The introduction to integers in a grade 7 classroom through an intentional teaching strategy

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A thesis submitted in fulfilment of the requirements for the degree, Master in Education in the Faculty of Education, University of the Western Cape, South Africa.

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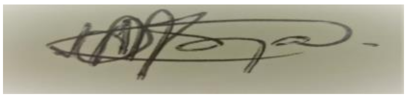
Declaration

I declare that ***the introduction to integers in a grade 7 classroom through an intentional teaching strategy*** is my own work, that it has not been submitted before for any degree or examination in any other university, and that all the sources I have used or quoted have been indicated and acknowledged by complete references.

Mncedisi. H. Soga Date: November 2017

Signed: ________________________



Acknowledgements

I wish to express my sincere thanks to my supervisors, Prof. Cyril Julie and Prof. Monde Mbekwa, for their encouragement, patience and professional guidance.

Special thanks go to my wife Ellen Liyema Soga for her steadfast support, understanding and encouragement. This study is especially dedicated to my late parents, Zithulele Spencer Soga and Mildred Somikazi Soga. The last word of appreciation goes to my school Principal, Mahlubi Ngqukuvana for his encouragement and unwavering support.

Abstract

This research investigated how grade 7 learners dealt with introductory aspects of integers when they are introduced through a temperature model. In particular, the study analysed the effect of an intentional teaching strategy on learners' engagement with integers. The idea of combining an intentional teaching strategy with the introduction of integers in grade 7 learners using a temperature model is what makes this study unique. A qualitative study was adopted. Data was collected by means of audio and video and also by means of learners' completed worksheets.

The results of the study indicate that the majority of learners could recognise, compare and order integers. It is recommended that the application of intentional teaching with a temperature model is a viable strategy to introduce grade 7 learners to integers.

Key words

Integers

Categories

Design-based research

Effective teaching

Qualitative research

Intentional Teaching model

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LIST OF ABBREVIATIONS AND ACCRONYMS

AfL	Assessment for Learning
ANA	Annual National Assessment
CAPS	Curriculum Assessment Policy Statements
CRA	Concrete Representational Abstract
DBE	Department of Basic Education
DoE	Department of Education
IQ	Intelligence Quotient
IQMS	Integrated Quality Management Systems
LoLT	Language of Learning and Teaching
LTSM	Learning Teaching and Support Material
SACMEQ	Southern and Eastern Africa Consortium for Monitoring Educational Quality
SC	Success criteria
STAR	search, translate, answer and review
TIMSS	Trends in International Mathematics and Science Studies
WALT	We Are Learning to
WCED	Western Cape Education Department
WILF	What I'm Looking for

CHAPTER 1

INTRODUCTION, MOTIVATION AND RESEARCH QUESTION

1.1 Background

Learners in grade 7 encounter negative numbers for the first time as they are used to working with positive whole numbers in their schooling career prior to their entry in the grade. Consequently, they find it difficult to understand integers as numbers and moreover operations with integers. The difficulty may arise from the incorrect methods of teaching integers, learners' errors when working with integers or the language of learning and teaching (LoLT) used. One such teaching method is the emphasis on memorisation as a learning technique. The researcher's assumption is that teachers do not normally allow learners to recognise integers, compare integers and order integers in a meaningful way, thus leading to difficulties learners experience with integers. One of the ways integers are being taught is, for example, to teach addition and subtraction separately. Teachers often take the shortest route in solving the problem of equipping learners with the necessary skills of dealing with integers by substituting their teaching of integer operations with teaching learners to memorise the rules involved.

Learners see their teachers as the main source of information while they are regarded as individuals without knowledge. They consequently decide to do as their teachers do by applying the shortest route of memorising the rules involved in integer operations without practicing the application of the rules. Niss (1993, p.9) says "to understand and master mathematics there is a need to provide genuine assistance to each individual learner in monitoring and improving his or her acquisition of mathematical insight and power." The genuine assistance that Niss (1993) talks about involves equipping learners with strategies to practice mathematics in their own time. The provision of genuine assistance that Niss (1993) talks about should come from mathematics teachers by way of differentiating learning. Differentiations of learning involve catering for each individual in class and not take for granted that every learner's cognitive level is the same.

Using one method of teaching such as chalk-and-talk, the dominant method most teachers use, is considered a contributing factor to learners' poor performance in mathematics. When the chalk-and-talk teaching strategy is constantly used, insufficient planning occurs. Also, unclear goals of what learners should understand at the end of the mathematics lesson are always ignored. Ultimately the practice leads to learners receiving no conceptual or procedural understanding of the mathematical concepts.

Planning thoroughly to teach integers purposefully to attain the intended goals may yield improved results and better learner performance.

1.2 Motivation

This study is motivated by the steady decline of learners' mathematics performance in most grades in South Africa, including the senior phase of schooling. South Africa has shown the biggest positive change, with an improvement of 87 points in mathematics, although it started with very low performance scores in TIMSS 2003, (Reddy, Visser, Winnaar, Arends, Juan, Prinsloo & Isdale, 2016, p. 2). South Africa is one of the lower performers of the 39 participating countries and has achieved a mathematical score of 372 and a standard deviation (SE) of 4.5, Reddy et al. (2016, p.5). The Department of Basic Education (DBE, 2013, p. 53) concurs with TIMSS 2003 by defining that "only 2-3% of grade 9 students in each year reached 'acceptable achievement'. According to Reddy et al. (2016, p.5), "between TIMSS 2003 and 2011, the mathematics scores improved by 67 and between 2011 and 2015 the mathematics scores improved by a further 20 points. In 2015, 34% of mathematics learners achieved a score of over 400 points however it is an interesting exercise to examine the change in the percentage of South African learners who performed above 400 point TIMSS benchmark for mathematics between 2003 and 2015, Reddy et al. (2016, p.6). Furthermore, according to Reddy et al. (2016, p. 6) "in 2003, a mere 10, 5% of mathematics learners achieved a score of above 400 points and that increased to 24, 5% in 2011 and to 34, 3% in 2015."

The 2012 annual national assessment (ANA) report indicate that there is a steady decline in learners' mathematics performance in the senior phase and a rise in the foundation phase. Learners start performing badly in mathematics from the grade of entry in the senior phase which is grade 7. The 2012 ANA guidelines indicate that mathematics learners perform worse in grade 9 than learners in grade 3. Table 1.1 of the report of ANA results shows a decrease of learners' mathematics performance in the senior phase and an increase in the foundation phase (Department of Basic Education, 2012, p.7). The table only shows mathematics in the main source. The table 1.1 shows that a high percentage of learners achieved scores at level 6 and 7 in grade 3 and 6 than in grade 9. The table show learners' performance in grade 9 and not in grade 7 as the ANA assessments are only measured in the last grade of the phase band.

Table 1.1: Performance spread across levels of achievement for grades 3, 6 and 9 (Department of Basic Education, 2012, p.7).

	L1 0-29%	L2 30-39%	L3 40-49%	L4 50-59%	L5 60-69%	L6 70-79%	L7 80-100%
Grade 3 Mathematics	31.3	16.0	15.9	15.1	10.5	6.6	4.5
Grade 6 Mathematics	65.4	13.6	10.2	5.4	3.2	1.4	0.8
Grade 9 Mathematics	91.9	3.8	2.1	1.1	0.6	0.3	0.2

When one studies the above table one would be able to reflect on how to teach mathematics better to grade 7 learners so that they enhance their mathematics performance. The decline in grade 9 mathematics performances might emanate from the teaching strategies applied in the phase and lower grades. Another rationale for this study is to provide a turn-around strategy in teaching mathematics so that learners understand and master the subject. One assumes that it is every teacher's wish to improve learners' performance in his or her subject by the end of the year. In this research project learners were encouraged to understand integers. Learners were also provided with an improved introduction of integers through using temperature model as a basic means of understanding integers.

Another indirectly motivating factor of this study comes from South African learners' poor performance in large scale national and international studies such as TIMSS and the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ), (Department of Education (DoE), 2005; Mothibeli, 2005).

1.3 Research aim

The aim of this study was to investigate how grade 7 learners deal with the introductory constructs of integers when they are introduced through a temperature model.

1.4 Research question

How do grade 7 learners deal with the introductory constructs of integers when they are introduced to it through a temperature model?

1.5 Key terms of this study and their definitions

1.5.1 Integers

The grade 7 curriculum and assessment policy statement (CAPS) document (Department of Basic Education, 2011, p.16) requires learners to be able to compare, order and recognise integers as skills that should be attained before adding and subtracting integers. Recognising and using commutative and associative properties of addition and multiplication for integers is regarded as a primary skill.

The solving of problems in contexts involving addition and subtraction with integers may be regarded as a secondary skill. Integers are all positive and negative whole numbers and zero.

1.5.2 Temperature model

The temperature model is one of the many models that may be categorised as a number line model. It is chosen in this study as an ideal model for grade 7 learners since they are beginners in dealing with integers. The temperature model as a number line model and not a neutralisation model will be further explained in the next chapter.

1.5.3 Intentional teaching

There are many teaching strategies such as chalk-and-talk mentioned earlier, the demonstration method, the experimental method, the observation method and so on. The intentional teaching strategy is chosen as it is deemed effective and produces good results. This teaching strategy requires teachers to set clear goals in advance so that all learners are able to know where the lesson is headed. More details about an intentional teaching strategy are discussed in the next chapter.

1.6 Organisation of the Study

This study is organised into five chapters. Chapter 1 deals with the introduction background and motivation of the study. The literature review which includes explanation of integers, literature and intentional teaching and literature on errors in learners' discussions or arguments are discussed in Chapter 2. The research methodology, sampling and data analysis is dealt with in Chapter 3. Chapter 4 deals with the presentation and analysis of results against the research question. The recommendations emerging from this study and the conclusions of the study are dealt with in Chapter 5.

CHAPTER 2
LITERATURE REVIEW

2.1 Introduction

This chapter reviews work on the teaching and learning of integers from other researchers. It discusses research on difficulties which learners experience with integers, research findings related to the use of models for teaching integers, and gives an explanation of intentional teaching.

2.2 Research on difficulties which learners experience with integers

Stephan & Akyüz's (2012, p.429) studied the history and development of integers by earlier mathematicians and declare that "students have similar difficulties, such as conceptualising numbers less than zero; creating negative numbers as mathematical objects and formalising rules for integer arithmetic, particularly the meaning for the opposite of a negative number being a positive number".

Many researchers have interviewed, surveyed and conducted teaching experiments with learners (Steffe & Thompson, 2000) as a way to document students' conceptions of integers (Bofferding, 2010) and (Gallardo, 2002; Vlassis, 2004, 2008). Learners find it easy to subtract 5 from 7 as this can be modelled by for instance having seven oranges and taking away five oranges. The answer is 2 oranges left. The difficulty arises when they have to subtract 7 from 5 as 5 – 7. Learners struggle with the problem of 5 – 7 and alter the problem to the usual one of 7 – 5 when trying to solve 5 – 7.

Ryan & Williams (2007, p.24) state the following about integers and the two operations of addition and subtraction of integers:

The integer is very commonly conceived of as a combination of the sign and the number. They further state that the minus is understood to mean smaller numbers than the plus, so -1, -2 and -3 come before +1, +2, +3, possibly in that order, giving …0, -1, -2, -3, +1, +2, +3, … or maybe -1, -2, -3, 0, +1, +2, +3…Then computations children make with these integers typically involve first combining the number and second manipulating the signs: thus -2 +8 = -10, and +3 - - 8 = + or -5.

These procedures are typically articulated as in I added the 2 and the 8 to get 10, then the signs are minus and plus, so it's minus …-10. There may be very little by way of justification of these procedures, it is just so.

According to Reid (undated) cited in Balbuena & Buayan (2015, p.15) four of the more confusing operations in integer mathematics are: (1) adding two negative numbers; (2) adding a positive integer and a negative integer; (3) subtracting a negative integer from a negative integer and (4) subtracting a negative integer from a positive integer.

Reid further states that these difficulties were also observed by the researchers in their own classrooms. Vlassis (2008) argues that the negative sign can take on at least three meanings in mathematics: unary, binary and symmetric functions. The unary operation identifies the quantity as a negative, in other words, the sign is "attached to the number" (Vlassis, 2008, p.561), as in negative 10 or -10. A second function of a negative sign and the most common way interpreted by elementary students (Bofferding, 2010), is binary. A negative sign functions as a binary operator when students interpret it as an action such as taking away, completing (as in how much more is needed to have 25, if you have 10) and the difference between numbers (Gallardo & Rojano, 1994). The third way to interpret the negative sign is as a symmetric function, where the symbol signifies taking the opposite of a number. For the expression – (- 10), the first negative number sign would signify the operation of taking the opposite of – 10. The above uses of subtraction as described by Gallardo & Rojano (1994) mean taking away, completing and the difference between two numbers. The three uses of subtraction will be described in terms of two numbers, 5 and 7.

In this case we also need to consider that subtraction is also from the position of its first perspective, so because 5 is smaller than 7, the answer is negative, i.e. 5 – 7 = -2. Completing considers 'what is missing from' (Vlassis, 2004). In other words, what is missing from 7 to give 5 or 7 +? = 2.

Badarudin & Khalid (2008, p.86) suggest the following:

> Many articles and research papers were found to describe studies in which teachers and students used different strategies in the teaching and learning of integers.

> Developing effective teaching strategies of integers has been on-going in many parts of the world. In order to make students understand integers we have to extend their knowledge, help them make logical connections with what they know and use appropriate strategies in learning.

Badarudin & Khalid's (2008, p.86) above suggestion emanates from the fact that teaching integers requires modelling of examples by the teachers and teaching from the known to the unknown. Integers are part of real numbers. Integers become more evident in cases where situations become less than zero in amounts. The above argument by Badarudin & Khalid (2008, p.86) is very relevant to this study as it emphasises on making use of an appropriate teaching strategy.

Bell (1983) studied students who were fifteen years of age. The aim of the study was to determine students' understanding of integers and then "design teaching which would provoke cognitive conflict with the misconceptions shown in the interviews" (Bell, 1983, p.67).

Twenty of these students were successful in all of the addition problems that dealt with integers. These students depended on some physical model such as the number line to get answers. Only ten of the students were successful with integer problems that dealt with subtraction and these students depended on the rules rather than on understanding. Students had a tendency to disregard signs of integers but joined the scales by referring to the operation sign.

According to a study done by Hativa & Cohen (1995), some types of problems showed a greater proportion of intuitive knowledge on the part of the fourth grade participants. For example, prior to instruction, more than fifty per cent of the students were successful in subtracting a positive number from zero. These authors also stated that using negative numbers in operations was particularly difficult for lower-achieving students. The problems that involved the most pre-instructional knowledge involved subtracting a positive number from a smaller positive number (Steiner, 2009, p. 37). More than fifty per cent of the non-treatment group and more than three-fourths of the experimental students did these problems correctly when asked in interviews prior to the experiment.

McCorkle (2001) conducted a study of seventh grade students who were taught addition and subtraction of integers using an experimental group that used an approach in which students operated a thermometer scale using hot cubes and cold cubes. The control group was taught in the traditional manner using procedures for addition and subtraction of integers. Two weeks of instruction was provided for each group. Students were given a post-test at the end of the two weeks and the same test was given three weeks later to determine the quantity that students recalled. The results suggest that students who learned using their conceptual understanding had higher scores on the post-test and were able to recall the material better than those who were taught in the old-fashioned manner.

Hackbarth (2000) conducted an eight-day study that involved sixty-eight seventh graders. The study engaged two experimental groups and one control group. One experimental group used plus and minus pieces while the other experimental group used two-coloured chips. The students in the control group were taught rules without using calculation. A post-test was given two months after the beginning of the study and no arithmetic differences were found between the groups.

Ferguson (1993) suggests that subtraction of integers is a challenge for students and there are five implications it has for instruction. Her first suggestion is that the instructions address the informal knowledge of integers that students already possess. Students should understand that negative integers do not always represent bad things. For example, in golf below par is good but "- "in the inspection of an account it is not good.

Secondly, Ferguson (1993) suggests that operations with integers should take advantage of student understanding of operation with whole numbers. The meaning of the operations in the set of whole numbers should be developed and related to integers.

Thirdly, Ferguson (1993) suggests that students should generalise the workings of integer operations rather than be given procedures for operating with integers.

Fourthly, Ferguson (1993) suggests that students need time to make generalisations about operations and these generalisations can be brought to fruition through practice.

Finally, Ferguson (1993) suggests that students who have misconceptions about integers need to have these misconceptions addressed in a manner that allows them to rebuild and strengthen concepts using prior knowledge and experience.

There are many errors that learners commit when adding or subtracting integers. According to Davis, McKnight, Parker & Elrick, (1979) "One type is referred to as symmetric subtraction, in which the students subtract the number with the lesser magnitude from one with the greater magnitude." Sometimes when children are taught how to subtract, the teacher may incorrectly tell them that one should always subtract the smaller number from a bigger number or that this cannot be done. In the study done by Davis et al., it was found out that symmetric subtraction explained about 50% of all the answers that were given by the 28 fifth graders involved in the study.

A third type of error that students commit is the sequential interpretation of symbol strings and this is related to the way students read. They go from the left to the right carrying out the arithmetic as they go. A child making this error would get -7 for the example -5 +2 because they would add the 5 + 2 and append a negative sign since this was given to them at the left-hand side of the problem (Steiner, 2009, p. 49).

According to the Department of Basic Education, (2011, p. 67) the focus of the CAPS document is on learners recognising, comparing and ordering integers. This study investigates whether the introduction of integers using a temperature model with the focus on recognising, comparing and ordering integers may bring learners' difficulties.

2.3 Models and metaphors used for teaching integers

Models can be categorised into two main groups, namely, neutralisation and number line models" (Stephan & Akyüz, 2012, p.431). Lytle (1994) cited in Akyüz, Dixon & Stephan (2012, p. 270) states that "the neutralisation models use physical objects such as coloured chips or tiles to represent positive and negative numbers and show the operations by manipulating them, number line models represent the operation by the direction of movement along the line and the number located based on its position and the distance." In this section, the researcher looks at each of the two models, by providing explanations of each model and how these models contribute to the teaching of integers in mathematics teaching.

A neutralisation model is one in which individual units are cancelled when they are opposites. For example, positive and negative charges are used in a model to indicate some operations with integers by using the fact that a positive charge cancels a negative charge. Some research was done on the use of neutralisation models on learners' dealing with integer arithmetic. The proponents of neutralisation model searched for a way to support students organising their thinking so that it can be modelled or inscribed in the form of physical tools and symbols. The neutralisation model described below endeavoured to build the unary, binary and symmetry functions of the negative sign.

An example of a neutralisation model is Battista's (1983) use of electromagnetic charges as context. Battista's model would have the student place 5 white chips (+ values) on a mat, to solve the problem of 5 - 8. Since there are not 8 positively charged chips available for subtraction, students would have to place down "zero-pairs" or 3 pairs of white and red chips (- values), which sum to zero. Now there would be 8 white chips and 3 red chips on the mat. The student would remove the 8 positive chips and have 3 negatives remaining for a solution of -3, to complete the operation. Battista (1983, p.31), produced the following results about the electromagnetic model of neutralisation.

The results were that "the model can also be used to illustrate important structural properties of the system of integers such as the commutative and associative properties, the existence of addition and multiplication identities and the existence of additive inverses.

The major advantages of using this model are twofold. First, the model is concrete. Many students who receive initial instruction on the integers need concrete representations of the concept involved. The charge model's closest "competitor," the number line is usually presented in a pictorial manner. Second, the model is complete. It can effectively represent all four basic operations on the set of integers. Students being introduced to the representations for multiplication and division can build on their knowledge of the representations for addition and subtraction, thus making the learning of the new operations more meaningful. So the completeness of the charge model serves to give students a more meaningful and coherent picture of the workings of the four basic operations on the set of integers.

Linchevski & Williams (1999) used an abacus with red (positive) and yellow (negative) dice in a game setting to develop addition and subtraction of integers. They found out that using a game situation created a better platform for most learners to understand addition and subtraction of integer and supported learners understanding of addition and subtraction operations when dealing with integers.

The model involves 4 games in each of which two teams of two children are throwing dice (e.g. a yellow and a red die in game 1) and recording team points on abacuses. The points for the yellow team are recorded as yellow cubes on the yellow abacuses and those for the red team are recorded as red cubes on the red abacuses. The students sit in two pairs, each having a member of each team and an abacus. On each pair's abacus, points for both teams are recorded and the team points on the two abacuses are added up. The students in turn throw the pair of dice, recording each time the points for the two teams on their abacus. "When the two abacuses combine to give one team a score of 5 points ahead of their opponents, that team wins the game. For instance, in game 1, if the yellow team at a certain point is 2 ahead and they get a score on the pair of dice, say 4 yellows and one red, then they can add 3 yellows to their existing score of 2 and so get 5 ahead so that they can win. In game 2 an extra die marked '*add*' and '*subtract*' is thrown", (Koukkoufis & Williams, 2006, p. 161). This die is called add/sub die. The introduction of this die allows for subtraction to come into play, instead of just addition, as in game 1. In game 3, formal mathematical symbols for integers are introduced.

The add/sub die is not used and the *yellow* and *red* die are replaced with an *integer* die giving one of the following results on each throw; -1, -2, -3, +1, +2, +3.

Positive integers are points for the yellow team and negative integers are points taken from the yellows, thus they are points for the reds. Here the mathematical voice is encouraged so that the children say “minus 3” and “plus 2” etc.

Maccini & Ruhl (2000) pilot tested an instructional strategy that combined Concrete-Representational Abstract sequence (CRA) with search, translate, answer and review (STAR) strategy for solving subtraction of integers. They used different coloured squares and algebra tiles to represent positive and negative numbers. For the word CRA, the C was represented by the algebra tiles, the R was represented by the drawings, and the A represented by numbers only. For the word STAR, the S stands for “search the word problem”, the T stands for “translate the words into a mathematical equation”, the A stands for “answer the problem”, and the R stands for “review the solution”. STAR incorporated several phases: (a) pre-test, (b) concrete applications, (c) representational applications and (d) abstract application. Instructional phases include teaching the first letter of the mnemonic STAR to cue students to perform steps. The participants in this study were three students with learning disabilities.

