

Implementing productive practice in a grade 9 Mathematics class: A design research study.

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A thesis submitted in fulfilment of the requirements for the degree, Master in Education in the
Faculty of Education. University of the Western Cape, South Africa.



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DECLARATION

I declare that **implementing productive practice in a grade 9 Mathematics class through a design research study** is my own work, that has not been submitted before for any degree or examination in any other university, and that all the sources I have used or quoted have been indicated and acknowledged by complete references.



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Date: June 2023

Signed:

A handwritten signature in black ink, appearing to read 'Luxolo Mlofane'.

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ABSTRACT

Several attempts have been made to encourage practice in Mathematics, but no consensus has been reached to implement practice to promote mastery of procedural fluency in integer arithmetic in a grade 9 Mathematics class. To determine the role of practice in Mathematics, the research addressed the significance of “productive practice” to promote mastery of procedural fluency in integer arithmetic in a grade 9 Mathematics class as forthcoming from the National Senior Certificate diagnostic reports on Mathematics. The study focused more on procedural fluency in integer arithmetic in a grade 9 Mathematics class. Hence, the research question of the study was: “Does productive practice promote mastery of procedural fluency in integer arithmetic?” In the study, I gave learners work related to procedural fluency in integer arithmetic and asked them to analyse the procedures and reflect on possible solutions. Observations of learners working with the designed activities and a teacher reflective journal were the types of data collection methods that I used in the study. The results were discussed per activity and the groups’ written responses to the questions in the five designed activities dealing with integer arithmetic in a grade 9 Mathematics content. The results emphasis was on how the groups responded to designed activities. The objective of this study is to contribute towards ways to improve achievement outcomes in school Mathematics.

Keywords: Productive practice; mass practice; distributed practice; deliberate practice; drill and practice; analysing and reading questions; understanding of concepts; procedural fluency with fundamental and mathematical computations.

TABLE OF CONTENTS

| | |
|-------------------------|----------|
| TITLE PAGE | |
| DECLARATION | i |
| ACKNOWLEDGEMENTS | ii |
| ABSTRACT | iii |
| TABLE OF CONTENTS | iv – vii |

CHAPTER 1

INTRODUCTION

| | | |
|-----|--|-------|
| 1.1 | Background | 1 |
| 1.2 | Motivation of the study | 2 – 3 |
| 1.3 | Research aim and research question | 3 |
| 1.4 | Significance of the study | 3 – 4 |
| 1.5 | Organisation of the study | 4 – 5 |
| 1.6 | Content of thesis | 5 – 6 |

CHAPTER 2

LITERATURE REVIEW

| | | |
|-----|----------------------------|---------|
| 2.1 | Role of practice | 7 |
| 2.2 | Mass practice | 7 – 9 |
| 2.3 | Distributed practice | 9 – 10 |
| 2.4 | Drill and practice | 10 – 12 |

| | | |
|------|---|---------|
| 2.5 | Deliberate practice | 12 – 14 |
| 2.6 | Mastery learning | 14 – 16 |
| 2.7 | Productive practice | 16 |
| 2.8 | A productive practice perspective | 16 – 18 |
| 2.9 | Integer arithmetic | 18 – 19 |
| 2.10 | Conclusion | 19 |

CHAPTER 3

RESEARCH DESIGN

| | | |
|-------|---|---------|
| 3.1 | Research approach | 20 – 22 |
| 3.2 | Learning trajectory | 22 |
| 3.3 | Instructional activities | 22 – 23 |
| 3.4 | Participants | 23 |
| 3.5 | Validity and Reliability | 23 – 24 |
| 3.5.1 | Internal and external validity | 24 - 25 |
| 3.5.2 | Internal and external reliability | 25 |
| 3.6 | Data collection | 25 – 27 |
| 3.7 | Data analysis | 27 – 30 |
| 3.8 | Ethical considerations | 30 – 33 |
| 3.9 | Conclusion | 33 – 34 |

CHAPTER 4

RESEARCH RESULTS

| | | |
|-------|----------------------------|---------|
| 4.1 | Results per activity | 35 |
| 4.1.1 | Activity 1 results | 35 – 41 |

| | |
|--------------------------------|---------|
| 4.1.2 Activity 2 results | 41 – 46 |
| 4.1.3 Activity 3 results | 46 – 50 |
| 4.1.4 Activity 4 results | 51 – 53 |
| 4.1.5 Activity 5 results | 53 – 55 |
| 4.2 Conclusion | 55 – 56 |

CHAPTER 5

DISCUSSION, REFLECTION, RECOMMENDATION AND CONCLUSION

| | |
|--|---------|
| 5.1 Introduction | 57 |
| 5.2 Discussion of results | 57 – 59 |
| 5.3 Reflection on implementation | 59 – 63 |
| 5.4 Recommendations for further research | 63 – 66 |
| 5.5 Conclusion | 66 |

| | |
|-------------------------|---------|
| REFERENCES | 67 – 70 |
|-------------------------|---------|

| | |
|-------------------------|----|
| APPENDICES | 71 |
|-------------------------|----|

| | |
|------------------|---------|
| Appendix A | 71 – 75 |
|------------------|---------|

| | |
|------------------|----|
| Appendix B | 75 |
|------------------|----|

| | |
|------------------|----|
| Appendix C | 76 |
|------------------|----|

| | |
|------------------|----|
| Appendix D | 77 |
|------------------|----|

| | |
|------------------|---------|
| Appendix E | 78 – 79 |
|------------------|---------|

| | |
|------------------|----|
| Appendix F | 80 |
|------------------|----|

LIST OF FIGURES

| | |
|----------------------------------|---|
| Figure 1: Layout of Thesis | 5 |
|----------------------------------|---|

| | |
|---|----|
| Figure 2: A Productive Practice Perspective | 17 |
|---|----|

| | |
|-------------------------------------|---------|
| Figure 3: Response of Group 1 | 35 – 36 |
| Figure 4: Response of Group 2 | 38 |
| Figure 5: Response of Group 1 | 42 |
| Figure 6: Response of Group 1 | 46 – 47 |
| Figure 7: Response of Group 1 | 49 |
| Figure 8: Response of Group 1 | 51 |
| Figure 9: Response of Group 1 | 53 – 54 |

LIST OF TABLES

| | |
|---|---------|
| Table 1: Summary of the Groups` Responses to Questions 1.2; 1.4 and 1.5 | 37 |
| Table 2: Summary of the Groups` Responses to Questions 1.1 and 1.3 | 39 |
| Table 3: Summary of the Groups` Responses in Activity 2 | 44 |
| Table 4: Summary of the Groups` Responses to Question 3.1 | 48 |
| Table 5: Summary of the Groups` Responses to Question 3.2 | 50 |
| Table 6: Summary of the Groups` Responses to Questions 4.1; 4.2 and 4.3 | 52 |
| Table 7: Summary of the Groups` Responses to Questions 5.1; 5.2 and 5.3 | 55 |
| Table 8: Summary of Responses of Groups for Different Activities | 57 – 58 |

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CHAPTER 1

BACKGROUND, MOTIVATION OF THE STUDY AND CONTENT OF THESIS

1.1 Background

Some learners in grade 9 Mathematics find integer arithmetic difficult. The difficulty sometimes arises from learners ignoring the signs of integers and operating with them as if they are natural numbers when working with integer arithmetic. A teaching method frequently used focuses on emphasising memorisation as a learning technique.

In my perspective, some teachers take the short route to teach integer arithmetic and do not allow learners to recognise integers. According to Mbekwa (2002) Mathematics education in South Africa is in difficulty when it comes to equity and quality achievement of learners' academic performance. Soga (2017) states that the chalk-and-talk teaching method mostly contribute to learners' low performance in school Mathematics.

When it comes to the chalk-and-talk teaching method, the learning outcomes that learners need to know at the end of the lesson are mostly not considered. Furthermore, the chalk-and-talk teaching method sometimes can lead to learners not developing procedural fluency and understanding of mathematical concepts.

There are three issues in Mathematics from the National Senior Certificate (NSC) Mathematics diagnostic reports that need to be addressed to enhance academic performance and this must happen from the lower grades (8 - 10) as stated by Julie (2021). The three issues are understanding concepts, analysing and reading questions and procedural fluency with fundamental and basic mathematical computations. Julie (2021) offers a productive practice perspective to address the above three issues. The next section deals with the motivation of the study.

1.2 Motivation of the study

The concern of the study is the learners' mastery of integer arithmetic in grade 9 Mathematics. The study investigated how learners dealt with activities to enhance their understanding of mathematical concepts and develop procedural fluency when productive practice was employed. The research study also investigated how spiral revision and deepening mathematical thinking address procedural fluency of integer arithmetic. Productive practice is a form of practice consisting of spiral revision and deepening mathematical thinking.

Julie (2021) states that productive practice focuses on the work already done and is used to cover the range of activities that test and extend learners' thinking. Productive practice is unique to fit the context within which South African teachers teach.

Julie's (2021) productive practice is the type of practice that focuses more in a distributed manner on the work that has been already taught. Productive practice helps learners to master prior knowledge and develop new skills. Julie's productive practice also assists learners to be able to explain and justify solutions, and learners are also able to reflect on their ways of working.

Spiral revision and deepening mathematical thinking address ways to improve school Mathematics achievement. Spiral revision and deepening mathematical thinking are embedded in the toolkits with activities that Julie proposes to assist learners consolidate and revise the work done in the classrooms. In general, spiral revision and deepening mathematical thinking help to identify mathematical concepts that learners did not master from work done in the classrooms.

Spiral revision and deepening mathematical thinking are implemented by teachers in classrooms not more than 10 minutes before the new lesson starts. Spiral revision and deepening mathematical thinking assist learners in mastering the work done previously. The research intends

to address the significance of productive practice concerning the following three issues from the National Senior Certificate (NSC) diagnostic reports (Department of Basic Education, 2021, p. 186).

- *Understanding of concepts* – which is about the comprehension of mathematical concepts, relations and operations.
- *Analysing and reading questions* – is about the capacity for logical thought, explanation and reflection.
- *Procedural fluency with fundamental and basic mathematical computations* – which is about carrying out procedures flexibly, efficiently, appropriately and accurately.

Julie (2021) states that in productive practice learners are expected to produce something on their own. Julie's productive practice seeks to improve Mathematics achievement in examinations. It is necessary to study this topic and the need for the study is to add value to the improvement of Mathematics achievement in schools. The research aim and research question are addressed in the next section.

1.3 Research aim and research question

This study investigated how productive practice promotes mastery of procedural fluency in integer arithmetic in grade 9 Mathematics. My research question is: Does productive practice promote mastery of procedural fluency in integer arithmetic? The significance of the study is dealt with in the next section.

1.4 Significance of the study

Public schools in South Africa are funded by the national government through the Department of Basic Education (DBE). Public schools are accountable to the national government by improving learners' results in examinations. According to the Department of Basic Education report some schools are underperforming (Department of Basic Education, 2021). School underperformance puts a lot of pressure on the resources of the basic education system. It is for these reasons that learners should consistently master integer arithmetic in school Mathematics through quality teaching and learning.

The findings of this study may help basic education employees and employers understand the importance of using productive practice in school Mathematics. The knowledge gained from this study may be used in public schools to advise on mastering procedural fluency in grade 9 Mathematics.

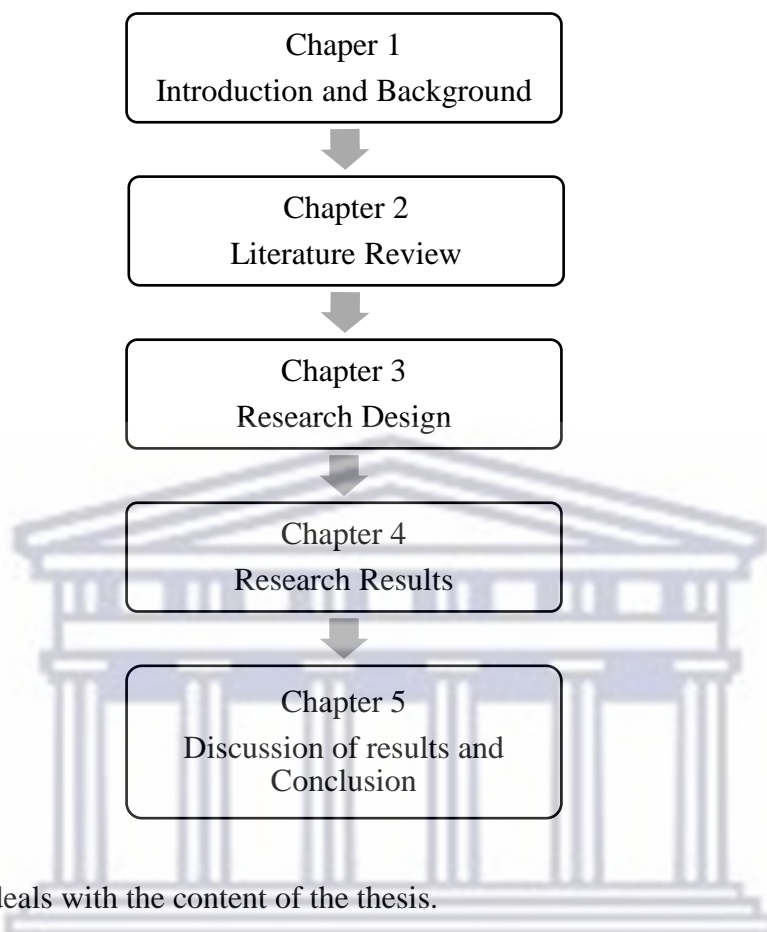
Furthermore, this study will make three main contributions. Firstly, it will promote mastery of integer arithmetic. In addition, insight gained from this study will advise on important ideas for implementing productive practice in school Mathematics. The outcome of this study will be a framework for the implementation of spiral revision and deepening mathematical thinking. Lastly, this study is novel to the proposed study setting, meaning new findings might contribute to existing literature. The next section deals with the organisation of the study.

