An investigation of strategies used by grade 4 learners to solve measurement type word problems

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Abstract

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The aim of this study was to investigate and highlight the problem-solving strategies used by grade 4 learners when solving measurement word problems. Furthermore, it aimed at revealing how learners go about using these strategies to arrive at meaningful solutions. The findings of this study would strengthen the argument that our current curriculum needs to place greater emphasis on problem solving. It would also enhance the claim that learners need to be exposed to a greater variety of problem-solving strategies to allow them to select the most suitable strategies when solving word problems. The study used a case study research design and a mixed methods approach was adopted. The sample of the study was a class of 42 grade 4 learners at a primary school in Cape Town. The instruments of the study were an observation protocol, measurement word problem activities and focus group interviews. The results revealed that the following singular strategies were used by the grade 4 learners: addition, multiplication, making a drawing, grouping, division, subtraction, logical reasoning, guessing, and making a table.

The two most frequently used problem-solving strategies were arithmetic calculations and making drawings. The study also revealed the use of various combination strategies by many learners, such as arithmetic strategies being used in combination with making drawings. Another finding was that, although many learners used the same strategies, the manners in which they carried out those strategies were very different. Drawings ranged from circles, to sticks, to reconstructing a given drawing in a more simplified form. There was also a range of different methods in which arithmetic calculations were carried out, with many learners using multiple operations in their solutions. The study also found that the stage of problem solving as mentioned by Polya (1957) which learners had most difficulty with was carrying out the plan. Learners often made errors in arithmetic calculations which resulted in incorrect answers.

Keywords:

Problem solving, mathematics education, teaching methods, incompetent, strategies, measurement word problems, Curriculum and Assessment Policy Statement

Declaration

I declare that: *An investigation of strategies used by grade 4 learners to solve measurement type word problems* is my own work, that it has not been submitted for any degree or examination in any other university, and that all the sources I have used or quoted have been indicated and acknowledged by complete references.

Full name Shabeer Rawoot   Date 31 October 2022

Signed. Rawoot

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CHAPTER 1: INTRODUCTION AND BACKGROUND

1.1. Introduction

The state of education in South Africa has deteriorated rapidly in recent years and change is desperately required (Mouton, Louw & Strydom, 2013). Tsanwani, Harding, Engelbrecht, and Maree (2014) concur by adding that South Africa ranks amongst the lowest countries in the world when it comes to learner performance in Mathematics. This was evident in 2011 when South Africa participated in the Trends in Mathematics and Science Study (TIMMS), which consisted of 21 middle-income countries. When the results were released, South Africa had the lowest performance of all the 21 participating countries. Msimango and Luneta (2015) add that South African primary school learners were the worst performing in mathematics and sciences during TIMMS 2011.

In 2006 South Africa participated in the Progress in International Reading Literacy Study (PIRLS). This study assessed learners' reading ability as well as their attitudes towards reading. Reading and word problems have a close relationship in that a learner must be able to read a problem with comprehension to understand what the problem requires of them. Even though the South African learners who participated were grade 5 and the learners from the other countries were all grade 4, South Africa yielded the worst results out of all 38 countries involved in PIRLS 2006 (Pooran, 2011). These results reflect that South African learners' reading abilities are well below standard in comparison to other countries, despite the South African learners being a grade higher in the study. This indicates the magnitude of work that needs to be done to improve the reading abilities of South African learners.

The researcher suggests that many learners who underperform in school can do Mathematics. The issue, however, is that they are almost forced to solve problems using the methods in the textbook and taught by the teacher. Many learners have different methods and strategies which they can use to solve Mathematical problems, but there is no provision made within the classroom environment to allow them to use methods which are not taught in class. Thus, if learners are allowed to explore problems and using the methods which come naturally to each one of them individually, they may have greater success in solving the problems. This could change the way Mathematics is taught and learnt at school; hence the researcher proposes that problem solving becomes a more integral and important aspect within the teaching and learning of Mathematics.

By allowing learners to select from a variety of strategies, problem solving aids learners in preparing them for solving real-world problems. This characteristic of problem-solving forms an important partnership with one of the chief aims of Mathematics within the currently curriculum, showing learners how Mathematics is related to everyday life and basing problems on real-world experiences to equip them with the necessary mental skills to tackle out-of-school problems. Hence, problem solving teaches learners that just as there are many different strategies which can be used to solve one problem in Mathematics, there are also many different paths to take to solve daily life problems. Swanson, Jerman and Zheng (2008) agree and add that word problems provide the platform on which learners can use the formal knowledge they have acquired and apply it to real-world scenarios. This is echoed by Peranginangin and Surya (2017) who mention that by being able to do Mathematics, children will also be able to solve real life problems.

The topic of measurement is one which is very closely related to the everyday lives of learners. Units of measurement are also found in problems across different content areas within the curriculum. Lubin, Houde and de Neys (2015) add that throughout the curriculum, learners are required to solve arithmetic word problems. This makes it an important skill which learners are required to master.

The researcher adds that measurement involves not only addition and subtraction, but also multiplication and division. It requires learners to be able to comprehend the relationships between different operations, as well as between different units of measurement. Measurement also involves being able to identify certain rules or ratios for converting between units of measurement. For example, there are 10mm in 1cm, 100cm in 1m, and 1000m in 1km.

1.2. Mathematical problem

Historically, there has been a long-standing clash regarding the definition of a mathematical problem (Xenofontos, 2014). One belief is that a mathematical problem is a routine exercise which is used to for the revision or consolidation of new mathematical techniques. On the contrary, other scholars believe that a mathematical problem is a non-routine exercise due to its complex nature (Xenofontos, 2014). A mathematical problem poses an objective in which the solution is not obvious. This study will respect both definitions as mathematical problems can be both routine as well as non-routine.

Tambychik and Meera (2010) report that problem solving is made up of two key aspects. Firstly, using the information in the problem and forming mathematical sentences from it.

Secondly, understanding which steps and operations are required to solve the problem and then executing these steps to attain the solution.

George Polya is regarded as the father of mathematical problem solving. Polya (1957) saw the concept of problem solving as an art which could be both taught and learned given the correct guidelines. He went further by outlining four steps that any problem solver should follow. Firstly, the problem solver should understand the problem and be aware of what the problem requires the problem solver to do After becoming familiar with the given information, the problem solver must then articulate a strategy or a set of strategies which can be used to solve the problem. The plan is then executed by making use of specific strategies and through altering and modifying strategies, a solution to the problem is eventually found. Lastly, the solution then is reflected upon by looking back at all the steps taken during the problem-solving process to test its validity (Polya, 1957).

The four-step framework will be utilized in this study to ascertain which problem-solving strategies grade 4 learners use when solving word problems involving measurement, as well as how they carry out these strategies when solving the problems. Using the "devise a plan" and "carry out the plan" steps, the researcher will be given insight into the various cognitive processes which learners go through leading up to selecting a solving strategy. Secondly, the researcher will be able to observe how learners utilize their selected solving strategies, as well as any common practices or misconceptions found in the solutions.

1.3. Word problems

According to Chapman (2003), the definition of a word problem is not always obvious. There is a school of thought that believes that they include problems which are usually represented symbolically, being expressed in words. For example, find the difference between 25 and 13. The other school of thought believes that word problems refer solely to problems which are written in the form of stories. In other words, they are verbal explanations of problem scenarios. Their solutions can only be attained by applying mathematical procedures using the numerical or algebraic data given in the problem (Chapman, 2003).

For this study, the concept of word problem solving will refer to reading and interpreting a particular word problem, then devising a strategy or method to come up with a solution to the problem. This process includes showing all the steps as well as the errors the learners went through on their journey to solving the problem.

1.4. The importance of measurement

According to Clements, Barret, and Sarama (2017), acquiring knowledge of quantity as well as quantifying strategies are essential for learners. Geometric measurement, to be specific, enables learners to understand the relationships between number and geometry. This is done through reasoning about quantity, which happens continuously in the curriculum as learners progress through their schooling years. Drake (2014) adds that the topic of measurement is very practical, and it allows learners to solve problems which interest them.

McDonough and Cheeseman (2014) mention that despite the importance of measurement as an element of mathematics education, it remains a topic in which children perform inadequately around the world. Therefore, there is a need for designing assessments which better demonstrate the learning of children. By being able to identify the areas within their thinking where they need help, appropriate intervention and assessments can then be constructed in order to help learners become proficient in measurement.

Measurement also forms a central part of scientific inquiry. Therefore, it is imperative to start developing an understanding of the measurement of physical quantities as early as possible (Clements et al., 2017). Drake (2014) emphasizes the significance of learning linear measurement. In addition to being one of the most common types of measurement, linear measurement is also used in various other facets of mathematics as well as science. Linear measurement forms the foundation upon which scales in measuring instruments are constructed. This reinforces the importance of learners learning and understanding measurement.

In the curriculum, measurement is initially introduced to learners by assisting them in perceiving as well as comparing the magnitude which needs to be measured (Montoro, Aguayo-Arriaga, and Flores, 2021). Learners also need to learn to use measuring instruments and recognise that measurements are always approximate. Thus, they will also develop different estimation skills. Magnitudes, in science, are seen as properties of matter and are categorised as either specific or general. Being proficient in measurement and its related processes is therefore also essential to perform experiments and conduct scientific research. McDonough and Cheeseman (2014) add that when children learn to measure, they also develop understandings of the core aspects of measurement attributes. These include making comparisons, assigning appropriate units, as well as the importance of converting to identical units.

Even before attending formal schooling, children begin to devise ideas of mass from as early as twelve months. Furthermore, McDonough and Cheeseman (2014) mention that there is evidence which indicates that children can exhibit awareness of mass between the ages of four to six years. At years six to eight they are able to distinguish between heavy and light objects, as well as to comprehend the relationship between the size of a unit and the number of units required to determine the mass of an object. Clements et al., (2017) add that children make comparisons between different magnitudes either instantly, or they identify patterns over a prolonged period. This is achieved by making quantifications of their observations and experiences.

1.5. Background

Since the inception of democracy in 1994, South Africa has undergone numerous changes with regards to its curriculum approaches in attempts to improve the numeracy and literacy performances of its learners (Chisholm, 2003). When Outcomes-Based Education (OBE) was implemented in 1998, it was aimed at serving as a curriculum approach which promoted democratic education to further show the country's efforts to achieve transformation. The problem which OBE presented was that instead of focussing on developing the conceptual understanding of learners and teaching them the required skills to solve problems, it placed great emphasis on requiring learners to achieve specific outcomes (Mouton et. al,.2013). This resulted in teachers toiling to get their learners to achieve all the required outcomes of OBE, meaning learners were unable to master many basic skills and lacked fundamental knowledge which ultimately called for a new curriculum approach. (Chisholm, 2003).

In 2002 the Revised National Curriculum Statement (RNCS) was introduced. This approach was meant to replace as well as improve on the shortcomings of OBE. However, it failed to yield improved results particularly in language subjects and Mathematics (Mouton et al., 2013).

South Africa's current curriculum approach, the Curriculum and Assessment Policy Statement (CAPS), had been earmarked as the approach which would rectify the errors of past approaches and finally produce improved learner performances particularly in core subjects such as Mathematics and Home Languages (Mouton et al., 2013). Unfortunately, there has still not been substantial improvement and South African education continues to be in a state of decline. (Maree, Aldous, Hattingh, Swanepoel and van der Londe, 2006)

The researcher is of the view that if effective changes are to be made to improve learner performance in Mathematics, then we first need to analyse and understand how learners think as well as what their conceptions and misconceptions of different mathematical content are. the researcher wants to focus particularly on the topic of problem solving of measurement word problems. Using the four-step framework of Polya will enable the researcher to be able to analyse what thinking patterns learners experience during problem solving as well as how they select and carry out their solving strategies. Doing so will provide insight into how learners think and understanding their cognitive processes will allow us to be able to identify precisely where and why learners are struggling. As a result, these answers will allow us to then develop appropriate remediation programs and tools to help learners overcome specific misconceptions, to be able to select appropriate solving strategies and be able to carry out these strategies coherently.

According to Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier, and Wearne (1996), there is a large gap within the schooling curriculum between the acquisition of knowledge and the application thereof. To resolve this concern, many schools have adopted the problem-based learning approach. This learning and teaching model emphasises that lessons begin with problems. Learners are exposed to sets of problems and utilise their own conceptions and skills to make meaning of the problems and by doing so, find their own ways to come up with solutions. By first giving learners the problems, it allows them to form their own understandings and construct their own knowledge, keeping them engaged throughout the learning process and creating a learning environment that is less instructive than the more traditional method of exposing learners to antique methods and almost forcing them to master only those methods (Hiebert et al., 1996).

These arguments further back up the importance of this study, which will allow learners to develop their own conceptions of measurement word problems without being confined to the methods prescribed by the teacher. Allowing learners to devise and carry out their own strategies will enable us to observe how learners interpret each problem and how they go about solving each problem. Furthermore, conducting focus group interviews will reveal the reasons learners chose their specific strategies as well as what thinking steps they were following during the solving process.

Maree et al., (2006) reported on a study in which a vast number of grade 6 learners within the Western Cape failed to carry out grade 4 level numeracy skills. Out of the 35 000 learners who wrote the test, only 15,6 % of them passed. These statistics reinforce the claim that South

African learners are underperforming in Mathematics and that changes are urgently required. The researcher feels that these changes must start at the primary school level.

As a grade 4 teacher with experience in the other intermediate phase grades, the researcher feels that solving word problems is one of the biggest obstacles to both learners as well as teachers who often do not comprehend the problem. A combination of these two factors as well as various other variables contribute to learners performing very poorly during the word problems sections in assessments and class work. In the Platinum Mathematics textbook, word problems are found at the end of each topic and are used as a tool to determine whether learners can apply the knowledge they have acquired in the topic to problems which are still on the topic but are now posed and described within a real-life scenario.

Furthermore, as a Mathematics teacher in the intermediate phase the researcher has also experienced and observed several trends. Teachers have the tendency to only use the prescribed method in the textbook when showing learners how to go about solving the word problem. They do not make provision in their lessons to accommodate for new ideas and methods, as many teachers do not realise that there are numerous methods that can be used to solve a single problem, with each method being relevant and can provide learners with the solution to the problem. Too many teachers view Mathematics as a subject which only has a single correct answer to a problem, and that each problem only has one method which can be used to solve I it. (Hiebert et al., 1996). Through their teaching, they then impose these views and patterns of thought unto their learners too.

1.6. Rationale

Cummins, Kintsch, Reusser and Weimer (1988) remark that word problems have always been viewed as being tough to solve because they are non-routine in nature, and they do not explicitly state which mathematics needs to be used to solve them. Furthermore, word problems can be stated in such a vast variety of ways that it is difficult for learners to formulate any pattern or steps which they can apply to all word problems. The difficulty of a word problem is also greatly influenced by the word choice in the problem, as some word problems use very intricate language which confuses many learners. Nortvedt (2008) adds that even some teachers struggle to comprehend and interpret word problems.

However, Hosseini, Hajishirzi, Etzioni and Kushman (2014) disagree, stating that word problems have a very distinct pattern. They start off by providing a description of a particular real-world scenario, which is followed by one or more specific details, and concludes with a quantitative question. It is suggested that learners identify what is required of them to do by finding the verb in the problem. These verbs can be put into different categories which help learners to determine which operations they need to do to solve the problem.

Using the problem-solving approach enables learners to choose from a luxury of different strategies when they are required to solve a particular problem or set of problems. As opposed to the traditional prescribed methods found in textbooks and the curriculum, the problem-solving approach will provide a better platform for learners who generally find it challenging to solve problems. More specifically, these learners will have the freedom of using solving techniques which they are more familiar with or to choose a different strategy from the one which is found in their textbooks, and which usually causes them great difficulty. Hiebert et al., (1996) concur, adding that instead of basing our curriculum on learning skills, we should allow for learners to solve problems using their own understanding and interpretations of the problems. Allowing learners to engage in problem solving means they can then explore the problem, find various ways to solve the problem and reflect on their work by finding and rectifying their own mistakes within the problem-solving process.

Furthermore, the variety of problem-solving techniques is not only restricted to those which the teacher exposes to the learners to, but it also includes any relevant methods which the learners come up with independently or traditional methods which were engrained in them by parents or other relatives. Thus, the problem-solving approach endorses the Constructivist learning approach as it enables learners to draw on their prior knowledge and using this knowledge as a platform on which they can build on expand to construct new knowledge (Lesh & Doerr, 2003).

Using the problem-solving approach for teaching and learning also ensures that a teacher within the South African context fulfils one of the key aspects of what it means to be a teacher in this country, catering for differentiated learning. It is important to acknowledge that learners are on different levels of cognition and the problem-solving approach falls in line with this by allowing learners to choose the problem-solving strategy which best suits their individual cognitive levels, unlike conventional practices which almost forces them to utilize a single method and restrict them.

1.7. Statement of the Research Problem

The topic of problem solving is frequently either overlooked or not focussed on sufficiently within the current teaching and learning curriculum (Thompson, 1985). Many learners are unable to carry out mathematics problems using the methods prescribed in their textbooks and taught to them by their teachers and they are then graded as underperforming. However, many of these learners have methods and ideas of their own which may also lead them to successfully solving a given problem if they were allowed the freedom of not following conventional textbook methods. Mathematics must be taught in such a way in which it shows learners that there are various routes which can be taken to arrive at the same solution. The topic of measurement is particularly stringent in grade 4 as it is taught in such a way that learners can only use one method to convert or work with units of measurement (Hurrell, 2015). However, a problem-solving approach will allow learners to use their own initiative and approach the problem in a more personal manner, using methods which they feel comfortable with individually. This is because learners possess background knowledge which they learn through family members, their cultures, and other sources. However, school does not acknowledge their background knowledge or even accommodate for it. the researcher therefore argues that it is the responsibility of the teacher to create opportunities within lessons for learners to share and make use of their background knowledge. This will allow learners to make their own meaning of the problem and by using the method of their choice their solution will be more meaningful to them. The teacher should therefore also facilitate learners when they select and carry out their own solving strategies. This should be done by simply guiding them and harnessing their skills so that they can carry out their solving strategies to the best of their abilities. This study will therefore investigate the problem-solving strategies grade 4 learners use when solving measurement word problems.

1.8. Research Questions and Objectives

This study will identify the methods used by grade 4 learners when measurement word problems at a primary school in Cape Town. It will, thus, aim to answer the following research question: What strategies do grade 4 learners use when solving problems involving measurement? Furthermore, the following subsidiary question will also be answered: How do learners use their strategies when solving problems involving measurement?

1.9. Significance of the study

This study attempts to make a valuable contribution to Mathematics education by shedding some light regarding which problem-solving strategies grade 4 learners use when solving measurement word problems. Moreover, it will also provide more information as to precisely how learners go about using the strategies they choose when solving these problems. The aforementioned information will serve as a platform with regards to which levels learners are on in terms of their problem-solving skills, thus giving teachers a starting point as to which level to pitch their lessons at as well as the range of new problem-solving strategies they need to introduce to learners. In a South African context, research into this topic is very limited which further emphasises the significance of this study (Jojo, 2019). The study can be used as a platform upon which further future studies can then be built on.

The current curriculum places great emphasis on the concept of solving word problems (DoE, 2011). However, the textbooks do not show learners or teachers the various and diverse set of methods which can be used to solve these word problems. Many of the word problems simply have the final answer in the teacher's memorandum and during assessments the Formal Assessment Tasks (FATs) memorandum simply has only the final answer accompanied by the common statement of "any other correct method." This is doing an injustice towards word problem solving as these problems are meant to allow learners to not only keep using the column and expansion methods to add and subtract in various topics of mathematics such as mass and measurement, but also to expose them to other relevant strategies. It may be argued that these problems allow learners to use their own initiative, but their performances in this section of Mathematics can be further enhanced if the teacher first makes them aware of and introduces them to a wide variety of different strategies and steps they can choose from when solving these word problems.

There are many studies which have been carried out which analyse different aspects of problem solving and solving word problems in mathematics. However, there is not as much literature which focuses specifically on word problems involving measurement. Therefore, this study could be highly significant in that it could add valuable knowledge to a topic in mathematics as the published work on problem solving appears to be mainly general in nature. This further may further strengthen the argument to conduct research on this important part of mathematics. The next chapter will focus on previous studies which have been conducted on problem solving and problem-solving strategies.

1.10. Outline of chapters

Chapter 1

In this chapter, definitions for the following terms which are used throughout the study are shown: mathematical problem, word problems, and problem solving. An outline of the problem-solving framework of Polya (1957) is then discussed as it formed an integral aspect of the theoretical framework for this study. Furthermore, the background to the study is explained to establish the importance of changing how problem solving is being taught. The argument is then posed that allowing learners to solve measurement type word problems independently will enable them to form their own conceptions of the problems. Thereafter, the research questions are stated: What strategies do grade 4 learners use when solving measurement type word problems? How do learners carry out their strategies when solving problems involving measurement? These questions then lead into the statement of the research problem and the significance of the study.

Chapter 2

This chapter begins by providing a summary of related research which has been done on problem solving and word problems. The theoretical frameworks for the study are then discussed. This study used two different frameworks as points of reference. The Constructivist Learning Theory, and Polya's Problem Solving Model. Polya's problem solving strategies are then explained in detail, and each strategy is accompanied by an appropriate example to demonstrate how the strategy is carried out when solving a problem using Polya's framework. A summary of research on problem solving strategies is then provided to further enhance the significance of this study.

Chapter 3

Chapter 3 provides a comprehensive description of the research methodology that was undertaken when conducting the study. The exploratory case study research design was adopted for the study, and the mixed methods approach was implemented. The date collection techniques included an observation protocol, document analysis, and an interview protocol. Findings would be recorded and an analysis of all three sets of data would be done to identify patterns, relationships, and to answer the research questions.

Chapter 4

This chapter presents and analyses all the data that was collected and collated throughout the study. The observations of the researcher are discussed, followed by examining the data from the document analysis which shows the strategy each learner used for each problem in the problem-solving activity they completed. Analysis of the focus group interviews is then conducted to understand why learners selected the strategies they did. Furthermore, the interviews also reveal the conceptions learners had of the various problems as well as the feelings they had towards problems they found easy and challenging. After the three sets of data were extensively studied, the research questions could then be answered.

Chapter 5

This chapter aimed to collate all the findings from the study into conclusions which accurately summarized the main themes which emerged. Learners' conceptions of measurement word problems were discussed. This is followed by the problem-solving strategies which were used most frequently, as well as relationships between the chosen strategy and the type of problem. Following this are the concluding remarks regarding how learners carried out their strategies. The chapter concludes by applying the Constructivist Learning Theory and Polya's Problem Solving model to the findings as these are the two theoretical frameworks the study utilized.

CHAPTER 2: LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.1. Introduction

This chapter will cite previous literature and studies which have been published on the topic of problem solving in Mathematics. In addition, it will cite literature on Constructivist Learning Theory, as well as the roles of the teacher and learners within a Constructivist classroom environment. This will be done to find possible trends in previous studies which can be used to inform this study and allow the researcher to make comparisons once data has been collected. It will highlight major findings, trends within the literature as well as the different viewpoints of the various studies. This chapter will also discuss the steps for problem solving according to Polya (1957) and explain some of the different problem-solving strategies found in Mathematics. After reviewing the literature, this chapter will also identify a possible gap which this study will aim to fill.

2.2. Related research on problem solving and word problems

In a study to identify the support structures required by teachers to implement a problem-solving teaching approach, Anderson (2005), set up interviews, surveys as well as workshops with teachers. A general pattern found within the results was that most of the teachers were open to and in favour of using a problem-solving teaching approach. A prominent issue brought up by teachers was they based their teaching methods on the textbooks which they were prescribed with. Furthermore, a general finding was that the teachers are open and willing to teaching using the problem-solving approach if textbooks were adapted to cater of this teaching and learning approach. The researcher feels that if teachers could place greater emphasis on problem solving and create more opportunities for learners to be exposed to different strategies, then it would improve both the teaching and learning processes. In addition, it will also improve the performance of both teachers and learners in Mathematics. Another important finding from the study conducted by Anderson (2005), most of the teachers welcomed the idea of undergoing further training and workshops to equip themselves with the appropriate skills and knowledge to teach using a problem-solving approach.

A study reported by Ollerton (2007) was conducted across four schools in the West Midlands in England. After using a problem-solving approach in the teaching of Mathematics to learners over an extensive period, Ollerton (2007) found that learners began to adopt a specific set of problem-solving strategies and then selected the most appropriate one to solve a given problem. These strategies included describing patterns, making drawings as well as finding all the possibilities.

Nortvedt (2008) set up a study to determine which reading and solving strategies learners utilised when working on a set of word problems. An instrument consisting of a group of 8-word problems was prescribed to 22 grade 8 learners. Learners were first asked to read out the problems aloud and verbally utter out their ideas as to how they would go about solving each problem. They then attempted to solve each problem using any solving strategies and methods of their choice. All of this was done in an interview styled process whereby the teacher took notes on both the verbal section as well as the written part of the study. When reading the problems out aloud and talking about their understanding of and strategies to solve each problem, it was found that majority of the learners tended to reread each problem several times. It was suggested that learners do this to fully conceptualise the problem and to try and identify the key statement in each problem to determine what the problem requires them to do.

A study was conducted by Tshabalala and Ncube (2013) to determine the factors which contribute to poor performance in mathematics. The research instrument was a questionnaire which learners answered. It was found that learners who stayed absent frequently tended to achieve much lower marks during mathematics activities and assessments. Thus, low class attendance contributed to learners performing poorly in mathematics. Another finding was that learners also performed below the passing standards as a result of them not committing all their attention to the teacher during lessons. Learners may, therefore, be contributing negatively to their own development. These findings are backed up by Tachie and Chireshe (2013) who found that learners cited their own lack of punctuality as a reason for them performing poorly in mathematics.

Tshabalala and Ncube (2013) also found that 85% of the learners who took part in their study felt that the teaching methods used by their teachers were very tough for them to understand and make sense of. These findings indicate the need for different teaching methods to be used to teach mathematics. The view of the researcher is that implementing a problem-solving teaching and learning approach would create a learning environment which learners will find more interesting as it will allow them to explore their methods of solving problems. Doing so may result in learners finding strategies they are more comfortable with and bring them more success in solving problems than what the methods in our current curriculum do. This view is echoed by Gakure, Mukuria and Kithae (2013) who mentioned that teachers who are trained and equipped with teaching methods which are more learner-centred will yield better performances and results from learners.

In a study to find the different strategies used by learners when solving geometry problems, Aydogdu and Kesan (2014) found that most of the learners who participated in the study used the strategy of making drawings at least once. This suggests that learners are more comfortable solving a problem when they can visualise the problem. Furthermore, the study also found that after learners were taught a specific problem-solving strategy by the teacher, they tended to adopt the strategy and use it during classroom as well as homework activities.

Lubin, Houde and de Neys (2015) conducted a study to determine the errors made by learners when solving arithmetic word problems. A total of 137 learners from grades 3, 4 and 5 at a school in Paris participated in the study. Each learner used French as their first language. Learners were each given four problems to solve. They were then shown the five-point rating scale. The scale ranged from not being sure at all (0), to being completely sure (4). Therefore, after solving the problems learners would then rate their own answers using the five-point scale. Results showed that 85% of the errors learners made were reversal errors where they subtracted when they should have added. When analysing the learners' rating of their answers, learners appeared very confident in their answers to the non-conflict problems. However, when rating their answers to the conflict problems, learners were far less confident.

Msimango and Luneta (2015) conducted a study in which instructional strategies, which improved the problem-solving skills of grade three learners in Mathematics, were explored. Three teachers and eighteen learners from the same primary school participated in the study. The learners were then placed in three groups based on their academic abilities. Thus, there was a group containing 6 high achievers, a group of 6 moderate achievers and a group of 6 low achieving learners. The 2014 ANA scripts of each of the groups of learners were then analysed. Msimango and Luneta (2015) found that all three teachers asked questions at a moderate level. Furthermore, the teachers did not take into consideration that different learners learn using different techniques. The study also found that teachers did not provide sufficient opportunities for learners to solve problems. As a result, many learners were unable to compare quantities such and see their relationships. For example, 23 is the same as 20 and 3.

A study was carried out by Erdogan (2015) to determine which strategies grade 6 learners use when solving an unknown problem Learners were prescribed with a set of problems which they completed over a five-week period. When the answers and calculations of the learners' tests were analysed, it was found that most of them made use of the strategy of finding a pattern and then making generalisations. In the problems which require learners to use multiple or compound strategies (problems where learners need to break up the problem and then solve each part using separate strategies) to solve them, learners generally struggled. Cummins et al., (1988) carried out a study to explore learners' understanding of word problems. When grade 1 learners were given a particular type of arithmetic to solve, all of them solved it successfully. However, when the same problem was presented to them in the form of a word problem, only 29% of them could solve it. This was due to learners lacking sufficient exposure to word problems. They are more comfortable solving problems which have only mathematical symbols and numbers as these types of problems have been taught to them extensively in class. However, when seeing problems in word form, they do not have a framework they can use which can help them to understand the problem and what it requires them to do. Furthermore, on a national scale, learners perform up to 30% worse on word problems than what they do problems which are in numeric form. According to Erdogan (2015), these limited abilities of learners to solve problems are due to the current curriculum, which has "finding a pattern" as its sole problem strategy that is exposed to learners. This is found under the Numeric and Geometric Patterns topic. It is thus suggested that exposing learners to a greater range of problem-solving strategies within the schooling curriculum would allow them to explore different methods to solve problems and find strategies which work for them as individuals.

Tong and Loc (2017) conducted a study to identify the errors of students in solving mathematical word problems, as well as their ability to identify errors in incorrect solutions. One hundred and sixty grade 3 students completed the activity which consisted of a set of word problems. The results indicated that although students largely comprehended the problems, many of them still made critical errors.

A study was carried out by Alamian and Baseri (2020), to determine the effect of manipulation on reducing errors of grade 4 mathematics learners when solving word problems involving fractions. The participants consisted of 48 female grade 4 learners from two primary schools. The study utilised a quasi-experimental research design by using a pre-test and post-test with a control group. The results of the study revealed that when teaching learners fraction word problem solving through manipulation, it led to a substantial decrease in the number of errors made by learners.

Abadi and Amir (2022) conducted an analysis of the difficulties of elementary school learners in solving perimeter and area problems. The study used a qualitative research design by way of a case study. Three groups of grade 5 learners participated in the study, with these groups being categorised into low, moderate, and high mathematical ability in solving perimeter and area problems. Tests and interviews were used for data collection. Results revealed that learners with low ability levels showed verbal difficulties as they did not follow instructions. Learners with medium ability levels had conceptual challenges and they tended to use the perimeter unit when required to use the area unit. High ability level learners struggled with formula usage, resulting in incorrect solutions.

The researcher has the view that Mathematics is best taught when there is regular interaction between the teacher and learners. In addition, teachers need to expose learners to as many different strategies to solve a particular problem as possible in galvanise the idea that there are many ways to solve a particular problem. For this reason, the researcher recommends the use of the problem-solving teaching and learning approach as it enables learners to choose the strategy they are most comfortable with when solving a specific problem and at times it may even lead to learners coming up with their own methods which can then be shared with others.

2.3. Theoretical Framework

The study will integrate the Constructivist Learning Theory with Polya's problem solving model. The Constructivist Learning Theory will be used to allow learners to make sense of the problems on their own, whilst using Polya's problem solving framework as the format in which to compile their solutions.

2.3.1. Constructivist Learning Theory

The Constructivist Learning Theory states that knowledge is built through observations and experimentation. Knowledge is generated based on people's own experiences and their interactions with the environment around them (Major & Mangope, 2012). This theory is founded on the notion that learners can manifest their own understanding of different concepts by expanding on their previous set of knowledge and experiences (Surgenor, 2010). In addition, Bada and Olusegun (2015) mention that the new knowledge of ideas generated by learners is then integrated into their existing knowledge set which allows them to develop a meaningful understanding of the world.

According to Taber (2011), using constructivism within the classroom environment takes the focus of the lesson away from the teacher and shifts it onto the learners. This means that learners are participating in lessons and tasks which have them as the focal point of the lesson and they use their own knowledge and previous experiences to learn independently. By making their own observations, learners are active participants in their own learning process and construct their own relations which enable them to construct a more personal understanding of the information they have been given (Surgenor, 2010). Major and Mangope (2012) add that this serves as a contrast to conventional teaching methods which have the teacher as the only knowledgeable being in the learning process, who then shares this knowledge with the learners who are regarded as being entities who cannot acquire knowledge on their own.

Within a constructivist classroom environment, knowledge is regarded by both the teacher and the learners as tool which is tentative and can constantly change within our constantly evolving universe. This manner of seeing knowledge can thus be explored and to construct new knowledge and thus expand learners' understanding of the universe and their surroundings (Taber, 2011).

A constructivist classroom setup is typically arranged in a manner which promotes active learner engagement within the lesson and allows them to postulate, interact as well as to use their imaginations. These activities further motivate learners to make reflections on previous experiences and relate them to the new experiences they encounter to expand their conceptions of the world (Gray, 2005).

The role of the teacher within a constructivist learning environment is of great importance to ensure that learners can investigate the problems at hand in such a way that they are able to independently make deductions and draw conclusions. The teacher merely facilitates the lesson and takes up the role of mediator. As the mediator, the teacher moves around the classroom and instead of giving answers, the teacher asks leading questions and makes guiding statements to aid the learners to arrive at the solutions by themselves. In doing so, the learners are building their own knowledge and become the constructors of their own lessons. The teacher continues by encouraging learners to make comparisons, investigate and making hypotheses to allow learners to make connections between concepts and eventually develop a new understanding of the topic of concept at hand (Brooks & Brooks, 1999). As a result, learners feel more in control of the lesson and develop greater confidence in their own abilities and skills.

2.3.2. Social Constructivism within the classroom

This form of constructivism focuses more on social interactions and is frequently linked to the Vygotsky, who accentuated the impact that social and cultural contexts have on learning (Surgenor, 2010). Furthermore, Vygotsky supported a learning model which was based on discovery as he believed that a learner gets equipped with the tools required for development by interacting with his/her culture. Such a discovery model enables the cognitive abilities of the learner to naturally develop by making discoveries through processes such as making comparisons and exchanging thoughts with peers. Vygotsky's most renowned work which revolves around social constructivism is his Zone of Proximal Development (ZPD), which refers to the gap between the level that a child can achieve independently, and the potential of the child. This ZPD is the area in which learning occurs most successfully. Vygotsky claimed that with the assistance from and interaction with more knowledgeable peers and adults, the level at which a child can perform activities independently can be surpassed (Taber, 2011).

2.3.3. Polya's Problem Solving Model

Problem solving techniques have varied throughout the history and development of mathematics. The most famous work written on problem solving techniques was by George Polya. In 1945, Polya published his book titled "How to Solve It" in which he outlines four basic stages of problem solving (Aljaberi, 2015).

2.3.3.1 Understand the problem

This step is fundamental because if students are not able to comprehend what the problem requires them to do, then they cannot solve it. When fully understanding the problem at hand, there are certain things students should be able to do (Shirali, 2014). Proust (2014) adds that having group discussions is a very good method as students can share their thoughts, and by doing so they are helping each other generate a greater understanding of the problem.

Firstly, students must understand the meanings of all the words in the problem. If there are any words or phrases which they are not familiar with, they should consult a dictionary or more knowledgeable person to provide them with the meanings. Secondly, students must be able to identify precisely what the problem requires them to determine or explain (Kaufman, 2010).

After understanding all the words in the problem and determining what the problem requires them to do, students must then be able to rewrite the problem in their own words. By doing so, they are paraphrasing the problem so that it is stated in words which they are accustomed to and which form part of their own daily vocabulary. This allows them to feel more familiar with the problem and it enables them to comprehend the problem on their own cognitive level (Shirali, 2014).

If the student is not able to fully interpret the problem at this point, then constructing a diagram can aid in attaining a more complete understanding of the problem (Kaufman, 2010).

2.3.3.2. Devise a plan

Once the student has interpreted the problem and established what the problem requires him/her to determine, a comprehensive plan must be developed which the student will follow to solve the problem. There is a variety of problem-solving strategies to select from when tackling a mathematical problem (In'am, 2014). Kaufman (2010) adds that the most appropriate strategy is determined by the information given in the problem, the missing pieces of information the student needs to determine and how these sets of known and unknown information relate to one another.

2.3.3.3. Carry out the plan

According to Polya (1957), this is the most challenging of the four steps as students may easily get stuck and find it difficult to modify or later their strategy. When executing the strategy, it is important that students keep a record of their steps and findings so that they can clearly identify where they went wrong if their strategy is unsuccessful. They can then make alterations to their strategy and continue to do this until they arrive at a valid solution. However, In'am (2014) states that many times the designated strategy does not get students to the solution. At this point, they need to devise a new strategy that will be more effective. Taking into consideration the shortcomings of their initial strategy, students can then choose a strategy which is more suitable to the nature of the problem at hand, and which does not carry the same deficiencies as their first strategy.

Aljaberi (2015) mentions that students may get frustrated when finding that after attempting several different strategies, they are still unable to arrive at any kind of rational solution. A common misconception which students have is that they assume they can only solve a particular problem if they have encountered a similar problem before. However, Proust (2014) adds that this is a false belief as students should realize that every problem is unique and that there is more than one method to solve any problem.

2.3.3.4. Look back

Polya (1957) emphasised the importance of reflection and evaluating one's solution after completing a problem. Thus, the process of looking back enables students to check their solutions to determine if the solution they came up with makes sense and satisfies all the conditions of the problem. Furthermore, it also helps them to check if they made any mistakes during the process of solving the problem (Shirali, 2014). It is very easy to add or subtract incorrectly or even to use the wrong operation at times, resulting in an incorrect answer (Aljaberi, 2015).

The next section will discuss Polya's problem solving strategies and list some of the most common strategies. Using suitable examples, each strategy will then be demonstrated.

2.4. Polya's Problem Solving Strategies

The researcher defines problem solving strategies as methodologies or plans carried out by learners when solving open-ended problems. There are several problem-solving strategies which are suggested by Polya (1957). These include looking for a pattern, creating an organised list, logical reasoning, guessing and checking, drawing a picture, using a formula, simulation and working backwards. These strategies will now be discussed with suitable examples shown for each case.

2.4.1. Looking for a pattern

This strategy typically involves using the data given in the problem to draw up a table. After analysing the tabulated data, learners then identify a common trend or pattern. After doing so, they can then extend the pattern to solve the problem (Freebern, 2006). Teacher Vision (2015) adds that learners look for events, numbers or procedures which are repeated. They then identify the pattern and determine where in the pattern the solution to the problem lies.

Finding patterns is particularly beneficial to learners in that it allows them to learn mathematical facts which they have not been taught yet.

For example: Given the problems 3 x 2 and 2 x 3

3 x 2 = 6; and 2 x 3 = 6.

After doing more examples such as 5 x 4 = 20 and 4 x 5 = 20, learners may begin to notice a particular pattern in that when changing the order of the numbers in a multiplication sum, the answer will still be the same.

Upon looking for the pattern learners can discover that 3 x 2 is the same as 2 x 3 (Teacher Vision, 2015). This is referred to as the commutative property of multiplication.

2.4.2. Creating an organised list

Many mathematical problems include a substantial amount of data which tends to confuse learners. However, making an organised list not only allows learners to arrange the data in an ordered manner, but it also enables them to then use this organised list to get a visual representation of the problem. Using this organised list, they can then make relations between the data and identify patterns (Hawkins & Stobie, 2006).

For example:

The following problem is given to learners:

Jacob has a fruit stall. On Monday he sells 102 bananas, 35 apples and 42 oranges. On Tuesday he sells 25 apples, 67 bananas and 90 oranges. On Wednesday he sells 12 oranges, 54 apples and 216 bananas.

How many of each fruit did Jacob sell in the three days?

This problem contains a lot of information in no particular order. Thus, some learners may order the information into a list that makes it simpler for them to work with.

Apples :

M – 35

T – 25

W – 54

Oranges:

M – 42

T – 90

W – 12

Bananas:

M – 102

T – 67

W – 216

The information has now been organised into a list where each type of fruit sold on the 3 days has been listed separately. Learners can now simply add up the numbers for each type of fruit to solve the problem.

