AN INVESTIGATION INTO THE USE OF A COMPUTER ALGEBRA SYSTEM FOR THE TEACHING OF INTRODUCTORY SCHOOL CALCULUS

OSMOND MONDE MBEKWA

A minithesis submitted in partial fulfilment for the degree M Phil in the Department of Didactics, University of the Western Cape.

Supervisor: Professor C Julie

January 1995

DECLARATION

I declare that AN INVESTIGATION INTO THE USE OF A COMPUTER ALGEBRA SYSTEM IN THE TEACHING OF INTRODUCTORY SCHOOL CALCULUS is my own work, that it has not been submitted before for any degree or examination in any other university, and that all the sources I have used or quoted have been indicated and acknowledged by complete references.

OSMOND MONDE MBEKWA DATE: JANUARY 1995.

SIGNED: ....

ACKNOWLEDGEMENTS

I wish to express sincere thanks to Prof. Cyril Julie, who motivated me and acted as my supervisor for this study, for his patience, encouragement, guidance and professional assistance in the preparation of this work.

Special thanks are due to Mr Patrick Nqolobe and Mr Matiso, the principal and the mathematics head of department and Std 9 students of Mfuleni Combined School for providing me with the opportunity to implement the lesson experiment in their school. Special thanks also go to my colleagues Larry Kannemeyer and Jerome Erentzen, whose assistance and participant observation were invaluable to the conclusion of this study.

Last, but definitely not least, I am grateful to my dear wife Nokwanda, for her support and encouragement and my sons and daughter for understanding.

TABLE OF CONTENTS

CHAPTER 1: THE CONTEXT AND RESEARCH QUESTION

1.1 INTRODUCTION

Mathematics education in South Africa is in a state of crisis. One manifestation of this crisis is that students in the former Department of Education and Training (DET), which catered for blacks in the former Apartheid government, fail dismally in the final matriculation examinations as compared to students in other departments. Whilst the process of amalgamating the different departments of education in the new dispensation is underway, this state of affairs remains. The poor matriculation results must be seen in the socio-historical context of Apartheid education which was introduced when the National Party assumed power in 1948. The Bantu Education Act of 1953 was promulgated, as part of the overall policy of apartheid, to ensure that blacks received a separate and inferior education. Verwoerd himself, the infamous architect of Apartheid, makes this crystal clear when he states that through Bantu Education:

> The Bantu must be guided to serve his own community in all its respects. There is no place for him in the European community above the level of certain forms of labour. (Christie & Collins, 1984:173).

Taken in this context one can deduce that Bantu Education was an instrument to legitimate the socio-political and economic restructuring of society to protect the selfish interests of whites in general and Afrikaners in particular.

Consequently, throughout the reign of the National Party, black matriculation results have remained atrocious as compared to those of other racial groups, in particular the white group. Whilst one may lay the blame squarely at the obnoxious policies of Apartheid, the educational struggles of the seventies and the eighties, which were characterised by school boycotts, compounded the problem. It is logical therefore that the picture of mathematics results would mirror the national matriculation results. The following table gives a comparative picture of national matriculation results for the different population groups from 1991 to 1993.

NATIONAL MATRICULATION RESULTS

	African	Coloured	Indian	White
1991. Candidates:	306480	22405	14258	65933
%Pass:	39	83	95	96
%Exemption:	10	22	50	42
1992 Candidates:	342848	24430	14485	66141
%Pass:	42	86	95	98
%Exemption:	10	21	49	42
1993 Candidates:	366241	25735	15203	63769
%Pass:	39,1	85,8	92,8	94,5
%Exemption:	7,9	21	45,1	41,7

Note: The percentage of passes also includes exemptions.
Source: Edusource, 1991, 1992, 1993.

The above table shows glaring disparities in matriculation results of black scholars as compared to other population groups. Without doing any statistical analysis of the above data, common sense would lead one to infer that mathematics results would closely correspond to the above results. This is indeed the case as the following table of mathematics results of the different education departments for the same period indicates. The results are differentiated into the higher and standard grades for 1991, and 1993 includes the conversion of passes from the higher to the standard grade and from the standard to the lower grade.

NATIONAL MATHEMATICS RESULTS

	African	Coloured	Indian	White
1991. Candidates:	10519	1127	3436	15399
%Pass HG:	10	63	80	89
Candidates:	13987	8804	6372	26695
%Pass SG:	13	65	68	90
1993.				
HG Candidates:	39305	1011	3698	14291
%Pass:	11	65	74	89
%Pass SG:	13	21	19	9
SG Candidates:	55861	9973	7231	26331
%Pass:	18	68	59	89
%Pass LG:	8	10	14	6

Source: Edusource.

One can also observe from the above data the gross disparities that exist in the mathematics results of black children as compared to other racial groups. Considering that blacks constitute approximately 75% of the total South African population, this is cause for concern. Also, if one views mathematics education as preparing students for participation in industry and commerce, then these results do not give them good prospects for professions in industry and commerce. This scenario is compounded by the fact that approximately 50% of black pupils drop out of school before standard three (Kahn, 1993:13). According to Laridon (1993:40):

> This has an immediate consequence in the illiteracy and innumeracy rate for adults approaching 65% in the black population and being over 50% of the population as a whole.

From the discussion above, one can deduce that one glaring and direct consequence of Bantu Education is that half the population is illiterate and innumerate. Mathematics education in particular is beset by a great shortage of qualified teachers and an increasing decline in the number of students who take mathematics as an option in their school curriculum. This is because mathematics is not a compulsory subject after standard seven in South Africa. Du Plessis et al., (1990) state that only approximately 28% of black children take mathematics in standard ten.

1.2 REFLECTION

Reflecting on the above discussion, I feel that part of the blame for this problem of poor mathematics results can be laid at the door of the policy of racial segregation, discrimination and economic deprivation of black people. But be that as it may, the historic and democratic South African elections of April 1994 have placed a new government of national unity at the helm which, it is hoped, will see to the eradication of past iniquities in all spheres of life and particularly in education.

Whilst moves are afoot to change education policy, other factors are at play which, if not immediately remedied, will tend to perpetuate the status quo. These are pedagogical issues, in

particular the curriculum which is still designed by experts chosen by the state. These experts, in turn, take the curriculum to syllabus committees who then draft the syllabus which is subsequently submitted to the appropriate education department. Teachers at school receive a completed syllabus which instructs them what to teach, when to teach and how to teach it. In this way teachers are alienated from their own work because they have not been part of the development of the curriculum and the syllabus which they have to adhere to (Julie, 1991:4-5).
From the inception of the curriculum to its implementation, one finds an authoritarian thread. In the classroom, students are taught mathematics as something that has been brought by the teacher and all that they have to do is to swallow and then regurgitate the mathematical knowledge in the examination room. This is the Freirean notion of banking education. This knowledge is taken as a given that is indubitable and unchallengeable. This approach to teaching produces students who are uninquiring, uncreative and apathetic. Hence alienated teachers produce alienated students.

Perhaps the retrogression in the mathematics results of students arises from a lack of self-esteem. This lack of self-esteem might in turn be a consequence of repeated failure which reduces motivation in students for good performance in mathematics. Lack of motivation, therefore, lowers performance. Larcombe (1985:7) states that once a student's performance has deteriorated, it further lowers motivation and thus creates a vicious cycle or a self-fulfilling prophesy. One can thus say that a student's performance in mathematics is a function of motivation or the student's positive feelings towards the subject. Hence to facilitate performance, the teacher should create such an environment as would boost the student's confidence and hence his/her self-esteem. Hjelle & Ziegler (1976:260) state that self-esteem contains:

> desire for competence, confidence, personal strength, adequacy, achievement, independence and freedom. An individual needs to know that he or she is worthwhile - capable of mastering tasks and challenges in life... in this case individuals need to be appreciated for what they can do; i.e. they must experience feelings of worth because their competence is recognised and valued by others.

I contend that confidence, worth and the competence of students is what teachers should promote if students are to do well in mathematics. One way of boosting the students' self-worth and motivation is the creation of a mathematics environment that mirrors social situations where students communicate with one another their understanding of mathematical concepts. The classroom situation should be one where equality and mutual respect for students' conceptions of mathematical ideas are promoted.

1.3 THE ROLE OF THE CURRICULUM

It is in the context of low achievement by students in mathematics that I think mathematics educators have proposed a revisiting of the mathematics curriculum with a view to its revision. This has also been part of the struggle to transform the education system as part of the struggle to transform society as a whole. The technological revolution in the world as a whole and in South Africa has necessitated a need to change the way mathematics is taught an hence the mathematics curriculum. The advancement in the field of technology has created a vast wealth of information. If education is going to stick to antiquated methods of information processing then the speed at which new information is created is going to leave us very far behind. Consequently educators are compelled to look at the new situation to see how to change the curriculum and teaching methodologies in the light of the available technology. Glencross (1991:8) describes the scenario as follows:

> This information explosion means that whereas the skill of memorising information used to be important in the past, in the future it will be far more critical to locate and utilize information in order to solve problems. Thus it is vital that the aims and objectives of any mathematics curriculum should be broadened and redefined in order to reflect the needs of society, the new knowledge available and recent advances in information technology.

Glencross then concurs with the idea that curriculum innovation and teaching methodology must meet the need of a changing society. Hence to ignore the computer revolution would be gross negligence on the part of mathematics educators. Students have to be prepared to be able to use the new technology to solve problems in the classroom and in real life situations. The new curriculum should be of such a nature that exploration of the mathematics environment is encouraged and that students have to move away from the memorization of meaningless information. Volmink (1993:34) states that in the new curriculum:

> The authoritarian, algorithmic rote, approach is de-emphasised and in its place comes a more exploratory approach in which children can construct their own way of looking at mathematics within a social context.

From the above statement, one would see the new curriculum as fostering social intercourse and interaction in the learning environment. This interaction would then involve communication, negotiation and consensual interpretation of mathematical ideas amongst peers and between the students and the teacher. It is in the context of the state of mathematics education and the proposals for a new curriculum and teaching methodology in the light of computer technology that this study is undertaken.

1.4 THE RESEARCH QUESTION

This study is concerned with the application of technology in the learning of mathematics in secondary schools. It is concerned with the response of students in a mathematics classroom in which computer technology is used for learning. In particular, the aim of the research is to investigate the use of computer algebra systems in the teaching and learning of an introductory school calculus course.

1.5 THE RATIONALE FOR THE RESEARCH

This research arises out of interest in the advancements in the field of technology and in particular the proliferation of computers with advanced programmes capable of doing complex algebraic manipulations. Technological innovations will radically affect the way that mathematics is taught and the way it is learnt. We find that nowadays computers are part of the world of work and the home and are increasingly becoming part of the teaching and learning environment especially in the First World. Although computer technology has not as yet become part of the day to day routine of learning, computers are increasingly being used in some schools and more so at university and other tertiary institutions. One can expect that in the near future, computers will be part of the classroom situation. It has thus become important for teachers of mathematics to investigate and become acquainted with available computer packages and see how they can serve as tools for the learning of mathematics.

1.6 COMPUTER USE IN MATHEMATICS

Computer learning programmes according Marsh (1989:21) are categorised into three types namely drill and practice, computer-aided instruction (CAI) and intelligent computer-aided instruction (ICAI).

1.6.1 *Drill and practice*

These programmes are used to assist students in rote learning of subject matter that has already been done at school. Students are given tasks of increasing difficulty to test their knowledge of work that has been done at school. The computer gives the student feedback on progress or competence in a particular area of work. An example of such a programme is SERGO which is used in South Africa to cover the mathematics syllabus up to standard 10.

1.6.2 *Computer-aided instruction*

This category of programmes is an advance on the SERGO-type of programme because in addition to the drill and practice facility, it also gives instruction on new information. This implies that a student can be engaged in a new mathematics topic and when she/he feels that she/he is ready to take the exercise, then she/he can do so. After finishing she/he can then proceed to another section of the work.

An example of such programmes are those on the PLATO system which have been used extensively at the University of the Western Cape. For quite a number of years the Goldfields outreach programme of the university has assisted secondary schools from the disadvantaged communities to improve their mathematics and science results. In addition university students have also used this system extensively.

1.6.3 *Intelligent computer-aided instruction*

Intelligent computer-aided instruction differs significantly from the two mentioned above. Whilst the drill and practice and the CAI packages assist the learner in revising already taught work with the CAI also giving instruction, in the ICAI programmes mathematical problems of increasing degree of difficulty can be solved. Whilst this occurs, students can be observed and the teacher can keep track of the student's progress as she/he works through the programme. The most advanced are also capable of graphic representation of functions. Computer Algebra Systems (CAS) like DERIVE fall under this group of programmes. The are capable of solving complex mathematical problems and it is left to the learner to reflect on the solution process and to conceptualise how the solution was arrived at. This is unlike the CAI which gives the student assistance by explaining how the solution was reached.

It is with respect to the ICAI systems and in particular computer algebra systems that this study is undertaken. The motivation for the research is consonant with the idea that:

> Teachers should use computers as tools to assist students with the exploration and discovery of concepts, with the transition from concrete experiences to abstract mathematical ideas, with the practice of skills, and the process of problem solving (NCTM:1987).

1.7 THE STRUCTURE OF THE MINI-THESIS

This mini-thesis has commenced in this chapter with a brief discussion of the state of mathematics education in South Africa

the consequential proposals for curriculum change especially pertaining to developments in the field of computer technology and its application to the mathematics classroom.
Chapter 2 elaborates on the background to the development of computer technology and its movement into the mathematics classroom. It elaborates on the nature of computer algebra systems and the surrounding debates on the desirability of these systems for mathematics education.

An exposition of the theoretical framework on which this study is based namely the constructivist theory of knowledge acquisition, is given in chapter 3. Constructivism is the view that knowledge is a human construction and thus does not exist sui generis.

In chapter 4, the research methodology is discussed. The methodology of interest is developmental research. This kind of research is similar to action research which studies practice as performed by practitioners in order to improve or change the practice for the benefit of both the practice and the practitioner. Whilst action research is carried out by practitioners themselves, in developmental research the practitioner and the researcher need not be the same person. This chapter also discusses particular data gathering techniques which have been employed in the research.

Chapter 5 is a discussion of a palmtop familiarization exercise that was conducted with a class of standard nine students at

Mfuleni Combined School at Mfuleni, one of several black townships in the Cape Peninsula. The computer programme with which the students were being familiarized is the CAS programme DERIVE on the palmtop computer HP95LX. The students were being familiarized with the manipulation of DERIVE to do algebraic solutions and graphical representation of simple functions. Discussion in this chapter includes my own familiarization with the computer.

Chapter 6 constitutes the core of the research. It is a case study of the implementation of the CAS programme DERIVE to introduce the basic procedures for the differentiation of power and polynomial functions. The chapter discusses how students used the programme to derive the basic rules of differentiation.

Chapter 7 is an evaluation of the whole research exercise, linking the theoretical framework and the research question to the implementation and the results of the data gathering exercise. Weighing these against one another, a conclusion is made with recommendations on these and further research proposals.

CHAPTER 2: THE HISTORY AND DEBATES ON COMPUTER TECHNOLOGY

2.1 INTRODUCTION

The aforegoing chapter highlights the rationale for this study as the appreciation of the advancements in the field of computer technology which has permeated modern society especially the First World. As such this is bound to affect the present mathematics curriculum if we have to meet the challenges of a changing society. Mathematics educators are thus seen as revisiting the mathematics curriculum with a view to locating it within the changing technology.

With regard to computer technology, it is stated that three main categories of computer use in education can be identified. These are the drill-and-practice programmes, computer-aided instruction (CAI) and intelligent computer-aided instruction (ICAI). The fist two types normally focus on revision of work already completed at school with the ICAI an advance on the CAI because of its capability of giving instruction on new work and keeping track of students' solution paths to problems, map these to expert solution strategies and give appropriate feedback. It is in respect of the ICAI that this study is undertaken because it is within the ICAI framework that computer algebra systems (CAS) can be located. This type of programme provides a mathematical environment which makes it possible for students to construct their own mathematical understanding.

This chapter elaborates on computer algebra systems and discusses the debates surrounding the introduction of computer algebra systems into the mathematics classroom.

2.2 *COMPUTER ALGEBRA SYSTEMS*

The term computer algebra systems, or computer mathematical systems refers to advanced computer programmes that solve algebraic problems for the mathematics student or for the mathematician. In addition, these programmes can represent functions graphically in two or three-dimensional space. Bollinger (1989:11) defines CAS as:

> Computer software packages which give the exact solution of mathematical problems in symbolic form in contrast to numerical approximations in conventional computer use. These interactive systems allow the user to define an expression, apply an operation and manipulate the output symbolically.

To explain what this means, if one wants to solve, for example, the quadratic equation $ax^2 + bx + c = 0$, then one would simply enter the equation into the computer and instruct it to solve for x in the equation. The computer would, in a few seconds, give the solution as $x = \frac{-b+\sqrt{b^2-4ac}}{2a}$ or x = $x = \frac{-b-\sqrt{b^2-4ac}}{2a}$. This is what is meant by an algebraic solution. In contrast, a conventional computer would not be able to give such a solution. It would need to have exact values for a, b and c to give exact values of x or numeric approximations of the solution of x.

2.3 *A BRIEF HISTORICAL BACKGROUND*

According to Hillel, e.a. (1992:119), CAS were developed in 1953 as a tool for the solution of problems in physics and chemistry in industry. Hence CAS were not initially conceived as educational tools. They appeared in the eighties firstly on main frame computers and later became adapted for personal or microcomputers. One can compare the development of CAS to the dvelopment of the electronic calculator. At the beginning of the century mechanical desktop calculators were used in industry (Bollinger, 1989:13). Later, four function calculators and scientific calculators became used generally in commerce and in schools although calculators were not allowed in examinations. It is later in the 1980's that electronic calculators were allowed in the examination room. For instance, the former Department of Education and Training (DET) allowed calculators into the standard eight mathematics examination room only in 1987. The permission was extended to the upper standards in the following two years (DET SYLLABUS, 1987, 1988, 1989). Permission to use the calculator does not extend to the programmable calculators. Education departments are still discussing the use of the calculator even in the 1990's as exemplified by Laridon (1993:41) when he states that:

> Proposals emanating from the Department of Education and Culture House of Assembly (DEC-HOA) see calculators being used from junior primary through to standard 10 in the not too distant future.

It is in the climate of the proliferation of electronic calculators and the debates whether or not the calculator is desirable in the mathematics classroom, that the graphic calculator and CAS came to the scene.

Graphic calculators are an adavnce on the scientific calculator by being able to sketch graphs of functions. CAS on the other hand have, in addition to calculator and graphic abilities, a feature to do algebraic manipulation and the solution of functions of tremendous complexity.

As mentioned at the beginning of this discussion, the earlier CAS were designed for mainframe or minicomputers. Some of the more popular ones were MACSYMA, REDUCE, SMP and were designed to solve problems relating to specific branches of physics (Bollinger, 1989:11). With the advent of micro or palmtop computers, there has been a modification in these programmes. The first such modified CAS for the educational setting was muMath. According to Bollinger (1989:11) muMath was designed for the mathematics classroom. CAS today have become more sophisticated and can do complex algebraic operations and the construction of graphical representation in two and three-dimensional space. Some of the more recent and popular systems are DERIVE, the Mathematics Exploration Toolkit, Mathematica, Milo and Theorist. These are compatible with IBM or MacIntosh computers (Dickey, 1990:106). These programmes can be used with desktop, laptop and palmtop computers. For this study the programme DERIVE was used. Derive is the only CAS that can currently be used on the palmtop computer HP95LX which was used in this study.