1.5 Organisation of the study

This thesis consists of five chapters, as depicted in Figure 1 below, which gives an introduction to the flow of this study.

Figure 1

Layout of Thesis



The next section deals with the content of the thesis.

1.6 Content of thesis

Chapter 1 is about the introduction background, motivation of the study, the research aims, research question, and the significance of the study and the content of the thesis. In Chapter 2 I describe the role of practice and different forms of practising. These forms are mass practice, distributed practice, drill and practice, deliberate practice and productive practice.

In Chapter 3 I describe the research approach, learning trajectory, instructional activities, participants, data collection, data analysis, ethical considerations, validity and reliability. Chapter

4 deals with the research results of the study. In Chapter 5 I discuss the research results addressed in Chapter 4 and reflect on the design of activities from the design research perspective. The next chapter deals with the literature review.



CHAPTER 2

LITERATURE REVIEW

In this chapter I discuss the role of practice and different forms of practising. These forms are mass practice, distributed practice, drill and practice, deliberate practice and productive practice. Furthermore, in this chapter I also discuss mastery learning and integer arithmetic. The chapter concludes with a discussion from a productive practice perspective and how it is linked to my research question.

2.1 Role of practice

The verb “practise” means to do something repeatedly. In my understanding, the role of practice in Mathematics class is to enhance sustainable performance. In other words, the role of practice is to ensure that learners provide responses to problems after mathematical topics have been taught. Practice can also help learners to develop creative thinking and reasoning. Furthermore, the role of practice is to give learners support in learning and encourage them to reach upper levels of expertise.

Dacey et al. (2018) allude that practice can assist learners to become successful in Mathematics class; practice enhances problem-solving skills and develops sense-making in Mathematics. The role of practice is to allow learners to gain a deeper understanding of Mathematics and make connections between mathematical concepts. There are different types of practice, namely mass practice, distributed practice, drill and practice, deliberate practice and productive practice. In the next section I discuss mass practice.

2.2 Mass practice

Murray and Udermann (2003) refer to mass practice as the type of practice with small breaks or a few rest periods of short duration during the work interval. Mass practice describes studying that is done less frequently, but similar activities are practised immediately after it was taught and then, normally before examinations (Murray & Udermann, 2003).

Rohrer and Taylor (2007) describe mass practice as the study that continues immediately after the material is presented. In mass practice learners are given more work to do, soon after what has been taught in a particular lesson and there is no break in between. Learners are also given time to do an activity on their own so that they can strengthen the concepts.

Mass practice is excellent for memorisation, good for learning simple skills and good for habitual responses. Another benefit of mass practice is to reinforce technique through repetition. Rohrer and Taylor (2007) argue that mass practice is a good type of study when the skill is simple and detached. O'Laughlin (2020) states that mass practice can be a good teaching strategy to use for learning new information in a short period.

The shortcoming of mass practice is that there is no time for feedback. Nguyen et al. (2019) state that mass practice takes more time to learn the same information in one lesson, it is too demanding and more tiring. Mass practice is less effective for developing understanding but the research has not determined whether it is less effective in all tasks that are taught to learners (O'Laughlin, 2020).

Another shortcoming of mass practice is that it comes down to learners cramming for the test and examination. Even though mass practice may feel efficient for learners to cram and pass examinations, it is not a satisfactory long-term solution to promote learners' performance. According to my experience, learners are likely to forget memorised information once the

examination is over. Therefore, what this implies is that mass practice is not a good study plan. Learners require more learning opportunities to acquire more skills and knowledge. In the next section I discuss distributed practice.

2.3 Distributed practice

Distributed practice is a form of studying whereby learners practise the work in sessions spaced out over time. Schutte et al. (2015) refer to distributed practice as the learning approach that describes a more spaced-out technique where studying is in intervals over time. In distributed practice, instead of studying all of the material for hours the night before an examination, learners study the previously taught material more regularly before examinations.

Rohrer and Taylor (2007) allude that distributed practice involves no more work than mass practice. In distributed practice, there is an interval for recovery and less intellectual pressure. The best way to use distributed practice is when the skill is complex. Overall, distributed practice correlates to a much higher academic success rate than mass practice.

The benefit of distributed practice is that it allows time for feedback between sessions. Rohrer and Taylor (2007) found benefits of spacing practice in Mathematics. For instance, distributing practice over time helps learners remember more. Learners benefit from distributing their study of the same material and no matter what such studying involves.

Distributing practice benefits learners by not to cram for examinations and for them to learn with understanding. When practice problems relating to a given topic are spaced across multiple sessions, a learner who is incapable of understanding during a lesson will be able to solve some of the difficulties in the next lesson, as stated in the study of Rohrer and Taylor (2007). The shortcoming of distributed practice is that it is “time-consuming”.

When it comes to distributed practice, a teacher may do a certain concept and allow learners to do activities related to the concept over many spaced-out sessions. Seabrook et al. (2005) state that knowledge from distributed practice is encoded into our understanding during sequential presentations of the information rather than just the first time. In distributed practice, learners are given more time to understand what has been taught to them in several sessions.

For instance, when a skill is practised on Monday and Tuesday and tested on Thursday, the retention interval equals three days and test performance is usually superior after distributed practice rather than mass practice. Distributed practice is less tiring than mass practice and that is why they are different. In a classroom setting, the study of Seabrook et al. (2005) found that distributed practice produces better learning than mass practice. Distributed practice is significant for learners to understand because retaining the knowledge of what learners learn in school is ultimately going to determine how successful they will be in their future careers. In the next section I discuss drill and practice.

2.4 Drill and practice

Lehtinen et al. (2017) define drill and practice as the simple repetition of tasks without any extra analyses of the quality of practice. Drilling is the formation of good or bad habits through regular practice. Kani and Sa`ad (2015) state that drill and practice is a method that is often used in the traditional teaching method to get learners to learn the first basics of Mathematics.

When it comes to the study of Kumar (2010), drill and practice is the method of instruction characterised by systematic practice problems and repetition of examples and concepts. Drill and practice involve an awareness of values, relationships and number names. Kumar (2010) alludes that learners must learn to count and be able to count the number of things in a group by patterns.

Drill and practice is a method of instruction characterised by systemic repetition of examples, mathematical concepts and practise problems.

Drill and practice can be interesting and beneficial to the learners if a teacher has the ingenuity to repeat mathematical concepts and examples in various ways (Kumar, 2010). Based on the study of Kumar (2010), it is clear that drill and practice is not always suitable for a classroom set-up. To make drill and practice more efficient in the Mathematics classroom, learners must practise the correct procedure and information. The correct practice makes perfect.

Kani and Sa`ad (2015) state that drill and practice allow learners to learn certain concepts effectively and quickly. The benefit of drilling is to provide basic skills for learners. For instance, learners learning algebra require to have mastered the basic skills of subtraction, addition, division and multiplication. Furthermore, drill and practice also allow learners to build on mastered skills.

Another benefit of drill and practice is to allow learners to learn by themselves. Drill and practice encourage learners to achieve skills and ensure perfect performance (Kani & Sa`ad, 2015). In drilling, there is specification and limit to what is learned. Reinforcement is also present in drill and practice.

According to Kumar (2010), drill and practice assist to identify the weakness of the learners. Drill and practice help learners to understand the procedural knowledge of Mathematics. Some learners can benefit from drill and practice since there is a repetition of examples to recall basic facts of mathematical procedure and attain higher order Mathematics skills.

Drill and practice might benefit learners to learn with understanding. Learners can use drill and practice with one another for shared learning opportunities. Drill and practice is also beneficial to assist a teacher to distinguish between not-strong and strong learners.

The shortcoming of drill and practice is that it is “time-consuming”. When learners rely on drilling heavily, learners may only be learning things for the sake of getting to the next level and not gaining a full understanding of mathematical concepts (Lehtinen et al., 2017). In other words, it is hard for the learners to focus when drilling is done too often. Drill is narrow in content and aim, and does not call for much understanding from the learners being drilled.

Kani and Sa`ad (2015) believe that when drill and practice is done in excess, it is most likely to destroy initiative. Drill and practice involves the more difficult task of promoting learners` reasoning and understanding. Mostly, drill and practice assist learners in understanding mathematical concepts. However, drilling and practising something more often when mathematical concept is not well introduced, cannot improve learners` understanding.

Another shortcoming of drill and practice is when some learners rely on just recalling to pass a test but without truly understanding the material properly. When the learning becomes too predictable, learners may not gain clear information about the skills they are supposed to master. Drill and practice do not lead to adaptive and flexible number knowledge (Lehtinen et al., 2017).

Drill and practice do not always lead to perfection. In drill and practice when the specific learning outcomes are not clarified and identified, the learners` needs may not be addressed. Another shortcoming of drill and practice is that it can only be effective if it is joyful and interesting to learners.

When there is no clarity in what is being practised, then it will not be effective for learners. In addition, if there is no systematic integration into the teaching and learning process, then drill and practice will not be effective. In the next section I discuss deliberate practice.

2.5 Deliberate practice

Ericsson (2016) defines deliberate practice as repetitive actions that are driven by the specific goal of improving someone's future ability to think. For Ericsson (2016), deliberate practice is a type of practice that is systemic and purposeful. Hambrick et al. (2014) state that deliberate practice is an evidence-based method of skill development.

According to Hambrick et al. (2014), deliberate practice is the type of practice with a clear awareness of the specific skill and competence that one aims to improve. Deliberate practice slightly fits into my study since it is invaluable for improving performance in the field such as teaching. Furthermore, deliberate practice is most suitable to be used in the sports and music industry. Lehtinen et al. (2017) state that deliberate practice is about the improvement of high-level performance.

According to Ericsson (2016), the benefit of deliberate practice is to modify and improve previously attained skills and also to build progressive skills on top of existing skills. Another benefit of deliberate practice is to focus on well-defined specific goals in order to improve some aspects of performance. Furthermore, deliberate practice turns potential into reality.

Deliberate practice also ensures that the fundamental skills are correct and well-developed. Deliberate practice is also beneficial to overcome many of the limitations that one might view as fixed. While engaging in deliberate practice, one always looks for errors or areas of weakness and establishes a plan to improve them, which is the benefit of deliberate practice.

The shortcoming of deliberate practice is to focus typically on developing skills that are already known by other people and that practice can sometimes be boring. Deliberate practice demands maximum effort which is not always entertaining. It also requires full attention and self-control.

However, deliberate practice is invaluable for improving learners' performance. The study of Lehtinen et al. (2017) demonstrates that lengthy engagement in deliberate practice is needed for learners to achieve their highest levels of performance. Therefore, based on the study of Lehtinen et al. (2017), lengthy engagement is a shortcoming of deliberate practice, especially in the field of teaching.

Another shortcoming of deliberate practice is the fact that it only aims to guide and direct future training (in the sport and music industry) to develop levels of performance. In general, deliberate practice is about how people prepare themselves for the future. Lehtinen et al. (2017) state that there are few studies that apply the concept of deliberate practice in formal educational contexts. Therefore, what this implies is that deliberate practice is not a good study plan. In the next section I discuss mastery learning.

2.6 Mastery learning

Crapnell (2020) defines mastery learning as an instructional strategy to eliminate and correct misconceptions in real time for "all learners" to gain Mathematics Education. Mastery learning is a strategy that aims to develop learners to have a deep understanding of Mathematics instead of memorising procedures and concepts. According to Davis and Sorrell (1995), mastery learning is the strategy of learning in which learners accomplish the same level of content mastery but at different time intervals.

Lengetti et al. (2020) define mastery learning as an educational approach that promotes individualised teaching method to support learner achievement. According to Lengetti et al. (2020) the objective of mastery learning is to attain success for all learners. The benefit of mastery learning is to help teachers cover the curriculum of Mathematics at deeper levels.

In Mathematics the benefit of mastery learning is for all learners to attain conceptual understanding to solve problems and show complex reasoning. Davis and Sorrell (1995) state that mastery learning is beneficial on the concept that “all learners” can learn when provided with instructions or conditions appropriate to them. While engaging in mastery learning, learners are given more support through peer discussion and testing or additional homework until mastery is met.

Another benefit of mastery learning is to assist learners in areas of attitude towards learning, enhance learning outcomes and maintenance of content. Mastery learning is also beneficial to divide subject matter into units that have determined expectations or objectives. Furthermore, mastery learning develops learners` thinking on one particular material or topic before moving on to a new topic. Mastery learning helps learners to identify what they have learned well and what they have not learned well.

