AN ETHNOMETHODOLOGICAL ANALYSIS OF STUDENTS' WAYS OF WORKING WITH ALGEBRAIC FRACTIONS IN HIGH-STAKES EXAMINATIONS: THE CASE OF LEVEL 3 MATHEMATICS STUDENTS AT TECHNICAL AND VOCATIONAL EDUCATION AND TRAINING (TVET) COLLEGES

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DECLARATION

I, Nwabisa Vivian Mbeki, declare that an ethnomethodological analysis of students' ways of working with algebraic fractions in high-stakes examinations, the case of Level 3 students at a TVET college is my work and that it has not been submitted before for any degree or examination in any other university. All the sources that I have used or quoted have been indicated and acknowledged by complete referencing.

Date: ____________________

Signed ____________________

ABSTRACT

The study investigates the National Certificate Vocational (NCV) Level 3 students' ways of working with rational algebraic fractions in a high-stakes examination. An ethnomethodological analysis was used to reveal the textures of examinees' work. Ways of working in this study refer to how examinees deal with algebraic fractions when simplifying them, including even those elements of their work, which are rough work in the sit-down examination.

Ethnomethodology is the study of ordinary actions by ordinary members of society. Ordinary action means that members regularly and recurrently do it with such automaticity that it is given little thought. Ethnomethodology is the study of how people use common sense, procedures, and considerations to gain an understanding of everyday situations (Garfinkel, 1967). In the context of mathematics education, ethnomethodology seeks to understand how examinees construe, construct and orient themselves to these norms that are usually seen but unnoticed (Garfinkel 1967).

The study is premised on a qualitative research paradigm that focuses on studying situations in their natural settings and applying an interpretive perspective. Data were collected from two colleges using students' examination scripts for the end-of-year NCV L3 mathematics exams. Guba and Lincoln's (1985) concepts of trustworthiness, credibility, transferability, dependability, and confirmability were used.

The study sought to ensure ethical principles were followed by applying to each college for permission to conduct research and collect data. Permission was granted. Ethical clearance from the University of the Western Cape was obtained before conducting any data collection. The researcher ensured confidentiality and the anonymity of the participants' scripts. The examination guidelines require that the examination scripts and mark sheets be kept at an institution for verification and cases of appeal. The institution keeps examinees' examination scripts, which are confidential. The researcher ensured that scripts did not leave the college and ensured the confidentiality of their information by making copies of the scripts and keeping the copies safe. The study poses no harm to the participants or the colleges.

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KEYWORDS

Ethnomethodological analysis

Students' ways of working

Rational algebraic fractions

NCV Level 3 students

TVET college

High-stakes examination

Resistance and accommodation

NCV L2 Examinees

Textures of mathematical work

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APP	Annual performance plan
DHET	Department of Higher Education and Training
ESSAS	External Summative Assessment
FET	Further Education and Training
ICASS	Internal Continuous Assessment
NCV	National Certificate Vocational
NATED	National Accredited Technical Education Diploma
NDP	National Development Plan
NCES	National Centre for Education Statistics
NCTM	National Council of Teachers of Mathematics
NRC	National Research Council
RSA	Republic of South Africa
SAQA	South African Qualification Authority
TVET	Technical and Vocational Education and Training
TIMSS	International Mathematics and Science Study
UNESCO	United Nations Educational, Scientific and Cultural Organization
UNEVOC	International centre for technical and vocational education and training

CHAPTER 1 INTRODUCTION AND BACKGROUND

1.1 Preamble

Mathematics, as a school subject in South Africa, is one of the most challenging subjects for some students. Research is one of the tools that can generate knowledge and understanding of the teaching and learning processes in mathematics education. This study seeks to add to the body of knowledge in the learning and teaching of mathematics by examining the students' ways of working with algebraic fractions in high-stakes examinations. Chapter one of this study discusses the background and motivation, the rationale, and the research questions for the study.

1.2 TVET Context in South Africa

The technical and vocational education and training (TVET) college sector in South Africa was established in 2002 through a merger process that transformed 152 technical colleges into 50 TVET colleges (Act 98 of 1998). The colleges were merged in terms of TVET Act 98 of 1998 to combine smaller and weaker colleges into stronger institutions, resulting in reduced costs and increased production and creating capacity within colleges to teach more students and offer a wider range of programmes. The TVET colleges were then moved from provincial management to the Department of Higher Education and Training (DHET). The move aimed to "address the levels of functionality and dysfunctionality in colleges and bring about comprehensive sustainable improvement in college performance" (DHET, 2012). The TVET curriculum aims to equip students with industry-relevant and specific skills. The TVET colleges provide both vocational and occupational training by offering a range of programmes in Engineering, Business Studies, Art and Music, and Food Services. The TVET sector offers two main programmes: the National Certificate Vocational (NCV) and the Report 191 National Accredited Technical Education Diploma (NATED). The NCV Level 2 to Level 4 programme was introduced in 2007 after the NATED curriculum review, which viewed the NATED syllabi as outdated. The introduction of NCV was intended to phase out the NATED N1-N6 programme, but that process is still in progress due to industry demands. Employers seemed to have confidence in the newly introduced NCV programme. In 2020, on October 21, the Parliamentary Portfolio Committee in Higher Education, Science, and Technology TVET curriculum review indicated that "On the industry buy-in for NC(V), over the 12 years of its implementation, things have changed and employers are now asking for more and more NC(V) students because of the rounded training they get. This buy-in has changed for the better in the recent past" (Parliamentary monitoring group RSA, 2020). NCV provides opportunities for students who have passed grade 9 to an industry-focused training alternative from the academic grade 10-12 schooling. Students are provided with an opportunity to obtain a vocational skill, enter the labour market, establish their businesses or further their studies with higher institution qualifications. The figure below shows the distribution of the 50 TVET colleges across the provinces.

Table 1.1 TVET Colleges in South Africa

Eastern Cape		**KwaZulu Natal**		**Northwest**	
• Buffalo City • Eastern Cape Midlands • Ikhala • Ingwe	• King Hintsa • King Sabatha Dalindyebo • Lovedale • Port Elizabeth College	• Coastal • Elangeni • Esayidi • Majuba	• Mnambithi • Mthashana • Thekwini • Umfolozi • Umgungundlovu	• Orbit	• Taletso • Vuselela
Free State		**Limpopo**		**Northern Cape**	
• Flavius Maleka • Goldfields	• Motheo	• Capricorn • Lephalale • Letaba • Mopane South East	• Sekhukhune • Vembe • Waterberg	• Northern Cape Rural	• Northern Cape Urban
Gauteng		**Mpumalanga**		**Western Cape**	
• Central Johannesburg • Ekurhuleni East • Ekurhuleni west • Sedibeng	• Southwest • Tshwane North • Tshwane South • Westcoast	• Ehlanzeni • Gert Sibande	• Nkangala	• Boland • College of Cape Town • False Bay	• Northlink • South Cape Peninsula University of Technology Westcoast

In South Africa, the TVET sector is viewed as one of the key role players in accelerating skills development. The government aims to have 3 million TVET student enrolments and produce 30,000 artisans annually by 2030 (DHET, 2013). For the government to achieve this goal, the enrolment and academic performance of students in TVET colleges should improve. Therefore, students' poor performance in South African Further Education and Training (FET) colleges is a concern for the country. This is highlighted in the Minister of Higher Education Science and Technology's 2019/20 Annual Performance Plan (APP), which indicates that student performance has not improved despite increased funding to Technical and Vocational Education and Training (TVET) Colleges. However, approximately 20% of colleges have consistently improved student performance, but this figure represents only a minority (DHET, 2020:15). In the DHET, APP 2019/20, improving academic performance in TVET colleges is stated as one of the key priorities.

The statement by the minister concurs with Papier (2009), who argues that despite the plans by the government and DHET to strengthen the TVET system and financial and human resources investment into creating a new identity for TVET colleges, the high failure rates and dropouts remain persistent.

1.3 NCV Mathematics and Student Performance

In South African TVET colleges, students with grades 9, 10, and 11 and those who failed or passed grade 12 can enrol in the courses offered by the TVET colleges. For engineering courses—National Certificate Vocational (NCV) specifically—students must take mathematics as one of the compulsory fundamental subjects. The inclusion of the fundamental subjects (Mathematics, mathematical literacy, Life orientation, and language of teaching and learning) was determined by the South African Qualifications Authority as the regulation requiring a fundamental component at the same level as the qualification itself to frame qualifications. This ensured that learners had sufficient general education to serve as a foundation for their learning and for further learning to progress to higher levels (Houston; Booyse & Burroughs, 2010). Umalusi concurred with SAQA's principle and maintained fundamental subjects.

The position taken by SAQA and Umalusi in making mathematics a fundamental subject is in line with the argument by the International Commission on Mathematical Instruction (ICMI), which says that mathematics is a fundamental part of human thought and logic. Specifically, geometry and algebra were two of the seven liberal arts in Greek as well as in medieval times, and such a historical role supports the notion that mathematics has provided the mental discipline required for other disciplines. Therefore, struggling with mathematics means it will be difficult for a particular student to complete the course, get a certificate for each level and finally graduate. This is different in the school system because in the FET phase (grade 10-12), learners can progress to the next grade without passing mathematics if it is the only subject they have failed, and they can continue to do mathematics in the next grade. The 2015 Trends in International Mathematics and Science Study (TIMSS) reports for grades five and nine show that learners fail mathematics (Mullis, Martin, Foy, & Hooper, 2015). Challenges with mathematics at the school level can impact the child's performance in college, as learners need a firm background of mathematical concepts to build upon as they learn college mathematics. Morgan and Ibrahim (2019) refer to global tests as transnationally fabricated tests that are designed by the designers who have constructed their ideal type of user, which is the student. The TMISS is criticized for reasons such as difficulty in achieving comparable samples, misleading forms of reporting, differences in the degree of the curriculum match, omission of results that would undermine policy, ecological fallacy, and others (Eckert, 2008).

However, competence in mathematics and science is important for social and economic development (TMISS, 2015), and South Africa is part of the global economy; hence, the results of these global assessments cannot be discarded. Grade nine mathematics students were ranked 38 out of 39 countries, and grade 5 was ranked 47 out of 48. On the five international benchmark scales, which are Advanced (above 625 points), High (550 to 625 points), Intermediate (475 to 550 points), Low (400 to 475), and Not Achieved (less than 400), only 1% of the grade 9 students achieved an advanced level (625 points), and 3% achieved a high level (550-625 points). "For grade nine, at advanced international benchmark level, students are expected to apply and reason in a variety of problem situations (fractions, percentages, proportions, geometry, averages, expected values, etc.), solve linear equations and make generalisations." (Letaba, 2017:2). Students who struggle with mathematics from grade 9 will face challenges with mathematics at the college level. In remedying the situation, the DHET has offered mathematics baseline assessments for the students who enrol for level 2 NCV mathematics for lecturers to identify gaps and devise intervention strategies. In addition, as part of the intervention, NCV L2 students engage in a practice referred to as "Ready Steady", which covers and revises some basic mathematical content. Despite different forms of intervention, the student's poor performance in mathematics remains a challenge.

The NCV curriculum has three fundamental subjects that are compulsory for students: English, Life Orientation, and Mathematics (Mathematical Literacy). For all engineering programmes, mathematics is a compulsory subject. Statistics show that the completion rates in these programmes that require mathematics as one of the fundamental subjects are very low. For example, in the 2016 examinations, Civil engineering and building construction was 27.4%; Electrical infrastructure construction 34%; Engineering and related design 29.8%; Mechatronics 33.8%; Information technology and computer studies 22.3% (DHET, 2015). Completion rates for the NCV have remained below 50% from 2011 to 2016, as shown in Figure 1.1 below.

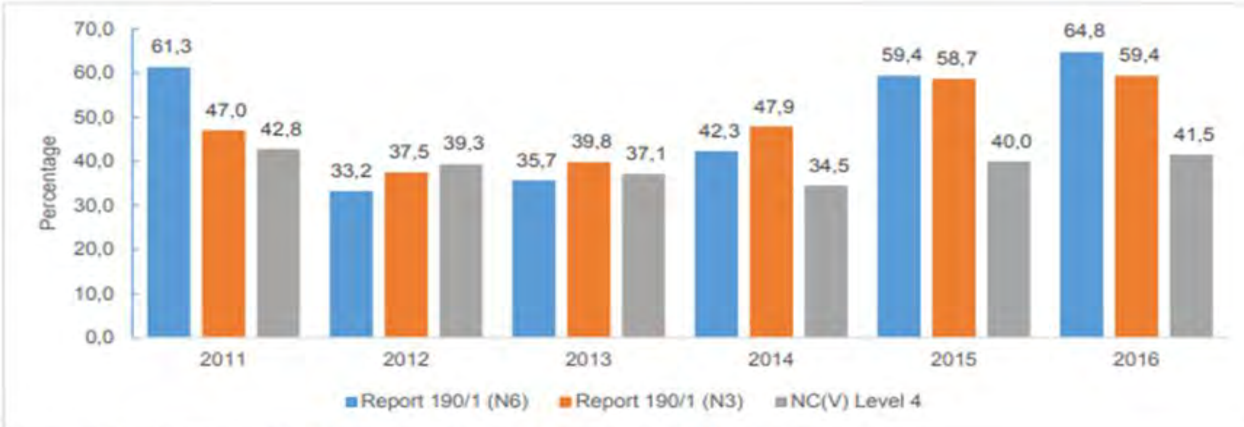

Figure 1.1 NCV Completion Rates 2011-2016

Source: DHET (Statistics on Post-School Education and Training in South Africa, 2016)

From 2011 till 2016, the completion rate for NCV L4 continued to be below 45%, with the highest percentage being 42.8% in 2011 and the lowest percentage of 34.5% in 2014. Such performance is a concern as the country needs many artisans, and the South African government spends much money improving the TVET college system.

The DHET statistics for the national results in NCV Mathematics show that the total pass percentage for Mathematics between 2016 and 2018 was just below 53%. The table below summarizes the 2016-2018 results for TVET colleges.

Table 1.2 National Results Mathematics NCV

NCV LEVEL	L2	L3	L4	TOTAL
2016	49.8%	62.2%	35.8%	50.6%
2017	52.3%	68.1%	38.6%	52.9%
2018	54.3%	52.1%	50.5%	50.5%

It should be noted that the percentages above are only for those students who qualified to sit for examinations, i.e., they obtained a minimum of 30% from their internal assessments (ICASS).

Some students did not qualify to write the examinations and can be counted as part of those who failed. The table below shows the enrolment for the subject and those who sat for examinations.

Table 1.3 Enrolments for subject and number of students who sat for the examination

NCV	L2		L3		L4		TOTAL	
	Enrolled	Written	Enrolled	Written	Enrolled	Written	Enrolled	Written
2016	34884	20687	16699	13683	11361	9593	63,441	44,263
2017	31221	18341	12886	9923	11235	9785	55637	38249
2018	29803	18067	14184	11277	7764	3920	53684	37266

Table 1.3 above shows that in 2016, 19 178 students who enrolled for L2, L3, and L4 did not write the examination, which is 30.23 % of the total enrolment. In 2017, 31,25 % of the student enrolment did not write examinations; in 2018, it was about 28.75 %. The number of students who enrolled but did not write could be a combination of those who dropped out or were deceased and those who could not meet the 30% ICASS mark requirement. If those who did not qualify to write were considered when calculating the pass percentage, the percentages would drop even further than the ones in Table 1.2.

Although many colleges have implemented special programmes to support underperforming students, this is not an ideal solution. The poor performance of students in mathematics can be associated with many factors, including poor learning foundations in literacy and numeracy, poor attendance (absenteeism), lack of motivation, fear, and negative attitudes toward mathematics (DHET, 2017; Ngoveni, 2018). The lecturer's capacity and teaching strategies are also mentioned as one of the reasons for the poor performance of students, including the lecturer's mathematical content knowledge. Ngoveni (2018), Sehole, Sekao, and Mokotjo (2023) attribute students' failure to teachers' instructional practices as a major contributory factor. Furthermore, a lack of exposure to instruction and assessment tasks that involved all representations of questions they must respond to hinders a deep conceptual understanding of the mathematical topic.

There is limited literature on students' performance in NCV mathematics in South Africa and the challenges students exhibit with mathematical problems, especially in working with algebraic expressions (including fractions), functions, and graphs. Therefore, the interest of this study is to explore the type of feedback one can get from students' ways of working with algebraic fractions to contribute to the body of knowledge in mathematics teaching and learning and assist lecturers in understanding some of the challenges that hinder their students from performing well in the rational algebraic fractions.

Research on NCV mathematics shows that students perform poorly in mathematics L2, L3, and L4 (Ngoveni, 2018; Dolley, 2015; Naicker, 2017; Rakhudu, 2017). The poor performance of students in mathematics translates to the need for more research on NCV mathematics to help both the lecturers and students understand the problem underlying the failure rate and suggestions on how the challenge can be alleviated. There is limited literature on mathematics education for the TVET sector in South Africa. This study is, therefore, significant in that it seeks to add to the body of knowledge available in TVET mathematics education. This study analyses examinees' ways of working with rational algebraic fractions in high-stakes examinations. An ethnomethodological analysis of student's work will be employed to highlight what is usually seen but ignored or unnoticed in the solution-seeking pursuance.

The envisaged results of the study are a closer understanding of the challenges students experience as they solve mathematical problems rather than focusing only on what they do right or wrong.

1.4 Statement of the Problem.

Students' poor performance in mathematics is a critical situation that requires attention. Many problems lead to poor performance in mathematics, and one study cannot address them all simultaneously. Challenges with fractional competency and algebraic manipulation are some of the factors contributing to poor performance. Algebraic fractions are part of the 20% mark for algebra in the final NCV L3 mathematics Paper 1 examination. Algebra is a key branch of mathematics that uses the basic building blocks of four operations of arithmetic (addition, subtraction. multiplication, and division) (Bernard & Zandy, 2013). Although competence in mathematics requires many skills, research shows that competence in algebra leads to success in mathematics. Algebra is considered a "gateway" to a sequence of mathematics courses. Students who proceed successfully in algebra have a higher chance of advancing to higher levels in college and university mathematics (U.S. Department of Education NCES, 2010). Algebraic fractions and the skills used to simplify them are embedded in other mathematics topics, such as functions and graphs, trigonometry, and others. There is a gap in the literature on the simplification of algebraic fractions by NCV mathematics students. This study seeks to fill that gap by examining how NCV L3 examinees simplify algebraic fractions in high-stakes examinations.

1.5 Purpose of the study

This study used an ethnomethodological analysis to analyse students' ways of working with algebraic fractions in high-stakes examinations.

1.6 Research question.

The study sought to find answers to the following question:

What are the L3 mathematics students' ways of working with algebraic fractions in high-stakes TVET College end-of-year national examinations?

1.7 Subsidiary research questions

(i) How do examinees respond to questions on algebraic fractions in a high-stakes examination setting?

(ii) What is the visible lived work in examinees' solution-seeking pursuance when doing algebraic fractions in a high-stakes examination?

1.8 Significance of the study

There is limited literature on NCV mathematics teaching, learning, and assessment in SA. This research will contribute to the available literature on the teaching, learning, and assessment of algebraic fractions in TVET colleges, specifically NCV L3 algebra.

The approach of this study seeks to reveal the textures of students' work as they simplify algebraic fractions in high-stakes examinations using an ethnomethodological analysis. The study will, therefore, contribute to the knowledge of applying ethnomethodological constructs in interpreting what students do as they respond to questions in a naturally organized examination setting.

The findings of this study can help lecturers, researchers, and other practitioners understand students' ways of working with algebraic fractions in an examination room and, therefore, adjust their pedagogies and assessment strategies to help students improve their performance in mathematics.

CHAPTER 2 LITERATURE REVIEW

2.1 Introduction

Using an ethnomethodological analysis, this study analyses the students' ways of working with algebraic fractions in a high-stakes examination. Therefore, this chapter provides an overview of the literature that relates to algebraic fractions as a key mathematics topic under analysis, assessment in mathematics as the focus is on high-stakes examinations, and ethnomethodology.

Literature on feedback after the assessment is discussed as a key element of the assessment, which also affects the teaching and learning of mathematics in classrooms. The topic under investigation is algebraic fractions, specifically looking at the students' ways of working with algebraic fractions. Hence, the available literature is presented on algebraic fractions, fraction competency, and its effect on learner performance in algebra and the students' ways of working with algebraic fractions. A brief discussion on ethnomethodological studies in mathematics education is also briefly discussed.

2.2 Assessment in Mathematics

Assessing means establishing quantity, quality, value, or characteristic (Sangwin, 2013). Assessment as a noun may refer to the instrument itself, such as an examination paper used to determine what examinees know and can do. Assessment in mathematics can either be a large-scale assessment or a classroom assessment. Both large-scale and classroom assessments have specific purposes. Classroom assessment usually gathers information and provides feedback to support individual student learning.

In contrast, large-scale assessment is used to monitor educational systems and increasingly plays a prominent role in the lives of students and teachers, as graduation or grade promotion often depends on students 'test results' (Suurtamm et al., 2016). According to the National Research Council (1993), assessment in mathematics can be used to inform teachers, students, parents, policymakers, and stakeholders about what students have learned, what mathematical terms they recognize and can use, the procedures they can carry out, the kind of mathematical thinking they do, the concepts they understand, and the problems they can formulate and solve. Furthermore, it provides information that can be used to award grades, evaluate a curriculum, or decide whether to review fractions of the curriculum.

Assessment in mathematics education concerns judging students' mathematical capability, performance, and achievement (from the lowest grade to the PhD level of study), whether as individuals or groups. Assessment, therefore, addresses the outcome of mathematics teaching at the student level (Niss, 1993). Teachers commonly use assessments such as tests, examinations and many others to judge students' performance in mathematics, monitor their mathematical learning, and inform their future instruction. Assessment is not primarily about grading or judging whether a student is ready to progress to the next grade, qualifies to enrol for a particular course, or can pursue a particular career, but assessment is vital in teaching and learning progress. Whether through internal continuous assessment or external examinations, the assessment feedback can be used to improve teaching and help students progress toward higher standards in their performance.

Niss (1993) explains a few reasons for assessment in mathematics. First, systematic observations should be carried out so that teachers can make judgments about the progress of the learning process. Second, assessment is an integral and fundamental part of teaching and learning. Teachers and students can improve their performance if they have assessed and identified areas that need improvement. Third, the assessment should consider attitudes and general procedures. Finally, assessment is not a goal but must be continuous and cater to individual differences. Considering all the above reasons, it can be highlighted that assessment is not only for students but also for both the student and the teacher and can also be for the school, policymakers, the state, and other stakeholders. However, the key components, the student and the teacher, should use assessment effectively for teaching and learning. The mathematics curriculum is interconnected. There is connectedness across mathematical strands, concepts, ideas, and interconnections with other subjects. (Annenberg Foundation, 2017) argues that connections between mathematics topics and concepts within and across grade levels, between mathematics and other subjects, and between mathematics and everyday life all contribute to making mathematics understandable and meaningful. The concepts and content covered in one level become very important and necessary for the content that the student will do in the next level. Therefore, each level's mathematical skills, competencies, and understanding must be mastered to progress smoothly to the next level. It becomes vital then that the feedback to assessment is not given attention at the level of formative assessment. However, feedback after an examination is important to inform the lecturer of how examinees deal with mathematical problems to inform the improvement of future teaching and the basis of how to address certain misconceptions and errors at the next level.

Assessment Standards for School Mathematics from the National Council of Teachers of Mathematics in the USA (NCTM, 2014) articulate that assessments should contain high-quality mathematics, enhance student learning, reflect and support equitable practices, be open and transparent, be such that inferences made from assessments are appropriate to the assessment purpose and that the assessment along with the curriculum and instruction should form a coherent whole. Suurtamm et al. (2016) argue that the principles mentioned by NCTM (2014) are still valid standards or principles for large-scale classroom assessment in mathematics education. Furthermore, assessment should reflect the mathematics that is important to learn and the mathematics that is valued. This means that both large-scale and classroom assessments should consider not only content but also mathematical practices, processes, proficiencies, or competencies. It, therefore, becomes vital that we look at how learners respond to questions in an assessment so that we can see the mathematical practices that learners apply when answering questions in an exam, as well as the processes involved.

2.3 Assessment of National Certificate Vocational

The NCV programme in TVET colleges offers a wide range of qualifications that aim to respond directly to the priority skills that will help the South African economy grow Branson, Hofmeyr, Papier and Needham (2015). They include theoretical and practical components in a particular vocational field. As previously stated, the programme has subjects referred to as fundamental subjects and are compulsory for all NCV students to do. Mathematics, as one of the fundamental subjects, becomes a stumbling block in completing the NCV certificate if students struggle with it. Branson, Hofmeyr, Papier and Needham (2015) argue that the NCV curriculum for fundamentals such as Mathematics proved to be more academically challenging than the National Accredited Technical Education Diploma (NATED) courses and might be tougher than the equivalent grades 10 – 12 materials in mainstream schools, which result in high subject failure rates and low certification rates. It, therefore, becomes vital to look at the assessment of Mathematics in NCV and how examinees respond to the questions asked in the question paper in pursuit of completing their NCV qualification.

According to the National Policy on the Conduct, Administration, and Management of the Assessment of the National Certificate Vocational, RSA, DHET (2017:7), "assessment means gathering of information to make a judgement about what a student knows, understands and can do". This includes a variety of assessment methods conducted by the provider, the outcomes of which count toward the achievement of a qualification and are thus inclusive of the Internal Continuous Assessment and examinations. The policy elaborates that the assessment is not only meant to judge what the student knows, but it can also describe the status of an individual's learning, and it should not be used in isolation but linked to teaching and learning. One of the aims of assessment is to provide supportive and positive mechanisms that help students improve their learning and lecturers improve their teaching (RSA, DHET, 2017). The policy also states that:

> Each assessment task should be designed to address the subject outcomes, learning outcomes, assessment standards, content competencies, skills, values and attitudes of the subject and to provide students, lecturers and parents with results that are meaningful indications of what the students know, understand and can do at the time of the assessment (RSA, DHET, 2017:3).

These statements show that assessment serves several purposes, including deciding whether a learner is ready to progress to the next level and assisting in the process of teaching and learning.

The mathematics NCV assessment framework comprises the internal continuous assessment (ICASS), which contributes 25% toward a student's final mark, and an External Summative Assessment (ESSAS), which contributes 75% toward a student's final mark. Assessment instruments for the ICASS can be assignments, projects, written tests, investigations, and others. The Department of Education (RSA, DHET 2017) conducts the external assessment according to the requirements specified in the Subject Assessment Guidelines for Mathematics. Two examination papers, Paper 1 and Paper 2, are set for the final examination, usually written once a year in November/December, and supplementary papers are set for those who will qualify to write supplementary examinations. The student needs to have a minimum of 30% of ICASS to qualify to sit for an external examination in mathematics, and the 25% of ICASS plus 75% of the examination should also be a minimum of 30% for a student to pass mathematics at that level.

The final examination papers are prepared by subject specialists according to the subject learning outcomes and comply with the relevant subject policies. Professional external moderators then moderate the papers according to the policies of the quality assurance bodies Umalusi and the South African Qualification Framework (SAQA).

According to the NCV L3 assessment guidelines, assessment in mathematics must consider that the process or method carries more weight than the final answer. This is more important; hence, this study seeks to uncover the methods employed to obtain the final answer when working with a mathematical problem rather than concentrating only on the final correct or incorrect answer.

