$

Triangle ABC is congruent to triangle XYZ

3.3.2) $\Delta ABC /// \Delta XYZ$

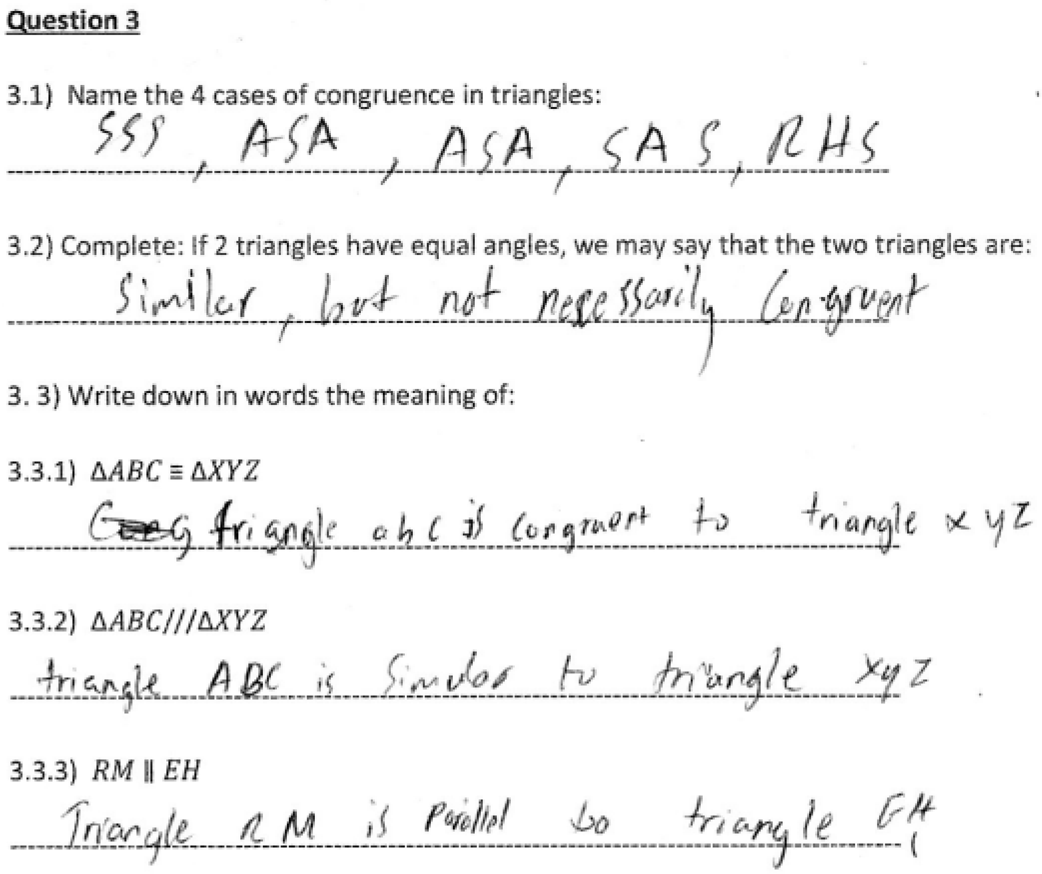
Triangle ABC is similar to triangle XYZ

3.3.3) $RM \parallel EH$

RM is parallel to EH

Figure 4.32: Learner 4 response to Question 3.1-3.3.3

L4 got all questions correct and clearly understood the theory with regards to congruency and similarity. The learners answered this question with relative ease. Such a learner should be exposed to higher order questions, which can really benefit the learner and test the learners understanding of congruency further. The learner shows a great level of competence when it comes to this question. Understands the concept of congruency and similarity.



Question 3

3.1) Name the 4 cases of congruence in triangles:

SSS, ASA, ASA, SAS, RHS

3.2) Complete: If 2 triangles have equal angles, we may say that the two triangles are:

Similar, but not necessarily congruent

3. 3) Write down in words the meaning of:

3.3.1) $\Delta ABC \equiv \Delta XYZ$

triangle abc is congruent to triangle xyz

3.3.2) $\Delta ABC /// \Delta XYZ$

triangle ABC is Similar to triangle xyz

3.3.3) $RM \parallel EH$

Triangle RM is Parallel to triangle EH

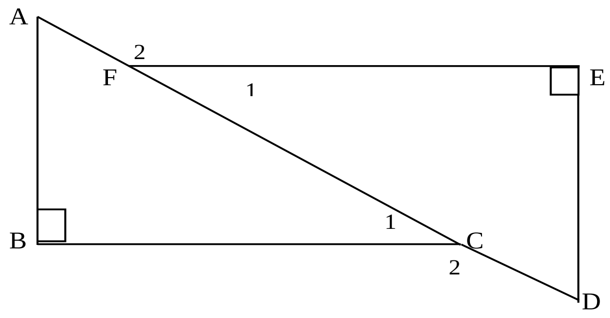
Figure 4.33: Learner 15 response to Question 3.1-3.3.3

Similar to L4, L15 got all questions correct and clearly understood the theory with regards to congruency and similarity. The learner shows a good competency level with regards to this question.

4.2.26. Analysis of constructions: Question 3.4 on investigation task

Question 3.4 in the investigation task reads as follows:

3.4) In the diagram below the following dimensions are given: ∠B = ∠E = 90°, ∠C1 = ∠F1 and AB = ED.



3.4.1) Prove: Δ ABC ≡ Δ DEF. (4)

STATEMENT	REASON

An expected solution for Question 3.4 is as follows:

3.4.1) Prove: Δ ABC ≡ Δ DEF. (4)

STATEMENT	REASON
∠B = ∠E=90°	Given
$\angle C_1 = \angle F_1$	Given
AB = ED	Given
∴ Δ ABC ≡ Δ DEF	∠, ∠, S

Figure 4.34: An expected solution for Question 3.4

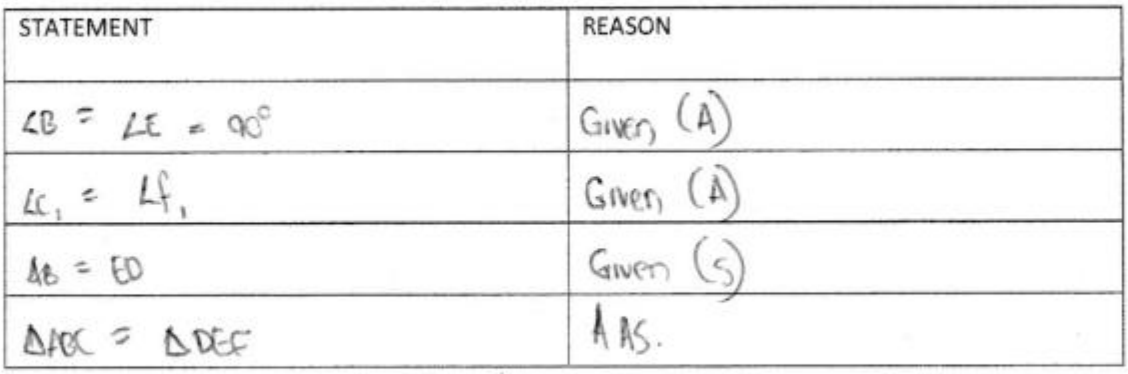
As illustrated in Table 4.10, analysis of learners' responses to Question 3.4 showed that 22 learners attempted the problem correctly, none of the learners attempted the problem but made errors, and none of the learners did not attempt Question 3.4.

Table 4.10: Analysis of Question 3.4

Learner response	**Attempted correctly**	**Attempted but made errors**	**Not attempted**
Number of learners	22	0	0
Learner number	1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17 18,19,20,21,22	None	None

As illustrated in the table above it is clear that learners performed well in this question. The reason why learners performed well in this question could be due to the fact learners found the video to be very helpful when answering this question. The video clearly explained on how to go about solving a congruency based proof problem by using the two column proof method. Learners found it easy to understand what a statement is and what a reason is, after watching the video

4.2.27. Exemplification of learners who attempted Question 3.4 correctly.



STATEMENT	REASON
$\angle B = \angle E = 90^\circ$	Given (A)
$\angle C_1 = \angle F_1$	Given (A)
$AB = ED$	Given (S)
$\Delta ABC = \Delta DEF$	AAS.

Figure 4.35: Learner 18 response to Question 3.4

L18, wrote the statement correctly and provided the correct reason for each statement. The learner also wrote down the correct case for these triangles to be congruent. The only thing the learner supposed to do was to write AAS under the reason section, otherwise the learner answered the question really well. This question was surprisingly one of the most answered questions. One would expect a proof question to be difficult for learners, even though the information was given. The learners displayed good levels of proof writing competence and will be able to cope with much more complex proofs. The learner was able to make a conjecture about the problem and also is able to justify the conjecture. The learner was able to use a proof in order to establish the validity of statements and explain why the conjectures are true. Learner was able to provide a warrant which justifies claims and supplies reasons and uses geometry property and relationship and provides a clear logic structure. There was clear evidence for a chain of reasoning because for every statement there was a valid reason. The student was able to create an argument consisting of several connected statements.

STATEMENT	REASON
6 $\angle B = \angle E = 90°$	Given (A).
$\angle G_1 = \angle F_1$	Given (A).
AB = ED	Given (S).
~~ΔABC ≡ ΔDE~~ ~~ΔABC ≡ ΔDEF~~	AAS.

ΔABC ≡ ΔDEF

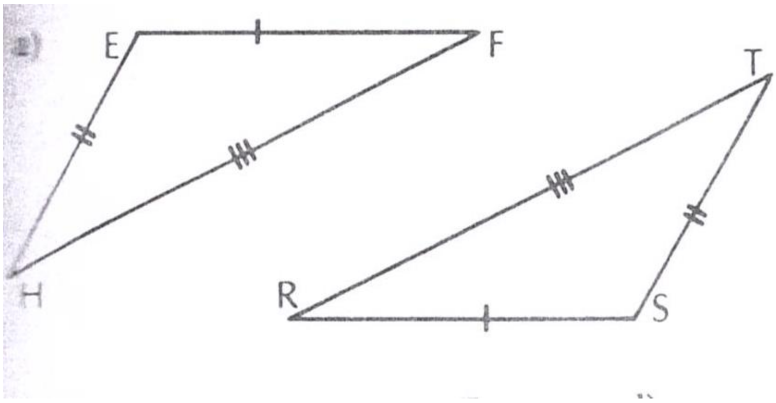
Figure 4.36: Learner 3 response to Question 3.4

Similar to L18, L3 wrote the statement correctly and provided the correct reason for each statement. The learner displayed good levels of proof writing competence and will be able to cope with much more complex proofs. The learner was able to make a conjecture about the problem and also is able to justify the conjecture. The learner was able to use a proof in order to establish the validity of statements and explain why the conjectures are true. Learner was able to provide a warrant which justifies claims and supplies reasons and uses geometry property and relationship and provides a clear logic structure. There was clear evidence for a chain of reasoning because for every statement there was a valid reason. The student was able to create an argument consisting of several connected statements.