Some learners with minimum prior knowledge of mathematical concepts may attain advanced achievement through mastery learning. Mastery learning assists learners that does not understand a lesson to be able to solve most of the difficulties in the next session. It also helps slow learners to increase the rate at which they learn. What this implies, is that mastery learning is very useful for basic skills.

The shortcoming of mastery learning is that it is “time-consuming”. As a result of the time it takes, mastery learning can have a negative impact when it comes to the time allocated for the syllabus. In mastery learning, there is a need to engage all learners appropriately. In mastery learning, educators are required to help and retain track of multiple learners who are at different stages of learning.

Another shortcoming of mastery learning is that learners who learn fast receive fewer instructions in comparison to their classmates. Learners who learn fast tend to finish some activities before the designated time. Furthermore, extra time may be required to provide slower-paced learners time to learn content. In the next section I discuss productive practice.

2.7 Productive practice

This type of practice has been used in research-based articles and as a theoretical framework (Julie, 2021). According to Wittmann (2019), productive practice is about explaining patterns with solutions to problems and the practice of skills. Wittmann (2019) states that learners should interpret the activities and apply the plan through mathematical procedures. Productive practice is more efficient to be used in the classroom.

Julie (2021) states that productive practice is about the work already done. Productive practice is used to cover the range of activities that test and extend learners' thinking (Julie, 2021). Julie's explanation of productive practice builds on Wittmann's productive practice but differs in certain aspects, as indicated in the first and second paragraphs of this section. In the next section I discuss the productive practice perspective of Julie (2021) and how it is linked to my research question.

2.8 A productive practice perspective

Julie's (2021) productive practice is the type of practice that focuses more in a distributed manner on the work that has already been taught. Productive practice assimilates some of the aspects of German productive practice (Wittmann, 2019). The distributive or spaced format is called "spiral revision" (Julie, 2021).

Julie (2021) states that his “productive practice” is unique because it combines the distributive rehearsal format with process aspects prioritised by Wittmann. Julie’s productive practice helps learners master prior knowledge and develop new skills. Another reason that Julie’s productive practice is unique is to fit the context within which South African teachers teach.

Julie’s productive practice also assists learners to be able to explain and justify solutions, and reflect on their ways of working. The diagram below captures and illustrates the principles of the elements of Julie’s productive practice perspective.



Figure 2: Productive Practice Perspective (Julie, 2021)

Julie (2021) alludes that if productive practice can be implemented consistently throughout the year then achievement results will improve. Based on my literature review, it is explicitly evident that there are few studies of educators in South Africa that have gone into attempts to address three issues from the NSC examination diagnostic reports on Mathematics using productive practice.

In the literature review, there are few clear-cut directions to address the three issues from the NSC examination diagnostic reports provided, and none using productive practice. This is part of what informed me to find a way to: “address the significance of Julie’s productive practice when it comes to three issues from National Senior Certificate reports”. I use the productive practice in my research to address the concerns of the National Senior Certificate examination diagnostic reports as I stated in Chapter 1.

Julie’s productive practice is my choice of practice and I found Julie’s productive practice linked to my research question because this type of practice speaks about “procedural fluency”. Hence, the research question for the study was and sought to understand: “Does productive practice promote mastery of procedural fluency in integer arithmetic?” In the next section I discuss integer arithmetic.

2.9 Integer arithmetic

Fuadiah and Suryadi (2019) refer to integer arithmetic as the first theory of integers. According to Fuadiah and Suryadi (2019) integer arithmetic is the collection of negative numbers and whole numbers. Soga (2017) states that integers are all negative and positive whole numbers, including zero. Based on the study of Soga (2017), a fraction and decimal numbers are not integers.

Cetin (2019) states that integer arithmetic is the basis of algebra learning in school Mathematics. Cetin (2019) believes that integer arithmetic help in computing the efficiency of positive or negative numbers in Mathematics. Soga (2017) refers to integers as the first number type that learners have to master to eliminate and correct mathematical concepts moving forward.

For Cetin (2019), the minus sign is significant in the development of understanding and using negative numbers. According to Fuadiah and Suryadi (2019), negative integers are the most

challenging as compared to positive integers due to the non-concrete nature of negative numbers. Fuadiah and Suryadi (2019) state that the impact of integer arithmetic difficulties continues when learners and Mathematics teachers discuss topics of a higher level. Therefore, the material of integer operation is an arithmetic skill that learners should learn more about to attain the basis of the next learning application called algebra. It is essential that learners have a good grasp of mathematical concepts before moving on to the next topic.

My research question aimed to minimise the complications and enhance sustainable performance in school Mathematics. Therefore, integer arithmetic links to my research question because integer arithmetic promotes the transition from concrete thinking to abstract thinking. Integer arithmetic also helps the learners to be efficient in school Mathematics.

2.10 Conclusion

In this chapter I discussed the role of practice and different forms of practising. The emphasis was on mass practice, distributed practice, drill and practice, deliberate practice and productive practice. Furthermore, in this chapter I also discussed mastery learning and integer arithmetic. The chapter concluded with a discussion of a productive practice perspective and how it is linked to my research question. The research design is dealt with in the next chapter.

CHAPTER 3

RESEARCH DESIGN

For Activity 1 – 5 based questions, I gave the groups approximately 10 minutes each. The groups proceeded with the designed activities with three learners per group, seated near each other. In this chapter I describe the research approach, learning trajectory, instructional activities, participants, data collection, data analysis, ethical considerations, validity and reliability. The research approach of the study is discussed below.

3.1 Research approach

Design-based research was the research approach of this study. Design-based research “*is a research methodology that aims at developing theories, instructional materials and an empirically grounded understanding of how the learning works*” (Drijvers, 2003, p. 19). Design-based research is about the knowledge gained through the experience of the work of the designer combined with the knowledge gained from the research.

Nieveen and Folmer (2013) state that design-based research is the systematic design, evaluation and analysis of educational involvements. Nieveen and Folmer (2013) state that design-based research mostly aims to intervene in addressing complex educational problems. Design-based research seeks to provide a qualitative explanation of what is actually happening inside the classroom.

For Dede (2004), design-based research is developed to report several matters fundamental to the study of learning; for example, the necessity to go beyond narrow measures of learning and the necessity to report theoretical queries about the nature of learning in context. Anderson and

Shattuck (2012) are of the opinion that design-based research was designed for teachers who seek to increase the translation of educational research into enhanced practice. Hence I researched whether productive practice in school Mathematics will be effective through the application of design-based research.

Another objective of the design-based research is to improve a situation, to understand the “how” question and not to explain. Furthermore, design-based research does not intend to verify that some innovation approach is better than some other approach, but rather to offer a grounded theory on how the suggested innovation works. According to Drijvers (2003) design-based research has a cyclic character whereby teaching and thought experiments are alternating.

In design-based research there are micro- and macro- research cycles. Micro-cycles are evidenced by the lessons and macro-cycles are indicated by teaching experiments. There are three phases of research macro-cycles, namely: the preparation and design phase, teaching experiments and retrospective analysis.

A school experiment and the additional learning resources to enhance the delivery of mathematical content was the implementation of a plan decided upon in this study to realise a specific aim. The efficiency of the applied teaching plan is determined by retrospective analysis as to whether the learners demonstrate an understanding of the subject matter. The aspects of macro-cycles I used are the preparation and design phase.

The design-based research approach was used in this study for a reason. The research question of this study starts with ‘How does...’. This demonstrates that I was interested not just in knowing whether productive practice promotes mastery of integer arithmetic and develops procedural fluency but specifically in understanding ‘how’. The character of my research question

links up with the general objective of design research. Design-based research meets the requirements of revising theories and instructional activities during subsequent research cycles. In the next section I discuss the learning trajectory and why it was chosen in the study.

3.2 Learning trajectory

Simon (1995) defines a learning trajectory as the path on which learning is expected to proceed. According to Simon (1995) a learning trajectory is made up of three components: the learning activities, the learning goals that define the direction and the learning process that is based on how learners' understanding and thinking will develop in the context of learning activities. A learning trajectory highlights the importance of having teaching decisions and a goal.

When it came to the learning trajectory, I developed a plan for classroom activities. The learning trajectory acknowledges and values the goals of the teacher for instruction and the importance of learners' learning processes. A learning trajectory involves the assessment of the starting level of understanding and the end goal.

The learning trajectory is not knowable in advance but characterises an expected tendency. In this study I condensed a learning trajectory into a table that contains the learners' responses that are supposed to lead to the next step in the learning process. In the next section I describe the instructional activities of this study.

3.3 Instructional activities

In this study I gave participants the work related to mathematical concepts and procedures, I asked the learners to analyse the concepts and procedures and reflect on possible solutions. In my perspective, the learners' thinking and understanding evolve in the context of the learning activities. The learning goal was for the participants to understand mathematical concepts and

develop procedural fluency. The design of instructional activities in this study was learning materials based on productive practice. An overview of the participants is discussed in the next section.

3.4 Participants

In this section I discuss the individuals who participated in the study and how they were selected. The target group of my study was a grade 9 class of Mathematics learners at a high school in a township in the Cape Peninsula. The school is situated in Mfuleni, where I currently work as an educator for Mathematics. The participants are between the age group 15 – 16 years old and they are females and males. I chose the school based on the fact that most learners in the school come from poor learning cultures or environments. The Mathematics assumed that participants are exposed to, is the one recommended by the National Curriculum and Assessment Policy Statement (CAPS). The participants were 42 learners to whom I teach Mathematics. In the next section I discuss the validity and reliability of this study.

3.5 Validity and reliability

In this section I discuss the validity and reliability of the research methodology. Bapir (2012) states that *validity* is concerned with the accuracy and truthfulness of the findings. According to Bapir (2012) *reliability* is concerned with consistency to obtain the same results every time. Measures of the research methodology to obtain validity and reliability were to gather data using the focus group discussions and observations of the participants` activities.

For Kirk and Miller (1986), validity is about the interpretation of the observations. Validity has the same meaning as when something is the truth. The use of learners` written responses ensured the validity of the data I collected. Since this was a qualitative study I did not focus on the

participants' correct or incorrect written responses to ensure validity but rather focused on the opinions of the learners when they were dealing with integer arithmetic.

When it comes to the reliability and trustworthiness of this study, the use of several sources and the keeping of an educator's journal was done. Furthermore, one educator colleague was asked to observe one lesson in the collection of data as part of the process of reliability.

In this study the checking of learners' written responses and making recommendations for the enhancement of data collection were done through the mediation of my supervisor. The internal and external validity and reliability are briefly discussed below.

3.5.1 Internal and external validity

Internal validity is the term used to state the extent to which research findings are a true reflection of reality. Drijvers (2003) refers to internal validity as the quality of data collection and the reasoning that leads to the conclusion. To ensure the internal validity of the data collection, I focused on what I wanted to know.

For the *internal validity* I also made use of the appropriate strategies such as member checks, peer review and triangulation. A comparison between different kinds of data collection methods (observations and teacher reflective journal) was done to see whether they corroborated one another. Bapir (2012) believes that internal validity is used to state the researcher's observations and the theoretical ideas that he develops. Therefore, in the study I analysed data with the intention to developing an alternative explanation of the findings.

External validity refers to the extent to which the results of a study are generalisable for the population that the sample is thought to represent. Given that the sample was my learners,

generalisations can only be assumed for other grade 9 Mathematics classes having more or less similar profiles and school quantile.

3.5.2 Internal and external reliability

Drijvers (2003) alludes that *internal reliability* refers to the reliability of the methods that will be used within the research. Internal reliability requires an observer to take notes of what is seen and heard during the teaching experiments. My methods to obtain internal reliability were systematic data through identifying prior key items in the learners' activities. I recorded the observations consistently and compared the analysis of the same data. For internal reliability, I also interpreted data and assessed the learners' responses using their written responses.

For *external reliability* Drijvers (2003) states that both transparency and justification of the choices that will be made within the study are required. External reliability refers to a point at which the study can be replicated. To ensure external reliability, I had to justify the decisions of learners' responses and I made the raw data available.

I also reflected on the learners' activities at the end of the lesson. According to Kirk and Miller (1986), the focus groups are appropriate for external reliability. Hence I decided to divide the participants into three learners per group to find out about their dissatisfaction and challenges with the activities that I used in the study. The data collection is discussed in the next section.

3.6 Data collection

In this section I discuss the type of data collection method I used and how I collected the data. In the study I made use of qualitative data collection procedures to ascertain the participants'

understanding of the designed activities. The mathematical concepts of the study were based on integer arithmetic.

The designed activities 1 and 5 were based on spiral revision and were meant to test whether the learners were able to simplify integers without the use of a calculator. The designed activity 5 was meant to test whether the learners were able to replace the symbols with the correct operational signs so that the mathematical statement was true. The designed activities 2, 3, and 4 were meant to test learners' deepening mathematical thinking.

In activity 3 learners were given a mathematical problem and its solution written by a certain learner, then the learners had to decide whether the solution was correct or not, giving a reason. The designed activities were chosen based on work already done in the previous school terms to consolidate what learners did not understand in Mathematics class. The designed activities were chosen based on the productive practice perspective.