To be eligible to participate in this study, students had to be: (a) in a secondary school, (b) diagnosed as having a learning disability, and (c) lacking knowledge in the targeted subtraction tasks. All three male participants meet the criteria for the study as they had IQ scores ranging from 70-104. The results were aided by the use of manipulative such as the coloured squares and algebra tiles.

Results indicated that this model has led to improved performance on problem solving with integers. They provided initial evidence that students with learning disabilities can learn to represent word problems involving integers with the use of concrete manipulative and pictorial displays.

Mutodi (2015) investigated the effect of the interchangeable use of the number, debts and assets and the chips model on grade 8 learners’ facility to perform the addition, subtraction, multiplication and division of integers.

He used a pre-and-post-test design. Forty learners were involved in the study.

The results of the study are summarised as the use of the 3 models yielded “a statistically significant increase in student performance” (Mutodi, p.434).

Akyüz, Dixon & Stephan (2012) conducted a study in the spring of 2009 in Central Florida using seventh grade classroom in a public middle school. There were twenty students in the classroom including thirteen boys and seven girls.

Data sources used in this study included audio-and video-tapes of the classroom sessions, field notes, teacher notes and a collection of students' artifacts. The data were used to describe the role of the teacher in supporting imagery during interactions with the students. A model of debts (negative), assets (positive) and net worth (an abstract quantity that one has when debts are taken out from assets) as a concept in story problems was used to support calculations with integers seemed to be particularly natural for modelling negative numbers as owed quantities. The context of the story involved determining a person's financial net worth where positive numbers are determined as owned properties or quantities and negative numbers are owed properties or quantities. However, a study by Mukhopadhyay, Resnick & Schauble (1990), emphasised that using story problems did not enhance students' mathematical performance especially when the story cues were misleading. During the classroom discussion students guessed different things that they may own such as houses, boats, loans as well as the things that they may owe money on such as car loans, credit cards and mortgages. Students were asked questions after they created their own net worth statements and find out their total net worth. That was followed by similar activities where students computed the net worth of other people. The activities were designed to support students' activity in this context as conceptualising asset as something owned, debt as something owed and net worth as an abstract quantity that you have when debts are taken out from assets. The first question the teacher asked was to find the net worth when the assets and debts are $940 000 and $850 000 respectively. The results were that student did not have difficulty finding correct answers when answering questions.

Schwarz, Kohn & Resnick, (1994) involved a model where quantities were regarded as money owed and money owned, through an activity of a game played in the classroom. In this study the quantities were considered fictitious, but money owed to another could also be thought of as concrete objects that would change hands at some point in the future. The rules that involved addition and subtraction as actions clearly indicated adding on and taking away. Owing or having an object was not a property of that object but rather the result of an action (giving or receiving). Therefore, the two states *owing* and *having* were not intrinsic properties of the objects and could not well exemplify two different states of a quantity.

According to Schwarz, Kohn & Resnick, (1994, p. 44), “the epistemological obstacles connected with using negatives to deal with debts (although not considering them true numbers) indicates that the link of debts to mathematical objects is not obvious, although even young children appear able to use negative numbers to describe debts (see Peled, Mukhopadhyay & Resnick, 1988).” According to Schwarz et al., (1994, p. 44) “children may not be able to think of the combination of two opposite quantities if they lack the understanding of two distinct types of quantities. Children may understand negative positions and a metric within a scale ranging from *having a lot* to *owing a lot* and moving within such an ordering may also be simple.” The researcher’s assumption is that children are familiar with the borrowing or lending situations that lead to owing a lot and having a lot. The result of this study was that learners found the answer by computing the difference between the assets and debts.

In this section the number line model is discussed. Some instances of the number line model are temperatures above or below 0˚C, elevators going above or below ground level and depths above or below sea level. The first of the three instances above can be displayed in horizontal or vertical number line systems while the last two instances can be displayed in vertical number line system. In the elevator model, the ground level is regarded as zero and all other floors below the ground level are regarded as negative integers while the floors above zero are regarded as positive integers. The elevator model allows learners to realise where the elevator is stationed and where it is destined so that teaching of integer addition or integer subtraction takes place. Even if the model about elevators is a good one as it is relevant to some urban grade 7 students, it was not used in this study as some learners have less knowledge about the context of elevators, especially some of the rural grade 7 learners.

According to Rousset (2010, p. 20) “number lines are often used to teach directed numbers, and are frequently included in textbooks. They are sometimes drawn free from any context, but there are also several different ‘real-life’ situations which are modelled by a number line, and with which students may already be familiar”. The number line is “arranged around an essential arbitrary zero, with the negative part mirroring the positive part” (Rousset, 2010, p. 20).

Resnick (1983) and Peled, Mukhopadhyay & Resnick (1989) concluded that the students relied on a number line model to support their thinking about negative integers.

They found that students either used a divided number line model, where students calculate to and from the zero point and interpret the positive and negative halves as separate, or a continuous number line model, allowing students to move easily between positive and negative numbers. The difference between a divided and a continuous number line is that a divided number line contains either only the positive or the negative numbers while a continuous number line contains both the negative and the positive numbers. Freudenthal (1973) and Fischbein (1977) agreed that number line models could be helpful for supporting integer addition and the National Council of Teachers of Mathematics (2000) recommends that students use a number line model to explore numbers less than zero. The divided and the continuous number lines are useful as they assist learners to be able to compare integers. The above claim is supported by Hativa & Cohen (1995) who found that fourth graders could successfully compare integers.

In cases where addition and subtraction are taught, the number line model can be a very good modelling practice. Some researchers such as Freudenthal (1983) treat the numbers as arrows and vectors to model out subtraction and addition. Below are some of several studies that involved number line models to build an understanding of integers and the operations performed on them.

Thompson & Dreyfus (1988) used a computer micro-world that stimulated a turtle walking along a horizontal number line. Students were asked to input commands that would make the turtle travel and then asked to predict their position on the number line. The prediction of the question that was asked from students in the computer micro-world required the position of the turtle which is what Stephan & Akyüz (2012, p.432) signified earlier. Stephan & Akyüz's (2012, p.431) signification earlier was that "research shows that all these contexts have strengths and when they are used, students demonstrate a significantly better understanding of negative numbers".

A similar research study to Resnick (1983) and Peled, Mukhopadhyay & Resnick (1989), mentioned earlier is by the following researcher. Peled (1991) studied two different types of number line conceptions.

One type involved a continuous number line model where numbers were ordered from lesser to greater while the other type of number line involved a divided number line that was disjointed at zero. A continuous number line is useful to me as it helps me teach the recognition of integers and comparing of integers. In my teaching a divided number line may be used as a basis to the introduction of integer number line. The two conceptions mean that a divided number line contains either only the positives or negatives while a continuous contains both the positives and the negatives. Students' actions were either toward or away from zero.

The students would decide how much was needed to get to zero and continue from there.The National Council of Teachers of Mathematics (2014, p. 198) describes Peled's (1991) results as follows:

For both mental models, Peled (1991) described four levels of understanding: Students at the first level of integer knowledge know the order of all integers, with larger numbers further to the right on the number line and the numerals ordered symmetrically around zero; at the second level, students can add positive numbers to any integer; at the third level, students can add or subtract two positive or two negative numbers; and at the fourth level, students can add or subtract any two integers.

Another research that involved number line models to build an understanding of integers and operations with integers is found in the following study. Yakes (2017) investigated the basics of the number line model and the coloured chip model for representing integers and their operations with the grade sevens. The comparison of the two models was performed through an assessment. The results were that "the number line model is an important one that should be taught alongside or in lieu of the chip model in order to deepen students' understanding operations with integers, rational numbers, and real numbers in general" (Yakes, 2017, p. 309).

Other research pertaining integer arithmetic was also conducted. Wilkins (1996) for example, examined the development of 16 sixth-grade (10-11 year olds) students' informal understanding of adding and subtracting positive and negative numbers before any formal integer instruction was provided. Students' knowledge of integers and ability to make use of a variety of relevant strategies and representations were measured using a clinical interview. Students were asked 43 questions dealing with positive and negative integers including symbolic and contextualised problems. Students were found to be able to answer more context questions correctly than symbolic questions. Students could use representations that matched problem structure from the context problems but did not show as skilful a use with symbolic problems. Students were also able to use a greater variety of strategies with the contextualised problems than with the symbolic ones.

The results were that “when students used conceptual approaches they were successful 64, 5% of the time and when they used the number line, students were successful 69, 6% of the time and when students used two-coloured counters they were successful 75, 3% of the time” (Steiner, 2009, p. 38).

In this study the researcher opted for a model that dealt with the recognition, ordering and comparison of integers. Addition and subtraction of integers form part of the questionnaires however they were not the researcher’s objectives. One model was therefore used in this study. The model dealing with recognition, ordering and comparison of integers was the one used in this study. The one that the researcher believed was familiar with the learners is a temperature model. The model about temperatures on a weather chart dropping or going up in certain areas was applied.

In this study the researcher investigated whether learners were able to order, compare or recognise integers as required by the CAPS document in the Department of Basic Education, (2011, p. 67).

2.4 Intentional teaching

This section discusses the meaning of an intentional teaching strategy. Intentional teaching is a form of teaching where the teacher teaches with a goal in mind in order to achieve the intended goal by the end of the lesson. Intentional teaching encompasses teacher-learner interaction but learner-learner interaction is more emphasised. It can also be regarded as one of the most effective teaching strategies. There are various definitions of intentional teaching but Epstein’s definition of intentional teaching provides a simple and sensible explanation. Epstein (2007, p.4) has the following to say about intentional teaching:

An intentional act originates from careful thought and is accompanied by consideration of their potential effects. Thus an “intentional” teacher aims at clearly defined learning objectives for children, employs instructional strategies likely to help children achieve the objectives, and continually assesses progress and adjusts the strategies based on that assessment.

Epstein (2007, p.4) declares that being intentional is "to act purposefully with a goal in mind and a plan for accomplishing it." The implication of this is that for every lesson there should be sufficient planning and understanding of what the teacher wants to achieve by the end of the lesson. The declaration also implies that during the lesson presentation the intended goals should always be clear and articulated in advance by both the teacher and the learners. The recommendation is that the teacher and learners should frequently check whether they are still within the framework of the intended goals during the progression of the lesson. To an intentional teacher the quote provides a contingency plan that if these goals are not achieved by the end of the lesson then that would provide feedback on the subsequent steps that should be taken.

Epstein further claims that "intentional acts originate from carefully thought actions and are accompanied by consideration of their potential effects." Effective teachers are intentional with respect to many facets of the learning environment; beginning with the emotional climate they create (Epstein, 2007, p.4). The above quote implies that an intentional teacher can assist learning by providing action words to guide learners' activities.

Epstein (2007, p.4) states that "the teacher who can explain why he or she is doing what he or she is doing then he or she is acting intentionally". She further states that this is "whether the teacher uses the strategy tentatively for the first time or automatically from long practice, as part of an elaborate set up or spontaneously in a teachable moment" (Epstein, 2007, p.4).

If teachers employ intentional teaching as their instructional teaching strategy, then it is likely that they will prove to be effective teachers. Various other researchers (Julie, 2013; Pianta, 2003, 2011) have contributed to the meaning of intentional teaching, its characteristics and its components.

According Julie (2013, p. 93) "intentional teaching comprises assessment for learning (AfL), which also needs to address the learning intentions."

Assessment for learning can be done during the lesson to check whether learners still understand what is taught and can also be done after completion of weeks', months' term's or years' work.

Assessments for learning can be formal or informal but whichever way assessments assist enhancing learning. The researcher agrees with the concepts of assessment for learning and learning intentions because they both serve as a yard-stick to ensure whether teachers, as transmitters of knowledge, have really conveyed the information through to learners.

The learning intentions for a unit of work must be clearly specified so that everyone involved in teaching and learning should know and are able to articulate them, Julie (2013, p.92). The question which must stay in the minds of those involved in the teaching and learning should be, "What are we learning to do?" The researcher is convinced that the learning intentions are congruent to the goals articulated by the above-mentioned researchers. The proper acronym to use for this question is WALT which means "We Are Learning to". The teacher should inform learners in advance what is to be learnt.

It will be through this introductory information of what we are learning that every stakeholder in classroom teaching will see where they are headed. The idea of imparting what is to be learnt keeps teachers and learners aware of the intended goals to be met and fairness is maintained between the two participants, namely teachers and learners. The same idea also shows that the teacher is well prepared for the lesson and understands what he or she wants to achieve by the end of the lesson.

According to Julie (2013) another component of AfL is the clear specification of the success criteria (SCs) to check whether the learning intentions have been achieved. Success criteria are the things that must be done to reach the goals. Success criteria can be those elements for which teachers award marks for tasks. For example, the success criteria for adding two fractions with different denominators are: (a) finding a common denominator, (b) writing the two fractions as equivalent fractions with common denominator (c) adding the numerators and (d) writing the answer as a fraction with the sum obtained in (c) as the numerator and the common denominator as the denominator.

The acronym to use in the success criteria is WILF that stands for What I'm Looking for. The WILF will provide mastery of the learning intentions by all concerned parties which are the learners and the teacher.

Pianta (2011, p.9) states that "teacher interactions with students stimulate critical thinking and convey new knowledge, organise attention and student effort, and motivate, engage and support".

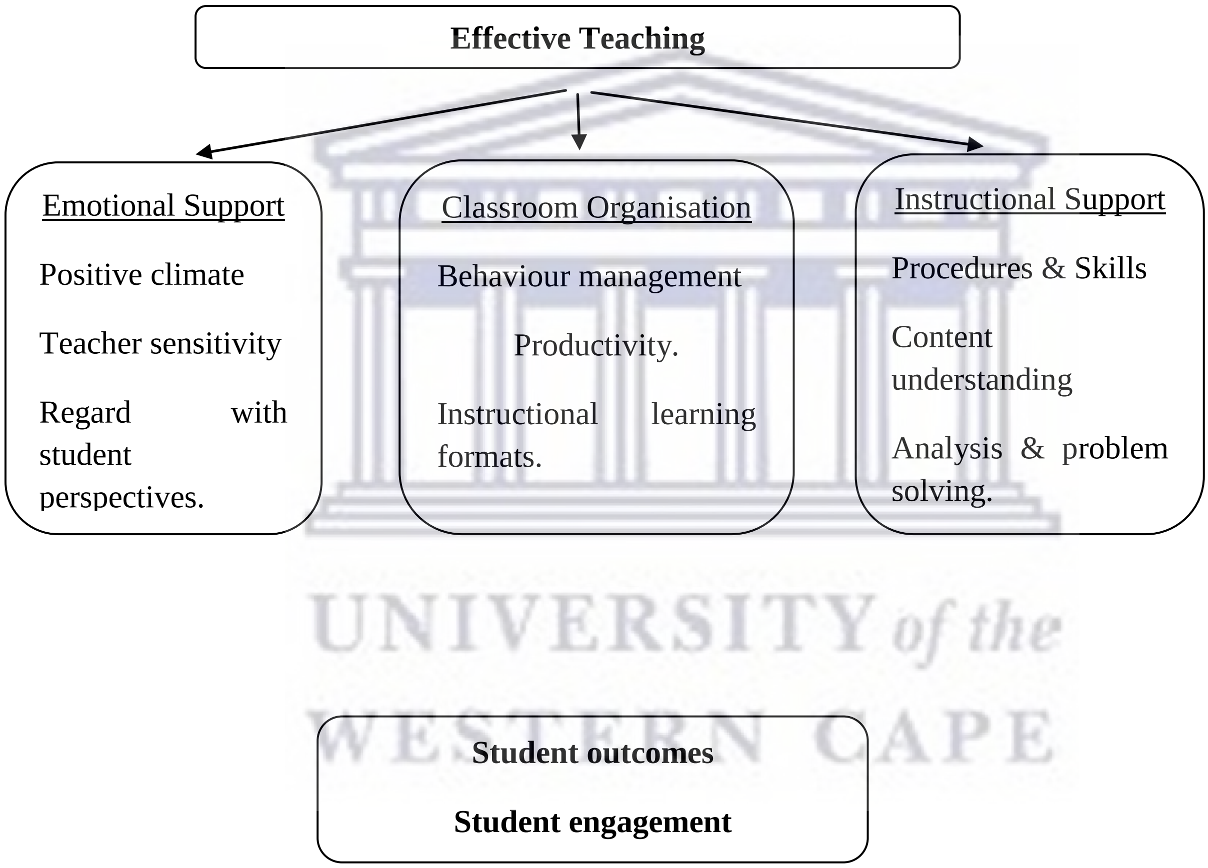
Therefore, an intentional teacher is obliged to use exact, clear and learner-understandable language during teaching so that all learners understand the instructions. An intentional teacher keeps in mind that not all learners learn the same way but differ in their levels of learning and therefore, have to be taught accordingly. Pianta (2003, p.5) defines intentionality as "directed, designed interactions between children and teachers in which teachers purposefully challenge, scaffold and extend children's skills." In the above definitions intentionality was mentioned to further emphasise its meaning. According to the above quote and the researcher's interpretation, the interactions between children and teachers are important but interaction between learners is even more important. The interactions between learners are important because they provide an opportunity for the teacher to scaffold and extend learners' skills.

In summary, intentional teaching is a transparent instructional teaching method because it indicates to all stakeholders, namely teachers and learners, what is required and whether it is achieved by the end of the lesson.

In this section the researcher looks at the three domains of effective instruction. Seidel & Shavelson (2007, p.459) declare that "investigated teaching effects by most studies on student learning in the past decade focused on intentional learning in an organised setting." The researcher's understanding after reading about intentional teaching reveals that intentional teaching is likely to be an effective teaching strategy. One therefore has to seriously consider application of teaching through interaction with learners to make a success of one's teaching. The researcher's observation reveals that interaction between teachers and learners is dropping from the fifth grade.

The drop in teacher-learner interaction causes learners not to take their studies seriously and causes school drop-out at an early age, Pianta, (2011, p.10). The researcher therefore believes that this standard of interaction between teachers and learners should be maintained at the highest level. If the standard of interaction is correctly maintained, then effective teaching will prevail. Pianta, (2011, p.10) represents effective teaching through interaction with learners by the following diagram:

The three domains of effective instruction



In summary of the above the researcher is convinced that good student outcomes will be achieved when the three domains are adhered to during the introduction of an effective teaching strategy. It is likely that when the three domains are adhered to then productivity will hopefully prevail and teachers will then be able to consider how well they manage their instructional time so that learners have maximum opportunity to be taught and learn at the same time.

2.5 Conclusion

In summary of this chapter the researcher discussed research on difficulties learners experience with integers. The emphasis was on how learners could be able to order, recognize and compare integers. The researcher also discussed models and metaphors used for teaching integers. The findings in this chapter informed this research study that an end to learners' difficulties when working with integers and teachers' hope in resolving these difficulties might be reached. The neutralisation and the number line models were discussed. The number line model in a form of a temperature model will be accepted for this study as it allows learners to manipulate the temperatures into integers. Intentional teaching was discussed as an underpinning teaching approach for the study.

The next chapter deals with the research design. Issues of ethical considerations, reliability and validity are also discussed.

CHAPTER 3

RESEARCH DESIGN

3.1 Introduction

This chapter provides an overview of the research design adopted for this study. The existing knowledge about the research and the nature of the research question posed guides the research methods. These include the target group of the study, the context, and methods of data gathering. Issues of ethical considerations, reliability and validity are also addressed in this chapter.

3.2 The Research Approach

In this study design-based research was used. Design-based research started out years ago and many authors (Freudenthal, 1991; Brown, 1992; Gravemeijer, 1993, 1994, 1998; Treffers, 1993; Leijnse, 1995; van den Akker, 1999; Gravemeijer & Cobb, 2001; Edelson, 2002; Drijvers, 2003; Bakker, 2004) contributed to it as a methodology.

Barab & Squire (2004) provide a broad definition of design-based research. They refer to design-based research as "a series of approaches, with the intent of producing new theories, artefacts and practices that account for and potentially impact learning and teaching in naturalistic settings." The researcher concurs with the definition as it emphasises different approaches, practices, learning and teaching.

According to Plomp & Nieveen (2013, p.15) design-based research means to design and develop an intervention (such as programs, teaching-learning strategies and materials, products and systems) as a solution to a complex educational problem as well as to advance our knowledge about the characteristics of these interventions and the processes to design and develop them, or alternatively to design and develop educational interventions (about for example, learning processes, learning environments and the like) with the purpose to develop or validate theories.

What is encapsulated in Plomp & Nieveen's (2013, p.15) definition is that design-based research is mainly aimed at intervening to address complex educational problems. It provides interventionist research that seeks to provide a qualitative description of what is really happening in classrooms. According to Collins, Joseph & Bielaczyc (2004, p.15) design-based research was developed to address several issues central to the study of learning, including the following:

- The need to address theoretical questions about the nature of learning in context.
- The need for approaches to the study of learning phenomena in the real world rather than the laboratory.
- The need to go beyond narrow measures of learning.
- The need to derive research findings from formative evaluation.

Design-based research "effectively bridges the chasm between research and practice" (Anderson & Shattuck, 2012, p.1). According to Anderson & Shattuck (2012, p.2) "Design-based research is a methodology designed by and for educators that seeks to increase the impact, transfer, and translation of education research into improved practice". The researcher thus accepts that the quality and effectiveness of intentional teaching will be revealed through the application of design-based research.

An important characteristic of design-based research is that it can be adjusted during the empirical testing of ideas when a specific idea is not working as anticipated (Baker & Van Eerde, 2013). Design-based research aims at improving a situation, in this case the teaching of the introduction of integers. The aim of design-based research is not to prove that some innovative approach is better than some other approach but "to offer a grounded theory on how the proposed innovative approach works" Gravemeijer (2001, p.43). The research adds to existing theory in order to make a worthwhile scientific contribution, (Baskerville 2001; Davis 1971). The research should assist in solving problems of practitioners, which are either current or anticipated.

Design-based research is cyclical in nature because thought experiments and teaching experiments alternate. In design-based research we observe macro and micro research cycles. Macro cycles are evident by teaching experiments and micro research cycles are indicated by the lessons. The cycles lead to a cumulative effect of small steps in which teaching experiments provide 'feed forward' for the next thought experiments and teaching experiments, (Gravemeijer, 1993; 1994).