2.4 High-stakes examination in TVET colleges

High-stakes examinations are "those assessments that have serious consequences attached to them" Nicholas and Berliner (2008: xv), such as failing and repeating the class or subject, adding more years to complete the course, incurring extra costs, and other consequences. For NCV mathematics, the end-of-year summative external assessment is the high-stakes examination determining whether a student is ready to progress to the next level. Mathematics level three (L3) external examination prepared nationally is a common examination written in all provinces of South Africa. The fraction (75%) of the external assessment and the internal continuous assessment (25%) gives a total percentage that decides if a student progresses to mathematics level four (L4), which is the last level for the National Certificate Vocational. The final examination in L3 has serious consequences for the students because failing Mathematics L3 means that those students will not receive a certificate for Level 3 and, therefore, will not be able to register for Mathematics L4. In addition, the student who fails Mathematics L3 will not receive funding for Mathematics L3 again but will have to pay for the subject by their own means and have an additional year added to complete the programme. This is unlike the Senior Certificate in the Department of Basic Education, where students can progress to the next level even if they fail the subject. For example, a student who fails mathematics in grade 11 but passes all other subjects can progress to grade 12 and do grade 12 mathematics. In the NCV, the students must pass the subject at the previous level to register for the next level. The NCV is a 3-year programme, but most students complete it after four or five years, and some drop out because they struggle with mathematics.

2.5 Feedback after the examination or assessment

Hattie and Timperley (2007) define feedback as information provided by an agent (e.g., teacher, peer, book, parent, self, experience) regarding aspects of one's performance or understanding; it is a consequence of performance.

> Feedback is information with which a student can confirm, add to, overwrite, tune, or restructure information in memory, whether that information is domain knowledge, meta-cognitive knowledge, beliefs about self and tasks, or cognitive tactics and strategies (Winne & Butler, 1994:5740)

William (1999) argues that there is no one right way to provide feedback, but feedback routines in each class will need to be thoroughly integrated into the daily work of the class, and teachers need to consider changing the kinds of feedback they give to maximise the effect on student performance in mathematics.

The main purpose of feedback is to reduce discrepancies between current understandings or performance and a desired goal (Hattie & Timperley, 2007). It only becomes effective when contextualized, meaning feedback should be addressed to a particular teaching and learning content (Hattie, 1999; Hattie & Timperley, 2007). In addition, for effective feedback, the following three questions should be answered: “Where am I going?”, “How am I doing?” and “Where to next?” (Hattie & Timperley, 2007). The above statements highlight the value of feedback and provide directions on ensuring effective feedback. Therefore, it can be argued that not all feedback effectively improves student performance, but it is important to set clear goals, the process to achieve the goals, and what needs to be improved or corrected to achieve the goals. Feedback, therefore, should be more specific to be effective.

Manley (2016) argued that students and teachers receive minimal feedback from high-stakes examinations compared with the effort they put into preparing for the examinations. Feedback to students usually focuses on the scores achieved and has no informative and constructive element (Manley, 2016). Temmerman (2018) criticised high-stakes examinations for failing to measure genuine learning constructively and comprehensively because they do not allow students to learn from their mistakes and improve. According to Temmerman (2018), teachers at all levels should facilitate the learning process, and a significant part of that process is providing feedback to students about their learning to help them improve. After writing a high-stakes examination, it is not common for students to be given feedback on their performance.

In South Africa, the TVET final examination papers are set by the Department of Higher Education and Training (DHET), and for levels 2 and 3, the scripts are marked and moderated internally by the colleges in their marking centres. Markers and moderators must complete the reports about the student’s performance for that examination paper and send them to the department for recording. Figure 2.1 below shows an extract from the National policy on the conduct, administration, and management of the national certificate (vocational) assessment, which indicates the processes to be followed in moderating and writing reports after the scripts are marked.

6. MARKING AND MODERATION OF MARKING DURING INTERNAL MARKING

All markers are required to complete marking reports and 10% of the scripts per paper written for a subject are to be moderated at the marking site for Engineering Studies N1 and NC (V) L2 and L3. Thereafter, each TVET College must collate and submit **one** marking and **one** marking moderation report per subject written across all examination centres registered with the college for Engineering Studies N1 and NC (V) L2 and L3 subjects **(*ANNEXURE H- MARKER'S / CHIEF MARKER'S REPORT*** and ***ANNEXURE I- INTERNAL MARKING MODERATOR'S REPORT*)** and submit these to the email addresses indicated in section 3.3 of this document.

Careful attention must be given to the completion of the section of the report dealing with ***student performance*** as the feedback provided in the marking reports will be used to inform subject teaching and learning. Comments such as "*a good question*" or "*students did well*" are not helpful. The comment should focus specifically on <u>student deficiencies</u> in the particular <u>curriculum content</u> assessed in each question. For example, in a subject such as Applied Accounting the following comment would be helpful to a lecturer- "*Income Statement – many students did not know how to show calculations in an income statement*".

The academic head of the TVET College must appoint a subject head/senior lecturer as the coordinator of the chief markers' reports for the TVET College and another subject head/senior lecturer as the coordinator of the internal moderators' reports per examination paper marked at the centre. These coordinators will be responsible for receiving the reports for an examination paper from each marking centre that is marking that paper and collating these reports into a single report for that TVET College. For example, where a college has five campuses all offering Applied Accounting Level 3, the coordinator for the chief markers' reports for the examination paper will receive five reports and then collate the information into a single report for the examination paper and submit this to the Department on behalf of the TVET College. Similarly, the internal moderator will consolidate the five moderation reports into a single internal moderator's report for submission to CDNEA. The consolidated reports for the marking and moderation processes must respectively reflect the examination centres that submitted reports to each of the coordinators.

The reports must be saved using the following naming convention: The full name and level of the subject being reported on and the full name of the college, **e.g. *Mathematics N1 Umbumbulu Marking report <u>and</u> Mathematics N1 Umbumbulu Moderation report.*** Reports not named in this manner will be ignored and will not be captured on the register of reports submitted by a college. These reports are handed to the quality assuror and the marks are standardised and approved on the basis of the content of these reports.

All consolidated Engineering Studies N1 and NC (V) L2 and L3 marking and marking moderation reports must be submitted electronically within 48 hours of the conclusion of the marking of a subject to the to the email addresses indicated in section 3.3 of this document by the Academic Head / Examination Officer of the College. The content of the reports must be captured electronically on the templates and emailed as an **editable Word** document. Do not ***pdf*** the documents. Scanned and faxed documents will be rejected. A covering sheet (***ANNEXURE J - COVER SHEET FOR SUBMISSION OF MARKER'S AND MODERATOR'S REPORTS)*** indicating the marking centre, centre number, levels and subjects for which reports are included must accompany the marking and marking moderation reports when submitting to CDNEA. This should be checked and signed off by the academic head of the TVET College prior to the dispatch thereof.

If a college fails to submit marking and moderation reports this will be declared an irregularity and due processes will follow. The centre will then have to explain to the candidates why their results are not released.

Figure 2.1 Marking and moderation during internal marking

Source: DHET (2015:9)

The markers and moderators' reports serve as feedback on the examinee's performance in the examination. In the report template that markers need to complete, they must reflect on the standard of the question paper and quality of the memorandum, comment on examinees' responses per question, and finally report on errors and misconceptions that examinees showed while responding to questions. The reporter must provide a detailed report and send it to the department's quality assurance, as indicated in Figure 2.2 below.

REPORT TEMPLATE FOR MARKING

higher education
& training
Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

TVET COLLEGES: MARKER'S / CHIEF MARKER'S REPORT

INSTRUCTIONS FOR COMPLETION

1. The report on marking will serve the following objectives:
   a) Provide feedback on the standard of the question paper.
   b) Provide feedback on the quality of the marking guideline.
   c) Provide an overview of student performance in the paper.
2. Sections 1, 2 and 3 are to be completed in consultation with the Internal Marking Moderator.
3. The report must be completed in detail and single word responses will not be accepted.
4. Where additional space is required, use a separate page which must be attached to this report.
5. All reports are to be submitted immediately via email on completion of the marking of a subject.

Figure 2.2 Extract of the reporting template for marking

Source: DHET (2015:28)

The DHET and the colleges use the report to inform the teaching of the subject. Subject teachers can use the report to inform their teaching and help students overcome misconceptions and errors.

2.6 Research in NCV Mathematics: What we Know Thus Far.

Research studies in vocational and training education have largely been quantitative and focused on the level of management and the potential contribution TVET education can make to the economy (Badenhorst & Radile, 2018; Powell, 2013). Both nationally and internationally, research has focused on policies and systems and neglected teachers' and students' teaching and learning (UNESCO-UNEVOC, 2014).

Rakhudu (2017) examined the challenges level 3 national certificate vocational students face in understanding hyperbolic functions in mathematics. Students demonstrated a lack of prerequisite knowledge required when doing hyperbolic functions and some difficulties in drawing a hyperbolic function, as performing calculations involved in working with hyperbolic functions showed that students have challenges in understanding functions and algebra in general.

Badenhorst and Radile (2018) explored the main stumbling blocks in improving NC (V) students' performance at TVET colleges as perceived by internal stakeholders: the students, lecturers, and management. The findings of their study indicate that college leadership attributes students' poor performance to the lack of meaningful orientation of new staff lecturers into the colleges, lack of monitoring and follow-up by management, and lack of subject expertise on the side of lecturers. The lecturer's view is that students perform poorly because they lack motivation; the programme (NCV) is pitched too high for the students compared to the school grade of the comparable level; absenteeism; lack of clear and specific selection criteria for the students admitted to the programme. Students, on their side, blame their lecturers for not having a passion for the teaching career and, as such, lack classroom management skills and motivation. Lack of dedication and discipline was also highlighted as one of the challenges that led to poor performance. It can be noted that this study did not focus on mathematics specifically but sought to find the general factors that could be the reasons for poor performance in the TVET colleges.

Ngoveni (2018) investigated the factors contributing to students' poor performance in mathematics level 2. The study's findings showed that students fail because of misconceptions, including false concepts, adding unlike terms and partial application of rules. False concepts and the addition of unlike terms indicate a lack of conceptual understanding. Fractions, simultaneous equations, and factorisation were found to be the most challenging topics for students. In his study, Ngoveni (2018:iv) categorises the following factors that can contribute to a student's poor performance in mathematics.

- Student factors: these included absenteeism, attitude and fear toward mathematics, lack of commitment and discipline, and poor mathematical background.
- College factors: these included "teaching strategies; enrolment, which takes place until the second term; students with different mathematical backgrounds in one class; and unverified entry qualifications" (Ngoveni 2018, iv).

This study used a qualitative approach to investigate factors contributing to the failure rate in mathematics level 2. The details of how students deal with mathematical problems in the classroom and during examinations were not covered. Ngoveni and Mofolo-Mbokane (2019) reported on the misconceptions held by National Certificate (Vocational) Level 2 mathematics students in Algebra. The report indicates that learners have difficulty working with algebra. Their errors result from failing to understand or know the principles necessary to solve a problem. Students were found to have committed arbitrary, executive, and structural errors (Donaldson, 1963). Structural errors were found to be dominant where learners struggled with the addition of unlike terms; factorization involves grouping only; the assumption that equations are only linear and negative signs not taken out in the second term when taking out a common factor.

Naicker (2017) investigated the extent of conceptual and procedural difficulties that NCV Level 4 students encountered when factorising and solving problems involving factorisation. The results indicated that NCV level 4 students experienced conceptual and procedural difficulties with all factorisations.

Sehole (2020) explored the conceptual and procedural difficulties experienced by NCV L2 students when learning the concept of functions. The results showed that due to possibly shaky mathematical foundations, lecturers and teaching methods, students have several procedural and conceptual errors.

More studies in NCV or TVET college mathematics have focussed on errors and reasons for failure rates (Dolly, 2015; Mofolo-Mbokane, 2012), and they are mostly on the topic of functions in mathematics. There is a gap in the literature on algebraic fractions in the TVET sector or NCV mathematics specifically. In addition, the available literature mostly investigates errors and possible reasons for high failure rates in Mathematics. This study focuses on how examinees produce their answers for the final examination paper from an ethnomethodological point of view.

2.7 Algebraic fractions

Algebraic fractions are fractions where either numerators or denominators or both are algebraic expressions (Dalla & McDonald, 2004). For example, $\frac{2x}{x^2+4}$ and $\frac{2a^2+4a}{2a}$ are algebraic fractions. An algebraic fraction whose numerator is a polynomial of a lower degree than the polynomial in the denominator is referred to as a proper algebraic fraction, such as $\frac{x^2}{2x^3+x}$ while an algebraic fraction whose numerator is the polynomial of a higher degree than that of the denominator is referred to as an improper algebraic fraction—for example, $\frac{ab^2+2b}{a+b}$. The letters a and x or any other letter that might be used are unknown, although there is no unanimous agreement in the mathematics community about the use of the terms *unknown*, *variable* and *placeholder*. This study uses letters or pronumerals as variables and placeholders.

Erly and Adams (2012) posit that when a letter is used as a variable, it represents a range of unspecified values, and a systematic relationship exists between two such sets of values. Variables can take on different values in different contexts. It is a symbol used to represent a number; letters are usually used for variables, but they can be anything other than numbers. For example, a certain number of lunch packs (a) for permanent employees (b) in a farm and contract employees (c) must be shared. The rational expression for sharing lunch packs will be $\frac{a}{b+c}$. The variables a, b and c can be any number. The placeholder means a letter representing a number provided in a particular problem or context. A placeholder is often called a given or a constant; in specific instances, it is a parameter or a coefficient.

An unknown is a letter that has a determinate value(s), which can be determined, for example, by solving an equation. These terms might seem confusing and difficult to differentiate, but we can differentiate by viewing an unknown as the value that is not known initially, which is being sought. It is usually used in equations. The variable can be any large set of values. It represents the generic element of a set—variables sometimes co-vary (Carlson et al., 2002) with another quantity. If one quantity is given, then the other variable becomes an unknown to be determined. In solving algebraic fractions, examinees are not seeking to determine the values of a, b, c, x, y or any letter used, but the intention is to write the expression in its simplest form.

The research shows that algebraic fractions are critical because they are the foundation for students to succeed in algebra and other topics in mathematics. As Wu (2001:1) says:

> The proper study of fractions provides a ramp that leads gently from arithmetic to algebra. But when the approach to fractions is defective, that ramp collapses, and students are required to scale the wall algebra not at a very gentle slope but at a ninety-degree angle, not surprisingly, many cannot.

Wu (2001) further argues that when students' knowledge of fractions is not good, they will struggle with algebra. In NCV mathematics L3, algebra weighs 35 out of 100 in the weighting value of the topics. In the final examination, it contributes 50 out of 200 marks in the mathematics final examination. Therefore, students who struggle with fractions will struggle with a significant part of the subject. The following discussion highlights what the research says about algebraic fractions. The discussion will cover the correlation between fraction competency and performance in algebra, errors and misconceptions, and students' ways of working with algebraic fractions.

2.7.1 Fractional competency and performance in algebra.

Competency in mathematics is seen as a multidimensional skill that cannot be achieved by being competent in one component or strand of mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001). Kilpatrick, Swafford, and Findell (2001:16) argue that mathematical proficiency can be achieved by achieving its five interwoven and interdependent strands or components, namely:

- *Conceptual understanding*: comprehension of mathematical concepts, operations, and relations.
- *Procedural fluency*: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.
- *Strategic competence*: the ability to formulate, represent, and solve mathematical problems.
- *Adaptive reasoning*: capacity for logical thought, reflection, explanation, and justification
- *Productive disposition*: a habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Proficiency cannot be achieved by only focusing on one strand; however, as learners progress from one grade to the next, they should increase their mathematical proficiency by being exposed to experiences that will help them develop these five strands. Therefore, mathematical content or topics are interrelated; in each topic, a learner develops certain levels and types of competencies necessary for another topic or mathematical content.

Fractional competency refers to performing basic operations involving fractions (Thomas, 2010). Students are fractionally competent if they add, subtract, multiply, and divide fractions. Performing these operations can involve other operations such as factorisation and cross multiplying in equations depending on the grade or level of study. When a student is competent in working with fractions at the college level, the student can work with operations involved in simplifying algebraic fractions. Competency with fractions is a prerequisite for success in algebra and beyond (Perkins, 2017). Some studies investigated the relationship between fractional competency and performance in algebraic fractions. Studies reveal that there is a correlation because the students who performed better in fractions also performed better in algebra. Thomas

(2010) investigated the importance of fractional competency to success in algebra. Thomas (2010) asserts that students who struggle with fractions cannot do well in algebra. "The more students struggle with fractional concepts, the more they struggle with algebra concepts"; fractions support the learning of algebra (Thomas, 2010:33). The results of the study proved the hypothesis to be true because there was a strong correlation between fraction competency and success in algebra. Fractional competency is, therefore, viewed as a predictor (Hurst & Cordes, 2018) and a prerequisite for success in algebra (Perkins, 2017). As such, students need to be taught fractions well at a young age before they reach adulthood. This is because learning fractions and their operations during adolescence still affects students' abilities and success in mathematics courses (Aldrich, 2015). However, Brown & Quin (2007) assert that fraction operations should not be taught to elementary algebra students but should rather be postponed to secondary school because young children perform operations from their instrumental understanding and have no conceptual understanding of what they are doing. Bracey (1996) blames the use of generalizations, such as using a piece of pizza or cake when teaching fractions, which he argues are not helping students develop fractional competency. Bracey advocates for deeper thought and understanding in the manipulation of fractions in real-life contexts that do not only focus on part-whole relationships like the pizza and cake examples.

The teaching and learning of algebraic fractions need attention as it is evident from the literature that deficiencies in algebra competency may have consequences for success in mathematics as a subject. Finding ways to help students do well in algebra is vital.

2.7.2 Students' ways of working with algebraic fractions.

Brown & Quinn (2007) and Welder (2006) argue that proficiency with rational numbers is related to success in algebra, which means that students who struggle with rational numbers will have a challenge succeeding in mathematics L3 because algebra is about 70% of the first examination paper and elements of algebra are also incorporated in other topics of level 3 NCV mathematics. Deficiencies with rational numbers may also contribute to students' difficulties with college-level mathematics (Yantz, 2013). Understanding fractions is critical to student success in more advanced mathematics courses (Booth & Newton, 2012; Braithwaite et al., 2017). Difficulties with fractional competency inhibit the proper development of algebra, trigonometry, and calculus (Bentley & Bossé, 2018). College students share the same difficulties, misconceptions, and errors that elementary school learners have on algebraic fractions (Bentley & Bossé, 2018). Therefore, learners carry their misconceptions of algebraic fractions through to college. The expectation is, therefore, not to find completely different errors or ways of working in college students than school learners. This study takes a different approach to how students work with algebraic fractions. The approach does not focus on errors and misconceptions that students commit as they simplify algebraic fractions but uses an ethnomethodological analysis. The ways of working discussed below are mostly those of learners in high school.

In a study by Yantz (2013), students struggled to simplify algebraic fractions. The results showed very low percentages for rational algebraic fractions compared to numerical rational fractions. Learners were given three sets of activities with three pairs to simplify, with each set having a numerical and algebraic fraction. Shown below is one of the three sets of problems.

$\frac{1+6}{2}+\frac{9}{2+3}-3$	$\frac{x+2}{4}+\frac{6x}{x+1}-3$

Figure 1. Numeric and Algebraic items in Problem Set A

$\frac{3}{3-1}\div\frac{9}{9-1}$	$\frac{x}{x-1}\div\frac{x^2}{x^2-1}$

Figure 2. Numeric and Algebraic items in Problem Set B

$\frac{1}{4\cdot 5}+\frac{3}{5\cdot 7}$	$\frac{1}{x^2-x-2}+\frac{x}{x^2-7x+10}$

Figure 3. Problem Set C

Figure 2.1 Algebraic fractions problem set c (Yantz, 2013:7)

The students' work exhibited serious deficiencies with rational algebraic fractions. Students who correctly answered the numeric item were 41.1 %, while those who correctly answered the algebraic item were 5.6 %. The students who correctly answered the numerical and algebraic fractions were 2.8%. Of all sets of questions, less than 14% of students answered the rational algebraic fractions correctly. The study sought to establish the extent of undergraduate students' algebra deficiencies in the context of rational expressions and determine if a correlation exists between algebraic procedures and proficiency with rational numbers (Yantz, 2013). The results indicate that students struggle with algebraic expressions and, hence, have problems working with algebraic fractions. However, there was no evidence of a correlation between proficiency with algebraic fractions helping the students do well in numerical fractions and vice-versa.

Hurst and Cordes (2018) used quantitative methods to investigate whether a link exists between rational number processing and algebraic ability. The results indicated some correlation between rational number understanding and algebra performance, but only depending on the type of knowledge being measured and the notation being used. "In particular, higher algebra fluency was associated with a higher fluency with symbolic decimal magnitudes and with stronger decimal ratio effects." (Hurst & Cordes, 2018:110). Mhakure, Jacobs, and Julie (2014) argue that algebraic fractions are multi-complex and present many difficulties for students in the early stages of learning algebra.

Students commit some errors as they work with rational algebraic fractions. The errors include cancellation errors, partial division, like term errors, equationisation, defractionalisation, confusing factors, dropping the denominator, no recognition of common factor, common denominator error, careless error, unable to factorise trinomial, and conjoining (Mhakure, Jacobs, & Julie, 2014; Figueras et al., 2008; Judah, Makonye & Nzima, 2016; Makonye & Khanyile, 2015). Baidoo (2019) found that conceptual, mathematical language, procedural, and application errors hinder grade 10 learners' appropriate understanding and the application of algebraic fractions. Permata et al. (2019) highlighted students' misconceptions that possibly lead to several errors they commit as they work with fractions. The misconceptions included integer addition, integer subtraction, integer division, multiplication with zero numbers, fractions addition, and fractions division. From the studies mentioned above, students commit numerous errors as they work with algebraic fractions, which emanate from the misconceptions that learners have about fractions and other mathematical procedures. Students need help to mitigate these errors so they can progress well in algebra and mathematics in general.

The literature review did not reveal enough about how NCV mathematics students work with algebraic fractions. However, Naiker (2017) analysed the conceptual and procedural difficulties that NCV L4 students displayed when working on factorization and solving mathematical problems that require factorizations. The results of the study showed that students experience both conceptual and procedural difficulties when working with any type of factorization. Problems included incorrect simplification of algebraic expressions, turning algebraic expressions into equations, failure to differentiate between an expression and equation, and failure to define in their own words the meaning of some foundational concepts such as factorisation, multiplication, expression, equation and others. If students have difficulty with factorization, they will experience challenges in simplifying algebraic fractions because factorization is a required procedure.

Studies investigating students' ways of working with algebraic fractions have focused on what learners are doing right or wrong, looking at types of misconceptions and errors that students commit as they simplify algebraic fractions and the possible reasoning behind the errors committed. This study's standpoint is different; the focus will not be on what is right or wrong, but from an ethnomethodological point of view, examinees' ways of working will be investigated, focusing on the aspects of work that are usually ignored.

2.8 Ethnomethodological Studies in Mathematics Education.

This study uses ethnomethodology as the founding theory to investigate examinees' ways of working with algebraic fractions. Therefore, looking at the ethnomethodological studies in mathematics education becomes imperative. Ethnomethodological studies analyse everyday activities as members' methods for making those same activities visibly-rational-and-reportable-for-all-practical-purposes, i.e., 'accountable,' as organizations of commonplace everyday activities. It concerns the methods people use to construct, account for, and give meaning to their social world. The function of ethnomethodological studies is to explain the methods and accounting procedures that members employ to construct their social order. Koschmann et al. (2004:4) posit that EM studies "are purely descriptive and cannot be used to form prescriptive judgments". The statement implies that, as DeMontingy (2017) stated, ethnomethodological studies are not directed at formulating or arguing correctives.

Ethnomethodological studies in mathematics education have focused more on analysing language use and classroom interaction in mathematics classrooms. Such studies have widely used conversation analysis, discursive psychology, and interactionism. Data is usually collected in natural settings by researchers who have naturalized themselves to understand better the natural setting and how people create their order. The data collection tools are usually videos and audio records of interviews or natural documents. Natural documents are explained by Cole (2005) as the records available to portray social life in written texts, photographs, and drawings that become part of the social record of events and historical profiles. Cole (2005) further mentions that the ultimate meaning of documents is understood in the social context in which they were produced and discovered. These documents contain notes that are critical and essential for documentary analysis in establishing the factuality of the claims through the authenticity, credibility, and representativeness of artefacts.

A different direction in the use of ethnomethodology in mathematics education has evolved. Some studies employed ethnomethodology to collect naturally occurring interactions within mathematics classrooms, and the analysis has focused on detailed transcriptions of these interactions (Ingram, 2018). Ingram (2018) further argued that ethnomethodological approaches focus much on what learners do as they work with mathematical problems rather than what they say and can be used to explore identity work and the construction of mathematical knowledge and expertise.

Researchers such as Pickering (1995), Merz and Knorr-Cetina (1997), Julie (2003), Greiffenhagen (2008), Roth (2011), and Simons (2016) have reported studies on the ways of doing mathematics using an ethnomethodological approach. The work of these researchers will be used to draw the conceptual framework for this study and hence will be discussed in the conceptual framework of the study.

2.9 Chapter Summary

This chapter discusses the literature reviewed for this study. The literature covered the discussion of assessment in mathematics. The meaning and purpose of both classroom and large-scale assessments were discussed, emphasizing the importance of examining the student's responses to questions in an examination.

The NCV Mathematics assessment process and how students qualify to sit for the final examination were explained. The high-stakes examination was explained, and, in reference to the NCV-level mathematics examination, the stakes of the exam have been stated. The feedback process for the NCV examinations is discussed.

Research in NCV Mathematics shows that there is a low pass rate in mathematics, and there are some factors or reasons that lead to students failing mathematics, such as conceptual and procedural difficulties and other difficulties with working with algebraic expressions. There is still a very limited number of studies on NCV mathematics. There is a gap in research on NCV algebra, which calls for more research or studies related to mathematics learning and teaching.

Algebraic fractions have been discussed as a key topic in mathematics and are seen as a key building block that can hinder the students' proficiency in algebra and mathematics if not fully grasped. The ways of working with algebraic fractions are mostly discussed in reference to the errors and misconceptions that school learners exhibit as they simplify algebraic fractions. The search did not reveal the students' ways of working with algebraic fractions in TVET colleges or NCV specifically.

The chapter ends with a discussion of ethnomethodological studies in mathematics. Ethnomethodological studies are not directed at formulating or arguing correctives but rather focus on the methods people use to make their social order. Studies in mathematics education research have mostly focused on analysing language use and classroom interaction. The next chapter discusses the theoretical underpinning of this study.

CHAPTER 3 THEORETICAL CONSIDERATIONS

3.1 Introduction

This chapter discusses the theoretical and conceptual framework of the study. A theoretical and conceptual framework explains the path of research and grounds it firmly in theoretical constructs (Adom, Hussein, & Agyem, 2018). For readers to follow the research study, both the theoretical and conceptual framework should align with all other parts of the research, such as the objectives of the study, methodology to be followed and others. Adom, Hussein and Agyem (2018) assert that research without a theoretical or conceptual framework makes it difficult for readers to ascertain the academic position and the factors underlying the researcher's assertions or hypotheses. The overall aim of the two frameworks is to make research findings more meaningful, acceptable to the theoretical constructs in the research field and ensure generalizability.