4.2.28. Analysis of Question 1.a on investigation task 2

Question 1.a in the investigation task 2 reads as follows:

Question 1.a: Prove that the following triangles are congruent:



STATEMENT	REASON

An expected solution for Question 1.a is as follows:

STATEMENT	REASON
$EF = RS$	Given
$EH = ST$	Given
$FH = RT$	Given
$\therefore \Delta EFH \equiv \Delta RST$	SSS

Figure 4.37: An expected solution for Question 1.a

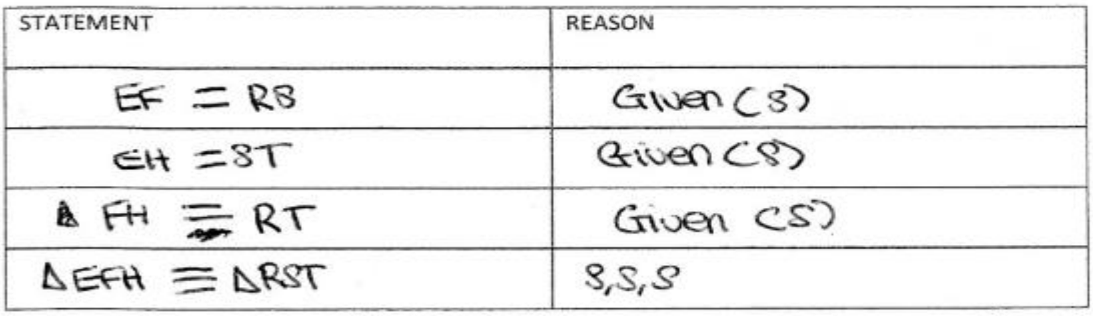
As illustrated in Table 4.11, analysis of learners' responses to Question 1.a showed that 22 learners attempted the problem correctly, none of the learners attempted the problem but made errors, and none of the learners did not attempt Question 1.a.

Table 4.11: Analysis of Question 1.a

Learner response	Attempted correctly	Attempted but made errors	Not attempted
Number of learners	22	0	0
Learner number	1,2,3,4,5,6,7,8,9,10,11,12,13,14 15,16,17,18,19,20,21,22	None	None

As illustrated in the table above it is clear that learners performed well in this question. The reason why learners performed well in this question could be due to the fact learners found the video to be very helpful when answering this question. The video clearly explained on how to go about solving a congruency-based proof problem by using the two-column proof method. Learners found it easy to understand what a statement is and what a reason is, after watching the video.

4.2.29. Exemplification of learners who attempted Question 1.a correctly.



STATEMENT	REASON
EF = RS	Given (S)
EH = ST	Given (S)
FH = RT	Given (S)
ΔEFH ≡ ΔRST	S,S,S

Figure 4.38: Learner 6 response to Question 1.a

L6, wrote the statement correctly and provided the correct reason for each statement. The learner also wrote down the correct case for these triangles to be congruent, which is SSS. The learners displayed good levels of proof writing competence and will be able to cope with much more complex proofs. The learner clearly benefited from watching the YouTube video based on congruency-based proofs. The learner was able to make a conjecture about the problem and can justify the conjecture. The learner was able to use a proof in order to establish the validity of statements and explain why the conjectures are true. Learner was able to provide a warrant which justifies claims and supplies reasons and uses geometry property and relationship and provides a clear logic structure. There was clear evidence for a chain of reasoning because for every statement there was a valid reason. The student was able to create an argument consisting of several connected statements.

STATEMENT	REASON
EF = RS	Given (S)
EH = ST	Given (S)
FH = RT	Given (S)
Δ EFH ≡ Δ RHS	S,S,S

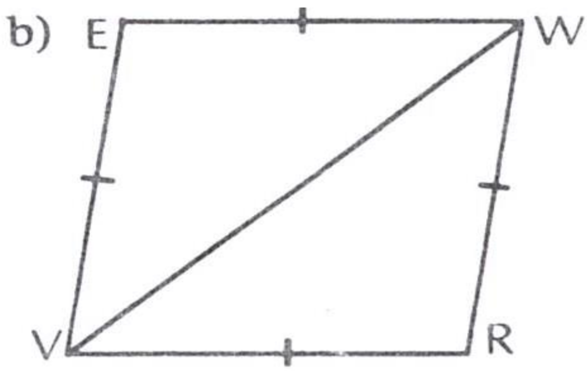
Figure 4.39: Learner 17 response to Question 1.a

L17, similar to L6, wrote the statement correctly and provided the correct reason for each statement. The learner also wrote down the correct case for these triangles to be congruent, which is SSS. The learners displayed good levels of proof writing competence and will be able to cope with much more complex proofs. The learner clearly benefited from watching the YouTube video based on congruency-based proofs. The learner was able to make a conjecture about the problem and is able to justify the conjecture. The learner was able to use a proof in order to establish the validity of statements and explain why the conjectures are true. Learner was able to provide a warrant which justifies claims and supplies reasons and uses geometry property and relationship and provides a clear logic structure. There was clear evidence for a chain of reasoning because for every statement there was a valid reason. The student was able to create an argument consisting of several connected statements.

4.2.30. Analysis of Question 1.b on investigation task 2

Question 1.b in the investigation task 2 reads as follows:

Question 1.b: Prove that the following triangles are congruent:



STATEMENT	REASON

An expected solution for Question 1.b is as follows:

STATEMENT	REASON
$EV = WR$	Given
$EW = VR$	Given
$VW \text{ is common}$	Common in both triangles
$\therefore \Delta EWV \equiv \Delta VWR$	SSS

Figure 4.40: An expected solution for Question 1.b

As illustrated in Table 4.12, analysis of learners' responses to Question 1.b showed that seven learners attempted the problem correctly, 15 learners attempted the problem but made errors, and none did not attempt Question 1.b.

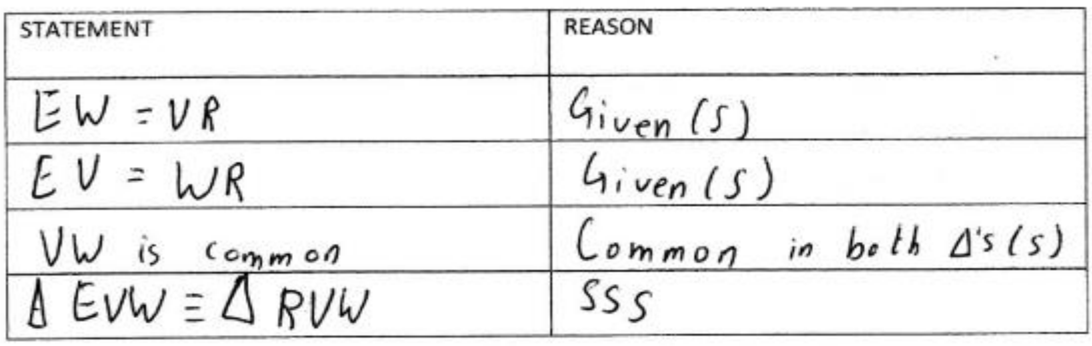
Table 4.12: Analysis of Question 1.b

Learner response	**Attempted correctly**	**Attempted but made errors**	**Not attempted**

Number of learners	22	0	0
Learner number	1,2,3,4,5,6,7,8,9,10,11,12,13,14 15,16,17,18,19,20,21,22	None	None

As illustrated in the table above it is clear that learners performed well in this question. The reason why learners performed well in this question could be due to the fact learners found the video to be very helpful when answering this question. The video clearly explained on how to go about solving a congruency-based proof problem by using the two-column proof method. Learners found it easy to understand what a statement is and what a reason is, after watching the video.

4.2.31. Exemplification of learners who attempted Question 1.b correctly.



STATEMENT	REASON
EW = VR	Given (S)
EV = WR	Given (S)
VW is common	Common in both Δ's (S)
Δ EVW ≡ Δ RVW	SSS

Figure 4.41: Learner 17 response to Question 1.b

L17 wrote the statement correctly and provides the correct reason for each statement. The learner also wrote down the correct case for these triangles to be congruent, which is SSS. The learners displayed good levels of proof writing competence and will be able to cope with much more complex proofs. The learner clearly benefited from watching the YouTube video based on congruency-based proofs. The learner was able to make a conjecture about the problem and also is able to justify the conjecture. The learner was able to use a proof in order to establish the validity of statements and explain why the conjectures are true. Learner was able to provide a warrant which justifies claims and supplies reasons and uses geometry property and relationship and provides a clear logic structure. There was clear evidence for a chain of reasoning because for every statement there was a valid reason. The student was able to create an argument consisting of several connected statements.

STATEMENT	REASON
EW = VR	Given(s)
EV = WR	Given (s)
VW is common	Common in both Δ's(s)
ΔEVW ≡ ΔRVW	SSS

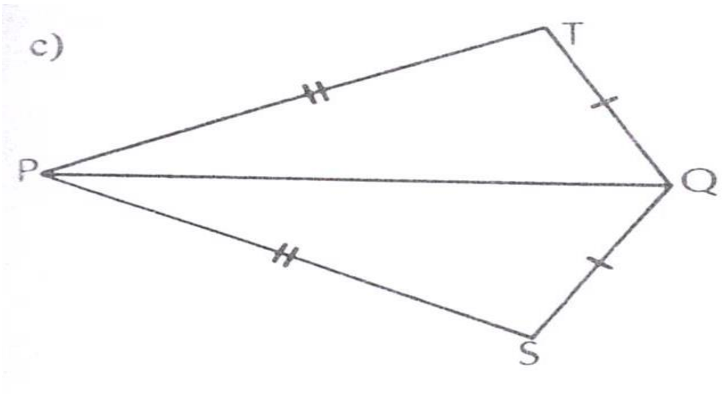
Figure 4.42: Learner 1 response to Question 1.b

L1, similar to L17, wrote the statement correctly and provides the correct reason for each statement. The learner also wrote down the correct case for these triangles to be congruent, which is SSS. The learners displayed good levels of proof writing competence and will be able to cope with much more complex proofs. The learner clearly benefited from watching the YouTube video based on congruency-based proofs. The learner was able to make a conjecture about the problem and can justify the conjecture. The learner was able to use a proof in order to establish the validity of statements and explain why the conjectures are true. Learner was able to provide a warrant which justifies claims and supplies reasons and uses geometry property and relationship and provides a clear logic structure. There was clear evidence for a chain of reasoning because for every statement there was a valid reason. The student was able to create an argument consisting of several connected statements.

4.2.32. Analysis of Question 1.c on investigation task 2

Question 1.c in the investigation task 2 reads as follows:

Question 1.c: Prove that the following triangles are congruent:



STATEMENT	REASON

An expected solution for Question 1.c is as follows:

STATEMENT	REASON
$QT = QS$	Given
$TP = SP$	Given
$PQ\ is\ common$	Common in both triangles
$\therefore \Delta PQT \equiv \Delta PQS$	SSS

Figure 4.43: An expected solution for Question 1.c

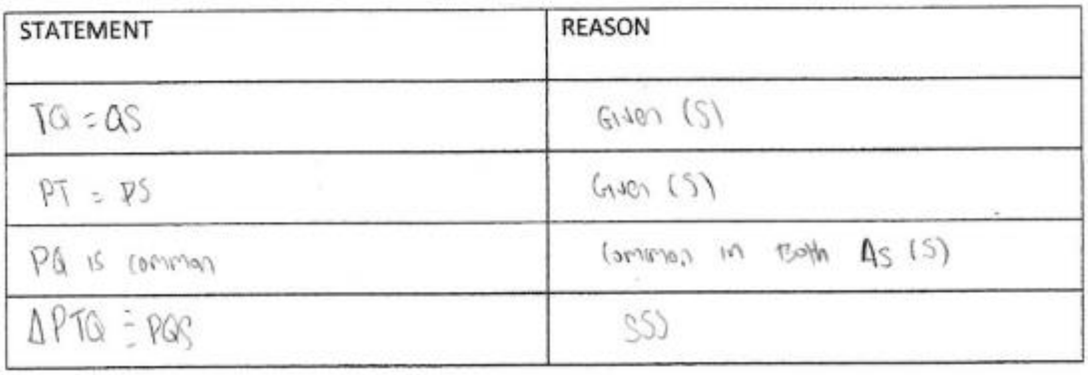
As illustrated in Table 4.13, analysis of learners' responses to Question 1.c showed that 22 learners attempted the problem correctly, none of the learners attempted the problem but made errors, and none of the learners did not attempt Question 1.c.