2.4.3. Logical and Direct reasoning

The logical reasoning strategy entails using rational thinking in order to form a systematic set of reasoning which will lead the learner towards the solution of a particular mathematical problem. Moreover, this strategy entails the organisation and analysis of data with the aim of arriving at a solution which is rational and comprehensive (Volce, 2015). During the process of using logical reasoning, every piece of information is viewed as a part of a larger puzzle. Hence, the puzzle will be built by assembling the different pieces of the puzzle (Edgar & Robin, 2000). Making use of charts, illustrations and Venn diagrams are all characteristic of logical reasoning.

Example

A cereal company is running a promotion in which they have put toy prizes in every box of their cereal brand. There are four different prizes, and John wants two identical toys. How many boxes of cereal does John need to buy to ensure that he gets two identical prizes?

Understand the problem

There are four different prizes that can be found in each cereal box. We need to determine the number of boxes John needs to buy to be certain of having two prizes which are identical.

Devise a strategy

The prizes will be called P, Q, R and S. We will now look at how many boxes must be bought until we have two prizes with the same letter.

Carry out the plan

Buying two boxes of cereal can result in John ending up with toys:

PP, PQ, PR, PS, and so onwards.

Thus, buying only two boxes will not guarantee him of getting identical toys.

Buying four boxes of cereal could yield in John ending up with prizes

P, Q, R and S. Hence, buying four boxes still does not ensure him of getting two identical toys. Buying a fifth box will however result in John getting one of the four toys which John already has because he already has one of each. (Fierro, 2013).

Look back

Is there perhaps a different method we could have used to solve this problem? Can we think of another strategy that would help us in a problem of this nature if logical reasoning does not work?

2.4.4. Guessing and checking

This is a very simple strategy whereby learners read the problem, make an educated guess regarding the solution and then review their guesses by checking if it satisfies the conditions of the problem. If the guess does not yield the solution to the problem, learners modify it until they arrive at the solution (Hawkins & Stobie, 2006). CK-12 Foundation (2012) adds that if the guessed answer is too small or too big, then the guessing and modification processes continue until the correct answer is found. Freebern (2006) adds that after guessing a solution, learners then test the solution. Using logical reasoning, learners then refine their guess. Although this strategy appears to be strenuous and time-consuming, making guesses can often also lead to learners recognising patterns which can then be utilised to make more accurate guesses.

Example

Solve the following problem:

$6 \times \boxed{} = 102$

Understand the problem

We need to determine the value of a number that, when multiplied by 6, gives an answer of 102.

Devise a plan

We can multiply 6 by different numbers until we get to 102.

Carry out the plan

6 x 10 = 60

6 x 12 = 72

6 x 15 = 90

6 x 16 = 96

6 x 17 = 102

Therefore, the missing number is 17.

Look back

We could have used other methods to solve the problem too, such as repeated addition or we could have used division and determine 102 ÷ 6.

Some learners may not be aware that they can use division to solve the problem as division is the inverse operation to multiplication. Some may also not be aware that repeated addition can be used to add 6s until getting to 120. Therefore, some learners will guess what the missing number is and use a bigger or smaller number each time depending on which number gets them closer to an answer of 102. By process of elimination learners can eventually get to the correct missing number which is 17.

2.4.5. Drawing a picture

Representing a problem through a picture is a very effective strategy for not only understanding the problem more comprehensively, but it can also lead to finding the solution. Watters and Logan (2000) add that constructing a diagram or drawing an image aids the learner in visualising the problem. Furthermore, this strategy is ideal for problems which involve physical scenarios.

In mathematics, visualisation refers to generating diagrammatic and/ or pictorial representations of problems to allow the learner to transform the mental picture of the problem onto paper (CK-12 Foundation, 2012). Thus, the strategy of drawing a picture is particularly effective for learners who lean more towards the visual learning style.

Example

Find the perimeter of a rectangular field with a length of 70m and breadth of 50m.

Understand the problem

We need to calculate the total length of the border of a field, in this case we need to find the total length of the four sides of a rectangular field.

Devise a plan

We will draw a picture of the rectangular field with the dimensions for each side. We can then add up the lengths of each side to give us the perimeter of the entire field.

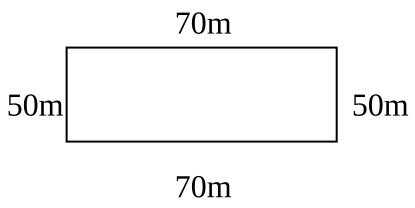


Figure 1: Rectangular field

Carry out the plan

Since the field is a rectangle, we can simply add up the lengths of the four sides to get the perimeter of the field.

50m + 70m + 50m+ 70m = 240m

Therefore, the perimeter of the rectangular field is 240m.

Look back

We could also have used the perimeter formula for a rectangle: $P = 2l + 2b$.

2.4.6. Using a formula

After critically analysing the problem at hand and identifying the set of given variables, learners then select the most suitable mathematical formula which can be used to solve for the unknown variable. This strategy is frequently used to solve geometric and algebraic problems (CK-12 Foundation, 2012).

Example

Calculate the area of triangle with a base of 6cm and a perpendicular height of 7cm.

Understand the problem

We need to find the area of a triangle using the given dimensions.

Devise a plan

We will use the area formula for a triangle to determine its area. We will plug the given values into the formula and calculate the area of the triangle this way.

Carry out the plan

We use the formula for the area of a triangle:

A = ½ bH

We were given dimensions:

base, b, = 6cm; and the perpendicular height, H, = 7cm.

Substituting these values into the formula: A = ½ (6cm) (7cm)

= ½ ($42cm^2$)

= $21cm^2$

Therefore, the area of the triangle is 21 cm^2.

2.4.7. Acting out the problem (Simulation)

It can be very helpful to act out or simulate an aspect of the problem, particularly if the problem is a complex one. This strategy is commonly used during probability problems (Fierro, 2013).

Example

Mr Smith has 24 learners in his class. To celebrate the end of term, he buys a bag of 120 sweets to share amongst the learners. How many sweets will each learner receive?

Understand the problem

We need to divide 120 sweets between 24 learners. We must determine the number of sweets each learner will receive.

Devise a plan

We will solve the problem by acting it out. Therefore, we will physically divide 120 sweets between 24 of our peers in the classroom. We will hand each learner 1 sweet until all learners have 1 sweet, then start at the first learner again and repeat the process until all the sweets have been handed out equally. We will then count how many sweets each learner is holding.

Carry out the plan

We now carry out the process as stated above. After handing 1 sweet to each of the 24 learners and repeating the process until all the sweets were shared, we then counted the number of sweets each learner is holding. After counting, we have concluded that each learner is holding 5 sweets.

Look back

We could also have used division or repeated addition to solve the problem.

2.4.8. Working backwards

It can sometimes work particularly well to break down the components of the problem and unwinding it to a solution. This process of undoing constituents of the given problem is referred to as working backwards (CK-12 Foundation, 2012).

According to Shapiro (2000), this strategy is implemented when dealing with problems which contain numerous related factors. Furthermore, certain information has been omitted from the problem. By starting at the answer and systematically working backwards, the missing information can be determined and hence the problem can be solved. Watters and Logan (2000) add that problems which require this strategy typically provides one with the final answer as well as the set of steps it took to get to the answer.

During the process of working on a mathematical problem, then all operations must be reversed when working backwards. In other words, the inverse operation is used when working backwards (Shapiro, 2000). If a particular part of the problem involves addition, then when working backwards requires the learner to use subtraction. Similarly, division problems will require multiplying upon working backwards.

Example

Mom buys a box of eggs containing 156 eggs. How many dozens of eggs are in the box?

Understand the problem

A box of eggs contains 156 eggs. We need to determine how many dozens of eggs are in the box, meaning how many dozens of eggs are found in 156 eggs. We know that 1 dozen = 12 eggs.

Devise a plan

We will start with 156 eggs, then subtract 12 each time. We will repeat this subtraction process until we cannot subtract anymore.

Carry out the plan

156 – 12 = 144

144 – 12 = 132

132 – 12 = 120

120 – 12 = 108

108 – 12 = 96

96 – 12 = 84

84 – 12 = 72

72 – 12 = 60

60 – 12 = 48

48 – 12 = 36

36 – 12 = 24

24 – 12 = 12

12 – 12 = 0

After counting the total number of 12s we subtracted, we ended at 13. Therefore, there are 13 dozen eggs in a box containing 156 eggs.

Look back

We must be careful not to make errors in calculations when subtracting so many times. Instead of working backwards, we could also have repeatedly added 12s until we got to 156. Alternatively, we could have used division, $156 \div 12$.

2.4.9. Using a variable

This strategy is particularly helpful when there are relationships between quantities which are unknown in the problem. It is also very effective when we can represent the problem by writing an equation to model it or when we want to find a general formula. Using this strategy also allows one to use algebra.

Example

In a triangle, the size of the largest angle is three times the size of the smallest angle. The size of the third angle is 35° greater than the size of the smallest angle. What are the sizes of each angle in the triangle? Use the fact that the sum of the interior angles of a triangle add up to 180°.

Understand the problem

Firstly, we must use the fact that the sum of the interior angles of a triangle add up to 180° to determine the sizes of the angles in the triangle. The problem also gives us information about the sizes of the angles compare to each other.

Devise a plan

By using a variable to represent the angles, we can express the size of each angle in algebraic form.

Let the size of the smallest angle be x

Then the size of the biggest angle would be $3x$

The third angle would have a size of $x + 35^\circ$

Carry out the plan

By adding up the three algebraic expressions, which represent the size of each angle, we can then solve for x.

$x + 3x + (x + 35) = 180$

$x + 3x + x + 35 = 180$

$5x = 180 - 35$

$5x = 145$

$x = 29^\circ$

Therefore, the size of the smallest angle is 29°

Solving for the size of the biggest angle

$3(29) = 87^\circ$

Solving for the size of the third angle

$(29) + 35 = 64^\circ$

Look back

We can check our solutions by seeing if the sizes of the three angles add up to 180°.

$29^\circ + 87^\circ + 64^\circ$

$= 116^\circ + 64^\circ$

$= 180^\circ$

Therefore, our solution is valid. However, are there perhaps other values which would satisfy the conditions of the problem?

2.4.10. Make a table

Putting data in tabular, organised form can help make it easier to find patterns. Moreover, a table is an effective method of organising data as the data is generated. It also makes it easier to summarise information as well as to list the values of any variables.

Example

A man wants to enclose a rectangular field of 100 square meters for his horse (Figure 2). He wants to use rectangles with dimensions which are all whole numbers. Which dimensions would require the least amount of fencing?

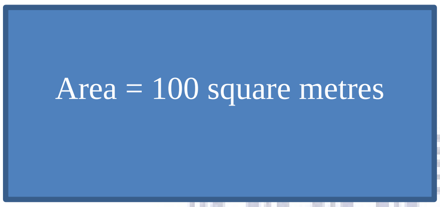


Figure 2: Enclosed rectangular field for the horse

Understand the problem

The area of the rectangle needs to be exactly 100 square metres. The dimensions must be only whole numbers. For the total amount of fencing to be as small as possible, the perimeter of the rectangle also needs to be as small as possible.

The area of a rectangle is written as A = l x w, while the perimeter of a rectangle is written as P = 2l + 2w. Therefore, we need to find the length (l) and width (w) which will yield the smallest perimeter.

Devise a plan

Which combinations of lengths and widths will result in an area of 100 square metres? We can draw a table and record all the possible dimensions.

Carry out the plan

Table 1 : Possible lengths and widths that will give an area of 100 square metres

l	w	Area = l x w	Perimeter = 2l + 2w
10	10	100	40
20	5	100	50
25	4	100	58
50	2	100	104
100	1	100	202

The only factors of 100 which are whole numbers are 1, 2, 4, 5, 10, 20, 25, 50 and 100. Hence these were the factors used in the table. The smallest perimeter is 40m and this happens when the rectangle has dimensions of 10m by 10m. With these dimensions, the shape is a square. However, a square is a special type of rectangle.

Look back

Will the perimeter always be the smallest when the shape is a square? We could try rectangles with different areas.

2.5. Worked examples using Polya's problem-solving strategies

The following section will demonstrate how different strategies can be used to solve problems. Each example will use Polya's problem solving framework to solve the problem.

2.5.1. Drawing a picture

Making drawings is synonymous with solving geometry problems as these problems usually relate to diagrams.

Example

A large cube is built by arranging 64 smaller cubes in a stack. Each of the small cubes has dimensions of 4 cubes by 4 cubes by 4 cubes. A box is constructed from cardboard with an open top so that the large cube fits very tightly into this open cardboard box. How many of the small cubes are not in contact with the bottom or the side of the box?

Understanding the problem

There are no gaps between the small cubes and the side of the cardboard box. Also, the box does not have a top. The problem requires me to work out how many of the 64 small cubes are not touching the side or the bottom of the cardboard box.

Devise a plan

To visualise the problem, draw a picture of the cubes inside the box. There are 64 cubes in the large 4 by 4 cubes, arranged in 4 layers [Figure 3(a)].

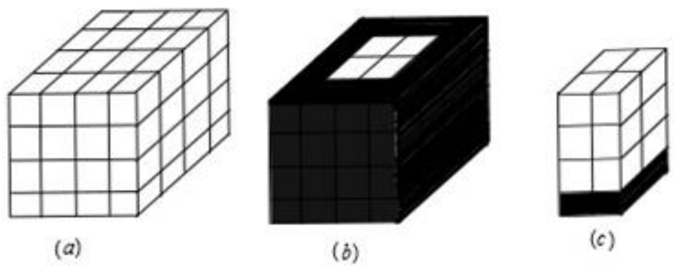


Figure 3:The cubes inside the box

Carry out the plan

We can now count the number of small cubes which are in contact with the side and the bottom of the box [Figure 3(b)]. To make this more visible on the drawing, we can shade these cubes. We shall call the stack of cubes still visible the “core” cubes. Now we shall remove this “nucleus of cubes” [Figure 3 (c)]. Some of these cubes only make contact with the bottom of the box, but not with the sides. We can now see that it is the top 3 layers of this core that touch neither the bottom nor the sides of the box. These 3 layers all contain 4 cubes each, thus they are a total of 12 cubes.

Look back

Another way of solving this problem would have been to subtract the number of cubes that touch the box from the total number of 64 cubes. What would have happened if the cube measured 6 cubes by 6 cubes by 6 cubes? How many small cubes would then not be in contact with neither the bottom not the sides of the box?

The nature of word problems tends to revolve around being given specific dimensions or measurements for a particular object. For example, when asked how many bottles of water of 1,5L will fill a container which can hold 30L. Some learners may have difficulty dividing or using repeated subtraction/addition. Therefore, they will guess numbers and then multiply that number by 1.5L until they find the number which yields in an answer of 30L.

2.5.2. Guess and check

This strategy is frequently the first one used by mathematicians both experienced and novices. Many times, the guess and check strategy may not get you all the way to the solution, but it assists you in getting a better understanding for the problem as well as its parameters.

These parameters set by the problem ensure that the guessing and checking are not random. You may start to notice a pattern after a few observations which will further aid you in limiting your guesses.

Example

Five friends were all seated at one side of a table. Gary sat next to Bill, while Mike sat next to Tom. Howard was seated in the third seat from Bill. Gary sat in the third seat from Mike. Who sat on the side of Tom opposite from Mike?

Understand the problem

All five friends sat at one side of the table, so the seats were all in a straight line. The problem requires us to allocate the five friends to the five seats and work out who, along with Mike, is seated next to Tom. The positions of the friends may vary but it may still satisfy the conditions of the problem.

Devise a plan

Make a drawing to show the arrangement of the seats [Figure 4(a)]. Use the first letter of each friend's name to represent them. Test a variety of different arrangements of the friends to suit the conditions of the problem to narrow down the guesses.

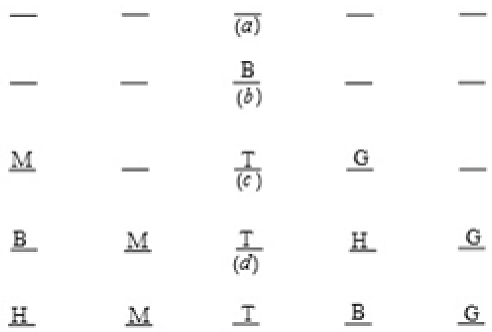


Figure 4: Arrangement of seats at the table

Carry out the plan

We can begin by testing a few possibilities for the middle seat as the problem stated that Howard sat in the third seat away from Bill. So now we could place Bill in the middle seat [Figure 4(b)]. We now observe that if Bill sits in the middle, then Howard cannot sit three seats away from him. The same challenge yields if we place Howard in the middle seat. The same issue arises with Gary and Mike. Gary must be seated in the third seat from Mike, therefore neither of them can be seated in the middle seat. Therefore. Tom must be the one in the middle seat.

We may now play around with some seating arrangements with Tom sitting in the middle seat. We first try placing Gary in the third seat from Mike, but now Mike is not next to Tom as the problem states [Figure 4(c)]. Next, we try assigning Mike next to Tom, while we place Howard and Bill in such a way that Howard is in the third seat from Bill [Figure 4(d)]. But now Gary is not sitting next to Bill. However, if we switch Howard and Bill then that issue is solved [Figure 4(e)]. This seating arrangement now satisfies all the conditions of the problem. Therefore, Bill is sitting on the other side of Tom.

Look back

Is there perhaps a different seating arrangement that would also satisfy the conditions of the problem? There is one other arrangement that would satisfy all the conditions: G-B-T-M-H.

This seating arrangement holds too as it also has Bill on the other side of Tom. Are there perhaps more such solutions? Why or why not?

Within the context of measurement word problems, learners may encounter problems in this study whereby they are unsure of the solution. As the researcher has already witnessed in previous assessments completed by these learners, when they are uncertain many of them resort to guessing the answers and then checking for its validity. Therefore, this is a strategy which is expected to be quite prominent when data will be collected for the study.

2.5.3. Looking for a pattern

Patterns are found in many aspects of mathematics through observation. This strategy is commonly combined with other strategies such as drawing tables.

Example

A row of squares can be formed by arranging toothpicks [Figure 5]. How many toothpicks would you need to form a row of 100 squares?

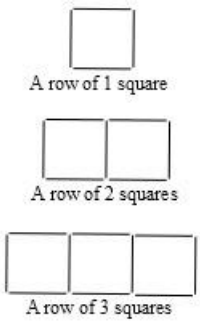


Figure 5: The first 3 rows of squares consrtucted using toothpicks

Understand the problem

The problem requires me to arrange the toothpicks in a straight line of squares. Therefore, the arrangement as shown in Figure 6 will not be valid. The problem then requires me to determine the number of toothpicks it would require to form a row of 100 squares.

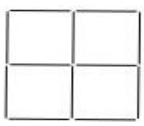


Figure 6: An invalad arrangement of the toothpicks

Devise a plan

The problem could be tackled by trying out a few additional rows of squares in hope of observing a pattern or a relationship between the number of squares and the number of toothpicks used [Figure 7].

Carry out the plan

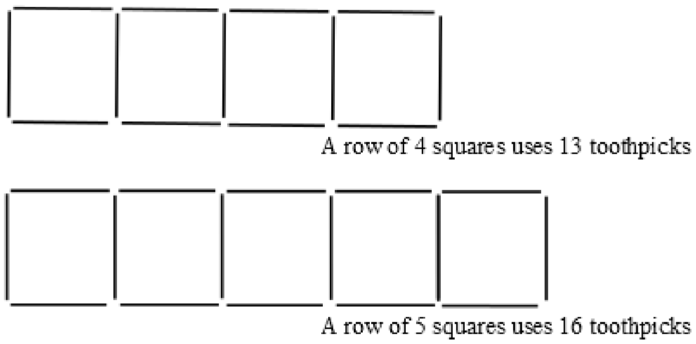


Figure 7: Pattern between the number of squares and the number of toothpicks

From Figure 6 and Figure 7, we can observe the following:

1 square uses 4 toothpicks

2 squares use 7 toothpicks

3 squares use 10 toothpicks

4 squares use 13 toothpicks

5 squares use 16 toothpicks

From these findings, for each new square that is added to the row an additional 3 toothpicks are needed. We can thus say that the row containing 100 squares will need 3 toothpicks more than the row containing 99 squares. However, counting such a large number of toothpicks might end in confusion. Therefore, we may try to identify a rule or pattern by observing the pictures. From Figure 8, a row containing 2 squares is made up of 2 sets of 3 toothpicks, as well as a single toothpick which is used to complete the final square.

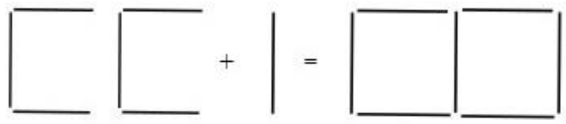


Figure 8: A row containing 2 squares

Furthermore, a row containing 3 squares is made up of 3 sets of 3 toothpicks and a single toothpick to complete the final square. Similarly, a row containing 4 squares is made up of 4 sets of 3 toothpicks and a single toothpick to complete the final square. This can be seen in Figure 8.

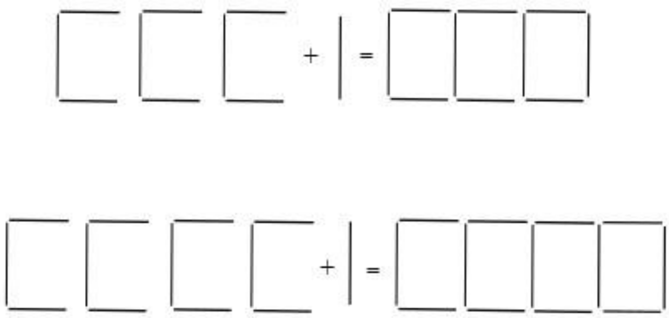


Figure 9: A row containing 3 squares and 4 squares

Using the pattern we have, a row containing 5 squares would then need 5 sets of 3 toothpicks and a single toothpick to complete the final square. So, 5 (3) + 1 = 16 toothpicks.

Applying this rule to a row containing 100 squares, we would need 100 sets of 3 toothpicks and a single toothpick to complete the final square. Therefore, the number of toothpicks needed to build a row containing 100 squares would be 100 (3) + 1 = 301.

Look back

We have identified a pattern for which we can try to express in a general formula. A variable could be used to generalise the pattern. Let the number of squares in a row be represented by n. Thus, the row would need n sets of 3 toothpicks plus 1 toothpick to complete the final square. The total number of toothpicks would then be n (3) + 1, which can be written as 3n + 1. This formula can now be used to find the number of toothpicks needed to form a row of any size.

This example will be very useful as it shows how patterns can be derived from drawings. Where possible, the patterns can also be expressed in general rules and then in formulae. This will be particularly helpful when working with measurement problems involving perimeter or area.

2.6. Related studies on problem-solving strategies - A synopsis of research

The following section will describe previous studies which have been conducted on problem solving and problem-solving strategies.

Chamot, Dale, O'Malley and Spanos (1992) carried out a study to investigate the mathematical problem-solving approaches of a group of elementary as well as secondary ESL students. These students were enrolled in a project which used the Cognitive Academic Language Learning Approach (CALLA). This approach makes use of curriculum content to teach students learning strategies with the aim of increasing the meta-cognitive awareness of the students. The primary objective of this study was to identify the learning and problem-solving strategies used students at high, middle, and low achievement levels used. These strategies would then be compared between the students in high implementation classes and those in low implementation classrooms. Furthermore, the study also wanted to see the difference in the number of problem-solving steps used by the two groups.

When analysing the results, it was found that students who were in the high mathematics achievement group performed better than those in the low achievement group. In addition, the students in the high-performance group also showed greater success in finding the correct solutions to the problems. Another finding was that a greater variety of problem-solving strategies were used by students in the high achievement group than those in the low achievement group. It was thus concluded that students who were high mathematics achievers and had more training in meta-cognitive strategies could apply themselves better in problems and show their steps more explicitly than low mathematics achievers, as well as be more successful in getting to the correct answers.

Sulak (2010) conducted a study to determine the effect of problem-solving strategies on problem solving achievement in primary school mathematics. Over a period of 14 school days, the research was carried out on grade 2 learners at a primary school. The experimental research approach was used. The experimental group was trained by the research with regards to mathematics problem solving strategies. On the contrary, the control group was continued to be taught in the classroom through conventional textbook problem-solving methods. Both groups then wrote two tests containing open-ended problems. Interviews were also conducted to get explanations from the learners regarding why they chose their specific strategies as well as how they went about carrying out their strategies.

The results showed that the learners who were trained in problem solving strategies performed far better than those who were taught through normal textbook methods. The strategies which the pre-trained group did particularly well in were making drawings, drawing a table, writing out sentences looking for a pattern, making lists, using logical reasoning, and the guessing and checking strategy. These findings indicate that with sufficient training and opportunities to explore different problem-solving strategies, learners will become better at solving problems. They will also be able to carry out their strategies more accurately.

A study was conducted by Bruun (2013) to investigate which problem-solving strategies were used most frequently by teachers when educating their learners on problem solving strategies. The participants in this study comprised of 70 teachers who taught in grades ranging from second to fifth grade. Interviews were used as the data collection technique.

The first major finding made by Bruun (2013) was that there was not a single teacher who utilized all the problem-solving strategies which were recommended by National Council of Teachers of Mathematics. Another finding was that the two most prominently taught problem-solving strategies were drawing of pictures as well as to locate the key information within a problem. The study concluded that there is a greater need to promote the instruction and educating of a wider variety of problem-solving strategies within mathematics lessons.

Intaros, Imprasitha and Srisawadi (2014) conducted a study to investigate the problem-solving strategies of students in a problem-solving mathematics classroom. The case study design was used. The study utilized an open teaching approach which was made up of four phases. These were giving the students open-ended problems, the self-learning of the students, having a discussion as a whole class, and finally making comparisons and summaries using the ideas and strategies the students came up with. The data collection techniques for the study were observations as well as audio and video recordings. The participants in the study were a group of six students in grade 1 who attended a school that was taking part in the Project for Professional Development for Mathematics Teachers using an open approach and lesson study.

From the results, it was found that the students used most of the problem-solving strategies in the self-learning phase. Another finding was that the students were able to work together and devise their own problem-solving strategies. It was therefore concluded that when learners solve open-ended mathematical problems within a classroom that promotes problem-solving, learners can come up with their own problem-solving strategies to solve mathematical problems.

Ramirez, Chang, Maloney, Levine and Beilock (2015) performed a study to determine the relationship between mathematics anxiety and mathematics achievement within early elementary school, as well as the role that problem solving strategies play within this relationship. The participants consisted of 564 learners: 256 from grade one and 308 from grade 2. All these learners attended the same elementary school, and their home language was English.

Learners were required to complete an anxiety questionnaire to determine their levels of anxiety towards different types of mathematics problems. The working memory of learners was then assessed by making use of forward and backwards letter span tasks. To measure the math achievement of learners, they were given a verified test to do which contained mathematical word problems of different difficulty and complexity. Finally, learners were then given a set of math problems to complete to determine the types of problem-solving strategies they utilised.

The study found that learners may not be able to make use of more advanced problem-solving strategies when they suffered from math anxiety. However, the study also showed that not all learners who suffer from math anxiety are unable to solve problems or use more complex solving strategies. Furthermore, learners with higher working memory capacities tend to avoid making use of advanced problem-solving strategies when their mathematics anxiety levels are high. As a result, these learners underachieve in mathematics as compared to learners with lower working memory capacities.

Saygili (2017) wanted to determine which problem-solving skills high school learners used when solving non-routine problems. In addition, the study also aimed to identify the problem-solving strategies these learners used when solving these problems. The descriptive research design was used, which made the research approach one that was qualitative in nature.

The participants were made up of 18 learners who volunteered to be part of the study. This process was done by having the researcher conduct interviews with 285 learners at Canakkale Kepez Vocational and Technical Anatolian High School to introduce them to the study. Due to not wanting to participate and some not feeling confident enough to do so, the group was narrowed down to 24 learners. This was then further reduced to 18 learners because of 6 learners submitting nearly blank test papers or changing their minds once they saw the questions in the test.

Of the 18 learners who then proceeded to take part in the study, they were categorised into three groups of 6 learners per group. These three groups were low success level, medium success level and high success level. The learners were given one hour during a lunch break period to complete the test administered by the researcher.

The researcher also solved each problem in the test using different solving strategies that were expected when analysing the papers of the learners. After consulting with the schoolteacher, a set of 8 non-routine problems were selected and formed the test that would be given to the 18 learners to complete. Results showed that in attempting to solve each problem, learners made use of at least three different problem-solving strategies per problem. Furthermore, the strategies which were used most often were making a list, looking for patterns, logical reasoning and making drawings.

It was concluded that learners may solve non-routine problems in a variety of different ways using various problem-solving strategies. Moreover, the results showed that the learners who participated in the study possess a medium skill level when it comes to their problem-solving skill proficiency. The study also recommended that if learners were educated more on the different problem-solving strategies and how to carry them out, then their skill level and success level at solving non-routine problems would increase.

Yasmeen (2019) ran a study which was aimed at identifying the problem-solving strategies of secondary level mathematics students. The descriptive research method was used for this study, accompanied by a survey which was used to narrow the school student population down to a smaller sample. The sample was eventually formed and made up of 250 social science students at the secondary school in Lahore. This consisted of 120 boys and 130 girls. For the research instrument, a test was developed by the researcher. This test consisted of 10 strategies, each then having 3 questions.

When analysing the results, Yasmeen (2019) found that the students performed best when using logical reasoning and simplifying the problem. However, the problem-solving strategy they performed the worst in was when having to use logical reasoning. The strategies where students performed moderately well in were making a drawing and acting out the problem.

A study by Susanti, Hapizah, Meryansumayeka, and Irenika (2019) attempted to describe the mathematical thinking of 13-year-old students by using problem-solving. The sample for the study consisted of six 13-year-old students. Interviews and tests were used as the research instruments. The test instrument was a problem-solving test which was prescribed to the students. A qualitative research approach was used to collect the data.

Data analysis found that the problem-solving strategy which was used most frequently by the students was deductive and logical reasoning. The findings add that this is due to the students being conditioned to thinking about which operations to use whilst reading the problem at hand. Another strategy that was used by students was the use of trial and error or guessing and checking until they arrived at an answer which they were satisfied with.

Additional findings show that when studying the mathematization of students, they tend to make connections and relations upon reading the problem and looking at the accompanying diagrams or pictures. Several students also made their own drawings, graphs, and other types of illustrations. Many of them also spoke to each other and shared ideas. In doing so they were showing how they interpreted the problem by engaging with their peers they were constructing meaning to the problem in a collaborative manner.

A study was conducted by Xin, Jitendra and Deatline-Butchman (2005) with the aim of investigating the differential effects of two problem solving instructional approaches. These approaches were schema-based instruction (SBI) and general strategy instruction (GBI). The research designed utilised a pre-test and post-test. The study consisted of 22 students with learning problems, with 18 of these students having been identified by the school as possessing learning disabilities. Furthermore, 3 learners were at risk of failing mathematics while 1 learner was identified as having severe emotional disorders. These learners attended a middle school in the United States of America.

The instructors for the study were two special education doctoral students as well as two teachers who had experience in special education. The first cohort of 8 students was taught by the two doctoral students, while the second cohort of 14 students was taught by the two teachers. Both the SBI and the GBI groups were given 12 instruction sessions each. The SBI did 4 sessions which focused on multiplicative compare problems as well as 4 sessions on proportion problems. This was followed by 4 sessions on solving word problems which were made up of both types. The GBI group also received a total of 12 instruction sessions. However, this group was required to solve problems of both types during each session. Moreover, the GBI group did not get any instruction being able to identify the difference between the two word problem types.

The results demonstrated quite a substantial difference between the SBI and GBI groups in the post-test, maintenance, follow-up as well as in the generalisation tests. In particular, the SBI group performed far better in the follow-up tests than in the post-tests. Xin, Jitendra and Deatline-Butchman (2005) attribute this to the SBI group having been exposed to a more coherent presentation of the word problems as well as internalising the schema-based strategy which was not prominent within the general strategy. Furthermore, results also showed that after the schema-based instruction, the SBI group was the only group to have improved performances on the generalisation measure.

Tambychik and Meerah (2010) carried out a study to determine the difficulties learners had in solving problems. Both qualitative and quantitative research methods were used during the data collecting process. Learners who performed on an average level were used in this study which spread across three schools in Malaysia. They were then instructed to complete a questionnaire which was made up of three components: demographics, mathematical skills, and the cognitive abilities of learners. Across all 3 schools, less than 40 % of the learners were competent in having all the necessary mathematical skills. These skills included being able to make connections between concepts and converting problems into appropriate mathematical number sentences.

Furthermore, learners also generally failed to be able to be able to visually represent problems. They found it highly challenging to illustrate problems and to make drawings. In terms of language, many learners struggled to understand basic mathematical terminology as the language to describe mathematical problems. With regards to the cognitive ability of learners, it was found that many learners who participated in this study were unable to readily memorise and recall mathematical information. It was concluded that if learners were not able to master the set of mathematical skills, then they would not be able to solve problems.

Lubin, Houde and de Neys (2015) conducted a study to detect the errors which children make when solving arithmetic word problems. 137 learners between the ages of 8 and 11 participated in the study. The learners came from grades 3, 4 and 5, from the same school in Paris. Furthermore, the first language of all the participants was French.

The learners all wrote the same test within their classroom. After solving each problem, learners had to rate the confidence of their answers using the given 5-point scale, where 5 was "very confident" of the answer and 0 denoted that they were "completely unsure" of their answer. The test consisted of 4 problems, 2 were which conflict problems while the other 2 were no-conflict control problems.

Results showed that despite the learners doing well in the test, they still found it hard to solve the conflict problems. On the contrary, in the non-conflict problems learners had little to no issues in solving them successfully. After analysing the errors learners made in the conflict problems, it was found that most of the answers containing errors were reversal errors. These are errors in which learners added when they needed to subtract and vice versa. The general conclusion made from this study was that when learners had erroneous answers, they were very sensitive about their errors by giving the answers low ratings which indicated they were very unsure about what they needed to do.

A study to investigate the mathematics problem solving ability of class VII learners in a school in Indonesia was conducted by Peranginangin and Surya (2017). The study made use of the descriptive qualitative research approach. The participants were 31 class VII learners, and the researchers used a problem-solving test as the research instrument.

Peranginangin and Surya (2017) found that most of the class VII learners were unable to solve almost all the problems from the test that the researcher prescribed to them. The study also found that when applying the stages of problem solving as designed by Polya (1957), learners performed poorly during the second stage which is to devise a plan to solve the problem. It is further added that the reason for such poor performances in their attempts to solve the problems is due to many of them being unable to come up with suitable strategies to solve the problem.

To investigate the which strategies grade 4 learners use when solving measurement type word problems, this study will use Polya's problem solving framework as the format in which learners are expected to show their solutions. The interview questions will also be focus on getting information on aspects of the framework, namely whether learners understood the problem, why they chose their specific strategy, how they carried it out as well as reflecting on the validity of their solutions. This framework will be combined with the Constructivist Learning Theory, which will enable learners to study, interpret and solve each problem independently and making use of their own background knowledge. The researcher will not instruct them on which strategies to use at all, not will he help them in any way.

The next chapter will detail the research design and methodology for the study. It will explicitly reveal how the researcher plans to carry out the study as well as the research approach and sample for the study. In addition, the research instruments will be discussed as well as the methods for collecting data.

CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

3.1.Introduction

This chapter will comprehensively outline the research methodology used in this study. It will explain the research design and approach as well as the reasons for selecting the specific design and approach. The data sources and sample will also be explained as well as how the data will be analysed and presented after data collection has been completed. The instruments used in the study will be discussed, as well as an explanation of the research intervention and the ethical statement.

Before commencing with the study, permission to conduct the study was first be requested from the Western Cape Education Department (WCED). Permission from the school principal was also requested to allow the learners of a specific grade 4 class to participate in the study. Refer to Appendix F and Appendix G.

3.2.Research Design

According to Eduafo (2014), a research design is the formal and detailed plan of action of a research project which explains how the research will be carried out. Kothari (2004) adds that it is imperative to formulate a suitable and relevant research design before proceeding with the research. This study used a case study research design. Rowley (2002) defines a case study design as inquiry which is experimental and aims to find a relationship between two or more variables. It also explores phenomena within their given contexts. In other words, a phenomenon can be studied naturally without the need to imitate it within a laboratory. This is what makes the case study design so unique, it provides us with data and knowledge that comes directly from the world around us. It helps us attain a better understanding of reality. Case studies also make use of a wide range of research sources such as observations, focus group interviews, and documents.

The researcher utilized the exploratory case study method. This type of case study is used when performing a study that will provide data which will serve as a basis for further studies to be conducted on the topic. In other words, it serves as a predecessor/ precursor for a bigger project. This study investigated the problem-solving strategies used by grade 4 learners when solving measurement word problems. Learners were required to complete a test consisting of Mathematics problems. The entire class of 42 grade 4 learners served as the sample of the study. Therefore, all learners completed the same activity which served as one of the research instruments. The activity consisted of problems which did not contain any specific instructions regarding the strategy or method they needed to use to solve each problem. This allowed learners to devise and select their own methods and problem-solving strategies. At the end of the test the researcher collected data from the activity and compiled graphical information to identify the different problem-solving strategies the learners used, as well as how they went about carrying out their strategies during the solving process.

After the data is analysed and conclusions are made, the information may form the groundwork for a larger scale project. The study will then be performed on a larger group of learners to determine whether similar results are yielded. The researcher is of the view that learners possess knowledge and use strategies to solve problems which are not found in textbooks or in the curriculum. Therefore, if the results of the study show that learners make use of strategies which are not taught within the class, then this study can be performed on a greater scale to investigate what other strategies learners use when solving problems.

Mathematics is unique in that a single problem can be solved in a variety of different ways (Maharani, 2014). For this study to be conducted correctly, a research approach is required which will allow the researcher to collect both quantitative and qualitative data. The next section will therefore discuss the research approach which will be used for this study.

3.3.Research Approach

The study used the mixed methods research approach. Tashakkori and Creswell (2007) define the mixed methods as a research approach in which data is collected and analysed, the findings are integrated and then conclusions are drawn from them. Furthermore, these conclusions are drawn using both quantitative and qualitative research approaches. For this study, it was decided that the mixed methods approach would be the most suitable approach. This approach was chosen because when investigating which problem-solving methods learners will use, getting results which show only the methods used by learners would still leave various unanswered questions such as what influenced the learners to use the specific method for the particular problem. To determine the reasons learners chose a particular strategy, the researcher therefore conducted focus group interviews after the learners completed the activities. In addition, the researcher compiled an observation protocol and recorded critical observations while the learners completed the activities. By then integrating all three these sets of data through triangulation, the researcher was able to find relationships between the different data sets and present a study which is more comprehensive with results and findings that were uncovered through a variety of extensive data sets.

To protect the identity of the learners, they were not required to write their names or any personal information on their answer scripts. The answer scripts were numbered from 1 to 42 and were distributed randomly to the learners. A copy of the activity and was prescribed to each learner. Learners were required to complete the activity under the supervision of the researcher. They were instructed to try to answer all the questions. The activity consisted of Section A and Section B, with each question requiring learners to solve an opened-ended word problem involving measurement. Learners were given a maximum time of 60 minutes to complete the activity. They were not allowed to consult with their peers, or the teacher, who in this study was also the researcher.

Learners were instructed to solve each problem using methods and strategies of their choice. Furthermore, they were instructed not to erase any mistakes they made. This enabled the researcher to get a perspective of the cognitive processes and different ideas learners had prior to arriving at their solutions to each problem.

This study integrated the qualitative research approach with the quantitative research approach. Quantitative inquiry analyses the effects of an independent variable on a dependent variable in manners which can be represented numerically. Causal inferences are made from observations, or from relationships established through statistical analysis. (Lakshman, Sinha, Biswas, Charles, and Arora, 2000). Qualitative inquiry is used where the effects of specific variables are not apparent or where there are not sufficient outcomes to be able to analyse statistically. Qualitative methods include interviews, observations, or the analysis of written material (Lakshman et al., 2000). The qualitative research approach was used when determining the problem-solving strategies learners used when solving the measurement type word problems. Moreover, the qualitative research approach was also used to determine the reasons learners used these strategies once focus group interviews were conducted. Thereafter, a quantitative research approach was adopted when comparing the number of learners who use each of the different problem strategies to determine which strategies were used most prominently.

The next section will detail the data sources, sampling and the data collection techniques used for this study.