The three phases of research macro cycles are:

- Preparation and design phase
- Teaching experiments
- Retrospective analysis.

A teaching experiment is the implementation of a strategy decided upon and the accompanying learning resources to improve delivery of mathematical content to realise a particular goal.

The effectiveness of the implemented teaching strategy is determined by retrospective analysis as to whether learners are demonstrating understanding of the subject matter they are exposed to. The researcher mentions only the above three phases of research macro cycles but might not apply them in this study. An overview of the participants who are learners in the study and the school situation is discussed below.

3.3 Participants

The researcher's own grade 7 learners (age group of 12-13 year olds) were the main participants in this study and hence the sample was a convenient one. "A convenient sample is formed when we select elements from a population on the basis of what elements are easy to obtain" (Taylor, 2015). Taylor (2015) further goes on to argue that:

Sometimes a convenient sample is called a grab sample as we essentially grab members from the population for our sample. This is a type of sampling technique that does not rely upon a random process, such as we see in a simple random sample, to generate a sample.

The manner in which we select our sample determines the type of sample that we have. Among the wide variety of types of statistical samples, the easiest type of sample to form is called a convenience sample.

Taylor (2015), states some issues about convenient samples as follows “convenient samples are definitely easy to obtain. There is virtually no difficulty in selecting members of the population for a convenient sample.”

The researcher’s own class of thirty-two learners provide a sample for this study. The sample was divided into four groups of research participants. Eight of these participants were classified as high performing learners in ANA results, another eight were middle performing learners in ANA results, the second last eight were low performing learners in ANA results and the last eight learners were those who participated voluntarily in the research study. Eighteen girls and fourteen boys formed part of the sample.

3.4 The School

The school has 20 teachers and 834 learners from grade R to grade 7. The school has 1 Principal, 1 Deputy Principal, 3 Departmental Heads and 15 post level one teachers. The school is situated in Gugulethu, a township 30 kilometres from Cape Town in the Western Cape of South Africa. The school started operating in 1959 and was moved from Retreat to Gugulethu the following year because of the then group areas act of 1960.The school is the oldest school in the township. Happiness Primary School is well-resourced and contains classrooms that are spacious although it is categorised as a section 20 school.

Section 20 schools are those schools that still depend on the Western Cape Education Department (WCED) for their budgets. The WCED intervenes by providing these schools with finances that would cater for Learning Teaching and Support material (LTSM), maintenance (repairs to school buildings), local purchases (cleaning materials) and commitments to the municipality (electricity bills, water and other municipal basic services). Classrooms at Happiness Primary School are well furnished with desks and chairs where each learner has his or her own chair.

Learners have continual access to textbooks and various learning materials meaning that the LTSM finances are adequately spent. The school is a no-fee paying school but only R3-00 fund-raising is paid weekly by parents to maintain the daily functioning of the school. The school is categorised as a quintile three school which a combination of a few is affording parents and a majority of non-affording parents. A quintile 1 school is a school situated in the poorest community, quintile 2 is a school situated in a poor community but not as in quintile 1 and quintile 3 is a school situated in the middle of the five quintiles. A quintile 5 is a school situated in prosperous communities but quintile 4 is just below the richest.

3.5 Data collection and instruments

The mathematical topic of this study focuses on integers which were introduced through a temperature model to grade 7 learners. The focus was mainly on whether grade 7 learners recognised integers, could arrange integers from the smallest to the biggest or vice versa and whether they could apply equal and unequal signs to make statements true. Data was collected by means of audio recordings, observation of classroom activities, video-recordings and the responses learners wrote on the worksheets. Learners were audio recorded when they were responding in groups to answers from the worksheets. The audio-recordings were transcribed from learners' chosen language of expression to the language of learning and teaching, in this case English. The classroom activities were controlled in a natural setting by the researcher. The researcher's activities were video recorded in such a manner that the focus was only on the researcher to show what was done and said during teaching. The main reason for the inclusion of video-recordings was to check whether the researcher applied an intentional teaching strategy during lesson presentation. Learners' answers on worksheets were used to determine whether they could handle the problems contained in the worksheets. For some problems, learners were required to provide reasons for their opinions when they agreed or disagreed, declaring statement to be true or false and deciding yes or no in order to determine whether they could justify the responses they provided.

Audio-recording, observation, video-recording and worksheets were the techniques used to collect data for this study. Although video-recording was used, it was not used to collect data from learners' activities but to collect data from the research teacher's teaching activities.

Audio-recording involved the use of a recorder to record the participants' conversations as clustered in groups in a research situation. Learners' recorded voices were transcribed from their original mother tongue language into English. The transcriptions were done to find out whether learners could arrange, compare and order integers. The audio-recording was a big advantage to the research teacher as it recorded and stored the conversations for re-visitation and further analysis.

Observation supplemented the other techniques of data gathering. Bailey (1987, p.40) refers to the application of observation as a method "… to discern on-going behaviour as it occurs and is able to record the salient features of the behaviour." During the lesson presentation and learners' discussions, the researcher moved around between different groups observing what they were doing and saying. This was done to find an appropriate opportunity to intervene and teach where necessary. The teaching was done in such a way that the researcher would pose a question as a build-up to the required answer. Sometimes the researcher would observe without making notes on what was encountered but intervened to assist the participants in enhancing their thoughts. Note taking was done after the lesson presentation by the researcher through choosing important highlights of the lesson.

Video-recording was used to capture the lesson as it happened. In this study the video-recording technique only focused on the teacher's actions and commands of classroom activities. Video-recording has the dual advantage of visual and aural recording. The researcher-teacher can therefore analyse the visuals and the verbal interactions done during classroom activities. Another advantage of video recording is that it captures data and stores it permanently so that it can be re-visited as many times as necessary for further analysis.

Worksheets were used as a technique for obtaining information about how participants dealt with integers when introduced by way of a temperature model. Three worksheets were used (see Appendix A). Altogether the three worksheets contained 19 questions. The content of the worksheet questions ranged through various competencies. Participants had to display certain competencies from the questions in the worksheets. The following competencies were contained in the worksheet questions; awareness of integers, ordering temperatures, comparing integers, identifying integers, saying integers out loud to distinguish between the minus (operational) sign and the negative (directional) sign.

3.6 Data analysis methods

The analysis was by focusing on documents, the completed worksheets, for this study although two kinds of analysis were active, namely the document analysis and the video analysis. The option of document analysis rather than the video analysis was chosen because the video analysis was only to check whether the research-teacher adhered to an intentional teaching strategy.

Data analysis was done by classifying the data from the worksheets into categories. Reading and re-reading of transcripts and frequent listening to the audiotapes allowed the researcher to become as familiar as possible with the data (Mbekwa, 2002, p.85). The answers written by participants on the worksheets were the data used for document analysis. The reading and re-reading of participants' written work was to develop categories that would lead to combined categories. The purpose was to get participants' common understanding of integers. Initial categories were developed and these were reclassified into broader categories. Data analysis focused on how the learners dealt with integers underpinned by a temperature model. Data analysis also focused on learners' strengths in dealing with integers as the strengths in comparing and identifying integers were gathered as categories.

The analysis generally focused on whether learners are aware of integers, can recognise integers, can compare integers and replace greater than or less than signs between integers rather than marking the responses correct or incorrect.

3.6 Reliability and Validity

3.6.1 Validity

The word validity has the same meaning as when something is being legal or true. The researcher had to deal with issues so that readers do not doubt the validity of explanations. Hammersley (1990: p.57) cited in Silverman (2000) says "by validity, I mean truth: interpreted as the extent to which an account accurately represents the social phenomena to which it refers". The use of the learners' written responses, audio- and video-recordings ensured the validity of the data.

Campbell & Stanley (1966) cited in Creswell & Miller (2000, p.125) have the following to say about validity in quantitative and qualitative research:

In quantitative research, investigators are more concerned about the specific inferences made from test scores on psychometric instruments (i.e. the construct, criterion and content validity of interpretations of scores) and the internal and external validity of experimental and quasi-experimental designs. In contrast, qualitative researchers use a lens not based on scores, instruments, or research designs but a lens established using the views of people who conduct, participate in, or read and review a study.

In this study, since it is also a qualitative study, the researcher did not concentrate on the learners correct or incorrect responses or scores to ensure validity but rather concentrated on the views of participants when they were dealing with arranging, comparing and recognising integers and temperatures. To further ensure validity in this study, triangulation as part of a validity procedure, audio- and video recording were also part of the study.

In validity two issues were observed. The issues of internal and external validity are addressed in this section. According to Seale (1998, p.134), "internal validity is the extent to which casual check statements are supported by the study and external validity is the extent to which findings can be generalised to populations or to other settings."

3.6.2 Reliability

When something is reliable it can be trusted. Researchers have to attempt to prove reliability of their research claims, research conduct and findings. Hammersley (1992: p.67) cited in Silverman (2000) says "reliability refers to the degree of consistency with which instances are assigned to the same category by different observers or by the same observer on different occasions." Research that is reliable becomes classic and is consistent in whichever instance it is referred to by other researchers. This research project is not different from reliable works as it contained sources and works of researchers that are reliable. To ensure reliability, credibility and trustworthiness of the study, a thorough recording of the data, the use of multiple sources and the keeping of a research journal were done. In addition, one teacher colleague was asked to assist in the collection of data as part of the process of triangulation. The data collected was therefore reliable. In this study the checking of the categories and making recommendations for the improvement of these categories was done through the intervention of other researchers. Learners' worksheets were genuine and reliable.

3.7 Triangulation

Triangulation refers to the application of multiple data collection methods (Denzin, 1978). These different techniques are used to substantiate the results of the research. The using of a variety of data gathering techniques is to ensure that their weakness balances out (Mouton, 1996). In this study the observation method was complemented by other data gathering methods such as audio-recordings and video-recording, learners' written work and worksheets.

3.8 Ethical Considerations

The preparation and design phase occurs when the researcher uses the teaching approach and the accompanying materials by way of inviting participants to take part in the research study. It requires seeking permission through completion of consent forms by guardians or parents if participants are minors and requesting permission from the WCED, principal of the school if participants are learners or students.

The fact that participants in this research project are children from the age-group 12-13 years old means that their rights have to be protected. These rights are enshrined in the United Nations' children's rights.

Researches consist of guidelines that give a code for ethical behaviour. This research is not different from the others as it used an example of the revised ethical guidelines for educational research. When adhering to these guidelines, Silverman (2010: p.155) states that:

Research staff and subjects must be informed fully about the purpose, methods and intended possible uses of the research, what their participation in the research entail and what risks, if any, are involved.

The above statement indicates that participants participated voluntarily in the research or received consent from their immediate superiors. The statement also showed that confidentiality and anonymity of respondents were at all times respected. A project information sheet (see Appendix B) was issued to parents to read before the start of the research. Permission to conduct research (see Appendix B) was obtained from parents, school principal and WCED. The researcher encouraged participants to take part in the study and promised them that they would benefit if they participated. The researcher requested permission from parents for the learners' participation in the study (also see appendix B). A letter from WCED granting permission to conduct the study is found in annexure A. The study pursued and respected integrity, honesty, confidentiality, voluntary participation and avoidance of learners' personal risk that formed part of ethical principles.

3.9 Conclusion

Research methodology and data collection for this study have been discussed in this chapter. Issues pertaining to qualitative research like reliability and validity have also been discussed. The results of this study are presented in the following chapter.

CHAPTER 4

Research Results

4.1 Introduction

In the previous chapter, the research design, sampling and data collection procedures were discussed. Chapter 3 also discussed the constant comparative approach method of analysis in qualitative research. This chapter presents the research results of the study. The results are presented per worksheet and the learners' responses to the questions in the worksheets and categories are described.

4.2 Results per worksheet

4.2.1 Worksheet 1 results

In Question 1 of Worksheet 1, learners were required to read and say temperatures out loud in words. In responding to this question, learners displayed **awareness that numbers in the context of temperatures are different from ordinary whole numbers**. For example, one learner in group 1 read the temperatures as "minus three" indicating the distinction by "minus". Another learner on the other hand, read the temperatures as "five, negative seven, zero" using "negative" to indicate the distinction. Some learners used "positive", some "plus" and some neither of the two. In group 1, four learners used neither "plus" nor "positive" when reading temperatures, three used "positive" and one used "plus" as a sign in front of the number when reading temperatures. Another example was when a few learners used neither plus nor positive while others used positive (+) and plus (+) in front of the number. The difference was primarily when there was a negative (-) sign and the sum was when there was a positive sign in front of the number.

Learners in groups 2, 3 and 4 also responded similarly to group 1 learners where there was mention of “minus three” or “negative three” and “positive” or “plus” before numbers when saying temperatures out loud in words. In group 2, three learners used “plus” while another three used “positive” and two learners used neither “plus” nor “positive”. In group 3, two learners used “plus” while another

two learners used “positive” and four learners used neither “plus” nor “positive when reading the temperatures out loud. In the last group, group 4, five learners used “positive” and three used “plus” when reading temperatures. The same thing applied when learners read temperatures with negative sign or minus sign in front of the numbers.

A majority of learners were aware that signs are used in front of integers as we realise that very few learners mentioned the temperatures as whole numbers by not using either positive or plus and negative or minus in front of these numbers when reading temperatures.

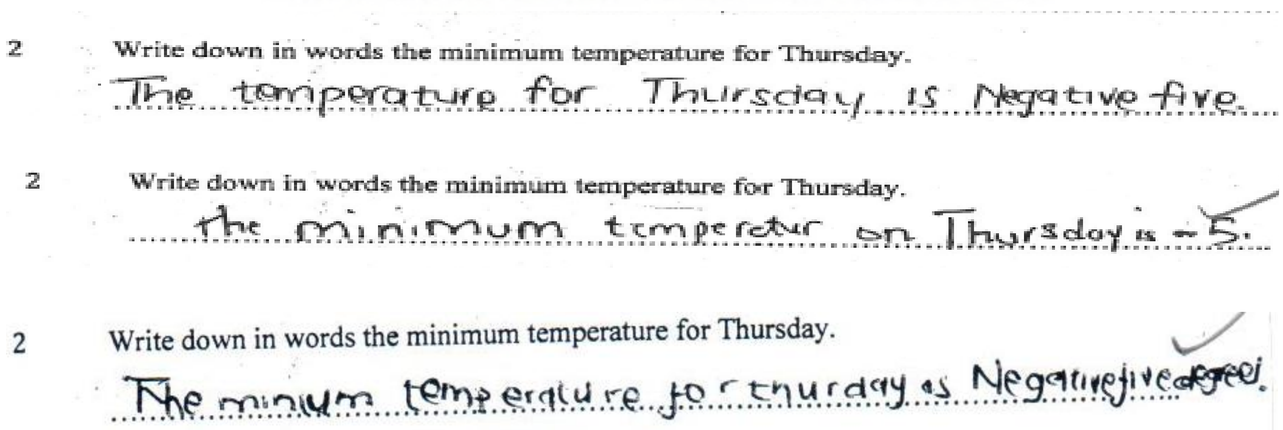
The majority of learners were aware that signs are used in front of numbers when saying temperatures out loud.

In responding to question 2 of worksheet 1, learners displayed **awareness that numbers in the context of temperatures are written differently from ordinary whole numbers**. Ordinary numbers like whole and natural numbers are written without writing the sign in front of the number but temperatures are written with a sign in front of the number. An indication of this awareness was realised from the written work that learners wrote.

Figure 4.1 below shows how four different learners answered question 2 of worksheet 1.

2. ***Write down in words the minimum temperature for Thursday***.

Answers


2 Write down in words the minimum temperature for Thursday.
The temperature for Thursday is Negative five.

2 Write down in words the minimum temperature for Thursday.
the minimum temperatur on Thursday is -5.

2 Write down in words the minimum temperature for Thursday.
The minum temperature for thurday as Negativefivedegree.

Figure 4.1: Examples of learners' written work

The learners' written work in group 1 provided evidence that learners are aware of writing temperatures as integers and not as whole numbers. The table below shows how learners wrote their responses.

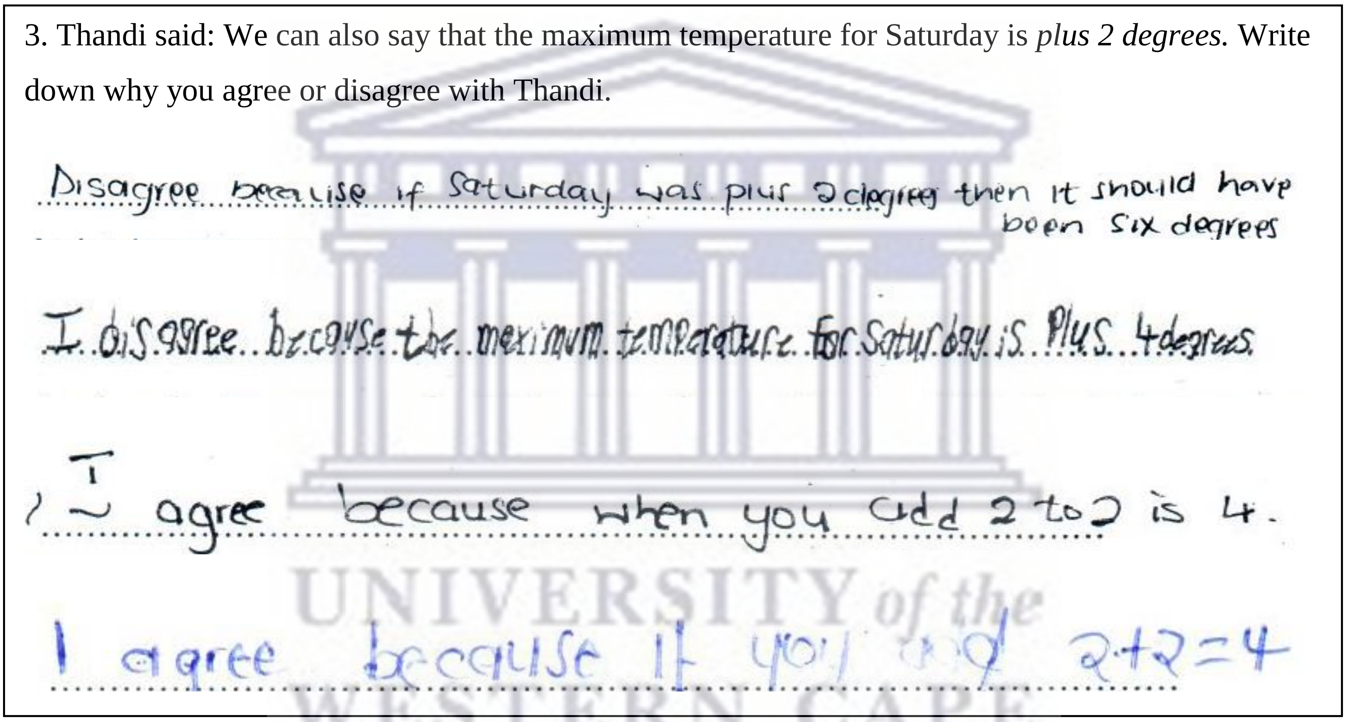
Table 4.1: Number of learners who displayed **awareness that numbers in the context of temperatures are written differently from ordinary whole numbers**

	Group 1	**Group 2**	**Group 3**	**Group 4**
Negative five degrees Celsius	6	5	3	4
Minus five degrees Celsius	2	2	3	1
-5 in symbols		1	2	1
No response				2
Total	**8**	**8**	**8**	**8**

Most learners in all four groups wrote either negative five or minus five degrees Celsius as compared to a few learners who wrote -5 while the least of these learners wrote neither negative five, -5 nor minus five degrees Celsius.

In responding to question 3 of worksheet 1, learners displayed a **tendency to get to the answer without focusing on the entire question statement.** Learners were guided by the words "plus 2" in the question and attempted to get the 2 thus displaying the tendency of attempting to get to the answer.

The support of this claim is evident in an argument that arose between learners. Some of the learners supported Thandi's claim when she said: "We can also say that the maximum temperature for Saturday is *plus 2 degrees"*. The learners based their agreement from what appeared in the question statement that read: "We can also say that the maximum temperature for Saturday is *plus 2 degrees"*. Learners were therefore not focusing on Thandi's claim but attempting to get the answer. Learners who agreed with Thandi's claim used a calculation approach as a technique. Figure 4.2 below shows how four different learners in different groups responded to Thandi's claim.


3. Thandi said: We can also say that the maximum temperature for Saturday is *plus 2 degrees.* Write down why you agree or disagree with Thandi.

Disagree because if Saturday was plus 2 degree then it should have been six degrees

I disagree because the meximum temperature for Saturday is Plus 4degrees

I agree because when you add 2 to 2 is 4.

I agree because if you add 2+2=4

Figure 4.2: Learners' responses to Thandi's claim

Learners took cognisance of the 2 degrees which was Friday's maximum temperature from the given table and added it to the "*plus 2 degrees*" mentioned by Thandi.