2.4 THE PROGRAMME DERIVE

DERIVE is a very powerful programme. It has the capability of doing all the algebraic computations, that in the past, students had to contend with from the basic algebra of the junior secondary to postgraduate mathematics. As Arney & Giordano (1992:142) put it:

> It can find derivatives, definite and indefinite intergrals, graphs in two and three-dimensions, manipulates matrics, determines Taylor polynomials and solves systems of equations, it does all the traditionally onerous computational tasks faced by a calculus student.

The power of DERIVE lies in it being able to solve algebraic equations and the sketching of functions without explaining to the student how the solutions have been obtained. It is the student who has to figure out how the solutions have been arrived at by looking at patterns emerging from the answers. The benefits of using DERIVE are stated by De Marois (1992:Preface) as follows:

- Use technology to remove computational drudgery so that students can concentrate on ideas rather than mechanisms.
- Present students with a series of guided experimental activities that result in their intuitive discovery of key concepts.
- Help students to make mathematics theirs through

active engagement in the construction of key ideas.

- Broaden student's ability to visualize mathematics ideas.

From these recommendations four key issues emerge. The first is that DERIVE provides an environment for experimentation. Secondly, students construct their own concepts from these experiments. Thirdly, the formation of concepts by students enhances the notion of creating knowledge that they can claim as their own. Such a conception would therefore create a powerful sense of self confidence in the students. Lastly, because of DERIVE'S multi-representational strategy i.e. the representation of mathematical ideas both in algebraic symbolic and graphic form, students are able to see the connection between the symbolic and the visual form of mathematical notions. Students are also able to appreciate that a single mathematical idea can be conceived from a multiplicity of representations.

2.5 MULTIPLE REPRESENTATIONS

Many mathematics educators endorse the idea that a multi-representational strategy promotes the learning of mathematics. They posit that students should be able to see the connection between the symbolic and the visual form. Any strategy therefore that enables the student to see this connection would presumably enhance the student's understanding of concepts. The move from a symbolic to a visual medium is critical for the student's appreciation of representation. In this regard Vonder Embse

(1990:90), states:

> To visualize an object, relationship, problem situation or concept means to form a mental image or images in such a way that the construct becomes understandable.

Embse argues that in order that students can better understand and create better constructions, they should make the necessary connections between the various forms of representation namely the symbolic, the numeric and the graphical or geometric representation. This implies that for instance, if a computer programme is able to sketch a graph of a function defined by $y = f(x)$, then a student will be able to see the relationship between the symbolic form of $y = f(x)$ and its graphical form. Also, for instance, if it is required that the roots of the function be determined, it would be immediately determined from the graphical form whether the function does have roots or not. Waits & Demana (1990:38) state that:

> The easy availability of geometric representation gives the students and teachers the opportunity to explore/exploit the connection between algebraic and geometric representation.

Without delving deep into the debate about the position of visualization relative to symbolic representation in the solution of mathematical problems, one finds strong arguments for the use of visualization in addition to the symbolic in mathematics education. Among others, Julie (1993:344) sees the relationship between the symbolic and visual representation as cyclic and that these two representations must be accorded an equal status. This proposal is against those who see the algebraic symbolic representation as being the only correct way to do mathematics. This view holds that visualization should only serve to confirm the results of an algebraic manipulation. One such confirmist of the symbolic is Polya (1957:Preface), noted as a "verbal" type of mathematician, who states that in the solution of a problem:

> First. You have to understand the problem... Draw a figure. Introduce suitable notation... Even if your problem is not a problem of geometry, you may try to draw a figure. To find a lucid geometric representation for a non-geometric problem could be an important step towards the solution. (Preface to 2nd edition).

Julie (1990:344) puts it that these result-confirmation arguments:

> Project the graphical / visual as a sort of bolstering mechanism for the symbolic / algebraic and so devalues the graphic / visual. Is this a case of a "manipulation-skill" mentality that refuses to accept other ways of working as legitimate? I believe it is and that any mathematics curriculum reconstruction effort ought to seriously consider the equality of ways of working, albeit the equality of the graphic/visual and the symbolic/algebraic.

From the above discussion one would deduce that visualization plays an important role in learning mathematics. The power of visualization can be seen in the Chaos theory of Mandelbrot, Tukey's methodology of data analysis which uses the number crunching and graphic ability of the computer (Laridon, 1993:352). One would see the application of a multi-representational strategy in mathematics as promoting the process of concept formation. Students, through the connections they make between the symbolic and the visual, would be able to create a new understanding of the dynamics of mathematics. This, therefore, would promote a constructivist way of learning.

2.6 *DEBATES ON THE DESIRABILITY OF CAS*

The advent of CAS and their encroachment into the classroom domain has generated intense debate. This debate goes along the same lines as that which occurred when the electronic calculator was introduced into the mathematics classroom. Generally the argument can be categorized into two broad streams, namely a pedagogical argument and a socio-economic argument. The pedagogical argument focuses on the educational outcome of the introduction of computers in schools. The question is whether the computer fosters learning or whether it stultifies it. The pending redundancy of the teacher also comes into this argument. The socio-economic argument looks at social problems of inequality that would further disadvantage those students belonging to the lower strata of society and whether or not the education authorities would be able to foot the bill of introducing computers in the classroom.

The mathematics education fraternity is split between those who advocate the use of CAS in the classroom and those against it. Advocates of the use of CAS posit that CAS enhances the development of conceptual insight.

The argument is mainly based on the observation that CAS can speedily solve and complete a large array of mathematical problems and thus save more time for the student to reflect on the solution process and deduce relevant concepts. (Bollinger, 1989:15). Notice is taken, however, of other variables that contribute to the effectiveness of learning, for example, the teacher, the type of student, the type of pedagogy and the kinds of problems that are given. Hillel et. al. (1992:124) in explaining the conceptualization and flexibility aspects of CAS, state that:

> (i) CAS make it possible to change the type and complexity of problems which students typically are asked to solve. Students may be given problems with more realistic data and which are not always amenable to analytical solutions. Since students don't need to do the actual manipulation, they can focus on planning and the interpretation of results.
>
> (ii) CAS allow, for a shift in emphasis away from learning techniques...towards trying to build better conceptual understanding.

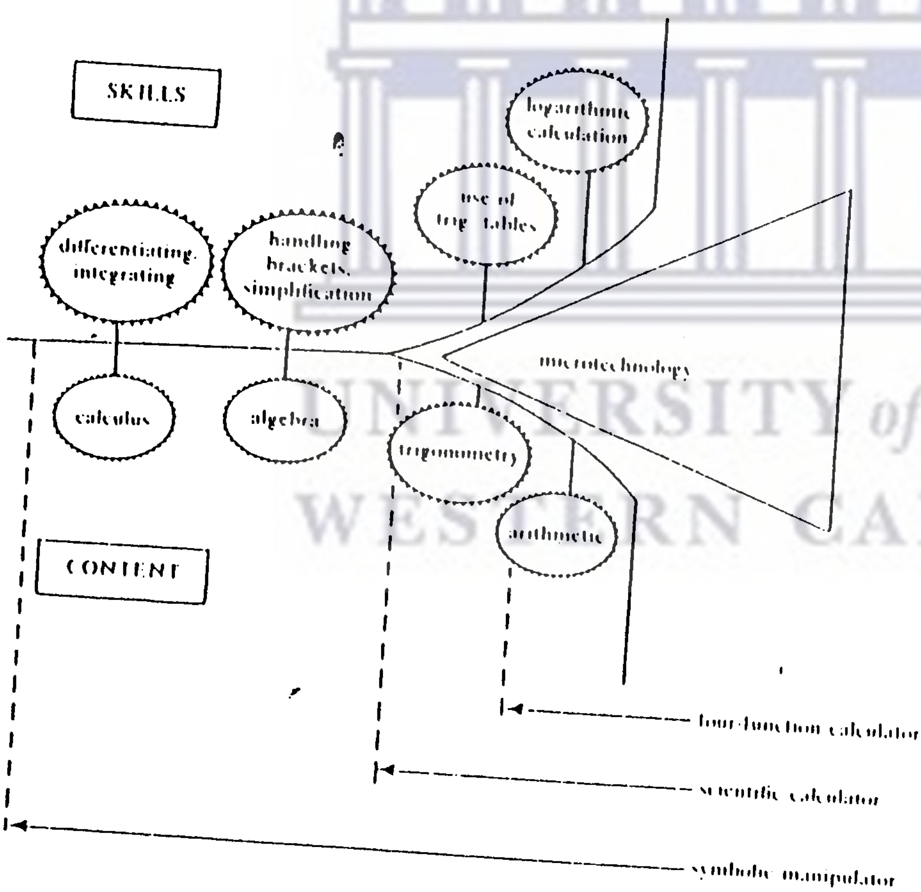
(iii) With CAS, students have the opportunity to try many examples and to receive immediate feedback. This can be exploited to foster a more experimental approach to learning mathematics, one which encourages students to search for patterns to anticipate results, to manipulate mathematical objects, and to employ mathematical processes.

The essence of the above quotation is that CAS promote the kind of environment that allows students to explore, experiment and interpret results and thus create their own knowledge about mathematical objects. It is an environment which transforms the traditional instructional role of the teacher. She/he becomes a facilitator rather than an instructor. She/he facilitates the interaction between the student and the computer programme. Once students gain confidence in the use of CAS, they seek less help from the teacher. This is corroborated by Julie (1991:108) in an experiment to investigate the learning of concepts by pre-service teachers in a computer environment. He states that:

> Initially students were uncomfortable... They wanted to know whether what they were doing was "correct". However, as they gained more experience the security-seeking behaviour disappeared.

The ability of CAS to carry out routine algebraic algorithms that has always been the preserve of humans in the past has raised questions as to whether computer technology is not rendering paper and pen computations redundant. Bibby (1991:42) sees micro-computer technology acting as a wedge between the content of mathematics and the traditional paper and pen computational algorithms. He illustrates this idea as depicted in the diagram below:

THE WEDGING EFFECT



(From Bibby (1991))

The idea of microcomputer technology acting as a wedge implies the separation of the mathematics content and the traditional mathematical calculation skills through the performance of the calculation by the microcomputer. Computer technology is now able to do very simple or complex algebraic calculations. The dilemma posed on the mathematics teacher is whether to pursue the learning of manipulation skills or whether to let the technology do the algorithmic manipulation for the student so the she/he can concentrate on concept building. Bibby (1991:43) puts it thus:

> ...if we can automate an algorithm, what does it imply for the teaching and learning of the equivalent "manual" algorithm? Most of the traditional manual algorithms (from "long multiplication" to "integration of x") are efficient but obscure: a pedagogical dilemma for many years has been to what extent one can reconcile the development of good plug-and-chug technique (i.e. the ability to apply an algorithm appropriately and reliably - "how") with the development of insight (i.e. the ability to appreciate exactly what makes the algorithm work - the "why").

With regard to the "how" and "why" mathematics, Bibby (1991:43) employs the Skempian notion of instrumental understanding and relational understanding. Instrumental understanding refers to the "how" of mathematics, that is the understanding of the procedure to be followed in solving a mathematical problem. Relational understanding on the other hand seeks to understand the "why" which refers to why an algorithm works. One can refer to instrumental understanding as algorithmic knowledge and relational understanding as conceptual knowledge. The process of using the computer to solve problems Bibby terms "black-boxing." The converse of "black-boxing," that is, taking an algorithm and checking why it works, he calls "unpacking." The "unpacking" of the black-boxed procedure, implies the unravelling of the "mystery" of what happens in the computer during its algorithmic manipulation. The unravelling leads to relational understanding or conceptual knowledge. Bibby proposes a balance between relational and instrumental understanding. He states that in the past there has been an over-emphasis on algorithmic operations at the expense of concept formation.

Proponents of the use of CAS prefer relational understanding as a priority to instrumental understanding whilst the opposition views instrumental understanding as making better mathematicians. The statement that the anti-CAS lobby makes is that computer mathematicians (if they can be called that) would in future be so dependent on the computer that they would not be able to do simple mathematics. Proponents of CAS education state that in any case, the technology already exists and as such it would be gross irresponsibility for mathematics educators to ignore what potentially can advance mathematical knowledge (Tanner, 1992:191). What is crucial is to exploit the technology to enhance our own understanding and an appreciation of the intricasies of mathematics. This implies then that CAS can serve the function of the appreciation of mathematics for its own sake and also as a tool for future researchers.

The anti-CAS lobby also puts forward strong arguments against their use in schools. The first argument is on the question of accessibility and expense. They state that the use of CAS needs expensive laboratories or pocket computers and these will not be easy to acquire for students to use on a regular basis (Waits & Demana, 1992:180). They also state that even if schools could obtain these computers for the school situation, the problem of disadvantaged students who cannot acquire a pocket computer to do out-of-school work, still remains. The proponents of the use of CAS counter this argument by saying that there will always be those students who are capable of obtaining the technology because of their superior economic bakcground and it is for this very same reason that there needs to be a struggle to obtain pocket computers for every student. It is actually to close the gap between those that have and those that do not have. With regard to this accessibility argument, Julie, (1991:124) makes the analogy that if the majority of people desire but cannot afford to buy Mercedes Benz cars it does not stop those who can afford them from buying and using them. Furthermore, mathematics is one of the least subsidized subjects at school. For mathematics, the government mostly supplies only books and chalkboard instruments. Subsidizing mathematics through the provision of computer laboratories or pocket computers would no more than bring it on par with other subjects like biology and science. The pedagogical argument given by opponents of the use of CAS in the classroom is that CAS solve problems for the student without hinting how the problem is solved. This is the point that pro-CAS mathematics educators are making! They argue that the computer must churn out the answers so that the student's role is to think about the process of solution.

The multi-representational capacity of CAS would assist the student on conceptualizing on the solution process. The whole aim of CAS is, therefore, not the reduction in the power of the intellect but to strengthen it. Atiyah (1986:46) bemoaning the solution of great problems in mathematics like the four colour problem, by computers, as stealing the thunder from the intellectual process, asks:

> Is this to be the future? Will more and more problems be solved by brute force? If this is indeed what is in store for us, should we be concerned at the decline of human intellectual activity this represents, or is that simply an archaic viewpoint which must give before the forces of "progress"?

Atiyah further postulates that if computers had been available, for example, in the fifteenth century, then mathematics would be a pale shadow of itself by now. One should point out, as a counter to this thinking, that the development of implements or tools in the development of humankind is an indication of the development of the human's intellectual capacity. Hence the development of computers as a tool in human learning, is an indication of the further sophistication in the development of the human intellect. In answer to Atiyah, therefore, one would be inclined to say that if computers had been developed in the 15th century then we would have reached the stars by now! As an answer to the anti-CAS lobby regarding the concern at the potential loss of technical skills as a result of computer technology, one can refer to the recent period of the application of electronic calculators in the mathematics classroom. What has been observed is that students have not lost their mathematical skills but rather, the calculator has assisted students to focus on rules and principles whithout bogging them down in laborious manual algorithms.

2.7 *CAS AND THE TEACHER'S ROLE*

The advent of the computer in the classroom is bound to change the relationship between the student and the teacher. This is due to the fact that the pedagogical interaction will increasingly be between the student and the machine. Perhaps the student will begin to develop more autonomous behavioural patterns through his/her relationship with the computer. The teacher's role would be to initiate the interaction between student and machine. The teacher then would observe and react to how the learning process occurs in this new environment.

One would thus tend to see the teacher as both a student and researcher in the new situation because firstly she/he has to gain competency in the new technology before she/he can familiarize the student with the new technology. In this way the teacher acts as a student and as a facilitator. She/he becomes a researcher in so far as she/he studies the learning process in the new environment and the exploration of new domains of enquiry by the student. Having gained the competence and confidence of handling a computer dominated environment, the teacher has to accept his/her new role as a facilitator of learning rather than as the informatiasive figure of the past.

Also, because technology is constantly developing and changing, the teacher will constantly be under pressure to learn new skills and to adapt to new programmes. Hence computer technology will force teachers to be constantly in inservice training as mathematics education researchers. (ICMI:27-28)

2.8 CONCLUSION

This chapter has given the background to the development of CAS and their proposed use in the mathematics classroom. The motivation for the use of CAS in the mathematics classroom is their perceived promotion of concept development through their de-emphasis in the traditional paper-and-pencil algorithmic manipulation of mathematical ideas. Furthermore CAS are seen as providing a multi-representational mathematical working environment which is conducive to a better understanding of mathematical concepts and the creation of new knowledge by students.

Debates around the use of CAS in the classroom have been discussed. It seems that the argument that computer technology exists and that therefore, it does not make sense to ignore it holds sway. What is needed is for mathematics educators to investigate ways through which CAS can be used as an effective pedagogical tool. Lastly, the discussion has highlighted the changing role of the teacher in a computerized mathematical environment. The teacher in this environment is no longer the dominant figure who, in the past, drove the lesson forward to a destination set by him/herself. The computer environment puts him/her in a less obstrusive position as a teacher and a more prominent one as a facilitator. His/her intervention will occur only when students require it. Thus as students gain mastery of the technology and thus more confidence, the teachers'role changes from that of instructor to facilitator and co-researcher.

CHAPTER 3: THE THEORETICAL FRAMEWORK

3.1 INTRODUCTION

The previous chapter gave an exposition of computer algebra systems. The discussion also focused on how these systems provided an appropriate mathematical environment in which students can create their own understanding of mathematical concepts. The notion of students creating their own knowledge is underpinned by a constructivist theory of knowledge acquisition.

Based on this understanding, the theoretical framework on which this study is based is the constructivist theory of knowledge acquisition. This chapter gives an exposition of the notion of constructivism and its implications for mathematics education. The discussion ends with an explanation why computer algebra systems can be considered as promoting a constructivist conception of mathematical knowledge acquisition.

3.2 AN EXPOSITION OF CONSTRUCTIVISM

The basic notion of a constructivist theory of knowledge is that all knowledge is constructed from our experience. According to constructivism, the human being cannot know anything that exists beyond his/her experiential reality. The experiential reality referred to is our immediate physical and social environment. Sensory information from the environment is the source of our experience.

Elaborating on this conception, Von Glasersfeld (1987:5) sees constructivism as:

> A second kind of knowledge apart from faith and dogma, a knowledge that fits observations. It is knowlege that human reason derives from experience. It does not represent a picture of the real world but provides stucture and organization to experience. As such it has an all important function: It enables us to solve experiential problems.

Confrey (1990:108) echoes this understanding of knowledge acquisition when she states that:

> Constructivism can be described as essentially a theory about the limits of human knowledge, a belief that all knowledge is necessarily a product of our own cognitive acts. We can have no direct or immediate knowledge of any external or objective reality. We construct our understanding through our experiences and the character of our experiences is influenced profoundly by our cognitive lenses.

From the above quotations, one can see constructivism as focussing on subjective understanding of reality. Hence, according to this conception, it is impossible to know anything about an objective external reality. An important feature of constructivism is that individuals respond to their environment through their senses. What is experienced through the senses is then organized in the person's existing mental frames.

If one looks at Confrey's formulation of constructivism, one does not fail to see its connection to cognitivism. Cognitivism derives from Piaget's cognitive psychology. This theory posits that every mental act is constructed on the basis of pre-existing mental structures. By cognitive stuctures is meant mental structures like thinking, memory and other information processing mechanisms. Learning, according to cognitivism, involves the processing of incoming stimulus information from which new knowledge, which fits a person's cognitive schemes, is constructed. Construction of new knowledge is an active process of learning. Even sensory processes like hearing and seeing which are usually regarded by people as passive processes are actually active processes of organizing information from the environment in order to make sense of it.

Constructivism does not focus on the construction of new knowledge only but is also a self-evaluative, inward-looking mechanism by which we seek to make sense or our own knowledge construction. This is what Von Glasersfeld (1987:11) refers to as reflection. This is the Lockean notion of the mind's ability to observe its own operations - knowledge that was not already available to the person generating it (Von Glasesrfeld, 1987:11). The generation of new knowledge comes through the interpretation of information. Interpretation involves sifting through many possiblities until one chooses one that fits one's schemata. In this way the choice that one makes is a rational one.