3.4. Data Sources and Sampling

The sample for this study was comprised of one grade 4 class which was made up of 42 learners as stated earlier.

3.5. Data Collection Techniques

The data was attained from the written activity which was completed by each of the learners in the grade 4 class. The strategy used by each learner for each of the questions in the activity was tabulated. Data was collected through document analysis, an observation protocol and focus group interviews.

3.5.1. Document Analysis

Document analysis was utilised in the form of a short activity containing measurement word problems. The activity consisted of Section A and Section B (Refer to Appendix and Appendix B). Bowen (2009) defines document analysis as a means for interpreting and reviewing data. In contrast to other research methods, document analysis requires data to be analysed to interpret it into knowledge which is meaningful. The data was attained from the written activity which was done by each of the learners in the grade 4 class. The strategy used by each learner for each of the 6 questions in the test was tabulated and comparisons were then made.

3.5.2. Observation Protocol

The researcher observed the learners whilst they were doing the activity and try to answer specific questions to help in answering the broader set of research questions. Refer to Appendix C for observation questions.

3.5.3. Interview Protocol

Learners were divided into focus groups after they have completed the activity and the researcher conducted an interview on each group to get reflective feedback from learners. Two sets of interview questions were used. The first set of questions was to determine how the learners applied the stages of problem solving to their problems as explained by Polya (1957). The second set of questions was to determine learners' conceptions of the problems, how they felt during the activity, as well as what they found easy and what challenges they had. Refer to Appendix E and Appendix G for interview questions.

The learners were divided into focus groups to be interviewed according to the groups below:

The "L" symbol was used to indicate "Learner" followed by the number assigned to the learner.

Group 1

- L8, L12, L14, L15, L25, L26, L31, 32

Group 2

- L5, L6, L9, L13, L33, L34, L40, L41

Group 3

- L1, L2, L10, L11 L19, L21, L37, L42

Group 4

- L3, L4, L18, L22, L24, L27, L29, L36

Group 5

- L23, L28, L30, L35, L38

3.6. Data Analysis Procedure

After the learners submitted their solutions to the activity problems, the researcher marked each script to collect data. Each question was then analysed individually, paying particular attention to the methods learners used when attempting to solve each problem as well as how they went about carrying out the methods they chose. These results were then presented in the form of tables to get a more visual representation of the data. The data was then thoroughly studied to find common methods learners used as well as how their choices were related to the nature of each question individually.

The researcher also had an observation protocol. This consisted of a set of questions which the researcher answered through observation of the learners whilst they were solving the problems. The researcher also hoped to find any new methods used by learners which might not have been taught to them in previous grades or from textbooks.

After the learners completed the problems from both sections, the researcher then divided them into focus groups. Each focus group was then interviewed by the researcher to get the thoughts of the learners behind the selection of their strategies, as well as to allow them to verbally discuss each problem and reflect upon their solutions according to the problem-solving steps of Polya (1957). After completing analysis of the data, conclusions were made and the researcher then posed recommendations for future studies on this topic.

3.7. Presentation of Results

The observation protocol of the research was presented with answers to each of the questions the researcher aimed to answer whilst observing the learners during the activity. The different problem-solving strategies used by the learners were identified for each question and were then tabulated to determine the number of times each strategy was used. The focus group interviews and responses from learners were then transcribed and presented. This enabled the researcher to compare the thought processes of the learners to one another as well as to determine their reasons for choosing their specific strategies. It also allowed the learners to reflect on the problems. Summaries were then written to analyse the relationship between the type of problem learners were answering and the strategy they selected to solve it.

3.8. Research Instruments

This study consisted of three research instruments. The first was the test which was comprised of two sections, Section A and Section B (Appendix A and Appendix B). The second instrument was the observation protocol of the researcher (Appendix C), and the third instrument was the questionnaire which was used during the focus group interviews. The interview questionnaire consisted of two parts, one for Section A and one for Section B (Appendix D and Appendix E).

3.9. Validity and Reliability

The instrument was validated through a peer review system to ensure that the questions were fair, reliable, and compatible with the research questions being investigated. This validation was done by receiving the approval of researcher's supervisor, who is an expert researcher and who confirmed that the instruments measure what they were designed to measure. According to Eduafo (2014), if the validity of the study and its instruments are threatened, then it will not be possible to establish the relationships between variables.

3.10. Research Intervention

This intervention was applied by engaging learners in solving measurement type word problems which were open-ended in nature. Instead of first doing examples on the topic to show learners which methods they can use and how to determine which order of operations to use when solving measurement word problems, the researcher simply engaged the learners in the activities without any explanations or examples to aid them. Doing so meant that the researcher did not manipulate the learners to form any pre-activity ideas regarding which methods and strategies to use when solving word problems on this topic. Therefore, learners needed to utilise their own knowledge and solving skills to both understand each problem as well as to come up with strategies to solve each problem.

3.11. Ethical Considerations

When conducting research, it was important to implement certain measures so that ethical beliefs are respected and acknowledged. These ethics include cultures and socio-economic circumstances to prevent results which are biased and unethical.

Firstly, permission was to conduct the study was requested from the WCED. Hence, a permission letter was written (Appendix F). To protect the identity as well as integrity of the school and the learners participating in the study, the names of the learners and the school were not mentioned. Before a child could participate in a study, permission was required. Thus, permission was requested from and granted by the school principal (Refer to Appendix G). No comparisons were done in terms of ethnicity as the study respects the values of racial equality in a democratic South Africa. Thus, the ethnicity of learners was not asked. Questionnaires were answered anonymously to protect learner sensitivity.

3.12. Assumptions and Limitations

It was assumed that all learners participating in the study could read and solve Mathematics word problems independently.

In the next chapter, the findings of the study are detailed. The problem-solving strategies learners used for each question of the activity are recorded. Thereafter, the strategies will be analyses to find any possible patterns. The focus group interviews are then analysed to investigate how learners carried out their strategies as well as the conceptions and feelings of the problems.

CHAPTER 4: ANALYSIS AND RESULTS

4.1. Introduction

In this chapter, the results of the study are represented. During this study, the objective was to answer the following research questions:

- What strategies do grade 4 learners use when solving measurement type word problems?
- After choosing their strategies, how do learners go about using the strategy to arrive at their solution?

The results are presented in two sections in correspondence with the research questions. In the first section, the results address the following research question: What strategies do grade 4 learners use when solving measurement type word problems? The results represent data which was received and analysed from 42 grade 4 learners who completed a problem-solving activity on measurement type word problems.

In the second section, the results address the following research question: After choosing their strategies, how do learners go about using the strategy to arrive at their solution? These results represent data taken from an observation protocol and semi-structured focus group interviews.

4.2. Observations

SECTION A

1. Sipho has 14 pieces of rope. Each piece if rope is 8 m long. What length of rope does Sipho have altogether?

2. Kate has 100 cm of ribbon. She cuts it into 4 equal pieces. How long is each piece of ribbon?

3. Ciaran walked ½ km, Alex walked 1/5 km and Calum walked ¼ km. How far did each learner walk in metres?

4. A pencil is 130 mm long. A ruler is 3 times longer than the pencil. How long is the ruler in centimetres?

5. On the first day John drives his car a distance of 75 km. On the second day he drove 12 500 m and on the third day he drove 13 km. What is the total distance John drove in metres?

6. Mary's mass is 48 kg. Her younger sister, Paula, has a mass equal to three-quarters of Mary's mass. What is Paula's mass?

4.2.1. Observations – Section A

1. Which methods were learners using?

Learners were primarily making drawings and using arithmetic calculations. It was also interesting to observe the variety of different ways learners went about carrying out these strategies. When doing addition, some learners used the column method, some use expansion, while others used grouping.

Another fascinating observation was that some learners chose to use addition while others use multiplication to solve the same problem. For example, in Problem 1 some learners did a multiplication procedure of 14 x 8m = 112m, while others added eight 14s together or fourteen 8s together.

Moreover, some learners made drawings to aid them their problem-solving. Learners tended to draw either sticks or circles and then used grouping to add them or divide them equally.

2. Did learners understand what the problem required of them to do?

Problem 1

Sipho has 14 pieces of rope. Each piece if rope is 8 m long. What length of rope does Sipho have altogether?

Most learners recognised that they needed to find the product of 8 and 14. However, the challenge for some learners lay in carrying out their strategy. Many of them had a good plan and strategy, but they made errors in counting which resulted in them having incorrect answers. On the contrary, there were also a few learners who did not understand what the problem required of them. As a result, they were simply guessing what they should do.

Problem 2

Kate has 100 cm of ribbon. She cuts it into 4 equal pieces. How long is each piece of ribbon?

Most learners comprehended this problem. They recognised that they needed to divide 100cm equally into 4 pieces. Some learners used arithmetic division, while others made drawings. There were even a few learners who were observed to be working backwards and adding a number together 4 times until they found the number which when added together 4 times, came to 100.

Problem 3

Ciaran walked ½ km, Alex walked 1/5 km and Calum walked ¼ km. How far did each learner walk in metres?

The purpose of this problem was to assess the ability of learners to be able to work with measurements which are written in fraction form.

It seemed that many learners found this problem very challenging. Many of them were confused because all the distances in the problem were given as fractions. Learners seemed to struggle when working with fractions. Another common mistake which was observed was that many learners forgot that there are 1000 metres in 1 kilometre. As a result, they were not able to convert the fractional distances into whole numbers.

Problem 4

A pencil is 130 mm long. A ruler is 3 times longer than the pencil. How long is the ruler in centimetres?

Learners appeared to be doing well in this problem. It seemed that the word "times" in the problem had been identified as the keyword which learners recognised to signify a multiplication procedure. Interestingly, instead of saying 130mm x 3 = 390mm, some learners also used repeated addition (130mm + 130mm + 130mm = 390m). However, a common error being made was that several learners did not convert their answer to centimetres as was required. This could be due to learners not being accustomed to being asked to do an additional step after getting to their answer. Therefore, many of them assumed that their work was complete when getting their answer in millimetres.

Problem 5

On the first day John drives his car a distance of 75 km. On the second day he drives 12 500 m and on the third day he drives 13 km. What is the total distance driven in metres?

Learners seemed to be aware that they needed to add the three distances together. It was observed that most learners were using addition strategies to do so. However, some learners seemed to be making errors when converting the distances from kilometres to metres, while others were adding them without making conversions resulting in them adding distances in metres with distances in kilometres.

Problem 6

Mary's mass is 48 kg. Her younger sister, Paula, has a mass equal to three-quarters of Mary's mass. What is Paula's mass?

Like Problem 3, it seemed learners were finding this problem very challenging. Again, the problem involved fractions, and this is a topic many learners struggle with. It seemed many learners recognised the first step of finding one quarter of 48kg. However, not many of them were able to do the next step of then finding 3 quarters.

3. Which problems did learners find easy to solve?

Learners were generally able to do Problem 1. Although many of them made errors in calculations which yielded incorrect answers, most of them at least recognised and comprehended what the problem required of them to do. Thus, they came up with good strategies. The format of the problem was also similar to those found in textbooks. They were told that each rope was 8cm in length, and that there were 14 pieces of these ropes in total. Therefore, they found it quite straightforward to visualise the problem and to recognize that they needed to use multiplication or repeated addition.

4. Which problems did learners find challenging?

Problem 3 and Problem 6 were the two problems which learners were finding very challenging. Both problems involved fractions, and it seemed learners were not very comfortable working with fractions yet. In grade 4, learners were taught fractions using a fraction wall as a visual aid. However, they were not given any visual aids for these two problems. As a result, it was more challenging for them to comprehend and compare different fractions.

5. Which items of stationery were learners using to solve the problems?

Learners appeared to only be making use of their pencils. They were instructed to show all their calculations as well as their detours and mistakes. Thus, they did not use erasers. A few learners made use of rulers when solving Problem 4. This problem involved finding the length of a ruler by increasing the length of a pencil by a scale of 3.

4.3. Observations

4.3.1 Observations - Section B

1. **Which methods were learners using?**

Learners used mostly arithmetic calculations in this section. It appears that the second most frequently used strategy is the making of drawings. Interestingly, in Question 4 and Question 5 which are accompanied by drawings, many learners are making their own drawings of these problems. Some are simply re-drawing the image in the question, while others are making more simplified versions to make the problem easier for them. It was also observed that a few learners draw tables and use grouping methods to solve the problems.

2. **Did learners understand what the problem required of them to do?**

Problem 1

408 L of water is poured into 4 containers so that each container holds the same amount. How much water is in each container?

Most learners understood that this problem required them to divide 408 litres of water equally into 4 containers.

Problem 2

A drum contains 70 L of petrol. The petrol tank of my small car can hold 6 L of petrol. How many times can I fill the tank from the drum and what amount is finally left in the drum?

Some learners knew what the problem required of them to do, but most of them seemed to be confused by this problem. Furthermore, those who knew what to do carried out their strategies, but they seemed to be thrown off by the fact that there was a remainder in this answer. This concept was still very new to grade 4 learners. In grade 4, learners were re-introduced to division where the dividend was always perfectly divisible by the divisor. They were only introduced to division problems which have remainders later in the academic year. Therefore, when learners realised that there was still a remainder left after solving the problem, they began to doubt their work.

Problem 3

How many 500 ml bottles can be filled from a can containing 4.5 L of water?

This problem was similar in principle to Problem 5. Thus, there were many learners who seemed unsure of what they needed to do. The main issue learners appeared to have with this problem was that the two quantities given in the problem were in different units of measurement. Learners were accustomed to working with measurement problems in which all the quantities were in the same unit of measurement, or the numbers were much smaller when given in different units of measurement. They, therefore, needed to perform a conversion to have both quantities in the same unit of measurement, and many learners were unable to execute this part. Furthermore, these numbers were quite big, which made it more intimidating for some learners to work with.

Problem 4a

Use the dimensions given in the drawing to find:

The area of the shaded part in mm^2

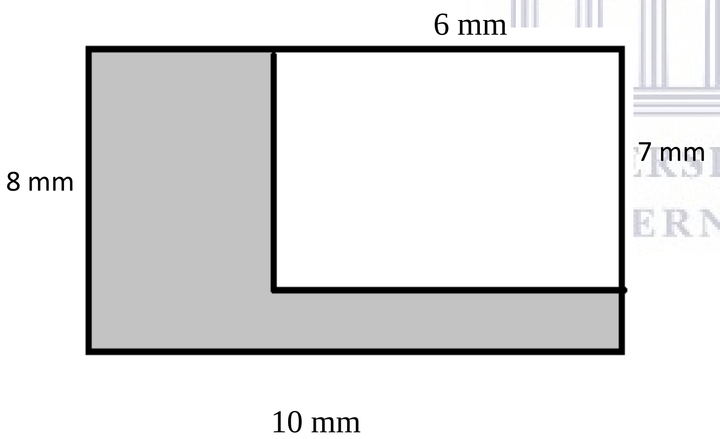


Figure 10: Shaded and unshaded parts of rectangle

Learners generally understood that the problem required them to find the area of a particular part of the shape. However, the shaded part was neither a square, nor a rectangle as these were the only two shapes they have learned about thus far with regards to area. Many of the learners were able to recognise that they could break the shape up into two rectangles and find the area this way. Some succeeded at doing so, while others got incorrect answers simply because of calculation errors they made along the way.

Problem 4b

Use the dimensions given in the drawing to find:

The perimeter of the shaded part

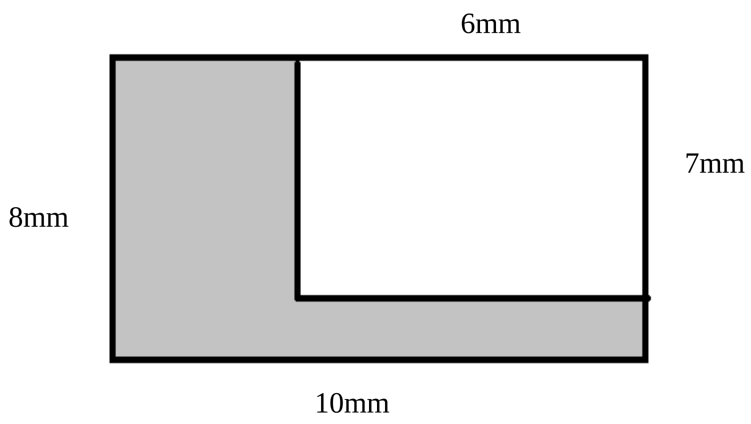


Figure 11: Shaded and unshaded parts of rectangle

Learners were familiar with the concept of perimeter. Thus, most of them seemed to easily understand that they needed to add the dimensions of all the sides of the shaded shape to find its perimeter.

Problem 5

Calculate the area of the shaded part of the diagram

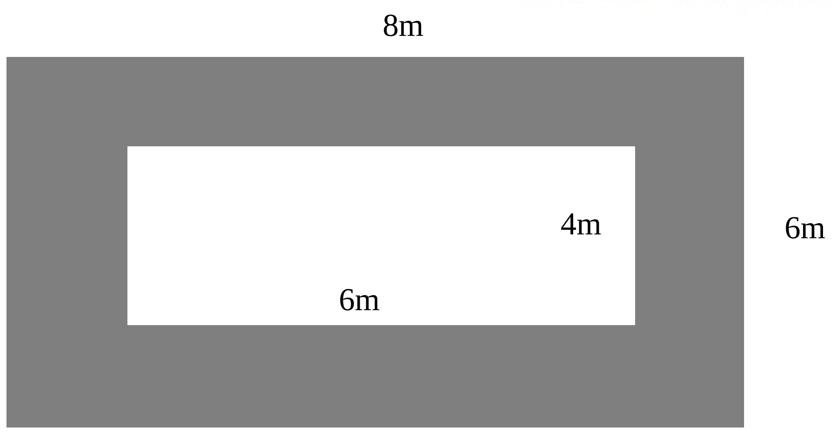


Figure 12: Shaded and ushaded parts of rectangle

This problem was similar in nature to Problem 4a. Learners, thus, understood that they needed to determine the area of the given shape. However, as with Problem 4a, the shaded shape in focus was neither a square nor a rectangle. Learners recognised that they could break the shape up into 4 rectangles and calculate the area of each one. A few learners also recognised there was another method in which they could subtract the area of the inner rectangle of the overall shape from the area of the outer rectangle, giving them the area of the shaded shape.

1. **Which problems did learners find easy to solve?**

Problem 1 seemed to be the problem that learners found easiest to do. This is possibly because it was a straight-forward problem that did not require any conversions. The problem was also written in a format which was familiar to them as it was similar to problems they encountered in textbooks. Furthermore, there was also a phrase in the problem which they were able to identify as a sight phrase for division: "so that each container holds the same amount."

2. **Which problems did learners find very challenging?**

Problem 4a and Problem 5 were the two problems which learners found the most challenging. This is possibly because they needed to determine the area of compound shapes, which is not what they are accustomed to doing yet. They had only recently been introduced to perimeter and area of 2D shapes. Moreover, up to now they had only worked with simple shapes. Therefore, when learners looked at the diagrams given for the area and perimeter problems, they were trying to identify the shape of the diagram to match it to its corresponding formulae. However, the shapes in these two problems were compound shapes (shapes which are constructed using 2 or more 2D shapes). This resulted in many learners becoming confused as they were encountering a diagram which did not appear to be in a form they had learned about before.

4.4. Research Question 1:

What strategies do grade 4 learners use when solving measurement type word problems?

NB: When conducting the focus group interviews, learners were assigned numbers and they were referred to according to their numbers during this study. For example, a learner who was assigned number 1 was referred to as L1.

4.4.1. Analysis of the problem-solving strategies used by the learners in Section A

Problem 1

Sipho has 14 pieces of rope. Each piece if rope is 8 m long. What length of rope does Sipho have altogether?

Table 2: Problem solving strategies used by learners to solve Problem 1

addition				multiplication		grouping		Making a drawing only	solutions	
addition only	making a drawing	grouping	multiplication	multiplication only	division	grouping only	making a drawing		38	
12	2	4	1	9	1	1	7		correct	incorrect
19				10		8		1	12	26

Table 2 indicates that 19 learners used addition strategies to solve to problem. Out of these 19 addition strategies, 12 solutions contained only addition, 2 solutions combined addition with making drawings, 4 solutions combined addition with grouping, while 1 solution combined addition with multiplication.

Furthermore, Table 2 shows that 9 solutions contained only multiplication strategies, while 1 solution made use of a combination of multiplication and division.

There were 7 solutions that combined grouping and making drawings, while 1 solution used only grouping as a strategy. One solution used the making of drawings as a strategy.

Table 2 also indicates that out of the 38 solutions for Problem 1, twelve solutions were correct while 26 were incorrect.

Most learners stated that they understood the problem when asked. The questions posed to learners are written in bold capital letters.

Did you understand what you needed to do?

L8: You must plus. You must add all the pieces of rope.

L15: You must add the numbers.

L14: You must multiply by 3.

L8: You must plus.

L28's strategy of using only addition can be seen in Figure 13.

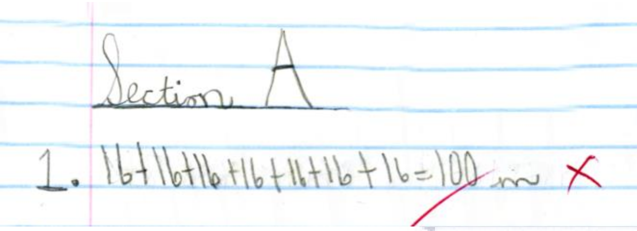


Figure 13: The addition strategy used by L28 when solving Problem 1

L28 said she understood the problem, but she also found it challenging because the numbers were big. Instead of adding 14 groups of 8, L28 used 7 groups of 16 explaining that doing it this way meant she had less numbers to add.

What made you out them in groups of 16 and add them like that?

L28: It was easier for me to count.

L15's strategy of combining addition and multiplication can be seen in Figure 14.

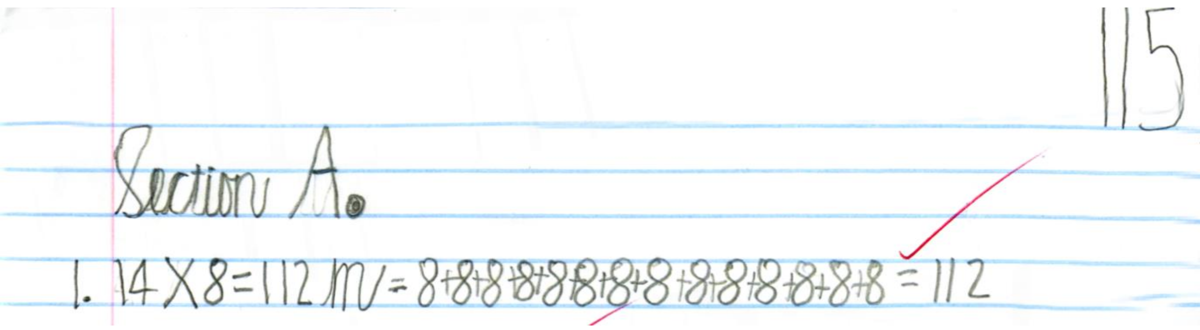


Figure 14: L15 using multiplication as well as addition when solving Problem 1

Figure 14 shows that learner L15 first used multiplication (14 x 8 = 112m), then checked the solution by using addition. Addition was carried out by adding 14 groups of 8. L15 got the correct answer.

L40’s strategy of combining grouping with making drawings can be seen in Figure 15.

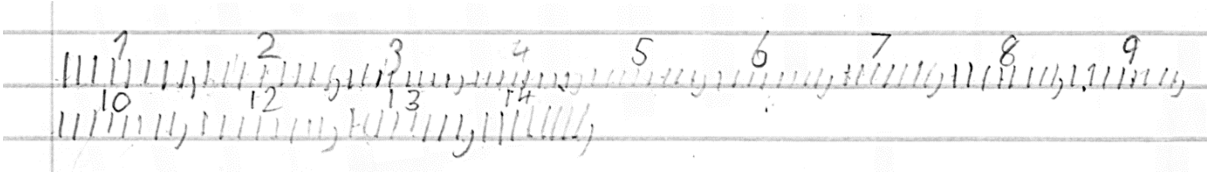


Figure 15: L40 drawing stick shapes in groups when solving Problem 1

Figure 15 shows that L40 made stick drawings, then grouped them together in groups of 8. However, the learner arrived at an incorrect answer. This is possibly due to the first group only containing 7 sticks instead of 8. The drawing was therefore inaccurate.

L34’s strategy of using only addition can be seen in Figure 16.

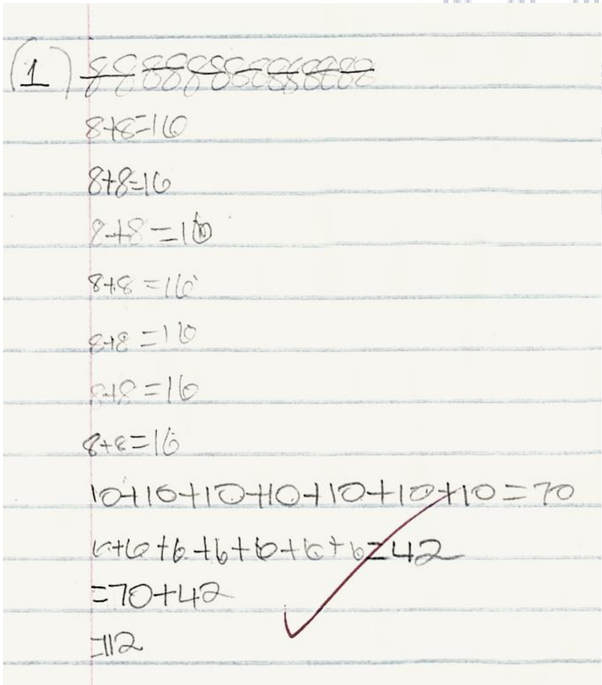


Figure 16: L34 using addition when solving Problem 1

Figure 16 shows that L34 used a different method of addition to that of learner 15. This learner did 7 sums of 8+8 = 16. After doing so, learner 34 then added the 16s together by breaking them up into tens and units first.

L21’s strategy of using only multiplication can be seen in Figure 17.

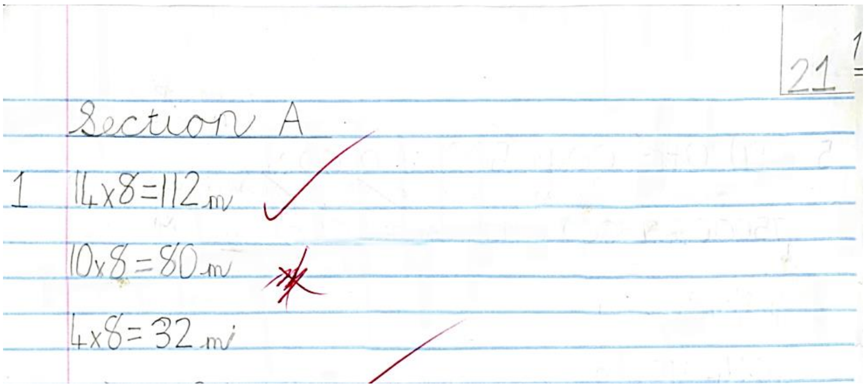


Figure 17: L21 using multiplication when solving Problem 1

Figure 17 shows that L21 used multiplication by breaking up the 14 into tens and units (10+4). The learner then multiplied 10 by 8, then 4 by 8, and finally added the two answers together to get the final answer. L21 got the correct answer.

Problem 2

Kate has 100 cm of ribbon. She cuts it into 4 equal pieces. How long is each piece of ribbon?

Table 3: Problem solving strategies used by learners to solve Problem 2

<table>
<tr><td colspan="3">addition</td><td colspan="2">division</td><td rowspan="3">subtraction only</td><td colspan="2">working backwards</td><td rowspan="3">grouping</td><td rowspan="3">guessing</td><td colspan="2">solutions</td></tr>
<tr><td>addition only</td><td>making a drawing</td><td>division</td><td>division only</td><td>making a drawing</td><td>addition</td><td>addition and division</td><td colspan="2">38</td></tr>
<tr><td>3</td><td>1</td><td>1</td><td>15</td><td>4</td><td>3</td><td>2</td><td>correct</td><td>incorrect</td></tr>
<tr><td colspan="3">5</td><td colspan="2">19</td><td>3</td><td colspan="2">5</td><td>2</td><td>4</td><td>24</td><td>14</td></tr>
</table>

Table 3 shows that 3 solutions contained only addition strategies, while 1 solution combined addition with making a drawing, and 1 solution combined addition with division.

Furthermore, Table 3 indicates that 15 solutions used only division strategies, while 4 solutions combined division with making a drawing. Three solutions used only subtraction as a strategy.

Table 3 also shows that 3 solutions combined working backwards with addition strategies, while 2 solutions combined working backwards with addition and division. Two solutions used grouping strategies while 4 solutions used guessing.

According to Table 3, out of the 38 solutions for Problem 2, twenty-four were correct and 14 were incorrect.

When asked if they understood the problem, learners generally had a sound conception of the problem and how they needed to go about solving it:

Did you understand what you needed to do?

L33: I added all of this (the numbers) together.

L6: I divided into 4 equal parts.

L41: Yes sir.

Tell me the problem in your own words

L34: You must find the pieces of rope (length).

L6: You must divide the 100cm.

L4’s strategy of combining division with making a drawing can be seen in Figure 18.

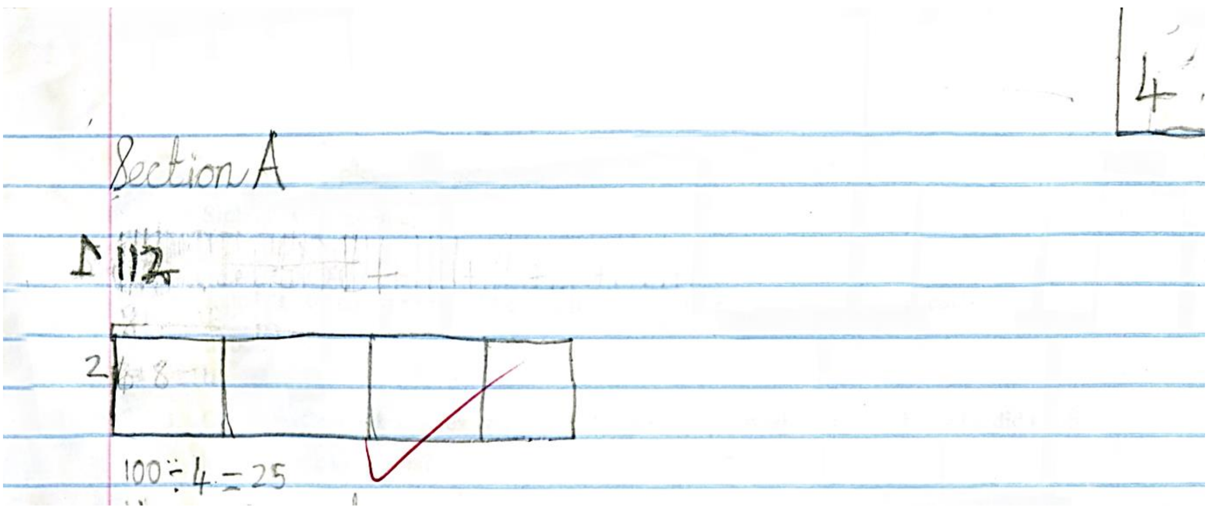


Figure 18: L4 making a drawing when solving Problem 2

Figure 18 shows a drawing made by L4 and combining it with a division strategy. From drawing a rectangle and then dividing it into 4 parts, the learner realises that the problem requires the 100 cm of ribbon to be divided into 4 equal parts. The drawing thus aided the learner in realising what operation needed to be used. L4 got the correct answer.

L2's strategy of combining working backwards and addition can be seen in Figure 19.

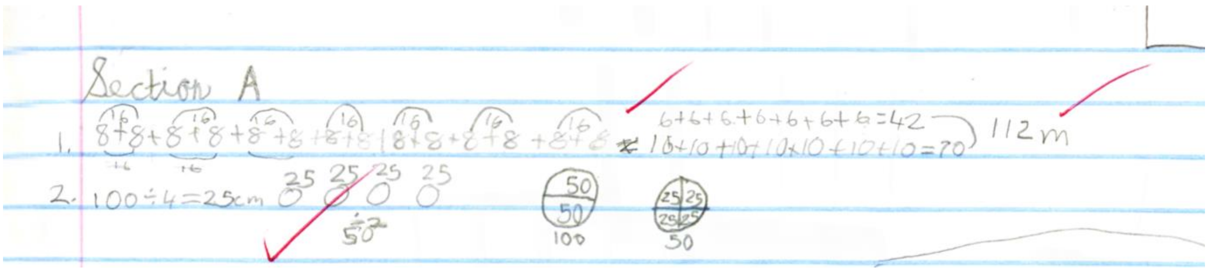


Figure 19: L2 working backwards when solving Problem 2

Figure 19 shows that L2 worked backwards. The learner started with the solution, then worked backwards using addition. Once the learner got an answer of 25cm, division was used to verify the solution. L2 got the correct answer.

Problem 3

Ciaran walked ½ km, Alex walked 1/5 km and Calum walked ¼ km. How far did each learner walk in metres?

Table 4: Problem solving strategies used by learners to solve Problem 3

addition		division		subtraction		logical reasoning only	making a table only	guessing	solutions	
addition only	logical reasoning	division only	logical reasoning	subtraction only	logical reasoning				38	
5	1	8	3	3	1				correct	incorrect
6		11		4		11	2	4	7	31

Table 4 indicates that 5 solutions used only addition strategies, while 1 solution combined addition with logical reasoning. Furthermore, 8 solutions used only division strategies, while 1 solution combined division with logical reasoning. Eleven solutions used only logical reasoning as a problem-solving strategy.

Table 4 also shows that 2 solutions only made use of drawing of tables, while 4 solutions used guessing strategies. According to Table 4, out of the 38 solutions, seven were correct and 31 were incorrect.

L37's strategy of making a table can be seen in Figure 20.

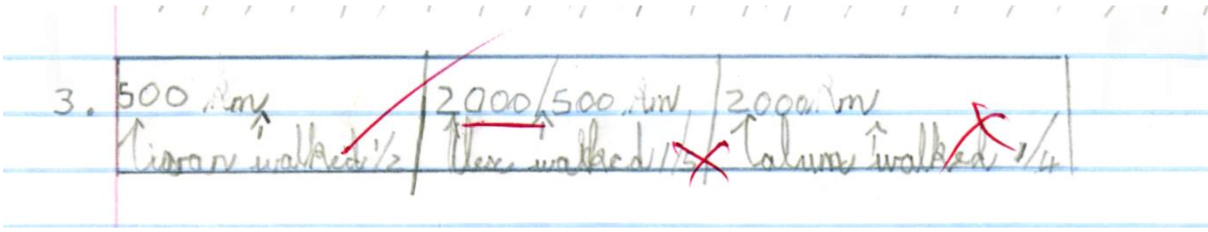


Figure 20: L37 drawing a table when solving Problem 3

Figure 20 shows that L37 drew a table to organise the information given about each child in the problem. The learner got the correct distance for the first child, but not for the second and third children.

L21's strategy of using division can be seen in Figure 21.

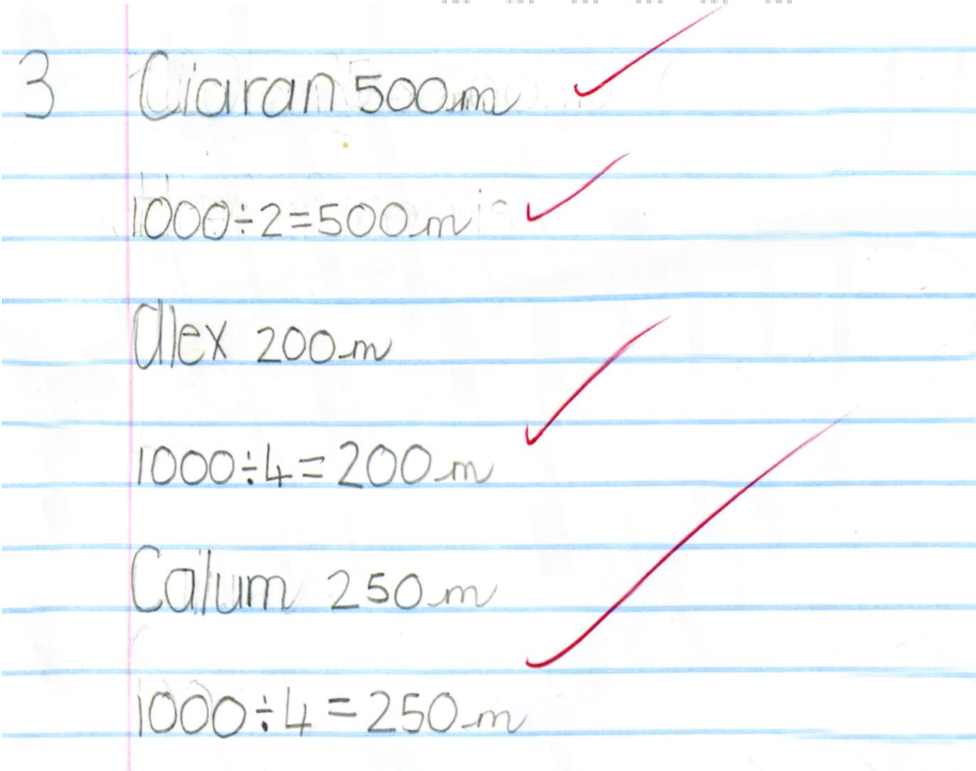


Figure 21: L21 using division when solving Problem 3

Figure 21 shows that L21 used division to get the distances travelled by each child in metres. L21 got the correct distances in metres for all 3 children.

Problem 4

A pencil is 130 mm long. A ruler is 3 times longer than the pencil. How long is the ruler in centimetres?

Table 5: Problem solving strategies used by learners to solve Problem 4

addition only	multiplication only	division only	guessing	solutions 33	
				correct	incorrect
10	17	4	2	8	25

Table 5 indicates that 10 solutions used only addition strategies, 17 solutions used only multiplication strategies, while 4 solutions used only division strategies. Furthermore, 2 solutions used guessing as a strategy. According to Table 5, out of the 33 solutions, eight were correct and 25 were incorrect.

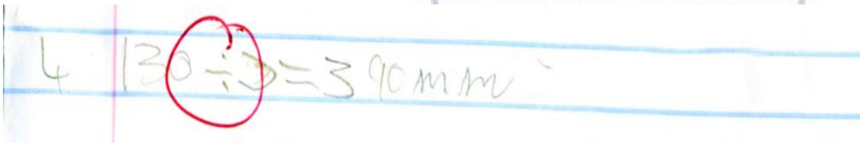


Figure 22: L12 using the division symbol when performing multiplication

Figure 22 shows that L12 used the division symbol instead of the multiplication symbol but got the correct final answer. This indicates that L12 had a misconception regarding what the operational symbols are for division and multiplication. When reflecting on this, L12 recognised the mistake:

L12, you said divide by 3 but you got the right answer, did you make a mistake by writing divide?

L12: Yes sir, I'm not supposed to divide.

L9’s addition strategy can be seen in Figure 23.

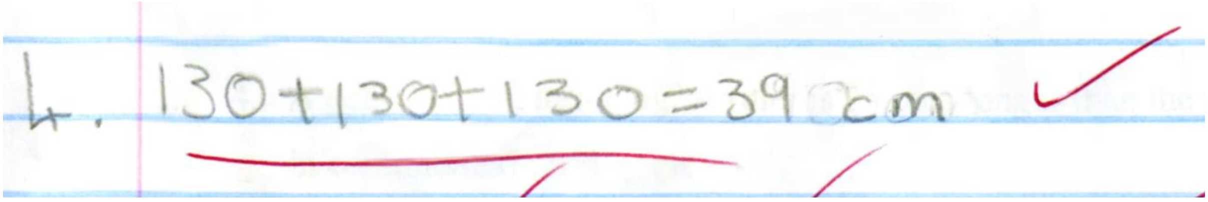


Figure 23: L9 using addition when solving Problem 4

Figure 23 shows that L9 added 130mm three times. L9 then wrote the answer immediately in centimetres without showing any conversions.

L34’s multiplication strategy can be seen in Figure 24.