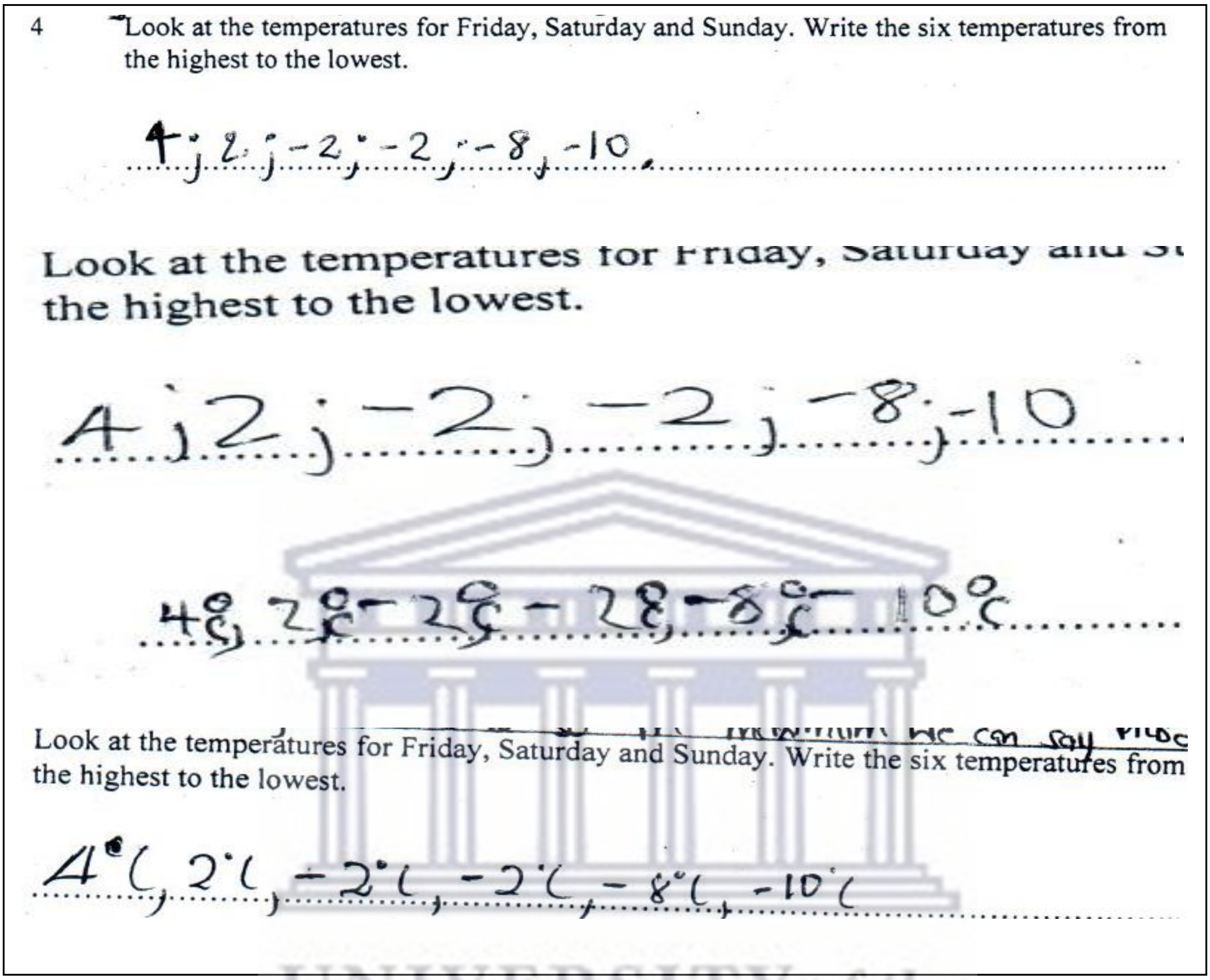
The learners were aware that if they add 2 and 2 they get the sum of 4. Learners' focusing on answering the question using a calculation approach created an atmosphere of not adhering to Thandi's claim but missed another important part of the question.

The missed part of the question was the provision of an appropriate reason why learners agree or disagree with Thandi. The figure above and the table below show how learners' answers support my category when they get to the answer without focusing on the entire question statement.

Table 4.2: Number of learners who displayed a **tendency to get to the answer without focusing on the entire question statement** when responding to Thandi's claim

	Group 1	**Group 2**	**Group 3**	**Group 4**
Agree with a reason	4	4	3	4
Agree without a reason	2	2	2	2
Disagree with a reason	2	2	1	1
Disagree without a reason	0	0	2	1
Total	**8**	**8**	**8**	**8**

In responding to question 4 of worksheet 1, learners displayed **competency in arranging temperatures in descending order** by being able to arrange Fridays', Saturdays' and Sundays' temperatures from the highest to the lowest. Figure 4.3 below shows how learners arranged the temperatures from highest to lowest.


4 Look at the temperatures for Friday, Saturday and Sunday. Write the six temperatures from the highest to the lowest.

4; 2; -2; -2; -8; -10.

Look at the temperatures for Friday, Saturday and Su... the highest to the lowest.

4; 2; -2; -2; -8; -10

4°C, 2°C -2°C - 2°C, -8°C 10°C

Look at the temperatures for Friday, Saturday and Sunday. Write the six temperatures from the highest to the lowest.

4°C, 2°C, -2°C, -2°C, -8°C, -10°C

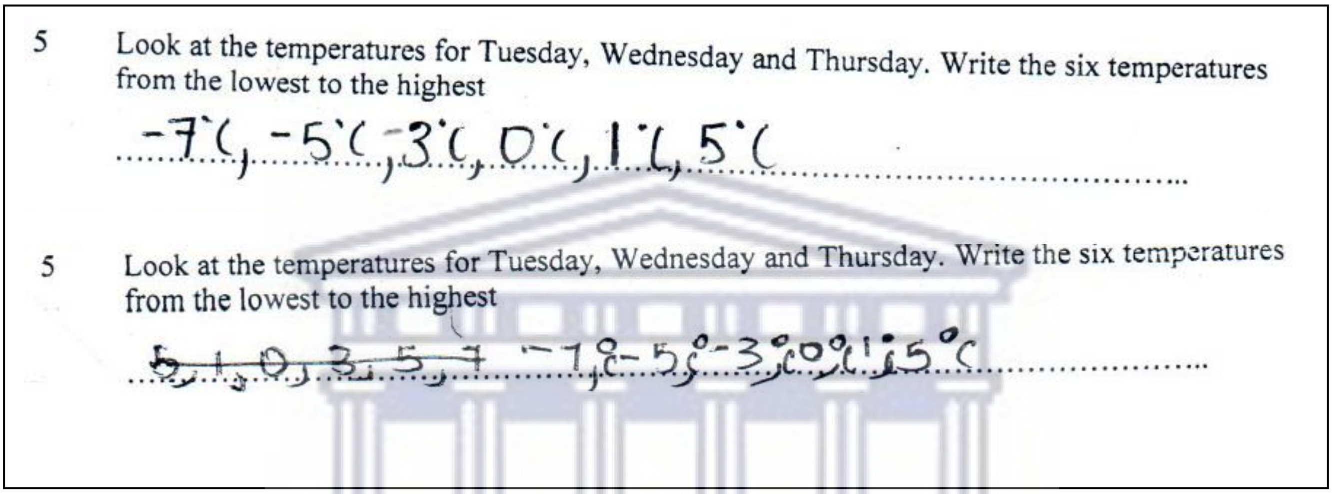
Figure 4.3: Learners' arrangement of temperatures in descending order

Table 4.3: Number of learners who displayed **competency in arranging temperatures in descending order**

	Group 1	Group 2	Group 3	Group 4
Able	7	6	3	5
Unable	1	2	4	3
Blank space			1	
Total	**8**	**8**	**8**	**8**

Learners' responses in question 5 of worksheet 1 indicate that learners possess **competency in arranging temperature in ascending order** and are able to display this category.

The arrangement of Tuesdays', Wednesdays' and Thursdays' temperatures from lowest to highest is proof that learners can display the category. Figure 4.4 below shows how two learners arranged the temperatures from the lowest to the highest.


5 Look at the temperatures for Tuesday, Wednesday and Thursday. Write the six temperatures from the lowest to the highest

-7°C, -5°C, -3°C, 0°C, 1°C, 5°C

5 Look at the temperatures for Tuesday, Wednesday and Thursday. Write the six temperatures from the lowest to the highest

~~5, 1, 0, 3, 5, 7~~ -7°C, -5°C, -3°C, 0°C, 1°C, 5°C

Figure 4.4: Learners' arrangement of temperatures in ascending order

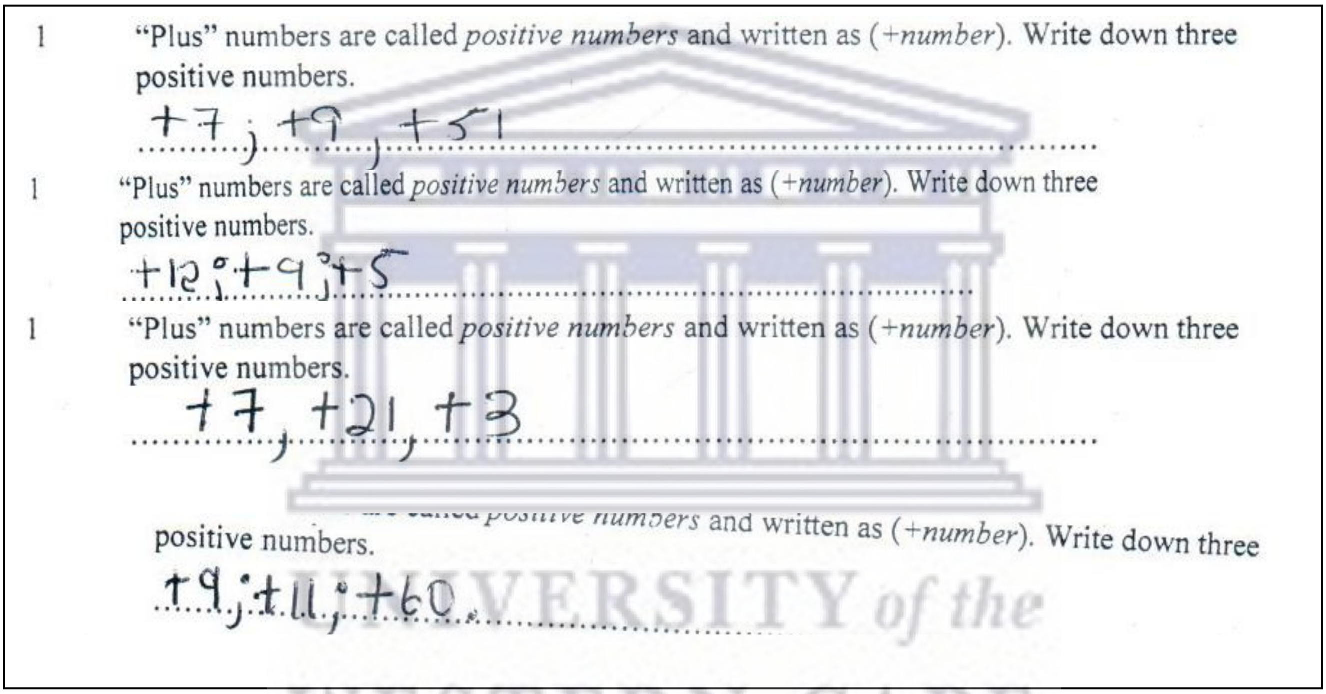
Table 4.4: Number of learners who displayed **competency in arranging temperature in ascending order**

	Group 1	**Group 2**	**Group 3**	**Group 4**
Able to arrange/write temperatures	7	7	2	7
Unable to arrange/write temperatures	1	1	6	1
Total	**8**	**8**	**8**	**8**

According to the above table most learners were able to arrange temperatures from lowest to highest as opposed to few learners who were unable to arrange temperatures from the lowest to the highest. The high percentage of learners who could arrange temperatures from lowest to highest provides proof that learners are aware of temperature arrangement in ascending order.

4.2.2 Worksheet 2 results

In responding to question 1 of worksheet 2, learners displayed **capability in writing positive numbers with a plus sign in front**. The figure below depicts how learners expressed their capability in writing positive numbers.


1 "Plus" numbers are called *positive numbers* and written as (+*number*). Write down three positive numbers.

+7 ; +9 ; +51

1 "Plus" numbers are called *positive numbers* and written as (+*number*). Write down three positive numbers.

+12 ; +9 ; +5

1 "Plus" numbers are called *positive numbers* and written as (+*number*). Write down three positive numbers.

+7 , +21 , +3

... called *positive numbers* and written as (+*number*). Write down three positive numbers.

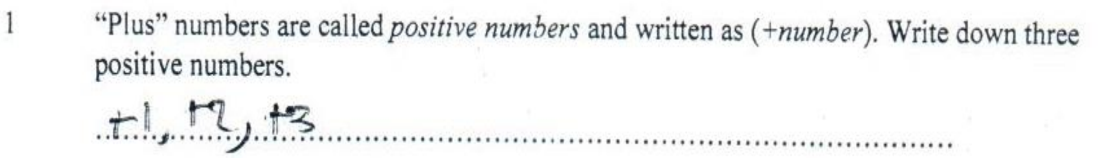
+9 ; +11 ; +60

Figure 4.5: Learners' capabilities in writing positive numbers

In all four groups learners wrote the positive numbers correctly by writing a positive sign or no sign in front of the number. A majority of learners wrote a positive or plus sign while few learners wrote

nei ... ber of learners

wh ... he two signs in


1 "Plus" numbers are called *positive numbers* and written as (+*number*). Write down three positive numbers.

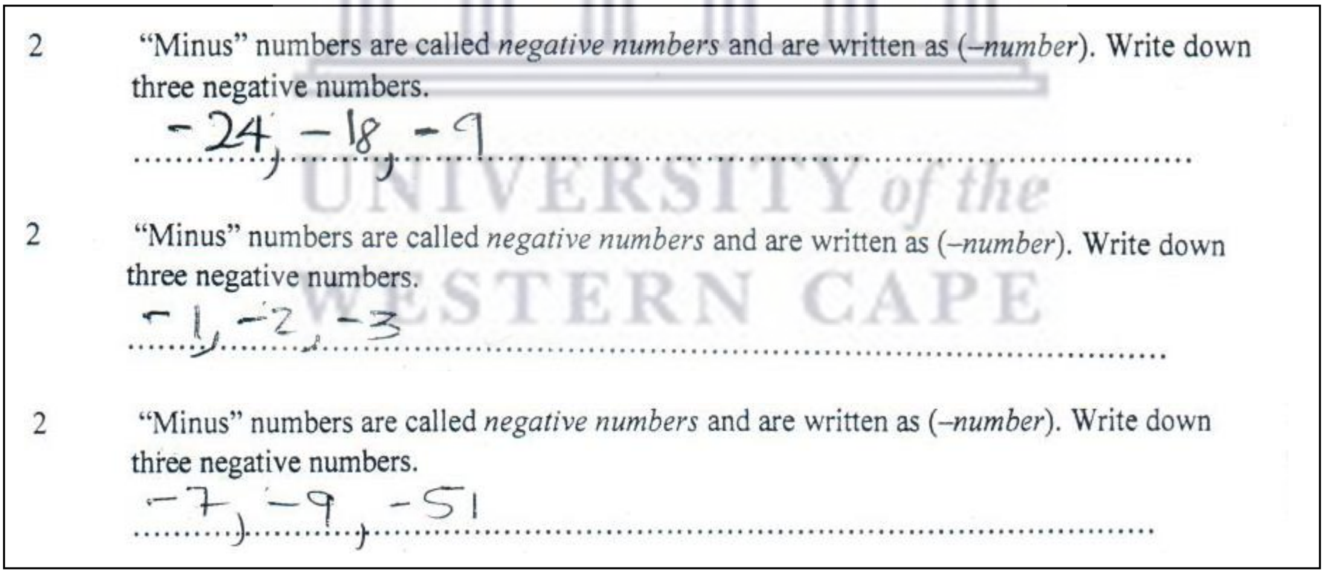
+1 , +2 , +3

front of the number.

Table 4.5: Number of learners who displayed **capability in writing positive numbers with a plus sign in front**

	Group 1	Group 2	Group 3	Group 4
Positive/Plus	8	6	6	6
No sign	0	2	2	2
Total	**8**	**8**	**8**	**8**

In responding to question 2 of worksheet 2, learners displayed **capability in writing negative numbers with a negative sign in front**. Learners in all four groups wrote numbers with a negative sign in front of each number. The figure below is an excerpt of how learners wrote negative numbers with a negative sign in front of each number.


2 "Minus" numbers are called *negative numbers* and are written as (–*number*). Write down three negative numbers.

–24, –18, –9

2 "Minus" numbers are called *negative numbers* and are written as (–*number*). Write down three negative numbers.

–1, –2, –3

2 "Minus" numbers are called *negative numbers* and are written as (–*number*). Write down three negative numbers.

–7, –9, –51

Figure 4.6: Learners' capabilities in writing negative numbers

After analysing the whole sample, I noticed that most of the learners wrote the negative sign correctly. The table below gives a summary of the learners' responses.

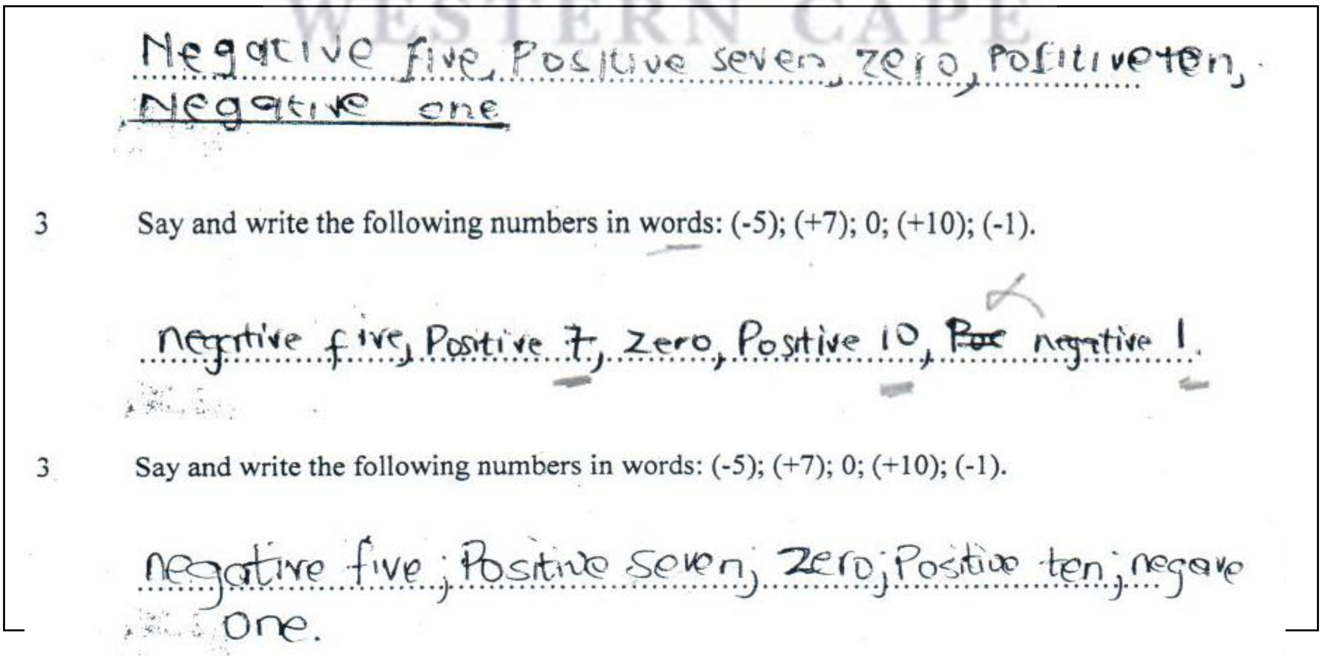
Table 4.6: Number of learners who displayed **capability in writing negative numbers with a negative sign in front**

	Group 1	Group 2	Group 3	Group 4
With a negative sign	8	8	7	8
Without a negative sign	0	0	1	0
Total	**8**	**8**	**8**	**8**

When analysing the table above I noticed that only one learner out of thirty-two learners failed to use the negative sign correctly before each negative number while the rest (thirty-one) used the sign correctly.

That was therefore an indication that a large majority of these learners are capable of writing negative numbers correctly.

In responding to question 3 of worksheet 2, learners displayed **capability of writing numbers that are in symbols into numbers that are in words.** The majority of learners wrote the symbols correctly in words as required by the question. However, some of the learners mixed the symbols with words, e.g. "positive 10" instead of "positive ten". The figure below is an excerpt of what three learners wrote.



Negative five, Positive seven, zero, Positiveten, Negative one

3 Say and write the following numbers in words: (-5); (+7); 0; (+10); (-1).

negative five, Positive 7, zero, Positive 10, ~~Po~~ negative 1.

3 Say and write the following numbers in words: (-5); (+7); 0; (+10); (-1).

negative five; Positive seven; zero; Positive ten; negative one.

Figure 4.7: Example of learners' written work from symbols into words

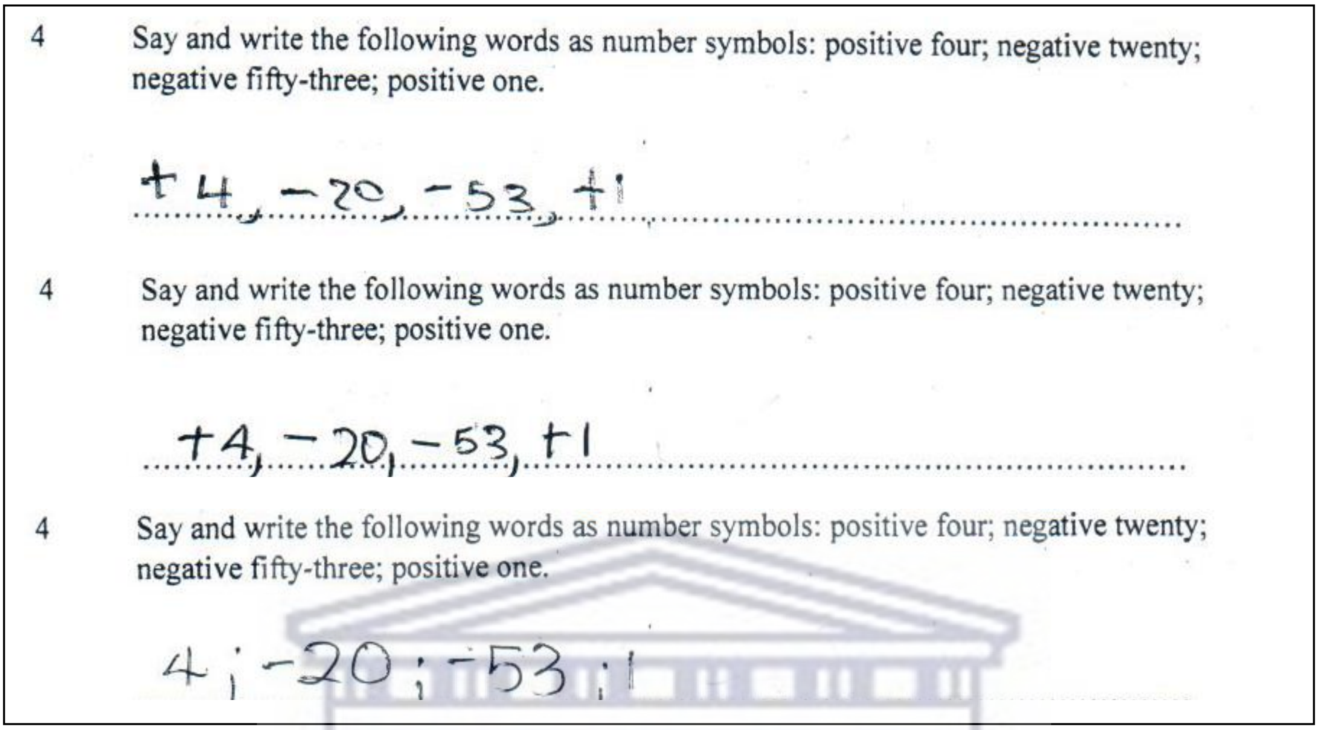
After analysing the entire sample, the results were summarised in the table below.