Whilst constructivists generally agree on the experiental basis of the construction of knowledge, there are differences of emphasis on constructivist principles. They, therefore, categorize themselves into different strands of thought within the general school of constructivism. There are three main variants of constructivism, naive, radical and social constructivism.

3.2.1 Naive Constructivism

Naive constructivism (trivial constructivism) is what Ernest (1993:168) refers to as information processing constructivism. Naive constructivism is common to all constructivists. Its basic assumption, according to Von Glasersfeld (1987:22) is that:

> Knowledge is not passively received either through the sense or by way of communication. Knowledge is actively built up by the cognizing subject.

This kind of constructivism (which might also be termed cognitivism) likens the mind to a computer which processes input data. The mind, like the computer, is responsible for the processing of environmental information through the application of various procedures in the organization and categorization of information. The successful processing of input data is dependent on stored information from previous experience (Ernest, 1993:169).

Hence, according to naive constructivism, knowledge is not simply acquired as a commoditized parcel or as an entity but is constructed through the organization of input data until the mind is able to make sense of it. This new understanding is what is known as new and viable knowledge.

3.2.2 Radical Constructivism

To explain radical constructivism means firstly to understand constructivism as an epistemology which is based on two principles. One of these principles is the one that underpins naive constructivism. This is the notion of knowledge as being actively constructed by the cognizing subject.

Radical constructivism, in addition to the first principle which underpins naive constructivism, embraces a second one which most other constructivists reject. This principle states that the process of knowing is a process of adaptation and not a discovery of reality that exists independently of the knower, Von Glasersfeld (1987:23) states that:

> 2a. The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability;
>
> b. Cognition serves the subject's organization of the experiential world, not the discovery of an objective ontological reality,

This second principle is what separates the radical constructivist from the naive constructivist. According to this adaptive principle the cognizing subject forms cognitive schemes which give guidance as to how the subject should interpret experience. Any new experiences will, therefore, be interpreted in terms of cognitive schemes from previous experience. Responses to new experiences will depend on how the new experiences fit with old experiences. If the new experience does not have exact "fit" with previous experience then it is either rejected or modified to suit the old experience.

Radical constructivism, in its rejection of an independently existing reality, proposes the abandonment of any search for objective truth. Knowledge, according to radical constructivism, exists only through cognitive construction. As the mind constructs knowledge, it negotiates with the world and internal cognitive frames to find the best possible fit and hence the best possible response to experiential situations. One can thus view radical constructivism as deriving from the Darwinian theory of evolution and the Piagetian notion of genetic epistemology. The Piagetian understanding of development, to which radical constructivism subscribes, is that cognition develops through adaptation to experience in the same way as organisms physically survive through adaptation to the environment. Radical constructivism sees the creation of new knowledge as possible through trial and error. A metaphor is made by Von Glasersfeld to explain the fit that new experience has to make with existing frames in the knower's mental structures. This metaphor is of a key which should not be taken as the image of a lock but one of many keys that can open a particular lock. Through trial and error one can find a key from a bunch which is able to fit and open the lock.

Hence radical constructivism views coming to know as a process of constructing knowledge through testing all alternatives at our disposal until we find the best possible solution. As Kilpatrick (1987:9) puts it:

> Out of the rubble of our failed hypotheses, we continually erect ever more elaborate conceptual structures to organize the world of our experience.

3.2.3 *Social Constructivism*

In the discussion on radical constructivism one sees the cognizing subject isolated in a closed world of his/her own. If cognition occurs in a privately constructed world, one cannot see how people would be able to communicate with one another because each individual has his/her private and unique understanding of his/her experiences. Radical constructivism has been criticised on this score. Social constructivism has arisen to address the shortcomings of radical constructivism.

According to social constructivist theory, the cognizing subject and his/her social milieu are indissolubly bound. The human being, as a social being, is in constant interaction and communication with other people. Social constructivism derives from sociological theories like symbolic interactionism and activity theory because of its emphasis on the role of language in the construction of meaning (Ernest, 1993:172). Communication occurs through the use of language and this is how learning occurs. Hence learning takes place in the context of society. The mind then constructs new knowledge not in a closed space but in a socially negotiated context. This implies that the meaning attached to an experience by a cognizing subject must be made to fit socially accepted assumptions.

If one analyses the discussion on social constructivism, one sees on the one hand the filling of the gap in radical constructivist's individual constructions which must then be negotiated with society. Secondly social constructivists speak of the cognizing subject as indissolubly bound to society and hence constructions occur in and with society. Ernest (1994:307) has pointed two forms of social constructivism to correspond with the two distinct trends discussed above. He terms the first social constructivism as having a Piagetian theory of mind and the other as having a Vygotskian theory of mind.

The first form derives from a radical constructivist framework with complementary social aspects. This implies an emphasis on the primacy of individual constructions which later become negotiated in social interaction. The Vygotskian form of social constructivism views the individual as indivisible from the social context. Hence any construction occurs contextually in social interaction through shared language and forms of life. Ernest (1994:309) says of the Vygotskian approach:

> This approach views individual subjects and the realm of the social as indisolubly interconnected, with human subjects formed through their interactions with each other (as well as by their individual processes) in the social context. These context are shared forms

of life and located in their shared language games.

The Vygotskian conception of the mind is of a social entity and thus conceptualisation is a social activity. Hence the development of the individual's mathematical knowledge comes from collective action, conversation and negotiation. This can clearly be observed in group problem solving.

Ernest (1993:172) sees social constructivism, because of its emphasis on dialogue and negotiations as lending itself to a democratic learning environment. He states that generally:

> Social constructivism naturally lends support to an emancipatory vision for mathematics education, one which embodies egalitarian values of social justice, and aims at a mathematically literate and empowered critical citizenry.

From the above discussion, one sees constructivism as an epistemology, that is, a discipline which is aimed at explaining how knowledge is acquired. Because of the constructivist emphasis on human construction, one deduces that the kind of knowledge that is sought is not certain nor is it objective. It is knowledge that depends on context. From this understanding, the connection between the way that knowledge is acquired and the nature of that knowlegde becomes apparent.

3.3 THE ABSOLUTIST-FALLIBILIST DEBATE

Constructivism arises out of philosophical debates on the nature of knowledge acquisition and thus also mathematical knowledge. One view of mathematics arising out of these philosophical debates is the absolutist understanding of mathematics. Absolutism holds that knowledge is an entity that exists independently of our existence. According to absolutism knowledge is therefore, unchangeable and unchallengeable. Hence mathematics, according to absolutism, is a paradigm of knowledge that is certain (Ernest, 1991:4).

Absolutist theorists propound that the way to expound truth is to systematically use logic, axioms, propositions and definitions. The implication for this view of mathematics is that no one can create new knowledge. It is knowledge of objective truth that exists out there that can be discovered. One variant of absolutism, Platonism, sees mathematics existing independently of human logic. David & Hersh (1968:318), explaining Platonic formulation state:

> Mathematical objects are real. Their existence is an objective fact quite independent of our knowledge of them. Infinite sets, uncountably infinite sets, infinite-dimensional manifolds, space filling curves - all members of the mathematical zoo are definite objects, with definite properties, some known, many unknown. These objects are of course, not physical or material. They exist outside the space and time of physical existence.

Hence according to absolutism, mathematics is an object that has an independent existence. I has only to be discovered and is not a human creation.

On the other hand is the fallibilist notion of mathematical knowledge which rejects the absolutist view of an objective existence, immutability and the certainty of mathematical knowledge. Fallibilism rejects the notion of a logically perfect mathematics. According to fallibilism mathematics is fallible and corrigible. According to Ernest (1991:18):

> The fallibilist thesis thus has two equivalent forms, one positive and one negative. The negative from concerns the rejection of absolutism: Mathematics is not absolute truth, and does not have absolute validity. The positive form is that mathematical knowledge is corrigible and perpetually open to revision.

Julie (1991:111) concurs with Ernest's fallibilist conception when he says:

> Mathematics is viewed as a product created by people. It is not seen as something that is "out there", waiting for the right moment and the right person through which it will be revealed. The Platonic view of the nature of mathematics is rejected.

From this quotation one deduces the conception of mathematical knowledge as a social construction. There are implications for the two opposing philosophical stances on the nature of mathematical knowledge. The implication of absolutism for the learning of mathematics is that no student can create new knowledge. Because mathematical knowledge exists independently of human logic, mathematical truth can only be discovered. Learning occurs by way of memorization of rules, formulate and pre-organized ways of proving. Students follow the example set by the teacher. On the other hand the fallibilist conception of knowledge promotes the seeking and construction of knowledge by the mathematics student in a social context. One can thus see the connection between fallibilism as a theory of mathematical knowledge and constructivism as an epistemological theory of the development of mathematical knowledge.

3.4 IMPLICATION OF CONSTRUCTIVISM FOR THE TEACHING AND LEARNING OF MATHEMATICS

A constructivist epistemology for mathematics education is that it allows the student to explore the mathematics environment. The teacher's role is to provide the mathematics environment, Confrey (1990:110) states:

> When I teach mathematics I am not telling students about the mathematical structures which underlie objects in the world, I am teaching them how to develop their cognition, how to see the world through a set of quantitative lenses which I believe provide a powerful way of making sense of the world..

From the above quotation one can deduce that constructivism thus provides students with the opportunity to create powerful constructions and also to assess and reflect on the constructions they have made. It is important to note that constructions that are made are not made in isolation, they are made in social contexts. If everyone had their own idividual and private constructions then it would be difficult to have a universal understanding of the world. It thus becomes important for students to communicate and negotiate their constructions with one another in order to come to a common understanding of their constructions. Hence new knowledge, to gain universal acceptance, must be a product of negotiation between stakeholders. These stakeholders constitute the mathematics community.

According to Confrey (1990:112) a teacher subscribing to constructivism should, as a goal of instruction:

> ...promote and encourage the development for each individual within his or her class of a repertoire for powerful mathematical constructions for posing, constructing, exploring, solving and justifying mathematical problems and concepts and should seek to develop in students the capacity to reflect on and evaluate the quality of their constructions.

Constructivism, therefore, promotes autonomy on the part of the student whilst the teacher's role is limited to giving guidance and direction. It therefore, becomes the student's responsibility to learn. In seeking to promote constructivist learning, the teacher provides problem solving scenarios that promote reflection and self-introspection on the part of the student. For maximum reflection and sense-making the teacher should present problems in a variety of representations. This should be a combination of the symbolic and visual strategies. In the process of problem-solving, students should communicate with one another and with the teacher about what they are doing.

3.5 COMPUTER TECHNOLOGY AND CONSTRUCTIVISM

Having briefly discussed the implications of constructivism for teaching and learning of mathematics, the question that one can pose is whether computer technology can promote a constructivist mode of teaching and learning of mathematics.

In seeking to answer this question, one needs to distinguish between drill and practice computer programmes and new programmes like DERIVE that can be used for concept formation. Julie (1990:228) argues that the drill and practice programme:

> Atomises knowledge, inhibits inquiry and promotes monological discourse. It is my contention that this mathematics - via computer experience denies students the opportunity to experience mathematics as a human creation with pragmatic sanction.

In trying to provide a solution to this problem, Julie (1990:233) proposes computer programmes to promote the construction of knowledge by students themselves. This is the kind of computer programme which would encourage constructivist learning. On this basis one would say that computers do provide the environment in which the teacher is in the background and thus allows for more independent student learning. The role of the teacher is in the development of learning material and the provision of guidance. Today's computer technology is of such a nature that there are sophisticated programmes like DERIVE that can do a wide variety of complex computations so that the student is freed from the rigours of algebraic or numerical calculations to concentrate on concept formation. DERIVE, furthermore, is able to solve problems in symbolic and graphic forms and this multiple-representation presents the student with the opportunity for reflection.

One can say that the nature of computer software is changing the nature of mathematics education. This is because now mathematics students can do exploration to search for mathematical patterns through manipulation in the same way that scientists do with experiments. Hence computer technology puts the students in a situation where he/she can control his/her learning environment.

Recapitulating then, there are computer technologies that offer students an ideal environment of constructivist learning because it encourages them to observe, question, explore, guess, learn from their errors, make connections, discover mathematical ideas on their own, work collaboratively, sharing ideas and talking mathematics with one another, read and write mathematics (Baxter, 1992:149).

3.6 CONCLUSION

With the starting point being that the teaching and learning of mathematics is influenced by the teacher's philosophy of mathematics, this chapter has located this research in a constructivist theoretical framework. It has given an exposition of constructivism and some of its variants including social constructivism. This research tends towards social constructivism as an epistemology of knowledge acquisition. I contend that since learning occurs in society, it is important that learning be a shared achievement of all humankind. Whatever constructions are made, are made with and in society as a whole. The implications for constructivist learning has been discussed, in particular the creation of a learning environment that enhances inquiry and which reduces the status of a teacher as an omniscient dispenser of knowledge. The student is seen as taking control of his/her learning.

Lastly, the role of the computer as providing the necessary mathematical environment for enquiry has been discussed. The computer's ability to present learning content in a multiple representaional form assists in the reflections made by students and thus promotes constructive learning.

CHAPTER 4: RESEARCH METHODOLOGY

4.1 INTRODUCTION

The research methodology of interest in this study is developmental research. In this chapter a brief discussion of the historical origins of developmental research is given. This is followed by an exposition of the nature of developmental research which have been used in this study.

4.2 BACKGROUND TO DEVELOPMENTAL RESEARCH

Developmental research began in the Netherlands in 1961 when the Dutch government established a commission on the modernisation of mathematics education (CMLW). This was followed a decade later by commissions in other school subjects. The CMLW because of these developments decided to set up a centre in order to co-ordinate the activities of all these different commissions. Having formed this co-ordinating commission, an Institute on Developing Mathematics Education (IOWO) was established under the auspices of CMLW. This institute had loose ties with Utrecht University (Freudenthal, 1991:162). The task of IOWO was to see to the pedagogical development of mathematics. Issues of the curriculum were no concern of theirs. A difference of approach to educational issues between IOWO and the government led to discord between them. The approach of IOWO was an integrated one with a generalized philosopphy of mathematics and a strong de-emphasis on specifics and educational objectives. On the other hand the government believed in strict compartmentalization of sectors within the education department. As Freudenthal

(1991:163) puts it:

> IOWO people could not answer questions about the curriculum or learning theories they adhered to nor could they produce calatogues of learning objectives, simply because they did not have any. The only things they had to show were a philosophy of mathematics and mathematics education...

Before the advent of IOWO the education department was demarcated into sectors such as the Training, Counselling, Retraining, Development, Test Production and Research sectors. Each of these sectors was administered or controlled separately. IOWO administered all of these as one unit. The government issued legislation to halt this integration and to ensure clear lines of demarcation between the sectors. Consequently IOWO detached itself from the government and became a fully fledged department in the faculty of mathematics at Utrect University. Today IOWO is known as the Institute for Research on Mathematics Education and Educational Computer Centre (OW & OC). The centre concentrates on developmental research in mathematics education (Freudenthal, 1991:163).

4.3 DEVELOPMENTAL RESEARCH: AN EXPOSITION

Before giving an exposition of what developmental research is, a brief definition of a kind of research to which developmental research is similar, namely action research, will be given. Thereafter developmental research will be discussed and this will show how it distinguishes itself from action research.

Briefly, action research can be defined as a self-reflective analysis of one's own practice with a view to enhancing or remedying it. Elliott (1981:1) succintly states that action research refers to "the study of a social situation with a view to improving the quality of action within it." From this one can see action research as identifying certain problems of practice within a particular situation whether it be educational, legal, political or otherwise. Hence the identification of the problem would then lead the practitioner to conduct a researh experiment to see in what way she/he can improve his/her practice.

Developmental research on the other hand focuses more on subject matter rather than the practitioner's shortcomings. This kind of research seeks to unpack the learning material so as to see in what way the material can be changed to effect the necessary improvements in the teaching-learning environment. Gravemeijer (1994:443) provides a fine distinction between curriculum development and developmental research by stating:

> In curriculum development the focus is on the instructional activities that embody the educational change; the emphasis is on the product, not on the learning process of the developer. .... In developmental research, knowledge gain is the main concern. The focus is on building theory, exploiting implicit theories.

One can then read in this that developmental research also involves the development of curriculum material. Any change in learning would be affected by curriculum material. Hence to change learning processes one has to change the existing material.

Freudenthal (1991:156) states that perhaps developmental research is not something new. It is actually the oldest kind of research although mathematics educators do not recognize it as such. What is significant to realize is that developmental research is aimed at change in the learning of a subject matter and its environment. In a nutshell, therefore, developmental research is aimed at curriculum change.

The developmental researcher is someone who observes current learning processes over a period of time in order to find out what would happen if the curriculum or the learning environment were to be changed. The developmental researcher does not necessarily have to be a practitioner in the situation that is being researched. She/he can be an outsider who has an interest in the subject in question. The researcher, nevertheless, can only do the research with the collaboration or co-operation of the practitioner in the field. By its very nature, developmental research is a collective or team effort. The research team collaborates in the development of generative material and in the implementation of the research. Through the investigation of how the new learning material changes the learning process, the researcher is able to contribute to the building of a new theory of learning. Streefland (1991:35) states that development research is:

- characterized by the depth of the investigation;

- determined by the subject matter;
- focused on forming a theory.

In this, therefore, Streefland is indicating that developmental research is responsinble for the creation of a new educatiotanl reality and this new reality would be illuminated through investigation. The extent of the illumination of the new reality depends on the depth of the investigation.

The new learning material that is developed in developmental research may be instructional material, instructional techniques or instructional programmes. Developmental research focuses on how the instructional material, instructional techniques or instructional programmes affect the learning process. In the context of this research the learning material is a combination of all of the above-mentioned material. Romberg (1992:58) mentions four stages in the developmental process, namely product design, product creation, product implementation and product use. One would suppose that product use is a post-research phase.

Gravemeijer (1994:454) states that an important distinguishing feature of developmental research is its qualitative nature as opposed to positivist research with its emphasis on using experimental results to generalize predictions. In developmental research, the experimental data is:

> not projected onto a mathematical-numeric system with the objective of doing analytical reasoning within that system...

In developmental research what is important is to make sense of the results of an experiment. The interpretation of the results is informed by what went on during the experiment. Hence in developmental research interpretation and sense-making is more important than the predictability of the experimental outcomes.

During the developmental process constant evaluation is made by the researcher to test for the appropriateness of the product and hence the succcess of the research. Four forms of evaluation are proposed by Romberg (1992:58) to correspond to the four stages of product development:

- Needs assessment: This is to decide whether the design of a new product is good. The researcher is checking whether there is a need for the product and whether the new product is going to fulfil the need.

- Formative evaluation: This refers to the determining of whether the product is of high quality and whether intended outcomes are reached. Unintended outcomes are also probed.

- Summative evaluation: This determines whether or not a created product is ready for use. The question that is asked is how the product is different from others, its performance and its cost.

- Illuminative evaluation: This involves the application of research methods like the case study ethnography, action research (I might add developmental research) to the the evaluation of new products. Illuminative research focuses on telling a story about the use of the product and making judgements about it.

These methods of evaluation are applied constantly to ensure that design, creation and implementation of the product are effective and the outcomes of the research are analysed.

4.4 STAGES IN DEVELOPMENTAL RESEARCH

One of the main characteristics of developmental research is that it is a phased process starting with the exploration of the problematic situation. This implies that the researcher has to identify the problem or what must be changed. An example of such a problem identification exercise would go thus: There is a new computer technology that is able to solve mathematical problems. Performance of students in mathematics is poor. Is it possible to use the new technology to assist student learning? Will the new technology suppress or will it facilitate concept formation in mathematics?

