AN ANALYSIS OF THE FEASIBILITY OF INCORPORATING PRODUCTION ACTIVITIES IN SCHOOL MATHEMATICS

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October 1996

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A mini-thesis submitted in partial fulfilment of the requirements for the degree of M.ED in the Department of Didactics, University of the Western Cape.

Promoter: Professor Cyril Julie

October 1996

Abstract

I taught for fourteen years, as a secondary school mathematics teacher, at schools in disadvantaged communities in South Africa. During this period pupils' attitude towards school mathematics textbooks drew my attention, as it also affects their progress at school. Textbooks are issued yet it seems, for one or the other reason, pupils very seldomly use the books. The excuses for not using the textbook made me realise that an actual problem might lie with the presentation of the textbook content. My impression is that pupils cannot take ownership of mathematics education, because the content of the textbook is far removed from their daily real-life experiences.

My research is about the development of mathematics concepts yielded through production activities of carpenters and seamstresses. The research is based on realistic and ethno-mathematics and is underpinned by the following ideas:

- The starting point should be in the real world.
- Mathematics should contribute to the cultural background of the student.
- The gap between knowing mathematics and using mathematics be bridged.
- Conceptual mathematics, the extraction of the appropriate concept from a concrete situation, is the focal point.
- Mathematics should not be separated from other sciences.

Lessons on quadrilaterals and symmetry were developed and a workshop was held to test the opinions of teachers and educators. This helped to improve the presentation of the lessons.

Acknowledgements

I wish to express my sincere thanks to professor Cyril Julie , my supervisor, for his insightful comments and support during the preparation of this work.

I am also grateful to my mathematics colleagues and friends Jerome Barnes (Lentegeur Secondary School), Mike Cameron (Senior Mathematics Subject-advisor), Llewellyn Dyers (Bellville College of Education), Louise De Koker (Mathematics Department, UWC), Professor Roland Fray (Mathematics Department, UWC), Jannie Isaacs (Protea College), Shaheeda Jaffer (REMESA, UWC), Clyde James (Belhar Secondary School), Larry Kannemeyer (Mathematics Department, UWC), Elroy Lawrence (FDE, UWC), Archie Lewis (Beaconhill Secondary School), Jansie Niehaus (REMESA, UWC), Professor Jan Persens (Mathematics Department, UWC), Derrick Pieterse (Woodlands Secondary School) and Ephne Williams (Bellville College of Education) for their much appreciated criticisms and suggestions.

My thanks go to my family for their continued strong support of my efforts to contribute to the development of the educationally marginalised people in South Africa. A special word of thanks to a very understanding and supportive husband, Roland, who through his example of academic commitment and perseverance encouraged me tremendously.

Last, but not least, I am grateful to the Almighty for strength to make this work a reality.

I declare that **AN ANALYSIS OF THE FEASIBILITY OF INCORPORATING PRODUCTION ACTIVITIES IN SCHOOL MATHEMATICS** is my own work and that all the sources I have used have been indicated and acknowledged by means of complete references.

Signed : _______________

Date : 9.3.97

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CHAPTER 1

THE STUDY IN PERSPECTIVE: INTRODUCTION

For fourteen years as a mathematics teacher at secondary schools, I have witnessed how pupils are grappling with mathematics. I came to realise that mathematics is perceived by students as a 'very difficult' subject. It also seems as if they pass on the notion of mathematics being difficult from one school generation to another. Hand in hand with this goes a growing negative attitude towards the subject mathematics. There were times when pupils thought that mathematics was a subject for 'clever' pupils which in turn make other pupils feel inferior and 'stupid'. Those pupils who did not perform so well in mathematics became 'mathematic introverts' fearing being labelled 'stupid' by fellow pupils. Pupils display fear for the subject and when unable to master the subject they prefer to shut themselves off at the start of a mathematics lesson. In most instances a relationship, that is not conducive for mathematics education and teaching, started to build up between pupils and the mathematics teacher.

Attempts by pupils were also made to overcome the problems described above. To avoid embarrassment and humiliation in class many pupils prefer to use only textbooks. Although provision of mathematics textbooks has always been priority in our schools, it did not seem to change the pupil's attitude and enthusiasm about mathematics. It has become increasingly clear to me that our mathematics textbooks do not serve the purpose they ought to fulfill in South African schools. Things that point to this are the following:

- pupils' hesitance to carry textbooks to school,
- books are in most cases used only on request of the teacher,

- pupils' nonchalant attitude when textbooks get lost or stolen, and very seldomly a mathematics textbook is reported stolen,
- many pupils display some kind of indifference when not receiving a mathematics textbook for a particular year,
- although the textbook serves as a resource, the pupils excuse themselves from using the textbook.

The reasons for the above state of affairs are many and varied. One can only speculate that the above-mentioned problems are symptoms of other underlying problems. One of the problems could be the diverse interests of author and user of mathematics textbooks among the disadvantaged people of South Africa. Authors of mathematics textbooks for instance very rarely reflected on activities practised by disadvantaged people. Due to segregation in South Africa, involvement in cross-cultural activities was nonexistent. The authors and most of the pupils represented two groupings in the Apartheid era with opposing political and educational standing. The real world of the one group was either avoided or interpreted by members of the other. First hand knowledge of the mutual group's livelihood, culture and activities was almost nonexistent.

I believe something can be done about the context in which the mathematics problems are embedded and the presentation of mathematics in mathematics texts. I also believe that the domain of activities among disadvantaged people of South Africa has not been investigated for generating of lesson material. Being a member of a disadvantaged group in South Africa myself, I must admit that the context of mathematics problems is rather foreign to many pupils. The real world of the disadvantaged people in South Africa must not be used to impede the pupil's educational progress, but to enhance the pupil's educational opportunities. The nature of mathematics lessons or explanations in textbooks is very theoretical. Very little effort was made to put mathematics over in practical terms. Dealing with mathematics entirely in the theoretical mode contradicts the universal use of the subject. It creates the impression that mathematics cannot account for practical applications.

This study is aimed at investigating activities from the world of work of the mainly disadvantaged people for developing mathematics lesson material. I will research two activities namely *mitring of corners* as applied by the carpenter and *the concept of symmetry* as applied by the seamstress. An analysis of these two activities will help me to decide the feasibility of incorporating production activities in school mathematics. The application of these activities is visible in the pupil's real world and the context from which the activity is taken should be appealing to the pupil. Recognising that the work of carpenters and seamstresses can be fruitfully used in mathematics may boost the confidence of the pupil. Carpentry and dressmaking are practices with which most pupils from disadvantaged backgrounds can identify and the activities embedded in these practices, might cause a change in the attitude of the pupil towards mathematics. Against this background I see my work as a reconstruction of certain themes in mathematics.

In this chapter I sketched the background against which this study should be seen. Our circumstances as disadvantaged people in South Africa seemed to play a pertinent role in the provision of education. I have chosen the activities of the carpenter and the dressmaker for investigation and development of mathematics lessons. The focus of chapter two will be on various educational ideologies and other factors that effect education. The second chapter will also highlight attempts from inside South Africa and countries abroad, which had similar educational problems, to transform the process of education.

CHAPTER 2

MOTIVATION FOR STUDY

2.1 *Need for reconstruction of mathematics education.*

Although I reflected on my personal experiences in the introduction, I do regard it necessary to cross-check with other sources if mathematics education at school level is in need of reconstruction. I will evaluate reconstruction of mathematics education against the background of what I deem inefficient and inappropriate in the education realm. I regard mathematics education in relation to providing life-skills, another important factor in deciding the need for reconstruction. What follows is a listing of some factors that shaped education, more importantly mathematics education, in South Africa.

2.1.1 *Educational ideologies*

Starting with the ideologies which drive education is appropriate. Educational ideologies are systems of beliefs and values about the purpose of education held by particular groups of educators which result in educational action. We can never view values and beliefs in isolation from their social function and the purpose they serve in society. Educational ideologies therefore express and transmit beliefs about the nature of social, economic, political and religious reality through formal and non-formal processes. The school is the agent where transmissions of these beliefs formally take place.

Christian National Education (CNE) in South Africa was the official ideological position of Afrikaner Nationalists on education until 1994. The Afrikaner Nationalists had been in power since the 1940's and for the past forty years they had a very important impact on educational policy and practice. Their educational policy has been the educational expression of Apartheid, the transmission of the Afrikaner Nationalist Government's belief for racial segregation. Two central features of this educational policy were:

(1) that education should be based on the Christian Gospel and,

(2) that the people in South Africa be divided into nations and that education should reflect these national differences. (Ashley, 1989)

In 1948 the Afrikaner Nationalists came into power. The state has then become the vehicle for the accomplishment of Christian National Policy. The years 1976 and 1980 marked widespread protest against inferior provision of education. Black people protested, among others, against Afrikaans being made a compulsory language for instructional delivery in Black schools. For Black people this meant learning some subjects through a third language. The Human Sciences Research Council was appointed, with terms of reference, to investigate provision of education on an equal basis. Despite this appointment, the then government deemed it fit to keep education under a separate portfolio with a White minister in charge.

Ideas about People's Education started to emerge. An important development followed the severe crisis in Black schools during 1985. A committee of parents and community leaders established the National Education Crisis Committee (NECC) to address the situation. Its activities resulted in the development and dissemination of the conception of a People's Education. This education is meant to:

(1) enable the oppressed to understand the evils of the Apartheid system, and prepare them for participation in a non-racial, democratic system;

(2) eliminate capitalist norms of competition, individualism and stunted intellectual development and encourages collective input and active participation by all, and stimulating critical thinking and analysis. (Ashley, 1989).

2.1.2 *Political Factors*

Our circumstances today are the result of our own history. Minority rule has laid the foundation for the unique pattern of South African inequality and underdevelopment. Economic, labour and social policies for development are ethnically-based. The National Party ethnically segregated schools and colleges and White central government retained ultimate control of funds and policy. Changes regarding political, economic, labour and social development policies in South Africa since April 1994 promised to be radically different from what it was before. The main theme for all these policies in South Africa, is nation-building.

Could these political circumstances have an influence on education? Searching beyond the boundaries of South Africa we find in British schools traces of anti-racist education. In the book Multicultural Mathematics - Teaching Mathematics from a Global Perspective (Nelson, Joseph and Williams, 1993) the authors state that anti-racist education emerged during the late 1960s in Britain. Parents, mainly of children of West Indian origin, became increasingly critical of British schools which they felt, were failing to educate their children to compete in the wider world on an equal footing with their white peers. David Nelson, George Gheverghese Joseph and Julian Williams, (1993), state that a multicultural approach to mathematics plays an important role in the creation of a just society. They best see this approach as part of a general strategy of making mathematics more accessible and less anxiety-arousing among the wider public. The implicit assumption is that the present society is unjust in certain important aspects and that prejudice is institutionalized. The notion of multicultural education of their book is:

(1) Education must incorporate material from several cultures.

(2) Education must incorporate material from several cultures to be effective.

(3) Education ought to incorporate such material to help *all* children in the future to negotiate more effectively in a multicultural environment.

They distinguish four objectives in pursuing a multicultural approach to mathematics:

(1) *Drawing on a child's own experience as a resource.*

Certain abstract concepts of mathematics can be given a concrete form by well-chosen examples which are familiar to the class. For children belonging to the Islamic faith time measurement involves the principles of constructing calenders and demarcation eras, including the role of lunar calendars. An ability to convert from one system of recording time to another would not only be a useful exercise in arithmetic, but would help all children to appreciate the diversity of their local environment.

(2) *Recognizing different cultural heritages.*

Modern mathematics evolved to its present form because of centuries of cross-fertilization of ideas from different cultures. All mathematics teachers should have at their fingertips a fund of interesting stories on the origins and development of various topics in mathematics. The source of such material should not be just the standard histories of mathematics, which tend to be eurocentric, but should include also the literature on non-European traditions.

(3) *Combatting racism.*

The Swann Report of December 1985 is one of a number of recent reports that have highlighted the existence of racism in both British society and British schools. The mathematics teacher is cautioned about the way in which racism enters the classroom and how it can be countered. Ways in which bias or insensitivity to minorities in Britain may creep, however unconsciously, into a mathematics lesson are highlighted in the following examples:

(A) *Classroom examples are unduly restrictive.* In statistics examples of ownership of pets are used in a class with a large number of Asian or African children, while keeping pets may be uncommon with these racial groups.

(B) *Certain ethnic groups are ignored or devalued.* In Britain the stereotype of a West-Indian child being 'not good' at mathematics compared to an Asian child is accepted.

Britain is not an isolated case where politics affects education. Gerdes' account of Mozambique's history is testimony to the fact that mathematics education is not neutral (Gerdes, 1994). During Portuguese domination in Mozambique, mathematics was taught in the interest of colonial capitalism, to only a small minority of African children. Those Mozambicans were taught mathematics so that they could calculate better the hut tax to be paid as well as the quota of cotton every family had to produce. They were taught mathematics to be lucrative "boss-boys" in the South African mines. Mozambique achieved Independence in 1975. Post-independence objectives were developed to teach mathematics in such a manner to serve the liberation and peaceful progress of the people. Mathematics is taught to place its applications within the reach of the worker and peasant masses. Stimulating the broad masses to take an interest and delight in mathematical creation is thus priority in mathematics education.

Eshiwani (1993) from Kenya is of the opinion that curriculum should serve the needs of the majority and not the interests of a small segment of the society. Curriculum programmes of the 1960s, particularly those in science and mathematics, have come under attack in Kenya because they seem to have served the needs of a small percentage of the school population and tended to ignore the needs of the greater percentage of the population. The majority of school leavers did not make it to the higher rungs of the educational ladder. This anomaly has been due largely to the fact that the curriculum changes that took place in science and mathematics in many countries of Africa were hastily introduced and were carbon copies of various curriculum packages in the West. One could almost say the prevailing attitude was that what was good for Europe or North America was also good for Africa. It is this assumption that is being questioned today. A parallel could be drawn between the situations in Kenya and South Africa. Against the background of transplantation of curricula an educational crisis developed.

2.1.3 *Impact of society and culture on education*

Ernest (1991) draws attention to the social and political values brought into the mathematics curriculum. He proposes that we should not teach pure abstract mathematics but mathematics related to society. He comments on the authoritarian-democratic ways in which mathematics is taught and observes that:

> *'The received view of mathematics is that it is neutral with regard to both culture and gender. However, the divorce of Mathematics from its social context which the above neutrality implies, leads to monoethnic, sexist and possibly even racist mathematics.'* (Shan, 1991, page 42)

Traditionally, mathematics has been seen as that paradigm of certain, absolute and value-free knowledge, comprising timeless and universal truths (Ernest 1991). An alternative conception of mathematics as a historical and social construct which is culturally embedded and having a political dimension has emerged and gained support. It is this latter perspective that underpins ethnomathematics. D'Ambrosio (1989,1990) defined ethnomathematics as the art of understanding, explaining, learning about, coping with and managing reality in different and diverse natural, social, political and cultural environments. The central tenet in ethnomathematics is that mathematics and mathematics education are seen as being inextricably tied to their cultural contexts.

Vithal (1993) identifies three strands of ethnomathematics research namely:

- research that challenges the conventional histories of mathematics and that has focused on the mathematical histories of cultures outside Europe
- research that focuses on the mathematics found in traditional societies
- research that focuses on the mathematics of different groups in societies including both

adults and children.

Jean Lave (1989) finds that cognitive activity is based on two different forms of knowledge. Her empirical observations of shoppers who went for 'best buys'reveals two dominant activities in the development of mathematical knowledge. The one activity related to the knowledge about the frames of the situations (apples, prices, storage possibilities,...), and the other activity related to knowledge about arithmetic. The resulting arithmetic activity is shaped by both these cognitive activities. She also refers to research done by Posner on the development of mathematical knowledge in two West African societies. The participants have been members of two ethnic groups, the Dioula and the Baoule, living in Central Ivory Coast. Historically, the two groups had different religious ties and distinct economic roles. All the activities of the Dioula involve the transfer of money. Buying and selling, measuring, collecting and counting out quantities are all typical daily activities, particularly for the unschooled child. The Baoule society, in contrast, is primarily agricultural, revolving around the cultivation of subsistence crops. Opportunities for learning and practising skills of addition are not commonplace in Baoule culture. The Dioula merchant culture promotes counting quite early.

It was found that what was acquired by the Dioula group via cultural inputs is achieved by means of formal education among the Baoule. Although there may be several factors which contribute to the development of mathematical competence in unschooled Dioula children, the most direct influence is their merchant culture. While parents do not generally teach their children methods for calculating directly, they transmit certain techniques indirectly. The children observe older Dioula counting objects in groups of two or five and adding by regrouping and model their own efforts accordingly. It is noteworthy that the extent to which children learn without instruction is dependent on cultural factors. Particular environmental "presses" such as the exigencies encountered in the arithmetic class or the African marketplace, promote the development of more economical methods of calculations, like the application of number facts or mental regrouping.