Table 4.7: Number of learners who displayed **capability of writing numbers that are in symbols into numbers that are in words**

	Group 1	Group 2	Group 3	Group 4
Mixing symbols and words	0	2	2	2
Writing symbols into words correctly	8	6	6	6
Total	**8**	**8**	**8**	**8**

When learners were saying the integers written in symbols, they all said these integers correctly. However, when they had to write the same integers, twenty-six of these learners correctly wrote the symbols into words while the other six learners mixed the symbols with words. That was an indication that these learners were capable of writing numbers that are in symbols to numbers that are in words.

In responding to question 4 of worksheet 2, learners displayed **capability of writing numbers that are in words into numbers that are in symbols**. All the learners in the four groups correctly said and wrote the numbers that were given in words into symbols.



4 Say and write the following words as number symbols: positive four; negative twenty; negative fifty-three; positive one.

+4, −20, −53, +1

4 Say and write the following words as number symbols: positive four; negative twenty; negative fifty-three; positive one.

+4, −20, −53, +1

4 Say and write the following words as number symbols: positive four; negative twenty; negative fifty-three; positive one.

4 ; −20 ; −53 ; 1

Figure 4.8: Example of learners' responses in writing symbols from given words

There was an argument between learners of group 3 when they have to verbally state positive 4. Some said plus four or positive four while others just said 4. One learner volunteered by asking another learner's opinion in group 2 to put an end to this disagreement and thus opening a debate. The learner in group 3 shouted thus inviting others to the debate:

Learner 1 of Group 3: "Four"

Learner 2 of Group 3: "No. We say positive four."

Learner 1 of Group 2: "Which is correct? Four or plus four?"

Learner 2 of Group 2: "Four and plus four, mean the same thing and you are correct whichever way you say it."

All in both groups: "Yes, it is the same thing."

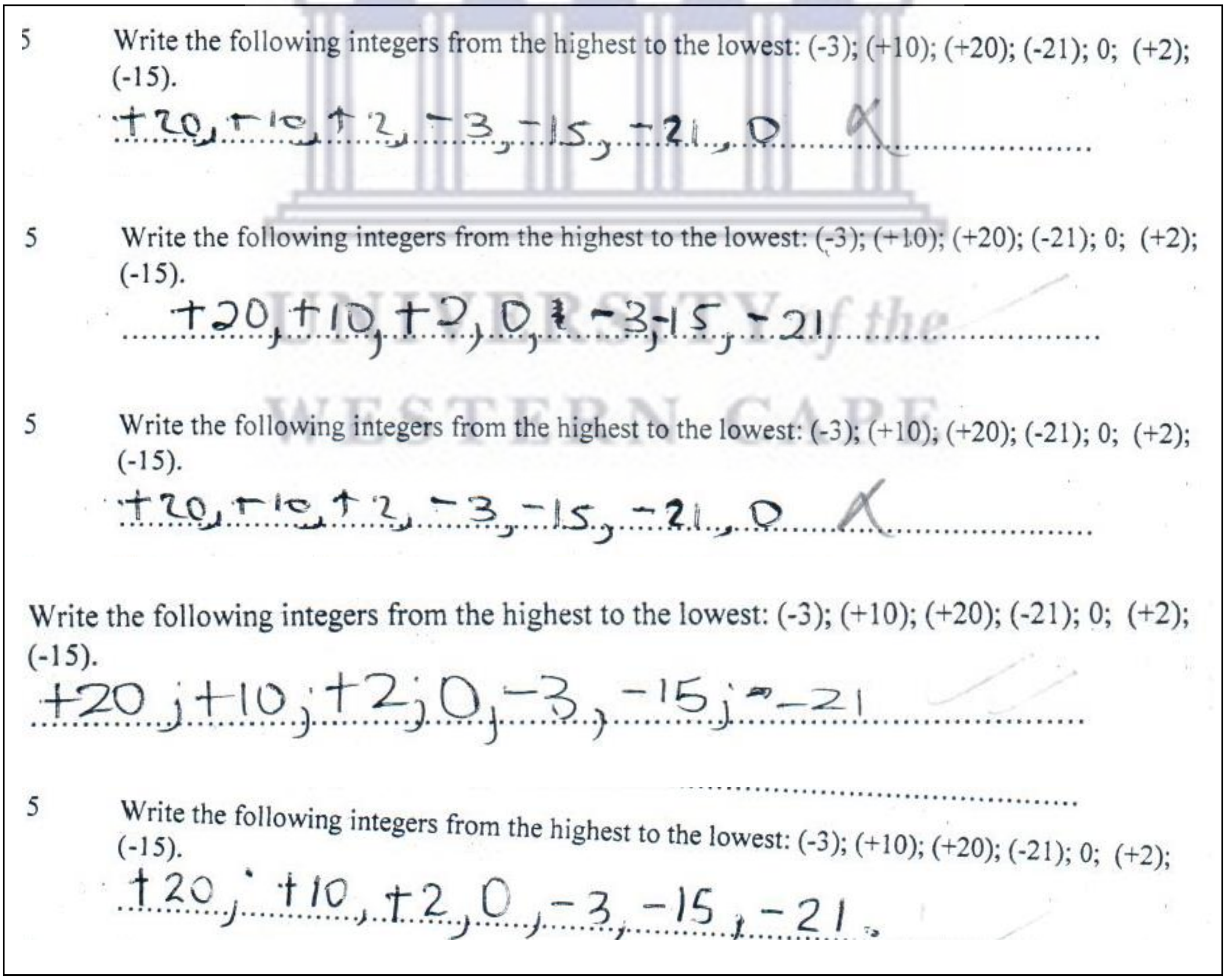
The above excerpt in figure 8 shows that some learners understand that when writing positive numbers, you can write the number with or without a sign, e.g. +4 or 4.

The table below provide an indication that all learners were capable of writing numbers that are in words into numbers that are in symbols.

Table 4.8: Number of learners who displayed **capability of writing numbers that are in words into numbers that are in symbols**

	Group 1	Group 2	Group 3	Group 4
4; -20; -53; 1	8 (100%)	8 (100%)	8 (100%)	8 (100%)

In responding to question 5 of worksheet 2, learners displayed **capability of writing numbers that are in words into numbers that are in symbols**. The figure below indicates how learners demonstrated their competencies.


5 Write the following integers from the highest to the lowest: (-3); (+10); (+20); (-21); 0; (+2); (-15).

+20, +10, +2, -3, -15, -21, 0

5 Write the following integers from the highest to the lowest: (-3); (+10); (+20); (-21); 0; (+2); (-15).

+20, +10, +2, 0, -3, -15, -21

5 Write the following integers from the highest to the lowest: (-3); (+10); (+20); (-21); 0; (+2); (-15).

+20, +10, +2, -3, -15, -21, 0

Write the following integers from the highest to the lowest: (-3); (+10); (+20); (-21); 0; (+2); (-15).

+20; +10; +2; 0, -3, -15; -21

5 Write the following integers from the highest to the lowest: (-3); (+10); (+20); (-21); 0; (+2); (-15).

+20; +10, +2, 0, -3, -15, -21.

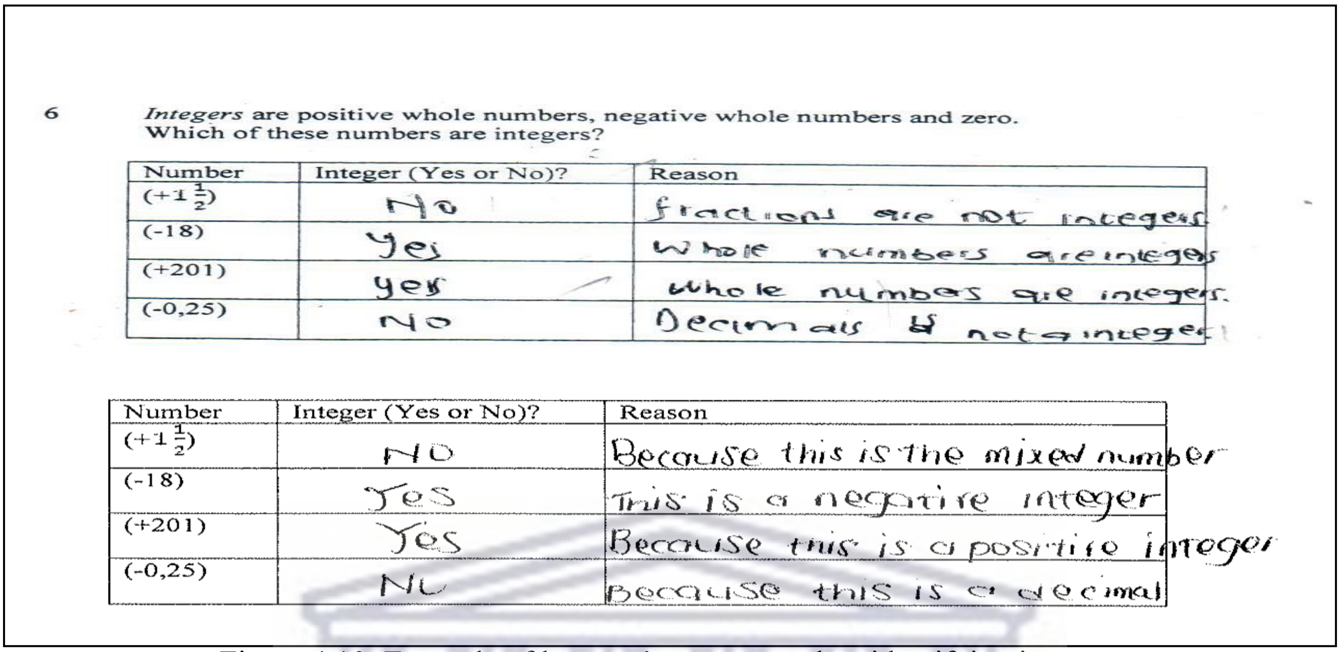
Figure 4.9: Examples of learners' arrangement in descending order

In this excerpt of figure 4.9 above, three learners arranged the integers correctly while two got it wrong. A closer examination of the answers of the two learners who did not do the arrangement correctly suggests that these learners seem to believe that zero is less than all the other numbers, even negative numbers. The concept that zero is the least integer when arranging temperatures is merely incorrect as it indicates more teaching. The competencies for the entire sample are summarised in the table below.

Table 4.9: Number of learners who displayed **capability of writing numbers that are in words into numbers that are in** symbols

	Group 1	**Group 2**	**Group 3**	**Group 4**
Arrangement of integers, correct.	7	7	5	6
Arrangement of integers, merely correct.	1	1	3	2
Total	**8**	**8**	**8**	**8**

In responding to question 6 of worksheet 2, most learners displayed **capability of identifying integers from other numbers**. However, some of these learners could not express reasons in an acceptable manner. This will be explained further after this excerpt.



6 *Integers* are positive whole numbers, negative whole numbers and zero.
Which of these numbers are integers?

Number	Integer (Yes or No)?	Reason
$(+1\frac{1}{2})$	No	fractions are not integers
(-18)	Yes	whole numbers are integers
(+201)	yes	whole numbers are integers.
(-0,25)	No	Decimals is not a integer

Number	Integer (Yes or No)?	Reason
$(+1\frac{1}{2})$	NO	Because this is the mixed number
(-18)	Yes	This is a negative integer
(+201)	Yes	Because this is a positive integer
(-0,25)	NO	Because this is a decimal

Figure 4.10: Example of learners' responses when identifying integers

The excerpt in figure 4.10 shows how learners differed when supporting their stances. In the above excerpt the learner supported his or her stance by writing "because this is a fraction". The learner at the bottom of the excerpt supported his or her stance by writing "fractions are not integers" when providing a reason.

The table below indicate the number of learners in each group which identified integers. The majority of learners agreed with a common reason namely "because this is a fraction". However, very few other learners supported their stances by writing "fractions are not integers". Learners were too consistent in arranging integers when completing this question. Learners in all four groups did not confuse temperature arrangement with integer arrangement as they used to in previous questions. The teacher intervention might also have contributed to their consistency as the teacher mentioned at some point in this question that learners should read the instructions carefully and write what is required.

Table 4.10: Number of learners who displayed **capability of identifying integers from other numbers**

	Group 1	Group 2	Group 3	Group 4
Agree with the reason, "because this is a fraction".	8	5	4	6
Agree with the reason, "fractions are not integers"	0	3	4	2
Total	**8**	**8**	**8**	**8**

In responding to question 7 of worksheet 2, half of the learners displayed **capability of identifying integers even when instructions are not clear.** The other half of these learners seem to have a problem with following the instructions of the question. Learners were required to write integers from the given set of numbers. The researcher was expecting them to write -16; 8; 0; +20; -101.

7 Most of the time the positive sign (+) is not written in front of positive number. Only 2 is written for (+2). Complete the following: The integers in the set of numbers (-16); $(-\frac{1}{2})$; 8; $2\frac{3}{5}$; 0; (+20); $(-\frac{5}{4})$; (-101) are

negetive numbers (−) and positive numbers, (−) and (+)

7 Most of the time the positive sign (+) is not written in front of positive number. Only 2 is written for (+2). Complete the following: The integers in the set of numbers (-16); $(-\frac{1}{2})$; 8; $2\frac{3}{5}$; 0; (+20); $(-\frac{5}{4})$; (-101) are

-16; 8; 0; +20; -101

Figure 4.11: Example of learners' responses when identifying integers

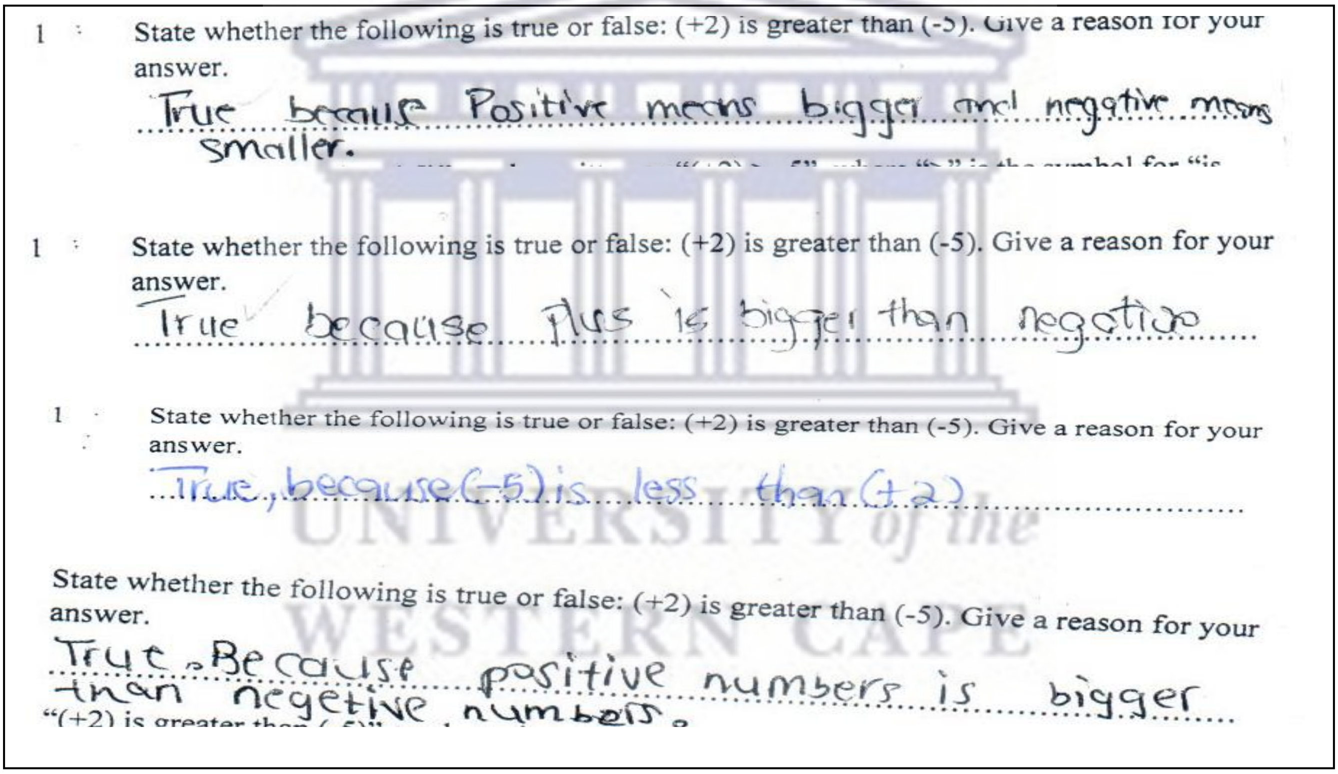
The above excerpt in figure 11 indicates how learners identified integers. However, some of these learners provided incomplete answers as they answered in short statements by writing only "positive and negative numbers" instead of the entire definition that "Integers are positive whole numbers, zero and negative whole numbers". The correct answer is positive and negative whole numbers and zero. The learners' responses are summarised in the table below.

Table 4.11: Number of learners who displayed **capability of identifying integers even when instructions are not clear**

	Group 1	**Group 2**	**Group 3**	**Group 4**
Capable of identifying integers even when instructions are not clear	50%	50%	50%	50%
Incapable of identifying integers even when instructions are not clear	50%	50%	50%	50%
Total	**100%**	**100%**	**100%**	**100%**

4.2.3 Worksheet 3 results

In responding to question 1 of worksheet 3, learners displayed **an understanding that positive numbers are always greater than negative numbers with some providing general although not completely convincing reasons why they think so**. Some learners wrote “positive means bigger and negative means smaller”. This reasoning means that positive numbers are always greater than negative numbers and in terms of the temperature context positive temperatures are always higher than negative temperatures. The excerpt below shows how some learners tried without mathematically convincing reasons to persuade readers.


1 State whether the following is true or false: (+2) is greater than (-5). Give a reason for your answer.

True because Positive means bigger and negative means smaller.

1 State whether the following is true or false: (+2) is greater than (-5). Give a reason for your answer.

True because Plus is bigger than negative

1 State whether the following is true or false: (+2) is greater than (-5). Give a reason for your answer.

True, because (-5) is less than (+2)

State whether the following is true or false: (+2) is greater than (-5). Give a reason for your answer.

True. Because positive numbers is bigger than negetive numbers.

Figure 4.12: Example of learners’ responses and reasoning

Some learners stated that the statement is true but provided reasoning that was not completely convincing why they thought so. The requirement that learners should provide reasons why they declare the statement to be true or false became a challenge. However, few learners provided reasons closer to what may be deemed appropriate justification that positive numbers are greater than negative numbers.

Learner 1 in group 1: “True because positive numbers are bigger than negative numbers”. The table below presents how learners responded when stating their reasons why they declare the statements to be true.

Table 4.12: Number of learners who displayed **an understanding that positive numbers are always greater than negative numbers with some providing general although not completely convincing reasons why they think so**.

		Group 1	Group 2	Group 3	Group 4
True with conviction	Not convincing reasons	7	7	8	8
	Convincing reasons	1	1	0	0
Total		**8**	**8**	**8**	**8**

In responding to question 2 of worksheet 3, learners displayed the **ability to write word sentences with correct inequalities from number sentences but expressed some differences when saying the word sentences**. For example, an argument arose when one learner asked whether to say “bigger than” or “greater than” and “less than” or “smaller than”. Another learner intervened that both words mean the same thing, so they can say either of the two. The excerpt in the table below shows how learners responded when answering this question in writing.

2 “(+2) is greater than (-5)” can be written as “(+2) > -5”, where “>” is the symbol for “is greater than”. The symbol “<” is the one used for saying “is less than”. Say and write in the following number sentences in words.

(a) (-1)<(+1): negative one is less than positive one

(b) (-4)>(-16): negative four is greater than negative sixteen

(c) (+2)>(-10): positive two is greater than negative ten

(d) 5<8: Five is less than eight.

2 “(+2) is greater than (-5)” can be written as “(+2) > -5”, where “>” is the symbol for “is greater than”. The symbol “<” is the one used for saying “is less than”. Say and write in the following number sentences in words.

(a) (-1)<(+1): Negative one is less than positive one.

(b) (-4)>(-16): Negative four is bigger than negative sixteen

(c) (+2)>(-10): Positive two is bigger than negative ten

(d) 5<8: five is less than eight.

