An investigation into pre-service teachers' mathematical behaviour

in an application and modelling context.

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SEPTEMBER 2006

A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery.
(Polya, 1957, p.v)
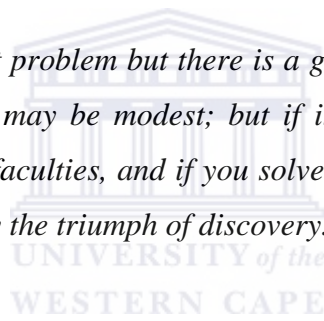
DECLARATION

An investigation into pre-service teachers' mathematical behaviour in an application and modelling context is my own work and all sources I have used and quoted have been indicated in the text and acknowledged by means of complete references.

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T.V. Lebeta

September 2006

DEDICATIONS

This thesis is dedicated to my mother, Makemiso Eusebia, and to my late father, Biggs Gregory, who raised and supported me and my siblings under difficult circumstances.

To my wife, Thandi, for being there when needed and my special thanks for taking care of the children during my periodic study leave.

To my children, Thapelo Thomas, Tumelo Christopher, Lebohang Faith Choeu and Kabelo Leloko, for their understanding.

To my sisters and brothers for encouraging and supporting me throughout my tertiary studies.

To all friends and colleagues for their support.

To the catholic family, especially late Fr Dieter Gahlen and late Fr Florian Makoro, for arranging financial support that helped me to start and pursue the career of my choice.

To Fr Xolile Tedy Khumalo and Fr Gregory Van Dyk for their support and best wishes.

ACKNOWLEDGEMENTS

KEY WORDS

Mathematical behaviour and strategies

Applications of mathematics

Mathematical modelling

Modelling context

Problems and dilemmas

Reasoning

Familiar context

Realistic considerations

Thematic approach

Cultural Historical Activity Theory

ABSTRACT

The field of Teacher Education is a frequent target of criticism both from inside and from outside and different views on how to improve it has been expressed (National Center for Research on Teacher Learning; 1992). In South Africa, the attempts to improve teacher education resulted in the development of Norms and Standards for Educators. The norms provide seven key roles and expected competencies for teacher preparation programmes. One of these seven roles focuses on teachers' knowledge, particularly looking at the teacher as a subject specialist. In this study, teachers' knowledge is identified as the key issue in teacher education (teacher preparation and in-service training). Acknowledging that measuring of teachers' knowledge does not depend on the degree obtained or the type of mathematics courses taken, the focus of this study is on describing students' ways of finding solutions to mathematical problems formulated from familiar contexts.

The aim of the study was to investigate the hypothetical view that the use of familiar social institutions in the formulation of mathematical problems by mathematics pre-service teachers will enable them to find solutions to problems by taking meaning, context and realities of a problem into consideration. The approach to investigate this hypothetical view was to describe the mathematical behaviour of pre-service teachers in an application and modelling context. This study, therefore, describes the strategies used to arrive at solutions for problems from real life situations that are familiar to the participants. In order to describe the participants' strategies in depth, several qualitative methodological considerations were made. Amongst others, interpretivism, hermeneutics and social constructionism were key in making sense of the participants' work.

The findings show that the participants' strategies utilise both social and mathematical rules. In these strategies the social rules play a dominant role. The dominance of the social

rules is attributed to the use of familiar social institutions in the themes that were used to collect data. It was evident that the use of familiar social institutions in the formulation of mathematical themes enabled the mathematics pre-service teachers to find solutions to problems by taking meaning, context and realities of a problem into consideration. These realistic considerations, that is, contextual interpretation of the problem and the solutions, transcend the traditional placement of the mathematics solutions as classrooms. Participants' solutions agitate for social change. The answer to the solution is seen as having a direct link to social change. The study argues that this kind of agitation has a potential to have them (participants) transformed in the process, that is, becoming changed agents.

This study has opened a window for looking at the ways student teachers would approach mathematical problems that have been formulated out of familiar contexts and being aware that their text reports or 'artifacts' will be presented to appropriate stakeholders for assessment or being aware that their work will become 'public entity'. The variety of strategies that emerged during the process of an investigation should not necessarily be assessed in terms of whether students managed or did not manage to find solutions to the problems but should be assessed in terms of richness and 'meaningful artifacts' (at least to them) the students' generated approaches bring to mathematics education.

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CHAPTER 1

INTRODUCTION

1.1   An overview of the study

This study focuses on mathematical behaviour of pre-service teachers in an application and modelling context. The study explores the way pre-service teachers work out solutions to problems and the way they apply mathematics in the social institutions familiar to them. In contrast to the traditional way of teaching high school mathematics to pre-service mathematics in a more abstract and context free way, the study uses students' social dilemmas to work out real life problems applying mathematical strategies. Guided by the Cultural Historical Activity Theory (CHAT) as its theoretical framework and using qualitative methods to analyse data, the study has found four strategies that pre-service teachers use in finding solutions to real life problems. These are knowledge driven, imaginary driven, example driven and students' contextual logical strategies. It also emerged that the participants in this study prefer mathematical problems from real life contexts and that there is a strong view that these types of problems help them to achieve a meaningful understanding of mathematics.

Increasingly, there are calls to bridge the gap between classroom mathematics and mathematics outside the classroom. The National Qualifications Framework (NQF) policy issues, as explained below under "Background", have added to these calls. This study has made a contribution in bridging this gap.

1.2   Definition of key concepts

The key concepts in this study are: mathematical behaviour, applications of mathematics and modelling context. The meaning of these key concepts is closely linked to other concepts that are presented in the next paragraphs. Firstly, I present the description of the

concepts related to the key concepts in this study, and I conclude the discussions in this section by providing the description of the key concepts.

The contextual description of these concepts is necessary, as different authors have tended to present them differently in different contexts. The significance of these concepts serves to describe and to build the desired mathematical environment for learning and teaching in pre-service teacher education. The desired environment in this study refers to a situation where students' familiar social institutions are closely linked to mathematics learning and teaching. The concepts are intended to fulfil three roles and there are overlaps in some cases. Firstly, some concepts serve the role of a criterion, e.g. what is mathematics? What do we mean by a problem? Further examples in this category are: meaning, understanding, model, mathematical model and mathematical power. The second category is evaluation, and examples are: realistic considerations, activity, human activity, and object of learning. The line between the first and second category is a thin one. The third category describes the process, and examples are: mathematization of activities, modelling, mathematical modelling, and activity systems.

1.2.1 Problem, mathematical problem, dilemmas

In this study, the distinction between a mathematical exercise and a mathematical problem is made. *The mathematical problem* refers to a situation where students are confronted with a task and they need to arrive at the solution to that problem as it relates to their life experiences. The students see a need to solve that problem as it may, in one way or another, affect them in their lives. *Mathematical exercises* are those tasks, which call for manipulative skills (e.g. multiplication, factorisation techniques). Lave's (1988) view is that a problem is a dilemma with which the problem solver is emotionally engaged and conflict is the source of dilemmas. She continues to say that the processes of resolving dilemmas are correspondingly deprived of their assumed universalistic, normative,

decontextualized nature. Furthermore, Lave (1988) argues that studies of math in practice have demonstrated that problems generated in conflict can be solved or abandoned, and often have no unique or stable resolution.   Lave's (1988) view encourages teacher educators and researchers to go beyond the 'no solution' responses as they have different messages. Forman (2003) also argues that if the problem is not answered, it does not necessarily mean that students do not have knowledgeable information about the problem, it may, amongst others, mean that students hold different beliefs that are in conflict with the given problem. Hayes (1981), Borba (1990) and Nickles (1981) also elaborate on this issue by further showing various methods of solving problems from real life contexts.

For a mathematical learning environment geared towards development, these descriptions of a problem are important and are taken as a guide in this study.

1.2.2 Reasoning/Reason

Reasoning, though difficult to describe, is the cornerstone of any understanding and sense making. It is not surprising that even in mathematics education, mathematical reasoning is at the heart of mathematical learning and teaching. Ball and Bass (2003), for instance, argue that any mathematical understanding is meaningless without reasoning. Different authors describe reason / reasoning from different perspectives. I here choose Donald (1999) and Simon (1983). Donald (1999) argues that reason is a biological product – it is a tool with inherently and substantially restricted power. He further says that it has improved how we do things; it has not changed the reasons why we do things. Donald (1999) also views reason as a tool, as described as follows:

To a teacher, it is an intellectual exercise for developing the minds of the young. To a

    lawyer, it is a way to confirm or refute testimony…to an economist, it is a means of

    allocating resources to maximise efficiency, utility and wealth (Donald, 1983, p.14).

On the other hand, Simon (1983) further says that reason is wholly instrumental, it cannot tell us where to go; at best it can tell us how to go there. "It is a gun for hire that can be employed in service for any goals we have - good or bad" (Simon, 1983, p.8).

From Donald's (1999) point of view, there are two types of reasoning, namely: informal and formal reasoning. Simon (1983), on the other hand, talks of cold and hot reasoning. Simon describes hot reasoning as the reasoning that "seeks deliberately to arouse strong emotions, often the

emotion of hate, a powerful human emotion" (Simon, 1983, p.30). In terms of cold reasoning, it can be argued that there is lack of passion and emotions are not really evoked in the process of doing something. These categories also contributed to the analysis of the data of this study. Furthermore, from these descriptions of a reason, I describe how groups used reasoning (a gun for hire) to facilitate the process of finding solutions to the given mathematical tasks.

1.2.3   Realistic considerations

Realistic considerations refer to a situation where calculations or steps or arguments are checked against the real life situation. In this case the logical reasoning or an outcome is validated in terms of the conditions in real life. Any other responses or reactions that are assessed to be out of context are referred to as unrealistic as opposed to 'wrong'.

1.2.4 Mathematisation of reality, mathematisation of activities/tasks

Mathematisation of reality or activities refers to conscious (sometimes unconscious) action oriented strategies to arrive at the solution to a problem through the application of mathematics. The word 'conscious' is used as there may be other ways of finding the solution, but in this case the route of mathematics as a tool is used.

1.2.5 Meaning

The students are said to have learnt mathematics meaningfully if they see mathematics as a relevant tool, connected to their lives and experiences, with which to make sense out of social phenomena (Gutstein, 2003).

1.2.6 Understanding

From Skemp's (1979) perspectives there are two types of understanding, *relational understanding* and *instrumental understanding*. Relational understanding refers to the

ability to deduce specific rules or procedures from more general mathematical relationships. Instrumental understanding refers to the ability to apply an appropriate remembered rule to the solution of a problem without knowing why the rule works.

Romberg (in his 1989 report) says that learning with understanding involves more than being able to produce correct answers to routine problems. He further argues that learning with understanding occurs when it becomes the focus of instruction, when students are given time to discover relationships, learn to use their knowledge, and when they reflect about their thinking and express their ideas. Learning with understanding is a product of interactions over time with teachers and other students in a classroom environment that encourages and values exploration of problem situations (Romberg, 1989).

Usiskin (1991) also says that understanding is doing, it involves skill and algorithm, understanding is knowing why. Envisaged 'understanding' or evidence of understanding in this study will, therefore, refer to the ability to show/discover relationships, the ability to give reasons and the ability to apply (skills) or deduce rules/algorithms required for the problems.
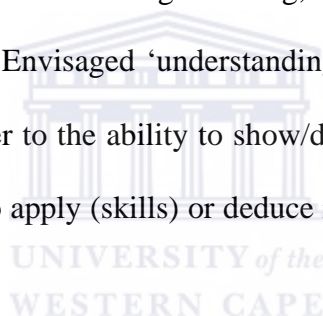
1.2.7   Mathematical power

Gutstein (2003) says that according to the Principles and Standards from NCTM (2000) (which was adopted as a working definition of mathematical power), students displaying *mathematical power* will have the following:

Students confidently engage in complex mathematical tasks…draw on knowledge from a
      wide variety of mathematical topics, sometimes approaching the same problem
      from different mathematical perspectives or representing the mathematics in
      different ways until they find methods that enable them to progress… are flexible
      and resourceful problem solvers…work productively …communicate their ideas

and results effectively …value mathematics and engage actively in learning it (Gunstein, 2003, p.46).

1.2.8   Activity

Davydov (1999) describes *activity* as a specific form of the societal existence of humans consisting of purposeful changing of natural and social reality. Furthermore, Davydov says that any activity carried out by a subject includes goals, means, the process of molding the object, and the results. Activity is, in principle open and universal; it should be taken as a form of historical and cultural creativity (Davydov, 1999). Davydov also argues that in fulfilling the activity, the subjects also change and develop themselves.

The notion of activity in the framework of dialectical materialism takes a particular form, as it is agued that the initial form of activity is the production of material tools that help people produce objects satisfying their vital needs (Davydov, 1999).

The significance of this concept is overarching in the sense that it creates focus and also the transformative process in the sense that in engaging in an activity, there is a deliberate intention to 'mould the object'. The description of this concept ties well with the concept of modelling, mathematical modelling and application of mathematics.

1.2.9   Mathematics, human activity

The South African Department of Education describes mathematics as follows:

Mathematics is the construction of knowledge that deals with qualitative and quantitative relationships of space and time. It is a **human activity** (my emphasis) that deals with patterns, problem solving, logical thinking, etc., in an attempt to understand the world and make use of that understanding. This understanding is expressed, developed and contested through language, symbols and social interaction.

(Department of Education, 1997, p.108)

There are no further explanations on what is meant by human activity. It could be deduced that it implies that mathematics is part of human day-to-day experiences or mathematics is something that relates to all human experiences. Davydov says that human activity uses features of one natural object as tools for acting on other objects, thus turning the former into an organ of activity. Davydov's description is very helpful as it links human activity to the concept of activity and, therefore, human activity is done with a purpose in mind. The issue of engagement in patterns, problem solving, etc is intended to understand the world and make use of that understanding, is emphasized.

1.2.10 Object of learning activity

Miettinen (1999) refers to Engestrom (1987) as saying:

> The *object of learning activity* is the societal productive practice, or the social life-world, in its full diversity and complexity. The productive practice…exists in this present dominant form as well as in its historically more advanced and earlier, already surpassed forms. Learning activity makes the interaction of these forms, i.e. the historical development of activity systems, its object (Miettinen, 1999, p.331).

The above quotations emphasise Leont'ev's (1978) view that an activity is defined by its object (Miettinen, 1999). The point made by Engestrom (1987) is that the meaningful object of learning activity cannot be confined to the school environment or be reduced to a university's or schools' text, but it could be found through interaction with the society in the real world. This concept is then very important in the context of application of mathematics and it is helpful for evaluative purposes for the desired learning and teaching environment.

1.2.11 Activity systems

Activity systems *are multi-voiced strategies. This is as a result of the collective nature of participation in fulfilling an activity. They are not situation bound, not linked to a particular text or object, they are relatively lasting new patterns of interaction and within*

activity systems (Engestrom, 1999a). Activity systems as described are very collective and the strategy is to have an outcome of which the impact has a lasting effect. This again emphasizes the view that activity has a goal and results.

1.2.12 Mathematical behaviour

Mathematical behaviour in this study refers to any strategy that is used to find the solution to the given mathematical theme. Within each strategy or combined strategy, the mathematical principles feature in varying degrees, that is, from a less prominent role (dominance of social rules) to a more prominent role. An investigation of pre-service mathematical behaviour, therefore, refers to what Collins, Brown and Newman (1989) call "students' generated algorithms".

1.2.13 Model and mathematical model

According to Christiansen (1994), a model *is a tripartite arrangement of* A, M, f. A *represents the reality under consideration;* f *is the mapping of some items in* A *into* M. *A model is, therefore, seen as the representation of something. Following the description of a model, Christiansen (1994) states that a model will be a* mathematical model, *if* M, *as indicated above, is the collection of mathematical objects. In short, a model mimics the relevant features of the situation studied and, therefore, a mathematical model is that model that mimics reality by using the language of mathematics (Bender, 1978). There are various ways of classifying mathematical models, for instance Kapur (1988) classifies the mathematical models in terms of subjects (e.g. mathematical model in biology), nature (e.g. deterministic) and technique (e.g. mathematical model through partial differential equations).*

1.2.14 Modelling

Following Christiansen's (1994) argument, *modelling* is a process through which a mapping or correspondence segment of reality and collection of objects is constructed.

1.2.15 Mathematical modelling and applications of mathematics

In this study *mathematical modelling* is understood to refer to the process through which the solutions to real world problems are found using mathematical strategies and the solutions are interpreted in terms of the original formulation. Applications of mathematics in the context of this study will refer to a situation where mathematics is used as a tool to find solutions to real life problems. According to Ormell (1991), there are twelve levels of applications of mathematics. In this study I only refer to eight levels, that is, level 1 to 8. In these levels (1-8), mathematics is useful in practice and mathematical modelling is applied to find solutions to real world problems. To show the overlapping roles or mathematical modelling and application of mathematics, Burghes (1980) is quoted by Christiansen (1994) as saying that mathematical modelling is a unifying theme for all applications of mathematics. Christiansen (1994) further refers to Niss (1990) as saying that any application of mathematics presupposes an explicit or implicit modelling (process). It has been difficult to practically separate the two concepts application of mathematics and mathematical modelling, and consequently, to a large extent, these terms are used interchangeably. It has also been noted that the use of problem solving is not different from mathematical modelling. For instance, in Schoenfeld's (1989) work, the concept of mathematical modelling assumes the same meaning as problem solving. For consistency, the concept of mathematical modelling has been used throughout.

1.3   Background to the study

The South African history since 1994 has been characterised by a number of policy formulations. In the educational arena, the Department of Education has come up with the National Qualifications Framework (NQF) which promises a bold new educational dispensation which includes learner centredness, the integration of education and training, the facilitation of lifelong learning and the development of the knowledge and skills required to carry South Africa into prosperity in the 21st century (Ensor, 1997). The translation of the NQF in different fields or learning areas is yet to be evaluated. As explained by Ensor (1997), what is different about the NQF compared to other educational policies both here in South Africa and internationally, is that it attempts to establish, through outcomes-based education, an equivalence between formal schooling and the knowledge which is produced and acquired in other settings. This is also clearly articulated in the Education White Paper of 1995 which states that the NQF seeks to achieve the unity of the mental and manual, the academic and vocational, an equivalence between school knowledge, work knowledge and everyday knowledge of domestic and recreational settings (Ensor, 1997). There are a number of concerns pertaining to the performance of learners at schools and the ability of learners to apply their 'acquired' knowledge in the place of work. My informal interaction with a number of persons and reviewed literature indicate that employers are relatively happy about the basic mathematical skills of new graduates, but appalled by their ability to translate a real problem into a mathematical problem. There is also an apparent inability to communicate their results to other people. As to who should carry the blame, varies from one person to another.

There is, however, an emerging consensus that the problem is not in the learning but in the teaching. Mogale (1998), for instance, has strongly argued that many

educationists believe that children do not have a learning problem; it is the teachers who have a problem teaching. While some may contest this view, particularly its generality in South Africa, it still remains necessary for this claim to be investigated. The pursuance of this issue is not within the scope of the study. Another issue widely acknowledged is that learning and teaching is a complex process and exact factors that contribute towards acceptable performance (ability to meet certain agreed standards, ability to communicate results, demonstration of understanding as opposed to memorisation) vary. The teaching problem as identified by Mogale (1998) may be influenced by other external factors.

In the field of mathematics education, some international researchers such as Greer (1997) and Reusser and Stebler (1997) have also argued that the lack of understanding and the tendency to ignore contextual meaning of the learning situation cannot be attributed to the cognitive deficit of learners. These authors argue that there is an emerging consensus that the explanation of this pattern can be attributed to the cultural context of the classroom and the nature of schooling itself. In fact Greer (1997) says that children act in accordance with the 'didactical contract'. The didactical contract refers to adherence by students to the restrictive 'culture' the teacher and students have adopted to work out mathematics problems in classroom situations. Within this unsigned contract students do not think beyond classroom challenges such as different representations or extra variables in a problem. If the teacher has worked with two variables x and y, students more often think of two variables and the approach used in class.

These issues (e.g. relevance and understanding) as raised in the NQF and many other issues that are put forward by some researchers, have made an increasing demand that teaching of mathematics at all levels should be contextualised and meaningful. This view is in line with the view expressed elsewhere in the United States wherein Presmeg (1998)

argues that there is a convincing literature on culture and connections between mathematics content and home, between mathematics and world of work and between mathematics content and the real world. This in some ways portrays the ideas advocated by the realist mathematics education movement as practised and explored in the Netherlands (see Gravemeijer, 1994). Further emphasis (as argued in the Realistic Theory) on the need to mathematise horizontally (mathematise reality) and vertically (mathematise mathematical activities) shows the relevance of considering contexts, meaning, interpretation of what we teach, and practical applications of mathematics. The issue of relevance, meaningful learning/teaching is not only a South African issue, but also a worldwide concern.

While strong advice has been given and earnest appeals have been made in a number of literatures, there are *very few studies* conducted in this area in South Africa. Most of the studies conducted largely concentrate on how to teach mathematics content without looking beyond the teaching of this content and also more importantly, the history, the nature of that content and the circumstances surrounding those who are taught. To put it more bluntly - the learning and teaching of mathematics seem to be an innocent process. There are, however, interrogative studies done on culture and mathematics (Mosimege, 1998), and also research projects such as those advocated by the Realistic Mathematics Education in South Africa (REMESA). Amongst others REMESA aims to:

> develop and research the impact of innovatory mathematics learning and teaching materials based on the premise that 'reality is the basis of and the domain of application of mathematics' (REMESA, n.d.).

I am of the opinion that studies in this direction emphasise Presmeg's (1998) view that in the growing diversity of student populations, mathematics is not value and culture free. Furthermore, I believe that studies/research of this nature will encourage conscious

inclusion of day-to-day experiences. Reference as to how mathematics relate to those experiences will create a rich classroom environment.

To a large extent this study is influenced by ideas expressed in the NQF, other related policy documents and research work done elsewhere on the gap between school mathematics (mathematics world) and mathematics needed for our working life (real world). In line with the argument expressed earlier by Mogale (1998) Greer (1997), and Reusser and Stebler (1997) concerning teaching, the study focuses on teacher education and investigates how mathematics could be taught to student teachers in such a way that the objectives of the NQF and Education White Paper (1995) are realised. The challenge facing those involved in teacher preparation is what should be taught and how? These questions and the views expressed in policy documents have been very influential in choosing the area of study.

1.4 Social context of the study

This study was conducted at a place where cultural values such as decorations, play, music, food and transport are still visible. The difference between urban settings and villages is not only in the physical location and resources but also in terms of physical appearances and social practices. The urban houses are typical modern houses (built with bricks and corrugated zinc / tile roofing) and the houses in the villages are more of the indigenous type of houses as shown in figure 1 below.

Figure 1: Village houses
Source:http://www.sa.venues.com/attractionsfs/thabamofutsanyana.htm

Figure 1 is an example of typical Basotho practices, play, decorations, dress, preparation of food and transport (horses). About 15 kilometres from the main township, there is a place called Basotho Cultural Village. The village is seen as the heritage for Basotho cultural practices.

From an ethnomathematics perspective and from general text analysis, figure 1 shows a lot of mathematics in this social context. The pre-service teachers who took part in this study are familiar with these types of contexts. The question that has never been explored, is how much do they bring from these contexts to mathematics learning environments and learning scenarios. In spite of the lack of information, the social context presented by figure 1 is important as it shows the social life-world of pre-service teachers or societal productive practices as argued by Engeström (1987) as quoted by Miettinen (1999). According to Engeström, these contexts (social life-world or societal productive practices) form part of the object of the learning activity.

1.5   Problem statement

The study looked at the hypothetical view that the use of familiar social institutions by mathematics pre-service teachers will substantially increase their knowledge on how mathematics can be taught by taking meaning, context and realities of a problem into consideration. The study investigated the mathematical behaviour of pre-service teachers

in an application and modelling context. This study therefore, describes the strategies

used in arriving at solutions for problems from real life situations. Amongst others, the

following are the key variables to be described: meaning given to the solution or steps,

demonstration of mathematical understanding, and contextual considerations in working

out mathematical problems. The issue of assumptions in shaping meaning or in finding

the solution to the mathematical problems was also looked at.

From here on the words pre-service teachers, students and participants will be used

interchangeably.

1.6   Questions guiding the study

- What are the students' strategies in arriving at solutions for problems in real life situations?

- What are the students' views on the nature of mathematics?

- What are the students' views on the use of familiar contexts in the application of mathematics?

1.7   The aim of the study

The purpose of this study centres on narrowing the gap between pure mathematics

(school mathematics) and mathematics out of school (real world) or what Lave (1988)

calls gap-closing arithmetic. The students who participated in this study were able to

apply their formal mathematical knowledge and skills learned at school and post school

education in real world situations. It was envisaged that at the end of this study, at least

the non-cognitive factors (beliefs), cognitive (understanding) and psychological (attitudes

towards mathematics) modes would have been attended to. This will be realised through

the following objectives:

- to prepare student teachers to be able to build on the learner's own mathematical thinking

- to prepare student teachers to ask realistic questions instead of training them how to answer posed questions

- to prepare student teachers to devise mathematical problems that ask learners to organise information and predict strategies that might be needed - this will enable students to design a mathematical plan/model

- to instil a positive attitude in students which will increase their level of involvement and decrease high levels of anxiety in mathematics learning and teaching.

- to prepare students to put calculations in context

- to prepare students to look more carefully at creating strategies for arriving at an answer and to encourage students to see mathematics as a set of ideas that make sense. Going through the project, it is hoped that the students will become adaptive rather than routine experts at solving real world problems as argued by Greer (1997).

1.8 Rationale

One challenge facing teacher education institutions is the development of a curriculum which embodies aspects relating to students' contexts and the role of these contexts in shaping meaning, which create learning environments where the gap between pure mathematics and applied mathematics is addressed. This study, therefore, hopes to make a contribution towards South African literature on the applications of mathematics in real life situations and the use of modelling contexts to influence the development of a mathematical curriculum in teacher education.

1.9   Chapter structure of the thesis

The study is divided into four main sections. The first section (chapter 1, 2 and 3) is historical and theoretical; the second section (chapter 4) deals with the design of the study and the frameworks for analysing data. The third section (chapter 5 and 6) focuses on the data and the interpretation of that data. The last section (chapter 7) contains the conclusion and recommendations for further investigations.

1.9.1   Chapter 1

This chapter starts by outlining the purpose of the study. It presents the background to the study, and how this background relates to the problem under investigation. Reference is made in particular to the National Qualifications Framework, which seeks to achieve the unity of the mental and manual, the academic and vocational, and an equivalence between school knowledge, work knowledge and the everyday knowledge of domestic and recreational settings (Ensor, 1997). The context in which the study takes place is described. The chapter offers an attempt to create unity among different settings as identified in the NQF. This attempt is outlined in the rationale of the study; and the chapter presents the objectives and the guiding questions based on the purpose and the statement of the problem. The penultimate section of the chapter discusses definitions of key concepts and how they relate to the study. Finally, the chapter ends by providing an overall picture of the study.

1.9.2 Chapter 2

This chapter focuses on teacher preparation in South Africa. It presents key factors such as models for teacher preparation, knowledge base for pre-service teachers, expected seven roles to be achieved across different learning areas, and policy challenges in teacher preparation. An overview of mathematics teacher education relating to teacher preparation is also presented. At the end the degree to which the constitutive order (political, social and economic structure) and arena (environment and physical structures) need to be considered, is suggested, whenever any study is undertaken as they may directly or indirectly influence the outcome of the study. In particular, the chapter indicates that the findings of the study on the students' ways of finding solutions to mathematics problems should also be understood in a broader context. In sum, the chapter shows that pre-service teacher preparation is a complex process.

1.9.3   Chapter 3

This chapter focuses on the literature review that captures empirical and theoretical debates related to the application of mathematics in a modelling context. It derives its major focus from the main research question that seeks to describe the mathematical strategies that students adopt when they find solutions to familiar social problems. The chapter has two major sections. Section 1 discusses related literature to the application of mathematics in a modelling context, the role of context in the teaching and learning of mathematics, and the issue of transferability across diverse problem domains. Section 2 discusses the theoretical framework that guides the study. The framework draws mainly on the work on the Cultural Historical Activity Theory (CHAT).

1.9.4 Chapter 4

This chapter discusses the methodology of the study. It is argued in this chapter that richness of data comes as a result of a multi methods strategy of investigation, collection and analysing. The chapter consists of section I and section II.

Section I starts by arguing that any research, whether qualitative or quantitative, is idea driven. To illustrate this point three ideas as argued by Wolcott (1992) are presented. These research ideas are theory-driven ideas, concept-driven ideas, and reform or problem-focused ideas. Secondly, the chapter focuses on the methodologies in the context of qualitative inquiry. Key methodologies discussed are intepretativism, hermeneutics and social constructionism. Thirdly, the focus is on research participants, research instruments and delivering 'credible data'.

Section II first presents the approach for analysing the data for the completed tasks as provided in Appendix 2. The approach is grounded in CHAT and other qualitative methods. In this section, other frameworks that were studied and are not explicitly referred to are briefly presented. These frameworks are drawn from the work done in Belgium, the Netherlands, and the United States of America. The first framework centres on the models by Elshout-Mohr, Van Hout-Wolters and Broekkamp (1999), Schoenfeld (1989) and Verschaffel and De Corte (1997).

The section then describes the model by Carlson (1997). This model was used for collecting and analysing quantitative data.

1.9.5   Chapter 5

The chapter presents the results of the three layers of data collection as discussed in the previous chapter. The presentation of the data is two-fold. The first presentation presents

the qualitatively analysed data and the second part presents the quantitatively analysed data. The former relies heavily on Elshout-Mohr, Van Hout-Wolters and Broekkamp (1999) and Verschaffel and De Corte (1997). The source of quantitative data mainly relies on the use of Carlson's (1997) model: Views About Mathematics Survey (VAMS) and Statistical Package for Social Sciences (SPSS) were used to do calculations.

1.9.6   Chapter 6

This chapter presents discussions based on the results in the previous chapter. The guiding questions for the study are revisited. In line with the guiding theoretical framework, the chapter presents discussions focusing on developmental trajectories in students' mathematical behaviour in an application and modelling context.

1.9.7   Chapter 7

Chapter 7 presents the conclusion, recommendations and suggestions for future investigations.

CHAPTER 2

INITIAL TEACHER EDUCATION: AN OVERVIEW OF THE SOUTH AFRICAN CONTEXT

2.1   Introduction

The goal of this chapter is to provide a broader context for teacher preparation (as one of the components of teacher education) of pre-service mathematics teachers in South Africa. The chapter presents key factors such as models for teacher preparation, knowledge base for pre-service teachers and policy challenges in teacher preparation. At the end the suggestion is made that constitutive order (political social and economic structure) and arena (environment and physical structures) need to be considered whenever any study on teacher education is undertaken as they may directly or indirectly influence the outcome of the study. In particular, the chapter states that the findings of the study on students' ways of finding solutions to mathematics problems should also be understood in a broader context.

To achieve the goal of this chapter, I rely on three main authors, namely: Adler (2004), Samaras (2002) and Ma (1999). Samaras (2002) provides a reflective model that needs to be considered in any study related to teacher preparation. She argues that any study related to teacher preparation should consider the following four components: (i) the teacher, or someone who instructs, (ii) the student, or the learner, (iii) the subject matter content, or knowledge, skills, values, attitudes, or ideas that are being presented and shared, (iv) the context or social milieu in which teaching occurs. She further explains that context includes the ethos of the school, the classroom, the norms, and the expectations of the students, teachers, community and the broader society.

Adler's (2004) work provides an overview of mathematics teacher education in South Africa, and Ma's (1999) work demonstrates that teacher preparation in terms of development of mathematical knowledge is part of a cyclic process that starts at school.

Firstly, the types of models for preparation and types of knowledge provided in these models for the teacher preparation process are discussed. Secondly, the chapter presents key aspects of policy that guide both pre-service and in-service education in South Africa. In discussing this overview, I look at the Norms and Standards for Teacher Educators and Adler's (2004) overview of research on mathematics teacher education in South Africa.

2.2   The context or social milieu in which teacher preparation occurs

2.2.1   Models for teacher preparation

Institutional approaches to teacher preparation in South African basically follow two models in preparing pre-service teachers to become qualified teachers. These models are the concurrent model and the consecutive model. The concurrent model is where a qualification programme is designed in such a way that both content and method of teaching are presented simultaneously (Wilson, 2002). An example of such a qualification in South Africa is the new Bachelor of Education (B.Ed.) degree. On the other hand, the consecutive model is described as a type of a teacher preparation programme wherein students are engaged in a qualification where they are first exposed to content and on completion of this qualification, the students enroll in a qualification that specifically deals with methods of teaching (Wilson, 2002). South African examples of this model will be a career path that moves from a three year degree (that meets the criteria of a certain minimum number of school subjects) such as Bachelor of Science to a one year Post Graduate Certificate in Education (PGCE). All qualifications, irrespective

of the model(s) followed by an institution, have an in-built internship programme. The internship programmes are intended to offer a pre-service teacher opportunity to practice teaching, to be exposed to school mathematical content and to the general day to day school activities. The main difference in the internship programmes in the two models, is the duration. For instance, in the consecutive model, the duration is about a month whilst in the concurrent model the internship programme takes about 18 to 24 weeks spread over a period of four years (the larger part of this duration is in the final year of study). The difference has become a point of contention, as some commentators in education believe that the shorter duration in internship compromises quality in training. Other commentators in education defend the consecutive model as saying that it gives prospective student teachers a sound content knowledge that will provide much-needed confidence in the prospective student teachers.

Up to 2001, the South African education system was such that (to a large extent) the colleges of education were following the concurrent model and the universities were following both models. Despite criticisms on methodologies used to teach students how to teach and criticisms on the models, especially the consecutive model, different tertiary institutions in different countries have retained (even after restructuring educational landscapes) the two models of teacher preparation and South Africa is not an exception to this. These two models for teacher preparation cover three major school phases, that is: Foundation Phase, Intermediate Phase and Senior Phase as well as the Further Education and Training Band. The grades and ages for these phases are given in table 2.1.

School Phases	Grades	Age range	Band
Foundation Phase	R, 1, 2, 3	6 – 9 yrs	General Education Band (GET)
Intermediate Phase	4, 5, 6	10 – 12 yrs	
Senior Phase	7*, 8, 9	13 – 15 yrs	
Further Education and Training (FET) Phase	10, 11, 12	16 – 18 yrs	Further Education and Training (FET)

Table 2.1: School phases and bands in South Africa. * In some schools, grade 7 falls in the intermediate phase.

In the following section I present general types of knowledge that are offered within these two models.

2.2.2 Types of knowledge

Mathematics curricula for pre-service teachers outline amongst other issues, knowledge about what is to be taught for which school phase and how this knowledge should be taught. Drawing from various theoretical foundations, institutions of higher learning include in their curricula/programmes aspects that will form a strong knowledge base for pre-service teachers. Both local and international literature has presented major components of initial teacher education programmes. For the purpose of this study, I only focus on the South African categorisation of these components and only a brief reference is made to the international perspective.

Julie (1998) appropriately captures the main types of knowledge, which South African teacher education providers focus on. These types are referred to as Pedagogical Content Knowledge (PCK) and Subject Matter Content Knowledge (SMK). Depending on a model followed by a particular institution, pre-service teachers will have one year or four years (full time) of intensive induction on professional issues of the teaching profession. Over and above the exposure to theoretical issues related to education, more time is spent on issues relating to tasks that student teachers will perform when they enter the teaching profession (Julie, 1998). This means that the sessions that focus on PCK basically offer the students an opportunity to learn how to teach the knowledge prescribed for school learning.

In the case of SMK, the students learn mathematical content that goes beyond the school curriculum. The intention is to provide students with an opportunity to know various

mathematical conventional principles in order to deepen their mathematical understanding.

In addition to the two types of knowledge as described above, Parker (2004), arguing from the South African perspective, has put forward a suggestion that there is a need for a third component, which focuses on learning mathematics education. Parker explains this component as the one wherein student teachers will learn about teaching and learning mathematics. Whilst some people might argue that the second component includes the third component, her distinction between the SMK and PCK makes the third category a necessity. She differentiates between practicing mathematics (learning mathematics as a discipline) and practicing mathematics teaching (learning a professional practice). In the third component, the focus will be on developing knowledge about teaching and learning. If one assumes that Parker's category includes beliefs in the learning and teaching of mathematics, then there is a place for this category in the types of knowledge in the South African context. The role of these types of knowledge is very critical in terms of the pre-service professional development as they lead to the seven expected roles as prescribed in the Norms and Standards for Teacher Educators of 2000 which is the key South African policy document in teacher education. These roles are described later in this chapter.

A cursory comparison of types of knowledge as described in Julie (1998) and Parker (2004) with what most Canadian tertiary institutions prescribe as minimum requirements for a professional programme, shows that there are similarities. This is given in table 2.2. Wilson's (2002) overview of types of knowledge in Canada shows a similar trend in the South African context. Table 2.2 shows a comparison of typical knowledge areas in Canada and in South Africa. The table shows that there are no major differences in the descriptions of the categories. Column 1 and 2 indicate the typical Canadian knowledge base structure as given by Wilson and column 3 provides the typical categorisation of

knowledge base in South African universities. The last column shows official

(government) categorisation of expectations of knowledge base. Descriptions of types of

knowledge are only provided in column 1. These descriptions are similar across the other

three columns, hence they are not repeated.

Grossman's knowledge base as cited by Wilson	*Churukian's knowledge base as cited by Wilson*	*Knowledge base: Institutional based expectations in South Africa*	*Norms and Standards for Teacher Educators: Applied competence – Official Expectations in South Africa*
Subject matter-*knowledge about content*	*Specialised content knowledge and skills*	*Subject Matter Content Knowledge (SMK)*	*Foundational competence (knowing that/what)*
General pedagogical knowledge-*knowledge about beliefs*		*Pedagogical Content Knowledge (PCK)*	*Reflexive competence (knowing why)*
Pedagogical content knowledge- *specific knowledge of what to teach and how to teach it*	*Generic teaching knowledge and skills*	*Pedagogical Content Knowledge (PCK)*	*Practical competence (knowing how)*
Context knowledge – *knowledge of the particular local context of the school and classroom, wider social, cultural and political context and how they affect classroom happenings*	*Foundational studies*	*Pedagogical Content Knowledge (PCK)*	

Table 2.2: Tabular comparison of knowledge areas

A knowledge base is critical in determining the level of competence and professionalism.

For instance, it is reported in the National Strategy for Mathematics, Science and

Technology Education in General and Further Education and Training of 2001 (p.13)

that in 1997, out of 85% of qualified educators teaching mathematics, only 50% of

teachers had specialised in mathematics and that about 8000, that is, 35% mathematics

educators needed to be targeted for in-service training to address lack of subject

knowledge (Department of Education, 2001).

This brief presentation of the types of knowledge offered in initial teacher preparation

shows the knowledge categorisation in teacher preparation and also the emerging

debates on these types of knowledge. The question is how much research has been done

on the impact of these types of knowledge in terms of classroom practice for mathematics

teachers. In other words how 'transferable' are these types of knowledge? As far as the

literature is concerned, there is no comprehensive research on these questions. The

forward thinking on questions such as these is found in Adler (2004) who suggests that

there is a need for continued work to understand the gap between theoretical and

practical knowledge of teaching, between teacher educators and teachers as agents in the

field of mathematics teaching and between research and practice. The issue of knowledge

and its application is further taken up in the next chapter. Elsewhere, research on teacher

knowledge contains numerous examples of a mismatch between the aims of teacher

programmes and prospective teachers' knowledge and beliefs (Kinach, 2002). Kinach

(2002) argues that, increasingly, teacher educators/researchers find that the subject

matter understanding brought to teacher education coursework by pre-service teachers is

not the sort of conceptual understanding that they will need to develop in their future

students. Furthermore, the study by Sloan et al. (2002) also reports a positive correlation

between learning styles of pre-service teachers and mathematics anxiety among

elementary pre-service teachers. These studies, though they do not refer to the South

African context, call for greater caution, critique and more research on issues such as

expected prospective teachers' knowledge, attitudes and beliefs during teacher

preparation of mathematics teachers and also all other prospective teachers in other

learning areas.

2.3 Context, ethos, norms and expectations

Following Samaras' (2002) reflective model, I now focus on the expectations and norms as articulated in the official Policy Document. This document was released in 2000 by the Department of Education (DoE) in South Africa and was regarded as a blueprint in teacher education. The development of this policy was a participative one as statements from the press release below show:

> *They (norms and standards) have emerged from a two year process of consultation, which involved intensive discussions with teacher unions, the South African Qualifications Authority, the Council on Higher Education, the Department of Labour, universities, technikons, and colleges of education. Workshops were held in all the provinces and over 1000 people made comments on draft versions of the policy (Department of Education, 2000b, p.1).*

In terms of norms and expectations in teacher education in South Africa, the process of directions is better summarized by the document on Norms and Standards for Teacher Educators of 2000. It spells out the seven roles and competences that are key to teacher preparation.

The seven roles and the required competences are listed below in table 2.3. It is not within the scope of this study to go into detail on roles and competences. The explanation of roles and competences are attached as Appendix 7. The crucial issue to this study is how they are to be achieved in a balanced way and to what extent the 'overarching role' is achieved.

Relevant quotes from the Norms and Standards for Education of 2000 (pp. 4-6) that relate well to this study are given below.

The roles are:

Learning mediator Interpreter and designer of learning programmes and materials Leader, administrator and manager Scholar, researcher and lifelong learner Community, citizenship and pastoral role Assessor Learning area/subject/discipline/phase specialist

Table 2.3: Seven expected roles for teachers

The roles are subdivided into three competences, namely: Practical Competence,

Foundational Competence and Reflexive Competence. Full details of these competences

are provided in Appendix 8.

Regarding the above competences, the Norms and Standards Policy Document states that:

a. The cornerstone of this Norms and Standards policy is the notion of **applied competence** and it's associated **assessment criteria**.
    Applied competence is the overarching term for three interconnected kinds of competence:
    Practical competence (knowing how)
    Foundational competence (knowing that/what)
    Reflexive competence (knowing why)

b. The seven roles and associated competences for educators for schooling provide the **exit level outcomes**. They are in effect the **norms** for educator development and therefore the central feature of all initial educator qualifications and learning programmes. The critical cross-field outcomes are integrated into the roles and their applied competence. Providers have the freedom and the responsibility to design their learning programmes in any way that leads learners to the successful achievement of the outcomes as represented in their associated assessment criteria.

c. The seventh role that of a learning area/subject/discipline/phase specialist, is the over-arching role into which the other roles are integrated, and in which competence is ultimately assessed. The specialisation can take a variety of forms. It can be linked to phase (for example, foundation phase), or to a subject/learning area (for example, mathematics or human and social sciences), or a combination thereof. Qualifications must be designed around the specialist role as this encapsulates the 'purpose' of the qualification and 'shapes' the way the other six roles and their applied competences are integrated into the qualification.

(Department of Education, 2000a, p. 4 – 6)

The three points above emphasise the Department of Education's view that norms and

standards for educators form a key foundation in the transformation of teaching in South

African schools and that they provide directions and guidelines for the pre-service and in-

service development of professional and competent educators (Department of Education,

2000a). These quotes/extracts are carefully chosen in order to highlight the emphasis of

the DoE's expectations on the types of competences as indicated in point a, and the

planned relationship between outcomes. Furthermore, point (c) identifies the role, which

forms the core in a qualification.

As a form of demonstrating relevance of this study to meet some of the curricular

expectations, particularly the subject specialist role, I present the extracts from the

Revised National Curriculum Statement of 2003. The Revised National Curriculum

(RNCS) specifies learning outcomes and assessment criteria for mathematics and

mathematical literacy and as a result I have relevant extracts from both mathematics and

mathematical literacy. The extracts are presented in table 2.4a and table 2.4b. Table 2.4a

captures the relevant outcomes in as far as mathematics is concerned.