Fasheh (1982) shed light on another factor which is important in the compilation of texts namely the relevance of mathematics. His experience was that despite his Masters degree and four years of mathematics teaching, his formal institutionalized education were of little use in practice. He remarked that while he was using mathematics to empower other people, it was not empowering for him. His illiterate mother benefitted immensely from mathematics to the extent that she created perfectly fitted clothing with few measurements and no patterns. He analysed the meaning of mathematics and found that to him mathematics was the subject matter he studied and taught while for his mother it was basic to the operation of her understanding.

He also came to realise that his mathematics had no power connection with anything in the community and the western hegemonic culture which engendered it. His mathematics was solely connected to symbolic power. Hegemonic education produces intellectuals who have lost their power base in their own culture and society and who have been provided with a foreign culture and ideology, but without a power base in the hegemonic society. He also warns that accepting western mathematics as universal is detrimental to creating a healthier and humane world. Mathematics needs to be treated in a critical way, not only at the implementation or application stage but also at the level of the basic premises and values governing conceptions, practices and production. In contrast to this, his mother's mathematics was deeply embedded in the culture and was connected to concrete and immediate needs and actions. The values of her kind of mathematics and knowledge were continuously discredited by the world around her, by the culture of silence and the cultural hegemony. He clearly distinguishes between praxis education and hegemonic education, the former being in line with the idea of ethnomathematics of D'Ambrosio (1985). In D'Ambrosio's ethnomathematical terms it means the fundamental right of every individual of society to understand, explain, cope and manage reality in his cultural environment. The main focus of this education is to raise the level of cultural consciousness and self-esteem through the unbiased promotion of the use of diversified modes of coping with, managing and explaining reality.

Diverse modes of surviving and transcending lead to different modes of thought. Bishop (1988) pointed out that mathematics was conventionally viewed as culture-free knowledge. Recently, research evidence from anthropological and cross-cultural studies has emerged which not only support the idea that mathematics has a cultural history, but also that from different cultures has come what can be described as different mathematics. One can cite the work of Zaslavsky (1979), who has shown in her book Africa Counts, the range of mathematical ideas existing in indigenous African cultures as did Gerdes (1985) in Mozambique. Just as all human cultures generate language, religious beliefs and food-producing techniques, so it seems do all human cultures generate mathematics. As each cultural group generates its own language and religious belief, so it seems that each cultural group is capable of generating its own mathematics. Since language and religious beliefs are not mutually exclusive, it is preposterously eurocentric to try to identify mathematics as a compartmentalized piece of knowledge.

Although Freire is mainly concerned with literacy, his ideas are also valid in mathematics education. His approach to education entails a system in which the locus of the learning process is shifted from the teacher to the student. This is a process that takes place not in a classroom but in a cultural circle. This shift signifies an altered power relationship not only in the classroom but also in the broader society. He proposes an anti-authoritarian pedagogy where teachers and students are teaching and learning together. Freire (1987) has provided a very practical and emancipatory model upon which to develop a radical philosophy of literacy and pedagogy. Freire rejects the the 'banking' approach to education according to which teachers open students' heads to the treasures of civilized knowledge. He firmly believes in education that begins with helping students achieve a grasp of the concrete conditions of their daily lives, of the limits imposed by their situation on their ability to acquire the meaning of the truism 'knowledge is power'.

2.1.4 *Syllabus*

In the light of the previous discussions let us look at the objectives of mathematics teaching prior to 1994. The objective of mathematics teaching as described in the mathematics syllabus is a matter of concern. I quote from the last education bulletin from the Apartheid era dated January 1985 regarding the standard grade mathematics syllabus for secondary schools:

' *Although the development of insight and understanding is important, more emphasis should be placed on mechanical reproduction of formal knowledge.* '

It appears as if the authors of textbooks take their cues regarding textbook content from the syllabus. Learning material that is not prescribed by the syllabus is mostly left out of textbooks. Teachers perceive the syllabus as an official document and they usually follow the prescriptions to the letter. The syllabus did not really encourage experiencing mathematics practically so that they had neglected this aspect. Over the years I noticed that teachers deem topics outside the range of the syllabus unimportant.

Mechanical reproduction of knowledge bars students of becoming critically conscious of reality, suppresses the curious nature of the pupils and as a result of these the pupils lose interest in mathematics as a subject. Mechanical reproduction of knowledge does not allow the students to reflect on the needs of their society through mathematics.

2.1.5 *Textbook authors' attitude*

Reading our mathematics textbooks one still finds that textbooks written by some South Africans exhibit a Eurocentric bias. Textbooks portray a sympathetic and pitiful attitude towards the disadvantaged majority, which is reminiscent of the Apartheid era. The following sum about rate is taken from a standard seven textbook, published approximately two years prior to the watershed national elections of 1994:

" A Standard seven class does badly in a test and the teacher decides to adjust all the marks using the following formula:

$G = \frac{11}{10}B$ where G is the new adjusted mark and B the original mark (weaker mark)"

(Dekker , Wilters , Coetzee , Coetzee ; 1992)

One must look beyond the calculations involved in the sum and try to understand the hidden messages. A suggestion to 'improve' weak marks in mathematics is put to the reader. Including problems of this nature in textbooks is not desirable. It gives a distorted idea of remedial work. Pupils should do remedial work, since progress is in their interest. It subtly brings over the idea to the pupil that adjusting the marks in this fashion is the norm. From the author's viewpoint the teacher should pity the pupil. From the pupils' perspective they might see that the teacher's reputation is at stake and that they oblige him or her to adjust the marks. These attitudes and perspectives are harmful to education overall and mathematics education in particular.

Education in South Africa is not practised in a vacuum. Politics have changed since the elections held in April 1994. South Africa has shed the ideology of Apartheid and domination by Whites has ended. It is now the moral responsibility of the new government to uplift the disadvantaged people in South Africa. We can achieve reconstruction and development of the South African human resources through an education system free of discrimination. The discussion in this section underlines the need for reconstruction in education overall and mathematics education in particular.

We must bear in mind that limited learning opportunities for the disadvantaged people contributed successfully to segregation in South Africa. We must scrutinize our mathematics curriculum for prejudices to ensure that mathematics education does not contradict the idea of nation building. Previous curriculum developers overlooked the world of work of the disadvantaged people where curriculum planning was concerned. The implication of the latter is that they do not take the world of work as a source for the learning of mathematics. Researchers, educators, authors of textbooks and curriculum designers owe it to all the people of South Africa to review the aims of mathematics education and the learning material that pupils need. Textbooks and study-aids play a very important role in teaching and they should contribute to establishing an equal and non-racial society in South Africa.

2.2 *Restructuring of education*

2.2.1 *Education policy as in White Paper*

The first policy document on education and training of democratic South Africa was read in parliament on 15 March 1995. Education and training for this democratically elected government are matters of national importance. The challenge of the government is to fulfill the vision to " open the doors of learning and culture to all ". I quote the national minister of education:

" South Africa has never had a truly national system of education and training, and it does not have one yet. This policy document describes the process of transformation in education and training which will bring into being a system serving all our people, our new democracy, and our Reconstruction and Development Programme." (Government Gazette, 15 March 1995, p.5)

Actual provision of education and training occurs primarily in schools, colleges, technikons and universities. These institutions bear the responsibility for managing the teaching and learning process. The curriculum, teaching methods and textbooks at all levels and in all programmes of education and training should encourage independent and critical thought, the capacity to question, enquire and reason. This process of transformation is aimed at building a system of education and training with which all people in South Africa can identify because it serves their needs and interests. Such a system *must be founded on equity and non-discrimination,* it must be owned and cared for by the communities and stakeholders it serves, and it must use all the resources available to it in the most effective manner possible. *Education and training are basic human rights.* The State has an obligation to protect and advance these rights so that all citizens, irrespective of race, class, gender, creed or age, have the opportunity to develop their capacities and potential, and make their full contribution to the society.

An *integrated approach* is favoured for education and training. This approach implies a view of learning which rejects a rigid division between " academic" and "applied", "theory" and "practice", "knowledge" and "skills", "head" and "hand". Such divisions have helped to reproduce occupational and social class distinctions. These distinctions have in the past been closely associated with ethnic structures and economic opportunity and power. The integrated approach is seen to be a prerequisite for successful human resource development, and can make a significant contribution to the reconstruction and development of our society and economy.

2.2.2 *New aims of mathematic syllabus*

With the new political dispensation and developments within mathematics, education demands that new educational aims be formulated. Recommendations were made to the National Education and Training Forum about mathematics education. It was felt that mathematics education needs to contribute towards society's reconstruction and development. It is therefore necessary to formulate aims about societal needs, general learning needs and mathematical needs.

The specific aims of mathematics education (Framework Commission: Mathematics, 1994) are the following:

a. to enable pupils to gain mathematical knowledge and proficiency,

b. to enable pupils to apply mathematics to other subjects and in daily life,

c. to develop insight into spatial relationships and measurement,

d. to enable pupils to discover mathematical concepts and patterns by experimentation, discovery and conjecture,

e. to develop number sense and computational capabilities and to judge the reasonableness of results by estimation,

f. to develop the ability to reason logically, to generalise, specialise, organise, draw analogies and prove,

g. to enable pupils to recognise a real-world situation as amenable to mathematical representation, formulate an appropriate mathematical model, select the mathematical solution and interpret the result back in the real-world situation,

h. to develop the ability to understand, interpret, read, speak and write mathematical language,

I. to develop an inquisitive attitude towards mathematics,

j. to develop an appreciation of the place of mathematics and its widespread applications in society,

k. to provide basic mathematical preparation for future study and careers.

l. to create an awareness of and an appreciation for the contributions of all peoples of the world to the development of mathematics.

2.2.3 *International development*

Curriculum changes and content of mathematics textbooks were under the spotlight in many countries for the past few years. Although the motivation for curriculum changes and revision of textbook content is different, it seems as if there is a common denominator namely the *relation between mathematics as subject and reality*. Politics of a country is also a very determining factor in this matter. The approach towards revision of textbook content depends on the circumstances that prevail inside the country. The trend nowadays with mathematics education is by embedding it in the reality of the students' lives. This relatively new approach is variously termed: *multicultural mathematics, ethnomathematics, antiracist mathematics, realistic mathematics.*

2.2.3.1 *Multicultural mathematics*

The multicultural approach to mathematics, as in Britain, is best seen as part of a general strategy of making mathematics more accessible and less anxiety-arousing among a wider public. It counters the view that mathematics consists of a sequence of unconnected skills, taught in isolation from the real world of application. This approach is valuable for the following reasons:

- By increasing the awareness of *all* pupils to different cultures, the teacher is helping to overcome the existing deep-rooted Eurocentric bias relating to the origins and practice of mathematics.

- Teaching of mathematics should relate both to a child's immediate experience of his or her everyday social and physical environment and to the wider society of which he or she is a part.
- A multicultural approach helps to promote a holistic view of learning.
- A multicultural perspective is an invaluable aid to an education in awareness of the heritage of pupils which contains components other than the European one.

Multicultural education is also seen as a moral necessity.

2.2.3.2 *Ethnomathematics*

"When pupils become self-confident in relation to the informal knowledge of their environment and classroom mathematics, they can develop their mathematical thinking more naturally and creatively." (Gerdes, 1994, p. 20).

2.2.3.3 *Antiracist mathematics*

Immigrants of many different cultures in Britain are required to conform to British standards. The antiracist approach is

"about enabling the pupils to understand how the imbalances of economic power are created"(Shan, 1991, page 16)

and how to challenge racism directly. Through this approach pupils can learn about the power mathematics can provide for analysing our world. Pupils can also explore how mathematics is used to perpetuate the political and social structures that control countries. Students also need to understand how inequality is systematically created and how people are disempowered by it.

2.2.3.4 *Realistic mathematics*

In 1959 a survey and a seminar were initiated in the Netherlands for improving mathematical education. The foundations for structuralist mathematics education were laid. However this approach met with criticism, among others the fact that the new curriculum had failed to relate mathematics to the real world, which led to the inception of realistic mathematics. Realistic mathematics subscribes to the following ideas:

- The mathematics curriculum should provide for the needs of all students.
- It should contribute to the cultural background of the general student and it should offer professional preparation for the future user.
- It should bridge the gap between knowing mathematics and using mathematics.
- Conceptual mathematization is stressed, i.e. extracting the appropriate concept from a concrete situation.
- Mathematics should not be separated from other sciences.

A. *Contextually-based mathematical activities.*

To answer the question what contextually-based mathematical activities are, let us look at the influence of Hans Freudenthal, who was the founder of so-called realistic mathematics education.

B. *Role of Freudenthal Institute*

The idea to approach innovation of mathematics education on a wide front - via training institutions of future teachers via in-service training, training of counsellors and instructors, research, development, via textbooks, and curriculum development - was an idea Hans Freudenthal of the Netherlands was most keen on. In 1971, on founding the IOWO, Institute for the Development of Mathematics Development, Freudenthal put Wiskobas on the track of realistic mathematics education. Reality is seen not only as an area of application but also as a source for learning. In his views he wanted to incorporate everyday reality emphatically in mathematics education. His other fundamental idea was to let the rich context of reality serve as a source for learning mathematics. He was not in favour of New Mathematics but he believed that the improvement of mathematics education was an urgent matter.

In mathematics instruction Freudenthal believes that the context is in the first instance an opportunity for the learner to mathematise it. Mathematising to him is the key concept meaning that a non-mathematical matter can be turned into mathematics, or a mathematically underdeveloped matter can be turned into more distinct mathematics. Treffers (1993) distinguished two components in the concept of mathematising, horizontal and vertical.

"Realistic learning strands start with the informal context bound working methods of children, in their personal reality. From there models, schemes, symbolisations are developed which serve as intermediaries to gradually bridge the gap between these start situations and the level of the formal, more general subject related operations" (Treffers, 1993, page 102).

2.2.3.5 *Developments in Belgium*

I quote from the introductory section of the book *Wiskunde vanuit Toepassingen:*

" Motivatie en vooral zingeving voor de abstractere wiskundige begrippen, meer aandacht voor echte begripsontwikkeling vanuit reele probleemcontexten, voor wiskunde als voorstellingsmiddel in allerlei gebieden kwamen de laatste jaren in de wiskunde-didactiek steeds meer aan de orde." (Roels, J., De Bock,D. et al, 1990)

It is made clear that new learning material, as with realistic mathematics, is not the main concern. Didactical development concerning mathematics education is about the learning process of mathematical ideas. Context plays a very important role in problem solving. All relevant aspects must be taken into consideration when solving a problem. The importance of context can be illustrated by looking at the problems encountered by a high voltage cable in the way of a Belgian oil rig, Yatzy, on route to Rotterdam on the river Schelde. To solve the problem several questions must be asked for instance:

- will the problem be solved by working on the height of the cable ?
- could the depth of the river be the problem?
- if the depth is the problem, will the middle, left side or right side have the required Depth?
- if the depth is established what will the tide be?

It is thus clear that the context in which the problem occurs must be understood to solve the problem. Understanding and analysing the context will reveal feasible strategies for problemsolving.

Three different uses of context can be distinguished:

- to introduce and develop a mathematical model or concept.
- a real world problem is presented to the student, and the student is expected to find the relevant mathematics, to organize and structure and solve the problem.
- if mathematical operations are embedded in contexts a simple transition from the problem to a mathematical problem is sufficient. The context in this situation is used to camouflage the mathematical problem.

Context use requires careful consideration as it may demotivate students. A quite trivial aspect is that the context should be motivating. It must also be kept in mind that students have also a personal idea about motivation. For younger students artificial contexts are acceptable and motivating while for older students contexts must be more realistic to be acceptable. To make sure that the context is motivating to as many students as possible, one should offer a whole range of different contexts. *Conceptual* conflicts, a conflict within the individual about possible different solutions to a problem, may be caused by difficult contexts. A controversial context for instance is an exercise in which the rates of abortion in different countries are compared.

A realistic context does not always mean that there has to be a direct connection to the physical world. The real world of the students imagination provides also a source for developing mathematical concepts.

The discussions in this chapter reveal a wide support for a review of mathematics education.

Researchers discovered that different modes of thinking originate from the concrete day to day situation of people (Bishop; 1988). Ernest views ethnomathematics and abstract mathematics in the following light:

> "The problem is to move from socially or concretely embedded mathematical situations to their theoretical content, without the loss of meaning and the switch into a new, disconnected realm of discourse" (Ernest; 1991; p 214)

Transplant of curricula from one culture to another, as with eurocentrism, aggravates the situation of provision of mathematics education. Central to meaningful mathematics education are the culture and activities of the people. In the next chapter two activities widely practised by the disadvantaged sectors in South Africa will be under the spotlight, namely *mitring* as applied by the carpenter and *symmetry* as used by the dressmaker. These activities are commonplace and will contribute to pupil participation in the lesson. The purpose of the lessons is to extract and develop mathematics concepts from the activities.