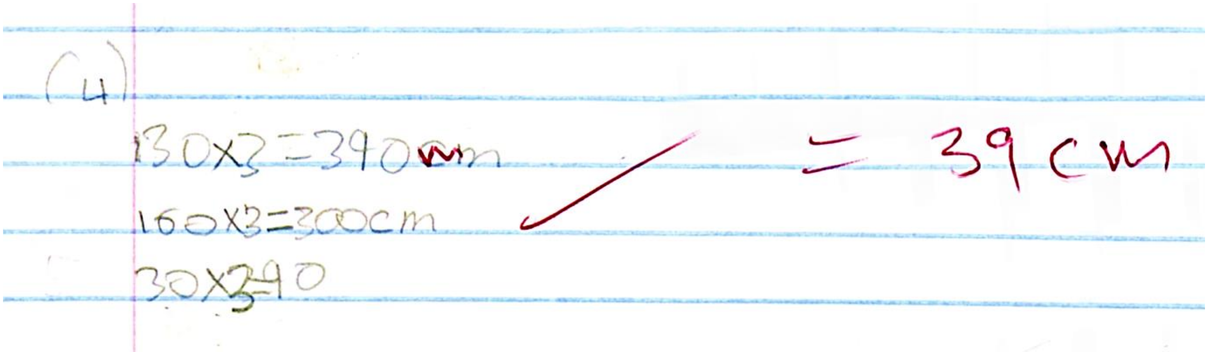


Figure 24: L34 using multiplication when solving Problem 4

Figure 24 shows that L34 used an expansion method to multiply the 130mm by 3 and get 390mm. The learner decomposed the 130 into 100 and 30, then multiplied each by 3 before adding the two answers to get 390mm. The learner did not convert the final answer to centimetres as the problem requested.

L11’s multiplication strategy can be seen in Figure 25.

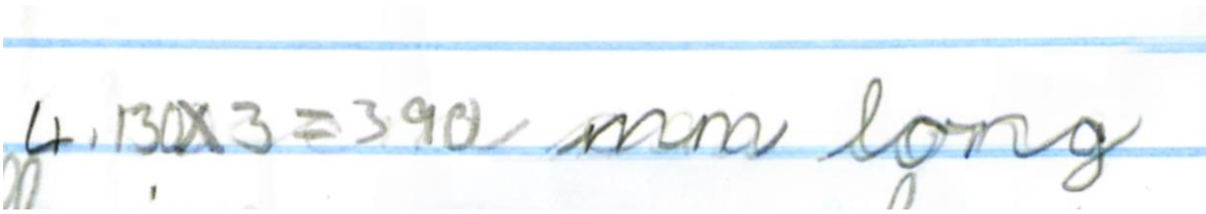


Figure 25: L11 using multiplication when solving Problem 4

Figure 25 shows that L11 used multiplication without any expansion. The learner multiplied the 130mm by 3 to get 390mm. However, the learner did not convert the answer to centimetres as was required in the question.

Problem 5

On the first day John drives his car a distance of 75 km. On the second day he drives 12 500 m and on the third day he drives 13 km. What is the total distance driven metres?

Table 6: Problem solving strategies used by learners to solve Problem 5

addition only	multiplication only	guessing	solutions 33	
			correct	incorrect
29	1	3	8	25

Table 6 indicates that 29 solutions contained addition strategies, 1 solution contained multiplication strategies, while 3 solutions contained guessing strategies. According to Table 6, out of the 33 solutions, eight were correct and 25 were incorrect.

Learners tended to struggle with Problem 5, with many of them stating that they found it particularly challenging:

Did you understand what you needed to do?

L29: No sir

L3: No sir

L26: Yes

L22: No sir

L20’s addition strategy can be seen in Figure 26.

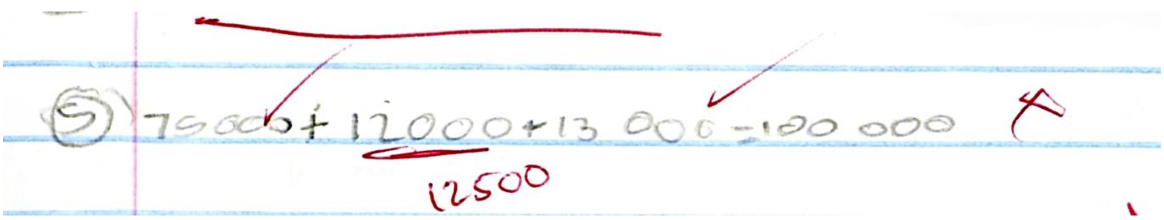


Figure 26: L20 using addition when solving Problem 5

Figure 26 shows that L20 used addition to add the three distances. Thereafter, L20 wrote down the distances in metres and then added them. However, L20 an error by writing down 12 000m instead of 12 500m. As a result, the answer was incorrect.

L29's addition strategy can be seen in Figure 27.

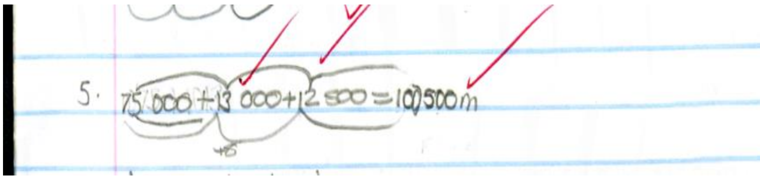


Figure 27: L29 used addition to solve Problem 5

Figure 27 shows that this learner was able to add the three distances without needing any calculations. The learner mentally converted all the distances to metres, then added them. L29's explained the method:

L29, you have a very different method, what do the lines mean?

L29: I plus (added) the thousands and then I added that (Explaining being aware that because all three distances were multiples of 1000, they could be added without the need to work it out on paper).

Problem 6

Mary's mass is 48 kg. Her younger sister, Paula, has a mass equal to three-quarters of Mary's mass. What is Paula's mass?

Table 7: Problem solving strategies used by learners to solve Problem 6

addition				multiplication only	division only	guessing	solutions	
addition only	multiplication	division	making a table				33	
6	1	5	2				correct	incorrect
14				2	10	7	11	22

Table 7 indicates that 14 solutions contained addition strategies. Of these 14 solutions, 6 contained only addition strategies, 1 combined addition and multiplication strategies, 5 combined addition and division strategies, while 2 combined addition strategies with making a table.

Furthermore, 2 solutions used only multiplication strategies, 10 solutions used only divisions strategies, while 7 solutions made use of guessing. According to Table 7, out of the 33 solutions, eleven were correct and 22 were incorrect.

Learners found it challenging to carry out division operations. This is evident in the feedback given by learners from Group 3:

Challenges and struggles

L37: To divide.

L19: I struggled with number 6.

What was difficult about number 6?

L10: To get the answer.

L42: To get the answer.

Why?

L1: Because you had to divide.

And you find dividing challenging?

L1: Yes sir.

L37: Yes.

L28's strategy of combining division and addition can be seen in Figure 28.

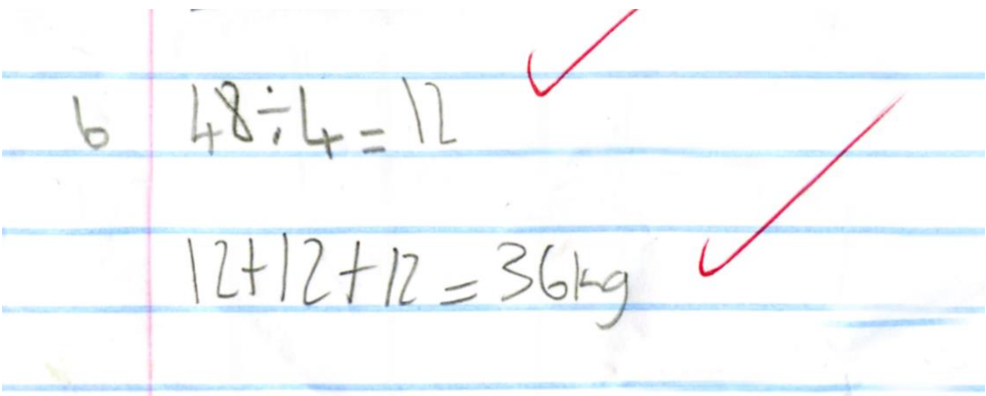


Figure 28: L28 using both division and addition when solving Problem 6

Figure 28 shows that L28 divided Mary's mass of 48kg by 4 to get 12kg. After finding one quarter of Mary's mass, L28 then added three masses of 12kg together to get three quarters of Mary's mass which was 36kg. This was the correct answer.

L13's strategy of combining addition with making a table can be seen in Figure 29.

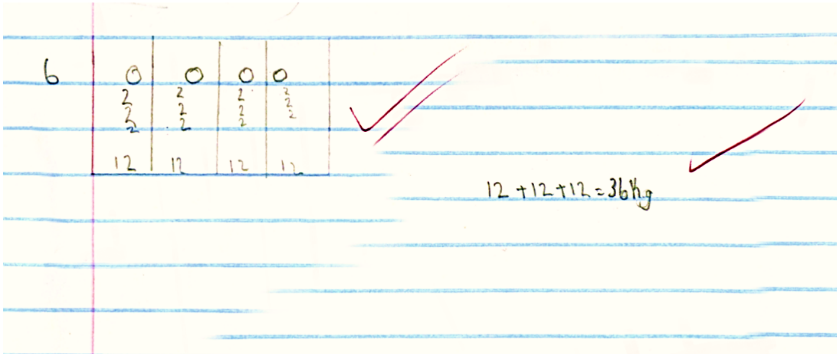


Figure 29: L13 constructing a table when solving Problem 6

Figure 29 shows that L13 also made use of a table. L13 tabulated four groups of 12 for each 12 to be a quarter of the total mess. Recognising that the problem required the learner to find three quarters of the sister's mass, L13 added three of the 12s together to get the correct answer of 36kg.

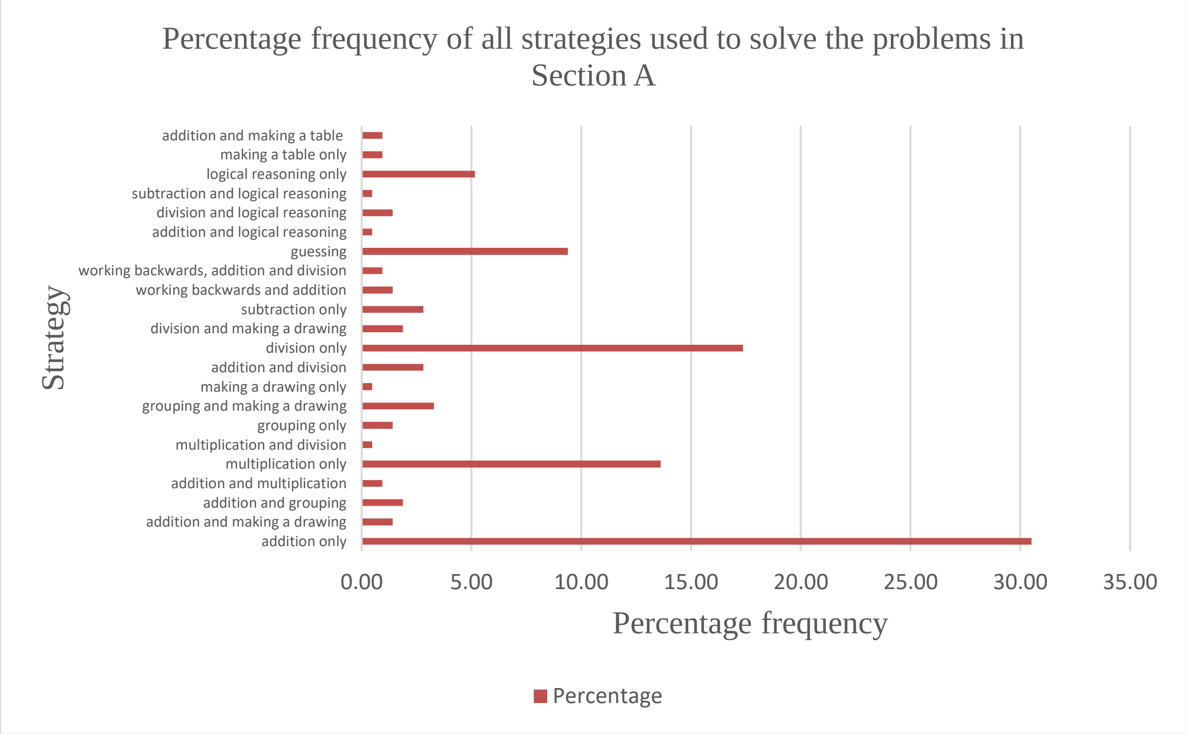


Figure 30: Percentage frequency of all strategies used to solve the problems in Section A

Figure 30 shows that students used a variety of problem-solving strategies to solve the problems in Section A. According to Figure 30, strategies using only addition were found in 30,52% of all solutions. Strategies made up of only division were found in 17.37% of all solutions, while 13,62% of all solutions contained only multiplication strategies.

An interesting finding was the usage of strategies which were not taught in the classroom, such as logical reasoning strategies which were found in 5,16% of all solutions. Even more intriguing was the variety of combination strategies which were used. Strategies such as combining addition with making a drawing, and combining addition with working backwards, each appeared in 1.41% of all solutions. This indicates that these learners were very innovative and that they recognised the need to use different aspects of mathematics to solve a problem.

4.4.2. Analysis of the problem-solving strategies used by the learners in Section B

Problem 1

408 L of water is poured into 4 containers so that each container holds the same amount. How much water is in each container?

Table 8: Problem solving strategies used by learners to solve Problem 1

addition			division			making a drawing only	guessing	solutions	
addition only	division	making a drawing	division only	making a drawing	grouping			39	
2	1	1	23	5	3			correct	incorrect
4			31			3	1	23	16

Table 8 indicates that out of the 4 solutions that contained addition strategies, 2 of these solutions used only addition strategies, while 1 solution used a combination of addition and division, and 1 solution combined addition with making a drawing.

Table 8 also indicates that out of the 31 solutions which contained division strategies, 23 of these solutions used only division strategies, while 5 solutions combined divisions with making a drawing, and 3 solutions combined division with grouping strategies.

Furthermore, Table 8 shows that 3 solutions used only drawings as strategies while 1 solution used guessing. Out of the 39 solutions for this problem, twenty-three were correct and 16 were incorrect.

L5's strategy of combining addition and division can be seen in Figure 31.

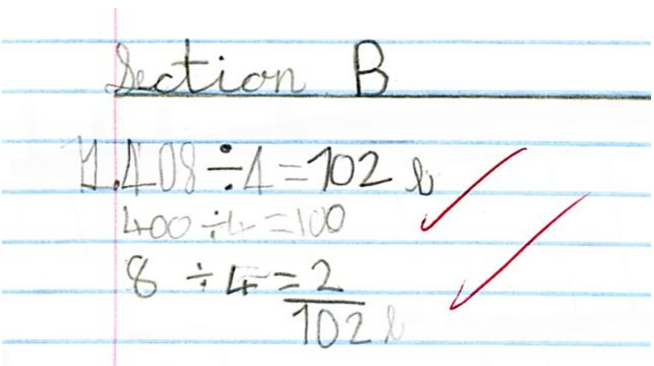


Figure 31: L5 using division and addition when solving Problem 1

Figure 31 shows that the L5 used a form of breaking up or expansion to divide the 408 litres into 4 equal quantities. L5 expanded the 408 into 400 + 8, then divided each value separately by 4 and then adding the two answers together to get 102 litres. This was the correct answer.

L36's division strategy can be seen in Figure 32.

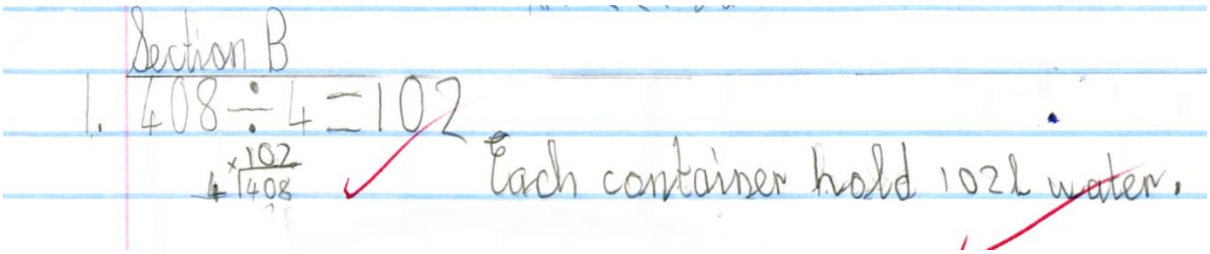


Figure 32: L36 using division when solving Problem 1

Figure 32 shows that L36 used short division as the strategy to solve this problem. L36 got the correct answer.

L19's strategy of combining division and making a drawing can be seen in Figure 33.

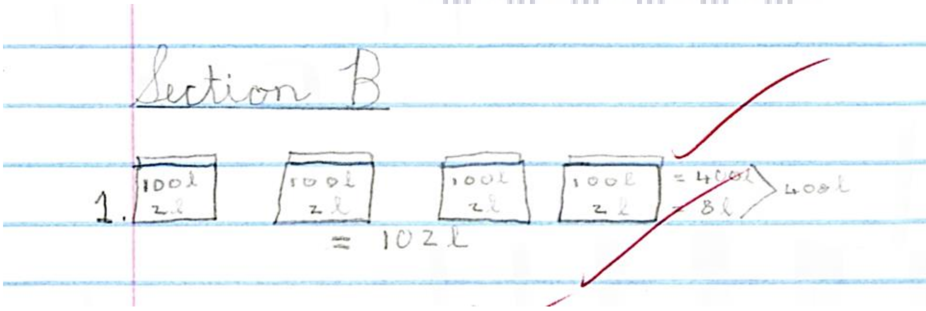


Figure 33: L19 making a drawing and using division when solving Problem 1

Figure 33 shows that learner L19 made drawings to illustrate the containers, splitting the 408 litres equally into 4 groups. L19 got the correct answer.

Problem 2

A drum contains 70 L of petrol. The petrol tank of my small car can hold 6 L of petrol. How many times can I fill the tank from the drum and what amount is finally left in the drum?

Table 9: Problem solving strategies used by learners to solve Problem 2

addition		subtraction		multiplication		division		guessing	solutions	
addition only	division and making a drawing	subtraction only	grouping	multiplication only	making a drawing	division only	making a drawing		39	
1	1	6	1	1	1	25	2		correct	incorrect
2		7		2		27		1	13	26

Table 9 indicates that out of the 2 solutions that contained addition strategies, 1 solution used only addition while 1 solution combined addition with division and making drawings. Table 9 also indicates that out of the 7 solutions that contained subtraction strategies, 6 of these solutions used only subtraction strategies while 1 solution combined subtraction with grouping.

Furthermore, Table 9 shows that out of the 2 solutions that contained multiplication strategies, 1 of these solutions used only multiplication while 1 solution combined multiplication with making a drawing. Table 9 also indicates that 8 solutions used only division strategies, while 2 solutions combined division strategies with making a drawing. One solution used guessing.

According to Table 9, out of the 39 solutions for the problem, thirteen were correct and 26 were incorrect.

L5’s subtraction strategy can be seen in Figure 34.

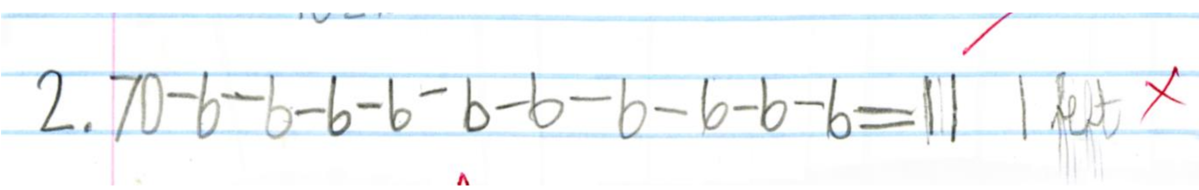


Figure 34: L5 using subtraction when solving Problem 2

Figure 34 shows that L5 subtracted 6 from 70 continuously until it was not possible anymore. L5 then counted a total of eleven 6's which were subtracted. This was correct, but L5 made an error when calculating the remainder.

L2's strategy of combining drawings with multiplication can be seen in Figure 35.

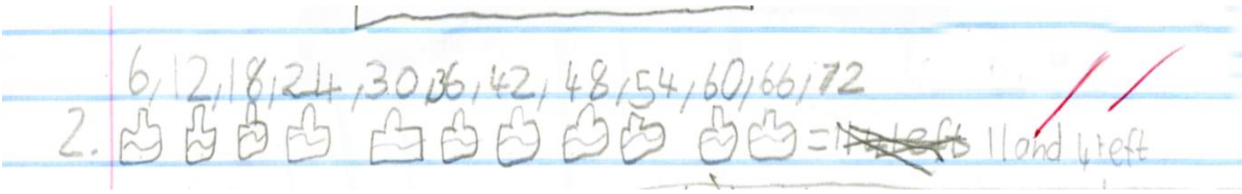


Figure 35: L2 using multiplication and drawings when solving Problem 2

Figure 35 shows that the L2 counted in sixes (used the 6 times table) to solve the problem. When L2 got to 72, the learner realised that this was too much, then went back by one multiple to end on 66. Thereafter, L2 counted eleven whole sixes and a remainder of 4, as 66 was 4 less than 70. This was the correct answer.

There was a general understanding amongst the learners with regards to what the problem required them to do. They realised that they needed to divide 100cm of rope into 4 equal parts. L9 divided the 100cm into 2, then divided the 50cm into 2 again. To aid her in this process, she drew a block to represent the 100cm piece of rope and then halved it twice. Similarly, L34 started off by drawing blocks, and after some trial and error realised that the 100cm must be halved twice. Similarly, L41 divided 100cm by 2, and then divided the 50cm by 2 again to arrive at the solution. When asked to explain why they used their respective methods, learners explained that they used the method which they felt was the easiest for them:

What made you decide to choose this method?

L9: It's easier.

L41: I halved the 100 into 50s, and then I halved the 50s into 25s.

So, you found it easier that way?

L41: Yes sir.

L6: I saw that it’s easier this way.

Problem 3

How many 500 ml bottles can be filled from a can containing 4.5 L of water?

Table 10: Problem solving strategies used by learners to solve Problem 3

addition		subtraction only	multiplication only	division only	solutions	
addition only	working backwards				33	
4	1				correct	incorrect
5		3	2	23	8	25

Table 10 indicates that out of the 5 solutions that contained addition strategies, 4 of these solutions used only addition strategies while 1 solution combined addition with working backwards. Furthermore, Table 10 shows that 3 solutions used only subtraction strategies, 2 solutions used only multiplication strategies, while 23 solutions used only division strategies.

According to Table 10, out of the 33 solutions for the problem, eight were correct and 25 were incorrect.

L23’s division strategy can be seen in Figure 36.

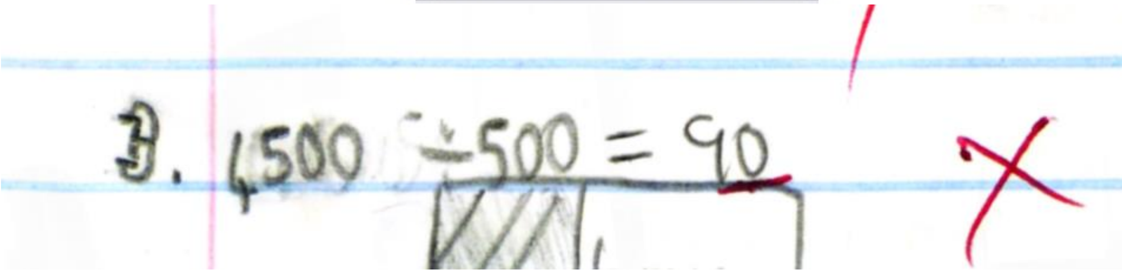


Figure 36: L23 using division when solving Problem 3

Figure 36 shows that the L23 used division to solve the problem. L23 recognised that there are 4500ml in 4.5 litres. Thereafter, L23 divided 4500 by 500. However, L23 made an arithmetical error which resulted in a final answer of 90 instead of 9.

Problem 4a

Use the dimensions given in the drawing to find:

The area of the shaded part in mm^2

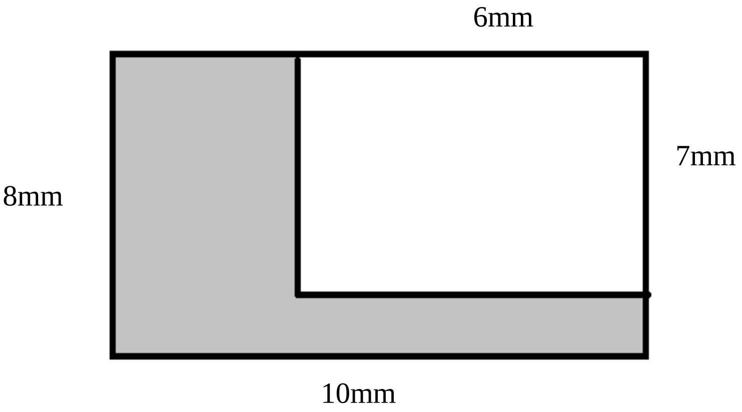


Figure 37: Dimensions of the rectangle

Table 11: Problem solving strategies used by learners to solve Problem 4a

<table>
<tr><td colspan="7">multiplication</td><td rowspan="3">division and making a drawing</td><td rowspan="3">making a drawing only</td><td colspan="2">solutions</td></tr>
<tr><td rowspan="2">multiplication only</td><td rowspan="2">addition</td><td rowspan="2">making a drawing</td><td rowspan="2">addition and making a drawing</td><td rowspan="2">addition and breaking up the problem</td><td rowspan="2">making a drawing and breaking up the problem</td><td rowspan="2">addition, making a drawing and breaking up the problem</td><td colspan="2">37</td></tr>
<tr><td>correct</td><td>incorrect</td></tr>
<tr><td>3</td><td>2</td><td>7</td><td>1</td><td>2</td><td>7</td><td>5</td><td rowspan="2">1</td><td rowspan="2">9</td><td rowspan="2">7</td><td rowspan="2">30</td></tr>
<tr><td colspan="7">27</td></tr>
</table>

Table 11 indicates that 27 solutions used multiplication strategies to solve the problem. Of these 27 solutions, 3 solutions used only multiplication strategies, 2 solutions combined multiplication with addition, 7 solutions combined multiplication with making a drawing, while 1 solution combined multiplication with addition strategies and making a drawing.

Furthermore, 2 solutions combined multiplication with addition and breaking up the problem, 7 solutions combined multiplication with making a drawing and breaking up the problem, while 5 solutions combined multiplication with addition, making a drawing and breaking up the problem.

Table 11 also shows that 1 solution combined division with making a drawing, while 9 solutions only made drawings as strategies. According to Table 11, out of the 37 solutions for the problem, seven were correct and 30 were incorrect.

L36's strategy of combining drawings with multiplication can be seen in Figure 38.

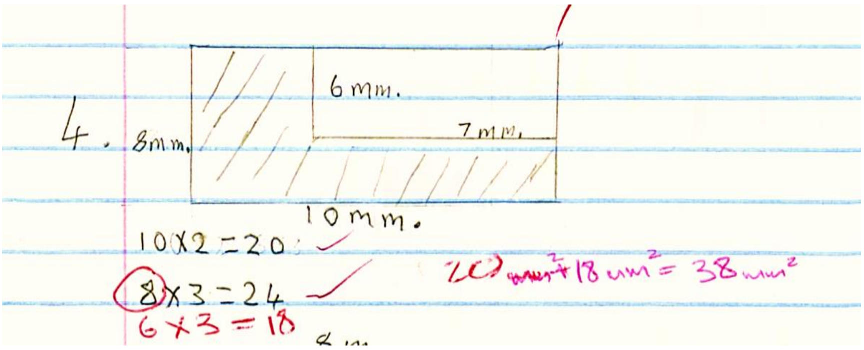


Figure 38: L36 making a drawing when solving Problem 4a

Figure 38 shows that L36 made a drawing of the problem. L36 then used multiplication to find the area of the shade part. It appears that L36 broke the shaded part up into separate rectangles then calculated the area of each one. Unfortunately, L36 made an error when trying to break up the shaded part into two smaller rectangles though. The calculation written as 8 x 3 = 24 should be 6 x 3 = 18. Thus, L36 forgot to subtract 2mm when finding the length of that rectangle. Furthermore, L36 did not perform the final step of adding the areas of the 2 rectangles together to get the total area of the shaded part.

L6's strategy of combining multiplication with addition and a drawing can be seen in Figure 39.

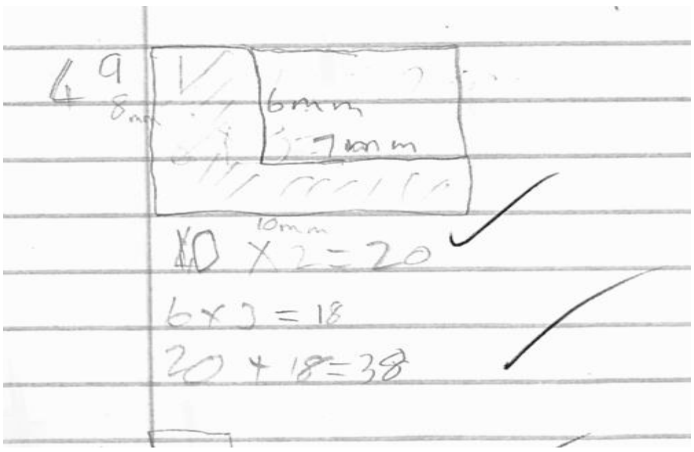


Figure 39: L6 making a drawing when solving Problem 4a

Figure 39 shows that L6 used the same method as L36. However, L6 completed the final step of adding the 2 areas of the rectangles together to get the final area of $38mm^2$.

Problem 4b

Use the dimensions given in the drawing to find:

The perimeter of the shaded part

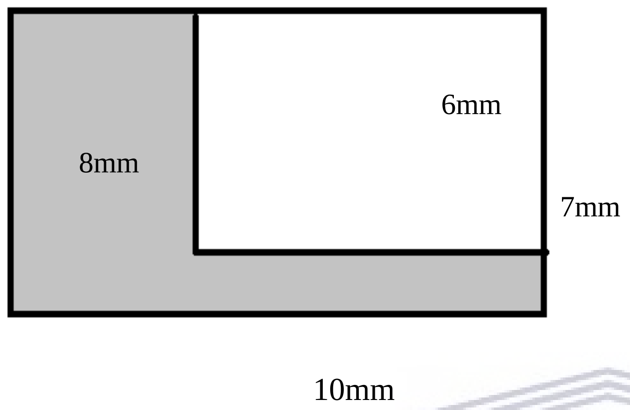


Figure 40: Dimensions of the rectangle

Table 12: Problem solving strategies used by learners to solve Problem 4b

addition			making a drawing only	multiplication only	division and making a drawing	guessing	solutions	
							28	
addition only	making a drawing	breaking up the problem					correct	incorrect
11	6	1	4	2	2	2	11	17
18								

Table 12 indicates that out of the 18 solutions that contained addition strategies, 11 of these solutions used only addition strategies, 6 solutions combined addition with making a drawing, while 1 solution combined addition with breaking up the problem.

Furthermore, 4 solutions made use of only making drawings, 2 solutions used only multiplication strategies, while 2 solutions combined division strategies with making drawings. Two solutions used guessing strategies.

According to Table 12, out of the 28 solutions for the problem, eleven were correct and 17 were incorrect.

L9's addition strategy can be seen in Figure 41.

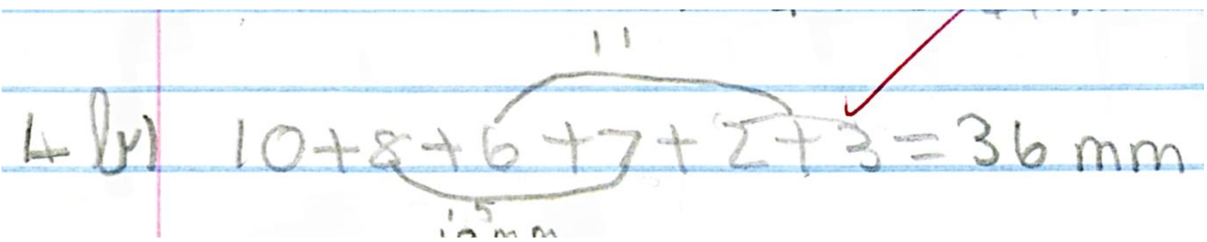


Figure 41: L9 using addition when solving Problem 4b

Figure 41 shows that L9 used addition to determine the perimeter of the shaded part. Thereafter, L9 added the lengths of all six sides of the shaded part of the shape and got the correct answer of 36mm.

Problem 5

Calculate the area of the shaded part of the diagram

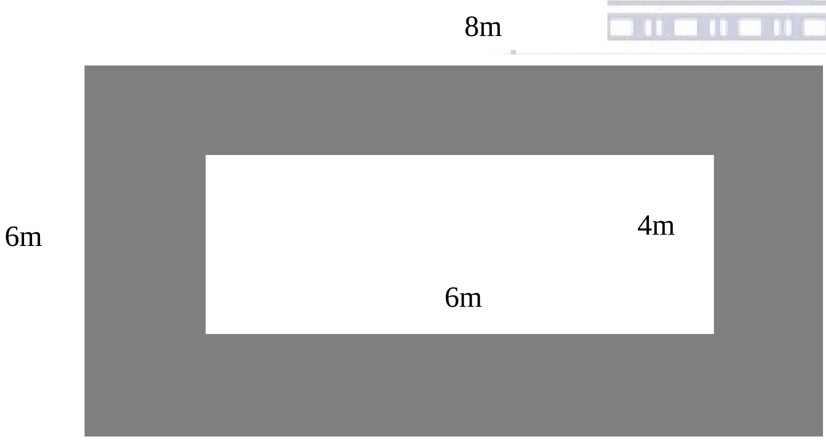


Figure 42: Dimensions of the rectangle

Table 13: Problem solving strategies used by learners to solve Problem 5

multiplication							addition and making a drawing	making a drawing only	solutions	
									35	
multiplication only	subtraction	making a drawing	subtraction and making a drawing	addition	addition and making a drawing	addition, making a drawing and breaking up the problem			correct	incorrect
4	2	7	1	3	3	1	2	12	6	29
21										

Table 13 indicates that 21 solutions contained multiplication strategies. Out of these 21 solutions, 4 solutions used only multiplication, 2 solutions combined multiplication and subtraction strategies, while 7 solutions combined multiplication with making a drawing. Furthermore, 3 solutions combined multiplication and addition strategies, 3 solutions combined multiplication with addition and making drawings, while 1 solution combined multiplication with addition, making a drawing and breaking up the problem.

Table 13 also shows that 12 solutions used only drawings, while 2 solutions combined addition strategies with making a drawing. According to Table 13, out of the 35 solutions for the problem, six were correct and 29 were incorrect.

Similar in nature to Problem 4, this problem required learners to determine the area of a complex shape. Thus, most learners used drawings and arithmetic calculations as their problem-solving strategies. Moreover, 12 of these learners combined the two aforementioned strategies by breaking the compound shape up into simpler shapes and then calculating their areas.

L21's strategy of making a drawing, combined with multiplication and subtraction can be seen in Figure 43.

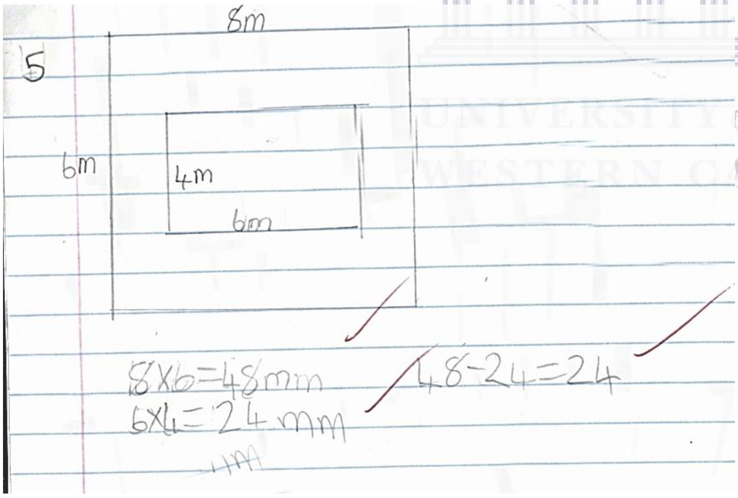


Figure 43: L21 making a drawing, then using multiplication and subtraction when solving Problem 5

Figure 43 shows that L21 made a drawing of the problem. Using the drawing, L21 calculated the areas of the outer and inner rectangles. After doing so, the L21 subtracted the area of the inner rectangle from that of the out rectangle to get the area of the shaded part. The learner got the correct answer.

Figure 43 also shows that L21 used multiplication and subtraction to solve the problem. After drawing the problem, the L21 calculated the areas of the big rectangle and that of the smaller rectangle, then subtracted the two from each other to get the area of the shaded part. The learner got the correct answer.

The low number of correct answers for this problem was explained in the responses from Group 2, where there was an overwhelming consensus that this was the problem they least enjoyed as they found it very challenging:

What did you enjoy the least about doing this activity?

L6: Number 5, because it wasn't so easy.

L5: Number 5, because it's not easy.

L13: Number 5 because it was very hard to times (multiply).

L40: Number 5 because it was too hard.

L41: Number 5 because it was making me confused.

Similar responses were given by several learners from Group 4:

L4: Question number 5, because it was difficult.

L36: I didn't enjoy number 2.

L27: Number 2, because it was difficult, and I didn't know what to do.

L18: I enjoyed everything because it's easy to get to the answers.

Is there anything that made you feel uncomfortable or frustrated?

L36: Yes sir. Number 5, I was getting tired and number 2. I didn't do it.

L4: I'd say number 5, it was very difficult.

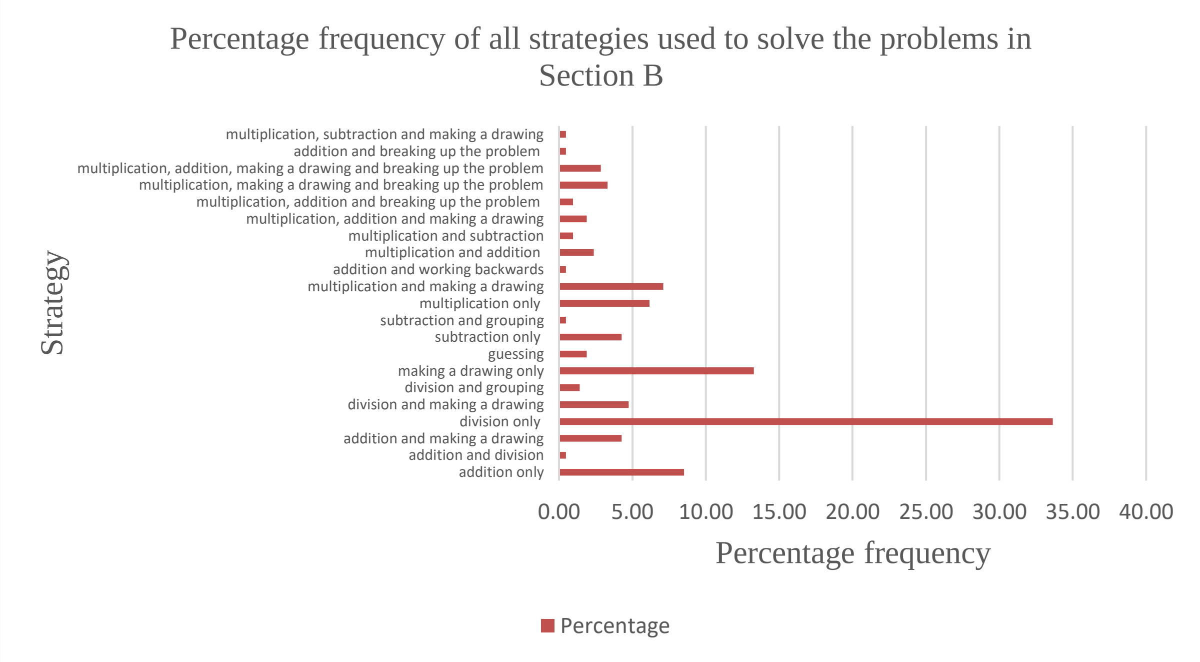


Figure 44: Percentage frequency of all strategies used to solve the problems in Section B

Figure 44 indicates that 33,65% of all solutions in Section B used strategies made up of only division. Making only drawings to solve the problems were found in 13,27% of all solutions, while the use of only multiplication strategies was found in 6,16% of all solutions. Furthermore, the use of only subtraction strategies was found in 4,27% of all solutions.