Learning outcomes	Assessment standards		
	Grade 10	Grade 11	Grade 12
Number and number relationships	10.1.4.Use simple & compound growth formulae $A = P (1 + ni)$ & $A = P (1 + I)^n$ to solve problems, including interest, hire purchase, inflation, population growth & other real life problems.	11.1.4. Use simple & compound growth formulae $A = P(1 - ni)$ & $A = P(1 - i)^n$ to solve problems (including straight line depreciation & depreciation on a reducing balance)	12.1.4.a) Calculate the value of n in the formula: $A = P(1 +/- i)^n$ b) Apply knowledge of geometric series to solving annuity, bond repayment & sinking fund problems, with or without the use of formulae
	10.1.6. Solve non-routine, unseen problems.	11.1.6. Solve non-routine, unseen problems	12.1.6. Solve non-routine, unseen problems
Functions and Algebra	10.2.6. Use mathematical models to investigate problems that arise in real-life contexts…(examples should include issues related to health, social, economic, cultural, political & environmental matters)	11.2.6. Use mathematical models to investigate problems that arise in real-life contexts…(examples should include issues related to health, social, economic, cultural, political & environmental matters)	

Table 2.4a: Selected expected outcomes from RNCS -school mathematics
 (Department of Education, 2003, p.21-31)

As shown in Table 2.4a, the RNCS amongst others put an emphasis on use of

mathematical knowledge in day-to-day life experiences. The similar emphasis is also

noted in Table 2.4b. The main emphasis in the Revised National Curriculum Statement

(RNCS) amongst other issues is found in the pre-launch press statement in which it is declared: "It is the Government's policy to replace the present emphasis on an academic approach, and a lack of emphasis on skills..." (Asmal, 2002, p.1). This study has therefore face curricular validity and can act as a resource in the pre-service mathematics programme.

Learning outcomes	Assessment standards		
	Grade 10	Grade 11	Grade 12
Number and number relationships	10.1.2.Relate calculated answers correctly and appropriately to the problem situation by: interpreting fractional parts of answers in terms of the context; reworking a problem if the first answer is not sensible, or if the initial conditions change; interpreting calculated answers logically in relation to the problem and communicating processes and results.	11.1.2. Relate calculated answers correctly and appropriately to the problem situation by: interpreting fractional parts of answers in terms of the context; reworking a problem if the first answer is not sensible, or if the initial conditions change; interpreting calculated answers logically in relation to the problem and communicating processes and results.	12.1.2. Relate calculated answers correctly and appropriately to the problem situation by: interpreting fractional parts of answers in terms of the context; reworking a problem if the first answer is not sensible, or if the initial conditions change; interpreting calculated answers logically in relation to the problem and communicating processes and results.
Functions and Algebra	10.2.1. Work with numerical data and formulae in a variety of real-life situations, in order to establish relationships between variables by: finding the dependent variable; finding the independent variable; describing the rate of change.	11.2.1. Work with numerical data and formulae in a variety of real-life situations, in order to establish relationships between variables by: finding break-even points; finding optimal ranges	12.2.1. Work with numerical data and formulae in a variety of real-life situations, in order to: solve, design and plan problems

Table 2.4b: Selected expected outcomes from RNCS –mathematical literacy
(Department of Education, 2003, p.16-23)

The key roles as stated earlier in this section provide clear governmental expectations on the type of exit level outcomes that prospective teachers need to possess. Those in the practice are also expected to demonstrate the abilities in these seven roles. The following from the press release by the DoE (2000b) shows the extent to which these roles are to guide professional development:

> *The roles will also be linked strongly to **developmental appraisal** (my emphasis), to career pathing and grading and to performance management. They will be used to identify the strengths and weaknesses of a teacher and indicate what kind of training is needed to improve their performance (Department of Education, 2000b, p.2).*

Like any other policy, there are challenges in implementing this policy. For instance, the research report provided by Graven (2002) on the roles the new curriculum presents to mathematics teachers indicates that these new expected roles are complex, comprising of multiple strands and emerge to be conflictual rather than complementary. Graven has further found that there were contradictions in terms of emphasis in as far as the incoming curriculum (official intended curriculum) and outgoing curriculum (which is still largely in implementation stage). Jansen (1999) has also raised issues concerning innovations in the South African curriculum (driven by the philosophy of Outcomes based Education). The issues that Jansen raises amongst other are: the complexity and contradictory language used, flawed assumptions about what happens inside the classrooms and a means end stance of the outcomes based education.

These are highlights of the challenges facing the implementation of curricular innovations at all levels of education in South Africa. These challenges and complexities must be seen in what Adler (2004) considers to be a complex and layered domain of practice that is enabled and constrained by its socio-cultural and political context, leading to varying policies and practices.

At the international level, Beyer (2002) raises some of the challenges and concerns that face programmes for teacher education. I have selectively chosen and paraphrased some challenges raised by Beyer (2002, p.310) below:

a. Standards Boards for Teacher Education (SBTE) focus on a technical-rational approach to teaching and largely ignores social, political and philosophical understanding.

b. Substantive changes in teacher education are externally driven and, therefore, deny professional judgment and intellectual inquiry for teacher educators and prospective teachers at local level.

In view of the above and other concerns, which he summarizes as "Politics of Standards and the Education of Teachers", he offers the following to teacher educators:

..have obligation to prepare their students in ways that go beyond the development of skills, apprenticeship-oriented activities, compliance with current trends, …Given the intellectual depth and moral sensitivity of education as a field and a process, teachers must engage with contrary positions and interpretations, with challenges regarding what is taken for granted, and being open to a range of political directions and normative possibilities. Ignoring larger issues and perspectives diminishes the intellectual and moral dimensions of education, which in turn undermines teaching as a genuine profession. (Beyer, 2002, p.311).

Beyer's (2002) views are further indications that teacher education is complex. The challenges that inhibit the progress in pre-service teacher education should be seen in this context. Furthermore any investigation that is conducted is done so within a complex system.

The question whether the rationale for teacher preparation should be based only on a technical approach, which ignores social, political and philosophical understandings or on both, is a very crucial one. Beyer (2002) argues that the focus should be on both as teacher educators must create and design programmes that enhance not only the quality of education, but also the quality of human life. Feinmann-Nemser and Buchmann (1986) emphatically state that it should be noted that teaching means helping people learn worthwhile things and it is a moral activity that requires thought about ends, means and their consequences.

The argument by the above authors is critical to this study as it centres on the mathematisation of contexts that students are familiar with and are directly affected by those contexts. In terms of a broader focus on mathematics teacher preparation, it needs

to be investigated whether the expectations are realistic. What challenges are there in focussing on both technical approach and philosophical understanding? How sustainable is this kind of milieu that is characteristic of taking expectations of the following: schools, students, educators, community and broader society? These are critical questions that need to be addressed in the development of teacher preparation programmes. In the next chapter, the issue of rationale is taken up and it is argued that rationales are important in shaping what one does and how it is done. This chapter is concluded by looking at highlights of the developments in mathematics teacher education and career paths for mathematics teachers. In doing so, I briefly present Adler's (2004) view on the developments in South African mathematics teacher education and Ma's (1999) view on becoming a competent mathematics teacher.

2.4. Trends in mathematics teacher education in South Africa

Adler's (2004) overview of trends in South African mathematics teacher education covers both pre-service and in-service dimensions. Her main source of data was the conference proceedings for Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE). As the focus of this chapter is on teacher preparation, I have only selected from her study those critical components that relate to initial teacher preparation. I will, therefore, refer to the following as raised in Adler's (2004) overview:

- *Massification of mathematical access at school level vs shrinking of interest in terms of prospective mathematics teachers.*
- *Few studies on pre-service teacher education.*
- *Gap between theoretical knowledge and practical knowledge.*
- *Invisibility of theoretical orientations in research.*

Adler's (2004) study on mathematics teacher education, which covered a period of ten years, shows that whilst there are concerted efforts by Government to create mathematical access at school level, the number of those who join the teaching profession is not increasing. Earlier to Adler's (2004) study, the Department of Education (2001) had attributed the low enrolment of prospective mathematics teachers in initial teacher education programmes to the few learners graduating in mathematics and science at school level resulting into few choosing teaching in these subjects as a career. In the Department of Education's view, this had resulted into a vicious cycle of undersupply of educators in these fields of study. In most cases those that offer these subjects have educators who are under qualified and unqualified (Department of Education, 2001). Adler (2004) suggests that there should be research on the status and growth of the profession.

Related to research on issues in mathematics teacher education, her study also shows that there are few research projects on pre-service teacher education. As Adler points out, the lack of research in teacher preparation at primary level is even more visible. The consequence of the inadequate research in the pre-service site does not only limit the understanding of the issues such as the gap between theory and practice, attitude and beliefs, but also hold back possibilities of support as systematic needs analysis will be lacking.

The issue of the gap between theoretical knowledge and practical knowledge also featured strongly in this overview. The reasons for this gap were not within her (Adler's, 2004) scope of study, neither are they within the scope of this study. This issue is discussed in the next chapter from a different angle. It suffices to say that there are a number of suggestions from literature that proposes how to deal with this problem. For

instance, Ball and Bass (2000) have offered ways of making mathematical knowledge usable in teaching. In making mathematical knowledge usable, Ball and Bass recommend that teacher educators need go beyond just probing what teachers need to know, but to require them to learn how that knowledge needs to be held and used in the course of teaching.

Finally, Adler's (2004) observation is that a number of conference papers show that the theoretical orientation of the research study is under-described. This, according to her, raises questions as to whether the research is theoretically informed. She says that even though this phenomenon may be a function of the space limitations for a paper in conference proceedings, this is one of the challenges facing teacher education research. As pointed out earlier, not all issues raised by Adler (2004) about mathematics teacher education over a period of ten years have been included in this discussion. The selected critical components show that there is a need for research in South African mathematics teacher education both on a small and large-scale in order to have a broader empirical base. The international study (which included South Africa) by Adler, Ball, Krainer, Lin, & Novotna (2005) also emphasise the need to have large-scale research in the field of mathematics education. Amongst the variety of issues that Adler et al. raise, there are also two other issues that I think are relevant to mathematics teacher education in South Africa especially in view of reform processes that are underway. These issues are: support to teachers and the question of who does research in mathematics education. As ideas are created and implemented, there is need to support those who implement (teachers) and there is also a need to continuously find out who does research in South African mathematics teacher education as this will help in finding balance between what Adler et al. call "nearness and distance" in the researcher's work. This will help to address the passion that that has potential to translate into biasness.

In the next section, I present some issues on the choice of a career path for mathematics teachers and Ma 's (1999) view on becoming a mathematics teacher.

2.5 Debates and controversies

In the previous sections in this chapter, the focus was on policy, research and curriculum issues. The question of who enters the teaching profession and the development of this career, did not feature much. In this section, the discussions highlight different views on the career path for mathematics teachers.

Chuene, Lubben and Newson (1999) argue that teaching, unlike other professions, is self-recruiting. These authors further say that prospective mathematics teachers have knowledge about mathematics teaching and learning from their own experience as students. Chuene et al. (1999) quote Eisner (1992) and Meier (1992) as saying that prospective teachers are socialised through continuous contact with teachers for about 15 years before starting to teach.

Feinman-Nemser and Buchmann (1986) call this duration of 15 years or so a teacher-watching period. The analysis of the watching period may have negative and positive effects. Negative in the sense that pre-service may equate teaching as an emulation of their 'model teachers', which may turn out to be repetition of practices without any innovation. The past is the only reference point and this may have potential for resistance to change. On the other hand, this may have positive effects as the students' experiences may turn to be useful resources.

Ma (1999) uses a similar notion of a watching period, but her approach is a developmental one. Figure 2 below, shows Ma's (1999) idea that explains three periods during which the teacher's subject knowledge develops:
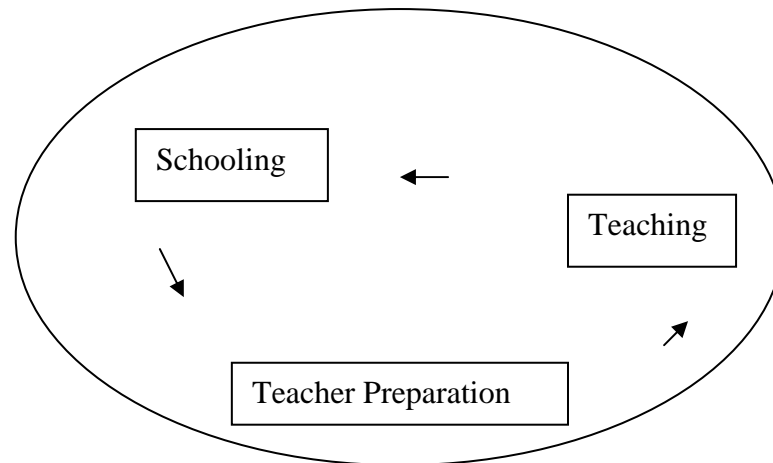
Figure 2: Ma's (1999) cycle for mathematics teachers' knowledge development

Ma (1999) views this period as part of a cyclic period within which the teacher's subject knowledge develops. This development is shown in figure 2 above. In China, Ma (1999) argues that the cycle spirals upward indicating that a teacher's subject matter knowledge improves during his or her teaching career.

There are two significant features that relate to Ma's (1999) cycle. First, the period is linked to teacher preparation; and second, Ma's (1999) cycle has outcomes, reflection and evaluation built in. That is, it is expected that students attain mathematical competence at school level (outcome), during pre-service teacher education programmes their mathematical competence begins to be connected to a primary concern about teaching and learning school mathematics (reflection) and during their teaching careers, as they empower students with mathematical competence, they (teachers) develop a teacher's subject knowledge which Ma (1999) calls the highest form of Profound Understanding of Fundamental Mathematics (PUFM). Teachers in Ma's (1999) context continue to learn mathematics and to refine their content understandings throughout their teaching careers (Ma, 1999). PUFM is also described in terms of depth, breadth, and thoroughness of the knowledge teachers need. "Depth" refers to the ability to connect ideas to the large powerful ideas of the domain, whereas "breadth" refers to connections among ideas of

similar conceptual power; thoroughness is essential in order to weave ideas into a coherent whole (Ball and Bass, 2000). I refer to this period as the evaluation and reflection session. In a South African context, this evaluation and reflection is not consciously planned and the pursuance of the subject matter often leads to mathematically educated teachers (obtaining higher qualifications in mathematics).

Becoming a teacher may be a wish to bring to life the image that one has built of one's role model (Chuene et al., 1999). The irony of the situation is that some pre-service and novice teachers have chosen to travel a journey that they should not have chosen, or they are in a path that they should not have chosen to follow. This is evident following Chuene et al's (1999) study and from the participants' view on teaching as a career. For instance, the biographical data of students involved in this study (chapter 4) show that the majority of them did not choose teaching as their primary career.
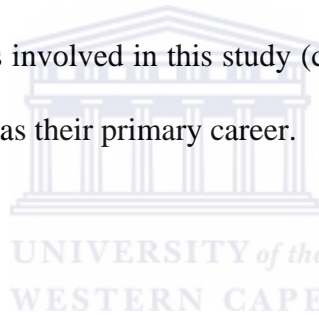
2.6 Summary

In this chapter the discussions were largely guided by Samaras' (2002) point of view. That is, understanding the context in which teacher preparation takes place, looking at knowledge and expectations of the broader society. The roles of the first two, that is the teacher, or someone who instructs, and the student, or the learner, are briefly dealt with in chapter 4.

The context for teacher preparation is dependent on policy directives and expectations. The knowledge base for and seven roles expected of teachers play an important role in teacher preparation and consequently determine the required competences.

The chapter has also shown that there are challenges in mathematics teacher education. There is a gap between theory and practice. One way of dealing with this gap is to adopt Ma's (1999) cyclic approach in which teachers' subject knowledge is continuously developed until the highest level of the Profound Understanding of Fundamental Mathematics (PUFM) is reached. This will raise the level of confidence in mathematics teacher education at both pre-service and in in-service levels. Most researchers will agree with Ma (1999) when she says that it is not that people are not willing to perform or are unable to perform, but that, at times, they don't know how to perform. This is an indication of the importance of knowledge base as a component of teacher development. Tied to the idea of knowledge base, is flexibility and adaptiveness that prospective teachers exercise when they are in real situations. The two concepts "flexibility" and "adaptiveness" are at the heart of Ma's (1999), and Ball and Bass's (2000) views on the use of mathematical knowledge. For instance, Ma (1999) says that teachers' knowledge of mathematics for teaching must be like an experienced taxi driver's knowledge of a city, whereby one can get to significant places in a wide variety of ways, flexibly and adaptively (Ball and Bass, 2000).

The other challenge is that pre-service teacher education remains a "black box". Whilst this seems to be a problem even in European countries, the problem needs to be attended. Cochran-Smith (2002) has strongly argued for the adoption of inquiry stance in teacher education in order to know what works and what needs to be improved. This is the route teacher educators in South Africa need to explore. This study provides a framework for analysis of how policy relates to teacher preparation and to the classroom practices. The challenges in mathematics education have been articulated and the need to address these challenges has been proposed.

The next chapter discusses the related literature and the theoretical framework guiding this study.

CHAPTER 3

LITERATURE REVIEW AND THEORETICAL FRAMEWORK

3.1   Introduction

This chapter focuses on the literature review that captures empirical and theoretical debates related to the application of mathematics in a modelling context. It derives its major focus from the main research question that seeks to describe the mathematical strategies that students adopt when they find solutions to familiar social problems. The chapter has two major sections. Section 1 discusses related literature on applications of mathematics in a modelling context, the role of context in the teaching and learning of mathematics, and the issue of transferability across diverse problem domains. Section 2 discusses the theoretical framework that guides the study. The framework draws mainly on the work on Cultural Historical Activity Theory (CHAT).
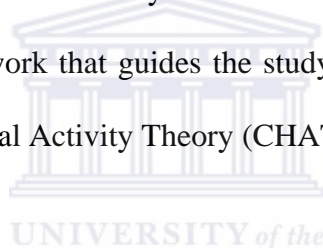
In sum, the questions that guide this chapter are as follows:

- How does a thematic approach relate to the applications of mathematics in a modelling context?

- What influence does the consideration of the rationale (social and instrumental) have on the working strategies of students in finding mathematical solutions to problems?

- What importance does mathematical modelling have in finding mathematical solutions to mathematical problems in a real life context?

- What is the role of context in the teaching of mathematics and what evidence has research produced?

- Are cognitive skills 'transferable' across diverse problem domains?

- What is the theoretical attitude that one should adopt when investigating mathematical behaviour of pre-service students in a modelling and application context?

The answers to these questions, drawn from the reviewed literature, are further suggestive of the process that the study has to consider documenting developmental trajectories of the research participants.

3.2   Background

As indicated earlier, there is limited literature in South Africa in the area of mathematical behaviour of pre-service teachers in an application and modelling context and the role the use of familiar contexts play in enhancing pre-service teachers' mathematical strategies.

The literature given below has been drawn from several countries. Even in these countries, most literature focuses on learners at schools, their perceptions about mathematics, and how mathematical modelling can be used to facilitate their understanding. The literature that focuses on mathematical modelling as used by student teachers is limited. In the literature there is an emerging consensus on the need for a participative atmosphere where there is communication and listening in class and careful selection of tasks and inquiry and, more importantly, a situation where learners are encouraged to think not just do problems (Adler, 1991).

3.3   Related literature

In keeping with the six questions that guide the discussions on literature related to the investigation of mathematical behaviour in an application and modelling context, this section is divided into six subsections. Under related literature, I focus on the thematic approach, social rationale and instrumental rationale, mathematical modelling and

application of mathematics, role of context and cognition. These subheadings constitute section 1 of this chapter.

3.3.1 Thematic approach

In this section the thematic approach in the teaching and learning of mathematics is discussed. The identification of this category came as a result of the formatting of research instruments for data collection in terms of four themes, namely: finance, transport, catering and academic programme delivery. As I did not find literature that describes the use of the thematic approach at teacher preparation level, I present arguments of literature that I suggest may be applicable to teacher education in general and I indicate how the literature relates to the focus of the study. According to Handal and Bobis (2003), thematic teaching of mathematics involves organising instruction around thematic units or projects. Furthermore, Handal and Bobis (2003) cite Freeman and Sokoloff (1996) as saying that a thematic unit is a collection of learning experiences that assist students to relate their learning to an important question. From this explanation, it can be concluded that thematic teaching will involve taking into consideration a collection of learning experiences that will assist students to relate their learning to the topic at hand and this will basically mean referring to situations that learners are very familiar to. According to Handal and Bobis (2003), Freeman and Sokoloff (1995) have argued that the thematic approach consists of the three components as shown in figure 3.1.
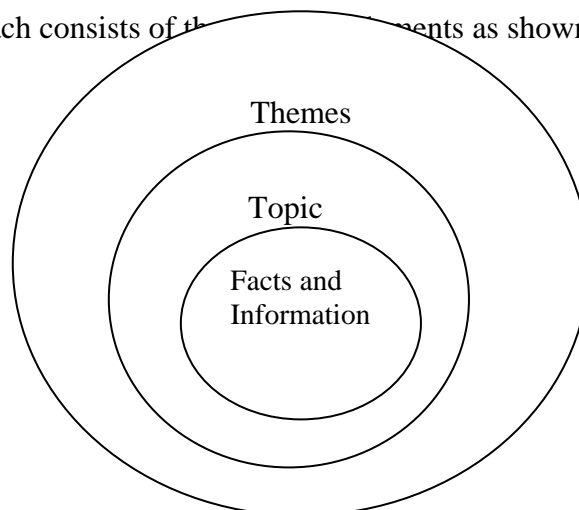
Fig 3.1: Freeman and Sokoloff's (1995) structure of a thematic unit as cited by Handal and
Bobis (2003)

Figure 3.1 shows that within a thematic unit, mathematical facts and topics are taught
within the context of an overarching theme and furthermore,
themes defined as broad existential questions, transcend disciplines, allowing learners to
integrate the information and the topic within the full range of human experience
(Handal & Bobis, 2003, p. 3).

It will be indicated below, as closer examination of the literature shows, that the thematic
approach, as described above, is not very frequently used in the teaching and learning of
mathematics except in a modelling context. Edwards (1999) supports this assertion by
saying: "Mostly, math has not been included in this approach…" This is apart from the
importance of the approach. Some of the highlighted benefits of the approach as put forth
by Edwards (1999) in method 1, paragraph 3 are:

- there is an increase in children's motivation
- individual differences are catered for
- children's confidence in their mathematical abilities grow
- mathematics is seen as part of "real life"

Handal and Bobis (2003) divide the thematic approach into three categories. These
categories are described as follows: In the first category of thematic approach the
mathematical content is taught though drill and rote learning and there are no applications
of mathematics in the classroom. In the second category the teacher introduces a
mathematical concept and "later presents applications of mathematics as a way to practice
the already taught mathematical concept" (Handal and Bobis, 2003, p. 7). In this category

there are, therefore, applications of mathematics but in a restricted dimension and there is no attempt to elicit ideas from real life situation. In the third category, the teacher is engaged with mathematical modelling. The teacher using this approach "typically begins from a real life situation involving experiential and hands on activities that eventually lead to formalization of the mathematical concept involved" (Handal & Bobis, 2003, p. 8). In this section, I only focus on one of these categories since it is related to the investigation tools of the study. This category involves introducing the thematic situation first, i.e. a real life problem, followed by a lesson structure that leads to the discovery of the mathematical concept or to the building of a mathematical model (Handal & Bobis, 2003). Furthermore, Handal and Bobis (2003) say that a teacher who uses this approach is engaged in mathematical modelling. Drawing from these sources (Edwards, 1999 and Handal & Bobis, 2003), I argue that the thematic approach is related to the focus of the study and helps in two major ways:

a. it brings to the fore that application of mathematics in a modelling context and requires careful analysis of students' solutions

b. it shows that whilst it is important for students to learn mathematical facts, the understanding of these facts is equally important and they need to be tested in various (educational) contexts that relate mathematics to real life situations.

The emphasis of a. above could also be seen in the following statement:

As a result of this style of teaching (*thematic approach that is discussed here*), students would probably be engaged in research assignments and mini-projects. The assessment is usually more complex than just pencil and paper tests [my emphasis].

(Handal & Bobis, 2003, p.13)

Though the study is not on teaching, I proceeded to investigate the mathematical behaviour of pre-service students in an application of mathematics in a modelling context, using the thematic approach to collect data in a more informed manner. Handal and Bobis (2003) also argue that the thematic teaching of mathematics is considered as belonging to the realm of situated learning because the content is embedded in themes that in turn serve as learning contexts. Following this line of argument, it can be argued that, in this study, investigating mathematical strategies in a thematic oriented research is considered to belong to that realm of situated learning because the content is embedded in themes that, in turn, serve as learning contexts. This view will be elaborated later in this chapter when presenting the theoretical framework for the study.

The next section focuses on what is considered to be drivers of learning and the point is made that behind the visible mathematical strategies of students or participants, there are invisible drivers that in some ways influence the actions that students take during the learning process.

3.3.2   Social rationale and instrumental rationale

The purpose of this sub-section is to present views on what is believed to be the influential drivers of learning. To fulfil this purpose, I draw on the work of Mellin-Olsen (1981, 1987) and Goodchild (1997) to look at the dichotomy between the social rationale of mathematics and instrumental rationale of teaching and learning mathematics. Mellin-Olsen's (1981, 1987) and Goodchild's (1997) arguments stem from Mead's (1934) and Schutz's (1967) work on social relations. According to Mellin-Olsen (1981), Mead has successfully made the point that the behaviour of an individual should be understood in relation to the generalised other (GO) and that Schutz's point is that the behaviour of each

person is based on his interpretation of the other. The importance of Mead's (1934) and Schutz's (1967) contribution is best put by Mellin-Olsen (1981) who says that both Schutz (1967) and Mead (1934) provide us with concepts and theories of how to look for rationales for learning in an educational context.

Mellin-Olsen (1981, 1987) argues that it is necessary to study the motives for actions in order to understand the actions in a social situation. In pursuance of this view, Mellin-Olsen (1981, 1987) identified two major rationales as drivers for school learning, namely: S-rationale (S for social) and I-rationale (I for instrumental). Mellin-Olsen's (1981, 1987) view is that

> I-rationale in its purest form will tell the learner that he or she has to learn, because it will pay out in terms of marks, exams, certificates etc. On the other hand, S-rationale will provide an incentive in learning a particular content that says that this knowledge has a value besides its importance for the external examination, it has an importance beyond its status as school knowledge (Mellin-Olsen, 1987, pp. 157-158).

In addition to these two rationales, Goodchild (1997) has presented the view that there is a third rationale referred to as P-rationale. This rationale stems from the notion that students get engaged in a classroom activity because of the expectations presented to them in that situation. As a result, they put "themselves in the place of the generalised other in that situation but without an active rationale for learning" (Goodchild, 1997).

P-rationale is then described as a rationale in which a student engages in a classroom activity when he or she has accepted the tasks prescribed by the teacher and takes part in these tasks from an underlying belief that is his or her role in that situation. Furthermore, Goodchild (1997) says that:

> Holding a P-rationale means that the student accepts the expectations of the practice and interprets these as her own goals: I use P to signify that this rationale arises from entering the 'practice' of the arena (Goodchild (1997, p. 91).

Goodchild (1997) also distinguishes P-rationale from I, and S rationales by indicating that P-rationale is a rationale for engaging in an activity and not a rationale for learning.

Whilst acknowledging the fact that the premise from which both Goodchild (1997) and Mellin-Olsen (1987) argued is classroom oriented, I argue that the issue about these rationales is not restricted to physical location but tied to intentions for engaging or refusing to engage in an activity. The "practice of the arena" that Goodchild (1997) talks about goes beyond the classroom environment. In fact, Goodchild (1997) argued as follows:

> Thus the arena consists of more than just the physical space, defined by the dimensions of the room, lighting, wall displays, size and arrangement of furniture, available resources including texts and writing materials. The arena is also socially and culturally constituted… (Goodchild, 1997, p.39)

Following the arguments presented by Mellin-Olsen (1987) and Goodchild (1997), the question is to what extent do the rationales that drive participants in a learning activity that is intellectual in nature, influence the way they solicit their solutions to the problems. The rationales emphasise the need to study motives for actions but, on the other hand, students' actions (working strategies) may indicate what the motives were and this becomes a two way process. This line of thinking is helpful in terms of this study when considering the fact that the analysis of students' behaviour in any arena, including finding solutions to mathematical problems in a modelling context, may be driven by a number of rationales. One problem with Mellin-Olsen's (1987) views is that he focuses on rationales without tackling issues of cognition. The next subsection looks at the literature that focuses on mathematical modelling as a process oriented strategy to find solutions to mathematical problems.

3.3.3 Mathematical modelling and application of mathematics

Handal and Bobis (2003) have argued that following a thematic teaching method in which the teacher begins from real life situations is being inherently engaged in mathematical modelling. In this third subsection, I briefly focus on the concept of mathematical modelling and argue that the concept fits well in this study, both from Handal and Bobis' (2003) point of view and in terms of views gathered from other authors.

The approach in this subsection is to present highlights in terms of the description of the concept of mathematical modelling, how it may act as a means for sense making and the importance of context in mathematical modelling. The concept of mathematical modelling is associated with a number of strategies in mathematics, such as problem solving or teaching in a setting where it follows constructivist principles. For the purposes of this study, I present only the work of authors who appear to be addressing issues related to the main focus of the study, i.e. mathematical behaviour in a modelling context. The following were found helpful: Townend (1993), Bassanezi (1994); Greer (1997); Gravemeijer (1997) and Reusser and Stebler (1997). In sum, all these authors describe mathematical modelling as the study of problems in real world situations with the use of mathematics as its language for their comprehension and resolution. In their arguments, the authors go beyond descriptive definitions. They also put forth arguments on how to operationalise the description. For instance, Bassanezi (1994) says that working with mathematical modelling does not simply mean an attempt to widen knowledge but also a means of developing a particular way of thinking and acting. Applications of mathematics and mathematical modelling also appear side by side, as explained in chapter one. Mathematical modelling is presented more as a process and application of mathematics as a strategy to complete that process. In drawing from Masingila's (1993) and Greer's (1997) work, there is also a strong view that regards mathematical modelling

or application of mathematics as a strategy to bridge the gap between informal mathematical knowledge and formal mathematics.

The descriptions of mathematical modelling and applications of mathematics, as drawn from the above authors, provided working borders for the focus of the study and helped me to see how the themes that I have administered to pre-service teachers relate to mathematical modelling.

Mathematical modelling is also viewed as a strategy that inherently encompasses sense making in learning mathematics. Several authors argue that making sense of mathematics should be part of training. For instance, the studies by Lampert (1985), Southwell (1994), Anderson (1994), De Lange (1993), Lamb (1991), Rawlins (1991), Mason (1989) and Gravemeijer (1999) put emphasis on this issue of sense making in mathematics learning. These authors also demonstrate ways of helping students on how to come to grips with formal mathematics using contexts. Gravemeijer (1999), for instance, shows how to learn calculus using examples from real life situations. The results of the studies conducted in the area of sense making in mathematics show that, by and large, students and teachers have not adequately shifted from traditional methods that do not encourage making sense in learning, as shown by Greer (1997) in figure 3.2.

From a teacher preparation perspective, some studies conducted elsewhere in the world demonstrate that the introduction of teaching mathematics in an applications and modelling context is needed for initial teacher training (Buffton, Oldknow & Regis, 1989). They argue that this approach is pragmatic and it encourages the spirit of exploration and variety of computing skills.

3.3.4 The role of context

Related to the issue of sense making, some studies show that, in some instances, the students' solutions are not reflective of the realities of the situations. For instance, the analyses of the research work by Chisko (1985), Boaler (1993), Stuessy (1993), Van Den Brink (1993), Verschaffel, De Corte & Lasure (1994), Verschaffel, De Corte & Borghart (1997), Treffers (1993), Schwartz & Yerushalmy (1995), Streefland (1993) and Masingila (1993) indicate the success and challenges that students encounter when engaged in courses, which are designed taking into consideration the context and problems related to daily life experiences. In these studies, a number of suggestions that could be used in developing courses or programmes for students in order to bridge the gap between school mathematics and mathematics outside schools are presented.

Verschaffel, De Corte & Lasure (1994), Verschaffel, De Corte & Borghart (1997, Greer (1997), Gravemeijer (1997), and Reusser and Stebler (1997) argue that word problems have a critical role to play in the learning and teaching of mathematics. They argue that their studies have revealed why learners present solutions to 'unsolvable' problems. The main reason for this tendency is that students concentrate on algorithms and neglect realistic considerations of a problem. Masingila (1993) and Greer (1997) strongly argue that the closing of the gap between school mathematics and out of school mathematics helps to bring meaningful learning into mathematics and addresses the discontinuities between school contexts and everyday life contexts. Furthermore, this will change the mode of finding solutions to word problems that Greer (1997) argues is taking place at schools, as shown in figure 3.2 below. In this diagram students' consistent tendency is to take given numbers and apply basic operations.

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┌─────────────────────────────────────────────────────────────────────────┐
│   ┌──────────────────┐                    ┌──────────────────┐            │
│   │   Problem text   │──────────────────▶ │  Select operation │           │
│   └──────────────────┘                    └──────────────────┘            │
│                                                    │                       │
│                                                    ▼                       │
│                                           ┌──────────────────────┐         │
│                                           │ Perform computation  │         │
│                                           └──────────────────────┘         │
│                                                    │                       │
│                                                    ▼                       │
│                                           ┌──────────────────┐             │
│                                           │   Report result  │             │
│                                           └──────────────────┘             │
└─────────────────────────────────────────────────────────────────────────┘
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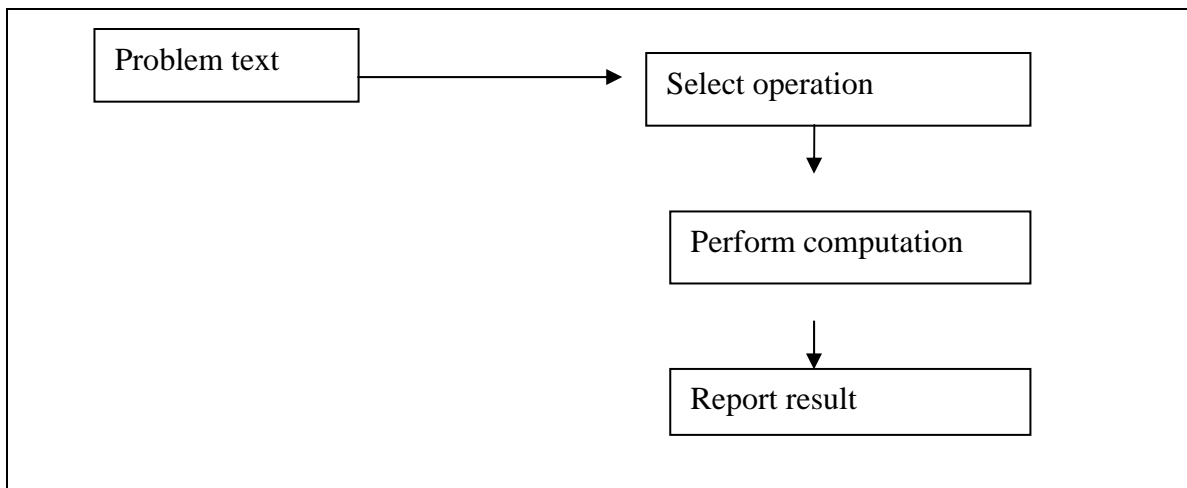
Fig 3.2: Diagrammatic representation of students' traditional methods

Contrary to Greer's (1997) observation, Rogoff (1984) argues that thinking is intricately interwoven with the context of the problem to be solved. Furthermore, context is an integral aspect of cognitive events, not a nuisance variable. The question is (in view of unrealistic considerations when finding solutions) why it happens that some mathematics students ignore the context or tend to assume that the problem is context free.

In view of the challenges such as ignoring realistic considerations in arriving at solutions for word problems, authors strongly argue for inclusion of more problems that are from real life contexts that will inculcate a disposition of reflection on the contexts. In line with this argument, Greer (1997) calls for renegotiation of didactical contract by the improvement of tasks formulation and variation of these tasks as the research has shown that variation tends to sensitize students to consider aspects of reality in giving solutions. There is a strong case from Verschaffel, De Corte & Lasure (1994) and Verschaffel, De Corte & Borghart (1997), Greer (1997) that more examples from more authentic settings will encourage the realistic considerations of the solutions that are presented, that is, whether the provided solution(s) meet the expectations of original real problem. Whilst

this recommendation is for teaching and learning, it is equally valid for research purposes that use tasks or themes as instruments for data collection.

At local level (South Africa), critical research has been conducted at the university of the Western Cape under the auspices of the Application of the Modelling in School Mathematics Project (AMSMAP). The goal with the project work of the AMSMAP, as explained by Julie (2002), is to investigate teacher mathematical modelling work and its relationship to a hypothesised school mathematical modelling activity system. The AMSMAP project is important in three ways, namely, its methodological approach, its theoretical orientations and its contextual relevance, as it is a South African based project. As it will be shown in chapters 4 and 6, the concept activity system will feature prominently in the study.

3.3.5   Cognition

The view that mathematics is an intellectual activity is probably one of the main reasons for studying students' mathematical behaviour from the perspective of cognitive psychology. From this point of view, I present an exploration of selected literature that captures the debates that dichotomise the difference between cognitive perspective and situated perspective in psychology. The goal of this section is to express a theoretical account in terms of cognition in practice (the application of knowledge) as it expresses itself in mathematical strategies of students when finding solutions to mathematical problems formulated from familiar social contexts. The approach is to present highlights of a broader picture in terms of cognition in practice and the identified key players in these debates. Finally, I present a perspective that I regard as a unifying theoretical position in terms of acquiring knowledge and applying it. This provides a stepping-stone to the theoretical framework for the study.

According to Greeno (1997), the cognitive perspective takes the theory of individual cognition as its basis and builds toward a broader theory by incrementally developing analyses of additional components that are considered as contexts. On the other hand, he says that situated perspective takes the theory of social and ecological interaction as its basis and builds toward a more comprehensive theory by developing increasingly detailed analyses of information structures in the context of people's interactions. Two groups of authors who have taken up the debates relating to the situative and cognitive perspectives are, on one side, Anderson, Reder and Simon (1996), and, on the other side, Greeno (1997).   One of the issues that these authors take up relates to knowledge transfer, i.e. the extent to which what is learned in one situation can be generalised to others or the idea that what is learned is specific to the situation in which it is learned. The research work

on knowledge transfer lacks generality. As this study is not so much on individual or group contribution in this research, the debate about cognitive and situative perspective is not dealt with in detail. The rest of the discussion is spent on the knowledge transfer.

There are instances where there is a dramatic transfer and where there is dramatic failure in terms of transfer (Anderson, Reder & Simon, 1996). Lave (1984, 1988), Nunes, Schliemann and Carraher (1993), and Rogoff (1988) show that contexts have a role to play in terms of knowledge transfer. These authors caution that cognitive skills are not easily transferable across widely diverse problem domains but consist rather of cognitive activity tied specifically to context. This view, according to Rogoff (1988), does not necessarily mean that cognitive activities are completely specific to the episode in which they were originally learned or applied. The caution highlights the fact that context plays a role to remove the assumption of broad generality in cognitive activity across contexts and focuses, instead, on determining how generalisation of knowledge and skills occurs (Rogoff, 1988).

In terms of difficulties in knowledge transfer, Resnick (1987) has argued that school is a special place and time for people, so the discontinuities should be seen in that context but she adds that there are ways to address these discontinuities. The above synopsis of literature provides an explanation of the challenges that may be encountered in making descriptive judgements on the solutions presented by students but one still finds a theoretical differentiation in these descriptions, as there are many variations that need to be considered in order to make a comprehensive assessment. The work that seems to offer a unified approach to the knowledge and transfer dichotomy is by Hebert, Carpenter, Fennema, Fusion, Human, Murray, Olivier and Wearne (1996). They argue that the

distinction between acquiring knowledge and applying it is problematic on its own, hence its failures. Building their arguments on Dewey's (1933) notion of "reflective inquiry", they argue that students should be given the opportunity to "problematize the subject": "in the sense that students should be allowed and encouraged to problematise what they study, to define problems that elicit their curiosities and sense-making skills" (Herbert et al., 1996, p.12).

Reflective inquiry driven by its key features (identifying problems, studying problems through active engagement, reaching conclusions) encourages students to look for problems and develop a sense of control over the subject matter (Herbert et al., 1996). Taking the reflective inquiry further, Herbert et al. (1996) further argue that problematising the subject leads to understanding and this problematising approach fits within two different views of mathematical understanding, i.e. a functional and a structural view. According to Herbert et al. (1996), from a functional perspective, understanding means participating in a community of people who practice mathematics and it, therefore, focuses on the activity of the classroom. On the other hand, from a structural perspective, understanding means representing and organising knowledge internally in ways that highlight relationships between pieces of information. This view focuses on what students take with them from the classroom.

The dichotomy between the acquired mathematical knowledge and applying it would have presented a number of problems for understanding students' mathematical strategies. Furthermore, the debate between cognitive and situated psychology would have presented some challenges, as the study is not interested in studying groups or individuals. The perspective by Hebert et al. (1996) has provided a theoretical account in which one can

justify the control over those variables (individual's or group's behaviour, and dichotomy between acquired knowledge and its application). Herbert et al.'s (1996) approach of problematising the subject and the notion of reflective inquiry fit in well with the investigative approach of this study in that students were allowed and encouraged to mathematically problematise their experiences in terms of transport, fees, catering and academic provisioning. It can be argued that during the problematisation process, students are challenged to find out which mathematical resources or principles or tools will be helpful in providing solutions to the problems.

Both the functional and structural perspectives provide a base from which to present the theoretical framework guiding the study, in that the students did not only find solutions but also presented their solutions to the head of the sections (finance, transport/taxi association, catering and academic division), thus extending 'context of discovery' to context of application. The approach to have students presenting their solutions to those in charge of the transport, finance, catering and academic sections presented an opportunity for students to mathematically defend if needs be. This process can best be described in terms of the model by Elshout-Mohr, et al. (1999). They provide eight episodes in a hierarchical form where the lowest level is type 1 and the higher one is type 8. Type 1 is characterised by reliance on memory, and application of knowledge is limited to conditions similar to those in which it was learned (near transfer). On the other hand, type 8 refers to a situation where students demonstrate the ability to apply knowledge in unfamiliar contexts (far transfer).

3.4 Theoretical framework

The purpose of this section is to present a theoretical framework that guided the investigation of applying mathematics in real life contexts. The consideration in

identifying the appropriate theoretical framework is a two-fold strategy. The section is influenced, in part, by those researchers who see connection between school environment and those who question the transferability of school knowledge to the contexts outside the school.

3.4.1. Factors influencing the choice of a theoretical framework in mathematics education

There are several factors that need to be considered when a choice of a theoretical framework is made. In this study, it is argued that the nature of mathematics and the categories of mathematics teaching environments at schools are very crucial in the choice of a framework.

3.4.1.a.   Nature of mathematics

The process of investigating mathematical behaviour is dependent, amongst other factors, on the nature of mathematics. There are at least two versions of what mathematics is. One is that it is a body of knowledge, which is the group of sciences (including arithmetic, geometry, algebra, calculus, etc), dealing with quantities, magnitudes, and forms, and their relationships, attitudes, etc, by use of numbers and symbols (Webster's New World Dictionary, 1971). On the other hand, mathematics is described as a human activity. In simple terms, this implies that mathematics is part of human day-to-day experiences or it is something that relates to all human experiences. The two concepts 'body of knowledge' and 'human activity' are loaded and often used in a pervasive way. As mathematics was earlier defined in chapter 1, for the purposes of reference in this section, the definition is restated below:

Mathematics is the construction of knowledge that deals with qualitative and quantitative

relationships of space and time. It is a human activity that deals with patterns,

problem solving, logical thinking, etc., in an attempt to understand the world and

make use of that understanding. This understanding is expressed, developed and

contested through language, symbols and social interaction

(Department of Education, 1997, p.108).

The use of 'human activity' in the above description further raises more questions. Of interest to this study is how the "patterns, problem solving, logical thinking, etc, in an attempt to understand the world and make use of that understanding" is to be achieved and on what theoretical guidance? Implied in this description is the issue of application of mathematics ("use of that understanding") in the real world. The description shows that mathematics is a cultural, historical phenomenon that has changed over time. As will be shown below, it has somehow changed from a homogeneous image to a more heterogeneous embodiment of social realities. This means that, initially, it was, by and large accepted as a unique entity and later, as this view received continual challenges, it embodied a variety of social peculiarities. These challenges have led to the use of different methods of teaching and the use of different media to facilitate learning and teaching. The emergence of more views on what mathematics is and how it should be learned and taught are still continuing.

3.4.1.b   *Categories of mathematics teaching environments*

The highlights of the historical dimension and critical reflections on mathematics and its teaching seems to lend into two categories. The first one is characteristic of the following: mathematics is the determined body of knowledge, neutral, static and is taught by

considering algorithmic strategies. Below, is a description of the nature of settings and beliefs in line with this category:

> The classroom is highly organized, the syllabus is rigid, and the textbooks are fixed. Maths is considered as a science that does not make mistakes…. There is one correct answer to every question and one meaning for every word and meaning is fixed for all people and for all times. 'Wrong' answers are not tolerated; students are usually punished if they make 'mistakes'. Teachers are also expected to perform according to a certain set of rigid expectations and they are punished if they don't (Fasheh, 1982, p.7).