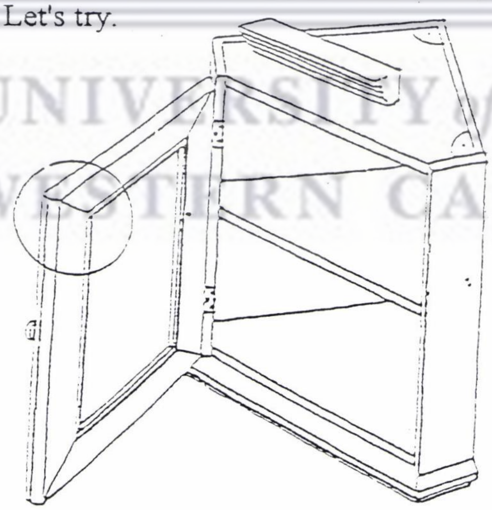
CHAPTER 3

CONSTRUCTION OF CONTEXTUALLY-BASED MATHEMATICAL ACTIVITIES I: QUADRILATERALS AND TRIANGLES

Many things are happening in the world around us which we take for granted. We have also become used to highly specialized fields of work that we sometimes forget information outside the specialist field can be used to solve a problem. The carpenter makes mitred corners so often, it almost comes naturally. First-hand experience of making a mitred joint will prove to us whether it is as easy as it seems. Pertinent questions of concern are:

- Is there perhaps more to learn through this activity besides physically performing mitring of the corner?
- Can the corner be mitred with the aid of secondary school mathematics?
- Can the pupil feel mathematically enriched after completing such an exercise?

Let us now look at the *joint* in the top right-hand corner of the door. The carpenter can make the joint with great ease - question is whether any person other than a carpenter can also successfully make the joint. Let's try.



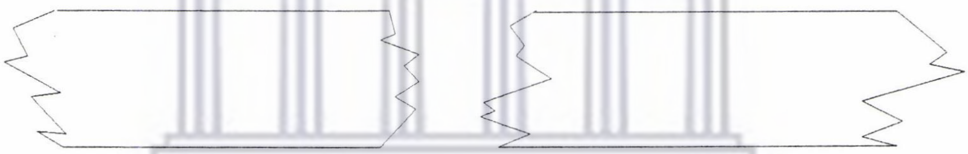
3.1 *Mitred corners*

We will try to reconstruct the steps that led to creating the joint. For our purpose we will use paper strips as a substitute for strips of timber used by the carpenter. Usually the carpenter mitre right angles, but instances also occur where angles, other than right angles, are mitred. Skirting boards are usually mitred if two walls do not form an angle of one hundred and eighty degrees. Usually walls meet at right angles, while on the other hand most facia boards are mitred at angles which are not right-angled. Timber used for mitring can be of equal width or different width. What we must investigate is how factors like *right angles, not right angles, equal width of both strips of timber* and *different width of both strips* affect our result.

3.1.1 *Mitring right angles using strips of equal width*

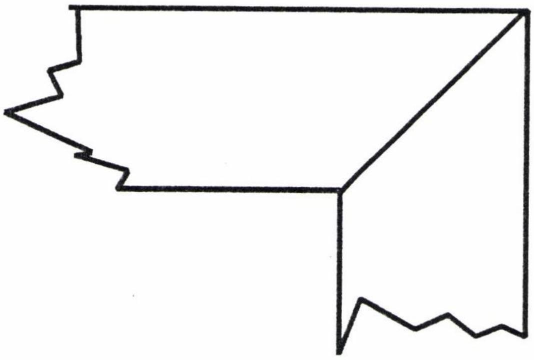
Before starting to mitre the angle the strips resemble the following sketch.

Fig. 3.1a



Comparing the initial strips in figure 3.1a with the end product in figure 3.1b, one can only imagine that a piece on both strips must be cut off to get the slanted edge. How does one know where to draw the line on which the cut is going to be made? Is the cutting line anchored between two specific points? How are these two points located on both strips in relation to each other?

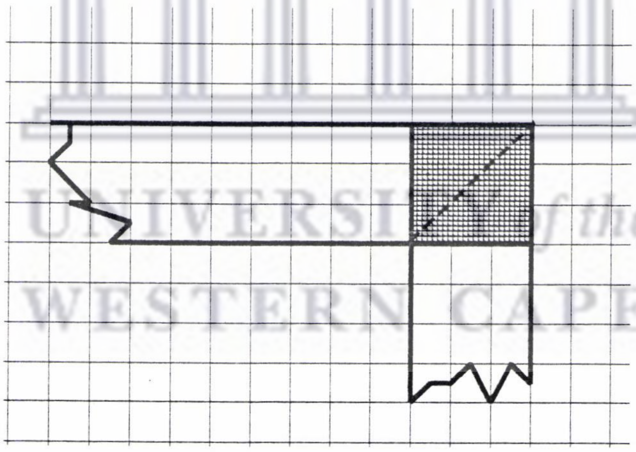
Fig 3.1b



Grid paper makes it easier for the reader to draw conclusions or to see certain effects - thus the illustrations on a grid. Starting the mitred angle, as shown in fig 3.1c,:

- *Overlap* the two strips *at a right angle.*
- *Outline the overlapped area on both strips.*
- *Join two opposite angles with a line, called the diagonal.*

Fig 3.1c



Having *four* sides and *four* angles the outline of the shaded area is called a QUADRILATERAL.

Adding extra features to the quadrilateral like

a. all four sides equal and

b. each angle equal to 90^0

we know that it is also called a SQUARE. Check the grid for these features and if you can answer positively to these features, you have successfully identified the overlapped area on the grid. Clearly, shaping the mitred corner depends on the construction that takes place *inside* the square. Move the two strips apart, cut on the diagonals and remove the off-cuts. When the two triangles are removed and the strips are shifted against each other we say the corner is *mitred, as shown in fig. 3.1d in the circle.*

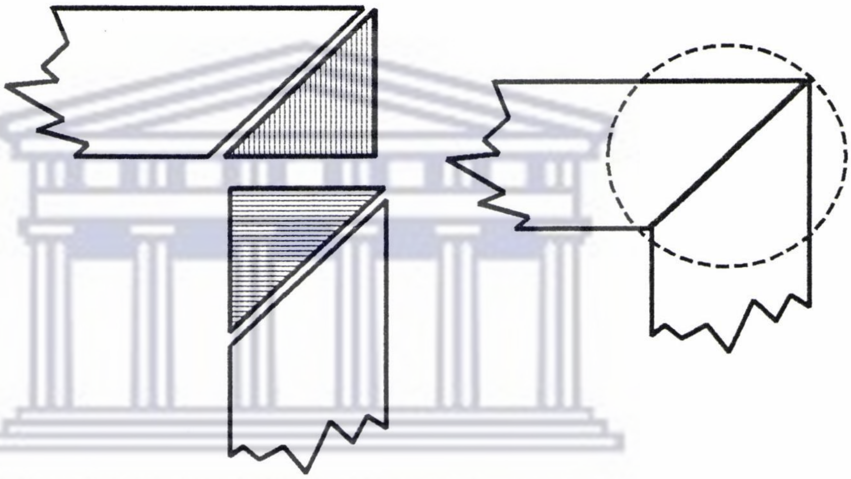


Fig. 3.1d

We outlined two squares, one on each strip, before mitring. Removing the two triangular off-cuts we must decide what is left of the two squares. By examining the off-cut triangles it is useful to do the following:

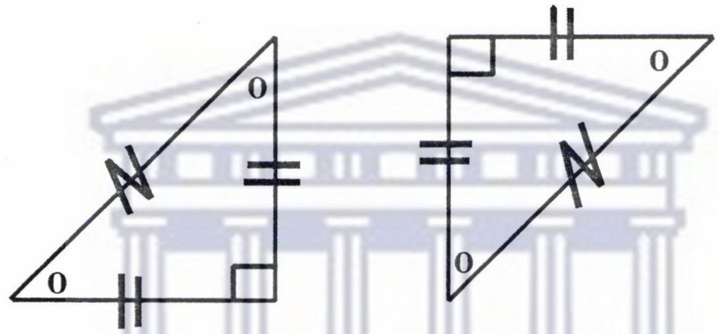
- Find out if the two triangles are of the same size or not. This can be done by putting the one triangle on top of the other one.
- Use the sides of the one triangle to measure the sides of the other triangle. If equal sides (sides of same length) are found, it must be marked with the same

sign.

- Use the angles in the one triangle to measure and mark the angles in the other triangle.
- It may happen that more than one pair of equal sides or equal angles appear in the same triangle. Describe the possible relationship between equal sides and equal angles in the same triangle.

In summary, the characteristics of the off-cuts are illustrated in fig. 3.1e.

Fig. 3.1e



It is known that the off-cuts are two triangles, but let us find out what *single figure was removed at the corner without altering the angle between the strips.* Making different shapes with the two triangles can help in deciding this. Keep one triangle fixed and move the other one around. The figures created are given below. The triangle containing the vertical lines is kept fixed. Fit these figures one at a time in the mitred corner.

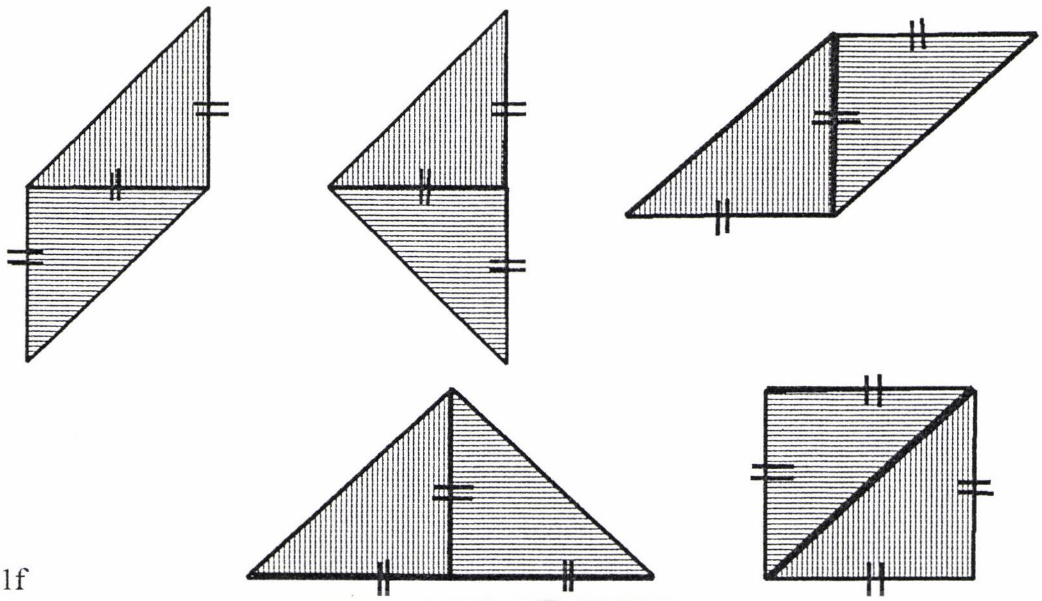


Fig. 3.1f

The last sketch in fig.3.1f, a SQUARE, fits precisely in the corner and the diagonal of the square also coincides with the joint. Be sure about this, *two identical triangular off-cuts make the square.* Identical triangles are also called *congruent triangles.* We will return to this topic later.

We discovered that if two strips *have the same width* initial planning and construction takes place inside a square. The combined size of the off-cuts matches the size of the initial square on each strip. THE OFF-CUT TRIANGLES CAN BE SHAPED in the form of a QUADRILATERAL, NAMELY A SQUARE. Through fitting the triangles onto each other, relationship between angles and sides are revealed as illustrated in fig 3.1e. Arranging the triangles in fig 3.1e in a square, we learn about:

- the *location of equal angles* inside the square. This in turn could start

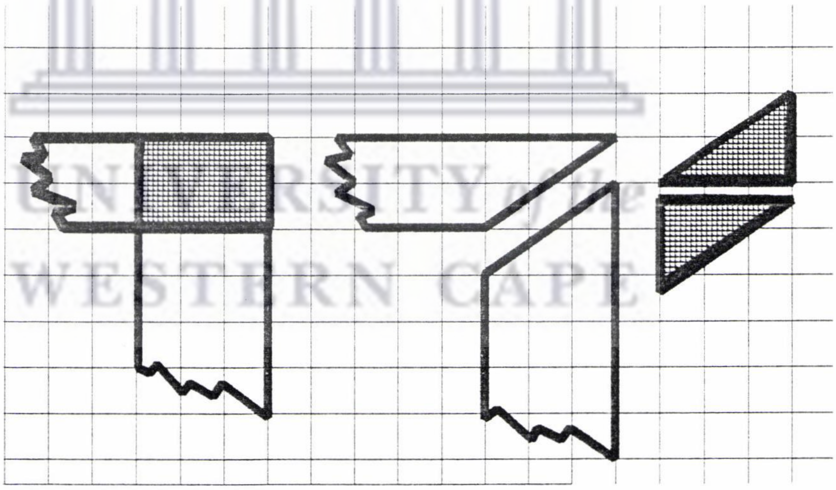
- discussions on parallel lines and alternate angles.
- *isosceles triangles* being created when drawing the diagonal of the square.
- the diagonal as *bisector of both opposite angles.*

3.1.2 *Mitring right angles using strips of different width*

Carpenters are sometimes commissioned with unusual designs, for example to mitre right angles using timber of different width. The procedure, as is the case with 3.1.1, entails the following steps: (refer to fig. 3.1g)

- overlap the two strips at right angles.
- outline the overlapped area on both strips.
- draw and cut on the diagonal

Fig. 3.1g



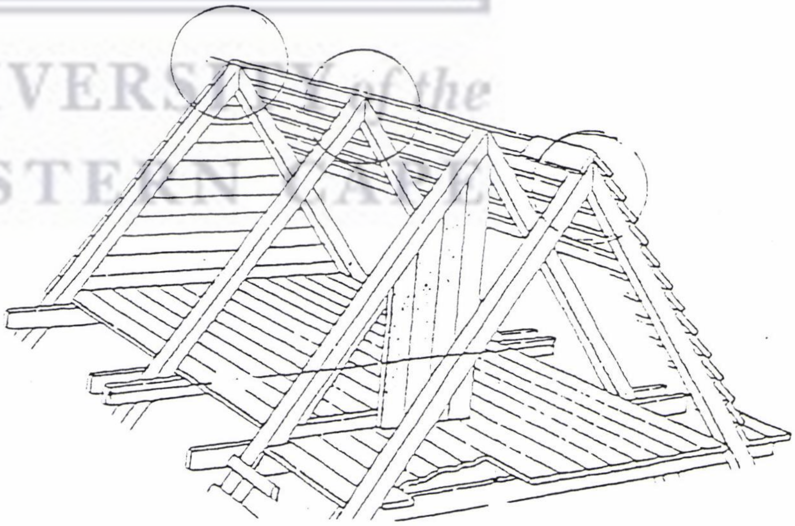
Fitting the two off-cut triangles onto the overlapped area one notices that jointly the triangles cover the area, a RECTANGLE, completely. Wanting to learn more about the characteristics of the rectangle, compare the two triangles (perhaps through fittng) to establish equal angles and equal sides. Mark the equal angles and equal sides with the same signs. Characteristics that were discovered with the square, emerge also in the case of the rectangle, for example:

- opposite angles being equal
- opposite sides of the rectangle are equal
- the diagonal *does not bisect* the opposite angles.
- existence of alternate angles inside the rectangle.

3.1.3 *Mitring angles that are not equal to 90 ° using strips of equal width*

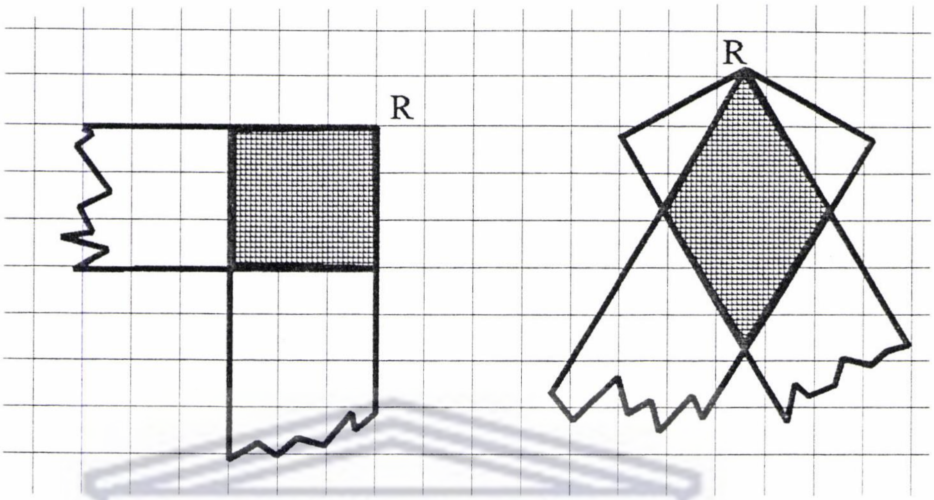
This technique is often applied when cutting facia boards. Facia boards are usually flat timber strips, covering the ends of rafters in a roof. The facia boards slope at the same angle as the roof and very seldomly is the pitch (slope) of the roof equal to 90°. The facia boards are mitred just below the ridge (junction line of the two sloping sides) of the roof (see fig.3.1h).

Fig. 3.1h



Following the same procedure as for the square and the rectangle one finds that mitring the boards, half of the total overlapped area gets cut off. If we overlap the two strips at a right angle, then the overlapped area is a square (sketch to the left in fig.3.1i). Swivelling one of the two strips around R, one vertex of the square, the overlapped area changes in shape (fig.3.1i).