Table 4.13: Analysis of Question 1.c

Learner response	Attempted correctly	Attempted but made errors	Not attempted
Number of learners	22	0	0
Learner number	1,2,3,4,5,6,7,8,9,10,11,12,13,14 15,16,17,18,19,20,21,22	None	None

As illustrated in the table above it is clear that learners performed well in this question. The reason why learners performed well in this question could be due to the fact learners found the video to be very helpful when answering this question. The video clearly explained on how to go about solving a congruency-based proof problem by using the two-column proof method. Learners found it easy to understand what a statement is and what a reason is, after watching the video.

4.2.33. Exemplification of learners who attempted Question 1.c correctly.



STATEMENT	REASON
TQ = QS	Given (S)
PT = PS	Given (S)
PQ is common	Common in Both Δs (S)
ΔPTQ ≡ PQS	SSS

Figure 4.44: Learner 10 response to Question 1.c

L10, wrote the statement correctly and provides the correct reason for each statement. The learner also wrote down the correct case for these triangles to be congruent, which is SSS The learner displays good levels of proof writing competence and will be able to cope with much more complex proofs. The learner clearly benefited from watching the YouTube video based on congruency-based proofs. The learner was able to make a conjecture about the problem and is able to justify the conjecture. The learner was able to use a proof in order to establish the validity of statements and explain why the conjectures are true. Learner was able to provide a warrant which justifies claims and supplies reasons and uses geometry property and relationship and provides a clear logic structure. There was clear evidence for a chain of reasoning because for every statement there was a valid reason. The student was able to create an argument consisting of several connected statements.

STATEMENT	REASON
PT = PS	Given (S)
TQ = SQ	Given (S)
PQ is common	common i~~s~~n both Δ's (S)
Δ PQT ≡ Δ PQS	SSS

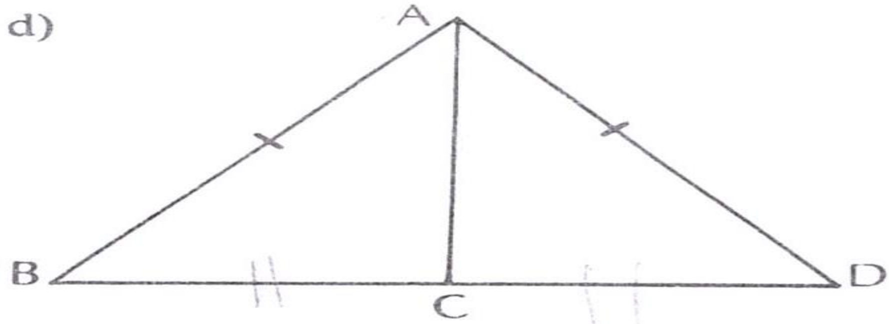
Figure 4.45: Learner 20 response to Question 1.c

L20 wrote the statement correctly and provides the correct reason for each statement. The learner also wrote down the correct case for these triangles to be congruent, which is SSS The learner displays good levels of proof writing competence and will be able to cope with much more complex proofs. The learner clearly benefited from watching the YouTube video based on congruency-based proofs. The learner was able to make a conjecture about the problem and is able to justify the conjecture. The learner was able to use a proof in order to establish the validity of statements and explain why the conjectures are true. Learner was able to provide a warrant which justifies claims and supplies reasons and uses geometry property and relationship and provides a clear logic structure. There was clear evidence for a chain of reasoning because for every statement there was a valid reason. The student was able to create an argument consisting of several connected statements.

4.2.34. Analysis of Question 1.d on investigation task 2

Question 1.d in the investigation task 2 reads as follows:

Question 1.d: Prove that the following triangles are congruent:



C is the midpoint of BD.
Prove Δ ABC $\equiv$ Δ ADC.

STATEMENT	REASON

An expected solution for Question 1.d is as follows:

STATEMENT	REASON
$AB = AD$	Given
$BC = CD$	Midpoint C divides BC and CD in two equal parts
$AC\ is\ common$	Common in both triangles
$\therefore \Delta ABC \equiv \Delta ADC$	SSS

Figure 4.46: An expected solution for Question 1.d

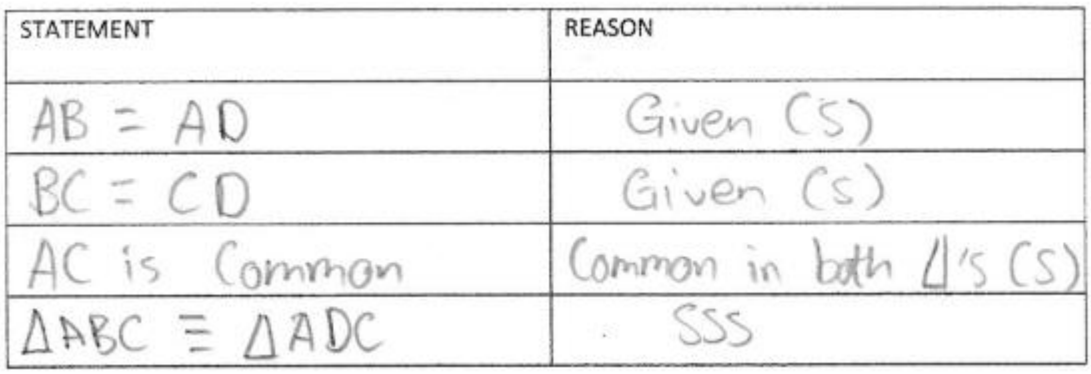
As illustrated in Table 4.14, analysis of learners' responses to Question 1.d showed that 22 learners attempted the problem correctly, none of the learners attempted the problem but made errors, and none of the learners did not attempt Question 1.d.

Table 4.14: Analysis of Question 1.d

Learner response	Attempted correctly	Attempted but made errors	Not attempted
Number of learners	22	0	0
Learner number	1,2,3,4,5,6,7,8,9,10,11,12,13,14 15,16,17,18,19,20,21,22	None	None

As illustrated in the table above it is clear that learners performed well in this question. The reason why learners performed well in this question could be due to the fact learners found the video to be very helpful when answering this question. The video clearly explained on how to go about solving a congruency-based proof problem by using the two-column proof method. Learners found it easy to understand what a statement is and what a reason is, after watching the video.

4.2.35. Exemplification of learners who attempted Question 1.d correctly.



STATEMENT	REASON
AB = AD	Given (S)
BC = CD	Given (s)
AC is Common	Common in both △'s (S)
△ABC ≡ △ADC	SSS

Figure 4.47: Learner 4 response to Question 1.d

L4, wrote the statement correctly and provided the correct reason for each statement. The learner also wrote down the correct case for these triangles to be congruent, which is SSS. The learner displayed good levels of proof writing competence and will be able to cope with much more complex proofs. The learner clearly benefited from watching the YouTube video based on congruency-based proofs. The learner was able to make a conjecture about the problem and is able to justify the conjecture. The learner was able to use a proof in order to establish the validity of statements and explain why the conjectures are true. Learner was able to provide a warrant which justifies claims and supplies reasons and uses geometry property and relationship and provides a clear logic structure. There was clear evidence for a chain of reasoning because for every statement there was a valid reason. The student was able to create an argument consisting of several connected statements.

STATEMENT	REASON
AB=DA	Given (S)
DC=BC	Given (S)
AC is common	Common in both Δ's
ΔABC ≡ ΔADC	SSS

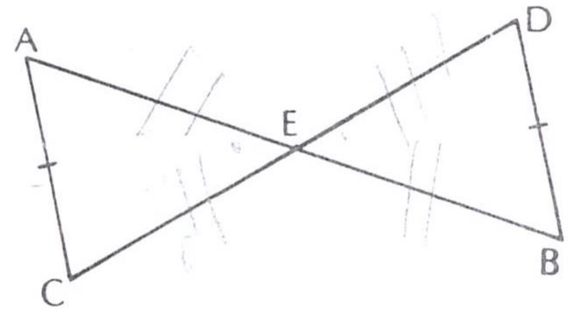
Figure 4.48: Learner 12 response to Question 1.d

L12, similar to L4 wrote the statement correctly and provided the correct reason for each statement. The learner also wrote down the correct case for these triangles to be congruent, which is SSS. The learner displayed good levels of proof writing competence and will be able to cope with much more complex proofs. The learner clearly benefited from watching the YouTube video based on congruency-based proofs. The learner was able to make a conjecture about the problem and is able to justify the conjecture. The learner was able to use a proof in order to establish the validity of statements and explain why the conjectures are true. Learner was able to provide a warrant which justifies claims and supplies reasons and uses geometry property and relationship and provides a clear logic structure. There was clear evidence for a chain of reasoning because for every statement there was a valid reason. The student was able to create an argument consisting of several connected statements.

4.2.36. Analysis of Question 2 on investigation task 2

Question 2 in the investigation task 2 reads as follows:

2.



AB and CD bisect each other at E.

Prove $\Delta ACE \equiv \Delta EDB$.

STATEMENT	REASON

An expected solution for Question 2 is as follows:

STATEMENT	REASON
$AC = DB$	Given
$A\hat{E}C = B\hat{E}D$	Vertically opposite angles equal
$AE = EB$	Given
$\therefore \Delta ACE \equiv \Delta EDB$	SAS

Figure 4.49: An expected solution for Question 2

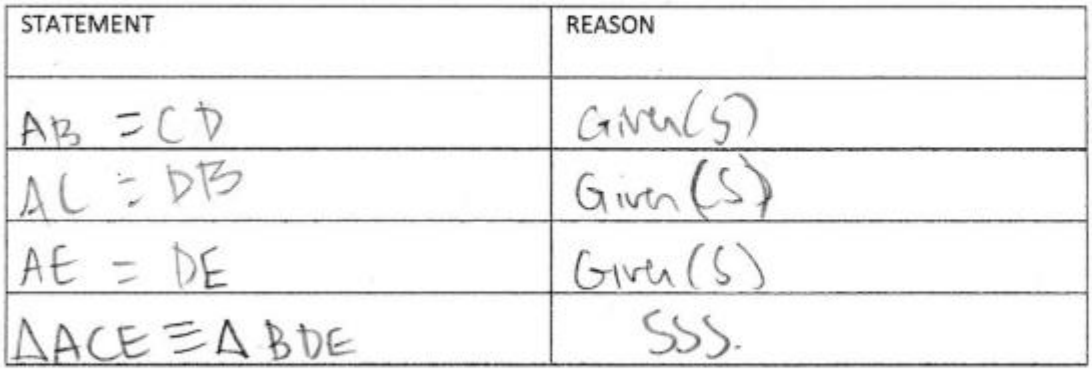
As illustrated in Table 4.15, analysis of learners' responses to Question 2 showed that seven learners attempted the problem correctly, 15 learners attempted the problem but made errors, and none did not attempt Question 2.