Breen (1993) has argued that the classroom scenario as described by Fasheh (1987) is applicable to South African context. It could be argued that, following the debates on the teaching of mathematics in South Africa, there are a significant number of classrooms that still resemble the observation by Fasheh (1982). Pre-service teachers as part of a continuous circle as discussed in chapter 2, may emulate this practice. Though much has been done in initiating different innovative methods, as reported earlier in chapter 1, most educators are still in the traditional, transmission and transitional stages of OBE.

In addition to reactions to a similar situation such as the one sketched above, Anderson (1990) talks of "Pedagogical Disasters in Mathematics Education". These disasters are divided into six categories: separation of arithmetic from algebra; teaching of mathematics without historical references; use of textbooks that are elitist and cryptic (mysterious); doing work and being tested as an individual as opposed to working and being tested as study groups; accepting the myth that mathematics is pure abstraction and, therefore, antithetical to one's cultural and working environment; and to memorize, memorize, memorize…(Anderson, 1990).

The second category leans on realistic theory, in this case, the approaches are very much evolving. The strategies are characterised by building on the students' own productions

and constructions, interactive character of the teaching process and intertwinement of learning strands. There is much more critical analysis in this category. Smythe (1998) argues that social settings are resources and should not be seen as obstacles to learning. This has emerged after dealing with literacy issues and it was found that settings might be used to facilitate learning. Adults, young people, and the poor may use their settings as resources for facilitating learning. This encourages creativity and ownership in a learning environment. This thinking has helped and guided the study and, at the end, a research project was developed within a framework in which a Social Contextual Model is dominant. This meant that I had to consider social issues and the context in which those social issues are evolving.

The second category, to a large extent, is a direct response to the first category. These are actions that could be taken in order to intentionally make a change in a classroom and outside. The extent to which a change in a classroom is achievable is seen in Anderson's (1990) argument when he says that action to improve, quantitatively and qualitatively, students' knowledge and appreciation of mathematics is real, do-able, and *theirs*. This line of persuasion emphasises the need to intentionally take actions that could bring changes in classrooms. There is, therefore, a necessity to conduct studies that will provide explanations as to why there are still practices in the first category and why some students are in the second category as explained.

I argue that in accepting mathematics as a human activity, its functions are defined differently and its content is dynamic as well as the methodologies for teaching. For instance, if its main function is to prepare students for adult life or place of work, the content in mathematics and its relation to place of work is important. Hakkarainen (1999)

describes different roles/consequences of play. These are: a play, depending on the purpose, may end on stage and what may remain is the memory of what happened. A comparison of this view to mathematics brings the following reflections: currently, there are situations where mathematics is 'done' and it ends there; what may remain are just static memories (not really helpful for future purposes). Other teaching and learning incidences lead to what I call 'pragmatic memories' (useful for practical purposes/references and are needed for future use). Pragmatic memories are helpful as they give support to learning, and link known situations to new situations. There are other incidences. I prefer to call these incidences 'contextual levels' in mathematics teaching and learning. They are contextual because they may vary from one context to the other.

Given the above scenario, the following key concepts as defined earlier in chapter 1 are central to this section: contexts, activity, human activity, activity system, learning activity, and possibilities for 'transfer'. This list may be extended, as McLeod (1989) argues that in doing problem solving (focussing on cognitive factors), affect is a very key factor and studies have shown that it is related to other factors such as beliefs, attitudes and emotions.

3.4.2. CHAT as a framework for the study

The theoretical framework that may adequately deal with these concepts in this study is the Cultural Historical Activity Theory (CHAT) or Activity Theory, for short (Engestrom, 1999a). Forman (2003) argues that it is also referred to as the sociocultural approach, Vygotskian theory, neo-Vygotskian theory or cultural psychology. There are various authorities on this theory stretching from the Vygotskian era to date. In this section, the

research that is cited emerged form the work of Engeström (1999a), Davydov (1999) and Lambert (2001) because of the relevance it has to this study.

The main reason of grounding this study on CHAT is that potential expansive learning as explained by Engestrom (1999a) is identified to be descriptive developmental growth of strategies employed by the participants in this study. This will explain what tools the participants use to find solutions to the mathematical tasks, rules applied during the process of finding the solutions, the sharing of responsibilities and collective object that the participants have. This is illustrated in the figure 3.3 below.
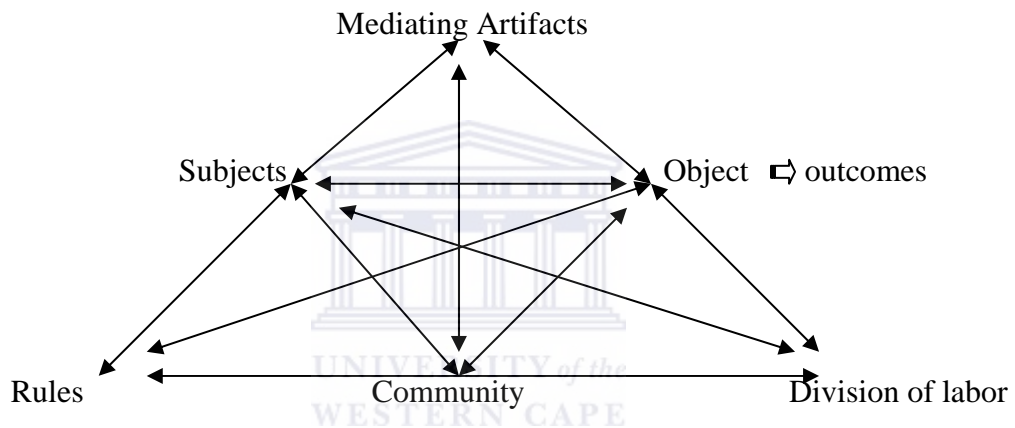
Fig 3.3: Engeström's (1999a) model of an activity system

The description of components within the activity system is presented in table 3.

Components	Description
Mediating Artifacts	Tools used toward the object
Subjects	Participants/actors
Rules	Regulate the subjects' actions
Community	Persons who share an interest/involved in the same object
Division of labor	Stipulates what's to be done by whom toward the object
Object	Societal productive practices-'end product'

Table 3: Description of components within activity system

According to Engeström (1999a), there are at least four arguments as to why CHAT is important for developmental and qualitative research. Firstly, the theory is deeply contextual and designed to understand historically specific local practices, their objects, mediating artifacts, and social organisation. Secondly, CHAT is based on a dialectical theory of knowledge and thinking, focused on the creative potential in human cognition. Thirdly, CHAT is a developmental theory that seeks to explain and influence qualitative changes in human practices over time. Furthermore, it seeks to bridge the gap between the individual and the social structure. This is pursued in order to change the view that an individual may be seen as an acting subject who learns and develops, but somehow the actions of that individual do not seem to have any impact on the surrounding structures (Engeström, 1999a). These four arguments by Engeström (1999a), particularly the last one on bridging the gap between the individual and the social structure, form the benchmark for this study. It is this point of reference that guided the investigation in this study to move from that the context of discovery to the level of the context of application.
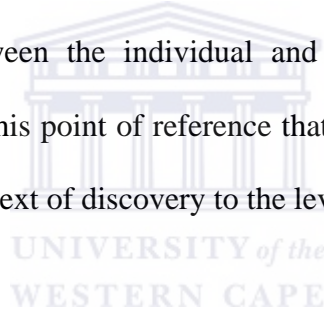
In chapter 4, a description will be presented on how this framework further helped in analysing the data. In the paragraphs that follow, a brief discussion is presented explaining the relevance of this theoretical framework in terms of the data collection instruments that were used in the study.

3.4.3. Theoretical framework and the instruments (themes) for data collection

Most mathematics teachers rely on the textbook to teach. The use of textbooks is so crucial that, in some instances, learners use those textbooks to validate their knowledge.

Unfortunately, this reliance on textbooks may lead to undesirable results, as described by Engestrom (1987) below.

> In school-going text takes the role of the object. The object is molded by pupils in a curious manner. The outcome of the activity is above all the same text reproduced and modified orally or in written form (Engestrom, 1987, p. 101)

In the situation described by Engestrom (1987) above, the text and the object exchange roles. The students tend to plan 'to master' the text rather than focusing on the main purpose of learning and the purpose of learning goes beyond individual successes as captured in the following quote.

Dewey and Child (1933) as quoted by Miettinen (1999) argue that:

> Education has the responsibility for training individuals to share in this social control instead of merely equipping them with an ability to make their private way in isolation and competition. The ability and desire to think collectively, to engage in social planning conceived and conducted experimentally for the good of all, is a requirement of good citizenship under existing conditions. Educators can evade it only at risk of evasion and futility (Miettinen, 1999, p. 332)

In adopting CHAT as framework, the focus shifts form the text as an object of learning towards the goal or the object of the study. The formulation of the mathematical themes as they will be described in the next chapter, are such that the subjects/participants are geared towards the common object of finding the solutions to unpaid fees, provision of transport to students, efficient utilization of the catering services and provision of tuition to both day and evening students. Following these contextual issues (unpaid fees, transport etc) to be addressed, the work of Engeström's (1999b) context of discovery and context of application becomes relevant. Engeström's (1999b) view on context of discovery and context of application is discussed in chapter 6. In finding solutions to these themes, students' strategies may be mediated by a number of factors such as interaction with experienced persons from finance and transport. Forman (2003) also argues that CHAT proposes that teachers need to understand the mathematical knowledge that

children bring with them to school from the practices outside of school as well as the motives, beliefs, values, norms, and goals developed as a result of those practices. In addition, she says that the assessment of mathematical learning needs to take into account children's out-of-school as well as in-school experiences. This approach is of relevance to this study as students bring with them their own experience relating to student debt, catering, academic and transport problems.

This framework, therefore, provides guidelines to analyse the extent to which the themes are appropriate to the objectives of the study or the object of the study and actual importance of those themes. For instance, using this framework, it is possible to assess the level of skills that are required in the projects or skills that participants display, and the knowledge (showing access to the tools, choice and use) the participants have. It has also helped to assess whether the themes or activities or projects do encourage a surface learning or deep learning process or to what extent the learners are engaged in types of learning (surface or deep). The potential expansive cycle addresses this point.

Operating within this framework, it has been possible to understand students' levels of application of mathematics (near transfer or far transfer), as it will be explained in chapter 5. The practical nature of application of CHAT enabled the investigation of meaning during the data collection to describe the analysis of mathematical behaviour of pre-service mathematics students in an application and modelling context.

3.4.4. Implications: CHAT as a theoretical framework

One of the challenges in terms of the theoretical approach that has been adopted in this study is that

a. the mathematical knowledge of the participants is assessed beyond what the textbooks say in terms of mathematical conventional representations. The focus is on the goal of the investigation of this study.

b. there should be an inherent strategy in the investigation process that promotes 'transformation' of these participants. This may entail deep assessment of their working strategies using a variety of methods of analysis in order to observe any emerges of developmental trajectories. CHAT, which is the guiding theoretical framework of this study, requires that the subjects/participants who go through the activity must, at the end, be transformed.

Furthermore, this framework implies that there should be implicit changes in the way thinking is organised, and to the approach to the work. Table 4.2 (steps in mathematical modelling and expansive cycle) in the next chapter demonstrates this point. The CHAT, with its expansive cycle, is inherently a fluid approach encouraging critical thinking and is action oriented. The framework, therefore, observes and encourages a disposition in thinking of the participants.

3.5. Summary

In this chapter, different categories of related literature were presented. It was argued that the six categories of literature provide the working benchmarks and are suggestive of the process that the study has to consider in documenting developmental trajectories of the research participants' work.

CHAT is arguably the appropriate theoretical framework for the study as it addresses the key issues of the study, namely: context, understanding of practices, their objects,

mediating artifacts, social organization and bridging of the gap between an individual and

the social structure.

The next chapter discusses how the investigations were carried out in order to collect data on students' strategies to find solutions to problems formulated out of familiar contexts.

CHAPTER 4

METHODOLOGY

4.1   Introduction

This chapter discusses the methodology used in this study. It is argued that richness of data comes as a result of a multi-method strategy of investigation, data collection and analysis. In pursuit of this argument, I have used a context-driven strategy to shift from the traditional triangulation method (cross checking data sources to see if the same pattern keeps recurring) to a multi-stage strategy. The context driven strategy necessitated data collection on various issues other than the data that responds to the three research questions. This idea was applied in this study as a result of the three selected components from Samaras' (2002) four components that are applicable to any teaching situation as discussed in chapter 2. The research for this study was viewed as a teaching process, not because of the methodologies used, but because of the sense in which learning was to be experienced. In line with Samaras' (2002) idea about the importance of knowing students, social milieu, expectations and subject matter, data were collected on these three components in order to back up theoretical claims in those contexts.

This chapter is, therefore, premised on the view that the methodology of a research project in a teaching situation invariably extends beyond simply indicating subjects, instruments and procedures used in a study. It requires one to elaborate on the context in which the study process took place. I used Samaras' approach to make this point.

The chapter has three sections. Section 1 briefly outlines theoretical research motives and short descriptions of key methodologies that are largely applied in the study. It starts by arguing that any research, whether qualitative or quantitative, is idea-driven. To illustrate this point, three research ideas, as argued by Wolcott (1992), are presented. These

research ideas are: theory-driven ideas, concept-driven ideas, and reform or problem-focused ideas. Secondly, the chapter focuses on the key methodologies in the context of qualitative inquiry. Key methodologies discussed are intepretativism, hermeneutics and social constructionism.

Section 2 presents the research process and issues of 'credible data'. Section 3 discusses the bottom-up approach that was adopted for analysing the data from completed themes. The discussion of the bottom-up approach has as its foreground the frameworks that were initially considered for analysis purposes but later used as theoretical bases for understanding the data.

4.2   Theoretical research motives

In this section a brief background on what drives different types of research is presented.

4.2.1 Idea-driven research

Wolcott (1992) argues that any research, whether qualitative or quantitative, is idea driven. The argument here is that problem setting (formulation of an idea) acts as a pivot for all science, social as well as not so social (Wolcott, 1992). Wolcott (1992) divides research ideas into three categories, namely: theory-driven ideas, concept-driven ideas and reform or problem-focus based ideas.   Theory-driven research includes those investigations that use theory as a guidepost or those investigations whose main purpose is to build theory (Wolcott, 1992). On the other hand, concept-driven research serves to orient the researcher. Though concept-driven research types lack precision, they are helpful in driving the fieldwork of researchers to fulfill their mission of searching out interpretations of data rather than seeking illustrations for the theory (Wolcott, 1992). Lastly, in case of reform or problem-focused driven research types, Wolcott (1992) says

that the declared purpose is to bring about change directed at improvement as the researcher assumes that things are not right as they are or most certainly, are not as good as they might be. There are therefore immediate concerns that drive this type of research idea

Wolcott's (1992) last category on problem-based driven research is closer to this study. The difference is that the idea of bringing about changes is not a priority in this study. There is certainly an arguable assumption that things are not right as they are. In this study, 'things' include the unrealistic consideration in working out mathematics problems, lack of ability to communicate mathematical results and lack of application of mathematics in the real world.

4.2.2 Methodologies

Different research projects require different research methodologies. In most cases, the researchers caution that the study is not exclusively qualitative or quantitative or critical. The appropriateness of the methodology depends on a number of factors. There is a trend showing a greater preference for the application of qualitative research methodology in educational research in South Africa. The qualitative methodology or inquiry is more comprehensible as a site or arena for social scientific criticism than as any particular kind of social theory, methodology or philosophy (Schwandt, 2000). Schwandt (2000) further says that, for some researchers, the site is a place where a particular set of laudable virtues for social research are championed, such as fidelity to phenomena, respect for the life world, and attention to the fine-grained details of daily life. On the other hand, some researchers seem to view the site as a place for experimentation with empirical methodologies and textual strategies inspired by postmodernist and poststructuralist thinking.

There are basically two main methodologies that are dominant in this study. The study has largely used the quantitative research method to develop the research instruments for data collection and qualitative research methods were dominant in the analysis of the data. There were, of course, instances where quantitative methods were also used in analysing data, e.g. the use of the computer software – Statistical Package for Social Sciences (SPSS). This involved the coding of statements and determining the frequencies. In the paragraphs that follow, qualitative methods selected for the study are briefly discussed.

The relevance of the qualitative inquiry in this study is discussed under three designs, which are central components of qualitative research. These are intepretativism, hermeneutics, and social constructionism. The adoption of the multi-qualitative methods became necessary, as I had accepted the view that richness of data comes as a result of a multi-methods strategy of investigation, collection and analysing. Each of the following methods fit in individually and collectively in the dissection of the data.

4.2.1.1 *Interpretivism*

Schwandt (2000) says that from the interpretativist point of view, what distinguishes human (social) action from the movement of physical objects is that the former is inherently meaningful. Thus to understand a particular social action (e.g. friendship, voting, marrying, teaching), the inquirer must grasp the meanings that constitute that action. Schwandt (2000) quotes Fay (1996) and Outhwaite (1975):

To say that human action is meaningful is to claim either that it has a certain intentional content that indicates the kind of action it is and /or that what an action means can be grasped only in terms of the system of meanings to which it belongs (cited by Schwandt, 2000, p. 191).

In light of the above quotation, Schwandt (2000) says that to find meaning in an action, or to say one understands what a particular action means, requires that one interprets in a particular way what the actors are doing and the process of interpreting or understanding is differentially represented. These differences could be grasped through a consideration of four ways, which define the notion of interpretative understanding. They are: emphatic identification, phenomenological sociology, language games, and shared features. These are briefly discussed below.

a   *Emphatic identification*

Emphatic identification refers to a situation wherein understanding the meaning of human action requires grasping the subjective consciousness or intent of the actor from the inside (Schwandt, 2000). Schwandt further says that emphatic identification is an act of psychological reenactment – getting inside the head of an actor to understand what he or she is up to in terms of issues such as motives, beliefs and thoughts. Adopting this kind of an interpretivist stance, Schwandt assures researchers that it does not make it impossible for the interpreter to transcend or break out of her or his historical circumstances in order to reproduce the meaning of the actor.

The emphatic identification was central in taking the conscious move to interpret the groups' intent and to understand the meanings provided from their perspective. The emphatic identification, therefore, calls for careful analysis of the texts.

b   Phenomenological sociology

According to Schwandt (2000), the main person behind the work on phenomenological sociology is Schutz (1962) and it is primarily concerned with understanding how the everyday, intersubjective world (the life world) is constituted. The aim is to grasp how we come to interpret our own and others' actions as meaningful and to "reconstruct the genesis of the objective communication of individuals in the social life-world", (Outhwaite, 1975 as quoted by Schwandt, 2000, p. 192). Two conceptual tools often used in that reconstruction are *indexicality* and *reflexicality* and these two conceptual tools are closely connected (Potter, 1996). According to Potter (1996) the former signifies that the meaning of a word or utterance is dependent on its context of use. The latter directs our attention to the fact that utterances are not just about something but are also doing something. Potter's basic viewpoint is that it is the combination of words and context that give the utterance sense.
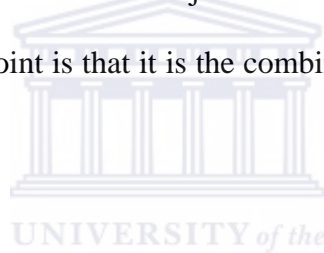
The importance of the phenomenological sociology in this study is the emphasis on the role context and how it may influence the meaning or the solution. I use Potter's two examples to illustrate the role of context in meaning construction and that descriptions are about something and they also do something. First, in the case of indexicality, Potter explains that the sense of the phrase by the child Sam who says, "my tummy hurts" can vary widely. For instance he argues that in the appropriate settings it might mean a plea for food, or given that Sam has already had a second apple and an ice cream, it might be taken as a plea for no more food. Furthermore it might be a sign that indicates pain and therefore it might mean a plea for medical attention.

Second, in terms of reflexivity, Potter uses description from a legal setting (small claims court) in which a complainant was claiming for compensation for his flat which was

damaged by water coming from the ceiling. This description is taken from Pomerantz's (1987) research work. The extract (Adj. Refers to adjudicator and Pla. Refers to plaintiff) from Pomerantz' work is as follows as quoted by Potter:

> Adj.: at two o'clock in the morning.
> Pla.: on the eleventh.
> Adj.: in March last year where early in the morning.
> (Potter, 1996, p.45).

Potter's point here is that the description " two o'clock in the morning" is not only about time but it appeals for sympathy as it is quite a bad experience to be woken up by water at that time of the night. The description from the above extract is therefore not just about time but it also 'does something' as it appeals for sympathy.

c   *Language games*

Human action is meaningful by virtue of the system of meanings. In brief, the emphasis here is on the use of language in the construction of meaning or analysis of meaning. I also found Freire and Macedo's (1987) view that pedagogy of literacy education involves not only reading the *word*, but also reading the *world,* to be more relevant in seeing the words groups used as one way of reading the world they describe.

d   *Shared features*

Here the acknowledgement of the contribution of human subjectivity (i.e. intention) is emphasized without sacrificing the objectivity of knowledge. In other words, interpretivists argue that it is possible to understand the subjectivity of meaning of action (grasping the actor's beliefs, desires, and so on) and yet do so in an objective manner. This was the guide at all times in subjecting the participants' written work to analysis.

4.2.1.2   Hermeneutics

Hermeneutics argue that understanding is a condition of human beings, and this understanding is participative, conversational, and dialogic (Schwandt, 2000). Schwandt (2000) further argues that understanding is always linked to language and is achieved only through the logic of question and answer. Understanding is something that is produced in dialogue, not something reproduced by an interpreter through an analysis of that which he or she seeks to understand. According to Aylesworth (1991), Bernstein (1983) and Gadamer (1941), the meaning one seeks in 'making sense' of a social action or text is temporal and always comes into being in the specific occasion of understanding (Schwandt, 2000). This view as noted by Schwandt (2000), is different from the interpretivists view that human action has meaning and that meaning is, in principle, determinable or decidable by the interpreter. From the hermeneutical point of view, interviews, as a follow up, are considered in order to get further understanding from the dialogue.
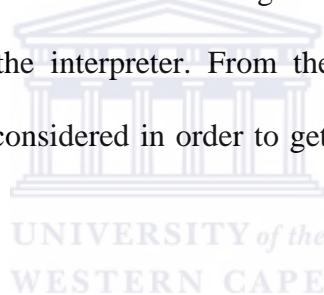
4.2.1.3   Social constructionism

People in different disciplines or within same disciplines use social constructionism differently. This is evident from Papert' s (1993), Potter's (1996) work and Schwandt's (2000) work. For instance, Schwandt's explanation of social constructionism is almost synonymous to the concept of constructivism and Papert and Potter's approaches are different. For the purposes of this study, the main focus is what it does rather than detailed description of what it is. The working definition in the case of this study is the view that social constructionism refers to an approach that aims to account for the ways in which phenomena are socially constructed (Anonymous, n.d.).

I now briefly and selectively present Potter's (1996) and Papert's (1993) views on the constructionism and the final point is to indicate the contribution of constructionism in this study. Potter' s argument is that the assumption on which (social) constructionism is based stems from the realisation that the worlds in which we all live are not just there, not just natural objective phenomena, but are constructed by a whole range of different social arrangements and practices. Schwandt (2000) has a similar view as he says:

> (w)e invent concepts, models, and schemes to make sense of experience, and we continually test and modify these constructions in the light of new experience. Furthermore, there is an evitable historical and sociocultural dimension to this construction. We do not construct our interpretations in isolation but against a backdrop of shared understandings, practices, language, and so forth (Schwandt, 2000, p. 197).

Potter further argues that the descriptions and accounts that construct the world, or at least versions of the world are themselves constructed and descriptions are human practices. Accordingly, this is the view that gives constructionism to be pragmatic and Potter is of the view that one needs to adopt a 'symmetrical' stance to knowledge that is treated as true and false. This symmetrical stance, Potter argues that "it frees the researcher from taking sides with particular groups whose beliefs are better established than others, more fundamentally, from deciding what should be counted as true or not" (p.12).

The above quote is very helpful in understanding how categories or models emerge from the interpretation of the data (written or verbal), and how sense is made as a result of experience and considerations for historical and sociocultural perspectives.

From Papert's (1993) view point, is that constructionism is built on the assumption that children will do best by finding for themselves the specific knowledge they need; organised or informal education can help most by making sure they are supported morally, psychologically, material, and intellectually in their efforts. Similar level of pragmatism that Potter associated with constructionism, is also seen in Papert and Harel

(1991) in which they argue that one can think of constructionism as learning-by-making. They argue that this learning-by-making or constructionism happens especially felicitously in a context where the learner is consciously engaged in constructing a public entity, whether it's a sand castle on the beach or a theory of the universe (Papert and Harel, 1991) and product that can be shown, discussed, examined, probed and admired (Papert, 1993).

Papert's (1993) emphasis is that there is a need for creation of an environment that is conducive for learning. In the context of this study, the type of problems and familiarity of the contexts of these problems constitute the environment and the 'public entity' is similar to text reports that were to be presented to appropriate stakeholders (transport, catering, finance, and academic division).

Constructionism/social[+] constructionism in this study particularly in this chapter, provides a window through which the research instruments of this study could be understood in terms of opportunistic environments that are created for building an artifact or a public entity. It also provides methodological frame from which one can operate when dealing text analysis or reports as these texts or reports may reflect more than understanding or perceptions of participants but may reflect descriptions of human practices. Furthermore, Papert's view on constructionism draws researchers' and educators' attention to the fact that 'diving into' situations rather looking at them from a distance, that connectedness rather than separation, are powerful means of gaining understand (Ackermann, 2001).

Considering intepretativism, hermeneutics, and social constructionism, it is not a matter of choosing the best of these three different philosophical methods, but the role they play in making sense of human actions (verbal, non-verbal and written). Schwandt (2000) captures this point well when he says

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[+] The word 'social' does not feature explicitly in Papert's work on constructionism, however it can be argued that its inclusion is implied.

...tendency to categorize and label complicated theoretical perspectives as either this or

that… is dangerous, for it blinds us to enduring issues, shared concerns, points of

tension that cut across the landscape of the movement, issues that each enquirer

must come to terms with in developing an identity as social inquirer… what we

face is not a choice of which label…. We are confronted with choices about how

each of us wants to live the life of a social inquirer (Schwandt, 2000, p. 205).

As indicated earlier in this chapter, the designs as discussed above formed an informative base for

interpreting and analyzing the data in the next chapter. They were used collectively to

determine the trends in the data. It was not a question of choosing, but of how the three

complement one another and other framework analyses.

4.3   Research process

In this section, I discuss the research process but I only limit the discussions to sampling

procedures and the procedure that was adopted in collecting data followed in this study.

4.3.1   Research participants

4.3.1.1   Biographical details of the research participants

The research participants of the study were pre-service teachers who were in the first year

of study. They were following the consecutive model, as described in chapter 2. The

curriculum for the students had the following as compulsory courses: mathematics,

computer science and English. The summary of the mathematics curriculum is attached as

Addendum 7. When the data were collected, it was at a time when the students had almost

finished all the courses. This was important in order to get a sense of where the students

were in terms of post secondary school mathematics knowledge development. This

knowledge and any other information they possessed were (by assumption) to act as tools for finding solutions to the themes provided. The participants' final examination results were not available at the time of data collection. There were 30 students, 44,8% of them was female and 55,2% was male. The majority of these students (99%) came from rural areas. Their ages ranged from 20 to 25.

In order to get to know the students, I conducted a survey asking the participants to indicate what they thought their adult careers would be. Out of 28 first year students, only two (7,7 %) regarded teaching as an adult career, while 26 students (92,9 %) gave different careers. In this 92, 9 %, the dominant career was computer programming (data administration, computer scientist, programmer, process controller) and one regarded being an actuary as an adult career. These results confirm the results of the survey done by Chuene et al. (1999) in which both pre-service and novice teachers did not originally identify positively with teaching as their adult roles. The reasons for choosing teaching as a career were extrinsically inclined (generally job security).

The outcome of this survey is very interesting because 99 % of these students come from rural areas and they are from areas where the most common professions are teaching, nursing and joining the police force. The question is whether this order in career choices may have any impact on teacher preparation in general and what the implications for their performance as teachers are. There is no conclusive answer to these questions except to speculate that student teachers react differently in different educational environments.

Furthermore, the results indicating students' preferences in terms of mathematical topics are presented and this is also compared to the preferences of the practicing mathematics

teachers. These statistics are important in this study as they give more information on what students think and whether there is any traceable link between the ways they behave in mathematical contexts and their preferences.

4.3.1.2   Brief profile for the selected practising mathematics teachers

Of the 28 surveyed practising mathematics teachers, 57, 1% was male and 42.9% was female and their ages ranged from 20 to 45 with the majority being between 30 and 40.

Teaching experience	FREQUENCY	Percentage
From 1 to 4 years	9	32,1
From 5 to 8 years	3	10,7
From 9 to 12 years	7	25
From 13 to 16 years	7	25
From 17 to 20 years	1	3,6
From 21 years and above	1	3,6
Total	**28**	**100**

Table 4.1: **Experience in teaching mathematics**

Table 4.1 shows that 32,1% have teaching experience of about 1 to 4 years and those who have 5 to 21 years teaching experience in mathematics constitute 67,9%. This 67,9% is important in terms of Ma's (1999) idea of developmental competence in school mathematics depending on the support and in-service programmes during these years.

4.3.1.3 Participants' preferences in terms of mathematical topics

In this section, the participants' preference is compared to the preference of the practicing mathematics teachers, referred to as seasoned teachers. Highlights of the reasons for practicing teachers are also presented.

Mathematics topics related to real life situations or contexts

	I Like them, good problems	Moderate, I don't like them that much	I don't like them
Pre-service students	66.7	26.7	6.7
Seasoned teachers	60.7	17.9	21.4
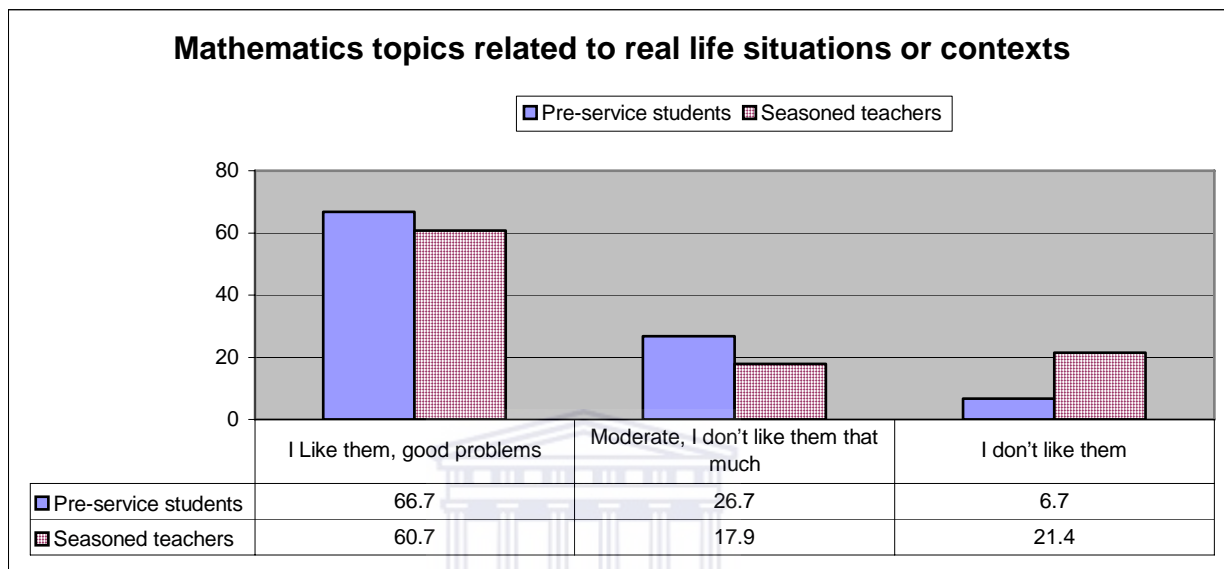
Figure 4.1: Mathematical tasks that are related to real life situations

Figure 4.1 shows that 66,7% of pre-service mathematics teachers is in favour of mathematical tasks that are related to real situations as compared to 60,7% of seasoned mathematics teachers. The gap (6,7 % vs 21,4%) is even bigger between those pre-service teachers who do not like these types of tasks and the seasoned teachers who do not like them. The reasons why seasoned teachers do not like this type of tasks are related to teaching as indicated by teacher number 13 and 15 below:

Teacher 13: I understand them but I can't teach them.
Teacher 15: Very difficult to make equations.

Mathematics problems with more than one answer depending on the context

	Pre-service students	Seasoned teachers	
	I Like them, good problems	Moderate, I don't like them that much	I don't like them
Pre-service students	66.7	26.7	6.7
Seasoned teachers	32.1	42.9	25
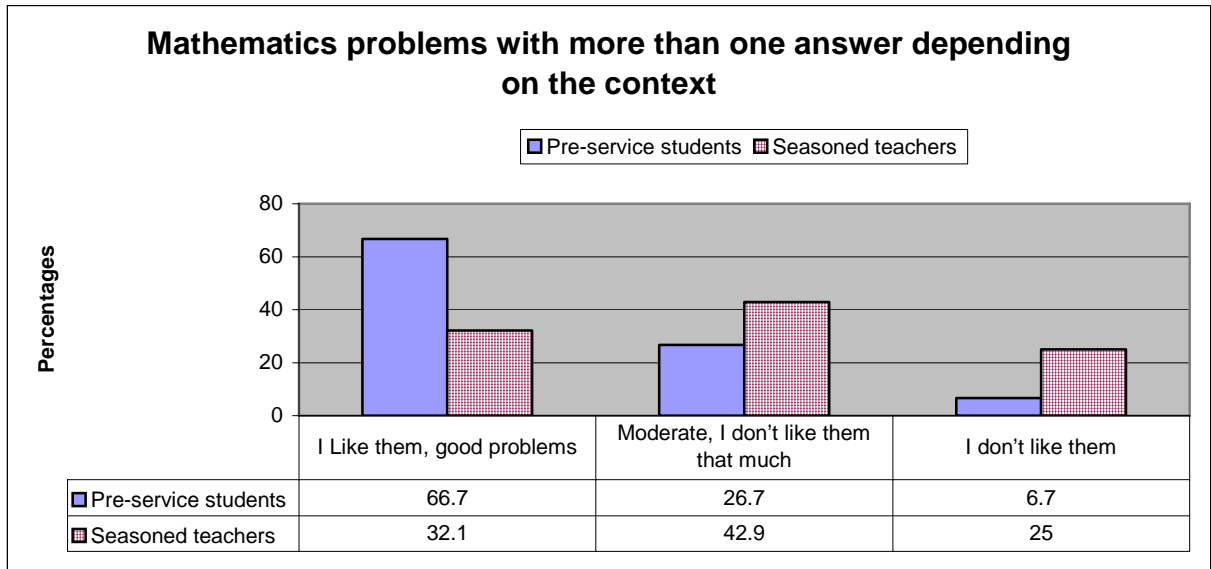
Figure 4.2: Mathematical tasks that require more than one solution

The emphasis in this type of tasks as compared to real life tasks was that, in this category, one would have open-ended problems. Figure 4.2 shows a similar pattern to the one in figure 4.1 above. This is not surprising as the problems of these types have a similar mathematical structure. The stance by seasoned teachers is even more profound in terms of their responses as shown in figure 4.2.

Abstract Mathematics- those problems that need proofs and conceptual understanding.

	Pre-service students	Seasoned teachers	
	I Like them, good problems	Moderate, I don't like them that much	I don't like them
Pre-service students	60	6.7	33.3
Seasoned teachers	53.6	21.4	25
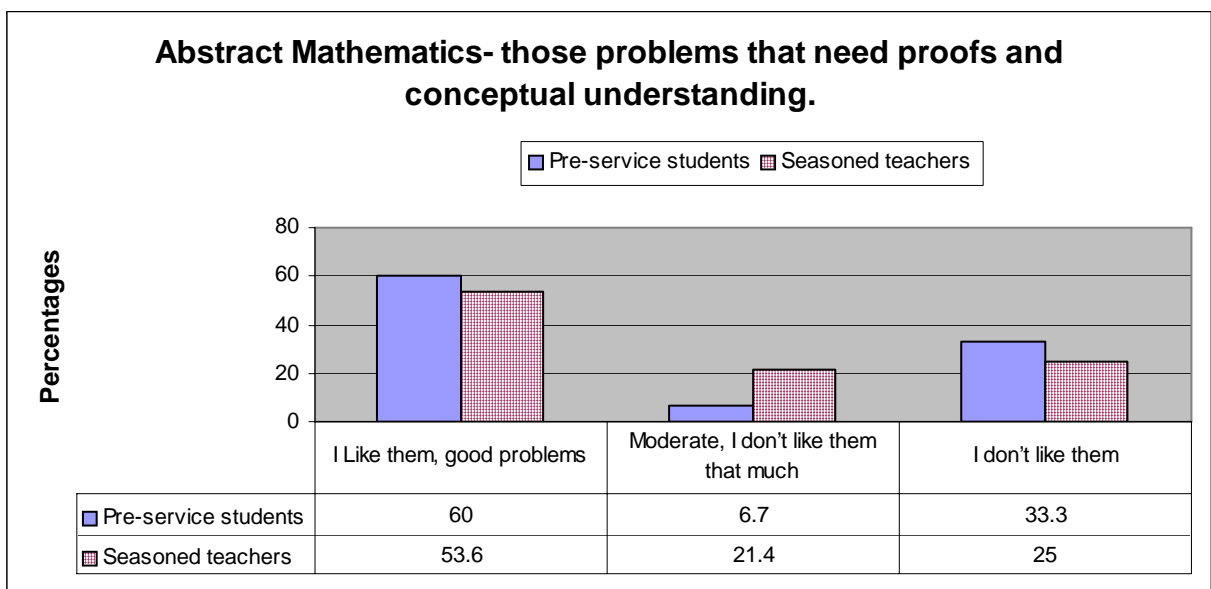
Figure 4.3: Abstract mathematics

These tasks were described as those that require proof and conceptual understanding. In terms of the frequencies, the seasoned teachers do not prefer these types of problems. The reasons given relate to learners' attitude and lack of methodology to teach them as indicated in the statements of the following teachers:

Teacher 14: Not able to teach them.
Teacher 15: Most of our learners don't like them.
Teacher 16: Very difficult for our learners to follow.

Mathematics problems with only one possible answer

Pre-service students — Seasoned teachers

	I Like them, good problems	Moderate, I don't like them that much	I don't like them
Pre-service students	36.7	23.3	40
Seasoned teachers	63	37	0
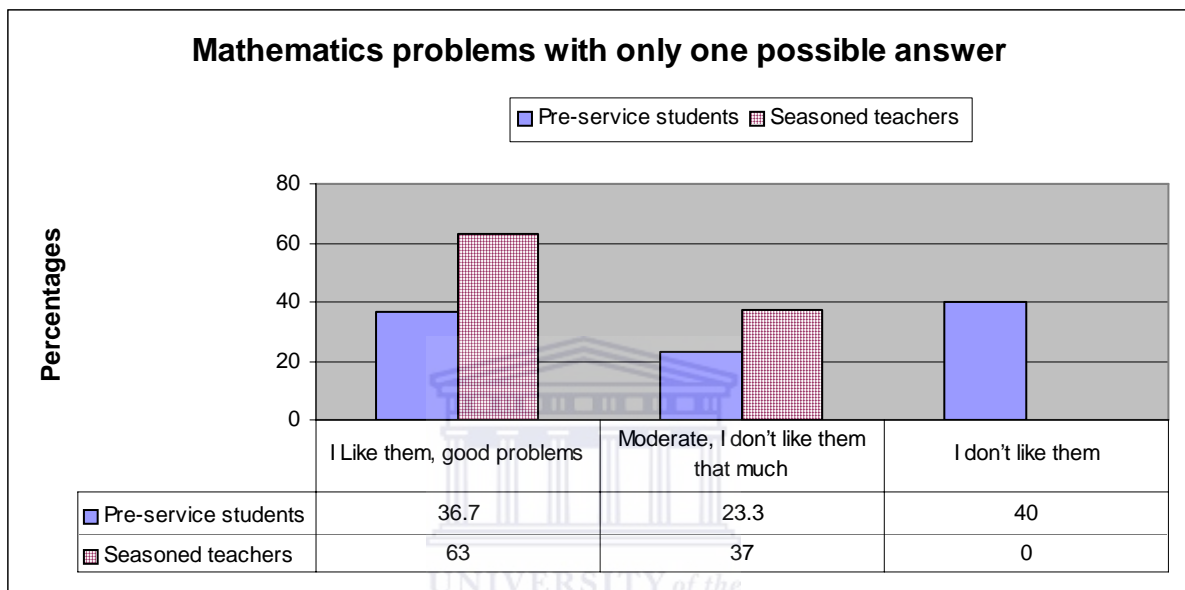
Figure 4.4. Mathematical tasks that require single solution

Figure 4.4 shows that the pre-service mathematics teachers do not prefer single solutions type problems. This is in contrast to the seasoned teachers who clearly prefer single solution type mathematical problems. Their preference ranges from moderate to good problems. None of them chose the category "I don't like them". Once more reasons are based on the needs of the classroom practices. The statements by the three teachers capture this view:

Teacher 15: Easy to solve.
Teacher 16: Easy to understand and easy to teach.
Teacher 19:...good for accuracy.

There are those who indicated that, even though they liked them, they are not helpful in terms of real life application. Teacher 14 and 17 explain this as follows:

Teacher 14: ... easy to solve, but difficult to relate to real life.
Teacher 17: Learners cannot relate them to real life situations.

Memory tasks - those require recalling of facts/rules/formulae

Pre-service students ▪ Seasoned teachers

	I Like them, good problems	Moderate, I don't like them that much	I don't like them
Pre-service students	43.3	26.7	30
Seasoned teachers	50	32.1	17.9
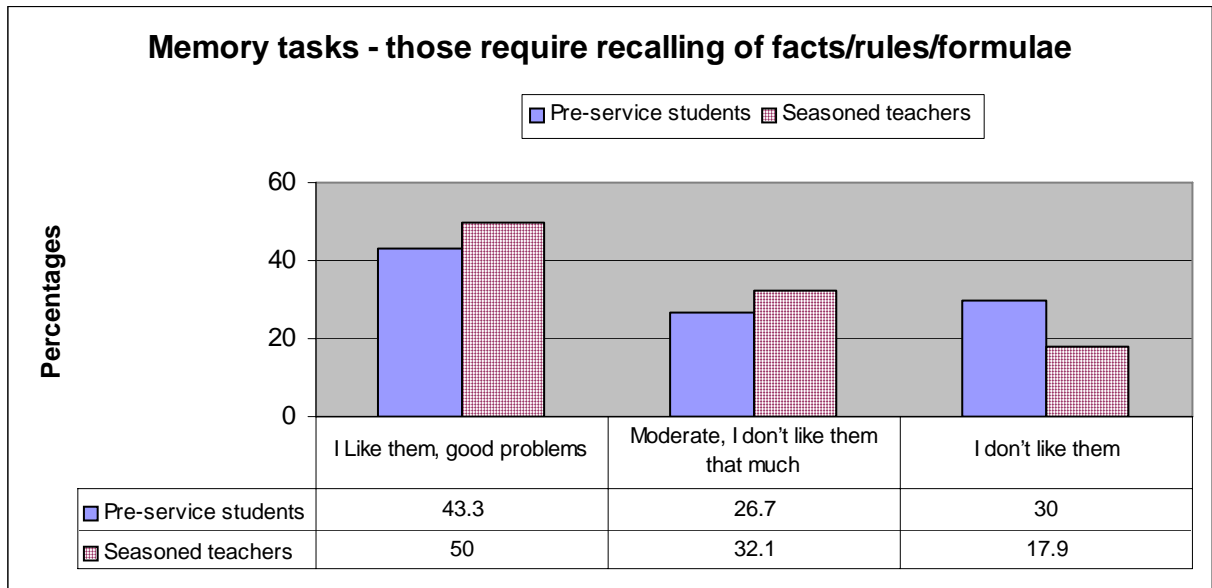
Figure 4.5: Memory tasks

The situation is not that different from the picture provided on the single solution type problems. The preference of the seasoned teachers is higher than those of the pre-service teachers. Some teachers have, however, mentioned that it is not "easy for learners to remember which to use" and is not easy to "apply something you have memorised".