Fig. 3.1i



All the characteristics of the overlapped area but one correspond with the characteristics of the square. This odd characteristic refers to the size of each angle. *Not one angle is equal to 90 °* and because of this distinction the quadrilateral is called a RHOMBUS (Sketch to the right in fig. 3.1i). Each block on the grid is a square. Should each side of the rhombus go through the *vertices of the grid squares* then the angles inside the rhombus will each be right angles.

3.1.4 *Mitring angles that are not 90 °, using strips of different width*

A repetition of the procedure of overlapping the strips at a required angle, as explained in 3.1.3, results in a quadrilateral, having all the characteristics of the rectangle except for one. Opposite sides are equal, opposite angles in the quadrilateral are equal, alternate angles are equal *but not a single angle is equal to 90 °*. These are the characteristics of the PARALLELOGRAM.

These activities performed by the carpenter are valued. Although we are familiar with certain characteristics of the quadrilaterals, new characteristics also came to light. We have not discover brand new characteristics of the four quadrilaterals, yet it gives meaning to the carpenter's activities. It shows that generalizations of mathematics can be applied in a specific context. By marrying the context of generalizations with a concrete context, the activity becomes meaningful.

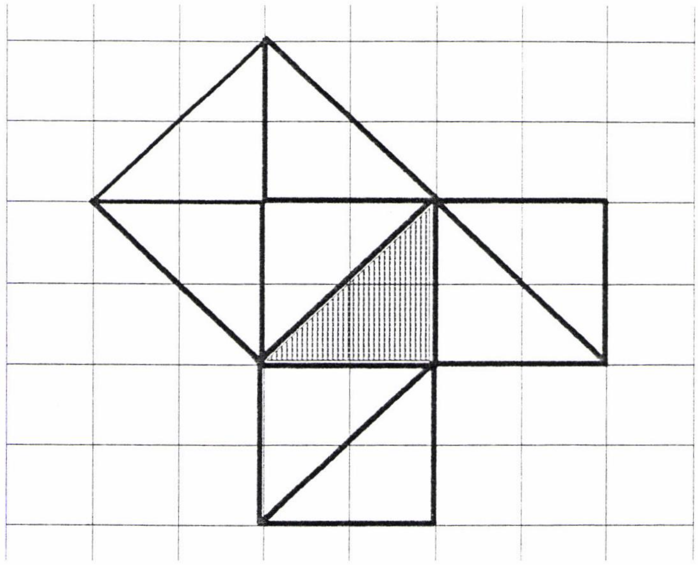
3.2 *Off-cuts of mitred corners*

3.2.1 *Off-cuts of the square*

When the carpenter mitres a corner, he overlaps the timber strips and outlines the overlapped area on both strips. He then joins, on both strips, the opposite angles (diagonal) and then cuts on the diagonal. I already showed that the overlapped area is very important and that half the overlapped area becomes wasted. The triangular off-cuts in the case of the *square overlapped areas* have yet another important use. Using the two off-cut triangles on grid paper, we can prove the theorem of Pythagorus. Firstly, we must bear in mind that there are TWO IDENTICAL TRIANGLES INSIDE THE GRID SQUARE. For the next exercise we will build squares using identical triangles only. The triangles may be turned around, as happens with puzzles, to make the sides fit. Keep also in mind that all the triangles are *isosceles.*

Secondly, one triangle will be kept fixed while the other triangle will be used as a building block in making a squares on each side of the fixed triangle.(fig. 3.2a)

Fig.3.2a



Copying the triangle twice on the vertical and the horizontal sides of the shaded triangle, we create a square on each of the two sides. To form a square on the hypotenuse, we must copy the square four times. If we focus on the grid squares, we have a (2X2) square on both these sides and together on these two sides are eight grid squares. Counting the grid squares in the big square on the hypotenuse, we get a total of eight grid squares. The result of this being:

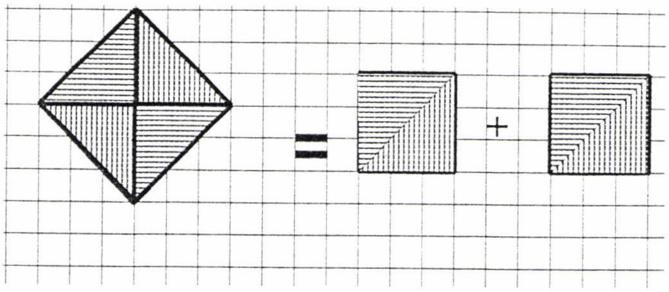
Total number of *grid squared on vertical and horizontal sides* = $2^2 + 2^2$

$= 8$

Total number of *grid squares on the hypotenuse* = 8

Using the squares formed with the other identical triangle, one notices one square each on the vertical and horizontal sides of the shaded triangle. Bearing in mind that two triangles form a square, one counts two squares on the hypotenuse. Again it is proven that the squares on the hypotenuse equal the total number of squares on the other two sides (fig.3.2b)

Fig. 3.2b



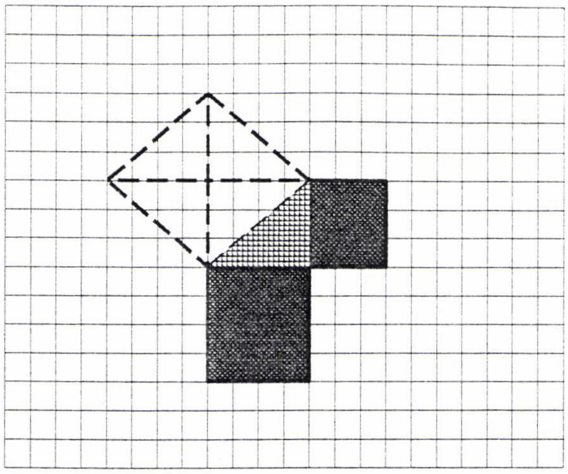
It is clear from the sketches on grid that squares on the vertical and horizontal sides can be fitted into the square which is on the hypotenuse. This reaffirms the very important theorem of Pythagorus.

3.2.2 *Off-cuts of the rectangle*

Drawing a square on the hypotenuse is easy if the hypotenuse connects the vertices of grid squares. This would be the case with the off-cuts of the square where the vertical and horizontal sides in the right-angled triangle have the same length. On the other hand, building squares on the sides of the off-cut of the rectangle is a little bit tricky. The two off-cut triangles produced are *right-angled triangles but not isosceles*. Using length 4 units and 3 units respectively on the vertical and horizontal sides, a (3X3) square and a (4X4) square are placed on these two sides.

Copying the off-cut triangle on the hypotenuse, in order to make a square, a *rhombus is created* (refer to fig. 3.2c). Compare fig.3.2b with fig.3,2c for the difference between the square and the rhombus.

Fig. 3.2c

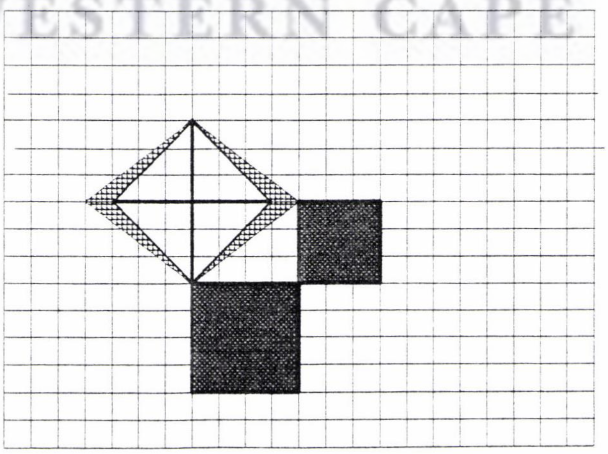


The features of this quadrilateral, a RHOMBUS, are:

- the sides are equal.
- the angles are not right angles (angles not equal to 90^0).
- the diagonals cut at right angles.

In the square on the vertical side of the triangle there are nine grid squares. In the square on the horizontal side of the triangle there are sixteen grid squares. To prove the theorem of Pythagorus in this case we must change the rhombus to a square. *Draw the biggest possible square, containing eighteen grid squares, inside the rhombus (fig. 3.2d)*

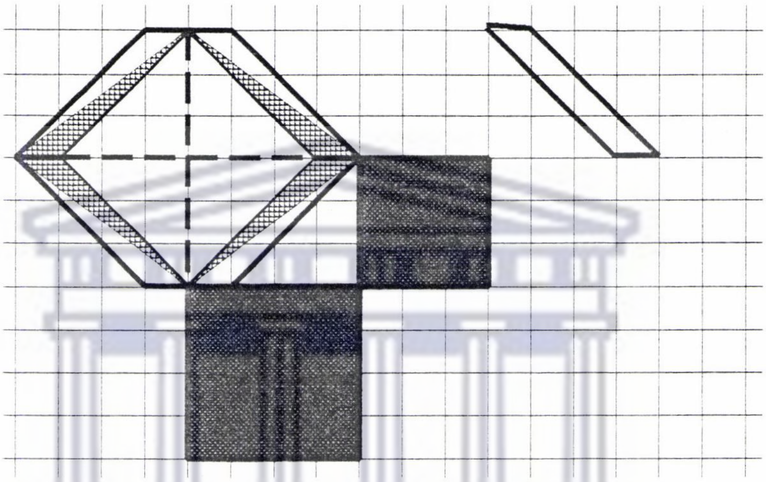
Fig. 3.2d



To create the square the four shaded triangles are copied on the outside of the rhombus.

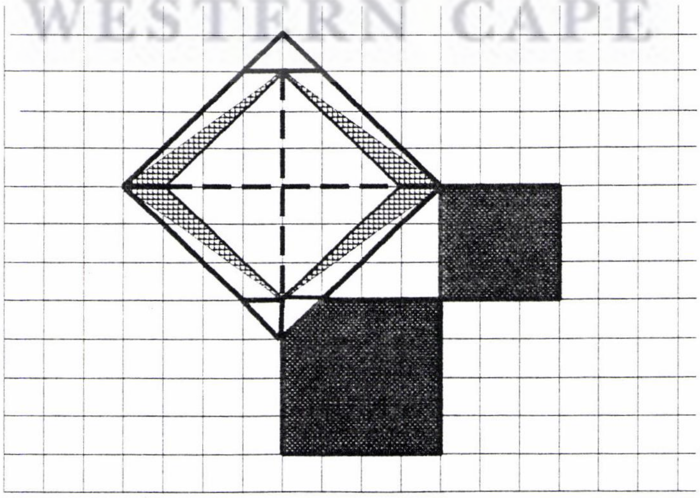
The area left out inside the rhombus (the shaded triangles) when creating the inner square, is added on the outside. The area of the rhombus is now displaced to the inner clear square plus the four clear triangles on the outside of the rhombus. It means the inner clear square plus the four clear triangles have the same number of grid squares as the rhombus. (Fig. 3.2e) The inset on fig. 3.2e shows the total number of grid squares covered by the two adjacent triangles, namely *three grid squares.*

Fig. 3.2e



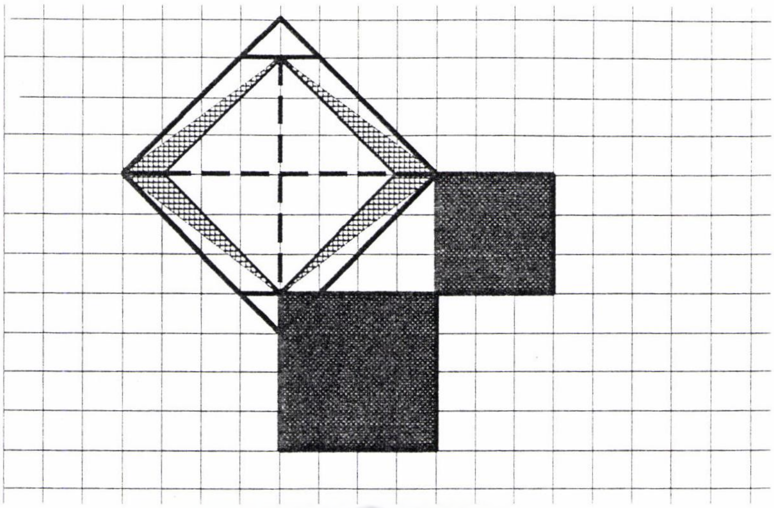
If the two opposite corners of the diamond-shaped figure are drawn, the *outer square* is complete. A full grid square is added at each corner, as shown in fig. 3.2f.

Fig 3.2f



However, by completing the square around the rhombus, *half a grid square is cut off the* *square on the horizontal side of the right-angled triangle (fig. 3.2g)*

Fig. 3.2g



Replacing this half grid square we must subtract half a grid square from the outer square. The adjacent clear half square must be seen as a replacement for this loss (fig.3.2h)

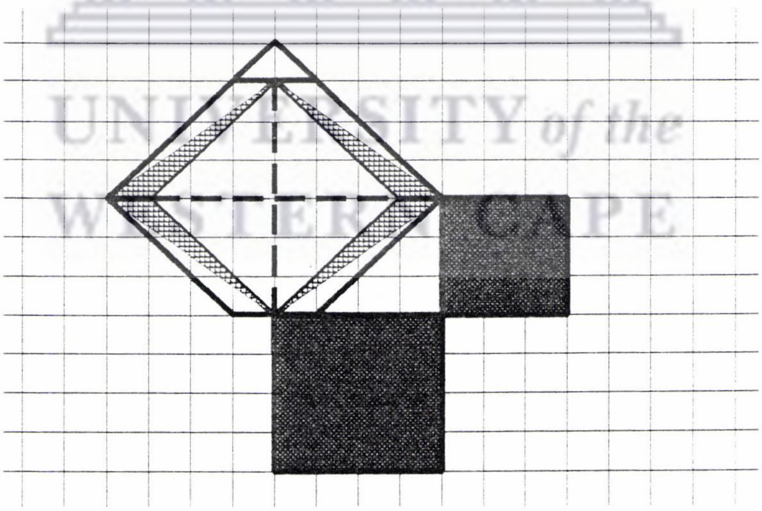


Fig. 3.2h

To refresh your memory the following :

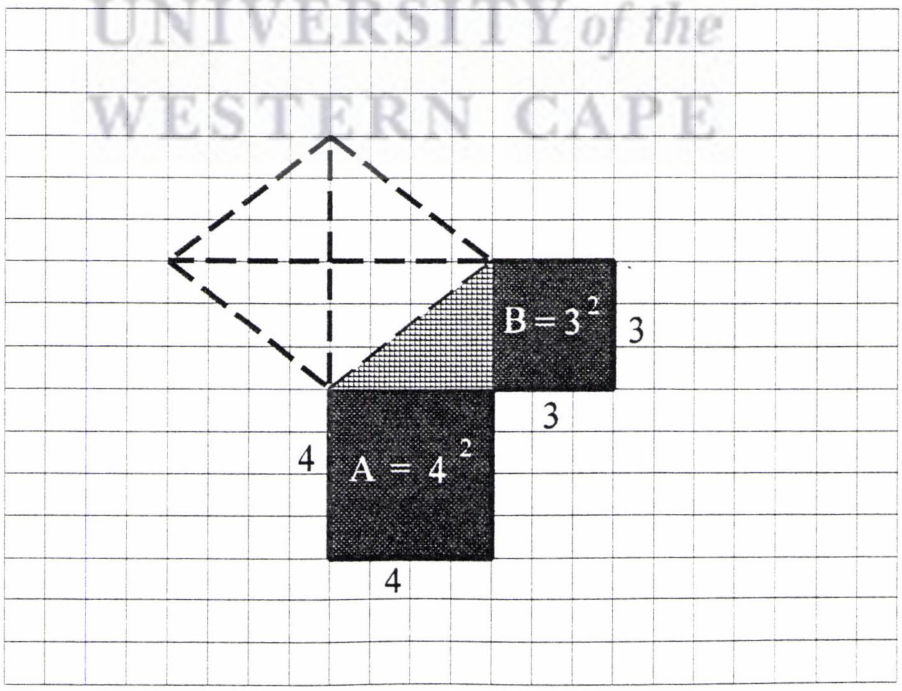
- the rhombus is situated on the hypotenuse

- the number of grid squares in the rhombus is equal to the number of grid squares in the clear inner square and the clear triangles on the sketch.

Adding the grid squares in the clear inner square and triangles we reach a number of twenty-five grid squares. Once more it is true that the number of squares on the hypotenuse equals the total number of squares on the other two sides. The theorem of Pythagorus is proven again.

Let us study fig 3.2i again. Square A has <u>four</u> grid squares down (vertical) and <u>four</u> grid squares across (horizontal). Without counting the grid squares the squares 'down' can be multiplied with the squares 'across'. We can simplify this even further. The squares 'down' and the squares 'across' in a square are always the same number. We can therefore square only one side in square A. That gives us the number of grid squares in square A. Applying the same method to square B nine grid squares are obtained.

