AN ANALYSIS OF LEARNERS' WAYS OF SOLVING LINEAR EQUATIONS: THE CASE OF GRADE 10 LEARNERS IN THE WESTERN CAPE, SOUTH AFRICA.

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Date: December 2023

DECLARATION

I declare that "*An Analysis of Learners' Ways of Solving Linear Equations: The case of Grade 10 Learners in the Western Cape,South Africa"* is my work, that it has not been submitted for any degree or examination in any other university, and that all the sources I have used or quoted have been indicated and acknowledged by complete references.

Jeanne d'Arc Muberarugo Date: December 2023

Signed

DEDICATION

To my husband Mr. Gilbert Ndayisaba, my daughter Nelissa Sugi Ndayisaba and Bernice Umwizerwa Ndayisaba, I dedicate this work.

ACKNOWLEDGEMENTS

It would not have been possible to complete my Masters's Education work without the grace of God and the support of the people around me. These people deserve to be acknowledged especially:

- I am grateful to my supervisor, Dr. Marius Simons, for his patience, encouragement, effort, and guidance throughout this study. This would not have been possible without your guidance.
- To my dear husband Gilbert, and my children Nelissa and Bernice. You have always been my inspiration to move a step higher in everything I do.
- To my late father Mr. Mwizerwa Ignace, and my father-in-law Mr. Ndayisenga Francois, I express my heart-felt gratitude for the encouragement and your positive attitude and inspiration even amid personal and academic challenges. I attribute this success to you.
- My mother Mrs. Mukamfizi Beatrice who is still encouraging me and my siblings, especially Mugorewera Marie Chantal, I thank you.
- To my colleague, sister, and friend Dr. Karangwa Ineza Claire, thank you for your support, motivation, and encouragement;
- Thanks to my lovely brothers and friends, Dr. Emmanuel Tuyishimire; Dr. Adrien Ndamyabera; and Dr . Jean Baptiste Ngilirabanga for all your support.

ABSTRACT

While there are several factors for poor performance (Mamba,2013), there is a tendency amongst learners to struggle to transform linear equations from other types of equations and thereafter follow a suitable solution-seeking path. This study presents a conceptual framework that gives a comprehensive idea of the research that was done in this field as well as formulating an analysis framework from it. The theoretical underpinning is further strengthened by the constructivist learning approach.

The objective of this study is to understand the types of errors and misconceptions learners make in their ways of working with Mathematics in an examination context. These errors are observable in the learners written responses in their answer-seeking pursuance when solving linear equations in the grade 10 school-based final examination. The study sought to find errors that will show learners' conceptual and procedural understanding of Mathematics when engaging with linear equations in Algebra.

The study seeks to give feedback, which will contribute to the teaching and learning of linear equations and how they may be derived from other types of equations or Mathematics problems. From this, the researcher noted that learners' learning difficulties are usually presented in the form of errors they show (Mamba,2013). Additionally, Mamba notes that some errors in procedures can be caused by faulty algorithms or "buggy algorithms". Other errors can have a conceptual basis and hence can be termed "misconceptions".

Results show that learners' examination scripts in the final grade 10 of the school-based examination in 2021 gave valuable insight into the errors of learners and strategies that teachers use in the classroom to correct these errors and misconceptions. The research employed qualitative methods and was conducted in the Metropole South Education District, an urban education district situated in Cape Town, within the Western Cape province of South Africa. The results indicate that learners tend to make procedural mistakes as they lack a complete understanding of the underlying structural properties involved in the transition from exponential equations to linear equations.

KEYWORDS

Algebra

Algebraic equation

Exponential equation

Teaching strategies

Examination

Errors

Misconceptions

Alternative ways of working

Feedback

Definitions of keywords

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CHAPTER 1
BACKGROUND AND MOTIVATION

1.1 INTRODUCTION

The challenge of understanding mathematics is a common problem faced by many countries worldwide (Lau, Hawes, Tremblay&Ansari,2022). In South Africa, learning and teaching mathematics remains a challenge in the education system (Feza, 2015). Since the dawn of democarcy in1994, the Department of Education has implemented various reforms to improve the quality of education, including the introduction of a new Curriculum 2005, National Curriculum Statement (NCS), Revised National Curriculum Statement (RNCS) and, most recently, the Curriculum and Assessment Policy Statement (CAPS) (Raoano, 2016).

The Department of Education hoped that changing the curricula would help learners better understand mathematics concepts, including solving algebraic equations. However, despite the change in curricula, education challenges persist. There are several factors contributing to learners' poor performance in linear equations and other types of equations(Raoano, 2016). Girley and Emybel (2019) argue that poor performance in mathematics can be attributed to teachers' failure to impart the necessary knowledge, skills, attitudes, and values to the learners.

Mathematics is a crucial subject that tests students' problem-solving abilities, memory retention, and mental agility, all of which are essential in daily life (Radatz, 1980). It's important to note that a teacher's effectiveness plays a significant role in students' performance too. To improve the learners competence in maths and science graduates, it's imperative to attract, produce, utilize, and retain excellent mathematics teachers (Mamba, 2013). For instance, the concept problem solving in mathematics are often challenging for students, and as such rfequires a positive attitude towards maths so as to improve their performance (Mingke & Alegre, 2019).

However, the CAPS document (2018) emphasizes that mastering mathematical concepts is a sequential process as failure to understand one concept has a trickle-down effect on subsequent concepts. Many students make mistakes in the early stages of learning mathematical concepts, leading to difficulties in higher-level mathematics (Veloo, Krishnasamy, & Wan Abdullah, 2015).

South Africa's Department of Basic Education (DBE) and other stakeholders are continually working to improve mathematics performance across all grades. Unfortunately, the grade 12 National Senior Certificate (NSC) has shown consistent low performance in mathematics results. According to the DBE's diagnostic reports, the number of applicants who wrote Mathematics during the grade 12 final examination decreased by 14178 nationally in 2018 to 2019.

According to a report by the Department of Basic Education(Department of Basic Education, 2019), there has been a decline in mathematics passrate as indicated above , with learners achieving 30% and 40% marks experiencing a decline of 3.4% and 2.1%, respectively. Additionally, there has been a decrease of 0.5% in learners achieving distinction (80%-100%) in mathematics. Only 54% of Mathematics exam candidates achieved a minimum passing score of 30%, and the total failure rate remained above 50% (Department of Basic Education, 2019).

To address this issue, the Department of Education in Western Cape has implemented various mathematical strategies to improve performance in mathematics, but the problem persists (Mashazi,2014). The diagnostic analysis reveals a lack of algebraic skills and mathematical competencies among learners, highlighting the need for teachers to address common errors and misconceptions and teach algebra in a more meaningful way(Diagnostic Report of Non-language Subject,2019). The report emphasizes the importance of understanding definitions and concepts, particularly in linear equations, to solve problems effectively.

10.2 OVERVIEW OF LEARNER PERFORMANCE IN PAPER 1

a. It was evident, from the marking process, that many candidates were better prepared to answer the routine questions and scored some marks in the majority of the questions. This is very encouraging going forward.

b. The algebraic skills of the candidates are poor. Most candidates lacked fundamental and basic mathematical competencies which could have been acquired in the lower grades. This becomes an impediment to candidates answering complex questions correctly.

c. Whilst calculations and performing well-known routine procedures form the basis of answering questions in a Mathematics paper, deeper understanding of definitions and concepts should not be overlooked.

Figure 1: Overview of learner performance in paper 1.

Thus, the researcher agrees with what is mentioned in the overview of learner performance in paper 1 above (Figure 1), which indicates that certain mathematical competencies should be acquired and understood in the early grades. Hence, the interest of this study is to investigate the errors and missunderstandings that are visible when learners are solving linear equations from other types of equations espacilly in algebra.

Algebra is a fundamental topic in school mathematics in South Africa, as outlined in the Continuous Assessment Policy Statement (CAPS). It serves as a prerequisite for higher mathematics, according to Lee and Bull (2018), who also studied the cognitive capabilities required for developing algebraic problem-solving skills. The importance of algebra in high school and tertiary education cannot be overstated, as understanding the reasons for learners' algebraic errors can help educators identify and overcome obstacles to learning (Mashazi, 2014).

Hariyani (2018) notes that learners often lack the creativity and logical thinking necessary for solving mathematical problems in real-world situations. These critical thinking skills are essential for systematic problem-solving. Despite extensive research on the challenges of constructing linear algebraic equations from word problems, learners' poor performance remains a persistent issue. This presents a challenge for researchers to discover strategies that can improve the teaching and learning of algebraic concepts in real-life contexts (Feza, 2015).

As a Mathematics tutor, I have observed that learners face significant challenges when it comes to linear equations originating from word problems. Recurring errors and misunderstanding often contribute to students' difficulties and hinder their proficiency. Through my investigations, I have gained valuable insights into the obstacles that learners face and the strategies that can help them overcome these challenges.

It has been noted that the lack of diverse question types in word problems in certain schools where data was collected could contribute to misconceptions and errors among students(Feza, 2015). Without exposure to a range of question types, students may struggle to identify patterns and apply appropriate problem-solving strategies. As a result, a more nuanced exploration into exponential equations was undertaken to determine if similar errors and misunderstaning/misconcepton persist in other mathematical domains or if distinct patterns emerge (Mamba, 2013).

By shifting focus to exponential equations, this presents an opportunity to not only address issues related to linear equations but also gain a comprehensive understanding of the broader spectrum of mathematical challenges students face. By delving into the errors and misconceptions that arise from their ways of working, we can better appreciate the difficulties learners experience in studying mathematics.

Studies have shown that learners often struggle with solving linear equations from other forms of equations on selected mathematical topics in their tests, examinations, assignments, and daily classwork(Dizha, 2021). It is crucial for schools to produce learners who can identify and solve problems and make informed decisions through critical and creative thinking. The development of general mathematical skills such as ductive reasoning, understanding of rational numbers, procedural fluency with computational skills, and advanced problem-solving skills is emphasized by the Department of Basic Education(DBE, 2011). By addressing mathematical misconceptions or missunderstanding early enough, learners can build a strong foundation to excel in mathematics at a higher level.

1.2. SIGNIFICANTS OF THE STUDY

1.2.1. Errors and misconceptions in mathematical work

Errors and misconceptions in mathematical work are significant for several reasons, both in terms of learning and application: errors and misconceptions provide valuable learning opportunities. When students or mathematicians make mistakes for instance, they often gain a deeper understanding of the concept through the process of identifying and correcting those errors (Ben-Hur, 2006). Within the realm of mathematics education, it is important to distinguish between errors and misconceptions, while recognizing their interconnectedness. As Ben-Hur (2006) explains, an error is characterized by an inaccuracy, whereas a misconception stems from a misunderstanding. Specifically, a misconception arises when a student accepts a false concept as true, whereas an error may be a result of said misconception (Muzangwa and Chifamba, 2012). This process can lead to a more profound grasp of the underlying principles. According to

Muzangwa and Chifamba (2012), errors and misconceptions help students in highlighting areas of weakness or misunderstanding in relation to mathematical knowledge. Also, recognizing these gaps is the first step in addressing and rectifying them. Without errors, it can be challenging to pinpoint areas that require improvement (Ball, Hill, and Bass, 2005). Mistakes often reveal underlying misconceptions about mathematical concepts.

By identifying these misconceptions or misunderstanding, individuals can work on developing a more accurate and robust understanding of the topic, which can lead to better problem-solving skills (Schoenfeld, 2014). Errors and misconceptions can help educators understand common stumbling blocks that students encounter. Brown and Campione (1986) argued that knowledge allows teachers to design better instructional strategies, anticipate potential misunderstandings, and tailor their teaching methods to address specific misconceptions.

Analysing errors and misconceptions in mathematical work promote critical thinking and problem-solving skills (Belecina and Ocampo, 2018). It encourages individuals to question their assumptions, evaluate their reasoning, and develop the ability to spot errors in their work and the work of others. Dealing with errors and misconceptions can foster resilience and persistence in problem-solving. It teaches individuals to persevere, even in the face of challenges and setbacks, which are essential qualities in mathematics and many other fields (Hill, Blunk, Charalambous, Lewis, Phelps, Sleep and Ball, 2008).

In practical applications of mathematics, such as engineering, finance, or scientific research, identifying and rectifying errors is crucial for ensuring the accuracy and reliability of results (Attaran and Deb, 2018). Errors can have significant real-world consequences, making it essential to catch and correct them. In fields like data analysis and statistics, errors and misconceptions can have ethical implications. Misleading or incorrect mathematical analyses can lead to biased decisions or misinformation, emphasizing the importance of accuracy in mathematical work.

Briefly, errors and misconceptions in mathematical work are not just setbacks but also valuable learning experiences. They help individuals improve their mathematical understanding, enhance problem-solving skills, and contribute to the advancement of mathematics and its applications in various domains.

1.2.2. Errors and misconceptions learners do when solving linear equations

Researchers have found that studying the common errors and misconceptions that students make while solving linear equations is significant (Booth & Koedinger (2008). Moreover, according to Booth and Koedinger (2008), this knowledge can help teachers understand the challenges their students face, and create better learning experiences. It's fascinating how even small mistakes can have a big impact on learning outcomes.

Therefore, understanding common errors and misconceptions helps educators design more effective teaching strategies. By knowing where students tend to go wrong, teachers can develop targeted interventions to address these issues, leading to better learning outcomes. Borasi and Rose (1989) said that identifying specific misconceptions that individual learners may have allows for personalized instruction. This can help educators tailor their teaching to the needs of each learner, leading to more efficient and effective learning. Research on errors and misconceptions can inform the development of curriculum materials and textbooks (Foster, Francome, Hewitt, and Shore, 2021). It allows curriculum designers to anticipate and address common stumbling blocks, making educational resources more user-friendly and impactful especially when writing tests and responding quizzes.

Tests and quizzes can be designed to not only evaluate knowledge but also diagnose misconceptions, providing valuable feedback to both students and teachers (Foster, Francome, Hewitt and Shore, 2021). Addressing these misconceptions can lead to a deeper comprehension of algebraic concepts, which can be foundational for more advanced mathematics and other disciplines.

In light of the above, the study of errors and misconceptions in how students solve linear equations is a significant area of research that contributes to the broader field of mathematics education (Trouche and Fan, 2018). It helps researchers investigate the origins of misconceptions, how they evolve as learners progress through their mathematical education, and the most effective pedagogical approaches to address them (Smith III, DiSessa, and Roschelle, 1994). By correcting errors and misconceptions, students' confidence in their mathematical abilities can be boosted, which can lead to a positive attitude towards mathematics as a whole.

Studying errors and misconceptions in mathematics education is important as it informs teaching practices, curriculum development, assessment methods, and research. The Department of Basic Education (DBE) has attempted to give feedback to teachers and stakeholders on the factors that cause learners to perform poorly in certain topics in Mathematics (DBE, 2011). This feedback is meant to help teachers improve the teaching and learning of mathematics from grade 8 to grade 12.

The diagnostic analysis provided by the DBE explicitly indicates to teachers the misconceptions and errors that learners have while solving mathematical problems, and suggests ways to improve teaching and learning of certain mathematical concepts and competencies in the early grades. With the diagnostic report, further feedback is given to teachers in the form of an in-depth question analysis, showing the performance of individual questions. This feedback is meant to help teachers focus on their teaching and learning of mathematics in the classroom. Moreover, recognizing a learner's mistake is essential for improving their performance in Mathematics (Fuchs, Fuchs & Prentice, 2004). Therefore, it is important to help teachers teach algebra in a meaningful way and address and rectify the common errors that learners produce (Mashazi, 2014). By doing so, learners and the education system in general will benefit from the findings of this research.