Table 4.15: Analysis of Question 2

Learner response	Attempted correctly	Attempted but made errors	Not attempted
Number of learners	14	4	4
Learner number	1,2,3,4,5,7,11,12,14,16,17,18 20,21	6,13,15,19	8,9,10,22

As illustrated in the table above it is clear that learners performed well in this question. The reason why learners performed well in this question could be due to the fact learners found the video to be very helpful when answering this question. The video clearly explained on how to go about solving a congruency-based proof problem by using the two-column proof method. Learners found it easy to understand what a statement is and what a reason is, after watching the video.

4.2.37. Exemplification of learners who attempted Question 2 correctly.



STATEMENT	REASON
AB = CD	Given (S)
AC = DB	Given (S)
AE = DE	Given (S)
ΔACE ≡ Δ BDE	SSS.

Figure 4.50: Learner 11 response to Question 2

L11, wrote the statement correctly and provides the correct reason for each statement. The learner also wrote down the correct case for these triangles to be congruent, which is SSS. The learners displayed good levels of proof writing competence and will be able to cope with much more complex proofs. The learner clearly benefited from watching the YouTube video based on congruency based proofs. The learner was able to make a conjecture about the problem and also is able to justify the conjecture. The learner was able to use a proof in order to establish the validity of statements and explain why the conjectures are true. Learner was able to provide a warrant which justified claims and supplied reasons and uses geometry property and relationship and provides a clear logic structure. There was clear evidence for a chain of reasoning because for every statement there was a valid reason. The student was able to create an argument consisting of several connected statements.

4.2.38. Exemplification of learners who attempted Question 2 but made errors.

STATEMENT	REASON
AC = DB	
CE = ED	

Figure 4.51: Learner 15 response to Question 2

L15, wrote the statement correctly but did not provide a reason for each statement. The learner does not complete the solution to the problem. The learner displayed poor levels of proof writing competence and will be not able to cope with much more complex proofs. The learner clearly did not understand the YouTube video based on congruency-based proofs. The learner is unable to use a proof in order to establish the validity of statements and explain why the conjectures are true. Learner was not able to provide a warrant which justifies claims and supplies reasons and uses geometry property and relationship and provides a clear logic structure. There is no evidence for a chain of reasoning because for every statement there was no valid reason. The student was not able to create an argument consisting of several connected statements. Clearly the learner displays poor levels of proof writing competency.

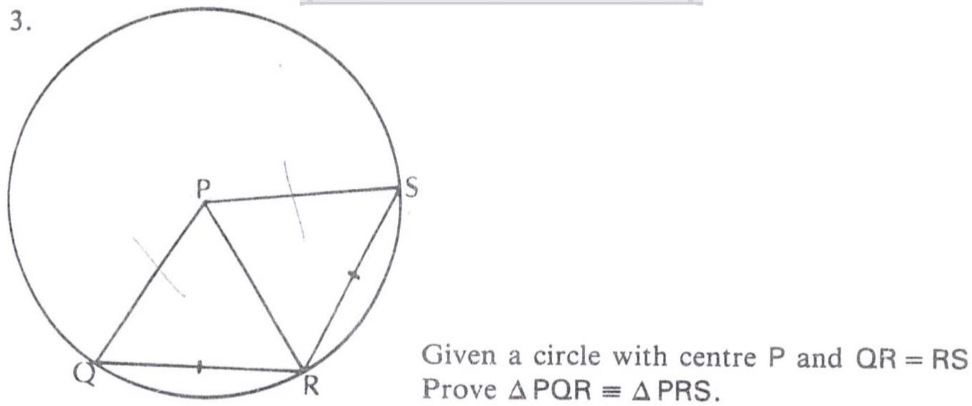
STATEMENT	REASON
AC = DB	Given (S)
AB = CD	Given (S)
E	

Figure 4.52: Learner 6 response to Question 2

L6, wrote the statement correctly but only provided a reason for the first two statements and does not provide enough information to prove that the triangles are congruent. The learner does not complete the solution to the problem. The learner displayed poor levels of proof writing competence and will be not able to cope with much more complex proofs. The learner clearly did not understand the YouTube video based on congruency-based proofs. The learner was unable to use a proof in order to establish the validity of statements and explain why the conjectures are true. Learner was not able to provide a warrant for every statement which justifies claims and supplies reasons and uses geometry property and relationship and provides a clear logic structure. There is not enough evidence for a chain of reasoning because statement there was only a valid reason for two statements and the learner needed more statements to build up an argument good enough to prove that the triangles was congruent. The student was not able to create an argument consisting of several connected statements. Clearly the learner displays poor levels of proof writing competency.

4.2.39. Analysis of Question 3 on investigation task 2

Question 3 in the investigation task 2 reads as follows:



STATEMENT	REASON

An expected solution for Question 3 is as follows:

STATEMENT	REASON
$PQ = PS$	Both radii
$QR = RS$	Given
$PR\ is\ common$	Common in both triangles
$\therefore \Delta PQR \equiv \Delta PRS$	SSS

Figure 4.53: An expected solution for Question 3

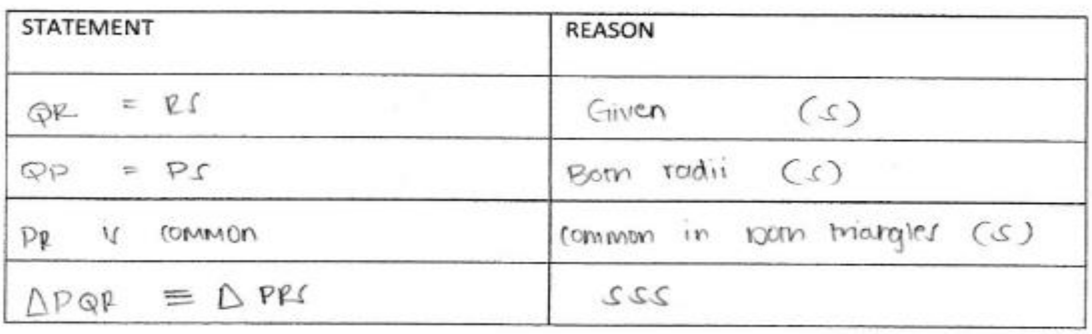
As illustrated in Table 4.16, analysis of learners' responses to Question 3 showed that 10 learners attempted the problem correctly, none of the learners attempted the problem but made errors, and 12 learners did not attempt Question 3.

Table 4.16: Analysis of Question 3

Learner response	**Attempted correctly**	**Attempted but made errors**	**Not attempted**
Number of learners	10	0	12
Learner number	1,2,4,5,6,13,15,16 18,22	None	3,7,8,9,10,11,12,14,17 19,20,21

As illustrated in the table above it is clear that learners performed poorly in this question. The reason why learners performed poorly in this question could be due to the fact that learners found it more difficult as each question increased with difficulty. This needs to be addressed by the teacher, the teacher could expose the learners to more higher order questions.

4.2.40. Exemplification of learners who attempted Question 3 correctly.



STATEMENT	REASON
QR = RS	Given (S)
QP = PS	Both radii (S)
PR is common	common in both triangles (S)
ΔPQR ≡ ΔPRS	SSS

Figure 4.54: Learner 2 response to Question 3

L2, wrote the statement correctly and provided the correct reason for each statement. The learner also wrote down the correct case for these triangles to be congruent which is SSS. The learner displayed good levels of proof writing competence and will be able to cope with much more complex proofs. The learner clearly benefited from watching the YouTube video based on congruency-based proofs. The learner was able to make a conjecture about the problem and also is able to justify the conjecture. The learner was able to use a proof in order to establish the validity of statements and explain why the conjectures are true. Learner was able to provide a warrant which justifies claims and supplies reasons and uses geometry property and relationship and provides a clear logic structure. There was clear evidence for a chain of reasoning because for every statement there was a valid reason. The student was able to create an argument consisting of several connected statements.

STATEMENT	REASON
QR = RS	Given
QP = PS	Both radii
PR is common	Common in both
PQR ≡ PRS	S,S,S

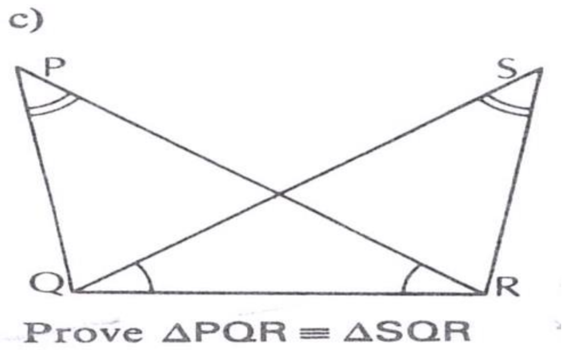
Figure 4.55: Learner 13 response to Question 3

L13, similar to L2, wrote the statement correctly and provided the correct reason for each statement. The learner also wrote down the correct case for these triangles to be congruent which is SSS. The learner displayed good levels of proof writing competence and will be able to cope with much more complex proofs. The learner clearly benefited from watching the YouTube video based on congruency-based proofs. The learner was able to make a conjecture about the problem and also is able to justify the conjecture. The learner was able to use a proof in order to establish the validity of statements and explain why the conjectures are true. Learner was able to provide a warrant which justifies claims and supplies reasons and uses geometry property and relationship and provides a clear logic structure. There was clear evidence for a chain of reasoning because for every statement there was a valid reason. The student was able to create an argument consisting of several connected statements.

4.2.41. Analysis of Question 4 on investigation task 2

Question 4 in the investigation task 2 reads as follows:

4.



An expected solution for Question 4 is as follows:

STATEMENT	REASON
$S\hat{Q}R = P\hat{R}Q$	Given
$Q\hat{P}R = Q\hat{S}R$	Given
$QR\ is\ common$	Common in both triangles
$\therefore \Delta PQR \equiv \Delta SQR$	AAS

Figure 4.56: An expected solution for Question 4

As illustrated in Table 4.17 analysis of learners' responses to Question 4 showed that 11 learners attempted the problem correctly, none of the learners attempted the problem but made errors, and 11 learners did not attempt Question 4.

Table 4.17: Analysis of Question 4

Learner response	**Attempted correctly**	**Attempted but made errors**	**Not attempted**
Number of learners	11	0	11
Learner number	1,2,4,5,6,7,13,15,16 18,22	None	3,8,9,10,11,12,14,17,19,20,21

As illustrated in the table above it is clear that learners performed poorly in this question. The reason why learners performed poorly in this question could be due to the fact that learners found it more difficult as each question increased with difficulty. This needs to be addressed by the teacher, the teacher could expose the learners to more higher order questions.

4.2.42. Exemplification of learners who attempted Question 4 correctly.

STATEMENT	REASON
∠PQR = ∠SRQ	Given
∠QPR = ∠QSR	Given
QR is common	Common in both triangles
ΔPQR ≡ ΔSQR	AAS

Figure 4.57: Learner 18 response to Question 4

L18 wrote the statement correctly and provides the correct reason for each statement. The learner also wrote down the correct case for these triangles to be congruent which is AAS. The learner displayed good levels of proof writing competence and will be able to cope with much more complex proofs. The learner clearly benefited from watching the YouTube video based on congruency-based proofs. The learner was able to make a conjecture about the problem and also is able to justify the conjecture. The learner was able to use a proof in order to establish the validity of statements and explain why the conjectures are true. Learner was able to provide a warrant which justifies claims and supplies reasons and uses geometry property and relationship and provides a clear logic structure. There was clear evidence for a chain of reasoning because for every statement there was a valid reason. The student was able to create an argument consisting of several connected statements.