Kerlinger and Lee (2000:11) define theory as a set of interrelated constructs (concepts), definitions, and propositions that present a systematic view of phenomena by specifying relations among variables, with the purpose of explaining and predicting the phenomena.

This definition says three things: (1) a theory is a set of propositions consisting of defined and interrelated constructs, (2) a theory sets out the interrelations among a set of variables (constructs) and in so doing, presents a systematic view of the phenomena described by the variables, and (3) a theory explains phenomena; it does so by specifying which variables are related to which variables and how they are related, thus enabling the researcher to predict from certain variables to certain other variables.

The theoretical framework defines, discusses, and evaluates theories relevant to the research problem. It explains the key concepts, models, and assumptions that guide the research project and shows that work is grounded in established ideas.

In research, each study has a vision and a plan of or structure of how the research study is to be conducted. The theoretical framework is, therefore, one of the key elements of a research study as it gives a clear vision and theoretical beliefs of the researcher. The theoretical framework is a blueprint for the research.

> [I]t is the foundation from which all knowledge is constructed (metaphorically and literally) for a research study. It serves as the structure and support for the rationale for the study, the problem statement, the purpose, the significance, and the research

> questions. The theoretical framework provides a grounding base, or an anchor, for the literature review, and most importantly, the methods and analysis (Grant & Osanloo, 2014:12).

It is a framework based on an existing theory in a field of inquiry that is related to and reflects the hypothesis of a study. It is a blueprint that is often 'borrowed' by the researcher to build their own house or research inquiry (Adom, Hussein, & Agyem, 2018). The theoretical framework provides the researcher or a reader with the selected theory or theories that guide the thinking with regards the understanding and planning of the research, as well as the concepts and definitions from that theory that are relevant to the topic (Grant & Onsloo, 2014). It gives the readers the lens from which to view the problem under study and the guiding principles for the research study.

The study employs Garfinkel's (1967) theory of "ethnomethodology": ethnomethodological analysis, a branch of sociological investigation, will be used to highlight the textures of students' work. The term ethnomethodology refers to the study of a particular subject matter, the body of common-sense knowledge and the range of procedures and considerations using which the ordinary members of society make sense of, find their way in and act on the circumstances by which they find themselves (Heritage, 1984). Research in mathematics education that analyses students' work usually focuses on assessing and identifying the errors they commit as they work with mathematical problems. This study adopts a framework from Garfinkel's theory of ethnomethodology, which does not focus on being judgemental of right or wrong but seeks to analyse the transitive nature of mathematical work.

From the theoretical framework, a researcher draws the concepts that form the basis of their inquiry. The concepts and how they relate to one another within a theoretical framework, therefore, form a conceptual framework of the study. Miles and Huberman (1994) defined conceptual frameworks as a system of concepts, assumptions, and beliefs that support and guide the research plan. Grant and Osanloo (2014) assert that a conceptual framework is not just simply a string of concepts but a way to ascertain and construct for the reader the epistemological and ontological worldview and approach to the topic of study.

The following discussion deals with ethnomethodology as the theory employed for this study, together with the concepts and constructs of ethnomethodology and the work of Pickering (1995) that form the conceptual framework of the study.

3.1.1 Ethnomethodology

Ethnomethodology is the study of ordinary actions by ordinary members of society. The definition of "ordinary" is anything people regularly and recurrently do with such automaticity that it is given little thought. (Vance & Noelle, 2019). Ethnomethodology is the study of how people use common sense, procedures, and considerations to gain an understanding of everyday situations (Garfinkel, 1967). Garfinkel re-specified Durkheim's view that:

> The objective reality of social facts is sociology's fundamental principle. Garfinkel's argument is that 'the objective reality of social facts, in that and just how it is every society's locally endogenously produced, naturally organized, reflectively accountable, ongoing, practical achievement, being everywhere always, only, exactly & entirely, members' work, with no time out, passing, postponement, or buyouts is thereby the sociology's fundamental phenomenon' (Garfinkel 1967:11).

According to Garfinkel, ethnomethodology focuses on understanding how actors experience social order and use their common-sense knowledge to create order in society.

The following discussion will give an overview of some of the principles of ethnomethodology, which the nature of the study will embrace.

Locally and endogenously: People create order as they interact with one another; they do not do it because social scientists or theories inform their actions. People collaborate and create visible accounts that can be interpreted in that situation or context. Local analysis is how people make sense of their actions. The analyst does not make generalisations about people's accounts. Ethnomethodologists do not speak on behalf of the people they are studying but ask relevant questions about the subject of their study, and the people themselves explain and make sense of how order is produced.

Naturally organized: People are doing what they are doing on natural occasions that were not organized in a specific way that will satisfy the expectations of the researcher. People should be in their natural setting and behave naturally as they would have behaved in the absence of the researcher. When collecting data, people must do what they do for themselves and not for the researcher because that cannot reflect the reality of the accounts for that particular setting or situation. For the purposes of this research, the study analyses students' work in a high-stakes examination. Because of the nature of the assessment and the procedures followed to ensure the credibility and authenticity of the examination, the researcher cannot be allowed in the examination room while students are writing their final examination. The researcher, therefore, follows a documentary analysis approach together with the method of accessing lived work and lived experiences without having to interview research participants (Livingston, 1986). The examination will continue in its naturally organized setting; the students will write the examination for their final year assessment, and there is no influence by the researcher or disturbance, which can result in students not behaving as naturally as they would have behaved in the exam room.

Reflexively accountable: Reflexivity refers to the fact that practices that occur within a given situation are both a product of the situation and constitute the situation (Vance & Noelle, 2019). Sociologists encounter reflexivity on the actual occasions of their inquiries as indexical properties of natural language. These properties are sometimes characterized by summarily observing that a description, for example, unavoidably elaborates those circumstances it describes and is also elaborated by them (Garfinkel & Sacks in Mc Kinney & Tiryakin, 1970). The analysis of action must consider the actor's use of common-sense knowledge, namely that the social constitution of knowledge cannot be analysed independently of the contexts of institutional activity in which it is generated and maintained (Heritage 1984). The reflexive relations between knowledge and action are very important. The description is part of the situation it describes. In mathematical work, the meaning of any work account or action, including the lived work (Livingston 1986), becomes evident by investigating how participants solve mathematical problems (Simons, 2019; Julie & Holtman, 2019). The students, as they seek solutions to the mathematical problems, are driven by what the question paper requires of them to solve. The methods or skills employed by students in solving a mathematical problem are influenced by their understanding of what the question requires. Hence, there is a reflexive relation between what the students know or skills the student must have to solve the problem, the understanding of what is required by the question paper, and how the student will apply the methods. After the student has answered the question, they revert to the question paper to check if the asked question was answered.

Ethnomethodological indifference: Ethnomethodological indifference as a procedural policy of ethnomethodology studies does not focus on arguing or formulating correctness but rather

> Ethnomethodological studies of formal structures are directed to the study of such phenomena, seeking to describe members' accounts of formal structures wherever and by whomever they are done while abstaining from all judgements of their adequacy, value, importance, necessity, practicality, success, or consequentiality (Garfinkel & Sacks, 1970:345).

Ethnomethodology deliberately ignores the established methods of researching or describing social order in formal analytics. Ethnomethodological indifference refers to the fact that in ethnomethodology, the description of the phenomena is not hypothesized; it cannot be based upon prior formal analytical studies like in normal social science research.

Ethnomethodological indifference in this study implies that the analysis of students' work is not focusing on what students do right or wrong but seeks to describe the accounts of students' ways of doing mathematics in the high-stakes exam setting; those textures that are usually taken for granted which students exhibit as they solve mathematical problems, specifically in this case algebraic fractions. Garfinkel (2002) states that ethnomethodological indifference is an instruction for those doing ethnomethodological studies not to decide in advance what phenomenon consists of based on prior formal analytic studies. Therefore, the study's outcome cannot be hypothesized.

3.2 Conceptual Framework

The introduction above defined the conceptual framework as a system of concepts, assumptions, and beliefs that support and guide the research plan (Miles & Huberman, 1994). It is the means to ascertain and construct for the reader the researchers' epistemological and ontological worldview and approach to the topic of study (Grant & Osaloo, 2014). The conceptual framework explains how the researcher intends to explore the research problem. The framework makes it easier for the researcher to specify and define the concepts within the problem of the study (Luse, Mennecke & Townsend, 2012). The presentation of the conceptual framework in a study can be graphical or narrative, showing the key variables or constructs to be studied and the presumed relationships between them (Miles and Huberman, 1994). The conceptual framework for this study draws from the work of the researchers Pickering (1995), Merz and Knorr-Cetina (1997), Julie (2003), Greiffenhagen (2008), Roth (2011) and Simons (2016). Concepts such as lived work, which talks about the importance of rough work that this study considers as an important element of students' work, a dialectic of resistance and accommodation, which are the ways that students employ as they simplify algebraic fractions in the case of this study and the other ways of working in mathematics will form the basis of the analysis for the study. The following discussion will explain these concepts and how they relate to the study.

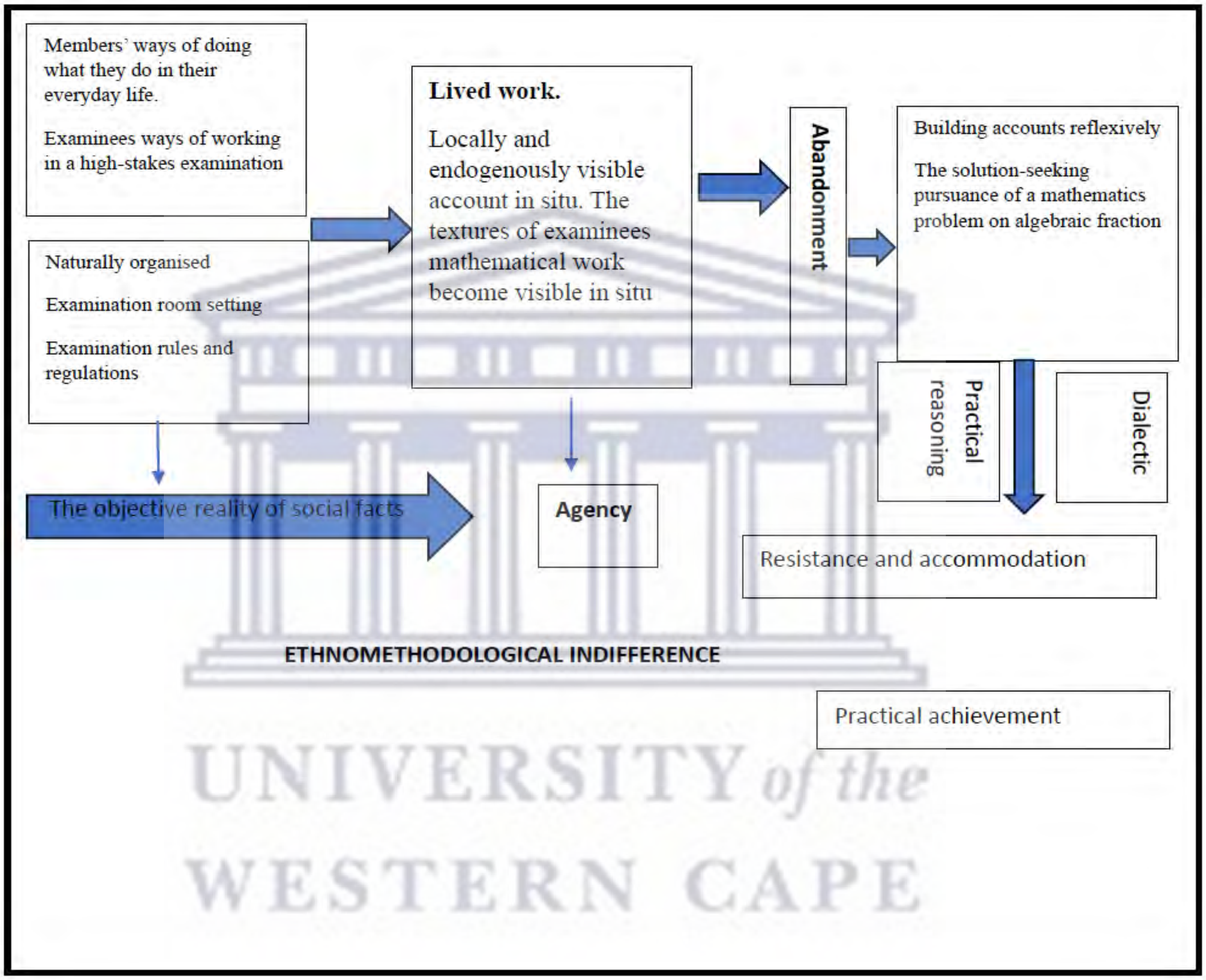

Figure 3.1 Conceptual framework

3.2.1 Lived work.

This study employs ethnomethodology, which is concerned with the methods people use to produce social order. Lived work is a method that mathematicians employ to bring out the textures of mathematical proofs. Livingston (1986) brings us closer to what mathematicians do as they work on mathematical proofs. The details of what a mathematician does—whether a formal presentation on the blackboard during a lecture or mathematical work in a paper—seek to arrive at the final proof and convince others about the proof. Livingston (1986) seeks to rediscover and exhibit the naturally accountable mathematical proof as a social achievement in its identifying detail for mathematicians. Livingstone's work intended to discover what happens when a mathematician works on the mathematical proof by analysing the "lived work" of the proof.

3.2.2 Practical Achievement

Roth (2011) articulated the difference between living/lived mathematical work and accounts of mathematical work from the perspective of perception (seeing), labelling and producing, which come about through the movement of the eye in different positions. The lived work of mathematics refers to that which a mathematician does to bring about the structures of a mathematical proof that can usually be presented in a book or any written form. The written accounts of a proof are usually aimed at sharing the proof with others such that they allow others to re-discover the proof in their own praxis, which is the objective nature of geometry (Roth 2011).

> The ideal (subjective) objects exist virtually in the world in written form and can therefore be produced at any time. However, the lived praxis (labour) within which this written account counts as the proof is not contained in the written account. It is precisely this lived work that we are interested in here and in ways of capturing it. (Roth, 2011:1)

In his research of the lived work of mathematics, Roth focused on elements not written as part of the final proof, such as the origin of the perception. What is underlying one seeing the combination of lines in a drawing as a cube is what he refers to as "lived work of seeing".

In Livingston's perspective, the lived work of proof and the account of proof are inseparable. It is both the account of a proof and the lived work of a proof that brings about a complete proof. In this study, the student will not necessarily be proving theorems. What we see written on the learners' scripts can be seen as a description of learners' work, while the processes and practices involved in producing the written work can be referred to as "Lived work".

The lived work of mathematics in this study is used to focus on what students do as they solve algebraic fractions, including those structures of work that are considered rough work and are not allocated marks when marking the scripts, but they say something about how the student came to the final product or the solution to the algebraic fraction problem. Julie (2002) asserts that it is common that during the construction of mathematical work, most, if not all, the mathematical work, the real work, is the actual scribbles, marks, diagrams, doodles and so forth that normally land up in the dustbin. Julie (2002) further asserted that what we read in journals, lecture notes, or any document is the cleaned-up reports and gives us little on the actual process of mathematical construction work. This study will examine the real construction of mathematical work and why the scribbles and those elements of work that are not allocated marks are important in how learners answer mathematical questions in an exam setting. The student's scribbles of such data provide traces of the behaviour that the participants were involved with at that stage and the sense of occurrences at the work site and workbench Julie, (2002). The study thus focuses on the practical achievement of mathematical work.

3.2.3 Dialectic of resistance and accommodation

Pickering (1995) explains working with mathematical and scientific problems. Pickering argues that finding a solution to the problem takes the form of a dialectic of resistance and accommodation, which he refers to as the dance of agency. When scientists, for example, construct a new machine, they model and make the machine and take a passive role to see if it performs the expected role. In the failure of the machine to perform, the scientist again takes an active role in revising the modelling of the machine. The process continues until the desired outcome is achieved. This process is illustrated in the case of building the "Bubble Chamber", which was invented in the early 1950s. The process of building the bubble chamber took the form of a dialectic of resistance in the sense that each time Glasers' technique failed to produce the desired results (resistance), his response was to devise some other tentative approach toward his goal (accommodation).

Referring to elementary algebra, Pickering (1995) posits that the agency of discipline—elementary algebra, for example, leads us through a series of manipulations within an established conceptual system. In the context of this study, working with algebraic fractions may take the form of a dialectic resistance and accommodation where the students seek to arrive at a solution, but some resistance happens, and other means are devised to arrive at a solution. The final solution does not reflect the accounts of the ways of working by the student. The analysis of the students' scripts will examine how learners exhibit the dialectic resistance and accommodation process as they seek to respond to the questions on algebraic fractions posed to them in the mathematics final examination paper.

3.2.4 Ways of working in mathematics

The ways of working in mathematics adopted as a conceptual framework in this study mirror the one adopted by Simons (2019) from studies such as Julie (2003) and Julie (2015). The ways of working identified in these studies were:

Compensation: Compensation in mathematics is a strategy of adding or subtracting a value to make a problem easier to solve. In this case, compensation refers to a pattern of work where it compensates for missing data either by using numbers derived from the assumed information or when the given information is adequate to replace missing data (Julie, 2003).

Shedding is referred to by Mason (1999) as mathematical work that points to the phenomenon of being stuck cited in Simons (2019:45)

The U-turn: This way of working is identified from students who, after attempting to solve a problem, go back to the start to solve a problem with a different interpretation of the strategy used or use a different strategy or just go back to the start because the results do not meet the expectation (Julie, 2013).

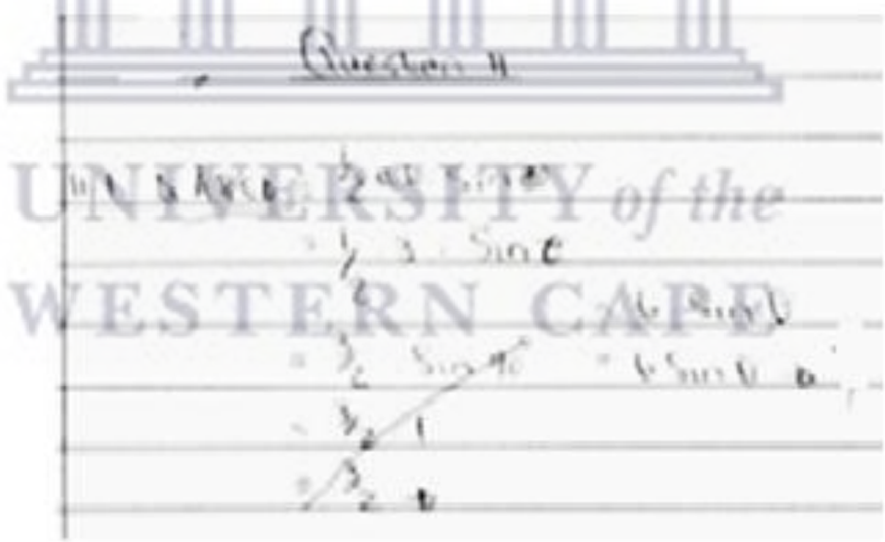

Figure 3.1 Making a U-turn

Source: Simons (2019:64)

Figure 3.2 above shows a student who, after getting to the final step of the activity, took a U-turn and tried a different approach.

Reversal: a way of working in which a calculated resistance was created, and the student removed the produced work by drawing a line through it, immediately starting again from a certain point (Julie, 2003)

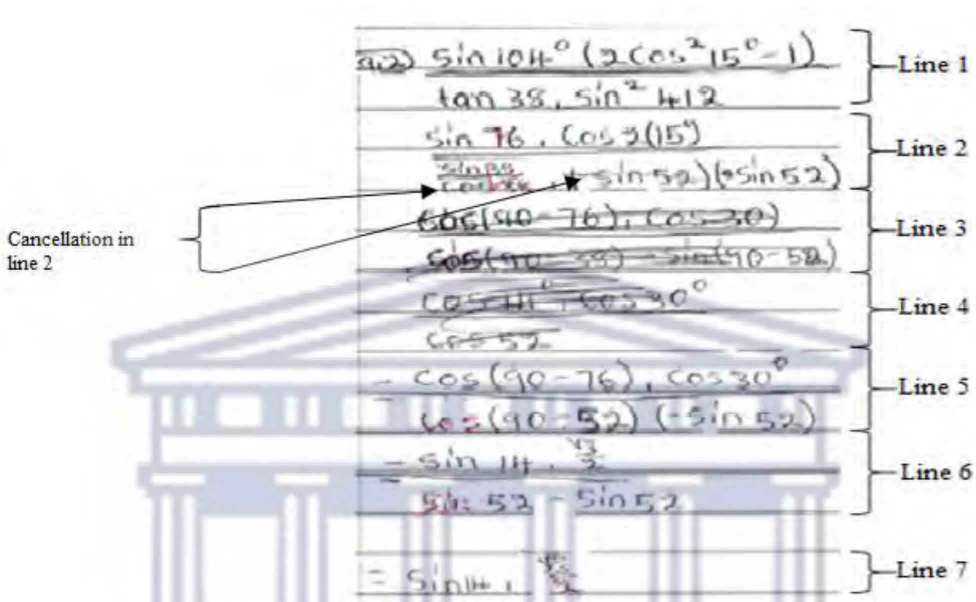

Figure 3.2 Reversal

Source: Simons (2019:74)

Figure 3.3 is an example of a reversal by a student, scratching out the work previously produced work and producing a different texture.

Convenience: This is a way of achieving a suitable objective for a mathematical problem using the wrong method. (Julie, 2003)

Abandonment: This way of working is usually seen when a student, after following a certain path to finding a solution to the question, decides to leave that path and take another after realising that it will not yield the intended outcome. The student either adjusts the previous path or starts a new path to a solution. Simons (2016), as discussed below, illustrates different forms of abandonment.

1. Abandonment after a nearly complete attempt
   "This way of working showed some form of working almost to the point of the final answer, and then an early abandoning of the path of pursuit and a recommencement of the solution-seeking process having made adaptations to the previous path of pursuit"

(Simons 2016:97). In this case, the student had a predetermined objective to get to a particular answer, and when the first attempted solution does not appear to be leading to the pre-determined objective, the solution is abandoned. When learners abandon the solution, it is usually shown by scratching, drawing a line through the work and then starting again to pursue a solution.

2. Abandonment of the solution

   This type of abandonment is in two ways; the first is abandonment and the construction of a new problem by "proving the given", and the second is abandonment and reconstruction of the same problem by "proving the procedural objective".

There is little data on ethnomethodological ways of working in mathematics linked with human agency. The ways of working discussed above will be used as a conceptual framework and, hence, the basis of analysis for the Level 3 Mathematics final examination scripts. The ways of working identified in the studies did not cover ways of working with algebraic fractions. This study examines ways of working with algebraic fractions and will report what will be identified as students' ways of working with algebraic fractions.

3.3 Chapter Summary

This chapter discussed the theoretical underpinnings and conceptual framework of the study. The study is premised on a qualitative approach and uses ethnomethodology to demonstrate students' ways of working with algebraic fractions in a high-stakes examination setting. The conceptual framework is drawn from the ways of working identified from the literature and will be analysed through the lens of the dance of urgency and the lived work.

The next chapter will discuss this study's research design and methodology.

CHAPTER 4 RESEARCH DESIGN AND METHODOLOGY

4.1 Introduction

The methodology shows how researchers formulate their problem and objective and present their results from the data obtained during the study period (Sileyew, 2009). "A research design is the process for collecting, analysing, interpreting and reporting data in research studies" (Cresswell & Plano Clack, 2007:58). MacMillan and Schumacher (2001:166) define research design as a plan for selecting subjects, research sites, and data collection procedures to answer the research question(s). This chapter explains the path or approach followed to conduct this research systematically. The process of how data was collected, analysed, interpreted, and reported is explained. This chapter also addresses how the study ensures reliability, validity and compliance with ethical considerations.

4.2 Research design

The study adopts a qualitative research approach, which considers the phenomena in its natural setting. The qualitative research approach is naturalistic in that it does not seek to manipulate the objects of the study (Patton, 2001) but focuses on the natural behaviour of the objects and describes their behaviour. It allows the researcher to collect data by observing and interpreting even the non-verbal communications from the participants by interacting with them in their own settings.

According to Denzin and Lincoln (2005), qualitative research is a situated activity that locates the observer in the world. It consists of a set of interpretive, material practices that make the world visible. These practices transform the world. They turn the world into a series of representations, including field notes, interviews, conversations, photographs, recordings, and memos to the self. At this level, qualitative research involves an interpretive, naturalistic approach to the world. This means that qualitative researchers study things in their natural settings, attempting to make sense of or interpret phenomena in terms of the meanings people bring to them (Denzin & Lincoln, 2005:3).

In this study, an ethnomethodological analysis of students' work as they solve algebraic fractions in a natural exam setting is used to expose those textures of work that are usually ignored. Qualitative researchers believe in the value of rich descriptions of the social world, and they use different tools, such as interviews, observations, texts and documents, to collect data from natural settings. The interpretive nature of the qualitative research allows for the detailed interpretation of the methods students employ when they simplify algebraic fractions.

Qualitative research for this study is relevant as it helps researchers understand human behaviour from the perspectives of the people involved in the phenomena (Welman, Kruger & Michel, 2007). This study employs an ethnomethodological approach, which is concerned with the methods used by people to construct, account for, and give meaning to their social world. The qualitative research method supports this because qualitative researchers stress the socially constructed nature of reality (Denzin & Lincoln, 2005). In addition, qualitative research makes sense of the meaning that participants have about the world instead of starting new theories (Creswell, 2014).

4.3 Data Sources and Sampling

4.3.1 Sampling

A sample is defined by Sapsford & Jupp (1996) as a set of elements selected in some way from a population. Sampling aims to save time and effort but also to obtain consistent and unbiased estimates of the population status in terms of whatever is being researched. Sampling in qualitative research is defined as selecting specific data sources from which data are collected to address the research objectives (Gentles, Charles, Ploeg, & Mckibbon, 2015). There are generally two types of sampling that researchers usually follow: probability sampling and non-probability sampling. Qualitative researchers usually follow a non-probability sampling type. Bhattacharjee (2012) asserts that researchers conduct non-probability sampling in non-random ways. Examples of non-random ways of sampling include convenience or opportunity sampling, quota sampling, purposive sampling, expert sampling, and snowball sampling.

In this study, a purposive sampling technique was used. The purposive sampling technique, also called judgment sampling, is the deliberate choice of a participant due to the qualities the participant possesses (Ilker, Sulaiman & Rukayya, 2016). The L3 NCV mathematics students have been selected purposively because they do algebraic fractions as part of their curriculum for mathematics. Mathematics L3 NCV students who obtained an ICASS mark of 30% and above were selected purposively because they qualified to write the national high-stakes examination for mathematics L3.

4.3.2 Participants

McMillan and Schumacher (2010) define participants as a group of people who participate in a study from whom data are collected. Decisions regarding selecting participants are based on the research questions, theoretical perspectives, and evidence informing the study; hence, the selection of participants in qualitative research is purposeful. Participants are selected who can best inform the research questions and enhance understanding of the phenomenon under study (Sargeant, 2012). The researcher selected NCV L3 mathematics students from two colleges: one TVET college in Cape Town in the Western Cape, South Africa, and one in Butterworth in the Eastern Cape, South Africa. The participants were selected on the basis that they do mathematics—which is the topic under study.