Procedural or routine tasks

Pre-service students ▪ Seasoned teachers

	I Like them, good problems	Moderate, I don't like them that much	I don't like them
Pre-service students	46.7	26.7	23.3
Seasoned teachers	14.3	64.3	21.4
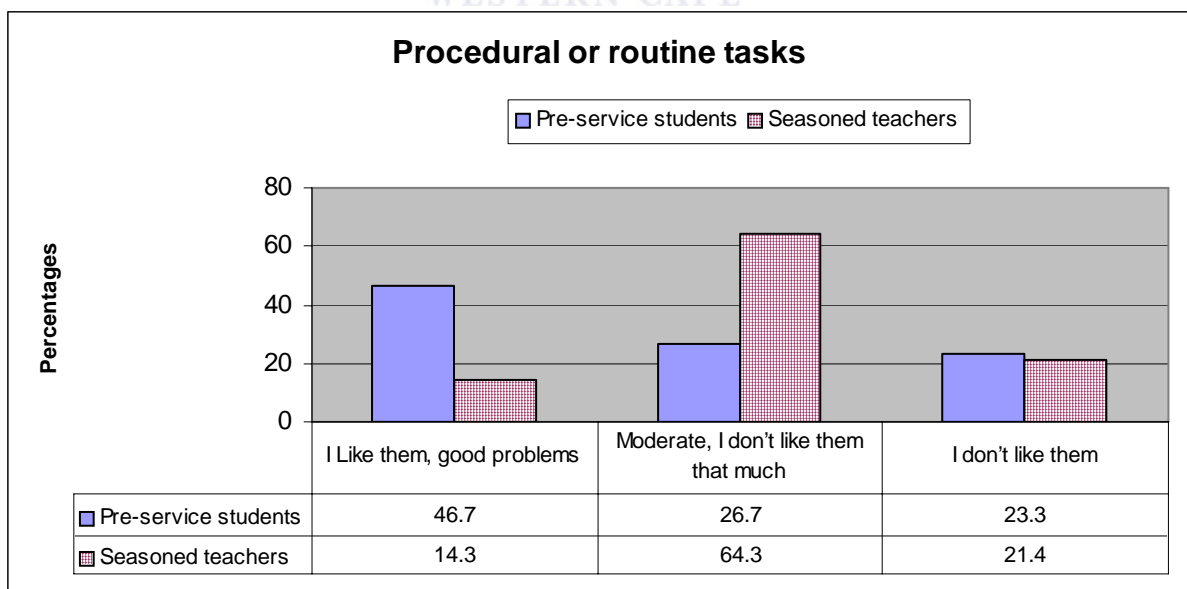
Figure 4.6: Routine tasks

The routine or procedural problems were described as those tasks that are context free or word problems that require one or two basic arithmetic operations. The former is the one that practicing teachers feel that, to some extent, routine problems are moderate. A total of 46,7% of pre-service teachers prefer these types of tasks, as opposed to 14,3% of seasoned teachers who think that these are good problems. The majority of practicing teachers are in the 'moderate' category. Once more, practicing teachers are consistent in terms of their reasons. The main reason is that learners do not understand these problems.

In summary, seasoned mathematics teachers who were surveyed preferred traditional problems with one solution. This is in contrast to the pre-service teachers' view, as they prefer a variety of problems with more than one solution. It may well be that the pre-service students were influenced by the type of tasks they were engaged in during the research. The validity of this assumption cannot be substantiated, as the pre-testing of their views on different types of tasks was not conducted. As indicated, the teachers' preference is largely driven by learners' attitude and to the ability to teach those sections.

4.3.2 Research instruments

As indicated earlier, a multi-stage data collection strategy was adopted in this study. In this section, I discuss three instruments that were used to collect data for the purpose of answering the three research questions posed in chapter 1. The first instrument was the four themes (refer to Appendix 3) that were given to pre-service students to solve. The second instrument was a questionnaire and is divided into two sections (refer to Appendix 4), which was to elicit students' views on mathematics and how they feel about doing problems from an application and modelling context, and finally, the reports/views from

sectional heads of divisions mentioned in the themes. A brief discussion of the questionnaires is given below.

4.3.3   Views About Mathematics Survey (VAMS)

Carlson (1997, 1999) presents an instrument referred to as: Views About Mathematics Survey (VAMS) and this instrument was adapted from the VASS (Views About Science Survey) instrument, which was developed by Halloun and Hestenes in 1996. VAMS is designed to assess students' views about knowing and learning mathematics. The instrument has six dimensions with two contrasting views. The six dimensions address issues relating to epistemological principles and pedagogical principles.

Under the epistemological principles we have the following dimensions:

- structure of mathematics

- validity of mathematical knowledge

- methods of mathematics

The pedagogical principles have the following dimensions:

- learnability of mathematics

- role of critical thinking

- personal relevance of mathematics
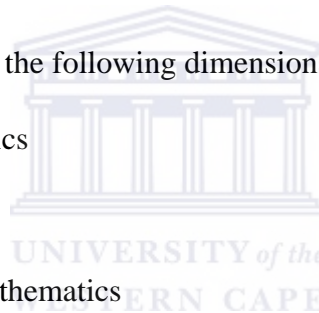
The first view of the two contrasting views of the six dimensions, which is the primary view, is the one that is commonly held among mathematicians (pure) and this is referred to as the expert view. The second view, which is taken as the secondary view, is the view contrasting the expert's view and it is attributed to the lay community and "naïve" students of mathematics. This view is referred to as the folk's view. Carlson's (1997) views (expert and folk) were arrived at by surveying mathematicians' views and the views were summarized to give an "expert's view". The opposite was then referred to as the "folk's view". In this study, Carlson's (1997) model has been slightly modified to suit the needs of the study and also to present a more simplified version to the students. This modification came as a result of the pilot study that was conducted.

For instance, the descriptions of the 'expert view' and 'folk view' have been modified. The folk view refers to those who show static representation of opinions or views; in short, those who hold to the traditional view. The expert view in this study is the view that corresponds with the expectations in an applications and modelling context. The simplistic view here was the consideration of the context in finding a solution and the consideration of the key steps in finding the solution to a mathematical problem. The 'expert view' provided here, is the summarized and simplified view from the reviewed literature on application and modelling. The 'expert view', therefore, could be taken as the 'popular view' in terms of the current debates and some views from the literature.

The individual items from Carlson's (1997) model have been modified and the VAMS's instrument used in this study has additional statements. Statements have not been paired; this was intended to check the consistency of responses. For instance, a student who will 'choose more of': (a) and (b) for: "When I experience a difficulty while studying mathematics: (a) I immediately seek help, or give up trying, (b) I try hard to figure it out on my own". The choice in the scale cannot be on the same level.

4.3.4 JUSTIFICATION FOR THE SELECTED MATHEMATICAL THEMES AS INSTRUMENTS DATA COLLECTION

Anderson (1994) declared that doing context bound and relevant mathematics is not only desirable but also necessary. She also argued that good mathematical tasks are ones that do not separate mathematical thinking from mathematical concepts or skills, that capture students' curiosity and that invite them to speculate and pursue their hunches.

The research participants in this study are in the environments as portrayed in the four themes. In their student careers, they had experienced them as problems to their lives and the lives of the general student population. The themes, therefore, appeared to appeal to their curiosity and right from the beginning (preparatory stage) of the discussion of themes, they were already speculating what the solutions would be and, clearly, these speculations were context based. They were so eager that the solutions should be presented to the authorities, that I felt like I was in the process of organising a protest action.

Using Anderson's (1994) criteria as stated above, the themes were found to be appropriate. Furthermore, the research conducted elsewhere provides evidence that shows that the participants who take part in the type of themes that involve modelling and have emphasis on social context, have potential for human development. For instance, in Gauvain's (1998) work, there is a link between cognitive development (referring to mathematics learning) and tasks that are grounded in a particular social context, Hatano (1997) emphasises the role of word problems in a modelling activity in terms of sense making and Gravemeijer (1997) and Toom's (1999) research work emphasise mathematics as human activity.

Furthermore, the discussions in chapter 2 about appropriate mathematical tasks and issues from reviewed literature in chapter 3, helped determine the choice and the structure of the themes presented. In line with Davydov's (1999) idea (CHAT paradigm), these themes were considered to be fulfilling the criteria of an activity as described in the first chapter and that in fulfilling these activities/themes, the participants would also change and develop themselves.

Finally, the themes as presented here, were consciously planned to be outside the 'hard sell' sales pitch. In this 'hard sell' sales pitch, the salesperson is always right and what he/she says does not entertain objections; the listener must accept the truth as presented. It was acknowledged that these themes need realistic solutions, i.e. responses were to be informed by realistic assumptions (not because of ambiguity – but considering different contexts) and these intentions were made very clear to the students. In collecting the data, the point by Gerofsky (1999) that students need to be allowed to give their opinions, was kept in mind and the task was planned accordingly.

4.3.5   PROCESS AND VALIDITY

There are different interpretations of what validity means in positivistic studies and qualitative studies. One description of validity is the one given by Schwandt (1997), which is fallibilistic validity. This refers to a test of whether an account accurately represents the social phenomena to which it refers. He argues that the defenders of this view argue that we judge validity of an account by checking whether it is plausible and whether it is credible given the nature of the phenomenon being investigated. Common means of establishing validity are analytical induction, triangulation, member check, and theoretical frankness.

Furthermore, Doyle (1983) argues that academic tasks may be presented in different ways and could be understood differently, while the intention of the tasks may also be received differently. To illustrate this point, he discusses four basic types of academic tasks and these are: memory tasks, procedural or routine tasks, comprehension tasks and opinion

tasks. In designing and administering the themes to the participants of the research, these types of tasks were taken into considerations.

The themes were presented to the students, then read, the explanations offered, students were given time to discuss them and ask questions. After questions were clarified, the students were then divided into groups of six. At this stage, there was an understanding that the intentions of the themes were understood in the same way. The following guidelines were presented to the students as the ideal actions to be taken.

4.3.6   Guidelines for the themes

Steps to be followed when solving mathematical problems (in sequential form)

a. Specify the real problem.
b. Set up a model.
c. Formulate the mathematical problem.
d. Solve the mathematical problem.
e. Interpret the solution.
f. Compare with reality.
g. Write the report.
NB. In doing the attached mathematical problems, note that every idea/comment/remark etc you have is important, so please write it down (even if it is a rough work)

Figure 4.7: Steps to be followed in mathematical modelling.

It is noted that the steps in figure 4.7 are almost similar to the steps suggested by Engeström (1991) as shown in table 4.2 below. The only difference identified in terms of this study was operationalisation of the planning stage. The planning stage was limited to organizational purposes and for clarification whereas in Engeström's (1991) steps, the stage is also used to critique the problem (including reasons for doing it) that needs to be solved.

Steps in mathematical modelling Column 1	Engeström (1991) Column 2
1. Planning stage 2. Model building 3. Reflection on the tools to be used 4. Looking at assumption, contextual needs looking at assumption, contextual needs 5. Realistic consideration of the solution	1. Questioning 2. Analyzing the situation 3. Modelling 4. Examining the model 5. Implementing the model

Table 4.2: Epistemic actions/Potential expansive cycle

Column 2 of table 4.2, as argued by Engeström (1991) forms an expansive cycle or elsewhere referred to as epistemic actions (Foot, 2001). One of the key issues about an expansive cycle is the time or duration of the cycle. It is more suitable for longitudinal studies. In this study the steps followed resemble those in column 2. It has the potential to realise some of the issues Engeström (1991) raises in column 2. I then argue that column (1) is a potential expansive cycle.

4.3.7   PILOT STUDY

A pilot study was conducted and there were nine questions. One of the nine questions in the pilot study was: There are 125 sheep and 5 dogs in a flock. How old is the shepherd? Other questions are reflected in Appendix 1. The piloting process was executed in two phases. One phase had only three student teacher participants. On interacting with colleagues and from presentations in conferences, it became clear that the pilot study should include learners from schools. In line with the study done by Verschaffel, De Corte and Borghart (1997) in which they involved the pre-service teachers and learners from schools on word problems (problematic and unproblematic), this strategy was implemented in the second round of the pilot study. The feedback from the pilot study shows that at times learners do not consider realistic contexts of the problems and that there are limitations within closed word problems. The extract from the responses illustrates this view as shown on page 89 to 91.

The contributions of the pilot study were two-fold. Firstly, I was able to restructure questions and move from closed questions to themes. Secondly, it helped me to think and plan for various analytic frameworks as it became clear that various responses would emerge from the data of the main study. A cursory glance of the extracts on page 89 to 91 illustrates this point. The analysis of the results of the pilot study came to three categories, namely: number manipulation, comparison between mathematics and reality, and working from reality to the world of mathematics. The first category involves number manipulations. Here, the students' focus is on the use of mathematical operational signs. Little attention is given to reality or the realistic consideration of the solution. In the second category, the mathematics world and real world are treated as different worlds, as

Number of dogs = 5
Number of sheep = 125

- If we take 125 sheep and 5 dogs in a flock and divide the number of sheep with the number of dogs, then we get 25
- That means 1 dog can look after 25 sheep, but we cannot determine the age of the shepherd because the question doesn't specify whether the shepherd is a man or a boy.

$$\frac{5x}{5} = \frac{125y}{5}$$

$$x = 25y$$

$$\frac{x}{y} = 25$$

1 year
1d → 5 s
xd → 125
5x = 125
x = 25

shown in extract 2. In the third category, realistic considerations are made and the way to find solutions is based on real life possibilities.

The direct translation of the last response is as follows: "The shepherd has 25 years because we have divided 125 by 5, then it gives us that answer"

Type 1: Non-realistic type of solutions

THE TWO RESPONSES ABOVE WERE CATEGORIZED AS NON-REALISTIC TYPES BECAUSE THE GROUPS SIMPLY FOCUSED ON NUMBER MANIPULATIONS BY USING DIVISION AND MULTIPLICATION WITHOUT CONSIDERING THE COMPONENTS OF THE PROBLEM.

Type 2: Comparison of classroom mathematics and reality

In this case both groups treated classroom mathematical operations as different from the real world.

Information is insufficient about the years of the shepherd, but if each dog can do its work perfectly, then little or old shepherd won't panic much.

P2. Classroom
- 8 pieces of rope would be needed to the poles together
Reality
- more than 8 pieces of rope would be needed because knots of ropes take some pieces.

P3. Classroom
5 friends of Carl + 6 friends of Sifiso = 11 friends of Carla and Sifiso.
Reality
If everyone at the party is the friend to each other

P.2 Mathematically he needs 8 pieces of rope but in real life he needs more than eight because he had tie knot when connecting the rope and immediately he tie knots he reduce the lengths of the ropes.

P.3 Mathematically we have 11 friends at the party but in real situation friends are 13 all in all, because Carl and Sifiso, also are friends to their friends.

Type 3: Working from reality to the world of mathematics
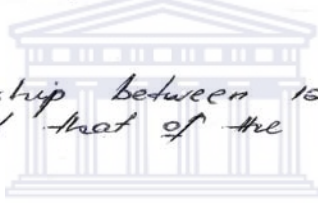
In this case reality is the entry and exit points. This means that the groups understood the problem from a real life context and the solution was interpreted as such. In this way, the dilemma was realised.

P.1   Data

sheeps = 125

Dogs   = 5

shepherD Age = ?

Real World

-Shepherd Age = M [if M > 0]

-It is not a child.

Ideal World

  0 < shepherd Age < + ∞

/ No relationship between 125 Sheep and 5 dogs and that of the sheperd's age

P.1. data given doesn't lead to the sheperd's age in any way whatsoever.

4.3.8   Peer debriefing

There are at least two reasons for using peer debriefing in qualitative research. Firstly it may imply a procedure whereby the field worker confides in trust and knowledgeable colleagues and uses them as a sounding board for one or more purposes. Secondly, it may imply sharing ideas about procedures and logistics in the field to get advice and check dependability of ways of proceeding, and it can involve sharing evolving attempts at describing and analysing qualitative data to achieve some consensual validation (Schwandt, 1997). In this study, the idea of debriefing took variety of forms that are similar to the second category as described here such as conference presentations, seminar presentations to community of mathematics education specialists and students, and a visit to a university in United States of America.

A more focused debriefing session involved six M.Ed. students in mathematics education who were presented with the four mathematical themes and groups' solutions. I then gave them the categories that I thought emerged from the data. The students were asked to comment on consistency of my analysis of the students' solutions.   In general, we were in agreement. We, however, had different opinions on the categories of the example driven strategy and the imaginary driven strategy. They were of the opinion that what I considered to be the imaginary driven strategy could be taken as an example driven strategy. We finally settled for keeping the two categories separate, as I explained that with the imaginary driven strategy, in the end, there is no solution to the problem, whereas, with example driven strategy, groups used 'living data' and this was more acceptable if used appropriately to yield a solution.

4.3.9 MY ROLE AS RESEARCHER

The purpose of the project is to describe the mathematical behaviour of pre-service students in an application and modelling context. I tried not to influence the way students should do the themes or bring suggestions to the process. To a large extent, I acted as a 'disinterested observer'. I, however, offered to help where students needed my assistance and I emphasized that I would not suggest any overriding of their opinions. Students welcomed this idea and they requested me to help with the material from time to time. An example was the need for information from certain offices, such as taxi offices, that were off campus.

I saw my role as one of

a. preparing students' participation during the four-week period, explaining the purpose and the role they will play

b. leading the discussion on the research work by Verschaffel et al. (1994; 1997), that is, engaging in the process of scaffolding

c. chairing presentation sessions and conducting semi-structured interviews

d. ensuring division of groups

e. monitoring attendance during the preparation period

f. setting rules and guidelines for doing the problems

g. monitoring the schedule (which was not strictly as per the students' request)

4.3.10 Procedure

The collection of data was done at three levels and data will also be presented in that way. The levels are as follows: At level A, students/participants completed themes; at

level B, I administered questionnaires (Views About Mathematics Survey-VAMS) that were adapted from Carlson's (1997) model; and, at level C, I acquired reports from

section heads i.e. reports from taxi associations, a head of a catering section, a manager at a university finance section concerned with student debt reduction and a head of an academic division related to both day and evening lectures.

4.4 STRATEGY FOR ANALYSING DATA

In this section, I discuss the strategy adopted for analysing data and how the bottom-up strategy came about. The continuous reflection on the data gathered and the interaction with other mathematics education community members, forced me to critically look at the literature that was reviewed for developing a strategy for data analysis. The key source for detail analysis of the data was the reliance on the theoretical framework, that is the Cultural Historical Activity Theory (CHAT). As a practical theory, CHAT enabled me to identify the unit of analysis and what the object of each of the four mathematical themes would be. The unit of analysis was identified as the application of mathematics in a modelling context, and different themes as an activity system. My focus was on how groups worked out (strategies) solutions to the mathematical themes. Engeström's (1999a) model of an activity system is illustrated in figure 4.1 and table 4.4 shows an expanded model of an activity system.

Mediating artifacts

Subjects   Object ⇨ outcomes

Rules   Community   Division of labor
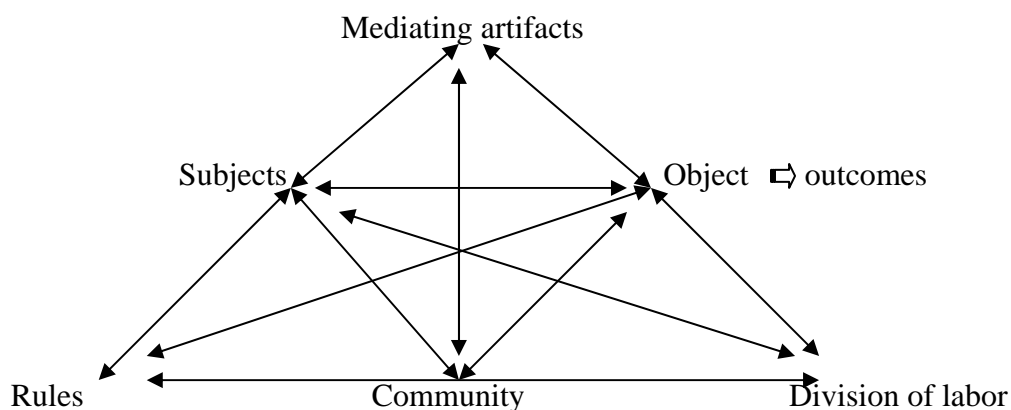
Figure 4.8: Model of an activity system – (Engestrom, 1999)

The description of components within the activity system is presented in table 4.3.

Components	Description
Mediating artifacts	Tools used toward the object
Subjects	Participants/actors
Rules	Regulate the subjects' actions
Community	Persons who share an interest/involved in the same object
Division of labor	Stipulates what's to be done by who toward the object
Object	Societal productive practices-'end product'

Table 4.3: Description of components within an activity system

Components	Description
Mediating artifacts	Mathematical knowledge, facts, procedures, maps, calculator, etc
Subjects	Pre-service teachers
Rules	Procedural rules – steps in approaching problems and social rules governing interactions of the group
Community	Taxi associations, kitchen staff, academic head, finance personnel
Division of labor	Groups were to have scribes, presenters
Object	Mathematical strategies by groups, that is reports to be presented to stakeholders
Outcome	Effective methods of transport; reduction of student debt model, effective utilization of the institution's catering system and access to tuition by both day and evening students

Table 4.4: An expanded description of emergent components within an activity system

Table 4.4 above shows how CHAT was used to facilitate analysis of the groups' work on the mathematical themes that were presented to them. Together with the CHAT approach in analysing data, the three qualitative methods (interpretative, hermeneutics, and social constructivism) discussed earlier were applied and the following were also considered.

Fennema, Carpenter, Franke, Levi & Empson (1996) have also used the Cognitive Guided Instruction (CGI) programme as a framework that focuses on helping teachers' to understand the development of mathematical thinking. The use of such understandings, according to Towers (2003), guide subsequent instruction and move teachers' instructional practices through a system of levels towards a fourth level. The guiding themes for CGI based on research by Fennema et al. (1996) are:

a. Children can learn important mathematical ideas when they have opportunities to engage in solving a variety of problems.
b. Individuals and groups of children will solve problems in a variety of ways.
c. Children should have many opportunities to talk or write about how they solve problems.
d. Teachers should elicit children's thinking.
e. Teachers should consider what children know and understand when they make decisions about instruction.

Secada and Brendeur (2000) support the success of this programme and they say that, over a period of 50 years, the CGI programme has shown how people solve arithmetic word problems and it has produced an impressive record. The developmental approach embedded in the CGI program significantly sharpened my thoughts on examining students' strategies. On the other hand, Towers (2003) says that, while he appreciates the contribution of the CGI, he finds the levels problematic to some extent. He then argues that "The Dynamic Theory for Growth of Mathematical Understanding" as argued by Pirie and Kieren (1994), is more appropriate for teacher development. The importance of the CGI in this study was to provide a frame of reference in terms of the minimum behavioural characteristics of learners when they are engaged in word problems. Even though this study is not about children, the CGI sets guiding principles from

which one may work in terms of understanding patterns of written work and exercise some forward thinking in terms of where the steps in the solutions lead to.

Hayes's (1981) strategy is to look at students' strategies to find solutions to problems in four ways. He argues that students use trial and error, proximity, fractionation, and knowledge. Though Hayes's (1981) strategies are not specifically meant for mathematics, they were found to be very helpful in understanding how students' strategies evolved.

Finally, Schoenfeld's (1989) framework as discussed in Carlson (1997), provides four components that are used in analysing students' problem solving strategies. These components are shown in table 4.5. This framework presented some thoughts on how data could be analysed.

Components	Evidence or indicators
Resources	checks mathematical facts and procedures that are accessible to the problem solver
Heuristics	broad range of general problem-solving techniques
Control	global decisions to select and implement resources
Beliefs	deep seated convictions e.g. it can only be done in this way

Table 4.5: Schoenfeld's (1989) theoretical framework for data analysis

In the end, rather than viewing the data against a particular framework (top-down approach), I opted for a bottom-up approach that was enriched by the above and the literature I reviewed. A pure grounded approach cannot be claimed.

Below, I briefly discuss the frameworks initially applied to the data and I indicate very briefly why I did not follow that route. The reason for including these in this chapter is that, though their application will be implicit, these frameworks exposed my thinking to various options in developing categories of data or labeling categories of data.

There are two categories of literature that I closely reviewed for analysing the data.

The two categories of sources consulted, but not explicitly mentioned in the data analysis, are briefly discussed in the following two paragraphs. The purpose of this is to show how each can enhance the analysis. In the first instance, we looked at Verschaffel and De Corte's model (1994, 1997). This model moves from the premise that mathematical problems can be divided into two forms, standard problems and problematic problems. The standard problems simply require the straightforward application of one or more arithmetic operations. The problematic problems refer to tasks that require realistic considerations when solving a mathematical problem. To assess the students' responses this model uses six subsections, namely expected responses, technical errors, realistic responses, non-realistic responses, and no responses. As the themes were open-ended, only two subsections became very relevant, namely, non-realistic/realistic considerations and technical errors. While this model helps in assessing realistic and non-realistic considerations, it is more suitable for closed problems, where there is an expected answer. As the mathematical themes were open- ended, the larger part of the model was not used except in consideration of realistic and non-realistic responses.

Another model that was considered in analysis of data is by Elshout-Mohr, et al. (1999). These authors provide a model with eight episodes (teaching segments of teaching activity). The model assesses a situation where teachers experience conceptual conflict. In this process, one must take care to select or construct contexts in which teachers can engage in observing, predicting, critiquing, and analysing the instructional process. The authors of this model argue that the episodes define four boundaries for the processes that foster deep processing and learning. These boundaries, which are not discussed in this

study are reproductive and productive, knowledge and skill, metacognitive and cognitive, and far transfer and near transfer. Elshout-Mohr, et al. (1999) describe the episodes in a hierarchical form of which the lowest level is type 1 and the highest level is type 8. Type 1 is characterised by reliance on memory, and application of knowledge is limited to conditions similar to those learned (near transfer). On the other hand, type 8 refers to a situation where students demonstrate the ability to apply knowledge in unfamiliar contexts (far transfer). The boundary that became relevant was near transfer and far transfer. By and large, Elshout-Mohr, et al's (1999) model appeared to be appropriate for assessing achievement or performance in a direct way and this was not the primary aim of the study. The larger part of the model was not used except the last boundary on near transfer and far transfer. Moreover, the thinking had shifted from dichotimising acquired knowledge and its application to a unified approach of reflective inquiry as discussed in the chapter 3.

4.5   Summary

To locate the research methods for the study, one has to identify the 'idea' driving the research. In the case of this investigation, the idea behind the research was identified as problem-focused or reform. The chapter suggests that there should be a variety of methods to investigate ideas of this nature. The chapter has also shown that qualitative methods are very important in analysing work in which the central focus is meaning and developmental trajectories. The chapter has identified intepretativism, hermeneutics, and social constructionism as three qualitative methods that could be used in analysing the data.

The bottom-up approach was used to analyse the groups' strategies to find solutions to the four mathematical themes. This kind of analysis was grounded in CHAT and other theoretical frameworks. A pure grounded approach is not claimed.

CHAPTER 5

FINDINGS

This chapter deals with the findings of the study. The data are analysed in response to three questions, namely:

1. What are the students' strategies in arriving at solutions for problems in real life situations?

2. What are the students' views on the nature of mathematics?

3. What are the students' views on the use of familiar contexts in the application of mathematics?

The question on the students' strategies was the main question of the study. The other two were subsidiaries.

Fennema et al. (1996) argue that individuals and groups of children solve problems in a variety of ways. This statement was used as a conjecture that anchors the analysis of the data. Though Fennema et al.'s context may be different to the context of this study, the spirit of the conjectures as mentioned above was very influential in reading and making sense of the data. The approach is to present the findings in terms of the three research questions. Firstly, the participants' views on the nature of mathematics are presented. This is followed by the findings on the strategies and lastly the findings on the participants' views on use of familiar contexts in the application of mathematics are presented. The order of the presentation of findings is to provide outward thinking, that is, who the students are in terms of their views on the nature of mathematics.

5.1 WHAT ARE THE STUDENTS' VIEWS ON THE NATURE OF MATHEMATICS?

The purpose of this question was to get a sense of pre-service teachers' views on the nature of mathematics. The importance of this question is based on the arguments of Ernest (1991) and Volmink (1993) that some of the solutions students offer to mathematical problems are largely influenced by the way they perceive mathematics. As explained in the previous chapter, the instrument that was used to solicit the participants' views was one designed by Carlson (1997). As per Carlson's (1997) strategy, the views are categorised into two principles, namely:

1. The epistemological principles (structure of mathematics, the validity of mathematical knowledge, methods of mathematics)

2. Pedagogical principles (learnability of mathematics, the role of critical thinking, personal relevance of mathematics)

The participants' views in terms of these principles are presented in the following sections in tabular form.

5.1.1 EPISTEMOLOGICAL PRINCIPLES

Table 5.1 to 5.5 below show participants' views on the structure of mathematics, validity of mathematical knowledge and methods of mathematics.

5.1.1.1 THE STRUCTURE OF MATHEMATICS

Mathematics is a coherent body of knowledge about relationships and patterns contrived by careful investigation rather than a collection of isolated facts and algorithms.

	Frequency	Valid Percentage
Expert view	23	76,7
Folk view	4	13,3
Mixed	3	10,0
Total	30	100,0

Table 5.1: Mathematics as a coherent body of knowledge

TABLE 5.1 SHOWS THAT IN TERMS OF THE STRUCTURE OF MATHEMATICS, PARTICIPANTS LARGELY HOLD

SIMILAR VIEWS (76,7%) THAT ARE CLASSIFIED UNDER THE CATEGORY "EXPERT" AS EXPLAINED IN CHAPTER 4, SECTION 4.5. THIS MEANS THAT IN GENERAL, PARTICIPANTS' VIEWS ARE POSITIVELY COMPARABLE TO THE 'CURRENT THINKING' IN TERMS OF MATHEMATICS.

5.1.1.2   METHODOLOGY

In terms of methods of mathematics, participants strongly think that mathematical modelling uses more than selecting formulas and doing number manipulations when finding solutions to mathematical problems. Those who think methods of mathematics are particular and situation specific, constitute 30%. The percentages per category are shown in table 5.2 and 5.3 below.

a) The methods of mathematics are idiosyncratic (particular) and situation specific rather than systematic and generic (general).

	Frequency	Valid Percentage
Expert view	9	30,0
Folk view	18	60,0
Mixed	3	10,0
Total	30	100,0

Table 5.2: Mathematics methods are systematic and generic

Table 5.2 shows that 60% of the participants take the folk view which is that of the layperson. Sixty percent, therefore, take the view that the methods are systematic and generic.

b) Mathematical modelling for problem solving involves more than selecting formulas for number manipulations.

	Frequency	Valid Percentage
Expert view	24	80,0
Mixed	6	20,0
Total	30	100,0

Table 5.3: Involvement of mathematical modelling in problem solving

The majority (80%) of participants take the expert view that mathematical modelling involves more than selecting formulas and only 20% took a mixed view about the statement.

5.1.1.3   Validation

a) Mathematical knowledge is validated by correspondence to the real world rather than by logical proofs

	Frequency	Valid Percentage
Expert view	6	20,0
Folk view	7	23,3
Mixed	17	56,7
Total	30	100,0

Table 5.4: Validation of mathematical knowledge by logical proofs

The majority (56,7%) of participants take a mixed opinion about the notion that mathematical knowledge is validated by correspondence to the real world. Table 5.4 shows that more or less the same number of participants think that mathematical knowledge is validated by correspondence to the real world than those who think that it is validated by logical proofs. There is no clear preference in terms of the expert view and folk views.

b) Mathematical knowledge is tentative and refutable (may be proven incorrect) rather than absolute and final.

	Frequency	Valid Percentage
Expert view	9	30,0
Folk view	16	53,3
Mixed	5	16,7
Total	30	100,0

Table 5.5: Tentativeness and refutability of mathematical knowledge

As shown in table 5.5, most students (53,3%) take the view that mathematical knowledge is absolute and final. On the other hand, 30% of students feel that mathematical knowledge is tentative and refutable.

5.1.2   Pedagogical Principle

In general, the views of participants on the pedagogical principle are in line with the views of the experts. There is, however, an indication of mixed feelings (63,3%) about the following statement: For meaningful understanding of mathematics, one needs to look for

discrepancies in one's own knowledge instead of accumulating new information. *Only 13,3% chose the expert view compared to 23,3% that chose a folk view.*

5.1.2.1   Learnability

Table 5.6 shows that 90% of participants hold the view that mathematics is learnable by any one who is willing to make an effort. No one took a different view except about 10% of the participants who took a mixed view on this statement.

a) Mathematics is learnable by any one willing to make the effort rather than by a few talented people.

	Frequency	Valid Percentage
Expert view	27	90,0
Mixed	3	10,0
Total	30	100,0

Table 5.6: Learnability of mathematics in terms of making an effort

b) Achievement depends more on persistence effort than on influence of the teacher or textbook.

	Frequency	Valid Percentage
Expert view	18	60,0
Folk view	10	33,3
Mixed	2	6,7
Total	30	100,0

Table 5.7: Factors influencing mathematical achievement

Table 5.7 shows that 60% is of the opinion that achievement in mathematics depends on persistence effort as opposed to 30% who think that the teacher or the textbook plays a part.

5.1.2.2   Critical thinking

Table 5.8 and 5.9 shows that the majority of participants took the view that, more systematic use of general thought processes, the exploration of situations in different ways and the reconstruction of new knowledge are crucial issues to consider for meaningful understanding of mathematics. On the other hand, the issue of looking for discrepancies

in one's own knowledge instead of accumulating new information in order to have

meaningful understanding has raised mixed feelings as shown in table 5.10.

a) For meaningful understanding of mathematics, one needs to concentrate more on the systematic use of general thought processes than on memorizing isolated facts and algorithms.

	Frequency	Valid Percentage
Expert view	26	86,7
Folk view	2	6,7
Mixed	2	6,7
Total	30	100,0

Table 5.8: Meaningful understanding of mathematics and general thought processes

Table 5.8 shows that the majority (86.7%) of participants supports the expert view that meaningful understanding of mathematics needs deep thinking and analysis rather than relying on memorisation of mathematical facts and procedures.

b) For meaningful understanding of mathematics, one needs to examine situations in many ways, and not feel intimidated by committing mistakes rather follow a single approach from an authoritative source.

	Frequency	Valid Percentage
Expert view	27	90,0
Folk view	2	6,7
Mixed	1	3,3
Total	30	100,.0

Table 5.9: Meaningful understanding of mathematics and examination of situations

Ninety percent of the participants are of the opinion that for meaningful understanding of mathematics, one needs to examine situations in many ways.

c) For meaningful understanding of mathematics, one needs to look for discrepancies in one's own knowledge instead of accumulating new information.

	Frequency	Valid Percentage
Expert view	4	13,3
Folk view	7	23,3
Mixed	19	63,3
Total	30	100,0

Table 5.10: Meaningful understanding of mathematics and discrepancies in one's knowledge

Table 5.10 shows that 63,3% of the participants take a mixed view about the idea that for meaningful understanding of mathematics, one needs to look for discrepancies in one's own knowledge instead of accumulating new information.

d) For meaningful understanding of mathematics, one needs to reconstruct new knowledge in one's own way instead of memorizing it as given.

	Frequency	Valid Percentage
Expert view	22	73,3
Folk view	6	20,0
Mixed	2	6,7
Total	30	100,0

Table 5.11: Meaningful understanding of mathematics and reconstruction of one's knowledge

Table 5.11 shows that 73,3% of the participants think that reconstruction of new knowledge in one's own way is needed for meaningful understanding of mathematics. Only 20% think that for meaningful understanding of mathematics, one needs to memorise knowledge as it is given.

5.1.2.3 Personal relevance

a) Mathematics and related technology are relevant to everyone's life rather than being of exclusive concern to mathematicians.

	Frequency	Valid Percentage
Expert view	23	76,7
Folk view	4	13,3
Mixed	3	10,0
Total	30	100,0

Table 5.12:Mathematics and everyone's life

The majority (76,7%) of participants think that mathematics and related technology is relevant to everyone's life.
b) Mathematics should be studied more for personal benefit than for just fulfilling curriculum requirements

	Frequency	Valid Percentage
Expert view	20	66,7
Folk view	4	13,3
Mixed	6	20,0
Total	30	100,0

Table 5.13: Mathematics and personal benefit

Table 5.13 shows that 66,7% of the participants think that mathematics should be studied for personal benefit. Only 13,3% take a different view.

Table 5.12 and 5.13 shows that mathematics should be regarded as being for personal development and that technology related to mathematics should not alienate mathematics from everyone's life.

5.1.3 SUMMARY OF STUDENTS' VIEWS ON THE NATURE OF MATHEMATICS

The participants' views in terms of the epistemological principle are summarised in figure 5.1.

Epistemological Principle

Legend: □ Expert view ▦ Mixed □ Folk's view

Categories (top to bottom):
- Tentativeness and refutability of mathematical knowledge
- Validation of mathematical knowledge by correspondence to world
- Involvement of mathematical modelling in problem solving
- Mathematics methods are particular and situation specific
- Mathematics as a coherent body of knowledge

X-axis: Percentages (0 to 90)

Figure 5.1: Epistemological principle

In sum, figure 5.1 shows that, in terms of epistemological principle, the expert view is prominent with regard to two dimensions. These dimensions are the structure of mathematics (mathematics as a coherent body of knowledge) and the dimension on methodology (the use of mathematical modelling as a problem-solving strategy). The participants take mixed views about whether mathematics methods are specific to a situation or systematic and generic, and whether mathematical knowledge is tentative and refutable or absolute and final.

Furthermore, the issue of the validation of knowledge, the majority (56,7%) of participants think that it is validated by logical proofs rather than by reference to the real world.
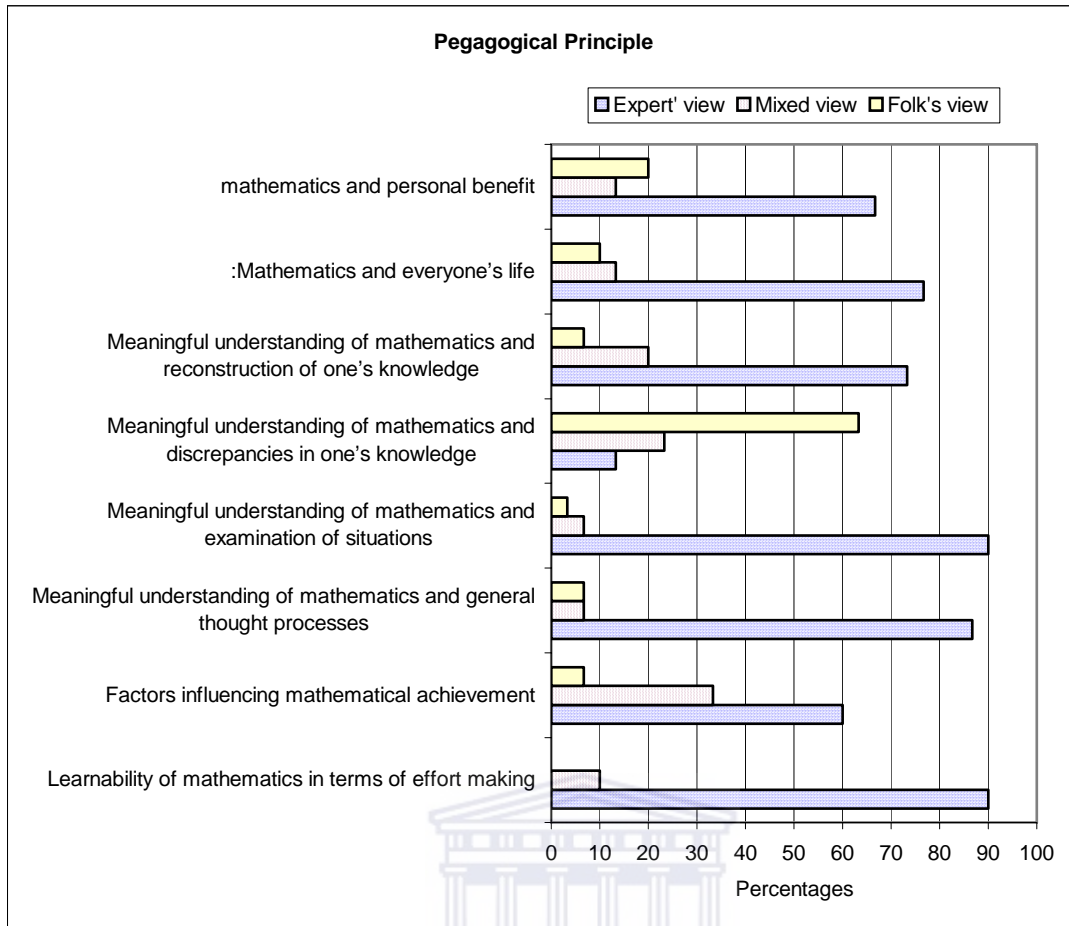
Figure 5.2: Pedagogical principle

Figure 5.2 shows that in terms of the pedagogical principle, almost all participants' views are similar to the expert view. The only difference is in terms of the dimension on critical thinking, in particular, the participants take the folk view in relation to the statement that for meaningful understanding of mathematics one needs to look for discrepancies in one's own knowledge instead of accumulating new information.

As mentioned earlier, the interest in the above results is to see how the participants' views on the nature of mathematics relate to the formulated strategies to find solutions to mathematical problems.

5.2 What are the students' strategies in arriving at solutions for problems in real life situations?

The summary of results is presented in terms of the four themes that were presented to the six groups of students. Each consisted of five students. The grouping was based purely on

the interaction that would maximise participation of each group member. This ultimately resulted in a situation where day students worked together and those staying in university residences worked together in finding solutions to the themes.

For the purpose of this chapter, only selected solutions are used to illuminate the groups' strategies. These selected solutions are representative of the strategies used by groups. Detailed and unedited group responses are provided in Appendix 5.

5.2.1   Background information: response rate per theme

As indicated in the previous chapter, the students were requested to present their solutions to themes according to the format that was provided to them (also refer to Appendix 2). It was clear that this request was not strictly adhered to. Some steps were not explicitly stated. This was particularly evident with regard to step 2 (set up a model) and 3 (formulate the mathematical problem). When asked why they did not follow the format, one group replied as follows: "Our report does cover the format, ja, it does".
In general the six groups attempted almost all themes. The groups' responses per theme differ as shown in the table 5.14 below. There were two major reasons for the incomplete solutions. These were, "no time to complete" and "no model" for the problem. In some instances two groups did not give reasons for not attempting the problems. The "no model" reaction was recorded as a response in line with Lave's (1988) argument that a problem can be resolved or abandoned. So the "no model" response was interpreted to mean that the group is saying that because the problem cannot be resolved, it was abandoned.

Themes	Response rate by the groups
Theme 1	100 %
Theme 2	83 %
Theme 3	67 %
Theme 4	50 %

Table 5.14: Response rate

The above background information is regarded as very relevant as I also wanted to see whether certain themes generated more interest than others and what the possible reasons are.

5.2.2 Categorisation of groups' working strategies

The groups' solutions to the mathematical themes were read several times. Following this "constant compare and contrast procedure", four ideal type categories were first identified in sections of the groups' work. These categories are: knowledge driven, students' contextual logical reasoning, example driven and imaginary strategies. There were also instances where combinations of strategies were prominent, and following this observation, two additional categories were formulated. The last two strategies are combinations, namely a combination of knowledge driven and imaginary driven strategies and combination students' contextual logical reasoning and example driven strategies. The criterion was that the strategy should feature prominently in order to stand on its own or to be combined. The descriptions and examples of these categories are presented below. These categories emerged as the way I interpret the groups' solutions. So they are more from my perspective (as an outsider). In terms of McMillan and Schumacher's (1993) types of categories, I regard these as etic rather than emic categories as the latter refer to the category/categories that emerge from participants' perspectives.

The approach in presenting the results in this section is to provide the description of the category and illuminate the categories by examples in terms of the extracts from the themes presented to the groups.

5.2.2.1 CATEGORY 1: KNOWLEDGE DRIVEN STRATEGY

The groups employed various mathematical strategies that involved using formulae, sections of mathematics such arithmetic series, linear programming, percentage, ratio, proportion, interest and statistics. Various mathematical topics are covered in this

category. These strategies are classified as 'knowledge driven strategies'. In this

category, students' reasoning tends to be formal and deductive.

Furthermore the students' strategies in this category exhibited reliance on mathematical

knowledge, suppositions and arguments supported by examples or by ordinary

communiqué logic, that is, using contextually based arguments in terms of the 'usual

social practices' in as far as certain locality is concerned. This is illustrated below.