The data were collected through observations, recording the learners' written responses, my teacher's reflective journal and focus group discussions whereby the participants had to answer the designed activities. The participants' written responses were recorded by taking pictures on a weekly basis for the duration of the study to gain insight into their views. I have controlled the designed activities in a normal classroom setting.

During implementation the participants were divided into groups of three and were requested to discuss the designed activities. The participants selected a group leader and submitted their written responses to each activity for three weeks. The designed activities that were used are given in Appendix A. Altogether the designed activities contained 18 questions.

According to Illeris (2007), learning can be seen in three parts: the conceptual process of interaction that occurs between the individuals and their materials, the result of the learning process, and social surroundings. Hence I made the intentions and expectations very clear to the participants. When the learners participated in group discussions and had disagreements, I generally remained an observer. The use of focus group discussions aimed to capture the description of the participants' internal experience and external reality.

During implementation I encouraged the participants to advance their discussions to higher levels of engagement. Before the end of each day of the data collection I recorded a short reflection of my experience in the journal. In my teacher reflective journal I decided to first look at a critical incident from previous lessons by reading the participants' scripts and my observation notes.

I also observed the participants' discussion without taking notes on what was encountered but mediated to assist the learners in improving their opinions. I have done note-taking after the lesson presentation by choosing important highlights of the lesson. The designed activities were used as a technique for obtaining information about how the participants dealt with integer arithmetic. In the next section I discuss the data analysis of this study.

3.7 Data analysis

In this section I discuss the qualitative techniques that I used to analyse data. The analysis was done by focusing on completed designed activities and I classified the data into categories. The data analysis focused on the main sources of the data collection such as the observations, focus group discussions, teacher reflective journal and recordings of the learners' written responses.

I reflected on some of the learners' written responses and analysed the learners' mathematical correctness. The first step of my analysis concerned elaborating on the data

collected. The selections from the participants' written responses were made by evident sampling. The criteria for the selections were the relevance of the fragment to the research question: *How does productive practice promote understanding of mathematical concepts and procedures?*

The data concerning what I wanted to know was mostly selected and there was a guideline I used to make the selection. For instance, I grouped all learners' written responses with the same or similar answers. Some of the participants did not always justify their methods. In such cases I summarised the learners' written responses into descriptions of what the participants had done wrong and the overall strategy they had used.

The long explanations given by the participants were summarised as well unless there were interesting learners' interventions. The recordings of the learners' written responses were not elaborated on when some of the participants did not complete the designed activities since the key items I wanted to know were missing in their written responses. Some of the learners' written responses were analysed; especially their responses on key items that I wanted to know.

The first category of my data analysis was the first attempt at the first activity. The first category of the analysis consisted of working through the procedures with an open approach that was stimulated by the constant comparative method. I noted the decisions on specific categories of the observations that were not foreseen between the categories.

The second category of my data analysis concerned looking for trends by sorting the learners' written responses with the same answers. The findings of the participants were summarised for each designed activity and exemplified by the ideal observations. I analysed the other data sources and in particular the learners' written responses to find the instances that confirmed or rejected the preliminary conclusions.

The analysis of the learners' written responses often motivated a reconsideration of the procedures. The analysis was continued in this way until a last designed activity, precisely activity 5 which meant that no new elements were added to the analysis and no conclusions were subject to change.

The third category of the data analysis was the response of the learners who struggled to do the designed activities. The change was added to the learners' written responses. For example, was the observed learner behaviour in the procedure neutral, correct or did it prevent the progress of the task in question? The learners' written responses will be presented in Chapter 4.

The fourth category in analysing the data was the interpretation of the findings and comparison with the preliminary expectations. The explanations for the differences between the expectations and findings were also developed. These conclusions and interpretations functioned as feedforward in the research. During the data analysis phase I tracked and made the decisions and findings in a chronological analysis journal.

In this study I show three learners' written responses to each designed activity and discuss three different responses presented by the participants. The designed activities that learners were presented with are given in Appendix A.

The purpose of reading and re-reading the participants' written responses allowed me to become as familiar as possible with the data I had collected and to get the learners' common understanding of integer arithmetic. The learners' written responses to the designed activities were the data used for a teacher reflective journal.

The data analysis focused on the learners' strong points in dealing with integer arithmetic and replacing the symbols with mathematical operations. In the next section I discuss the ethical considerations of this study.

3.8 Ethical considerations

In this section I briefly explain how I conducted the research and refer to the confidentiality of the learners, anonymity and ways to avoid threats to the learners, informed consent from the school management, parents and learners and adherence to Covid-19 protocols. According to Akaranga and Makau (2016), ethics guide the norms and standards of behaviour of people and some relationships with each other. The section on ethics is about the formation of social norms which focus on the behaviour that a person is anticipated to maintain in a certain study.

Fleming and Zegwaard (2018) highlight some ethical considerations commonly encountered in the research such as confidentiality, anonymity and informed consent. Research ethics require a researcher to protect the dignity of the participants and to publish well the information that is researched.

Akaranga and Makau (2016) state that *confidentiality* means any identifying information is not made available to anyone but the programme coordinator. When it comes to ethical confidentiality, I had to make sure that the rights and interests of anyone affected by my work were safeguarded. I had to obey the legislation on human rights and data protection.

To safeguard and protect the data, I stored the data collected on my USB flash drives and computer files which were password protected. Furthermore, I had to make sure that my password was hard to determine and be protected as wisely as confidential data. I did not share my password

nor leave it on mistakes of paper at work places or desks. Furthermore, to safeguard and protect the data, my USB flash drives and computer were protected from theft and unauthorised use.

My computer was configured to “lock out” after 10 minutes of inactivity to reduce the risk of theft and unauthorised use of the data collected. I had to ensure an adequate level of confidentiality of the research data. There was a need for me to consider the identity of the learners` confidentiality, protect their names and avoid the use of self-identifying information given by the learners.

Another ethical consideration was *anonymity*. According to Akaranga and Makau (2016), anonymity is based on keeping the identity of the participants unknown to the research team. To ensure anonymity, I had to disguise the learners` real identities and make sure that the learners could not be traced and identified. In addition, I was very careful not to use the learners` names, in order to preserve their privacy and anonymity. When there was a need I decided to refer to their names as “Learner X” or “Mrs. Y”. I tried to keep the real and ‘code’ names in the notebook so that I did not get confused. Furthermore, I had to hide the name of the school to avoid any potentially sensitive issues.

In my study *avoiding threats to the learners* was also considered. In my teacher reflective journal and observation, I had to respect the learners` views and ideas. When the learners were participating in the activities I observed them and kept my thoughts to myself. I tried not to disagree with the learners violently because that might have ended up stopping them from expressing their views for fear of “getting something wrong”. At the end of each day I had to acknowledge all the learners that were participating in my research as a way of not making them feel threatened. I had to avoid any overstatement about the objectives of the research. Any type of communication when

it comes to the research was done with transparency and honesty. The use of offensive language was avoided in the focus group activities.

Informed consent was also significant and considered prior to the study. According to Fleming and Zegwaard (2018), informed consent means that the people taking part in the evaluation are fully informed about the research being conducted. To gain informed consent I had written a letter of assent for the learners that were taking part in my research.

The learners were fully informed of what was expected of them, how the data would be used, and what (if any) consequences there could be. The letter also explained their right to access their information and their right to withdraw at any point. Providing sufficient information allowed the learners to understand the implications of participation and reach a fully informed decision without the exercise of any pressure. These letters agreeing to take part in my research, were signed by the learners.

Furthermore, I had also written a consent letter for the parents, school management and the Western Cape Education Department (WCED) so that they could sign the consent letters to give me permission to conduct research. Their awareness of my research was very important and an indication that they agreed on behalf of the learners. The learners' parents returned 42 signed consent letters. The completed signed consent letters and assent letters were secured in one file portfolio and are kept by me in a safe place. A copy of the parents' consent letter is in Appendix B and a copy of the learners' assent letter is in Appendix C.

Ethical clearance was obtained from the University of the Western Cape and this document is attached as appendix. The university's ethical clearance is presented in Appendix D. The

research approval letter from the WCED is in Appendix E and the consent letter for the school management team is in Appendix F.

For *adherence to Covid-19 protocols* I had to make sure that all the learners complied with the rules and guidelines of the country about the Covid-19 pandemic. In the classroom I had to remind the learners about the five golden rules at the start of every day. The learners had to wear the masks correctly covering their nose and mouth at all times. I encouraged the learners to keep the windows and door open and report windows that did not open. The learners were not allowed to share stationery, books and other items. I had to rearrange the activities to minimise sharing of books and other materials. When there was a need for the learners to share stationery, I consistently reminded them to disinfect it after each use.

To protect all the learners and myself I had to ask the learners to sit at the same desk every day until the end of my data collection. The desks were arranged in rows facing forward and as far apart as the classroom allowed. I moved between the classroom for the observations and data collection, instead of the learners. Furthermore, I had to ensure that there was hand sanitiser available in the classroom for the learners and myself to sanitise our hands. The study respected integrity, confidentiality, honesty, avoidance of personal risk to the participants and voluntary participation that informed part of the ethical principles. The next section is a brief summary of what was discussed in this chapter.

3.9 Conclusion

In this chapter I discussed the research approach and methodology. The main focus was on the issues about qualitative research such as the learning trajectory, instructional activities,

participants, data collection, data analysis, validity and reliability. The chapter concluded with a discussion of the ethical considerations. The next chapter deals with the results of this study.



CHAPTER 4

RESEARCH RESULTS

In this chapter I discuss the research outcomes of this study. The results are discussed per activity and the groups' written responses to the questions in the designed activities. In this chapter I only show one of the groups' written responses to each designed activity. Furthermore, in this chapter I describe the categories. Research results per activity will now be examined.

4.1 Results per activity

4.1.1 Activity 1 results

On the first day of implementing the designed activities, I asked the 14 groups to do questions 1.2; 1.4 and 1.5. In questions 1.2; 1.4, and 1.5 of activity 1, groups were required to simplify integers without the use of a calculator. Most of the participants were able to share their ideas in classroom discussions, to assist one another with the end goal of the designed activities.

The groups have done their responses to the different parts of the activity on the printed page and I am focusing on three responses. The groups' written responses were presented and motivated by the representatives of each group on the whiteboard. Each of these solutions was reflected upon and debated in terms of mathematical correctness by myself and some learners.

Figure 3 below shows how one group of three learners responded to questions 1.2; 1.4 and 1.5 of activity 1

Figure 3

Response of Group 1

ACTIVITY 1: (Spiral Revision)

1. Simplify the following without the use of a calculator:

1.1 $5 - (-3 + 2) - 11$

1.2 $-5 - (-12) + 7$
 $-5 + 12 + 7$
 $7 + 7$
 $= 14.$

1.3 $-12 + 4 - 3(2)$

1.4 $-22 \div 11 - 5 - (-3)$
 $-22 \div 11 - 5 - (-3)$
 $-2 - 5 - 3$
 $-2 - 2$
 $= 0$

1.5 $\frac{5 - (-5)}{2 - (-6)}$
 $\frac{5 + 5}{2 + 6}$
 $= \frac{10}{8}$

Group 1 has done question 1.2 correctly. Looking at the solution of group 1 in question 1.4, group 1 did not simplify $-22 \div 11 - 5 - (-3)$. Group 1 invoked $-2 - 5 - 3 = 0$ and that was incorrect. When they presented this solution on the whiteboard they said “ $-2 - 2 = 0$.” Other groups were so interested to know how they came up with “0” but they were not able to tell. In question 1.5 this group did not simplify $\frac{10}{8}$.

Group 2 merely added 5 and 19 without taking into consideration the negative sign of -5 in “ $-5 + 19$ ” because their final solution was 24 which was incorrect. There were some misconceptions about how to add and subtract integers. Group 2 did not subtract 5 from 19, they added 5 and 19. That is how they started to do the calculations incorrectly. Group 2 did questions 1.2 and 1.4 incorrectly. In question 1.5 they did not simplify $\frac{10}{8}$.

Group 3 has done questions 1.2; 1.4 and 1.5 correctly. The methods used to do their calculations were correct. In questions 1.2; 1.4 and 1.5, I conducted the discussions on the responses to resolve issues I wanted to draw the groups' attention to.

I emphasised that the participants were not allowed to make use of a calculator. I concluded questions 1.2; 1.4 and 1.5 by collecting the written responses from all 14 groups even though I am focusing on only three responses. There was a positive environment for peer discussion and sharing of ideas when I implemented activity 1. In activity 1 not all groups have done questions 1.2; 1.4 and 1.5 correctly.