4.3.3 Data collection procedures

Data was collected from the final NCV mathematics examination for L3. A pack of answer scripts from a group of L3 engineering and related design students and boiler-making students at the two colleges was used as data for this study.

4.4 Validity and Reliability

Patton (2002) argues that the concepts of validity and reliability are important for qualitative researchers to consider when designing the study, analysing results, and judging the quality of research. Joppe (2000) defines reliability as the extent to which results are consistent over time, while an accurate representation of the total population under study is referred to as reliability. If the results of a study can be reproduced under a similar methodology, then the research instrument is considered reliable. Joppe (2000) continues to argue that validity is a form of determining whether the research truly measures what it intended to measure or how truthful the research results are.

Guba and Lincoln (1985) substituted reliability and validity with the parallel concept of "trustworthiness," consisting of four aspects: credibility (in place of internal validity), transferability (in place of external validity), dependability (in place of reliability) and confirmability (in place of objectivity).

Credibility answers the question of how congruent the results are with reality, i.e., a true picture of the phenomena under scrutiny is being presented (Shenton, 2004). The implementation of the credibility criterion is twofold. Firstly, the researcher must carry out the inquiry so that the probability that findings will be found to be credible is enhanced, and secondly, the researcher must demonstrate the credibility of the findings by having them approved by constructors of the multiple realities being studied. Shenton (2004) argues that to increase the probability of high credibility in the field, research methods that are well-established in studies of the same nature as the study should be considered. This research collected data from the mathematics high-stakes examination, a method widely used by many researchers in education research. A purposive sampling was used, but the sample selection is not based on the students' characteristics or demographics such as age, level of intelligence or academic performance, gender, race or any other criteria. The selection is based solely on the fact that the students are candidates for the mathematics examination under study.

Another form of ensuring credibility is triangulation, which is the application of multiple data collection methods. According to Guba (1989), using different methods in concert compensates for their limitations and exploits their individual benefits. If one method has a limitation, the other can cover for the limitation while the researcher is not losing the former's advantage. In this research, two methods of data collection were used.

Some researchers use transferability in preference to external validity or generalizability, which are concerned with the extent to which the results of the study can be applied to other situations. Regarding transferability, the concern demonstrates that the research findings can be applied to a wider population. For qualitative research, it is impossible to specify that the results and conclusions apply to other situations or larger populations. Therefore, a thick description can be provided to address the transferability of the research findings, which is necessary to enable someone interested in making a transfer to conclude whether the transfer can be considered a possibility. Therefore, the responsibility of the researcher is to provide a database that makes transferability judgements possible on the part of the potential appliers. Purposive sampling can be used to address transferability. In this research, a detailed explanation of the setting where data was collected and how data was collected and analysed is provided so that a reader or researcher can be able to make decisions about the transferability, considering the contextual factors.

Dependability: In quantitative research, it is possible to employ techniques to ensure the possibility of the consistency or repeatability of measures. This is not easy in the case of qualitative research, considering the changing nature of phenomena scrutinised by qualitative researchers. To address the dependability issue more directly, the processes within the study should be reported in detail, thereby enabling a future researcher to repeat the work, if not necessarily to gain the same results. Thus, the research design may be considered a "prototype model" (Shenton, 2004:72).

Confirmability: Confirmability is the qualitative investigator's comparable concern to objectivity. Here, steps must be taken to help ensure, as far as possible, that the findings are the result of the experiences and ideas of the informants rather than the characteristics and preferences of the researcher. It must be clear that the findings emerge from the data and not researcher-predisposition (Shenton, 2004). The researcher must have measures to avoid researcher bias (Sheton, 2004). In this study, the data collected was kept safe as evidence and verification of the results.

4.5 Ethical Considerations

Schurink (2005) asserts that ethical issues are the concerns and dilemmas that arise over the proper way to execute research, specifically, not to create harmful conditions for the subjects of inquiry in the research process. According to McMillan et al. (2006), ethics deals with a belief or guidelines about right or wrong, proper or improper, good or bad, from a moral perspective. Ethical conduct in educational research is very important because it involves people. In adhering to the research code of ethics, the researcher applied for ethical clearance from the University of the Western Cape to conduct research. A letter requesting the use of students' final examination scripts was submitted to the colleges' principals to grant permission; permission was granted. Codes of ethics insist on safeguards to protect people's identities and those of research locations (Denzin & Lincoln, 2005). Confidentiality is an undertaking that a participant will not be identified or presented in an identifiable form. Anonymity is a promise that even the researcher cannot tell which responses came from which respondent (Sapsford & Jupp, 1996). The names of participants and the colleges have been kept anonymous; no form of identification that can reveal the names of students or the colleges was included in the excerpts used for analysis. The analysis of data has been done in a manner that does include any manipulation or false reporting.

Considering that examination scripts are critically important documents that must be kept safely by public colleges for any future enquiries or audits of the students' results, the students' scripts were not taken out of the institution, and the copies of any students' work were safeguarded with a security code.

4.6 Chapter Summary

This chapter outlined the research design and methodology employed in this study. It discussed the data sources and sampling and outlined the validity, reliability and ethical considerations of the research. Chapter five will present the data analysis.

CHAPTER 5 DATA ANALYSIS

5.1 Introduction

Flick, Metzler, and Scott (2014) view data analysis as the central step in qualitative research because, whatever the data are, their analysis decisively forms the research outcome. Data analysis is "the classification and interpretation of linguistic (or visual) material to make statements about implicit and explicit dimensions and structures of meaning-making in the material and what is represented in it. Meaning-making can refer to subjective or social meanings" (Flick, Metzler, and Scott, 2014:4).

Ten Have (2004) asserts that ethnomethodology is unique among many schools, perspectives and traditions within social sciences that use or even favour qualitative research methods. Therefore, the analysis of this research employs this distinctive qualitative research method. Ethnomethodology is geared to study the local accountability of any kind of practice (Ten Have 2004). The analysis seeks to reveal the examinees' methods of simplifying algebraic fractions, find the accountability, reflexivity, and indexicality, and determine how the examinees' work reveals the dance agency.

Data displays facilitate valid analysis focused enough to permit viewing a full data set in one location and are systematically arranged to answer the research question (Huberman & Miles, 1994:432). In this study, the data was organised so that ways of working identified in the examinees' scripts were coded to simplify analysis. The analysis revealed learners' ways of working with algebraic fractions in an ethnomethodological way. Ten Have (2004) asserts that members' applied knowledge—the practical notions taken for granted in everyday life—is the foundation of social order.

The examinees' scripts with answers to question 2 of 2020 and 2019 NCV L3 Mathematics Paper 1 were analysed to determine the examinee's ways of working with algebraic fractions. The analysis sought to reveal how examinees simplify algebraic fractions in high-stakes examinations. Forty-eight scripts from the 2019 high-stakes examination were analysed, as well as 27 scripts from the 2020 high-stakes examination exam.

The analysis began with the first deductive approach—identifying and coding the ways of working that the study's conceptual framework reveals. After several rounds of reading through and identifying the ways of working from the conceptual framework, an inductive approach was used to identify other ways of working that the conceptual framework did not reveal.

The 2019 high-stakes final examination paper required the examinees to respond to the question below:

2.4 Simplify each of the following.

2.4.1. $\frac{a^2-a}{a-2} \cdot \frac{a+1}{a^2+4a} \div \frac{a^2-3a-4}{a^2-16}$

2.4.2. $\frac{y}{y^2-16} - \frac{y+1}{y^2-5y+4}$

The 2020 high-stakes final examination paper required the examinees to respond to the following question:

Simplify the following expressions:

2.4.1. $\frac{4x}{3x+1} + \frac{2x}{1-3x} + \frac{5x^2}{9x^2-1}$

2.4.2. $\frac{3x}{x-3} - \frac{2x}{x^2-6x+9}$

2.4.3. $\frac{1}{x} + \frac{2}{y} - \frac{2}{z}$

The following discussion will give the findings from the analysis of examinees' work. The working methods that were first identified and coded are briefly explained below.

Not attempted: Not attempted is a way of working that shows no visible textures of work. An examinee does not attempt to answer the question but leaves or does not leave space to return to it later.

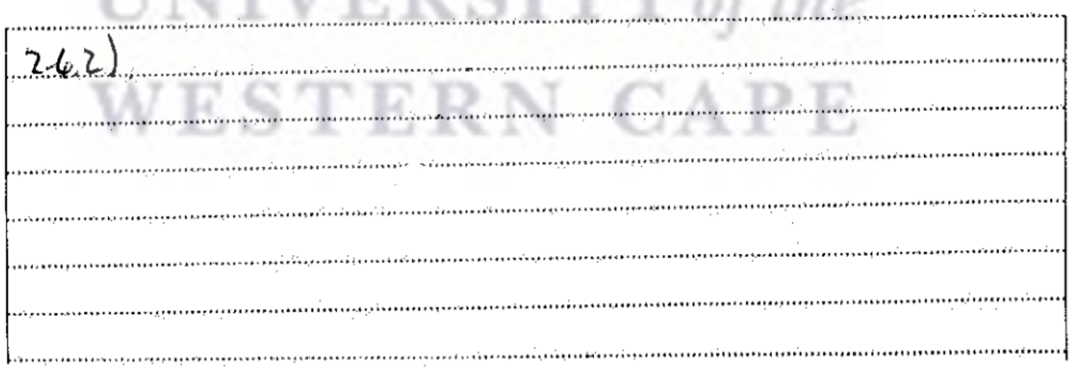

Figure 5.1 Not Attempted

Convenience: A way of working in which an examinee uses a mathematical procedure that does not comply with the dictates of the mathematical context to pursue a desired practical achievement.

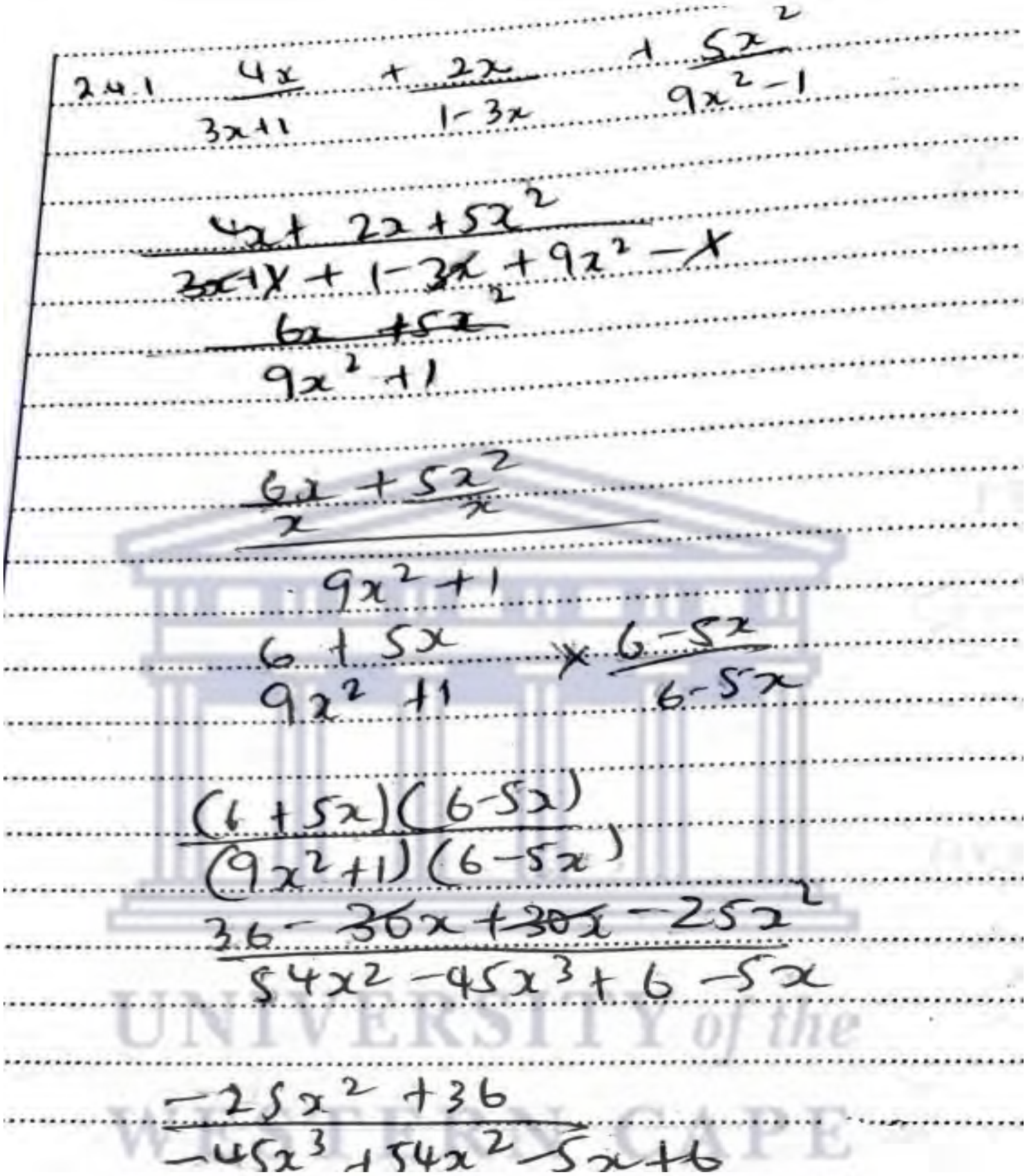

Figure 5.2 Convenience

Reversal: Reversal shows a way of working where an examinee begins a pursuit to produce a practical achievement. However, before achieving that practical achievement, a calculated resistance is encountered; then, the examinee abandons the produced work and starts again at a certain point with adjustments.

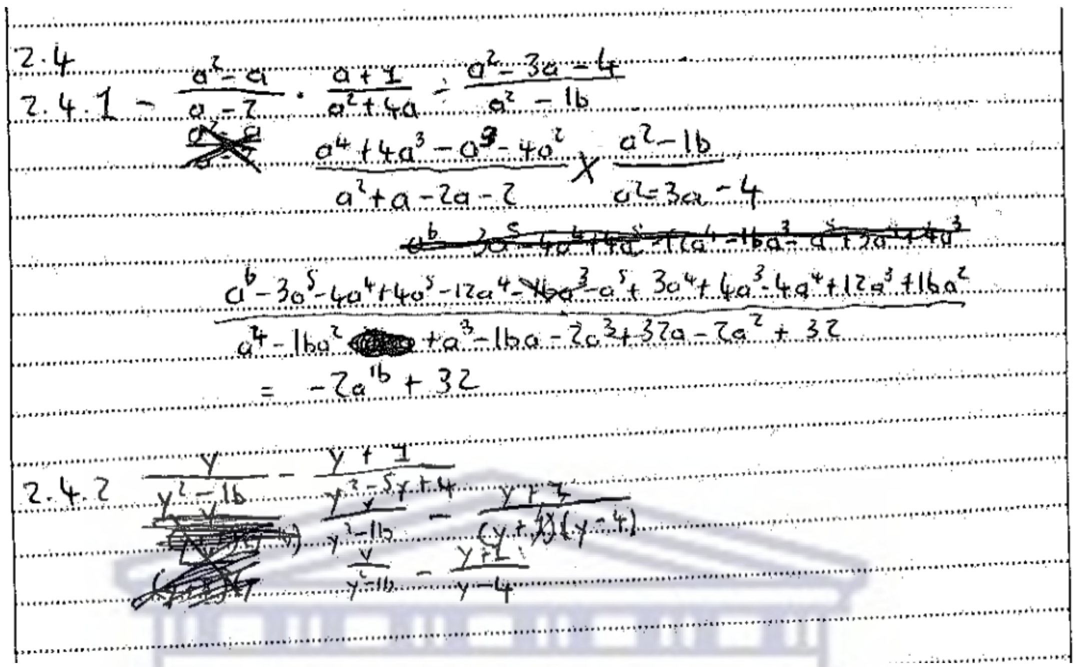

Figure 5.3 Reversal

Abandonment: Abandonment refers to the commencement of a solution-seeking path, and after or before completing the pursuit, the solution is abandoned; the student starts a new way of working on the problem or creates a new problem (Simons, 2019).

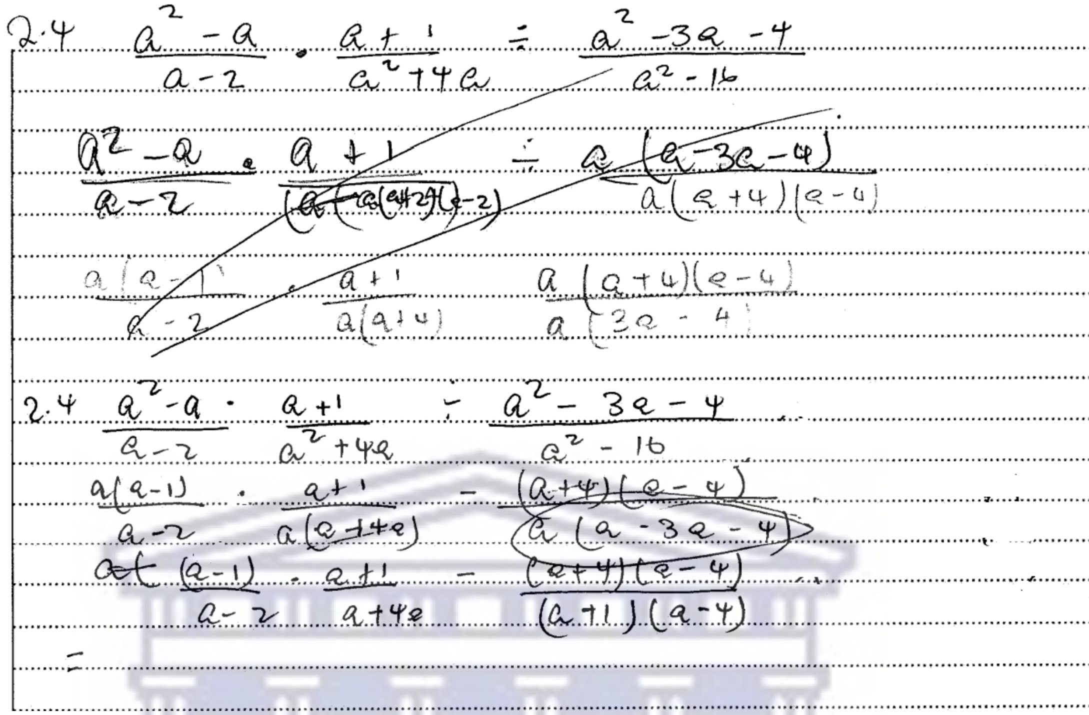

Figure 5.4 Abandonment

5.2 Analysis

5.2.1 Not Attempted:

Not Attempted means an abandonment of a complete question before attempting a solution. The examinee either writes the question number with or without the problem text and leaves the blank space if intending to attempt later or leaves no space and continues to write the next question. Not attempted means the examinee did not show any attempt to answer the question. This type of abandonment shows no visible textures.

Jacobs et al. (2014) explain this phenomenon as “not attempted”. According to them, this is when the ways of working showed no indication in the script that the examinee had answered the question. The examinee either only writes down the item number, but the script indicates no further work with it, or the item does not appear at all. Mullis et al. (2004) and Jacobs et al. (2014) report the “not attempted” in their studies as a custom of reporting diagnostic statistics for large-scale testing. In this study “not attempted” is reported because it indicates a resistance in seeking solutions to a mathematical question in an exam setting. It also talks to the agencies, such as time, that the high-stakes assessments exert on the examinees. This way of working can be divided into two; the first is "no attempt" with the intention to return to the question later, and the second is "no attempt" without the intention to return to the question.

5.2.1.1 Problem 1: No attempt with intention to return to the question.

The examinee writes down the question number and the algebraic fraction to be simplified but does not show any attempt to solve the problem. The examinee shows evidence of the intention to return to the question by leaving a space for it. This way of working shows that the examinee writes down the problem to be solved and abandons the work by not attempting any solution path. The examinee reserved a space for the solution, showing the intention to complete the solution-seeking pursuance.

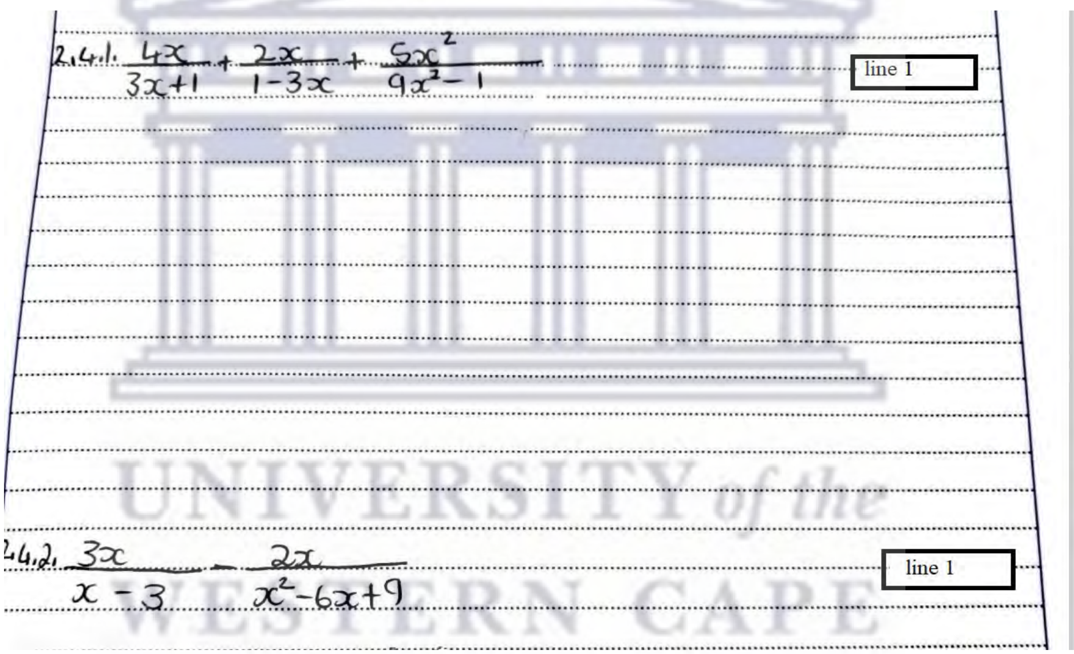

Figure 5.5 Not attempted with the intention to return to the question

Figure 5.5, line 1 above, shows an excerpt from the examinee script. The examinee copied the expression in 2.4.1 and left it without a visible attempt to answer it. The examinee left a space of 12 lines and wrote down the next problem to be solved 2.4.2 of Figure 5.5. The 12 open line spaces left open by the examinee show that this space is enough to accommodate the solution path for the expression in line 1, thereby indicating the intention to return to the question later.

Part of the examinee's historicized mathematical knowledge (Simons, 2019) is the common instruction teachers and invigilators give examinees: "If you are not sure about a question, do not waste time on it; you can always go back to it". This shows a reflexive way of working to complete the time-restricted examination through the exertion of the mathematically historicized self. In this instance, it is systematically produced *in situ*. Thus, due to the pressures of high-stakes, time-restricted examination, the examinee experienced resistance, and the delay in finding a way of firing the resistance-exerted time agency and the historized self about the examination may lead to abandonment, with the intention to prioritise those questions that exert no resistance and return later to the question with resistance.

5.2.1.2 Problem 2: No attempt with no intention to pursue a solution.

This way of working shows the examinee only writing down the question number and abandoning the question without showing any attempt to answer the question. The examinee's way of working shows no visible textures.

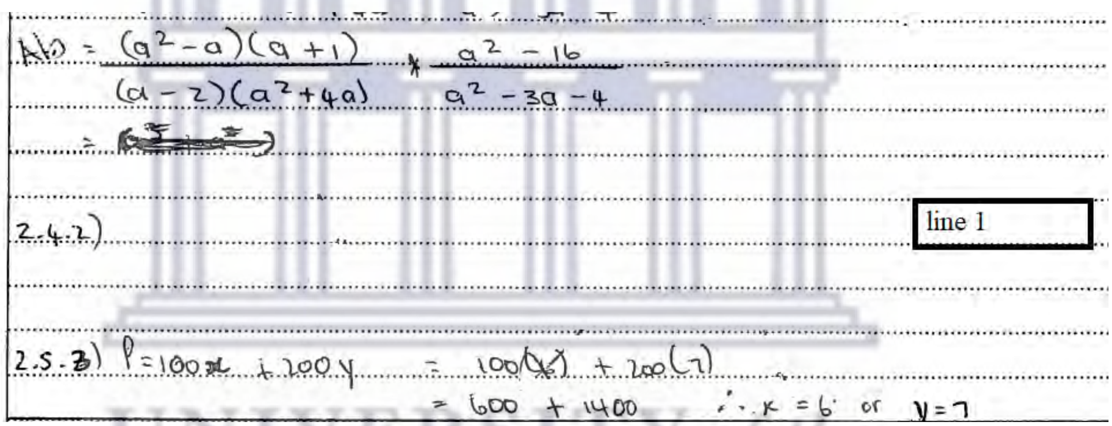

Figure 5.6 No attempt with no intention to pursue a solution

Figure 5.6 above shows the examinee wrote down question number 2.4.2, line 1. Only the question number was written, which shows that the examinee had no intention to pursue a solution to this problem. This is further indicated by the examinee leaving no space for the solution-seeking path.

5.2.1.3 Problem 3: No attempt with no clear evidence of an intention to return to the question later.

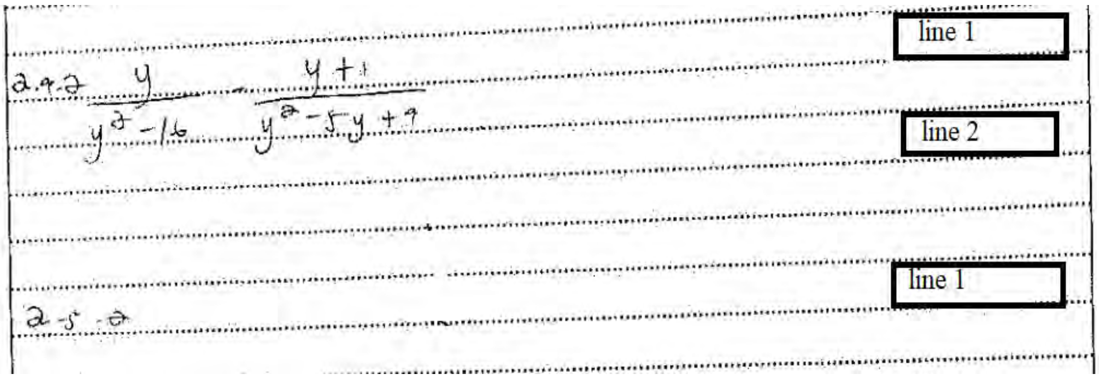

Figure 5.7 No attempt with no clear evidence of an intention to return to the question later

Figure 5.7 above shows the examinee's work where only the question number and the problem statement in line 1 of 2.4.2 were written, and three lines were left before writing another question, 2.5.2 line 1. It is unclear if the examinee intended to come back and only write the first step of the solution, which would be to factorise the denominators. The space left does not show evidence that the examinee intended to return to the problem and answer it in full, as the space of three lines would not be enough to simplify the expression completely. This way of working in Figure 5.7 above shows no visible textures of the examinee's work.