1.3. AIM OF THE STUDY

This study aims to identify the strategies learners follow when solving mathematical linear equations from other types of equations in their grade 10 final examinations by recognising exhibited errors in their responses. This was done to give comprehensive feedback to learners and educators, to advance the Mathematics teaching and learning .

1.4. RESEARCH QUESTIONS AND SUB-QUESTIONS

The following is the main question that this study sought answers for:

What are the errors and misconceptions visible in the learners written answers in their solution-seeking pursuance when solving linear equations from other types of equations related to algebraic equations?

Sub-questions:

What are the strategies used in the learners written answers in their solution-seeking pursuance when solving linear equations from other types of equations related to algebraic equations?
What are the factors that contribute to these errors and misconceptions?

.

1.5. OUTLINE OF THE CHAPTERS IN THE STUDY

The thesis consists of five chapters which are arranged to provide deeper insight into problems raised and to provide answers to the research questions that guided the current study. Chapter 1 provides an overview of the study, the background and motivation of the study, the significance of the study , the aim ,research questions, sub-questions ,and key terms of the study .

Chapter 2 provides a comprehensive review of related literature to the study.

Chapter 3 provides a theoretical framework and a conceptual framework for the study. From the conceptual framework the analytical framework was developed.

Chapter 4 provides an outline of a research methodology that guided the current study. This chapter addresses issues relating to the research design that was employed in this study; the study population, and the sampling techniques that were used in this study. In addition, instrumentation, data collection procedures, data analysis techniques, and issues relating to ethical considerations are also addressed.

Chapter 5 is a presentation and analysis of the collected data against the research question.

Chapter 6 is the concluding chapter. It presents the findings, conclusions, and recommendations for further study based on this research.

CHAPTER 2

LITERATURE REVIEW

2.1. Introduction

In the study of Borasi from 1996, it was observed that the educational field has not adequately analyzed the concept of error-making in education. As a result, the community of education has yet to discover innovative ways of utilizing errors constructively within formal instruction. Borasi (1996) emphasizes that errors can serve as valuable tools for identifying learning difficulties, planning curriculum and teaching materials accordingly, and gaining insight into conception of students and learning processes .

The objective of this chapter is to provide a comprehensive overview of research conducted on errors in Mathematics. The aim is to enhance our understanding of different error types and develop an analytical framework that can be used to classify misconceptions and errors . The specific emphasis of this discussion and review will be on solving linear equations from other forms of equations, as outlined in the national curriculum statement for grade 12 Mathematics in South Africa.

2.1.1. The importance of Algebra

Algebra uses symbols for generalizing Arithmetic, these symbols have different meanings and interpretations in different situations (Samo, 2009). Samo (2009) points out that students have different perceptions about these symbols, letters, and signs.

Algebra is a core component of the Mathematics curriculum for all students (Ahmad and Shahrill, 2014). Algebra has been considered a key aspect in forming the base courses in advanced Mathematics and Science coursework in Secondary and Post-Secondary Education, as argued by Muchoko, Jupri and Prabawanto (2019).

According to Ziegler and Stern (2014), Algebra has been considered as a gate keeper to students' further education and career. Additionally, Muchoko et al. (2019) argued that for students to be successful in secondary school algebraic topics such as equations and functions, and in post-secondary courses such as Calculus and Algebra, they need to overcome difficulties that they have in the basic aspects of algebraic operations.

Algebraic equations serve as the foundation for numerous areas of modern mathematics, having been extensively studied by researchers. According to Muchoko et al. (2019), students often struggle with visualizing algebraic forms and applying the associative and distributive properties of algebraic expressions when simplifying or factoring algebraic terms. Şengül and Üner (2010) argue that the acquisition of abstract thinking is crucial for students when learning complex concepts. Additionally, they note that the measurement of logical ability has been used as a criterion for scientific achievement.

2.1.2. Equations in Mathematics Education

Equations have a rich historical background in mathematics education, playing a fundamental role in problem-solving and mathematical reasoning. The study of equations can be traced back to ancient civilizations, where early mathematical systems emerged (Boyer, 1991). In ancient Egypt and Babylon, basic equations were already being solved. These civilizations developed methods for solving linear equations, often in practical contexts such as land measurement and quantity calculations (Katz, 1998).

The ancient Greeks made significant contributions to the understanding of equations. Mathematicians like Euclid and Diophantus explored various types of equations and their solutions. Euclid, known for his work in geometry, also addressed linear equations (Heath, 1908). Diophantus considered the "father of algebra," introduced the concept of algebraic symbols and symbolic notation to represent unknowns (Katz, 2007).

During the Islamic Golden Age, scholars like Al-Khwarizmi made substantial advancements in algebra. Al-Khwarizmi's book "Kitab al-Jabr wal-Muqabala" introduced systematic methods for solving linear and quadratic equations (Sesiano, 2000).

In the 16th and 17th centuries, symbolic algebra revolutionized the study of equations. Mathematicians like François Viète and René Descartes introduced algebraic notation, enabling the manipulation of equations using symbols and letters (Katz, 1998). The 18th and 19th centuries witnessed further advancements in equation solving techniques. Mathematicians like Euler and

Gauss expanded the understanding of equations, exploring concepts like polynomial equations, systems of equations, and transcendental equations (Boyer, 1991).

In the 20th century, with the advent of computers, equation solving became more accessible and efficient. Numerical methods and algorithms were developed to solve complex equations (Burden & Faires, 2010). Additionally, computer algebra systems (CAS) emerged, enabling students and mathematicians to solve equations using computational tools (Sutherland et al., 2004).

Equations continue to be a central topic in modern mathematics education. They are taught at various grade levels, serving as a foundation for understanding algebraic structures and mathematical modeling (National Council of Teachers of Mathematics, 2000). The historical development of equations in mathematics education reflects the ongoing evolution of mathematical thinking and problem-solving strategies. From ancient methods to symbolic notation and computational tools, equations have played a vital role in mathematical exploration (Burton, 2010).

2.2. Types of equations in mathematics education

In mathematics education, several types of equations are studied and solved. These include linear equations, quadratic equations, polynomial equations, exponential equations, logarithmic equations, trigonometric equations, rational equations, radical equations, absolute value equations, and systems of equations (Young,2023). Each type of equation has its own characteristics and solution methods, and they find applications in various mathematical disciplines and real-world problem-solving scenarios. Understanding these different equation types is essential for developing mathematical reasoning and problem-solving skills. Some of the key types of equations include:

2.2.1. Polynomial Equations

Polynomial equations are a versatile type of equation in mathematics education. They involve expressions with multiple terms, each having a different power of the variable. Polynomial equations can include linear terms (power 1), quadratic terms (power 2), cubic terms (power 3), or even higher-order terms. These equations can be written in the form "a_nx^n + a_{n-1}x^{n-1} + ... + a_1x + a_0 = 0," where "a_i" are constants, "x" represents the variable, and "n" is the highest power of the variable in the equation. Solving polynomial equations often requires factoring, synthetic division, or numerical methods like Newton's method. Polynomial equations arise in various mathematical contexts, including algebra, calculus, and mathematical modeling, making them essential for understanding mathematical functions and their properties (Young,2023).

2.2.2. Linear Equations

Equations in which the highest power of the variable is 1. They can be expressed in the form "ax + b = 0," where "a" and "b" are constants and "x" is the variable. Linear equations are a fundamental type of equation in mathematics education. These equations represent straight lines on a graph, and their solutions are the values of "x" that make the equation true. Linear equations have widespread applications in fields such as physics, economics, and engineering, making them essential for understanding mathematical modeling and problem-solving (Larson et al., 2019).

2.2.3. Quadratic Equations

Quadratic equations are another important type of equation in mathematics education. They are equations that follow the form "ax^2 + bx + c = 0," where "a," "b," and "c" are constants, and "x" represents the variable. Quadratic equations commonly give rise to parabolic curves when graphed. They typically have two solutions or roots, though it is possible for them to have one or zero solutions as well. Solving quadratic equations involves various methods, such as using the quadratic formula, completing the square or factoring . Quadratic equations find applications in numerous areas, including physics, engineering, computer graphics, and optimization problems (Larson et al., 2019; Anton et al., 2018). Understanding quadratic equations is essential for developing algebraic problem-solving skills and mathematical modeling abilities.

2.2.4. Exponential Equations

In the realm of mathematics education, there exists a specific type of equation known as exponential equations. These equations feature the variable as an exponent and take the form of "a^x = b," where "a" and "b" are constants and "x" represents the variable. Solving exponential equations typically involves using logarithms, which are the inverse of exponential functions. By taking the logarithm of both sides of the equation, we can transform the exponential equation into a more manageable form. Logarithms aid in determining the value of the variable that satisfies the equation. Exponential equations have wide-ranging applications in finance, biology, physics, and population growth modeling, making them critical for comprehending exponential functions and their real-world implications (Larson et al., 2019; Stewart, 2015).

2.2.5 .The importance of equations exponential

Exponential equations are essential in many fields of study because they provide a basic tool for simulating processes of growth and decay over time. They are frequently used in situations like population expansion, radioactive decay, bacterial proliferation, and asset depreciation, according to Shiflet & Shiflet (2014). Broverman (2010) explains how investments change with periodic compounding and highlights their importance in compound interest computations. Exponential equations are useful in economics for understanding inflation, investment dynamics, and economic forecasting, as noted by Blanchard and Fischer (1989). Exponential equations are an efficient way to represent physics and natural processes, such as fluid flow and radioactive decay (Niemeyer, 1995). According to Theis and Wong (2017), exponential growth theories—most notably, Moore's Law in computer power—are the foundation of technology. Zacks (2012) provides evidence of the applicability of exponential distributions in probability and statistics, particularly in reliability engineering and risk assessment. In environmental science, Kuglerová et al. (2017) showcase how exponential equations model processes like pollutant decay, vegetation growth, and species spread. Exponential growth in biological systems, such as cell replication and disease spread, finds applications in pharmacokinetics and epidemiology (Austin et al., 1998). Finally, exponential functions play a crucial role in data analysis and trend forecasting, as highlighted by Gardner Jr (1985), enabling the examination of trends in sales, website traffic, and other phenomena characterized by proportional rate changes.

2.2.6. Logarithmic Equations

Logarithmic equations are a specific type of equation encountered in mathematics education. They involve logarithmic functions and can take different forms. Logarithmic equations can be expressed as "$\log_a(x) = b$," where "a" is the base of the logarithm, "x" represents the variable, and "b" is a constant. In these equations, the variable may appear within the logarithm or as the argument of a logarithmic expression. Solving logarithmic equations typically involves using properties of logarithms, such as the logarithmic identity or the change of base formula. By applying these properties, we can manipulate the equation and determine the value of the variable that satisfies the equation. Logarithmic equations find applications in various fields, including finance, physics, signal processing, and exponential growth modeling (Larson et al., 2019; Stewart, 2015). Understanding logarithmic equations is crucial for working with logarithmic functions and their practical implications.

2.2.7. Trigonometric Equations

Trigonometric equations are a specific type of equation encountered in mathematics education. They involve trigonometric functions such as sine (sin), cosine (cos), or tangent (tan). Trigonometric equations can take various forms, including equations where the trigonometric function is equal to a constant or another trigonometric function. These equations often arise in geometry and physics applications, where angles and triangles are involved.

Solving trigonometric equations requires applying trigonometric identities, properties, and special angle relationships. These tools help manipulate the equations to find the values of the variable that satisfy the equation. Trigonometric equations have diverse applications, including engineering, navigation, wave analysis, and modeling periodic phenomena (Larson et al., 2019; Stewart, 2015). Understanding trigonometric equations is essential for working with trigonometric functions and their practical applications in various fields.

2.2.8. Radical Equations

Rational equations are a specific type of equation encountered in mathematics education. They involve rational expressions, where the variable can appear in the numerator, denominator, or both. Rational equations can be expressed as "p(x)/q(x) = r(x)," where "p(x)," "q(x)," and "r(x)" are polynomial expressions, and "x" represents the variable. Solving rational equations often involves finding a common denominator for the rational expressions involved. By multiplying both sides of the equation by the common denominator, the equation can be simplified and transformed into a polynomial equation. Solving this polynomial equation then provides the solutions to the original rational equation. It is important to check for extraneous solutions, as some solutions may not be valid when plugged back into the original equation. Rational equations find applications in various fields, including physics, engineering, and economics. They are also important in understanding asymptotes, domain restrictions, and solving real-life problems involving rates and proportions

(Larson et al., 2019; Anton et al., 2018). Mastery of rational equations is crucial for developing algebraic problem-solving skills and mathematical modeling abilities.

2.2.9. Absolute Value Equations

Absolute value equations are a specific type of equation encountered in mathematics education. They involve the absolute value of the variable. Absolute value equations can be expressed as "|x| = a," where "a" is a constant and "x" represents the variable.

Solving absolute value equations often results in multiple solutions. This is because the absolute value function is non-negative, and an equation of the form "|x| = a" implies that "x" can take on either the positive or negative value of "a." Thus, the equation typically has two solutions, namely "x = a" and "x = -a."

To solve absolute value equations, we consider both the positive and negative cases separately. We set the expression inside the absolute value bars equal to the constant "a" and its negation "-a," and solve for "x" in each case. It is important to check the solutions obtained by substituting them back into the original equation to ensure they satisfy the given condition.

Absolute value equations find applications in various areas, such as physics, engineering, and optimization problems. They are also important in understanding the concept of distance and working with inequalities (Larson et al., 2019; Stewart, 2015). Mastering the solution techniques for absolute value equations is essential for developing algebraic problem-solving skills and understanding the properties of absolute value functions

2.2.10. Systems of Equations

A set of two or more equations with multiple variables. The solutions to these equations are the values that satisfy all equations in the system simultaneously. Simultaneous equations are usually one of the challenging topics to be taught in school because participants usually struggle to understand the concepts and just prefer to memorise steps and methods for the sake of getting through tests or exams (Johari and Shahrill, 2020). Also, Johari and Shahrill (2020) highlighted that simultaneous equations are an integral part of algebra which is needed in most mathematical topics or even other learning areas of the 21st century such as computer, sciences or even engineering to name a few. According to Nordin et al. (2017), usually, the students are expected to solve a system of simultaneous equations involving two linear equations and two unknown variables.

Additionally, the common methods of solving simultaneous equations problems are by using substitution or elimination, solving simultaneous algebraic equations, requires the students to be able to manipulate and solve basic algebraic equations (Nordin et al., 2017).

2.3. Learners' ways of constructing equations

In mathematics, there are a lot of different equations, with different methods to construct them, but learners are struggling to change issues of vocabulary and mathematical language used in mathematical word problem solving as highlighted by Sepeng and Madzorera (2014). Also, further asserts that the totality of a mathematical message is often embedded in the context of a three-way relationship involving mathematical words, symbols, and numerals and that these three components define the mathematical language as argued by Boulet (2007).

More recent research on mathematics learning has focused on how students construct equations , negotiate meanings, and participate in mathematical communication (Moschkovich, 2002).

According to Ballard (1980), the act of constructing an equation involves a geometrical process that produces line segments equivalent to the equation's roots. However, learners often encounter obstacles in terms of counting, knowledge transfer, and comprehending mathematical language and visual perception. These difficulties can result in an inability to grasp mathematical concepts and ultimately lead to failure in the subject.

2.4. Learners' ways of solving equations

The main goal of Mathematics learning is to allow students to be able to solve everyday problems (Baiduri, 2018). Additionally, Baiduri (2018) highlighted that it can be said that Mathematics is a tool to train students to solve problems and build the process of thinking which nurtures the skill to solve non-Mathematical problems. Students seem to consider that problems can only be solved in one single method, especially certain types of problems taught at school as mentioned by Baiduri.