Fig. 3.2i



Expressing the findings about right-angled triangles in words and then working towards a uniform formula, would be :

squares on hypotenuse = squares on second side + squares on third side

i.e.

$$\boxed{(\text{hypotenuse})^2 = (\text{2nd side})^2 + (\text{3rd side})^2}$$

$$(\text{hypotenuse})^2 = (4)^2 + (3)^2$$
$$= 16 + 9$$
$$= 25$$

Interpreting this answer we can say there are <u>twenty-five</u> grid squares in the square on the hypotenuse. One can however take this a step further and work out the actual length of the hypotenuse. *One number must have been multiplied twice (been squared)* to give us the number twenty-five. *If that number is found, the SQUARE ROOT has been determined and we know the length of the hypotenuse.* The square root refers to the number which has been squared.

Contextually-based mathematics activities are clearly valuable sources for building a better understanding of mathematics. The off-cuts of squares and rectangles through mitring helped in establishing the number of squares on the hypotenuse.

3.2.3 *The role of the off-cuts of square, rectangles, rhombus and parallelogram in explaining congruency*

The off-cuts, created through mitring, play another important role. In mathematics information can be conveyed to the reader in different forms namely through written sentences, sketches and symbols. Equal angles and sides in the mitred corner can be identified and marked by fitting the off-cut triangles in the mitred corner (fig. 3.1c - fig 3.1e). *Congruency* is a means of determining whether triangles are identical. It is merely a comparison of the two triangles which is made after calculations have been done and angles and sides have been marked. It boils down to comparing two triangles *looking for the same arrangement of marked angles and sides.*

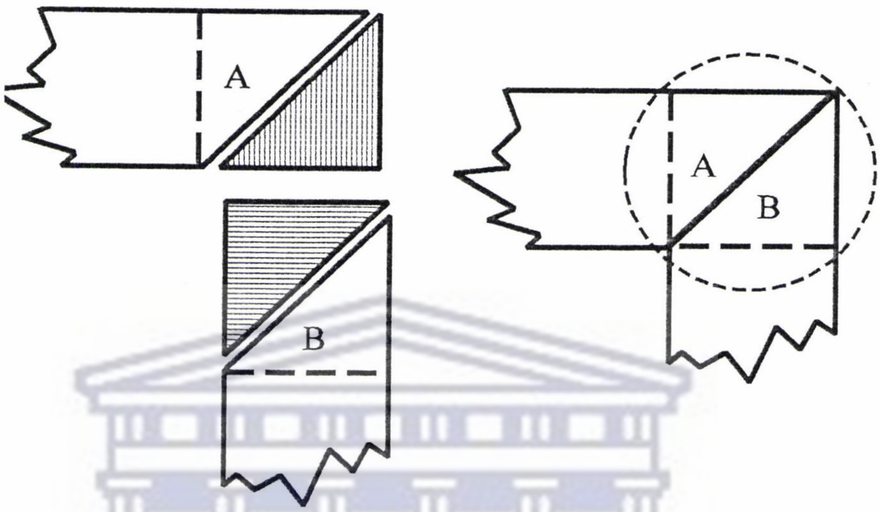
In each triangle there are three sides and three angles. Four categories for arranging the angles and sides are:

- *three sides.* In fig 3.2e one need not fit the triangles physically. By focussing on the sides only one can decide if the triangles are congruent or not. Precisely the same markings appear on the sides of both triangles in fig. 3.2e , which we already know are identical, and therefore the triangles are congruent.
- *two sides and one angle.* Reference in this case is made to an angle. *The two sides enclosing the angle and the angle itself* is to be focussed upon. In fig. 3.2e one finds in both triangles an angle where the sides forming the angle, and the angle itself carries the same markings.
- *Two angles and one side.* Any two angles and one side in the triangle can be considered. Knowing the placement of the side is however crucial (opposite an angle, adjacent to an angle etc.) in relation to at least one angle mentioned. If the angles and side in the other triangle display the same placement, the triangles are congruent.
- *A right angle, hypotenuse and side.*

3.3 *From a practical to a theoretical perspective*

It is all right to discover the characteristics of quadrilaterals practically, but how can we make the connection between a practical and a theoretical approach?

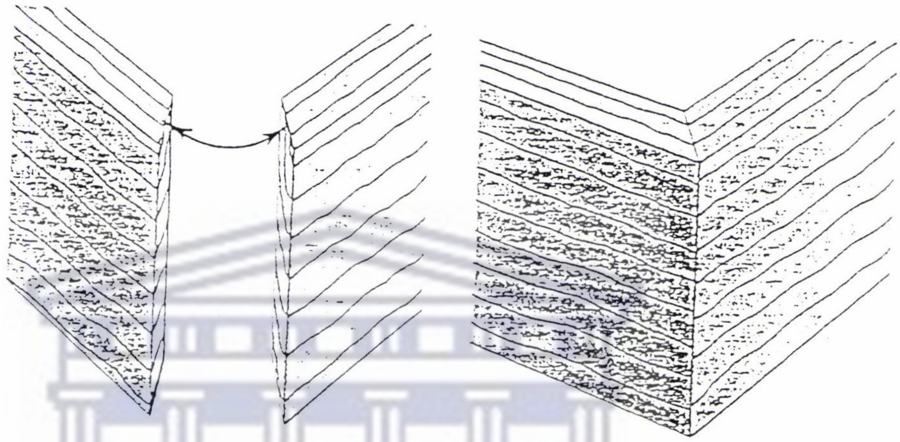
Fig.3.3a



To me, the answer to the question lies in a person's perception of and his / her attitude towards the activity. If the involvement of a person stops at admiring the end product without looking at the activity critically, the person will view that activity as irrelevant where mathematics is concerned. A classical example of this is the compilation of the school curriculum where boys must choose between the subjects woodwork and mathematics, and girls between needlework and mathematics. The mathematics educator's task is also to show the pupil that the subject is not only meant for mental development, but its relevance for people is also very important. Theoretical work is usually the result of the former and relevance of mathematics encompasses practical applications.

What sparks off special attention to the mitred corner? Looking at the mitred corner, as shown in fig.3.3a, in the circle, the slanted cut makes one wonder. The first guess could be that a triangle could have been removed from a rectangular strip of timber. One will however not reach this conclusion unless there is certainty from which side you are looking at the mitred corner (top view or side view etc.) In fig.3.3b.

Fig.3.3b

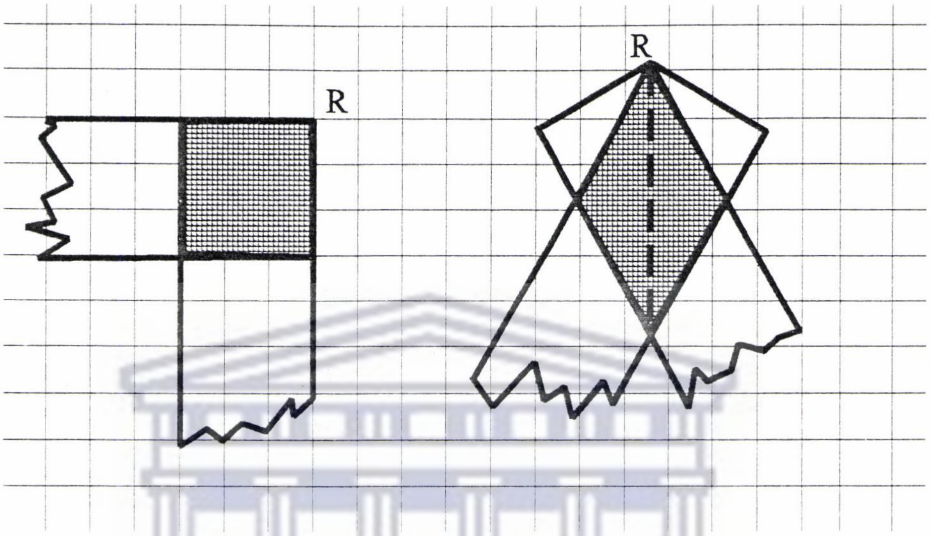


When one starts wondering about the view, a 'mental picture' is taking shape. *Visualizing, to me, is the first important step to link practical mathematics with theoretical mathematics as it can present one with different sketches. Lines* (outline of vertical and horizontal planes) in the 'mental picture' or image are the focal point and other characteristics take second place. One can visualize the triangular off-cuts as puzzle pieces fitted in various positions to the sides of the overlapped area (Refer to fig.3.3a). Knowing also the equal sides and angles on both the triangular off-cuts and the overlapped area (area used for mtring), enables one to formulate questions about the sketches produced. The following questions have been created in this way.

3.3.1 *Questions resulting from mitering on diagonal of a rhombus*

A close-up look at the case where mitering was done on the diagonal of the rhombus shows that two right-angled triangles have been cut off each strip(fig.3.3c).

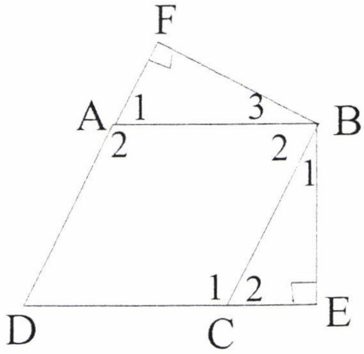
Fig.3.3c



By dislocating different sections of the sketch and moving it to another place in the sketch, gives us fresh questions. The questions in the following exercise are created in this manner.

EXERCISE

1. ABCD is a *rhombus* , FB $\perp$ FA and BE $\perp$ CE.



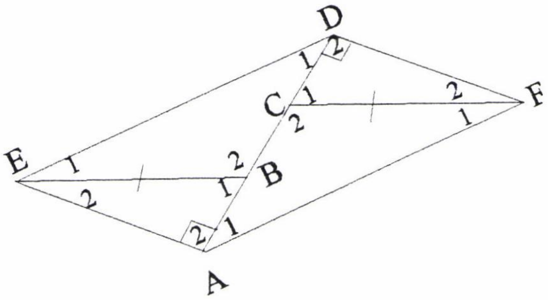
Prove : 1.1 $\angle A_1 = \angle C_2$

1.2 $\triangle$ FAB $\equiv$ $\triangle$ BCE

1.3 FD = DE

2\. $BD = AC = CF = EB$, $EA \perp AD$

and $DF \perp AD$.



Prove : 2.1 $DF \parallel EA$

2.2 $AB = CD$

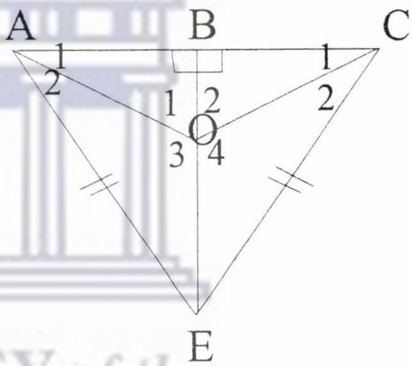
2.3 $\triangle EAB \equiv \triangle DCF$

2.4 $\triangle EBC \equiv \triangle ACF$

Deduce that: 2.5 $ED \parallel AF$

2.6 What type of quadrilateral is EAFD ?

3\. $\angle ABE = 90^0$, $\angle CBE = 90^0$ and $AE = CE$.



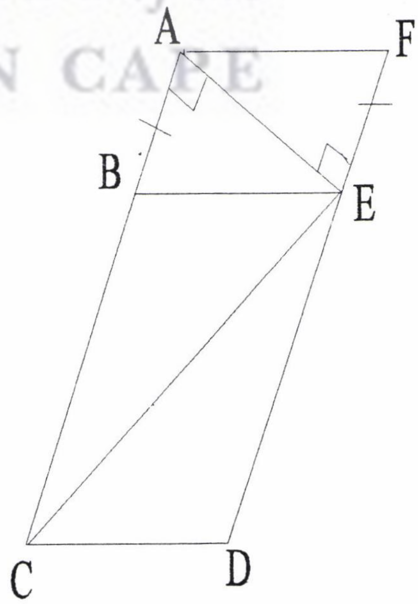
Show that : 3.1 $\angle BAE = \angle BCE$

3.2 $AO = CO$

Deduce that : 3.3 $\angle A_2 = \angle C_2$

3.4 $\angle O_3 = \angle O_4$

4\. BCDE is a rhombus and $AB = FE$.



4.1 Prove that

$\triangle ABE \equiv \triangle AFE$

4.2 Show that $AF \parallel BE$

4.3 Deduce that ABEF is

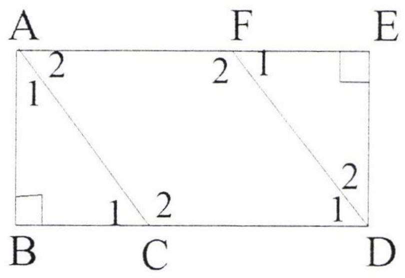
a parallelogram.

4.4 Prove that ACDF is a

parallelogram

4.5 Show $\angle$ BEC = $\angle$ CED

5. ACDF is a rhombus , $\angle B = 90^0$ and $\angle E = 90^0$. AFE and BCD are straight lines.



Prove : 5.1 $\angle C_1 = \angle F_1$

5.2 AE = BD

6. AFCE is a parallelogram and BF = AE = ED. $\angle A_1 = 35^0$ and $\angle A_2 = 20^0$.

BFA and DEC are straight lines.

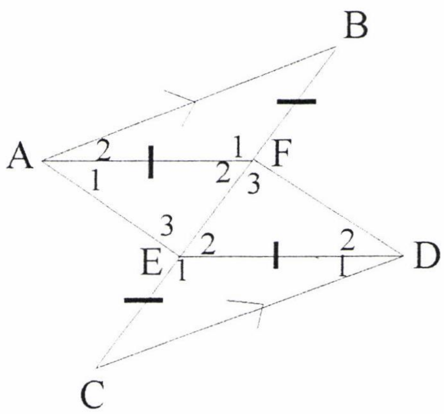
Determine : 6.1 $\angle C_3$

6.2 another right angle

6.3 four angles each equal to 110^0

Prove that : 6.4 $\angle B + \angle C_1 = 70^0$

7. Given AB || CD , $\angle D_1 = 35^0$ and AF = FB = CE = ED .



Calculate : 7.1 $\angle A_2$

7.2 $\angle E_3$

7.3 $\angle F_3$

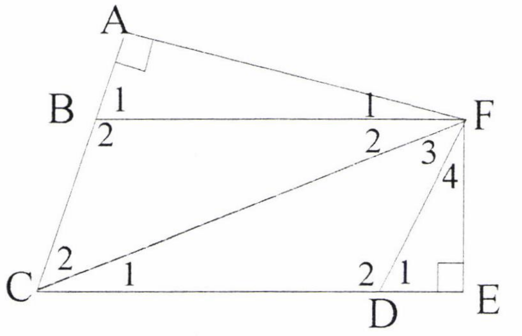
Prove : 7.4 $\triangle AFE \equiv \triangle FED$

3.3.2 *Questions resulting from mitering on the diagonal of a parallelogram.*

1. BCDF is a parallelogram. ABC and CDE are straight lines with $AF \perp AC$ and $FE \perp CE$. $\angle B_1 = 60^0$ and $\angle F_2 = 40^0$.

Calculate : 1.1 $\angle D_2$

1.2 $\angle F_4$



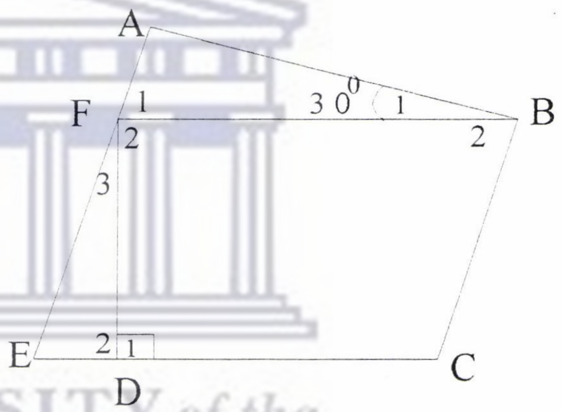
2. FECB is a parallelogram and AFE and EDC are straight lines. $\angle D_1 = 90^0$;

$\angle B_1 = 30^0$ and $\angle B_2 = 2 \angle B_1$.

Determine : 2.1 $\angle C$

2.2 $\angle A$

2.3 $\angle F_3$



The activity of mitring is indeed a valuable source for mathematics education. Researching the little mitred corner empowered me a lot and I can now 'see' meaning in the sketches. I could develop questions because I understood the context of the problem. The hypothesis to start in the real world and extract mathematics concepts for further and deeper mathematics activity, was very rewarding. Not only is the obvious mathematics under discussion, it also creates opportunities to uncover the 'hidden' mathematics. The theorem of Pythagorus is an example of the latter. Chapter four will be dealing with production activities that mainly operate on symmetry. The feasibilty for developing mathematics lessons will be investigated.

CHAPTER 4

CONSTRUCTION OF CONTEXTUALLY-BASED MATHEMATICS ACTIVITIES: II

SYMMETRY

Symmetry is a simple and common technique which we employ in production of various things. Understanding symmetry and its application are very stimulating for the creative mind and can also be applied with very little effort. To a large extend, fashion design, building construction, the printing industry and many other industries are using the principle of symmetry extensively. In mathematics we do not emphasize symmetry to the same extent as with various industries. I believe a proper investigation about the application of symmetry as technique will reveal its significance in mathematics. We should not prejudge the importance of symmetry. Getting the hang of it can possibly help the pupil to grasp certain themes like congruency in geometry and graphs in trigonometry much better. A very quick and easy way to form symmetrical parts is through *folding*.