Table 1

Summary of the Groups' Responses to Questions 1.2; 1.4 and 1.5

| | Question 1.2 | Question 1.4 | Question 1.5 |
|---------------------------------------|---------------------|---------------------|---------------------|
| Correct answers per question | 6 groups | 3 groups | 14 groups |
| Incorrect answers per question | 8 groups | 11 groups | None |
| Total number of groups | 14 | 14 | 14 |

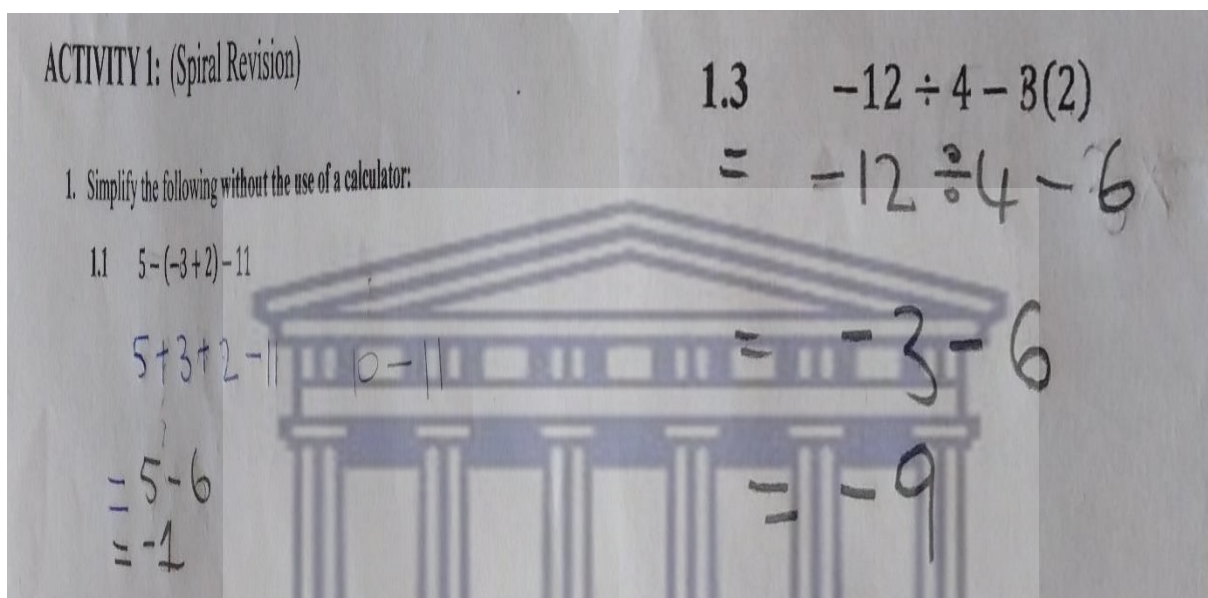
Some groups in activity 1 showed a tendency to get to the answer without focusing on the entire instructions. Some groups did not leave their answers in simplest form since they were not allowed to use a calculator in activity 1.

In responding to questions 1.1 and 1.3, some of the groups were able to answer the questions because they displayed an awareness that they already had a background of how to add and subtract integers. Questions 1.1 and 1.3 were the continuations of adding and subtracting the

integers. Figure 4 below shows how one group of three learners responded to questions 1.1 and 1.3 of activity 1

Figure 4

Response of Group 2



Group 2 did not do question 1.1 correctly. Group 2 merely added “ $5 + 3 + 2$ ” – 11. In question 1.3, group 2 has done calculations correctly.

Unlike group 2, group 1 found the correct answer to question 1.1. Even though their first two steps were not mathematically correct, group 1 showed an understanding of how to add and subtract the integers. For instance, looking at questions 1.1 and 1.3, group 1 has done question 1.1 correctly, unlike group 2. In question 1.3 groups 1 and 2 were both correct. It would be fruitful to revise the addition and subtraction of the integers with learners.

Group 3 had many mistakes in questions 1.1 and 1.3 their calculations were incorrect. Group 3 did not consider that a negative number is always less than a positive number. According to the groups' written responses, the implementation of questions 1.1 and 1.3 was not better than questions 1.2; 1.4 and 1.5. Even though I gave 14 groups feedback on Activity 1, I did not see any improvement in their calculations.

By inspection it was easy for some groups to predict the answer; hence they made more mistakes in their calculations. When I implemented the next activity I asked the participants to form new groups so that they could mix up with their classmates to ensure that they were at the same level of understanding of integer arithmetic.

Table 2

Summary of the Groups' Responses to Questions 1.1 and 1.3

| | Question 1.1 | Question 1.3 |
|---------------------------------------|---------------------|---------------------|
| Correct answers per question | 3 groups | 4 groups |
| Incorrect answers per question | 11 groups | 10 groups |
| Total number of groups | 14 | 14 |

Most groups did not get to the answers to questions 1.1 and 1.3 without using a calculator. For normal teaching, in the first week of implementation I was teaching squares and cubes of integers without variables. I told the learners that when an exponent is an even number then the answer will have a positive sign. For example $(-3)^2 = 9$ but it does not mean the same as " $-3^2 = -9$." If the exponent is an odd number then the answer will have a negative sign. For instance $(-3)^3 = -27$.

The learners were expected to know how to simplify squares and cubes of integers, leaving answers in the simplest forms. I emphasised that when multiplying two positive numbers, the answer will be positive. It was also important to inform the learners that when multiplying two negative numbers, the result will be positive.

When learners understood squares and cubes of integers, I taught them how to simplify integers in algebra. I informed the learners that integers play a very significant role as numerical coefficients in algebraic expressions. During my lessons I taught the learners how to add and subtract integers with the same variables in algebra. For normal teaching, I also showed the learners how to calculate the fractions without variables. The following is one of the activities I have done in class with the learners.

Activity:

1. Calculate the following without the use of a calculator, leaving your answer in the simplest form.

(a) $-5 - (-2) - 3$

(b) $\frac{-3 - (-5)}{-2 + (-3)}$

(c) $-2x^2 + 14y - 7x^2 - 3y - 8y^2 + 11y^2$

The changes I made to my teaching on integer arithmetic was to ask the learners to write a class test just for 30 minutes whereby I encouraged them to do the given test (to revise the work done on integer arithmetic). The learners marked their scripts. By doing so, I was assisting the learners to understand integer arithmetic and build their confidence in Mathematics. That is how I checked what they knew and did not know before they did the next activity.

After the implementation of activity 1, I made sure that each learner had a copy of the activity so that they could prepare themselves for the test. To address the issue of groups' misconceptions, I gave them feedback based on designed activity 1. To show what I expected of them to do, I encouraged the groups to read the instructions and make sure they understood what needed to be done.

The following are the changes I made before the implementation of activity 2. I asked the groups to try by all means to stick to the time allocated for activity 2. Before they started with activity 2, I requested them to make sure that they had the necessary stationery so that they did not disturb my plan of implementation.

I asked the participants to participate in activity 2. Every learner was to bring his own opinion about how to do activity 2. I asked the participants to have one learner write, one timekeeper and another one present, when necessary. I requested the groups to remember the work done in activity 1 (subtraction and addition of integers). They should not be afraid to make mistakes.

The groups were not supposed to use a pencil in activity 2 but only a blue pen and a black pen. When one group was done with activity 2 before time, I asked the participants to keep quiet and allow other groups to finish their work too. I also informed the groups that designed activities do not require them to be always correct; an incorrect answer is also accepted.

4.1.2 Activity 2 results

In activity 2, I implemented a Deepening Mathematical Thinking question type. For peer discussion, the learners were separated into groups of at most three learners per group as was done in activity 1. However, in activity 2 I am focusing on four groups instead of three groups.

In activity 2 the groups had to state whether the given mathematical statement is “always true”, “sometimes true” or “never true”. In total I was working with 14 groups in activity 2 but I am focusing on two groups with “sometimes true” and two groups with “always true” written responses. I compared their reasons and checked why their answers were different. Figure 5 below shows how one group of three learners responded to question 2.1 in activity 2. A written response from group 1 with “sometimes true” is given below.

Figure 5

Response of Group 1

ACTIVITY 2 (Deepening Mathematical Thinking)

2. Mathematical statements can be sometimes true, always true or never true. Mark with a cross (X) the correct block for the given mathematical statement.

| Mathematical statement | Always True | Sometimes True | Never True |
|---|-------------|----------------|------------|
| When two integers are multiplied, the product is larger than any of the two integers. | | X | |

2.1 Give reasons with possible examples for your choice:

When you multiply two integers the product is not always larger than any of the two integers.

For example:

- $-3 \times 4 = -12$ ∴ the product is smaller than the integers.
- $2 \times 3 = 6$ ∴ the product is larger than the integers.

Groups 1 and 2 merely said when you multiply two integers, the product is not always larger than any of the two integers. They also stated their reasons for their choices. The first group gave two examples, as follows:

- $-3 \times 4 = -12$. “The product is smaller than the integers.”
- $2 \times 3 = 6$. “The product is larger than the integers.”

One can observe that group 1 understood the mathematical statement given to them. Furthermore, they considered a multiplication sign rule between negative and positive integers.

According to their examples, it is easy to understand why they said the statement is “sometimes true.”

Group 2 gave four examples as follows:

1. $1 \times 1 = 1$. “*The product is equal to any of the two integers.*”
2. $9 \times 1 = 9$. “*The product is equal to one of the two integers.*”
3. $6 \times 0 = 0$. “*The product is equal to one of the two integers.*”
4. $5 \times 2 = 10$. “*Here the product is larger than the two integers.*”

The only thing that was missing in the examples given by group 2 was to state which one is larger between two integers and the product. The second group only used the numbers to support their reasons. In their examples they had a situation whereby the product was larger than any of the two integers and the other way around.

Groups 3 and 4 said when you multiply two integers the product is always larger than any of the two integers. They also stated their reasons for their choices. Group 3 gave two examples as follows:

1. $-7 \times 3 = -21$
2. $7 \times -10 = -70$

Group 3 did not consider that a negative number is always less than a positive number. According to their first example “-21” is larger than “-7 or 3” which is incorrect. The same misconception applies to their second example; they did not consider the operation before their conclusion.

Group 4 did not give any examples but they used an explanation for their choice. They said the following: *“It is always true because if you multiply a number with another number, the product will be larger. It does not matter whether it is a negative number or a positive number.”* One can observe that group 4 did not understand the mathematical statement given to them. In terms of their reasons, group 4 merely said the mathematical statement “is always true” without considering a multiplication sign rule between negative and positive integers.

In my findings, groups 1 and 2 have done activity 2 correctly as shown in Figure 3 above. The implementation of activity 2 was indeed better than activity 1. I was more disciplined in monitoring the actual time (10 minutes). I concluded activity 2 by summarising, using the question-answer approach. Some groups participated very well in activity 2. Some groups have done activity 2 correctly.

Table 3

Summary of the Groups` Responses in Activity 2

| | Always True | Sometimes True | Never True |
|-------------------------------|--------------------|-----------------------|-------------------|
| With a reason | 8 groups | 6 groups | 8 groups |
| Without a reason | 6 groups | 8 groups | 6 groups |
| Total number of groups | 14 | 14 | 14 |

Some groups` responses were incorrect with the possible correct examples. For normal teaching, I was teaching multiplication and division of integers in algebra. Learners were expected to know how to multiply and divide integers. I taught them the multiplication rules and division

rules. For instance, $+$ \times $- = -$; $-$ \times $+$ $= -$; $-$ \times $- = +$; $+$ \times $+$ $= +$. The same applies when you divide integers $+$ \div $- = -$; $-$ \div $+$ $= -$; $-$ \div $- = +$; $+$ \div $+$ $= +$.

The multiplication rules and division rules assist learners to understand that when you multiply or divide two integers, the product is not always larger than any two integers. Furthermore, multiplication and division rules allow learners to understand how principles of dealing with integers feature in algebra.

The learners were expected to know how to simplify integers in algebra. When learners understood multiplication and division rules, I told learners that integers play a significant role as numerical coefficients in algebraic expressions. During my lessons I taught the learners how to multiply and divide integers with the same variables in algebra. For normal teaching I also showed the learners how to calculate common fractions with variables. The following is one of the activities I have done in class with the learners.

Activity:

1. State whether the following are true or false. Give a reason for your answer.

(a) $\frac{7}{3} \times \frac{2}{5} = 12$

(b) $-2x \times -5x = -7x^2$

(c) $15x \div -3 = 5$

(d) $-\frac{6}{5} \div -\frac{9}{10} = 2$

The changes I made to my teaching on integer arithmetic was to ask the learners to write the corrections of the activity on the whiteboard, immediately after 10 minutes when they are done. The reason was to ensure that they could discuss and share their ideas in class. The learners had to

mark their workbooks. That is how I checked what they knew and did not know before they did the next activity.

In addressing the issue of groups' misconceptions, I gave the groups feedback based on designed activity 2. The changes I made when I implemented the next activity were the same as I did before implementing activity 2. In activity 3, I requested the groups to remember the work done in activity 2 (multiplication of integers). The groups had to use the same stationery they used in activity 2 (a blue pen and a black pen).

4.1.3 Activity 3 results

In question 3.1 one group showed an understanding that positive numbers are always bigger than negative numbers. Before attempting to answer activity 3, one group asked the following questions: *“Are we supposed to write the first expression or the other way around?”*, *“Is it necessary to use brackets when you subtract an expression from the next?”*, *“Can you do one example for us?”* I said they must follow the instructions and do what they thought should be done. In activity 3 results I am focusing on three groups.

Some groups did not finish question 3.1 within 10 minutes and I had to add three minutes so that they could finish writing their responses. A written response from one group is given below.