5.2.2 Reversal

"Reversal occurs when a calculated resistance is created, and a learner removes the produced work by drawing a line across it and subsequently begins again from a certain position" (Julie 2003:121). This texture and way of working is shown when a student follows a particular path in responding to the question before arriving at a desired solution and where the process is seen that its performance will not yield the desired result. The student observes this performance and decides to adapt and reverse back to a particular stage of the process to start again.

5.2.2.1 Problem 1: Multiple reversals in one solution–seeking path.

The way of working in Figure 5.8 below reveals the reversal after a calculated resistance was experienced. The examinee accommodated by cancelling part of the pursuit and beginning again.

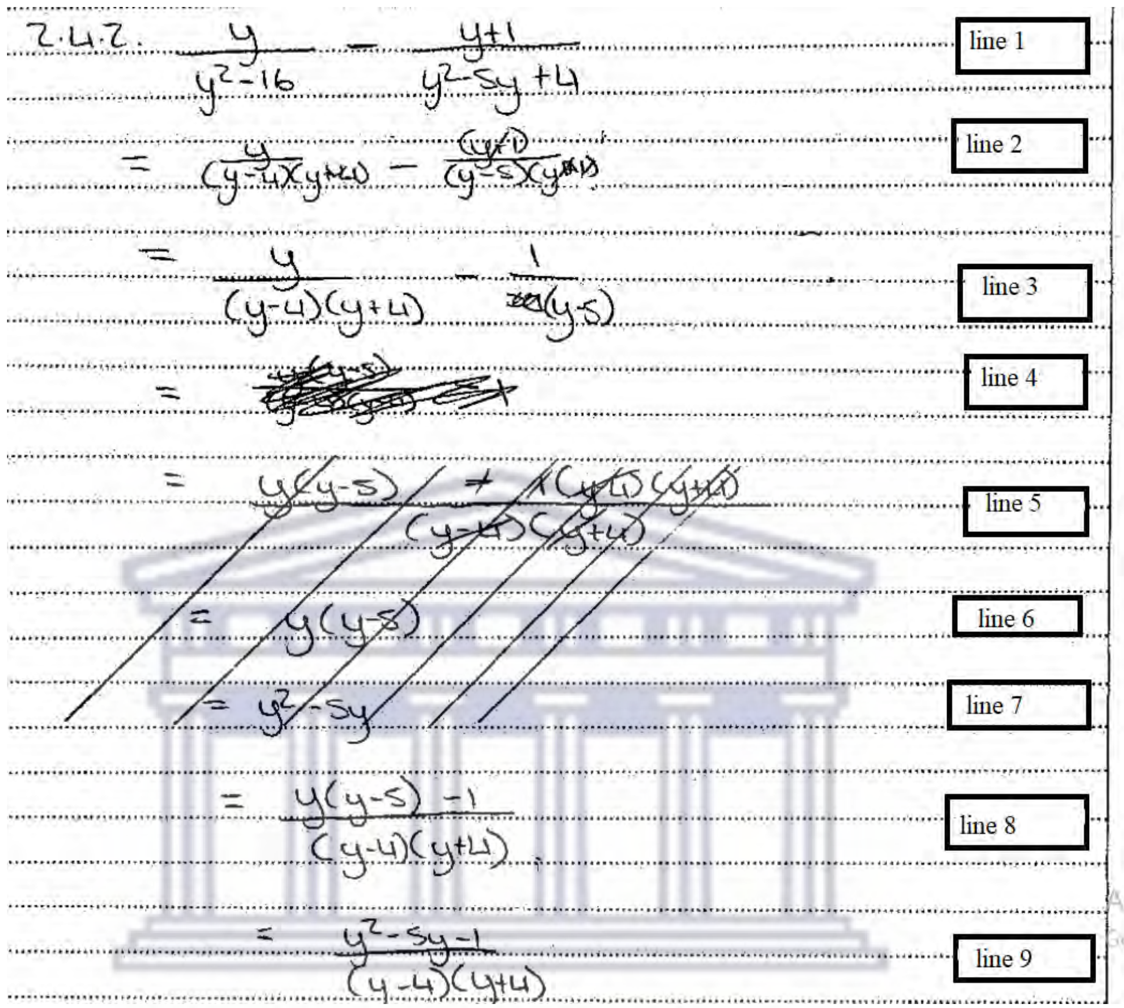

Figure 5.8 Multiple reversals in one solution-seeking path

In Figure 5.8, the solution pursuance begins by writing down the algebraic fractions expression to be simplified in line 1 of 2.4.2. The pursuit continues in line 2, where the examinee factorised the expressions in the denominators $(y^2 - 16)$ and $(y^2 - 5y + 4)$. The common factor in the second fraction $(y + 1)$ is then cancelled. In line 3, the solution from line 2 is written down with a small slip writing x and then cancelling it.

When students work with algebraic fractions in class, x is the most commonly used variable. The examinee continues in line 4 and multiplies the numerator y with $(y - 5)$ from the denominator of the second fraction yielding to $\frac{y\,(y-5)}{(y-4)(y+4)} - 1$. However, a resistance is experienced, and line 4 is scratched out, and the examinee starts again in line 5. The pursuit shows that the examinee attempted to produce a common denominator that would help add the two fractions. The examinee cross multiplies the two fractions and uses the denominator of the first fraction $(y-4)(y-5)$ as a common denominator, a procedure commonly used when solving rational algebraic equations. With no equal sign between the two fractions, cross-multiplying does not yield the expected outcome. The examinee then cancels $(y-4)(y+4)$ in the numerator with that of the denominator as if they are common factors in the fraction yielding $y(y-5)$ written in line 6. In line 7, the examinee simplifies by multiplying out the bracket from line 6 and gets $y^2 - 5y$. The outcome is written down and expanded by opening the bracket. Before completing that pursuit, another resistance was encountered, and the part of the work was cancelled, reversing back to line 3, and to produce $\frac{y(y-5)-1}{(y-4)(y+4)}$ in line 8 in the same solution-seeking pursuit. The examinee rewrites the produced work in line 8 as the result in line 9.

5.2.2.2 Problem 2: Reversal with abandonment by pursuing the solution to a single procedure.

The way of working refers to the commencement of a pursuit to find a solution to a mathematical problem. The attempt shows some form of working up to a point where a procedure is produced and completely abandoned, and following on a path adjacent to the abandoned work, the pursuit shows an abandonment of the new attempt followed by a continuation moving back to the initial solution-seeking path.

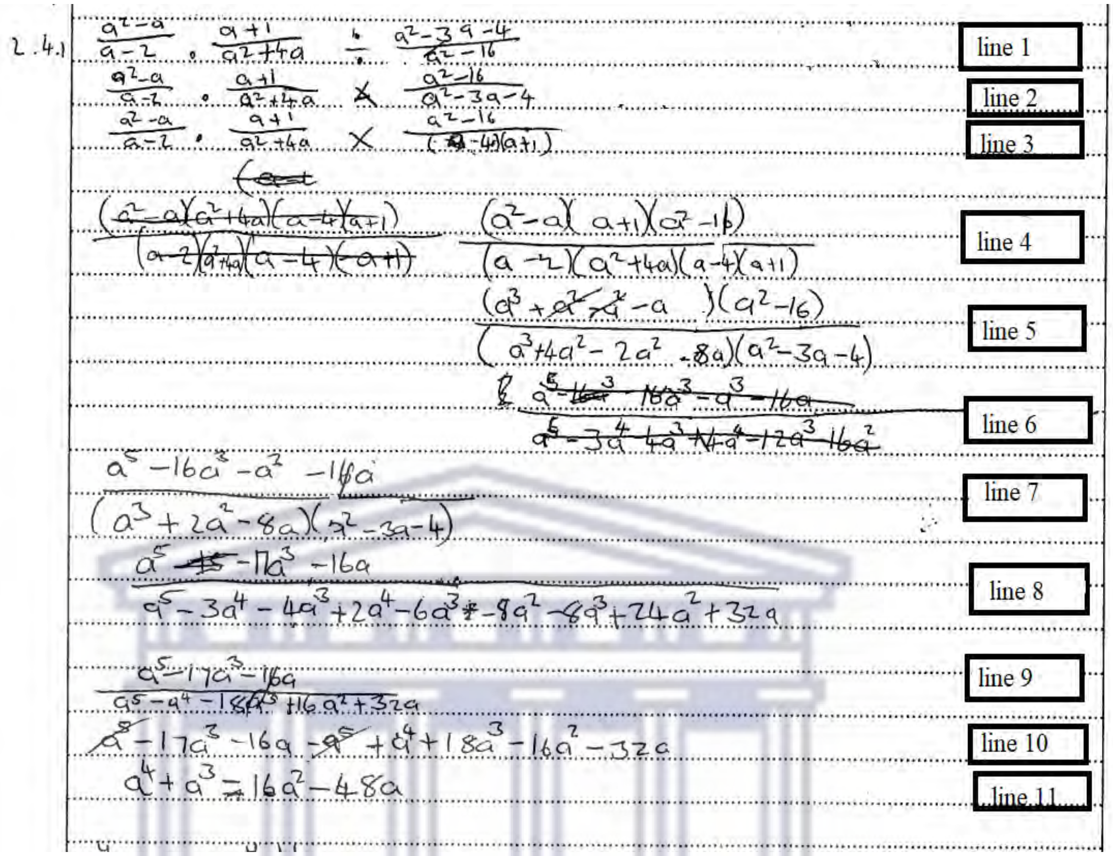

Figure 5.9 Reversal with abandonment by pursuing the solution to a single procedure

Figure 5.9 above shows an examinee's way of working, which started in line 1 of 2.4.1, where the examinee wrote down the problem text. The solution pursuance begins in line 2 by changing the division sign to multiplication by inverting the fraction $\frac{a^2-3a-4}{a^2-16}$ to be $\frac{a^2-16}{a^2-3a-4}$. The pursuit continues in line 3, and the examinee factorises the expression a^2-3a-4 in the denominator (line 2) to $(a-4)(a-1)$ but left the other quadratic expression un-factorised; at this point, a calculated resistance occurred. Thus, a non-firing of resistance occurs, and the examinee's way of working continues to cross-multiply the expression without factorising. The way of working used in line 4 was later abandoned, and, adjacent to the abandoned attempt, the student reversed to try a new path where all the numerators of the fractions were multiplied and the denominators multiplied to get $\frac{(a^2-a)(a+1)(a^2-16)}{(a-2)(a^2+4)(a-4)(a+1)}$ (first reversal). The examinee continued to simplify by opening the brackets and multiplying the first two factors of the numerator and multiplying the first two factors of the denominator with each other and the last two factors of the denominator. The work done in line 5 was cancelled, and the examinee reversed back to the factors that were in line

4 (second reversal). The solution-seeking pursuance continues in line 5 and the factors are multiplied and like terms cancelled. Cancelling like terms is another method that members of the school mathematics community socially share. This way of working is understood to mean the subtraction of like terms that have the same coefficient, which results in a zero answer. In line 6, the examinee opened the brackets in both the numerator and the denominator but realised a resistance that occurred in line 5, where like terms in the denominator were not added. The examinee, therefore, abandoned the work done in line 6 and reverted to firing the resistance experienced in line 5, producing line 7. The solution pursuance continues to line 11.

5.2.3 Retracing

Retracing refers to abandoning a solution after assiduous efforts towards a specific goal, removing and adjusting, and going back over the same path to the source that did not comply with the dictates of the mathematical context.

The excerpt in Figure 5.10 from the examinee's work shows another example of retracing.

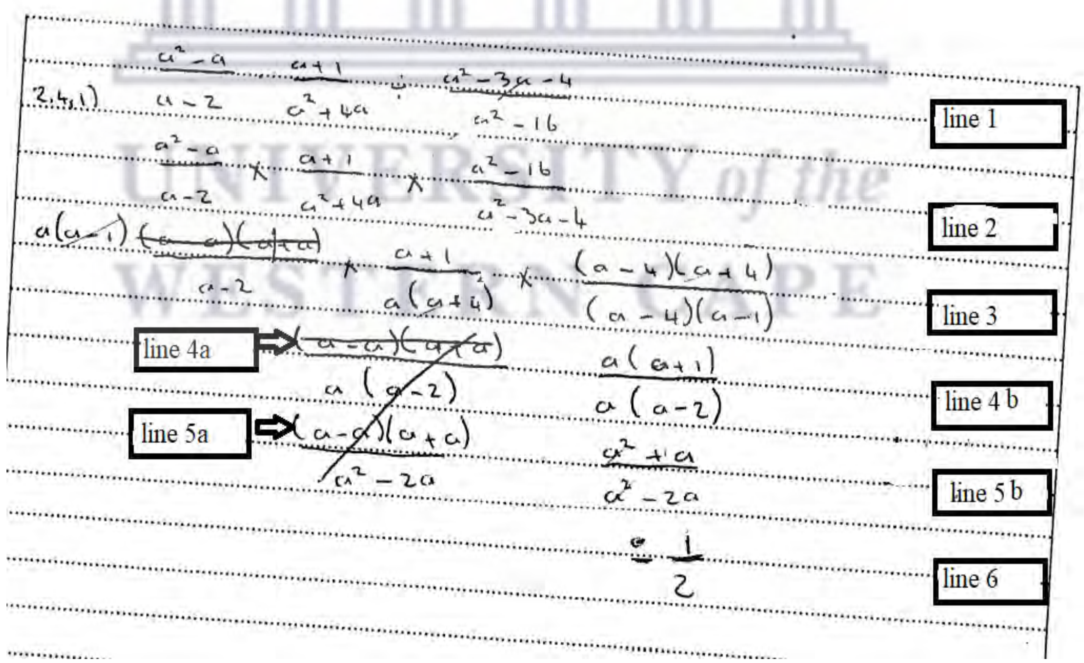

Figure 5.10 Retracing

In Figure 5.10 above, the examinee started in line 1 of 2.4.1 by writing the problem text. In line 2, the solution pursuance begins by changing the division sign to multiplication and inverting the fraction $\frac{a^2-3a-4}{a^2-16}$. In line 3, the examinee begins factorising the quadratic expressions. $a^2 - a$ is first factorised to $a(a-1)$ a resistance is encountered, and the way of working is then abandoned and cancelled, which is shown by a line through the work. The examinee continues to adjust and factorise $a^2 - a$ as the difference of two squares and gets $(a-a)(a+a)$ and then continues to write the second fraction $\frac{a+1}{a(a+4)}$ with $a^2 + 4a$ factorised correctly to $a(a+4)$. The last fraction is also written in line with its quadratic expressions in the numerator factorised, $a^2 - 16 = (a-4)(a+4)$ and the denominator $a^2 - 3a - 4 = (a-4)(a-1)$. The resistance in sign (-1) for factors of $a^2 - 3a - 4$ was not noticed, and the examinee continued with the solution pursuance in the same path with non-firing of the resistance. In line 4a, the examinee continues with the way of working that did not comply with the mathematical context and writes the outcome after cancelling the common factors $(a-4)(a+4)$ and $(a-1)$ in line 3. The outcome written in line 4a is $\frac{(a-a)(a+1)}{a(a-2)}$. The solution pursuance continues in line 5a, and the examinee opens the brackets in $a(a-2)$ and gets $a^2 - 2a$. The solution path does not yield the desired results; the examinee reverses to line 3, retracing the first way of working abandoned when a U-turn was taken. The examinee removed the source of mathematically calculated resistance in line 3 $(a-a)(a+a)$. The quadratic expression $a^2 - a$ that was factorised as $a(a-1)$ and then cancelled is now used in line 4b, and the factors $(a-a)(a+a)$ are abandoned and cancelled with an indication of a line crossed through in line 3 and 4a. Another line is cancelled through lines 4a and 5a to indicate that all that solution path is abandoned. The solution pursuance begins again from line 4b, and the factor $a(a-1)$ is used. In line 5b, the examinee simplifies by opening the brackets and gets $\frac{a^2-a}{a^2-2a}$. In line 5b, the examinee cancels the common terms in the fractions like common factors. This way of working is referred to as convenience and will be discussed later. $a^2{'}s$ and $a's$ are cancelled, leaving the solution as $\frac{1}{2}$. The solution seemed to be the expected outcome from the examinee and was left as the final solution.

5.2.4 Convenience

Convenience is explained by Julie (2003) as a way of working where an examinee creates a situation using a faulty method to simplify a mathematical problem to get a suitable objective. In this study, convenience is observed in the examinees' work where a mathematical operation is changed, dismissed or incorrectly applied to find a convenient way of simplifying the expression.

In Figure 5.8 reversals, line 10 of reversal with abandonment, the examinee changed the division sign to addition to make adding or subtracting like terms conveniently easy. In Figure 5.8, line 5b, terms are cancelled as common factors, making it conveniently easy to eliminate the variables in the fraction and leave a numerical answer.

5.2.4.1 Convenience problem 1: cross multiplying the difference of two fractions

Convenience is also noticed in Figure 5.11 below, where the examinee started in line 1 of 2.4.2 by copying the problem text and began the solution pursuance by deciding to cross multiply the numerator with the denominator of the given two fractions. In line 1, a cross is used to symbolise the cross multiplication. The minus sign in between the two fractions is ignored. In line 2, the student multiplies the numerator of the first fraction $3x$ with the denominator of the second fraction $x^2 - 6x + 9$ and also the numerator of the second fraction $2x$ with the denominator $x - 3$. The outcome of the multiplication, which is opening the brackets, is written in line 3 as $3x^3 - 18x^2 + 27x + 2x^2 - 6x$. The solution path continues, and the examinee adds the like terms from line 3 to get the expression $3x^3$-16$x^2 + 21$ as the final solution. The expression is underlined by a line with an arrow at the end indicating the end of the solution path.

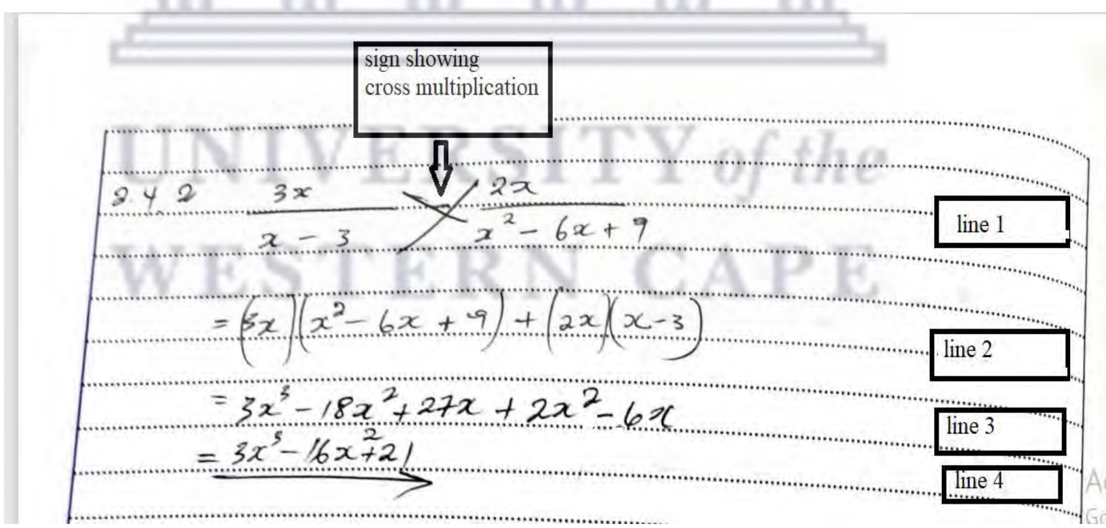

Figure 5.11 Convenience problem 1: cross multiplying in the difference of two fractions

5.2.4.2 Convenience: problem 2 uses quotient rule and factorising individual second degree terms as difference of two squares.

Figure 5.12 shows the examinee applied several procedures to simplify the algebraic fractions. The mathematical procedures applied are applicable when finding solutions to different mathematical questions or problems but do not apply to the specific problem text given in this examination question.

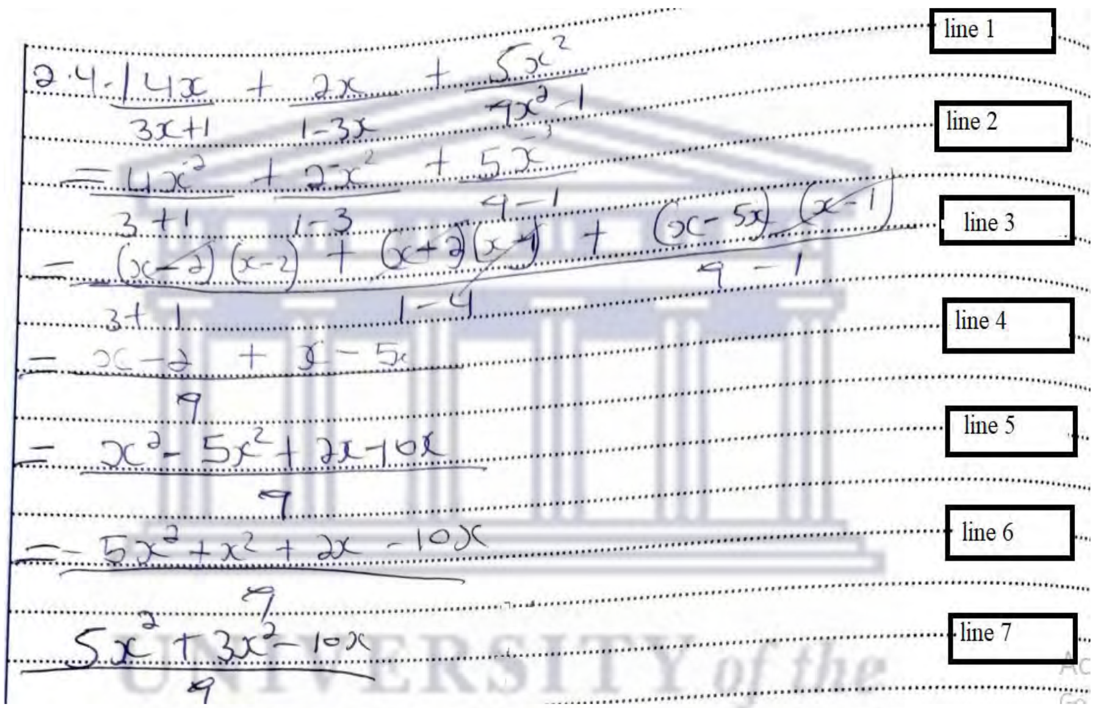

Figure 5.12 Convenience problem 2: using quotient rule and factorising individual second degree terms as difference of two squares

The pursuit of finding a solution begins in line 1 of 2.4.1 in Figure 5.12 by writing the problem text or algebraic fraction to be simplified. In line 2 the examinee uses the exponential laws to eliminate the x-variable in the denominator. The quotient rule of exponents allows for subtracting the exponent of a variable in the denominator from the exponent in the numerator if the bases are the same, i.e., for any unknown variable a; $\frac{a^m}{a^n} = a^{m-n}$. This rule only applies to terms multiplied or divided, i.e., factors, not to terms separated by a plus or minus. The examinee continues this way of working and uses x from $3x + 1$ in the denominator, subtracting exponents, $\frac{4x}{3x+1} = \frac{4x^{1-1}}{3+1} = \frac{4x^{-2}}{3+1}$. A resistance, 1-1, in the exponent is left without firing it, and -2 is written as the outcome of the subtraction. The same solution-seeking path is applied to all the fractions, $\frac{2x}{1-3x} = \frac{2x^{1-1}}{1-3} = \frac{2x^{-2}}{1-3}$ and the last one $\frac{5x^2}{9x^2-1} = \frac{5x^{2-2}}{9-1} = \frac{5x^{-3}}{9-1}$. The resistance in subtracting the exponents is left without firing, and the solution pursuance continues to line 3.

The goal is to find the quadratic expressions so that they can be factorised. In line 3, the examinee also uses another convenience by factorising the terms in the numerator, $2x^{-2} = (x-2)(x-2)$; $2^{-}x^2 = (x = 2)(x-1)$ and lastly $5x^{-3} = (x-5x)(x-1)$. The three fractions are then conveniently added together without finding a common denominator. The denominators are kept the same as from line 2. The solution-seeking pursuance continues in line 3, where the examinee cancels the common factors in the numerator. When simplifying algebraic fractions, one can factor the numerator and the denominator so that any common factors may be cancelled, reducing the fraction to the lowest terms. In this case, the examinee cancels common factors that are only in the numerator. In line 4, the remaining factors $(x-2)$ *and* $(x-5x)$ are written down. In line 5, the examinee does not add the like terms but multiplies $(x-2)$ *and* $(x-5x)$ to get $x^2 - 5x^2 + 2x - 10x$. Working with sign seems to give some resistance several times. In Line 6, the examinee re-arranges the expression in the numerator and continues to line 6. In line 6, the unlike terms are conveniently added $x^2 + 2x = 3x^2$. The final solution is left as an algebraic fraction $\frac{-5x^2+3x^2-10x}{9}$.

5.2.4.3 Convenience problem 3: adding like terms in the numerator and unlike terms in the denominator

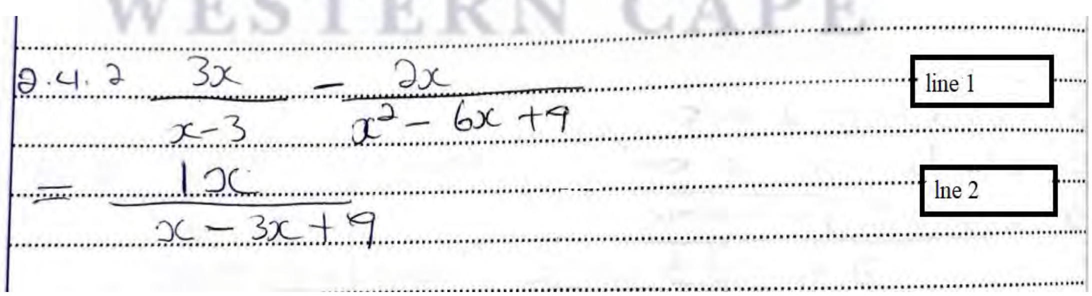

Figure 5.13 Convenience problem 3: adding like terms in the numerator and unlike terms in the denominator

In Figure 5.13 above, convenience is observed in the examinee's way of working, in which the examinee began by writing down the problem text for 2.4.2 in line 1. In line 2, an outcome from adding the numerators $3x$ *and* $-2x$ and the denominators $(x-3)$ *and* (x^2-6x+9) is written. The examinee experienced resistance in simplifying the two algebraic fractions and accommodated for the resistance by using convenience.

5.2.4.4 Convenience problem 4: Adding like terms in the numerator and denominator.

In Figure 5.14 below, the examinee's way of working reveals textures of convenience. The examinee wrote the problem text in line 1 of 2.4. In line 2, the examinee started the solution path by factorising the fraction $\frac{5x^2}{9x^2-1}$ as $\frac{(5x)(x)}{(3x-1)(3x+1)}$. The solution path continues in line 3, where convenience was applied by adding the like terms in the numerator $4x + 2x = 6x$ and leaving $5x^2$. In the denominator $3x - 3x = 0; 1 - 1 = 0, 3x + 3x$ was also $= 0$ and $-1 + 1 = 0$, which left the final answer for the denominator as 0. The examinee did not write the zero in the denominator, and the final solution was left as $6x + 5x^2$.