Chamberlin(2005) argued that teachers need to interpret their students' logic which can result in effective reactive instructional decision-making, such as selecting and designing mathematical assignments in problem-solving activities. In such situations, contradictions, misconceptions, perturbations, and surprises lead to cognitive conflict that eventually leads to the restructuring of schemas through the processes of assimilation and accommodation (Sepeng and Madzorera, 2014).

2.4.1. The importance of equations in mathematics

It is noteworthy to mention that a mathematical equation has recently garnered significant attention within the South African new curriculum. The integration of this equation into the curriculum has allowed for a more comprehensive and structured approach to problem-solving among students. This recent development is undoubtedly an exciting prospect for the academic community, as it offers a promising opportunity to improve mathematical learning outcomes. The National Curriculum Statement (NCS) was implemented at different times for different grades. It was implemented in in grades 9 and 12 in 2008 ; in grades 8 and 11 in 2007; in grades 7 and 10 in 2006 ; in grades 4, 5, and 6 in 2005 and grades 1, 2, and 3 in 2004. A study was conducted to analyze the results of the National Curriculum Statement (NCS) during its implementation period. The discussion mainly focuses on the envisioned key outcomes of the NCS. Since then, the NCS has undergone improvements and has been renamed as the National Curriculum and Assessment Policy (CAPS). It was implemented in grade 12 in 2014, grades 4 to 11 in 2013, and grade 10 and grades R to 3 in 2012.

According to the Department of Education (2003), the national curriculum declaration states that learners should be proficient in problem recognition, critical thinking, decision-making, and original thinking. This declaration is reiterated in the new CAPS document (DBE, 2010). The national curriculum statement emphasizes problem-solving in mathematics throughout its content:

- As an element of the explanation of mathematics: mathematical equation allows us to recognize the world and create use of that in our everyday lives.
- Mathematics improve learners self-concepts with respect to the abilities to solve problems and also make them aware of the problem-solving strategies,
- It is widely acknowledged that learners should actively engage in the process of identifying and solving unseen problems in the field of mathematics. Such an approach fosters a deep understanding of mathematical concepts and principles, and allows learners to develop their problem-solving skills.

In the world of mathematics, equations play a crucial role. They help learners develop problem-solving skills, which are essential for tackling mathematical challenges they may face in their day-to-day schooling. The curriculum and assessment policy statement (CAPS) emphasizes the importance of mathematical problem-solving, not only in helping us understand the world around us but also in fostering creative thinking (CAPS, 2010).

2.4.2. The teaching and learning of equations

The instruction of equations to students has encountered several impediments, including their challenges in determining the roots of first or second-degree equations, as contended by Brandt and Baccon (2015) in their scholarly work.

Recently the use of concrete models in teaching solving equations has become a more common practice to help students develop a conceptual understanding of equality (Joffrion, 2007). Learners must see the equals sign as relational, denoting either side has equal value as mentioned by Joffrion(2007).

2.4.3. Feedback in mathematics teaching and learning

According to the online definition, feedback is information given to the learner about the learner's performance relative to learning goals or outcomes. It should aim to improve students' learning. Feedback redirects or refocuses the learner's actions to achieve a goal, by aligning effort and activity with an outcome. Also, feedback is one of the essential elements found in the oral questioning process, where feedback acts as a response given to the students after the students answer the questions posed to them as highlighted by Mahmud and Yunus (2018).

Providing feedback to students during oral questioning is a vital component in enriching their comprehension of mathematics. This practice allows educators to keep track of their progress, offer guidance, and facilitate successful teaching and learning (Mahmud, 2021). Timely and constructive feedback empowers students to pinpoint areas of strength and weakness, contemplate their learning strategies, and make necessary changes to enhance their performance(Mahmud, 2021). By integrating feedback, educators can utilize a valuable tool to promote effective mathematics education.

2.4.4. The importance of feedback

Effective feedback is a powerful tool that can significantly impact learning and achievement. However, it's important to note that feedback can have both positive and negative effects, as pointed out by Hattie and Timperley (2007). In this study, the focus is on feedback as a means of providing information about the content and understanding of students' learning experiences, rather than a behaviourist input-output model. Additionally, feedback has been shown to play a crucial role in professionalizing teaching at the higher education level, as noted by Ahea, Ahea, Kabir, and Rahman (2016).

It is also significant to distinguish what the learners think. It is not enough to just give learners feedback on theirperformance and progress . Teachers should encourage learners to give criticism on their own familiarity of learning as higlited by Godden (2012). Additionally, it is the case that feedback is not only given by teachers, learners, peers, and so on, but can also be sought by learners, peers, and so on, and detected by a learner without it being intentionally sought (Hattie and Timperley, 2007).

Also, Ferguson (2011) argued that feedback is considered a vital approach to facilitating learners' development as independent learners to monitor, evaluate, and regulate their learning. (Hattie and Timperley, 2007), effective teaching not only involves teaching information and understanding to learners but also involves assessing and evaluating students' understanding of this information so that the next teaching act can be matched to the present understanding of the learners.

This study stems from a study that sought to identify and analyse the errors that candidates commit in the final mathematics examination paper in grade 10 with an outlook to use these errors and misconceptions as an instrument by teachers for more effective teaching. The findings show that most teachers are not applying multiple teaching methods and assessment approaches which are necessary to prepare learners to participate in a developing economy.

CHAPTER 3

THEORETICAL AND CONCEPTUAL FRAMEWORK

3.1. Introduction

In mathematics education research, a theoretical framework serves as a conceptual lens that guides the study, providing a foundation for understanding and interpreting the research findings. It encompasses a set of concepts, principles, and assumptions that shape the research design, inform research questions, guide data analysis, and contextualize the study within the broader field of mathematics education (Boaler, 2016; Lerman, 2001). By providing a theoretical framework, researchers in mathematics education establish a theoretical grounding for their work, enabling them to investigate and explore the complexities of teaching and learning mathematics (Pape & Tchoshanov, 2001). The significance of theoretical frameworks lies in their ability to offer conceptual clarity, guide research design, provide contextual understanding, facilitate the development of hypotheses, and contribute to the integration and building of knowledge in the field (Schoenfeld, 1992; Steffe and Thompson, 2000). What will follow is a theoretical and conceptual lens that underpinned the research.

3.2. Constructivism

The theory that underpins this study is based on Constructivism. Constructivist learning theory is concerned with the view that learners control their learning as they construct their knowledge instead of passively receiving it. Owusu (2015) argued that constructivism is an epistemological view of knowledge acquisition that emphasises knowledge construction rather than knowledge transmission and the recording of information conveyed by others. In this regard, constructivism helps learners to ask questions and construct and express their knowledge and views based on their physical and mental activities without limitations (Reys, Suydam, Lindquist and Smith, 1998). Learners come to class with some already existing ideas. According to Mamba (2013), a constructivist framework challenges teachers to create environments in which learners are encouraged to think and explore.

This may imply that the learner's existing ideas have consequences for learning anything new. Knowing so, it becomes possible to teach mathematics more effectively if an account is taken of the learner's existing ideas (Timperley, Wilson, Barrar, and Fung, 2008). It is always important to understand that learners learn well if the knowledge being taught is based on what they know already. It is, therefore, recommended that teaching should proceed from known to unknown and from simple to complex. When it is given in that way, then learning will have value.

Constructivism is an epistemological view of knowledge acquisition that emphasizes knowledge construction rather than knowledge transmission and the recording of information conveyed by others (see, Section 1.9.2). It is aligned with active learning and promotes the comparison of new ideas with prior knowledge (Von Glasersfeld, 1995; Steffe, 1991; Goldin, 1990; Vygotsky, 1978; Piaget, 1973).

The educational theory of constructivism holds that learners actively interpret knowledge and understanding from their experiences. Furthermore, as Von Glasersfeld (1995: 19) explains, 'our knowledge of things only exists in our conceptual world as relatively viable permanent entities that we have constructed'. Effective communication and justification of ideas play a crucial role in enabling learners to develop their problem-solving abilities, as noted by Piaget (1973).

Ensuring accurate mathematical language, explaining and exchanging concepts with others (Ball and Bass, 2000) holds great significance. Learners can derive meaning in mathematics through individual objects or the guidance of others (Von Glasersfeld, 1995). By solving problems independently (Wood, Cobb, and Yackel, 2012) and inquiring about diverse strategies applicable to mathematical topics, learners can enhance their mathematical aptitude and minimize errors they might make (Fuson, Wearne, Hiebert, Human, Murray, Olivier and Fennema,1994).

3.3 . Behaviourism

Behaviorism is unable to deal with complex human behavior. According to the online definition (2022), behaviorism is a theory of learning based on the idea that all behaviors are acquired through conditioning, and conditioning occurs through interaction with the environment. The behaviorist or connectionist theory of learning relates to an empiricist philosophy of science that all knowledge originates in experience (Olivier, 1989). Behaviorists believe that our actions are shaped by environmental stimuli. In fact, learning occurs with the acquisition of new behavior (Faryadi,2007).

Also, Faryadi argued that Behaviourists rely only on observable behavior to learn. Additionally, Behaviourism, therefore, assumes that pupils learn what they are taught, or at least some subset of what they are taught, because it is assumed that knowledge can be transferred intact from one person to *a*nother. However, this research focused more on constructivism.

3.4. Conceptual Framework

The conceptual framework is established through several ideas and hypotheses from the literature, to construct an investigation, grant a way forward, and lead the data collection and its analysis. The leading hub of the conceptual framework in this research is on the clarity of non-identical errors recognized for mathematics. The investigation of the origin of the mistakes and implementation of the upshot from the research in the procedure of organizing the teaching and studying of Mathematics could supply valuable awareness for mathematics teaching.

Olivier (1989) argued that once teaching new concepts or processes, the realization of mistakes tells the teacher what to pay attention to, how to negotiate and what to elucidate, the understanding of new expression in order not to repeat the same mistakes and how to apply positively the already recognized errors as the foundation for mathematical comprehension. An error is a mistake, slip, blunder, or inaccuracy and a deviation from accuracy (Luneta and Makonye, 2010).

Mathematics has an inmost logical form and its concepts are enhanced on the foundation of other ideas. So, the assistance of studying mathematics requires upholding this concern. A small gap in understanding generates more mistakes that rely on one another, which from time to time are disclosed in an error avalanche. An unidentified mistake is embedded in the student's brain, and therefore it is a vital threat to the building up mathematical familiarity of students.

Therefore, Ciosek (1992) said that an exposed and elucidated error may be extremely useful together for teachers and learners, the understanding of why students make errors can be interpreted in terms of learning theories. The constructivist theory is considered in this study. This conceptual framework demonstrates errors and misconceptions identified by various authors.

Application errors: Learners make these mistakes when they know the concept but cannot apply it to a specific situation or question as Godden, Mbekwa, and Julie(2013)argues. This is in contrast to procedural errors where learners apply the incorrect procedure.

Example: A triangle base (length) is triple the size of the height. Find the sides of the rectangle if its area is $120m^2$. One learner responds to this by writing that 3x plus x is equivalent to 120. At that point $\frac{1}{2}(4x = 120)$ which is equivalent to (2x=60) the student then divides by 2 into both sides of the equation to get x = 30, then completes the problem with x equal to 30. This error was summarised as an application error because the learner did not use the right formula to find an area of a triangle, the learner added base and height instead of multiplying them.

Careless Errors: Godden (2012) defined careless errors as changing answers, missing answers, from the correct ones to incorrect .Careless errors in the framework, and together with analysing the scripts, a more precise description for careless errors was derived. It is needless errors made by learners.

Example of careless: This example of a careless error is taken from one of my learners. While busy with a question the error was made when solving for x:

x^2- 4x- 5= 0 (step a)

(x-5)(x+1) = 0 (step b)

(x-5)=0 (x+1)= 0 (step c)

The careless error is made when the equation x-5 = 0 in step (b) was miscopied in the next step (3) and written as x +5 = 0 while in the process of solving x

Procedural Errors: (Godden, 2012), clear definition happens when a learner uses incorrectly the method.

Example a learner solved the problem x^2 -3x +2 = 0 as follows,

(x -2) (x-1)= 0 (1),

x = 2 or x = -1 (2),

When the answer is given as for then step (1) was properly done then in step (2) the law for the process for answering the quadratic equation was incorrectly used .

Calculation Errors: These errors are often the result of carelessness or short attention span as Simons(2012) said .The below answered question shows that the work out on the left-hand side of the equation was unnoticed, and errors in multiplication, division ,addition and subtraction of numbers when using the quadratic formula. One of the students I tutored make this calculation error.

Solve: $x^2+2x = 7+3$ (step a)

$x^2+2x= 4$ (step b)

$x^2+2x-4= 0$ (step c)

A calculation error is found in (step b) (**7+3→11**) of the equation, in the third step of working out, moving the terms to the left hand side of the equal sign, was done in right way.The right process was used when solving the question, only the calculation was incorrect. This error focuses on the basic operation.

Non-completion Error: This error is committed when the learners attempt to solve the inequality, but stop after the factorizing and do not follow the procedure to complete the question.

This is an example of a question that is not completed, provided by one of the learners I tutor.

solve for y:

$3(2y-1) \geq 32$

$6y-3 \geq 32$

$6y \geq 32+3$

$6y \geq 35.$

This learner was on the way to get the value of y, but she/he did not manage.

Comprehension Error: Simons (2012) said that comprehension error happens when the learner has been capable to read all the key words in the problem, but has not grasped the overall meaning of the words and therefore, is unable to proceed further along an suitable problem-solving steps.

Example: Consider a student who encounters a word problem involving fractions. Despite having learned the procedural steps for adding fractions, the student consistently makes errors because they fail to grasp the underlying concept of fractions as parts of a whole. This comprehension error hinders their ability to accurately solve fraction-related problems and demonstrates a need for deeper conceptual understanding.

Reading Errors: According to Simons (2012), a reading error is an error that should be distinguished as reading if the learner could not read symbol or a keyword in the written question to the extent that this prevented her /him from proceeding additional along an appropriate problem-solving steps.

Blanks: The learner did write the question down but did not try to answer the question or they provided the number of the questions.

Arbitrary Errors: (Simons, 2012), In mathematics, an arbitrary error refers to a mistake or inaccuracy made during a mathematical process or calculation that is not based on any logical or valid reasoning. It is an error that does not stem from a misunderstanding of a concept or a procedural mistake but rather from a random or arbitrary choice.

An arbitrary error can occur when a student or individual attempts to solve a mathematical problem without a clear understanding of the underlying principles or without following established rules and procedures. This type of error often arises when a person makes haphazard decisions or guesses without any systematic approach or logical justification.

Consider a learner who is asked to find the solution to the equation $4x + 10 = 20$. Instead of correctly subtracting 10 from both sides and then dividing by 4 to isolate x, the student decides to randomly add 10 to both sides of the equation.

Conceptual Errors: This error is made due to a misunderstanding or misconception about a mathematical concept and rules (Simons, 2012).

This is an example of an error made by one of the learners, I tutored:

Two road runners, Zara and Tamsin, set off at 07:00 from the same place but in opposite directions. Zara runs at an average speed of 12km/h and tamsin at 8 km/h. At what time will they be 90 km apart?

In this example, it will be useful to let the time at which they were 90km apart be x o'clock.

Let x be the time in hours taken for the distance between them to become 90km.

Use the relationship distance= speed ×time

	Speed in km/h	Time in hours	Distance in km
Zara	12	x	12x
Tamsin	8	x	8x

After x hours, the distance between the girls is 90km.