4.1 *Garment construction through symmetry*

The work of the seamstress for clothing manufacturing is based on symmetry. Let us review the construction of a garment's bodice pattern piece as shown in fig.4.1a. The bodice of a dress is the part above the waist and consists of two different pattern pieces, namely a *bodice front* and a *bodice back*. Concentrating on the *bodice back* one requires only half of the pattern piece for the construction purposes. One

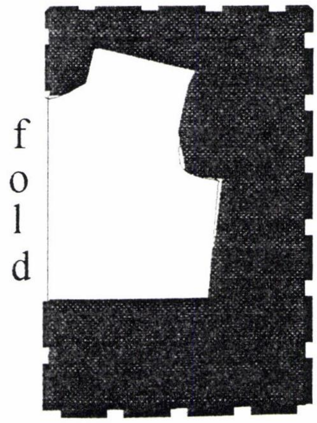


FIG. 4.1.a

line on this pattern piece is very important and deserves special attention.

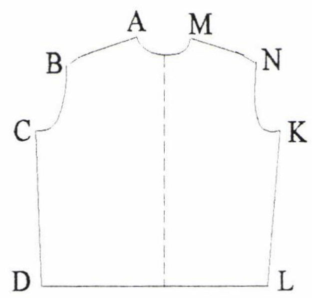


FIG. 4. 1b

4.1.1 *Axis of symmetry*

The line from where crucial horizontal measurements are made is called *centre back*. *Centre back*, illustrated in fig. 4.1b, is the vertical lacerated line on the pattern piece that separates two symmetrical pieces on the *bodice back*.

Cutting out the back of a bodice, the seamstress folds the material to form a rectangular double layer. Only half of the pattern piece for the back is needed and *centre back* is taken *as the axis of symmetry*. It means that points B and N on the slope of the shoulder are the same distance away from the lacerated line. The assumption is also that the line through B and N is perpendicular to the lacerated line (*centre back*). For the same reason are points M, K and L equidistant from A, C and D respectively. If no cut is made on the centre back line and the pattern piece is opened it will produce the full *bodice back*.

4.1.2 *Mirror Images*

What lesson is involved here? *Measuring the same distance, in opposite directions, on the line perpendicular to the centre back line, produces symmetrical points.* Applying this rule to every point on the outline of half the pattern piece, results in duplicating of the pattern piece. This explains why the seamstress folds the material to form a double layer. Duplication around *centre back* displays also other interesting features namely *mirror images*. Referring to the two halves of fig.4.1b we see that they appear back-to-back on the sketch. We call this kind of positioning a mirror image.

4.2 *Creating a parabola through paper folding.*

Through the construction of the bodice back the importance of the axis of symmetry is self-evident, but there are also other production activities where the use of symmetry is less obvious. However, the basic characteristics of the axis of symmetry as displayed in the work of the seamstress have far reaching effects on:

- the acoustic of cathedrals or buildings with parabolic ceilings,
- reflectors in electric heaters,
- reflectors in car headlamps,
- television dish aerials.

Let us make parabola by folding paper (Hobbs, 1993, page 43). You will need a sheet of plain paper.

- Mark a dot about 2 cm from the edge, and about *half way* down as seen in fig. 4.2a. What do you understand by *half way*.

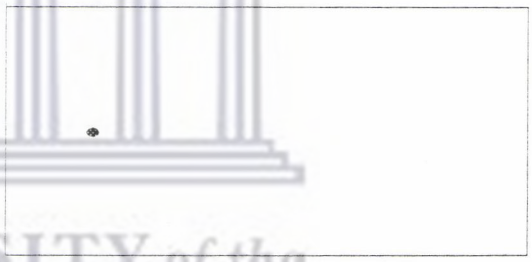


Fig. 4.2a

- Fold the paper over so that the edge near the dot falls exactly *on* the dot, as in fig. 4.2b

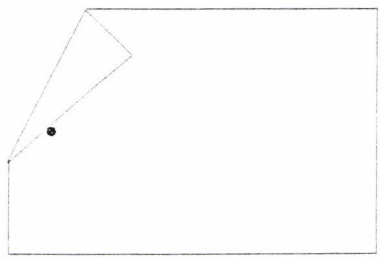


Fig. 4.2b

- Unfold the paper and draw a line along the crease, as shown in fig.4.2c. The crease line is running between two lines forming an angle. On the one leg of the angle the crease starts close to the vertex while on the other leg the crease stops far from the vertex.

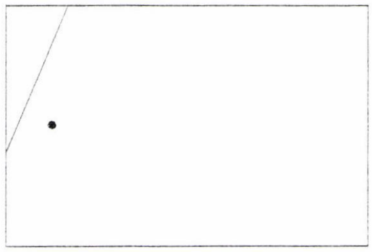


Fig. 4.2c

- Do this folding and drawing several times. As more lines are drawn, you should see a curve forming (fig. 4.2d)

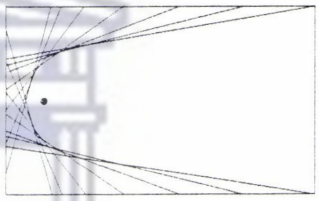


Fig. 4.2d

Try it once more on a piece of paper but mark the dot about 7 cm from the edge and about half way down. Your curve should resemble the accompanying sketch in fig. 4.2e

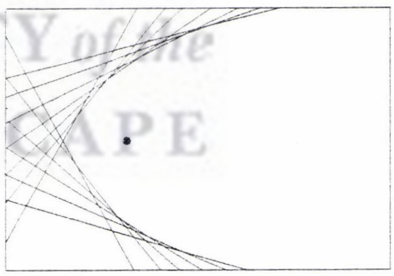


Fig. 4.2e

The curve you have made from straight lines is called a *parabola*. The dot you made for folding the paper is called the *focus* and is in a very special position. Fold the two long sides of the rectangle onto another and open the page again. Fold the two short sides onto one another and open the page.

Make three more creases

parallel to the short sides (fig. 4.2f).

The broken lines represent the creases.

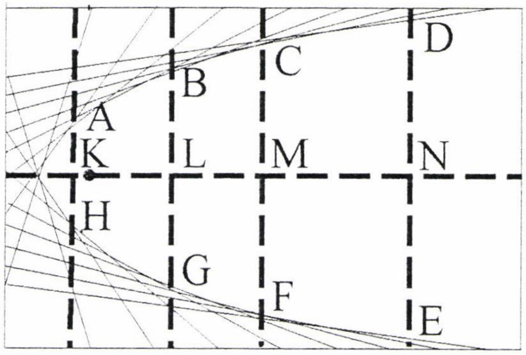


Fig. 4.2f

Measure now the following linesegments and see if any conclusions can be made about the lengths:

DN = ...units NE = ... units

CM = ... units MF = ... units

BL = ... units LG = ... units

AK = ... units KH = ... units

You should have got the following results:

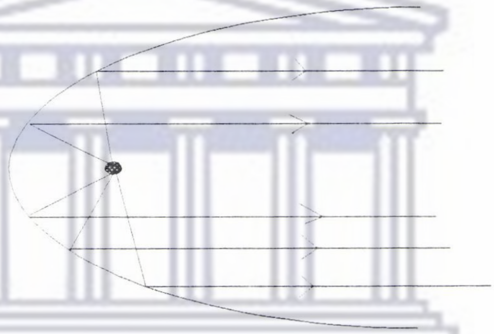
DN and NE having the same length :	DN = NE
CM and MF having the same length :	CM = MF
BL and LG having the same length :	BL = LG
AK and KH of equal length :	AK = KH

If the angles AKL, BLM, CMN and DNM are measured you will find that each one is equal to 90^0. Each vertical broken line is therefore *perpendicular* to the horizontal broken line KN.

According to the results :

- the section above the broken line KN is *a mirror image* of the section below the broken line.
- the line KN separates the two sections *symmetrically* and is called *the axis of symmetry.*
- to the left of K on the broken line KN the curve changes direction. That point on the broken line is called the *turning point.* No part of the curve goes beyond this point.

Fig. 4.2g



Let us return to the *focus,* in fig. 4.2g, which we learnt is on the axis of symmetry. This point plays an important role in reflectors and is used in electric heaters, car headlamps and television dish aerials. In an electric fire, rays of heat from the focus strike the shiny surface. They are then reflected so that they all come out parallel to each other to make a beam of heat. Physics deals with this in detail.

We created curves practically, through folding of paper, but are there other ways of making curves?

4.3 *Curves from straight lines*

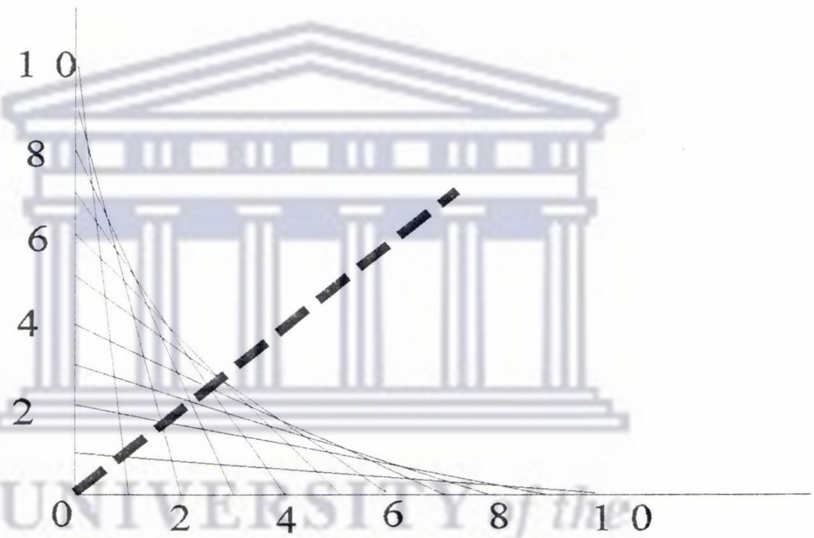
Curves can be made by joining a point on one arm of an angle with another point on the other arm of the angle. Draw two lines to form an angle, as seen in fig. 4.3a:

- On each line make marks 1cm apart and number them.
- Join the marks with straight lines in the following way :

  1 to 10 ; 2 to 9 ; 3 to 8 and so on.

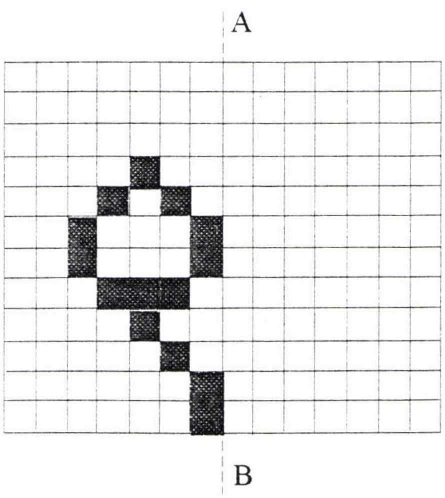
Explain why you think the dotted line is the axis of symmetry.

Fig. 4.3a



EXERCISE

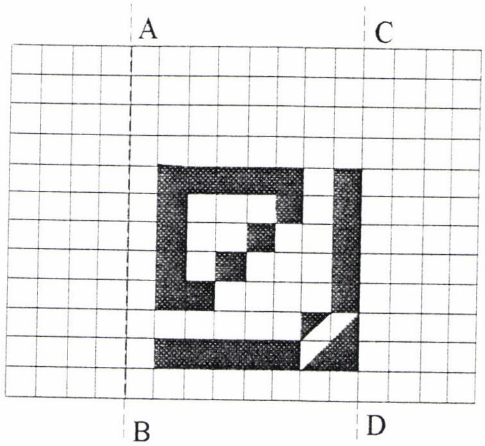
Use your knowledge about symmetry and mirror images to show the designs for the various undertakings.



1. In the accompanying sketch use AB as the axis of symmetry and complete the

other half of the embroidery design.

2. The accompanying diagram is needed for screenprinting and forms a border if printed.

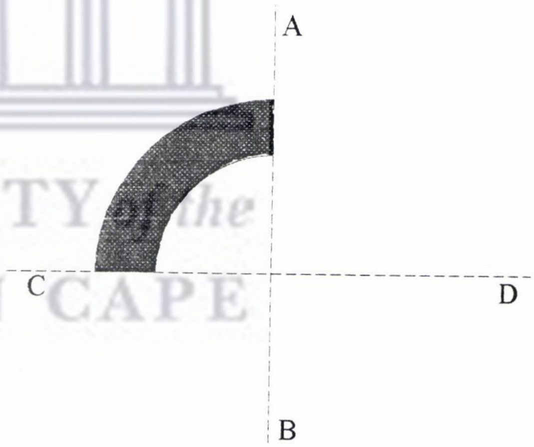


a. Use AB as the axis of symmetry and mirror image the diagram.

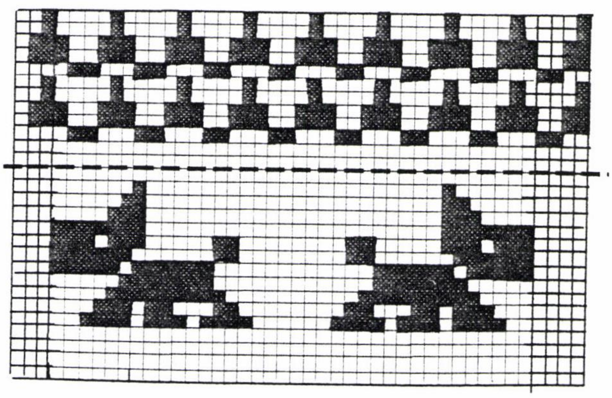
b. Use CD as the axis of symmetry and mirror image the sketch.

c. Take AB as the beginning of the diagram obtained in (b) and repeat the sketch twice more.

3. The letter 'O' is to be cut out on perspex for a sign board. The letter must be constructed with precision and only a quarter of it is provided. Use symmetry and the two dotted lines to make a template of the letter. Give full details how you made the template .



4. The accompanying diagram shows a design punched out for machine knitting. The broken line divides the design into



two sections. Study the design carefully and then answer the questions that follow :

a. How many horizontal lines can be drawn in the top section to create symmetrical sections.

b. Determine the number of vertical axes of symmetry in the top section.

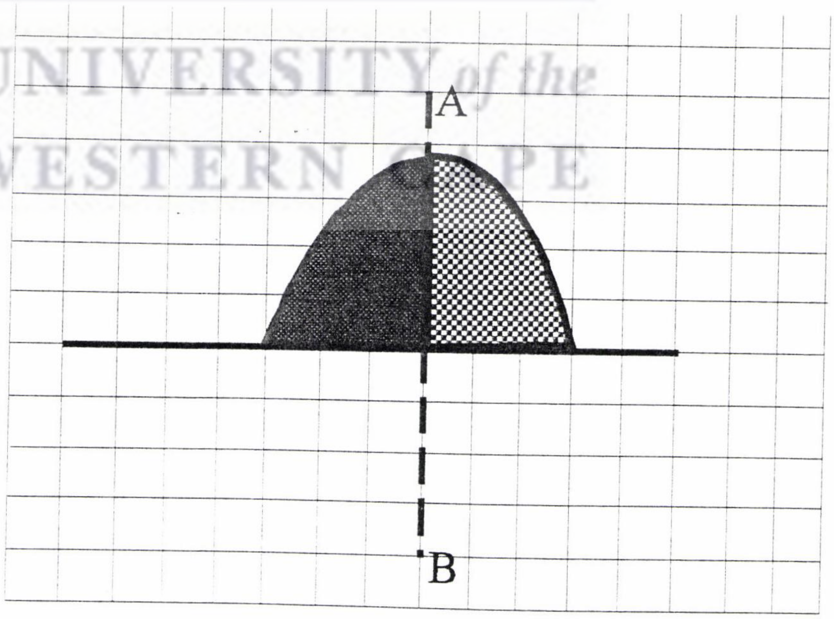
c. Find the number of horizontal axes of symmetry in the bottom section.

d. Describe where the vertical line in the bottom section must be drawn so that the dog on the left is a mirror image of the one on the right.

4.4 *From a practical to theoretical perspective*

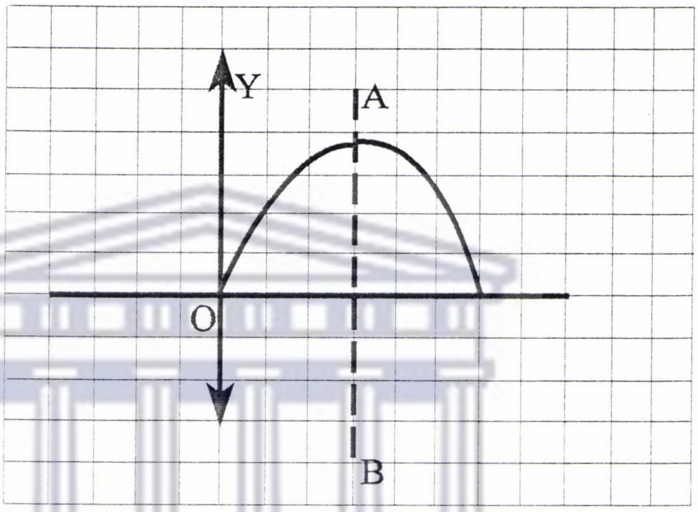
Knowing how to draw symmetrical sketches, as with the previous four questions, is a great advantage when one moves from a practical perspective to a theoretical one in mathematics. Using AB as axis of symmetry in fig. 4.4a, the mirror image will be drawn to the right of AB.