Similar to the findings in Section A as shown in Figure 30, Figure 44 also indicates the use of a variety of combination strategies to solve the problems in Section B. Strategies which combined multiplication, making a drawing, and breaking up the problem were found in 3.32% of all solutions, while a combination of multiplication, addition, making a drawing and breaking up the problem was found in 2,84% of all solutions. A combination of multiplication and addition strategies were also found in 2,37% of all solutions. These findings are further evidence of the innovation of learners to use different aspects of mathematics in order to help them solve the problems.

4.5. Research Question 2: After choosing their strategies, how do learners go about using the strategy to arrive at their solution?

4.5.1 Examples of how learners used their chosen problem-solving strategy to arrive at their solutions Section A

Problem 1

Sipho has 14 pieces of rope. Each piece if rope is 8 m long. What length of rope does Sipho have altogether?

L28 – repeated addition

L28's strategy of using repeated addition can be seen in Figure 45.

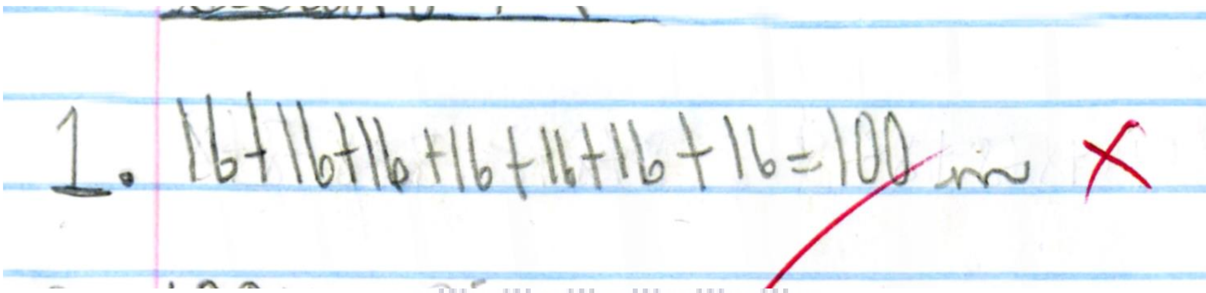


Figure 45: L28 using repeated addition when solving Problem 1

Figure 45 shows that learner L28 made use of addition by grouping the 8s into 16s and then adding 7 groups of 16. However, the answer was incorrect due to an error in calculation.

However, L28 understood what the problem wanted, and this is evident when L28 was asked to state the problem in her own words:

Can you state the problem in your own words?

L28: They are asking that, can you put 14 ropes on the floor and then you count it in 8.

L15 – addition and multiplication

L15's strategy of combining addition and multiplication can be seen in Figure 46.

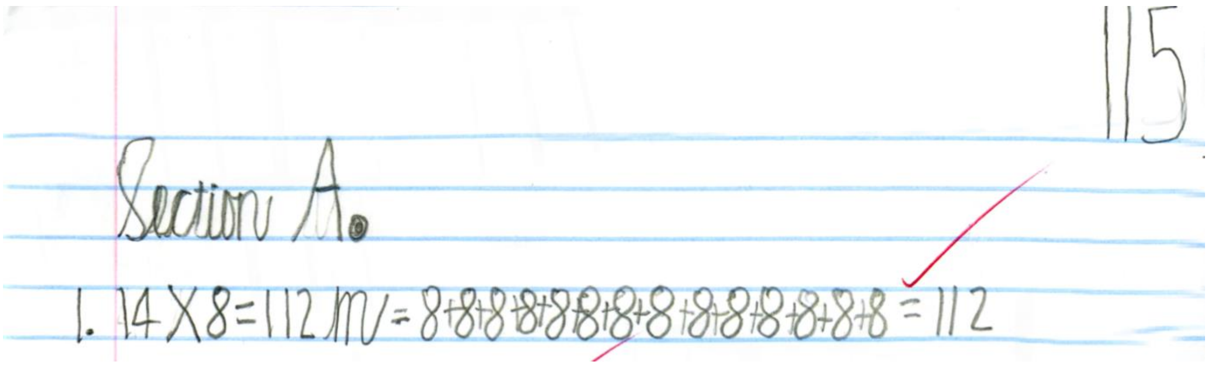


Figure 46: L15 using multiplication as well as addition when solving Problem 1

Figure 46 shows that learner L15 first used multiplication (14 x 8 = 112m), then checked the solution by using addition. Addition was carried out by adding 14 groups of 8. This learner got the correct answer.

L31 – drawing circles and addition

L31's strategy of combining circle drawings with addition can be seen in Figure 47.

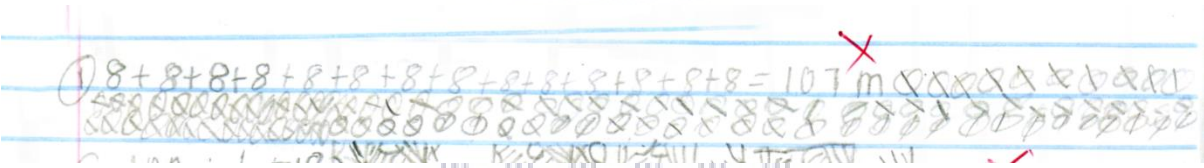


Figure 47: L31 drawing circles and using addition when solving Problem 1

Figure 47 shows that L31 made drawings of circles and then used elimination when adding the circles. However, the learner made an error when adding.

L40 – drawing groups of sticks

L40's strategy of drawing sticks can be seen in Figure 48.

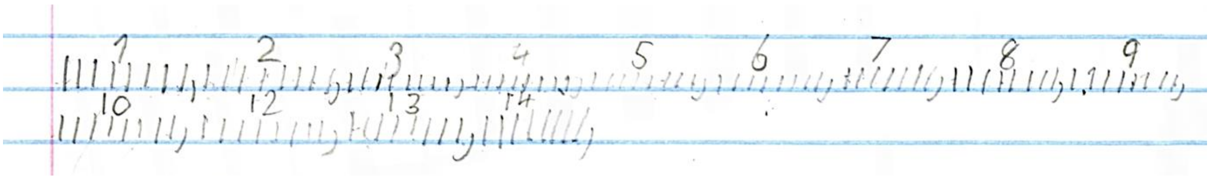


Figure 48: L40 drawing stick shapes in groups when solving Problem 1

Figure 48 shows that L40 made stick drawings, then grouped them together in groups of 8. However, the learner arrived at an incorrect answer. This is possibly due to the first group only containing 7 sticks instead of 8. The drawing was therefore inaccurate.

L34 – addition

L34’s strategy of breaking up the addition can be seen in Figure 49.

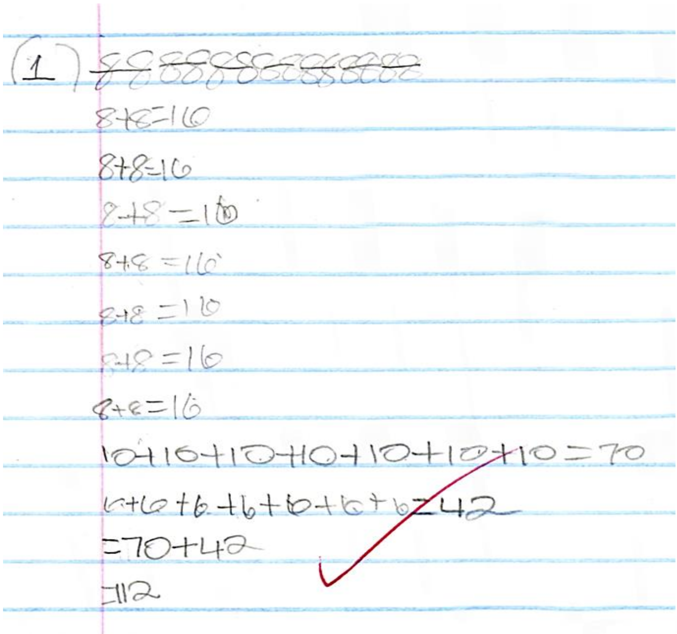


Figure 49: L34 using addition when solving Problem 1

Figure 49 shows that L34 used a different method of addition to that of learner 15. This learner did seven addition operations of 8+8 = 16. After doing so, learner 34 then added the 16s together by breaking them up into tens and units first.

L21 – multiplication

L21’s strategy of multiplication through decomposition can be seen in Figure 50.

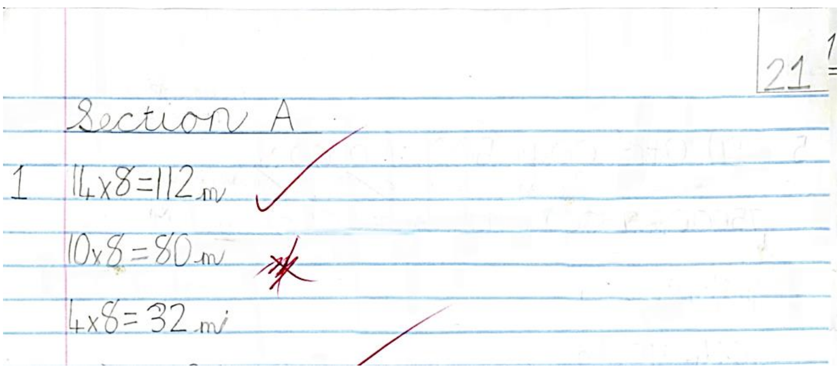


Figure 50: L21 using multiplication through decomposition when solving Problem 1

Figure 50 shows that L21 used multiplication by breaking up the 14 into tens and units (10+4). The learner then multiplied 10 by 8, then 4 by 8, and finally added the two answers together to get the final answer. This learner got the correct answer.

L14 – drawing circles

L14's strategy of drawing circles to assist with addition can be seen in Figure 51.

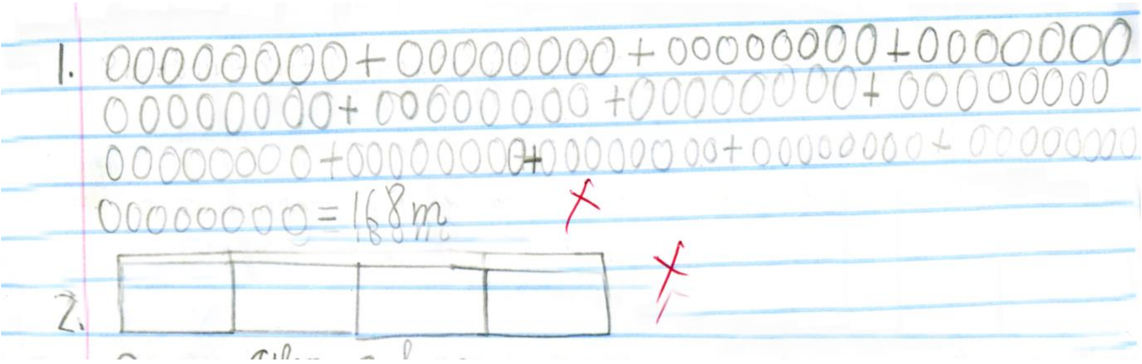


Figure 51: L4 drawing circular shapes in groups, then adding them when solving Problem 1

Figure 51 shows L4 drew balls in groups of 8. The learner then attempted to add all the groups of 8 together. Unfortunately, the L4 made an error in calculation.

L4 – drawing different shapes

L4's strategy of drawing different shapes can be seen in Figure 52.

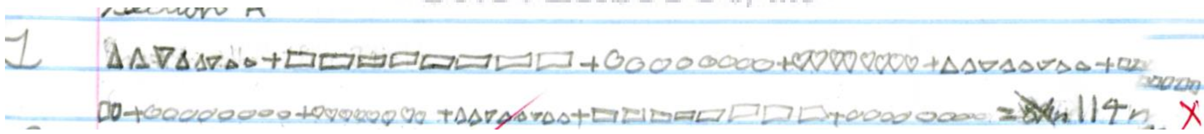


Figure 52: L4 drawing shapes in groups when solving Problem 1

Figure 52 shows that L4 made drawings of different shapes and then grouped them. The learner then attempted to use addition to add all the shapes.

L25 – drawing faces

L25's strategy of drawing faces can be seen in Figure 53.

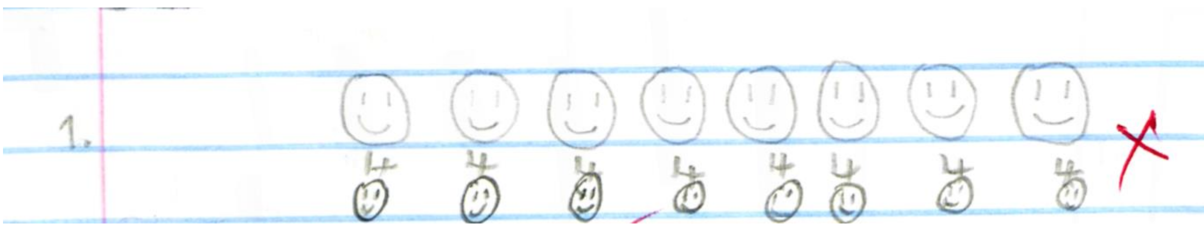


Figure 53: L25 drawing circular-shaped faces when solving Problem 1

Figure 53 shows that L25 made drawings of faces. It appears that each face represented groups of 4. Unfortunately, the learner could not arrive at a solution after making the drawings.

L2 – Grouping and pairing

L2’s grouping and addition strategy can be seen in Figure 54.

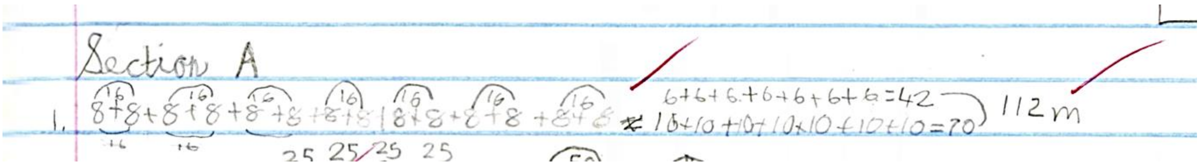


Figure 54: L2 using seven groups 16 when solving Problem 1

Figure 54 shows that L2 made groups of 8, the paired up the 8s into groups of 16 before adding up all the pairs.

L9 – Repeated grouping

L9’s strategy of repeated grouping can be seen in Figure 55.

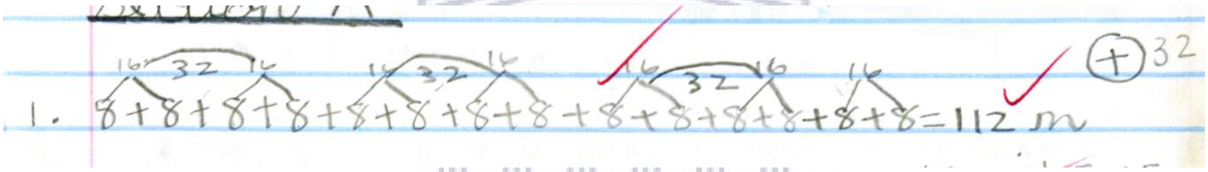


Figure 55: L9 using groups of 8 and then further group them into groups of 16 when solving Problem 1

Figure 55 shows that L9 also used grouping. This learner wrote down 14 values of 8 to add, then paired them into 16s before pairing the 16s into 32s. The learner then added the three sets of 32 as well as the remaining unpaired 16 to get an answer of 112.

L41 – grouping

L41’s grouping strategy can be seen in Figure 56.

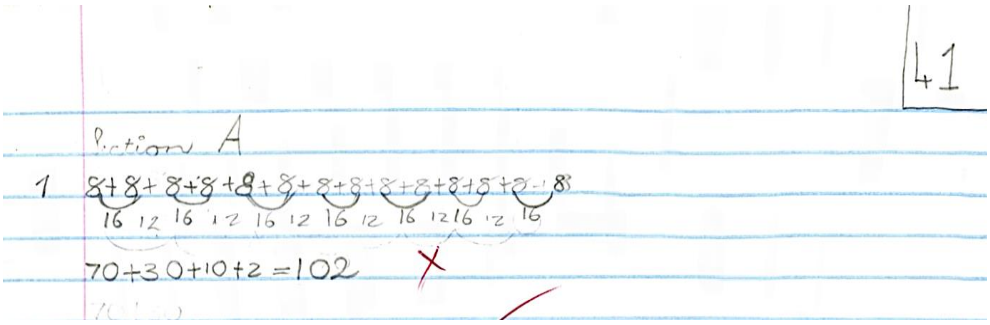


Figure 56: L41 using a form of grouping when solving Problem 1

Figure 56 shows how learner 41 grouped the 8s together into 7 groups of two 8s per group. The learner then used addition to add the 16s. The learner got the wrong answer due to an error in calculation.

Problem 2

Kate has 100 cm of ribbon. She cuts it into 4 equal pieces. How long is each piece of ribbon?

L4 – drawing and dividing up a rectangle

L4's strategy of combining division with making a drawing can be seen in Figure 57.

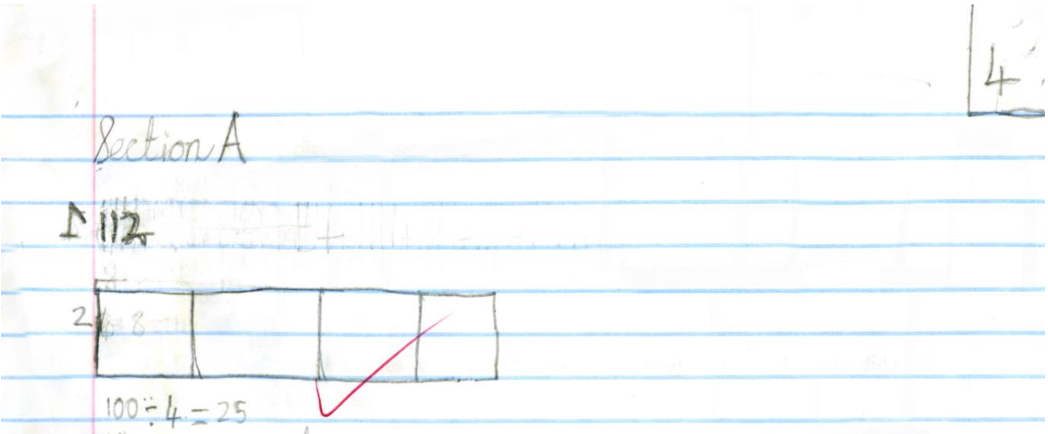


Figure 57: L4 making a drawing and using division when solving Problem 2

Figure 57 shows a drawing made by L4 and combining it with a division sum. From drawing a rectangle and then dividing it into 4 parts, the learner realises that the problem requires the 100 cm of ribbon to be divided into 4 equal parts. The drawing thus aided the learner in realising what operation needed to be used. The learner got the correct answer.

L9 – drawing and dividing up a rectangle

L9's strategy of drawing and dividing up the rectangle can be seen in Figure 58.

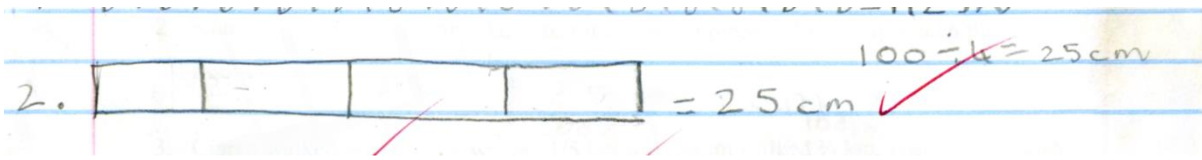


Figure 58: L9 using a drawing when solving Problem 2

Figure 58 shows that L9 used the same drawing strategy as learner 4, which enabled the learner to realise that division was required to solve the problem.

L28 – division

L28’s division strategy can be seen in Figure 59.

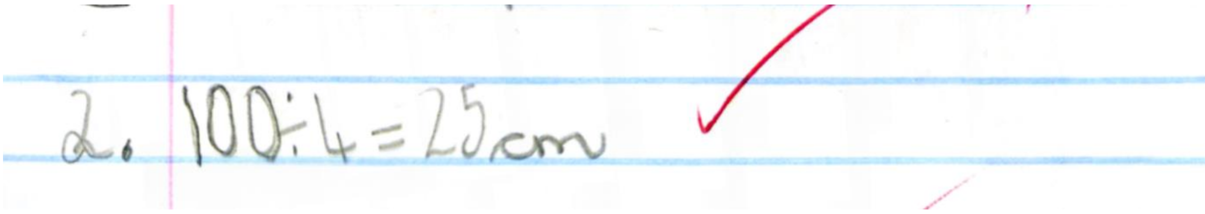


Figure 59: L28 using division when solving Problem 2

Figure 59 shows that learner 28 used division to solve the problem. The learner got the correct answer.

L33 – division

L33’s division strategy can be seen in Figure 60.

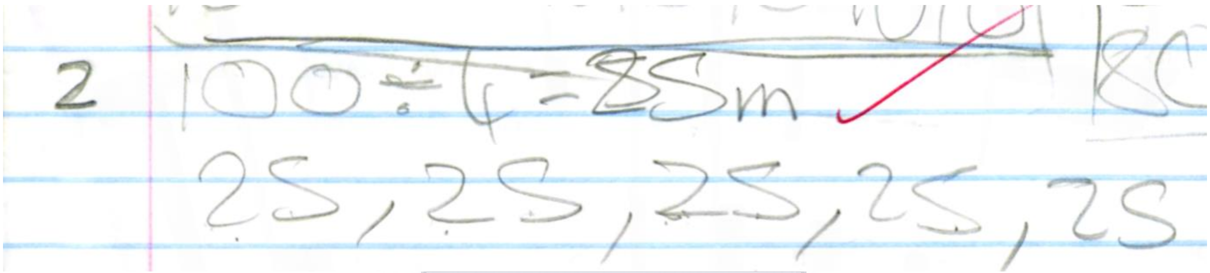


Figure 60: L32 using division when solving Problem 2

Figure 60 shows that L32 divided the 100 by 4 and got an answer of 25. The answer was correct, but the learner omitted the units of measurement.

L42 – repeated addition and working backwards

L42’s repeated addition strategy to work backwards can be seen in Figure 61.

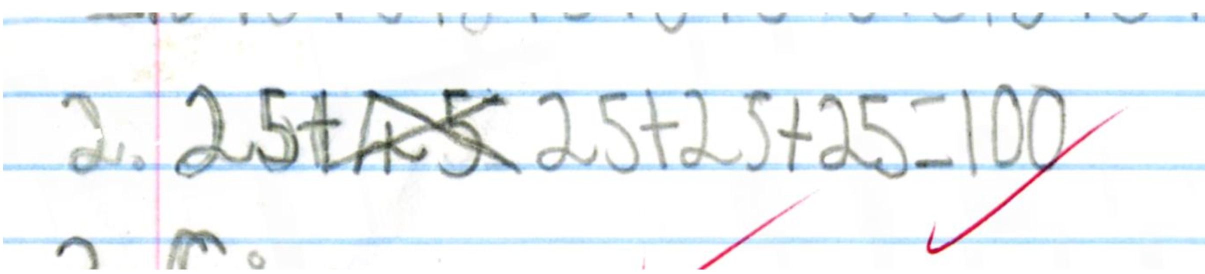


Figure 61: L42 using repeated addition when solving Problem 2

Figure 61 shows that L42 worked backwards and used addition to solve the problem. The learner got the correct answer, although the units of measurement were omitted.

L2 – working backwards through division

L2’s strategy of working backwards by using division can be seen in Figure 62.

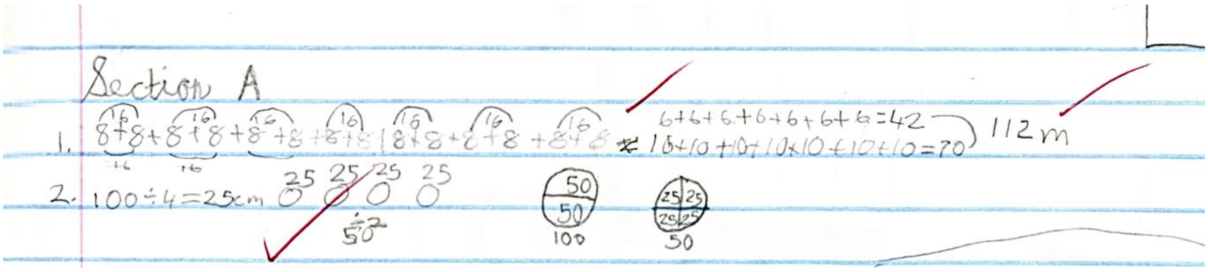


Figure 62: L2 working backwards when solving Problem 2

Figure 62 shows that L2 worked backwards. The learner started with the solution, then worked backwards using addition. Once the learner got an answer of 25cm, division was used to verify the solution. The learner got the correct answer.

L41 – working backwards through addition

L41’s strategy of working backwards using addition can be seen in Figure 63.

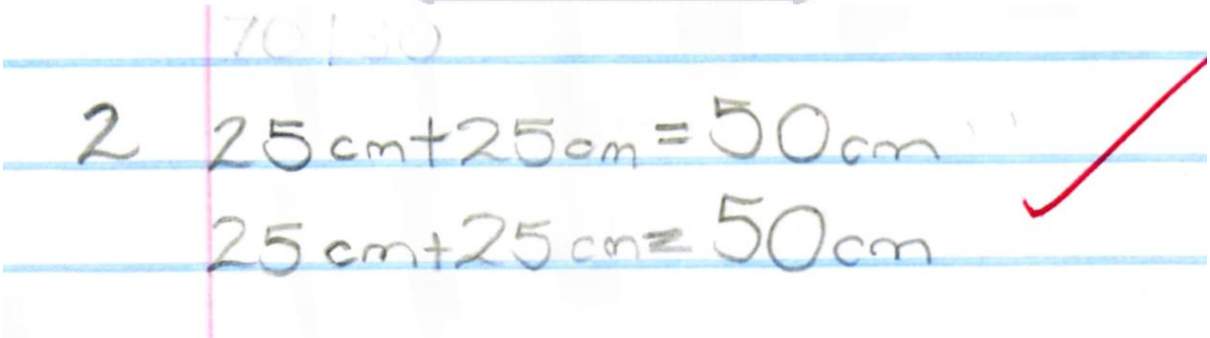


Figure 63: L41 working backwards and using addition when solving Problem 2

Figure 63 shows that L41 used addition and worked backwards to solve the problem. The learner started with 25cm measurements and added them together to get to two sets of 50cm, which together came to 100cm when being added together.

L29 – grouping via a table

L29’s table strategy can be seen in Figure 64.

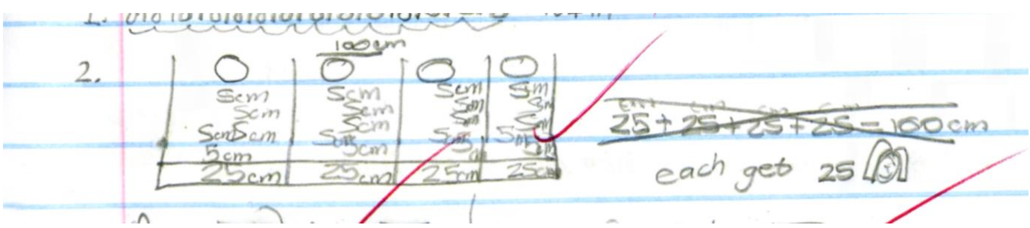


Figure 64: L29 using grouping and a table when solving Problem 2

Figure 64 shows that L29 used a form of grouping, dividing the 100 cm into 4 groups of 25cm each. This method is similar to what was taught in the Foundation Phase, also making use of a table when grouping the numbers. The learner got the correct answer.

Problem 3

Ciaran walked ½ km, Alex walked 1/5 km and Calum walked ¼ km. How far did each learner walk in metres?

L34 – reasoning by writing out a sentence

L34's logical reasoning strategy can be seen in Figure 65.

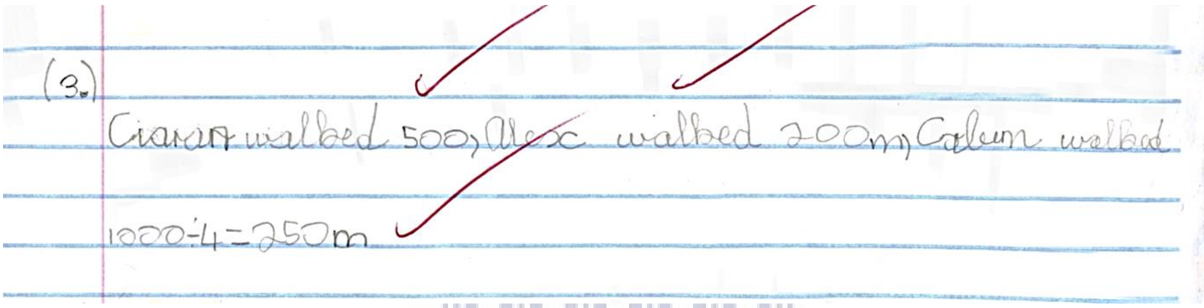


Figure 65: L34 using logical reasoning when solving Problem 3

Figure 65 shows that L34 used reasoning and logic to write out the distances travelled by each child in metres. This comes as a result of the learner already knowing the answers of distances which are fractions of a kilometre, e.g., 1/5km = 200m. The learner was able to solve almost the entire problem without calculations. However, the learner used division only to determine the distance covered by Calum. The learner got the correct answer.

L26 – reasoning by writing out a sentence

L26's logical reasoning strategy can be seen in Figure 66.

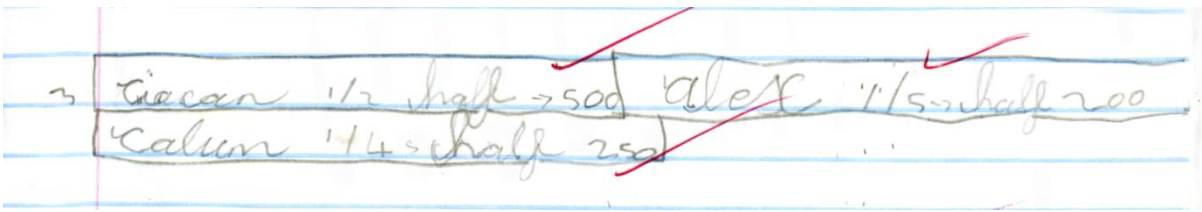


Figure 66: L26 using logical reasoning when solving Problem 3

Figure 66 shows that L26 also used logical based on the learner's prior knowledge of fractional distances. The learner got the correct answer

L21 – division

L21’s division strategy can be seen in Figure 67.

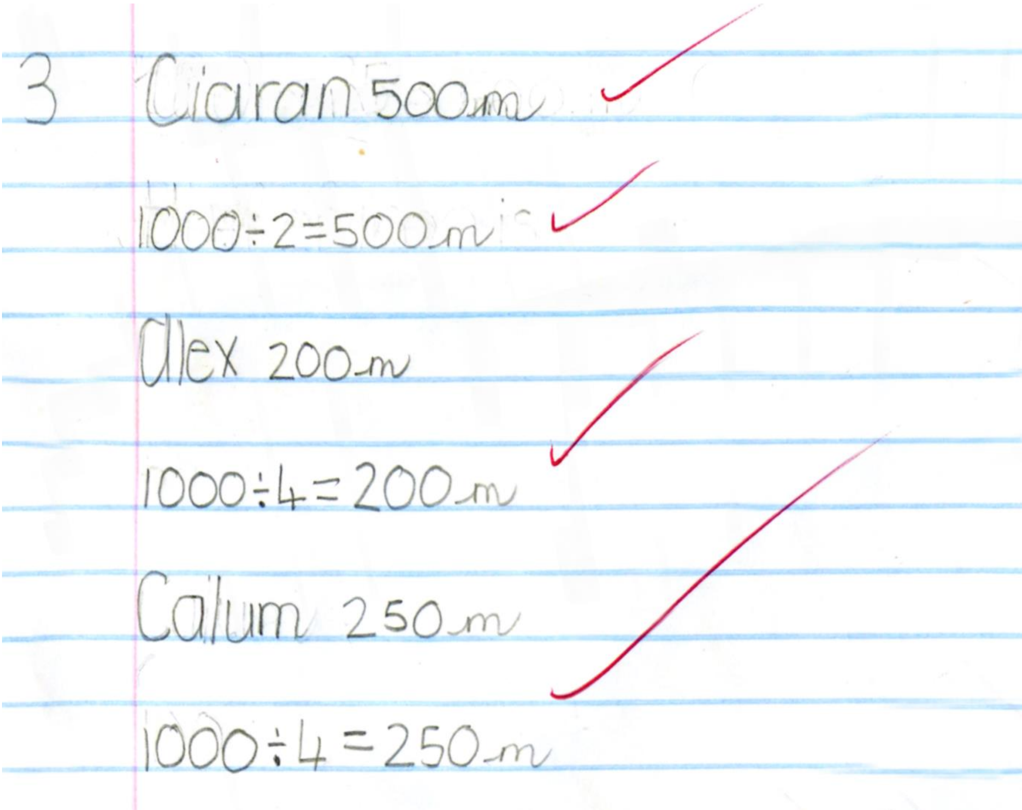


Figure 67: L21 using division when solving Problem 3

Figure 67 shows that L21 used division to get the distances travelled by each child in metres. The learner got the correct distances in metres for all 3 children.

L37–making a table to separate information

L37’s table strategy can be seen in Figure 68.

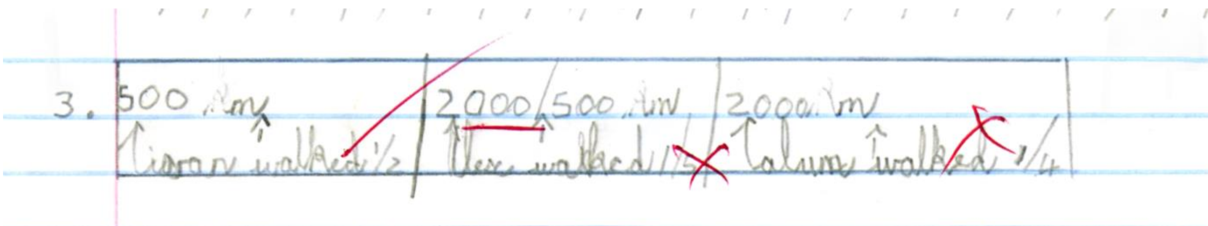


Figure 68: L37 drawing a table when solving Problem 3

Figure 68 shows that L37 drew a table to organise the information given about each child in the problem. The learner got the correct distance for the first child, but not for the second and third children.

L29 – making a table

L29's table strategy can be seen in Figure 69.

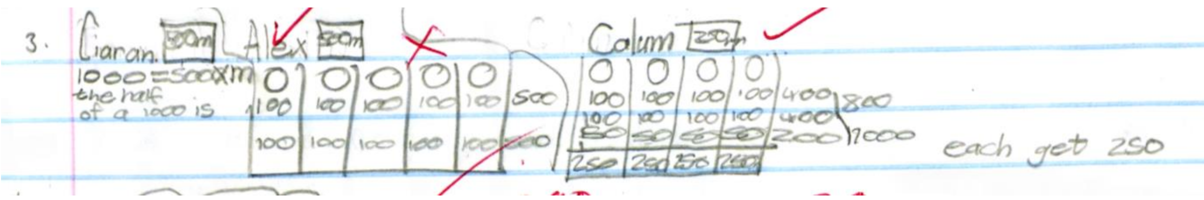


Figure 69: L29 drawing a table when solving Problem 3

Figure 69 shows that L29 also constructed a table to be able to work out the distances travelled by each learner in isolation. This table contained quite a lot of information, including grouping numbers as well as evidence of reasoning for Ciaran. The learner was able to successfully convert the fractional distance of Ciaran and Calum, but not for Alex. 7

Problem 4

A pencil is 130 mm long. A ruler is 3 times longer than the pencil. How long is the ruler in centimetres?

L9 – repeated addition

L9's repeated addition strategy can be seen in Figure 70.

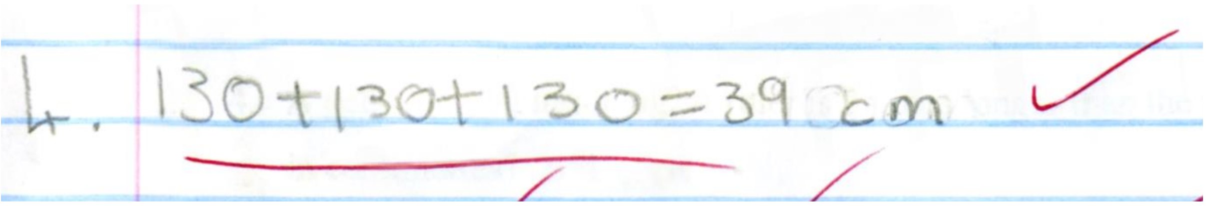


Figure 70: L9 using addition when solving Problem 4

Figure 70 shows that L9 added 130mm three times. The learner then wrote the answer immediately in centimetres without showing any conversions.

L34 – multiplication by decomposition

L34’s strategy of multiplication by decomposition can be seen in Figure 71.

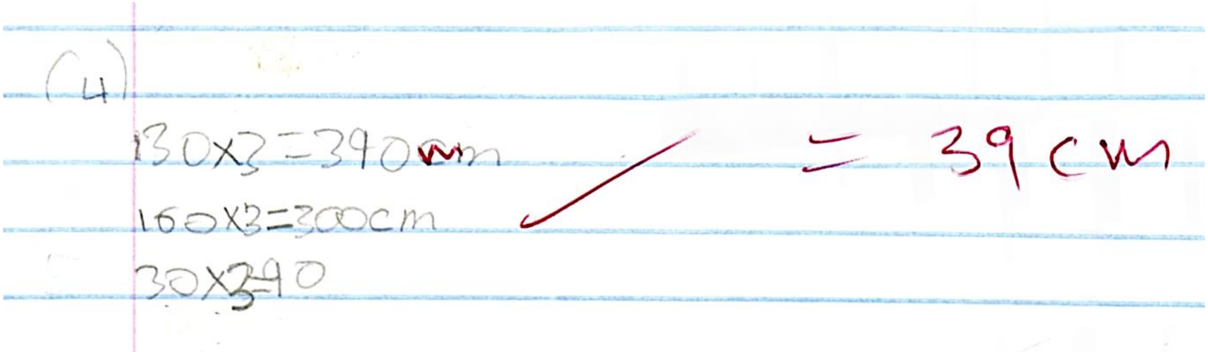


Figure 71: L34 using multiplication when solving Problem 4

Figure 71 shows that L34 used an expansion method to multiply the 130mm by 3 and get 390mm. The learner decomposed the 130 into 100 and 30, then multiplied each by 3 before adding the two answers to get 390mm. The learner did not convert the final answer to centimetres as the problem requested.

L26 – multiplication by decomposition

L26’s strategy of multiplication by decomposition can be seen in Figure 72.

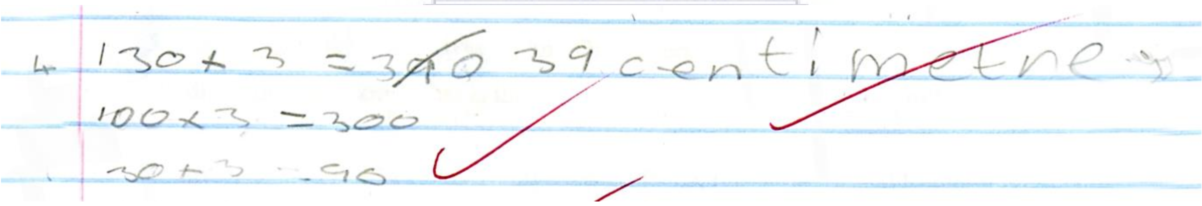


Figure 72: L26 using expansion multiplication and then addition when solving Problem 4

Figure 72 shows that L26 used the same decomposition method as L34 to break up the 130, then multiply by 3. However, this learner converted the final answer from millimetres to centimetres as the problem requested.