$12x + 8x = 90$

$20x = 90$

$x = 4\frac{1}{2}$ hours

$x = 2$ hours

2 hours after 07:00 will be 09:00.

The runners will be 90 km apart at 09:00.

In the above steps 4 is multiplied by a ½ to obtain the answer 2. The answer shows a mix-up of the concepts of the fraction that was used to specify time.

3.4.1. Conclusion

In conclusion, the conceptual framework established for this research serves as a vital tool for understanding and addressing various types of errors encountered by learners in the field of Mathematics. The focal point of this framework is the exploration of non-identical errors, particularly those related to the clarity of mathematical concepts. The insights derived from investigating the origins of these errors, as well as the implications drawn from the research, contribute valuable knowledge for the improvement of Mathematics teaching and learning.

Olivier (1989)'s perspective underscores the significance of recognizing and comprehending students' mistakes as a guide for effective teaching strategies. The framework integrates diverse error categories, such as application errors, procedural errors, careless errors, calculation errors, non-completion errors, comprehension errors, reading errors, blanks, arbitrary errors, and conceptual errors. Each category sheds light on specific challenges students face, offering a nuanced understanding of their learning processes.

Application errors, as illustrated in the example of misapplying the formula for finding the area of a triangle, emphasize the importance of not only knowing concepts but also being able to apply them correctly. Careless errors, on the other hand, highlight the need for attention to detail, as seen in instances where learners inadvertently miscopy information or skip crucial steps in problem-solving.

Procedural errors pinpoint cases where learners apply processes incorrectly, showcasing the need for a clear understanding of mathematical procedures. Calculation errors emphasize the role of careful attention to basic operations, while non-completion errors underscore the importance of seeing problems through to their conclusion.

Comprehension errors reveal the significance of grasping the underlying concepts, as learners may struggle even when they can read and follow procedures. Reading errors and blanks indicate challenges in understanding written problems or completing assignments. Arbitrary errors emphasize the importance of logical reasoning and systematic approaches to problem-solving.

Conceptual errors, as demonstrated in the example involving a misunderstanding of fractions, highlight the necessity of addressing misconceptions and promoting a deeper conceptual understanding.

In essence, this comprehensive conceptual framework provides a roadmap for educators to navigate the diverse landscape of errors students may encounter in their mathematical journey. By recognizing, elucidating, and addressing these errors, educators can tailor their teaching approaches to foster a more effective and meaningful learning experience for students.

CHAPTER 4
METHODOLOGY

4.1. Introduction

According to Bamberger (2000), two broad approaches are usually implemented by researchers to gather data, namely, qualitative and quantitative approaches. This study made use of a qualitative research approach. According to Bleakley (2005), qualitative research is defined as using various techniques to socially interact, and it is aimed at making sense of interpreting or reconstructing an interaction. This chapter provides pertinent details concerning the research approach adopted for this study. Knoblauch and Schnettler (2012) stated that the data analyzed for qualitative research comes from observations, interviews, and written documents like tests. The methodology chapter explores the research approach used in the research, including the research design, data collection methods, data analysis strategies, and research ethics.

4.2. THE RESEARCH APPROACH

This research utilised a qualitative method. According to Mohajan's (2018) definition, qualitative research is a method of delving into and comprehending the significance that individuals or groups attach to a social or human issue. This methodology involves gathering data from participants within their natural setting, followed by the researcher's interpretation of the data, based on naturally occurring themes that emerge from it. The cause for selecting a qualitative model is because this method is best right to comprehend the phenomenon in this field of research. According to Mohajan (2018), qualitative research is a form of social action that stresses the way people interpret and make sense of their familiarities to comprehend the social reality of people. Qualitative researchers are interested in people's beliefs, experiences, and meaning systems from the perspective of the people (Mohajan, 2018).

4.3. Research setting

In the methodology chapter of a research study, the research setting refers to the physical, social, cultural, or organizational context in which the research is conducted. It encompasses the specific location, population, or group of individuals being studied, as well as the temporal and spatial dimensions of the study.

The research setting is crucial as it provides the backdrop against which the research questions are explored, data are collected, and results are interpreted. It influences the generalizability of the findings and helps in understanding the contextual factors that may affect the outcomes of the study. Furthermore, the research setting provides insights into the feasibility and applicability of the study's methods, as different settings may require specific adjustments or considerations. Creswell (2014) When describing the research setting in the methodology chapter, researchers typically provide details about the selection criteria for the setting, the rationale for its choice, and any potential limitations or challenges associated with it. This information helps readers understand the scope and applicability of the study's findings Robson,(2011).

This research took place in the Metropole South Education District, which is an education district in an urban area, in Cape Town, in the Western Cape province of South Africa. This is a diverse cultural community where English and Afrikaans are the predominant languages of use. This district has a large Christian and Muslim community.

4.3.1. Analysis of Document in qualitative research

Document examination is a rigorous procedure for scrutinizing and appraising documents. While this method is typically used in conjunction with other research techniques, it is a preferred approach for this particular study. Previously, the analysed document was utilized as a stand-alone process (Bowen, 2009). As with other qualitative research methods, the aim of document analysis is to extract meaning, examine, and interpret data to gain insights (Corbin, 1990). For example:

- According to Quade's (1970) definition, document analysis is a methodical, deliberate, and purpose-driven process that involves organizing and examining documents in a systematic manner. In this study, the document under analysis is the final script for grade 10 mathematics learners in 2021. Like other qualitative research methods, document analysis has both advantages and limitations. One clear advantage is that it is a highly efficient method that saves time compared to other research techniques.
- The analysis of profitable documents does not cost more than other research methods.

- Document analysis helps the resaercher to focus on the questions he/she might ask in interviews and also helps in the understanding of what to look out for with participant observation.
- Access to restricted accessibility documents can be rejected intentionally for bias and selection of incomplete unformation (Yin, 2003).
- In the presence of stability, researchers do not change and manipulate the results being considered.

4.3.2. Sampling

According to Godden (2012), sampling is a subgroup of a population which is selected based on the representativeness of the population. The sub-group selected must possess characteristics of the entire population. The sample for this study was obtained from 10 learners in a high school from the Metropole South Education District of the Western Cape. Salihu (2017) argued that a sample is a representation of a larger population from the target population.

For this study, I used a convenient sample as the school was nearby my place of residence and also this type of smapling involves using participnats who are convinient to the resarcher . Convenience sampling involved the selection of participants when they were generally willing and available. In, convenience sampling tends to be the sampling technique favored by the resaecher because it is inexpensive and there is no pattern in aquiring participnats (Ackoff, 1953). Convenience sampling often helped overcome many of the limitations associated with research.

Table 4.1. The learners's sample doing mathematics in the schools conveniently located

Schools in the district	Number of grade 10 learners	Number of learners doing Mathematics in Grade 10
School A	192	108
School B	226	74
School C	193	67
Total	611	249

4.3.3. Data collection

In fulfilling the objectives and attempting to answer the research questions for this paper, the researcher utilised the documenatry review to collect the data. According to Bernard (2002), documentary review is a data collection method that involves analyzing and evaluating documents to gather information for research purposes. It is a qualitative research method that can be used to gain insights into learner's mathmatics problem solving and how they interprete the equations (Patton, 1990; Tongco, 2007).

In this study, data was obtained from the schools that offer mathematics as a subject in grade 10 as well as learners who are doing mathematics as a subject in grade 10. The table above (Table 3.1) represents the number of grade 10 learners at three schools in the Metropole South District doing mathematics. The total number of scripts analysed were 249. Of the grade 10 mathematics final examination scripts of 2021 were selected and , only questions on linear equations from other forms of equations were selected from the scripts for analysis which will be on linear equations from exponential form.

4.4 .DATA ANALYSIS

This research aims to explore the possibilities and discuss the application of qualitative content analysis as an interpretation method in case study research. Case studies are widely used in organizational studies and the social sciences. Some experts suggest that the case study method is increasingly relied upon as a strategy for rigorous research. Hartley (1994) and Hartley (2004), as well as Stake (2000) agree, stating that case studies have become one of the most popular ways to conduct qualitative surveys.

For this study, grade 10 learners' responses to questions about linear equations from other types of equations were explored, focusing on linear equations from exponential form. The analytical framework, table 3.2 guided the analysis of the data, which was created from the conceptual framework as well as reflecting on the main research question and sub-questions. The process followed during the analysis of learners' responses was to find the misconception and errors emanating from their conducts of working. Errors that are not mentioned in the analytical framework are labeled with a particular definition to explain the way of working.

Table 4.2. Analytical Framework

Errors	**Extensions**	**Definition**
Careless	Incorrect transcribe	Incorrect transcribe a symbol or number from the question paper.
	Incorrect Transfer	Incorrect transfer from a previously-obtained Result.
Calculation	Basic Calculation	When simple addition,subtraction, multiplication, division, errors are made.
	Calculator	Errors when by means of a calculator .
Application	Substitution	Where substitution was done incorrectly.
Procedural	Wrong method	Wrong procedure when transforming exponential equations into linear equations.
Non-Completion,	Non-Completion	When the question started with right steps, but not finished to answer the question.

All errors in the answer scripts, focussing on questions dealing with exponential and linear equations theories, were labeled ,categorised and identified, according to definitions provided in the analytic framework,as Godden (2012)argued.

4.5. RELIABILITY AND VALIDITY

Taherdoost (2016) refers to reliability and validity as the accuracy and consistency of the survey/questionnaire, which forms a significant aspect of the research methodology. In addition, validity and reliability are conceptualized as quality ,rigor ,trustworthiness and rigor, in the qualitative model (Golafshani,2003). According to Salih (2017), reliability means checking if a test is a good test. A test is reliable if one gets consistent test results even though other researchers administer it, while validity helps the researcher to have the main study suitable, unmistakable, and effective for the study (Salih, 2017). For this study, I made use of an inter-rater agreement to ensure the trustworthiness and validity of my interpretation of the data and the study. The inter-rater agreement was conducted through the help of two colleagues who were asked to give their analysis of learners' work. For this, a sample of 10% of the data was used.

The mention of an inter-rater agreement in the study adds an additional layer of rigor and validity. By involving two colleagues in the analysis of learners' work, the researcher seeks to enhance the trustworthiness of the interpretations. The use of a sample of 10% of the data is a practical approach, allowing for a manageable yet representative assessment of inter-rater consistency. The choice to incorporate inter-rater agreement aligns with the broader goal of ensuring trustworthiness in qualitative research. Multiple perspectives contribute to a more comprehensive understanding of the data, and agreement among raters enhances the reliability and validity of the findings.

In summary, reliability and validity are critical considerations in both quantitative and qualitative research methodologies. They serve as benchmarks for the accuracy, consistency, and appropriateness of research instruments. The use of inter-rater agreement in this study exemplifies a commitment to enhancing the trustworthiness and validity of the data interpretation, ensuring the research outcomes are robust and dependable.

4.6. Ethical considerations

Raghav and Saxena (2009) emphasize the importance of adhering to key ethical principles and actions when conducting research. These include protecting participants from harm, respecting their right to self-determination, privacy, anonymity, and confidentiality. Researchers must prioritize the physical and mental well-being of participants, as some studies may cause temporary discomfort during human interaction.

To follow ethical guidelines, the resaecher obtained permission from the Western Cape Education Department, the University of The Western Cape, and the schools involved. I provided all institutions, parents, and learners with information about the study and research. I considered parental consent essential and prioritized the voluntary ascent of the learners. Participants were informed of their right to withdraw from the study at any point. We obtained clearance from the superior of any prospective participant who requested it before the interview. The researcher ensured that all legal and ethical requirements for research involving respondents were followed.

To ensure accuracy and prevent fraudulent analysis, the resarcher carefully interpreted and analyzed all data. I can assure you that no answers were altered to suit my research. Additionally, I value confidentiality and have taken measures to protect the identity of learners, parents, teachers, principals, and schools. The data has been securely stored in a locked cupboard and will only be handled by the researcher and supervisor. All involved parties have signed a consent letter agreeing to remain anonymous. I will keep this information confidential for five years before securely discarding it.

4.7. SUMMARY

This chapter discusses the methodology used in a research study and explains the study procedures employed. The methodology is crucial because it helps the researcher provide answers to research questions. The chapter provides a detailed explanation of the research design, including sampling techniques, data collection methods, data analysis processes, and a statement of ethics that guided the research. The study gave names to the errors that occurred in the written answers of learners, guided by an analytical template. The study used a qualitative research method, which allowed for the analysis of data generated in a natural setting - the final Grade 10 mathematics exam of 2021. The learners wrote in a controlled environment under the school's supervision.

The scripts analyzed in this study were collected from three high schools located in the Western Cape. The data was scrutinized using a template-based approach, which was developed by defining appropriate errors to establish a framework for identifying and analyzing mistakes made by students. The study takes into account the validity, reliability, and ethical considerations. In the following chapter, the study's findings are presented in detail.

CHAPTER 5

RESEARCH FINDINGS

5.1. Introduction

In this chapter, I will be presenting the findings and analysis of our research study which aimed to identify the various strategies and tactics utilized by learners when solving mathematical linear equations, as opposed to other types of equations presented to them in their final grade 10 examinations. My goal was to identify the errors presented in their answers and classify them according to the analytical framework in the conceptual framework. To achieve this, we analyzed a sample of 119 grade 10 scripts from 2021 repeatedly, in order to classify the misconceptions and errors found in the learners' approaches.

5.2. Data interpretation

Validity, ethical issues, and data analysis methods were explained. The analysis of this data gave me insight into the kinds of errors and misconceptions that learners are likely to produce in introductory algebra. A wide-ranging description of the data collection tools and how they were used and discussed in detail. An analysis of the grade 10 learner's final scripts, is provided in this chapter and the main findings are summarised. Documentary analysis of the actual scripts rendered that learners committed careless, procedural, calculation, and application errors as described in the literature.

The questions that my analysis focused on in final grade 10 Mathematics scripts of 2021 from school A, school B and school C different high schools in Western Cape were the following:

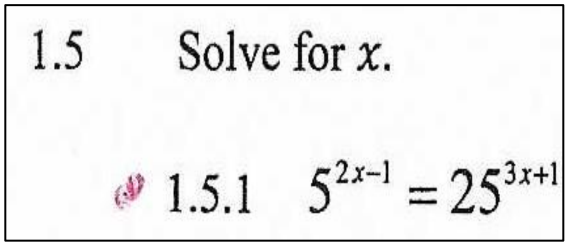

Figure 2: Part of question 1 from School A

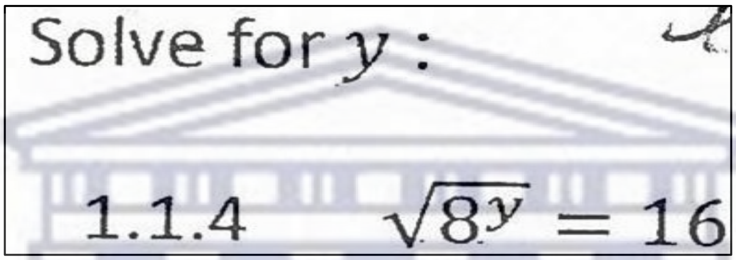

Figure 3: Part of question 1 from School B

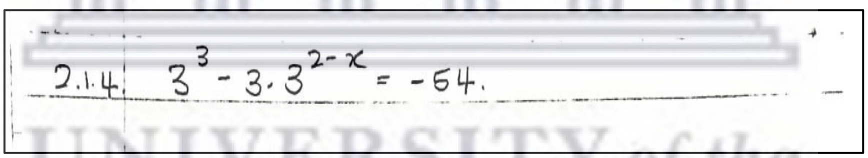

Figure 4: Part of question 2 from School C

Misconceptions and Errors that could not be placed in a category named in the template were given a different classification. The errors and misconceptions that were identified were classified according to the analytical framework template designed in the literature review chapter. The grade 10 Mathematics scripts were analysed, focusing on errors made by the learners in the final school base mathematics examinatiuon. This study only focused on exponential equations.