---

1. *Increasing debt on student fees (real problem)*
2. $A = P(1-r/100)^n$
   A =?
   P =16000
   R = 5%
   N = 12
3. *The solution above will decrease so that the student can afford to pay his/her fees.*
4. *We can use the compound interest formula to identify the money that student will pay.*
5. *The management of the institution must disagree to allow student to graduate while he/she still owes the university.*
6. *The student must not be re-admitted the following year if he/she owes the past year.*
7. *They must disallow the student to be a resident student if they agree to admit him/her and if he was a resident student previous year.*

NB. The Principal amount of 16 000 is the amount allocated by National Student Financial Aid Scheme (NSFAS) as a student loan.

Figure 5.3: An illustration of the knowledge driven strategy

In the above extract (figure 5.3), the group relied on the related topic "interest". The group

used the formula for decreasing of debt compounded monthly.

The formula is simply provided but not used in arriving at the solution. The solution is

arrived at by providing conditions for resolving the problem i.e. numbers 3 to 7 of the

extract in figure 5.3 where the group says that:

The students must not be re-admitted the following year if he/she owes the past year and

that they (management) must disallow the student to be a resident if they agree to

admit him/her and if he was a resident student previous year.

5.2.2.2 CATEGORY 2: IMAGINARY DRIVEN STRATEGY

SECONDLY, SOME GROUPS PROVIDED SOLUTIONS BY

MAKING A SUPPOSITION AND PRESENTING IMAGINARY

DATA. THE REAL DATA OR THE ACTUAL DATA FROM

THAT INSTITUTION/SECTION ARE NOT USED. THE

CATEGORY IS CLASSIFIED AS THE 'IMAGINARY DRIVEN

STRATEGY'. TWO ILLUSTRATIONS OF THE IMAGINARY

DRIVEN STRATEGY ARE GIVEN IN FIGURE 5.4 AND

FIGURE 5.5.

The model to be set up, it should b subtraction to provide reduction of cause
Suppose initially we have 200 students, each student pays R200,00 but there are those who do not
have adequate money to pay. The total pay of the students (190) be R38 000,00 and the total pay
of the students (100) be R1000,00
Therefore Debt = 1000
To curb the increases is for the management to admit equal number of students who are able and
not able to pay. But the amounts should be proportional to that stated above, e.g. if they admit 10
students for next year

Figure 5.4: Illustration 1 of the imaginary driven strategy

The solution extract presented in figure.5.4 is one of those classified under the imaginary

driven strategy. This group used an imaginary number (not used anywhere in the data) of

students in a particular institution. In figure 5.4 it is stated that: "suppose we have 200

students…"

The group then concludes the solution by suggesting that:

> *"To curb the increases (meaning debt) is for the management to admit equal number of students who are able and not able to pay. But the amounts should be proportional to….".*

Another example is presented in figure 5.5 where the model structure is suggested but does not link to the data from the institution. In the first instance the group first presents the different classifications of students and then the solution for the payment of fees is presented in terms of parents' salary scales as presented in figure 5.5.

SOLUTION: **These should be a contract between the parents and the university and the parents should be held responsible for paying the universityvarsity for a parent to pay directly to varsity.** *All the invoices should be forwarded to the parents reminding them him/her of the amounts due. Since university fees are so high, some parents might not be able to pay on or before the due date. In this case parents should be classified according to their salariesor scaled n terms of their salary. Consider the following table:*

Parent	Annual Salary	Quarter Fees	Semester Fees	Annual Payment	Date of payment
1	144 000,00	5000,00			30 days after due date
2	180 000,00		10 000,00		60 days
3	250 000,00		20 000,00		90 days

This table is designed in such a way that parents with salaries belowy falling R150 000.,00 pay 25% at the fee quarterly and those with income above R150 000.00 but below R200 00000 pay 50% of the fees every six months, while those who earn salaries of more than R200 000,00make a once of payment of 100% of the fees .However, this can create problems as it is very difficult for parents to make a once off payment of such a large amount, so it better to arrange monthly payments of R1 700,00 per month.

Figure 5.5: Illustration 2 of the imaginary driven strategy

5.2.2.3   *CATEGORY 3: STUDENTS' CONTEXTUAL LOGICAL REASONING DRIVEN STRATEGY*

In the third category the groups provided the solutions by giving logical reasons. These reasons are bound to the context. The reasons are mostly general and not very mathematical. The solutions to the problems are arrived at by fixing or by giving

conditions. For instance, some groups argued that not allowing students' to graduate or not re-admitting them to the institution would achieve the reduction of students' debt. These are classified as **'students' contextual logical reasoning'**. The examples in this category are presented in figure 5.6 and figure 5.7.

CLASSIFICATION OF STUDENTS
1.Students with no parents
  Student loan
                    } Pay institution
  Bursary

 2. Students duly assisted by pensioners + limited salary
   Student loan
                    } Pay institution
  Bursary

3. Students with affording parents
The problem emerges where parents give students fees for university payment, and these students use money for their own benefit instead at paying the varsity

Figure 5.6:Classification of students using students' contextual logical reasoning

In figure 5.6, the students used their contextual knowledge of social strata in terms of students' profiles to arrive at three sub-groups of students.

The group further gives guidelines on how debt could be managed. In the next extract in figure 5.7 the group places the responsibility on the credit manager. There is a strong case for a credit policy, admission of students and average length of time between the due date of payment and collection of debts. The two factors determining the outstanding debts are identified as:

1. The number of students admitted

2. The average length of time between the due date of payment and collection of debts.

*This can be expressed as follows: Total debts = (students admitted per faculty * fees payable per year or per semester). To emphasise their point, the group argues that the institution should encourage "high volume" to be admitted and efforts should be made to*

"minimise the length of collection period". To justify their arguments they make a

reference to the University of Oxford and they argue that this system is a success.

DEBTORS MANAGEMENT
The management of debts starts with the decision to grant credit. This is the responsibility of the credit manager. In order to establish whether corrective action is needed, the credit manager would need an effective debts control system. A control system monitors the application of the credit policy. Without Credit Policy debts may build up, cash flows decline, and the bad debts arise.
The total amount of debts outstanding is determined by two factors:
1.The volume of students admitted
2.The average length of time between the due date of payment and collection of debts.
This can be expressed as follows:
Total debts = (students admitted per faculty * fees payable per year or per semester)
As, within reason, the institution would encourage a high volume of students admitted, the credit manager needs to concentrate efforts on minimising the length of collection period. For example, university of oxford was admitting students on terms of say 3/15 net 30, so its students on average appear by 3% discount. The 20-day collection period is less than 30-day credit period. Note, however, that most are students are paying within 15 days to take advantage of the discount while others could be taking longer than 30 days to pay their accounts. To identify these accounts, an ageing scheduled used to analyze the outstanding accounts. For example, University of Oxford is ageing scheduled is shown below...
Management would used to analyze the institution's average collection period and it's ageing schedule in comparison with other institutions, averages, recent trends and the institution's credit terms to see how effectively the credit department is managing debts. If the results appear to be not consistent and effective or efficient. This could mean that the credit manager is not enforcing standards closely enough, or the collection policy being too lax.
Credit policy involves making decisions regarding:
  Credit worthiness
  The collection policy and settlement discounts (3/15 net 30)

Figure 5.7: An illustration of students' contextual logical reasoning driven strategy

The extract in figure 5.7 is a further illustration of how the students' contextual logical

reasoning driven strategy is used to present a solution to the problem. The arguments

advanced on the unavailability of a credit policy and the responsibility of the credit

manager is more institutionally based (context based) than a general approach to financial

policy on debt reduction.

In the next category, I present a typical example driven strategy that was presented as a

response to theme 1.

5.2.2.4   CATEGORY 4: EXAMPLE DRIVEN STRATEGY

In this fourth category groups used examples of data or figures (living data) from real situations to demonstrate how solutions could be arrived at. The approaches seem to be practical, explanations are clear and formulae are implicit. Validation is largely through reference to some daily events or known events. These are classified as **example driven strategy**.

Figure 5.8 shows that the group used the data provided by the institution and the argument is made of how debt could be paid over a period of time by different groupings. The strategy used here is classified as the example driven strategy, since it uses actual data.

G31. Theme 1: Student fees
*Debt * 10% = Interest*
Debt: Debt + Interest = Total Debt
G31 (a) UNEMPLOYED AND UNREGISTERED STUDENTS
RANGE 1
*53 342,00 * 10% = 5 334,28; 53 342,00 + 5 334,28 = 58 676,28*
*30 000.00 * 10% = 3 000.00; 30 000.00 + 3 000.00 = 33 000.00*
Actual range is 33 000.00 to 58 676.28. In this range 10% is payable per annum
10% of 58 676.28 = 5 867 63 per annum for 10yrs
And 33 000.00 at 5 867.63 will be paid for 6yrs. Therefore paying range is 6 to 10yrs
G31 (b)
RANGE 2
*29 999.00 * 10% = 2 999.00; 29 999.00 + 2 999.00 = 32 998.90*
*10 000.00 * 10% = 1 000.00; 10 000 + 1 000.00 = 11 000.00. Therefore, actual is 11 000.00 to 32 998.90*
In this range 17.4% is payable per annum
17.4% of 32 998.90 = 5 741.81 for 6yrs & 11 000.00 at 5 741.81 will be paid for 2yrs
Therefore, paying range is 2 to 6yrs
G31 (c)
RANGE 3
*9 999.00 * 10% = 999.90*
*1.0   * 10% = 0.10*
Therefore Actual Range is 1.10 to 10 998.90
Sub Range 1

3 000.00 to 10 998.90 should pay in 2yrs, instalments ranging between 125.00 and 458.29 per month
Sub Range 2
1.10 to 2 999.99 should pay within 1yr, instalments ranging between 1.10 to 250.00 per month
Interest of 10% is an estimate
Remark: Ranges 1 and 2 pay almost the same amount, this is to ensure that no group has an advantage over the other and that the university collects the debt in the shortest possible period.
G31 (d) UNREGISTERED AND EMPLOYED STUDENTS
In this section we concentrate on the amount the student earns. We also consider whether the student will be able to live after paying the instalment.
SALARY SCALE 1: 10 000.00-34 999.00
Will pay 60% of the salary per annum
*10 000.00 * 60% = 6 000.00*
*& 34 999.00 * 60% = 20 999.40*
Therefore, Paying Range will be 6 000.00 to 20 999.40 p.a.
G31 (e) SALARY SCALE: 35 000.00-59 999.00
Will pay 65% of the salary per annum
*35 000.00 * 65% = 22 750.00*
*& 59 999.00 * 65% = 38 999.35*
Therefore, Paying Range will be 22 750.00 – 38 999.35 p.a.
G31 (f) SALARY SCALE: 60 000.00 AND HIGHER
Will pay 70% of the salary per annum
G31 (g) SALARY SCALE: 9 999 AND L0WER
Will pay according to the ranges of the unemployed (as if they are unemployed).
Remark: People earning R10 000,00 are expected to pay 60% of their salary per annum, this is to ensure that we don't have double standards. We expected a person who is unemployed to pay 5 334.25 per annum so it would not make sense to have a person who is employed to pay anything less than that (5 334.25) with this plan the university will recover the money within 10yrs.

Figure 5.8: An illustration of the example driven strategy

In the extract in figure 5.8, 10% is charged for the outstanding amounts and a student pays

10% of his or her outstanding amount per annum. This group indicated a number of sub-

groupings; in the above extract only two ranges are shown. Range 1 covers those students owing between R30 000,00 and R53 342,00 (as provided in the institution's records) and range 2 is for those who owe between R10 000,00 and R29 999,00. The group demonstrates how larger amounts could be paid over a period of time. From the above extract it is clear that the smaller the outstanding amount, the shorter the repayment period and the larger the outstanding amount, the longer the period of repayment.

A similar approach is used to demonstrate how debt could be reduced in other sub-groupings such as employed not registered in different salary scales in terms of their income.

In the following section, the intention is to make a point that even though there were four ideal types of categories of strategies, the group's solution did not have a single strategy but there was a mixture of strategies. I illustrate this point in two ways. I firstly present the table (table 5.15) showing the spread of categories in different themes and I also present the table (table5.16) showing groups' strategies per theme. Secondly, I present two examples of combination or mixture of strategies as pursued by groups.

Table 5.15 shows the summary of the etic categories in terms of themes.

Theme 1	Theme 2	Theme 3	Theme 4
Knowledge driven strategy, Imaginary driven strategy, Students' contextual logical reasoning strategy, Example driven strategy, Students' contextual logical reasoning and example driven strategy	Students' contextual logical reasoning strategy, Knowledge driven strategy, Imaginary driven strategy, Example driven strategy, Combination of knowledge and students' contextual logical reason driven strategy	Knowledge driven strategy, Example driven strategy, Students' contextual logical reasoning strategy, Students' contextual logical reasoning and example driven strategy	Knowledge driven strategy Students' contextual logical reasoning driven strategy

Table 5.15: Types of strategies per theme

From table 5.15 it is clear that, in general, the groups have used various ways or strategies to arrive at the solutions. In some cases a combination of strategies was used. The strategies that featured in almost all four themes are the knowledge driven strategy and students' contextual logical reasoning strategy. Table 5.16 shows the four ideal category types that were identified in the sections of the groups' work. The combinations of strategies used by the groups are not reflected in this table. The "no model" response was categorised under the students' contextual logical reasoning driven strategy.

Groups	Theme 1	Theme 2	Theme 3	Theme 4
1	K, I			
2	S, K	S	S	
3	E, S	K, E	S, E	
4	K, E	K, S	K, S	K, S
5	K	S		K, S
6	S	I	K, S	S

Table 5.16: Summary of groups'/ students' strategies per theme
K = Knowledge driven, S = students' contextual logical reasoning, E = example driven, I = imaginary strategies.

Table 5.16 shows that the knowledge driven strategy and the students' contextual logical reasoning dominated the approaches in finding solutions to the themes. Furthermore, in instances where strategies were combined, the knowledge driven and the students' contextual logical reasoning strategies featured much stronger as a combination than others such as knowledge and imaginary and knowledge and example driven strategies. Later in chapter 6 the leaning towards social rules (students' contextual logical reasoning) will be discussed.

Finally, in the next sections (5.2.2.5 and 5.2.2.6) I present two examples of categories of strategies that involved a combination of strategies.

5.2.2.5 Category 5: Combination of knowledge driven and students' contextual logical reasoning strategies

In this category students used a combination of the knowledge driven strategy and students' contextual logical reasoning to arrive at a solution. There were also a few

elements of the imaginary driven strategy but these elements were not as prominent as the other two. The extract below provides an illustration of this category.

THEME 2: "USAGE OF UNIVERSITY CATERING SECTION BY STUDENTS"
Institution D has its own catering section for residential students. It has, however, realised that the sustainability of this section is under threat given the inconsistent usage of this section by students and the general financial constraints faced by the institution.
Develop a mathematical model, which you will sell to the staff of the catering section to address the formulated problem.
Use the attached data as historical information.

The solution shown in figure 5.9 is a combination of the knowledge driven strategy and students' contextual logical reasoning. The students' approach to this theme was to use certain suppositions concerning numbers of students who are using might use the dining hall. In the process of presenting their solution, actual prices (local prices of that institution) for items such as breakfast, lunch, supper, bread, et cetera were used. The group made a mistake in calculating the 'quantity of food to be prepared for a day'. The group indicated that $Z=Y/X$, but in the substitution of $Z=25$ and $X=400$, instead of finding for Y, they found that $X=16$. This is interpreted as a technical error as the interpretation of the group's solution is correct. The statement: "*The quantity of food per student for 400 students (bookings must be made on time) is 16*" clearly illustrate this point.

Financial constraints faced by the university.
Let students (No. of students who use the Dh) be X [Dh refers Dining hall]
Quantity of food prepared per day be Y
And the budget per day be Z
Therefore Z=Y/X
Estimation: *If X=400.00 and Y will be determined by estimating Z*
Then Z=(Based on each Student) Breakfast + Lunch + Supper
NB: This model is for the catering section not the tuck-shop section.
Let Breakfast=P, Lunch=Q and Supper=R.
Z=P+Q+R
And P=6.50, Q=9.50 and R=9.00
Therefore Z=6.50+9.50+9.00
*   =25.00*
25=X/400. Therefore, X=16
The quantity of food per student for 400 students (bookings must be made on time) is 16.
And if for every student 20.00-25.00 can be spent to cover the quantity which means more funds can be saved.
Remark: *It is claimed that food ends up rotting because no one buys, and this is where money is wasted. The staff must try as hard as can to work on the number of the bookings made per day.*
TUCKSHOP / FAST FOOD SECTION

> *This is where profits can/is made. Looking at the prices e.g. Brown Bread: (normal price around QwaQwa)=R3,.00 and at the DH 3,40. From the bakery a loaf is sold at R2,80 and that means the Dh get 0.60 which is 0.40 more than the normal price, therefore this section can claim no financial constraints.*

Figure5.9: An illustration of a combination of the knowledge driven strategy and students' contextual logical reasoning

Furthermore, the group suggests that bookings should be done in order to avoid wastages. This initial planning will help in sorting out losses by kitchen management.
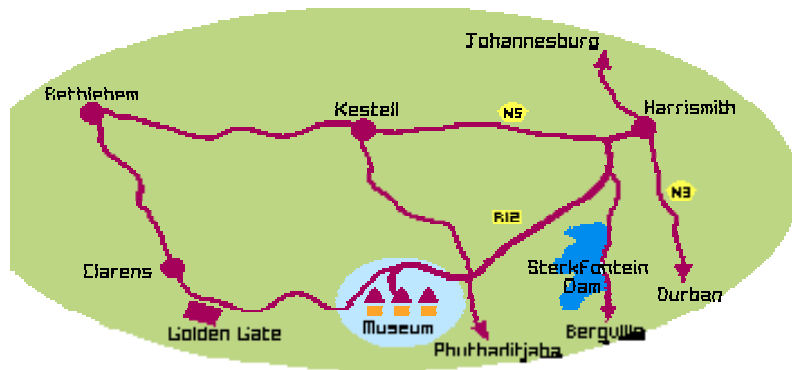
Referring to figure 5.9, the group feels that there is actually no loss as the section is charging more for items than the normal price and *"therefore this section can claim no financial losses"*. The students' contextual knowledge contributed to the arrival at this conclusion.

5.2.2.6 Category 6: Combination of students' contextual logical reasoning and the example driven strategy.

THEME 3: TRANSPORT FOR STUDENTS

Students from Institution B have raised a concern with the management of that institution regarding transport to and from the campus. The issue of transport affects both day and evening lectures. Identify the problem and develop a mathematical model, which the management may discuss with the taxi association (or an alternative plan) in order to resolve this issue.

The number of students attending day lectures who are in need of transport is 213 females and 149 males and for evening lectures 124 females and 138 males.

The following are the routes and estimated kilometres from the institution.

Routes	Kilometres	Remarks
Phuthaditjhaba	12	Estimates are return trips
Harrismith	94	
Bethlehem	156	
Clubview	12	
Bluegumbosch	15	
Elite	18	
Riverside	16	
Tseki	51	

Road Map: QwaQwa

Theme 3: TRANSPORT FOR STUDENTS

Solution:

Total number of students: *362-Day students*

10 people to Bethlehem

15 people to Harrismith

37 people to {Clubview and Bluegumbosch} (A1)

100 people to Phuthaditjhaba

200 people to {Elite, Riverside and Tseki} (A2)

Those, which are grouped, are grouped for convenience and the fact that it costs the same amount to go to any of them.

Normal price to A1 and A2 by bus is R7,60 return

7.60*14.5%=1.10 discount. Therefore 237(number of students)*6.50=1 540.00 per day for the buses.

Normal price for Phuthas by bus is3.80 return

3.80*21.1%=0.80 discount. Therefore 100*3.00=300.00 per day for the buses

Normal price for Harrismith is 21.00 return. 21.00*14.3%=3.00 discount

Therefore 15*18.00 =270.00 per day for a 16-seater taxi. Normal price for Bethlehem is 38.00 return.

38.00*21.1%=8.00. Therefore 10*30.00=300.00 per day for the bus

In a day the bus service can make 2 140.00. The discount is to ensure that the students become interested in this service and it's negotiable between the parties.

ROUTES (LOCAL)

3 bus-trains will be used, each carrying approximately 110 people. If buses were to used, four would be needed, so bus-trains are more cost-effective.

Route 1

Tseki via Elite, de Bult and Makgaolaneng.. 120 people will be transported.

Route 2

Tseki via Phuthaditjhaba, Riverside and Bolata (No 4). 120 people will be transported.

Route 3

Phuthaditjhaba via Bluegumbosch and Clubview. 97 people will be transaported

Route 4

Harrismith-We negotiate with the taxi association for a 16-seater taxi that would normally leave QwaQwa for Harrismith at 17:30 travelling a **'dead-mile'**, *we use for our students*

Route 5

Bethlehem-We negotiate with the bus service to provide a bus that leaves Qwa-Qwa for Bethlehem at 17:30. Our students will be discounted.

Figure 5.10: Illustration 1 of a combination of students' contextual logical reasoning and the example driven strategy

The above solution for theme 3 is classified under the example driven strategy and students' contextual logical strategy because of the reasons shown in figure 5.10. Data

used were the existing data in that section and that part of the solution relies on logical conclusions in the context of the problems. It is argued in the solution that the purpose of a discount is *"to ensure that the students become interested in this service and it's negotiable between the parties"*

The point illustrated in the above paragraph could be seen in the practical categorisation of routes and the step-by-step presentation of the solution in figure 5.10. From the road map of Qwa-Qwa, the group constructed routes to different destinations. The routes were constructed taking the distance between each destination and the institution into account.

It is this step-by-step categorisation of routes that enforces the idea of negotiations that comes up frequently in the students' presentation of the solutions in which a strong contextual logical reasoning is also used. For Route 4, for instance, the group says that between QwaQwa and Harrismith, "We negotiate with the taxi association for a 16-seater taxi that would normally leave QwaQwa for Harrismith at 17: 30 travelling a **'dead-mile'** we use for our students". The concept of a 'dead mile' is used in the South African Taxi Industry to mean a vehicle travelling between two points without picking up any passengers along the way. The passengers are transported from one point to the other or collected from one point to the other. It is used under special circumstances.

A similar approach is used for the students who attend evening lectures. Furthermore, it is suggested that the proposal will be sustainable if the students use the proposed service and use monthly tickets. This point is explained under remark 1 in Fig. 5.11. The students also used actual bus timetables and the data (bus rates) referred to, were those used by the bus company at that time.

Remark 1: *The key here is to get students to use these services and to get them to want to leave the campus at the same time (17:30 and 19:30). One other thing that we should enforce is that students should buy monthly tickets. If we tell the bus service that they are going to make R 2 140,00 (per day) +R 1 558,00 (per night) =R 3 728,00 per day. That is approximately R 111 840,00 per month and for approximately 10 months =R 1 118 400,00 - there is no way they will refuse.*

> **Remark 2:** *through this, students must pay the transport fare on hand to the institution on weekly basis. And this will solve problems of not getting taxis to school and not having day's fare.*

Figure 5.11: Illustration 2 of students' contextual logical reasoning and example driven strategy

The emphasis and positive consequence on monthly tickets is further made clear under remark 2 in figure 5.11.

5.2.3   Summary on groups strategies

The way the strategies are spread over the four themes is indicated in table 5.15. In the previous section a selected few solutions were presented in order to illuminate the etic categories of the study. The full solutions through which the strategies of the groups are shown are provided as Appendix 4.

5.2.4   Reality as an exit point to groups' solutions

The previous section looked at how groups found solutions to the problems from familiar contexts. These were real life contexts. Since reality was the point of entry in this investigation of mathematical behaviour of pre-service students in a modelling and application context, it was planned that reality should be the exit point as well. This is also consistent with Engeström's (1987) idea the object of a learning activity as a societal productive practice. I also considered Engeström's argument that the projected outcome is no longer supposed to be momentary or situational, but that it should rather consists of a new societally important objectified meaning and new, relatively lasting patterns of interaction. In line with this thinking, further collaboration was needed.

In the sections below, I present the highlights of the reactions from the section heads of the catering, finance and transport sections/taxi association.

5.2.5.1   Responses to the students' solutions

Out of four expected reactions to the students' solutions, three reports were received from experienced heads of sections. These sections were: finance, transport and a catering

company. The head of the academic division promised to send the report but one was

never submitted. At the time of the analysis of the data the head of the academic division

had terminated his/her services in the institution following higher education

restructuring. Attempts were made to follow up via e-mail and telephone, but nothing was

forwarded to me. The new team could not really assist me as they said that they were not

familiar with the nature of the problem.

Below is the synopsis of the reactions from the different heads of sections. The full texts

are included in Addendum 2. The reports from section heads are also reflected in the next

chapter.

a	*Student fees*

The response from the head of finance was that some recommendations by the groups

may work in educational institutions and some may not work. This is well captured in the

statements from the general comment:

> Institutions cannot control debt through admission. Therefore, more concerted efforts
> should be made in collecting debt rather than preventing debt. **Option 6** seems to be
> looking from this angle, if combined with parts of **Option 3**, something good can come
> from this.

b	*Catering*

The manager seems to concur with some of the suggestions made by the groups. The

manager is, however, somewhat skeptical about the solution as food prices may change.

The following quote summarises the point:

Yes, if students utilize the D. Hall everyday for three meals a day, i.e. breakfast at R9, 50 not (R6,

50) lunch and supper at R15, 00, this will mean each student will use R39, 50 per day.

Also we should be mindful of the fact that food is expensive and prices are going up every

month. In conclusion it is really necessary to keep both fast food section and the booking

system to cover everybody.

c	*Transport*

The representative from the taxi industry was of the opinion that the suggestions were mainly for use of the bus sector. The report states:

> In conclusion, the bus services are the one's that will help as far as transportation of students is concern. Looking at the fact that they are subsidised by the government, the buses are always in good condition and are always on time...

Earlier in the report the taxi association representative mentioned the fact that their taxis are not on time due to the fact that they need to wait for the taxi to fill up first and as well as the fact that the roads are generally in 'n bad condition.

5.3 What are the students' views on the use of familiar contexts in the application of mathematics?

The results that are presented below were obtained through the use of Carlson's (1997) modified questionnaire as explained in the previous chapter.

In this question, the pre-service teachers' views are analysed to find the extent to which they think that familiar contexts help to solve mathematics problems.

5.3.1. In solving mathematics problems, familiar contexts, e.g. student fees, transport, catering issues help me to get involved in a meaningful way

	Frequency	Valid Percentage
Agree	20	66,7
Neutral	8	26,7
Disagree	2	6,7
Total	30	100,0

Table 5.17: Familiar contexts in getting involved in a meaningful way

From the table 5.17 it is obvious that the majority of participants feel that familiar contexts should be used in solving mathematics problems in order to get involved in a meaningful way.

5.3.2. In solving mathematics problems, familiar contexts, e.g. student fees, transport, catering issues help me to devise or develop good mathematical contexts that expect of learners to organize given information and identify skills that are needed

	Frequency	Valid Percentage
Agree	22	73,3

	Frequency	Valid Percentage
Neutral	6	20,0
Disagree	2	6,7
Total	30	100,0

Table 5.18: Familiar contexts and devising of good mathematical contexts

Table 5.18 reveals that, to a large extent, the participants agree that familiar contexts are very helpful in organizing given information and identifying needed skills.

5.3.3. When I experience a difficulty while studying mathematics I try to relate the problem to real life situations and try again.

	Frequency	Valid Percentage
Agree	22	73,3
Neutral	8	26,7
Total	30	100,0

Table 5.19: Studying mathematics and relating to real life situations

Table 5.19 indicates that in relating the content under study to real life situations is helpful as a studying strategy in order to understand.

5.3.4. When I experience a difficulty while studying mathematics I immediately give up.

	Frequency	Valid Percentage
Agree	1	3,3
Neutral	2	6,7
Disagree	27	90,0
Total	30	100,0

Table 5.20: Giving up when experiencing difficulty in studying mathematics

The table 5.20 shows that nearly all participants do not immediately give up when they experience difficulties while studying mathematics.

5.3.5. When I experience a difficulty while studying mathematics I try hard to figure it out on my own.

	Frequency	Valid Percentage
Agree	21	70,0
Neutral	4	13,3
Disagree	5	16,7
Total	30	100,0

Table 5.21: Resolution of a difficulty in studying mathematics by figuring it out

When experiencing difficulties in mathematics, the majority (70%) of participants try to understand the problem on their own as opposed to 16,7% who disagrees.

5.3.6. In solving mathematics problems, familiar contexts, e.g. student fees, transport, catering issues help me to have a positive attitude towards the teaching and learning of mathematics.

	Frequency	Valid Percentage
Agree	23	79,3
Neutral	4	13,8
Disagree	2	6,9
Missing	1	
Total	30	100,0

Table 5.22: Familiar contexts and positive attitude towards teaching and learning

Table 5.22 shows that the use of familiar contexts helps most of the participants to develop a positive attitude towards teaching and learning of mathematics.

5.3.7. In solving mathematics problems, familiar contexts, e.g. student fees, transport, catering issues help me to put calculations into context

	Frequency	Valid Percentage
Agree	22	73,3
Neutral	5	16,7
Disagree	3	10,0
Total	30	100,0

Table 5.23: Familiar contexts and calculations into context

From table 5.23 it is clear that 73,3% of the participants think that the use of familiar contexts helps to put calculations into context. This means that the use of familiar contexts helps them to interpret the solutions in a meaningful way.

5.4 SUMMARY OF STUDENTS' VIEWS ON THE USE OF FAMILIAR CONTEXTS IN FINDING SOLUTIONS TO MATHEMATICS PROBLEMS

In sum, the response to question three indicates that the use of familiar contexts in the application of mathematics plays an important and significant role. The participants' views show that familiar contexts in particular contribute positively to the following: in studying, in finding solutions to the problem, in attitudinal reactions to learning and teaching and in interpreting the solution in terms of real life situations. The comparative

representation of the views on the role of familiar contexts in finding solutions to mathematics problems is shown in figure 5.12.
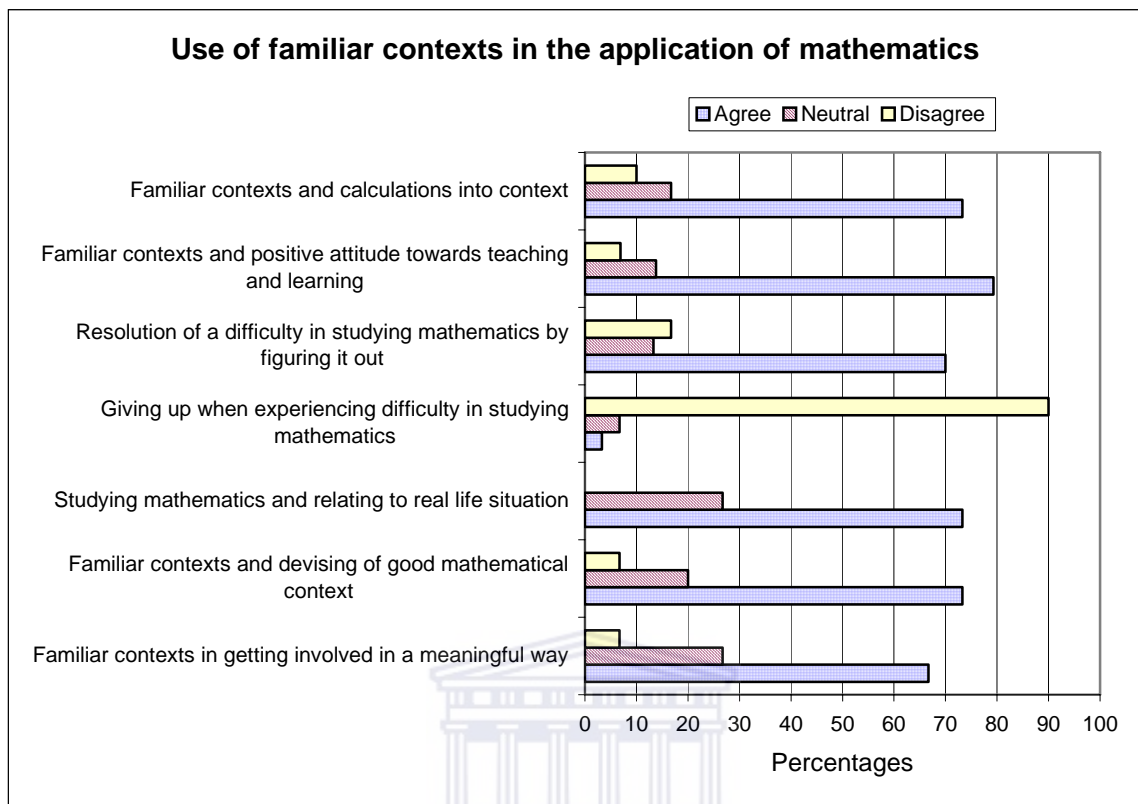
Figure 5.12: Views on the use of familiar contexts

5.5   CONCLUSION

In this chapter, the findings to the three key research questions that are central to the study were presented. The participants' views on the nature of mathematics are very similar to those of experts. This means that most of their views relate well to those that are in some of the literature and are dominating the current debates. On the other hand, there are mixed and different views on other issues such as the validation of mathematical knowledge and the description of mathematical methods. On the validation of mathematical knowledge it is regarded as being validated by logical proofs rather than by correspondence to the real world. The participants' views do not support the view that

mathematical methods are situation specific, they think that these methods are systematic and generic.

The participants used various strategies to find solutions to the four themes chosen from familiar contexts. From the etic perspective, four different strategies emerged. These are as knowledge driven, students' contextual logical reasoning, imaginary driven and example driven strategies. In some themes, one strategy was used and in some themes strategies were combined. It was observed that the knowledge and imaginary driven strategies were frequently used together and the example and the contextual logical reasoning strategies were also frequently used together or that the one easily led to the other.

On the question of the role of familiar contexts in the application of mathematics, the response indicated that the link brings a positive influence to the process of finding solutions to mathematical problems.

From the data, there was no statistical proof to show how students' views on the nature of mathematics influenced the way they approached the mathematical themes. However, the frequencies did show that there was a link. From these frequencies it could then be argued that it is a necessary condition but that is cannot elaborate on the sufficient conditions.

The next chapter will discuss the strategies, the students' views and how the strategies fit into the theoretical framework guiding the study.

CHAPTER 6

DISCUSSION

6.1   Introduction

This chapter discusses the findings of the previous chapter. The chapter discusses the students' strategies in finding solutions to problems using familiar social contexts, students' views on the nature of mathematics, and students' views on the use of familiar social contexts in the application of mathematics.

In presenting this chapter, the approach is largely shadowing Secada and Brendefur's (2000) strategy-driven approach in examining students' understanding of a function. Secada and Brendefur's (2000) strategy-driven approach is bottom up and explores students' developmental trajectories for mathematical reasoning. Furthermore, in Secada and Brendefur's (2000) approach categories of students' reasoning and strategies emerge from the work the students have presented. This approach is particularly helpful for my study as it is descriptive and not evaluative or investigative of students' achievements. In presenting the discussion, the following key concepts are used interchangeably: task, activity and problem, and group, participants and students.

The chapter has one major goal and this goal is to help teacher educators to understand strategies (mathematical behaviour) that are used by pre-service mathematics teachers in arriving at solutions for problems from real life contexts. The Cognitive Guided Instruction (CGI) for professional development programmes drives the goal of this chapter – in particular the fact that individuals and groups of learners will solve problems in a variety of ways. As described in chapter 4, the CGI programme also focuses on children's thinking about mathematics and the use of such understandings to guide subsequent instruction (Towers, 2003). There is also an argument that there is

increasing evidence that knowledge of children's thinking has a powerful influence on teachers as they consider instructional change (Fennema et al., 1996). Secada and Brendefur (2000) further say that the CGI is based on extensive research on how people solve arithmetic word problems, and explains in simple terms the logical, semantic and syntactic structures of arithmetic word problems. Use of the CGI in this discussion is also in line with the theoretical framework (CHAT) for the study as argued in chapter 3. The linking reference between the CGI programme and the theoretical framework in this instance is the sense of developmental orientation and situative perspectives in mathematics education research, which focuses on developing identities of students as mathematical knowers and learners (Siegler, 2003). According to Siegler (2003), this perspective involves describing processes such as problem solving, communicating, reasoning, and understanding mathematical connections as aspects of social practices.

In demonstrating how the above goal is realised, reference is made to the main research question that lies at the heart of this study. That is, what are the students' strategies in arriving at solutions for problems in real life situations? The findings of the two sub research questions are not discussed in this chapter as these findings point to the implications and recommendations on mathematical instruction. The issues on the implications and recommendations are dealt with in the next chapter.

The approach to discuss the groups' strategies is a bottom up approach. In discussing the groups' / students' solutions, categories that emerged from the data are discussed and potential instructional opportunities arising from this study in terms of the students' solutions are presented. These opportunities are linked to the students' views on the nature of mathematics and the role of rich contexts in applications of mathematics and modelling contexts. The challenges will be referred to in the context of a classroom

environment and on out-of-classroom environment. Furthermore, the strategies are discussed within the theoretical framework in which three CHAT's theoretical perspectives are considered to understand and locate the students' work. The section will be concluded by arguing that the resolution of mathematical themes have led to boundary crossing.

6.2   To help teacher educators to understand strategies that pre-service mathematics teachers use in arriving at solutions for problems from real life contexts

The larger part of the analysis in this study was based on the analysis of the solutions offered by groups. In reading and studying the groups' solutions I had to consciously consider the theoretical framework (CHAT) and the methodological suggestions by Schwandt (2000) as described in the previous chapter. I had to consider how to, in a complementary manner, use the approaches followed by interpretativism, hermeneutics and social constructionism in the analysis of data, in order to get as much meaning out of the data as I can.   In discussing these strategies by the groups, the diagrammatic representation in figure 6.2 is used in order to describe the process followed by the groups and the way in which the strategies emerged. It is the representation of what happened in the process of finding solutions to the themes using the Moerlands's (2002) model. This representation is an adapted form of Moerlands's (2002) model that was intended to show process and understanding of finding mathematical solutions in a Realistic Mathematics Education (RME) context. Moerlands's (2002) model is shown in figure 6.1.
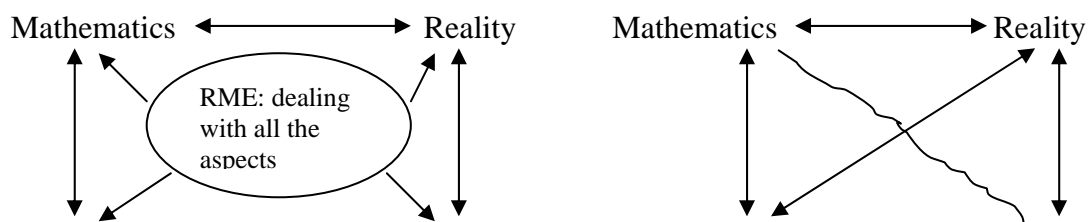
Learning ⟷ Infrastructure   Learning ⟷ Infrastructure
Child   Student

Figure 6.1: Moerlands's (2002) model – learning in RME Figure 6.2: Modification of Moerlands' (2002) model

Description of the four basic elements in mathematics education.

Reality – where mathematics originates and is used

Mathematics – field of study with potential learning

Infrastructure – resources in teaching (what and how)

Learning student - the input and prior experience

Table 6.1: Description of basic elements in mathematics education in Moerlands' (2002) model

By and large, figure 6.2 is the subset of the Moerlands's (2002) model. In short, I'm saying that Moerlands's (2002) model is complete in the context of realistic mathematics. Figure 6.2 shows what happened in the study in relation to this model and highlights some paths followed by students following what Greer (1997), refers to as 'didactical contract' as described in chapter 2.

My emphasis in figure 6.2 is to show that in some cases one gets the relationship between a learner and mathematics. So this is out of context as there are no realistic considerations. Furthermore, what learners are confronted with are not problems or mathematics problems, but mathematical exercises that require manipulation. The cursive line between infrastructure and reality is to indicate non-consideration of reality in choosing tools to be used to find the solution to the problem. This consideration (realistic or nonrealistic) contributes to the way solutions are structured and the model formulations of the problems. From the analysis of the solutions, it was clear that problem space (steps/moves taken to find a solution) varied from one group to another. The evidence of this is the detailed and less detailed solutions that were provided. There

are number of possible reasons for the problem space. Amongst others, the closer

examination of the implications of the Schoenfeld's (1989) model and Hayes' (1981)

search model may offer a broader understanding of these possible reasons. For instance,

using the Schoenfeld's (1989) model to analyse the student's solution, one is able to

assess the access that the student has to the mathematical facts and procedures, and

general problem-solving techniques. Similarly, Hayes's (1981) search model may show

which search models were used and as argued by Hayes (1981), some search models are

better than others and, therefore, the space problem would be indicative of the type of

search methods that the problem solvers have been able to identify.

Before discussing the students' strategies, I briefly discuss the students' space problem in

order to show what Moerlands (2002) calls 'the capacity of a floating iceberg', that is, the

observation of the significance and the power of the process that leads to an end of the

solution.

In general the analysis of the texts indicates that the models were restricted to what

Edwards and Hamson (2001) refer to as deterministic models and stochastic models.

Edwards and Hamson describe deterministic models as those models in which outcomes

are a direct consequence of the initial conditions of the problem. This directness is not

affected by any arbitrary external factors or particular, random factors. On the other

hand, the term "stochastic model" is reserved for those situations where a random effect

plays a pivitol role in the problem investigation. Examples given by Edwards and Hamson

(2001) centre on models involving queues, random service times at supermarkets, and

random arrivals at bus stops. In short, Edwards and Hamson (2001) refer to stochastic

models as kinds of 'next-event' models. Looking at the groups' solutions, a pattern

emerged that indicated a tendency to fix the initial conditions in order to arrive at the

solution to the problem. The indications are there to illustrate an application of a

deterministic model type. Examples are found in some solutions in themes on catering,

students' fees and on tuition to day and evening students. However, there are overlaps.

Edwards and Hamson (2001) also say that care should be taken as there are many

situations within the same model, and some features are random, whereas others are

deterministic. The use of deterministic models featured prominently in situations where

students used logical reasoning and this logical reasoning was facilitated by their

familiarity with the problem. The following extracts are typical examples from the

students' solutions wherein the deterministic model was used:

a)   …..to curb the increases (*debt*) is for the management to admit equal number of students who are able and not able to pay. But the amounts should be proportional to that stated above, e.g. if they admit 10 students for next year….

b)   The key here is to get students to use these services and get them to want to leave the campus at the same time (17h30 and 19h30). One other thing that we should enforce is that students should buy monthly tickets. If we tell the bus service that they are going to make 2 140.00(day) + 1 558(night) = 3 728.00 per day, that is approximately 111 840.00 per month and for approximately 10 months = 1 118 400.00, there is no way they will refuse.

In extract (a), the group is of the opinion that in order to prevent increasing debt;

management must restrict the number of students to be admitted to be proportional to

those who are paying. Their argument is that the number of students who are able to / or

who do pay, should be more than those who are not able to pay as required. So the group

fixes the number of students and the solution is derived from that fixed number. Similarly

in (b), the group indicated that by fixing the departure times and expecting of students to

use the bus it will result in the bus company agreeing to help, as the offer will be too good

to refuse. The other examples are given later in this chapter. Edwards and Hamson

(2001) was helpful in classifying students' strategies in terms of the types of models to

which they belong.

In terms of the process of formulating models, Hayes (1981) provides a better description and he identified, what he refers to, as search heuristics and search algorithms. He describes search heuristics as procedures that are useful but not guaranteed to produce the solution to the problem (Hayes, 1981). Examples of search heuristics are: hill climbing (looking one step ahead), means-ends analysis (takes a sequence of steps to reach a goal and each step reduces the distance to the goal) and fractionated methods (dividing the problem into auxiliary problems and sub-goals). There were instances where a group would suggest a formula (stating one step ahead) to solve a problem but not proceed further than that. Also, there were instances where several sequential steps were used to solve the problem and also dividing the problem into parts. For example sub-ranges in the 'student fees problem' and different routes in the 'transport problem'. The examples are found in figure 5.8 and figure 5.10.

On the other hand Hayes (1981) says that problem solvers use procedures that, if applied correctly, may lead to 'right solutions'. These procedures are referred to as search algorithms. These are knowledge based search methods where problem solvers may use information already known to them (Hayes, 1981). In the study, some groups identified formula used in Compound Interest and Linear Programming as a topic to be used in the theme on provision of tuition to day and evening students. These are examples of search algorithms.