Having these questions in mind, the researcher would reflect on possible solutions by proposing certain hypotheses. She/he could start by discussing the effects of using the new technology with peers or consult theoretical writings and documents on what has happened in the past and present. She/he can then decide that research would be essential to test the applicability of the new technology.

The second phase of the research is the reconnaisance of the research. This would involve checking where and how to obtain equipment, the cost of equipment and so on. Also the target of the research is checked out - who is going to assist in the research and negotiations with relevant authorities and so on. This plan also includes time scales and any other constraints to the implementation of the plan.

The third phase is when the plan in put into operation. This is the critical stage of data collection including the participation of people who act as co-researchers and observers who would have their own perspectives and analyses of the execution of the research plan. At the end of the phase of execution an evaluation of the process occurs which involves the analysis and interpretation of data. After this, the researcher would reach a particular conclusion regarding the whole research process. McKernan (1991:28) states that at the conclusion of the research:

> the critical research group seeks to understand what effects have been and what has been learned as a result of the action.

4.5 DATA GATHERING

Various data gathering techniques are employed in developmental research. The following are the techniques that have been used in this particular study.

4.5.1 Field Notes

Field notes are written notes about the observations of the researcher on a research exercise. The researcher continually makes notes of what happens during the course of a lesson in the research. She/he records impressions, interesting occurrences and generally the progress and direction which the developmental research takes. This can be made during the course of the lesson if possible or soon thereafter. The recording should be done while the impressions are still fresh in the mind. Field notes can focus on a particular issue in the classroom or can form a general picture of the lesson under investigation.

Hopkins (1985:59) states that field notes can do three things:

- they can focus on a particular issue or teaching behaviour over a period of time,
- they can reflect general impressions of the classroom and its climate,
- they can provide an ongoing description of an individual child that is amenable to interpretation and use in case study.

4.5.2 Student Diaries

The student diaries are similar to field notes but as the name suggests, they are the student's notes of what is happening in the classroom. They serve to corroborate or contrast the researcher's observation of the learning situation. It is important to note that students' diaries are students' possessions and as such the researcher can only gain access to them with their consent. The researcher can then compare his/her observations with those of the students. Hence students' diaries give the students' side of the story about the research and the general atmosphere in which the research occurs.

4.5.3 Peer Observer

This refers to an observer or co-researcher who has been asked by the researcher to come and observe the progress of the research. Otherwise peer observers are part of the collective research team. The co-researcher or observer can write his/her own impressions on interesting aspects of the process. These observations are shared with other members of the team.

The role of the peer observer is the same as that of the student insofar as comparisons will be made of notes of other stakeholders in the research. Out of these comparisons a conclusion can be made about the effectiveness of the research.

4.5.4 Document Analysis

Document analysis refers to the analysis of documents that provide relevant information on the research. In the case of developmental research this may include the student's books or written work, work cards, assignment sheets and so on.

Student's written work provides part of the essential evidence in the analysis and evaluation of the research. In the context of this research, the focus has been mainly on student's written responses in worksheets.

4.5.5 Audio-tape recording

Audio or video-tape recording of a lesson situation can be made during developmental research. In this research a tape recording was made of one lesson during the familiarization exercise and once during the implementation. The aim was to tape discussions amongst a group of students during the two lessons. A recording of this nature is useful in following up on communication and reasoning trends in students during a learning process.

The researcher can use the audio-tape recording as a rich source of information after transcribing the recorded information. Relevant sections of the transcriptions can then be compared with relevant written observations of the researcher, observers and students. The portable battery-driven micro-recorder is more convenient for developmental research because the researcher can carry it around so that it does not take students' attention from what they are doing.

4.6 TRIANGULATION

Triangulation is not a data gathering technique. It is a technique used in qualitative research to validate data. Elliot (1981:19) states that:

> the basic principle underlying the idea of triangulation is that of collecting observations/accounts of a situation (or some aspect

> of it) from a variety of angles or perspectives and then comparing and contrasting them. For example as a teacher one can compare and contrast accounts of teaching acts in the classroom from one's own, the pupil's and an observer's point of view.

The comparing and contrasting can be weighed against collected evidence like the recording of the lesson and notes taken by different components in the research situation.

The process of triangulation can occur during the process of data gathering itself or at the evaluation stage of the research. Triangulation does not necessarily refer to the observations made by different people in the research situation. It can also refer to the comparison of different sets of data collected by the same person.

All the data gathering occurs during the implementation stage of developmental research. The fourth stage now comes to the fore. This is the evaluation of the research by considering its successes and failure.

4.7 *EVALUATION*

Evaluation in developmental research comes through a process of reflection during and after the analysis and interpretation of data. In the process of evaluation the researcher is also trying to build a theory of learning. It should be understood that the process of evaluation is constantly being applied throughout the research and the various phases referred to are not distinct and separated in time but flow into one another as a continious process. As Streefland (1991:35) posits, contribution of developmental research to the theory of learning occurs through "didactic deliberation, discussion and discourse...." The notion of didactic deliberation is similar to the idea of triangulation. Central to didactic deliberation is the posing of pedagogical arguments on the process and hence the outcome of the research. It is these compelling arguments which sift the wheat from the chaff, that result in a tangible conclusion being made about the success or failure of the research. The process of didactic deliberation also serves the function of justifying the outcome of the research or the new theory that is being built. Gravemeijer (1994:453) states that in positivist research justification occurs through empirical evidence but in developmental research, interpretation of the data serves this function. Hiccups that occur during developmental research which result in an inconclusive ending usually result in a new cycle of the research being proposed.

Because real life is never smooth, one therefore is not surprised that developmental research rarely goes smoothly and hence is cyclic in nature. At the end of each cycle there is dissemination or transmission of information to others on its success or failure to assess whether the research will need another cycle of implementation. Thus, dissemination and didactic deliberation is a way of authenticating the data in order to arrive at an authentic conclusion.

Freudenthal (1991:161) states that developmental research should be reported in such a manner that no one should doubt its authenticity. As he states:

> developmental research means experiencing the cyclic process of development and research so consciously, and reporting on it so candidly that it justifies itself and that this experience can be transmitted to others to become like their own experience.

The idea of candid reporting is found in the notion of dissemination. Dissemination refers to the idea of constant communication between colleagues and stakeholders in the research. This should occur at the outset. Students, in particular, should not be treated as guinea pigs - they should be equal partners in the research and the research should be geared at their benefit. According to Freudenthal (1991:162) developmental research should be aimed at the development of teacher training and retraining, guidance, testing and the development of everybody in the education sphere. This extends to teachers, students, working groups, text book writers and so on. Developmental research is aimed at long-term learning but does not exclude short-term benefits for the learners.

4.8 CONCLUSION

This chapter has discussed developmental research as an instrument of exploration to test how change can be effected through the introduction of generative learning material into the learning environment. The researcher then evaluates the outcome of students' interaction with the learning material. It is the students' own constructions, that is, the production of their own knowledge from the material presented to them, that will enable the researcher to develop his/her own theoretical formulation regarding the new learning material and its feasibility within the context of the total education environment. What is also important to note about developmental research is that generative material is situated in reality (Streefland, 1991:31).

CHAPTER 5: THE HP95LX FAMILIARIZATION PROCESS

5.1 INTRODUCTION

The previous chapters are concerned with theoretical issues that impact on this study. These are in the main the constructivist theoretical framework and developmental research in the context of which this study is located. This chapter discusses the commencement of the nitty gritty of this research. This refers to the computer familiarization process which occurs prior to the application of the technology in the teaching experiment which seeks to respond to the research question. What is to be answered through this study is the teaching and learning outcomes that will be yielded by the application of the CAS programmes in the teaching and learning of an introductory course in differential calculus in secondary schools.

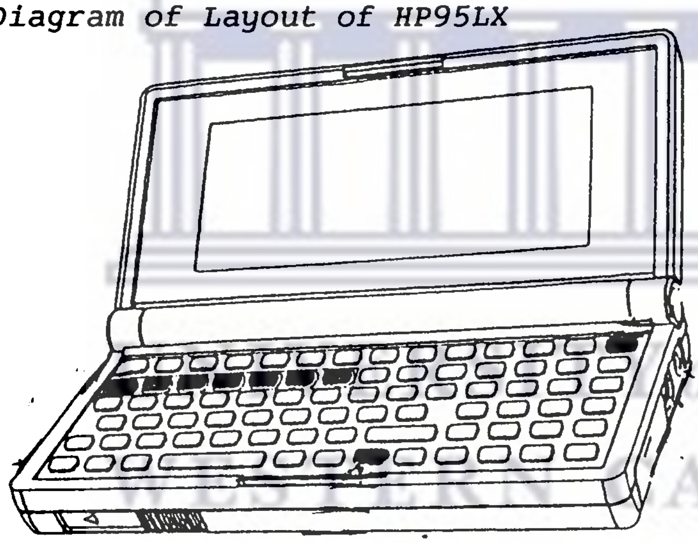
The need for familiarization is informed by the understanding that any person who has never had any contact with a particular technology is not automatically adept at using the technology. It becomes therefore, imperative to acquaint the individual with the relevant technology before she/he can use it in the solution of particular problems. It is after gaining competency in the technology that one can gain the confidence to use the technology towards whatever goals the technology has been designed for. This is true of any technological device and in this instance, computer technology.

This chapter, therefore discusses the familiarization of a class of students at a Secondary School at Mfuleni, a Cape Peninsula township, with the palmtop computer HP95LX incorporating the CAS programme DERIVE. Before discussing this process, it is important to discuss what the HP95LX looks like and how the programme DERIVE works.

5.2 THE HP95LX AND DERIVE

The palmtop or pocket computer HP95LX is approximately the size of a handheld calculator but is a bit heftier. The following diagram shows the layout of the HP95LX.

5.2.1 Diagram of Layout of HP95LX



Like any other computer, the HP95LX is divided into two areas, namely the keyboard at the base and the screen on top. In the HP95LX the keyboard is used to key the appropriate commands to operate the programme DERIVE. The screen in DERIVE is divided into a further two areas, namely the working area and the command area. The working area shows results to algebraic operations including graphical representation of functions. Below the working area is the command area showing a menu of appropriate commands for algebraic or graphical manipulaltion. The algebra menu for DERIVE looks like this:

Author Build Calculus Declare Expd Fact Help
Jump Solve Manage Options Pwt Quit Remove Simp
Transfer moVe Window Approx (Enter option)

To enter a command one simply presses the key with the capital letter in the command. For example, to command the programme to solve an equation, one would simply press L in soLve since it is the capitalized letter for the solve command. The computer then solves the given equation. The solution is then shown in the working area.

The most important keys that were used in the familiarization and implementation stages of the research were the following:

(i) All the capitalized letters of the approriate commands e.g. A for Author
C for Calculus
F for Factor and so on.

(ii) The Esc key used for escaping problematic situations where one is unsure about how to proceed and would therefore like to start at the beginnng. The Esc key takes one to the menu screen.

(iii) The F1 key changes a window, i.e. it changes the algebra window to the graphic window and vice versa.

(iv) The backspace and delete keys function to delete terms or expressions in the command area.

(v) The arrow keys help in moving cursor to the origin of the cartesian plane in the graphics window.

(vi) The shift (🡅) keys is used for the typing of the orange coloured symbols like ^ which is used to type exponents.

All expressions or equations are typed in the command area by first using the Author key and then entering them into the working area. One first presses A (for Author) and type the expression onto the screen. The following is an example of how one operates a window in DERIVE. Suppose that one wants to use DERIVE to differentiate the polynomial $2x^3 + 3x^2 - x + 2$. The following procedure is followed to manipulate the computer to do the differentiation:

1. Operator: Press A (for Author)

   Computer: Writes Author expressions.

2. Operator: Presses 2x^3 + 3x^2 - x + 2

   Computer: Shows expression in the form

   2x^3 + 3x^2 - x + 2 in the command area. Enter expression appears below expression shown above. (for^, one holds down the shift key and then presses 6.

3. Operator: Presses Enter

   Computer: Shows $2x^3 + 3x^2 - x + 2$ in the working area.

4. Operator: Presses C (for Calculus)

   Computer: Shows menu: Differentiate Integrate Limit Product Sum Taylor. Enter option appears below the menu.

5. Operator: D (for Differentiate)

Computer: Calculus Differentiate expression #1. Cursor shows below 1.

6. Operator: Enter

Computer: Calculate Differentiate: variabe: x Enter variable.

7 Operator: Enter

Computer: Calculus Differentiate: Order 1. This is for the first derivative. Enter.

8. Operator: Enter

Computer: Shows in the working area

$$\frac{d}{dx}(2x^3+3x^2-x+3)$$

The command area now shows the menu as at opening screen of the algebra window. Below this appears: Enter option. Dif (1,x) implying differentiating expression 1 with respect to x.

9. Operator: Presses S (for Simplify)

Computer: Simplify expression #2. Enter.

10. Operator: Enter

Computer: $6x^2 + 6x - 1$. Shows at the end: Computer has o,o seconds. Command area shows opening menu.

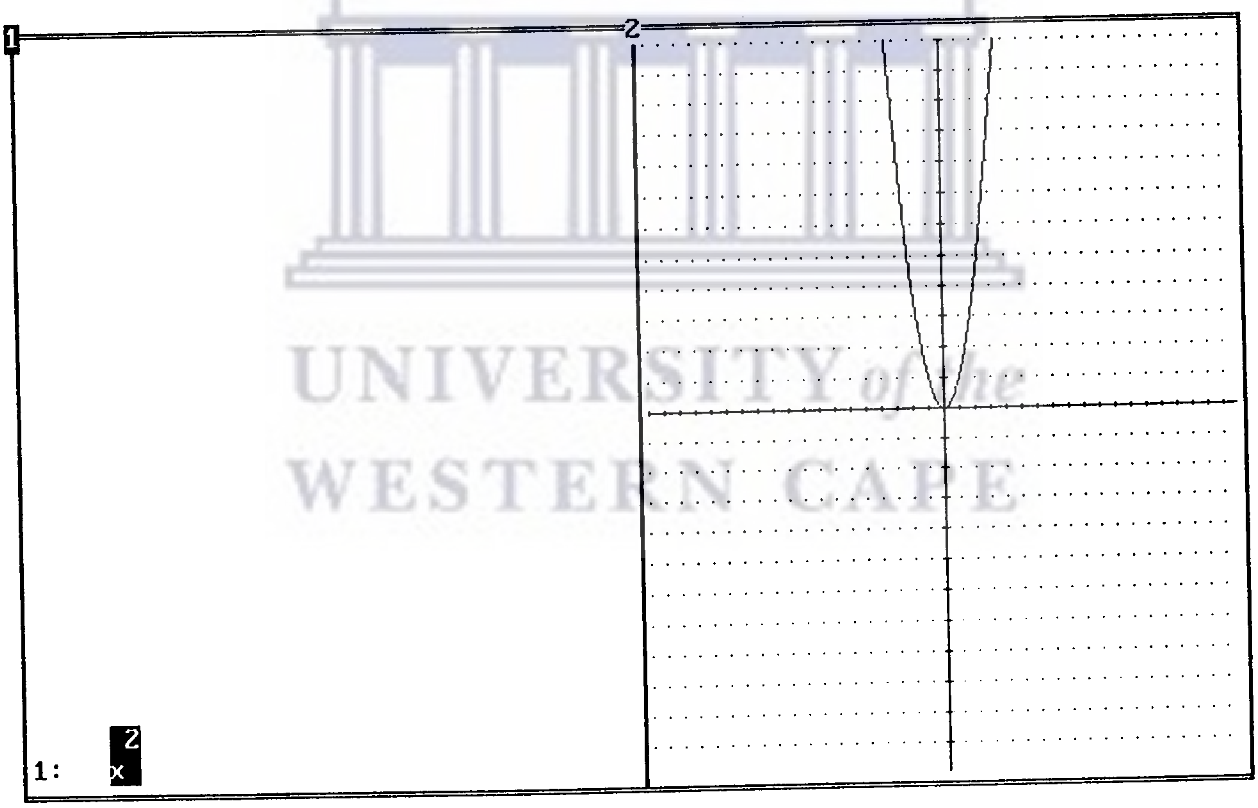
This is but one example of the many algebraic operations that DERIVE is capable of doing as mentioned in chapter 2. As mentioned already, DERIVE is able to draw graphs in two and three dimensions. To draw a graph one has to open a graphics window.

5.2.2 To Open and Operate a Graphics Window

To open a graphics window one has to press W (for Window) followed by S (for Split), V (for Vertical). This is followed by Enter to split the window in the middle. This vertical partition is at column 20. Either half of the screen can serve as the graphics window and the other half will serve as the algebra window. To make the right hand side window the graphics window one has to press F1, otherwise the lefthand side window will function as the graphics window. An active window is always highlighted at the top left hand corner as number 1 or number 2 as the case may be. It must be noted that one can have more than two windows in DERIVE. Supposing that one wants to make window 2 the graphics window, one presses F1 followed by W (for Window), D (for Designate) and 2 (for 2-D plot). A set of co-ordinate axes appears in the graphics window along with a small cross called cursor. To set the axes one presses T (for Ticks), 1 to indicate 1 tick for 1 unit on the X-axis, TAB (   ) to shift cursor to the Y values and then 1 to indicate 1 tick per unit on the Y-axis. One can then press S (for scale) and set the scale for the x and y axes accordingly. The cross hair in the cartesian plane is then moved with the arrow keys (

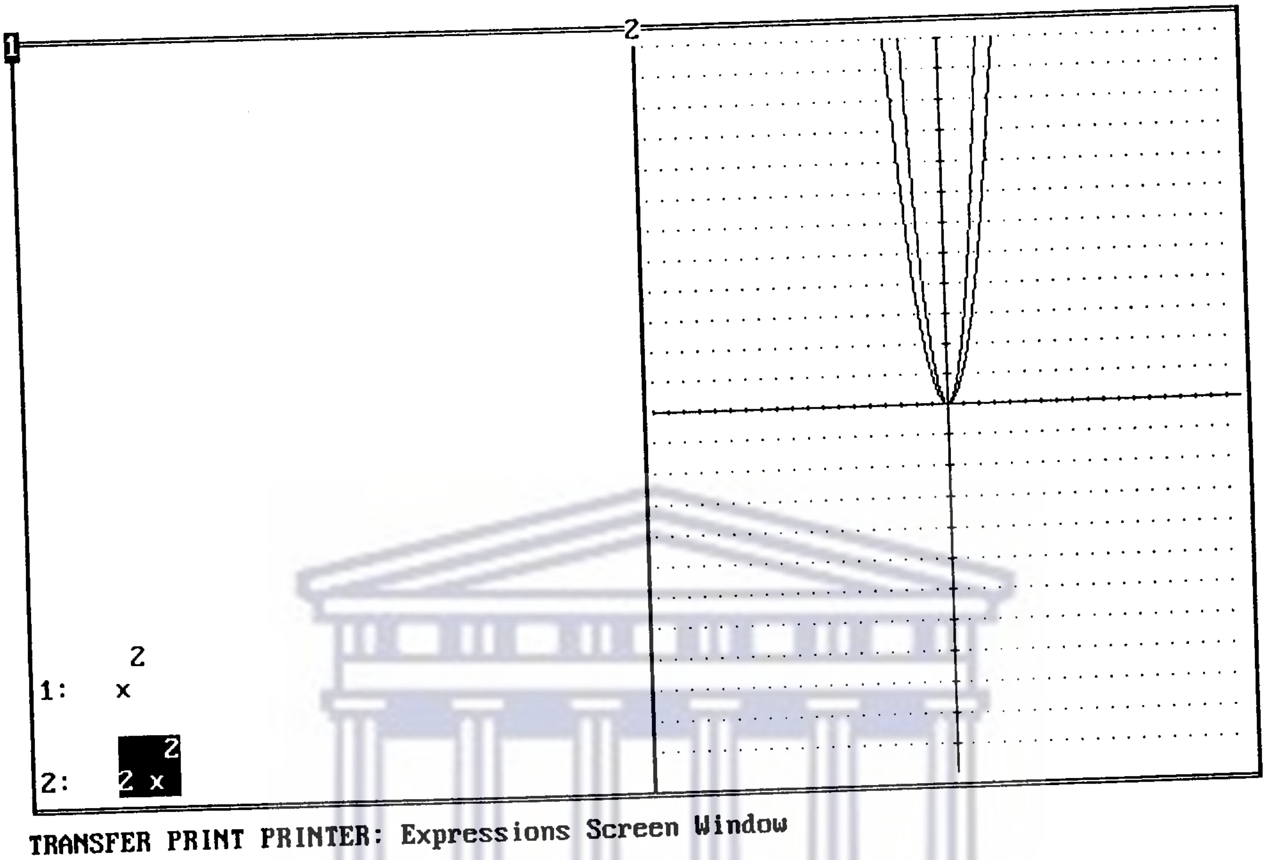
and ▼) until it is centered at the origin. One can then begin to plot the required function.