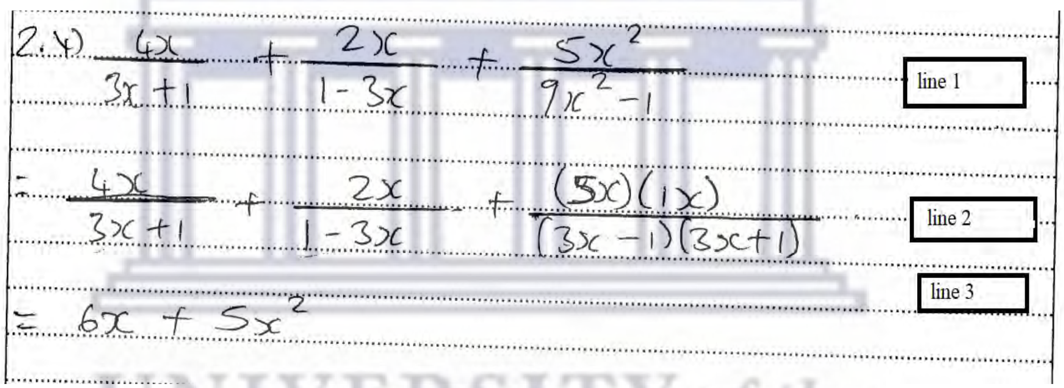

Figure 5.14 Adding like terms in the numerator and denominator

Examinees use the learned procedures and rules to solve mathematical problems in a solution-seeking pursuit. In the examination setting, the examinee has no access to any material that helps remind them of the rules or procedures in cases where an examinee experiences a resistance in finding the applicable procedure to solve a problem. The agency of time pushes the examinee to apply the procedure they think will work. Demby (1997) and Kieran (2007) argue that the terminology and rules of algebra offer little meaning to many learners, resulting in learners memorising algebraic rules with little or no conceptual understanding. Usman (2012) asserts that some learners only learn the manipulation of rules without reference to the meaning of the expression being manipulated. Students who have memorised the manipulation of rules may experience resistance in remembering the rules or procedures when faced with time agency and other exam factors, such as the pressure to pass and fear of the consequences of the examination.

A student, therefore, can accommodate resistance by finding a procedure that can help find the solution.

5.2.5 Abandonment

As explained, abandonment refers to the commencement of a solution-seeking path and after or before completing the pursuit, the solution is abandoned; the student starts a new way of working on the problem or creates a new problem (Simons, 2019).

5.2.5.1 Problem 1: Abandonment and reversing back to restart the solution pursuance with some adjustments.

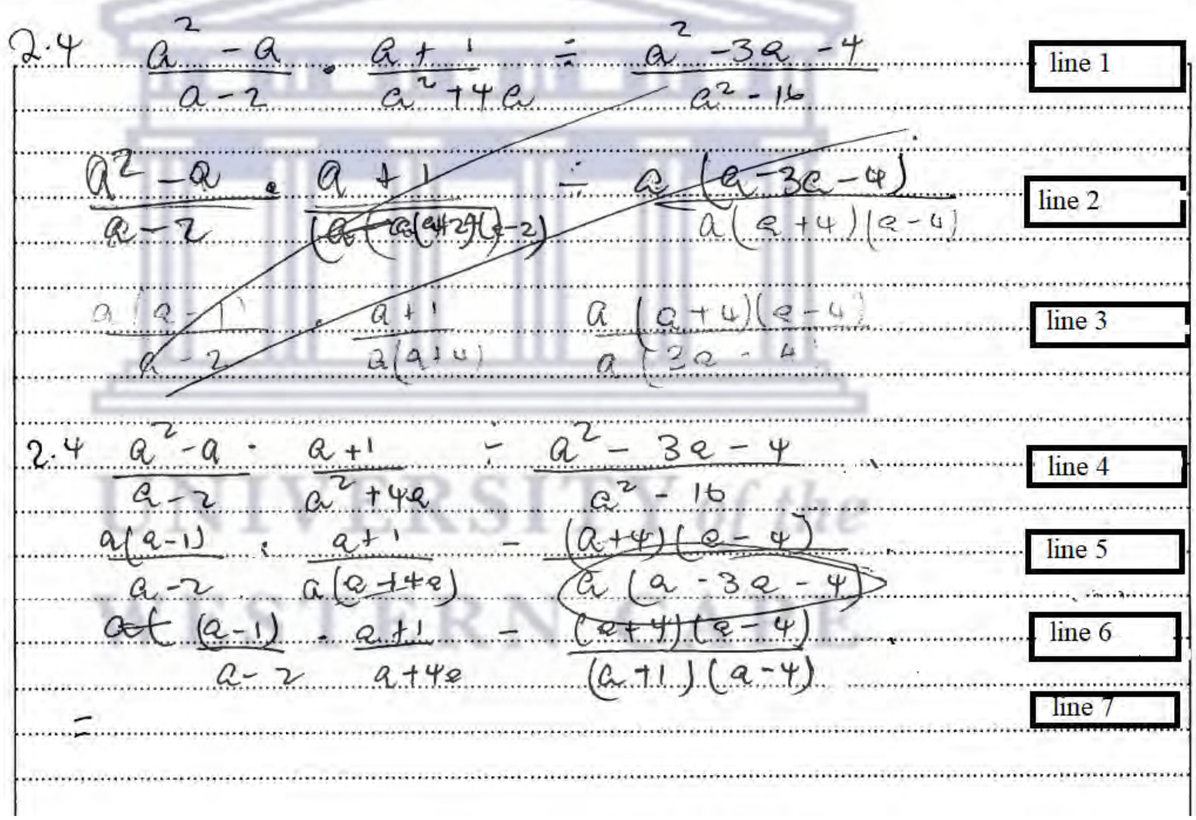

Figure 5.15 Abandonment and reversing back to restart the solution pursuance with some adjustments

Figure 5.15 shows the examinee's way with visible textures of abandonment. An examinee started in line 1 of 2.4 by writing the problem text. A solution-seeking path commences in line 2 by first factorising the quadratic expressions $a^2 + 4a$ in the denominator of the second fraction, $a^2 - 3a - 4$, in the numerator of the third fraction, and $a^2 - 16$ in the denominator of the last fraction. The way of working in factorising the fraction follows two procedures—factorising by taking out the common factor and factoring the difference of two squares. In line 2, when factorising the expression $a^2 + 4a$, resistance is experienced, and the examinee first takes out the common factor a. The dance of agency leads to taking a step back to observe the performance.

There is resistance, and before completing the factorisation, the way of working is abandoned, $a(a$ is cancelled. The examinee starts again and factorises by the difference of two squares which now becomes $a^2 + 4a = (a + 2)(a - 2)$. The third fraction in line 2 is also factorised—numerator and denominator. In the numerator $a^2 - 3a - 4 = a(a - 3a - 4)$., the common factor is only factored out in the first term a^2; the other term $3a - 4$ is left the same. The denominator $a^2 - 16$ is factorised by using the difference of two squares and taking the common factor a. The expression $a^2 - 16$ did not have a common factor, but the examinee's way of working showed convenience because, with a as a factor in the numerator and denominator, it would then be easy to cancel it out. In line 3, the solution pursuit continues, and $a^2 - a$ is factorised by taking out the common factor.

After factorising this expression, the examinee makes a U-turn, reconsiders how $a^2 + 4a$ was factorised and redid the factorisation by taking out the common factor. In line 3, the examinee further changes the division sign to multiplication by inverting the fraction $\frac{a^2-3a-4}{a^2-16}$ which was factorised as $\frac{a(a-3a-4)}{a(a-4(a+4)}$ is inverted to be $\frac{a(a+4)(a-4)}{a(3a-4)}$. The factors of $a^2 - 3a - 4 = a(a - 3a - 4)$ are also reconsidered and changed to $a(3a - 4)$. After line 3, the whole solution is abandoned. The examinee cancels it by drawing two lines across the solution. The solution pursuit commences again from line 4 by starting from the beginning, writing the problem number and text. In line 5, the examinee factorises the quadratic expressions, $a^2 - a = a(a - 1)$, and the expression $a^2 + 4a = a(a - 4a)$. The resistance in taking out the common factor from $4a$ is not fired, and the solution pursuit continues by changing the division sign to subtraction and inverting the fraction. $a^2 - 16$ is factorised as $(a - 4)(a + 4)$ and $a^2 - 3a - 4 = a(a - 3a - 4)$.

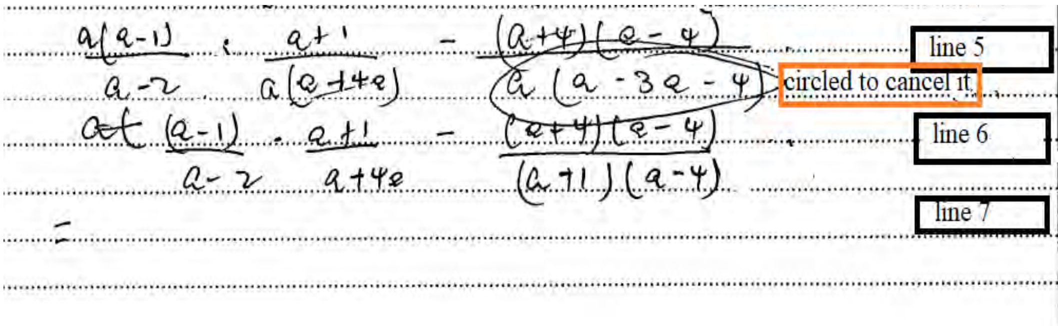

Figure 5.16 Abandonment and change of the applied procedure

Figure 5.16 above shows lines 5, 6 and 7 of Figure 5.14. In line 9, the way of working in factorising $a^2 - 3a - 4$ is abandoned, and the factors $a(a - 3a - 4)$ are changed to $(a + 1)(a + 4)$ and written in line 6. In $\frac{a(a-1)}{a-2} \cdot \frac{a+1}{a(a+4a)}$ the common factor a is cancelled and solution $\frac{a-1}{a-2} \cdot \frac{a+1}{a+4a}$ is written in line 6. In line 7, an equal sign is written, which shows the examinee has not reached the final step of the solution, but the solution-seeking path is abandoned and left incomplete.

5.2.5.2 Problem 2: Abandonment and restarting following the same way of working

Figure 5.17 below shows an example of abandonment observed in an examinee's script. The examinee in line 1 wrote the problem text. In line 1, the solution-seeking path begins, the quadratic expressions are factorised, and the division is changed to multiplication by inverting the fraction $\frac{a^2-3a-4}{a^2-16} = \frac{(a-4)(a+4)}{(a-4)(a+1)}$. Further in line 2, the examinee cancels the common factor *a* from $\frac{a(a-1)}{a-2} \cdot \frac{a+1}{a(a+4)}$. The solution is written in line 3 as $\frac{a-1}{a-2}$ but this solution path was abandoned before it was written in full, and the examinee cancels line 2 and line 3 and uses an arrow to direct the marker to line 4, where the solution pursuance commences after abandoning the first attempt. The solution pursuance follows the same path that was taken in the previous attempt, but this time the expression $a^2 - 16$ is not factorised. The path continues in line 5, and now all the quadratic expressions in the fractions are factorised, and all common factors are cancelled. The final step of the solution is written in line 6.

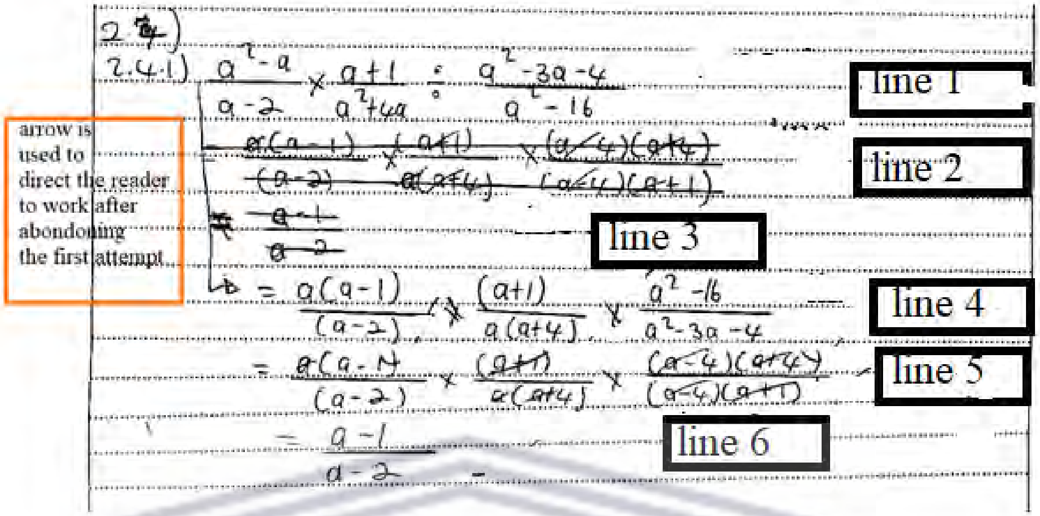

Figure 5.17 Abandonment and restarting following the same way of working

5.2.5.3 Problem 3 Abandonment after the application of a single procedure in solution pursuance

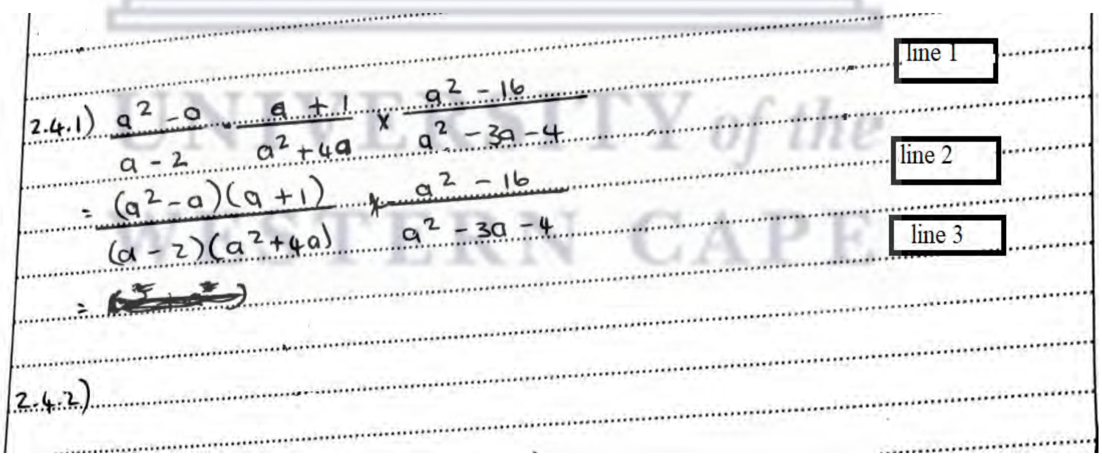

Figure 5.18 Abandonment after applying a single procedure in solution pursuance

The examinee in Figure 5.18 above started in line 1 of 2.4.1 by writing down the problem text and changing the division sign to multiplication by inverting the fraction $\frac{a^2-3a-4}{a^2-16}$ *to be* $\frac{a^2-16}{a^2-3a-4}$. In line 2, there are no procedures applied in simplifying the fractions, but the examinee uses the brackets for multiplication of the first two fractions $\frac{(a^2-a)(a+1)}{(a-2)(a^2+4a)}$. The examinee attempted to simplify the fractions by only changing the division sign to multiplication and got stuck. The examinee abandoned the solution pursuance. In line 3, the examinee attempted to simplify by opening the brackets but realised it would not yield the desired outcome. The examinee faced resistance and could not find a method to accommodate the resistance. The examinee decided to abandon the solution, leaving no space for returning and attempting later.

5.2.6 Rough work

The way of working is used to test the solution-seeking path to determine if it will produce the intended results before writing it formally as the solution to the question or problem. The students usually do their rough work on the side of the answer script and draw a line that separates it for the work to be marked, or they write their rough work on the last page of the answer script or on the question paper.

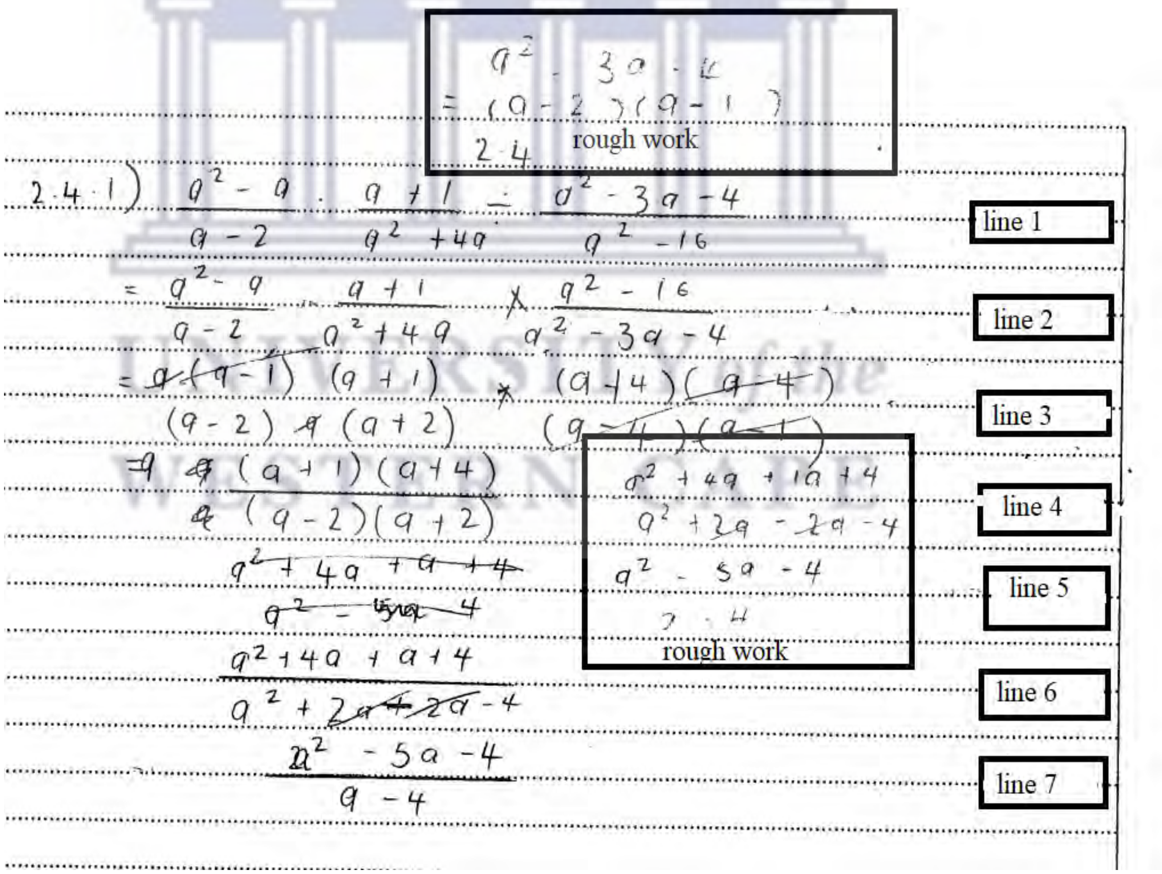

Figure 5.19 Rough work

In Figure 5.19 above, the examinee used rough work as a way of testing if the factors for the expression $a^2 - 3a - 4$ will work using a lead pencil. The rough work is written above line 1 of 2.4.1, which is the problem to be simplified. $a^2 - 3a - 4$ is factorised in the rough work as $(a - 2)(a - 1)$, but the attempt did not work out, as the examinee did not use the factors in line 3 where the quadratic expressions for the fractions were factorised. In line 3, $(a - 4)(a - 1)$ is written as factors for $a^2 - 3a - 4$. In line 4, the examinee used rough work again to multiply out the brackets for the numerator and the denominator of the fraction $\frac{(a+1)(a+4)}{(a-2)(a+2)}$ next to the factors $(a + 1)(a + 4)$ the examinee wrote $a^2 + 4a + 1a + 4$ and next to $(a - 2)(a + 2)$ = $a^2 + 2a - 2a - 4$. The examinee changed the signs when writing the sum of the terms $a^2 + 4a + 1a + 4$ and the outcome is written as $a^2 - 5a - 4$, but there was no negative sign in the factors. The examinee possibly reverted to the expression and realised the resistance that was not fired and accommodated for the resistance by changing the sign in the expression. For examinees, rough work tests a particular solution path to determine if it yields the desired result before it can be written as work to be marked.

5.2.7 Shedding

According to Mason (1999), shedding is a way of working that shows a situation of being stuck. As students pursue to simplify algebraic fractions in an exam setting, the time agency pressures them to complete within a specified time limit. When students find themselves stuck and unsure of the procedures or operations to apply in simplifying an algebraic, they usually abandon the expression.

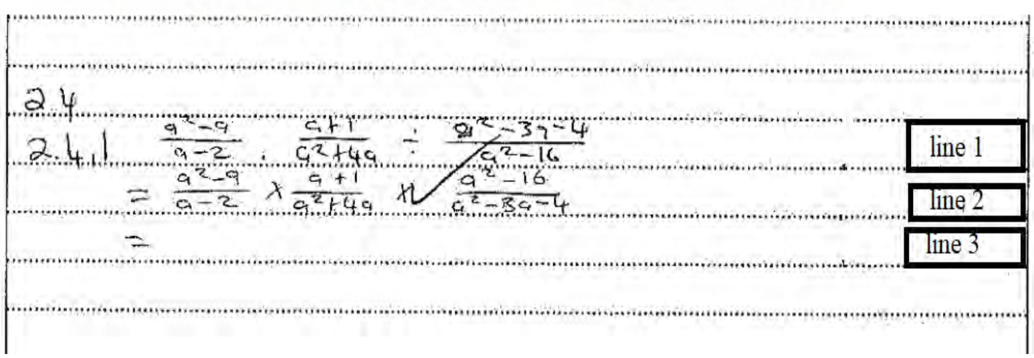

Figure 5.20 Shedding

Figure 5.20 above shows the texture of work where an examinee started in line 1 of 2.4.1 by writing the problem text. In line 2, the examinee changed the division to multiplication. In line 3, an equal sign is written showing that the solution pursuit continues. Nevertheless, the examinee was stuck and could not find the procedure to apply in continuing with simplifying. The solution was then abandoned.

5.2.7.1 Shedding problem 2: shedding after many attempts to find factors.

In Figure 5.21 below, the examinee copied down the problem text to be simplified in line 1 of 2.4.2. In line 2, the examinee started to simplify the fractions by factorising the expression $x^2 - 6x - 9$. Resistance was experienced, and the examinee accommodated for the resistance by cancelling the factors $3x(x-3)(x(3-x)$, which were first produced and the factors $x(-3x+3)$ were used as factors for the expression. The examinee observed that the performance of the factors $(x-3)(3x\ )$ did not yield the expected practical achievement; this way of working was also cancelled and re-factorised the denominator as $x(-3-x+3)$. In the numerators, the examinee subtracted, which gave x. But this way of working was also cancelled, and the examinee returned and wrote the difference $3x - 2x$ in the numerator of line 3. The solution path is then abandoned, and the examinee continues to complete the solution.

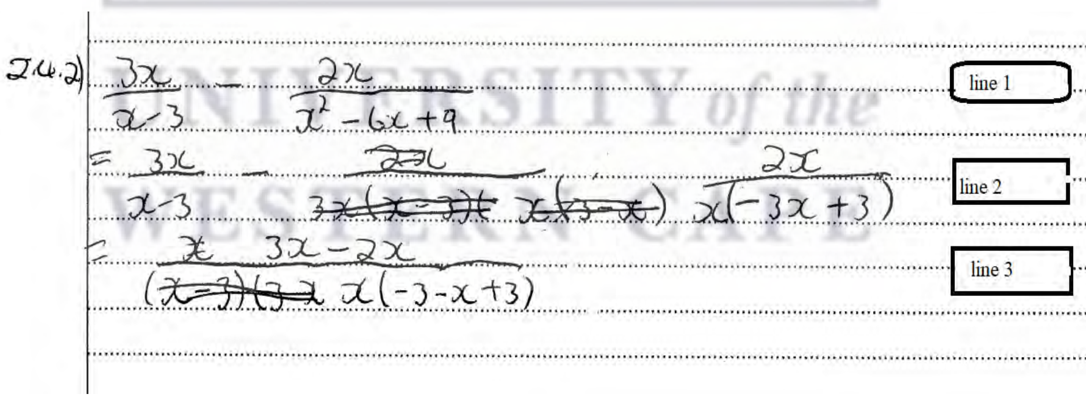

Figure 5.21 Shedding after many attempts to find factors

Textures of examinees' ways of working are summarised in Table 4.1 below.

Table 5.1 Textures of examinees' ways of working

Texture name	Symbol used in analysis	No of occurrences	Percentage
Reversal	RE	28	16%
Retracing	RT	12	7%
Convenience	CO	23	13%
Abandonment	AB	47	27%
Rough work	RW	6	4%
Not attempted	NA	6	4%
Shedding	SH	23	13%

There were 75 scripts analysed with two questions in the 2019 exam, which makes $48 \times 2 = 96$, plus $27 \times 3 = 81$. The total number of questions which were analysed for textures was 177.

Of the 177 questions analysed, abandonment was the highest occurrence, with 47 examinees abandoning questions during the exam. Rough work and not attempting questions had the lowest occurrence. Some examinees use their question papers to do their rough work, which can be the reason for only a few occurrences of examinees showing their rough work on the answer sheet.

5.3 Chapter Summary

This chapter discussed the textures of work identified in students' examination scripts. The ways of working identified are comparable to those discussed in the conceptual framework but have additional textures that are unique to this study. The next chapter will discuss the findings.

CHAPTER 6 DISCUSSION OF THE RESULTS

6.1 Introduction

The discussion is framed within the objectives of the study and gives an account of the textures of examinees' ways of working with algebraic fractions in high-stakes examinations using the ethnomethodological lens and dance of agency. The discussion highlights that examinees' ways of working with algebraic fractions are reflexively accountable, intelligibly, indexically expressible and understandable. Ways of working are reflexively accountable in the sense that they are observable and reportable, but also examinees' accounts of simplifying algebraic fractions are contingent accomplishments (Garfinkel, 1967).

The purpose of this study was to investigate how examinees solved algebraic fractions questions in the mathematics L3 TVET high-stakes examination in November 2019 and 2020. As stated in the chapter on methodology, a convenient sample of examination scripts from students at two TVET Colleges was used.

All the scripts were examined and assessed to find those with obvious textures, such as deletions and other elements, suggesting that the examinees did not use a linear method to answer algebraic fraction problems. This data selection procedure resulted in 75 scripts for detailed analysis. The textures are classified according to their relevance to the research question. The textures were categorised based on the researcher's interpretation. The analysis revealed eight ways of working, including gaps in the answer scripts displaying a visible texture of partial or no response. The researcher recognised these as textures in how the questions were numbered, and the examinee was expected to respond to all the questions in a question paper. The gap shows a texture in the flow of the structured problem text; it is evidence of the way of working in the context of a time-restricted examination. Two basic theoretical approaches guided the category identification procedure. The first is ethnomethodology and its constructs (Garfinkel, 1967), and the second is the mangle of practice (Pickering, 1995), with an emphasis on finding the dialectic of resistance and accommodation. The textures of the examinees' ways of working in search of a practical achievement or solution become visible through the dialectic resistance, accommodation, and ethnomethodological constructs.