In all these search methods, there were some 'breakthroughs' and 'dead ends'. These search methods led to the students' strategies that are discussed in the next sections. The approach is to discuss these strategies and use one of the solutions as a reference point in these discussions. The focus is on the research question**:** What are the students' strategies in arriving at solutions for problems in real life situations?

I have chosen theme 1 (on student s' fees) and the solutions of two groups. The selected solutions contain a variety of search methods and on the basis of this diversified approach they are arguably the most appropriate ones to use in this discussion.

The categories of mathematical strategies from chapter 5 indicate that students used different strategies and different assumptions, which result in different solutions. For the purposes the discussions, the strategies are restated below.

    a.   the imaginary driven strategy

    b.   the example driven strategy

    c.   the knowledge driven strategies

    d.   students' contextual logical reasoning

The categories demonstrate different ways used by students in these open-ended problems. The solutions below show how students approach an open-ended problem from different perspectives. As mentioned earlier, there is no intention to generalise with these solutions. The intention is to show reasoning within these problems, communication of ideas, and problem-solving strategies of students in the context of the study.

6.3.1   The example -driven strategy

The first solution (figure 6.1) is indicative of the category of the example driven strategy ('using living data').

The data provided to the groups are reflected in table 6.2 and group one regrouped the data by changing the class intervals as reflected in table 6.3.

The group subdivided the students who are in debt into three categories, namely: unregistered and unemployed, and unregistered and employed, and registered. In terms of the last category, the suggestions include awarding bursaries to deserving students and

also employing students on a part time basis. The next two examples focus on the first two categories.

From the given information (table 6.2), the group used table 6.3 to categorise students' debts into range 1, range 2 and range 3. In this discussion, range 3 is used to illustrate the group's strategy. The class intervals for the amounts as reflected in table 6.3 have further been changed after adding 10% interest and the group refers to them as 'actual range' as shown in fig. 6.1.

Highest amount in student debt:R53342,83	
Number of students	Amount range
3	40 000-xxxxxx
5	35 000-39 999
15	30 000- 34 999
29	25 000- 29 999
81	20 000- 24 999
240	15 000- 19 999
379	10 000-14 999
1120	05 000- 09 999
2112	01 000- 04 999
928	1 to 999
727	Credits

Table 6.2: Number of students and debt

Highest amount in student debt: R53342,83	
Number of students	Amount range
23	30 000 – 53 342
729	10 000- 29 999
4160	01 – 09 999
727	Credits

Table 6.3: Group 1's table on students' debt

In figure 6.1 below the group makes use of the sub-goal approach to find the solution to debt reduction. In this approach range 3 has been divided into a sequence of steps as sub-ranges 1 and 2 in order to do what Hayes (1981) refers to as the reduction of the distance between each step and the goal (final solution). The goal in range 3 is to find payment mechanism of the amounts ranging from R1, 10 to R10 998,90

G31 (c) RANGE 3
R9 999,00 * 10% = R999,90
1.0* 10% = 0.10
Therefore the actual range is R1,10 to R10 998,90
Sub-range 1
R3 000,00 to R10 998,90 should pay in 2 years, instalments ranging between R125, 00 and R458, 29 per month….
Sub-range 2
R1, 10 to R2 999,99 should pay within 1 year, instalments ranging between R1, 10 and R250, 00 per month

---

Figure 6.3: An illustration of the example driven strategy

In sub-range 1 in figure 6.3, the payment period is 24 months whereas in sub range 2 the payment period is 12 months with monthly instalments ranging from R125.00 to R458.29 and R1.10 to R250.00 respectively.

6.3.2 The imaginary driven strategy

Figure 6.4 and 6.5 show a step-by-step use of categorisation of different salary scales.

This hypothetical situation (actual data were not used) systematically looks at students

who are employed but unregistered.

G31 (d) UNREGISTERED AND EMPLOYED STUDENTS
In this section we concentrate on the amount the student earns. We also consider whether the student will be able to live after paying the instalment.
　　　　SALARY SCALE 1: R10 000.00 - R34 999.00
Will pay 60% of the salary per annum
R10 000,00 * 60% = R6 000,00
& R34 999,00 * 60% = R20 999,40

Therefore, Paying Range will be 6 000.00 to 20 999.40 p.a.

Figure 6.4: Illustration 1 for the imaginary driven strategy

In figure 6.4 the group shows that minimum to be paid by the students in the earning bracket of R10 000,00 to R34 999,00 is R6 000,00 per annum, which is R500,00 per month and the maximum is R20 999,40 per annum, which is R1 749,95 per month. From the group's argument, students in the salary bracket R10 000,00 to R34 999,00 will be able to pay this amount and be "able to make a living after having paid the instalment".

G31 (e) SALARY SCALE: R35 000,00 - R59 999,00
Will pay 65% of the salary per annum
R35 000,00 * 65% = R22 750,00
& R59 999,00 * 65% = R38 999,35
Therefore, paying range will be R22 750,00 – R38 999,35 per annum
G31 (f) SALARY SCALE: R60 000,00 AND HIGHER
Will pay 70% of the salary per annum

Figure 6.5: Illustration 2 for imaginary driven strategy

In figure 6.5 and figure 6.6, the same argument as in figure 6.3 is used. The only

difference is that the students in this earning bracket (R35 000,00 to R59 999,00 per

annum) pay 65% of their salary per month. The remark in figure 6.4 explains why the

percentages differ. This is an example of what Donald (1999) refers to as emotional or

self-preservation in addressing their motives (why question). This emotionally clouded

drive seems to influence how fair solutions could be found as indicated in the figure 6.6.

Remark: People earning R10 000,00 are expected to pay 60% of their salary per annum; this is to ensure that we don't have double standards. We expected a person who is unemployed to pay R5 334,25 per annum so it would not make sense to have a person who is employed pay anything less than that (R5 334,25) **with this plan the university will recover the money within 10years.**

Figure 6.6: Illustration 3 for the imaginary driven strategy

6.3.3   Knowledge and Students' contextual logical reasoning driven strategy

Figure 6.7 shows that the students have heavily relied on known information and their reasoning and this typifies reliance on contextual knowledge. In this contextual logical reasoning mode, 'living data' has been used as supportive tools to indicate the *problem of increasing students' debt*.

The key variables in figure 6.7 are course fees, residence and meals and total student's fees. The familiar structure of a mathematical function that has emerged typifies what psychologists call Kendler's (1968) formula as explained by Jordaan and Jordaan (1998). Kendler's formula says that if $Y = f(X_1, X_2, X_3, X_4, X_5)$ then Xs cause Y hence Y is a dependent variable and Xs are independent variables. In this extract, student's fees are a function of multi variables or factors such as course fees; fees for meals and residence fees and, therefore, individual student fees are dependent on these multi variables that occur simultaneously. What is implicit in this relationship is that this only applies to those students who stay in the university residences. According to Hilgard (1987) as quoted by Jordaan and Jordaan (1998), the measurement of the function Y could be regarded as an important objective observation of the phenomenon (Jordaan and Jordaan, 1998). In this case the phenomenon being the student fees. The kind of objective observation of the phenomenon is seen in figure 6.7.

G31 (h) **PART B (STUDENT FEES)**
PROBLEM: Increasing Debt (Student fees)
Let X, represent student
Y, represents fees
If X owes 67 000.00 and has to register for another semester on: course + residence + meals
Where course = 4 295.00 (let course be S)
Residence = 2 790.00 (let residence be T)
Meals = 700.00 (let meals be R)
Therefore Y=S+T+R
= 4 295.00+2 790.00+700.00
= 7 785.00
When we add Y to the debt of 6 700.00 (and let debt be Z)
Therefore Y+Z
       7 785.00 + 6 7000.00 = 74 785.00
Total Debt per X=Z (74 785.00)
If X pays 20% of 67 000.00 *(university usually requires down payment of 20% before a student registers for the following semester or year)*
Then 67 000.00-13 400.00 = 53 600.00
X still owes a huge amount
Since Z = 53 600.00+Y
       = 61 385.00
From 67 000.00 we minus Z. ***Therefore, the difference is 5 615.00, which still leaves the student with a huge debt*** (my emphasis)

Figure 6.7: An illustration for knowledge and students' contextual logical reasoning

The group shows that the multi variables: course fess (S), residence fees (T) and meal fees (R) are also impacted on by the debt that a student has from the previous registration (s). The total amount of student debt is a transformation of different functions that results in a new function as the student registers. In general, if one analyses the group's solutions, a step function that follows defines the total student debt per student X:

$$Student\ debt = \begin{cases} Y + Z \text{ if debt is not 0} \\ \\ Y \text{ if debt} = 0 \\ Z \text{ if } Y = 0 \text{ i.e. if student pays 100\% of Y} \end{cases}$$

If debt (Z) is not zero, the student's fees (Y) changes as it is added to the student debt as shown in figure 6.7. This then results in increasing student debt as identified by the group in the heading of the solution. On the basis of the calculations, the group argues that the debt is "huge" and as a result external assistance is recommended as explained in figure 6.8. In figure 6.8 the group demonstrates how the 'huge debt' of R67 000,00 may be paid within two years.

Figure 6.8: Students' recommendation to finance section

As mentioned in figure 6.7, the 20% referred to in the above recommendation refers to the local university arrangement for students to register if they have an outstanding balance. The group shows that the debt could be reduced if the students are granted the maximum allocation from National Students' Financial Aid Scheme (NSFAS).

On the other hand, the solution below shows a different approach in arriving at the solution for theme 1. The approach applied in this case is an example of the knowledge driven strategy.

> 1. *Increasing debt on student fees (real problem)*
> 2. $A = P (1-r/100)^{n}$
> 3. $A = ?$
> 4. P =16000R = 5%
> 5. *N = 12*
>
> 6. *The solution above will decrease so that the student can afford to pay his/her fees.*
> 7. *We can use the compound interest formula to identify the money that students will pay.*
> 8. *The management of the institution must disagree to allow student to graduate while he/she still owes the university.*
> 9. *The student must not be re-admitted the following year if he/she owes the past year. They must disallow the student to be a resident student if they agree to admit him/her and if he was a resident student previous year.*NB: The principal amount of 16 000 refers to the allocation from NSFAS as a student loan

Figure 6.9: An illustration of the knowledge driven strategy

Figure 6.9 indicates the challenges concerning connectedness between reality and mathematics as a field as represented in Moerlands's (2002) model in figure 6.2. This lack of connectedness is further compounded by inaccuracies in some instances. For instance, in figure 6.7, the formula that is intended to "identify the money that the student will pay" is incorrect, the operational sign should have been a positive sign (+) instead of a negative sign (-) as the interest is added to the student debt (as they are penalised for not paying on time). The purpose of finding decreasing debt gives an impression that the sign should be a negative sign (-).

The solutions in the example driven strategy are divided into three categories, namely, the unregistered and unemployed, the unregistered and employed, and registered students. In case of the latter category, whether they are employed or not, the conditions set for employed and unemployed will apply as indicated in figure 6.4. The group has indicated step by step how the debt could be reduced for unregistered unemployed students within 10 years (for larger amounts), 6 years and 2 years. In advancing the arguments, the group has used the statistics given by the local institution. On the other hand, the other group that used the knowledge driven strategy (in this case they used their knowledge in

simple interest and compound interest), gave a formula and numbers and argued that the compound interest formula could be used to "identify the money the students should pay" and from the other reasons are advanced. It is observed that the formula was not applied beyond this statement in arriving at the solution.

In terms of solutions given from in figure 6.3 through 6.9, there is a strong argument to support Lave's (1988) view that familiar contexts serve as calculative devices for problem solving. Through the students' knowledge or understanding of the context, they used the data in a meaningful way to present their argument for reducing student debt over a maximum period of 10 years. This specific example shows that some of the mathematical behaviour that students had displayed in their approach could be explained in terms of the existing literature. This point is further elaborated in the paragraph 6.4 below.

6.4   LITERATURE REVIEWED AND STRATEGIES

In the previous two chapters, it was indicated that the approach to categorise the students' working strategies was a bottom-up approach. This bottom-up approach meant that the students' working strategies were not categorised according to a certain predetermined criterion. However, these students' strategies (imaginary driven strategy, example driven strategy, knowledge driven strategy and students' contextual logical reasoning) are in some ways largely consistent with some of the categories or approaches used by some authors of the reviewed literature related to this study. The previous section 6.3 (6.3.1 to 6.3.3) shows the theoretical explanation of the strategies used by the students. In particular, the work by Hayes (1981), Schoenfeld (1989) and De Lange (1999). De Lange's (1999) work provides broad categories that embrace contexts in which students' work may fit. These categories are: real context (drawn from real life experiences), virtual context (drawn from idealized contexts, not from physical or social

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or practical contexts), and artificial context (these include fairy tales, non-existent

objects, or constructs). These three types of contexts may help in understanding examples,

suppositions or justifications given by students in this study. Similar categories to those

by De Lange (1999) are found in Verschafel et al., (1994; 1997) in which they refer to

realistic considerations and non-realistic considerations.

In terms of this study, imaginary driven strategies fall within the virtual context as

described by De Lange (1999). On the other hand, the example driven and students'

contextual logical reasoning strategy will fit in well with De Lange's (1999) real context.

The real context, example driven and students' contextual logical reasoning are

characterised by the use of real life experiences, social rules/social practices and specific

real (e.g. specific price of bread) data.

SCHOENFELD (1989) PROVIDES A FRAMEWORK FOR ANALYSING
DATA IN TERMS OF STUDENTS' MATHEMATICAL BELIEFS AND
BEHAVIOUR. THE FRAMEWORK, AS DISCUSSED IN CHAPTER 4,
HAS FOUR COMPONENTS. COMPARING THE STUDENTS'
STRATEGIES IN THIS STUDY TO SCHOENFELD'S FRAMEWORKS
SHOWS THAT THE COMPONENT OF 'RESOURCES' IS RELATED TO
KNOWLEDGE DRIVEN STRATEGIES AS THE STUDENTS TRY TO
USE MATHEMATICAL FACTS AND RULES IN BOTH CASES, THAT
IS, IN THE CASE OF THIS STUDY AND IN THE CASE OF THE
SCHOENFELD (1989) STUDY.

Finally, the students' approaches to divide the solutions into sub-categories appropriately

match Hayes' (1981) search methods for solutions to the problems very well. For

instance, Hayes's search method on fractionation (dividing problems into auxiliary and

sub goals) explains why students tended to divided students' fees into sub-categories such as those owing money between R1 and R9 999,00, between R10 000,00 and R29 999,00 etc. and dividing students into social strata, e.g. unemployed students and employed students. The social strata as presented as much as they are more of classification of students, the search method here is to sub goals for each stratum.

In the next section, I present discussions on how the strategies related to the theoretical framework.

6.5 THEORETICAL FRAMEWORK AND STRATEGIES

In relating the four main strategies to the theoretical framework, two perspectives as presented by Engeström (1997) are considered. These are Davydov's theory of moving from abstraction to the concrete and Engeström's theory of expansive learning. These perspectives have been drawn from Engeström's (1991) work that focused on the strategy toward overcoming the encapsulation of school learning. In this study, I have selected two of the three perspectives used in Engeström's (1997) study. Whilst Engeström (1997) states that the perspectives are not supposed to be seen as necessarily linear in form, in this study, because of the assumptions and consequently the design, the linearity has become evident and Davydov's theory is at the entry point and Engeström's (1997) theory of expansive learning is at the exit point.

Engeström (1997) describes these theories in the context of using school learning to understand the world as follows:

> *Davydov's theory suggests that the school learning may be used to understand the world by teaching students theoretical and dialectical thinking, embodied in "kernel concepts" of the given curricular subject, seen as powerful cognitive tools that transcend the boundaries of school learning (Engeström, 1997, p. 243).*

Furthermore Engeström argues that the theory of expansive learning is the one that

suggests that the object of learning should be radically widened to include the context of criticism, the context of discovery, and the context of application of the given contents (Engeström, 1997, p. 243).

The approach is to reflect on the traditional approach in teaching of mathematics and present an alternative approach to the teaching of mathematics. The argument is put forward in terms of the degree to which each of the above theories are applicable in the way solutions to the mathematical themes were found by the groups. While these theories / perspectives are presented separately, the main purpose is to show how they complement each other (one is presented as an extension of the other) in demonstrating how to break the "encapsulation" of institutional based learning (such as teacher preparation programmes) and link these to communities outside teacher education institutions.

Before presenting the discussions that relate the strategies to the two perspectives, I first present a traditional scenario where institutional learning is separated from the other 'daily' experiences.

6.5.1   Traditional learning: separation of institutional learning from the rest of the experience

There is a sense that traditional intuitional based learning is characteristic of the form in which learning models are complete, static and not reconstructed for the purposes of being tested by students. The second characteristic is that the textbook is the object of learning. This is why, in most cases, the textbooks validate students' knowledge, and reference to the real life situations is not at the heart of knowledge validation. Furthermore, the application of knowledge in real life situations or far transfer is very limited and Volmink (1991) summarises it well when he says that the application only entails the application of a principle or a concept. In some instances, as reported in the pilot study, some students blindly apply these principles and concepts and end up solving

unsolvable problems. The example in question is the question put to high school learners and pre-service teachers during the pilot study about determining the age of a shepherd based on the number of sheep and dogs. The categories of responses that emerged from the pilot as briefly presented in chapter 4, clearly show the situations where calculations or steps or arguments are not checked against real life situations and the object of learning is not the societal productive practice or social life, as these responses contradict the fact in real life there is no relationship between the age of the shepherd and the number of sheep and dogs. In figure 6.10 I present a traditional model adapted from Engeström (1997) that depicts traditional learning.

In the figure 6.10, the teaching of word problems is textbook bound and in most cases the textbook only presents the closed ended problems. The students regard the textbook as validation of their answers. As explained earlier, the textbook takes becomes the object instead of a tool or an instrument for learning.

Figure 6.10: Traditional model for the learning of word problems

The outcome is whether students fail or pass the lecturers' questions in the test or examinations.

In the next sections, I discuss how I think traditional learning was turned around.

6.5.2 Davydov's (1988) theory

One of the assumptions made in approaching this study was that students who took part in this project brought along mathematical knowledge that is almost equivalent to the mathematics at first year university level. The content covered in the first year level of their programme is attached as Addendum 7. It was further assumed that the students would be able to access this knowledge in finding the solutions to the problems in a variety of ways. That is, they would not use similar methods. The application of mathematical knowledge will be evidenced by the mathematical power that would entail different identifiable relationships, constructing relationships from the known subject matter. Furthermore, through the deductive process, they would ultimately arrive at the solutions to the given problems.

In the process of finding solutions to the problems, the groups' utilisation of search algorithms (e.g. statistics, linear programming) was evident in some steps in their solutions. The extent to which mathematical knowledge used or accessed, differed from one group to another. The demands of the theme also contributed to the choice. In the process of utilising the known information two scenarios emerged. Some groups were able to arrive at and explain their solutions, some groups used the known information to construct hypothetical examples (suppositions) in order to arrive at the solutions, but some solutions never progressed further than suppositions. Following Hayes' (1981) argument, the solutions in this case were not presented. These two ways of utilising search algorithms led to knowledge categorized as knowledge and imaginary driven strategies. Figure 6.11, as adapted from Engeström (1997) is used to argue that, to some extent, Davydov' (1988) theory of ascending from the abstract to the concrete is applicable in the groups' process of finding solutions to the problems.

Instruments:

Definitions, theorems, theory etc

Subject: The
Groups'
explanations

Object: The discovery →
theoretical knowledge theoretical

Outcome

Rules
Code of behaviour

Community
Lecture hall

Division of labour
Lecturers lecture and control, students study

Figure 6.11: Davydovian theory of ascending from the abstract to the concrete
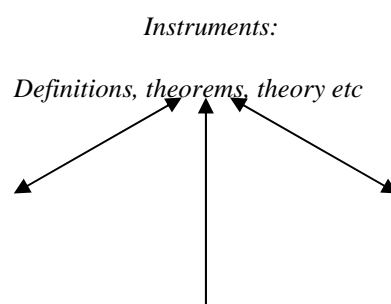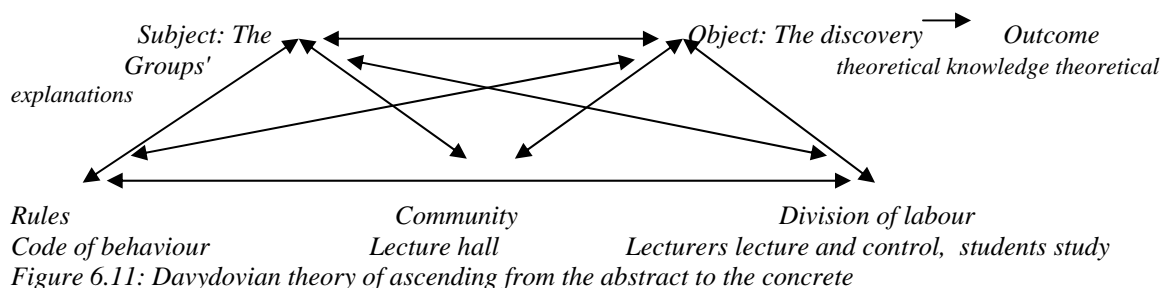
Figure 6.11 shows the extent to which Davydov's (1988) theory, as presented in Engeström's (1997) research works. Although an attempt was made, the last three learning actions were not successfully concluded. That is, the activities/themes could not be resolved by a general mode, there was a lack of monitoring of preceding actions and evaluation of the assimilation of the general mode. What is different between figure 6.10 and figure 6.11 is the object of the learning activity. The discovery and application is the object of learning in figure 6.11 (Davydovian model) as compared to the mathematics textbook as the object of learning in figure 6.10.

The context of discovery as the object of learning gets students to interact with the discoverers of the past – an action through which the students may be empowered (Engeström, 1997). In the context of this study, students were allowed and encouraged to refer to some of the work or theories they had studied. During the planning stage it was emphasised that they could use any theory or formula. The extent to which this was done is not known, but the student had opportunity to look back and to scrutinise the theories/discoveries of the past in order to find the appropriate relationships within and between mathematical topics. This was the kind of interaction expected. The use of the formula involving compound interest and the use of linear programming served as an indication that discoveries of the past were scrutinised.

Before presenting the next perspective, I briefly present some influence of social rules on mathematical knowledge. The point here is that, whilst the students may have

mathematical power, identifying relationships and constructing relations, they do that within a particular (social) context. This is demonstrated in the following paragraphs.

6.5.3   Analysis of the solutions using social rules

The analysis of this particular angle or approach, showed levels of interest and motivation from students. It is here that they incorporated their experiences from their communities. The social realities were mathematised and understanding was linked to a context. The use of indexical terms such as "4+1" and "dead mile" is evidence of understanding that is socially constructed. This largely led to the students' contextual reasoning driven strategy and to some extent, to the example driven strategy. For instance, in calculating the transport needs for 262 students, one group gave the following steps:"…262/15 = 17 + (4+1) taxis". According to their explanation, one taxi in their area carries 15 passengers. Dividing 262 by 15 gives 17 fifteen seater taxis and one small five seater taxi.

In another context, 262/15 may result in a different answer. In the students' context, the answer is acceptable in terms of the existing and common mode of transport. Their familiarity with the context contributed to the use of the students' contextual reasoning strategy. That is, students' reasoning or justification was based on the social practices and they were using their background knowledge to assist and drive their solution-seeking procedures. Mathematical justifications to find solutions to a problem or to provide an explanation to a problem did not feature as much as the usage of background knowledge from social practices.   It was also noted that students prefer the use of '4+1' as it is used in the communities where they reside rather than 4:1 or 1:4. From discussions with them, it was clear that in '4+1', the driver is in charge and responsible for 4 passengers. In '1:4 or 4:1' they said that it only shows the relationship between 1 and 4. The group was asked to compare the '4 +1' teacher/learner ratio. Some felt that

teacher/learner ratio of 1:32 shows that one teacher is responsible for 32 learners and some felt it could be the same as 32+1. There is a sense of power attached to the taxi driver in a '4+1' scenario and not in the teacher/learner ratio.

In this contextual reasoning strategy, the solutions are obtained by largely fixing initial conditions or by enforcing some sanctions. For instance, in order to reduce student debt, some solutions indicated that management should start by not admitting students who can't pay the required fees. This exhibits the use the stochastic model as put by Edwards and Hamson (2001).

*From the above discussions, it is clear that the students' contexts or backgrounds played an important role in their solutions. This type of using context featured prominently in the students' contextual logical reasoning strategy and it was clear that this background stimulated interest in working out the solutions. This approach is in contrast to Skovsmose's (1996) argument that students' background should not be used as contexts. . In the context of this study, the background played a major role, as indicated in table 5.15 of the frequency of strategies per theme. What may be acceptable is the fact that Skovsmose (1996, p. 413) says, "students interest cannot be described in terms of the background **only**" (my emphasis). Otherwise it is clear that in some instances background cannot be downplayed, as Skovsmose (1996) would prefer.*

6.5.4   *Engeström's (1997) theory*

As stated in chapter 4, steps followed when finding solution to the problem did not exactly match the steps that Engeström (1991) suggests in terms of the epistemic actions in an expansive cycle. For instance, the point was made about the differences between the process that was followed in this study and the planning and the implementation stages as advocated by Engeström (1997). In this study, problems were designed based on students' 'real' experiences and then the practitioners considered the solutions.

Furthermore, during the planning stage, the interaction with communities of practice was intended to get explanations and actual data (living data) that may be helpful in finding solutions to the problems.

Also, the implementation stage in this study was intended to provide a platform to students to present the solutions in the form of recommendations to these communities of practice and get the reactions from these communities. It was seen as using the real world to validate their findings and also using the real world as an exit point.

It is this (interaction with the community) implemention stage that widened the object of learning. The key factors that characterised the widening of the object of learning were the exploration of possibilities of transforming or 'bringing change' to problems in student debt, the transport section, the catering section and the provision of tuition to both full time students (day students) and to the part time students (evening lectures) and the criticisms of the students' reports by the heads of sections such as finance, transport and university catering section. As was demonstrated earlier during the discussions on students strategies about the use of mathematical content and social practices as tools for finding solutions, later in this section it will further be demonstrated how the exploration of possibilities to bring change reorganises the role of students from traditional providers of pencil-and-paper solutions to social agents of change. The point is then made that the two processes outlined above had in some ways presented an opportunity to students to be engaged in what Engeström (1997) refers to as theory of expansive learning. In the paragraphs that follow, I present attempts that emerged towards realisation of this theory of expansive learning.

One obvious criterion that was not satisfied in terms of the theory of expansive learning as explained earlier, was the critical questioning or evaluation of the problem and the solution at the end or at the exit point of reality. Engeström's (1997) critical question

starts at the entry point. I, therefore, acknowledge that the context of criticism as Engeström (1997) calls it, was limited. As explained earlier, the entry point of the study assumed Davydov (1988) principles in approaching problems from real life situations. However, during the briefing session (planning stage), there were comments to the effect that in the past they (general student body) were not involved in the resolution of problems of that nature (e.g. fees and transport) as the Student Representative Council (SRC) used to take up the issues with the university management. In relation to Engeström's (1997) context of criticism, one can argue that this was an indication of the critical analysis of the practice in terms of these problems. Though none of the students/participants suggested that the former strategy should be maintained, two unintended difficulties were eminent in which some students could have abandoned the study as they thought that the SRC would pursue those matters or some could have remained with reservations as in terms of the institutional operations the problems are not really within their terrain. According to Engeström (1997), Bateson (1972) argues that this stage of opening up wider context of the problems (he calls level III learning) can be dangerous and some fall by the way side. Indeed, following the critical analysis of the problem that limited the questioning was characteristic of what Engeström (1997) calls highlights of contradictions, debates, questioning and powers resisting.

The next level of interaction between groups and communities of practice focused on the section heads' scrutiny of the students' recommendations. An analysis of the students' recommendations indicates that the social awareness issues dominated the recommendations. The object of learning turned out to be a critical analysis of social issues for the purposes of improving those social conditions. Figure 6.4 summarises the Engeström's (1997) modified model of expansive learning.

Using Moll et al.'s (1990) view, one can argue that the students had in a real sense seen transport, debt reduction, catering problems, provision of tuition to day and evening students as educational settings in which a major function was to apply knowledge that enhanced the survival of its dependents. The level of empathy was high. The social awareness displayed by groups appropriately falls in one of Julie's (1998) categories, namely, activism potential. Julie's (1998) second category (utilitarian potential) was not that explicit. The following examples illustrate this point. I have selected just a few examples at random to highlight students' challenging thoughts that take the solutions to the level of continuous debate.

6.6 Activism potential

What is very striking in the students' activism is the shift of focus from students to the institution as shown in the next quotations. The role to be played by students is minimal. On the transport issue, one of the recommendations is that students should buy monthly tickets to "solve problems of not getting taxis to school and not having day's fare". According to the students, there are students who at times do not attend lectures because they do not have sufficient funds for transport and it is much too far to walk from their place of residence to the institution. I followed up on this matter during the presentations at which students confirmed that there are, indeed, students who are unable to attend some lectures because of the transport fare. One student from the group added that, "If a student has no money and had an evening lecture, he or she will ask a friend for accommodation in res" *(res refers to university residence). The recommendation emphasises management's responsibility to negotiate with the transport industry as indicated below:*

> *The students should be grouped according to routes (places) and times (day/evening). And this is for the convenience and the fact that it costs the same amount to go to any of them. For the places with the large number of students buses will be used and for the smallest taxi will be used.* ***And the management should negotiate with transport services***

for a discount in every route that is outlined (my emphasis). Students should be urged to buy monthly tickets and also be urged to leave the campus at almost the same time. And for taxis, students should pay transport fare on hand to the institution on weekly or monthly basis, as this will solve the issue of no taxis and no money for transport (recommendation 3; 2002, see Appendix 2)

Further examples are:

The management of debts starts with the decision to grant debts, which is the full responsibility of the credit manager who will need an effective debts control system. In many cases where there's no credit policy, debts may build up, cash flows decline and the bad debts arise. Therefore the institution should have its credit policy. **And then the university should encourage the high volume of students admitted and credit manager should focus on minimizing the collection period. A certain discount can be given to the students who will pay within the first 15 days** *(my emphasis). The management should analyze the average collection period and its ageing schedule in comparison with other institution's averages, recent trends and the institution's credit terms to see how effectively the credit department is managing debts. (Recommendation 1; 2002, see Appendix 2)*

The above quotation indicates the kinds of actions management must take in order to bring about change regarding students' debt. In the above quotation, the students act as campaigners of 'efficient delivery' of debt collection and consistent management of debt collection across different institutions.

In the following, the focus is on planning and affordability.

If 400 students utilise the D.Hall everyday for three meals a day i.e. breakfast (R6, 50), lunch (R9, 50) and supper (R9, 00), this sums up to R25, 00 per student a day. And this amount or less of it can be used to cater one student a day. **Or if R20.00 is used to cater one student a day then R5.00 is saved and that is a lot of profit on the management's side.** *Nevertheless, the staff must try as hard as they can to work on the bookings made a day before catering is done.* **For the fast food part, there can never be claims of financial constraints because that section is always more expensive that shops outside the campus.** *(Recommendation 2; 2002, see appendix 2, my emphasis)*

From the above three quotations, three critical issues are highlighted. Firstly, the students in their activist approach saw an opportunity to use the application of mathematics as a tool for analysing social strata, e.g. those who cannot afford the taxi fare and unemployed students. Secondly, it afforded them an opportunity to raise equity issues and effectiveness. In recommendation 1 as quoted above, the students emphasise

the importance of comparing the current trends in other institutions and the effectiveness

of debt management in those institutions. Finally, in this activism, the students

emphasized the issues of equity and affordability. For instance, from the last quotation

above (recommendation 2), the students stressed that the fast food section is expensive

and over-priced (not affordable) as compared to fast food establishments off campus.

Figure 6.12 shows Engeström's (1997) model of expansive learning in the context of this

study.

Engeström (1999a) at some stage articulated the need for methodological change that

does not separate the study of socioeconomic structures from the study of individual

behaviour and human agency. He argued that this dualistic framework does not help to

understand today's deep social transformation. To this end he proclaimed:

> *More than ever before, there is a need for an approach that can dialectically*
> *link the individual and the social structure. From its very beginnings, the*
> *cultural-historical theory of activity has been elaborated with this task in mind*
> *(Engeström, 1999a, p.19).*

Instruments

Questioning, critical analysis, etc

Subjects: Groups of students
Taxi Ass. University Teams

reduction,
transport etc

Object
Discovery
& application

Outcome
a) Math knowledge
& debt

b) Collaboration

Student

School text

Rules
Complementary

Community
Networking with communities

Division of labour
Designed in terms of
objects and context

Code of behaviour

lecture hall

Lecturers lecture and
control, students
study

Figure 6.12: Modified Engeström's (1997) model of expansive learning

The application of model of expansive learning to this study, has afforded an opportunity to link pre-service students to the socioeconomic structures in the communities (taxi industry, university finance section, academic division and catering section). The two basic processes of activity theory (CHAT), namely, internalisation and externalisation that operate at every level of human activities were selectively followed to suit the structure of the study. This then implies that in this study the modified expansive learning model as advocated by Engeström (1997, 1999) was implemented. As indicated in chapter 4 in terms of the steps that were followed in finding the solutions (see table 4.2) to the problems, the internalisation process entailed description of what modelling and mathematical modelling mean and explanation of the steps to be taken (with examples) in mathematical modelling. This then involved mastery of key concepts in mathematical modelling, socialisation (division of groups) and making sure that all members of the six groups 'were competent' to start the routine process of finding solutions. On the other

hand externalisation took place when groups explored different mediating tools/thinking tools for the four themes. The externalisation process reached its peak point when students presented their 'model solutions' to heads of sections such as finance and taxi association.

During the expansive learning model, it can be argued that the object of learning had crossed the boundaries of the university environment. The activity systems (four mathematical themes) became the voice of the people in the transport section, catering section, academic and finance section, and had potential lasting patterns of interactions (e.g. dialogue among communities of practices on issues of transport) in a collective way. The solutions were not institutionally based nor textbook based.

From the analysis of students' strategies that are used to find solutions to the assigned problems, it was clear that mathematics is utilised as a tool to find solutions to social problems and is different from the way mathematics would be used for instance in other subjects such as biology and physics. In some instances, mathematics moved beyond being mere body of knowledge regulated by conventional rules, most students tended to be more practical than presenting rigorous mathematical proofs. On the other hand some students pursued a theoretical approach and this tended to provide incomplete solutions as the strategies involved suppositions and hypothetical scenarios. The collective presentation (which is the combination of individual innovations) of solutions to the themes indicates some kind of 'mathematical power' in the sense that groups approached same problem from different perspectives, combining mathematical knowledge and social rules and presenting solutions in a 'goal-directed way' or fractionated way in order to make progress (see figure 5.8 and 6.4). The different types of strategies that came out, present possibilities to analyse mathematical trajectories for groups. It is not within the scope of this study to present these trajectories, however, since the framework of this

study (CHAT) is developmental and as Davydov (1999) argues that in fulfilling an

activity, the subjects also change and develop themselves, I now briefly highlight some

mathematical dispositions that can be associated with this model of expansive learning.

Furthermore, Boaler (1993) argues that context within learning experiences plays a

major role in the development of positive dispositions in mathematics. To present the

highlights, I use Perkins, Jay and Tishman's (1993) triad model of dispositions, namely,

inclination, *which is how a learner feels towards a task;* sensitivity towards an occasion

or the learner' alertness towards a task*; and* ability*, which is the learner's ability to follow*

through and complete an actual task. Applying this triad model to the earlier discussions

(section 6.2) on the analysis of students' views on the nature of mathematics and on the

use of familiar contexts in the applications of mathematics, it can be concluded that

results show positive dispositions on almost all statements. However, this argument

cannot be generalized across different contexts and it may well be that the issue of

developmental mathematical trajectory is, amongst others, context specific. That is, it may

depend on how exciting or provocative are the problems or as Nickles' (1981) argues

whether participants see the demand to search for solutions.

6.7   Summary

In this chapter main goal anchoring the discussions was identified as discussing students'
strategies in finding solutions to the problems whose contexts are familiar to them. The
students' mathematical strategies were linked to the theoretical framework. From these
discussions, the chapter has shown how two theories Engeström's (1997) and Davydov's
(1988) as cited by Engeström (1997) were used to understand students' strategies.

It is argued that these two theories are important in understanding students' ways of finding
solutions in the application of mathematics in a modelling context. In view of the

students' strategies, it is argued that there are identifiable mathematical growth paths and fundamental factors that influence these paths. One of these factors that influence the mathematical growth or dispositions is the use of familiar contexts in the applications of mathematics.

CHAPTER 7

SUMMARY, CONCLUSION, RECOMMENDATIONS AND SUGGESTIONS FOR FURTHER INVESTIGATION

7.1   Introduction

This chapter presents the summary of the study, conclusion, recommendations and suggestions for future investigations. In the summary, I focus on three concluding arguments based on the discussions from the previous chapter. First, I argue that a teaching professional identity is emergent. Second, I argue that when student teachers are exposed to problems that have been formulated out of the familiar contexts, their mathematical behaviour is dependent on their understanding of these contexts rather than on how they perceive mathematics to be. Finally, I also present an argument that there are four distinct students' generated algorithms (strategies) that emerge when students are engaged in finding solutions to problems that are part of their social dilemmas. The classification of these students' working strategies lend themselves in a tension between epistemological and postepistemological paradigms and I posit that this tension, in terms of mathematics education, does not result in a choice of either of the two, but in a possible unification (not a merger) of the two.

These concluding arguments also constitute my learning experience as a result of this study. It must however be stated that even though the concluding arguments are presented in a general form in this study, this generalisation is not necessarily transferable across all contexts. The chapter is concluded with the educational implications for the study and recommendations for future investigation.

7.2   Summary

7.2.1   Focus of the study

The study's main focus was on the description of the strategies used to arrive at solutions for problems from real life situations that are familiar to the pre-service mathematics teachers (participants). During the process of investigation, a multi-layered approach was adopted. This multi-layered approach amongst others involved the use of the biographical information on who the student teachers want to be in terms of their primary career choice. Secondly, it was to analyse the relationship (pattern) between the perceptions the students hold about what mathematics is and the ways solutions to problems in a modelling context are found. Thirdly, it was to find out student teachers' views about the use of familiar contexts in the application of mathematics. Finally it was to analyse student teachers' strategies.

7.2.2. Methodology

The study relied on the use of the quantitative and qualitative research approaches. A quantitative research approach was largely used to develop the research instruments for data collection and qualitative research approaches were dominant in the analysis of the data. The research instruments used involved four themes and two questionnaires.

To present the key findings of the study, I focus on the concluding arguments emergent from the data analysed as a scholarly contribution to teacher preparation in the Further Education and Training phase.

7.2.3 Professional identity

7.2.3.1   Career path and students' mathematical tasks preferences as part of *teaching professional identity formation.*

In terms of the career path for mathematics student teachers, the majority (92%) of the participants did not choose the teaching profession as their first choice. This kind of late identification with a teaching profession is consistent with one of the South African study on mathematics pre-service teachers that Chuene et al. (1999) conducted. Even though as

argued in chapter 2 that the prospective teachers are exposed to teaching processes (social and cognitive) for about 15 years during their schooling period, the decision to take teaching as a career is delayed. It can therefore be argued that the teaching profession has an identity that is very much an emergent process than a continual or an uninterrupted process. The implications of this emergent identity formation are speculated later in this chapter.

From the analysis of which topics participants preferred to teach it came out that the participants were more interested in mathematical word problems and problems formulated from real life contexts. The most dominant reasons advanced for this kind of preference seem to be the relevance of word problems to real life situations and the practical nature that they have as compared to abstract problems.

This preference by the pre-service mathematics teachers differs from the view of the practicing mathematics teachers. The practising teachers' reasons for not preferring word problems or problems formulated out of real life contexts are dominated by the difficulty to understand them and lack of strategies to teach problems formulated out of real life situations. Furthermore even though the pre-service and seasoned mathematics teachers had largely comparable levels of mathematical content knowledge (main difference was age and teaching experience), the pre-service teachers showed more interest in open-ended problems as compared to seasoned mathematics teachers. Once more the reasons for the lack of interest from the seasoned mathematics teachers relate to methodologies.

The factors contributing to this pre-service mathematics teachers' bent were not pursued, as they did not form part of the unit of analysis. What was more critical was to determine the extent to which the participants' views on the nature of mathematics are linked to the

ways in which solutions to the selected themes were found. This was in pursuance of the idea that some of the solutions are largely influenced by the way mathematics is perceived to be, as argued by Ernest (1991) and Volmink (1993).

The characteristic difference between pre-service and practicing teachers shows that within teaching itself, there are issues that may influence or shape teachers' identity or association with other sections. These influences are based on the practical and implementation issues such as difficult to teach and lack of relevance to human life (also refer to Appendix 6 on why students prefer different types of tasks). In view of the reasons from the pre-service and practicing teachers, I posit that an identification or association with certain mathematical topics emerges out of certain experiences and is not necessarily a continual process.

7.2.3.2 *Perception of what mathematics is versus students' strategies*

The concept of what mathematics is and how it relates to the way solutions are presented constitutes a complex conundrum in view of Ernest's (1991) and Volmink's (1993) argument.

What came out clear was that the degree of familiarity of the context of the problem elicits more strategies than what mathematics is perceived to be. In contrast to Ernest's (1991) and Volmink's (1993) views, this study has revealed that the use of contexts that students understand are more related to the way solutions are found than the way mathematics is perceived.

7.2.2.3 *The epistemological and postepistemological nature of students' strategies.*

This study has revealed that there are four distinct strategies (i.e. knowledge driven strategy, students' contextual logical reasoning strategy, example driven strategy and

imaginary strategy) that students use to present solutions to the mathematical problems that have been formulated out of contexts familiar to them.

For the purposes of highlighting mathematical trajectories and general scholarly contribution brought in as a result of these four strategies, I decided to position these strategies in two (rather unexplored) educational philosophical paradigms in order to make theoretical conclusions on these strategies. These are epistemological and postepistemological paradigms and to present the key points from these two paradigms, I draw on from the work of Underwood (2003); Holzman (1997) and Newman & Holzman (1997). In simple terms epistemology is concerned with how we know and what the rules are that enable us to know (Underwood, 2003) and postepistemology deals with rejection of the notion that knowing is a path to a better life and/or better world or progress or growth (Holzman, 1997). My understanding of the latter is that, finding solutions to problems or progress in human life does not necessarily rely on the application of rules or existing laws but relies on the social interpretation of the problem. Working in an epistemological paradigm therefore involves use of certain rules to explain what we know and working in postepistemological paradigm requires understanding of immediate circumstances, performing the required task and accounting for its successful performance. The latter relies heavily on the interpretative approach rather than application of laws or rules in an explanatory way. From Newman and Holzman's perspective, there is a tension between epistemology (knowing) and postepistemology (end of knowing).

In this study, there was a sense in which strategies relied in the usage of general mathematical laws, e.g. in knowledge and imaginary driven strategies and there was a sense in which strategies involved a practical approach and very narrative approach, e.g. in the example and students' contextual logical reasoning strategies. As explained in the

previous chapter, in the case of example and students' contextual logical reasoning driven strategies, it can be argued that knowing the mathematical laws in some ways played a role. There was also a strong sense of indication (which Newman and Holzman would classify as a postmodernists view) that meaning comes from ordinary day-to-day conversation, that is, the 'language created' in the community. This was evident in the way arguments were advanced in the students' contextual and logical reasoning and example driven strategies.

This trend differs from the postmodernist's view that "human world structuring is linguistic rather than cognitive" (Newman and Holzman; 1997, p8). It may be that human world structuring is more linguistic but at times it is mediated by some sort of knowing as argued by Newman and Holzman.

In sum, this thesis argues that the strategies employed by students when they find solutions to problems that are formulated out of the contexts familiar to them, these strategies fall within epistemological and postepistemological paradigms.
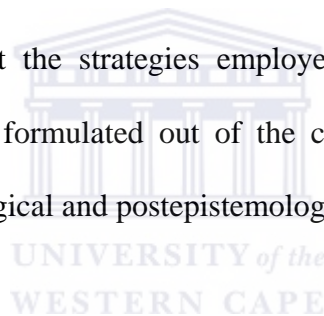
7.3   Concluding remarks

In view of the findings in this study, the following conclusions are presented.

The use of familiar social institutions in the formulation of mathematical problems by pre-service mathematics teachers, enable them to find solutions to problems by taking meaning, context and realities of a problem into consideration. These realistic considerations, that is, contextual interpretation of the problem and the solutions, take various forms. Students view the problems from different angles and consequently have different goals in terms of the solutions to the problem. The end of the solution is seen as having a direct link to social change. For instance, those students who likened the mathematical themes to their life experiences, tended to be advocates for social change.