L29 – addition

L29’s addition strategy can be seen in Figure 73.

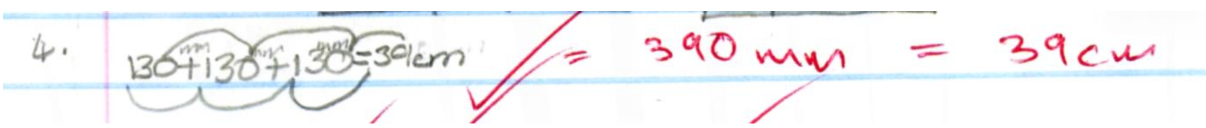


Figure 73: L29 using repeated addition when solving Problem 4

Figure 73 shows that L29 used a method of repeated addition. The learner repeated added three values of 130 to get to the solution. It can also be seen that the learner immediately wrote the solution in centimetres, so the conversion process was not shown on paper.

L32 – multiplication by decomposition

L32's strategy of multiplication by decomposition can be seen in Figure 74.

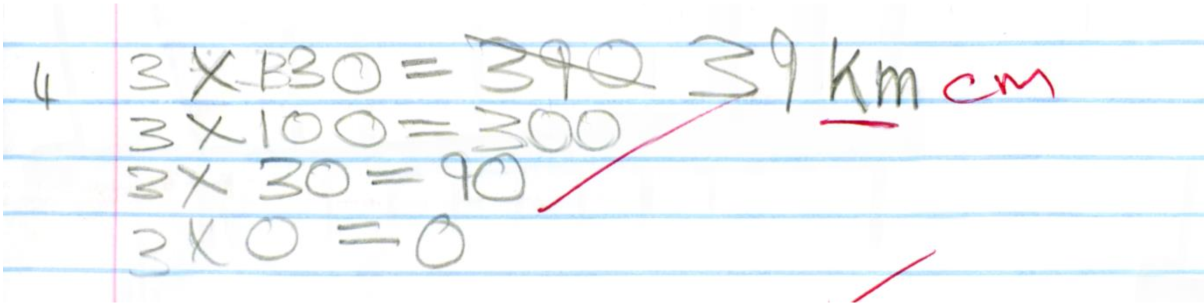


Figure 74: L32 using multiplication when solving Problem 4

Figure 74 shows that L32 used multiplication. The learner multiplied 130mm by 3 by first expanding the 130. This is referred to as the breaking up or expansion method in grade 4. The learner got an answer of 390mm, then converted it to centimetres to get 39cm. The learner made an error by writing the abbreviation for centimetres as "km" instead of "cm."

L11 – multiplication without expansion

L11's multiplication strategy can be seen in Figure 75.

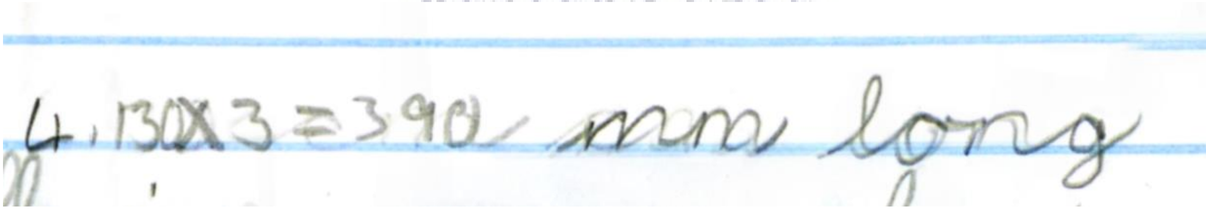


Figure 75: L11 using multiplication when solving Problem 4

Figure 75 shows that L11 used multiplication without any expansion. The learner multiplied the 130mm by 3 to get 390mm. However, the learner did not convert the answer to centimetres as was required in the question.

Problem 5

On the first day John drives his car a distance of 75 km. On the second day he drove 12 500 m and on the third day he drove 13 km. What is the total distance John drove in metres?

L20 – addition

L20’s addition strategy can be seen in Figure 76.

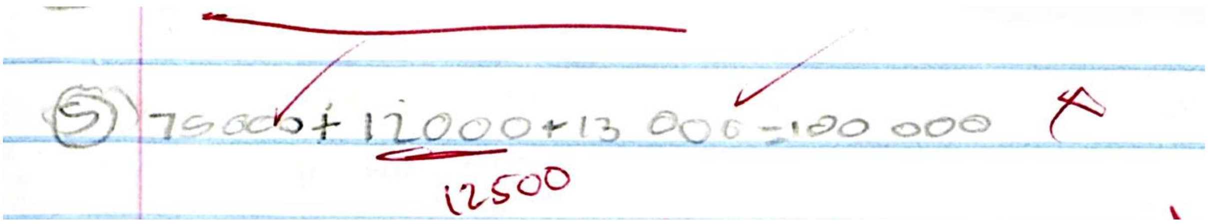


Figure 76: L20 using addition when solving Problem 5

Figure 76 shows that L20 used addition to add the three distances. The learner wrote down the distances in metres and then added them. However, the learner made an error by writing down 12 000m instead of 12 500m. As a result, the answer was incorrect.

L14 – addition without converting all distances to the same unit of measurement

L14’s addition strategy can be seen in Figure 77.

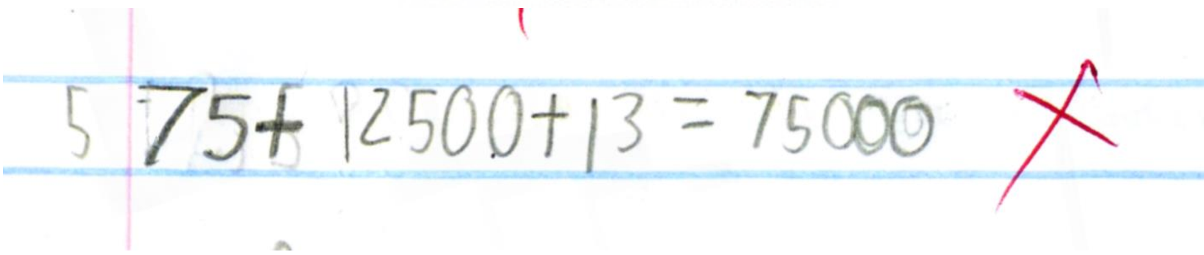


Figure 77: L14 using addition when solving Problem 5

Figure 77 shows that L14 used addition. However, the learner did not convert two of the distances from kilometres to metres. As a result, the learner added distances which were not all in the same unit of measurement.

L10 – addition

L10’s addition strategy can be seen in Figure 78.

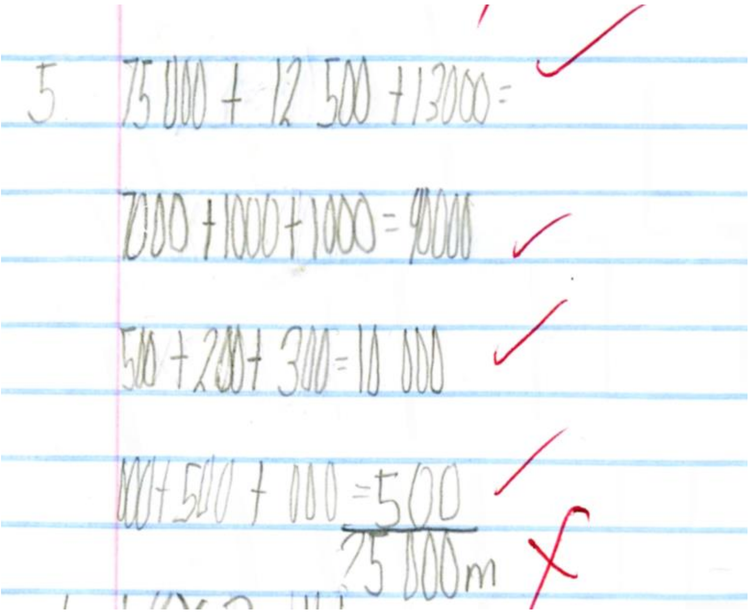


Figure 78: L10 using addition when solving Problem 5

Figure 78 shows that L10 wrote down all three distances in metres, then added them together using the breaking up method. However, the learner made an error when adding which resulted in an incorrect answer.

L29 – addition

L29’s addition strategy can be seen in Figure 79.

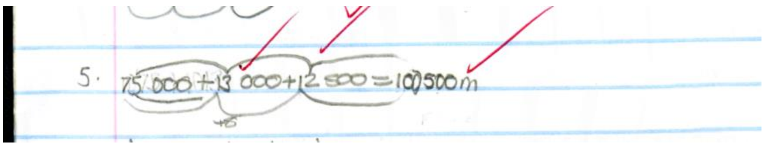


Figure 79: L29 used addition when solving Problem 5

Figure 79 shows that L29 was able to add the three distances without needing any calculations. The learner mentally converted all the distances to metres, then added them.

L29 explained the method:

L29, you have a very different method, what do the lines mean?

L29: I plus (added) the thousands and then I added that (Explaining being aware that because all three distances were multiples of 1000, they could be added without needing to work it out on paper).

Problem 6

Mary's mass is 48 kg. Her younger sister, Paula, has a mass equal to three-quarters of Mary's mass. What is Paula's mass?

L28 – division and addition

L28's strategy of combining division and addition can be seen in Figure 80.

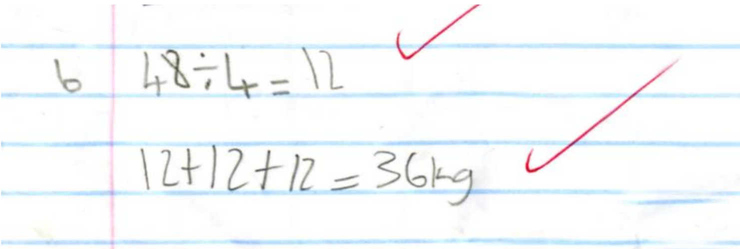


Figure 80: L28 using both division and addition when solving Problem 6

Figure 80 shows that L28 divided Mary's mass of 48kg by 4 to get 12kg. After finding one quarter of Mary's mass, the learner then added three masses of 12kg together to get three quarters of Mary's mass which was 36kg. This was the correct answer.

L16 – division

L16's division strategy can be seen in Figure 81.

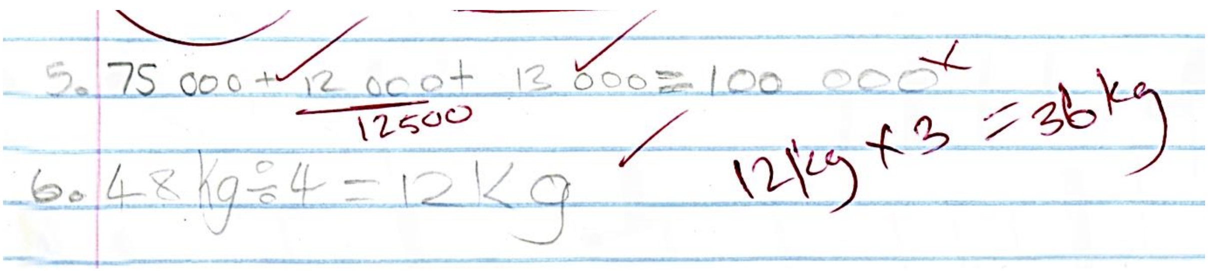


Figure 81: L16 using division when solving Problem 6

Figure 81 shows that the L16 used division to find one quarter of Mary's mass. However, the learner stopped there without then finding 3 quarters of the mass.

L21 – Division and repeated addition

L21's strategy of combining division and addition can be seen in Figure 82.

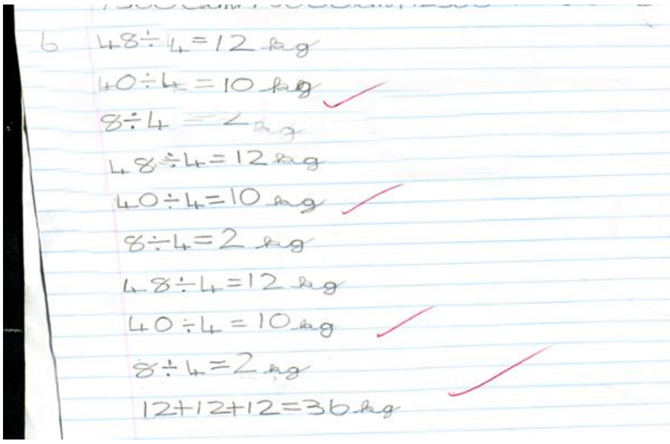


Figure 82: L21 using division and repeated addition when solving Problem 6

Figure 82 shows that L21 first used division to find one quarter of 48kg. After getting an answer of 12kg, she then added three 12kgs together to determine three quarters of 48kg. she got a final answer of 36kg, which is the correct answer.

L21 comprehended the problem well and explained the strategy which resulted in the correct final answer:

L21: I divided 48.

I said 40 divided by 4 equals 10,

8 divided by 4 equals 2.

L21 repeated these steps 3 times to get three 12's, then added them together to get 36.

L29 - addition and making a table

L29's strategy of drawing a table to assist with division can be seen in Figure 83.

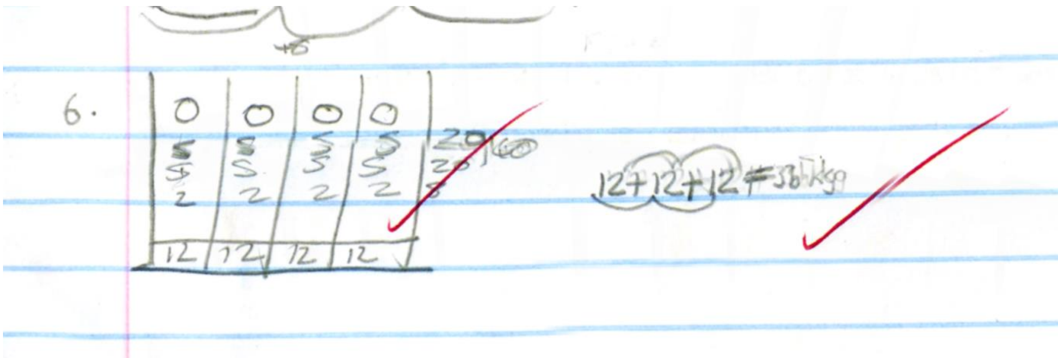


Figure 83: L29 drawing a table when solving Problem 6

Figure 83 shows that L29 constructed a table to equally organise the four sets of 12kg which each represented one quarter of the 4kg. L29 then added three masses of 12kg together to find three quarters of Mary's mass. The learner got a final answer of 36kg, which was the correct answer.

4.5.2. Examples of how learners carried out their chosen problem-solving strategy to arrive at their solutions in Section B

Problem 1

408 L of water is poured into 4 containers so that each container holds the same amount. How much water is in each container?

L4 – drawing containers

L4's strategy of drawing containers can be seen in Figure 84.

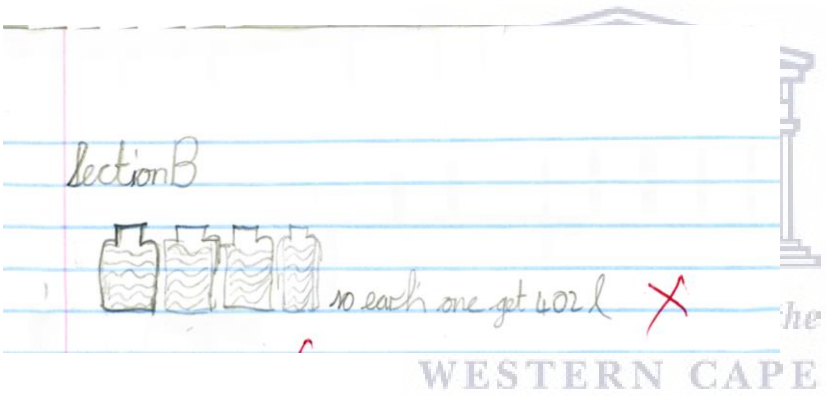


Figure 84: L4 making a drawing when solving Problem 1

Figure 84 shows that L4 drew 4 containers to represent the 4 equal quantities that the 402 litres needed to be divided into. However, the learner seemed to have gotten confused thereafter.

L5 – division by expansion

L5's division by expansion strategy can be seen in Figure 85.

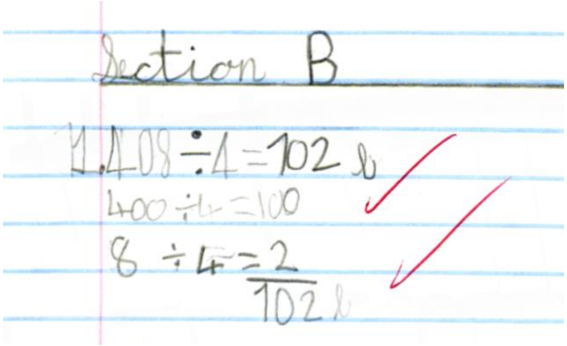


Figure 85: L5 using division when solving Problem 1

Figure 85 shows that the L5 used a form of breaking up or expansion to divide the 408 litres into 4 equal quantities. The learner expanded the 408 into 400 + 8, then divided each value separately by 4 and then adding the two answers together to get 102 litres. This is the correct answer.

L36 – short division

L36's division strategy can be seen in Figure 86.

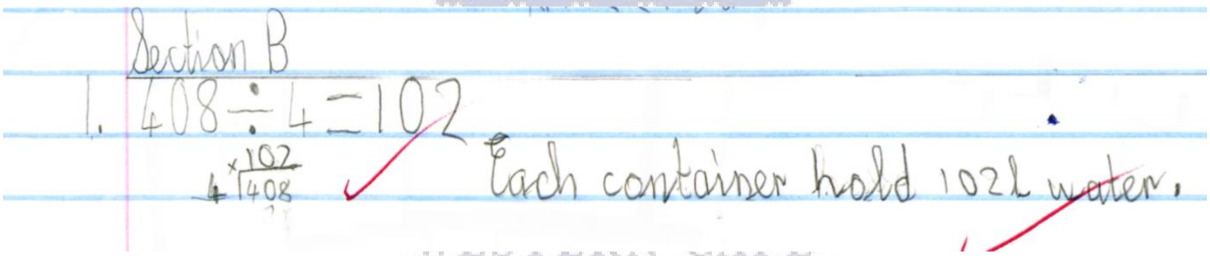


Figure 86: L36 using division when solving Problem 1

Figure 86 shows that L36 used short division as the strategy to solve this problem. The learner got the correct answer.

Grouping

L19 – grouping through drawings

L19's drawing strategy can be seen in Figure 87.

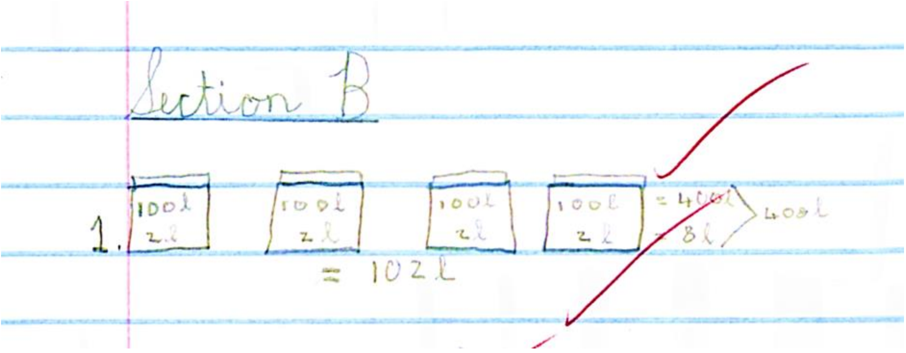


Figure 87: L19 using grouping when solving Problem 1

Figure 87 shows that learner L19 also used a type of grouping, splitting the 408 litres equally into 4 groups. The learner then used addition to check the validity of the solution, the answer was correct.

Problem 2

A drum contains 70 L of petrol. The petrol tank of my small car can hold 6 L of petrol. How many times can I fill the tank from the drum and what amount is finally left in the drum?

L5 – repeated subtraction

L5's subtraction strategy can be seen in Figure 88.

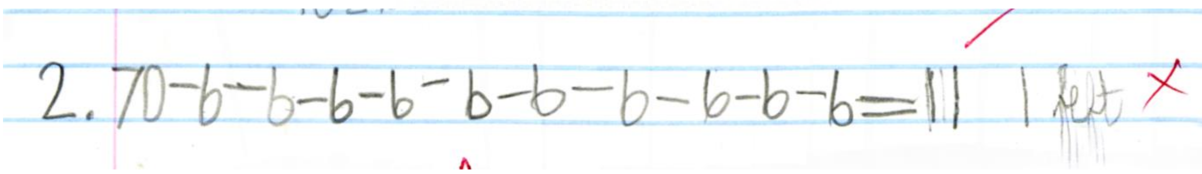


Figure 88: L5 using subtraction when solving Problem 2

Figure 88 shows that L5 subtracted 6 from 70 continuously until it was not possible anymore. The learner then counted a total of eleven 6's which were subtracted. This was correct, but the learner made an error when calculating the remainder.

L13 – division

L13's division strategy can be seen in Figure 89.

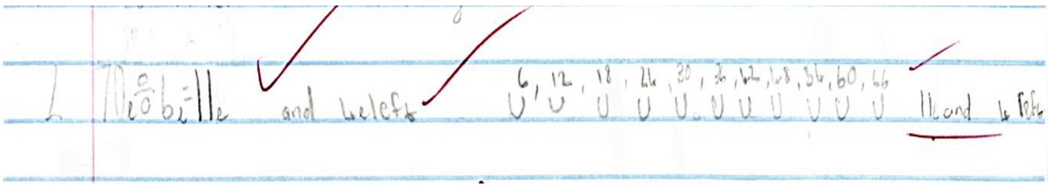


Figure 89: L13 using division when solving Problem 2

Figure 89 shows that L13 used division to solve the problem. The learner got the correct answer and correct remainder. It can also be seen that the learner counted in sixes to get the answer.

L2 – multiplication using multiples of 6

L2's multiplication strategy can be seen in Figure 90.

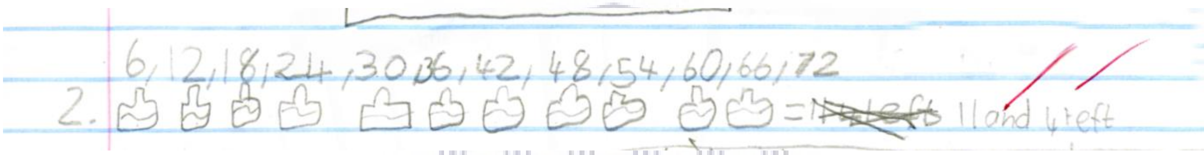


Figure 90: L2 using multiplication when solving Problem 2

Figure 90 shows that the L2 counted in sixes (used the 6 times table) to solve the problem. When the learner got to 72, the learner realised that this was too much, then went back by one multiple to end on 66. The learner then counted 11 whole sixes and a remainder of 4, as 66 was 4 less than 70. The learner got the correct answer.

Problem 3

How many 500 ml bottles can be filled from a can containing 4.5 L of water?

L23 – division

L23's division strategy can be seen in Figure 91.

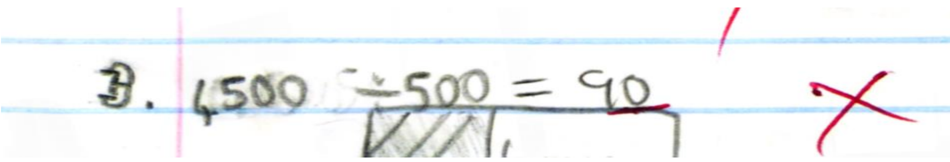


Figure 91: L23 using division when solving Problem 3

Figure 91 shows that the L23 used division to solve the problem. The learner recognised that there are 4500ml in 4.5 litres. The learner then divided 4500 by 500. However, the learner made an arithmetical error which resulted in a final answer of 90 instead of 9.

L21 – division without converting

L21's division strategy can be seen in Figure 92.

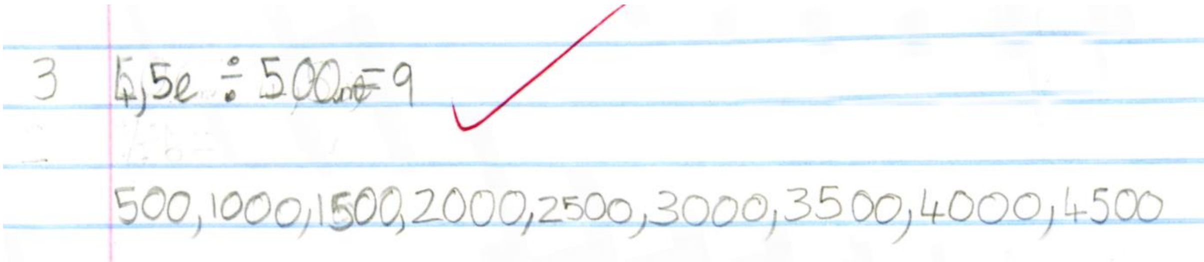


Figure 92: L21 using division when solving Problem 3

Figure 92 shows that the L21 did not do any conversion. The learner simply divided 4,5 litres by 500ml and got the correct answer

Problem 4a

Use the dimensions given in the drawing to find:

The area of the shaded part in mm²

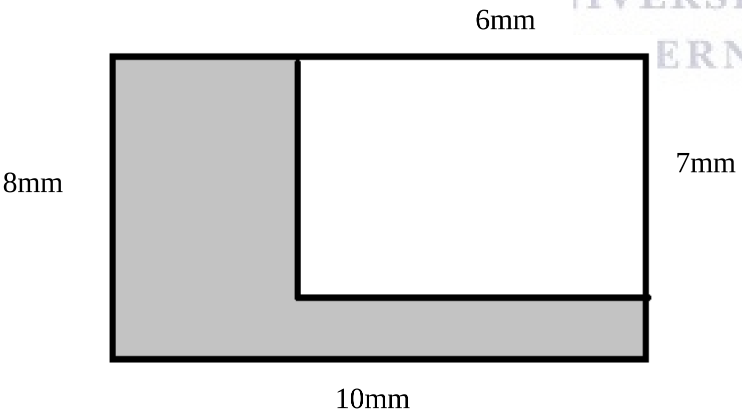


Figure 93: Dimensions of the rectangle

L36 – Multiplication and making a drawing

L36's strategy of using a drawing and multiplication can be seen in Figure 94.

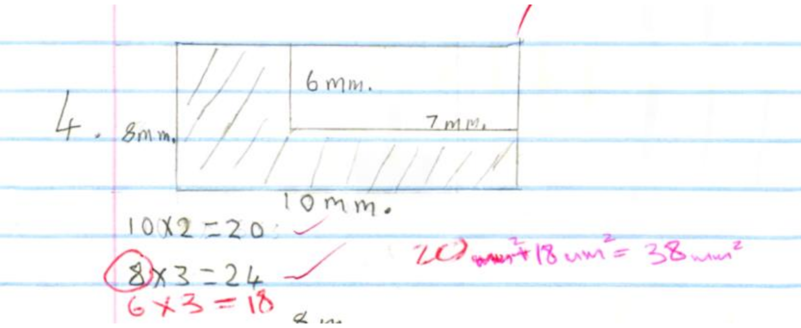


Figure 94: L36 making a drawing when solving Problem 4a

Figure 94 shows that L36 made a drawing of the problem. The learner then used multiplication to find the area of the shade part. It appears the learner broke the shaded part up into separate rectangles then calculated the area of each one. The learner made an error when trying to break up the shaded part into two smaller rectangles though. The calculation written as 8 x 3 = 24 should be 6 x 3 = 18. Thus, the learner forgot to subtract 2mm when finding the length of that particular rectangle. Furthermore, the learner did not perform the final step of adding the areas of the 2 rectangles together to get the total area of the shaded part.

L6 – Multiplication, addition and making a drawing

L6's strategy of combining a drawing with multiplication and addition can be seen in Figure 95.

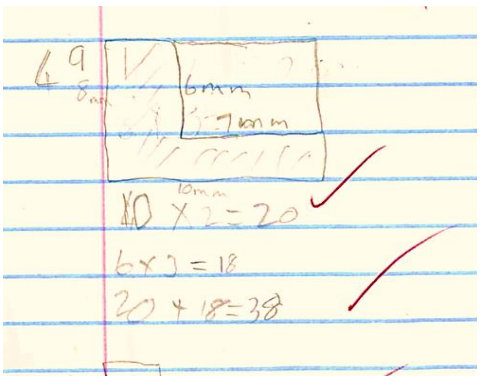


Figure 95: L6 making a drawing when solving Problem 4a

Figure 95 shows that L6 used a similar method to L36. However, L6 did the final step of adding the 2 areas of the rectangles together to get the final area of 38mm^2.

Problem 4b

Use the dimensions given in the drawing to find:

The perimeter of the shaded part

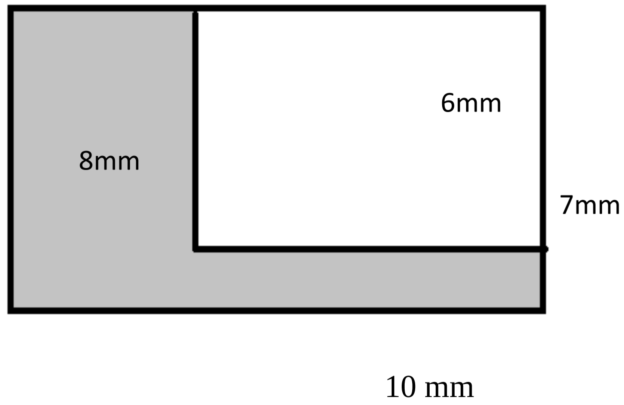


Figure 96: Dimensions of the rectangle

L9 – addition

L9's addition strategy can be seen in Figure 97.

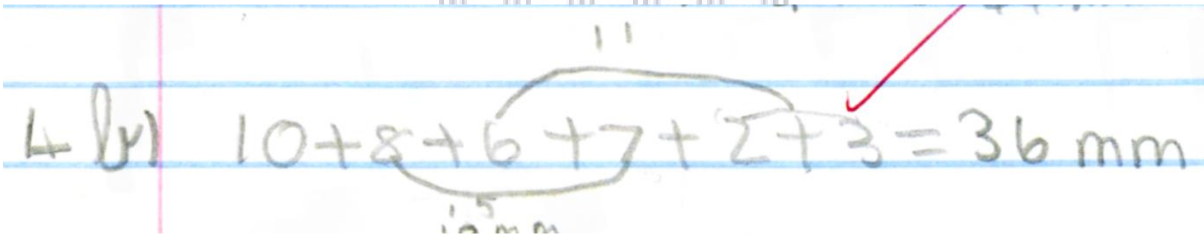


Figure 97: L9 using addition when solving Problem 4b

Figure 97 shows that L9 used addition to determine the perimeter of the shaded part. The learner added the lengths of all six sides of the shaded part of the shape and got the correct answer of 36mm.

L10 – simplifying the problem with a drawing

L10's strategy of combining addition with a drawing can be seen in Figure 98.

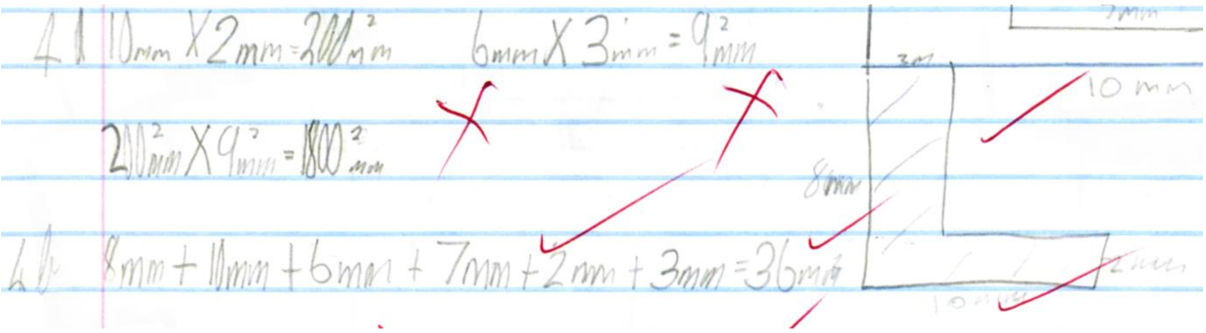


Figure 98: L10 using making a drawing when solving Problem 4b

Figure 98 shows that L10 also used addition to find the perimeter of the shaded part. However, this learner first constructed a drawing to show the shaded part on its own. After doing so, the learner could then add the lengths of all six sides. It is also worth noting that unlike learner 9, learner 10 was consistent in writing down the units of measurement for all the lengths of the shape.

Problem 5

Calculate the area of the shaded part of the diagram

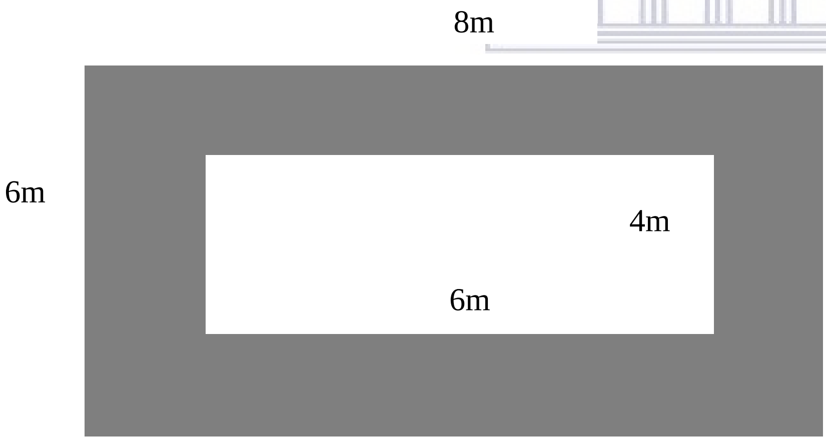


Figure 99: Dimensions of the rectangle

L21 - Multiplication, subtraction and making a drawing

L21's strategy of making a drawing and using addition and subtraction can be seen in Figure 100.

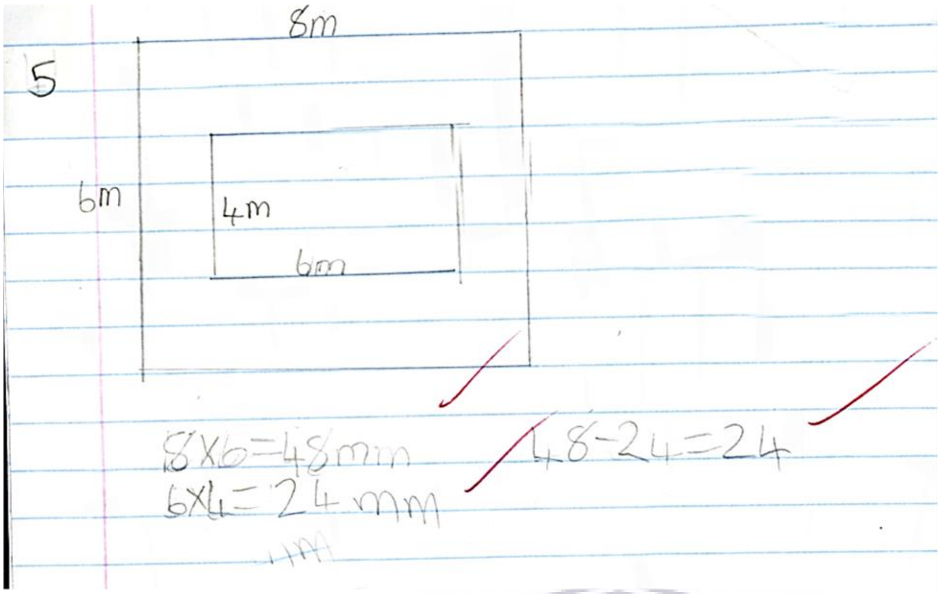


Figure 100: L21 making a drawing when solving Problem 5

Figure 100 shows that L21 made a drawing of the problem. Using the drawing, the learner calculated the areas of the outer and inner rectangles. After doing so, the learner subtracted the area of the inner rectangle from that of the out rectangle to get the area of the shaded part. The learner got the correct answer.

Figure 100 also shows that L21 used multiplication and subtraction to solve the problem. After drawing the problem, the learner went the route of calculating the areas of the big rectangle and that of the smaller rectangle, then subtracting the two from each other to get the area of the shaded part. The learner got the correct answer.

4.6. Summary of findings

In this chapter, results were presented in three sections; the observations made by the researcher, data tables of the problem-solving strategies used by each learner for each problem, and the feedback focus group interviews which were conducted after the learners completed the problems.

The strategies used most prominently by the learners were arithmetic calculations (addition, subtraction, multiplication, and division strategies). These are the basic operations which form the foundation of the mathematics curriculum content. Learners are accustomed to using these operations when solving mathematics problems, these operations have been taught to them throughout their foundation phase years and in each schooling year, the mathematics curriculum content begins with Numbers, Operations and Relationships as per CAPS (2011). It is therefore understandable that these strategies are the preferred strategies when solving mathematics problems.

Making drawings was also found to be a very frequently used strategy, and similar findings were made in the study by Susanti et al., (2019) who found that learners tend to make drawings or other forms of illustrations which helped them make better meaning of the problems. These findings were also made in the study by Aydogdu and Kesan (2014), where making drawings was a frequently used strategy.

A substantial variety of combination strategies was used when solving the problems. This was quite fascinating as it agreed with the views of Tshabalala and Ncube (2013), who stated that a problem-solving approach allowed for learners to solve problems using strategies which were based on their own conceptions of the problems. Other strategies which were used included logical reasoning, making a table, and grouping.

Results from the problem-solving activity also showed that most of the learners had good conceptions of each problem. Many of them also devised appropriate strategies for each problem. However, incorrect solutions resulted due to arithmetic calculation errors. In other words, learners knew what they needed to do and formulated valid problem-solving strategies, but during the solving process many of them made errors when using arithmetic or when converting between metric units. These findings are similar to those made in the study by Lubin et al., (2015), who also found that many learners were simply unable to solve problems due to arithmetic and reversal errors.

Observations revealed that learners tend to be more confident and perform better when solving problems which are familiar in format to them. These are problems which are posed in similar styles to those they do during lessons. Problems involving fractions were very challenging to many learners. Working with fraction problems in word problem form is still a relatively new topic for these grade 4 learners. Moreover, finding fractions of big numbers made it even more challenging. Alamian and Baseri (2020) report that one factor causing learner difficulties in solving fraction problems is making the transition from integers to fractions. This is due to learners' informal knowledge of fractions being largely based on whole-number strategies, leading to solutions which are incorrect.

The other type of problem which learners struggled with were perimeter and area problems. Findings indicate that learners have sound knowledge of how to calculate perimeter and area of 2D shapes. However, when they are presented with compound shapes they struggle to identify that these shapes can be broken up into the shapes which they are familiar with. These are similar to the findings made by Erdogan (2015), who reported that learners found it very challenging when they were presented with problems which required them to use combination strategies to solve. Perimeter and area also involve adding and/ or multiplying different dimensions of the 2D shapes together. Unfortunately, errors in arithmetic often resulted in incorrect answers. The study of Abadi and Amir (2022) found similar learner difficulties in that many learners do not understand the difference between length unit and area unit, resulting in confusion and misconceptions.

Furthermore, the results indicate that when given problems on perimeter or area, learners tend to make their own drawings to simplify their problem and make it visually easier for them to understand. These findings again corroborate with those found by Susanti et al., (2019), as well as those of Aydogdu and Kesan (2014) who also concluded from their study that making drawings is the strategy most often used by learners.

When posed with a word problem that had accompanying diagrams, learners tended to redraw these diagrams as part of their solving process. The study by Nortvedt (2008) revealed a similar finding in that learners tended to re-read the problem to full comprehend what it required them to do. This could also be a possible reason learners re-draw the diagrams given to them.

Another finding which the study revealed was that many learners enjoyed being able to try different strategies as well as realizing that different strategies can be used to solve a single problem. These findings are evident from the learners' responses:

What did you enjoy the most about doing this activity?

L19: Different ways to get the answer.

L11: Dividing numbers.

L10: Different methods can be used to work out the problems.

L2: I enjoyed drawing the and putting the numbers in columns.

L21: I enjoyed question number 4 because it was easy.