DERIVE plots graphs of functions authored in the algebra window. One has to firstly make the algebra window active by pressing F1. Taking the example of $f(x) = x^2$, one authors the expressions x^2 or $f(x): = x^2$ or even $y = x^2$ in the algebra window. One then presses F1 to make the graphics window active followed by P (for Plot). One can also plot by pressing P twice without pressing F1 to change the algebra window. The graph of $f(x) = x^2$ will then appear as follows:



TRANSFER PRINT PRINTER: Expressions Screen Window

One can instruct DERIVE to plot another graph on the same system of axes, for instance, $y = 2x^2$ and this will appear like this:



As many curves as one wishes can be plotted on this system.

5.2.3 *Clearing and Closing A Graphics Window*

One can delete all or some of the graphs that appear on the graphics window. To do so one presses D (for Delete) and the computer shows DELETE: All Butlast First Last. One can then choose any option indicating the erasure of all or any particular graph. All erases all graphs, Butlast erases all except the last graph. First and Last are obvious.

One can also simply close the graphics window by pressing W (for Window), C (for Close) and Enter. This of course will be possible only if the window is active. The operator should take note of this. The graphics window will then be closed leaving only the algebra window open.

To clear the algebra window one presses R (for Remove). The computer shows REMOVE: Start: Number End: Number. One has to indicate from which expression one wants to erase. One then erases the number which is shown by the cursor at Start by using the backspace or the DEL key. One then types the particular number of the expression from which the erasure should occur. The TAB key is pressed and moves to the number at End. Enter is pressed and all the relevant expressions are deleted. (De Marois, P. 1992:1-6).

5.3 PREPARING FOR IMPLEMENTATION: THE NEGOTIATION-FAMILIARIZATION PHASE

Preparations for the research commenced in August 1994 when I obtained the HP95LX from my supervisor, Prof. C Julie, in order to familiarize myself in its functioning. I was just given the HP95LX and a manual. I also had to begin to negotiate with a school where I would implement the research. It would be more convenient to find a school with one standard 9 mathematics class. The reason for settling for a standard nine class is that the research would take place in the third or fourth term of the academic year and we did not want to disturb the standard 10 students who were preparing for their final examinations. Furthermore the standard 10 class had already completed the calculus section of their syllabus. We wanted this to be a new lesson experience in terms of the objectives of the research. A single standard 9 class would save time for implementation.

A test that would be set at the end of the research would take less time than would be the case for two or more classes.

I made a telephonic survey of all secondary schools in the Peninsula under the jurisdiction of the DET. This was on a matter of access and convenience for me, as this is a department that employs me. Almost all schools surveyed had more than one standard 9 mathematics class with the exception of Mfuleni Combined School.

5.4 MFULENI COMBINED SCHOOL.

I chose Mfuleni Combined School using the criterion of a single standard 9 class. Also the number of students in this class is quite small according to DET standards.

Mfuleni is the name of the black township in which the school is located. The name Mfuleni is a Xhosa word meaning "in the stream." I presume this is taken from the Kuils River passing the township, which is also the name of the town under whose jurisdiction Mfuleni falls. Mfuleni is approximately 30 kilometres east of Cape Tpwn. The township was established in 1976 from mostly male contract workers from the homelands and "squatters" removed from Somerset West, Waterkloof and Eerste River (CPP: 1994:2-3)

The population, according to the latest survey stands at 11 441 of which approximately 23,9% have formal family housing. The rest occupy shacks and hostels. (CPP: 1994:2-3). Mfuleni

Combined School, the only school, serves this community. Since its establishment in the seventies, it has always been a primary school until 1991 when a standard 6 class was introduced. This year (1994) the secondary school section has standard 9 and next year the school will present its first matric class. A Combined School therefore, is a school with different levels of teaching. Mfuleni Combined School has both primary and secondary schools in the same buildings. Sometimes even a primary school may be termed a Combined School because it has a junior and senior primary in the same school.

Mfuleni Combined School has a pupil population of 1 500 with 510 in the secondary sector. The one standard 9 mathematics class in which this research was undertaken, has 25 students comprising 11 boys and 14 girls. There are 32 teachers in the school, 12 of whom teach in the secondary school section. It is quite interesting to note the symbolism in the construction of the school. The secondary school is located on an elevated plane and the primary section is lower down. One needs to use a flight of stairs to reach the secondary school.

5.5 NEGOTIATIONS

Negotiations to use Mfuleni Combined School were undertaken in September after most schools in the Peninsula were found to be unsuitable. As already stated most of them had too many mathematics classes and others were to commence their final examinations for the year in early October.

I went to Mfuleni on the 13th of September where I met the principal, Mr Patrick Nqolobe (coincidentally a teacher who had taught me at secondary school 24 years earlier). I was warmly received and the head of department of mathematics was called to facilitate negotiations between me, the class teacher and students. I explained, in a meeting with the class teacher and the Head of Department (HOD) what the research was all about and the amount of time I would require. This would be two to three one hour familiarization sessions with the students and one week for the implementation of the research experiment. The teacher's co-operation and collaboration in the whole excercise was requested to which she consented. The HP95LX familiarization excercise with the students was to commence the following Tuesday, 20th September 1994. The HOD was asked to contact the target class and negotiate their co-operation. I requested the mathematics teacher to participate in a familiarization workshop on Friday the 16th of September 1994. She agreed.

5.6 PERSONAL FAMILIARIZATION

My own familiarization in the use of the HP95LX started in August when my supervisor gave me the computer and a manual on DERIVE by De Marois (1992). When I arrived home I started to fiddle with the keys to author and enter expressions into the computer, with the manual as reference. I must confess that initially I struggled as I had not been workshopped in the use of the HP95LX. I had been thrown into the deep end and had to find my way to shore. The manual was my lifebuoy. If I authored expressions incorrectly in the command area and finding it difficult to use the correct key to erase it I would resort to the help key. Pressing H, which I thought was for Help, immediately erased the whole programme from the computer's memory! I could not programme it because there was no disk in the computer. I had to make an appointment with my supervisor to reprogramme the HP95LX. It is then that I learnt that, as far as he knew, the programme DERIVE was the only one in the country and as such he was guarding it jealously. To reprogramme the computer one presses CTRL-ALT simultaneously and then the DEL (for delete) key. One then presses DERIVE from the options that appear on screen and then the DERIVE opening screen appears. I made two more such trips to my supervisor when the computer went off programme. This occured once when I pressed Q (for Quit) instead of Esc (for Escape) and also when the battery got very low. All in all I workshopped myself intensively until I had full confidence of handling the technology.

On the 16th September 1994 I had my first practice run by giving the mathematics teacher an hour's workshop in the use of the computer. After showing her the basic ways of entering and performing algebraic manipulation and graphical representation by the HP95LX, I let her do a few examples. At the end of the hour I gave her one HP95LX and a manual prepared by my supervisor and a workshop exercise prepared by me to work at home. We were to meet on Tuesday the 20th September for the familiarization excercise with the students. On Monday the 19th I had another practice run by giving a workshop to my M.Ed class colleagues. The excercise that I gave them after showing the functions of the basic keys of DERIVE is the same as that which I had given the mathematics teacher.

5.7 STUDENT FAMILIARIZATION

5.7.1 Session 1 (20.09.94)

I arrived at the school at 08h00. Students were at morning assembly. I met the HOD and the subject teacher who guided me to stardard 9A, when assembly had ended. I found a class of 20 students to whom I was introduced by the HOD. The subject teacher excused herself as she had to attend another class. She was to join us later. I was then left alone with the class.

I reintroduced myself in the context of my experience as a mathematics teacher and student. I explained to them what the research was all about and what we want to investigate. I then asked who had seen a computer before. Approximately more than half had seen one though there was not one who was computer literate, I briefly related the history of technology as relating to the mathematics classroom starting with counting stones, the abacus, four figure mathematical tables, the four function calculator, the scientific calculator and how each of these contributed to make calculation a less laborious excercise. The advent of the graphic calculator and the palmtop personal computer was then mentioned. Thirteen HP95LX were issued so that students could see how these looked like. They could see that the HP95LX is a miniaturized version of the desktop computer and that it is capable of doing everything that its bigger counterpart is capable of doing. Students asked whether at the end of the experiment they would be allowed to take computers into the examination room. I explained to them that the present education policy is that programmable calculators and computers are not allowed but that investigations will be made regarding the possibility of having computers as part and parcel of the examination room.

We then commenced with the familiarization. I wrote on the blackboard and explained the functions of the different keys. I wrote down a number of promblems on the chalkboard on factorization, solving and simplification. The objective of the excercise was to ensure that students were able to enter and do correct algebraic operations by using the appropriate key commands of the computer. This was to ensure that when the implementation of the main lesson experiment occured students would be competent in using the computer.

I explained the operating procedure for each problem to guide the students. The subject teacher joined us in the middle of the lesson. The first lesson generally went well with less than 40% of students needing constant attention. I found out that the teacher was not of much assistance and that she was not yet confident in the use of the computer. I, therefore, had to do the explaining without her assistance. I noticed that three groups were doing particularly very well. One was a group of two boys and a girl and the others were of two boys each. They were doing the operations without asking for any assistance from me.

One boy even stood up to assist others who were struggling. I was wondering what motivated this altruism. I found out later during an informal chat that he was a member of the students' representative council. The end of the period came without some of the students having finished all problems. I told students that we would have a follow up session on an alternate day. On the following day, I decided I would have another workshop session with the subject teacher.

The session with the subject teacher went well because she was able to do the required operations with the computer. We proceeded to the graphics and she operated the computer to draw a few graphs. She took home one HP95LX and a workshop manual to practice at home. The session took an hour.

5.7.2 Session 2 (22.09.94)

During this session, I was assisted by Jerome Erentzen, a colleague in our masters class. I met the mathematics teacher. Her husband had fiddled with the unit and pressed all the wrong keys and caused DERIVE to bomb out. She said she had to attend to her other classes. Jerome had brought an audiotape to tape some of the groups in the class.

I decided to divide the class into groups of four to encourage communication and mutual co-operation in big groups. I told the class, after I had introduced Jerome, that on this day I would not go back to revise the computer procedure. I would simply give them problems so that they could use the computer to work through the problems. The problems given in this session were basically similar to those given in the previous workshop session.

HP95LX Orientation session 2

1. Use the computer to factor

 (i) $x^2 - 9y^2 + x - 3$

 (ii) $(y + 1)^2 - 8$

 (iii) $2x^3 + x^2 + x - 1$

2. Solve for x:

 (i) $6x^3 - 17x^2 + 14x - 3 = 0$

 (ii) $\frac{5}{x-2} - \frac{4}{x} = \frac{3}{x+3}$

 (iii) $6^x - 6^{x-1} = 180$

3. Given $f(x): = 4x^3 - 8x^2 - 9x + 18$

 Calculate $f(2) - f(-1)$

4. Given $f(x): = -x^2 + 3x + 6$

 Evaluate $f(4)$

The exercise was geared to strengthen students' understanding of operations on the computer. These are the key procedures for authorising, entering and pressing the appropriate keys for factorising, solving and simplifying. I found the students gaining in confidence. There were still some students having problems and Jerome and I went to their assistance. Problems were arising in question 2 (iii) where they had to press the shift ( ) key to type the exponent x - 1 in parentheses.

Otherwise the equation $6^x - 6^{x-1} = 180$ appears as $6^x - 6^x - 1 = 180$ which would hence yield an absurdity. Similarly in the equation $\frac{5}{x-2} - \frac{4}{x} = \frac{3}{x+3}$ students were making mistakes in not typing x - 2 and x + 3 in parentheses. This then appears in the computer as $\frac{5}{x} - 2 - \frac{4}{x} = \frac{3}{x} + 3$ and this different equation would yield a different solution. These are the kind of typing errors we had to rectify. In the last two problems we had to emphasize that DERIVE accepts f(x): = and not f(x) =, i.e. there should be a colon after f(x).

An observation I made was the intense discussion that occurred in the groups. A transcript of the audiotape was difficult to make because of the background noise caused by group discussions. Some groups nevertheless, were audible on tape. I could transcribe snippets of discussions as follows:

- What are you doing?
- Coming back to the menu.
- What should we do now?
- Press the capital L in solve
- How did you get this?
- Look at it here. Enter - it's simple.
- Andiswa, is this right? - What we have obtained
- What have you obtained? (I presume the questioner was inquiring from another group)

- Look here. This is how I do it.
- Sir how do we enter this exponent -1?
- Sisebenzis' amathamb' engqondo (Xhosa literally translated: We are using the bones of the brain). This means: We are using our mental faculties.
- We've got the same answer Sir! We can really calculate! (This is after one group asked me to verify their answer).

What all the above snippets of discussions show is that the answers produced by the computers occur after much discussion, questioning and negotiating in the groups. Where the group was unsure of their operations they called out to the teacher to assist or corroborate a solution. There was also much intergroup cross reference and verification of answers. Some of the groups went straight ahead to the finish without calling for help. The group of boys I noticed during the previous session was still excelling, doing all the allotted problems within a short space of time. There were others, however, still struggling with basic keys. I observed that some of these students were not present during the first session and as such were not aware of the basic key procedures. To offset some of these problems, I dispersed these students among the groups that were progressing.

It is quite interesting to note some of the comments made by students. For example, the comments that they are using their mental faculties and that they are really calculating. This highlights the notion that the production of the correct answer comes through their mental capability and their ability to manipulate the computer. Hence students view this process as a process of constructing their own knowledge. It is not the computer that is calculating, they are really calculating! They see the computer as an implement to use to produce answers. One can deduce from the animated discussions within the groups that the computer facilitates social interaction and thus social learning. We could also note that there was a tendency for boys in mixed groups to dominate the discussion and also to monopolize the handling and the operation of the technology. I did however request these groups to rotate the handling of the computer.

We proceeded with the opening of the graphics window. This also went well but some had to be assisted in setting the co-ordinate axes and moving the cursor to the centre of the axes. The period ended and we had to stop. Schools were going to close the following day for the third term and it would be ten days before I could meet the students again. I decided to negotiate another session during the holidays. Students readily agreed and stated that they would be prepared to spend the whole period of the holidays working on computers. Nevertheless I felt that only one more session would suffice. We settled on September the 27th as a suitable date.

5.7.3 Session 3 (27.09.94)

On this day Jerome, who had assisted me in the second session, did not arrive. A communication problem between us was the cause of his non-arrival. Fortunately Larry, another colleague, came. The teacher whom I had also requested to come did not come. In fact I never saw her again in class even during the implementation period.

There were 14 students present with two new faces. The previous session had 22 students. I started the session by giving them some problems relating to work that we had done in the previous two sessions. This was to consolidate work in the algebra window. They worked through the problems but there were others still lagging behind. I found out that one of the groups who were struggling contained the two new faces that I had noticed earlier. I found out from others that they belonged to the History class group. They had come to see what this wonderful new machine could do!

I issued out a worksheet detailing how to open and operate a graphics window. I directed them in the process of opening the graphics window, setting up axes and the plotting operation. Students then proceeded to apply the procedures.

Larry made observations and gave me these in writing (see Appendix B). Basically three observations can be noted. Firstly Larry felt that I would need to give more time to recap the basic functioning of the computer to assist those who had been absent and those who had forgotten before tackling the one exercise that we had to do in the implementation. As he put it:

> Initially, some time needs to be invested in pupils

gaining some expertise in using the gc (sic) so that this does not become a major "detraction factor" in later exercises.

This is correct but I feel that in the first two sessions the necessary groundwork insofar as the basic functions are concerned had been done. For me, struggling, grappling with the technology was a good learning process. The other factor which I noticed much later was the "shifting population" in the classroom. By this I mean those students who were absent in a previous session were the ones who tended to have problems in later exercises. I must confess, I tend to be less patient with students who miss my classes!

The other observation that Larry made which I also noted, is that some students wrote down instructions for basic commands of the computer which they consulted in later exercises. Larry also noted the dominating nature of boys in mixed groups. I agree that in future this needs to be more consciously handled and gender domination to be curtailed. The other thing is that some groups take the answers given by the computers as correct on faith without checking on their correctnes. Of course, a computer will yield a correct result only insofar as it was correctly fed. This implies that incorrect manipulation procedures will yield incorrect answers. It is through checking the correctness of the results that better conception in ensured.

An important observation made by one group is that when one has completed a plotting procedure on DERIVE, there is a time lapse between the end of the plot steps and the plotting of the graph on the axes by DERIVE. A dark spot appears at the top left corner of the graphic window before DERIVE begins to draw a graph of the given function. One boy asked what was happening in the "quiet" period. I explained to him that in the same way that one has to calculate the relevant cartesian points in the conventional paper-and-pencil algorithms, so does the computer programme during the "quiet" period. It is in a point calculating mode. Interestingly, Larry observed this question in another group. As he states:

> One of the members of the group thought it quite strange that the GC (sic) took "a long time" to draw a graph! This raises the question about what the gc (sic) is doing when drawing the graph, and hence what it means to draw (a) graph... It seems as if we as instructors have to be aware of the possible questions that can arise or at least respond to these questions in such a way that stimulates pupils' further interest in this kind of "participatory" learning.

5.8 CONCLUSION

There were positive outcomes about the familiarization process. Firstly, I, as a researcher, had gained confidence and competency in handling the technology. This was also true of students. It also gave a fascinating insight into students' thoughts and their way of communicating, questioning and their gaining of an understanding of the power of computer technology. Furthermore, the students were viewing the outcomes of the manipulation of the computers as largely a result of their own efforts and these results as their own creation. Their ability to press the correct keys and giving the correct commands they regarded as an intellectual achievement. They were viewing their competency as power over the technology. One student shouted (on the audiotape): "We are commanders now - we are commanding!"

Observations made by my colleagues noted sense-making and concept formation resulting from students' questions about what the computer was doing in the "black-box" period. It is questions like these which show that students think when manipulating the computer. It is not simply "plug-and-chug." Computers will not stop students from thinking.

I felt that at the end of these sessions of familiarization, students were ready to go ahead to the implementaion stage.

CHAPTER 6: IMPLEMENTATION

6.1 INTRODUCTION

The familiarization process which took place at Mfuleni before schools closed for the third term set the stage for the core of the research. This is the stage of implementation or data gathering. This chapter is about this crucial stage of the research process.