As mentioned earlier, the object of learning turned to be critical analysis of social issues for the purposes of improving those social conditions. This is an example of what Julie (1998) calls "potential activism". These types of students tend to jointly use mathematical knowledge, social practices and other contextual considerations in their solutions. In the end the students tend to be more practical in their approach. This approach has potential for creating a 'continuous debate' that may result in 'boundary crossing' and a 'potential expansive cycle'. On the other hand, some students find solutions to the open-ended problems by seeking to apply known mathematical knowledge and hypothetical scenarios or suppositions. In the end, students tend to be theoretical in their approach. This kind of an approach where students ascended from theory to practice had challenges such as technical errors in some formulae and simplification of the solutions in terms of the assumptions. It is clear that the question of application of knowledge or 'transfer of knowledge' particularly 'far transfer' (application of mathematical knowledge in an unfamiliar context) is an issue to be carefully considered in describing students' strategies and application of mathematics in a modelling context.

What echoes loudly and clearly in this study, is the fact that in practice (when students work out mathematical problems), there are signs of unconnectedness between reality and mathematics as a field of study. The interaction is limited between student and mathematics. Using Moerlands's (2002) model in learning mathematics in the context of realistic theory, this lack of connectedness creates a potential for students to use strategies that present solutions that do not require reference to the world as a validation tool for knowledge. The students improvise by making suppositions or creating what De Lange (1999) would call a virtual context (non physical world). In strict terms this approach does not present a *final* solution to the problem as presented to them. For instance, in the

following, the final solution is yet to be presented. There is a need to go beyond suppositions, for the solution to be presented.

> *Suppose initially we have 200 students, each student pays R200,00 but there are those who do not have adequate money to pay. The total pay of the students (190) be R38 000,00 and the total pay of the students (100) be R1000,00*
> *Therefore Debt = 1000*

To curb the increases is for the management to admit equal number of students who are able and not able to pay. But the amounts should be proportional to that stated above, e.g. if they admit 10 students for next year

On the other hand, when students are exposed to problems that appeal to their emotions or needs (familiar social institutions), the use of contextual logical reasoning is very strong and justifications or validations of knowledge are viewed in terms of their life experiences. The examples seem to fit in well with the students' arguments once more shown in this extract:

> *The key here is to get students to use these services and get them to want to leave the campus at the same time (17h30 and 19h30). One other thing that we should enforce is that students should buy monthly tickets. If we tell the bus service that they are going to make 2 140.00(day) + 1 558(night) = 3 728.00 per day, that is approximately 111 840.00 per month and for approximately 10 months = 1 118 400.00,* **there is no way** (my emphasis) *they will refuse*

I do not claim that these echoes are universal, or that they are common across different contexts. It was not the intention of the study to make generalisations on the findings, however, it may well be that these echoes are in existence in non-reverberating learning environments. The way to trigger the sounds is to present students with mathematical themes that encourage or challenge learners to realise the problem or dilemma or the challenge in that context. This study offers a perspective in that line of the theme approach.

7.4   Critique and limitations

In dealing with investigations about the applications of mathematics, I grappled with the idea whether I was to find mathematics in authentic social problems, use that mathematics to find the solutions or use mathematics as an 'imported tool' to find solutions to the authentic social problems. The downside of the latter was that the approach tended to be more achievement based. In case of the former, the instruments tended to be too open-ended or general to realise the role of mathematics or its operationalisation. In some cases mathematics was 'invisible'. A way out of this tricky situation was to turn to Herget's (1984) approach.

Herget (1984) identified the following scenarios involving the application of mathematics.
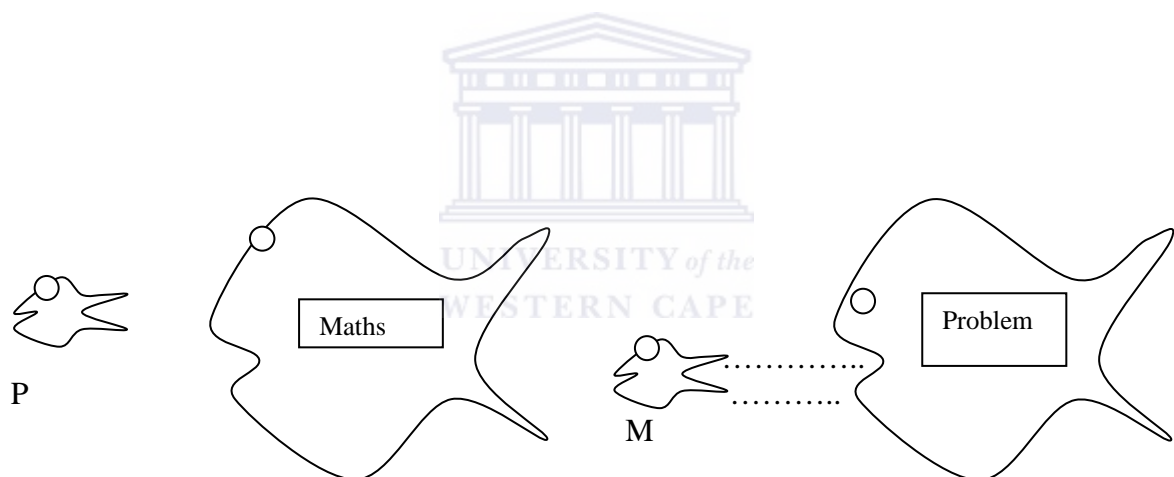
Figure 7.1: Herget's (1984) model – Math as a tool   Figure 7.2: Herget's (1984) model-
                                                                                                      Math within a problem

In figure 7.1, Herget (1984) explains that mathematics is seen as a powerful tool to find solutions to real world problems, and in the end these problems tend to be small and harmless ones. On the other hand, figure 7.2 shows the big problem in the real world, participants see nothing but a little bit of mathematics.

In this study, it was the combination of the two in a linear form starting from mathematics within a problem (figure 7.2) to mathematics as a tool (figure 7.1). My initial plan was to use mathematics as a tool or mediating instrument, but as the discussions continued,

focusing on the purpose of the project and what the project entailed, it became clear that the starting point was reality (a problem) and students were expected to apply their mathematical knowledge in finding the solutions to the four themes. This was also evident in the way the students presented their textual reports, and I was, therefore, more convinced that the starting point is as described in the scenario in figure 7.2. That is, students were thrown into a large set of social problems and saw little mathematics. This feeling lead to "constant compare and contrast procedure" in order to describe the way students find solutions to problems that come from familiar social institutions. I had a strong feeling that mathematics, as a tool, was invisible. I soon realised that my approach was dominated by assessment (achievement) type of thinking and that the analysis frameworks were too judgemental. The move from figure 7.2 to 7.1 contributed to finding the balance.

This was further addressed by making a shift to a more developmental type of approach in analysing the way students find solutions (method of finding solutions and mediating artifacts used). This made a big difference, as I examined what the students were able to do instead of looking at what they cannot do. At the centre of this shift was the CHAT as the theoretical framework, Samaras's (2002) four components in every teaching situation as discussed in chapter 2, and the realisation that mathematic exists in many fundamentally different environments (Steen, 1999) and that success in one context does not necessarily lead to success in other contexts (Lave & Wenger, 1991). This afforded students to make use of their social experiences.

Another limitation, which emerged, is that being immersed in social issues, the transition from 'informal knowledge' to 'formal knowledge' is not easily identifiable. It can be argued that in terms of Nonaka and Takeuchi's (1995) model of collaborative knowledge

in problem-solving context as presented by Engeström (1999b), the link between tacit knowledge and explicit knowledge is not that strong. Nonaka and Takeuchi's (1995) model is argued to be cyclic. On the other hand Engeström (1999b) has argued that Nonaka and Takeuchi's (1995) argument that their matrix as shown in figure 7.3 could become a cycle is questionable. He argues that the components in each matrix exist on their own or exist as separate components.

Tacit knowledge	to	Explicit knowledge
Socialisation Sympathised knowledge		Externalisation Conceptual knowledge
Internalisation Operational knowledge		Combination Systemic knowledge
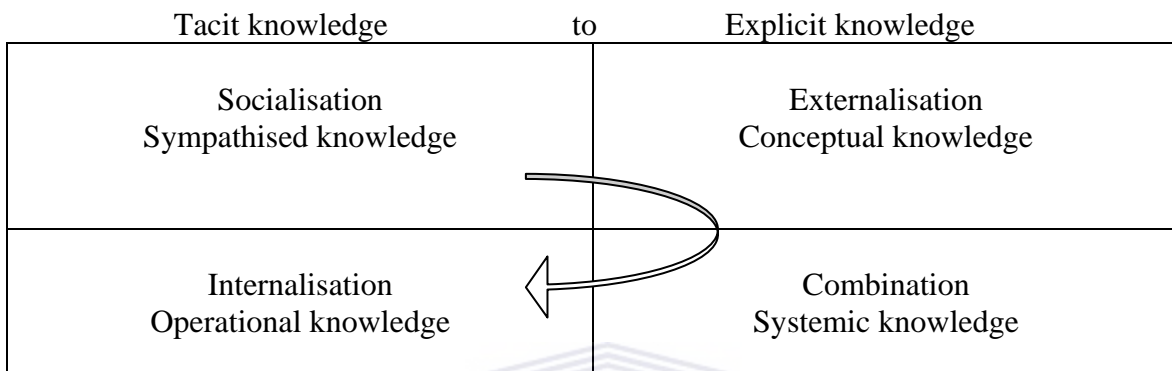
Figure 7.3: Nonaka and Takeuchi's (1995) adapted cyclic model of knowledge creation in problem solving as cited in Engeström (1999b)

Whilst this gap may be seen as the downside of the approach that ascends from dense social issues,at the end it depends on the angle from which one looks.

What is necessary, however, is that some intervention is needed in order to 'push' participants to the level of concept creation and the justification of concepts (explicit knowledge). Otherwise there is a possibility of remaining at an implicit or tacit level. During the investigation, it was not possible to intervene, since my role in the research was non-participative.

7.5   Recommendations and implications

The findings of this study need to be taken to classrooms in order to enhance the reform-based strategies that are being suggested for teacher preparation in South Africa.

The following opportunities have emerged from this study and I argue that they offer an entry point to theme-oriented teaching in the teacher preparation context or these opportunities may offer reinforcement of the theme approach in the teaching of mathematics for pre-service mathematics educators.

7.5.1   Development of programmes for pre-service mathematics teachers

Programmes for pre-service mathematics teachers need to be flexible in order to accommodate student generated algorithms / strategies. Furthermore, there should be a deliberate effort to understand pre-service strategies as these strategies have implications for future classroom instruction. Research elsewhere has shown that understanding learners' thinking, guide subsequent instruction (Towers 2003).

Close scrutiny by teacher educators of the general types of strategies that have emerged from this study will, therefore, help them to develop programmes that take the students' needs, thinking and understanding into consideration.

7.5.2 Integrated approach

Given the complexities in teacher preparation as discussed in chapter 2, there is a need for the providers of teacher preparation to adopt a more integrated approach. Some of the key lessons learned from this study are that: (a) one cannot use an 'isomorphism measure model[*]' for instance to trace the impact of perception of what mathematics is in terms of the way solutions are presented (b) problem solvers hold different views (may be at different moments or stages) as to what contributes to the solution of a problem. To address the following questions: what knowledge do students bring? reasoning questions (how do they find solutions to the problem?), understanding questions (how did they do

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[*] Nunes et al. (1993) use an isomorphism measure model concept in situations where only two variables are involved, each value on one variable corresponds to one value on the other variable.

that?), and finally emotional needs or self-preserving questions (why do they do what they do?) need an integrated approach.

The responses to these questions constitute critical part for teaching as argued by Green (1971) in Mewborn (2004). According to Green (1971), as cited by Mewborn (2004), the purpose of teaching is to modify students' belief systems in order to have an ideal belief system. Mewborn (2004) has argued that Green's (1971) notion of an ideal belief system should be extended to teacher preparation in order to help pre-service teachers to develop an ideal belief system with regard to the teaching and learning of mathematics. The idea, as argued by Mewborn (2004), is not to indoctrinate pre-service teachers but to help them create a reasoned framework of the way people believe.

The focus on these questions will also help in understanding prospective teachers' views they bring to the profession and possible identities (word problems teacher, geometry teacher etc).

7.5.3   Fostering of abilities that mathematics students' need

McLone (1984) argues that a mathematics student needs at least four basic abilities: technical knowledge, discovery, criticism and communication. From the strategies that emerged from the participants' solutions (knowledge driven strategies, students' contextual logical reasoning, example driven and imaginary strategies) and subsequent discussions on these strategies, it became clear that in teaching, the four abilities as presented by McLone (1984) should be considered. It is, therefore, recommended that these four abilities be part of the outcomes for teacher preparation of mathematics teachers.

7.5.4   Face validity in terms of the curriculum

The findings of this study offer a reference guide to teacher educators on how to realise the achievement of key curricula outcomes. Relevant extracts quoted from the Revised National Curriculum Statement of 2003 that support this view was presented in chapter 2 (figure 2.4(a) and 2.4(b)).

The four strategies (identified in this study) will, therefore, provide mathematics teacher educators with something to refer to in terms of curriculum implementation. While these strategies may not sufficiently cover all the range statements as depicted in table 2.4a and 2.4b, the findings from this study certainly offer critical issues that need to be taken into consideration in implementing those appropriate curriculum expectations. This study has, therefore, face validity in terms of the new curriculum. As argued earlier, the strategies that emerged from the students' solutions go a long way in addressing the key abilities (technical knowledge, discovery, criticism and communication) expected from a mathematics student.

7.6 Suggestions for further study

7.6.1 Large-scale research on pre-service teachers' mathematical behaviour

The National Qualifications Framework, which is one of the policy documents on the transformation of the South African education system, has articulated the vision that promises to carry South Africa into prosperity in the 21st century (Ensor, 1997). Ensor (1997) further explains that what is different about the National Qualifications Framework (NQF) as compared to other educational policies both here in South Africa and internationally, is that it attempts to establish, through outcomes-based education, an equivalence between formal schooling and the knowledge which is produced and acquired in other settings. This is also clearly articulated in Education White Paper of 1995 which states that, the NQF seeks to achieve the unity of the mental and manual, the academic

and vocational, an equivalence between school knowledge, work knowledge and the everyday knowledge of domestic and recreational settings (Ensor, 1997).

In order to achieve the narrowing the gap between pure mathematics (school mathematics) and mathematics out of school (real world) or what Lave (1988) calls gap-closing arithmetic, this type of a research needs to be conducted on a much larger scale than this study. This will allow the possibility of good practices filtering down to other areas. This study can only offer opportunities for the improvement of exemplary driven practices.

7.6.2 From a graduate to practising teacher: issues of changing behaviour and tension between theory and practice

In chapter 2, it was clear from Adler's (2004) research that a number of issues in mathematics teacher education, particularly in pre-service teacher education, remain un-researched and that there seems to be a gap between theory and practice.

From the findings of this study as presented in chapter 5 and 6, there seems to be a gap between pre-service students' views about certain mathematical topics and the teaching of those topics at school. This gap seems to be linked to 'theorising group' and the 'group in practice'. The message from the group in practice seems to be the following: certain ideas don't work at schools because of a lack of methodologies or due to the fact that learners do not like those topics. Institutions of higher education as providers of teacher education have long inducted (through philosophy and psychological approaches) their teacher trainees into new ways of handling different topics. Why do we still have mathematics teachers (no matter how few) who for instance take the view that certain topics cannot be taught because of a 'lack of methodologies'? Is it a question of being overwhelmed by what Goodchild (1997) calls "post office behaviour", that is, behaviour that conforms to the usual practices in those teaching contexts? Adoption or acceptance of this type of

behavioural thinking makes it difficult for one to start something new or to do something extra. This needs to be investigated.

7.6.3 Interpretation of policy guidelines: the radicalisation of concepts

In chapter 1, key concepts such as mathematical problem, dilemmas, mathematics as a human activity, activity, object of learning, mathematical modelling and meaningful understanding were described. In this study, the meaning of these concepts has been problematised in order to encourage critical analysis in the presentation of solutions. In the Revised National Curriculum Statement, for instance, the phrase "to solve problems" frequently appears in learning outcomes. Learning Outcome 1 on Number, Operations and Relationships reads ad follows:

The learner is able to recognise, describe and represent numbers and their relationships and can count, estimate, calculate and check with competence and confidence in solving problems (National Curriculum Statement- Grade 3, Parents Guide, 2005, p.63).

Assessment standards indicated under this learning outcome, focuses on what Volmink (1993) calls "interpretation of symbolic information", the presentation of "situated problems" or some kind of demands on the problem solver without being emotionally engaged (Lave 1988). The above quotation from the parents' guide necessitates the following questions: What problems are they expected to be able to solve? What is a problem? In this study, in order to design the themes as instruments for data collection, conscious decisions had to be made on what is meant by a problem and for this I relied on Lave's (1988) description of a problem and descriptions of problems by other researchers

such as Nickles (1981), as discussed in chapter 1. There is, therefore, a need for an investigation into what the problem is and what problems (within the school environment and outside of the school environment) learners are expected to solve? What does it mean when one says learners can't solve problems? These questions help to interpret the policy guidelines and its implementation.

7.6.4   Framework

Finally, the nature of a study that seeks to understand ways in which learners find solutions to problems in application of mathematics in a modelling context, lands itself in cognitive and situated cognition perspectives. In this study, I have argued that CHAT was adopted because of its developmental nature. This was to address to pertinent issues that relate to knowledge transfer. The issue of knowledge transfer is very fundamental in an investigation that looks into issues of application of mathematics. Questions such as: What mathematical knowledge should they (problem solvers/students) possess in order to find solutions to situated problems? Is knowledge transferable? These are not new questions, but have certainly not been explored to the fullest in order to inform the community of mathematics educators in South Africa.

7.7   Final word

Investigating situations from life experiences are complex in a number of ways. Whilst it is clear from this study that students' backgrounds helped them to understand problems and also to enrich their responses with social rules, researchers in this area need to be mindful of Skovsmose's (2005) argument that "meaning not only represents the past. It also represents the present and the future" (p.8). This requires that one should also consider the foreground (aspirations) of the students. In the case of this study, we need to

look beyond the students' familiar contexts and explore their aspirations. In this exploratory process, researchers need to challenge their participants on their content and the structure of their aspirations. This will inevitably lead to a situation where participants not only become sources of data, but become sensitised in the process or become involved in a debate as to why certain things are to be done and not others.

It is also evident from this study that the process of finding solutions to the problems that were formulated out of the familiar contexts to the participants, invoked emotions of the participants. The 'potential activism' that was referred to in the previous chapter provides an evidence of this claim. This type of engagement has a potential to bring in what Simon (1983) calls 'hot reasoning' in mathematics and with this approach (thematic approach involving familiar contexts), mathematics pre-service teachers may realise that mathematics is not only about 'cold cognition' (characteristic of passive learning also refer to Simon, 1983) but about problems that excite and motivate learners. Furthermore, problems that challenge learners and excite learners offer an opportunity for opening up avenues for variety of mathematical strategies from various perspectives (cognitive and social).

In sum, given the mathematical dispositions that were highlighted in the previous chapter, the activity of the 'arena' (in the classrooms/lecture halls or research field) is likely to be perceived to be personally or socially relevant to the students thereby satisfying Mellin-Olsen's (1987) S-rationale as explained in chapter 3, that is the classroom activity assumes kind of importance that is beyond its status as school content knowledge. There is an indication that the types of problems that one presents inevitably influence mathematical behaviour or the way the students would approach the problems.

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APPENDIX 1: PILOT STUDY

P1. There are 125 sheep and 5 dogs in a flock. How old is the shepherd?

P2. A man wants to have a rope long enough to reach between two poles 12m apart, but he only has pieces of rope 1.5m long. How many of these pieces of rope would he need to tie together to reach between the poles?

P3. Carl has 5 friends and Sifiso has 6 friends. Carl and Sifiso decide to give a joint party. They invite all their friends. All friends are present. How many friends are at the party?

P4. What will be the temperature of water in a container, if you pour 1litre of water at 80 degrees and 1 litre of water at 40 degrees into it?

P5. Mr Letlaka, the butcher had 26 kg of meat in his shop, and orders 10kg more. How much meat does he have now?

P6. This year a shop sold 32 umbrellas in February. About how many do you think it will sell altogether in June, July and August this year?

P7. Suzan and Thabo need 750g of flour and 50g of yeast in order to bake cakes for 20 friends. How many grams of flour and yeast are they going to need for baking cakes for 40 schoolmates?

P8.

| Kestel |
| 9>> |
| Phuthaditjhaba |
| << 15 |

What is the distance between Kestel and Phuthaditjhaba according to the road signs?

P9.Thapelo and Tumi attend same school. Thapelo lives 1.2km from the school and Tumi 0.7km from the school. How far apart do they live?

APPENDIX 2: OUTLINE AND PROCEDURE

The collection of data was done within three levels and data will also be presented in that way. The levels are:

Level A- students/participants completed tasks,

Level B- Presentations and groups' recommendations which later complemented with questionnaires (Views About Mathematics Survey-VAMS) which were adapted from Carlson's Model and
Level C- Reports from sectional heads of the researched scenarios. i.e. report from taxi association, head of catering section, university management/finance section-student debt reduction plan and head of academic division on day and evening lectures. The interviews were abandoned after several aborted appointments with the groups; only two groups honoured the appointments. The questionnaires were complementary to the interviews but were largely substitutes to the interviews and the two questionnaires more broader views than finding how they approached the problems. These questionnaires (though administered from non participant point of view) were found to be richer than the interviews.

FORMAT WHICH WAS GIVEN TO GROUPS.

STEPS TO BE FOLLOWED WHEN SOLVING MATHEMATICAL PROBLEM

(in sequential form)

1. Specify the real problem.

2. Set up a model.

3. Formulate the mathematical problem.

4. Solve the mathematical problem.

5. Interpret the solution.

6. Compare with reality.

7. Write the report.

In doing the attached mathematical problems, note that every idea/comment/remark etc you have is important, please write it down(even if it is a rough work)

APPENDIX 3: THEMES AND TYPES OF MATHEMATICAL TASKS

Theme 1: Student fees.

Institution A has reported on the increasing debt on student fees. They had indicated attempts to address this problem; you have been approached to assist.

Develop a Mathematical Model referred to as: "Student Debt Reduction Plan", to be used by the management of that institution to address the debt backlog (recollection) and to curb further increases. The attached data may be considered.

THEME 2: "USAGE OF UNIVERSITY CATERING SECTION BY STUDENTS"

Institution D has its own catering section for residential students. It has however realized that the sustainability of this section is under threat given the inconsistent usage of this section by the students and general financial constraints faced by the institution.

Develop a mathematical model, which you will sell to the staff of the catering section to address the formulated problem.

Use the attached data as historical information.

THEME 3: "TRANSPORT FOR DAY STUDENTS"

Students from institution B have raised a concern with the management of that institution on as far as the transport to attend lectures. The issue of transport affects both day and evening lectures. Identify the problem and develop a mathematical model, which the management may discuss with the taxi association (or an alternative plan) in order to resolve this issue. The number of students attending day lectures and in need of transport is 213 females and 149 males and evening lectures is 124 females and 138 males.

The following are the routes and estimated kilometres from the institution.

Routes	Kilometres	Remarks
Phuthaditjhaba	12	Estimates are return trips
Harrismith	94	
Bethlehem	156	
Clubview	12	
Bluegumbosch	15	
Elite	18	
Riverside	16	
Tseki	51	

Road Map: QwaQwa

THEME 4: "EVENING AND DAY LECTURES"

Institution C is offering courses during the day and in the evenings for the working students. This has however brought some challenges as it has limited human resources and is working on a restricted budget, so it is difficult to employ additional staff for the evening lectures. In some cases, courses are offered both in the evening (session 2) and during the day(session1). The institution has committed itself on the principle of " educating people on their jobs". Use the data below to develop a mathematical model which will solve the institution's problem.

	Number involved	Remarks
Day lectures(session 1)		
Evening lectures (session 2)		
Courses duplicated		
Non Duplicated courses		
Total number of lecturers		
Distribution of lecturers per courses		No. of lecturers in affected courses only.

VIEWS ABOUT TYPE MATHEMATICS TASKS

This questionnaire is about views on types of mathematics tasks

Complete it as honest as you can.

Please circle the appropriate number describing your feeling about the statement.

1. Gender: Female / Male

2. Age: below 20yrs; between 20 and 25; between 25 and 30; 30 –35, 35-40, 45-50, 50+

Give rating of the following according to your preference: 1=Good problems I like them, 2=Moderate, don't like them that much and 3= I do not like them, If I had a choice I'll leave them out.

Types of Mathematical Tasks	Good = 1	Moderate =2	Dislike them =3	Possible reason(s)
Mathematics topics related to real life situations or contexts	1	2	3	
Mathematics problems with only one possible answer e.g. closed-ended problems (linear equations)	1	2	3	
Mathematics problems with more than one answer depending on the context e.g. open-ended problems	1	2	3	
Abstract mathematics – those which need proofs and conceptual understanding e.g. theorems	1	2	3	
Memory tasks- those that require recalling of facts/rules/formulae	1	2	3	
Procedural or routine tasks	1	2	3	

APPENDIX 4: VIEWS ABOUT MATHEMATICS SURVEY (VAMS)

QUESTIONNAIRE 1

This questionnaire is on Views about Mathematics

Complete it as honest as you can. No name required.

Please circle the appropriate letter describing your feeling about the statement.

1.Gender :   Female / Male

2. Age : below 20yrs; between 20 and 25; between 25 and 30; above 30

Structure	Agree=A	N=Neutral	Disagree=D
Mathematics is a coherent body of knowledge about relationships and patterns contrived by careful investigation-rather than a collection of isolated facts and algorithms.	A	N	D
Methodology			
Mathematics and related technology are relevant to everyone's life rather being of exclusive concern to mathematicians.	A	N	D
The methods of mathematics are idiosyncratic (particular) and situation specific -rather systematic and generic (general).	A	N	D
Mathematical Modelling for problem solving involves more than selecting formulas for number manipulations.	A	N	D
Validity			
Mathematical knowledge is validated by correspondence to the world -rather than by logical proofs	A	N	D
Mathematical knowledge is tentative and refutable(may be proven incorrect)-rather than absolute and final.	A	N	D
Learnability			
Mathematics is learnable by any one willing to make	A	N	D

the effort-rather than by a few talented people.			
Achievement depends more on persistence effort-than on influence of the teacher or textbook.	A	N	D

Critical Thinking

For meaningful understanding of mathematics, one needs to: concentrate more on the systematic use of general thought processes-than on memorizing isolated facts and algorithms.	A	N	D
For meaningful understanding of mathematics, one needs to: Examine situations in many ways, and not feel intimidated by committing mistakes rather follow a single approach from an authoritative source.	A	N	D
For meaningful understanding of mathematics, one needs to: Look for discrepancies in one's own knowledge instead of accumulating new information.	A	N	D
For meaningful understanding of mathematics, one needs to: Reconstruct new knowledge in one's own way instead of memorizing it as given.	A	N	D

Personal Relevance

Mathematics and related technology are relevant to everyone's life rather being of exclusive concern to mathematicians.	A	N	D
Mathematics should be studied more for personal benefit than for just fulfilling curriculum requirements	A	N	D

QUESTIONNAIRE 2

This questionnaire is on views about how you feel about doing problems from an application and modelling context.

Complete it as honest as you can. No name required.

Please circle the appropriate letter describing your feeling about the statement.

1.Gender :   Female / Male

2. Age : below 20yrs; between 20 and 25; between 25 and 30; above 30

	Agree =A	Not sure=N	Disagree =D
For me, making unsuccessful attempts when solving a mathematics problem is: a natural part of my pursuit of a solution to the problem	A	N	D
For me, making unsuccessful attempts when solving a mathematics problem is: an indication of my incompetence in mathematics.	A	N	D
In solving mathematics problems, graphic calculators or computers help me: understand underlying mathematical ideas	A	N	D
In solving mathematics problems, graphic calculators or computers help me:	A	N	D
In solving mathematics problems, graphic calculators or computers help me: obtain numerical answers to problems.	A	N	D
In solving mathematics problems, familiar contexts, e.g. student fees, transport, catering issues help me: to learn mathematics in a meaningful way.	A	N	D
In solving mathematics problems, familiar contexts, e.g. student fees, transport, catering issues help me: to put calculations into context	A	N	D
In solving mathematics problems, familiar contexts, e.g. student fees, transport, catering issues help me: to have	A	N	D

positive attitude towards the teaching and learning of mathematics.			
In solving mathematics problems, familiar contexts, e.g. student fees, transport, catering issues help me: make realistic assumptions about the problem and its solution	A	N	D
In solving mathematics problems, familiar contexts, e.g. student fees, transport, catering issues help me: to devise or develop good mathematical context that ask learners to organize given information and identify skills that are needed	A	N	D
In solving mathematics problems, familiar contexts, e.g. student fees, transport, catering issues help me: to get involved in a meaningful way	A	N	D
When I experience a difficulty while studying mathematics: I immediately seek help	A	N	D
When I experience a difficulty while studying mathematics: I immediately give up.	A	N	D
When I experience a difficulty while studying mathematics: I try hard to figure it out on my own.	A	N	D
When I experience a difficulty while studying mathematics: I try to relate it to real life situation and try again.	A	N	D
My score on a mathematics exam is a measure of how well: I understand the covered material	A	N	D
My score on a mathematics exam is a measure of how well: I can do things the way they are done by the lecturers or course materials	A	N	D
I study mathematics: to satisfy course requirements.	A	N	D
I study mathematics: to learn useful knowledge	A	N	D

APPENDIX 5: GROUPS' SOULTIONS PER THEME

Reponses per groups.

GROUP1

Theme 1

G11.

    i.   The real problem is that the amount of fees are not specified hence the solution is to first make the assumption

    ii.   The model to be set up, it should b subtraction to provide reduction of cause

    iii.   Suppose initially we have 200 students, each student pays R200.00 but there are those who do not have adequate money to pay. The total pay of the students (190) be R38 000.00 and the total pay of the students(100) be R1000.00

        Therefore Debt = 1000

        To curb the increases is for the management to admit equal number of students who are able and not able to pay. But the amounts should be proportional to that stated above, e.g. if they admit 10 students for next year

GROUP 2

THEME 1
STUDENT DEBT REDUCTION PLAN
G21.

1. The institution must see to it that it's important employ some of the financially needy students. By so doing the institution should take a certain percentage of the student's salary every month and put into student's account as to reduce the student's debt. The mathematical problem for this is as follows.

$SD = S – (\% \text{ of } Y) \ldots$ "1"

Where SD is student debt, S is money owed by student and Y is salary earned by student per month.

2. The other way of reducing student debt is whereby the institution will need to go out and search for the bursaries only for the students who are financially needy and performing academically well. In this case the mathematical model does not differ from and it will be as follows:

$SD = S - B_r \ldots$ "2"

Where B_r is money offered by the bursary fund.

3. The other way of reducing the student debt is a way of reducing certain percentage from the fees. The formula or mathematical model is as follows:

Tuition fee = R - %
Student fee = R - %

Student levy = R - %
Student Insurance = R - %
Lodging (for Residence students) = R - %
Total amount = R_n - %

THEME 2: " USAGE OF UNIVERSITY CATERING SECTION BY STUDENTS"

G22

I would like to regard the problem of institution "D" with the problem that UNIQWA has with its catering section. If the catering section has two sections namely fast food and booking section, the institution must eliminate the fast food section and remain with the booking section. This is because in the fast food section we don't know how many students will buy Russians and others and this will lead to the waste of food and money. In the booking section the advantage is that students book a day before, and in that case it is known how many students are interested in stuff that is prepared for the following day. Hence it will be known that how many plates should be taken out and how much the institution gained.

$S_n = P_n(Z_k)$

THEME 3 "TRANSPORT FOR DAY STUDENTS"

G23 (a)

For day lectures we find that the institution has 362 students that need transport.
According to the data given it is found that students are traveling from 8 places.
Therefore, if we divide the number of the students by this number of places we find there are about 45 students in each place.
i.e.
$nS/nP = 362\ S\ /\ 8\ P$
$= 45\ S.p^{-1}$

This is for the evening lecture. It is found the total number of students is 262 over 8 places.
i.e.
$nS/nP = 262\ S\ /\ 8\ P$
$= 32.78\ S.p^{-1}$

Therefore the transport will be distributed as follows:

G23(b) <u>DAY LECTURES</u>

PHUTHADITJHABA & ELITE	>>>	BUS TRAIN
TSEKI & RIVERSIDE	>>>	BUS TRAIN
CLUBVIEW & BLUEGUMBOSCH	>>>	BUS TRAIN
HARRISMITH	>>>	IVECO & TAXI
BETHLEHEM	>>>	IVECO & TAXI

G23 (c) <u>EVENING LECTURES</u>

PHUTHADITJHABA & ELITE	>>>	BUS
TSEKI & RIVERSIDE	>>>	BUS
CLUBVIEW & BLUEGUMBOSCH	>>>	BUS
HARRISMITH	>>>	TWO & TAXIS

BETHLEHEM >>> TWO & TAXIS

The matter will be negotiated with transport companies.

GROUP 3

G31. Theme 1: Student fees

Debt * 10% = Interest

Debt
Debt + Interest = Total Debt

G31 (a) UNEMPLOYED AND UNREGISTERED STUDENTS RANGE 1

53 342.00 * 10% = 5 334.28
53 342.00 + 5 334.28 = 58 676.28

30 000.00 * 10% = 3 000.00
30 000.00 + 3 000.00 = 33 000.00

Actual range is 33 000.00 to 58 676.28
In this range 10% is payable per annum

10% of 58 676.28 = 5 867 63 per annum for 10yrs
And 33 000.00 at 5 867.63 will be paid for 6yrs

Therefore paying range is 6 to 10yrs

_____

G31 (b)
RANGE 2

29 999.00 * 10% = 2 999.00
29 999.00 + 2 999.00 = 32 998.90

10 000.00 * 10% = 1 000.00
10 000 + 1 000.00 = 11 000.00

Therefore actual is 11 000.00 to 32 998.90

In this range 17.4% is payable per annum

17.4% of 32 998.90 = 5 741.81 for 6yrs
& 11 000.00 at 5 741.81 will be paid for 2yrs

Therefore paying range is 2 to 6yrs

_____

G31 (c)

RANGE 3

9 999.00 * 10% = 999.90
2.0   * 10% = 0.10

Therefore Actual Range is 1.10 to 10 998.90

Sub Range 1

3 000.00 to 10 998.90 should pay in 2yrs, installments ranging between 125.00 and 458.29 per month

Sub Range 2

1.10 to 2 999.99 should pay within 1yr, installments ranging between 1.10 to 250.00 per month
Interest of 10% is an estimate

Remark: Ranges 1 and 2 pay almost the same amount, this is to ensure that no group has an advantage over the other and that the university collects the debt in the shortest possible period.

_____

G31 (d) **UNREGISTERED AND EMPLOYED STUDENTS**

In this section we concentrate on the amount the student earns. We also consider whether the student will be able to live after paying the installment.

SALARY SCALE 1: 10 000.00-34 999.00

Will pay 60% of the salary per annum
10 000.00 * 60% = 6 000.00
& 34 999.00 * 60% = 20 999.40
Therefore, Paying Range will be 6 000.00 to 20 999.40 p.a.

G31 (e) SALARY SCALE: 35 000.00-59 999.00

Will pay 65% of the salary per annum
35 000.00 * 65% = 22 750.00
& 59 999.00 * 65% = 38 999.35
Therefore, Paying Range will be 22 750.00 – 38 999.35 p.a.

G31 (f) SALARY SCALE: 60 000.00 AND HIGHER

Will pay 70% of the salary per annum

G31 (g) SALARY SCALE: 9 999 AND L0WER

Will pay according to the ranges of the unemployed (as if they are unemployed).

Remark: People earning 10 000.00n are expected to pay 60% of their salary per annum, this is to ensure that we don't have double standards. We expected a person who is unemployed to pay 5 334.25 per annum so it would not make sense to have a person who is employed to pay anything less than that (5 334.25) with this plan the university will recover the money within 10yrs.

_____

G31 (h) PART B (STUDENT FEES)

PROBLEM
Increasing Debt (Student Fees)

Let X, represent student
Y, represent fees
If X owes 67 000.00 and has to register for another semester on: course + residence + meals

Where course = 4 295.00 (let course be S)
Residence = 2 790.00 (let residence be T)
Meals=700.00 (let meals be R)

Therefore Y=S+T+R
=4 295.00+2 790.00+700.00
= 7 785.00
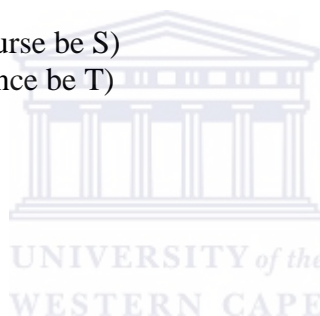
When we add Y to the debt of 6 700.00 (and let debt be Z)
Therefore Y+Z
- ⇨ 7 785.00+6 700.00
- ⇨ 74 785.00

Total Debt per X=Z (74 785.00)

If X pays 20% of 67 000.00
Then 67 000.00-13 400.00 = 53 600.00
X still owes a huge amount

Since Z=53 600.00+Y
=61 385.00
From 67 000.00 we minus Z
Therefore the difference is 5 615.00, which still leaves the student with a huge debt.

G31 (i) RECOMMENDATION

If every student who owes the institution more than 16 000.00 which is the maximum allocation from NSFAS can be allocated the maximum (16 000.00) and pay 20% of the money owed to the institution then,
e.g. 67 000.00-16 000.00=51 000.00
And from this we minus 20%, i.e. 20% of 51 000.00=10 200.00
The student will owe 40 800.00

And if this is for a period of 3yrs then
16 000.00*2=32 000.00

From 32 000.00+20%
32 000.00+6 400=38 400

Within 2yrs 38 400.00 will be paid to the institution which leaves the student with 2 400.00 of debt to the institution.

NB: And to achieve this student must cooperate with the Financial Aid Scheme (office) through (by) applying for NSFAS in time.

THEME 2: USAGE OF UNIVERSITY CATERING SECTION BY STUDENTS

G32

Problem(s)
Financial constraints faced by the university.

Let students (No. of students who use the DH) be X
Quantity of food prepared per day be Y
And the budget per day be Z

Therefore Z=Y/X

Estimation: If X=400.00 and Y will be determined by estimating Z
Then Z=(Based on each Student) Breakfast + Lunch + Supper
NB: This model is for the catering section not the tuck-shop section.

Let Breakfast=P, Lunch=Q and Supper=R.
Z=P+Q+R
And P=6.50, Q=9.50 and R=9.00
Therefore Z=6.50+9.50+9.00
        =25.00

25=X/400
Therefore X=16

The quantity of food per student for 400 students (bookings must be made on time) is 16. And if for every student 20.00-25.00 can be spent to cover the quantity which means more funds can be saved.

Remark: It is claimed that food ends up rotting because no one buys, and this is where money is wasted. The staff must try as hard as can to work on the number of the bookings made per day.

TUCKSHOP/FAST FOOD SECTION

This is where profits can/is made. Looking at the prices e.g. Brown Bread: (normal price around Qwaqwa)=3.00 and at the DH 3.40
From the bakery a loaf is sold at 2.80 and that means the DH get 0.60 which is 0.40 more than the normal price, therefore this section can claim no financial constraints.

THEME 3: TRANSPORT FOR STUDENTS

G33 (A)

Total number of students: 362-Day students
10 people to Bethlehem
15 people to Harrismith
37 people to {Clubview and Bluegumbosch} (A1)
100 people to Phuthaditjhaba
200 people to {Elite, Riverside and Tseki} (A2)

Those, which are grouped, are grouped for convenience and the fact that it costs the same amount to go to any of them.
Normal price to A1 and A2 by bus is 7.60 return
7.60*14.5%=1.10 discount
Therefore 237(number of students)*6.50=1 540.00 per day for the buses

Normal price for Phuthas by bus is 3.80 return
3.80*21.1%=0.80 discount
Therefore 100*3.00=300.00 per day for the buses

Normal price for Harrismith is 21.00 return
21.00*14.3%=3.00 discount
Therefore 15*18.00 =270.00 per day for a 16-seater taxi

Normal price for Bethlehem is 38.00 return
38.00*21.1%=8.00
Therefore 10*30.00=300.00 per day for the bus

In a day the bus service can make 2 140.00

The discount is to ensure that the students become interested in this service and it's negotiable between the parties.
ROUTES (LOCAL)

G33 (b)
3 bus-trains will be used, each carrying approximately 110 people. If we were to use buses, we would use 4, so bus-trains are cost-effective.

Route 1

Tseki via Elite, de Bult and Makgaolaneng.
120 people will be delivered.

Route 2

Tseki via Phuthas, Riverside and Bolata (No. 4)
120 people will be delivered.

Route 3

Phuthaditjhaba via Bluegumbosch and Clubview
97 people will be delivered

Route 4

Harrismith-We negotiate with the taxi association for a 16-seater taxi that would normally leave QwaQwa for Harrismith at 17h30 travelling a 'dead-mile', we use for our students.

Route 5

Bethlehem-We negotiate with the bus service to provide a bus that leaves Qwaqwa for Bethlehem at 17h30. Our students will be discounted.
TOTAL NUMBER OF STUDENTS: 262 'NIGHT' STUDENTS

9 people to Bethlehem
9*30.00=270.00

11 people to Harrismith
11*18.00=198.00

71 people to Phuthaditjhaba
71*3.00=213.00

170 people to Tseki, Elite, Clubview, Bluegumbosch and Riverside
170*6.50=1 105.00
ROUTES (LOCAL)
G33 (c)
2 bus-trains will be used each to carry 121 people.

Route 1

Riverside via Bluegumbosch, Clubview and Phuthaditjhaba.

Route 2

Tseki via Elite, de Bult and Makgaolaneng.
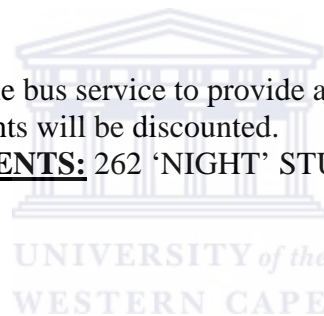
Route 3 & 4

Bethlehem and Harrismith we use the same strategy we used for the day students. Departure time at 19h30.

Remark: The key here is to get students to use these services and get them to want to leave the campus at the same time (17h30 and 19h30). One other thing that we should enforce is that students should buy monthly tickets. If we tell the bus service that they are going to make 2 140.00(day) + 1 558(night) = 3 728.00 per day, that is approximately 111 840.00 per month and for approximately 10 months = 1 118 400.00, there is no way they will refuse.

PART B

G33 (d)

FOR TAXI ASSOCIATION AND MANAGEMENT

Students (females) = 213, males 149 for day attendance and 124 females for and 138 males for the evening.

Total no. (day) =213 + 149
          = 362
Total no. (evening) = 124 + 138
              = 262
And the overall no. for both = 362 + 262
                  = 624
This 624 must be divided according to different routes and here 8 routes are given

We may say 624/8 = 78
This is to say for every route 78 students are estimated.
However different bills are charged per route.

PHUTHADITJHABA = R4.00 RETURN
HARRISMITH = R20.00
BETHLEHEM = R36.00
CLUBVIEW = R8.00
BLUEGUMBOSCH = R8.00
ELITE = R8.00
RIVERSIDE = R8.00
TSEKI = R8.00

TOTAL BILL R100.00 for all 8 routes return
Day students (624)
If 78 students pay R4.00 = R1560
For students who pay R4.00 then is R20.00 per student a week
And for 78 is R1560

And therefore the management should discuss with the Association (Taxis) to pay between R1200.00 and R1000.00 per week to cater for 78 students on R4.00 basis. And this will push the Association to organize a coaster or taxis to fetch students at a certain point to the institution and vice versa.

For R8.00 on 78 students

R40.00 per student a week (40.00 * 78 = 3 120). And there should be a spot or different spots where students must be fetched and dropped. And between R2 600.00 and R2 800.00 should be agreed upon.

For R20.00 on 78 students
R100.00 per student a week (100.00 * 78 = 7 800.00)
A taxi from school straight to Harrismith can be organized and paid R7000.00

For R36.00 on 78 students
R180.00 per student a week (R14 040.00)
A taxi from Bethlehem to the institution can be organized at R13 800.00
Remark: through this, students must pay the transport fare on hand to the institution on weekly basis. And this will solve problems of not getting taxis to school and not having day's fare.