Fig. 4.4a



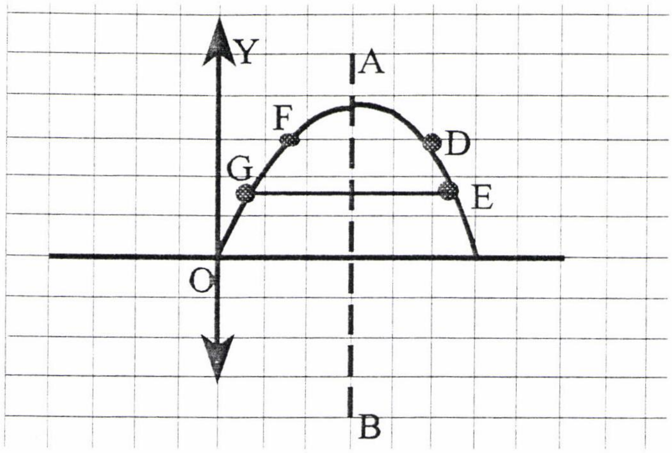
The principles applied in this instance are the same principles used for constructing the full bodice pattern. Should the line OY be drawn in on the sketch and *only the outline* of the total shaded area be given , we would have a parabola on the sketch (refer to fig.4.4b). What is then the real reason for a pupil not being able to finish the parabola in fig.4.4b, but being able to complete the sketch in fig. 4.4a? My opinion is that we do not encourage pupils to present us (the teachers) with their different perspectives of the same sketch. From their answers one can determine how they view the sketch. It might be that the pupil is making a two-dimensional or three-dimensional

Fig. 4.4b



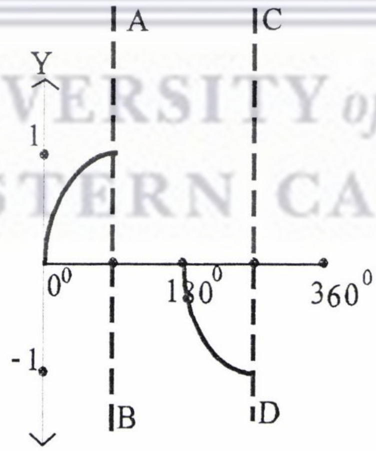
association with the sketch. We should develop the pupil's ability to view a two-dimensional presentation as a three-dimensional structure. By doing this we are narrowing the gap between mathematics and woodwork, especially where woodwork drawings are concerned. Although the standard six and seven-pupil will have a very limited idea of the parabola, they can flip sections of the parabola around a line. Let us take this one step further and put point D on the curve in fig.4.4c.

Fig.4.4c



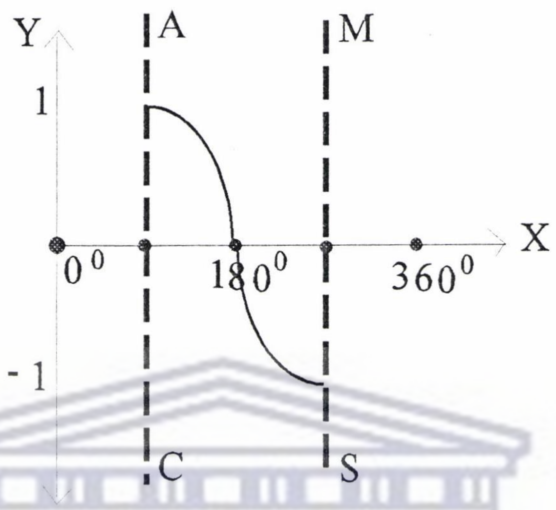
Recalling the discussion of mirror images we are reminded that points on the outline of the curve are the same distance away from the axis of symmetry. The only requirement is that the straight line through these two points must form a right angle with the axis of symmetry. If point D is 3 units to the right of AB, the axis of symmetry, point F is positioned 3 units to the left of AB. Similarly, if point G is 3,5 units to the left of AB, then point E is also 3,5 units to the right of AB. Using the practical situation of the seamstress, we can gradually introduce important characteristics of the curve. This kind of exercise lays the foundation of determining coordinates of points on the curve.

Fig.4.4d



Stepping up the degree of difficulty, an incomplete sine curve between 0° and 360° can also be completed using two axes of symmetry, AB and CD. The name of the curve need not be mentioned. Using numbers on the x- and y-axis, would also make the sketch appear more complicated. Displaying the curve on a grid (fig. 4.4c) or omitting the grid, as in fig. 4.4d, might be a problem to the pupil, and it is therefore necessary that we deal with both presentations.

Fig. 4.4e



To test if pupils understand the notion of mirror images one must refrain from drill and practice method. Giving sections of the curve at different positions on the sketch help to counter the method of drilling. Examples as shown in fig.4.4d and fig.4.4e are therefore very important It is better to use the practical examples as an aid to explain problems of a theoretical nature.

In chapters three and four the activities have been investigated with the objective to develop mathematics lessons. The lessons in the form of worksheets for pupils have been drawn up (appendix B and C) and educators' responses to the lessons have been requested. The next chapter will deal with the responses of the mathematics educators to the activities.

CHAPTER 5

RESPONSES BY MATHEMATICS EDUCATORS TO THE ACTIVITIES

5.1 *Response to activities*

I started my research on mathematics texts for secondary school pupils. The objective was to research the feasibility of developing mathematics lessons from contextually-based activities. I worked on two activities that stem from commonplace contexts, and to which pupils can relate. The two activities are *mitring as performed by the carpenter* and *symmetry as applied by the seamstress.* The motivation behind this idea is also to try and raise the level of interest in mathematics activity and mathematics as a school subject.

The research is based on realistic and ethno-mathematics and is underpinned by the following ideas:

- The starting point should be in the real world.
- It should contribute to the cultural background of the general student.
- It should offer professional preparation for the future user.
- It tries to bridge the gap between knowing mathematics and using mathematics.
- Conceptual mathematization, the extraction of the appropriate concept from a concrete situation, is stressed.
- It should not be separated from other sciences.

Being aware that mathematics teachers are important role-players in this regard, I invited a group of mathematics educators to a workshop for a critique of the lessons developed. The workshop took place on 8 December 1995 at the Gold Fields Resource Centre, University of the Western Cape (UWC) from 12h00 until 16h30. The group of educators consists of:

- three mathematics teachers,
- a mathematics subject advisor,
- two mathematics lecturers from the Bellville College of Education,
- four lecturers from the Mathematics Department of the University of the Western Cape,
- a principal of a secondary school,
- two representatives of Realistic Mathematics Education in Southern Africa Project,
- a mathematics teacher at a technical school enrolled in the program Teacher Advancement In Mathematics Teacher Advancement in South Africa, at Iniversity of the Western Cape,
- the vice-principal of Protea College, a college of careers.

The educators were issued with the lessons concerning *mitring* and *symmetry*. They worked through the activities, filled out a questionnaire related to each set of activities. The five questions of the questionnaire are:

- Do you think the presentation of the learning material will stimulate the pupil to greater mathematics activity?
- How successful are the lessons in shedding the notion that mathematics is abstract and meant for intelligent people?
- What is your opinion about the attempt to integrate mathematics with other school subjects?

- Does this approach help to narrow the gap between knowing mathematics and using mathematics?
- How successful was this approach in extracting mathematics from concrete situations?

Provision was also made on the questionnaire for further comments.

5.2 *Responses on the questions*

The overall opinion about the presentation of the learning material was very favourable. The educators felt strongly positive about the question of bridging the gap between knowing mathematics and 'doing' mathematics. On the question of the success of extracting mathematics from concrete situations, responses were as follows:

- The attempts have been successful.
- Concrete situations were used effectively.
- This approach could be very successful at school level. There is very little school mathematics which we cannot approach in this way.
- The approach is very realistic.
- The method of deriving the characteristics of the quadrilaterals is good.
- The approach will better the understanding of these concepts and retention of these concepts will last longer.
- A lot of mathematics from concrete situations was extracted.
- The pupil will look at life more mathematically.

About integrating mathematics with other school subjects the educators expressed great enthusiasm about the attempts. Comments on the worksheets are as follows:

- The worksheets are very well presented.
- Very good idea but it depends on the student's interest in the other subject as well.
- Very good. The respondent also suggests an even more rigorous approach.
- It was a good attempt. The hope is expressed that links with other school subjects will also be made.
- Learning is a holistic experience and we should not classify subjects to the extend that its operation / area of application becomes disjoint. This attempt is applaudable.
- This attempt is part of relating mathematics to the pupil's 'real' world. Other subjects are part of this world and provide cross-reference.
- Pupils get the picture that his / her subjects are interrelated and not entities standing on its own.
- Students need to see where their knowledge of mathematics is applicable.
- This approach gives the mathematics class the opportunity to draw on the pupil's experiences.
- Respondent favours this approach because 'mathematics needs other subjects and other subjects need mathematics'.

Recommendations were invited for implementing on quadrilaterals and symmetry. Recommendations made by the respondents are as follows and the worksheets have been adjusted, where feasible.

- Caution must be taken that the pupils understand the context of the lesson.
- Logical order of developments must be maintained.

5.3 *Conclusion*

The responses obtained through the workshop suggest a few important ideas, among others a review of the current approach in mathematics. We should adopt a cross-cultural approach to reflect the composition of our school communities. Educators should not deny any pupil the opportunity to be duly educated. Persisting with an approach in mathematics that frustrates many pupils is therefore not right.

The South African disadvantaged communities have been dealt severe blows in many ways. People have been classified and labelled and the deep-rooted problems of South Africa originated from these labels. At school level not only have pupils been labelled 'clever', 'stupid' or 'bright' but they have also categorised school subjects. We owe it to our pupils and the school community as a whole to eradicate these labels and instill a strong self-image. Teachers must take the initiative to fit real-life experiences of pupils in their approach. So doing, the lesson will be more interesting to the pupil and make him / her part of the education process. Mathematics teachers must be vigilant of the context in which problems are presented. Familiar contexts should be used as stepping stones to unfamiliar and abstract contexts.

Mathematics has yet another role to play. I believe that we should not use mathematics in isolation since mathematical principles and ideas permeate other school subjects. Allowing people to categorise subjects as important and others as less important, is unfounded discrimination. Very few, if any, subjects at school are without mathematics application. The responsibility rests on the mathematics teacher to find the links and common areas between subjects. I, and the educators involved in this endeavour, saw the need for reaching out to other subjects and I want to recommend that we engage in this type of study.

Paulus Gerdes, concerned with ethno-mathematics in Mozambique, and Arthur Noble (1990), a former professor of Mathematics Education at Rhodes University, have been working on real world problems. Their work however deals with analizing the structure of the real world problem. Realistic mathematics education goes beyond point where the problem is taken from the real world, ample opportunity given for reflection and application of realistic mathematics education will also take place in the real world. Not merely is *thinking* as such the main concern, but also the application of *common sense.* The integration of common sense and mathematics is nothing else but reflection. By the activity of mathematization in realistic situations, interaction with others and reflection take place and common sense is continually brought to a higher state. My wish would be to see change in mathematics education in South Africa taking place against this background.

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APPENDIX A

LETTER OF INVITATION TO MATHEMATICS WORKSHOP

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Suid-Afrika
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Teleks: 526661
Telefoon: (021) 959-2911

Dir. line/lyn ....

Dept. ....

Ref./Verwys. ....

13 November 1995

Dear ....

I am presently researching the development of mathematics concepts yielded through production activities. The research is based on realistic and ethno-mathematics and is underpinned by the following ideas:

- the starting point should be in the real world.
- it should contribute to the cultural background of the general student.
- it should offer professional preparation for the future user.
- it tries to bridge the gap between knowing mathematics and using mathematics.
- conceptual mathematization, the extraction of the appropriate concept from a concrete situation, is stressed.
- it should not be separated from other sciences.

I have extracted mathematics concepts from the world of work of the carpenter and the seamstress. However, I am very keen to get the opinion of other mathematics teachers on the work done. I wish to invite you to partake in the mathematics workshop which will be held on **Friday 8th December 1995 from 12h00 till 14h00. The workshop will take place at the** Gold Fields Resource Centre, UWC. An honorarium to the amount of R30 will be contributed to your transport expense.

Pleae contact Mrs R. Petersen at 959 2322 before or on 5th December to confirm if you will attend the workshop. Your cooperation will be appreciated.

Yours truly

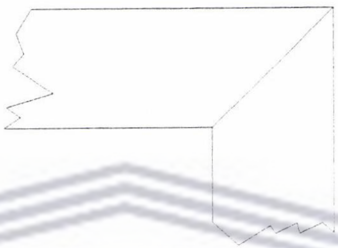
Benji Fray

APPENDIX B

PUPIL WORKSHEET ON QUADRILATERALS AND TRIANGLES

1.1 Two strips for mitring are supplied. You may use the strips to reconstruct the mitred corner as shown in figure 1.

Fig. 1



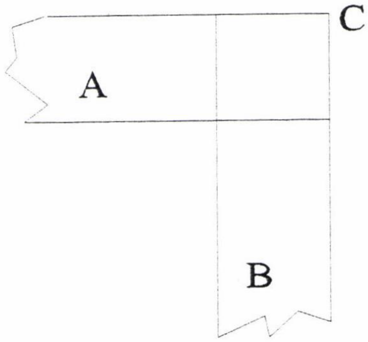
It is also advisable to work together in small groups and exchange your ideas. Assume also the following:

- The opposite edges of the strips are parallel.
- the strips are of equal width.

Our discussion will be based on what you will do next.

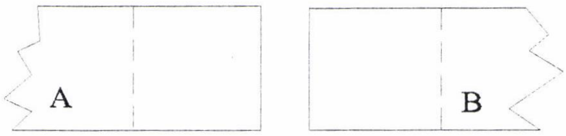
a. Form a right angle with the two strips by overlapping strips A and B (Fig. 2).

Fig. 2



b. On both strips mark the outline of the overlapped area and move the strips apart. (Fig. 3)

Fig. 3



Use the two sketches in figure 2 and figure 3 to answer the following questions. Write your answers in the spaces below the question.

Discuss in your group and consult the dictionary, if necessary, the definition of aQUADRILATERAL

Explain how you will go about measuring the sides of the outlined area.

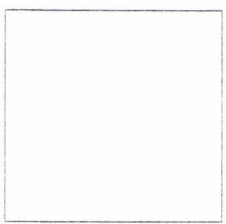
Describe the side's length in relation to each other. Do not use rulers to measure.

Determine the size of each angle in the overlapped area. Do not use any protractors.

Only the overlapped area appears in the space below.

(a) Show on the sketch what you discovered through answering the previous questions.

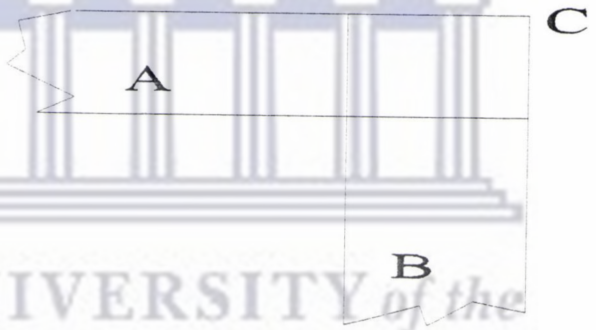
(b) Consult fellow pupils, teachers, librarian, dictionaries or mathematics books about the name of the four-sided figure. Take the characteristics of the figure into consideration and write next to the sketch on the line.



To confirm what you should have discovered, is a SQUARE with all four sides equal and each angle a right angle. The strips are positioned perpendicular to each other, therefore the angle at C in figure 1 is a right angle (90^0). The opposite sides of the strips are parallel, therefore the co-interior angles in the overlapped area are also right angles.

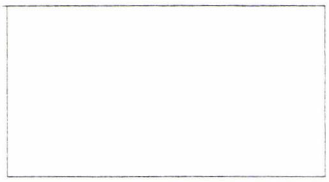
1.2 In 1.1 we have been working with two strips of same width and the overlapped area turned out to be a SQUARE. Will we get the same result if *two strips of different width* be used? Concentrating on the same questions as for 1.1 see if you can find the characteristics of the overlapped area.

Fig. 4



(a) Use the sketch to illustrate what you discovered about the sides and angles in fig. 4

(b) Summarise the characteristics in the space next to the sketch under the headings indicated.