Figure 6

Response of Group 1

ACTIVITY 3: (Deepening Mathematical Thinking)

3. Answer the following questions:

3.1 A learner is asked to subtract $3 - x^2 - 5x$ from $2x^2 - 12 + 5x$. Here is her solution:

$$\begin{aligned} 2x^2 - 12 + 5x - 3 - x^2 - 5x \\ = 2x^2 - x^2 + 5x - 5x - 12 - 3 \\ = x^2 - 15 \end{aligned}$$

Explain why you agree or not with the way she did it?

~~Disagree~~ Disagree because the statement is not true and its product

$$\begin{aligned} 2x^2 - 12 + 5x - (3 - x^2 - 5x) \\ 2x^2 - 12 + 5x - 3 + x^2 + 5x \\ (2x^2 + x^2) - (12 + 3) - (5x + 5x) \\ 2x^4 - 15 - 10x \\ - 3x^5 \end{aligned}$$

Group 1 disagreed with a solution of a learner. However, their explanation was incorrect. They did not add the like terms correctly. For example, one of their mistakes is when they said “ $2x^2 + x^2 = 2x^4$ ” which was incorrect. Their final answer was also incorrect.

Group 2 did not state whether they agreed or disagreed with a learner’s solution. When it comes to their calculations they had two options and both were incorrect. Group 2 did not add the like terms and a multiplication sign rule was not considered. Their final answer was $3x^2 + 10x + 9$ and a constant term was supposed to be -15 instead of $+9$. For the desired results it would assist to ask the groups to do three more questions on how to add and subtract integers in algebra.

During implementation I had 14 groups in total, but group 3 was the only group to do question 3.1 correctly. Group 3 has done all the steps correctly. Their first attempt was to say they did not agree with a learner’s solution and followed this with their reasoning.

Based on other groups’ written responses, the implementation of question 3.1 was not better than questions 1.1 and 1.3. Although I gave them feedback on Activity 1 I did not see any improvement in their calculations for activity 3.1. I had to go back to my class and give them more

questions about the integers in algebra so that the groups could understand what was required and avoid any careless mistakes when it comes to adding or subtracting the like terms.

By inspection, the groups merely said “a solution given by that learner is correct,” until they had to do some calculations and then compare their answers with one given on the printed page.

Table 4

Summary of the Groups` Responses to Question 3.1

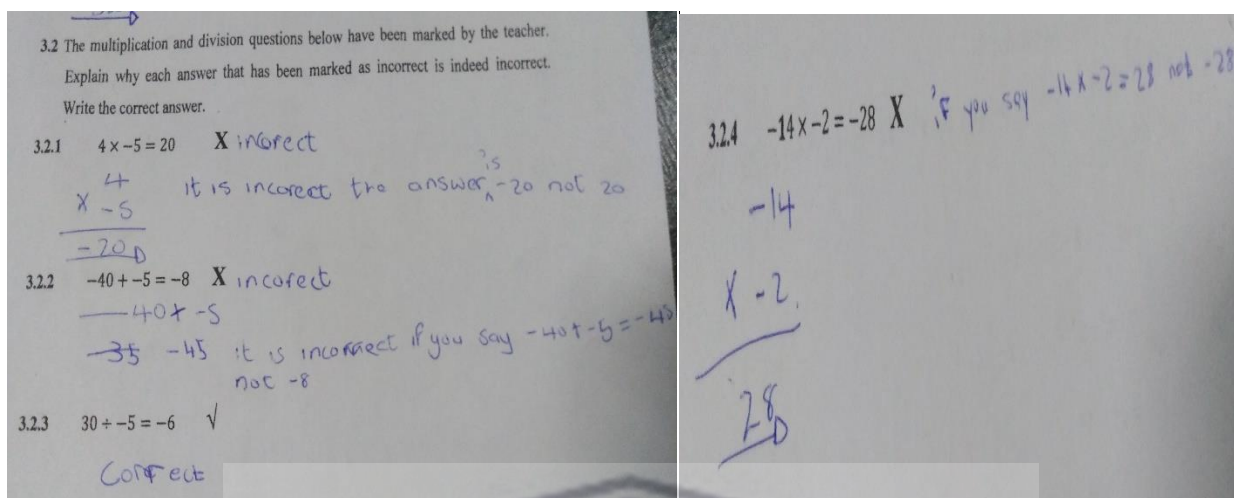
| | Question 3.1 |
|---------------------------------------|---------------------|
| Correct answer with a reason | 1 group |
| Incorrect answer with a reason | 13 groups |
| Total number of groups | 14 |

Some groups in question 3.1 showed a propensity to get to the answer without focusing on the entire instructions. In question 3.2 most groups showed an awareness of how to add and multiply the integers.

In question 3.2 the groups had to explain why each answer that had been marked as incorrect was indeed incorrect. I focused on three groups to compare their solutions and to check what went wrong for them to have different answers, if applicable. During implementation, some groups asked the following questions: “*Must we show calculations?*” “*Are we supposed to explain in words?*” “*What are we supposed to do when one question has been marked correctly?*” I asked the groups the following questions: “*Can you do the first one without my assistance?*” “*Can you read the question again?*” The groups used the printed page to answer the questions. A written response from one group is given below.

Figure 7

Response of Group 1



Group 1 has done question 3.2 correctly. Question 3.2 was similar to the first activity they did on the first day of implementation. In question 3.2, group 1 seemed to understand a designed activity.

Group 2 also has done question 3.2 correctly. Their explanations were so appropriate. They merely considered what to do when you multiply, add or subtract the integers. A multiplication sign rule was also taken into consideration. When it comes to their explanations and calculations there was nothing I discovered incorrectly. For the desired results, at this point of implementation some groups seemed to understand integers.

Group 3 also did question 3.2 correctly. Their explanations and calculations were similar to the ones given by groups 1 and 2. For activity 3.2 there was no need for me to give groups more questions; they have done what was supposed to be done.

Based on the groups' responses, I discovered that the implementation of question 3.2 was better than question 3.1. The reason is that I gave groups feedback on question 3.1 and I requested

them to link activity 1 with activity 3. Most groups have done questions 3.2.1; 3.2.2; 3.2.3 and 3.2.4 in activity 3 correctly.

Table 5

Summary of the Groups' Responses to Question 3.2

| | Question 3.2.1 | Question 3.2.2 | Question 3.2.3 | Question 3.2.4 |
|--|----------------|----------------|----------------|----------------|
| Correct answer with a reason | 14 groups | 12 groups | 13 groups | 14 groups |
| Incorrect answer without a reason | None | 2 groups | 1 group | None |
| Total number of groups | 14 | 14 | 14 | 14 |

Most groups in question 3.2 got the answers with correct reasons. For normal teaching, I was still teaching integers in algebra. The following is one of the activities I have done in class with the learners.

Activity:

1. State whether the following are correct or incorrect. Give a reason for your answer.

(a) $-2x \times 3 = 6$

(b) $-2 \times -5x = -10$

(c) $12x \div -3x = 5$

(d) $-20x + (-3x) = 2$

To answer the question “*What will I change?*” when I implement the next activity, I concluded designed activity 3 by summarising, using the question-answer approach. The changes I made were the same as I did before implementing activity 2.

4.1.4 Activity 4 results

In responding to activity 4, most groups displayed an understanding of comparing integers. After ten minutes I immediately asked all groups to put their pens down so that the group leaders could submit their responses. In activity 4 results I am focusing on three groups. A written response from one group is given below.

Figure 8

Response of Group 1

Activity 4 (Deepening Mathematical Thinking)

4. Check whether the answers in pair are equal. Explain why they are the same or different.

4.1 $5 - 22$ and $22 - 5$

= -17 and 17

= The pairs are not equal

= Signs are not the same

4.2 $-17 - (+12)$ and $+12 - (-17)$

= -29 and 29

= The pairs are not equal

= signs are not the same

4.3 $-8 - (-19)$ and $-19 - (-8)$

= 11 and -11

= The pairs are not equal

= sign are no the same

Handwritten calculations for 4.1: $5 - 22 = -17$ and $22 - 5 = 17$

Handwritten calculations for 4.2: $-17 - (+12) = -29$ and $+12 - (-17) = 12 + 17 = 29$

Handwritten calculations for 4.3: $-8 - (-19) = -8 + 19 = 11$ and $-19 - (-8) = -19 + 8 = -11$

Group 1 has done question 4.1 correctly with reasons for their answer. Questions 4.2 and 4.3 were also done correctly. Group 1 had an understanding of what they were supposed to do in questions 4.1; 4.2 and 4.3. On the right-hand side of their written responses they showed their

calculations. When it comes to their explanations and calculations, there was nothing I discovered that was incorrect.

Group 2 has done question 4.1 correctly. Group 2 explained why the answers in pairs were equal. However, in question 4.2 their answers in pairs were "5 and - 5" and that was incorrect. The correct answers in pairs were supposed to be " - 29 and 29". Group 2 did not multiply and add the integers as they were supposed to do in question 4.2. Questions 4.1 and 4.3 were the only questions they have done correctly.

Group 3 has done questions 4.1 and 4.2 correctly. Their explanations and calculations were similar to the ones given by group 1. The only thing missing in their responses was to explain question 4.2 and state why the answers in pairs were equal or different. In question 4.3 their answer was also incorrect. Group 3 said the answer in question 4.3 is "27 and - 27" instead of "11 and - 11". Based on the groups` responses, the implementation of activity 4 was better than activities 1 and 2. The feedback on designed activities 1; 2 and 3 might be the reason why most groups have done activity 4 correctly.

Table 6

Summary of the Groups` Responses to Questions 4.1; 4.2 and 4.3

| | Question 4.1 | Question 4.2 | Question 4.3 |
|---------------------------------------|---------------------|---------------------|---------------------|
| Correct answers per question | 12 groups | 9 groups | 8 groups |
| Incorrect answers per question | 2 groups | 5 groups | 6 groups |
| Total number of groups | 14 | 14 | 14 |

Most groups in questions 4.1; 4.2 and 4.3 displayed an understanding of what to do to get to the answer. For normal teaching, I was teaching algebraic equations. The learners were expected to know how to find the value of any variable in algebraic equations. The following is one of the activities I have done in class with the learners.

Activity:

1. Find the value of a and b . Show your calculations.

(a) $2a + 3 = 5$

(b) $-5 \times b = -5$

(c) $13 - 2a = 7a + 10$

(d) $-25b + (-3b) = 7$

To answer the question “*What will I change?*” when I implement the next activity, I concluded a designed activity 4 by using the question-answer approach. The changes I made were the same as I did before implementing activity 2.

4.1.5 Activity 5 results

In responding to activity 5, the groups showed awareness to display an understanding of replacing the symbol with the correct operational sign. In activity 5 results I am focusing on three groups. A written response from one group is given below.

Figure 9

Response of Group 1

Activity 5 (Spiral Revision)

5. In the mathematical statements below the symbols #, *, Δ and □ stand for one of the operational signs +, -, × and ÷. Replace the symbol with the correct operational sign so that the mathematical statement is true.

5.1 $1 * (-2) \# 2 = 5$

$1 - (-2) + 2$
 $1 + 2 + 2$
 $= 5$ →

* = -; # = +

5.2 $1 \Delta (-2) \square (-2) * 2 = -1$

$1 \div (-2) \times (-2) - 2$
 $1 \div (-2) \times (-2) - 2$
 $= -1$ →

Δ = ÷; □ = ×; * = -

5.3 $1 \# (-2) \Delta (-1) * 2 = 1$

$1 + (-2) \div (-1) - 2$
 $1 - 2 \div (-1) - 2$
 $= 1$ →

= +; Δ = ÷; * = -

Group 1 has replaced symbols using operational signs correctly. One can observe that they knew what to do and the left-hand side (LHS) is equal to the right-hand side (RHS), as they also showed some calculations.

Group 2 has replaced symbols with operational signs correctly. The only thing they did differently in comparison to group 1 was to say “Δ = ×, □ = ÷” They also did not show calculations but rather replaced symbols with operational signs. Groups 1 and 2 had an understanding of how to do activity 5. Groups 1 and 2 were able to do activity 5 in less than 10 minutes.

Group 3 has done activity 5 correctly. However, they did not show their calculations to explain why the LHS is equal to the RHS for each question. Just like the second group they said “Δ = ×, □ = ÷”. By inspection group 3 merely predicted the operational sign they may use to do questions 5.2 and 5.3. Based on the groups` responses, the implementation of activity 5 was better than activities 1; 2 and 3.

Table 7

Summary of the Groups' Responses to Questions 5.1, 5.2 and 5.3

| | Question 5.1 | Question 5.2 | Question 5.3 |
|---------------------------------------|---------------------|---------------------|---------------------|
| Correct answers per question | 9 groups | 6 groups | 4 groups |
| Incorrect answers per question | 5 groups | 8 groups | 10 groups |
| Total number of groups | 14 | 14 | 14 |

Some groups in questions 5.1, 5.2 and 5.3 displayed an understanding of replacing the symbol with the correct operational sign. For normal teaching, I was still teaching algebraic equations. The learners were expected to know how to solve complex algebraic equations by removing the brackets and finding the value of any variable. The following is one of the activities I have done in class with the learners.