STATEMENT	REASON
∠PQR = ∠SRQ	Given (A)
∠QPR = ∠QSR	Given (A)
QR is common	Common in both triangles
ΔPQR ≡ ΔSQR	A,AS

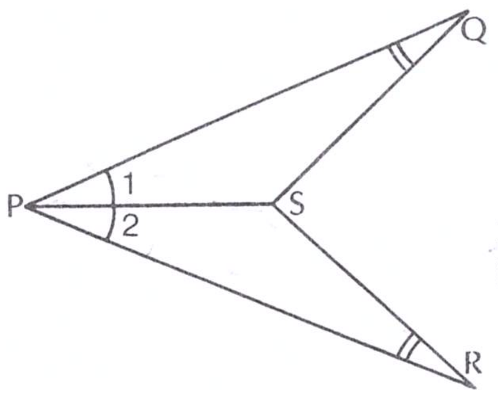
Figure 4.58: Learner 6 response to Question 4

L6 wrote the statement correctly and provides the correct reason for each statement. The learner also wrote down the correct case for these triangles to be congruent which is AAS. The learners displayed good levels of proof writing competence and will be able to cope with much more complex proofs. The learner clearly benefited from watching the YouTube video based on congruency-based proofs. The learner was able to make a conjecture about the problem and can justify the conjecture. The learner was able to use a proof in order to establish the validity of statements and explain why the conjectures are true. Learner was able to provide a warrant which justifies claims and supplies reasons and uses geometry property and relationship and provides a clear logic structure. There was clear evidence for a chain of reasoning because for every statement there was a valid reason. The student was able to create an argument consisting of several connected statements.

4.2.43. Analysis of Question 5 on investigation task 2

Question 5 in the investigation task 2 reads as follows:

5.



Prove that PQ = PR (Hint, first prove ΔQPS ≡ ΔPRS)

STATEMENT	REASON

An expected solution for Question 5 is as follows:

STATEMENT	REASON
$\hat{P}1 = \hat{P}2$	Given
$P\hat{R}S = P\hat{Q}S$	Given
PS is common	Common in both triangles
$\therefore \Delta QPS \equiv \Delta PRS$	AAS
$\therefore PQ = PR$	$\Delta QPS \equiv \Delta PRS$

Figure 4.59: An expected solution for Question 5

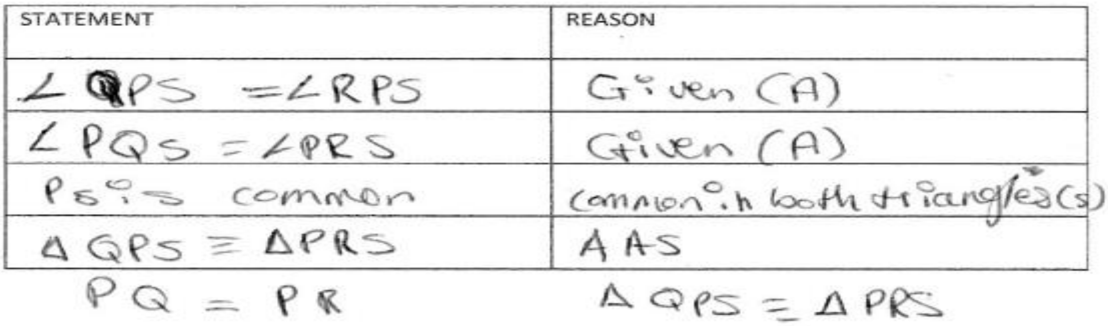
As illustrated in Table 4.18, analysis of learners' responses to Question 5 showed that 10 learners attempted the problem correctly, none of the learners attempted the problem but made errors, and 12 learners did not attempt Question 5.

Table 4.18: Analysis of Question 5

Learner response	**Attempted correctly**	**Attempted but made errors**	**Not attempted**
Number of learners	10	0	12
Learner number	1,2,4,5,6,8,13,15,16,22,	None	3,7,8,9,10,11,12,14,17,19 20,21

As illustrated in the table above it is clear that learners performed poorly in this question. The reason why learners performed poorly in this question could be due to the fact that learners found it more difficult as each question increased with difficulty. This needs to be addressed by the teacher, the teacher could expose the learners to more higher order questions.

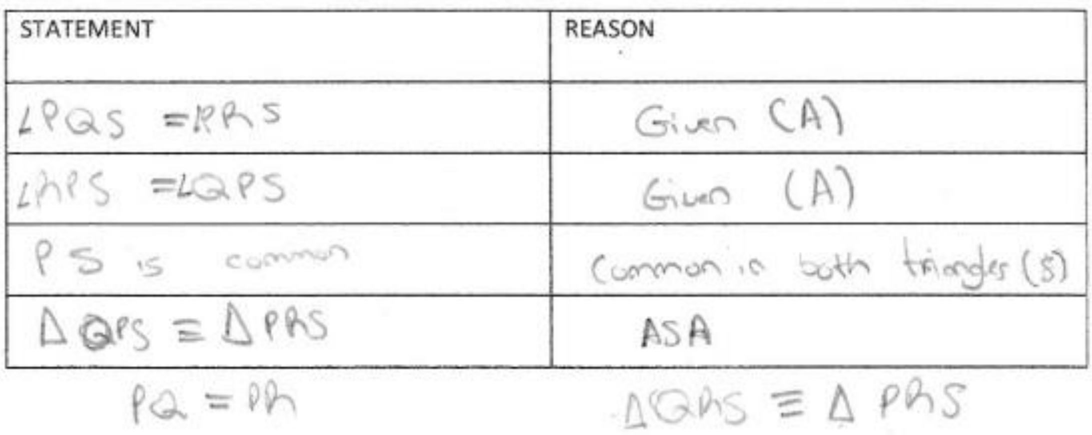
4.2.44. Exemplification of learners who attempted Question 5 correctly.



STATEMENT	REASON
∠QPS = ∠RPS	Given (A)
∠PQS = ∠PRS	Given (A)
PS is common	common in both triangles (S)
ΔQPS ≡ ΔPRS	AAS
PQ = PR	ΔQPS ≡ ΔPRS

Figure 4.60: Learner 1 response to Question 5

L1, wrote the statement correctly and provides the correct reason for each statement. The learner also writes down the correct case for these triangles to be congruent which is AAS, the learner also manages to deduce that if the triangles are congruent then PQ = PR, the learner sees this with relative ease. The learner displayed good levels of proof writing competence and will be able to cope with much more complex proofs. The learner clearly benefited from watching the YouTube video based on congruency-based proofs. The learner was able to make a conjecture about the problem and is able to justify the conjecture. The learner was able to use a proof in order to establish the validity of statements and explain why the conjectures are true. Learner was able to provide a warrant which justifies claims and supplies reasons and uses geometry property and relationship and provides a clear logic structure. There was clear evidence for a chain of reasoning because for every statement there was a valid reason. The student was able to create an argument consisting of several connected statements.



STATEMENT	REASON
∠PQS = ∠PRS	Given (A)
∠RPS = ∠QPS	Given (A)
PS is common	Common in both triangles (S)
ΔQPS ≡ ΔPRS	ASA

PQ = PR ΔQPS ≡ ΔPRS

Figure 4.61: Learner 16 response to Question 5

L16, similar to L1 wrote the statement correctly and provides the correct reason for each statement. The learner also writes down the correct case for these triangles to be congruent which is AAS, The learner also manages to deduce that if the triangles are congruent then PQ = PR, the learner sees this with relative ease. The learner displayed good levels of proof writing competence and will be able to cope with much more complex proofs. The learner clearly benefited from watching the YouTube video based on congruency-based proofs. The learner was able to make a conjecture about the problem and was able to justify the conjecture. The learner was able to use a proof in order to establish the validity of statements and explain why the conjectures are true. Learner was able to provide a warrant which justifies claims and supplies reasons and uses geometry property and relationship and provides a clear logic structure. There was clear evidence for a chain of reasoning because for every statement there was a valid reason. The student was able to create an argument consisting of several connected statements.

4.3. Learner's responses to the task questions

To support the task that was given to the learners and to fill in the gaps in research, interviews were used. The information collected from the interviews was presented in a narrative form, which includes the description and analysis of data.

The main purpose of the interviews was to find out how the learners felt about the task and the questions that were asked. The interviews were done with the same learners that agreed to take in this part of the research. The interviews were done with a group of 5 learners.

Question 1

1.1 How did you find the sketching of triangles?

1.2 What was the approach that you used to sketch the triangles?

1.3 How did you find comparing congruent triangles and non-congruent triangles?

Figure 4.62: Interview questions based on Question 1

Question 1, Interviewee 1 said that they struggled with drawing the triangles to scale and did not find the videos via the web based learning really helpful and would rather prefer that the teacher does the sketching with them. Interviewee 2 stated that the approach which was used to sketch the triangles was of that learnt in primary school and grade 8, so techniques from previous grades was used. Interviewee 3 found that stating which triangles are congruent or not was surprisingly the most challenging question. Comparing the triangles was not easy because certain triangles looked completely different from others.

Question 2

2.1 How did you find distinguishing between the four cases of congruency?

2.2 How did you find distinguishing between the two cases of similarity?

Figure 4.63: Interview questions based on Question 2

Question 2, Interviewee 4 found distinguishing between the 4 cases of congruency to be easy and completing the tables was also easy and stated that the video used via a web based learning system really helped to remember the 4 cases of congruency clearly. Interviewee 5 found that distinguishing between the two cases of similarity was easy and also stated that the videos were helpful, and it is easy to know the difference between congruency and similarity.

Question 3

3.1 How did you cope with distinguishing between congruency, similarity and parallel lines?

3.2 How did you find the congruency-based proof question?

3.3 What approach did you use to complete the congruency-based proof problem?

Figure 4.64: Interview questions on based on Question 3

Question 3, Interviewee 1 found that distinguishing between congruency, similarity and parallel lines was easy and had no trouble with answering the questions. The interviewee could also remember the correct mathematical symbols for each property. Interviewee 2 found the congruency-based proof to be relatively easy as it had a lot of information needed in order to solve the problem. This increased confidence in the learner's ability to do the actual proof. Interviewee 3 coped really well with the congruency-based proof problem; the interviewee's approach was to use all of the given information to complete the two-column method proof. The interviewee also stated that the video watched via a web-based learning system really helped.

Question 4

4.1 Did you understand the videos based on congruency-based proofs?
4.2 How did the videos help you solve the various congruency-based proof problems?
4.3 How did the videos help you to find out which sides and which angles were equal between the triangles?

Figure 4.65: Interview questions on based on task-based activity 2

Question 4, Interviewee 4 clearly understood the videos based on congruency-based proofs, also stating that the videos made it easier to solve problems and the videos clearly highlighted which sides and which angles were equal. Interviewee 5 found that the videos demonstrated how to answer the congruency-based proof problems and showed clearly how to answer the questions and it also showed the learner the different symbols needed to figure out the question. Interviewee 1 also agreed that the videos explained the congruency-based proofs well and the multiple examples that the videos had really helped. Similarly, to Interviewee 4, Interviewee 1 agreed that the videos highlighted which sides and angles were equal.

Question 5

5.1 Did you have a better understanding of congruency-based proofs after watching the videos?
5.2 Would you recommend that other learners watch videos to solve congruency- based proofs?
5.3 Will you use videos to solve congruency-based proof problems in future?