De Vos (1998:203) defines data analysis as the analyst's decomposition of data into constituent components to gain answers to research questions. The study data does not answer the research question on its own. De Vos (1998:203) further contends that interpretation aims to reduce facts to a comprehensible and interpretable form. By doing so, the relationship between concepts, constructs, or variables in terms of patterns or trends may be recognised or separated. The research problem can be explored and tested, and conclusions can be reached through established themes in the data.

In so doing, the relation in terms of patterns or trends between concepts, constructs or variables can be identified or isolated. As with established themes in the data, the research problems can be studied and tested, and conclusions drawn. The textures formed by the ways of working identified from the examinees' written responses on Algebraic Fractions in the 2012 NSC Mathematics Examination are presented in the analysis chapter, and this chapter will discuss the results from the analysis.

6.2 Responses to Algebraic Fractions Questions in High-Stakes Examination Setting

The results of the study reveal that examinees exhibit several ways in which they attempt to answer or answer the questions during the examination. The textures of ways of working identified in students' work were: not attempted, reversal, retracing, convenience, abandonment, rough work, and shedding.

6.2.1 Not attempted.

As mentioned in the previous chapter, "not attempted" is a phenomenon from Jacobs et al. (2014), which is explained as a way of working that shows no indication in the script that the examinee had answered the question. It is when the examinee leaves the question without showing an attempt to answer it. The texture of this way of working is indicated by writing the question number and leaving a space or just writing the question number and moving to the next question, leaving no space for that question's solution. This way of working without any visible texture happens in the phase of selecting what items the examinee decides to attempt right at the start of the exam writing process. Examinees undergo a selection process of which questions to answer first and which should be answered later. This form of selection is also applied for each question as the examinee continues to answer questions.

In this study, the examinees' way of working revealed this texture in three ways, one of which is referred to as "not attempted with the intention to return to the question". This way of working is noted in this study as showing no textures of the solution pursuance but as one form of members' produced order. The examinee writes a question number and leaves a space with a certain number of lines that are enough for the solution. Figure 5.5 shows the examinee's work where only the question number and the problem text were written, and the space with 12 lines was left. The examinee experiences resistance in trying to find the solution to the question and struggles to find a way to fire the resistance; the examinee leaves the question intending to come back later. The examination must be completed within a time limit of 3 hours, exerting the urgency of time to the examinees. Each examinee seeks to complete the examination and provide any practical achievements possible to give to each question posed in the mathematics paper within the time limit given. The time constraints in the examination exert anxiety on the examinees and can impact whether the examinees can show their mathematical skills and abilities in the examination. Time constraints raise the question about what the high-stakes mathematics examination seeks to assess. Jakwert, Stancavage and Reed (1999) found that time limits highly influenced the nonresponse rate to the questions for a mathematics test. The authors further argue that even high-performing students omitted questions due to time limits, and most of the students who did not make question selection of questions to answer first but answered all questions in order of numbering in the question paper had omitted questions because they did not finish the test.

Morony (2015) asserts that "the tests and examinations make important that which is assessable, rather than making that which is important assessable". The examination tests mainly the examinees' ability to respond to questions within a specified time limit. The assessment does not mainly test the examinees' mathematical knowledge and skills. This study does not aim to discuss the effects of time constraints in high-stakes examinations, but the examinees experiencing time urgency during an examination produce certain features of social order. The question number written down and the number of lines left as blank space can be understood by both the examinee and the examiner or marker as meaning that the examinee left the question intending to return to it later if time allowed. Examinees respond to time urgency during an examination by not spending much time on the question they are unsure about but rather starting with those they have a clear idea of the solution path to follow. This order is sometimes a result of the lecturer's examination preparation techniques for students before they write examinations. Lecturers emphasise that students should start with questions they know and are confident they will get marks from and later attend to those that give them difficulty.

In a local setting, the examination room has a watch in position for all examinees to see during the examination, and the invigilator usually writes the times on the board in intervals of 30 minutes or 60 minutes. After each 30 or 60 minutes of the examination time passes, the invigilator scratches the time as a reminder of the time left before the examination time ends. Such a local setting does not allow much time to recall the learned skills and knowledge necessary to answer mathematics questions during the examination.

The second form of not attempted is “not attempted with no intention to pursue the solution. This is indicated in Figure 6.1.

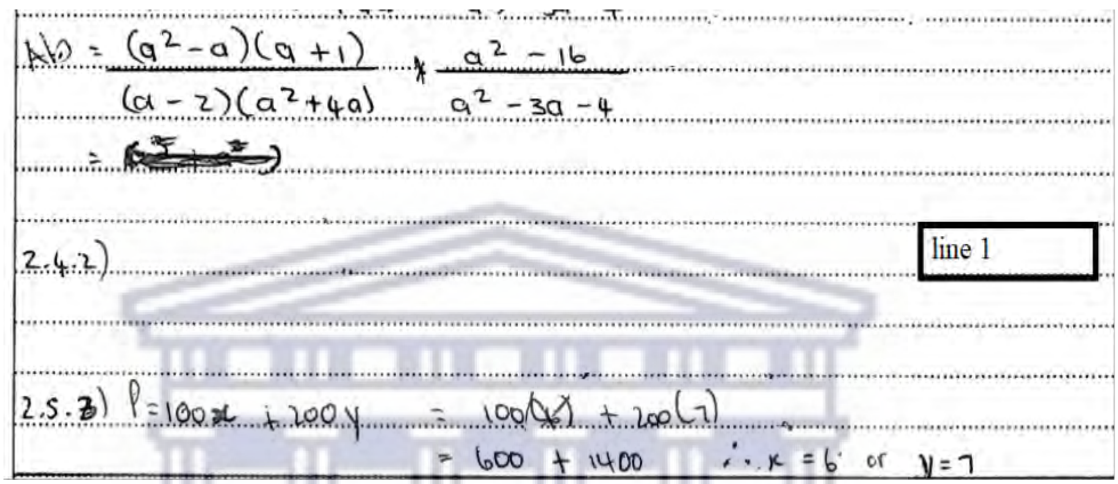

Figure 6.1 Not attempted with no intention to pursue the solution.

In Figure 6.1 above, the examinee did not attempt to answer 2.4.2. Only the question number is written and left with no textures of the solution pursuance. The examinee moved to 2.5.3, leaving no space for the solution for 2.4.2. This way of working indicates that the examinees did some mental calculations and experienced resistance, and the examinee abandoned the whole question and left it with no visible textures of attempt. Spring (1990) blames the teaching and learning methods of mathematics in that the teacher and the textbook are the authorities in a classroom, and mathematics is not viewed as a subject to be created or explored. Spring (1990:35) further asserts that the mathematical “truth is given in the teacher’s explanations and the answer book; there is no zigzag between conjectures and arguments for their validity, and one could hardly imagine hearing the words maybe or perhaps in a lesson”. The students in class are not exposed to the culture of making assumptions, explaining their reasoning, and validating their assertions. The examinees doubt the correctness of the assumptions they have in their minds about their mathematical knowledge.

The third form of not attempted is “not attempted with no clear evidence of an intention to return to the question”. This way of working is shown in Figure 5.7. An examinee only writes the question number and the problem text and leaves a few lines below the problem text, which can be enough space for the solution of the problem. The number of lines left does not give clear evidence of whether the examinee decided to leave the question with no intention to come back to it or the examinee left the question with the intention of coming back to find the factors for the quadratic expressions in the algebraic fractions. This way of working, therefore, shows no visible textures and does not reveal a particular order of the examinee's ways of doing mathematics.

6.2.2 Reversal

Reversal, as explained in this study, is a way of working where an examinee begins a solution pursuance following a particular procedure, resistance is experienced, and the examinee accommodates for the resistance by cancelling all or part of the work and starts again at a certain point with adjustments. Two ways of reversal were identified in this study. One is the multiple reversals in one solution-seeking path, and the second is reversal with an abandonment by pursuing the solution to a single procedure.

Figure 5.8 indicates this phenomenon in the examinee's work. After writing the problem text, the examinee started the solution pursuit by factorising$(y^2 - 16)$ and $(y^2 - 5y + 4)$ and continued by cancelling the common factors. The resistance in finding the factors for $(y^2 - 5y + 4)$ was left un-noticed, and the examinee continued with the factors $(y - 5)(\,y + 1)$ instead of $(y - 4)(y - 1)$. After cancelling the common factors, the examinee tried to simplify by cross multiplying the two fractions yielding $\frac{(y(y-5)}{(y-4)(y+4)} - 1$ in line 4. A calculated resistance is experienced in line 4 of Figure 5.8, and the examinee takes a step back to observe the performance of the solution pursuance and realises that it does not yield the desired practical achievement. The examinee, therefore, cancels the work produced in line 4 and starts again in line 5, still following the same solution-seeking path, which is to cross-multiply the two fractions. This time an adjustment is made by multiplying $(y - 4)(y + 4)$ with 1, which was not done in line 4. The examinee then cancelled the like terms in the fraction $\frac{y(y-5) - 1(y-4)(y+4)}{(y-4)(y+4)}$. The examinee could not recognise the resistance that the fraction could have been solved using the lowest common denominator but rather used cross multiplication, a procedure usually used in solving rational algebraic equations. The literature reveals that the ways in which algebraic fractions are taught to students are mostly procedural rather than conceptual (Mangwende, 2021) and as such, examinees experience difficulties in selecting the correct procedure to apply when simplifying algebraic fractions. Another resistance not fired was cancelling like terms in the fractions as if they were common factors. Mhakure, Jacobs and Julie (2014) argue that students rely on visual cues when they simplify algebraic fractions. When they see the same numbers, variables or factors in the numerator and denominator, that becomes a visual cue that tells them that they need to cancel without conceptually understanding what the procedure of cancelling means.

The examinee continued in line 6 and line 7 and reached a practical achievement of $y^2 - 5$. In observing this achievement, the examinee was unsatisfied with the outcome and cancelled all the work from line 5 to line 7, reversed back to line 4 and made adjustments.

From the "dance of agency" situation, which relates to human beings making the machines, Pickering (1995) argues that during the process, human beings first take an active, intentional role to tentatively construct some new machine. They then adopt a passive role, monitoring the machine's performance to see whatever capture of material agency it might affect. If the outcome is not the desired result, they reverse roles, take an active role again and make changes to produce the desired outcome. The moment of a passive role and observing the machine's performance can be at any stage.

When working with algebraic problems in an exam setting, the examinee begins a pursuit with the determined expected outcome; the journey to the outcome can be observed step by step. At some point in observing the progress, one can identify that a part of the process will not yield the expected outcome, meaning resistance is experienced. At this point of realisation, a student reverses to start again at a particular stage of the solution to fire the resistance by making changes, which is accommodation.

In the case of multiple reversals in one solution-seeking path, the examinee accommodates different forms of resistance by adjusting the solution by changing the procedure used to reach the final solution or a practical achievement.

The second type of reversal identified in this study is reversal with an abandonment by pursuing the solution to a single procedure. Figure 5.9 shows the examinee's work with this type of reversal. Reversal with an abandonment by pursuing the solution to a single procedure is characterised by textures of work where a solution pursuance is commenced and continues up to a particular point where resistance is experienced. The examinee, therefore, fires the resistance by abandoning part of the produced work and starts again, next to it, another solution pursuit which still follows the same procedure. Algebraic fractions are multifaceted, and to simplify algebraic fractions, examinees need to apply an understanding of mathematical concepts like division, variables, equations, perfect squares, exponent, factorisation, and rational numbers (Baidoo, 2019; Zulfa et al., 2020). Applying these different operations to yield the desired practical achievement gives several calculated resistances to examinees. Simplifying the algebraic fraction in Figure 5.9 required examinees to recall and apply factorisation of quadratic expressions, multiplicative inverse or reciprocal, which will help them invert the fraction. In line 2 of Figure 5.9, the examinee did notice that the division can be changed to multiplication by inverting the fraction $\frac{a^2-3a-4}{a^2-16}$ and had $\frac{a^2-a}{a-2} \cdot \frac{a+1}{a^2+4a} \times \frac{a^2-16}{a^2-3a-4}$. The examinee in line 3 factorised only the trinomial quadratic expression and experienced a calculated resistance in factorising other quadratic expressions, which were binomials. Factorisation of quadratic expressions alone involves several operations, which sometimes lead to more resistance in the process of solution pursuance. The examinee continued in line 4 by cross-multiplying but, after looking at the process, cancelled the produced work in line 4, reversed it to start again adjacent to the cancelled work, and adjusted it by only multiplying the numerators and denominators. The solution pursuance proceeded without factorising the quadratic expressions that were binomials. This non-firing of resistance makes the algebraic fractions more difficult and complex for the examinee to simplify. The examinee continues in the same pursuit that follows a single procedure in simplifying the fraction, several calculated resistances, which lead to several reversals and abandonment of produced work. Algebra involves several related procedures to be applied in a particular order to achieve an expected practical accomplishment. The failure to apply one procedure correctly affects the process, and the machine does not produce what is required.

The student's cancelled work shows a way of working that is usually ignored by the lecturers and receives no mark allocation because, by cancelling, the student communicates to the lecturer that the work should be ignored. This shows an intelligible and accountable social order because it is a socially shared procedure. The lecturer, the student and the marker understand that when the line or lines are drawn through the work, it means that ignoring that work was an error or mistake, which, in terms of accommodation and resistance, a student experienced a resistance and abandoned that work and sought a different way of arriving to the desired solution. In an ethnomethodological view, the activities of scientists are in many ways like those of ordinary lay activities, and ethnomethodology has a deep interest and respect for practical rationality and accountability of the commonplace of ordinary activities (Ten Have, 2004). This way of working is like most common activities that ordinary members of society use. Ten Have (2004) further argues that ethnomethodology can do two things at the same time: one is to show how a professional practice is embedded in quite ordinary competencies, and the other is to elaborate how it is special, in the sense of being part of a particular local version of a more generalised professional culture.

6.2.3 Retracing

As explained in Chapter 5, retracing is another form of reversal, a way of working where an examinee commences a solution-seeking path, and after several efforts to get to the desired solution, part of the produced solution is abandoned, and the examinee retraces back to a particular point. The abandonment happens after the realisation that the procedures applied to reach a desired outcome do not satisfy the dictates of the mathematical context. In Figure 5.10, an examinee commenced a solution-seeking pursuance and experienced resistance in factorising $a^2 - a$. At first, the examinee correctly factorised by taking out the common factor and produced $a(a-1)$ but abandoned this produced work and changed it to $(a-a)(a+a)$, which is factorising the difference of two squares. Another expression in the same question which needed a similar mathematical procedure to factorise is $(a^2 + 4a)$ was factorised by taking out the common factor $a(a+4)$. When examinees solve mathematical problems in an examination setting, they either draw from their sequences of mental or concrete actions (procedural knowledge) to achieve a specific goal, or they draw from a network of general facts, concepts, and principles (conceptual knowledge) (Rittle-Johnson, Schneider & Star 2015). In terms of conceptual knowledge, a web of relationships needs to be connected; the rich links and relationships are equally vital as the separate bits of information they join (Hurrell, 2021). When an examinee factorised the expression $a^2 - a$ and $(a^2 + 4a)$ that required the same mathematical procedure to factorise but doubted an expression that has a negative sign, this showed that the lack of connection on the learned principles and concepts lead to a resistance when seeking a practical solution for the mathematical questions posed in an examination paper.

The examinee continued with the solution pursuance with factors $(a-a)(a+a)$ until a situation of being stuck was reached; the produced practical achievement did not comply with the dictates of the mathematical context. The examinee then observed the production process and reversed back to the previous way of working. The factors $(a-a)(a+a)$ were abandoned together with the solution that proceeded from them, and then $a(a-1)$ was used. In the context of ethnomethodology, this shows local and time-bound features of phenomena. The examinees, who can be referred to as members in an examination room and a local setting, respond to examination questions that are time-bound and produce certain features that are visible and accountable. In this case of retracing, we observe that in an examination which is limited to three hours, examinees do their best to get as many marks as possible to succeed in a high-stakes examination. Examinees commence a solution pursuance using their acquired knowledge, either procedural or conceptual, to respond to the examination question. As they proceed with the solution pursuance, they observe if the produced work complies with the mathematical dictates of the context or whether the process leads to the desired outcome. If the practical achievement is not what the examinee expected, they revert and trace whether the non-firing of resistance occurred, and after finding the resistance, they fire it by either changing the procedure used or follow the same path with adjustments.

Examinees communicate to the examiners or markers through symbols such as drawing a line through the produced work and can refer the marker to a particular work they have produced without writing in words, but, as members of the mathematical community, they all understand the social order. These symbols reveal the "lived work" (Livingstone, 1986) of examinees, which is recipient-designed. According to Livingstone (1986), the written proof is not written for everyone but is meant for those who understand and are familiar with mathematical principles and processes needed for that proof. It becomes a similar case when responding to algebraic fractions questions in an examination. Examinees' written work is meant for those who understand the mathematical procedures and principles and the order of implementing them to produce a desired practical achievement.

6.2.4 Convenience

Convenience is a way of working where an examinee applies an incorrect mathematical procedure to make it conveniently easy or possible to solve a mathematical problem. In this study, the examinees' way of working revealed multiple forms of convenience in simplifying algebraic fractions. Examinees used cross multiplication in subtracting two algebraic fractions. The procedure of cross-multiplying is used to simplify two rational algebraic equations, but in this study, examinees cross-multiplied instead of finding a common denominator that would help them subtract the two fractions. Another convenient way of working used by examinees, as shown in Figure 5.12, was using the quotient rule, subtracting the exponents of the variable in the denominator from the exponents of the variables in the numerator. In Figure 5.12, the examinee factorised a second-degree term as a difference of two squares. The other convenient way of working is observed in the examinee's work, where terms of the numerators on three fractions that the examinee needed to add were added by adding like terms of the numerator and the denominator's terms, even without like terms.

What is observable from the learner's way of working, which applies convenience, is that examinees use convenience when they are stuck, where the solution-seeking path does not seem to yield the desired outcome.

In Figure 5.13, where an examinee added like terms in the numerator and denominator, the solution-seeking began by factorising the quadratic expressions, but the examinee experienced resistance and could continue to add fractions in line 2. The examinee then used convenience to add like terms, which eliminated the denominator, and the two unlike terms 6x+5x^2 remained as the final solution.

The other observation is that examinees tend to be confused about which method is applicable to reach the necessary practical achievement of a simplified algebraic fraction. This could emanate from the ways in which algebraic fractions are taught to them. The teaching does not help learners build sense-making of the procedures they apply as they simplify rational algebraic fractions. Learners do not understand mathematics as one about inquiry, sense-making, and understanding how and why mathematical ideas fit together the way they do Li and Schoenfeld (2019). The disconnection in knowing how mathematical ideas fit together results in applying mathematical procedures where they do not apply.

What is observable and of interest in this study is the lived work of simplifying the algebraic fraction. The students did not necessarily get the answer correct, but they made their way of working visible, and the examiner and any reader could follow their way of working. The lived work and practical account of simplifying the algebraic fraction are visible. The symbols used, for example, in Figure 5.11 line 1, as repeated below in Figure 6.2, the examiner used a cross to show an act of cross multiplying the two algebraic fractions.

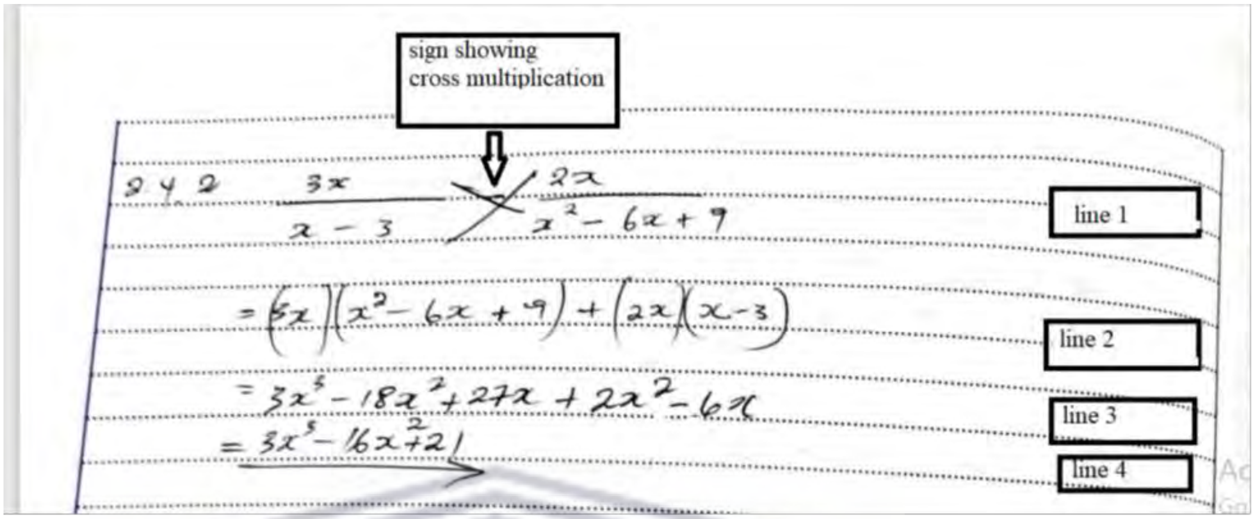

Figure 6.2 Convenience problem 1: cross multiplying the difference of two fractions

Furthermore, examinees use the procedures they have learned and hope they will achieve the desired practical outcome.

6.2.5 Abandonment

Abandonment is another way of working observed in examinees' answer scripts with solutions to algebraic fractions questions. Abandonment, as already explained in the previous chapter, refers to the commencement of a solution-seeking path and after or before completing the pursuit, the solution is abandoned; the student starts a new way of working the problem or creates a new problem (Simons, 2019). The results of this study show that the examinee's solution-seeking path is not a linear path that starts from the beginning by applying mathematical procedures and lands the mathematical practical answer. Solution seeking is a process whereby examinees read the questions and, drawing from different types of knowledge and understanding, interpret what they think is required of them, the solution to the problem, and which path and mathematical procedures must be applied to get to the solution. The solution pursuance then begins by applying the mathematical procedures the examinee thinks are applicable to solve the problem. As the solution pursuance continues, the examinee observes the performance of these procedures if they lead to the desired practical achievement. If, at some point, the examinee notices that the solution path does not lead to the desired outcome, the solution path is abandoned, and a new way of working is pursued, or the examinee adjusts in an attempt to move to the right path.

Examinees respond to agency and make visible the accounts of material agency. Sometimes, examinees begin with some future destinations in view (Pickering, 1995), and they plan how the path should be to reach the destinations. The plans and goals are transformable in encounters with non-human agency. The process takes the form of a dance of agency (Pickering, 1995), which applies dialectics or resistance and accommodation. Examinees begin the solution pursuit with a goal in mind. They apply mathematical procedures to solve an algebraic fraction, observe the performance of the non-human agency, and realise it does not conform with the mathematical dictates of the solution. The examinee experiences resistance, and such resistance is fired by accommodating it by abandoning the solution and starting a new solution path or adjusting.

In this study, examinees abandoned the solution and accommodated resistance in different ways. One of the ways was abandoning the solution pursuance and reversing back to starting from scratch. The solution-seeking pursuance with some adjustments is shown in Figure 5.14. The figure shows the second way as abandoning a part of the solution and adjusting only that one part of the solution. In Figure 5.15, an examinee abandoned the way in which the algebraic expression $a^2 - 3a - 4$ was first factorised as $a(a - 3a - 4)$ and changed factors to be to $(a + 1)(a + 4)$. The third abandonment observable in examinees' scripts was abandoning the solution and restarting from the beginning but still applying the same way of working, resulting in the same practical achievement as the abandoned work. The third form of abandonment was an examinee's way of making the lived work of simplifying the algebraic fraction more visible and clear. In the first solution that was abandoned, as shown in lines 1-3 of Figure 5.16, the examinee wrote the factors of $a^2 - 16$ in the denominator and cancelled the common factors without first writing the inverted fraction $\frac{a^2-16}{a^2-3a-4}$ as $\frac{a^2-3a-4}{a^2-16}$. In the new solution-seeking path, the examinee inverted the fraction before factorising and then cancelled the common factors. The last form of abandonment observed in this study is abandoning the solution while seeking pursuance after applying a single procedure. The examinee experienced a situation of being stuck and unable to continue with the solution pursuit.

6.2.6 Rough work

Rough work is the examinee’s way of working that is used to test a particular procedure or solution-seeking path. The rough work is done to test if the solution will lead to the desired practical achievement. Rough work as the examinees’ way of working is reflexively accountable in that the examinee or any other mathematicians will understand the examinees' work as rough work. The examinees do not write explanatory notes for examiners to recognise their rough work. Rough work is written with a lead pencil or pen either at the back of the answer script or opposite the answer that is being tested.

It is mutually recognisable by all in that local setting. The understandability and expressibility of the members' actions are sensible and essential to the visible account of rough work. Doing rough work on an examination paper is recognisable as just that.

6.2.7 Shedding

One of the examinees' ways of working identified in this study is what Mason (1999) calls shedding. As previously explained, Shedding is a way of working where an examinee experiences resistance in the solution pursuance and struggles to find a way of firing the resistance and then becomes stuck. Time and the pressure to achieve a particular percentage for a pass exert urgency on the examinees and push them into a corner, which results in them abandoning the way of working that seems not to reach the desired practical achievement or is taking time to reach a practical achievement. In Figure 5.19, the examinee started the solution pursuance for simplifying $\frac{a^2-a}{a-2}.\frac{a+1}{a^2+4a} \div \frac{a^2-3a-4}{a^2-16}$ by first changing the division sign to multiplication and inverting the fraction to be $\frac{a^2-a}{a-2}.\frac{a+1}{a^2+4a} \times \frac{a^2-6}{a^2-3a-4}$. After this step, the examinee could not continue to factorise and complete simplifying the fraction.

The equal sign in line 3 of Figure 5.19 shows that the examinee wanted to continue with the solution pursuance but was stuck and could not find a way to continue. The solution pursuance was then abandoned. This is a local and time-bound way of working. Members engaging in the process of responding to mathematics examination questions do so within the limits of time. The stakes for getting as many marks as possible to pass are high; the examinees, therefore, do not spend much time trying solutions that are difficult to find. In Figure 5.20, the examinee simplifying $\frac{3x}{x-3} - \frac{2x}{x^2-6x+9}$ first factorized the expression $x^2 - 6x + 9$ as $3x(x-3)$ cancelled this way of working and factorised as $x(3-x)$. The whole way of factorising was abandoned, and a new way of factoring $x^2 - 6x + 9$ was applied, which was $x(-3x+3)$. The examinee experienced resistance in finding the correct factors and had several attempts to fire that resistance. With the non-firing of the resistance, the examinee continued to simplify the fraction in line by subtracting the two fractions. The way of working also did not yield the desired results, and the examinee abandoned the solution-seeking pursuance after multiple attempts.

6.3 Reflexivity

Garfinkel explains reflexivity as "members' accounts, of every sort, in all their logical modes, with all of their uses, and for every method for their assembly are constituent features of the settings they make observable" (Garfinkel, 1992: 8). Reflexivity refers to the fact that practices that occur within a given situation are both a product of and constitute the situation.