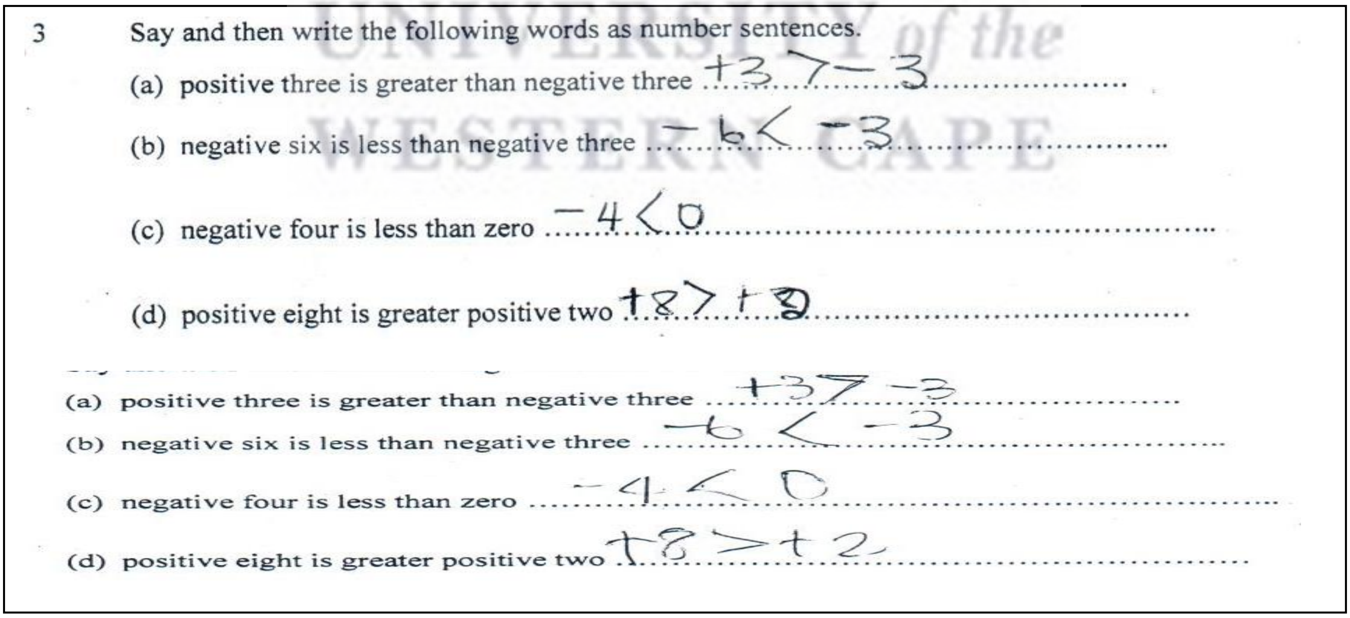
Figure 4.13: Example of learners' written responses

The majority of learners were able to say and write the number sentences using a combination of "greater than" and "less than" while the minority of these learners used another combination of "bigger than" and "smaller than". The table below shows how many learners were able to do so.

Table 4.13: Number of learners who displayed the **ability to write word sentences with correct inequalities from number sentences but expressed some differences when saying the word sentences**

	Group 1	**Group 2**	**Group 3**	**Group 4**
Greater/Less	7	6	5	7
Bigger/smaller	1	2	3	1
Total	**8**	**8**	**8**	**8**

In responding to question 3 of worksheet 3, learners displayed **ability in saying and writing the word sentences into number sentences using the relevant inequalities**.


3 Say and then write the following words as number sentences.

(a) positive three is greater than negative three $+3 > -3$

(b) negative six is less than negative three $-6 < -3$

(c) negative four is less than zero $-4 < 0$

(d) positive eight is greater positive two $+8 > +2$

(a) positive three is greater than negative three $+3 > -3$

(b) negative six is less than negative three $-6 < -3$

(c) negative four is less than zero $-4 < 0$

(d) positive eight is greater positive two $+8 > +2$

Figure 4.14: Excerpt showing examples of learners' responses using relevant inequalities

In the above excerpt the two learners were able to say and write the words as number sentences. The majority of learners in all four groups were able to say and write the words as number sentences. There were few not so important differences when learners used less than/smaller than and greater than/bigger than.

Learner 1 of group 2: "negative six is smaller than negative three".

Learner 2 of group 2: "negative six is less than negative three".

These differences are not so important because one can use either word of the two as they are mathematically acceptable and correct.

The table below shows how many learners said these combinations in each group and in each category.

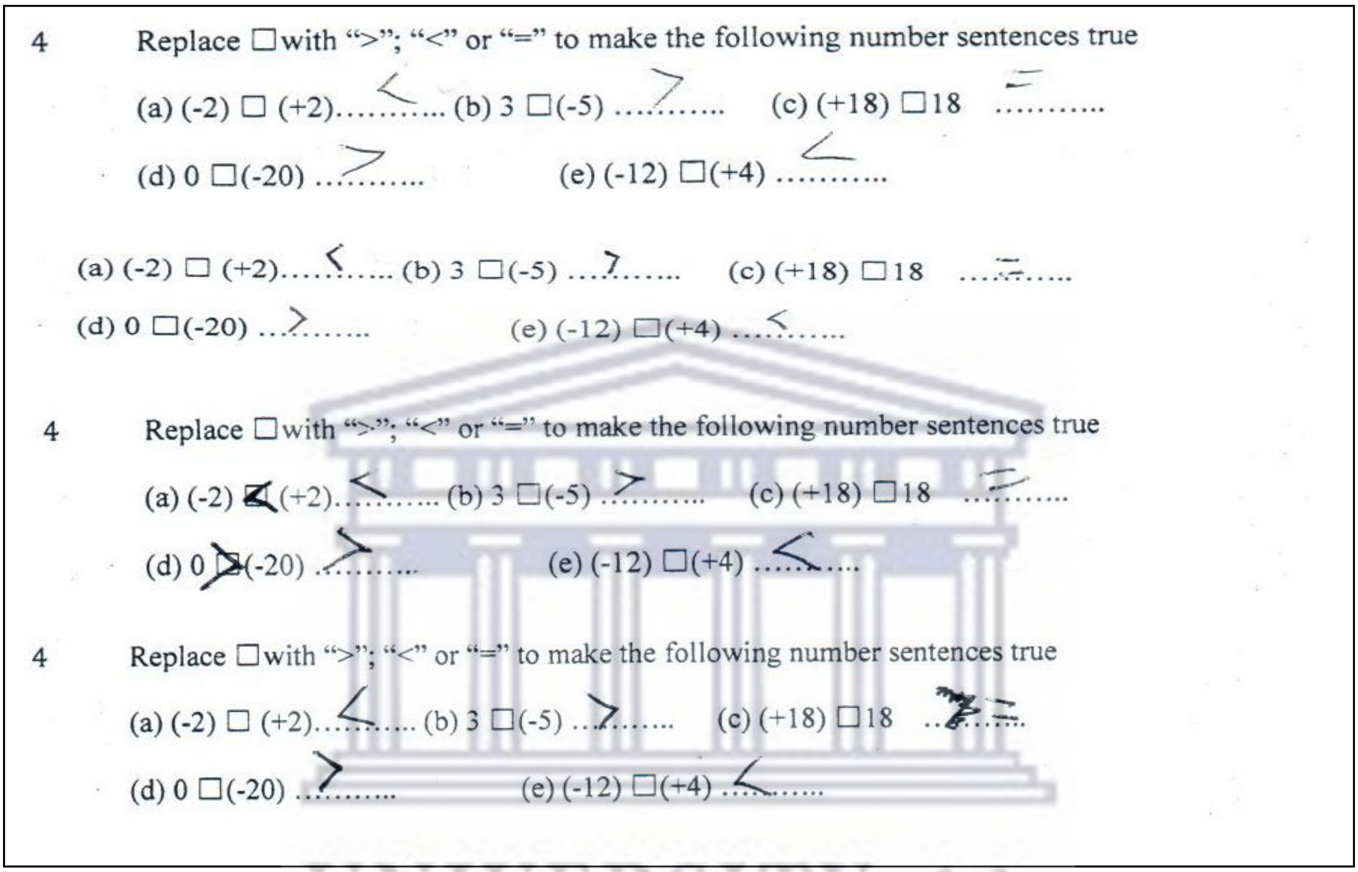
Table 4.14: Number of learners who displayed ability **in saying and writing the word sentences into number sentences using the relevant inequalities**

		Group 1	**Group 2**	**Group 3**	**Group 4**
Combinations	Less/Greater	8	7	5	6
	Smaller/bigger	0	1	3	2
Total		**8**	**8**	**8**	**8**

In the writing part of the words into number sentences all learners in all four groups were able to write the number sentences using the correct inequalities.

In responding to question 4 of worksheet 3, learners displayed an **ability to replace an empty space with the correct equality or inequality**. Learners used the knowledge gained in worksheet 1 and 2 to be able to complete this question.

The knowledge gained was arranging and identifying integers and temperatures. The following excerpt provides evidence that the majority of learners were able to replace an empty space with the correct equality or inequality.


4 Replace □with ">"; "<" or "=" to make the following number sentences true

(a) (-2) □ (+2)……<…… (b) 3 □(-5) ……>…… (c) (+18) □18 ……=……

(d) 0 □(-20) ……>…… (e) (-12) □(+4) ……<……

(a) (-2) □ (+2)……<…… (b) 3 □(-5) ……>…… (c) (+18) □18 ……=……

(d) 0 □(-20) ……>…… (e) (-12) □(+4) ……<……

4 Replace □with ">"; "<" or "=" to make the following number sentences true

(a) (-2) < (+2)……<…… (b) 3 □(-5) ……>…… (c) (+18) □18 ……=……

(d) 0 > (-20) ……>…… (e) (-12) □(+4) ……<……

4 Replace □with ">"; "<" or "=" to make the following number sentences true

(a) (-2) □ (+2)……<…… (b) 3 □(-5) ……>…… (c) (+18) □18 ……=……

(d) 0 □(-20) ……>…… (e) (-12) □(+4) ……<……

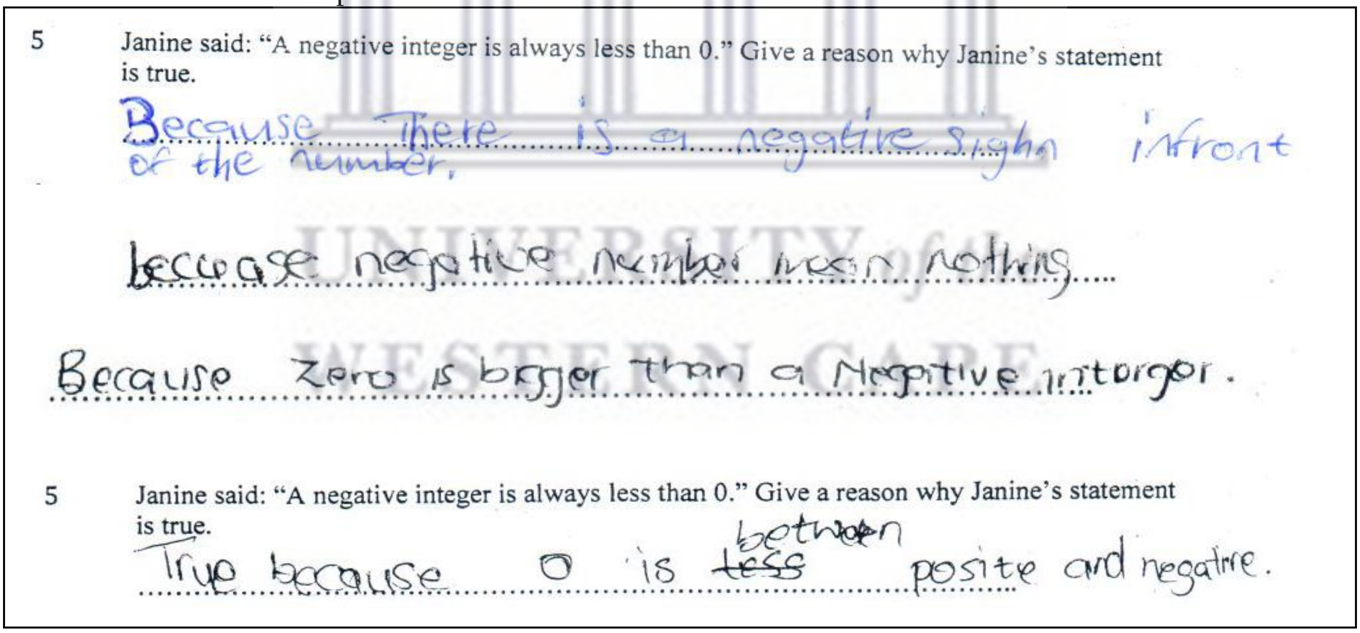
Figure 4.15: Example of learners' responses

Learners used the previously discussed knowledge where they argued in worksheet 2 that a number with a plus sign in front is equal to the same number without a plus sign in front. The table below shows how many learners were able to replace an empty space with an equal or inequality sign.

Table 4.15: Number of learners who displayed an **ability to replace an empty space with the correct equality or inequality**

	Group 1	Group 2	Group 3	Group 4
Able to replace □ with =; > and <.	8	8	8	8
Total	**8**	**8**	**8**	**8**

In responding to question 5 of worksheet 3, learners displayed an **understanding that negative integers are always less than 0 but inability to convince with reasons why they think so.** The excerpt below shows how learners put their reasons.


5 Janine said: "A negative integer is always less than 0." Give a reason why Janine's statement is true.

Because There is a negative sighn infront of the number.

because negative number mean nothing

Because Zero is bigger than a Negative intorger.

5 Janine said: "A negative integer is always less than 0." Give a reason why Janine's statement is true.

True because 0 is ~~less~~ between posite and negative.

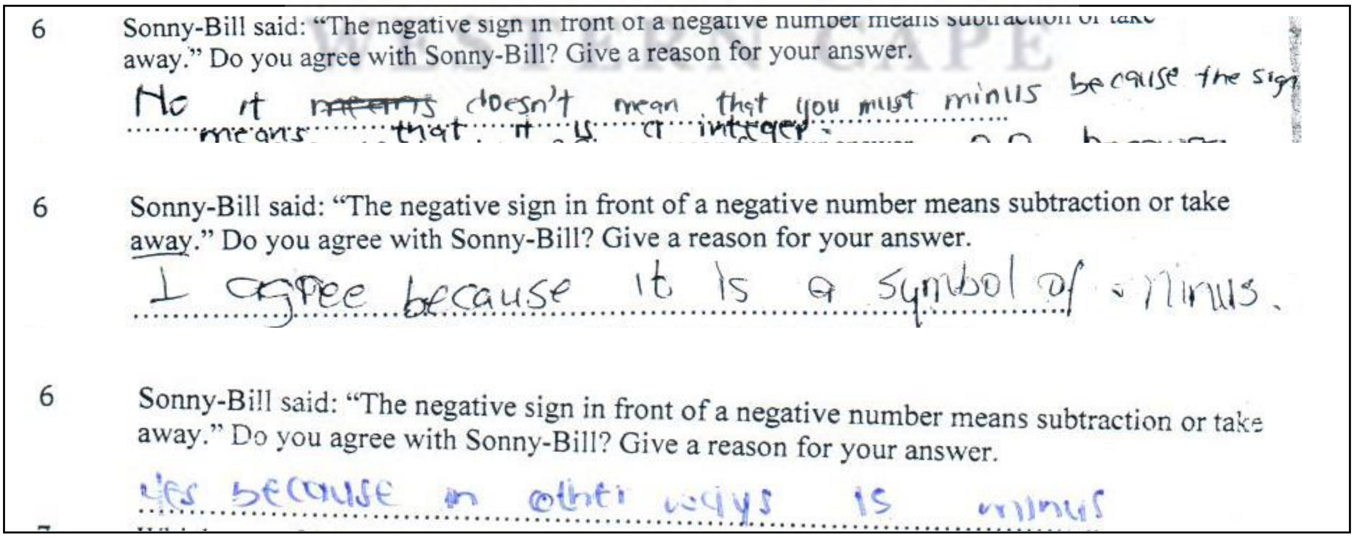
Figure 4.16: Example of learners' reasoning

The table below shows how many learners were able to provide convincing reasons or not mathematically convincing reasons.

Table 4.16: Number of learners who displayed an **understanding that negative integers are always less than 0 but inability to convince with reasons why they think so.**

		Group 1	Group 2	Group 3	Group 4
True	Convincing reason	0	0	0	0
	Not convincing reason	8	8	8	8
Total		**8**	**8**	**8**	**8**

In responding to question 6 of worksheet 3, learners displayed an **inability to explain the meaning of a negative sign in front of a negative number**. The instruction statement read as following: "The negative sign in front of a negative number means subtraction or take away." The above statement might mean the following: - - 8.


6 Sonny-Bill said: "The negative sign in front of a negative number means subtraction or take away." Do you agree with Sonny-Bill? Give a reason for your answer.

No it ~~means~~ doesn't mean that you must minus because the sign means that it is a integer.

6 Sonny-Bill said: "The negative sign in front of a negative number means subtraction or take away." Do you agree with Sonny-Bill? Give a reason for your answer.

I agree because it is a symbol of minus.

6 Sonny-Bill said: "The negative sign in front of a negative number means subtraction or take away." Do you agree with Sonny-Bill? Give a reason for your answer.

Yes because in other ways is minus

Figure 4.17: Example of learners' explanations

The statement would be better understood if it stated: “The negative sign in front of a number means subtraction and that would be the following: - 8. The researcher is using 8 as an example number to clarify the meaning of the ambiguity of the statement. The statement from Sonny-Bill was ambiguous as it contained more than one meaning because his intentions were different from what was written. It might be because of Sonny-Bill’s statement that the mixed reaction between learners occurred.

The table below shows how many learners were able to provide convincing reasons and how many could not to convince completely.

Table 4.17: Number of learners who displayed an **inability to explain the meaning of a negative sign in front of a negative number**

		Group 1	**Group 2**	**Group 3**	**Group 4**
Agree	Convincing reasons	0	0	0	0
	Not convincing reasons	4	4	4	4
Disagree	Convincing reasons	0	0	0	0
	Not convincing reasons	4	4	4	4
Total		**8**	**8**	**8**	**8**

In responding to question 7 of worksheet 3, learners displayed **an inability to explain using convincing reasons why other numbers are not integers**. These learners understood other sets of numbers but could not make a link between what they already know and integers. Learners could recognise that the numbers used in question 7 are decimals but they could not further explain why decimals are not necessarily integers. The excerpt below shows how learners provided convincing reasons why other numbers such as decimals may not necessarily be regarded as integers.

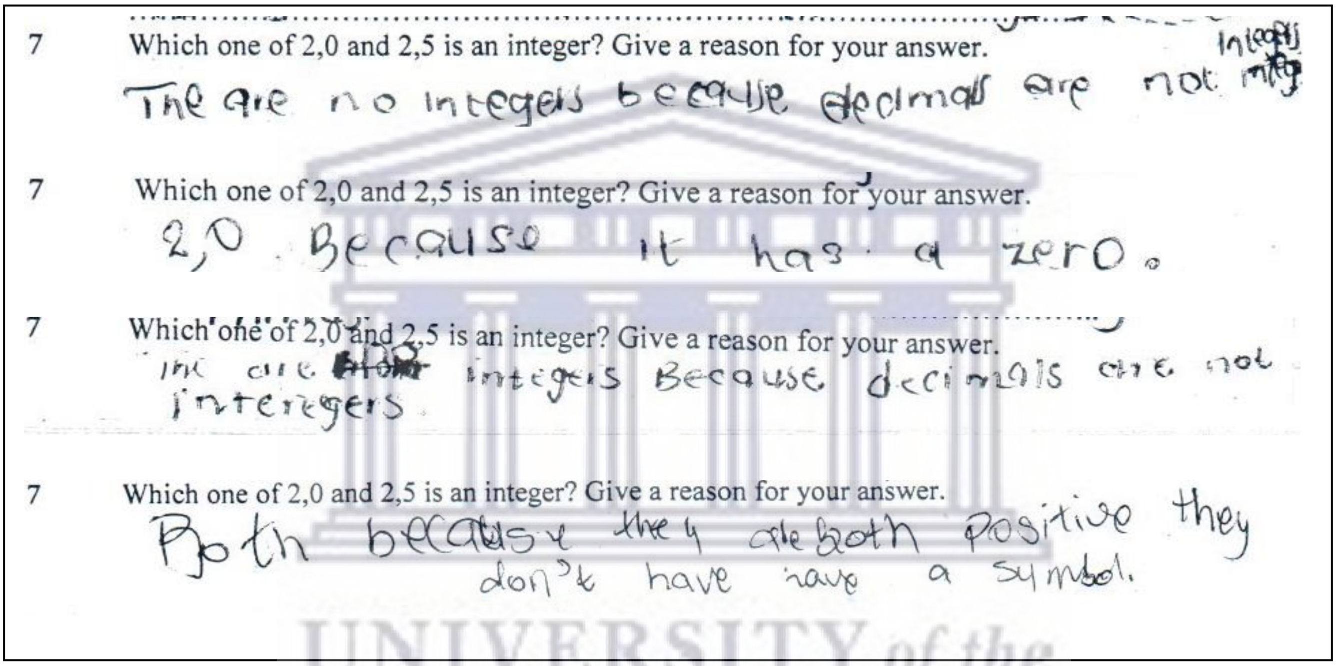


Figure 4.18: Example of learners' attempts to provide convincing reasons

The table below shows the number of learners who provided convincing reasons for their responses.

Table 4. 18: Number of learners who displayed **an inability to explain using convincing reasons why other numbers are not integers**.

		Group 1	Group 2	Group 3	Group 4
Both are integers	Convincing reasons	0	0	0	0
	Not convincing reasons	0	3	2	3
None are integers	Convincing reasons	0	2	2	1
	Not convincing reasons	8	3	4	4
Total		**8**	**8**	**8**	**8**

4.3 Summary

There were altogether 19 question statements that identified 19 categories. The 19 categories were further reduced to 10 categories for all 3 worksheets. The categories identified were:

Worksheet 1 categories

1. Awareness of the difference between ordinary numbers and numbers in temperature context.
2. Awareness that numbers in the context of temperatures are written differently from ordinary whole numbers
3. Tendency to get to the answer without focusing on the entire question statement
4. Competency in arranging temperatures in descending order
5. Competency in arranging temperature in ascending order

Worksheet 1 combined categories

- Awareness that numbers in the context of temperatures are different from ordinary whole numbers and are written differently.
- Tendency to get to the answer without focusing on the entire question statement
- Competency in arranging temperatures in ascending and descending order.

Worksheet 2 categories

1. Capability in writing positive numbers with a plus sign in front
2. Capability in writing negative numbers with a negative sign in front
3. Capability of writing numbers that are in symbols into numbers that are in words
4. Capability of writing numbers that are in words into numbers that are in symbols
5. Capability of writing integers in descending order
6. Capability of identifying integers from other numbers
7. Capability of identifying integers even when instructions are not clear

Worksheet 2 combined categories

- Capability in writing positive and negative numbers with a plus or negative sign respectively in front of a number.
- Capability of writing numbers in symbols into words or words into numbers.
- Capability of writing integers in descending order.
- Capability of identifying integers even when instructions are not clear or from other numbers.