GROUP 4
THEME 1/1
G41 (a)
CLASSIFICATION OF STUDENTS
   1. Students with no parents
        o Student loan
                               } Pay institution
        o Bursary

   2. Students duly assisted by pensioners + limited salary

        o Student loan
                               } Pay institution
        o Bursary

   3. Students with affording parents
        o The problem emerges where parents give students fees for university payment, and these students use money for their own benefit instead at paying the varsity

SOLUTION
These should be a contract between the parent and the varsity for a parent to pay directly to varsity.
All the invoices should be forwarded to the parent reminding him/her of the amount that is due.
Since university fees are so high, some parents might not be able to pay on or before the due date. In this case parents should be classified or scaled In terms of their salary.
Consider the following table:

PARENT	ANNUAL SALARY	QUARTER FEES	SEMESTER FEES	ANNUAL PAYMENT	DATE OF PAYMENT
1	R144 000.00	R5 000.00			30 DAYS AFTER DUE DATE
2	R180 000.00		R10 000.00		60 DAYS
3	R250 000.00			R20 000.00	90DAYS

This table is designed in such a way that parents with salary falling R150 000.00 pay 25% at the fee quarterly and those with income above R150 000.00 but below R200 000.00 pay 50% of the in six months time, so those with R200 000.00+ pay 100% of the fees annually.

However, this can be a problem since it is very difficult at some point for parents to pay huge amount at a go as indicated, so it always better to arrange monthly payment of which each parent will be paying R1700.00 a month.

Let's again consider students having a current debt at varsity.

G41 (b) <u>DEBTORS MANAGEMENT</u>
The management of debts starts with the decision to grant credit. This is the responsibility of the credit manager. In order to establish whether corrective action is needed, the credit manager would need an effective debts control system. A control system monitors the application of the credit policy. Without Credit Policy debts may build up, cash flows decline, and the bad debts arise.

The total amount of debts outstanding is determined by two factors:
1. The volume of students admitted
2. The average length of time between the due date of payment and collection of debts.

This can be expressed as follows:
Total debts = (students admitted per faculty * fees payable per year or per semester)

As, within reason, the institution would encourage a high volume of students admitted, the credit manager needs to concentrate efforts on minimizing the length of collection period. For example, university of oxford was admitting students on terms of say 3/15 net 30, so its students on average appear by 3% discount. The 20-day collection period is less than 30-day credit period. Note, however, that most are students are paying within 15 days to take advantage of the discount while others could be taking longer than 30 days to pay their accounts. To identity these accounts, an ageing scheduled used to analyze the outstanding accounts.

For example, university of Oxford is ageing scheduled is shown below:

Age accounts (days)	Percentage of Total Value of Debts
0-15	70% }
15-30	20% } Therefore 90% paid in 30 days
31-60	6% }
Over 60	<u>4%</u> } Therefore 10% paid in more than 30 days
Total	<u>100%</u>

It is apparent that 90% of the debtors are paying their accounts on time. However, while the average collection period is within the credit terms, 10% of the debtors (students) are overdue, 6% being up to 30 days overdue and 4% being more than 30 days overdue.

Management would used to analyze the institution's average collection period and it's ageing schedule in comparison with other institutions, averages, recent trends and the

institution's credit terms to see how effectively the credit department is managing debts. If the results appear to be not consistent and not effective or efficient. This could mean that the credit manager is not enforcing standards closely enough, or the collection policy being too lax.

Credit policy involves making decisions regarding:
- o Credit worthiness
- o The collection policy and settlement discounts (3/15 net 30)

THEME 2

G42

The real problem can be specified as the sustainability of the catering section which is under threat given the inconsistent usage of the catering section by students and the general financial constraints faced by the institution.

If the catering section spends R5000.00 for plates and R3000.00 for fast food & R1500.00 students buy fast food per day and 500 students buy plates per day.

e.g. Price (plate) = R10.00
Fast-food e.g. chips = R2.50
Bread = R4.00

Then let X be number of students who buy fast food
& Y be the number of students who buy plates

X + Y <= R2000.00
Y<= R5000.00
X<= R3000.00

500Y + 300X <= R8000.00
1500X + 500Y <= R2000.00

Y = 8/5 if X=0
  = 1,6
8/3 * 3/5X = 8/5 * 5/3 if Y=0
        X = 8/3
        X = 2,6

Y = 4 if X=0
X = 4/3 if Y=0
Profit for plates = R2000.00
Profit for fast food = R1000.00

Then P = 2000X + 1000y
      = (2000 * 9) / 10 + (1000 * 12) / 10
      = 1800 + 1200
      = 3000

- o If students visit the catering section maybe once a week or occasionally tat means food becomes decayed and cannot be eaten so must be dumped which is a waste.
- o When coming to toppings, people visit so many shops and restaurants and they eat very delicious food, then if they found that the food they get from catering section is not the kind of food they are expecting, there is no way in which they can go to catering section and buy food.
- o The price is something that people take into consideration, meaning that they compare before they buy.

THEME 4

G44	
DAY LECTURERS	5
EVENING LECTURERS	5
DUPLICATED COURSES	4
NON DIPLICATED COURSES	1
TOTAL NUMBER OF LECTURERS	5
Distribution of Lecturers per course	1

G44

Let x and y represent day and evening lectures respectively. Let's say maybe we have 5 lecturers for the day, these lecturers know more than one course, meaning that they will be able to lecture again in the evening.

We assume that 4 courses are duplicated, that means lecturers responsible for those courses during the day will be responsible for them in the evening. For non-duplicated course one lecturer left, will take it, as he will be knowing more than one course like other lecturers. From that we can say:

$X <= 5$

$Y <= 5$

$X + y <= 5$

Each lecturer can work approximately 8 hours

From the total number of lecturers we can say $5x + 5y <= 10$ to determine the number of courses a lecturer can offer each day.

THEME 3

G43

Transport for day and evening students

G43 (a) **FOR DAY LECTURES**

Let x be the number of female students attending day lectures and equal to 213.

Let y be the number of male students attending day lectures and equal to 149.

$213x + 149y <= 362$

For 362 students at least 24 taxis are required.

From Phuthaditjhaba to that institution 6 taxis are suitable for 90 students.

From Harrismith to that institution 3 taxis are suitable for 45 students.

From Bethlehem to that institution 1 taxi are suitable for 15 students.

From Tseki to that institution 5 taxis are suitable for 75 students.
From Riverside to that institution 5 taxis are suitable for 75 students.
From Elite to that institution 1 taxi are suitable for 15 students.
From Clubview and Bluegumbosch to that institution 3 taxis are suitable for 45 students.

G43 (b) **FOR EVENING LECTURES**

Let x be the number of female students attending day lectures and equal to 124.
Let y be the number of male students attending day lectures and equal to 138.
$124x + 138y \leq 262$

For 262 students at least 17 taxis are required.
From Phuthaditjhaba to that institution 6 taxis are suitable for 90 students.
From Harrismith to that institution 2 taxis are suitable for 30 students.
From Bethlehem to that institution 1 taxi are suitable for 15 students.
From Tseki to that institution 2 taxis are suitable for 30 students.
From Riverside to that institution 3 taxis are suitable for 45 students.
From Elite to that institution 2 taxis are suitable for 30 students.
From Clubview and Bluegumbosch to that institution 2 taxis are suitable for 30 students.

No taxi driver can be able to carry only one person from anyone of these places to that institution because of the great distance between them. It is better for the students to group themselves so that it becomes easier for the taxi driver to transport them.

GROUP 5

THEME 1: STUDENT DEBT REDUCTION PLAN

G51

8. Increasing debt on student fees (real problem)
9. $A = P (1-r/100)^n$

 $A =?$

 $P =16000$

 $R = 5\%$

 $N = 12$

10. The solution above will decrease so that the student can afford to pay his/her fees.
11. We can use the compound interest formula to identify the money that student will pay.
12. The management of the institution must disagree to allow student to graduate while he/she still owes the university.
13. The student must not be re-admitted the following year if he/she owes the past year.
14. They must disallow the student to be a resident student if they agree to admit him/her and if he was a resident student previous year.

THEME 2: USAGE OF UNIVERSITY CATERING SECTION BY STUDENTS

G52

1. The inconsistent usage of catering section by the students and general financial constraints (real problem)
2. There is no model that can be used or formula but that institution needs income from it's catering section.
3. In reality it is clear that institution will expect benefits because they are willing to benefit as they are selling.

GROUP 6

G61. A report is hereby presented after a specified research has been done. Although these consist primarily of estimations, but the exact solutions could be obtained if followed thoroughly. The solution for the first question was found to be as follows: The real problem was that the actual amount of the money the students had to pay was not specified hence the assumption was made. Debt was found being equal to the total amount paid by the students subtracting half amount paid by other students, hence the formula was:

Debt = total amount paid – half amount paid

(the model was subtraction in this case)

The suggestion was that the management should see to it that students to be admitted be proportional to the previous admitted ones in number so to curb on further increases of debt. For the calculations, please refer to the attached paper – where rough work was done.

G62. The solution for the second question / problem:

The real problem was found to be the actual amount of money the management uses for

catering which was not specified / stated. Hence the assumption was made also for

this problem. Following this sort of specimen the exact solution could be obtained

for this particular problem.

Catering money was found to be equal to the total money paid by the students for catering divided by number of months then subtract 10 percent of the total amount. The 10 percent subtracted is for the transport used for buying those foods. For more details and other steps to be followed, then the attached paper serves as the guidelines for solution number 2.

G63. The solution for the third problem:

The real problem was found to be about the data which was given below as a guide, the

main problem about it was that the number of students staying at those different

places was not known. But because the number of the student attending at those

different time scheduled was specified, hence the calculations further info, refer to

the attached paper where calculations were being worked out.

G64. The solution for the fourth problem:
The real problem was that the number of the lecturers and courses was not specified hence

the speculations were done for this particular problem.

The number of lecturers to offer a specific course was determined to be equal to the total number of lecturers divided by the total number of courses offered for the steps followed, then the attached paper serves as a reference.

Elaborations/Illustrations

G61 (i)
   a) The real problem is that we do not the actual amount of money the students have to pay, therefore the supposition should be applied.
   b) The model to be used is subtraction.
   c) Debt = Total amount – Half amount (model: subtraction)
   d) Suppose the institution has 200 students and each student has to pay R100.00 for the fees. But it there happens that only 190 students are able to pay full amount and to students are able to pay half of the required money
        1 : 100
        190 : x
        19000 = x ; x=> Total amount for 190 students
        1 : 50; 500 = x => Total amount for 10 students
Therefore Total amount = 19 000 + 500 = 19500
If then all paid full amount, then
1 : 100
200 : x => x=20000
Therefore debt = 20000 – 19500 = 500
Therefore to curb on further increases the management to see to it that when they admit students to what they had i.e if say there come 20 students then at least 5 students should be able to pay half and the rest of 15 should be able to pay full amount. By so doing, it will mean that the rate of debt increase would be curbed and hence it will be constant.
Debt = Totalamount – Halfamount .
   e) The totalamount subtract halfamount equals debt.
   f) If we compare this with reality: If you increase the number over the other that has subtracted something then it should be subtracted by the same number so to maintain proportionality.

G62 (ii)
a.   The real problem is that we do not actually know how much management uses for catering the students in institution D. Hence we need to assume /to make estimations. Supposing the institution D has 200 students and each student has to pay R100.00 for the meals covering six months   1:100
        200:x Therefore R20000.00 => total amount the students will be paying
        Therefore catering money=total money (paid by students)/6 months- 10% of
R2000.00
        = R20000.00 – R2000.00
        = R18000.00   {R2000.00 for transport}
            Year = 365 days
            Number of days in a year/moths of the year = 365/12

$$= 6.5$$

365/2 = Number of days in 6 months

182.5/3 => 61 of eaten times per semester

Therefore 18000/60=300

18000/182.5=99 => amount which should be used per day.

G63 (iii)

a) The real problem when considering the below data is that we do not know how

many students are staying at those different places stated below.

b) Proportionality should be used as a model.

c)

T:15   t=15

Xt:362   xt=262

d) The total number of the day students =213+149=362

362/8=46 (rounded off) => groups of students.

Number of students = 362/15 = (rounded off) => number of taxis needed

T:15

Xt:262

15xt/15t=262t/15t

=17+(1*(4+1)taxis)

G64 (iv)

a. The real problem is that the number of lecturers and courses offered is not specified

b. Division should be used as the model

c. Number of lectures offer 1 course =Total number of lecturers/number of courses

d. Supposing the institution has 10 lecturers and 5 courses then, number of lecturers to offer one course =Total number of lecturers/number of courses=10/5=2 therefore one lecture will teach during the day and the other one during the day.

The reason why division was used:

➢ Division makes it possible for us to give equal responsibilities to the number of lecturers found, therefore division brings a balance of courses, therefore there will be an elimination of inconsistence in the institution.

➢ The reason why proportionality was used is because the number of passengers to be taken by a taxi was known and it is 15 and this.

SUMMARY OF GROUPS' RECOMMMENDATIONS (SOLUTIONS)

THEME 1. STUDENT DEBT REDUCTION PLAN

REC 11.

OPTION 1. DEBTORS MANAGEMENT

The management of debts starts with the decision to grant debts, which is the full responsibility of the credit manager who will need an effective debts control system. In many cases where there's no credit policy, debts may build up, cash flows decline and the

bad debts arise. Therefore the institution should have its credit policy. And then the university should encourage the high volume of students admitted and credit manager should focus on minimizing the collection period. A certain discount can be given to the **students who will** *pay within the first 15 days. The management should analyze the average collection period and its ageing schedule in comparison with other institution's averages, recent trends and the institution's credit terms to see how effectively the credit department is managing debts.*

REC 12. <u>OPTION 2.</u>

To curb the increase in debts, the number of students admitted should be proportional to those who are currently enrolled with the institution i.e. if there 20 students then 15 students should be able to pay full amount and at least 5 be able to pay half amount.

REC 13. <u>OPTION 3.</u>

There should be strict rules in admitting the continuing student who are able to pay their fees.

> ➤ *Student owing the institution should be denied the chance to graduate until all debts are settled.*
> ➤ *Owing student must not be re-admitted the following year unless they first settle their debts.*
> ➤ *If the student was a resident student then if he/she admitted still owing then he/she must be allowed to stay in the residences.*

REC 14. <u>OPTION 4.</u>

The institution should employ financially needy student and a certain percentage be deducted at the end of every month as the payment to their debts. Or the institution should search for external/internal bursaries for academically performing and financially needy students.

REC 15. <u>OPTION 5.</u>

<u>UNREGISTERED STUDENTS</u>

People earning R10 000.00 p.a. are expected to pay 60% of their salary per annum, this being to ensure that double standards are avoided. And the person who is unemployed is expected to pay R5 334.25 p.a. and through these the institution will recover the debts within a period of 10 years.

REC 16. <u>REGISTERED STUDENTS</u>

If every student who owes the institution more than R16 000.00 which is the maximum allocation from NASFAS can get the maximum allocation and pay 20% of the debt and registration for the year then the institution will recover debts in a short period of time.

REC 17. <u>OPTION 6.</u>

There should be a contract between the parent and the institution to pay directly into the institution's account and all invoices be forwarded to the parent as a reminder of the amount that is due to the institution. Earning parents should arrange with the institution as to how are they going to pay the debts by installments.

THEME TWO
THE USAGE OF UNIVERSITY CATERING SECTION BY THE STUDENTS
REC 21. OPTION 1.

If the catering section has two sections that is fast food and booking section then the management should close down the fast food section. The reason being that fast food section has NO guarantee on the quantity of items (Russians, chips, etc) that will be sold a day. And if they are prepared and not sold then it is a waste and a loss at the same time. However, in the booking section bookings are made a day before and this leaves the catering staff with enough time to work on what and how they are going to cater for the next day.

REC 22. OPTION 2.

If 400 students utilize the D.Hall everyday for three meals a day i.e. breakfast (R6.50c), lunch (R9.50c) and supper (R9.00), this sums up to R25.00 per student a day. And this amount or less of it can be used to cater one student a day. Or if R20.00 is used to cater one student a day then R5.00 is saved and that is a lot of profit on the management's side. Nevertheless, the staff must try as hard as they can to work on the bookings made a day before catering is done. For the fast food part, there can never be claims of financial constraints because that section is always more expensive that shops outside the campus.

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THEME 3.
REC 30. TRANSPORT FOR DAY/EVENING STUDENTS.

The students should be grouped according to routes (places) and times (day/evening). And this is for the convenience and the fact that it costs the same amount to go to any of them. For the places with the large number of students buses will be used and for the smallest taxi will be used. And the management should negotiate with transport services for a discount in every route that is outlined. Students should be urged to buy monthly tickets and also be urged to leave the campus at almost the same time. And for taxis, students should pay transport fare on hand to the institution on weekly or monthly basis, as this will solve the issue of no taxis and no money for transport.

_____

THEME FOUR
DAY AND EVENING LECTURES
REC 41. OPTION 1.

The number of courses should be divided according to the lecturers available, e.g. if there are 10 lecturers and five courses then there will be two lecturers per course. Therefore one lecturer will teach during the day and the other during the evening or vice versa.

REC 42. **OPTION 2.**

DAY LECTURERS	5
EVENING LECTURERS	5
DUPLICATED COURSES	4
NON DIPLICATED COURSES	1
TOTAL NUMBER OF LECTURERS	5
Distribution of Lecturers per course	1

If we have 5 lecturers and five lectures for the day, with lecturers knowing more than one course, then this means that they will be able to lecture again in the evening.

We assume that 4 courses are duplicated, that means lecturers responsible for those courses during the day will be responsible for them in the evening. For non-duplicated course one lecturer left, will take it, from knowledge of more than one course like other lecturers.

Each lecturer can work approximately 8 hours.

Reactions to Recommendations : Usage of university catering section by the students : Sectional head : Managing Director.

RES 11. Option 1 (typed as given)

It is of paramount important for the catering section, particularly the Dining Hall to use both fast food section and the booking section, the motivating factor being that not all student can afford a full meal on daily bases, but could afford as little as possible to buy a packet of chips with bread, or a loaf of bread with milk. It is of course partially true that the fast food section has no guarantee of how many students will buy every day, but it is also gives the company in charge to market itself vigorously.

RES 12. Option 2

Yes, if students utilize the D. Hall everyday for three meals a day, i.e. breakfast at R9.50 not (R6.50) lunch and supper at R15.00, this will mean each student will use R39.50 per day. Also we should be mindful of the fact that food is expensive and prices are going up every month. In conclusion it is really necessary to keep both fast food section and the booking system to cover everybody.

Input and response by Queen & Catering Services

Managing Director :

Reactions to Recommendations on Transport theme : Joint report by the Taxi Association

Response type as it was reported.
RES 30.

According to Itshokolele Taxi Association, the recommendation is good but it can mainly be used for bus services. This is because in Taxi Associations there is no discount. Primarily because there is no subsidy from the government that is directed to the taxi industry. Secondly, taxi drivers have a tendency of not keeping time due to various reasons like having to pick some loads, bad conditions of the vehicle and the problem with the individual drivers and students.

The other point is that people/owners of the best taxi (conditions) are in most cases not willing to help desperate people. They are always complaining of bad roads and others things. Thus, they will not be in a position of picking and dropping student at their prospective places. Some will not let their taxis pick and drop evening student fearing the recent hijacks.

Owners of taxis that are in good conditions might be willing to help but student won't be satisfied. As it is known that students do not want to use bad cars that can easily brake down even before the final destination.

In conclusion, the bus services are one's that will help as far as transportation of students is concern. Looking at the fact that the are subsidized by the government, the buses are always in good conditions and they are always on time!!!.

(QwaQwa taxi Association didn't respond as they have promised)

APPENDIX 6: RESPONSE SHEET TYPES OF TASKS

Participants	Gender	Age	Math Topic 1	Math Topic 2	Math Topic 3	Math Topic 4	Math Topic 5	Math Topic 6
1	F	20-25	1-For calculating statistics & economic matters	2-They do not enable me to explore	1-Avoid being stereo typed	2-I don't like cramming proofs	1-Rules and formulae are unique that is simply problems.	1-Easily understood
2	F	20-5	1-Because one can refer or relate to real life when solving a problem	3-They do not allow one to come up with other ways of solving a problem	1-because they do not restrict, there are always alternatives to solve a problem	1-because u just don't get it right you have to understand the problem	2-because they can be forgotten easily especially formulae	No reason
3	F	20-5	1-good because they develop a person to be able to analyse even the real life problems consistently	3-Dislike them because one is channelled to get the correct answer. This at some stage forces one to ever forge the problem so to get the right one.	1-Good because gives people freedom to express themselves and to prove different procedure of getting to the answer.	1-they give chance to people to use their minds constructively and sufficiently	1-Do not put much stress on people as long as one understands the problem then one should just recall the formula	2-some how these problems are interesting and somehow are not.
4	F	20-25	3-I see no reason to relate them in real life	1-They allow you to make fast when coming to answer	2-they do not restrict you, there are more possible answers	1-they do not need you to be sure. They train your mind in order to have knowledge	2-you can forget them easily	2-they make u not to think of anything else
5	M	20-5	1-always more practical	2-You always have to remember the methods to get to the answer	1-interesting	1-they are better exercising your mind	1-reflect your mathematical knowledge	2-tedious
6	M	20-5	1-when everything brought to practicality makes sense	1-using the right formula at the right time	1-using other mathematical formulae to find other ways of solving it	1-for understanding the basics	1-facts,rules and formulae help to solve problems easily	1-leads to correct answer
7	M	20-5	1-u can apply mathematics to solve problems	2-Need to concentrate in one solution and not to think of other possibility	1-I think it is open in order to find possible answers in many ways	2-need to understand what you are doing not always formulas in order to solve	2-u can forget formulas even if you know how to solve problem	1-Good because u need to know where you are going to start
8	F	20-5	1-Because u don't have to	3-Dislike cause not	2-either open ended	1-Good cause one must give	1-Good cause in memory	2-procedures

			concentrate only on the memorising isolated facts	closed ended problems	problems depending on one answer	proofs so that there will be conceptual understanding	one have to recall formulas	are always needed
9	F	20-5	1-They are practical	2-they are much difficult and they need much time to practice	1-they are easy	1-they train your mind	1-they are interesting	1-easy to remember
10	F	20-5	1-can apply maths in real life to solve problems	3-It will make me to be stereo-typed to concentrate on one solution	1-It gives opportunity to solve a problem in many ways	3-They need lot of practise and they are making Maths a difficult subject	1-they used to be easy to understand	1-Everything in life should go in procedure
11	M	b-20	2-They do not give good thought of attempting a problem	1-presenting them is confidential	1-Good and challenging if one understand proof-written attributes	1-One have to make effort be willing to do	1-Good especially when facts are proved precisely	They channel one's mind
12	F	20-5	1-They are practical	3-They make people stereo-typed	1-It makes your mind to be broad	1-They help you to be logical	2	1-They give a clear picture about what you are doing
13		20-5	1-Helps to solve problems that can cause people pain	2-Is not all problems pertaining to life that have only one answer	1-They had to be relevant to the situation involved	1-improves creativity	3-make most people to leave mathematics	3-Does not give you chance to think
14	M	20-5	1-Enable one to use critical thinking and broaden his/her mind	3-They stereo type one's mind or channel his/her thinking capacity	1-One can tackle a problem from different angles or try different methods to solve it. It does not channel one's mind	1-One is able to express more about solution and even decrease the rate of impossible solutions	3-If one is unable to recall he/she cannot solve a problem	3-Disgusting
15	F	20-5	1-the numbers and real situation combination? Interesting learning	1-You just know when the answer is correct, one cannot just go wrong	2-The possibility off more than one answer might be confusing and you are at risk	1-They are lovely to work with. It just proves that you know if you give reasons to you answer.	2-You might mix formulae and move likely to go wrong	3-Boring and unchalleng ing
16	M	20-5	1-they help to solve life problems	3-One has to struggle for only one	1-they give a variety of solution and	3-Understanding a concept and	3-That's cramming which does	2-they are ok but one needs to

				solution which is stressing	every solution is possible	proving need a clear understanding and a lot of practicing and memorising theorems	not improve one's understanding	follow the procedure which is tiring sometimes
17	M	20-5	2-Because there are many possible answers.	1-You can be be sure about the correct answer.	2-You cannot be so sure about the correct answer	1-because theorems do not fail	3-They channel the memory	1-Gives challenge
18	M	20-5	1-They are easy to solve because they relate to real life situation	1-They help me to know what type of an answer must I get	1-They make me be able to choose different methods when solving them	1-They enlarge my understanding	1-They make my memory to work fast	NO ANS
19	M	20-5	1-Prepares for the life situation regarding maths	3-Cause if u get the answer wrong that is u are wrong	1-Cause no matter how you tackle the problem you gonna get the answer	3-If you do not know the theorem you wont solve the problem	3 you forget the formula you are dead	1-You only need procedure
20	M	20-5	1-They are practical	3-They are not challenging	1-They encourage one to explore	3-They are very difficult for me	1-They train my memory	1-They guide me
21	F	20-5	2-They don't really relate to life situations	3-Why should one be restricted to what they should answer	1-They give one a chance to explore, try something from many different approaches	1-Teaches one to memorise	1-Teaches one to be able to use memory	2-too much restriction
22	M	20-5	To know what you are really talking about put it practically	2-Channeling problems are not that much needy, you had to be broad	1-Here people should be careful because they are determined to use their critical thinking	1-Solve the problem and be able to explain what the answer means	1-Be sure what the problem really needs	Problems should be solved accordingly
23	M	20-5	1-Because Mathematics applies everywhere eg for drafting, budget, inside our public transport etc.	1-Because one have to endevour in order to obtain the exact answer, it somehow make sense	3-No one can claim to obtain a wrong answer of which is a mess.	1-It is where one can figure out or show up what's she/he is capable of	3-I hope its all about cramming and hence I cannot show up my intelligence	3-They are boring
24	M	20-5	2-It can be related to life	1-It helps to solve a	2-If there is more than	1-They help a person to be	1-It helps to keep your	1-One needs to

			situation or not	problem in many ways but to find one solution	one answer people will think their way	creative	mind thinking	follow procedure
25	M	20-5	1-The make one feel like you're contributing to solving human problems	3-When one has failed to answer the problem, one feels incompetent in Maths	1-They reveal one's intellectual, know how on approaching a problem and solving it	3-Why should I prove something that is written by somebody else	1-They seek one's knowledge of past lectures, recalling what was once taught, how far can one remember	1-The information stays because you do the same thing everytime
26	M	20-5	2-'Cause I've never yet experienced such situation	3-I always want to expose my opinion to any problem	2-So that I can have more choices related to that question	3-This is one that I am currently doing and it irritates me because I lack books	2-In maths u cannot memorize all the formulae, rules etc	2-Yes procedures need to be followed
27	F	20-5	1-practical	2	2	3-Too difficult	2-	1-Train memory
28	F	20-5	2-Life situations may or may not be related to Mathematics	3-They channel me to one possible answer	1-I can solve in many different steps or methods as I like	3-Some theorems are hard to proof	3-If it happens that I forget the formula, I may not be able to solve it	NO ANSWER
29	M	20-5	2	1	2	3-theorems	3-I hate them	2-Boring
30	M	20-5	3	1	3	3	3	3

APPENDIX 7: FIRST YEAR MATHEMATICS SYLLABUS

MAT 111 Algebra (credit units: 16)
Polynomials; the remainder and factor theorems; polynomial equations and inequalities, especially linear, quadric and cubic; domain and zeroes of rational function; partial fractions; curve sketching of polynomial and rational functions.

The principles of mathematical induction and its application to properties of natural numbers; permutations and combinations; the binomial theorem for any index and applications; sequences, series arithmetic progressions, geometric progressions, limits and sums to infinity; first and second differences of a sequence.

Addition, subtraction, multiplication and division of complex numbers; fundamental theorem of algebra (statement only); the argand diagram; De Movre's theorem; n^{th} roots of complex numbers. Introduction to m x n matrices where m,n ≤ 5; elementary operations on matrices and applications to solution of linear equations; elementary properties of determinants of at most 3 x e matrices.

Transformations of the plane: translation, reflection, rotation, magnification, shear, composition of transformations – invariant points and lines.

Lectures: 4 hrs per week
Tutorials/Practicals :4 hrs per week

Examination: 1 paper of 3 hrs

MAT 112 Vector Algebra (credit units: 16)

Equations of lines and planes, conic sections – circle, parabola, hyperbola, ellipse.
Vectors in R^2 ,R^3; scalar product, vector product, triple product; applications.
Derivatives of hyperbolic functions, inverse circular/hyperbolic functions;
Method of taking the logarithm before differentiating, successive differential of implicit functions and functions like $e^{at}\sin (bt + c)$; approximations.
Hard integration; further techniques-evaluation of $f\{t^2 + a^2\}^{-1}dt$, integration of irrational functions, integration via harder substitution, e.g. trigonometric substitutions, integration by parts.

Lectures: 4hrs per week
Tutorials/Practicals: 4 hrs per week

Examination: 1 paper of 3 hrs

MAT 121 Fundamentals of Linear Algebra

Systems of linear equations, Gauss Jordan, Euclidean plane and Euclidean space and vectors in them, Points and lines in the plane, lines and planes in the space, Linear transformation in the plane, Simplex method, scalar product, Vector product, Complex numbers.

MAT 122 Analysis

Definition of limit and continuity, Convergence and divergence of sequences and series, Definition of derivative integral, Differentiation and integration rules, Fundemental theorem of calculus, Mean value theorem and maxima, minima, Taylor expansion, Introduction of higher functions

Lectures: 4 hours hours per week
Tutorials/Practicals: 4 hours per week

APPENDIX 8: Extracts from Norms and Standards for Teacher Educators
APPENDIX 8: Extracts from Norms and Standards for Teacher Educators **Learning mediator**

Practical competences
(Where the learner demonstrates the ability, in an authentic context, to consider a range of possibilities for action, make considered decisions about which possibility to follow, and to perform the chosen action.)
Using the language of instruction appropriately to explain, describe and discuss key concepts in the particular learning area/subject/discipline/phase.
Using a second official language to explain, describe and discuss key concepts in a conversational style.
Employing appropriate strategies for working with learner needs and disabilities, including sign language where appropriate.
Preparing thoroughly and thoughtfully for teaching by drawing on a variety of resources; the knowledge, skills and processes of relevant learning areas; learners' existing knowledge, skills and experience.
Using key teaching strategies such as higher level questioning, problem-based tasks and projects; and appropriate use of group-work, whole class teaching and individual self-study.
Adjusting teaching strategies to: match the developmental stages of learners; meet the knowledge requirements of the particular learning area; cater for cultural, gender, ethnic, language and other differences among learners.
Adjusting teaching strategies to cater for different learning styles and preferences and to mainstream learners with barriers to learning.
Creating a learning environment in which: learners develop strong internal discipline; conflict is handled through debate and argument, and learners seek growth and achievement.
Creating a learning environment in which: critical and creative thinking is encouraged; learners challenge stereotypes about language, race, gender, ethnicity, geographic location and culture.
Using media and everyday resources appropriately in teaching including judicious use of: common teaching resources like text-books, chalkboards, and charts; other useful media like overhead projectors, computers, video and audio (etc); and popular media and resources, like newspapers and magazines as well as other artefacts from everyday life.
Foundational competences
(Where the learner demonstrates an understanding of the knowledge and thinking which underpins the actions taken.)
Understanding different explanations of how language mediates learning: the principles of language in learning; language across the curriculum; language and power; and a strong emphasis on language in multi-lingual classrooms.
Understanding different learning styles, preferences and motivations.
Understanding different explanations of how learners learn at different ages, and potential causes of success or failure in these learning processes.
Understanding the pedagogic content knowledge – the concepts, methods and disciplinary rules – of the particular learning area being taught
Understanding the learning assumptions that underpin key teaching strategies and that inform the use of media to support teaching.
Understanding the nature of barriers to learning and the principles underlying different strategies that can be used to address them.

Understanding sociological, philosophical, psychological, historical, political and economic explanations of key concepts in education with particular reference to education in a diverse and developing country like South Africa.
Exploring, understanding, explaining, analysing and utilizing knowledge, skills and values underpinning ETD practices.

Reflexive competences
(Where the learner demonstrates the ability to integrate or connect performances and decision making with understanding and with the ability to adapt to change and unforeseen circumstances and explain the reasons behind these actions.)
Reflecting on the extent to which the objectives of the learning experience have been achieved and deciding on adaptations where required.
Defending the choice of learning mediation undertaken and arguing why other learning mediation possibilities were rejected.
Analysing the learning that occurs in observed classroom interactions and in case studies.
Making judgements on the effect that language has on learning in various situations and how to make necessary adaptations.
Assessing the effects of existing practices of discipline and conflict management on learning.
Reflecting on how teaching in different contexts in South Africa affects teaching strategies and proposing adaptations.
Reflecting on the value of various learning experiences within an African and developing world context.
Reflecting on how race, class, gender, language, geographical and other differences impact on learning, and making appropriate adaptations to teaching strategies.
Critically evaluating the implications for schooling of political social events and processes and developing strategies for responding to these implications.
Critically reflecting on the ways barriers to learning can be overcome.
Critically reflecting on the degree to which issues around HIV/AIDS have been integrated into learning.
Analysing the strengths and weakness of the ways in which environmental, human rights and other critical cross-field issues have been addressed.

Interpreter and designer of learning programmes and materials

Practical competences
(Where the learner demonstrates the ability, in an authentic context, to consider a range of possibilities for action, make considered decisions about which possibility to follow, and to perform the chosen action.)
Interpreting and adapting learning programmes so that they are appropriate for the context in which teaching will occur.
Designing original learning programmes so that they meet the desired outcomes and are appropriate for the context in which they occur.
Adapting and/or selecting learning resources that are appropriate for the age, language competences, culture and gender of learning groups or learners.
Designing original learning resources including charts, models, worksheets and more sustained learning texts. These resources should be appropriate for subject; appropriate to the age, language competence, gender, and culture of learners; cognisant of barriers to learning.
Writing clearly and convincingly in the language of instruction.

Using a common word processing programme for developing basic materials.
Evaluating and adapting learning programmes and resources through the use of learner assessment and feedback.

Foundational competences
(Where the learner demonstrates an understanding of the knowledge and thinking which underpins the actions taken.)
Understanding the principles of curriculum: how decisions are made; who makes the decisions, on what basis and in whose interests they are made.
Understanding various approaches to curriculum and programme design, and their relationship to particular kinds of learning required by the discipline; age, race, culture and gender of the learners.
Understanding the principles and practices of OBE, and the controversies surrounding it, including debates around competence and performance.
Understanding the learning area to be taught, including appropriate content knowledge, pedagogic content knowledge, and how to integrate this knowledge with other subjects.
Knowing about sound practice in curriculum, learning programme and learning materials design including: how learners learn from texts and resources; how language and cultural differences impact on learning.
Understanding common barriers to learning and how materials can be used to construct more flexible and individualised learning environments.

Reflexive competences
(Where the learner demonstrates the ability to integrate or connect performances and decision making with understanding and with the ability to adapt to change and unforeseen circumstances and explain the reasons behind these actions.)
Reflecting on changing circumstances and conditions and adapting existing programmes and materials accordingly.
Critically evaluating different programmes in real contexts and/or through case studies both in terms of their educational validity as well as their socio-political significance.

Leader, administrator and manager

Practical competences
(Where the learner demonstrates the ability, in an authentic context, to consider a range of possibilities for action, make considered decisions about which possibility to follow, and to perform the chosen action.)
Managing classroom teaching of various kinds (individualised, small group etc.) in different educational contexts and particularly with large and diverse groups.
Constructing a classroom atmosphere which is democratic but disciplined, and which is sensitive to culture, race and gender differences as well as to disabilities.
Resolving conflict situations within classrooms in an ethical sensitive manner.
Promoting the values and principles of the constitution particularly those related to human rights and the environment.
Maintaining efficient financial controls.
Working with other practitioners in team-teaching and participative decision making.
Accessing and working in partnership with professional services and other resources in order to provide support for learners.
Respecting the role of parents and the community and assisting in building structures to facilitate this.

Foundational competences

(Where the learner demonstrates an understanding of the knowledge and thinking which underpins the actions taken.)

Understanding approaches to problem-solving, conflict resolution and group dynamics within a South African and developing world context characterised by diversity.

Understanding various approaches to the organisation of integrated teaching programmes and team teaching.

Understanding various approaches to the management of classrooms, with particular emphasis on large, under-resourced and diverse classrooms.

Knowledge of available professional and community support services and strategies for using their expertise.

Understanding current legislation on the management of learners and schools.

Knowledge of educators' unions, the South African Council for Educators and other relevant professional bodies.

Understanding constitutional commitments to human rights and the environment.

Reflexive competences

(Where the learner demonstrates the ability to integrate or connect performances and decision making with understanding and with the ability to adapt to change and unforeseen circumstances and explain the reasons behind these actions.)

Reflecting on strategies to assist educators working on integrated teaching programmes and in team teaching.

Critically examining a variety of management options, making choices based on existing and potential conditions, and defending these choices.

Adapting systems, procedures and actions according to circumstances.

Community, citizenship and pastoral role

Practical competences

(Where the learner demonstrates the ability, in an authentic context, to consider a range of possibilities for action, make considered decisions about which possibility to follow, and to perform the chosen action.)

Developing life-skills, work-skills, a critical, ethical and committed political attitude, and a healthy lifestyle in learners.

Providing guidance to learners about work and study possibilities.

Showing an appreciation of, and respect for, people of different values, beliefs, practices and cultures.

Being able to respond to current social and educational problems with particular emphasis on the issues of violence, drug abuse, poverty, child and women abuse, HIV/AIDS and environmental degradation. Accessing and working in partnership with professional services to deal with these issues.

Counselling and/or tutoring learners in need of assistance with social or learning problems.

Demonstrating caring, committed and ethical professional behaviour and an understanding of education as dealing with the protection of children and the development of the whole person.

Conceptualising and planning a school extra-mural programme including sport, artistic and cultural activities.

Operating as a mentor through providing a mentoring support system to student educators and colleagues.

Foundational competences

(Where the learner demonstrates an understanding of the knowledge and thinking

which underpins the actions taken.)
Understanding various approaches to education for citizenship with particular reference to South Africa as a diverse, developing, constitutional democracy.
Understanding key community problems with particular emphasis on issues of poverty, health, environment and political democracy.
Knowing about the principles and practices of the main religions of South Africa, the customs, values and beliefs of the main cultures of SA, the Constitution and the Bill of Rights.
Understanding the possibilities for life-skill and work-skill education and training in local communities, organisations and business.
Knowing about ethical debates in religion, politics, economics, human rights and the environment.
Understanding child and adolescent development and theories of learning and behaviour with emphasis on their applicability in a diverse and developing country like South Africa.
Understanding the impact of class, race, gender and other identity-forming forces on learning.
Understanding formative development and the impact of abuse at individual, familial, and communal levels.
Understanding common barriers to learning and the kinds of school structures and processes that help to overcome these barriers.
Knowing about available support services and how they may be utilised.
Knowing about the kinds of impact school extra-mural activities can have on learning and the development of children and how these may best be developed in co-operation with local communities and business.
Reflexive competences
(Where the learner demonstrates the ability to integrate or connect performances and decision making with understanding and with the ability to adapt to change and unforeseen circumstances and explain the reasons behind these actions.)
Recognising and judging appropriate intervention strategies to cope with learning and other difficulties.
Reflecting on systems of ongoing professional development for existing and new educators.
Adapting school extra curriculum programmes in response to needs, comments and criticism.
Reflecting on ethical issues in religion, politics, human rights and the environment.
Reflecting on ways of developing and maintaining environmentally responsible approaches to the community and local development.
Adapting learning programmes and other activities to promote an awareness of citizenship, human rights and the principles and values of the constitution.
Critically analysing the degree to which the school curriculum promotes HIV/AIDS awareness.
Critically analysing the degree to which the school curriculum addresses barriers to learning, environmental and human rights issues.
Scholar, researcher and lifelong learner
Practical competences
(Where the learner demonstrates the ability, in an authentic context, to consider a range of possibilities for action, make considered decisions about which possibility to

follow, and to perform the chosen action.)
Being numerically, technologically and media literate.
Reading academic and professional texts critically.
Writing the language of learning clearly and accurately.
Applying research meaningfully to educational problems.
Demonstrating an interest in, appreciation and understanding of current affairs, various kinds of arts, culture and socio-political events.
Upholding the principles of academic integrity and the pursuit of excellence in the field of education.
Foundational competences
(Where the learner demonstrates an understanding of the knowledge and thinking which underpins the actions taken.)
Understanding current thinking about technological, numerical and media literacies with particular reference to educators in a diverse and developing country like South Africa.
Understanding the reasons and uses for, and various approaches to, educational research.
Understanding how to access and use common information sources like libraries, community resource centres, and computer information systems like the internet.
Understanding and using effective study methods.
Reflexive competences
(Where the learner demonstrates the ability to integrate or connect performances and decision making with understanding and with the ability to adapt to change and unforeseen circumstances and explain the reasons behind these actions.)
Reflecting on critical personal responses to, literature, arts and culture as well as social, political and economic issues.
Reflecting on knowledge and experience of environmental and human rights issues and adapting own practices.

Assessor

Practical competences
(Where the learner demonstrates the ability, in an authentic context, to consider a range of possibilities for action, make considered decisions about which possibility to follow, and to perform the chosen action.)
Making appropriate use of different assessment practices, with a particular emphasis on competence-based assessment and the formative use of assessment, in particular continuous and diagnostic forms of assessment.
Assessing in a manner appropriate to the phase/subject/learning area.
Providing feedback to learners in sensitive and educationally helpful ways.
Judging learners' competence and performance in ways that are fair, valid and reliable.
Maintaining efficient recording and reporting of academic progress.
Foundational competences
(Where the learner demonstrates an understanding of the knowledge and thinking which underpins the actions taken.)
Understanding the assumptions that underlie a range of assessment approaches and their particular strengths and weaknesses in relation to the age of the learner and learning area being assessed
Understanding the different learning principles underpinning the structuring of

different assessment tasks.

Understanding a range of assessment approaches and methods appropriate to the learning area/subject/discipline/phase.

Understanding language terminology and content to be used in the assessment task and the degree to which this is gender and culturally sensitive.

Understanding descriptive and diagnostic reporting within a context of high illiteracy rates among parents.

Reflexive competences

(Where the learner demonstrates the ability to integrate or connect performances and decision making with understanding and with the ability to adapt to change and unforeseen circumstances and explain the reasons behind these actions.)

Justifying assessment design decisions and choices about assessment tasks and approaches.

Reflecting on appropriateness of assessment decisions made in particular learning situations and adjusting the assessment tasks and approaches where necessary.

Interpreting and using assessment results to feed into processes for the improvement of learning programmes.

Learning area/subject/discipline/phase specialist

Practical competences

(Where the learner demonstrates the ability, in an authentic context, to consider a range of possibilities for action, make considered decisions about which possibility to follow, and to perform the chosen action.)

Adapting general educational principles to the phase/subject/learning area.

Selecting, sequencing and pacing content in a manner appropriate to the phase/subject/learning area; the needs of the learners and the context.

Selecting methodologies appropriate to learners and contexts.

Integrating subjects into broader learning areas and learning areas into learning programmes.

Teaching concepts in a manner which allows learners to transfer this knowledge and use it in different contexts.

Foundational competences

(Where the learner demonstrates an understanding of the knowledge and thinking which underpins the actions taken.)

Understanding the assumptions underlying the descriptions of competence in a particular discipline/subject/learning area.

Understanding the ways of thinking and doing involved in a particular discipline/subject/learning area and how these may be taught.

Knowing and understanding the content knowledge of the discipline/subject/learning area.

Knowing of and understanding the content and skills prescribed by the national curriculum.

Understanding the difficulties and benefits of integrating this subject into a broader learning area.

Understanding the role that a particular discipline/subject/learning area plays in the work and life of citizens in South African society – particularly with regard to human rights and the environment.

Reflexive competences

(Where the learner demonstrates the ability to integrate or connect performances and

decision making with understanding and with the ability to adapt to change and unforeseen circumstances and explain the reasons behind these actions.)
Reflecting on and assessing own practice.
Analysing lesson plans, learning programmes and assessment tasks and demonstrating an understanding of appropriate selection, sequencing and pacing of content.
Identifying and critically evaluating what counts as undisputed knowledge, necessary skills, important values.
Making educational judgements on educational issues arising from real practice or from authentic case study exercises.
Researching real educational problems and demonstrating an understanding of the implications of this research.
Reflecting on the relations between subjects/disciplines and making judgements on the possibilities of integrating them.