Figure 4.66: Interview questions on based on task-based activity 2

Question 5, Interviewee 2 had a better understanding of congruency-based proofs after watching the videos and so did the rest of the interviewees. All Interviewee's agreed that they would recommend that other learners watch videos to solve congruency-based proofs and all interviewees also agreed to use videos in order to solve congruency-based proof problems in future. Overall, all interviewees were clearly in support of the use of videos when it comes to solving congruency-based proof problems, which is really encouraging.

4.4. Teacher's observations

4.4.1. Learners understood the videos based on congruency-based proofs

The learners clearly understood the videos based on congruency-based proofs and did not need the videos to played repeatedly in order to understand the concept being explained by the video. When the teacher asked learners whether they understood the video they all agreed that they did, even if the teacher did explain the video to the learners.

4.4.2. Learners could use the videos to solve congruency-based proof problems

Learners could easily use the videos to solve the congruency-based proof problems, since the videos explained the two-column proof method quite well and learners could understand the two-column proof method. The teacher observed that learners would make notes on the videos that were played, and this helped learners to solve the congruency-based proof problems.

4.4.3. Learners were able to apply what was learnt via the videos

Teacher observed that learners were able to apply what was learnt via the videos, since most learners displayed evidence of applying the two-column proof method and they could easily pick up which case of congruency each problem had. Learner could use the videos to develop a proof in order to solve a particular congruency-based proof problem.

4.4.4. Learners were interested in congruency-based proof problems

Teacher noted that about 11 learners did not complete questions 3 to 5 task-based activity 2, which showed that there were learners who were not entirely interested in the congruency-based proof problems. This was a concern for the teacher because videos were used in order to stimulate learner interest. This shows that videos are not always effective in order to stimulate learner interest, which is why it is important for the teacher to guide the learners throughout the task-based activity.

4.4.5. Teacher could interact with learners and use the videos as a guiding tool

The teacher could easily interact with the learners at the beginning of the video and also at the end of the video. The teacher would occasionally pause the video in order to explain a concept which learners did not understand or what was completely new to the learners. The pause and rewind option made it easier for the teacher to use the video as a guiding tool.

4.4.6. Learners could interact with each other and discuss the congruency- based proof problems

The teacher allowed learners to interact with each other and discuss the congruency-based proof problems whilst learners were solving the problems. The teacher observed this was very effective and it built learner confidence in solving the problems. Learner could easily discuss the two-column proof method and apply what was learnt.

4.4.7. The videos could promote a learning environment conducive for learners to solve congruency-based proof problems.

The videos could easily promote a learning environment that was conducive for learners. Teacher observed that the videos stimulated learners and promoted problem solving skills learners could use in order to solve the congruency-based proof problems. The videos promoted a proof friendly environment for learners by highlighting the proofs that learners needed to know in order to solve each problem.

4.4.8. Learners could use the methods provided to solve congruency- based proof problems effectively

The teacher observed that learners could use the two-column proof method effectively in order to solve the congruency-based proof problems. The reason for this is that the video clearly explained the two-column proof method effectively, by explaining the statement column and the reason column clearly. Learners could easily distinguish between what is the statement and what is the reason of the two-column proof method.

4.4.9. Videos were interactive and presentable for learners to watch.

Learners were visually stimulated by the videos, which made it easy for learners solve the congruency-based proof problems. The videos were clear, colourful and very creative, which aroused learner interest. Teacher observed that learners were captivated by the videos and learners could easily interact with the videos and ask questions based on what was being illustrated by the videos.

4.4.10. Congruency-based proof problems could challenge learners thinking

All of the congruency-based proof problems could challenge learners thinking and various cognitive levels were tested. Learners were exposed to a variety of congruency-based proof problems which increased in difficulty with each question. Learner could use their critical thinking skills in order to solve each congruency-based proof problems. The level of difficulty also led to 11 learners not completing questions 3 -5 of task-based activity 2, which was a trend amongst these 11 learners.

4.4.11. General

In general, the teacher observed that the videos visually stimulated the learning experience of learners when it came to solving the congruency-based proof problems. This was achieved through the videos being interactive and presentable. Most learners could solve the congruency-based proof problems with relative ease. This was really encouraging for the teacher and the teacher along with the learners would recommend that all mathematics learners globally should use videos in order to solve congruency-based proof problems in future.

4.5. Conclusion

This chapter discussed the analysis of data and the findings of the study on how grade 9 learners reasoned geometrically. Some learners had the answer without a supporting reason. About 11 learners failed to complete the task and they argued that they did not have time to do it or they did not know how to answer specific questions. What was interesting is that if learners could not solve the congruency-based proof problem, they would not attempt it at all. The teacher concluded that this group of learners clearly were reluctant to make any errors and rather chose not to complete a question if they did not know what to do.

The next chapter of this study presents the interpretation and discussion of the findings with some recommendations and a conclusion.

CHAPTER 5

DISCUSSION OF RESULTS, RECOMMENDATIONS AND CONCLUSION

5.1. Introduction

The aim of the study was to explore the geometrical reasoning of grade 9 learners via a web-based learning system. In essence, the research explored how a web-based learning system could be implemented to assist how learner geometrically reason congruency-based proofs. The significance of this research lies in its contribution to a better understanding of how learner's reason geometrically. This chapter includes discussions based on the tasks given to learners to be completed. It also makes a recommendation for teaching and concludes the thesis and most importantly it aims to answer the research questions.

5.2. Summary of findings in relation to research questions.

As indicated earlier the first research question pursued in this study was posited as:

What proof writing competencies do grade 9 mathematics learners demonstrate when attempting to prove triangles congruent in a web-based learning environment?

The findings associated with the research question showed that:

When attempting to prove that triangles are congruent in a web-based learning environment grade 9 learners demonstrated many proof writing competencies. These proof writing competencies can be explained by the proof writing competency model which identifies seven dimensions that needs to be prevalent in student's actions towards the development and writing of proofs in geometry (Mcrone & Martin ,2011).

The first proof writing competency which was making conjectures, was displayed clearly through learner's actions of making claims and asking questions about the relationships that exists between the congruency problem and the correct case of congruency. The YouTube videos clearly showed learners what a conjecture is and how to develop it. The teacher guided the whole process and through thorough explanations helped further enhance the learners understanding of conjectures

The second proof writing competency displayed by learners was justification and was shown through student's actions of justifying all conjectures or refuting them. It was clear that learners had the ability to justify their conjectures. There was a thorough explanation from the YouTube videos on how to justify conjectures and this really helped the learners. Added to this the teacher explanations also helped learners understand the concepts much better and played a critical role in achieving the aim of the lesson.

The third competency which was the role of proof that was clearly shown through learner actions of establishing the validity of statements and explaining why the conjectures are true. Most students could develop a two-column proof in order to prove that the triangles were congruent, by associating the correct reason with the correct statement in in each proof. The YouTube video which learners watched clearly explained what the two-column proof method was and how to structure a proof, learners found this to very helpful and effective as they solved each problem. The teacher also clearly wrote on the board on what the two-column proof method looks like and through explanation helped learners understand this method effectively, the more the teacher did the two-column proof method the more confidence learners gained in using this method two solve the congruency-based proofs.

The fourth proof writing competency which was learners' ability to provide warrants, was essentially shown through learner's ability to justify claims or supply reasons for others claims by making use of a geometry property or relationship and having a clear appeal to logic structure. Most learners could clearly provide reasons for each statement and there was a clear structuring of the proof and a flow to solving the congruency-based proof problem. With the aid of the YouTube video the learners could develop warrants because it showed them clearly how to go about providing a reason for each statement. The videos emphasized on the importance of providing the correct reason for each statement

The fifth proof writing competency was to a build chain of reasoning, and this was displayed through learner's ability to create an argument consisting of several connected statements. It was clearly evident from the findings of the research that most learners could develop proofs which consisted of interconnected statements that satisfied specific cases of congruency and their argument was strong as they had enough information to prove that the triangles were indeed congruent. Each worked example displayed on the YouTube video clearly showed learners how to build a chain of reasoning and the researcher picked up that learner confidence to solve each problem grew as each YouTube video was played. What was the teacher's role?

The sixth proof writing competency was standards for reasoning, and this was displayed through student actions by them having reasons that met certain standards in order to be valid. This proof writing competency was displayed by learners having reasons that was accurate enough to be valid. The YouTube videos clearly showed the standards that had to be met for a reason to be valid, this proved to be very effective and led to most learners having the ability to reason properly.

The seventh proof writing competency was to use a diagram. Learners displayed the ability to use the diagram to identify relationships and illustrate reasoning, through the observations of the teacher it was clear that learners used the diagram in order to solve the congruency-based problems and to even structure a proof. Most learners would even make notes on the diagram in order to solve the congruency-based proof problem and this was clearly effective based on the high success rate of students solving each problem. Learners were also visually stimulated with the use of the YouTube video on congruency-based proof problems and could use the diagrams effectively.

As indicated earlier the second research question pursued in this study was posited as:

To what extent can grade 9 learners use the congruency- based proofs to make mathematical deductions in a web-based learning environment?

The findings associated with the research question showed that:

The extent to which grade 9 learners used congruency-based proofs to make mathematical deductions in a web-based learning environment was clearly evident as a result of the findings of the research. It was clear that most learners could use the YouTube videos based on congruency-based proofs in order to prove that the triangles were congruent. This was displayed through learner's ability to extract the two-column proof method from the YouTube videos and apply it with each congruency-based proof problem that they had to solve. Learners could make mathematical deductions with the use of the two-column proof method by clearly having a reason for each geometric statement.

It was also evident from that there were a group of learners that could not solve every proof problem, they refused to even attempt the problem, which is a phenomenon on its own. Thus, there were learners whom did not make mathematical deductions using congruency-based proofs with the aid of the YouTube videos. The researcher concluded that due to the difficulty level of congruency-based proof problems that increased with each question, there were learners who refused to attempt these problems. This could be due to a lack of confidence or gaps in knowledge, even though learners could use the YouTube videos.

5.3. Interpretation of data

The research task was conducted in the researcher's class with 22 learners, and all learners gave consent for their scripts to be used, therefore analysis was done with only 22 scripts. Learners were given a task to complete, and the following is the interpretation of the task.

Question 1 was a construction of triangles question, learners found it difficult to answer and most learners had the incorrect constructions of triangles. Learners are more interested in whether their sketch is right and less interested in the process of drawing the correct sketch of each triangle.

In question 1 it was already picked up that some learners geometrical thinking are not sophisticated because they could not answer the question 1.7 pertaining to which triangles were congruent and which were not. One could argue that they did not understand fully what they were doing A few learners made mistakes by writing down the incorrect pair of congruent triangles. During the group discussion, one of the members in the group explained that the sketching of triangles was the most challenging of questions to do and most learners were unsure. Lack of exposure to sketching triangles played a role here because learners mostly got confused by which triangles were congruent or not.

In Question 2 learners found it easy to state which conditions satisfied the properties of congruency. There was also a clear understanding of the difference between congruency and similarity. Most learners could answer these questions with relative ease and could also discuss this in the group discussions or interviews.

All learners completed the table correctly and stated that triangles are not necessarily congruent if three angles are equal, because it satisfied the conditions for similarity. During group discussions, in the one group, 1 learner used the video to complete the question because it clearly highlighted the differences between congruency and similarity. So this indicates that there is an advantage to using a web based learning system.