1. An understanding that positive numbers are always greater than negative numbers with some providing general although not completely articulated reasons why they think so
2. Ability of writing word sentences with correct inequalities from number sentences but expressed some differences when saying the word sentences

3. Ability in saying and writing the word sentences into number sentences using the relevant inequalities
4. Ability to replace an empty space with the correct equality or inequality
5. Understanding that negative integers are always less than 0 but inability to convince with articulated reasons why this is so
6. Inability to explain the meaning of a negative sign in front of a negative number
7. Inability to explain using convincing reasons why other numbers are not integers

Worksheet 3 combined categories

- Ability of replacing, saying and writing word sentences with correct equality or inequality.
- Inability to explain the meaning of a negative sign in front of a negative sign and use convincing reasons why other numbers are not integers.
- An understanding that positive numbers are always greater than negative numbers or negative numbers are always less than 0 but unable to provide articulate reasons why.

4.4 Conclusion

This chapter presented the results forthcoming from the analysis of learners' work for the three worksheets dealing with the introduction of integers to a grade 7 class. Nineteen categories were originally identified. These were reduced to ten categories. As given in the summary section. The reduced categories as per worksheet are given in Table 4.19 below.

Table 4.19: Calculation of percentages per reduced category

Worksheet	Category	Number of learners giving satisfactory responses	%
1	Awareness that numbers in the context of temperatures are different from ordinary whole numbers	26	41
	Tendency to get to the answer without focusing on the entire question statement	23	72
	Competency in arranging temperatures in ascending order	44	69
2	Capability in writing positive and negative numbers with a plus or negative sign respectively in front of a number.	57	89
	Capability of writing numbers in symbols into words or words into numbers	58	91
	Capability of writing integers in descending order	25	78
	Capability of identifying integers even when instructions are not clear or from other numbers	48	75
3	Ability of replacing, saying and writing word sentences with	96	100

	correct equality or inequality		
	Inability to explain the meaning of a negative sign in front of a negative sign and use convincing reasons why other numbers are not integers	13	20
	An understanding that positive numbers are always greater than negative numbers or negative numbers are always less than 0 but unable to provide articulate reasons why	7	11

The satisfactory responses where categories were combined were obtained by adding the number of learners who responded satisfactorily for all of the combined satisfactory responses by the total number of learners for the combined categories. For example, in the category "Competency in arranging temperatures in ascending and descending order" 44 learners in total responded satisfactorily. Two categories were combined and the total number of learners was 64 for the two categories giving the percentage at 69%.

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CHAPTER 5

Discussion, Recommendation and Conclusion

5.1 Introduction

In this chapter the results reported in Chapter 4 are discussed. The research focused on the introduction of integers. It specifically focused on "recognise, compare and order integers" (Department of Basic Education, 2011, p.67) as stated in the curriculum and assessment policy statement (CAPS) document. A temperature model was used to introduce integers and the research was based on how the temperature model assisted or did not assist the introduction of integers to grade 7 learners. The results are discussed in relation to the notions of recognition, comparing and ordering of integers.

5.2 Discussion of results

The combination of intentional teaching strategy with the applied temperature model was used to assist in the introduction of integers and did contribute to learners' better understanding of integers. The percentages shown in table 4.19 of the combined categories of worksheet 1, 2 and 3 in chapter 5 are evidence that learners were able to recognise, compare and order integers. There were some few learners who were not able to recognise, compare and order temperatures although the majority of them were able to exhibit these skills.

The problem arose when learners had to explain the meaning of a negative sign in front of a negative sign. Another difficulty was also encountered when learners had to explain why positive numbers are always greater than negative numbers. Some of these learners thought that zero is the smallest number and all other numbers whether negative or positive are always bigger than zero. These learners still maintained their previous view that zero is the smallest number without meaning and the first number in the set of whole numbers.

Learners engaged in arguments in trying to support their statements why they think negative numbers are always smaller than zero.

Learners were at liberty to express their views through discussions in the language of their choice but the first additional language or the language of learning and teaching was preferred. The discussions were thereafter transcribed into the preferred language as presented in chapter 4. Learners' expressions in English seemed to be a serious threat and challenge as it proved them to be less competent in recognising, ordering and comparing integers. This was because these learners could not express themselves by providing reasons why they recognise integers or why they arranged them; ordered and compared integers in the manner they have arranged them.

A total of nine combined categories out of ten combined categories from table 4.19 in the previous chapter were positive categories to support the claim that there was more learners' understanding of integers. The positives were that learners displayed awareness, competency and capability when responding to the questions and that further proved that learners' understanding of integers. The high percentages of over 50% achieved in these combined categories by learners providing satisfactory response also support the above claim. In the 9 combined categories only 2 categories stood alone with 32 learners. The remaining 7 combined categories had 64 learners and 6 of them with over 50% of learners responding satisfactorily. Only 1 of the 7 categories had 41% of learners responding satisfactorily.

The only negative category in all 10 combined categories was the combined category of the inability to explain the meaning of a negative sign in front of a negative sign and use convincing reasons why other numbers are not integers. The category is deemed negative by the researcher as it required learners to support their reasoning by explaining the meaning of a negative sign in front of a negative sign. The explanation is where the challenge came from as it required learners' intelligent thinking. Further discussions about recognition of integers, comparing integers and ordering integers will be conducted as derived from table 4.19 shown in the previous chapter.

5.2.1 Recognition of integers

The alternative words for "recognise" are identify, know, distinguish, be familiar, make out, be aware of or be in familiar terms with. Learners demonstrated three times that they are able to recognise integers in this research as shown in table 4.19 in the previous chapter.

The first demonstration comes from the combined categories as per table 4.19 for the combined categories of worksheets 1, 2 and 3 in the previous chapter. The combined category displaying the ability to write numbers in symbols into words or numbers in words into symbols became the top category with the highest number of learners who provided satisfactory responses. In this combined category 91% of the 64 learners provided satisfactory responses. The fact that these learners were able to write a negative number from words to symbols and words to a negative number in symbols meant that they could recognise integers. Learners recognised integers with temperature in everyday situations and in mathematical domain.

The second combined category which verified that learners demonstrated the ability to recognise integers is found in their display of being capable of identifying integers. Learners could also identify integers from a given set of numbers. When presented with -16; -½; +8; $+2^{3}/_{5}$; 0; +20; -¾ and -101, 50% of the learners correctly selected -16; +8; 0; +20 and -101 as integers in the set.

This category demonstrated that the learners were able to recognise integers. In the combined category, "capability of identifying", learners showed that they were capable of recognising integers. In this category 75% of the 64 learners demonstrated the capability to recognise integers.

The third combined category attesting that learners were able to recognise integers came from the category displayed as awareness that numbers in the context of temperatures are different from ordinary whole numbers.

Some of the less satisfactory responses provided included learners' writing of "negative numbers, positive numbers and zero" as an answer rather than only identifying by selecting integers from the provided set of numbers.

The above three indications provide an indication that these learners demonstrated the skill to recognise integers as a required learners' skill by the grade 7 assessment standards on integers.

Stephan & Akyuz, (2012, p.429) argues that "students need to be able to interpret the minus sign in multiple ways." This means that if learners recognise integers they might as well be able to demonstrate skills relating to negative numbers, like the meaning of take away, the opposite of positive, direction or a sign in front of a number. Learners' ability to demonstrate the above skills means that they developed an understanding of how to apply the negative sign when solving integer problems or integer calculations. Chapter 2 discussed that learners find difficulty in solving problems such as -4 – (-8). In this situation the learners should be able to apply the meanings of (-8) as "negative eight" and it appears that working through the recognition of integers as in the resources used in this study might alleviate the difficulty referred to by Stephan & Akyuz.

5.2.2 Comparing integers

The word "compare" is synonymous with equates, liken, equal, associate, link, match or parallel.

The first of the two combined categories from table 4.19 in worksheet 3 displays an understanding that positive numbers are always greater than negative numbers and negative numbers are always less than zero, attested to the fact that these learners were capable of comparing integers. In this combined category 63% of the 64 learners demonstrated that they were able to compare integers as they provided satisfactory responses. The remaining 37% of the 64 learners could not provide satisfactory responses. Learners' disagreements with Janine related to the thought that zero is the least number in the set of integers and is regarded as an unsatisfactory response.

Some learners cannot conceptualise zero to be bigger than negative numbers and to them zero means nothing or it is the first number.

The inability to provide satisfactory responses might be because of learners regarding zero as the smallest number of all positive and negative numbers.

This is supported by Stephan & Akyuz's (2012, p.429) who assert that "students have similar difficulties, such as conceptualising numbers less than zero".

The second of the two combined categories that demonstrated that the learners were able to compare integers is found in the only three combined categories of worksheet 3. The combined category is displayed as the ability to replace, say and write word sentences with correct equality or inequality. The 32 learners who provided satisfactory responses in each of the first 2 categories even before the combination of the 3 categories is evidence that the learners were able to compare integers. Few learners encountered difficulties in the third category displayed as ability to replace an empty space with the correct equality or inequality but this was resolved by the teacher's interaction with the learners so that they reach the correct responses. After the teacher-learner interaction there was an overwhelming majority of learners who could replace an empty space with the correct equality or inequality.

In this combined category all 96 learners attested to be able to compare integers by providing satisfactory responses. The saying and writing part of what was already written in front of learners was much easier than the replacement of an empty space by less than, equal to, or greater than sign. The difficulty was in the replacement part as learners had to think and make an informed decision about which integer is less than, equal to, or greater than, between the 2 integers in a given number sentence. Learners reached their correct responses through teacher-learner interaction when they struggled with providing the correct response. The teacher-learner interaction involved the teacher asking the learner to name the coldest area between an area with the temperature of positive 2 degrees Celsius and an area with negative 2 degrees Celsius.

When the teacher asked this question the learner considered the situation and subsequently came up with the answer.

The teacher waited for an appropriate opportunity to ask this question. The opportunity is when the teacher realises that the learner struggles to reach the correct answer and is kept busy thinking. That is when the teacher takes advantage of the situation to build on what the learner thinks.

An integer is very commonly conceived of as a combination of the sign and the number (Ryan & Williams, 2007, p.24). According to Ryan & Williams 2007, learners have to understand that negatives are smaller numbers than positive numbers to be able to compare integers. This therefore means that -1, -2 and -3 are less than +1, +2, and 3, in that order. Ryan & Williams (2007) further affirm that children's comparison of integers precedes integer computation.

The ability to compare integers using words and symbols flexibly applying the equal, less than and greater than signs should be the primary objective while the computations with integers might be the secondary objective in the development of learners' conceptual understanding of integers.

The above two combined categories are indications demonstrated by the learners that they were able to compare integers as one of the requirements of the grade 7 assessment standards on integers.

5.2.3 Ordering integers

The synonyms for the word "ordering" are: collation, organisation and collection, gathering and assembling. In the above meanings collation means to put something in a definite order such as ascending or descending.

The third of the three combined categories was the first to attest that learners were able to order integers. This combined category is found in worksheet 1 and displayed as competency in arranging temperatures in ascending and descending order. In this combined category 69% of the 64 learners responded with satisfactory responses.

In this category learners had not only collated temperatures but also put them in ascending and descending orders as well The collation and understanding of temperatures required learners to have mastered the previously discussed points of recognition and the comparing of integers. The temperatures used in the worksheets lead to the introduction of integers hence the two terms temperature and integers where the latter is used in relation to integers.

The second category that attested to learners being able to order integers came from worksheet 2 as the only category that is not combined with others. The category is displayed as the capability to write integers in descending order according to table 4.19. In this category 78% of the 32 learners provided satisfactory responses. The learners understanding of how to write integers in ascending and descending orders was an indication that they have mastered the recognition of integers through being able to write integers fluently. It was mentioned earlier that ordering integers may only be achieved through having recognised and compared integers first.

Learners' ability to compare, order and recognise integers will contribute to the enhancement of their conceptual understanding of integers. The above three indications displayed by these learners were evidence that they were able to order integers as a required learners' skill by the grade 7 assessment standard on integers.

5.3 Recommendations

During this research the researcher learnt about helpful and productive techniques employed in the application of an intentional teaching strategy. Intentional teaching strategy taught the researcher to stay focused on the intended goals. The application of an intentional teaching strategy results in adherence to the assessment for learning (AfL) as discussed in chapter 2.

The assessment for learning further gives rise to clear success criteria that the researcher monitored throughout the duration of the lesson.

Sustaining the focus of the success criteria right through the lesson helps the teacher to articulately achieve the intended goals. The researcher learnt that a teacher using an intentional teaching strategy should constantly ask the learners the following questions: “we are learning to?” and “what I am looking for?” When these questions are subconsciously kept in the teacher and the learner’s minds then the intended objectives are reached.

Teaching using an intentional teaching strategy was the researcher’s first experience of this approach and was perceived to be effective and productive. The researcher ended up by discarding previous teaching strategies and deciding to transform to intentional teaching. The researcher’s assumption is that learners’ understanding of integers in grade 7 improved as compared to when the other teaching strategies like the chalk and talk strategy, textbook method, group-work, pair work, were used previously in the teaching of the same concept. The learner’s engagement during discussions about temperatures and integers improved relatively to previous approaches used to introduce integers. There was more of an atmosphere of belonging in class and participation in class activities when an intentional teaching approach was employed. The teacher videoing the lesson was also impressed by the lesson introduction and presentation when commenting later to the researcher. The videoing teacher commented that this was the best lesson he ever listened to and would score the researcher high in Integrated Quality Management Systems (IQMS) if he were to evaluate it.

5.3.1 Recommendations for teaching

The implementation of an intentional teaching strategy when introducing integers is highly recommended by the researcher as it provided successful results in its first trial. The researcher commends teachers’ efforts in encouraging learners to freely discuss challenges encountered in mathematics problems. The researcher always supports the notion of preparation and planning before the lesson starts and robustly encourages the idea of adhering to such principles. The researcher therefore highly recommends thorough preparation and planning that would lead to innovative use of quality resources in class activities. These enhanced resources could ultimately lead to improved learners' conceptual and procedural understanding of mathematical concepts.

To teach using an intentional teaching strategy requires that teachers should support one another in doing away with old practices. These old practices include teachers making prompt feedback to learners' work. The old practice of marking becomes difficult to unlearn when a teacher has practiced it for long.

An example of this scenario is when the writer left learners on their own to mark their homework while doing something else. This old practice is not ideal as it tends to leave learners unsure by not concentrating on their errors so as to build on their conceptual understanding of mathematics. Another common old practice includes teachers not carefully reading instructions given in the learning resources.

Pre-planning is very important as the researcher encountered a situation in this research study of handing out worksheets at the same time without reading the instructions first. This cursory perusal led the researcher to take for granted that learners were requested to respond on integers while they were required to respond on temperatures. The two may slightly be said and written the same with the positive and the negative signs in front but temperatures are always written with a degree's Celsius sign at the end which is not applicable with integers.

Learners' activities and worksheets are written by somebody else and therefore human error is possible. It is thus recommended that the teacher should prepare prior to track down these errors. The discussion in chapter 2 indicates that teachers do not generally review the assessment questions that they use and do not discuss them critically with peers, so there is little reflection on what is being assessed. In worksheet 1, question 3, learners were asked to either agree or disagree with Thandi when she said "We can also say that the maximum temperature for Saturday is plus 2 degrees".

I was not too sure about the answer as the 2 may be added to the maximum which was 2 the previous day or the minimum which was -10 of the same previous day. The researcher assumed that the plus 2 is added to the maximum of Friday.

The researcher therefore recommends that teachers should frequently mark learners' work and make prompt feedback. The researcher also recommends that teachers should carefully read learning resources' instructions so that learners answer accordingly. The pre-reading of these activities and worksheets will inevitably ease the uncertainties for other expected and unexpected responses.

5.3.2 Recommendations for the duration of the lesson

The duration of the lesson was satisfactory as it took two periods of 30 minutes each to finish the entire lesson. This means that for effective application of an intentional teaching strategy to take place one has to reserve a double period. The application of this strategy might not work well in single periods of 30 minutes. The teacher should pre-plan the lesson as the phase contains 4.5 hours of teaching time per week. Time for the introduction and consolidation of the lesson should be allocated in the planning of a lesson when the intentional teaching approach is employed.

5.3.3 Recommendations for the worksheets

The worksheets were clear and comprehensible to all learners. The resources used in this research project were of high quality but the writer recommends the adding of colours to the worksheet with the weather chart. The success criteria chart was hung up and introduced at the commencement of the lesson. Learners were made conscious of what and where they should be by the end of the lesson through the success criteria. This research study on its own enhanced the researcher's approaches to the teaching of mathematics.

5.3.4 Recommendations for the research

As articulated in Chapter 4, learners found it difficult to engage in discussions with one another. This might be due to them coming from a culture of mathematics teaching where they essentially have to follow procedures that a teacher has demonstrated. The researcher recommends that research be done on ways to facilitate discussions on mathematical ideas amongst learners.

The findings articulated in Chapter 4 as reduced categories might have provided just a glimpse of a bigger problem and therefore a bigger target group is recommended for the provision of more specifics in data analysis. The researcher recommends a further study which would expand the target group in order to enrich data analysis and interpretation. Another study could look at what happens when learners are given an opportunity by the teacher reading the questions for the learners in order to provide clarity and more understanding of what is being asked. This is of course a study related to teaching.

The researcher would recommend a comparative study between the two different models mentioned in chapter 2 and the temperature model. A comparison of an elevator model, a beach model and the temperature model might produce the most effective model of the three models.

An example of an elevator model can be the creation of worksheets around this example of elevator model such as the following: Mr. Peter went to his car parked down 27m from his office in the parking bay. He drove 10m from the parking garage to the street level. To facilitate discussion ask a learner to state how far above the street is Peter's office? Ask learners to give reasons for the answer.

An example of a beach model can be the creation of worksheets around a beach model in a situation like the following; Melisa and Jessica were playing with sand at the beach. Melisa dug a hole that was 22cm below the surface and Jessica built a tower that was 15cm high above the surface. Ask learners to agree or disagree with statements such as; the difference of the depth and the height of their constructions is 7cm. This would be done to check which of these models is most effective.

The temperature model has been used as a model for this study. An example of a temperature model can be a creation of worksheets around a situation such as the following: the average temperature in Calgary, Canada, is 22°C in July and -2°C in January. Mention other cities around Canada with different temperatures. Ask learners to replace equal, greater than and less than signs between the temperatures of different cities.

The researcher recommends an experimental study between the three models to determine which model produces enhanced learners' understanding of integer concept. Choosing three separate schools where each school will be randomly allocated with one of the three models as a lesson and investigate which school learners attained better learners' understanding of the concepts involving integer arithmetic. This would be done to check which of the three models best produces better learners' understanding of integer concept when taught through the implementation of an intentional teaching strategy.

5.4 Conclusion

In grade 7 learners encounter integers for the first time in their schooling career and this first encounter often brings about learners' misinterpretation of the integer concept. The implementation of an intentional teaching strategy and the temperature model in this study is therefore highly significant as it supports learners' better understanding of integers. The researcher has taught integers for years in grade 7 and never previously received such an overwhelming support of the learners' understanding of this concept.

Learners' discussions encountered in Chapter 4 are some of the exciting highlights brought about by this study. Previously, the researcher's classroom discussions were only restricted to teacher to learner discussions but during the implementation of an intentional teaching strategy discussions were extended to learner to learner. The latter point is in fact the cornerstone of an intentional teaching strategy.

To experience the exploration of the implementation of an intentional teaching strategy and the effectiveness of this strategy was an eye opener to the researcher. It was also striking to see the number of learners who could comprehend integers when the intentional teaching strategy was employed than when it was not employed. The main question that lingered in the researcher's mind was why he was not exposed earlier to such a productive strategy as it would have increased my learners' understanding of integers.

The researcher wishes to make the following suggestions regarding the introduction of integers in grade 7. The suggestions include the following. Firstly, of all is that the teachers should try to assist learners in getting to understand what is asked in the worksheets rather than leaving them on their own. Such assistance would create a situation whereby learners answer what is required.

Secondly, that integers should always be introduced with a model, be it a temperature model, beach model, elevator model or any other appropriate model mentioned in Chapter 2. Thirdly, integers should be introduced in small steps but be allocated more time in the curriculum. If the curriculum considers 2 periods of 30 minutes each equalling to 1 hour, then 4 periods equalling to 2 hours should be considered.

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Appendix A

Learners' classwork

INTEGERS and INTEGER ARITHMETIC

LESSON 1

GOALS/INTENTIONS

In this lesson learners will learn to

- Use temperature to learn about positive and negative numbers
- Place positive and negative numbers in order from smallest to largest and largest to smallest
- Write positive and negative numbers
- Give and use a definition of integers
- Compare integers using the equal (=); greater than (>) and less (<) signs

SUCCESS CRITERIA

Learners will be successful if they can:

1 Write positive and negative numbers

2 State and write down what integers are

3 Choose integers from a given set of numbers

4 Arrange integers from the smallest to the largest and from the largest to the smallest

5 Correctly place the symbols "=", ">" and "<" between two integers

MATERIALS NEEDED

The workbooks of the learners, a copy of each of the worksheets (the size of the worksheets can be reduced dependent on your knowledge of your learners' ability to read smaller print sizes), glue stick to paste the activities in their workbooks, coloured crayons, pencils or kokis and newsprint to make a poster.

You will also need a sheet with the goals/intentions of the lessons on it to use as display and reduce the time taken to write down goals/intentions on the chalkboard.

INTRODUCTION

- This lesson deals with the **concept** of an integer and elements related to the concept.
- Before the lesson learners do the readiness activity to get a sense of what ordinary terms/words about positive and negative numbers learners already know. You can use these terms/words they use to develop the ideas of "positive" and "negative".

  The 'readiness' activity is also used to ascertain whether learners can arrange temperatures from the highest to the lowest and vice versa. This is a precursor to ordering integers.
- Allow learners to work in groups of 2 to 4 learners with minimum disruption to impact on teaching and learning time.