SIDES: ____

____

____

ANGLES: ____

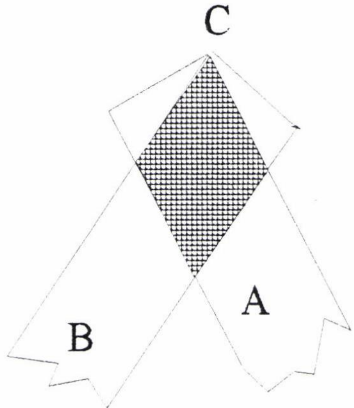
____

NAME OF FIGURE: ______________________________

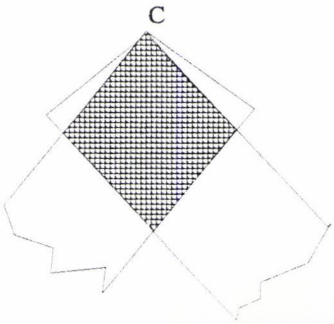
The figure under discussion in 1.2 is a RECTANGLE with opposite sides equal and also parallel (from the outset we said the sides of the strips are parallel). Each angle in the rectangle is a right angle.

1.3 Let us have another look at figure 2 and swivel strip A around strip B through the fixed point C. The overlapped area being outlined on both strips seems to have changed in shape(fig.5).

Fig.5



Use one side of the outlined area to measure the other sides. From figure s 2 and 4 it becomes clear that if the two strips overlap with no part of the strip extending beyond two adjacent sides of the overlapped area, the two strips are perpendicular. Measure the angles in a similar way and list your discoveries.



SIDES: ____________________

____

____

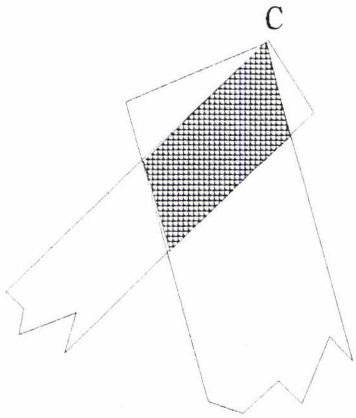
ANGLES: ____________________

____

____

NAME OF FIGURE: ____________

Indeed you have been working with a RHOMBUS. All the characteristics but one of this quadrilateral correspond with the characteristics of the square. The only difference is that no angle is equal to ninety degrees. All four sides are also equal. Follow now the same procedure if we take the strips in figure 4 and swivel the one strip around the other one through the fixed point C. Figure 4 will then change to the following sketch.



SIDES: ____________________

____

____

ANGLES: ____________________

____

____

____

Fig. 6

NAME OF FIGURE: ____________

1.4 By the look of it, figure 6 is different from figure 5. The obvious difference is in the length of the sides. We must again emphasise the extension of the strips on all four sides of the overlapped area, meaning that no angle is equal to ninety degrees. The sketch you have been concentrating on is a PARALLELOGRAM. You should have found that the opposite sides are equal, the opposite sides are parallel as was stated about the strips at the beginning and the opposite angles are also equal.

1.5 We have been introduced to the different quadrilaterals. However , if a corner gets mitred a piece of the strip gets removed and coincidentally the cut is made inside the overlapped area. Let us now focus on the the section of the overlapped area that will be removed. The cut in the cases of the square, rectangle, rhombus and the parallelogram is made in the same way.

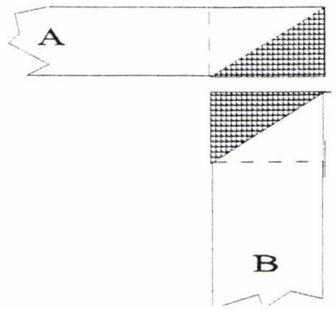


Fig. 7

Use figure 7 to answer the following questions. Write your answers in the spaces below the questions.

(a)	**Find out what do we call the line on which the cut was made (It is a very prominent line in the overlapped area).**
(b)	**Compare the sides, angles and size of the two shaded triangles on both strips and give your comments under the headings provided.**
(a)	______
(b)	**Vertical sides** in the shaded triangles: ______
	Horizontal sides in the shaded triangles: ______
	Slanted side in the shaded triangles: ______

Size of the triangles (you may fit the triangles): ______________

____

____

Find the meaning of congruency in the dictionary and decide if your comments under (b) fit the meaning of congruency:___________

____

____

____

Compare the off-cut triangles of the other quadrilaterals and draw some conclusion: ______________________________

____

____

We have found that the off-cut triangles are identical (congruent). You will now be provided with the overlapped area (square, rectangle, rhombus and parallelogram). Take each quadrilateral's off-cut triangle(s) and produce a sketch which you think can be used in a mathematics lesson. You may fit the off-cut triangles to the sides of the overlapped area. Make sure that you mark some key equal sides and angles. Use the same marks for eequal sides as well as equal angles.

An example is given in figure 8. The one triangle is fitted on top of the overlapped area and the other triangle is fitted on the outside of the area.

APPENDIX C

PUPIL WORKSHEET ON SYMMETRY

Symmetry is a simple and common technique that is used in production of various things. This technique is common practice in production of clothes. We will look at the way the dressmaker plan the bodice, more specifically the *bodice back*, of a dresspattern. Let me first explain what a bodice pattern piece is. The bodice of a dress is the part above the waist and consist of two different pattern pieces, namely a *bodice front* and a *bodice back.*

We are dealing with two important questions.

1. How are the *bodice back* pattern supplied?
2. What is the connection between the construction of the bodice back and mathematics?

To answer the first question, the supplier includes only one half the *bodice back pattern.* The seamstress overcome this problem through *folding* of the material. *When* the seamstress cut out the back of the bodice, she folds the material to form a double layer. The *centre back* line of the pattern is placed on the fold of the material (fig.1). If no cut is made on the centre back and the material pattern piece is opened, it shows the full back pattern piece (fig.2).

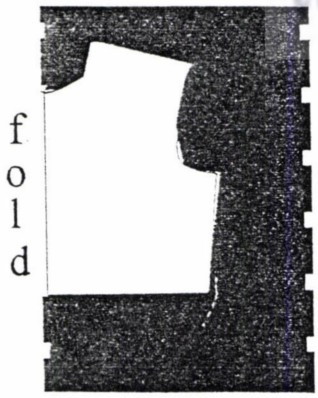


Fig. 1

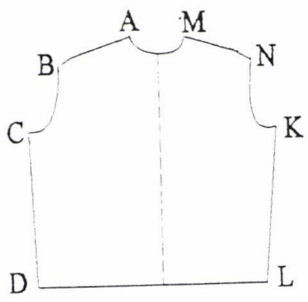


Fig. 2

Examine fig.1 and fig.2 carefully so that you can contribute to the discussion.

The division of the broken line creates what kind of section? (ask the Biology teacher what the division of the torso in fig.2 is called).
____
The name of the broken line is derived from the way the broken line divides the pattern piece. Find out from mathematics teachers or textbooks what the broken line is called.
____
The dictionary describes a mirror image as 'an identical image, but with the structure reversed'. **(a) Do you agree that the two sections in fig.2 are mirror images of each other?** **(b) If your answer in (a) is 'yes', what would you describe as the 'mirror'?**
______________________________ ______________________________ ______________________________
Line DL is <u>perpendicular to the dotted line,</u> therefore D is the mirror image of L. Write down two more sets of mirror images.
______________________________ ______________________________

The terms *symmetrical* and *mirror images* come into play only if <u>the line of division</u> causes two sections to be identical but facing in opposite directions. The reason why the seamstress fold the material is to 'measure' at once on both sections the same distance from the *centre back* (axis of symmetry). The *axis of symmetry* is without doubt the most important construction line on the pattern piece.

Through the construction of the bodice back the importance of the axis of symmetry is self-evident, but there are also other production activities where the use of symmetry is less obvious. However, the basic characteristics of the axis of symmetry as displayed in the work of the seamstress have far reaching effects on:

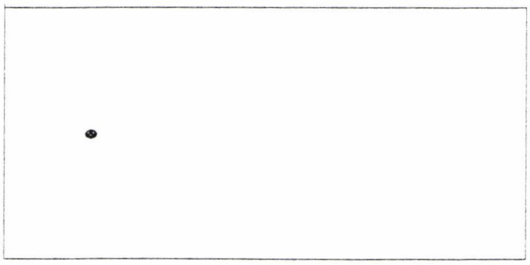
- the acoustic of cathedrals or buildings with parabolic ceilings,
- reflectors in electric heaters,
- reflectors in car headlamps,
- television dish aerials.

<u>Creating a parabola through paper folding.</u>

Follow the instructions and create a parabola through paper folding. You will need a sheet of plain paper.

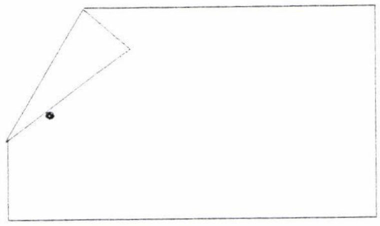
- Mark a dot about 2 cm from the vertical edge, and about *half way* between the two horizontal edges as seen in fig. 3.

Fig. 3



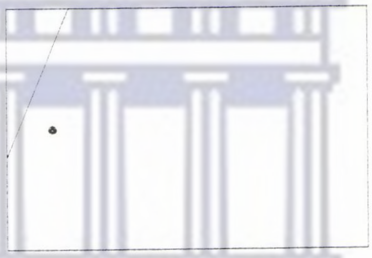
- Fold the paper over so that the edge near the dot falls exactly *on* the dot, as in fig. 4

Fig. 4



- Unfold the paper and draw a line along the crease, as shown in fig. 5 The crease line is running between two lines forming an angle. On the one leg of the angle the crease starts close to the vertex while on the other leg the crease stops far from the vertex.

Fig. 5



- Do this folding and drawing several times. As more lines are drawn, you should see a curve forming (fig.6)

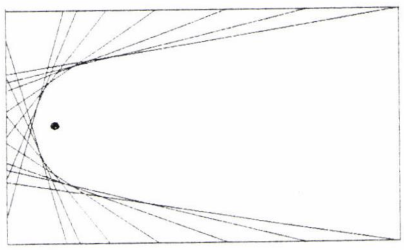
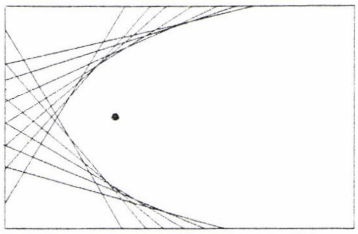


Fig. 6

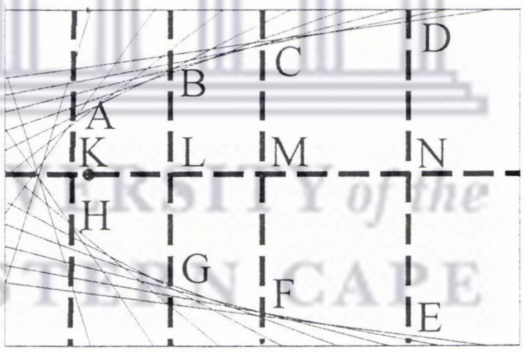
- Try it once more on a piece of paper but mark the dot about 7 cm from the edge and about half way down. Your curve should resemble the accompanying sketch in fig. 7

Fig. 7



- Fold the two long sides of the rectangle onto another and open the page again. Fold the two short sides onto one another and open the page. Make three more creases parallel to the short sides (fig. 8). The curve you have made from straight lines is called a *parabola* and the broken lines represent the creases.

Fig. 8

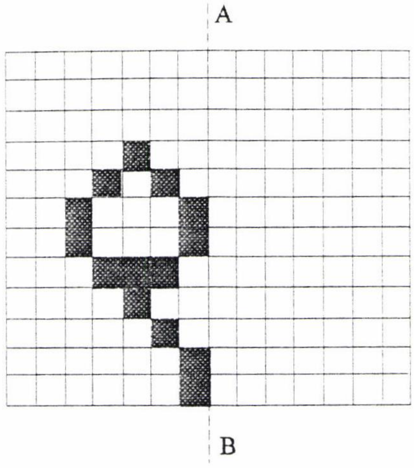


What is line KN called? Explain where KN got its name from.

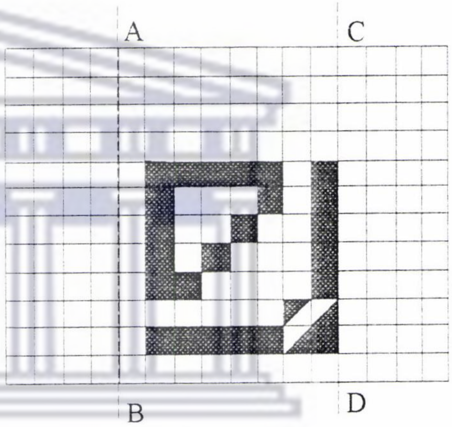
Show fig.8 to the physics teacher and ask **(a) what do we call the very important point on the horizontal broken line.** **(b) What role does that point play in reflectors, television dish aerials etc.**
(a) ________ (b) ________
Explain why B and D in figure 8 are mirror images of G and E respectively.
____

Be creative and use your knowledge about symmetry, mirror images and the axis of symmetry to complete designs for embroidery, knitting machines and screen printing. Use graph paper for your designs and use one sheet per design.

In the accompanying sketch use AB as the axis of symmetry and complete the other half of the embroidery design.



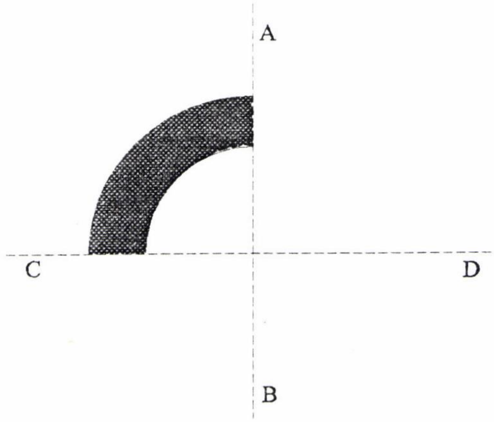
The accompanying diagram is needed for screenprinting and forms a border if printed.



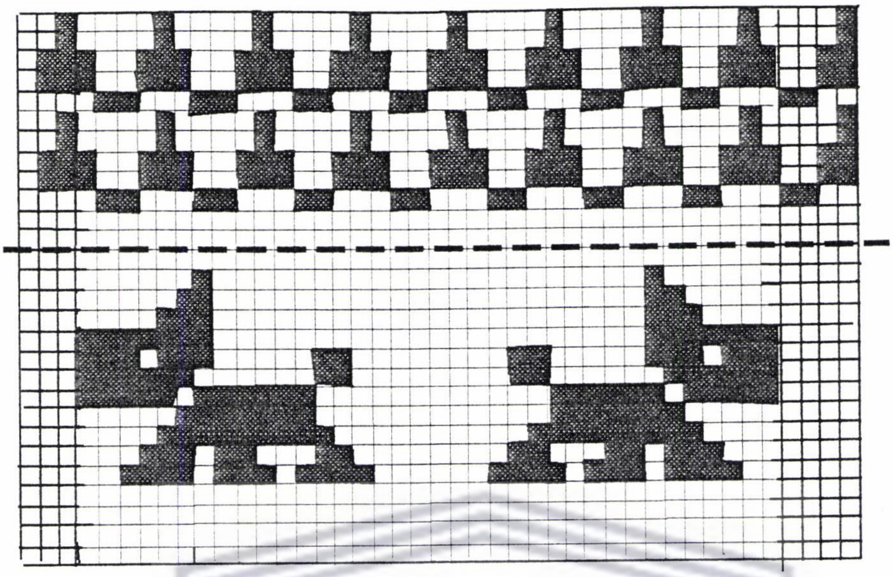
a. Use AB as the axis of symmetry and mirror image the diagram.

b. Use CD as the axis of symmetry and mirror image the sketch.

c. Take AB as the beginning of the diagram obtained in (b) and repeat the sketch twice more.

The letter 'O' is to be cut out on perspex for a sign board. The letter must be constructed with precision and only a quarter of it is provided. Use symmetry and the two dotted lines to make a template of the letter. Give full details how you made the template . 
The accompanying diagram shows a design punched out for machine knitting. The broken line divides the design into two sections. Study the design carefully and then answer the questions that follow : a. How many horizontal lines can be drawn in the top section to create symmetrical sections. b. Determine the number of vertical axes of symmetry in the top section. c. Find the number of horizontal axes of symmetry in the bottom section.

d. Describe where the vertical line in the bottom section must be drawn so that the dog on the left is a mirror image of the one on the right.



APPENDIX D

WORKSHOP PARTICIPANT QUESTIONNAIRE

About the lessons on SYMMETRY and QUADRILATERALS reply on the following questions:

1. Do you think the presentation of the learning material will stimulate the pupil to greater mathematics activity?

____

____

____

____

2. How successful are the lessons in shedding the notion that mathematics is abstract and meant for intelligent people?

____

____

____

____

____

3. What is your opinion about the attempt to integrate mathematics with other school subjects?

____

____

____

____

____

4. Does this approach help to narrow the gap between knowing mathematics and using mathematics?

5. How successful was this approach in extracting mathematics from concrete situations?

6. Any other comments?