L37: It was easy. You could use your head and find new ways to get to the answer.

Learning that problems could be solved in a variety of ways was also a theme that learners took from this study. This can be seen in some of their responses below:

What did you learn from doing this activity?

L10: You can use different methods to solve the problems.

L42: I learnt you can times (multiply) and add.

L37: It gives you the chance to add more.

L2: I learnt that it's fun using different methods.

L19: There are different kinds of ways to get the answer.

L11: I learnt different ways to divide, plus (add), minus (subtract) and times (multiply).

L1: I could use different methods.

The interview responses indicate that learners enjoyed being able to solve problems their own way, by selecting their own strategies and carrying out these strategies in their own unique ways. A trend which emerged from the data was that learners feel very comfortable using drawings to aid them in solving measurement word problems. A variety of different types of drawings were documented and discussed, and often these drawings guided learners towards successfully solving the problem correctly. This emphasizes the importance of visual learning in mathematics which stems from how the subject is first taught during the Foundation Phase years of schooling.

4.7. Look Back – Learners' reflections on the problems

Tell me about the strategy you felt most comfortable using

Group 1

L8: Drawing things because it's much easier.

L15: Drawing things too, then it's not difficult.

L12: Making drawings to work out the answers.

Does the drawing help you?

L12: Yes sir.

L26: Yes sir, drawings are an easier way.

L14: Yes because I think it makes it easy.

Group 2

L13: Adding, because if you add you write the number down that you need.

L9: Drawing because you can get answer more easily.

L5: Drawing, because you can get it easier and you can get the right answer.

L6: Division because you can get the answer easier.

L34: Writing it out because it makes it much when you write it out.

L33: Adding because it's easier for me.

Group 3

L21: I preferred to make a pattern and then I just add all the drawings.

L2: I preferred to break it up. That is very nice.

L1: I prefer to write it out.

L10: I prefer to break it up because it's easier.

L19: To draw because then I can see the problem better.

L37: To add and to subtract.

Group 4

L27: Adding.

L29: Drawing because it's easier to get to the answer.

L36: To draw. It is easier because then I can see the problem with my eyes.

L4: Adding because it's easier.

Group 5

L28: Grouping was my favourite, because grouping was much easier for me, but adding was a bit difficult.

L30: Drawing, because when you draw the things you get it easier.

L38: I drew circles because it makes it easier.

L23: Drawing helped me to do my sums.

When learners were asked about the strategy they felt most comfortable using, there was quite a resounding response in favour of making drawings. There are also a substantial number of learners who preferred using arithmetic strategies. These two types of strategies were also found to be the most used strategies as illustrated in Figure 30 and Figure 44. It could therefore be deduced that many learners tended to use arithmetic strategies and drawings as they were comfortable using these strategies. Similar findings were made in the study by Aydogdu and Kesan (2014) who found that majority of the learners in their study used made drawings at least once when solving word problems. These findings further enhance the argument that learners are more comfortable solving a problem when they have a visual depiction of the problem.

The next chapter will summarise all findings of the study and emphasise the main themes which emerged. Moreover, the chapter will answer each research question based on the findings of the study. Finally, it will be concluded that these findings can be used to help improve how problem-solving is taught. It may also provide sufficient information to encourage more studies into this topic in the future.

CHAPTER 5: DISCUSSION AND FINDINGS

5.1. Introduction

This chapter will collate all the findings of the study and highlight the main findings. It will also answer the research questions of the study and find relationships. Furthermore, after discussing these points, recommendations will be made for future studies. These recommendations may be used to conduct future research on the topic on a larger scale. The findings from this study can also help us understand not only how learners solve measurement word problems, but also why they choose the strategies they do. Finally, actions can then be put in place to help improve the way measurement word problems are taught to generate maximum understanding within learners.

5.2. Learners' conceptions of measurement word problems

From making observations, analysing the problem-solving activity answers of learners as well as the focus group interviews, it can be concluded that most of the learners had very good conceptions of the problems. Most of them could state the problem in their own words and could explain what the problem required of them when asked if they could explain the problem to a friend.

The one aspect learners struggled to understand was when asked to determine the perimeter and area of compound shapes. Many of them were unable to comprehend the concept of breaking up the compound shape into more simper shapes before then determining the perimeter and area of these simpler shapes. This is due to learners having not been exposed to compound shapes yet. They have only recently been taught the perimeter and area of 2D shapes and have so far only encountered 2D shapes.

In summary, learners had good conceptions of most of the problems, but what let them down was executing their strategies. Too many errors were made, particularly in arithmetic which resulted in incorrect solutions. These findings agree with those found by Tong and Loc (2017), whose study revealed that although learners had strong knowledge and comprehension of word problems, many of them continued to carry out inaccurate calculations.

However, it was very encouraging to see that learners chose appropriate strategies, and their intentions were more than commendable. In summary, most learners had good conceptions of the measurement word problems, but they lacked the mathematical skills required to solve the problems. These skills include arithmetic ability as well as being able to successfully carry out their chosen problem-solving strategy.

5.3. Problem-solving strategies used by learners when solving measurement word problems

Through observation and analysis of the problems solved by the learners, the problem-solving strategies used by learners were revealed. These strategies were:

Singular strategies

- addition
- multiplication
- making a drawing
- grouping
- division
- subtraction
- logical reasoning
- guessing
- making a table

Combination Strategies

- addition and making a drawing
- addition and grouping
- addition and multiplication
- multiplication and division
- grouping and making a drawing
- addition and division
- division and making a drawing
- working backwards and addition
- working backwards, addition and division
- addition and logical reasoning
- division and logical reasoning
- subtraction and logical reasoning

- addition and making a table
- addition and division
- division and grouping
- subtraction and grouping
- multiplication and making a drawing
- addition and working backwards
- multiplication and addition
- multiplication and subtraction
- multiplication, addition and making a drawing
- multiplication, addition and breaking up the problem
- multiplication, making a drawing and breaking up the problem
- multiplication, addition, making a drawing and breaking up the problem
- addition and breaking up the problem
- multiplication, subtraction and making a drawing

The singular strategies were primarily strategies that have been taught to learners during the Foundation Phase of their schooling. Prior to this study, the researcher had only exposed them to arithmetic strategies (addition, subtraction, multiplication, and division strategies), and making drawings. It therefore indicates that learners tended to use the strategies which were revealed to them during Foundation Phase as well as the first few months they have had in grade 4. It was also interesting to note that many learners tended to use drawing strategies taught in Foundation Phase as the basis of their solutions, then building on these drawings to make them applicable to the problems at hand. Bruun (2013) made similar findings, reporting that making drawings was one of the most prominently used strategies in the particular study. This finding was also echoed in the study of Yasmeen (2019), who reported that learners performed moderately well when making drawings to solve problems.

Arithmetic strategies and making drawings were the two strategies which were used most frequently across all problems. Furthermore, learners also mentioned that they enjoyed using these two strategies in conjunction with one another to solve problems.

Learners made use of a range of different strategies when solving the word problems. Similar findings were made during the study conducted by Sulak (2010). During that study, it was found that learners used strategies such as making drawings, drawing a table, writing out sentences looking for a pattern, making lists, using logical reasoning and the guessing-and-checking strategy.

The study also revealed that although most learners had good conceptions of the word problems and they devised appropriate strategies, many of their solutions were incorrect simply due to arithmetic errors. Therefore, if more effort is put into ensuring that learners improve their arithmetic skills, they will become more successful at solving word problems. This is also backed up by Lubin et al., (2015) who emphasised that it is imperative for learners to master arithmetic skills as arithmetic word problems are found throughout the mathematics curriculum.

Moreover, learners must be commended for using such a broad range of strategies as well as the various unique ways in which they carried out these strategies. These findings agree with the findings of Intaros et al., (2014) who found that learners were able to work devise their own problem-solving strategies when solving open-ended word problems. Often, learners also combined strategies when solving a problem.

5.4. The use of combination strategies to solve a problem

Using combination strategies to solve the word problems was a fascinating finding. Even more interesting was the different combinations learners used. Some combinations brought together two strategies, while there were also instances where three or even four different strategies were used to solve a single problem. Saygili (2017) made similar findings, reporting that learners solve non-routine problems in a variety of different ways using various problem-solving strategies.

5.4.1. Combining different arithmetic strategies

Learners also tended to combine different operations when doing arithmetic calculations. When performing multiplication, some learners would decompose the different number values of the number. They would then multiply each numeric value by the specific number and then regroup the answers by addition to get the final answer. Similarly, learners would multiply big numbers by using repeated addition. This method was used by learners who felt more comfortable using addition than multiplication.

5.4.2. Combining arithmetic strategies and making drawings

When devising their strategies, many learners also combined strategies to solve the problem. This indicates that learners recognise that it is possible to intertwine strategies to help them in making the problem-solving process easier. As previously mentioned, the most frequent pair of strategies which were combined was arithmetic calculations and making drawings. Learners would construct drawings of the problem at hand based on their understanding of the problem and use this drawing to determine the appropriate arithmetic operations that would yield them the solution.

5.5. How learners went about carrying out their strategies

Through analysis of the problems solved by the learners as well as their responses during the focus group interviews, the manners in which learners carried out their problem-solving strategies were revealed. When given a problem with an accompanying drawing, learners tended to redraw the problem either as it was in the question, or they constructed their own visual interpretation of the problem. This further demonstrates the importance of drawings and diagrams as being a key aspect of grade 4 learners' problem-solving strategies.

When given a problem containing a complex drawing, many learners were able to construct simplified versions of the shape and break it up into simpler shapes which they could then calculate the perimeters and areas of. A topic of concern which surfaced from these area and perimeter calculations was that some learners were unable to distinguish between the two, while others did not know how to calculate area and perimeter. Hence, after breaking up the drawings many learners could not carry on further with the problem. This is possibly due to the topic of perimeter and area being relatively new to learners. They started off being taught perimeter and area using gridlines which allowed them to measure these shapes using the grid blocks as guides. Thereafter, they were exposed to the formulae of the perimeter and area of various 2D shapes which were all simple. However, with the problems in this study being made up of compound shapes, most learners were unable to see that these were simply two simple shapes put together which meant they could have broken up the compound shape and worked with the 2 simple shapes it consisted of.

The results also showed that learners use a variety of different techniques when using arithmetic calculations. When using multiplication strategies, some learners would make use of expansion methods while others would use repeated addition to get to their solution. When using addition strategies, some learners would use expansion while others made use of place value tables.

When analysing the strategies used by the learners, two strategies were utilised more frequently than the others: making drawings and arithmetic strategies. This makes sense as during Foundation Phase, basic operations and visual learning through drawings form the core of the Mathematics Curriculum (CAPS, 2011). During the focus group interviews, there was also a strong theme that learners made use of these two strategies because it came to them instinctively because of them being taught using these strategies. These are the two strategies that the researcher had exposed them to so far during their first few months of grade 4. Furthermore, in the Foundation Phase teachers tend to teach operations and arithmetic with the aid of drawings to give learners a visual representation of the problem. These two strategies are therefore deeply engrained within learners.

However, even within these two strategies, there appears to be great variation in the types of drawings learners make as well as how they go about carrying out their arithmetic calculations. Drawing sticks or circles for addition and multiplication strategies was a common trend. Another trend within the drawing strategy was grouping their circles or sticks together into equal groups. The study conducted by Susanti et al., (2019) reflected similar findings whereby results showed that learners made their own drawings, graphs, and other types of visual depictions when solving open-ended word problems.

When reflecting on their own solutions, majority of the learners were satisfied with their attempts. There was also a consensus that certain problems were found challenging because they contained large numbers. Another frequent response within the interviews was learners stating that they did not understand the problem. This finding is similar to that of Xin, Jitendra and Deatline-Butchman (2005), who mentioned that reading comprehension may be a factor which contributes to learners being unable to solve problems. While this cannot simply be proven from the study, it does make a strong claim for more research to be done on the effect that reading comprehension has on the problem-solving ability and success of learners.

5.6. Conclusions

5.6.1 Applying Constructivist Learning Theory to the study

Major and Mangope (2012) mention that within this learning theory, knowledge is built through observation and people's experiences. Within this study, learners were allowed to work independently. By doing so, learners were able to form their own understanding of the problems, make relationships and devise their own strategies based on their conceptions of each problem. Through allowing learners to do the activity in a constructivist environment, the results showed not only the variety of strategies learners came up with, but it also showed the diverse range of ways they carried out their strategies. Basic arithmetic operations were carried out in different ways by learners, showcasing the different ways learners interpret a problem and thus the different manners in which they carry out their strategies. Working within a constructivist classroom environment also enabled learners to rectify many of their own mistakes as they formed their own conceptions of each measurement word problem.

The focus group interviews allowed learners not only to reflect on their own solutions, but also to share their thoughts with their peers. Sharing their ideas and thoughts in groups enabled learners to discuss the problems with other individuals on the same cognitive level. This made communication and understanding much easier as they used language and vocabulary that all of them understood. Group reflection also allowed learners to learn from each other and in sharing these thoughts, they can build onto their own knowledge using knowledge of their peers. Reflecting on problems also forms a core part of the problem-solving framework of Polya (1957), which will be discussed in the next section.

5.6.2. Applying Polya's stages of problem-solving to the study

Understand the problem

Majority of the learners who participated in the study understood most of the problems. They were generally able to restate the problem in their own words and explain what the problem required them to do. The problems which most learners understood and could solve were those which were posed in similar fashion to problems they have encountered in lessons. These problems typically contained sight words which would help them identify which operations to use to solve the problem.

Even when problems involved fractions or working with different units of measurement, learners generally knew which operations they needed to use to solve these problems. However, many of them tended to make arithmetic or conversion errors which yielded in incorrect answers. Therefore, in general, most learners had a good understanding of the problems and what each one required of them. The issue for learners was simply their errors in arithmetic which is an aspect that can be corrected with suitable remediation.

The one type of problem that many learners did not seem to comprehend was when being required to determine the perimeter and area of complex shapes. However, this is understandable as these grade 4 learners had not been exposed to perimeter and area of compound shapes prior to this study. On the contrary, it was also quite impressive to observe the solutions of those who were able to solve these problems successfully.

Devise a plan

Learners came up with appropriate and valid strategies to solve the problems given to them. It was quite fascinating to see the variety of different ways learners added, multiplied, subtracted, and divided. Furthermore, the range of different types of drawings was quite interesting. Another interesting finding was that even at grade 4 level, learners already combine strategies when solving a problem. These different methods used by learners reinforced the concept that there are many ways to solve a problem. Learners themselves mentioned that from these problems, they learned that a variety of different strategies can be used to solve a problem.

Carry out the plan

Overall, less than half of the learners were able to successfully carry out their strategies to arrive at the correct solutions. This was largely due to errors in arithmetic by most learners who used arithmetic calculations as their problem-solving strategies. It re-emphasizes the argument that learners require more training in the skills of doing arithmetic calculations. Another factor which resulted in incorrect solutions was that after making drawings, learners were unsure as to how to use their drawings as a means of assisting them in solving the problem. Some learners made accurate visual depictions of the problems, but they were unable to apply the mathematics which was required to solve the problems. These findings are similar to those made by Peranginangin (2017), who found that learners struggled to carry out their plan and to think meta-cognitively when executing their strategies.

The results of this study also showed that despite the wide range of drawings learners made when trying to solve the measurement word problems, many of their drawings were also either inaccurate or they did not quite match the problem. These results show similarities to those in the study by Tambychik and Meerah (2010), who found that learners are unable to make accurate and appropriate visual representations of word problems because they are unable to comprehend the problem upon reading it.

Look back

Upon reflecting on their solutions, learners gave honest and realistic feedback. Many learners were happy with the strategies they selected and how they carried them out. However, there were also several learners who felt they could have used different strategies had they been given the problems again. When sitting together in the different groups during the focus group interviews, many learners who were unsuccessful at solving a particular problem came to realisations as to which methods would have worked better for them by seeing and hearing what their peers did. Another reflection made by learners was their realisations that there are different strategies which can be used to solve a particular problem which all result in the same correct solution. To hear learners come to this conclusion on their own is remarkable as it is after all one of the primary objectives of this study. Finally, learners who used different strategies such as grouping, drawing tables and logical reasoning are all using valid strategies. However, they just require some guidance and training to show them how to use those strategies correctly as they are not taught these in the classroom or in textbooks.

5.7. Recommendations

A more concerted effort should be made to expose learners to a wider range of problem-solving strategies. Too often, learners struggle to carry out or make sense of the strategies taught to them by the teacher. It is then assumed that learners are unable to solve these problems. This is echoed by Bruun (2013), who emphasized the need to promote the teaching of a wider variety of problem-solving strategies within mathematics lessons.

However, it is also possible that learners will simply perform better if they used a different strategy to solve the problem. The study has shown that there are learners making use of unique drawing and arithmetic methods which are not seen in textbooks. These learners must be recognised for their methods, so that they feel confident enough to use these methods regularly without doubting themselves just because they are different to the methods that the teacher taught to them in class. Learners are almost forced to use a specific method when solving a problem. A similar finding was made by Intaros et al., (2014), who reported that when learners solve open-ended mathematical problems in a classroom that promotes problem-solving, they can devise their own problem-solving strategies to solve mathematical problems.

We need to be more open-minded to the possibility that these learners are not able to succeed using the taught method because they are naturally more adept at using a different method. Doing so may well lead to an improvement in learner mathematics performance and increase learner interest in the subject. Furthermore, if these additional strategies can be recognised and even taught, then it can also enable teachers to help learners in harnessing their skills to master their preferred strategies.

Another finding which needs to be addressed is the number of errors learners make when performing basic arithmetic calculations. Being proficient in arithmetic calculations is one of the core skills required to be able to solve mathematics problems. It is therefore paramount that learners are taught to master the skills of performing basic arithmetic calculations. Doing so would result in many more learners arriving at the correct solutions to problems as this study has shown that most learners use appropriate arithmetic calculation strategies, but their execution of their strategies let them down.

The study also shows that more emphasis needs to be placed on problem solving within the curriculum and to allow learners to solve problems independently without instructing them with precisely which method they should use. Some learners might not feel comfortable using the method prescribed in textbooks or by the teacher, resulting in them being unsuccessful at solving the problems. Teachers tend to then assume that the learner is struggling with the concept at hand, without considering the possibility that the learner might be much more capable and comfortable when using a different strategy.

As previously mentioned, learners make use of strategies which are not taught in the classroom or found in textbooks. These strategies include working backwards, drawing a table and logical reasoning. Finally, the study also showed comprehensive evidence of learners using more than one strategy and combing them to solve a problem. Once again, this needs to be promoted within the classroom. The time has come for learners as well as educators to see the relationships between different strategies and different basic arithmetic operations. They have been seen as separate entities for far too long.

5.8. Autobiographical Reflection

Conducting this study was a very enriching and insightful experience. It demonstrated to me what learners were capable of doing when allowing them to form their own conceptions of problems, and then devising their own problem-solving strategies to solve the problems. It showed me that just because learners are unable to solve problems using methods prescribed in textbooks, it does not necessarily mean that they cannot solve the problem. It may be that they simply find comfort doing it a different way, Mathematics is after all a subject in which there is a wide variety of methods that can be used to solve the same problem.

Furthermore, the study showed me that within the different problem-solving strategies which have been document there are so many ways a single strategy can be carried out. These findings have changed my perception of how I teach Mathematics. I will make a concerted effort to promote a wide range of methods which learners can use during problem-solving and be open-minded to the idea of some learners devising strategies which are different but still equally valid.

References

Abadi, M.A.S., & Amir, M.F. (2022). Analysis of elementary school students difficulties of in solving perimeter and area problems. *Jurnal Ilmiah Pendidkan Matematika (JIPM),* 10(2), 396-408. Retrieved from: http://e-journal.unipma.ac.id/index.php/jipm/article/view/11053/3928

Alamian, V. & Baseri, L.K. (2020). The effect of manipulation on reducing the errors of fourth-grade elementary students while solving fraction word problems. *Archives of Pharmacy Practice*, 11(1), 80-87. Retrieved from: https://archivepp.com/storage/models/article/MFBIdOjbQUZxCPeOBl3SscT7WbAAkqkxpbiH7sBDLS2jkS6qMARbRvfJO1FD/the-effect-of-manipulation-on-reducing-the-errors-of-fourth-grade-elementary-students-while-solvin.pdf

Aljaberi, N.M. (2015). University students' learning styles and their ability to solve mathematical problems. *International Journal of Business and Social Science*, 6(4), 152-165.Retrievedfrom: http://ijbssnet.com/journals/Vol_6_No_4_1_April_2015/18.pdf

Anderson, J. (2005). Implementing Problem Solving in Mathematics Classrooms: What Support do Teachers Want? Retrieved from: http://www.merga.net.au/documents/RP42005.pdf

Aydogdu, M.Z. & Kesan, C. (2014). A research on geometry problem solving strategies used by elementary mathematics teacher candidates. *Journal of education and instructional studies in the world,* 4(1), 53-62. Retrieved from: http://www.wjeis.org/FileUpload/ds217232/File/07a.aydogdu.pdf

Bada, S.O. & Olusegun, S. (2015). Constructivist Learning Theory: A Paradigm for Teaching and Learning. *IOSR International Journal of Research & Method in Education*, 5(1), 66-70. Retrieved from: https://vulms.vu.edu.pk/courses/edu201/downloads/edu%20201%20(assignment%202).pdf

Bowen, G. (2009). Document Analysis as a Qualitative Research Method. Retrieved from: https://www.researchgate.net/publication/240807798

Brooks, J. G., & Brooks, M. G. (1999). *In search of understanding: The case for constructivist classrooms*. Ascd.

Bruun, F. (2013). Elementary teachers' perspectives of mathematics problem solving strategies. *The Mathematics Educator,* 23(1), 45-59. Retrieved from: https://openjournals.libs.uga.edu/tme/article/view/1991

Chamot, A.U., Dale, M., O'Malley, J.M.& Spanos, G.A. (1992). Learning and problem solving strategies of ESL students. *Bilinigual Research Journal,* 16(3), 1-28. Retrieved from: https://www.tandfonline.com/doi/abs/10.1080/15235882.1992.10162635

Chapman, O. (2003). Teachers' Conceptions of Mathematical Word Problems: A Basis for Professional Development. *International Group for the Psychology of Mathematics Education,* 2(1), 197-204. Retrieved from: https://files.eric.ed.gov/fulltext/ED500938.pdf

Chisholm, L. (2003). The state of curriculum reform in South Africa: The issue of Curriculum 2005. *State of the nation: South Africa, 2004,* 268-289. Retrieved from: https://books.google.co.za/books?hl=en&lr=&id=Ybz9Ei5TVaIC&oi=fnd&pg=PA268&dq=curriculum+reform+in+south+africa&ots=zXj1VOCaQ1&sig=ciAHaMQxM8fudWHsNtxNkuwY9n4&redir_esc=y#v=onepage&q=curriculum%20reform%20in%20south%20africa&f=false

CK-12 Foundation. (2012). *Problem-solving strategies: Guess and check, work backward*. Retrieved from: http://www.ck12.org/book/CK-12-Algebra-I-Second-Edition/r17/section/2.6/

Clements, D.H., Barret, J.E., & Sarama, J. (2017). Measurement in Early and Elementary Education. Children's measurement: A longitudinal study of children's knowledge and learning of length, area, and volume. *Journal for Research in Mathematics Education, Monograph Series,* 16, 3-68. Retrieved from: https://www.nctm.org/Handlers/AttachmentHandler.ashx?attachmentID=LqBpwR2zmQk%3D

Cummins, D. D., Kintsch, W., Reusser, K., & Weimer, R. (1988). The role of understanding in solving word problems. *Cognitive psychology*, *20*(4), 405-438.

Department of Education. (2011), Curriculum and Assessment Policy Statement (CAPS), Mathematics: Intermediate Phase. Pretoria, South Africa.

Drake, M. (2014). Learning to measure length.: The problem with the school ruler. *Australian Primary Mathematics Classroom*, 19(3), 27-32. Retrieved from: https://www.semanticscholar.org/paper/The-problem-with-the-school-ruler-Ever-wondered-why-Drake/ca378c980022d13a077c121272ba0f42826f462b

Edgar, C. & Robin, J. (2000). *Algebra Readiness*. Retrieved from: https://books.google.co.za/books?id=d3w9F5Cw2AEC&pg=PT78&lpg=PT78&dq=problem+solving+strategy+use+logical+reasoning&source=bl&ots=UUVYbxILvu&sig=1DCYxR_cUk8Wwh7ivEv8Ga5jnrM&hl=en&sa=X&ved=0CD0Q6AEwBzgKahUKEwiOl_zCjOjHAhXCph4KHX_CD5M#v=onepage&q=problem%20solving%20strategy%20use%20logical%20reasoning&f=false

Eduafo, A.B. (2014). Effects of problem-solving approach on Mathematics achievement of diploma in basic education distance learners at University of Cape Coast, Ghana. Retrieved from: http://ir-library.ku.ac.ke/bitstream/handle/123456789/11982/Effects%20of%20problem%20solving%20approach%20on%20mathematics%20achievement%20of%20diploma%20in%20basic%20education%20distance%20learners%20at%20university%20of%20cape%20coast,%20Ghana.pdf?sequence=1

Erdogan, A. (2015). Turkish primary school students' strategies in solving a non-routine mathematical problem and some implications for the curriculum design and implementation. *International Journal of Mathematics Teaching and Learning.* Retrieved from: http://www.cimt.plymouth.ac.uk/journal/erdogan.pdf

Fierro, R. (2013). *Mathematics for elementary school teachers.* Retrieved from: https://books.google.co.za/books?id=IJBCCMaufWEC&pg=PA19&lpg=PA19&dq=problem+solving+strategy+be+ingenious&source=bl&ots=rM0Fgv4R6t&sig=j97oexjVvKIopJGA52-gFKxw6P0&hl=en&sa=X&ved=0CDMQ6AEwBGoVChMI_Jqyld7qxwIVAVUUCh31UAr9#v=onepage&q=problem%20solving%20strategy%20be%20ingenious&f=false

Freebern, N. (2006). *Guess and Check*. Retrieved from: http://www.studyzone.org/testprep/math4/d/guesscheck4l.cfm

Gakure, R.W., Mukuria, P., & Kithae, P.P. (2013). An evaluation of factors that affect performance of primary schools in Kenya: A case study of Gatanga district. *Academic Journals*, 8(13), 927-937. Retrieved from: http://www.academicjournals.org/article/article1379769520_Gakure%20et%20al.pdf

Gray, H. (2005). Constructivist Teaching and Learning (Doctoral Dissertation). Retrieved from: http://methodenpool.unikoeln.de/partnerarbeit/httpwww.principalspartnership.comconstructivism.pdf.pdf

Hawkins, J. & Stobie, M. (2006). *Survive Math Five. Problem Solving*. Retrieved from: http://www.openschool.bc.ca/k12/pdfs/math5/m5-p4-problemsolving.pdf

Hiebert, J., Carpenter, T.P., Fennema, E., Fuson, K., Human, P., Murray, H., Olivier, A., & Wearne, D. (1996). Problem Solving as a Basis for Reform in Curriculum and Instruction: The Case of Mathematics. *American Educational Researcher*, 25(4), 12-21. Retrieved from:http://journals.sagepub.com/doi/abs/10.3102/0013189X025004012

Hosseini, M.J., Hajishirzi, H., Etzioni, O., & Kushman, N. (2014). Learning to Solve Arithmetic Word Problems with Verb Categorization. *Proceedings of the 2014 Conference on Empirical Methods in Natural Language Processing (EMNLP)*, 523-533. Retrieved from: https://pdfs.semanticscholar.org/b83e/5dcf01eaf5b8a81c4a616b8511ebcc2e892a.pdf

Hurrell, D. (2015). Measurement: Five considerations to add even more impact to your program. Australian Primary Mathematics Classroom, 20(4), 14-18.

In'am, A. (2014). The Implementation of the Polya Method in Solving Euclidean Geometry Problems. *International Education Studies*, *7*(7), 149-158.

Intaros, P., Imprasitha, M. & Srisawadi, N. (2014). Students' problem solving strategies in problem solving – mathematics classroom. *Procedia – Social and Behavioral Sciences*, 116, 4119-4123. Retrieved from: https://www.researchgate.net/profile/Pimpaka_Intaros/publication/275543259_Students'_Problem_Solving_Strategies_in_Problem_Solving-mathematics_Classroom/links/556e840708aeccd7773f6f55/Students-Problem-Solving-Strategies-in-Problem-Solving-mathematics-Classroom.pdf

Jojo, Z. (2019). Mathematics education system in South Africa. *Education systems around the world*, 129-140.

Kaufman, C. (2010). *Strategies to improve the solving of word problems*. Brookes Publishing. Retrieved from: http://archive.brookespublishing.com/newsletters/downloads/kaufman_word-problems.pdf

Kothari, C.R. (2004). *Research Methodology methods and techniques*. New Age International (P) Limited, Publishers. New Delhi. Retrieved from: https://www.modares.ac.ir/uploads/Agr.Oth.Lib.17.pdf

Lakshman, M., Sinha, L., Biswas, M., Charles, M., & Arora, N. K. (2000). Quantitative vs qualitative research methods. *The Indian Journal of Pediatrics*, *67*(5), 369-377. Retrieved from: https://link.springer.com/article/10.1007/BF02820690

Lambdin, D. (2009). *Benefits of teaching through problem solving*. Retrieved from: https://www.google.co.za/search?q=Lambdin%2C+D.+(2009).+Benefits+of+teaching+through+problem+solving&rlz=1C1SAVG_enZA580ZA580&oq=Lambdin%2C+D.+(2009).+Benefits+of+teaching+through+problem+solving&aqs=chrome..69i57.442j0j7&sourceid=chrome&es_sm=122&ie=UTF-8

Lesh, R. A., & Doerr, H. M. (2003). *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Routledge.

Lubin, A., Houde, O., & de Neys, W. (2015). Evidence for children's error sensitivity during arithmetic word problem solving. *Learning and Instruction*, 40(1), 1-8. Retrieved from: http://www.sciencedirect.com/science/article/pii/S0959475215300190

Maharani, H. R. (2014). Creative thinking in mathematics: Are we able to solve mathematical problems in a variety of way. In *International Conference on Mathematics, Science, and Education*, 23(1). 120-125. Retrieved from: http://research.unissula.ac.id/file/publikasi/211313016/402521.pdf

Major, T.E. & Mangope, B. (2012). The Constructivist Theory in Mathematics: The Case of Botswana Primary Schools. *International Review of Social Sciences and Humanities*, 3 (2), 139-147. Retrieved from: http://irssh.com/yahoo_site_admin/assets/docs/15_IRSSH-155-V3N2.202200518.pdf

Maree, K., Aldous, C., Hattingh, A., Swanepoel, A., & van der Linde, M. (2006). Predictors of learner performance in mathematics and science according to a large-scale study in Mpumalanga. *South African Journal of Education*, 26(2), 229-252. Retrieved from:http://repository.up.ac.za/bitstream/handle/2263/2191/Maree_(Predictors)2006.pdf?sequence=1

McDonough, A., & Cheeseman, J. (2014). Young learners' understandings about mass measurement: insights from an open-ended task. In P. Liljedahl, S. Oesterle and C. Nicol (Ed). *Proceedings of the 38th Conference of the International Group for the Psychology of Mathematics Education and the 36th Conference of the North American Chapter of the Psychology of Mathematics Education*. Canada: International Group for the Psychology of Mathematics Education. Pp. 193 – 200. Retrieved from: https://acuresearchbank.acu.edu.au/item/8q5xz/young-learners-understandings-about-mass-measurement-insights-from-an-open-ended-task

Montoro, A.B., Aguayo-Arriagada, C.G., & Flores, P. (2021). Measurement in Primary School Mathematics and Science Textbooks. *Mathematics*, 9 (17), 1-19. Retrieved from: https://www.mdpi.com/2227-7390/9/17/2127

Mouton, N., Louw, G.P., & Strydom, G. (2013). Critical Challenges Of The South African School System. *International Business & Economics Research Journal*,12(1), 31-44. Retrieved from: https://www.researchgate.net/publication/297722032_Critical_Challenges_Of_The_South_African_School_System

Msimango, K., & Luneta, K. (2015). Exploring Instructional Strategies That Enhance Problem Solving Skills In Mathematics Grade 3. 191-206. Retrieved from: http://uir.unisa.ac.za/bitstream/handle/10500/22424/Kgomotso%20Msimango%2C%20Kakoma%20Luneta.pdf?sequence=1&isAllowed=y

Nortvedt, G. (2008). Reading word problems. Retrieved from: http://www.mai.liu.se/SMDF/madif6/Nortvedt.pdf

Ollerton, M. (2007). Teaching and learning through problem solving. Retrieved from: http://www.atm.org.uk/write/MediaUploads/Journals/MT201/Non-Member/ATM-MT201-03-05.pdf

Peranginangin, S.S.& Surya, E. (2017). An analysis of students' mathematics problem solving ability in VII grade at SMP Negeri 4 Pancurbatu. *International Journal of Sciences: Basic and Applied Research*, 33(2), 57-67. Retrieved from: https://www.researchgate.net/profile/Siska_Peranginangin/publication/317416492_An_Analysis_of_Students'_Mathematics_Problem_Solving_Ability_in_VII_Grade_at_SMP_Negeri_4_Pancurbatu/links/5939fd46aca272bcd1e295a3/An-Analysis-of-Students-Mathematics-Problem-Solving-Ability-in-VII-Grade-at-SMP-Negeri-4-Pancurbatu.pdf

Polya, G. (1957). *How to Solve It*. Princeton University Press. Retrieved from: http://www.math.utah.edu/~pa/math/polya.html

Pooran, A.(2011). An exploration of the teaching and learning of mathematics word problems in English to second language grade 8 learners in the Mfukuzela-Gandhi Circuit. Masters Dissertation. Durban University of Technology. Retrieved from: http://ir.dut.ac.za/handle/10321/749

Proust, M. (2014). *We need to talk about maths problem solving.* Retrieved from: www.into.ie/ROI/Publications/InTouch/FullLengthArticles/FullLengthArticles2014/WeNeedToTalk_ProblemSolving.pdf

Ramirez, G., Chang, H., Maloney, E.A., Levine, S.C., & Beilock, S.L. (2015). On the relationship between math anxiety and math achievement in early elementary school: the role of problem solving strategies. Retrieved from: https://reader.elsevier.com/reader/sd/pii/S0022096515001812?token=C4FBDC0F38A78EBC824548517373DB1E5FECC492360EF4737B9A512AEBD66366846E15E4ED28468DFEA6455CF57427D3

Rowley, J. (2002). Using case studies in research. *Management research news,* 25(1), 16-27. Retrieved from: http://psyking.net/HTMLobj-3843/using_case_study_in_research.pdf

Saygili, S. (2017). Examining the problem solving skills and the strategies used by high school students in solving non-routine problems. *E-International journal of educational research*, 8(2), 91-114. Retrieved from: http://www.e-ijer.com/en/download/article-file/338828

Shapiro, S. (2006). *Solve that problem. Skills and strategies for practical problem solving*. Retrieved from: https://books.google.co.za/books?id=9kJz6TXJbiMC&pg=PA20&lpg=PA20&dq=working+backwards+sharon+shapiro&source=bl&ots=hbmWG_s3io&sig=C1ZP3E66Glq6ckfNs8Wp07GFGyE&hl=en&sa=X&ved=0CCEQ6AEwAWoVChMI57qSoc33xwIVrI_bCh2iuAXk#v=onepage&q=working%20backwards%20sharon%20shapiro&f=false

Shirali, S. (2014). George Polya and Problem Solving. An appreciation. *Resonance, 19(4), 310-322.* Retrieved from: http://www.ias.ac.in/resonance/Volumes/19/04/0310-0322.pdf

Sulak, S. (2010). Effect of problem solving strategies on problem solving achievement in primary school mathematics. *Procedia Social and Behavioral Sciences, 9 (1), 468-472.* Retrieved from: https://www.researchgate.net/publication/238003877_Effect_of_problem_solving_strategies_on_problem_solving_achievement_in_primary_school_mathematics

Surgenor, P. (2010). *Teaching Toolkit. How Students Learn 4*. Retrieved from: http://www.ucd.ie/t4cms/UCDTLT0019.pdf

Susanti, E., Hapizah, H., Meryansumayeka, M & Irenika, I. (2019). Mathematical thinking of 13 years old students through problem-solving. *Journal of physics: Conf. Series* 1318. Retrieved from: https://iopscience.iop.org/article/10.1088/1742-6596/1318/1/012103/pdf

Swanson, H.L., Jerman, O., & Zheng. X. (2008). Growth in Working Memory and Mathematical Problem Solving in Children at Risk and Not at Risk for Serious Math Difficulties. *Journal of Educational Psychology*, 100(2), 343-379. Retrieved from: http://psycnet.apa.org/doi/10.1037/0022-0663.100.2.343

Taber, K.S. (2011). Constructivism as educational theory: Contingency in learning, and optimally guided instruction. *Educational Theory*, 1 (1), 39-61. Retrieved from: https://camtools.cam.ac.uk/access/content/group/cbe67867-b999-4f62-8eb7-58696f3cedf7/Educational%20Theory/Constructivism%20as%20Educational%20Theory.pdf

Tachie, S.A., & Chireshe, R. (2013). High Failure Rate in Mathematics Examinations In Rural Senior Secondary Schools in Mthatha District, Eastern Cape: Learners' Attributions. *Stud Tribes Tribals*, *11*(1), 67-73. Retrieved from: http://www.krepublishers.com/02-Journals/T%20&%20T/T%20&%20T-11-0-000-13-Web/T%20&%20T-11-1-000-13-ABST-PDF/S-T&T-11-1-067-13-290-Tachie-S-A/S-T&T-11-1-067-13-290-Tachie-S-A-Tt.pdf

Tambychik, T., & Meerah, T.S.M. (2010). Students' Difficulties in Mathematics Problem-Solving: What do they say? *Procedia Social and Behavioral Science*, 8(1), 142-151. Retrieved from: http://www.sciencedirect.com/science/article/pii/S1877042810021257

Tashakkori, A., & Creswell, J.W. (2007). Editorial: The new era of mixed methods. *Journal of Mixed Methods Research*, 1(1), 3-7. Retrieved from: https://journals.sagepub.com/doi/pdf/10.1177/2345678906293042

Teacher Vision. (2015). *Problem solving: Find a pattern*. Retrieved from: https://www.teachervision.com/problem-solving/teaching-methods/48900.html

Thompson, A. G. (1985). Teachers' conceptions of mathematics and the teaching of problem solving. Teaching and learning mathematical problem solving: Multiple research perspectives, 281-294.

Tong, D.H., & Loc, N.P. (2017). Students' errors in solving mathematical word problems and their ability in identifying errors in wrong solutions. *European Journal of Education Studies*, 6(3), 226-241. Retrieved from: https://www.oapub.org/edu/index.php/ejes/article/view/741/2100

Tsanwani, A., Harding, A., Engelbrecht, J., & Maree, K. (2014). Perceptions of teachers and learners about factors that facilitate learners' performance in mathematics in South Africa. *African Journal of Research in Mathematics, Science and Technology Education*, *18*(1), 40-51.