TIME NEEDED

10 minutes to complete worksheet 1 before the lesson (possibly homework), 10 minutes for worksheet 2; 10 minutes for worksheet 3 and about 10 minutes to produce a poster.

CLARIFICATION NOTES for WORKSHEET 1

The integers are introduced using a temperature model. Research shows that there is no one model that is comprehensive in the sense of allowing it be used for all the constructs to be taught and learned for integers and operations related to integers. The temperature model is selected because of its "obviousness" in terms of familiarity and linkage to "correctness" related to ordering of integers.

Question	Intention of the question
1 (Study the table and read aloud to someone the highest and lowest temperature for the week.)	To get of sense of whether learners are aware of the terminology used with ‘directed’ numbers and a sensitisation to the magnitude of ‘directed’ numbers.
2 (Write down in words the minimum temperature for Thursday.)	Writing number symbols in words. This is to assess whether learners are aware of a sign accompanying a number. Ultimately this is to deal with the unary meaning of ‘minus’ in in ‘directed or signed’ numbers and the notation of (+number) and (-number). [The signs are ‘attached’ to the numbers and do not indicate an operation.]
3 (Thandi said: We can also say that the maximum temperature for Saturday is *plus 2 degrees*. Write down why you agree or disagree with Thandi.)	This is a follow-up on the previous question and introducing an equivalent way for denoting whole with which learners are conversant.
4 (Look at the temperatures for Friday, Saturday and Sunday. Write the six temperatures from the highest to the lowest.)	This is intended to introduce the ordering of the integers
5 (Look at the temperatures for Tuesday, Wednesday and Thursday.	This is intended to introduce the ordering of the integers

Write the six temperatures from the lowest to the highest.)	

Introduction to lesson

Give each learner a copy of worksheet 1 (*What Do You Know About Temperatures?*) to complete either as homework or before the lesson. If the activity is done in class before the lesson, you should refrain from 'helping' the learners because you want to know what learners already know and what they can already do.

Assessing learners' responses for worksheet 1

Collect learners' responses. Do not 'mark' learners' work. Make notes of the "words" and "notations" learners are using. Discuss these with the learners and allow them to discuss it with another. Keep in mind that you are interested to know whether learners know "words" such as "minus", "plus" and that they indicate some direction with respect to zero. Some questions and prompts you can give are given in the table below. (These are based on our interpretations of how learners might respond. Add to the list so that you can use them in forthcoming years or share with other teachers.)

Issue	Questions and prompts
Interpreting 0 as the smallest number (Question 1). The lowest temperature is read "0".	Is a temperature of 0 degrees colder or warmer than a temperature -5 degrees?
Focusing only on the numerical part of the number (Ignoring the unary idea of the 'minus' sign) (Question 1). The highest temperature is "-10"	Is a temperature of -10 degrees warmer or colder than a temperature of 1 degree?
Only the numerical part is written in words for negative temperatures.	Is - a word or a symbol?

(Generally "1" is not read "plus one" in weather matters. Do not focus on this issue. Accept "one".) (Question 2) "-five" is written.	
Response is positive but reason is not provided. (Question 3) Examples: I agree. Yes. **Response is negative but reason is not provided. (Question 3)** Examples: I disagree. No	Also write down where you have read or heard it. Have you ever heard of "*plus 2 degrees*"? If no, tell learners "2 degrees can be written as "*plus 2 degrees*".
The ordering is by only considering of numerical parts of the negative temperatures (Questions 4 and 5) Example: 10; 8; 4; 2; 2; 2 for descending ordering.	Draw learners' attention to think about temperatures from the 'warmest' to the 'coldest' and vice versa.
Negative numbers are ordered next to their positive counterparts (Questions 4 and 5) Example: -7; -3; 0, 1; -5; 5 **Negative numbers are treated as zero and the numbers are placed to the right of zero. (Questions 4 and 5)** Example: 0; -7; -5; -3; 1; 5 **Zero is used as anchor and numbers placed on both sides of zero. (Questions 4 and 5)**	Develop questions for learners to answer around 'temperatures'. Also give write some of the orderings on the chalkboard and allow learners to discuss the orderings.

Example: 1; -3; 0; -5; 5; -7	

CONTINUING THE LESSON I (WORKSHEET 2)

Hand out worksheet 2 (What are *integers* 1?). Let learners read the opening paragraph. Judge whether learners grasp the second sentence of paragraph.

CLARIFICATION NOTES for WORKSHEET 2

Question	Intention of the question
1 ("Plus" numbers are called *positive numbers* and written as (+*number*). Write down three positive numbers.)	The term 'positive number' and its way of writing is introduced. "Positive" is introduced so that the confusion with "plus" as addition is addressed. The "( )" notation is used to maintain the unary meaning of the signs when dealing with negative numbers.
2 ("Minus" numbers are called *negative numbers* and are written as –*number*. Write down three negative numbers.)	The same as for 1 above.
3 (Say and write the following numbers in words: (-5); (+7); 0; (+10); (-1))	The focus is that learners practice reading and writing "positive" and "negative"
4 (Say and write the following words as number symbols: positive four; negative twenty; negative fifty-three; positive one.)	Same as for 3 but from word-form to symbol-form.
5 (Write the following integers from the highest to the lowest: (-3); (+10);	Spiral revision of ordering.

(+20); (-21); 0; (+2); (-15).)	
6 (*Integers* are positive whole numbers, negative whole numbers and zero. Which of these numbers are integers?)	Definition of integers and selecting integers from a set of numbers containing ordinary and decimal fractions.
7 (Most of the time the positive sign (+) is not written in front of positive number. Only 2 is written for (+2). Complete the following: The integers in the set of numbers (-16); $(-\frac{1}{2})$; 8; $2\frac{3}{5}$; 0; (+20); $(-\frac{3}{4})$; (-101) are…)	Alert to the representation of positive integers as whole numbers and consolidation the identification integers.

Issue	Questions and prompts
Difficulty in grasping the second sentence in the opening paragraph.	Read aloud the maximum and minimum temperature for Monday. If only "three" and "minus one" is said: Is there something that goes with it?
Only the numerical part is written down. (Question 1) Example: 2	What are all the things that are in "(+number)"?
The negative number is written without the brackets. (Question 2) Example: -1	What are all the things that are in "(-number)"?
"Plus" and/or "minus" are used to read	What are the words that we used with

the numbers (Question 3). Example: (-5) is read as "minus 5".	*number* in questions 1 and 2?
Ordering of numbers (Question 5)	See the issues for ordering in worksheet 1 exercises.
Confusing integers with temperatures (Question 6). Example: Integers are positive and negative temperatures.	Are numbers always connected to things such as temperatures?
Ignoring the fractional parts and focusing only 'whole' number part (Question 6). Example: $(+1\frac{1}{2})$ is an integer because there is a (+1).	What kind of number is $(+1\frac{1}{2})$?
Not accepting that positive whole numbers are integers (Question 7). Example: 8 is not part of the selected list of integers.	What do you understand by "2 is sometimes written for (+2)"?

CONTINUING THE LESSON II (WORKSHEET 3)

Hand out worksheet 2 (Comparing integers) for learners to do.

CLARIFICATION NOTES for WORKSHEET 3

Question	Intention of the question
1 (State whether the following is true or false: (+2) is greater than (-5). Give a reason for your answer.)	To assess whether learners can compare integers by linking it to their experiences with ordering of integers.

2 ("(+2) is greater than (-5)" can be written as "(+2) > -5", where ">" is the symbol for "is greater than". The symbol "<" is the one used for saying "is less than". Say and write in the following number sentences in words.)	Reading inequality statements. Translating inequality statements from symbol-form to word-form.
3 (Say and then write the following words as number sentences.)	Same as for 2 but from word-form to symbol-form.
4 (Replace ☐ with ">"; "<" or "=" to make the following number sentences true)	Consolidation of the use of the symbols for comparing pairs of integers.
5 (Janine said: "A negative integer is always less than 0." Give a reason why Janine's statement is true.)	Assessment of learners' understanding of 0 in relation to negative integers.
6 (Sonny-Bill said: "The negative sign in front of a negative number means subtraction or take away." Do you agree with Sonny-Bill? Give a reason for your answer.)	Assessment of whether learners grasp the unary nature of signed numbers.
7 (Which one of 2,0 and 2,5 is an integer?)	Spiral revision of definition of an integer.

Issue	**Questions and prompts**
Learners ignore the sign of (-5) (Question 1).	If you arrange the integers from (+2) up to (-5) from the smallest to the largest, which one

False 5 is greater than 2	is the smallest?
Words such as "bigger"; "smaller"; etc are used. (Question 2). Example: negative one is smaller than positive one	What did you read above about the symbol "<"?
Symbols and words are mixed. (Question 2). Example: (-1) is less than (+1)	How are the words we say when we read (-1)?
Words and symbols are mixed. (Question 3). Example: negative four < zero	How do we write "negative four" in number-form?
The incorrect symbol is place between the two integers. (Question 4). Example: 0 **<** (-20).	See question and prompts for question 1.
Zero is taken as the smallest number (Question 5). Example: Of all numbers zero is always the smallest.	See question and prompts for question 1.
The unary use of the minus sign is interpreted as the binary use (Question 6). Example: The minus sign means take away	Does "-5$°C$" mean "take away" 5$°C$?
The decimal (rational number) form of an integer is not recognised.	Are the number 2 and 2,0 the same?

Example: 2,0 is a decimal number.	

CONCLUDING THE LESSON

Hand out the newsprint, coloured crayons, pencils or kokis and ask to make a poster of "What they have learned in the lesson." They must give examples of the things they write on the poster. Encourage them to feel free and use their creativity to design their poster. If possible, show them examples of posters produced other learners. (An example of a poster is given after the worksheets.)

After they have completed their posters, paste your sheet with the intentions/goals and success criteria on the board and go through them with illustrating examples.

Worksheets

WORKSHEETS 1 - 3

What Do You Know About Temperatures?

Sutherland is in the Northern Cape. The South African Large Telescope (SALT) is near Sutherland. It is generally believed that during winter the coldest temperatures in the country are recorded in Sutherland.

The following maximum and minimum temperatures were recorded for Sutherland during a specific winter for a week.

	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Maximum ($^\circ C$)	3	5	0	1	2	4	-2
Minimum ($^\circ C$)	-1	-3	-7	-5	-10	-2	-8

1 Study the table and read aloud to someone the highest and lowest temperature for the week.

……………………………………………………………………………………………………..

2 Write down in words the minimum temperature for Thursday.

……………………………………………………………………………………………………..

3 Thandi said: We can also say that the maximum temperature for Saturday is *plus 2 degrees*. Write down why you agree or disagree with Thandi.

……………………………………………………………………………………

4 Look at the temperatures for Friday, Saturday and Sunday. Write the six temperatures from the highest to the lowest.

……………………………………………………………………………………………………..

5 Look at the temperatures for Tuesday, Wednesday and Thursday. Write the six temperatures from the lowest to the highest

……………………………………………………………………………………………………..

WORKSHEET 2

What are *integers?*

You have seen above that temperatures are recorded as "plus", "minus" or "zero" temperatures. If we only use the 'plus' numbers, 'minus' numbers and zero and not the degrees for the temperatures then these numbers are called *integers.*

1 "Plus" numbers are called *positive numbers* and written as (+*number*). Write down three positive numbers.

………………………………………………………………………………………

2 "Minus" numbers are called *negative numbers* and are written as (–*number*). Write down three negative numbers.

……………………………………………………………………………………………

3 Say and write the following numbers in words: (-5); (+7); 0; (+10); (-1).

……………………………………………………………………………………………

4 Say and write the following words as number symbols: positive four; negative twenty; negative fifty-three; positive one.

……………………………………………………………………………………………

5 Write the following integers from the highest to the lowest: (-3); (+10); (+20); (-21); 0; (+2); (-15).

……………………………………………………………………………………………

6 *Integers* are positive whole numbers, negative whole numbers and zero.

Which of these numbers are integers?

Number	Integer (Yes or No)?	Reason
$(+1\frac{1}{2})$		
(-18)		
(+201)		
(-0,25)		

7 Most of the time the positive sign (+) is not written in front of positive number. Only 2 is written for (+2).

Complete the following: The integers in the set of numbers (-16); $(-\frac{1}{2})$; 8; $2\frac{3}{5}$; 0; (+20); $(-\frac{3}{4})$; (-101) are

………

WORKSHEET 3

Comparing integers?

1 State whether the following is true or false: (+2) is greater than (-5). Give a reason for your answer.

…………………………………………………………………………………………………

2 "(+2) is greater than (-5)" can be written as "(+2) > -5", where ">" is the symbol for "is greater than". The symbol "<" is the one used for saying "is less than". Say and write in the following number sentences in words.

(a) (-1)<(+1): ………………………………………………………………………………

(b) (-4)>(-16): ……………………………………………………………………………

(c) (+2)>(-10): ……………………………………………………………………………

(d) 5<8: ……………………………………………………………………………………

3 Say and then write the following words as number sentences.

(a) Positive three is greater than negative three …………………………………….

(b) Negative six is less than negative three ……………………………………………..

(c) Negative four is less than zero ……………………………………………………….

(d) Positive eight is greater positive two …………………………………………………

4 Replace ☐ with “>”; “<” or “=” to make the following number sentences true

(a) (-2) ☐ (+2)……….. (b) 3 ☐ (-5) ……….. (c) (+18) ☐ 18 ………..

(d) 0 ☐ (-20) ……….. (e) (-12) ☐ (+4) ………..

5 Janine said: “A negative integer is always less than 0.” Give a reason why Janine’s statement is true.

…………………………………………………………………………………………..

6 Sonny-Bill said: “The negative sign in front of a negative number means subtraction or take away.” Do you agree with Sonny-Bill? Give a reason for your answer.

…………………………………………………………………………………………..

7 Which one of 2,0 and 2,5 is an integer? Give a reason for your answer.

Appendix B

PROJECT INFORMATION SHEET

UNIVERSITY *of the* WESTERN CAPE

Project information sheet:

The study proposed here intends investigating the implementation of intentional teaching in grade 7 learners when teaching integers. The aim of this study will be to investigate the effectiveness of applying an intentional teaching model as a teaching strategy to conceptual and procedural understanding of integer arithmetic. The study will be conducted over a two months' period which will consist of the integer topics prescribed in the curriculum for grade 7 mathematics.

Benefits:

The study will benefit the WCED and all educators in the improvement of mathematics results. The study will also benefit the learners in their conceptual and procedural understanding of mathematics.

Confidentiality and Consent:

Confidentiality will be ensured during the transcription of interviews as well as during analysis, reporting and publication. Learners will have the right to their identity being concealed with regard to all information collected in the course of the research. The same regard will be upheld with respect to the school involved. Learners will be fully informed of the nature of the research without being coerced to be involved. Learners also have the right with respect to written reports and informal discussions about the research.

If you have any questions concerning this research feel free to call Mr Soga at 0216376419 (mncesoga@gmail.com) or Prof C. Julie at 0219592861 (cjulie@uwc.ac.za) and Prof M. Mbekwa at 0219592957 (mmbekwa@uwc.ac.za)

Signed by Supervisors: Prof C. Julie and Prof M. Mbekwa

Letter to Teacher

UNIVERSITY *of the* WESTERN CAPE

Vuyani Public Primary School

NY 58 Guguletu

Cape Town

7750

18/August/2015

Consent letter

Vuyani Public Primary School

NY 58 Guguletu

Cape Town

7750

Dear Colleague

RE: REQUEST TO ASSIST IN RESEARCH CONDUCT

This serves to request your assistance in data collection during research conduct at Vuyani Primary school. Your assistance will involve writing notes, recording of interviews and class organization.

I hope that this request is given full consideration and undivided attention. If you need more clarity feel free to consult M. Soga on 0216376419 (mncesoga@gmail.com) and supervisors Prof C. Julie at 0219592861 (cjulie@uwc.ac.za) and Prof M. Mbekwa at 0219592957 (mmbekwa@uwc.ac.za)

Faithfully yours

M.H. Soga

_______________ (signed)

Letter to parent requesting permission for child to participate in the study

UNIVERSITY of the WESTERN CAPE

Vuyani Public Primary School

NY 58 Guguletu

Cape Town

7750

18/August/2015

Consent letter

Dear Parent/Guardian

RE: REQUEST FOR YOUR CHILD'S PARTICIPATION

I am a Master's student conducting a research project titled "The implementation of an intentional teaching model to investigate grade 7 learners' engagement with integer operations". I humbly request your child to participate in a study during data collection that will take place at his/her school. Your child will be expected to participate in individual interviews which will be taking place at his/her school. Observation will also be conducted in this research project.

Notes to the parent/guardian

- Your child's identity will not be divulged under any circumstances.
- All the responses will be treated with strict confidentiality.
- Fictitious names will be used to represent participants' names.
- Participation is voluntary therefore learners are free to withdraw at any time.

• Learners will not be forced to disclose what they do not want to reveal.

My contact: cell: 0711941538; Email address: mncesoga@gmail.com or 3115662@myuwc.ac.za

I thank you in advance. My supervisors contact details are cjulie@uwc.ac.za on 0219592861 and mmbekwa@uwc.ac.za on 0219592957.

Yours sincerely

Mncedisi Help Soga

____

If you understand and agree to participate, please sign a declaration form.

DECLARATION FORM

1……………………………………………. (Full names of the parent/guardian) hereby confirm that I have read and understood the contents of this document and the nature of the research project, and consent to let my child participate in the research project.

I understand that my child is at liberty to withdraw from the project at any time should he/she so desire.

SIGNATURE DATE

………………………………….. ………………………………………..

Letter to Principal

Vuyani Public Primary School

NY 58 Guguletu

Cape Town

7750

18/August/2015

Consent letter

The School Principal

Vuyani Public Primary School

P.O Box 359

Gatesville

7766

Dear Sir/Madam

RE: REQUEST TO CONDUCT THE RESEARCH AT YOUR SCHOOL

I am a Master's in education student at the University of the Western Cape conducting a research project titled "The implementation of an intentional teaching model to investigate grade 7 learners' engagement with integer operations. I humbly request permission to conduct my study at your school. Learners will be taught and interviewed but the interviews will be treated with high level of confidentiality.

Learners will participate voluntarily and be free to withdraw from the study any time they so wish. Fictitious names will be used to represent participants' names. The information will not be used for any purpose other than for this study. I hope that the results from this study will be beneficial to the school and the education district at large. My contact details and are provided below for further clarification of this study. Thank you in advance.

Yours faithfully

Mncedisi Help Soga (Mr) ____________________

Telephone 0216376319 (w) Cell 0711941538/0832471442

Email: mncesoga@gmail.com/3115662@myuwc.ac.za

Supervisors: Prof C. Julie, email: cjulie@uwc.ac.za telephone (w): 0219592861.

Prof M. Mbekwa, email: mmbekwa@uwc.ac.za telephone (w): 0219592957.

Consent form

1………………………………………………… (Full names) hereby confirm that I understand the contents of this document and the nature of the research project, and that I allow the researcher to conduct the research project at my school.

Signature Date/ stamp

……………………………….. ………………………………………

Letter to Western Cape Education Department (WCED)

Vuyani Public Primary School

NY 58 Guguletu

Cape Town

7750

18/August/2015

Consent letter

The Director of Research Services

Western Cape Education Department

Private Bag 9114

Cape Town

8000

Dear Sir/Madam

RE: REQUEST TO CONDUCT A RESEARCH AT VUYANI PRIMARY SCHOOL

This serves as a request for permission to conduct a research project at Vuyani Primary School, a school in the Metro Central Education District. I am a Master's student at University of the Western Cape conducting a research project titled "The implementation of an intentional teaching model to investigate grade 7 learners’ engagement with integer operations". The study is an academic study of which teachers and the Education Department might benefit as it will help teachers understand intentional teaching and how to improve learners’ mathematics performances. The estimated duration for the study is eight weeks.

My supervisors for the study are Prof C. Julie and Prof M. Mbekwa on contact details 0219592861 and 0219592957 respectively. The email addresses are cjulie@uwc.ac.za for Prof C. Julie and for Prof M. Mbekwa is mmbekwa@uwc.ac.za

Thanking you in advance for your cooperation.

Yours faithfully

M.H. Soga

____

Telephone: 0216376419 (w) and cell 0711941538 or 0832471442

Email: mncesoga@gmail.com or 3115662@myuwc.ac.za

ANNEXURE A.

Education

Audrey.wyngaard@westerncape.gov.za
tel: +27 021 467 9272
Fax: 0865902282
Private Bag x9114, Cape Town, 8000
wced.wcape.gov.za

REFERENCE: 20151015-4254
ENQUIRIES: Dr A T Wyngaard

Mr Mncedisi Soga
77 Paulina Street
Victoria Mxenge
Phillipi
7800

Dear Mr Mncedisi Soga

RESEARCH PROPOSAL: THE IMPLEMENTATION OF AN INTENTIONAL TEACHING MODEL TO INVESTIGATE GRADE 7 LEARNERS' ENGAGEMENT WITH INTEGER OPERATIONS

Your application to conduct the above-mentioned research in schools in the Western Cape has been approved subject to the following conditions:

1. Principals, educators and learners are under no obligation to assist you in your investigation.
2. Principals, educators, learners and schools should not be identifiable in any way from the results of the investigation.
3. You make all the arrangements concerning your investigation.
4. Educators' programmes are not to be interrupted.
5. The Study is to be conducted from **18 January 2016 till 30 June 2016**
6. No research can be conducted during the fourth term as schools are preparing and finalizing syllabi for examinations (October to December).
7. Should you wish to extend the period of your survey, please contact Dr A.T Wyngaard at the contact numbers above quoting the reference number?
8. A photocopy of this letter is submitted to the principal where the intended research is to be conducted.
9. Your research will be limited to the list of schools as forwarded to the Western Cape Education Department.
10. A brief summary of the content, findings and recommendations is provided to the Director: Research Services.
11. The Department receives a copy of the completed report/dissertation/thesis addressed to:

The Director: Research Services
Western Cape Education Department
Private Bag X9114
CAPE TOWN
8000

We wish you success in your research.

Kind regards.
Signed: Dr Audrey T Wyngaard
Directorate: Research
DATE: 16 October 2015