This type of structural error was noted frequently in the sample and to give prominence to this mistake it was thus decided to name it an operation selection error. If learners chose the wrong operation, it showed a lack of knowledge of the mathematical principles which was necessary to solve the problem.

5.3. Learners ways of working

5.3.1. Careless Error

Here , I analyse how a careless error occurs when a student makes a mistake due to oversight as shown in the figures 5.1,5.2 and 5.3(Simons,2012;Godden,2012; Godden, Mbekwa and Julie, 2013). Figures 5.1, 5.2 and 5.3 provide examples of the kind of careless errors which are incorrect transfer from the previous steps done by the learner when he/she was trying to answer the question given that was identified in the work of the learners, from all three different high schools in Western Cape.

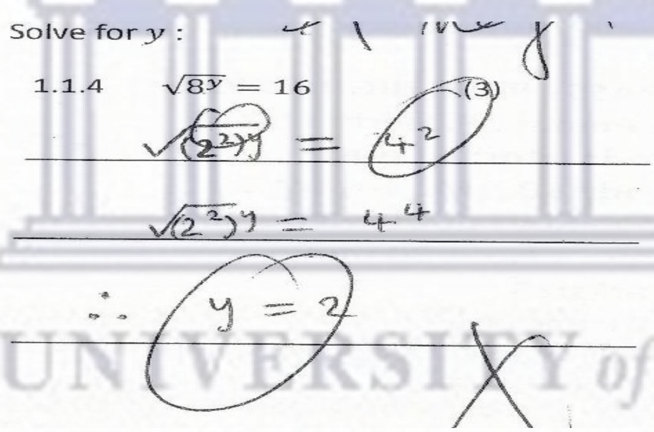

Figure 5.1: Careless Error

In Figure 5.1., the learner was able to work with exponential equations but in the second step, the learner did not manage to write 8, in exponential form, while in the next stepe, the learner was able to answer the question the right way.

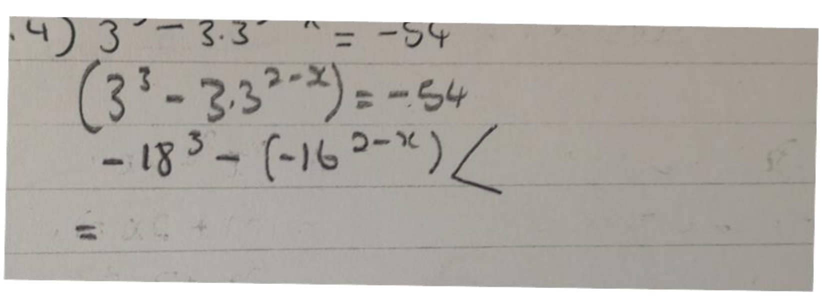

Figure 5.2: Careless Error(Incorrect transfer)

In Figure 5.2., the learner was not able to transform the given equation into the right form of exponential form, which caused him not to finish it in the right way.

The above findings demonsrates how learners struggle to understand the questions and end up doing it in the wrong way. The figure below is an example, of how learners make incorect transfers or make careless errors which in this case is a consequence of misconception.

Figure 5.3: Careless Error (Incorrect transfer)

In Figure 5.3., the learner failed to transform correctly the given equation into the same bases, which caused him to get the wrong value of variable (x).

In 116 scripts collected in three different high schools,12(10.3%) of the scripts were about careless errors, and the learners failed to continue with the correct step.

5.3.2. Calculation Errors

Calculation errors happen when a learner makes mistakes in performing mathematical operations. This is a specific type of calculation error that involves mistakes in fundamental arithmetic operations as highlighted by Godden (2012). There two different calculation errors were identified calculation errors and misconception. The misconception in Figures 5.4,5.5 and 5.6 are examples of basic calculation errors where the learner knows how to make the same bases to be able to work with exponential equations but applies the process wrongly.

The leaner was supposed to multiply the exponent of the new base he/she got on the right side of the equation with the given one, unseated he/she added them in (5.4); in 5.5, he/she misconceived and divided into both sides by 2, instead of multiplying it with its exponent. This misconceived error will be named the basic calculation error. Figure 5.6, the learner changed the positive sign into the negative sign. From three schools 16 scripts, learners made calculation errors are 16(13%)of the total collected scripts.

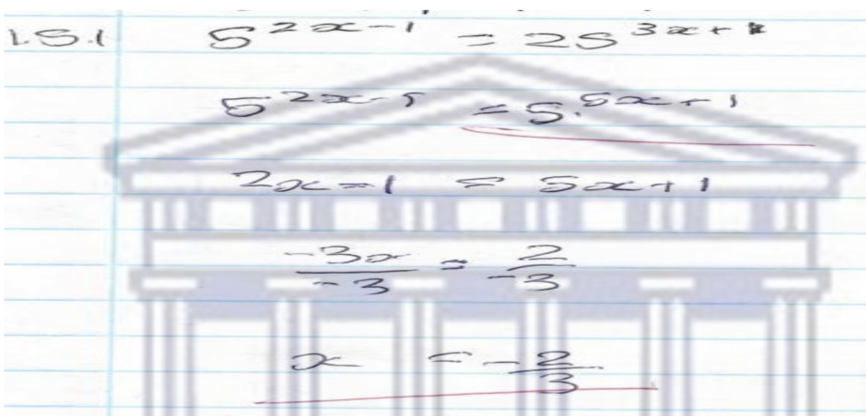

Figure 5.4: Basic Calculation Error

In Figure 5.4, the learner failed to apply the right sign instead of multiplying exponents on the second side of the exponential equation, the learner used the addition sign, which caused him/her to get the final wrong answer.

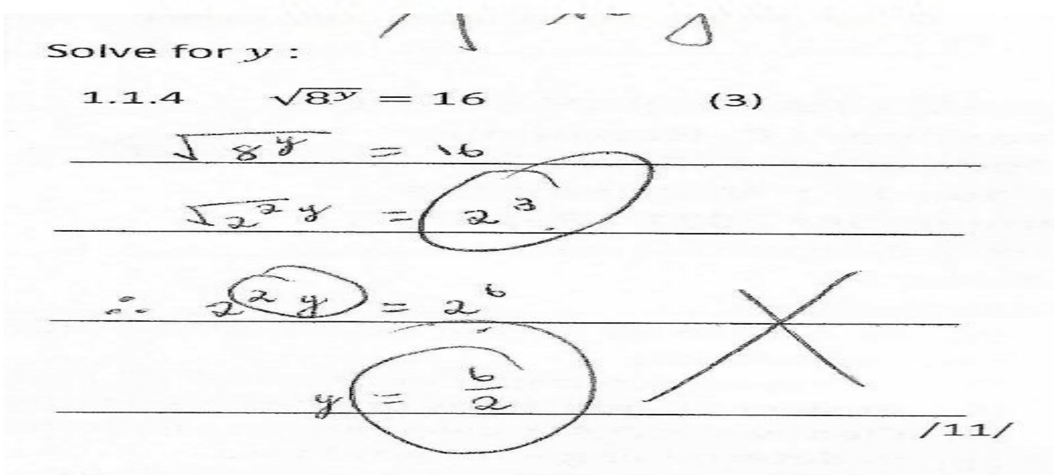

Figure 5.5: Basic Calculation Error

The learner started by dividing 2 into both sides instead of making all bases 2, the reason why in step two of the learner's calculations he got 2^{2y} instead of 2^{3y}, which caused the learner to end up with the wrong answer.

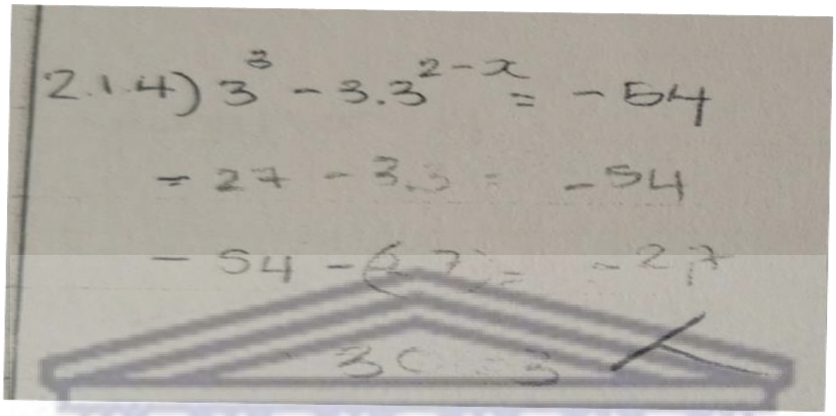

Figure 5.6: Basic Calculation Error

In Figure 5.6, the learner changed the positive sign into to negative sign on the first term of the initial equation sign from step one to step two, which drove the learner to end up with the wrong answer.

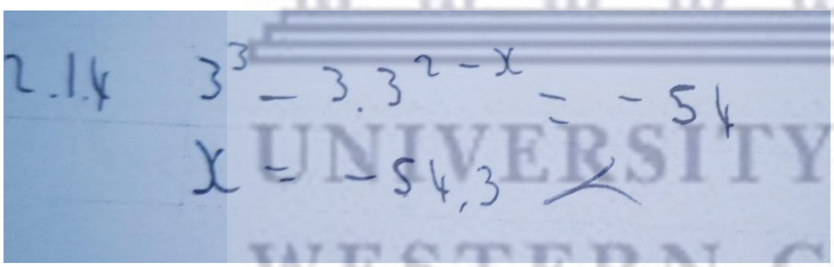

Figure 5.7: Calculation Error

The learner made an error using the calculator, the correct answer is -1. The learner failed to create the same bases first, to enable self to compare the exponents. It is observed that, careless errors and calculation errors are the errors made by the learners because of misconception and failure to attention to the basics they have to apply in answering the questions.

5.3.3. Procedural Errors

Findings show that, the procedural error occurred due to the learner's lack of understanding of the structural properties inherent in the transition from exponential equations to linear equations. For example, in the process of eliminating the square root from the expression (Figure 5.8), the correct approach required squaring both sides. However, the learner instead divided 8 by 2 and associated the square root sign with the value of 2, leading to the erroneous expression $4^y = 16$ (Figure 5.9). This approach indicates the learner's inability to effectively transform an exponential equation by applying the appropriate laws of exponents to solve for y (Figure 5.10).

Additionally, the learner demonstrated a lack of understanding of the procedure for establishing common bases. The methodology employed, as evidenced in Figure 5.8, reveals an attempt to equalize parts without adhering to established mathematical rules. This deviation from procedural norms is further emphasized by the lack of equivalence between the two sides of the equation. The resulting expression $4^y = 16$ does not align with correct mathematical transformations, highlighting the learner's challenge in applying the laws of exponents accurately to solve for y.

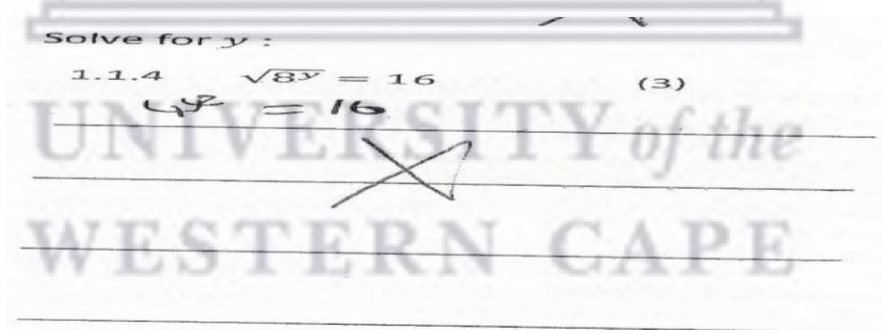

Figure 5.8: Procedural Error

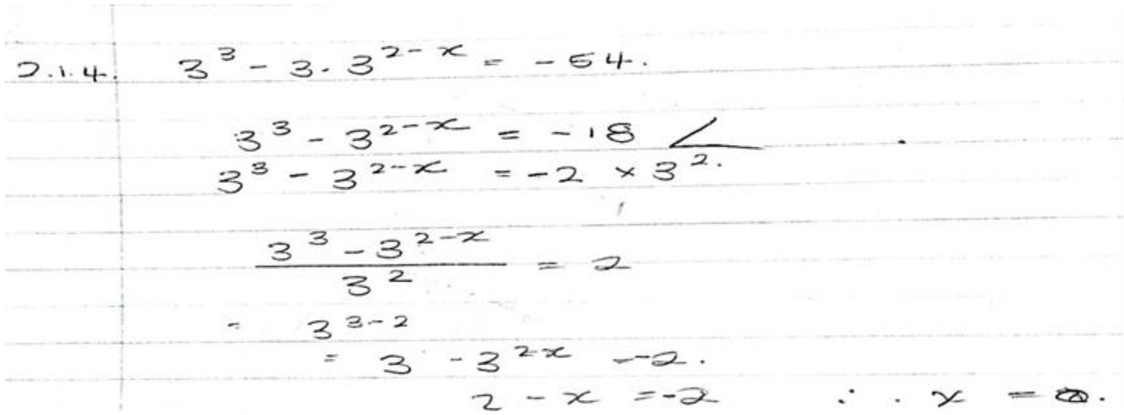

Figure 5.9: Procedural Error

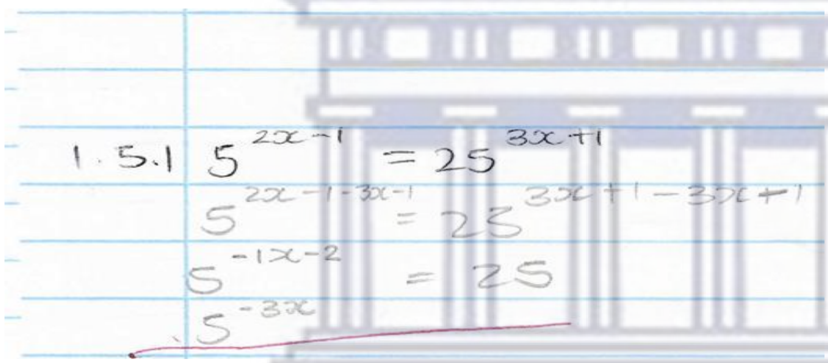

Figure 5.10: Procedural Error

While analysing the Mathematics scripts, it was observed that when the application of exponential rules was necessary to answer certain questions, learners consistently failed to employ them accurately, resulting in incorrect linear equations.

5.3.4 . Application Errors

Learners frequently commit errors when they possess a conceptual understanding but encounter challenges in applying it effectively to specific situations or questions. In instances where there is a conceptual gap, learners may struggle to recall the precise principles or rules required to solve a given problem. According to Moru, Qhobela, Wetsi, and Nchejane (2014), application errors are characterized by a failure to execute manipulations correctly. Figures 5.11, 5.12, and 5.13

exemplify situations where learners, as observed in this study, failed to apply the power of powers rules accurately. The inaccuracies in the application were particularly noticeable in the selection of incorrect signs for absolute numbers.

This research identifies distinct application errors, including scenarios where learners:

- Attempt to create the same bases but execute the process incorrectly.
- Understand how to transition from exponential equations to linear equations but encounter difficulties in arriving at the correct linear equation.
- Initiate a question but fail to complete it, resulting in an incomplete response.