For Question 3 there was similar answers and reasoning when answering the questions. This was a good question because all of them who were interviewed explained the difference between congruency, similarity and parallel lines. The most surprising problem which was answered with confidence was Question 3.4 on congruency-based proofs. Many learners managed to prove that the triangles were congruent, this highlights the high level of proof competency that learners have. Some learners also indicated that the online video help them and made it easy for learners to do the two-column method proof. As a researcher it is encouraging that learners could answer the most important question of this study with confidence, despite the flaws that learners had showed in previous questions, such as the sketching of triangles.

In the most important task-based activity which was task-based activity 2 which just had congruency-based proof problems, the researcher picked up that all 22 learners could solve question 1a, 1b, 1c, 1d and 2 with relative ease, however 11 learners did not even attempt questions 3-5 which was a major concern for the researcher. The researcher picked up that if learners did not know how to solve a particular problem, they would not do it at all. The researcher picked up that learner attitude could be the reason as to why they did not attempt these problems and needs to be addressed by the teacher. The encouraging thing was that if learners attempted the problem, in most cases they would get the answer correct. Learners were comfortable with using the two-column proof method in order to answer all problems pertaining to the congruency-based proof problems of task-based activity 2.

The researcher first wanted each learner to work on their own because mostly with group work there is always one who depend on others. When it came to the group discussions and learners had differences, one or two in the group would suggest making a comparison to check if the answers were right. This put many learners at ease because when they were uncertain, they could use the comparison between sketches and solutions. In one group a few learners just left out the answer when they did not understand. During group discussions, they did not participate and copied down answers from the others. This is what happens frequently in most classes I have been involved in. When learners do not understand, they do not tell the teacher or ask for assistance. Consequently, when they get home, they are lost and do not try to do their homework and hence during corrections and feedback they write down answers without understanding the work.

The researcher also discovered that as teachers we assume that there are certain things learners should know when they are in grade 9. For example, we assume that learners should know by this stage on how to sketch triangles, when this is a gap in knowledge that learners have because it was not properly addressed in earlier grades. It is this gap in knowledge that could lead to the depletion of learner confidence in proving congruency-based proof problems. The researcher encourages all teachers and learners to use videos in order to build learner confidence when it comes to solving congruency-based proof problems.

There were five main findings in this study:

- Firstly, learners who struggled to sketch the triangles and who could not state which triangles were congruent or not at the beginning, ultimately struggled to solve the congruency-based proof problems. This highlights the importance of prior knowledge of congruency that learners must have before doing proofs and it is the teacher's responsibility to make sure that learners get the basics right and have prior knowledge of the topic.
- Secondly, with regards to the proof writing competencies, learners who successfully solved the congruency-based proof problems displayed all 7 proof writing competencies and could apply themselves well even though the problems increased with difficulty, the more problems they solved the greater confidence they had. From making conjectures to displaying a well-structured chain of reasoning, it was evident that the web based learning environment helped learners to actually display all 7 proof writing competencies. Based on learner responses, the learners found the videos to be really helpful when understanding how to solve congruency based proof problems.
- Thirdly, the extent to which learners made mathematical deductions in a web based learning environment by using congruency-based proof problems was shown clearly through their understanding of the two-column proof method, which was fundamental. Several learners associated the correct reason with the correct statement, which was encouraging for the researcher. Learners found the two column proof method to be fairly easy to understand and useful when solving congruency based proof problems.
- Fourthly, the researcher found that a web-based learning environment is ideal for solving congruency-based proofs as it visually stimulates learners and thus caters for learners of various learning styles. Visual learning helps to stimulate an environment that is conducive for effective mathematics teaching and learning.

- Lastly, it was found that learner confidence is of vital importance when solving congruency-based proofs, it is vital for the teacher to build learner confidence, scaffolded worksheets and web-based learning could help with this, by making lessons more learner friendly. The more congruency-based proof problems there were, the more learners grew in confidence. Teachers should create an environment conducive for learners to solve congruency- based proof problems, in doing so it will lead to better results.

5.5. Limitations of the research

The were five major limitations of this study, firstly the researcher could not present all of the data due to the poor quality of handwriting of certain learners. Some learners have learning disabilities, which makes it difficult for one to view. Another limitation was that most learners have high levels of mathematical competency, which leads to lack of consistency in the results which was produced from this study. Thirdly time management was an issue, due to Coronavirus pandemic, a lot of learners were disadvantaged because not all learners have access to the internet and there is a limitation with face-to-face learning due to the pandemic. Fourthly, a limitation was that the sample size was small, and the researcher was limited on exploring learner differences in understanding congruency based proofs. Lastly lack of learner prior knowledge of the topic of congruency-based proofs was another limitation as it was not taught extensively in earlier grades.

5.5. Conclusion

In the beginning of the research, it was stated clearly that one of the major issues that learners encountered was their geometrical reasoning skills in mathematics, which was not written appropriately as one would expect. Web based learning was used so that learners could understand clearly what the content of the study was all about. This research revealed that some learners lacked understanding with regards to congruency-based proofs. It is important for learners to gain the correct understanding of concepts in the beginning. Learners were not aware of their errors and kept on making the same procedural errors, such as associating the wrong proof with the correct theorem. Learners also lacked conceptual knowledge and therefore could not check their solutions. The results of the study basically revealed that there are gaps in knowledge with regards congruency-based proof problems of grade 9 learners which could have been addressed in earlier grades, this could be due to learners having limited prior knowledge of the topic from previous grades and not having exposure to web-based learning. Therefore, it is important to use web-based learning during a lesson in order to further enhance learners understanding of mathematical concepts. The role of teachers when it comes to using web based learning is critical, because it is the teachers whom facilitate the teaching and learning processes. With relation to the research questions, it was clearly evident from the results of the data that web-based learning allowed the grade 9 learners to use the 7 dimensions of proof writing competencies and also most grade 9 learners could make mathematical deductions by using congruency-based proofs. In order to improve learner performance in this particular topic, the researcher suggests that it is imperative that web-based learning be used at grade 8 level already in order to teach the topic of congruency. If this is done it would help learners build their prior knowledge of congruency much better, thus making it easier for them to understand how to solve congruency-based proof problems in grade 9. I recommend that research be done on importance of learner's prior knowledge of congruency before attempting to solve congruency-based proofs, if this is done it could help remove many misconceptions with regards to congruency-based proofs that grade 9 learners have with the topic under investigation. Further research can also be done on how web-based learning can help improve the quality of teaching geometry at both primary and high school levels and why it is important to use web-based learning pre grade 9 level to further enhance the teaching and learning of congruency at grade 9 level. It is also recommended that research has to be done on the link between sketching triangles and congruency-based proofs, which was brought to light in this study. I personally believe that web-based learning can help remove many misconceptions that grade 9 learners have with regards to congruency-based proofs due to it improving the quality of teaching and learning of the topic of congruency-based proofs. Also, I firmly recommend that web-based learning be used in every mathematics classroom, if possible, especially in South African schools, because there is a need for a change in the right direction when it comes to the teaching and learning of mathematics. The researcher hopes that this research has contributed positively towards mathematical literature and mathematics in general.

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APPENDIX 1: TASK BASED ACTIVITY 1 - WORKSHEET

CONGRUENCY INVESTIGATION TASK

Congruent shapes are shapes which are identical in all respects. For example, in the case of two triangles, we will have the corresponding sides and angles equal.

To determine whether figures are congruent, all sides and angles must be measured and compared. The same would apply for congruent triangles.

We are now going, by accurate constructions, try to find out what is the minimum conditions needed to have congruent triangles. Each time use only the information provided. You will now determine what sets of measurements will give only one possible triangle. Use a ruler, compass and protractor to construct the following triangles.

Instructions:

1) Write your name and surname in the spaces above.
2) All questions must be answered on the question paper.
3) Show all calculations.
4) You may use an approved calculator.
5) You must do your own work.
6) Check your answers.
7) Show the units of measurement where applicable.
8) All answers must be rounded to one decimal place unless stated otherwise.

NAME OF LEARNER: ……………………………………………………………

Question 1: Minimum dimensions are given each time:

1.1) If three sides are given: side, side, side (S,S,S): (4)

ΔDEF with DE = 7 cm, DF = 6 cm and EF = 5 cm.

1.2) If three angles are given: angle, angle, angle (∠,∠,∠): (4)

ΔABC with ∠A = 80°, ∠B = 60° and ∠C = 40°.

1.3) If one side and two angles are given: side, angle, angle (S, ∠,∠): (4)

∆GHI with GH = 8 cm, ∠G = 60° and ∠H = 30°.

1.4) If two sides and an including angle are given: side, angle, side (S,∠,S):

∆JKL with JK = 9 cm, ∠K = 130° and KL = 7 cm. (4)

1.5) If two sides and an angle which are not included are given: side, side, and angle

(S, S, ∠): ∆MNP with MN = 10 cm, ∠M = 50° and PN = 8 cm.

(4)

1.6) If a right angle, the hypotenuse and another side is given: (90°, hypotenuse, S):

ΔTRS with TR $\perp$ RS, RS = 7 cm and TS = 8 cm. (4)

1.7) Compare your triangles with the triangles of three of your class mates. Which of your triangles are:

1.7.1 congruent to theirs?

----

(2)

1.7.2 not congruent?

----

(2)

[28]

Question 2:

Complete the table below. Write down if congruent triangles can be constructed if the following conditions are given:

Conditions	Congruent (Yes/
3 sides (SSS)	
2 sides (SS)	
3 angles (∠,∠,∠)	
2 angles and a side (∠,∠, S)	
2 sides and an angle that is not situated between the sides (S, S, ∠)	
2 sides and an angle that is situated between the two sides (S, ∠, S)	
A right angle with a hypotenuse and another side (90°HS) or (90°, hypotenuse, side)	

[7]

Question 3

3.1) Name the 4 cases of congruence in triangles:

----

[4]

3.2) Complete: If 2 triangles have equal angles, we may say that the two triangles are:

----

[1]

3. 3) Write down in words the meaning of:

3.3.1) $\Delta ABC \equiv \Delta XYZ$

----

(2)

3.3.2) $\Delta ABC /// \Delta XYZ$

----

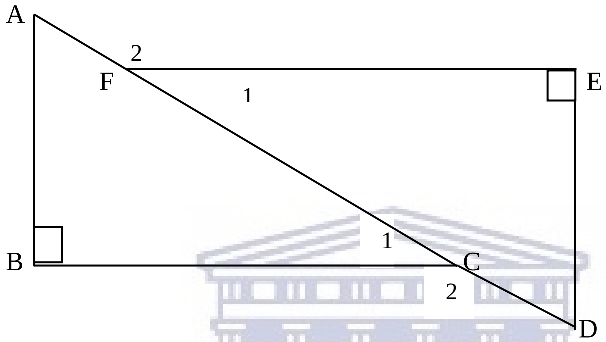
(2)

3.3.3) $RM \parallel EH$

----

(2) [6]

3.4) In the diagram below the following dimensions are given: ∠B = ∠E = 90°, ∠C1 = ∠F1 and AB = ED.