As stated in the first chapter, the aim of this study is to investigate the impact of computer algebra systems in the teaching-learning process of introductory differential calculus in secondary schools. The study sought to establish whether students would be able to deduce the basic rule of differential calculus through the use of the CAS programme, DERIVE. This basic rule states that given a power function $f(x) = x^n$, then its derivative, $f'(x) = nx^{n-1}$. Having deduced this, students would be required to extend it to the differentiation of polynomial and product functions.

6.2 MATERIALS DEVELOPMENT AND PLANNING

The learning material for this exercise in the form of worksheets was developed by my supervisor, Prof Cyril Julie in consultation with me. The material together with resposnes from students appear in appendix A and are constantly referred to in this chapter. I was called in regularly to make comments on material in preparation. I must say that my own contribution was not much except for the initial familiarization.

We had envisaged that the implementation would comprise four sessions of one hour each and the familiarization. Our plan and time allocation was quite good because we could finish the experiment in the time we allocated it and only the familiarization needed an extra session. The implementation process should have commenced on the 5th October 1994 when school reopened for the last term. Unfortunately my supervisor, who had to be part of the process was out of the country, attending a mathematics conference. We, thus, could only commence on the 12th when he came back.

On the 11th October I went to the school to negotiate periods that we could use for the research. This was so because in this particular school periods are only thirty minutes long. We needed double periods and thus we had to negotiate with teachers with adjacent periods to mathematics in the days which had a single period for mathematics. I observed doubt and reluctance on the part of the teacher to co-operate with us on the research. She was worried about the teaching time that was being lost to the research. I tried to reassure her that what we wanted to do was not out of the syllabus because it constitutes a large part of standard 10 work. Furthermore I could, at the end of the implementation process, set a question that could be incorporated in the students' mathematics question paper in the final examination. I told her that I had negotiated this with the students and they had accepted the proposal. She agreed. I decided to tell my supervisor about this so that he could reassure her about the research exercise. My supervisor did meet her on the day of implementation of the lesson experiments. He explained to her about the research and what it aimed to achieve. She seemed to be reassured by the explanation.

6.3 THE IMPLEMENTATION

6.3.1 Session 1 (12.10.94)

Activity 1

On the 12th of October we commenced with the implementation. The aim of the first lesson was to let the students find, using DERIVE, the derivatives of power functions from x to x^4. Extending this to find a pattern from the differentiating x^5 to x^{10} without DERIVE, students would be guided to deduce the general rule $f'(x) = nx^{n-1}$ given a power function $f(x) = x^n$. Activities 1.1 and 1.2 in appendix A were given to each of them to test their understanding of power functions and the notion of the gradient of a straight line. As one can observe, this was to check whether students understood the definition of power functions given at the top of the worksheet and also the notion of the gradient m given the gradient-Y-intercept form $Y = mx + c$ of a linear equation. The activities were completed by students in a period of five minutes.

6.3.1.1 The Results

Looking at the 20 responses I obtained from students I found that all 20 were able to write down two more power functions. Futhermore 2 of the students wrote down Y as a variable instead of x given in the example. Whilst the example of power functions gave the sequence x^2; x^3; x^4;..., students generally added the power functions x^5 and x^6 to the sequence. Two students did not simply continue through the list but gave as examples x^6 and x^7 and another wrote x^9; x^6.

The question asked on whether x is a power function was answered correctly by 17 of the 20 respondents. Students were asked to furnish a reason for their answer. Eight of the 17 correct responses stated that the power of is 1, a natural number. And hence x is a power function. Nine could not furnish a reason. The three who responded in the negative stated that there is no exponent in x like x^2 or x^3 and hence one cannot regard x as a power function.

In 1.2 all students except 1 could identify the gradient in the list of linear equations given in the table. The one incorrect response wrote the gradient of $Y = 4 - 3x$ as 3 instead of -3. One can regard this as more of a writing error than a conceptual mistake.

Activity 2

The second activity, a crucial activity for the whole implementation process, involved the use of DERIVE to deduce the derivative (which we defined as a derived function) of the power function $f(x) = x^2$. The activity and process of deduction involved a combination of a graphical and algebraic strategy to deduce the equation of derivative of x^2.

Before coming to class I had set up the graphics window and co-ordinate axes so that the class would not be bogged down with this exercise. The equation t(a): = tangent (x^2,x,a) had also been entered into the computers. This is the formula that would be employed to deduce the equations of tangent lines at various points on the curve of x^2. The strategy of deducing the derivative of x^2 was to plot the graph of x^2 in the graphics window. Secondly students had to derive equations of tangent lines to the curve of x^2 for $x \in \{-3;-2;-1;0;1;2;3\}$. The procedure for deriving the equations and plotting these tangents on the curve of are as stipulated in activity 1.3 of appendix A. For the first two examples students worked with me assisted by my supervisor. The first two examples of equations of tangents are given in the table. The student would then consider the gradient of each tangent and use this in combination with the value of x = a as the point [a m]. For example the gradient of the tangent line Y = 6x - 9 at the point of contact where x = 3 is 6. The student now had to author the point [3, 6] and enter it. Making the graphics window an active window, the student would then plot it. Using the various values of x students produced via the computer, a series of equations of tangents as stipulated in the table 1.3.3. Points yielded by x = a and the gradients are (3, 6), (2, 4), (1, 2), (0, 0), (-1, -2), (-2, -4), (-3, -6). All these points were then plotted on the computer screen.

This exercise took the better part of an hour to complete. Students had problems in following instructions and consequently step by step explanations were given by me and the supervisor as elaborated in the worksheet. Ultimately students deduced the equations of tangents and their gradients which they had to write in a table. The graphs of tangent lines had also been plotted.

Asking the students what form the points on the graphics window took, they noted that the points form a straight line. I asked them to use their knowledge of straight lines to derive the equation of the straight line that passes through these points. It was 5 minutes before the end of the period and they could not derive this equation before the period ended. I left this problem as homework.

6.3.2 Session 2 (13.10.94)

Activity 2 Continued

On this day there were 23 students in class. I asked whether any student had been able to derive the equation of the line from the points we had the previous day. None had. I wrote the points down on the chalkboard. I asked them to look for a pattern in the values of Y relative to the values of x. Students could discern that in the point (3; 6), 6 = 2 X 3, in (2, 4) 4 = 2 X 2 and so on. Generally they deduced that every value of Y is twice the value of x. Writing this algebraically they wrote y = 2x. Hence they had written the equation of the line passing through the indicated points. I also tried to find out whether they knew of any other methods of deriving and equation of a straight line given a set of points. It emerged from them that they had not been taught this section in the previous standard. I did not want to venture into controvercy with regard to the veracity of this statement. But be that as it may, the students had demonstrated that whether they had been taught the theory of lines or not, they were able to observe and conceive a pattern in a set of points. From this pattern they were able to deduce the desired equation. To verify and demonstrate that this indeed was the required equation, I told students to use DERIVE to plot the graph of $Y = 2x$. The graph passed through all the points!

Having written the equation $y = 2x$ in 1.3.4, students had to write how they had derived the equation. They all explained in terms of the pattern they observed between the values of x and y, that is the value of y is always twice the value of x.

The worksheet then states that the function $y = 2x$ is the derived function of $y = x^2$. It proceeds to let students into the notation for differentiation, i.e: $\frac{d}{dx}x^2 = 2x$.

Students had to complete this part, which they did. This completed activity 2.

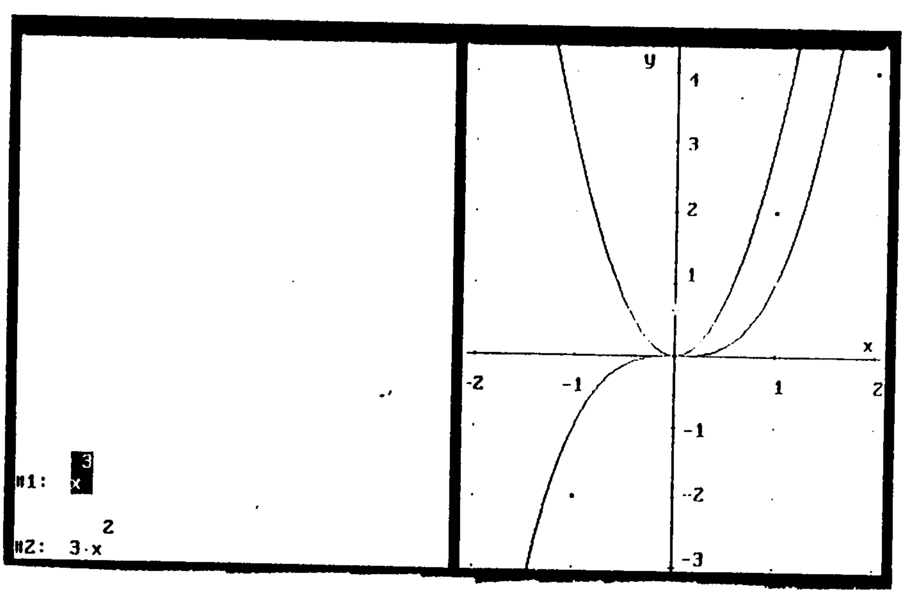
I explained to them the implications of the concept of a derivative. This is that given a function $f(x) = x^2$ there is another function that we can derive, known as the derived function or derivative $f'(x) = 2x$. Students also took note that the power of the derivative of a power function is always 1 less the power of the original function.

Activity 3

This activity involved the use of the computer to find derivatives of the functions x^3, x^4 and x. From these derivatives, students had to look for a pattern in order to deduce a rule for differentiation. For the function $f(x) = x^3$, I felt that the method of activity 2, using tangent lines and sets of points would produce some hurdles in terms of the length of time involved. I felt that a different approach was needed. Students were asked to plot the curve of $f(x) = x^3$ and then find

$\frac{d}{dx}x^3$ using DERIVE. The computer calculated $\frac{d}{dx}x^3$ to be $3x^2$.

This was then plotted on the same axes as the curve of x^3. This looks as follows:



The same process followed for the curves of x and x^4. Students were then given activity 1.7 in appendix A to fill in the derived functions of the given functions in the table and then complete the rest of the worksheet.

From the pattern in the table students could easily find that

$\frac{d}{dx}x^{1994} = 1994x^{1993}$. Using the same pattern students could

observe that the derivative of a power function is equal to the product of the exponent of the original function and a power function whose exponent is one less than that of the original function i.e. $\frac{d}{dx}x^{n} = nx^{n-1}$.

6.3.2.1 Results

The results of the exercise of filling in the table in the worksheet yielded 20 correct responses except 3 who wrote the derivative of x as 1x. Of the 20, 3 wrote the derivative of x as $1x^0$. Otherwise the rest of the worksheet was filled in correctly.

6.3.3 Session 3 (17.10.94)

Having derived the basic rule for the differentiation of power functions, this session aimed at extending this rule to the differentiation of a constant times a power function and also functions to which constants have been added i.e. the functions cx^n and $cx^n + a$, with $a, c \in R;\ n \in N$.

Activity 1

The focus of this activity was functions of the form cx^n. Activity 2 of appendix A was issued to the 24 students present.

Firstly students were required to write down 2 more functions similar to $2x$, $3x^2$, $\frac{1}{2}x^3$. Students were then required to fill in the table 2.2 by first completing the second column from their own knowledge of differentiation and then column 4 using DERIVE. From the table they were to write down in words a rule for the differentiation of functions of the form cx^n. Lastly without using DERIVE they had to complete table 2.3. After completing this table they had to verify their answers by using DERIVE.

6.3.3.1 Results

The following table gives a simple breakdown of responses to the writing of two examples of the function cx^n.

	CORRECT	INCORRECT	DID NOT WRITE
No. of students	19	2	3

The two incorrect responses wrote x^5 and x^6/x^4 and x^6 as examples of functions of the form cx^n. In hindsight, I think the worksheet should have indicated that $c \neq 1$. Otherwise the examples given are correct, technically speaking, because the students would argue that $c = 1$ in the cases they had given. But mathematically speaking $c = 1$ would give a trivial case for all cx^n and hence would not be admissible in this case of products of power functions and constants.

Coming to table 2.2 all students except 1 filled column 2 correctly. The 1 exception got all derived functions wrong. One suspects that this student was absent in the previous session where the rule of differentiation was deduced. It is even strange that she/he did not consult her/his group. Column 4, which required the use of DERIVE, was correctly filled by all students.

Students were then required to write down a rule for the differentiation of the function cx^n. Eleven students wrote the rule, correctly that $\frac{d}{dx}(cx^n) = c\frac{d}{dx}x^n = ncx^{n-1}$. In words this was put as: "one has to differentiate the power function without the constant and then multiply the derivative by the constant." Eight students wrote sentences which did not make sense like "The derived function is being multiplied by a power constant is equal to a derived function." Five did not write down anything.

With regard to the unclear articulation of the rule, I noticed that although the students could not state the rule in words, they could apply the rule when they had to differentiate these functions without using DERIVE. This is a clear case of non-verbal competence. Students are able to conceptualize - the problem is the language.

Results of differentiation without using the computer in table 2.2 are as follows:

	CORRECT	INCORRECT	NOT COMPLETED
STUDENT NUMBERS	16	6	2

Also, when I observed the incorrect responses, it was not a case of mathematical misconceptions but arithmetic or calculation errors. For an example a student would write the derivative of $\frac{1}{4}x^{16}$ as $64x^{15}$. One would immediately note that the error here is one of arithmetic rather than conception. Other examples that can be cited are $\frac{d}{dx}(-12x^4) = -44x^3$ and $\frac{d}{dx}(3x^2) = 9x$.

Otherwise these results indicate that students had grappled and grasped the principle of differentiating power functions.

Activity 2

The second activity of this session was on the application of the differentiation rule to functions with a constant added to them i.e. functions of the form $cx^n + a$, a being a constant.

Students had to complete activity 3.1 of appendix A. The results of this exercise are as follows:

	ENTRIES	
	CORRECT	INCORRECT
COLUMN 2	22	2
COLUMN 4	20	4

The two (2) incorrect entries in column 2 had the derivative of $4x^{15}$ as $60x^{16}$. Each of the 4 incorrect entries in column 4 had one error. For example 2 had the derivative of $-2x^3 + 1$ as 22. I think this is more of an incorrect manipulation of the HP95LX - a case of blind faith in the computer. One student had the derivative of $-2x^3 + 1$ as $-6x$. Interestingly the second column of this student had the derivative of $-2x^3 + 1$ as $-6x^2$. I, therefore, attributed the incorrect entry of this student in column 4 to carelessness. The last incorrect entry just reentered the derivative of $4x^{15} + 16$ as $4x^{15} + 16$. Again I regarded this as a concentration lapse.

Students had to write down a rule for this differentiation. I could classify responses into four categories. The first category stated that to differentiate functions of the form $cx^n + a$ with respect to x one had to just differentiate the first term and ignore the constant a i.e. to make it 0. Eight students responded this way. The second category to which 7 students belong, stated that the derivative of a function plus a constant is the same as the derivative of the function without the constant. Five students' formulation did not make sense. An example: "We multiply the power by a constant and then we add the constant."

Four students left this section blank. Students had to fill in table 3.2 of appendix A without using the computer. Nineteen students differentiated correctly with 5 students making the arithmetic error that $\frac{d}{dx}(5-x^3) = 3x^2$. This again shows that the basic principle of differentiation is grasped. This again nullifies the notion of the correct verbal articulation as an indication of concept formation. Students are able to conceptualize without being able to verbally articulate their conception.

6.3.4 Session 4 (18.10.94)

This was the last session of the implementation. The aim was to extend differentiation to polynomials and product functions.

Activity 1

Students had to use DERIVE to find a rule for the differentiation of polynomials. Activity 4 of appendix A was dealt with. As one can observe, examples of polynomials in x are given and students were required to write down their own examples. Out of 21 students 20 wrote 2 correct polynomials and 1 wrote only 1 correct polynomial.

Using DERIVE, students completed table 4.2 which required derivatives of the given polynomials. Seventeen students obtained correct answers and 4 obtained 1 incorrect answer each. These I attributed to faulty computer manipulation by students or carelessness. For instance one student wrote the derivative for number 4 as $12x^5 + 15 - 12x^2$ leaving the x^4 in $15x^4$. Similarly another wrote $12x^5 - 15x^4 - 12x^2$ and another $12x^5 + 15x^{-4} - 12x^2 + 2$. The last one wrote the derivative of number 2 as $36x^2 - 12$. This clearly is the second derivative of $3x^4 - 6x^2 + 12x$. These mistakes clearly justify my assertion of faulty computer manipulation. The majority of students obtained correct answers.

In the articulation of the rule for differentiating polynomials 15 students stated that each term is differentiated separately, 5 had nonsense wording and 1 student left it blank.

The worksheet also requires that each student should write down three polynomials and then exchange his/her worksheet with a neighbour so that each could calculate the derivatives of the others' polynomials. These are then returned to their original owners to check for correct differentiation.

Results of this exercise show that all 21 students wrote polynomials correctly. One would expect that this be the case, considering it as a repeat of the introduction in the worksheet.

The table below gives the scores for the differentiation. Only 18 students did the differentiation.

SCORES	3	2	1	0
STUDENTS	9	2	2	5

Six of these students who did not obtain all derivatives correct made a mistake in the differentiation of the constant. In all these cases students found the correct derivatives of the non-constant terms. What happened is that when it came to the constant term they simply added to a constant derivative. An example is this case: $f(x) = 7x^2 + 2x + 9$ then $f'(x) = 14x + 2 + 9 = 14x + 11$. This implies that the student correctly differentiates $7x^2 + 2x$ then adds 9 to $14x + 2$. Generally I felt that the basic understanding of differentiation was kept and these minor problems can be ironed out by revision and drill-and-practice.

Activity 2

This was concerned with finding the derivatives of products written in factor form like $(2x + 3)(2x - 3)$. The objective was to make students aware that if one is given this product one would have to expand it first before finding the derivative. Students had to do activity 5 of appendix A.

In table 5.1 students are given products. They are then required to guess the derivative of each product and then use DERIVE to find the derivative. The idea then was to use DERIVE to verify students' guesses. Ultimately the students would see from answers provided by DERIVE that to find the derivative of a product function one has to first expand the product before differentiating. The scores of students for column two stand as follows:

SCORE	5	4	3	2	1	0
NO OF STUDENTS	7	4	3	4	1	3

Of the seven who obtained 5 correct answers in column 2 it became apparent that 5 had first used DERIVE to fill in column 4 and then used this to fill in column 2. This is because their column 2 and column 4 answers were identical i.e. $6x(x - 1)$, $2(x + 2)$ etc. This is DERIVE's way of writing the derivative of products functions. If these students had first expanded the functions, their derivatives would not appear the same as those of DERIVE. For instance the function $(2x^2 + 3)(x^3 - 1) = 6x^5+3x^3-2x^2-3$ whose derivative is $30x^4 + 9x^2 - 4x$. DERIVE would write it as $x(30x^3 + 9x - 4)$.

In writing the rule for the differentiation of product functions, 19 responses stated that one needs to first expand and then differentiate. Three worksheets came back blank in this section.

Lastly students had to complete table 5.3 without DERIVE, the scores stand as follows:

SCORES	4	3	2	1	0
NO OF STUDENTS	4	11	5	1	1

Most of the errors I observed were calculation errors. All students had firstly expanded the products and then differentiated.

6.4 *CONCLUSION*

At the end of the session, I thanked the students for their co-operation in the exercise. I asked them to give me written comments about their feelings on the exercise. Here are some of the comments:

- I feel very happy and I learn too much. So I wish it to (be) continued because it is easy to calculate with computer.
- From the day I started working with this computer I felt good. I hope I can work with this computer next year. Andiswa.
- Working with this computer gives me interest. I would love to continue with this work next year. Nempucuko.
- This computer makes me very existed (sic) in Mathematics and my mind becomes very quickly (sic).
- It would be great to our studies if the government, as the people who have been deprived for a long time, can see a great need that we reach such complex machines, like the people from the opposite side. Thembani.