Activity:

1. Solve the following algebraic equations.

(a) $2(y - 3) = y - 3(y + 2)$

(b) $3(b + 4) = 6(b - 1) - 5(b - 2)$

(c) $-4(2x - 5) = -4x + 10$

To answer the question “*What will I change?*” when I implement the next activity, I concluded a designed activity 5 by using the question-answer approach. The changes I made were the same as I did before implementing activity 2.

4.2 Conclusion

In this chapter I discussed the results from the analysis of groups` work for five designed activities dealing with integer arithmetic in a grade 9 Mathematics content. The emphasis was on how the groups responded to designed activities. The next chapter deals with a discussion of the research results, reflection on implementation of designed activities from the design research perspective, recommendations for further research and the conclusion.



CHAPTER 5

Discussion of results, Reflection on implementation, Recommendations for further research and Conclusion

5.1 Introduction

In this chapter I discuss the research outcomes stated in Chapter 4 and reflect on the design of activities from the design research perspective. The study investigated the implementation of productive practice in a grade 9 Mathematics class. The study specifically focused on the designed activities of integer arithmetic whereby groups had to do computations without using a calculator. The next section discusses the results.

5.2 Discussion of results

Overall, the results show that there was an improvement in participants' mastery of integer arithmetic. There was a good transition in the groups' written responses when I implemented the other designed activities in comparison to the first activity. For instance, when I implemented designed activity 1, the participants seemed not to understand integer arithmetic. When I implemented the designed activities 2, 3, 4 and 5, I noticed an improvement in participants' mastery of integer arithmetic as shown in Tables 5 and 6 in Chapter 4.

The participants encouraged each other to think and support their interpretations of the designed activities. Participants' collaborations assisted them in enhancing their competence with integer arithmetic.

Table 8

Summary of Responses of Groups for Different Activities

| Activities | Number of groups without convincing reasons | Number of groups with convincing reasons | Number of groups with correct answers | Total number of groups |
|-------------------|--|---|--|-------------------------------|
| 1 | 9 groups | 5 groups | 5 groups | 14 |
| 2 | 8 groups | 6 groups | 7 groups | 14 |
| 3 | 5 groups | 9 groups | 10 groups | 14 |
| 4 | 4 groups | 10 groups | 11 groups | 14 |
| 5 | 3 groups | 11 groups | 13 groups | 14 |

The results in Table 8 above show that the designed activities had a positive role in participants' mastery of integer arithmetic. The implementation of designed activities in a grade 9 Mathematics class assisted participants to have a better competence with integer arithmetic. The results show that group collaboration offered chances for the participants to exchange their ideas and extract their existing knowledge of integer arithmetic.

The participants were permitted to express their opinions through discussions in the language of their first choice although the linguistic of teaching and learning was preferred. The research results were transcribed into English, as presented in Chapter 4. The positives were that participants showed capability and alertness when responding to designed activities.

Participants' ability to do computations without using a calculator contributed more to enhancing their competence with integer arithmetic in a grade 9 Mathematics class. Overall, the

results show a direct significance of productive practice in school Mathematics to promote mathematical concepts and procedures. The next section deals with reflection on the implementation of designed activities.

5.3 Reflection on implementation

After implementing the designed activity 1, I noticed that some participants did not master integer arithmetic. On the following day of implementation, I then decided to give them a test to write for 10 minutes. A class test is not in line with design-based research and a class test was probably not an appropriate way to deal with the problematic issue after I implemented designed activity 1.

What I did not do correctly in line with design-based research is when I asked the participants to write a test after I implemented designed activity 1. When I asked the participants to write test was not in line with design-based research because design-based research tries to pinpoint the issue that participants struggled with and then adapt the designed activity to ascertain if participants dealt with the designed activity in a better manner. Design-based research is about the knowledge gained through the experience of the work of the researcher combined with the knowledge gained from research.

Based on other groups' written responses, the implementation of question 3.1 was not better than questions 1.1 and 1.3. The following day I decided to give participants feedback on question 3.1 and I think that was important for the learning goal. In terms of design-based research, feedback assists in intervening to address the complex educational problem.

In designed activity 2, some participants did not finish their written responses within 10 minutes, so I gave participants three minutes extra time for them to conclude their written

responses. Three minutes of added time was a correct decision in line with design-based research because my end goal was to provide a qualitative explanation of what was happening inside the classroom.

Before I implemented designed activity 3, I asked participants to form new groups. I think that was a correct decision in terms of design-based research. When I asked participants to form new groups, I intended to encourage sharing of ideas among themselves. Furthermore, I saw the necessity to go beyond narrow procedures of learning and address the nature of learning in context.

When some participants kindly asked me to explain and do one example for them, I asked participants to read the instructions carefully and read them two times when necessary. I refused to give them answers for any designed activity and I reminded participants there was nothing wrong when their written responses were incorrect. Refusing to provide participants with answers was correct in terms of design-based research. My main objective was to increase the translation of educational research into improved practice, and improved practice is in line with design-based research.

I think I revealed the effectiveness of designed activities and productive practice through the application of design-based research. When I implemented the designed activities I think I improved measures of learning to understand the “how” question and not to explain. During implementation, I did not try to prove to the participants that productive practice is better than any other teaching strategy and I think that was in line with design-based research.

When I implemented designed activities a productive practice perspective improved the delivery of mathematical content to realise a specific goal. For instance, on the following day of implementation for each designed activity I ensured that participants presented their written

responses on the whiteboard to enhance mastery of integer arithmetic. The efficiency of designed activities ensured participants were demonstrating competence with integer arithmetic. My designed activities linked up with the general objective of design-based research.

The duration of the designed activities was adequate because it took 10 minutes for some participants to finish each designed activity. This means that for the effective implementation of productive practice to take place, one person has to reserve 10 minutes of the lesson. The implementation of productive practice might not work well in less than 10 minutes.

The designed activities were clear and understandable to all participants. The resources used in this research study were of high excellence. I made the participants conscious of what and where they should be at the end of each designed activity. This study enhanced my approach to the teaching of Mathematics.

In designed activity 1 the designated time was too short for some participants to finish the designed activity 1. I honestly think the designed activity 1 was too long to be done in 10 minutes. For instance, in designed activity 1 the participants had to do five questions without using a calculator.

What might have contributed to participants exceeding my designated time is the fact that the designed activity 1 had 5 questions. In designed activity 1 the participants would take a shorter time if the designed activity 1 was reduced to three questions instead of five questions if the activity becomes used in the classroom procedure.

In designed activities 2, 3 and 4, the designated time was not too short and designed activities 2, 3 and 4 were not too long to be done for 10 minutes. From my perspective, what contributed to some participants exceeding my designated time is the novelty of designed activity

2. The participants had to make an understanding of the mathematical statement first before answering designed activity 2.

What might have contributed to some participants exceeding my designated time in designed activity 3, is the fact that I did not consistently do similar questions to designed activity 3 in classroom procedure. In designed activity 4 some participants did all the questions in less than 10 minutes and some participants did all the questions within 10 minutes. As a result, my designated time in designed activity 4 was neither too short nor too long. The participants would use the shorter time when doing activities similar to designed activities 2 and 3 more often in classroom procedure.

From my perspective, the designated time was too short for some participants to complete designed activity 5. However, the designed activity 5 was not too long. Participants might have exceeded the designated time because of the novelty of the activity. The participants had to carefully replace the symbols with the correct operational signs so that the mathematical statement was true and I think the mathematical statements also contributed to participants exceeding the designated time.

Some participants seemed unsure whether they had to use the operational sign once when answering designed activity 5. I think the participants will use shorter time in the classroom procedure when doing similar questions to the designed activity 5. As a researcher, I noticed that for some participants to have done all five designed activities without using a calculator under my undivided observation came as a surprise and that might also have contributed to some participants exceeding my designated time.

Designed activities helped me to stay focused on the intended outcomes. As the researcher, I learned that it is significant to request the participants to read the instructions carefully before they proceed with designed activities. When the instructions are read with understanding by the participants, then the intended outcomes are reached. The next section deals with some recommendations for further research.

5.4 Recommendations for further research

For further research, I recommend that participants should not be asked to write a test after implementing the designed activity because a test is not in line with design-based research. A test is not an appropriate way to deal with the problematic issue that participants are struggling with and hence I do not recommend a class test to be used in research when participants struggled with designed activity. Participants should be given feedback the following day of implementation for the learning goal and to enhance participants' academic performance.

When some participants do not finish the designed activity within the designated time, I recommend participants be given three minutes to complete the designed activity and write their conclusion to ascertain the purpose of the research and provide a qualitative description of what was happening inside the classroom. I recommend participants form new groups before implementing the next designed activity. When participants form new groups, the participants will be able to share ideas amongst themselves. I recommend a researcher go beyond narrow measures of learning and address the nature of learning in context.

When some participants ask for assistance from a researcher, I recommend that participants should be requested to read the instructions carefully and always be reminded that there is nothing wrong when their written responses are incorrect. When participants are given a chance to do

designed activities, a researcher increases the translation of educational research into improved practice. During implementation I recommend that a researcher should always be careful not to try to prove to participants that a certain teaching strategy is better than any other teaching strategy.

For further research I recommend that additional learning resources should always be made available during implementation to enhance the delivery of mathematical content and to realise a specific aim. I recommend that all designed activities should be clear and comprehensible to all participants. Before implementation a researcher should be considerate not to have more questions in each designed activity.

Questions asked in each designed activity should always be in line with the designated time. When some participants finish the designed activity before time, I recommend that they should be given another activity to do so that they do not interrupt other participants. I recommend a researcher be considerate of the novelty of designed activities so that participants do not exceed the designated time. When a designed activity needs to be done in not more than ten minutes, I recommend a researcher give participants at least three questions.

A researcher should ensure that the instructions are read with understanding by the participants so that the intended outcomes are reached. I recommend the implementation of designed activities when teaching integer arithmetic as it provided the desired results in my study.

I recommend the idea of planning and preparation before the lessons start and I recommend the notion of adhering to such principles. Furthermore, I recommend thorough planning and preparation should lead to the advanced use of more quality resources in classroom activities. The enhanced resources might lead to enhanced participants' procedural understanding of mathematical concepts.

More spaces should be added to the designed activities for the participants' written responses. The participants should be made conscious of what and where they should be by the end of each designed activity. I recommend that research be done in ways to facilitate discussions of mathematical ideas among participants.

I recommend integer arithmetic to be taught in small steps and be allocated extra time in a grade 9 Mathematics class. For instance, if participants have to complete integer arithmetic within three weeks then it should be extended to 4 weeks. When it comes to the introduction of integer arithmetic in a grade 9 Mathematics class, I would recommend designed activities to master integer arithmetic. Such assistance in the designed activities would help participants to answer what is required.

I recommend a further study that would expand the target group into at least two classes instead of one classroom. Another study could look at what would happen when one classroom is daily making use of productive practice without any assistance from the researcher and the other one has been given an opportunity when the researcher reads the questions for the participants to provide more understanding and clarity on what is being asked. My study was allied to productive practice, therefore, I recommend a comparative study between the two different classes.

I recommend an experimental study between two classes to enhance participants' mastery of integer arithmetic. This would involve choosing two different classes whereby one class will be randomly allocated to make use of productive practice and the other one without productive practice. An experimental study would be done to check which classroom attained a better understanding of mathematical concepts and procedures involving integer arithmetic.

The objective of my recommendation in the previous paragraph seeks to know which class would best produce more mastery of integer arithmetic when taught through the implementation of productive practice.

Furthermore, I recommend research that will encourage participants to discuss questions that include perspectives and different discussions when it comes to mathematical concepts and procedures. Sometimes it is even better to use more open questions to ascertain participants' discussions where there are great possible answers and perhaps more than just one correct one to increase the learning ability through group discussions. The next section deals with the conclusion of the study.

5.5 Conclusion

In Chapter 5 I discussed the research results reported in Chapter 4 and reflected on the design of five activities dealing with integer arithmetic in a grade 9 Mathematics content from the design research perspective. Recommendations for further research were discussed based on reflection on implementation. I will include the uses of non-programmable calculators for integer arithmetic from a productive practice perspective in further research. When participants do computations in a grade 9 Mathematics content they should be allowed to use a non-programmable scientific calculator since it is stated in the pilot General Education and Training (GET) examinations. It was remarkable to see some participants master integer arithmetic in a grade 9 Mathematics class when designed activities were implemented and how productive practice perspective was implemented as part of “normal” teaching.