To provide clarity on the various application errors identified, examples are presented. For instance, the error illustrated in Figure 5.11 serves as a representative instance of an application error wherein the learner understands when to create the same bases but misapplies the process. This type of error aligns with the classification of an application error, as emphasized by Moru et al. (2014).

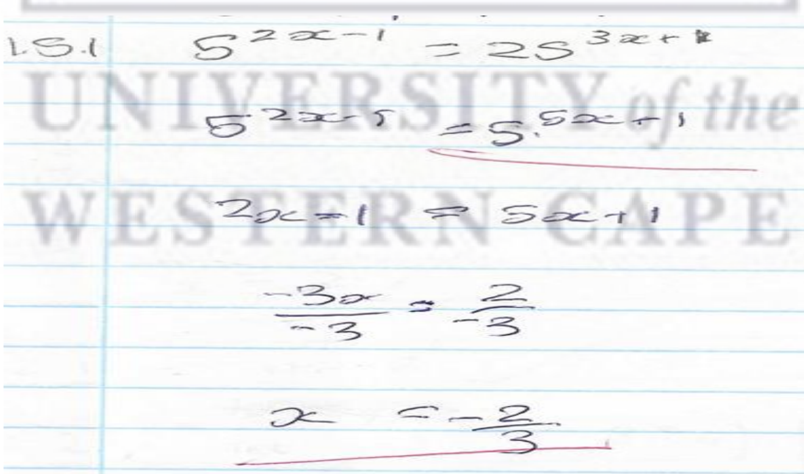

Figure 5.11: Application Errors

The following application error was obtained in a case where the learner knew how to answer a linear equation from an exponential equation, but failed to get the right one. This example shows the application error.

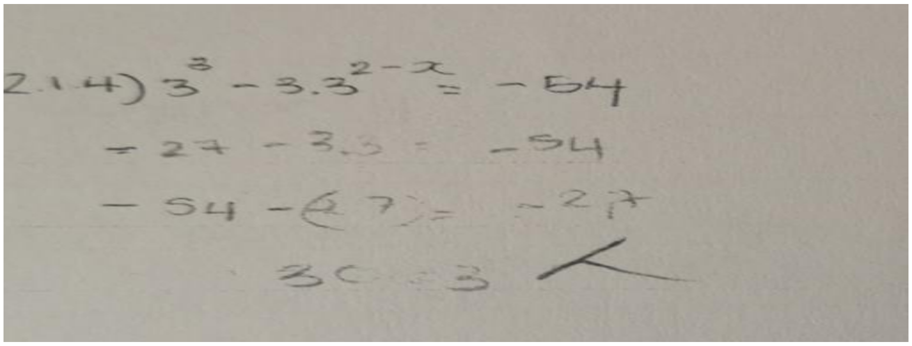

Figure 5.12: Application Error

The last one, the application error stated upstairs was found numerous times. During the study of the scripts, looking precisely at exponential equations solving x or y, where the right linear equation was calculated, the substitution of the equation was all done correctly. It was also evident that when there was a mistake made with the linear equation.

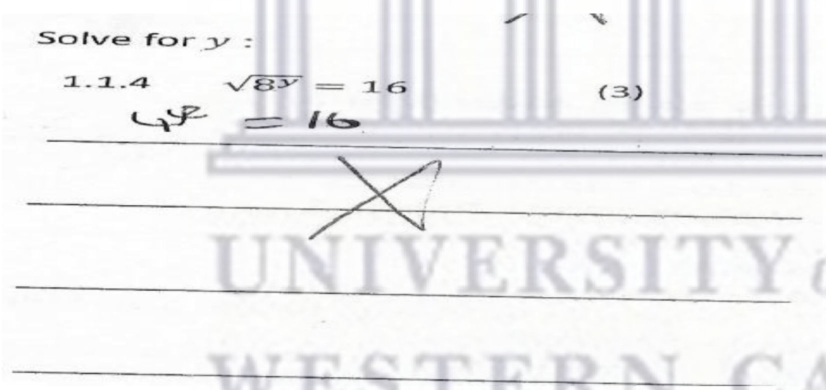

Figure 5.13: Application Error

5.3.5 . Non-Completion Errors

In figure 5.14, the learner managed to create the same bases but failed the procedure to come up with the right answer (Schoenfeld, 2010).

The down error was identified separately and was part of the analytic framework. This error can be seen as a non-completion error. This error is committed when the learners attempt to solve the equation, but stop after creating the same bases on both sides and do not follow the procedure to complete the question. In Figure 5.14 below, learners stop at the section where he/she was supposed to drop bases and compare the exponents.

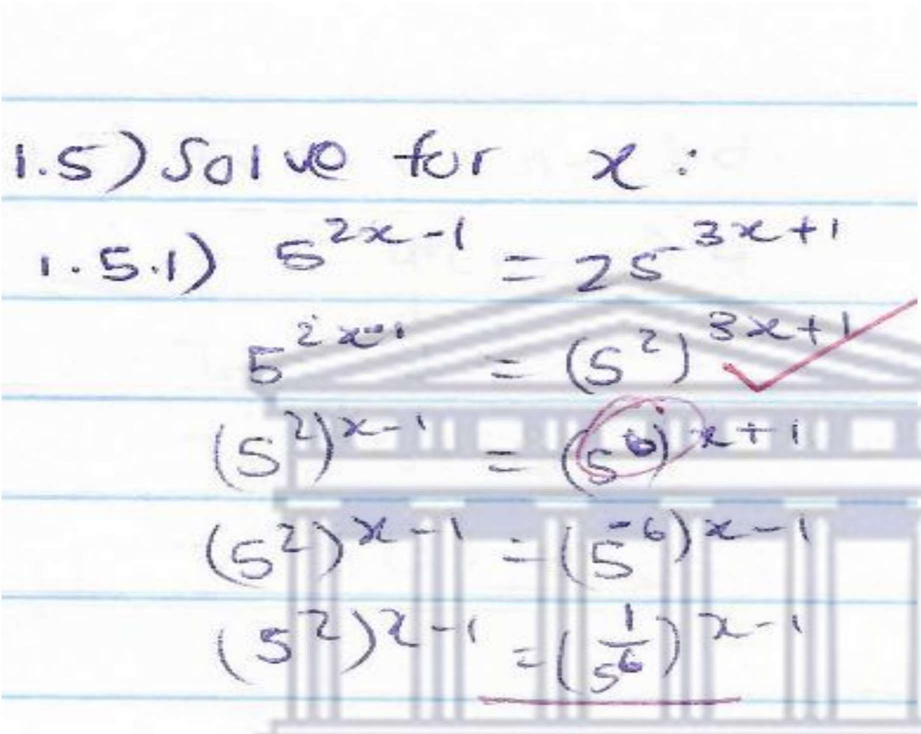

Figure 5.14: Non-Completion Error

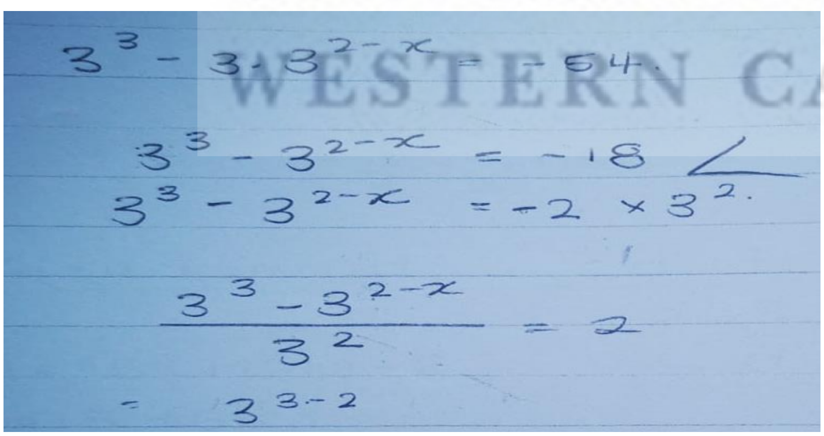

Figure 5.15: Non-Completion Error

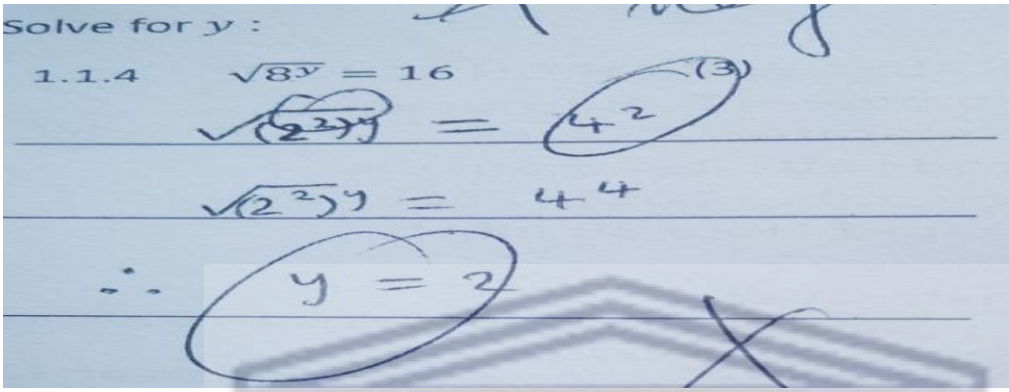

Figure 5.16: Non-Completion Error

5.3.6. Other ways of working: No Responses

This research cores on the learners' way of working. Then, it was necessary to study instances where there were no answers by learners since this is seen as a way of working. No reasons can be put forward as to why students leave questions without answers. One can only speculate the causes for learners not answering questions:

The learners might not know the answer to the question.

The learners might decide to answer the question last but he/she did not have enough time.

The learners might not have enough time to finish the exam.

According to the analysis, there were different categories of blank errors summarised in two different types and defined as follows:

Blank: Learner did write the question down but did not try to answer the question or they provide the number of the questions.

Jump: No number, no attempt, or no space, was not provided by the learner.

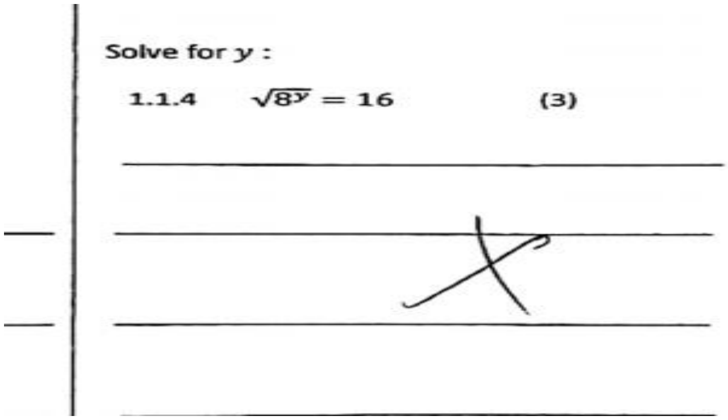

Figure 5.17: (Blank)

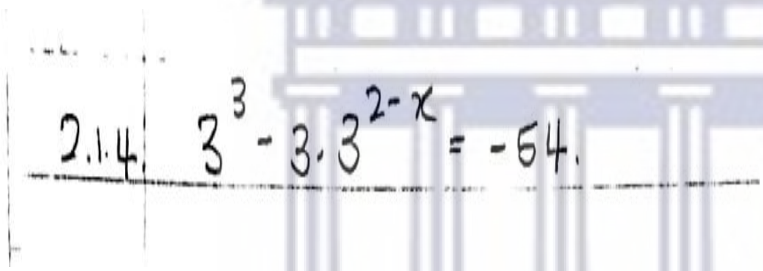

Figure 5.18: (Blank)

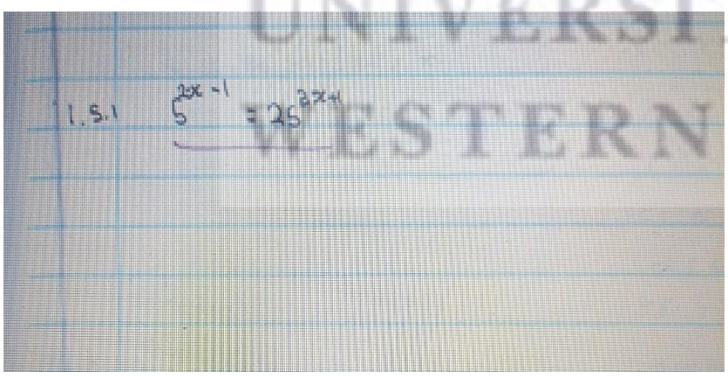

Figure 5.19: (Blank)

-**Jump**: No number, or no attempt, or no space, was shown on the answer script.

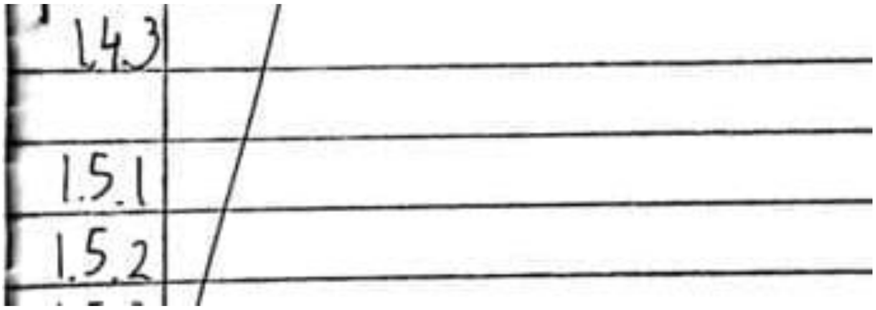

Figure 5.20: Other ways of working: Jump the question

2.1.4

Jump

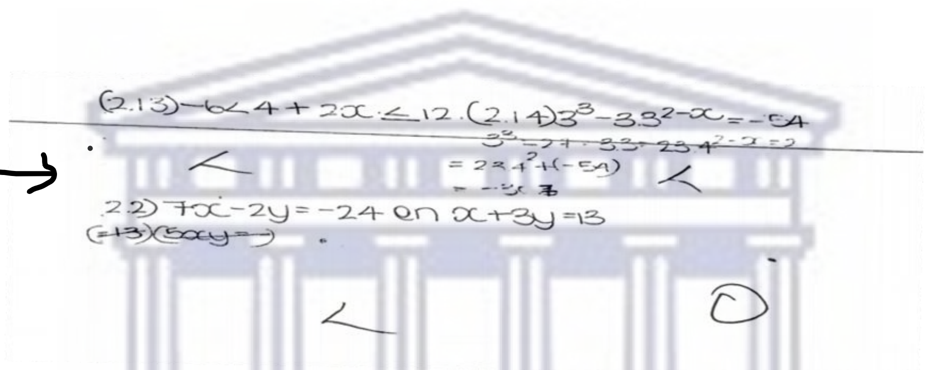

Figure 5.21: Jump (no space, no attempt.)

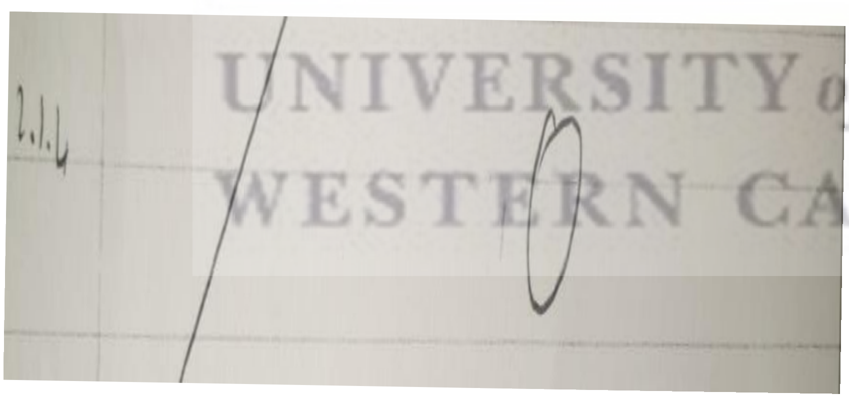

Figure 5.22: Other ways of working(only write the number down.)