From this study, the examinees' solution pursuance demonstrated reflexive accountability in that as examinees produce solutions for the questions on algebraic fractions, they read what the question requires and draw from the knowledge and skills they possess to answer the question. The solution pursuance made visible is an account of what they think the solution should be and the methods and procedures they think apply to reach it.

The knowledge and skills in simplifying algebraic expressions become a sketch map of the path that leads to the expected practical achievement. Understanding what the question requires (finding the simplest form of the sum, difference, or product of algebraic fractions) is more like knowing where the map leads. As examinees navigate their solution path, they refer to the skills and knowledge with which they come to the examination. Along the solution path, when resistance is experienced, and examinees are either stuck and cannot move on to the destination, they abandon the solution pursuance or move back to the previous position in the solution or at the beginning and restart the solution path. The solution-seeking path explains the accounts or ways of working with algebraic fractions, but the ways of working come from the knowledge and skills that the examinee has. Examinees reflect this in the cases of reversal, multiple reversals, abandonment, rough work, and in other ways. The multiple reversals and retracing discussed in the analysis chapter are observable and reportable accounts of the examinees' ways of working. It can be observed that examinees, in endless ways, simplify algebraic fractions by applying the skills and knowledge they have, and as they continue with the solution path, they keep looking back and forth to ascertain if the solution leads to the desired practical achievement.

The cancelling and scratching of abandoned solutions are done with no extra explanation because members do it as a contingent accomplishment; that is, an action carried on under the auspices of and is made to happen as events in the same ordinary affairs that they describe. Parties do the practices to those settings whose skill with knowledge of, and entitlement to the detailed work of that accomplishment-whose competence-they obstinately depend upon, recognise, use, and take for granted (Garfinkel, 1957). It, therefore, becomes a member's way of organising their social order and making accounts reflexively accountable. The examiners and examinees know the difference between the scratching that indicates abandonment and that which means the division of the same factors.

The other reflexive situation is the matter of examination time limit and how examinees respond to questions. How examinees decide on which questions to answer first or whether to spend more time seeking a solution is informed by the amount of time examinees have to answer all questions in the examination paper. The strategies that lecturers give to examinees in preparation for time management during examinations also form part of the account that examinees make visible. Examinees in this study abandoned some of the questions with a limited number of attempts and others with no attempts. Some examinees left spaces for the abandoned solutions, which may mean the examinee hoped to return to them. In other cases of abandonment and no attempt, there was no space left, which may mean that the examinee had no intention to attempt the question later. The examinees made visible their exam time management strategies through abandonment and no attempt, but abandonment and no attempt also made visible the resistance experienced in finding the right strategies that led to the final achievement. The difficulty in firing the resistance consumes time, and to accommodate the urgency of time, the examinee abandons the solution or does not attempt it.

6.4 Dialectic of Resistance and Accommodation

The examinees' ways of working with algebraic fractions exhibited a dialectic of resistance and accommodation. As discussed in the theoretical considerations chapter, the dialectic of resistance and accommodation is a phenomenon or process scientists undergo as they seek to produce machines or discipline-specific products. The example of building the bubble chamber took the form of a dialectic of resistance in the sense that each time Glasers' technique failed to produce the desired results (resistance), his response was to devise some other tentative approach towards his goal (accommodation).

The analysis of the examinee's scripts shows that a discipline-specific dance of agency can be accountable, expressible, and understandable. Examinees' ways of working exhibited that as they simplify algebraic fractions, they undergo a process of resistance and accommodation. Examinees read the question from the question paper and write it down in the examination answer script. They decide on the strategies and procedures to implement to get to the solution. As examinees apply the procedures, they have expected outcomes from applying a mathematical procedure. After applying a mathematical procedure, the examinee observes the performance. If the result does not produce the expected outcome or does not align with the mathematical dictates of the solution-seeking path, i.e., resistance is experienced, then the examinee seeks ways to accommodate the resistance. Such a process continues until the final practical achievement is reached or a solution-seeking path is abandoned due to the difficulty in finding resistance.

In this study, examinees exhibited resistance and accommodation in four ways:

The first was that examinees experience resistance and accommodation in their minds as they plan the path to reach solutions or strategies that will effectively solve an algebraic fraction. Data shows that examinees abandon the solution and do not even write the question number. The examinee seeks the first procedure to apply, and they experience resistance. The strategy to accommodate the resistance is not found and the question is left not attempted. Such accounts are not written down in the examinees' answer script, but the ethnomethodology's lens makes them visible and accountable.

The second exhibition of the dialectic of resistance and accommodation is demonstrated in solutions that were attempted only with one or few procedure(s) or step(s), and then the solution is abandoned without completing it. Examinees apply their first procedure for the solution-seeking pursuance, and they face resistance; the non-firing of the resistance leads to abandonment.

The third situation is exhibited in multiple reversals, retracing, abandonment and restarting following the same or different procedure. The analysis shows examinees' ways of working where a solution-seeking pursuance is commenced following a particular procedure, the solution is then cancelled before or after reaching the final practical achievement, and the examinee starts from the beginning or from a particular point following a new strategy or the same strategy previously applied with some adjustments. Examinees begin with the intention to produce a particular practical achievement; they devise strategies to get to the solution, apply such strategies and observe if they lead to the desired outcome. If the performance is not producing the desired practical achievement, some adjustments are made and observed again to determine if the final product is what was expected. This process happens at each step of the solution path, sometimes resulting in multiple reversals. For example, in Figure 5.14 on abandonment and reversing back to restart the solution pursuance with some adjustments, the examinee started the solution-seeking pursuance from line 1 by writing down the algebraic fractions to be simplified and continued in line 2 by factorising some algebraic expressions in the fractions, leaving others not factorised. In factorising $a^2 + 4a$ a resistance was experienced, and the examinee accommodated for the resistance, scratching the factors $a(a + 4)$ and used the factors $(a + 2)(a - 2)$. In line 3, all the examinee factorised all algebraic expressions in the fractions but realised a resistance that was not fired in line 2 when factorising $a^2 + 4a$ as $(a + 2)(a - 2)$. The examinee adjusted and factorised as $a(a + 4)$. After writing the solution in line 3, the examinee observed the performance and realised a resistance in the factors of $a^2 - 16$ were left as $a(a + 4)(a - 4)$. The solution path from line 1 to line 3 was abandoned, and the examinee restarted to simplify the fractions in line 4. As the solution path continues, the examinee keeps making adjustments where the solution-seeking pursuance does not produce the expected outcome.

A similar situation is observed in cases of retracing in Figure 5.10, reversal with abandonment in Figure 5.9, abandonment and restarting in Figure 5.16 and more.

The fourth exhibition is observed from the rough work. In the rough work in Figure 5.18, the examinee made visible a way of working where, before writing down a particular mathematical procedure, they first tested if it would work using a lead pencil. The examinee tested the factors of $a^2 - 3a - 4$ before writing them as the solution. The factors first tried were $(a - 2)(a - 1)$. After testing, the examinee realised that the factors fulfil the mathematical dictates of the solution and accommodated by using $(a - 4)(a + 1)$. The factors $(a - 4)(a + 1)$ were also tested in rough work using a lead pencil. After testing the examinee observed that the factors worked and continued using them in the solution-seeking pursuance.

In this study, therefore, examinees made it visible that the solution-seeking pursuance in simplifying algebraic fractions is the process of dance of agency. It follows resistance and accommodation. Examinees undergo a process of looking and recalling from the knowledge and skills they have learned before coming to the examination room the strategies and mathematical procedures that can lead to a correctly simplified algebraic fraction. The strategies or procedures are applied in the solution-seeking pursuance and observed if they lead to the desired practical achievement. If a resistance is experienced, examinees make adjustments to fire the resistance. They abandon the solution when they do not find a strategy to accommodate the resistance. A solution can be abandoned after a series of trials and visible textures or without showing visible textures or accounts that indicate the attempts to find the solution.

6.5 Chapter Summary

This chapter discussed the results of the study. The discussion is framed within the objectives of the study and gives an account of the textures of examinees' ways of working with algebraic fractions in high-stakes examinations using the ethnomethodological lens and dance of agency.

The discussion highlights that examinees' ways of working with algebraic fractions are reflexively accountable, intelligibly, indexically expressible and understandable. Ways of working are reflexively accountable in that they are observable and reportable, but also, members' accounts of simplifying algebraic fractions are contingent accomplishments (Garfinkel, 1967). Ways of simplifying algebraic fractions are constituent features of how they are produced and are constituted by the setting in which they are produced. There are reflexive relations between knowledge and skills examinees possess and action in producing an expected practical achievement. They are indexical in that the expressions depend on the context to be mutually understood. Different or similar ways of cancelling are mutually understood based on the context of each mathematical problem being solved. Members in the mathematics community can differentiate between scratching, which means the division of the same factors and scratching, which means abandoning the solution. Examinees, when abandoning the solution, do it in different ways, either by drawing a line diagonally or straight through the solution, or they can scribble over the part being abandoned. Any situation is mutually understood as to what the examinee is doing without further explanation required.

The chapter also discussed the processes of solution-seeking as the cycle of resistance and accommodation.

The next chapter will conclude the study, highlight recommendations for further study, present the study's limitations, and briefly reflect on the use of ethnomethodology as the study's theoretical foundation.

CHAPTER 7 CONCLUSION

7.1 Summary of Findings

The results of the study have shown how examinees engaged with the problem texts, resulting in various ways of working along solution-seeking paths. In addition, the study adds value to the type of feedback generated within the high-stakes NCV mathematics examination. It illustrates an alternative type of feedback, which can be extracted from the learners' ways of working with algebraic fractions. The study results show that examinees' ways of working with algebraic fractions are rational and exhibit ethnomethodological constructs such as reflexive accountability, indexicality, intelligibility and explicability. The textures of examinees' ways of working identified in this study are:

1. *Not attempted* means that an examinee did not show any visible textures of an attempt to find a practical solution to the question. The examinee either writes the problem statement and leaves the space or no space for the solution to the mathematical problem. In other cases, the examinee only writes the question number and leaves without any attempt to answer the question.
2. *Abandonment* is the texture observed when examinees commence a solution-seeking path, and after or before completing the pursuit, the solution is abandoned; the student starts a new way of working on the problem or restarts the same way of working with some adjustments.
3. *Reversal* is the way of working where an examinee begins a solution pursuance following a particular procedure, a resistance is experienced, and the examinee accommodates for the resistance by cancelling all or part of the work and starting again at a certain point with adjustments.
4. *Retracing* is a form of reversal, a way of working where an examinee commences a solution-seeking path, and after the realisation that the procedures applied to reach a desired outcome do not satisfy the dictates of the mathematical context, part of the produced solution is abandoned, and the examinee retraces back to a particular point.
5. *Convenience* is a way of working where an examinee applies an incorrect mathematical procedure to make it conveniently easy or possible to solve a mathematical problem.
6. *Rough work* is a way of working where an examinee tests a particular procedure on the side or at the back of the answer sheet before writing it as the answer to be marked.

7. *Shedding* is a way of working that shows a situation of being stuck.

The analysis indicates more textures of abandonment than all other textures. Examinees abandon solutions for several reasons that may be known and unknown.

7.2 Recommendations for Further Studies

The study analysed examinees' ways of working with algebraic fractions in a high-stakes examination using examinees scripts only. Due to Covid 19 pandemic, it was difficult to interview the examinees after writing the exams. It is recommended that the study be done where the researcher can be in the examination room during the examination to observe the examinees and immediately after the exam have a focus group discussion with examinees reflecting on their experience of working with algebraic fractions in the examination room.

According to the findings of this study, the method of doing mathematics in a high-stakes examination resembles a mathematician-like way of working. As suggested by Schoenfeld (1987), it should, therefore, be the aim of mathematics educators to introduce learners to such mathematics practices. Julie (1992) goes further, arguing that students should be given the opportunity to practise this mathematic-like way of doing mathematics at their own level.

Furthermore, lecturers need to see examples, such as the abandoned work presented in the analysis, as a useful object of teaching and learning. When lecturers allow students to make sense of this solution-seeking process, it will foster a greater understanding of their own ways of working in a time-restricted examination. According to Lampert (1990), the content of mathematical lessons should expose students to mathematical contexts that expose them to strategies that support or reject solutions instead of simply searching for the answer. Furthermore, this way of doing mathematics should encourage interaction between students to discuss their solution-seeking strategies. The resulting meaning-making will give students a greater understanding of their own ways of working. Getting students to analyse, explain, and interpret when they abandon an approach will also provide them with more direct feedback, improving their understanding of their work in a specific mathematical context.

Watson and Mason (2006) contend that using such mathematical objects, even in highly structured situations where students seek the same answers or practical accomplishments using the same data, may provide students with experiences. It would allow lecturers to plan their teaching from the students' perspectives.

Future research may encourage collaborative group work among students to ascertain the strategies students employ when analysing and interpreting responses taken from school-based assessments, such as class tests and examinations, as well as responses from high-stakes examinations.

7.3 Limitations of the Study

Dimitrios and Antigoni (2019) define limitation as an 'imposed' restriction, which is, therefore, essentially out of the researcher's control. They further argue that "[l]imitations of any particular study concern potential weaknesses that are usually out of the researcher's control, and are closely associated with the chosen research design, statistical model constraints, funding constraints, or other factors" (Dimitrios & Antigoni, 2019:156). This was conducted within a high-stakes examination, which has several restrictions in terms of who and what is allowed in the examination room during the examination. Another factor is that the data collected were the scripts of the examinees who wrote the examination during the pandemic. The number of examinees who qualified to write examinations was low, which can be attributed to the effects of the pandemic on teaching and learning time, as well as other social factors that affect students as they study from home. Lastly, the institutions are liable for securely storing the examination scripts; however, the institutions could not find all the examination scripts for both years that were requested for the study.

The limitations, as mentioned earlier, impact the quantity of data collected but have no impact on its quality and ability to expose examinees textures of ways of working with algebraic fractions in a high-stakes examination.

7.4 Ethnomethodology as the Theoretical Foundation of the Study

As previously discussed, ethnomethodology focuses on studying members' ways of producing social order and how members of society utilise mundane knowledge and reasoning in a specific setting (*in situ*). Ethnomethodological studies analyse everyday activities as members' methods for making those same activities visibly-rational-and-reportable-for-all-practical-purposes, i.e., "accountable," as organisations of commonplace everyday activities (Garfinkel 1967). Ethnomethodological indifference as a procedural policy of ethnomethodology studies does not focus on arguing or formulating correctness but rather "Ethnomethodological studies of formal structures are directed to the study of such phenomena, seeking to describe members' accounts of formal structures wherever and by whomever they are done while abstaining from all judgements of their adequacy, value, importance, necessity, practicality, success, or consequentiality" (Garfinkel and Sacks, 1970:345).

Through the lens of ethnomethodology, this study focused on how examinees make the textures of their ways of working with algebraic fractions visible. The study did not focus on whether the solutions examinees produced were correct or incorrect. The markings that indicated that the examinee was right or wrong were removed to ensure abstinence from being judgemental. Ethnomethodology's perspective helped in learning how examinees produce their solutions in a high-stakes examination. Such a body of knowledge can be useful for teaching and learning, preparation of examination papers and how examinees are prepared for mathematics examinations.

7.5 Conclusion

Using an ethnomethodological analysis, the study analysed students' ways of working with algebraic fractions in high-stakes examinations. The analysis revealed that examinees produce their solutions in diverse ways that are rational, reflexively accountable, indexically and influenced by dance of urgency. The examination room as a local setting and what is at stake for examinees exert urgency and play a reflexive role in how examinees respond to algebraic fractions in the examination. The kind of mathematical knowledge and skills that examinees come to the examination with becomes the map to which examinees refer to find the destination of practical achievement, and it also helps examinees to envision the destination.

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APPENDICES

Appendix A Ethics Approval UWC

UNIVERSITY *of the* WESTERN CAPE

60 YEARS *of hope, action & knowledge*

26 May 2021

Mrs NV Mbeki
SMME
Faculty of Education

HSSREC Reference Number:	HS21/2/31
Project Title:	An ethnomethodological analysis of student's ways of working with algebraic fractions in high stakes examinations. The case of Level 3 students at a TVET college.
Approval Period:	24 May 2021 – 24 May 2024

I hereby certify that the Humanities and Social Science Research Ethics Committee of the University of the Western Cape approved the methodology and ethics of the above mentioned research project.

Any amendments, extension or other modifications to the protocol must be submitted to the Ethics Committee for approval.

Please remember to submit a progress report by 30 November each year for the duration of the project.

The permission to conduct the study must be submitted to HSSREC for record keeping purposes.

The Committee must be informed of any serious adverse events and/or termination of the study.

Ms Patricia Josias
Research Ethics Committee Officer
University of the Western Cape

Director: Research Development
University of the Western Cape
Private Bag X 17
Bellville 7535
Republic of South Africa
Tel: +27 21 959 4111
Email: research-ethics@uwc.ac.za

NHREC Registration Number: HSSREC-130416-049

FROM HOPE TO ACTION THROUGH KNOWLEDGE.

Appendix B Letter of Consent February 2021 - Participant

FACULTY OF EDUCATION

Private Bag X17, Bellville, 7535
South Africa
Tel: +27 (0) 21 959 2861
Fax: +27 (0) 21 959 3358
Email: mdsimons@uwc.ac.za

Letter of Consent
Dear Participant

Project details and information

I am currently busy with me towards a PhD in mathematics education at the University of the Western Cape, Faculty of Education and would be conducting research in your college. I will collect the data in mathematics L3 NCV from the final national examination that I request you to participate in. I request to use your final examination script for my data collection .

My topic is: An ethnomethodological analysis of student's ways of working with algebraic fractions in high stakes examinations. The case of Level 3 NCV students at a TVET college.

The purpose of the study: The purpose of the study is to analyse students' ways of working with algebraic fractions in the high stakes' examinations using an ethnomethodological analysis.

Consent

Should you agree to take part in this research, you will be asked to sign this letter of consent. Two copies are required, one for our records and one for your records.

You will be aware that data collected during this research might result in research which may be published, but your name will **not** be used and any information you disclose will be kept confidential.

You may also refuse to participate in this study if you are not comfortable with.

You may withdraw from this study at any time.

Date: 09 February 2021

Learner's Name: ....

Learner's Signature: ....

University Official's Name: Dr M.D Simons

University Official's Signature...

Researcher Name: Nwabisa V. Mbeki

Researcher Signature: ....

If you have any questions concerning this research, be free to contact call Ms N.V Mbeki at (076941 address:3406582 @myuwc.ac.za or my supervisor, Dr M D Simons, email address: mdsimons@uwc.a

UNIVERSITY *of the*
WESTERN CAPE

Appendix C Authorisation for use of Northlink data

FACULTY OF EDUCATION

Request for authorization to use Northlink college data

Dear Mr Maggott

My name is Nwabisa Vivian Mbeki a PhD student at the University of the Western Cape, Department of Science and Mathematics Education.

The consent form herewith asks for your permission to use NCV L3 2019 Mathematics final examination scripts for Bellville campus. This will support me with the data collection process for the study. **All information will be anonymous and no names will be mentioned in any reports or discussion documents.**

My topic is: An ethnomethodological analysis of student's ways of working with algebraic fractions in high stakes examinations. The case of Level 3 NCV students at a TVET college.

The purpose of the study is to analyze students' ways of working with algebraic fractions in the high stakes' examinations using an ethnomethodological analysis.

Should you give permission for learners to take part in this research, I will have the opportunity to work with learners and hence collect the data needed to this study. The learner may refuse to answer any questions that he/she is not comfortable with.

The data collected might result in research which may be published, but no name of the student or college will be used.

Date: 09 February 2021

Principal Name: Mr N. Maggott

Principal Signature: ....

Researcher/Interviewer Name: Nwabisa Vivian Mbeki.....................

Researcher/ Interviewer Signature: ....

If you have any questions concerning this research, free to call Ms.N.V MAPHINI at (0769416323), or my supervisor, Dr M D Simons, email address: mdsimons@uwc.ac.za

Please liaise with Mr J.R. Robinson of our Educ & Training Unit when wanting to come on campus and receive data. 10/2/2021

UNIVERSITY *of the* WESTERN CAPE

		d)	
		e)	
3.3	Participate in focus group discussions/workshops	Expected participants	Number of participants
		a)	
		b)	
		c)	
		d)	
		e)	
3.4	Complete standardised tests (e.g. Psychometric tests)	Expected participants	Number of participants
		a)	
		b)	
		c)	
		d)	
		e)	
3.5	Undertake observations (please specify)		
3.6	Other (please specify)	Copies of L3 Mathematics final exam scripts will be made.	

2. SUPPORT NEEDED FROM THE COLLEGE

Please indicate the type of support required from the College (Please tick relevant option/s)			
Type of support		Yes	No
4.1	The College will be required to identify participants and provide their contact details to the researcher.		x
4.2	The College will be required to distribute questionnaires/instruments to participants on behalf of the researcher.		x

4.3	The College will be required to provide official documents. Please specify the documents required below.	x	
	Mathematics L3 final examination scripts for 2019 and 2020		
4.4	The College will be required to provide data (only if this data is not available from the DHET). Please specify the data fields required, below		X
4.5	Other, please specify below		
	L3 Mathematics exam scripts for 2019 and 2020. The scripts will not be removed from campus but rather copies of the questions under study will be made		

3. DOCUMENTS TO BE ATTACHED TO THE APPLICATION

The following 2 (two) documents must be attached as a prerequisite for approval to undertake research in the College	
5.1	Ethics clearance Certificate issued by a University Ethics Committee
5.2	Research proposal approved by a University

4. DECLARATION BY THE APPLICANT

I undertake to use the information that I acquire through my research, in a balanced and a responsible manner. I furthermore take note of, and agree to adhere to the following condition:

a) I will schedule my research activities in consultation with the said College/s and participants in order not to interrupt the programme of the said College/s

b) I agree that involvement by participants in my research study is voluntary, and that participants have a right to decline to participate in my research study.

c) I will obtain signed consent forms from participants prior to any engagement with them.

d) I will obtain written parental consent of student under the 18 years of age, if they are expected to participate in my research.

e) I will inform participants about the use of recording devices such as tape-recorders and cameras, and participants will be free to reject them if they wish.

f) I will honour the right of participants to privacy, anonymity, confidentiality and respect for human dignity at all times. Participants will not be identifiable in any way from the results of my research, unless written consent is obtained otherwise.

g) I will not include the names of the said College/s or research participants in my research report, without the written consent of each of the said individuals and/or College/s

h) I will send the draft research report to research participants before finalisation, in order to validate the accuracy of the information in the report.

i) I will not use the resources of the said College/s in which I am conducting research (such as stationary, photocopies, faxes, and telephones), for my research study.

j) Should I require data for this study, I will first request data directly from the Department of Higher Education and Training. I will request data from the College/s only if the DHET does not have the required data.

k) I will include a disclaimer in any report, publication or presentation arising from my research, that the findings and recommendations of the study do not represent the views of the said College/s or the Department of Higher Education and Training.

l) I will provide a summary of my research report to the Head of the College/s in which I undertook my research, for information purposes.

I declare that all statements made in this application are true and accurate. I accept the conditions associated with the granting of approval to conduct research and undertake to abide by them.

SIGNITURE	
DATE	02/07/2021

FOR OFFICIAL USE

DECISION BY HEAD OF COLLEGE

Please tick relevant decision and provide conditions/reasons where applicable		
Decisions		Please tick relevant option below
1	Application approved	
2	Application approved subject to certain conditions. Specify conditions below	
3	Application not approved. Provide reasons for non-approved below	
NAME OF COLLEGE		
NAME AND SURNAME OF HEAD OF COLLEGE		
SIGNITURE		
DATE		

Appendix D Letter of Consent April 2021 – Participant

FACULTY OF EDUCATION

Private Bag X17, Bellville, 7535
South Africa
Tel: +27 (0) 21 959 2861
Fax: +27 (0) 21 959 3358
Email: mdsimons@uwc.ac.za

Letter of Consent
Dear Participant

Project details and information

I am currently busy with my studies towards a PhD in mathematics education at the University of the Western Cape, Faculty of Education and would like to conduct research in your college. I will collect the data from mathematics L3 NCV the final national examination that I request you to participate in. I request to use your final examination script for my data collection.

My topic is: An ethnomethodological analysis of student's ways of working with algebraic fractions in high stakes examinations. The case of Level 3 NCV students at a TVET college.

The purpose of the study: The purpose of the study is to analyse students' ways of working with algebraic fractions in the high stakes' examinations using an ethnomethodological analysis.

Consent

Should you agree to take part in this research, you will be asked to sign this letter of consent. Two copies are required, one for our records and one for your records.

You will be aware that data collected during this research might result in research which may be published, but your name will **not** be used and any information you disclose will be kept confidential.

You may also refuse to participate in this study if you are not comfortable with.

You may withdraw from this study at any time. There will be no adverse consequences if you do not consent to participation in the study or withdraw.

Date: 12 April 2021

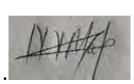
Learner's Name: ....

Learner's Signature: ……………………………………………….

University Official's Name: Dr M.D Simons

University Official's Signature……………………………………………

Researcher Name: **Nwabisa V. Mbeki**

Researcher Signature: ……………………………………………………………

If you have any questions concerning this research, be free to contact call Ms N.V Mbeki at (0769416323) email address:3406582 @myuwc.ac.za ; my supervisor, Dr M D Simons, email address: mdsimons@uwc.ac.za or the HSSREC, Research Development, Tel: 021 959 4111, email: research-ethics@uwc.ac.za

Appendix E Letter of Consent April 2021 – Parent

Private Bag X17, Bellville, 7535
South Africa
Tel: +27 (0) 21 959 2861
Fax: +27 (0) 21 959 3358
Email: mdsimons@uwc.ac.za

FACULTY OF EDUCATION

Letter of Consent
Dear Parent

Project details and information

I am currently busy with my studies towards a PhD in mathematics education at the University of the Western Cape, Faculty of Education and would like to conduct research in your child's college. I will collect the data from mathematics L3 NCV the final national examination that I request your child to participate in. I request to use the final examination script of your child for my data collection.

My topic is: An ethnomethodological analysis of student's ways of working with algebraic fractions in high stakes examinations. The case of Level 3 NCV students at a TVET college.

The purpose of the study: The purpose of the study is to analyse students' ways of working with algebraic fractions in the high stakes' examinations using an ethnomethodological analysis.

Consent

This consent asks for your permission to allow your child to participate in this project.

Should you agree to let her/him to take part in this research, you will be asked to sign this letter of consent. Two copies are required, one for our records and one for your records.

You will be aware that data collected during this research might result in research which may be published, but the name of your child will **not** be used. The child will not be compelled to participate in this research and may withdraw from the study at any time. There will be no adverse consequences if the student or parent does not consent to participation in the study or withdraws.

Date: 12 April 2021 ………………………………………………………

Parent's Name: …………………………………………………………

Parents's Signature: …………………………………………………………

University Official's Name……………………………………………………………….

University Official's Signature……………………………………………………

Researcher Name: …NWABISA VIVIAN MAPHINI………………………

Researcher Signature: …… ……………………………………………………

If you have any questions concerning this research, free to contact call Ms N.V Mbeki (0769416323) at email address: 3406582 @myuwc.ac.za ; my supervisor, Dr M. Simons at email address: mdsimons@uwc.ac.za or the HSSREC, Research Development, Tel: 021 959 4111, email: research-ethics@uwc.ac.za

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