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APPENDICES**APPENDIX A**

THE DESIGNED ACTIVITIES ON INTEGER ARITHMETIC

Activities 1 – 5**ACTIVITY 1: (Spiral Revision)**

1. Simplify the following **without the use of a calculator**:

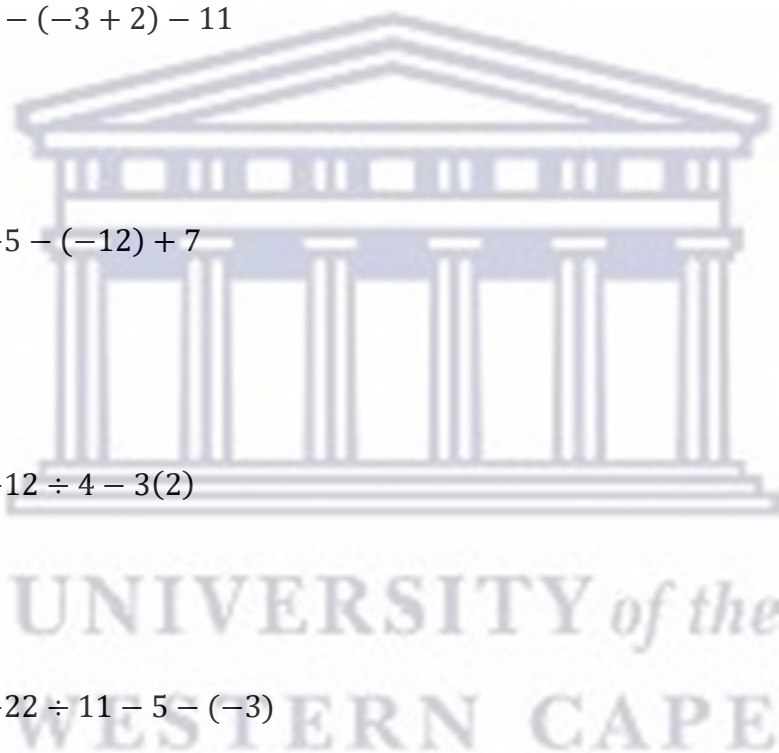
1.1 $5 - (-3 + 2) - 11$

1.2 $-5 - (-12) + 7$

1.3 $-12 \div 4 - 3(2)$

1.4 $-22 \div 11 - 5 - (-3)$

1.5 $\frac{5 - (-5)}{2 - (-6)}$



ACTIVITY 2 (Deepening Mathematical Thinking)

2. Mathematical statements can be sometimes true, always true or never true. Mark with a cross (X) the correct block for the given mathematical statement.

| Mathematical statement | Always True | Sometimes True | Never True |
|---|----------------|-------------------|---------------|
| When two integers are multiplied, the product is larger than any of the two integers. | | | |

- 2.1 Give reasons with possible examples for your choice:

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ACTIVITY 3: (Deepening Mathematical Thinking)

3. Answer the following questions:

- 3.1 A learner is asked to subtract $3 - x^2 - 5x$ from $2x^2 - 12 + 5x$. Here is her solution:

$$2x^2 - 12 + 5x - 3 - x^2 - 5x$$

$$= 2x^2 - x^2 + 5x - 5x - 12 - 3$$

$$= x^2 - 15$$

Explain why you agree or not with the way she did it?

3.2 The multiplication and division questions below have been marked by the teacher.

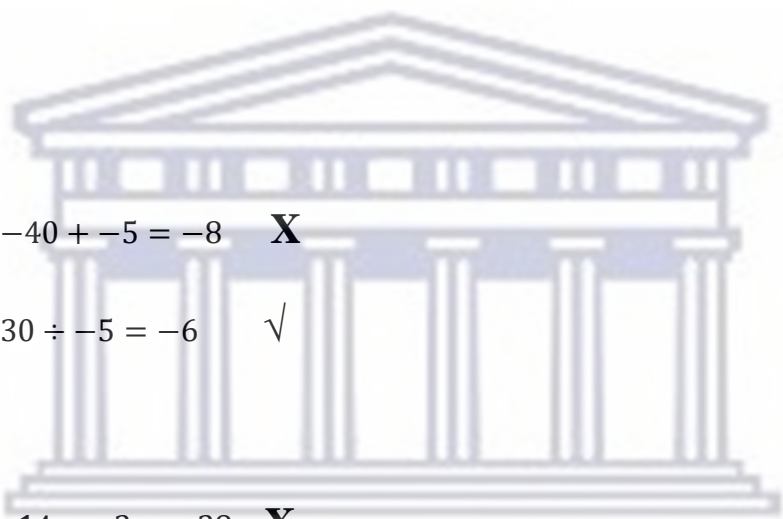
Explain why each answer that has been marked as incorrect is indeed incorrect. Write the correct answer.

3.2.1 $4 \times -5 = 20$ **X**

3.2.2 $-40 + -5 = -8$ **X**

3.2.3 $30 \div -5 = -6$ \checkmark

3.2.4 $-14 \times -2 = -28$ **X**



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Activity 4 (Deepening Mathematical Thinking)

4. Check whether the answers in pair are equal. Explain why they are the same or different.

4.1 $5 - 22$ and $22 - 5$

4.2 $-17 - (+12)$ and $+12 - (-17)$

4.3 $-8 - (-19)$ and $-19 - (-8)$

Activity 5 (Spiral Revision)

5. In the mathematical statements below the symbols #, *, Δ and \square stand for one of the operational signs +, -, \times and \div . Replace the symbol with the correct operational sign so that the mathematical statement is true.

5.1 $1 * (-2) \# 2 = 5$

* =; # =

5.2 $1 \Delta (-2) \square (-2) * 2 = -1$

Δ =; \square =; * =

5.3 $1 \# (-2) \Delta (-1) * 2 = 1$

=; Δ =; * =

APPENDIX B
Parent/Caregiver consent letter

Research title: Implementing productive practice in a grade 9 Mathematics class: A design research study.

| Name and Degree | Phone Number | E-mail |
|--|--------------|---------------------|
| Mr. Luxolo Mlofane, M.Ed. in Mathematics | 063 353 8246 | 3375617@myuwc.ac.za |

Dear Parent/Caregiver

I am investigating a way of teaching to find out whether it will assist with the improvement of your child's results in examinations in Mathematics. Your child will not be taught topics that are different from what I must teach. The only change I will make is to, during my normal lessons, ask learner to do specially designed mathematical exercises on work that has already been completed.

I will, from time to time, write notes on, audio- and video record how your child is working with normal (as in their textbooks) and the specially designed mathematical exercises on work that he or she have already completed. This is to assist with the improvement of the mathematical activities so that your child will be able to deal with similar or related problems in examinations.

I will also require your child to periodically complete a questionnaire on his/her experiencing of the teaching of Mathematics. The questionnaire will be completed under my supervision. The learner has the right not to participate and may at any time withdraw from completing the questionnaire. Under no circumstances will the name be revealed and she/he will not be asked to write her/his name on the questionnaire. Furthermore, I will display no image to identify of your child when I use the information obtained from him or her for report and research purposes.

There are no risks to your child.

Should you want your child or the child in your care **not** to participate in the information collection, kindly indicate your decision below and return the letter to me not later than {DATE}.

I thank you for your willingness to allow your child or the child in your care to participate in this important activity.

Yours sincerely,

Mr. Luxolo Mlofane

I, (name and surname), do give permission for (name child or the child in your care)..... to participate in the information gathering activity.

For any further issues related to the ethics of this project you can contact the HSSREC, Research Development, Tel: 021 959 4111, email: research-ethics@uwc.ac.za

Signature: (Signed)

APPENDIX C
Learner assent letter

Research title: Implementing productive practice in a grade 9 Mathematics class: A design research study.

| Name and Degree | Phone Number | E-mail |
|--|--------------|---------------------|
| Mr. Luxolo Mlofane, M.Ed. in Mathematics | 063 353 8246 | 3375617@myuwc.ac.za |

Dear Learner

I am investigating a way of teaching to see whether it will assist you to improve your results in examinations. You will not be taught topics that are different from those I am must teach. The only change that I will make is to, during our normal lessons, do exercises on work that have already been completed. I will, from time to time, write notes on, audio- and video record how you are working with normal (as in your textbooks) and other specially designed mathematical exercises on work that you have already completed. This information will be used by me to assist with the improvement of the mathematical activities so that you will be able to deal with problems similar or related to those in examinations.

Under no circumstances will your name be revealed or any image to identify of you be displayed when the information obtained from you is used for report and research purposes.

Kindly complete the part below to indicate that you understand the above and that I have explained it to you.

Yours sincerely,

Mr. Luxolo Mlofane

I, (name and surname), herewith acknowledge that the conditions above has been explained to me, that I understand them and willingly participate in the project.

Signature: **(Signed)**



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Department of Institutional Advancement

University of the Western Cape

Robert Sobukwe Road

Bellville 7535

Republic of South Africa



10 December 2021

Mr. L Mlofane
School of Science and Mathematics
Faculty of Education

HSSREC Reference Number: HS21/10/21

Project Title: Implementing productive practice in a grade 9
Mathematics class: A design research study.

Approval Period: 09 December 2021 – 09 December 2024

I hereby certify that the Humanities and Social Science Research Ethics Committee of the University of the Western Cape approved the methodology, and amendments to the ethics of the above mentioned research project.

Any amendments, extension or other modifications to the protocol must be submitted to the Ethics Committee for approval.

Please remember to submit a progress report by 30 November each year for the duration of the project.

For permission to conduct research using student and/or staff data or to distribute research surveys/questionnaires please apply via:

<https://sites.google.com/uwc.ac.za/permissionresearch/hom>

The permission letter must then be submitted to HSSREC for record keeping purposes.

The Committee must be informed of any serious adverse events and/or termination of the study.

*Ms. Patricia Josias
Research Ethics Committee Officer
University of the Western Cape*

Director: Research Development

University of the Western Cape

Private Bag X 17

Bellville 7535

Republic of South Africa

Tel: +27 21 959 4111

Email: research-ethics@uwc.ac.za

NHREC Registration Number: HSSREC-130416-049

APPENDIX E RESEARCH APPROVAL LETTER



Directorate: Research

meshack.kanzi@westerncape.gov.za

Tel: +27 021 467 2350

Fax: 086 590 2282

Private Bag x9114, Cape

Town, 8000 wced.wcape.gov.za **REFERENCE:** 20220117-9055

ENQUIRIES: Mr. M Kanzi

Mr. Luxolo Mlofane
E653 Europe Street
Nyanga
Cape Town
7750

Mr. Luxolo Mlofane

RESEARCH PROPOSAL: IMPLEMENTING PRODUCTIVE PRACTICE IN A GRADE 9 MATHEMATICS: A DESIGN RESEARCH STUDY.

Your application to conduct the above-mentioned research in schools in the Western Cape has been approved subject to the following conditions:

1. Principals, educators and learners are under no obligation to assist you in your investigation.
2. Principals, educators, learners and schools should not be identifiable in any way from the results of the investigation.
3. You make all the arrangements concerning your investigation.
4. Educators' programmes are not to be interrupted.
5. The Study is to be conducted from **17 January 2022 till 30 March 2022**.
6. No research can be conducted during the fourth term as schools are preparing and finalizing syllabi for examinations (October to December).
7. Should you wish to extend the period of your survey, please contact Mr. M Kanzi at the contact numbers above quoting the reference number.
8. A photocopy of this letter is submitted to the principal where the intended research is to be conducted.
9. Your research will be limited to the list of schools as forwarded to the Western Cape Education Department.
10. A brief summary of the content, findings and recommendations is provided to the Director: Research Services.
11. The Department receives a copy of the completed report/dissertation/thesis addressed to:

**The Director: Research Services
Western Cape Education
Department**

**Private Bag X9114
CAPE TOWN
8000**

We wish you success in your research.

Kind regards,
Meshack Kanzi

**Directorate: Research
DATE: 17 January 2022**



1 North Wharf Square, 2 Lower Loop Street,
Foreshore, Cape Town 8001
2531 wcedonline.westerncape.gov.za

Private Bag X 9114, Cape Town, 8000
Safe Schools: 0800 45 46 47 tel: +27 21 467



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APPENDIX F
School Management Team consent letter



Private Bag X17, Bellville, 7535

South Africa

Tel: 021 959 2861

Fax: 021 959 3358

Research title: Implementing productive practice in a grade 9 Mathematics class: A design research study.

The Research Team:

| Name/Degree | Phone Number | Department | E-mail |
|--|--------------|--|---------------------|
| Mr. Luxolo Mlofane, M.Ed. in Mathematics | 063 353 8246 | School of Science and Mathematics at UWC | 3375617@myuwc.ac.za |

Project worker/investigator: Mr. Luxolo Mlofane

Dear School Management Team (SMT)

The LEDIMTALI project intends to implement a project to improve the examination results in Mathematics in your school. The project is explained in the attached information sheet.

The teacher or a project worker will, from time to time, write notes on, audio- and video record how learners are working with normal (as in their textbooks) and other specially designed mathematical exercises on work that they have already completed. This information will be used by the project to assist with the improvement of the mathematical activities so that learners are able to deal with similar or related problems in examinations.

Learners will also be required to periodically complete a questionnaire on their experiencing of the teaching of Mathematics. The questionnaire will be completed under the supervision of an experienced project worker. The learner has the right not to participate and may at any time withdraw from completing the questionnaire. Under no circumstances will the name be revealed and learners will not be asked to write their names on the questionnaire. Furthermore, no image to identify learners will be displayed when the information obtained from them is used for report and research purposes.

There are no risks to learners.

Should you **not** want learners in your school to participate in the information collection, kindly indicate your decision below and return the letter to the project worker not later than {DATE}.

We thank you for your willingness to allow learners in your school to participate in this important activity.

For any further issues related to the ethics of this project you can contact the HSSREC, Research Development, Tel: 021 959 4111, email: research-ethics@uwc.ac.za

Yours sincerely
Mr. Luxolo Mlofane.

Signature: (Signed)