Learners are struggling to answer linear equations from exponential equations, in the collected scripts, the below table is showing the summary of :

The learners might not have had enough time to complete the examination.

The learners might have decided to come back to the question but ran out of time.

The learners might not have known the answer to a question.

Table 5.1: Other ways of working (No response)

Errors	**Other ways of working**	**School A**	**School B**	**SchoolC**	**Total**
Jump	No number Written down	2	3	1	6
Blank	No attempt	1	0	0	1
	No space leave	4	1	0	5
		8	3	1	12
Total		**15**	**7**	**2**	**24**

Table 5.1 shows the number of learners who failed to give the answer which presents the percentage of 21% of the scripts collected, while the other errors present 79%.

CHAPTER 6

DISCUSSION, RECOMMENDATIONS, AND CONCLUSION

6.1. Introduction

The objective of this study was to identify the strategies and tactics that learners follow when solving mathematical linear equations from other types of equations posed to them in their final grade 10 examinations 2021 by identifying errors exhibited in their responses. The following are the questions set out for this study:

- What are the errors and misconceptions visible in the written answers of learners in their solution-seeking pursuance when solving linear equations from other forms of equations on questions given to them in the final grade 10 mathematics examination?
- What are the strategies and tactics visible in the written answers of learners in their solution-seeking pursuance when solving linear equations from other types of equations related to algebraic equations?
- What are the factors that contribute to these errors and misconceptions?

The conceptual framework presented in this research shows consistent and systematic ideas and patterns that have been used by various authors to identify errors and misconceptions in mathematics. These systematic ideas and patterns made it easy to employ a qualitative document analysis method.

The papers analysed were the learner's examination scripts, focusing on the linear equations from exponential equations and specifically questions 1.1.4.; 1.5.1 and 2.1.4; from three different schools. The target questions that were given to learners in the final Grade 10 mathematics examinations of 2021 are set out in Chapter 4. A conceptual framework was used to set up a coherent set of ideas or categories of errors that would assist in the identification and analysis of errors and misconceptions in the learner's work. It is a process of reviewing students' written work by looking for patterns of misconception or misunderstanding (Simon,2012).

These errors were identified and named based on the analytic framework that was compiled in Chapter 2. The study also revealed misconceptions and errors outside the framework that were not part of the initially constructed framework but contributed to a vast number of errors that were identified, namely non-completion errors and exponential equation solution non-completion errors. This chapter discusses the results, makes recommendations for teaching practice, points to further research in the area, and concludes the thesis.

6.2. Analysis of errors and misconceptions

This comprises possible mathematical descriptions for the misconception and errors that were recognised.Misconception and Errors are normally part of the process of constructing knowledge and in fact, may be a necessary corrective step when it comes to teaching and learning. The conceptual framework presented in this study shows consistent and systematic ideas and patterns that have been used by various authors to identify errors and misconceptions in mathematics. Error analysis, also referred to as error pattern analysis, is the study of errors in learners' work to find explanations for these reasoning errors (Herholdt and Sapire,2014).

These systematic ideas and patterns made it easy to employ a qualitative document analysis method. Document analysis is a method commonly used to identify the causes of students' errors and misconceptions when they make consistent mistakes.

6.2.1. Careless Errors

Careless errors are those instances where learners incorrectly transcribe a symbol or number from the question paper to his/her script or a prior-obtained number or symbol is incorrectly copied and miscopied (Godden, 2012). A careless error is deemed to be an error that was made on one occasion but not on another similar occasion. Normally this does not result in learners losing a large number of marks in the final exam of grade 10 in 2021 it shows that some of the learners were capable of transforming linear equations from exponential equations but failed to continue with the other next steps to on the final answers.

Figures 5.1,5.2 and 5.3 in chapter 5, are examples of questions from three different schools of careless errors examined in the collected scripts. In the 116 scripts analysed, it was found that only 8% of errors can be recognised as careless errors. Of the number of errors found in 89, only careless errors were 10% which was a small number according to Godden, Mbekwa, and Julie (2013).

Even if the calculation showed that the number of careless errors was small in the data collected but still the learners' marks will be reduced, however, those errors can be caused by the disorder of the learner during that period of the exam. This can be a problem because learners are not used to practicing with similar methods of copying the questions before answering them, some of them start providing their answers before they copy the questions.

6.2.2. Calculation Errors

The error can be corrected if the learner goes over his answer sheet when finished writing (Godden,2012). Calculation errors occur when a learner makes mistakes in addition, subtraction, multiplication, and division of numbers. Two types of calculation errors were found In this research; basic calculation errors, where the learner managed to make the same bases by answering linear equations from exponential equations, but failed to proceed with other steps in the right way, and calculation errors, where learners used a calculator and ended up with the wrong result.

Of the total number of calculation errors analysed, 18% were calculation errors. The cause factor of this error can be attributed to how learners assimilated the basics as many learners started grade 10 with a lack of basic knowledge from previous grades. Since mathematics is a systemic subject, shortage of maths teachers is believed to be the cause of poor performance and errors since there is not enough time to asist learners to practice. This is collaborated with Makonye and Fakude (2016), who argue that poor performance in mathematics contributes to shortages of mathematics teachers in South Africa. Another factor can also be learners who do not take enough time to revise their work at a time because many families are not educated, and even if they are, some of them do not have enough knowledge in Mathematics.

The calculator error which is stated above was identified in questions where the learner used a calculator to answer exponential equations. (Refer to Figure 5.7 in Chapter 5). The total numb**er** of calculator errors was 16. This constituted 14% of the total scripts analysed. This error can be caused by the lack of knowledge of scientific calculators, as many schools in South Africa are not allowed to use calculators in Mathematics up to grade 9. Another cause of this error can be that some teachers did not train learners on how to use calculators.

6.2.3. Procedural Errors

A procedural error was committed when the learners tried to answer the questions but failed to use the right method to come out with the right answer. In questions 1.1.4;2.1.4 and 1.5.1 learners (refer to Figures,5.8;5.9; and 5.10) respectively, had to keep the focus on what the question predicted of them. Learners failed to follow the question method. One can argue that students did not know the procedure to answer the question. The cause of this can be that teachers do not give the complex questions related to the resolution of exponential equations to the learners for more practice.

In mathematics education, several challenges often surface in students' approach to procedural tasks, particularly in the context of high-stakes examinations. One prevalent issue is the overreliance on memorization, where students memorize procedures devoid of a profound understanding of the fundamental concepts Jones, Jones, and Hargrove (2003). This tendency can lead to critical errors when students encounter variations of problems that necessitate adaptability and a deeper comprehension of the underlying principles. Another common hurdle is the confusion in formulas or rules, where students may inadvertently mix up or misapply formulas, leading to procedural errors in problem-solving.

Furthermore, limited feedback opportunities in high-stakes exams pose a significant obstacle, hindering students from promptly rectifying misconceptions. As we delve into specific instances related to exponential equations, it becomes evident that misapplication, carelessness, and inflexibility in handling exponential processes are notable contributors to procedural errors in this mathematical domain. This compilation explores these challenges, shedding light on the intricacies students face and emphasizing the importance of targeted instructional strategies to enhance procedural proficiency in high-stakes assessments.

Overreliance on Memorization: Students may memorize procedures without fully understanding the underlying concepts. This can lead to errors when faced with variations of problems that require adaptation of procedures.

Confusion in Formulas or Rules: Confusion or conflation of formulas, rules, or the application of the wrong procedure to a specific problem type can lead to procedural errors.

Limited Feedback Opportunities: High-stakes exams often provide limited opportunities for students to receive immediate feedback on their procedural errors. This can hinder their ability to correct misconceptions promptly.

Misapplication of Exponential Rules: Students might misapply the rules governing exponential equations, such as errors in understanding how to simplify expressions with exponents, combine like terms, or solve equations with exponential variables.

Carelessness in Exponential Calculations: Procedural errors in exponential equations may result from carelessness in calculations, such as making mistakes in exponentiation or multiplication involving exponential terms.

Failure to Follow Exponential Solution Steps: Students might neglect specific steps in solving exponential equations, such as isolating the exponential term, applying logarithmic functions, or using properties of exponents correctly.

Incomplete Handling of Exponential Processes: In the context of exponential equations, students may provide incomplete solutions by not fully addressing the exponential processes involved, leading to procedural errors.

Lack of Flexibility in Exponential Problem-solving: Students may struggle to adapt exponential procedures to different types of exponential equations, especially if they lack a flexible understanding of the underlying concepts.

Inadequate Preparation for Exponential Equations: Inadequate preparation for dealing with exponential equations, including understanding the rules and properties, can contribute to procedural errors in high-stakes exams.

6.2.4. Non-Completion Error

This error occurs when the learner tries to answer the question, and the correct procedure is followed but she/he does not finish the question. There are some possible reasons for that, it could be that the learner did not have sufficient time to complete the question, or the learner did not check the answer sheet before he/she handled it to check if all questions were answered.

As also mentioned by Godden (2013), the learner did not have sufficient time to complete the question, or the learner possibly got stuck with the question, moved on, and forgot to come back or did not have time to come back. There can be logical reasons, but if the learner is not interrogated, it is hard to draw the right reason caused that.

6.2.5. Application Errors

Application error is committed when a learner knows the method but he/she applies it incorrectly. This error was made when learners tried to make the same bases and did manage but made mistakes in writing the wrong exponents or wrong bases. It could be caused that, the learners who made this error were may be focused on what the question expected of them than on how to read the rule needed to answer the asked question (refer to figure 5.11;5.12 and 5.14)in chapter 5.

6.3. Conceptual Understanding in Mathematics

According to Van de Walle, Karp, and Bay-Williams (2015) conceptual understanding in mathematics involves grasping fundamental mathematical principles, theories, and procedures. It includes a deep comprehension of concepts such as algebraic rules, geometric properties, and arithmetic operations.

6.4 .Challenges in Applying Mathematical Concepts

Mathematics education requires not only understanding mathematical concepts but also the ability to apply them to solve problems. Learners often encounter challenges when transitioning from abstract mathematical concepts to practical problem-solving situations. In the realm of mathematics education, the imperative emerges that it is not sufficient for learners merely to understand mathematical concepts; rather, the essence lies in their capacity to proficiently apply these concepts to problem-solving scenarios. Mathematics education transcends rote memorization, emphasizing the cultivation of robust problem-solving skills, critical thinking, and the ability to reason mathematically. This approach seeks to establish a seamless connection between theoretical knowledge and practical application, with an emphasis on the transferability of skills across diverse mathematical domains and real-world contexts (National Research Council, Donovan and Bransford (2005). Recognizing the significance of mathematics as a tool for problem-solving in various disciplines, including STEM fields, education in this domain incorporates real-world contexts and assessments that go beyond traditional testing methods. Ultimately, the goal is to equip learners with the skills not only for immediate application but also to foster a foundation for life-long learning and adaptation to new mathematical challenges throughout their academic and professional journeys.

6.5. Procedural Fluency vs. Conceptual Understanding:

There can be a discrepancy between procedural fluency (knowing how to execute mathematical procedures) and conceptual understanding grasping the underlying principles. Learners may memorize procedures without fully understanding the reasons behind them, leading to errors in application.

6.6. Problem-solving and Critical Thinking

Effective mathematics education emphasizes problem-solving and critical thinking skills. Learners need to be able to analyse problems, identify relevant mathematical concepts, and apply appropriate strategies to arrive at accurate solutions.

6.7. Transfer of Mathematical Knowledge

The challenge lies in the transfer of mathematical knowledge from one context to another. Learners might struggle to recognize when and how to apply specific mathematical concepts in different problem-solving scenarios.

6.8. Real-World Applications

Mathematics education aims to prepare learners for real-world applications. Errors often arise when learners struggle to connect abstract mathematical concepts learned in the classroom to practical situations that require mathematical reasoning and problem-solving skills.

6.9. Effective Pedagogical Approaches:

Educators play a crucial role in facilitating the application of mathematical knowledge. Employing teaching strategies that encourage active engagement, such as hands-on activities, collaborative problem-solving, and real-world examples, can enhance learners' ability to apply mathematical concepts effectively Frenc (2005).

6.10. Diagnostic Feedback and Error Analysis:

Teachers can use errors as diagnostic tools to identify misconceptions and areas of weakness. Providing constructive feedback that addresses both the procedural and conceptual aspects of errors helps learners understand and rectify their mistakes.

6.11. Mistakes as Learning Opportunities:

Mathematics education should foster a growth mindset, where mistakes are viewed as opportunities for learning and improvement. Encouraging learners to reflect on their errors and understand the reasons behind them contributes to a deeper and more enduring understanding of mathematical concepts (Boaler,2015). The challenges in applying mathematical concepts underscore the importance of a holistic approach to mathematics education, integrating both conceptual understanding and practical application. Effective teaching strategies and a supportive learning environment can contribute to bridging the gap between theoretical knowledge and its real-world application in mathematics(National Research Council, Donovan and Bransford ,2005).

6.12. Other ways of working: No response

When students leave answers blank in a mathematics examination paper, several factors could be at play, and educators need to consider various aspects:

6.12.1. Understanding vs. Confidence:

Leaving a question blank may not always indicate a lack of understanding. It might be due to a lack of confidence in their ability to solve the problem or uncertainty about the correct approach Wiberg, Lyrén, and Ramsay (2021, July).

6.12.2. Test Anxiety:

Some students may experience test anxiety, leading them to skip questions even when they know the answers. Anxiety can hinder their ability to recall information and perform to the best of their ability (Ramirez, Gunderson, Levine, and Beilock, 2013).

6.12.3. Time Management:

According to Cavalcante and Huang (2022), learners might run out of time before completing all the questions. It's crucial to consider whether the blank responses are a result of poor time management during the exam.

6.12.4. Misinterpretation of Instructions:

Students may misinterpret or misunderstand the instructions for a particular question, leading them to skip it. It's essential to assess whether the blank responses are a result of confusion or misinterpretation.

6.12.5. Fear of Making Mistakes: Some students might be afraid of making mistakes, especially in high-stakes exams. Fear of getting the wrong answer could lead them to leave questions unanswered (Boaler, 2015).

6.12.6. Lack of Motivation:

Students might leave questions blank due to a lack of motivation or interest in the subject matter. Understanding their level of engagement with the material is crucial.

6.13. Conceptual Gaps:

Blank responses could indicate a genuine lack of understanding or knowledge of the topic. It's essential to differentiate between students who are hesitant but knowledgeable and those who lack the necessary understanding.

In mathematics education, several factors contribute to students leaving questions unanswered. A lack of motivation or interest in the subject matter may lead students to bypass certain questions, underscoring the importance of gauging their level of engagement with the material. Blank responses can also be indicative of conceptual gaps, reflecting a genuine deficiency in understanding or knowledge of the topic (Chaman, Beswick, and Callingham,2014). It becomes crucial for educators to discern between students who are hesitant but knowledgeable and those who lack the necessary understanding. The assessment approach plays a pivotal role, and educators are encouraged to consider whether partial credit is allowed. When students are aware that partial understanding is acknowledged and credited, they may be more motivated to attempt every question. Embracing a growth mindset is paramount, emphasizing that mistakes are integral to the learning process and encouraging students to tackle all questions, even when unsure while communicating the value placed on effort. Additionally, fostering a supportive environment is essential, where students feel at ease seeking clarification on challenging questions, ultimately reducing the likelihood of leaving questions unanswered due to uncertainty (Nofriyandi and Andrian, 2022).