The essence of these comments is the excitement raised in students by these computers because it makes it easier to do mathematics. The computer handles the algorithm whilst students concentrate on conception.

At the end of the implementation I met the subject teacher, the HOD and the principal to thank them for allowing us to conduct the research at this school. I asked the subject teacher to include a 20 mark question in the final examination. I wanted to see how students performed in the calculus section if they were not using DERIVE. The subject teacher agreed. I duly set the question and faxed it to her with the memorandum (See Appendix D). After the examination and having established with the principal that marking had been completed, I went to the school to see how the students had fared. To my deepest disappointment the subject teacher told me she had decided not to include the question in her question paper because it was not in the standard 9 syllabus. I told her of my disappointment that she did not have the courtesy to inform me of her decision. I could have arranged to give a short test to the students who, I felt, were positively disposed to the idea when I talked to them.

I consoled myself because I knew that things do not always go according to one's wishes. In my own mind the implementation was concluded successfully and had given me and the students rich insights.

CHAPTER 7: FINAL REFLECTION and CONCLUSION

7.1 INTRODUCTION

As stated in the first chapter, this study is an investigation into the use of computer algebra systems in the teaching and learning of differential calculus in secondary schools. The focus of interest is concept formation by students whilst the computer is doing the necessary algorithmic calculations and graphing. The CAS programme DERIVE was used in a standard 9 class to investigate this concept formation process.

This chapter constitutes a conclusion of the research report. It gives an interpretation of the implementation process and locates the results of the implementation in the context of theory espoused in the preceeding chapters. An evaluation of the whole research and proposals for future research are given.

7.2 INTERPRETATION OF RESULTS

In interpreting the results of the research into concept formation I can begin by saying that the familiarization of students with the technology is essential if students are not going to be bogged down with manipulation problems during the stage of implementation. It became clear during the implementation that the students were familiar with the computer technology as we were not distracted from the task at hand by students needing assistance with the keying procedures. To obviate problems with the setting up of axes and scaling in the graphics window, I set them up myself before lessons started so as to save time. No serious manipulation problems were observed during the experiment.

With regard to the formation of the concept of the derivative, I can say that the understanding of preconcepts is necessary in any process of learning new concepts. By preconcepts I am referring to concepts that serve as building blocks of new concepts. For an example the concept of gradient is a building block of the derivative. Similarly students ought to know what a tangent line is before embarking on a process of creating knowledge of the derivative. Consequently, in terms of this understanding, the concept of the gradient of a straight line was tested in the first worksheet and students exhibited no problem in their knowledge of the concept. Similarly with the tangent line which was verbally asked from the students, they had the definition and understanding of the concept from geometry and thus did not have problems in translating this notion to curves of functions.

Whilst there were initial difficulties of following procedures to yield equations of tangent lines points constituted by values of x at points of contact of tangent lines with the curve of $y = x^2$ and the gradients of the tangent and the derivation of the equation of the line passing through these points, there is evidence that students were able to construct the concept of derivative and then apply it to various situations. Having derived the rule that $\frac{d}{dx}x^n = nx^{n-1}$ students were able to complete table 1.7 of worksheet. The implication for this is that these students were now able to differentiate power functions without the use of computer technology. The computer had assisted them in constructing this knowledge and now they were able to utilize this knowledge in new situations.

From this one can deduce that concept formation has been successful. Students had been provided with both a graphical and algebraic representation and from this students were able to build an understanding of the derivative as a function derived from another function. Like any function, the derivative can be represented both graphically and algebraically. The computer provided the environment so that students could switch back and forth between the symbolic and visual representation.

Having grasped the concept of the differentiation of power functions students moved to other functions including functions of the forms cx^n , $cx^n + a$ where $a, c \in R;\ n \in N$ and polynomial functions. Evidence shows that students were able to extend their conception of differentiation to these functions with the aid of the computer. Students, through the observation of patterns, derived rules for the differentiation of these functions and then articulated them in words. Evidence shows that even those students who are not verbally competent have an operational conception of the rules of differentiation of these functions. This is shown in their ability to apply these rules in exercises where they did not use the computer. Hence one can deduce that concept formation is not dependent on verbal ability especially when the medium of instruction is not the students' mother tongue.

Generally, one can say that the research did show that students can use computer technology to form concepts and apply the knowledge they have gained in new situations. Furthermore, transcripts of the audiotape show students in communication with one another. Students who do not understand, ask questions. Those who can provide answers readily do so, failing which they ask other groups or the teacher. One can therefore deduce that knowledge in this research occurred in a socially charged environment. Answers provided arose from discussion and negotiation. When there were disputes I was called to intervene and settle the points of difference. The informal atmosphere and group discussions corroborate the view that children learn easily from peers.

Findings of the research experiment corroborate several points in the theoretical framework of this research. One is that computer technology removes "computational drudgery so that students can concentrate on ideas rather than mechanics." (De Marois, 1992. Preface). This is indeed the case as can be observed when DERIVE computed all the tangent lines required for the deduction of the set of points leading to the derivation of the derivative. Hence the computer facilitates concept formation. As one student put it "it is easy to calculate with the computer."

The other point is that the computer provided the experimental environment in which students looked for patterns in order to deduce the basic rule of the differentiation of power functions. This fits well with Hillel e.a. (1992:124) who see CAS as facilitating an experimental environment. It is in this atmosphere that students explored, experimented and interpreted the results of their experimentation. The result in the form of rules, was applied in other situations. Students were then able to gain conceptual understanding of mathematical ideas through the use of the computer.

The notion of the constructivist theory of knowledge acquisition fits in well with how students formed the concept of the derivative. Students were generally left to their own designs in terms of interacting with the computer. Guided steps led to their being able to deduce their own knowledge of rules. These they were able to articulate and apply them to problems. These constructions were made in a social interactionist environment. Hence a social constructivist mode of learning was encouraged.

It is interesting to note the roles of both the student and the teacher in the computer environment. Initially the teacher is more visible whilst the students are still unfamiliar with the technology. As the students become more familiar with the technology the teachers' role becomes less prominent as instructor and becomes a facilitator and an arbiter of problematic situations. At the beginning students tended to call the teacher regularly to assist in the manipulation of the computer but as they became more proficient the teacher is "pushed aside." This is what Julie (1991:108) refers to as the disappearance of security-seeking behaviour. Nevertheless, the gain in confidence and competence in the use of computer technology by students does not imply that the teacher has become redundant. She/he will still be needed to explain concepts that are unclear or to give background information that is essential for a better understanding of new concepts. For instance in this research, if students had never encountered the concept of the tangent, it would have been necessary for the researcher to provide students with the knowledge of what a tangent is. But be that as it may, computer technology is transforming the traditional role of the teacher as instructor and the teacher-student relationship. The teacher has become much more of a catalyst in the interaction between student and computer. The learning process is centered around the student. The previous focus on the teacher as bestower of knowledge is a thing of the past. What is also important to note is that students do not see themselves subsumed under the domination of the computer. On the other hand they see themselves in control once they have gained mastery of the manipulation of the appropriate keys. As mentioned in chapter 5, one student shouted "We are commanders, we command!" This shows that this student has an understanding or perception of control over the machine.

7.3 EVALUATION OF THE RESEARCH

In evaluating the reseach process, I want to start by giving a brief distinction between the computer enhanced method of teaching and learning differential calculus and the traditional instructional method. The major distinction between the two methods is that the instructional method begins with the concept of a limit and uses this concept to define the derivative as the gradient of the tangent to the curve at a particular point. DERIVE on the other hand makes it possible for students to grasp the notion of the derivative without having to conceptualize the notion of a limit. Students see the derived function as being generated by a set of points (n,m) where n is the value of x at the point of contact of the tangent and the curve and m is the gradient of the tangent at that point.

In evaluating the whole implementation process, I believe that whatever its limitations or shortcomings, it has had definite benefits as indicated in the aforegoing discussion.

I feel that the first shortcoming of the research was its timing. The fact that it was undertaken in the fourth term whilst preparations were underway for the final examination, exerted pressure on everyone and the subject teacher in particular. This might explain her tension and reluctance to co-operate with the research. She might have perceived us as eating into her teaching time. Consequently, though I had negotiated and set a test to check an individuals' retention of concepts, she did not include the question in her examination question paper. I could not, therefore, obtain the test scores I required to gauge against students' performance in the classroom.

Research shows that test results are not always a good indicator of pupils' ability. Nevertheless, I could have used these results together with other data for triangulation purposes. If the research had been conducted early in the year I would have had ample time for damage assessment and control. Also if the research and preparation for it had taken longer time I would have had more control over data gathering including shadow studies. More observers could have been called and thus more data gathering techniques so that the outcomes of the experiment could be regarded as being more reliable. Sometimes I was alone in the class and thus relied on my own observations which are definitely biased. Nevertheless students' work and worksheets provided the counterbalance to my own biases. The participant observation by two of my colleagues and supervisor have also assisted to put my own interpretation of the research on an even keel.

I feel that in a second cycle of this study more graphical representation would be useful in building an understanding of the concept of the derived function. In deducing the derived function of $y = x^2$, the graphing capability of DERIVE was used. A set of tangent lines was sketched at various points on the curve of $y = x^2$ and using the gradients of these tangents and the various values of x on the curve $y = x^2$, a set of points was formed. It was then deduced that this set of points lies on a line $y = 2x$ which is then defined as the derivative of $y = x^2$. The problems that I encountered in deducing the function $y = 2x$ with the students made it necessary for me and my supervisor to change this graphing strategy and opt for a more algebraic approach. Nevertheless with more time available for implementation, the graphing strategy would have to be used so that students could deduce from a set of points, the derived functions of other power functions like $y = x^3$, $y = x^4$, $y = x$ and so on.

The other observation I made is that the class was big although it had no more than 25 students as the audiotape recording of a group was nearly impossible because noise from the discussions occurring in surrounding groups tended to drown the discussion in the group of interest. One could only get snippets of the discussion. I feel therefore that in addition to the audiotape a videotape could add to making sense of the discussions. Also a shadow study of one group could assist in obtaining a more microscopic view of the research.

7.4 *CONCLUSION*

In conclusion, therefore, one can say that despite the shortcomings of the research there is evidence that computer algbra systems can provide a mathematics environment in which students can construct new mathematical knowledge. The case that we have investigated has shown that students are able to learn new calculus concepts which they are able to apply in problems.

This research is only a miniscule part of research that should be ongoing to transform the mathematics curriculum and teaching methodology in the light of new computer technology which provides for constructivist learning. This kind of research, developmental research, although it does have short term-benefits, has long-term objectives. The long-term objectives refer to the change in the mathematics curriculum. The change which is envisaged by this research is one that will ensure that each school is provided with a laboratory or a set of palmtop computers with a CAS programme like DERIVE which would facilitate mathematical learning and teaching.

Because of the cyclical nature of developmental research, one would view this study as constructing the first cycle in a spiral that would have firm and conclusive evidence on the ability of CAS to assist concept-formation. Although research has been conducted overseas, more research needs to be conducted in South Africa on the feasibility of the introduction of computer based mathematics education incorporating computer algebra systems. The time is ripe for new proposals on account of the socio-political transformation of the country which would perhaps be more receptive to these proposals.

I propose that having analysed what limitations may exist in this study, a further cycle be conducted to iron out these glitches so that these findings may be corroborated or contradicted. Another proposal is the use of CAS to the whole of the differential calculus syllabus including the plotting of cubic functions and optimization problems.

BIBLIOGRAPHY

Arney, D C and Giordano, F R (1992). How Computing is changing core mathematics. **Proceedings of the Fifth Annual Conference on Technology in Collegiate Mathematics.**

Atiyah, M F (1986). Mathematics and the computer revolution. Hawson, A G and Kahane, J P (eds). **The influence of computers and information on mathematics and its teaching.** Cambridge: Cambridge University Press.

Baxter, N (1992). Learning mathematics using a lab-based constructivist approach. **Proceedings of the Fifth Annual Conference on Technology in Collegiate Mathematics.**

Bibby, N (1991). Wherefore "plug-and-chug." Computer Algebra Versus A-level Mathematics. **The mathematical Gazette,** Number 75. pp 40 - 48

Bollinger, G (1989). Computer algebra in high schools. **Journal of computers in mathematics and science teaching.** Volume 8, Number 3. pp 11 - 16

Christie, P and Collins, C (1984). Bantu Education: Apartheid Education and Labour Reproduction. In Kallaway, P (Ed.) **Apartheid and Education: The Education of Black South Africans**. Johannesburg. Ravan Press.

Community Partnership Project, (1993). **Report on Mfuleni Survey.**

Confrey, J (1990). What constructivism implies for teaching. Davis, R, Maher, C and Nodolings, N (Eds). **Constructivist views on the teaching and learning of mathematics.** Monograph 4. pp 107 - 122

Davis, P J and Hersh, R (1986). **The mathematical experience.** Harmondsworth: Penguin Books Ltd.

De Marois, P (1992). **College algebra laboratories using Derive.** Urbana, Illinois: Mathware.

Department of Education and Training, (1987, 1988, 1989). **Standard 8, 9 and 10 Syllabus.**

Dickey, E (1990). Computer algebra in high school: muMath and Derive. **Proceedings of the Third International Conference on Technology in Collegiate Mathematics.**

Du Plessis, A, Du Pisani, T and Plekker, S (1990). **Education and manpower development.** Research Institute for Educational Planning. Bloemfontein: University of the Orange Free State.

Edusource, (1990-1994). **National matriculation and mathematics results.**

Elliott, J (1981). **Action-research: A framework for self-evaluation in schools.** Cambridge: Cambridge Institute of Education.

Ernest, P (1991). **The philosophy of mathematics education. London:** The Farmer Press.

Ernest, P (1993). Constructivism and the problem of the social. Julie, C, Angelis, D and Davis, Z (Eds). **Political dimensions of mathematics education II: Curriculum reconstruction for society in transition.** Cape Town: Maskew Miller, Longman (Pty) Ltd.

Ernest, P (1994). What is social constructivism in the psychology of mathematics education? **Proceedings of the Eighteenth International Conference for the Psychology of Mathematics** Education. Volume 2.

Freudenthal, H (1991). **Revisiting mathematics education: China lectures.** Dordrecht: Kluwer Academic Publishers.

Gravemeijer, K (1994). Educational development and developmental research in mathematics education. **Journal for research in mathematics education.** Volume 25, Number 5. pp 443 - 471.

Glencross, M (1991). A new mathematics curriculum for a new South Africa. **Pythagoras.** Number 25. pp 25 - 28

Hillel, J, Lee L, Laborde, C and Linchevski, L (1992). Basic functions through the lens of computer algebra systems. **Journal of mathematical behaviour.** 11(2). 119 - 158.

Hjelle, L A and Ziegler D J (1976). **Personality: theories, basic assumptions, research and applications.** Tokyo: McCraw - Hill Kogakusho, Ltd.

Hopkins, D (1985). **A teacher's guide to classroom research.** Milton Keynes: Open University Press.

Howson, A G & Kahane, J P (Eds) (1986). Computers as an aid for teaching and learning mathematics. **Proceedings of the International Commission on Mathematics Instruction.** Strasbourg, 1985.

Julie, C (1991). Equations of inequality: challenging the school mathematics curriculum. **Perspectives in Education.** Volume 13. Number 1. pp 3 - 10

Julie, C (1991). Learning novel mathematics concepts in a computer-enriched environment. **British Journal of Educational Technology.** 22(2). pp 99 - 109.

Julie, C (1991). People's mathematics, graphic calculators and the development of mathematical culture. **Mathematics proceedings: 14th National Convention on Mathematics and**

Natural Science Education. Cape Town: University of Cape Town.

Julie, C (1993). Graphic calculators: beyond pedagogical and socio-political issues. Julie, C, Angelis D and Davis, Z (Eds). **Political dimensions of mathematics education II: curriculum reconstruction for society in transition.** Cape Town: Maskew Miller, Longman (Pty) Ltd.

Kahn, M (1993). **Building the base: report on a sector study of science and mathematics education.** Pretoria: Commission of the European Communities.

Kilpatrick, J (1987). What constructivism might be in mathematics education. **Proceedings of the 11th Annual Meeting of the International Group for the Psychology of Mathematics Education**, Montreal.

Larcombe, T (1985). **Mathematical learning difficulties in the secondary school: pupil needs and teacher rules.** Philadelphia: Open University Press.

Laridon, P (1993). Technology appropriate to the learning of mathematics in South Africa. Julie C, Angelis D and Davis Z (Eds). **Political dimensions of mathematics education II: curriculum reconstruction for society in transition.** Cape Town: Maskew Miller, Longman (Pty) Ltd.

Laridon, P (1993). Towards transforming mathematics education in South Africa. **Pythagoras.** Number 31. pp 40 - 45.

Marsh, T (1989). Microcomputer applications in mathematics education. **Pythagoras.** Number 19. pp 20 - 23.

McKernan, J (1991). **Curriculum: Action research - a handbook of methods and resources for the reflective practitioner.** London: Kegan Page.

NCTM News Bulletin. November 1987.

Polya, G (1957). **How to solve it.** New York: Doubleday and Company, Inc.

Romberg, T A (1992). Perspectives on scholarship and research methods. Grouws, D A (Ed) 1992. **Handbook of research on mathematics teaching and learning.** New York: Mac Millan Publishing Company.

Stickney, A (1990). Introduction to DERIVE with classroom applications: A "Hands-On" workshop. **Proceedings of the Third International Conference on Technology in Collegiate Mathematics.**

Streefland, L (1991). **Fractions in realistic mathematics education.** Dordrecht: Kluwer Academic Publishers.

Tanner, D (1992). Using computer algebra to teach the foundations of calculus. **Proceedings of the First DERIVE conference.** Austria.

Volmink, J D (1993). A different mathematical education for a different South Africa. **Pythagoras:** Volume 31. pp 32 - 37.

Vonder Embse, C (1990). Visualization in precalculus: Making connections using parametric graphs. **Proceedings of the Third International Conference on Technology in Collegiate Mathematics.**

Von Glasersfeld, E (1987). Learning as a constructivist activity. Janvier, C (Ed). **Problems of representation in the teaching and learning of mathematics.** Hillsdale, N J: Lawrence Erbaum Associates.

Waits, B K and Demana F (1990). New models for teaching and learning mathematics through technology. **Pythagoras.** Number 22. pp 36 - 39.

Waits, B K and Demana F (1992). A case against computer symbolic manipulation in school mathematics. **The mathematics teacher.** 85(3). pp 180 - 183.

1 DERIVED FUNCTIONS of POWER FUNCTIONS

1.1 **Power functions** are functions such as x^2; x^3; x^4;... In general, they are functions of the form x^n, where n is a natural number.

Write down two more power functions.

...x^6........ ...x^7.........

Is x a power function? Give reasons for your answer.

.x is a power function because the power of x is one.

1.2 The gradient of a line with equation y = mx + c is m. Complete the following:

Equation of line	Gradient
y = 2x + 1	2
y = 4 - 3x	-3
y = x + 2	1
y = 0,5x + 7	0,5
y = 2	0
y + x = 1	-1

1.3 The derived function of $y = x^2$

1.3.1 In this activity DERIVE will be used. First set up the screen as follows:

Press W (for Window)
S (for Split)
V (for Vertical)
ENTER (to indicate that the window is to be split at column 20)

F1 (to make window 2 the active window)
W (for Window)
D (for Designate)
2 (to make window 2 the graphic screen)

T (for Ticks)
1 (to set Rows equal to 1)
TAB (▹) (to move to Columns)
1 (to set Columns equal to 1)
ENTER

S (for Scale)
.5 (to set the markings on the x-axis to .5)
TAB (▹) (to move to y scale)
2 (to set the y scale equal to 2)
ENTER

1.3.2 Preparation to work with $y = x^2$

Press F1 (to make window 1 the active window)
A (for Author)

Type
t(a):= tangent(x^2,x,a)

Press ENTER

Press A (for Author)

Type
x^2

Press ENTER

Press P (for Plot) twice (to plot $y = x^2$)

Press F1 (to make window 1 the active window)

1.3.3 Finding and drawing the tangent lines

The tangent line of x^2 at $x = a = 3$

Press E (for Expand)
Delete everything and type
t(3)
Press ENTER
The entry for the tangent line in the table at x= 3 is given.