3.4.1) Prove: Δ ABC ≡ Δ DEF. (4)

STATEMENT	REASON

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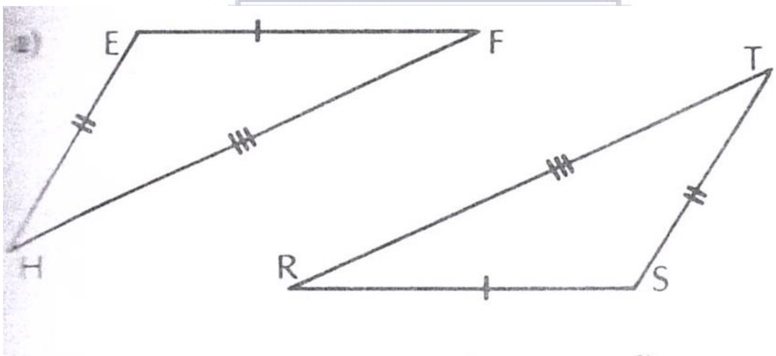
APPENDIX 2: TASK BASED ACTIVITY 2 - WORKSHEET

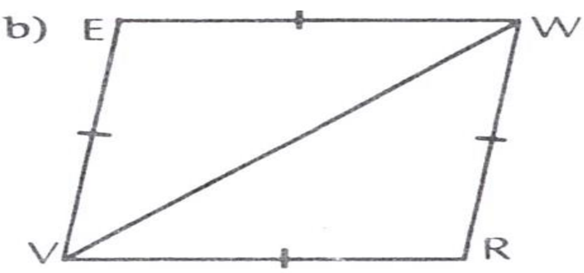
SOLVING AND PROVING CONGRUENCY RIDERS

Name & Surname: ______________________________ **Date:**________________

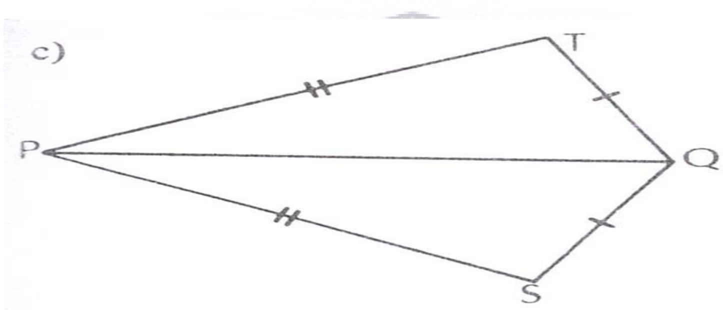
<u>Question 1: Prove that the following triangles are congruent:</u>



STATEMENT	REASON

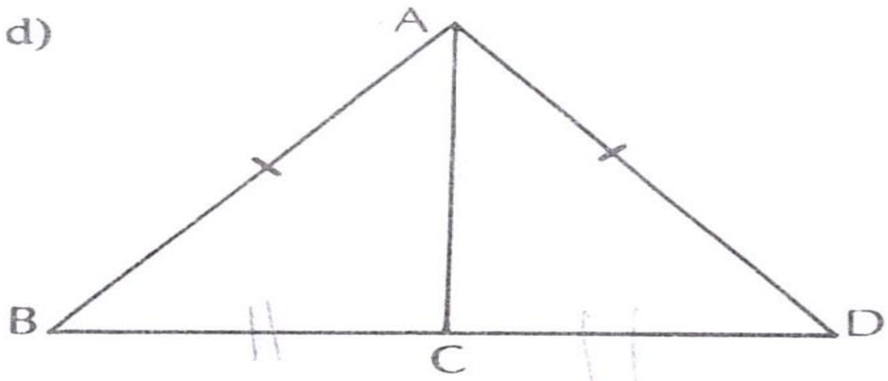


STATEMENT	REASON



STATEMENT	REASON

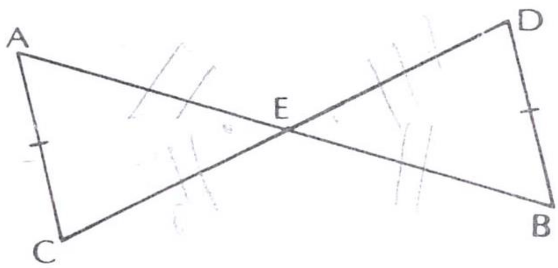
d)



C is the midpoint of BD.
Prove Δ ABC $\equiv$ Δ ADC.

STATEMENT	REASON

2.

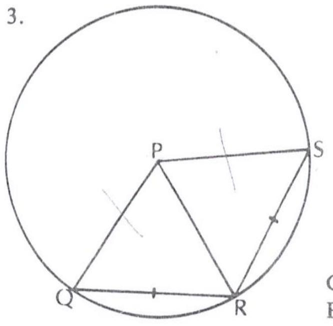


AB and CD bisect each other at E.

Prove Δ ACE $\equiv$ Δ EDB.

STATEMENT	REASON

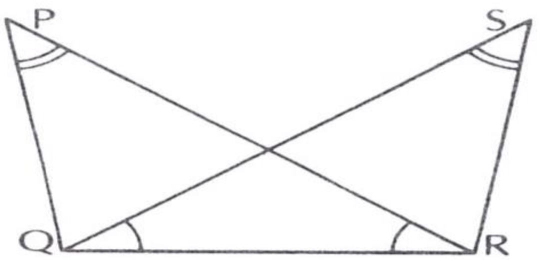
3.



Given a circle with centre P and QR = RS
Prove $\Delta PQR \equiv \Delta PRS$.

STATEMENT	REASON

4.

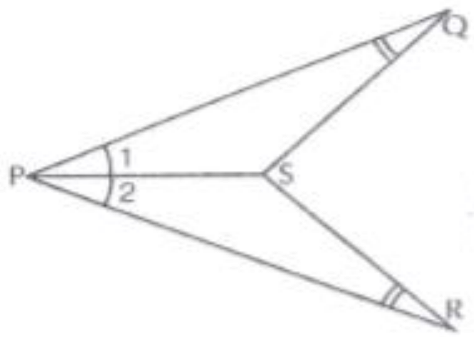


Prove $\Delta PQR \equiv \Delta SQR$

STATEMENT	REASON

.

5.



Prove that PQ = PR (Hint, first prove ΔQPS ≡ ΔPRS)

STATEMENT	REASON

APPENDIX 3: OBSERVATION SCHEDULE

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OBSERVATION SCHEDULE

TEACHER OBSERVATION

NAME: ______________________________

DATE: ______________________________

LEARNERS INTERACTION WITH VIDEOS AND CONGRUENCY BASED PROOF PROBLEMS	
Learner Actions	**Comment**
1. Learners understood the videos based on congruency-based proofs	
2. Learners could use the videos to solve congruency based proof problems	
3. Learners were able to apply what was learnt via the videos.	
4. Learners were interest in congruency-based proof problems	
5.. Teacher could interact with learners and use the videos as a guiding tool.	

6. Learners could interact with each other and discuss the congruency- based proof problems	
7. The videos could promote a learning environment conducive for learners to solve congruency- based proof problems.	
8. Learners could use the methods provided to solve congruency- based proof problems effectively	
9. Videos were interactive and presentable for learners to watch.	
10.Congruency-based proof problems could challenge learners thinking	
11. General	

APPENDIX 4: FOCUS GROUP INTERVIEW QUESTIONS

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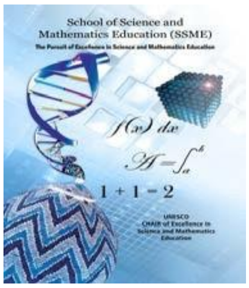

INTERVIEW SCHEDULE

Name of school ______________________

Gender _________________Grade ______________

Interviewer __________________

Interviewee _________________

Date ___________________

Question 1

1.1. How did you find the sketching of triangles?
1.2. What was the approach that you used to sketch the triangles?
1.3. How did you find comparing congruent triangles and non-congruent triangles?

Question 2

2.1 How did you find distinguishing between the four cases of congruency?
2.2 How did you find distinguishing between the two cases of similarity?

Question 3

3.1 How did you cope with distinguishing between congruency, similarity and parallel lines?

3.2 How did you find the congruency-based proof question?
3.3 What approach did you use to complete the congruency-based proof problem?

Question 4

4.1 Did you understand the videos based on congruency-based proofs?
4.2 How did the videos help you solve the various congruency-based proof problems?
4.3 How did the videos help you to find out which sides and which angles were equal between the triangles?

Question 5

5.1 Did you have a better understanding of congruency-based proofs after watching the videos?
5.2 Would you recommend that other learners watch videos to solve congruency-based proofs?
5.3 Will you use videos to solve congruency-based proof problems in future?

APPENDIX 5: ETHICAL CLEARNACE CERTIFICATE

UNIVERSITY *of the* WESTERN CAPE

YEARS *of hope, action & knowledge*

25 January 2022

Mr T Chetty
School of Science and Mathematics
Faculty of Education

HSSREC Reference Number: HS21/10/52

Project Title: Exploring grade 9 mathematics learners learning of congruency- based proofs in geometry via a web-based learning system.

Approval Period: **25 January 2022 – 25 January 2025**

I hereby certify that the Humanities and Social Science Research Ethics Committee of the University of the Western Cape approved the methodology, and amendments to the ethics of the above mentioned research project.

Any amendments, extension or other modifications to the protocol must be submitted to the Ethics Committee for approval.

Please remember to submit a progress report by 30 November each year for the duration of the project.

For permission to conduct research using student and/or staff data or to distribute research surveys/questionnaires please apply via:
https://sites.google.com/uwc.ac.za/permissionresearch/home

The permission letter must then be submitted to HSSREC for record keeping purposes.

The Committee must be informed of any serious adverse events and/or termination of the study.

Ms Patricia Josias
Research Ethics Committee Officer
University of the Western Cape

Director: Research Development
University of the Western Cape
Private Bag X 17
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Republic of South Africa
Tel: +27 21 959 4111
Email: research-ethics@uwc.ac.za

NHREC Registration Number: HSSREC-130416-049

APPENDIX 6: WCED RESEARCH APPROVAL LETTER

Western Cape Government
Education

Directorate: Research

meshack.kanzi@westerncape.gov.za
Tel: +27 021 467 2350
Fax: 086 590 2282
Private Bag x9114, Cape Town, 8000
wced.wcape.gov.za

REFERENCE: 20220303-353
ENQUIRIES: Mr M Kanzi

Mr Taariq Chetty
20 Octupus Crescent
Strandfontein
7798

Dear Mr Taariq Chetty,

RESEARCH PROPOSAL: EXPLORING GRADE 9 MATHEMATICS LEARNERS' LEARNING OF CONGRUENCY-BASED PROOFS IN GEOMETRY VIA A WEB-BASED LEARNING SYSTEM.

Your application to conduct the above-mentioned research in schools in the Western Cape has been approved subject to the following conditions:

1. Principals, educators and learners are under no obligation to assist you in your investigation.
2. Principals, educators, learners and schools should not be identifiable in any way from the results of the investigation.
3. You make all the arrangements concerning your investigation.
4. Educators' programmes are not to be interrupted.
5. The Study is to be conducted from **3 March 2022 till 31 May 2022.**
6. No research can be conducted during the fourth term as schools are preparing and finalizing syllabi for examinations (October to December).
7. Should you wish to extend the period of your survey, please contact Mr M Kanzi at the contact numbers above quoting the reference number.
8. A photocopy of this letter is submitted to the principal where the intended research is to be conducted.
9. Your research will be limited to the list of schools as forwarded to the Western Cape Education Department.
10. A brief summary of the content, findings and recommendations is provided to the Director: Research Services.
11. The Department receives a copy of the completed report/dissertation/thesis addressed to:

The Director: Research Services
Western Cape Education Department
Private Bag X9114
CAPE TOWN
8000

We wish you success in your research.

Kind regards,
Meshack Kanzi
Directorate: Research
DATE: 3 March 2022

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