To address these issues, educators can implement strategies such as providing clear instructions, offering time management tips, discussing test-taking strategies, and creating an atmosphere that encourages risk-taking and learning from mistakes. Additionally, individual discussions with students to understand their reasons for leaving questions blank can provide valuable insights into their learning experience.

This type of phenomenon was not defined in the analytical framework, which was also given the name of other ways of working, where I tried to divide them into two types: Blank and Jump questions. Blank, the learner wrote the question down with no attempt to answer the question or provide the number of the questions.

6.14. Jump

Jump is when there were, no question numbers, no attempt, or no space. Figure 5.17; 5.18; 5.19, 5.20, 5.21 and 5.22.

The assumption

Cognitive Load and Understanding: In the realm of educational research, much attention is paid to the idea of cognitive load. This term relates to the manner in which learners absorb and retain new information, as well as the most effective teaching methods to facilitate this process. For instance, certain individuals may possess a stronger aptitude for numerical concepts than others. As a result, some learners may find a question to be overly intricate or, if they lack comprehension of the underlying principles, they may opt to bypass the question altogether(Van Merrienboer and Sweller,2005).

Time Management Strategies: Literature on educational assessment and testing frequently discusses the importance of time management during exams. Students may strategically choose to skip questions to ensure they have enough time to answer other questions.

Motivation and Engagement: The literature on motivation in education can shed light on why students may skip certain questions. Lack of motivation or engagement with the subject matter can lead to avoidance behaviors, including skipping questions (Renninger and Hidi,2015).

6.15. RECOMMENDATION

Mathematics is a subject that requires the learners to have a succession of concepts and rules from previous grades, to help them provide the right answers when they are tested or examined. It was recognised in the responses of learners that there is a big gap where a large number of learners are not capable of creating the same bases for them to be able to solve linear equations. This may be a factor in how they thought or assimilated the related basics in the previous grades.

As I am a personal tutor in Mathematics from grade 6 up to grade 12 and first-year university and college, I started to do my research in 2020, where I chose a topic of why learners are struggling to construct linear equations from word problems, as mentioned that in first chapter 1.2. However, I was surprised when in the three schools where I collected the data, no school prepared the question on how learners can construct and answer linear equations from word problems. This created some sort of judgment towards teachers because it looks like, either they do not teach that to learners.

This led me to advise teachers, to follow the government curriculum when they are teaching as well as testing the learners, or practicing in classes to let the learners to get used it.
This caused me to shift to linear equations from exponential equations in the same grade as well as the same school, but the ways learners are struggling to answer these types of equations are very bad.
Another factor I recognised in my tutoring sessions was that some of the teachers limited their learners to some chapters to revise, or gave them questions they would work on in exams instead of revising the whole work they have done in their class according to the curriculum. This can be a cause of failing to answer the questions correctly because their teachers did not train them to work systematically with all the concepts or rules they met in their previous grades. This can be the factor that caused the learners to struggle with constructing and solving linear equations from other forms of equations. Teachers need to help learners revise mathematics concepts regularly and practice them daily, and they should be supervised very seriously.

6.16. CONCLUSION

The purpose of this research was to identify common errors made by students when solving mathematical linear equations in their final grade 10 exams. By analysing these errors and classifying them, I aimed to provide an effective method for pinpointing specific areas where students struggle with constructing and solving linear equations from other forms of equations.

It is important for students to take their assessments seriously and learn from their mistakes. This study serves as a valuable resource for classifying different types of errors that students may encounter when solving linear equations from other equation types.

Moving forward, it is important for future researchers to determine how best to provide assistance to students struggling with mathematics. This is a crucial step in ensuring that all students have the opportunity to succeed in this subject.

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APPENDICES

UNIVERSITY *of the* WESTERN CAPE

Private Bag X17, Bellville 7535, Cape Town, South Africa

Telephone :(021) 959 3858/9 Fax: (021) 959 3849

Humanities and Social Sciences Research Ethics Committee at 021 959 4111 or email research-ethics@uwc.ac.za

Information letter to Principal/ SGB

My name is Jeanne d'Arc Muberarugo and I am a Masters of Education student from the University of the Western Cape (UWC). **I am conducting research titled: An analysis of grade 10 learners' ways of working when constructing and solving algebraic equations from word problems: The case of three high schools in the Western Cape**. The research is intended to investigate the strategies and tactics learners follow when solving mathematical word problems posed to them in their final grade 10 examinations by identifying the errors and misconceptions exhibited in their responses. The University of Western Cape (UWC) has approved ethical clearance for this research study.

Research plan and method:

In this research, a case study method will be used to gain an in-depth understanding of the cause of errors visible in the written answers of learners in their solution-seeking pursuance when solving word problems related to algebraic equations. The qualitative analysis will seek to understand and give feedback on learners understanding of linear algebraic equations through the responses in the final grade 10 school based examination. No learner's name will be included in my records and no names will be mentioned of either the school or learners' in any chapter of my thesis. Thus, confidentiality and anonymity is a priority in my study.

This study has been motivated by the general performances in Mathematics in the NSC examination, which are written at the end of grade 12 year by all schools in South Africa. The Department of Basic Education use this examination to reflect of the state of curriculum delivery in all subject. From the NSC examination a diagnostic analysis are send out to teachers to give them a sense of where the focus in Mathematics should be in the teaching and learning of the subjects in all grade.

It is on this background and evidence from local literature that I am pursuing this research. I believe that the findings of this study can benefit the school, teachers and learners of Mathematics.

Thank you for taking the time to read this information. Please let me know if you require any further information. You can also contact the Humanities and Social Science Research Ethics Committee via Mr.P Josias at 0219594111 or email research-ethics@uwc.ac.za if you have any concerns or complaints that have not been adequately addressed by me. I look forward to your response as soon as is convenient.

(Jeanne d'Arc Muberarugo) (Dr Simons)

Cell: 0734911171 (021) 9592441

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Humanities and Social Sciences Research Ethics Committee at 021 959 4111 or email research-ethics@uwc.ac.za

SCHOOL PRINCIPAL/SGB

Dear Sir/Madam

My name Jeanne d'Arc Muberarugo, I am a student at the University of the Western Cape. I write this letter to you to ask permission to do my research at your school. The study I am doing is titled: **An analysis of grade 10 learners' ways of working when constructing and solving algebraic equations from word problems: The case of three high schools in the Western Cape.** As the researcher in this study, I want to ensure the following principles regarding the way I will work throughout this study.

- The involvement of the school and participants is voluntary.
- You may decide to withdraw the school's participation and or learners work at any time without penalty.
- Learners in respective Mathematics classes will sign an informed assent for their permission to analyse their examination scripts
- The parents of learners whose scripts will be analysed will be approach for their informed consent. .
- Only learners who parents signed consent will participate in the project.
- All information obtained will be treated in the strictest confidence.
- The school and learners' names will not be mentioned in any written report
- A report of the findings will be made available to the school.

If you need any further information, please do not hesitate to contact my supervisor or me.

Student name: Jeanne d'Arc Muberarugo Supervisor name: Dr. Simons Marius

Tel: 0734911171 Tel: (021) 9592441

Email: mdsimons@uwc.ac.za

I the Principal of the school ***give permission*** to Jeanne d'Arc Muberarugo to conduct research at the school**:** ________

I the Principal of the school ***do not give permission*** to Jeanne d'Arc Muberarugo to conduct research at the school**:** _______

Supervisor name: Dr. Simons Office no: (021) 9592441, Email: mdsimons@uwc.ac.za

Researcher: Cell: 0734911171, Email: 3760569@myuwc.ac.za

Principal Signature

____________________________ Date ________________________

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Humanities and Social Sciences Research Ethics Committee at 021 959 4111 or email research-ethics@uwc.ac.za

Dear Parent

My name is Jeanne d'Arc Muberarugo and I am a Masters of Education student from the University of the Western Cape (UWC). I am conducting research titled**: An analysis of grade 10 learners' ways of working when constructing and solving algebraic equations from word problems: The case of three high schools in the Western Cape**. The research is intended to assist in the improved teaching of word problems in schools in the Western Cape. The University of Western Cape (UWC) as well as the school principal has approved for me to collect data at your child's school. My supervisor is Dr. Simons Marius.

The data will be kept confidential as they will stay secured on my password-protected computer. All research data will be destroyed with 5 years after completion of the project. The research activities will not interfere in any way with your child's schoolwork. The permission is being sought to analyse parts of their final grade 10 maths exam. The learners themselves will not be involved in the study, only the scripts of learners will be analysed. If you grant your child permission to participate in this research, please complete and return the attached form.

Thank you for taking the time to read this information. Please let me know if you require any further information. You can also contact the Humanities and Social Science Research Ethics Committee via Mr.P Josias at 0219594111 or eresearch-ethics@uwc.ac.za if you have any concerns or complaints that have not been adequately addressed by me. I look forward to your response as soon as is convenient.

Note, all COVID protocol will be observed.

Student name: Jeanne d’Arc Muberarugo

Tel: 0734911171

Email: 3760569@myuwc.ac.za

Supervisor name: Dr. Simons Marius

Tel: (021) 9592441

Email: mdsimons@uwc.ac.za

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PARENT CONSENT FORM

I…………………………. (Full name of parent) hereby confirm that I received and understand the information letter as to the research study as well as the nature of the research project, and I assent/do not assent (scratch out where applicable) to my child…………….............(full name of child) participating in the research project.

I understand that my child is at liberty to withdraw from the project at any time without any negative consequences, should I/he/she so desire.

Signature of Parents: Date:

FACULTY *of* EDUCATION
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Humanities and Social Sciences Research Ethics Committee at 021 959 4111 or email research-ethics@uwc.ac.za

LEARNER ASSENT FORM

I ……………………………………………………….…………………………...

(Please write full name and surname)

Accept to participant in the study entitled:

An analysis of grade 10 learners' ways of working when constructing and solving algebraic equations from word problems: The case of three high schools in the Western Cape.

I have received a satisfactory explanation of the general purpose and process of this study and the conditions that I will be exposed to.

I understand that my participation in this study is voluntary and I will receive no payment for participating.

I know that this research will not interfere with my learning and that I can stop participating in this study at any time.

Signature of learner: ……………………………………………………….

Date: ………………………………………

Researcher's names: Jeanne d, Arc Muberarugo (Masters Student, UWC).

Signature: …………………………………………………………………………...

Date: …………………………………………

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Humanities and Social Sciences Research Ethics Committee at 021 959 4111 or email research-ethics@uwc.ac.za

PARTICIPANT INFORMATION SHEET

TITLE OF THE RESEARCH STUDY: An analysis of grade 10 learners' ways of working when constructing and solving algebraic equations from word problems: The case of three high schools in the Western Cape.

RESEARCHER'S NAME: Jeanne d'Arc Muberarugo

ADDRESS: Radiant Mansions, 15 East Street, Grassy Park, 7941, Cape Town.

What is this research study all about?

This research project is being conducted by Jeanne d'Arc Muberarugo, a student at the University of Western Cape. The purpose of this research is to identify the strategies and tactics learners follow when solving mathematical word problems posed to them in their final grade 10 examinations by identifying the errors and misconceptions exhibited in their responses. We hope that the research will provide possible suggestions and recommendations to the government, relevant authorities, and all stakeholders involved in introducing the Mathematics word problems in South African Schools.

Note the following:

Your participation in this research is voluntary

You can withdraw from the research at any time without consequences

Your name will not be mentioned and all information will be kept confidential

There is no monetary contribution from the researcher for your participation

The research will benefit teacher and learners when dealing with algebraic equations from word problems

If you not clear about thing pertaining the study, feel free to ask

If you are not interested in taking part in this study, that is fine! Nothing will happen to you at school or in class. It will not affect your marks or reports in any way. Just simply tick 'no' in the box below.

Note, all COVID protocol will be observed

Do you understand this research study and are you willing to take part in it?

YES NO

Do you understand that you can pull out of the study at any time?

YES NO

______________________________ ______________________

Signature of Child Date

Western Cape Government
Education

Directorate: Research

meshack.kanzi@westerncape.gov.za
Tel: +27 021 467 2350
Fax: 086 590 2282
Private Bag x9114, Cape Town, 8000
wced.wcape.gov.za

REFERENCE: 20220120-9150
ENQUIRIES: Mr M Kanzi

Mrs Jeanne Muberarugo
15 East Street
Grassy Park
Cape Town
7941

Mrs Jeanne Muberarugo,

RESEARCH PROPOSAL: AN ANALYSIS OF LEARNERS' WAYS OF DOING WHEN CONSTRUCTING AND SOLVING LINEAR EQUATIONS FROM WORD PROBLEMS: THE CASE OF GRADE 10 AT A HIGH SCHOOL IN WESTERN CAPE.

Your application to conduct the above-mentioned research in schools in the Western Cape has been approved subject to the following conditions:

1. Principals, educators and learners are under no obligation to assist you in your investigation.
2. Principals, educators, learners and schools should not be identifiable in any way from the results of the investigation.
3. You make all the arrangements concerning your investigation.
4. Educators' programmes are not to be interrupted.
5. The Study is to be conducted from **20 January 2022 till 31 March 2022.**
6. No research can be conducted during the fourth term as schools are preparing and finalizing syllabi for examinations (October to December).
7. Should you wish to extend the period of your survey, please contact Mr M Kanzi at the contact numbers above quoting the reference number.
8. A photocopy of this letter is submitted to the principal where the intended research is to be conducted.
9. Your research will be limited to the list of schools as forwarded to the Western Cape Education Department.
10. A brief summary of the content, findings and recommendations is provided to the Director: Research Services.
11. The Department receives a copy of the completed report/dissertation/thesis addressed to:

The Director: Research Services
Western Cape Education Department
Private Bag X9114
CAPE TOWN
8000

We wish you success in your research.

Kind regards,
Meshack Kanzi
Directorate: Research
DATE: 20 January 2022

1 North Wharf Square, 2 Lower Loop Street,
Foreshore, Cape Town 8001
tel: +27 21 467 2531

Private Bag X 9114, Cape Town, 8000
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17 December 2021

Mrs J Muberarugo
School of Science and Mathematics
Faculty of Education

HSSREC Reference Number:	HS21/10/29
Project Title:	An analysis of learners' ways of doing when constructing and solving linear equations from word problems: The case of grade 10 at a high school in Western Cape.
Approval Period:	**16 December 2021 – 16 December 2024**

I hereby certify that the Humanities and Social Science Research Ethics Committee of the University of the Western Cape approved the methodology, and amendments to the ethics of the above mentioned research project.

Any amendments, extension or other modifications to the protocol must be submitted to the Ethics Committee for approval.

Please remember to submit a progress report by 30 November each year for the duration of the project.

For permission to conduct research using student and/or staff data or to distribute research surveys/questionnaires please apply via:
https://sites.google.com/uwc.ac.za/permissionresearch/home

The permission letter must then be submitted to HSSREC for record keeping purposes.

The Committee must be informed of any serious adverse events and/or termination of the study.

Ms Patricia Josias
Research Ethics Committee Officer
University of the Western Cape

Director: Research Development
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Email: research-ethics@uwc.ac.za

NHREC Registration Number: HSSREC-130416-049

FROM HOPE TO ACTION THROUGH KNOWLEDGE.