Press P (for Plot) twice
The tangent line is drawn

Press F1 (to make window 1 the active window)
Press A (for Author)
Type [3,6]
Press ENTER (Note that 3 is the x-value and 6 the gradient of the tangent line at x = 3)
Press P (for Plot) twice to plot the point (3; 9).

The tangent line of x^2 at $x = a = 2$

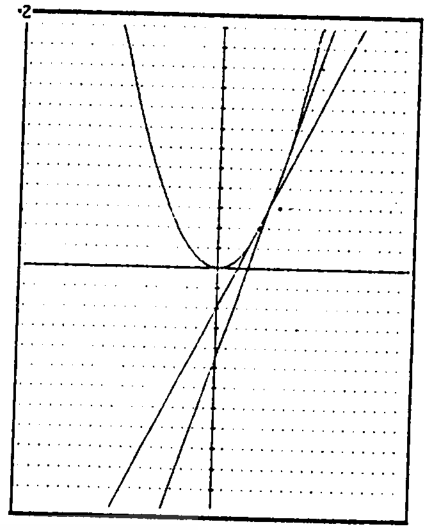
Make the text screen the active window.
Press E (for Expand)
Delete everything and type
t(2)
Press ENTER
The entry for the tangent line in the table at x= 2 is given.

Press P (for Plot) twice
The tangent line is drawn

Press F1 (to make window 1 the active window)
Press A (for Author)
Type [2,4]
Press ENTER (Note that 2 is the x-value and 4 the gradient of the tangent line at x = 2)
Press P (for Plot) twice to plot the point (2; 4).

Complete the table and graph for the other values

$f(x) = x^2$		
Tangent line	x = a	Gradient of tangent line
6x - 9	3	6
4x - 4	2	4
2x - 1	1	2
0	0	0
-2x - 1	-1	-2
-4x - 4	-2	-4
-6x - 9	-3	-6



1.3.4 Consider the points plotted on the graph. Find the equation of the graph that will go through all the points.

Equation of graph: ..2x..........................

Briefly explain how you obtained the equation.

Comparing the co-ordinates and seeing
y as double x

The function that you obtained is called the **derived function** of x^2.

∴ .... is the **derived function** of x^2.

Notation: The derived function of y = x2 is written as

$$\frac{dy}{dx} \text{ or } \frac{d}{dx}x^2$$

$$\therefore \quad \frac{d}{dx}x^2 = 2x$$

1.4 The derived function of x^3

1.4.1 Preparing the graphics screen

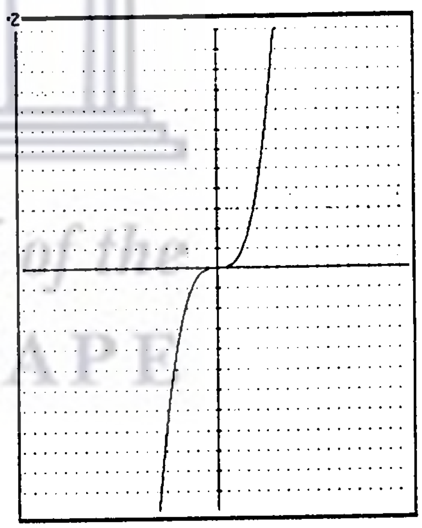
Make the graphics screen the active screen and delete all graphs as follows:
Press D (for Delete)
A (for All)
Press $ (for Scale)
Press TAB (⊳)
Type 6 (to make y scale 6)
Press ENTER

Press F1 to make the textscreen the active screen. Remove all text as follows:
Press R (for Remove)
Delete all numbers and
Type 1 (for the first expression to be removed.)
Press TAB (⊳)
Delete all numbers and type the number of the last expression on the textscreen.
Press ENTER.

1.4.2 Prepare the textscreen to work with $y = x^3$ in the same as it was prepared for $y = x^2$.

Use the table below to find the derived function of $y = x^3$.

$f(x) = x^3$		
Tangent line	x = a	Gradient of tangent line
	3	
	2	
	1	
	0	
	-1	
	-2	
	-3	



For y = x3 the derived function, $\frac{d}{dx}x^3$ = ...

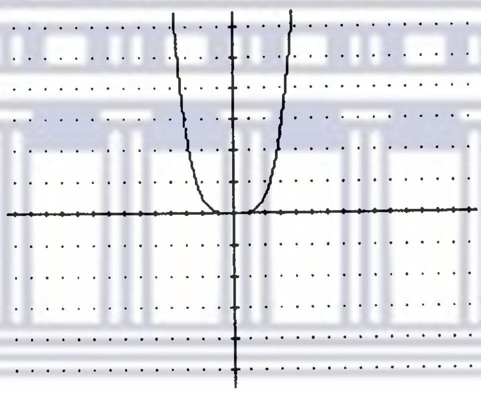
1.5 The derived function of $y = x^4$

Clear the graphics and make the y scale 8.
Make the text screen the active window and remove the expressions in the text screen.

Press A (for Author)
Type x^4
Press ENTER
Press P (for Plot) twice
Press F1 (to activate the text screen)
Press C (for Calculus)
D (for Differentiate)
Press ENTER (to indicate that x^4 must be differentiated)
Press ENTER (to indicate that the variabke is x)
Press ENTER (to indicate that the order is 1)

You should see $\frac{d}{dx}x^4$

Press P (for Plot) twice.
Draw the graph on the sketch below.



What is the equation of the graph of $\frac{d}{dx}x^4$?

Explain how you found the equation
....
....
....
....

Make window 1 the active window and highlight $\frac{d}{dx}x^4$.

Press S (for Simplify)

Press ENTER (to indicate that $\frac{d}{dx}x^4$ must be simplified)

Is the answer the same as the equation you found for the graph of $\frac{d}{dx}x^4$?

Complete the following:

For $y = x^4$, the derived function $\frac{d}{dx}x^4$ = ...

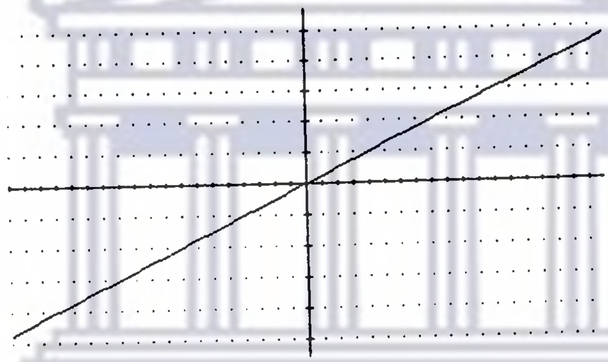
1.6 **The derived function of y = x**

Clear the graphics and make the y scale 1.
Make the text screen the active window and remove the expressions in the text screen.

Press   A (for Author)
Type   x
Press   ENTER
Press   P (for Plot) twice
Press   F1 (to activate the text screen)
Press   C (for Calculus)
    D (for Differentiate)
Press   ENTER (to indicate that x must be differentiated)
Press   ENTER (to indicate that the variabke is x)
Press   ENTER (to indicate that the order is 1)

You should see $\frac{d}{dx}x$

Press   P (for Plot) twice.
Draw the graph on the sketch below.



What is the equation of the graph of $\frac{d}{dx}x$?

Explain how you found the equation
....
....
....
....

Make window 1 the active window and highlight .
Press   S (for Simplify)

Press   ENTER (to indicate that $\frac{d}{dx}x$ must be simplified)

Is the answer the same as the equation you found for the graph of $\frac{d}{dx}x$?

Complete the following:

For y = x, the **derived function** $\frac{d}{dx}x$ = ...

1.7 Complete the following table assuming that the pattern that was obtained for x, x^2, x^3 and x^4 holds.

FUNCTION	DERIVED FUNCTION
x	1
x^2	$2x^1$
x^3	$3x^2$
x^4	$4x^3$
x^5	$5x^4$
x^6	$6x^5$
x^7	$7x^6$
x^8	$8x^7$
x^9	$9x^8$
x^{10}	$10x^9$

Complete the following:

$$\frac{d}{dx}x^{1994} = 1994x^{1993}$$

In general, for the power function x^n, $\frac{d}{dx}x^n = nx^{n-1}$

2 A CONSTANT X A POWER FUNCTION

2.1 Recall that x; x^2; x^3;... are power functions
Now $2x$; $3x^2$; $\frac{1}{2}x^3$;... are examples of
a constant X a power function

Write down two more examples of
a constant X a power function

$4x^4$ .... $5x^5$ ....

In general **a constant X a power function** is indicated by cx^n; where c is a real number and n a natural number.

2.2 DERIVE will be used to complete the table below.

First complete the second column.

Close the graphic screen.

To complete the first row, do the following:

Press	A (for Author)
Type	2x and press ENTER
Press	C (for Calculus)
	D (for Differentiate)
	ENTER (to indicate that the highlighted 2x must be differentiated)
	ENTER (to indicate that x is the variable)
	ENTER (to indicate that the order is 1)

You should see $\frac{d}{dx}(2x)$

Press S (for Simplify)

ENTER (to indicate that $\frac{d}{dx}(2x)$ must be simplified)

2 is the derived function of 2x.

Complete the table by following the same procedure for the other rows

Power Function	Derived Function	Constant X power function	Derived function of constant X power function
x	1	$2x$	2
x^2	$2x$	$2x^2$	$4x$
x^3	$3x^2$	$4x^3$	$12x^2$
x^8	$8x^7$	$\frac{1}{2}x^8$	$4x^7$
x^5	$5x^4$	$-6x^5$	$-30x^4$

Study the entries in column 2 and column 4. Write a rule that you can use to find the derived function of a constant X a power function

When we multiply the function by the constant, the deriditive is also multiply by constant

2.3 Use your rule to complete the following:

Function	Derived Function
$3x^2$	$6x$
$2x^4$	$8x^3$
$-x^3$	$-3x^2$
$\frac{1}{4}x^{16}$	$4x^{15}$
$-12x^4$	$-48x^3$

Use DERIVE to check your answers.

3 ADDING A CONSTANT

3.1 Complete column 2.

Function	Derived Function	Function + Constant	Derived function of Function + Constant
$4x^2$	$8x^1$	$4x^2 + 3$	$8x$
$-2x^3$	$-6x^2$	$-2x^3 + 1$	$-6x^2$
$6x^4$	$24x^3$	$6x^4 - 2$	$24x^3$
$4x^{15}$	$60x^{14}$	$4x^{15} + 16$	$60x^{14}$

Use the Calculus, Differentiate option of DERIVE to find the derived functions of the functions in in column 3. Study the entries in column 2 and column 4. Write down a rule to find the derived function of a function with a constant added to it.

When you have a function added with a constant, your derived function of function + constant is the same as the ~~old~~ dirived function.

3.2 Use your rule to complete the following:

Function	Derived Function
$x + 2$	1
$3x^2 - 6$	$6x^1$
$2x^4 + 5$	$8x^3$
$5 - x^3$	$-3x^2$
$4x^{10} - 14$	$-40x^9$

Use DERIVE to check your answers.

WORKSHEET 5

4 DERIVED FUNCTIONS of POLYNOMILAS

4.1 $x^2 + 3x + 2$; $3x^4 - 6x^3 + x^2 - 12x$ are examples of polynomials in x.

Write down two more polynomials in x.

$x^3 + 2x + 1$ .... $4x^3 + 3x + 5$ ....

4.2 Use DERIVE to find the derived functions of the polynomials in the table below:

Polynomial function	Derived Function
$x^2 + 3x + 2$	$2x + 3$
$3x^4 - 6x^2 + 12x$	$12x^3 - 12x + 12$
$x^{12} - 3x^5 + 7x^3 - 13x^2$	$12x^{11} - 15x^4 + 21x^2 - 26x$
$2x^6 + 3x^5 - 4x^3 + 2$	$12x^5 + 15x^4 - 12x^2$

Write down a rule you think can be used to find the derived function of a polynomial function.
When you differeciate, you derived each of the polynomial terms seperately.
....
....

4.3 Write down three polynomial functions.

$x^{10} + 4x^2 - 11x$ .... ~~$40x^9 + 4 - 11$~~ $10x^9 + 8x - 11$

$x^9 - 5x^3 + 6x$ .... $9x^8 - 15x^2 + 6$

$2x^5 - 3x^6 - 6x^5 + 12x^2$ $10x^4 - 18x^5 - 30x^4 + 24x'$

Exchange wit a friend and find the derived of his or her polynomial functions.

Return your friend's and collect yours. Check whether the derived functions are correct.

$x^2(2x-3)$

$2x^3 - 3x^2$

$6x^2 - 6x$

$6x$

$x^2 + 2x + 2x + 4$

$x^2 + 4x + 4$

WORKSHEET 6.

5 **Derived functions of product functions**

You know how to find the derived function of functions such as x^3, $-12x^5 + 6$, and $3x^4 + 4x^3 - 7x$. This activity is about finding the derived function of a product function - functions made up by multiplying two functions. $x^2(2x - 3)$ is an example of a product function.

5.1 Make a guess of what you think the derived function of the product function in column 1 is. Write your guess in column 2. Use the Calculus, Differentiate function of DERIVE to find the derived function and write this function in column 3.

Product Function	My guess for the derived function	The derived function with DERIVE
x^2x^3	x^5	$5x^4$
$x^2(2x-3)$	$6x^2 - 6x$ ✓	$6x(x-1)$
$(x+2)(x+2)$	$2x+4$ ✓	$2(x+2)$
$(2x+3)(2x-3)$	$8x$ ✓	$8x$
$(3x-4)(x+3)$	$6x+5$ ✓	$6x+5$
$(2x^2+3)(x^3-1)$	$10x^4 - 4x + 9x^2$	$10x^4 + 9x^2 - 4x$

5.2 Study the derived functions found with DERIVE and find a general rule to find the derived function of a product function. Write your below.

you first multiply and then you find the Deredetive function

5.3 Use your rule to complete the following table.

Function	Derived Function
$2x^3.x^5$	$16x^7$
$(2x-1)(x-2)$	$4x-5$
$(x^2-1)(x^2+1)$	$6x^3-1$ ✓
$(3x+2)(2x-3)$	$12x-5$

Use DERIVE to check your answers.

WORKSHEET 7

f
b
th
ar
en
the
arou

3.
- ed
mana
the p
much
howev
check

Monde Mbekwa

Field Notes of HP95LX familiarization workshop with a STD 9 Mathematics class at Mfuleni Combined School

Day 1. 20/09/94

No. of pupils: 25

Subject teacher: Mrs Abrahams

* introduced to class by HOD who then left -teacher also for another cl;ass
* explained to class the purpose of the intended exercise
* asked whether they had seen computers before
* some indicated that they had
* explained to them about computers and the development of computer technology in the classroom and consequently the aim of the research.
* issued out the HP95LX - one per desk-students eargerly opened them even before being instruction to do so.
* had to warn them not to start fiddling
* students asked whether they would be allowed to take computers into the examination room-told them that research is being done on this eventuality
* started with the workshop starting with algebraic manipulations
*Mrs Abrahams joined us in the middle of session
* workshop went relatively well- less than 40% had serious problems-otherwise I struggled since the teacher was also not ofay with the HP95LX-had to go from desk to desk to observe and to assist where necessary
*noticed three groups working extremely well -one boy even stood up to assist others who were struggling(found out later in an informal chat that he was a member of the SRC)
* noticed that teacher was still trying to learn manipulations.
*could not finish working through all problems-decided to have 2nd session.
* decided to meet teacher the next day to assist her

DAY 2(21/09/94)

*met Mrs Abrahams from 12 to 13h00.
*session went well
*gave her HP95LX to practise at home (with manual)

DAY 3(22/09/94)

*assisted by Jerome (who had brought an out -of-programme HP95
* Mrs Abrahams had same problem(her husband had decided to assist her at home and pressed all the wrong keys!)

* session generally went well with the exception of a few students who were experiencing problems - teacher still not of assistance- Jerome assisted (2 girls in front row still far behind -in any case progress of class differential)
* boys noticed on day 1 still excelling - in groups that are mixed boys dominating the girls (to look into this later)
*introduced the graphics window - some struggling in setting axes and scale/centering- time up.
*asked students about another session during the holidays.
*students said they would be prepared to spend the whole school holiday working.
*set appointment to meet them on Tuesday 27th at 10h30

DAY 4(27th/09/94)

* Larry came to assist- Jerome did not(I failed to confirm appointment with him)
* 14 students present (2 do not belong to the maths class-I hear they come from the history class! - nevertheless they seem to be enjoying themselves even assisting in the group after having grasped the manipulations)
*commenced immediately with graphics-went generally well except those still struggling with setting up axes -assisted them
* decided to have another session on graphics considering that less than half the class attended- this to occur when schools reopen.

OCT-20-94 08:55 DERIC TEL: P:01

TT: MRS ABRAHAMS
909 3812 (FAX)

MFULENI COMBINED SCHOOL.

OCT/NOV. 1994

MATHEMATICS STD 9
PAPER 2

QUESTION X

X 1. Complete: $\frac{d}{dx}[x^4] =$ (1)

X 2. Now find $\frac{d}{dx}$ in the ff.

(i) 3 (ii) $3x^5$ (iii) $x^{1/2}$ (7)

(iv) x^{-3}

X 3. Find the derived functions of the ff polynomials

(i) $-6y^4 + \frac{1}{3}y^3 - 2y$ (3)

(ii) $2x^3 + 4x^2 - 3$ (2)

X 4. Calculate the derivatives of the following

(i) $(x^2 - 1)(x - 2)$ (7)

(ii) $(3x + 2)(2x - 5)$ (5)

[25]

NB: You can reduce or add more problems to suit your mark allocation.

MEMORANDUM

QUESTION X (1)

nx^{n-1}

X 2 (i) 0 (2)

(ii) $15x^4$ (2)

(iii) $\frac{1}{2}x^{-\frac{1}{2}}$ (2)

(iv) $-3x^{-4}$

X 3 (i) $-24y^3 + y^2 - 2$ (3)

(ii) $6x^2 + 8x$ (2)

X 4 (i) $f(x) = (x^2 - 1)(x - 2)$

$= x^3 - 2x^2 - x + 2$ (7)

$\frac{d}{dx} f(x) = 3x^2 - 4x - 1$

(ii) $f(x) = (3x + 2)(2x - 5)$

$= 6x^2 - 11x - 10$ (5)

$\frac{d}{dx} f(x) = 12x - 11$

[25]

REVISION QUESTIONS

FIND THE DERIVATIVES OF THE FOLLOWING

1. (a) 5 (b) $6x^4$ (c) x^{-2}

(d) $x^{1/4}$

2. (a) $7x^3 - 3x^2 + 2x - 3$

(b) $\frac{1}{4}x^4 - \frac{2}{3}x^3 - 6x$

(c) $t^{13} - 15t^2 + 3$

(d) $(b+2)(b-4)$

(e) $(4q-3)^2$

(f) $3y^{-2} + 5y^{-2}$

(g) $(t+1)^2(t-2)$

(h) $x^3 - 4x^2 + 6x + 5$

(i) $(2x-3)(5x+1)$

You can assist them with the answers to these questions.