Tshabalala, T., & Ncube, A.C. (2013). Causes Of Poor Performance Of Ordinary Level Pupils In Mathematics In Rural Secondary Schools In Nkayi District: Learners' Attributions. *Nova Journal of Medical and Biological Sciences*, *1*(1), 4-14. Retrieved from: http://novaexplore.com/NJMBS/wp-content/uploads/sites/4/2014/02/N.JMBS_.4-14.pdf

Volce, J. (2015). *Use logical reasoning (another problem-solving strategy)*. Retrieved from: https://janiciavolce.wordpress.com/2015/03/02/use-logical-reasoning-another-problem-solving-strategy/

Watters, S. & Logan, P. (2000). *I can solve problems*. Retrieved from: http://www.greenfaulds.n-lanark.sch.uk/Greenfaulds%20Department/mathematics/ICanSolveProblemsComp.pdf

Xenofontos, C. (2014). Defining mathematical problems and problem solving: Prospective primary teachers' beliefs in Cyprus and England. *Mathematics Education Research Journal*. 26(2), 279-299. Retrieved from: https://www.researchgate.net/publication/259470940_Defining_mathematical_problems_and_problem_solving_Prospective_primary_teachers'_beliefs_in_Cyprus_and_England

Xin, Y.P., Jitendra, A.K., & Deatline-Butchman, A. (2005). Effects of word problem-solving instruction on middle school students with learning problems. *The journal of special education*. 39(3), 181-192. Retrieved from: https://www.researchgate.net/publication/249834124_Effects_of_Mathematical_Word_Problem_Solving_Instruction_on_Middle_School_Students_with_Learning_Problems

Yasmeen, A. (2019). Identifying the problem-solving strategies of mathematics students of secondary level. *International conference on statistical sciences*, 33(1), 127-146. Retrieved from: https://www.researchgate.net/profile/Asad-Ali-62/publication/332686469_A_GENERALIZED_REGRESSION-PRODUCT_ESTIMATOR_IN_PRESENCE_OF_NON-RESPONSE/links/5cd6a33fa6fdccc9dd9f77aa/A-GENERALIZED-REGRESSION-PRODUCT-ESTIMATOR-IN-PRESENCE-OF-NON-RESPONSE.pdf#page=133

Appendix A – Problem solving activity
SECTION A

1. Sipho has 14 pieces of rope. Each piece if rope is 8 m long. What length of rope does Sipho have altogether?

2. Kate has 100 cm of ribbon. She cuts it into 4 equal pieces. How long is each piece of ribbon?

3. Ciaran walked ½ km, Alex walked 1/5 km and Calum walked ¼ km. How far did each learner walk in metres?

4. A pencil is 130 mm long. A ruler is 3 times longer than the pencil. How long is the ruler in centimetres?

5. On the first day John drives his car a distance of 75 km. On the second day he drives 12 500 m and on the third day he drives 13 km. What is the total distance driven in metres?

6. Mary's mass is 48 kg. Her younger sister, Paula, has a mass equal to three-quarters of Mary's mass. What is Paula's mass?

Possible strategies for SECTION A

Possible strategies for Problem 1

1. **Logical and direct reasoning**

Understand the problem

Sipho has 14 pieces of rope. We need to find the total length of the 14 pieces if each piece is 8m long.

Devise a plan

If one piece of rope is 8m in length, then 2 pieces will be (8m+8m) = 16m in length. We can use this reasoning until we get to the length of 14 pieces of rope.

Carry out the plan

1 piece of rope = 8m

2 pieces of rope = (8m+8m) = 16m

3 pieces of rope = (16m+8m) = 24m

4 pieces of rope = (24m+8m) = 32m

5 pieces of rope = (32m+8m) = 40m

6 pieces of rope = (40m+8m) = 48m

7 pieces of rope = (48m+8m) = 56m

8 pieces of rope = (56m+8m) = 64m

9 pieces of rope = (64m+8m) = 72m

10 pieces of rope = (72m+8m) = 80m

11 pieces of rope = (80m+8m) = 88m

12 pieces of rope = (88m+8m) = 96m

13 pieces of rope = (96m+8m) = 104m

14 pieces of rope = (104m+8m) = 112m

Therefore, the total length of the 14 pieces of rope is 112m.

Look back

This strategy took quite some time. Looking back now we can see a pattern of adding 8s each time.

2. **Drawing a picture**

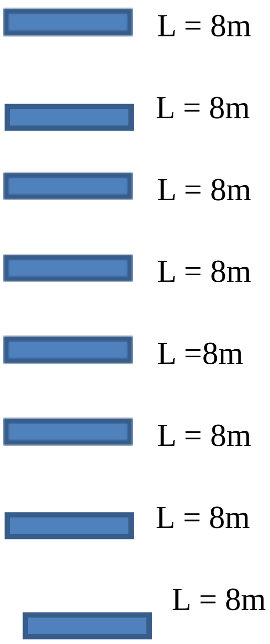
Understand the problem

Sipho has 14 pieces of rope. We need to find the total length of the 14 pieces if each piece is 8m long.

Devise a plan

We can draw pictures of the 14 pieces of rope. We can then draw a picture of all the ropes together in order to help us with adding the lengths.

Carry out the plan



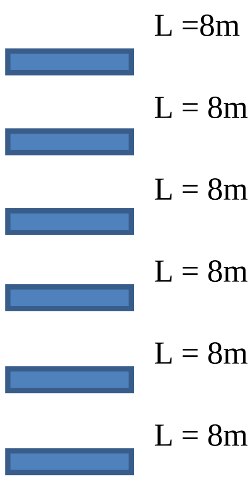


Figure 101

Putting all the pieces of rope together:

Figure 102

8m + 8m + 8m + 8m + 8m + 8m + 8m + 8m + 8m +8m + 8m + 8m + 8m + 8m = 112m

Therefore, the total length of the 14 pieces of rope is 112m.

Look back

Instead of using repeated addition, multiplication could have been used (8m x 14 = 112m).

3. Make a table

Understand the problem

Sipho has 14 pieces of rope. We need to find the total length of the 14 pieces if each piece is 8m long.

Devise a plan

We will use a table to record the total length each time we add another piece of rope.

Carry out the plan

Table 14

Number of pieces of rope	Total length in metres (m)
1	8
2	16
3	24
4	32
5	40
6	48
7	56
8	64
9	72
10	80
11	88
12	96
13	104
14	112

Therefore, the total length of the 14 pieces of rope is 112m.

Look back

The table made it very easy to keep track of our calculations as we worked towards the total length of 14 pieces of rope. However, looking at the table we realize that we could also have used multiplication as an alternate strategy.

Possible strategies for Problem 2

1. **Make a drawing**

Understand the problem

There is a ribbon of 100cm in length. We need to determine the length of each piece if we cut the ribbon into four equal lengths.

Devise a plan

We can make a drawing to represent the ribbon. This will help us in seeing precisely how to cut it into four equal parts. We can then use division to calculate the answer.

Carry out the plan

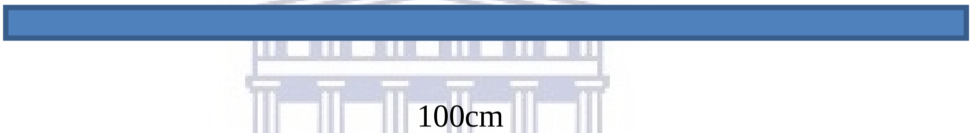


Figure 103: Length of ribbon

First, we cut the ribbon in half.

Figure 104: Length of ribbbon pieces cut in half

100cm ÷ 2 = 50 cm

Now, we divide it into quarters

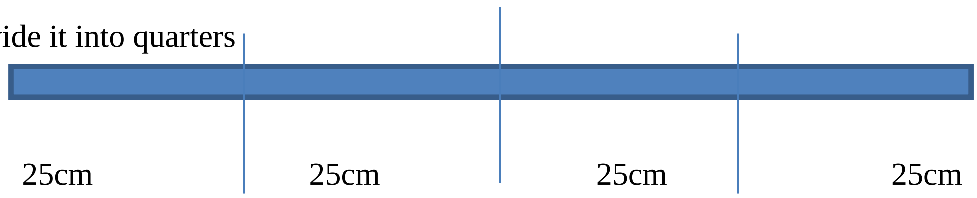


Figure 105: Length of ribbon pieces divided into quarters

50cm ÷ 2 = 25cm

Therefore, each of the 4 pieces of ribbon will have a length of 25cm.

Look back

We could have divided the 100cm piece of ribbon immediately into four. However, first halving it then repeating the halving process again helped to make the problem simpler to understand.

2. **Logical and direct reasoning**

Understand the problem

There is a ribbon of 100cm in length. We need to determine the length of each piece if we cut the ribbon into four equal lengths.

Devise a plan

If the ribbon has a length of 100cm and needs to be cut into four equal smaller pieces, then it means we are dividing the ribbon into quarters.

Carry out the plan

Dividing the ribbon into quarters means we must divide the 100cm by 4.

100cm ÷ 4 = 25cm

Therefore, the length of each of the smaller pieces will be 25cm.

Look back

We could have gone straight into the division operation, but using reasoning helped us to first realize that we needed to do division.

Possible strategies for Problem 3

1. **Logical and direct reasoning**

Understand the problem

Ciaran walked ½ km, Alex walked 1/5 km and Calum walked ¼ km. How far did each learner walk in metres?

Three people each walked different distances. Their walking distances are given in kilometres, but we need to determine these distances in metres.

Devise a plan

We can convert each distance from km to m as we know that 1km equals to 1000m.

Carry out the plan

If Ciaran walks ½ km, then half of 1000m = 500m

If Alex walks 1/5km, then one fifth of 1000m = 200m

If Calum walks ¼ km, then one quarter of 1000m = 250m

Therefore, Ciaran walked 500m Alex walked 200m and Calum walked 250m.

Look back

Could we have solved the problem using a different strategy? We may also have used a table to record the distances in km and another column for the converted distances in m.

2. **Make a table**

Understand the problem

Three people each walked different distances. Their walking distances are given in kilometres, but we need to determine these distances in metres.

Devise a plan

We will use a table to record the distances in km, then in the same table have an additional column to convert their distances to m.

Carry out the plan

Table 15

Name	Distance walked in km	Distance walked in m
Ciaran	½ km	½ x 1000m = 500m
Alex	1/5 km	1/5 x 1000m = 200m
Calum	1/4 km	¼ x 1000m = 250m

Therefore, Ciaran walked 500m Alex walked 200m and Calum walked 250m.

Look back

The table made it easier to organise the information. We could see the information (distances walked) of each learner in one graphic and make comparisons visually from the table.

Possible strategies for Problem 4

A pencil is 130 mm long. A ruler is 3 times longer than the pencil. How long is the ruler in centimetres?

1. **Make a table**

Understand the problem

The ruler length must be determined using the pencil length of 130 mm as a reference point. The ruler is 3 times the length of the pencil, and the answer needs to be in centimetres.

Devise a plan

The problem is stated in such a way that it seems a bit confusing. Therefore we will use a table to help us understand and the determine the length of the rule as "3 times longer than the pencil."

Carry out the plan

We can put the information into a table

Length of pencil (mm)	Length of ruler (mm)
130	130 x 3 = 390

1cm = 10 mm

So 390 mm = ….. cm

390 ÷ 10 = 39cm

Therefore, the length of the ruler is 39cm.

Look back

We followed logical steps. We calculated the length of the ruler by using the given information of it being 3 times longer than the pencil. We then converted this length to centimetres.

2. **Logical and direct reasoning**

Understand the problem

The ruler length must be determined using the pencil length of 130 mm as a reference point. The ruler is 3 times the length of the pencil, and the answer needs to be in centimetres.

Devise a plan

We will use logical reasoning and arithmetic calculations to determine this problem. First we shall calculate the length of the ruler as being 3 times bigger than the length of the pencil (given). We will then convert this length from millimetres to centimetres as the problem require the solution to be in centimetres.

Carry out the plan

Length of pencil: 130mm

Length of rule: 3 times length of pencil

Therefore, length of ruler: 130mm x 3 = 390mm

Converting 390mm to a length in centimetres:

1cm = 10mm,

Therefore, to convert from millimetres to centimetres we divide by 10:

390 mm ÷ 10 = 39 cm

Therefore, the length of the rule in centimetres is 39cm.

Look back

Using reasoning and arithmetic made the problem quite simple to solve.

Possible strategies for Problem 5

On the first day John drives his car a distance of 75 km. On the second day he drives 12 500 m and on the third day he drives 13 km. What is the total distance driven in metres?

1. **Logical and direct reasoning**

Understand the problem

The total distance driven by a car over three days must be calculated. The final distance must be converted to metres.

Devise a plan

We will use reasoning and arithmetic calculations to solve this problem. We will convert all the distances to metres, then add them together.

Carry out the plan

We will use the rule that 1km = 1000m

Then, 75km = 75 x 1000 = 75 000 km

13km = 13 x 1000 = 13 000 km

The 12 500 m is already in metres

Now, to find the total distance driven by the car over the three days we use addition:

75 000 m + 12 500 m + 13 000 m = 102 500 m

Look back

The solving process took quite a few steps. It would be very easy to make I mistake when converting the units of measurement or adding the distances. Therefore, we had to be very careful and work slowly,

2. **Make a table**

Understand the problem

The total distance driven by a car over three days must be calculated. The final distance must be converted to metres.

Devise a plan

We will use a table to record the distances the car drove each day and then convert all the distances to metres.

Carry out the plan

Table 16

	Day 1	Day 2	Day 3
km	75km		13km
m	75x1000 = 75000	12 500m	13x1000 = 13000

Total distance in metres = 75 000 m + 12 500 m + 13 000 m

= 102 500 m

Therefore, the total distance the car drove in metres is 102 500m.

Look back

We could also have converted the 12 500 m to kilometres, then add it to the other two distances given in kilometres. We could then convert this final distance to metres.

Possible strategies for Problem 6

Mary's mass is 48 kg. Her younger sister, Paula, has a mass equal to three-quarters of Mary's mass. What is Paula's mass?

1. **Make a drawing**

Understand the problem

Paula's mass must be determined. Her mass is three-quarters of 48kg.

Devise a plan

We can draw 48 balls and divide them into quarters. We can then divide the 48 balls into 4 equal groups. We can then take 3 of the 4 groups and count the total number of balls for these 3 groups as these would be three-quarters of 48.

Carry out the plan

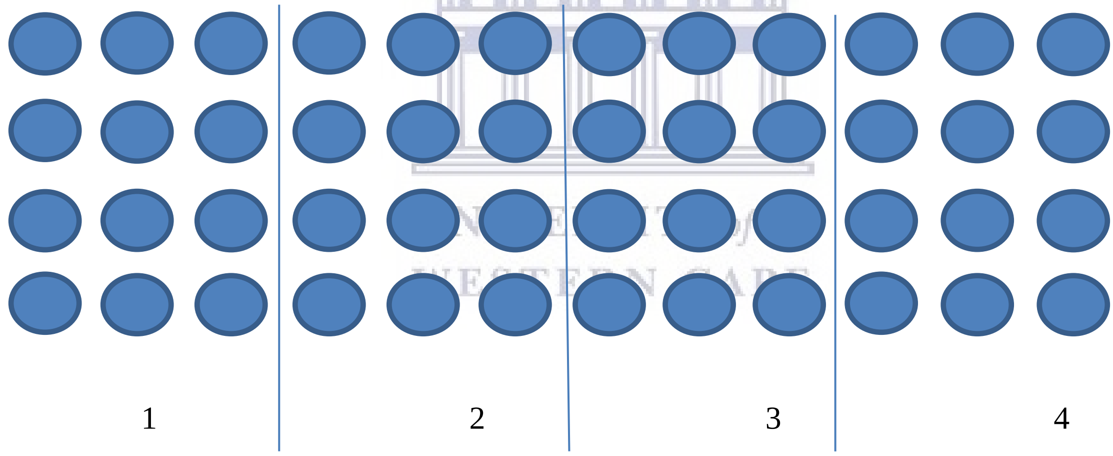


Figure 106: Dividing the 48 balls in quarters

Each group has 12 balls. After counting the number of balls for 3 groups (12+12+12) we arrived at a total of 36 balls.

So three-quarters of 48 is 36.

Therefore, Paula's mass is 36kg.

Look back

Drawing and counting balls was quite a time-consuming process. Perhaps we could have tried another method, such as arithmetic and doing calculations with fractions.

2. **Logical and deductive reasoning**

Understand the problem

Paula's mass must be determined. Her mass is three-quarters of 48kg.

Devise a plan

The mass of Mary is 48kg. Paula's mass is equal to three-quarters of Mary's mass, meaning her mass is three-quarters of 48kg. We can use reasoning and arithmetic to solve this problem.

Carry out the plan

¾ of 48 kg = …… kg

¼ of 48 kg = 48 kg ÷ 4

= 12 kg

Thus, ¾ of 48 kg = 12 kg x 3

= 36 kg

Therefore, Paula's mass is 36 kg.

Look back

We used reasoning to deduce that ¾ of a mass is the same as finding ¼, then multiplying it by 3.

Make a table

Understand the problem

Paula's mass must be determined. Her mass is three-quarters of 48kg.

Devise a plan

To help us get a more structured understanding of this problem, we will draw a table and divide the 48kg into quarters.

Carry out the plan

Table 17

Fraction values of 48kg	Calculations
¼	48kg ÷ 4 = 12kg
2/4	48kg ÷ 4 = 12 kg; 12kg x 2 = 24kg
¾	48kg ÷ 4 = 12kg; 12kg x 3 = 36kg

Appendix B – Problem solving activity

SECTION B

1. 408 L of water is poured into 4 containers so that each container holds the same amount. How much water is in each container?

2. A drum contains 70 L of petrol. The petrol tank of my small car can hold 6 L of petrol. How many times can I fill the tank from the drum and what amount is finally left in the drum?

3. How many 500 ml bottles can be filled from a can containing 4,5 L of water?

4. Use the dimensions given in the drawing to find:

   a. The area of the shaded part in mm^2
   b. The perimeter of the shaded part

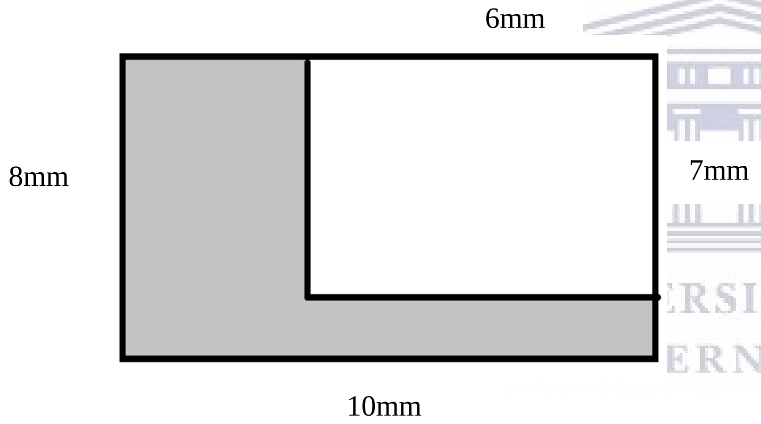


Figure: Dimensions of rectangle

5. Calculate the area of the shaded part of the diagram

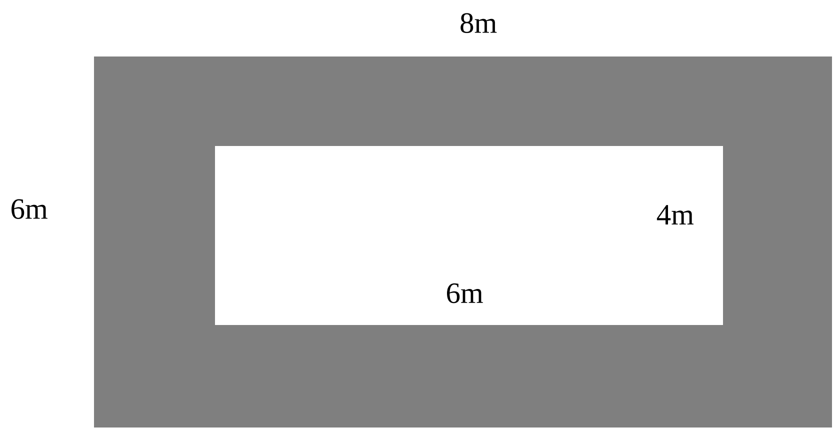


Figure 107: Dimensions of rectangle

Possible strategies for Section B

Possible strategies for Problem 1

408 L of water is poured into 4 containers so that each container holds the same amount. How much water is in each container?

1. **Logical and deductive reasoning**

Understand the problem

We need to divide 408 L of water into 4 containers so that each container has the same amount of water in it.

Devise a plan

If the water is being shared into 4 containers of equal amounts, then it means we can simply divide 408 L into 4 to determine the amount of water in each container.

Carry out the plan

408 L ÷ 4 = 102 L

Therefore, each container has 102 L of water in it.

Look back

We could also have divided the 408 L by 2, and then by 2 again to make it simpler.

2. **Break up the problem**

Understand the problem

We need to divide 408 L of water into 4 containers so that each container has the same amount of water in it.

Devise a plan

We can use division because sharing the water in 4 containers of equal amounts means we must divide the 408 L of water into 4. However, 408 is a big number so we could first divide it by 2 and then divide by 2 again.

Carry out the plan

First, we will find the half of 408 L

408 L ÷ 2 = 204 L

Now, we will find the half of 204 L

204 L ÷ 2 = 102 L

Therefore, each container has 102 L of water in it.

Look back

Breaking up the problem made it easier to work with and it is also much easier to divide by 2 and by 4.

Possible strategies for Problem 2

A drum contains 70 L of petrol. The petrol tank of my small car can hold 6 L of petrol. How many times can I fill the tank from the drum and what amount is finally left in the drum?

1. **Working backwards**

Understand the problem

We need to determine how many 6 L cans of petrol can be filled from a drum containing 70 L of petrol. Furthermore, we need to determine what amount is left after we are unable to fill any more 6 L cans.

Devise a plan

We can start at 70 L and work backwards by subtracting 6 each time until we cannot subtract 6 anymore.

Carry out the plan

70 – **6** = 64

64 – **6** = 58

58 – **6** = 52

52 – **6** = 46

46 – **6** = 40

40 – **6** = 34

34 – **6** = 28

28 – **6** = 22

22 – **6** = 16

16 – **6** = 10

10 – **6** = 4

We subtracted 6 a total of eleven times. Therefore, we can fill the tank from the drum using 6 L cans a total of 11 times, while there will be 4 L of petrol remaining in the drum.

Look back

Although this strategy was time-consuming, it allowed us to repeat the same procedure multiple times which made the solving process easier. Perhaps we could have used division instead if we wanted to solve it more quickly.

2. **Arithmetic calculations – division**

Understand the problem

We need to determine how many 6 L cans of petrol can be filled from a drum containing 70 L of petrol. Furthermore, we need to determine what amount is left after we are unable to fill any more 6 L cans.

Devise a plan

We will use division and divide 70 into 6, as well as determine the remainder.

Carry out the plan

70 ÷ 6 = ….

Counting in 6's:

6, 12, 18, 24, 30, 36, 42, 48, 54, 60, **66**, 72

We can count in 6's until we get to 66 as 72 is bigger than 70.

Therefore, 6 goes into 70 a total of 11 times and there is a remainder of 4.

Therefore, 70 ÷ 6 = 11, remainder 4

So, we can fill the tank from the drum using 6 L cans a total of 11 times, while there will be 4 L of petrol remaining in the drum.

Look back

Using division was effective, but we could easily have made an error if the numbers were bigger, or we could have calculated the remainder incorrectly.

3. **Guess and check**

Understand the problem

We need to determine how many 6 L cans of petrol can be filled from a drum containing 70 L of petrol. Furthermore, we need to determine what amount is left after we are unable to fill any more 6 L cans.

Devise a plan

We will use the guessing and checking strategy and try to multiply 6 by various numbers until we get to a number that is equal to or close to 70.

Carry out the plan

6 x 2 = 12

6 x 4 = 24

6 x 10 = 60

6 x 15 = 90

Thus, we will find our answer between 10 and 15

6 x 11 = 66

6 x 12 = 72

72 is more than 70, so 6 x 12 is just slightly too much, therefore we use 6 x 11 = 66.

Therefore, 6 goes into 70 a total of 11 times.

The remainder will be 70 – 66 = 4

Therefore, we can fill the tank from the drum using 6 L cans a total of 11 times, while there will be 4 L of petrol remaining in the drum.

Look back

We managed to solve the problem using guessing and checking, but this strategy might not have worked with bigger numbers.

Possible strategies for Problem 3

How many 500 ml bottles can be filled from a can containing 4,5 L of water?

1. **Logical and deductive reasoning - division**

Understand the problem

We need to determine the number of 500 ml bottles that can be filled with water when using a can that contains 4,5 L of water.

Devise a plan

We will use division and divide 4,5 L by 500 ml. But the quantities are in two different units if measurement, so we will first convert the 4,5 L to millilitres as well.

Carry out the plan

1 L = 1000 ml

Therefore, 4,5 L = 4,5 x 1000

= 4 500 ml

Now we can divide:

4 500 ml ÷ 500 ml = 9

Therefore, we can fill up 9 bottles of 500 ml each from a 4,5 L can.

Look back

We could also have divided the 500 ml to 0,5 L and then done the division (4,5 L ÷ 0,5 L = 9).

2. **Working backwards**

How many 500 ml bottles can be filled from a can containing 4,5 L of water?

Understand the problem

We need to determine the number of 500 ml bottles that can be filled with water when using a can that contains 4,5 L of water.

Devise a plan

We will start by converting the 4,5 L to millilitres, then we will use repeated subtraction by working backwards and subtracting 500 ml each time until we cannot subtract anymore.

Carry out the plan

1 L = 1000 ml

Therefore, 4,5 L = 4,5 x 1000

= 4 500 ml

Now we will work backwards by using repeated subtraction:

4 500 ml – **500 ml** = 4000 ml

4000 ml – **500 ml** = 3 500 ml

3 500 ml – **500 ml** = 3 000 ml

3 000 ml – **500 ml** = 2 500 ml

2 500 ml – **500 ml** = 2 000 ml

2 000 ml – **500 ml** = 1 500 ml

1 500 ml – **500 ml** = 1 000 ml

1 000 ml – **500 ml** = 500 ml

500 ml – **500ml** = 0 ml

We subtracted 500 ml a total of 9 times.

Therefore, we can fill up 9 bottles of 500 ml each from a 4,5 L can.

Look back

Working backwards and subtracting 500 ml repeatedly made the process easy to follow.

3. Arithmetic - Repeated addition

How many 500 ml bottles can be filled from a can containing 4,5 L of water?

Understand the problem

We need to determine the number of 500 ml bottles that can be filled with water when using a can that contains 4,5 L of water.

Devise a plan

We will first convert the 4,5 L to millilitres (4,5 L = 4 500 ml), then use repeated addition to add 500 ml repeatedly until we get to 4 500 ml. We will then count the number of 500 ml we have added together.

Carry out the problem

4,5 L = 4 500 ml

We will now add 500 ml repeatedly until we arrive at 4 500ml.

500 ml + **500 ml** = 1 000 ml

1 000 ml + **500 ml** = 1 500 ml

1 500 ml + **500 ml** = 2 000 ml

2 000 ml + **500 ml** = 2 500 ml

2 500 ml + **500 ml** = 3 000 ml

3 000 ml + **500 ml** = 3 500 ml

3 500 ml + **500 ml** = 4 000 ml

4 000 ml + **500 ml** = 4 500 ml

When counting the number of 500 ml we added to get to 4 500 ml, we get a total of 9.

Therefore, we can fill up 9 bottles of 500 ml each from a 4,5 L can.

Look back

Using repeated addition made the problem simple to follow. It was also made easier in that adding 500s is not very complicated.

Possible strategies for Problem 4

Use the dimensions given in the drawing to find:

a. The area of the shaded part in mm^2
b. The perimeter of the shaded part

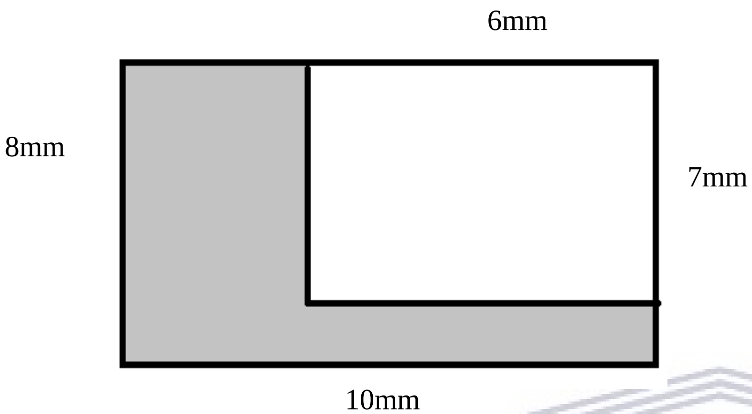


Figure 108: Dimensions of rectangle

1. **Make a drawing and breaking up the Problem 4a**

Understand the problem

The problem shows us a diagram of a rectangle. Part of the rectangle is shaded while another part is unshaded. We are required to determine the area of the shaded part of the rectangle.

Devise a plan

We will first find the area of the big rectangle, then we will determine the area of the unshaded part which is also a rectangle. We will then subtract the area of the unshaded rectangle from the area of the big rectangle. The difference will be the area of the shaded part of the big rectangle.

Carry out the plan

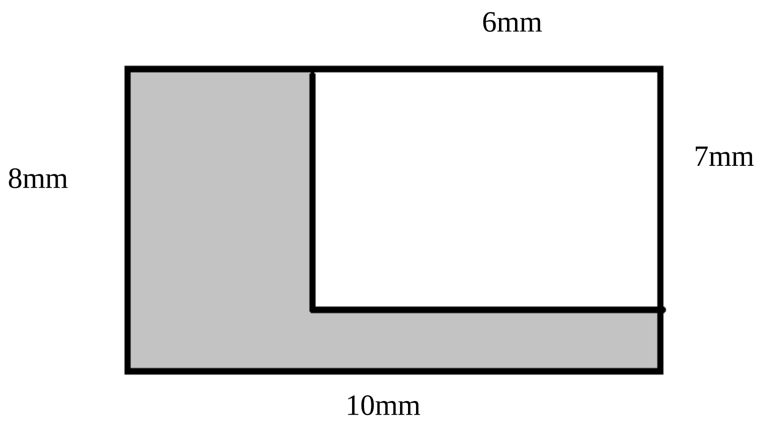


Figure 109: Dimensions of rectangle

Determining the area of the big rectangle

Given: length = 10 mm; breadth = 8 mm

A = l x b

= 10 mm x 8 mm

= 80 mm^2

Determining the area of the unshaded rectangle:

Given: length = 7 mm; breadth = 6 mm

A = l x b

= 7 mm x 6 mm

= 42 mm^2

Area of shaded part = Area of big rectangle – Area of unshaded rectangle

= 80 mm^2 - 42 mm^2

= 38 mm^2

Therefore, the area of the shaded part of the rectangle is 38 mm^2

Look back

This problem was quite complicated because we needed to first find the areas of two other rectangles before being able to calculate the area of the shaded part.

2. **Making a drawing and breaking up the Problem 4b**

Understand the problem

The second part of the problem requires us to determine the perimeter of the shaded part of the rectangle.

Devise a plan

We will break up the shaded part into two rectangles. We will then use the given dimensions as well as the drawing to determine the dimensions of the two shaded rectangles. Thereafter, we will calculate the perimeters of each of the two shaded rectangles and then add them together to find the perimeter of the entire shaded part.

Carry out the plan

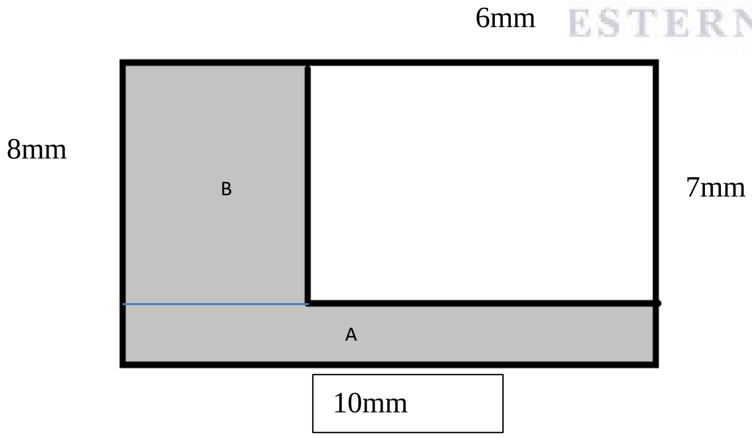


Figure 110: Dimensions of rectangle

After breaking up the shaded part into two rectangles, we have labelled these rectangles A and B.

Rectangle A

Breadth:

We can see that the width of Rectangle A is the difference between the width of the big triangle and the unshaded triangle.

Therefore, the width of Rectangle A will be:

8 mm – 6 mm = 2 mm

Length:

The length of Rectangle A is equal to the length of the big rectangle - Given

Therefore, length = 10 mm

We can now calculate the perimeter of Rectangle A:

P = 2l + 2b

= 2 (10 mm) + 2 (2 mm)

= 20 mm + 4 mm

= 24 mm

Rectangle B

Breadth

We can see that the breadth of Rectangle B is equal to the breadth of the unshaded rectangle.

Therefore, breadth of Rectangle B = 6 mm

Length

We can see that the length of Rectangle B is equal to the difference between the length of the big rectangle and the length of the unshaded rectangle.

Therefore, length of Rectangle B = 10 mm – 7 mm

= 3 mm

Perimeter of Rectangle B:

P = 2l + 2b

= 2 (3 mm) + 2 (6 mm)

= 6 mm + 12 mm

= 18 mm

Possible strategies for Problem 5

Calculate the area of the shaded part of the diagram

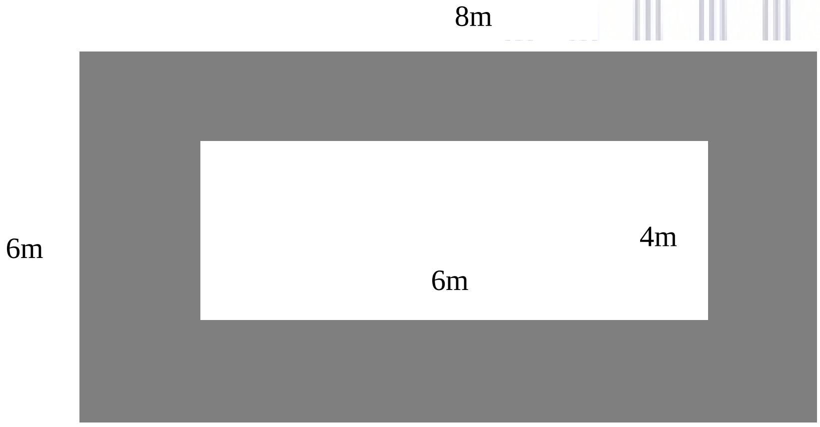


Figure 111: Dimesions of rectangle

1. **Making a drawing – breaking up the problem**

Understand the problem

The problem gives us a diagram consisting of a big rectangle which is shaded, but within this big rectangle is also a smaller unshaded rectangle. We are required to find the area of the shaded section of the diagram.

Devise a plan

We can see that this diagram consists of two rectangles – the big rectangle and then the smaller, unshaded one within the big one. If we calculate the area of the big rectangle, then the area of the smaller unshaded one then the difference between these 2 areas will be the area of the shaded section of the diagram.

Carry out the plan

Area of big rectangle:

Length = 8 m; breadth = 6m

A = l x b

= 8m x 6m

= 48 m^2

Area of smaller, unshaded rectangle:

Length = 6m; breadth = 4m

A = l x b

= 6m x 4m

= 24 m^2

Area of shaded part of diagram = Area of big rectangle – Area of smaller, unshaded rectangle

= 48 m^2 – 24 m^2

= 24 m^2

Therefore, the area of the shaded part of the diagram is 24 m^2

Look back

It was quite tricky to figure out a method to solve this problem, but when we eventually devised a strategy it was simple to solve. Perhaps we could also have worked with only the shaded part of the diagram and broken it up into four rectangles. We would then have calculated the area of each rectangle and the sum of the four areas would give us the total area of the shaded part of the diagram.

Appendix C
Observation Questions

1. Do learners understand the questions?

2. Which strategies are learners using to solve each problem?

3. Which problems are learners finding easy to solve?

4. Which problems are learners finding very challenging?

Appendix D
Interview Questions for Section A

1. Did you understand what you needed to do?

2. State the problem in your own words

3. What method did you use?

4. What made you choose this method?

5. How did you go about carrying out this method?

6. Describe any “aha” moments you had during the solving of the problems.

7. Describe and struggles and challenges you had during the solving of the problems.

8. Why do you feel your answer makes sense to you?

9. If you were given the problems again, would you use a different method? Explain.

Appendix E
Interview Questions for Section B

1. If you had to tell your friend about the activity, what would you tell them?

2. What strategy did you feel most comfortable using?

3. What did you enjoy the most about doing this activity?

4. What did you enjoy the least about doing this activity?

5. Is there anything that made you feel uncomfortable or frustrated?

6. What did you learn from doing this activity?

Appendix F – Permission letter to WCED

FACULTY OF EDUCATION
UNIVERSITY OF THE WESTERN CAPE
Private Bag X17, Bellville, 7535, South Africa

PERMISSION LETTER: THE WESTERN CAPE EDUCATION DEPARTMENT

Dr Audrey Wyngaard

Directorate Research

Western Cape Education Department

P/Bag X9114

Cape Town

Dear Dr Wyngaardt

Re: Permission to conduct research

My name is Shabbeer Rawoot. I am a Masters in Education (M.Ed) student (part time) at the University of the Western Cape. I am specialising in Mathematics Education. In order for me to be able to complete my degree, I am required to conduct research at a school which will form part of my Masters thesis.

I am also currently working as a teacher at Woodville Primary School in Mitchells Plain. I am requesting permission from the Western Cape Education Department (WCED) to carry out my study which aims toidentify the methods used by grade 4 learners when solving measurement word problems.The study will be conducted within one school day and will comprise of 42 grade 4 learners from my class to complete a test on measurement word problems.

My study is designed in such a manner that all learners involved will remain anonymous in order to protect their integrity and keep their identities confidential. I will also ensure that my questionnaires are answered anonymously to further protect learners. Furthermore, I will omit the name of the school from my research to prevent any ethical issues from arising. I assure you that I will conduct my research in a professional manner whereby I will respect all participants involved and comply will all the regulations required for me to be able to proceed with this study.

Therefore, I am sincerely requesting permission to conduct my research so that I am able to make a small contribution towards Mathematics Education.

Should you wish to find out more about the research, you are welcome to contact my supervisor, **Prof Rajendran Govender,** whose contact details are provided below.

Yours sincerely,

Researcher: Mr Shabbeer Rawoot

Contact number: 063 196 1439

Email: 3100547@myuwc.ac.za

Supervisor: Prof. Rajendran Govender

Tel: 021-9592248

Email: rgovender@uwc.ac.za

Signature of the researcher: ………………………………… Date: …………………

Appendix G – Permission Letter to Principal

FACULTY OF EDUCATION
UNIVERSITY OF THE WESTERN CAPE
Private Bag X17, Bellville, 7535, South Africa

PERMISSION LETTER- PRINCIPAL

The Principal
Woodville Primary School
Cnr. Juno Road & Mitchell's Avenue
Woodlands
Mitchell's Plain
7785
South Africa

Dear Principal

Re: Permission to conduct research at your school

Dear Sir

I, Shabbeer Rawoot (student number 3100547) am currently completing my Masters degree in Mathematics Education (Module MEE 801) at the University of the Western Cape. Part of my course requires me to conduct research and I will be doing a study on the following topic: **Investigating the methods used by grade 4 learners when solving measurement word problems.**

I am therefore requesting your permission to allow my grade 4 class to participate in my study which consists of a written test and focus group interviews. I assure you that the identities of all learners will be kept anonymous when doing the tests and the data collected from the study will be used strictly for research purposes only.

Regards

Shabbeer Rawoot

I, ……………………………………., the principal of ………………………………. grants grade 4B permission to participate in the study which will be conducted by Mr. Rawoot.

Principal's signature

………………………………

Date

………………………………

Yours sincerely,

Researcher: Mr. Shabbeer Rawoot

Contact number: 063 196 1439

Email: 3100547@myuwc.ac.za

shab.rawoot@gmail.com

Supervisor: Prof. Rajendran Govender

Tel: 021-9592248

Email: rgovender@uwc.ac.za

Signature of the researcher: ……………………………… Date: …………………

Appendix H – Ethical Clearance Certificate

UNIVERSITY of the WESTERN CAPE

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22 October 2019

Mr S Rawoot

Faculty of Education

Ethics Reference Number: HS19/2/14

Project Title: An investigation of strategies used by grade 4 learners to solve measurement type word problems

Approval Period: 17 October 2019 – 17 October 2020

I hereby certify that the Humanities and Social Science Research Ethics Committee of the University of the Western Cape approved the methodology and ethics of the above mentioned research project.

Any amendments, extension or other modifications to the protocol must be submitted to the Ethics Committee for approval.

Please remember to submit a progress report in good time for annual renewal.

The Committee must be informed of any serious adverse event and/or termination of the study.

Ms Patricia Josias
Research Ethics Committee Officer
University of the Western Cape

HSSREC REGISTRATION NUMBER - 130416-049

FROM HOPE TO ACTION THROUGH KNOWLEDGE