EXPLORING THE INCORPORATION OF MENTAL ARITHMETIC INTO PRIMARY SCHOOL MATHEMATICS: A CASE OF OSHANA REGION, NAMIBIA

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KEYWORDS

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Mathematics for social justice

ACADEMIC EDITOR

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05 April 2019

To Whom It May Concern

Dear Sir or Madam

Re: Editing of Frans Haimbodi 's Dissertation

This is to confirm that I have electronically edited Frans Haimbodi's Dissertation, *Exploring the Incorporation of Mental Arithmetic into Primary School Mathematics: A Case of Oshana Region, Namibia*.

I further confirm that the editing covered the language (grammar), philosophical soundness of ideas, relevance of literature, typing errors, formating style, citations (adherence to standard styles), and proof-reading.

The editing did not, however, cover the list of references, as these are easier checked on a hard copy.

I have 16 years' experience of reviewing and editing written academic work, such as theses, dissertations, research project reports and journal articles. Currently, I am the Chief Editor for the Namibian Educational Research Association (NERA) Journal. I am also the Managing Director of the company, Dynamic Education Solutions, for which editing of academic work is some of its core business.

Even though I edited the mentioned dissertation, the authorship and final responsibility for the edited draft of the mentioned dissertation lie with my client.

Please do not hesitate to contact me at the above-mentioned address, should you require additional information regarding the editing of the dissertation.

Thank you very much.

Sincerely yours,

Dr. Helena Miranda

Signature

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DECLARATION

I Frans Ndemupondaka Haimbodi declare that "*Exploring the incorporation of mental arithmetic into primary school mathematics: a case of Oshana region, Namibia*" is my own work, that it has not been submitted for any degree or examination in any other university, and that all the sources I have used or quoted have been indicated and acknowledged through complete references.

Full name: **Frans Ndemupondaka Haimbodi** Date: **18 /11/2019**

Signed:

DEDICATION

To my beloved children; Tonata & Etuna – may you grow to love mathematics.

To *Meme* Victoria Ndamana *Mukwahepo wa*Hayoonga. For the love that never wavered. For the faith that stood storms.

ABSTRACT

This study, *"Exploring the incorporation of mental arithmetic into primary school mathematics: A case of Oshana region, Namibia*", explored the execution of mental arithmetic strategies in senior primary schools in the northern Namibia.

Informed by the Critical Mathematics Education theory, the study explored the state of mathematics education in primary schools by looking at the computation strategies used by teachers and learners during classroom mathematics sessions at senior primary grades. Mathematics at the senior primary school level is expected to develop learners' functional numeracy and logical thinking in order to be able to apply mathematics in their everyday lives. However, learners recorded low academic achievements in mathematics over the years despite numeral reforms that attempted to address the situation. This study was guided by the research question: How is the senior primary mathematics curriculum incorporating mental arithmetic?

The study sampled 10 schools in the Oshana region in northern Namibia. Senior primary school teachers and learners at these schools were observed during mathematics lessons. The computation procedures used by both teachers and learners were recorded. The study utilised a mixed methods approach in two phases. The first phase consisted of classroom observations, individual teacher interviews and a psychometric mathematics test (pre-test) for the Grade 7 learners. The second phase consisted of a critical intervention for teachers and learners. The intervention involved series of workshops that trained teachers on mental computation strategies. The workshops were followed by a focused group interview for teachers. Out of the 10 schools, four were sampled (two experimental and two control) and the learners in the experimental schools were exposed to mental arithmetic strategies. A mathematics post-test was then administered to all the four schools. The study evaluated the effectiveness of the mental arithmetic approaches on learners' performance.

The teachers and learners mostly practiced standard algorithms even when learners displayed difficulties in comprehending these strategies. In most of the cases, the teachers restricted learners to the use of standard algorithms. It was observed on several occasions that teachers and learners practiced standard algorithms without logic or reasoning. Learners, while adhering to teachers' instructions of using standard algorithms, made use of tallies/counters marked on pieces of papers to aid computation. These findings suggest that an alternative to standard algorithms is needed.

The individual interviews with teachers sought to determine how teachers understood the concept mental arithmetic and their views on its incorporation in the senior primary school curriculum. Teachers had different views of what mental arithmetic is and what it entails. Some teachers believed that mental arithmetic is about memorizing numerical facts and multiplication tables. It emerged that teachers do not practice mental computation strategies as most of them were not aware of these strategies. The prescribed textbooks rigidly made use of standard algorithms. The critical mathematics education pedagogy targets to question the education systems via critical analysis of their practices in order to determine the 'taken for granted' aspects in mathematics education and address such. The study thus identified teachers' shallow understanding of mental computation strategies and hence the lack of mental arithmetic strategies in classroom practices as the aspects the education systems has not taken note of, which may have an influence on the numeracy development and ultimately the performance of learners. Hence, the study identified the need to run a critical intervention – a series of workshops to introduce teachers to mental computation strategies. A quasi-experimental control group design was conducted to test the effects of the mental computation strategies on learners' understanding.

At the end of the intervention, a focus group discussion was conducted with the teachers. The teachers who participated in the intervention expressed hope that the mental computation strategies they got exposed to during the critical intervention can be helpful to develop numeracy in the senior primary schools. The teachers pointed to discrepancies in the Namibian education system where they believe there are contents, such as mental computation strategies, which they were never taught, neither in their school years, nor during teacher training, and yet they are expected to teach such content in the current senior primary curriculum.

The findings from the psychometric tests suggest low level of numeracy among learners. Seven out of the 10 schools scored an average score below 50%. These results concur with earlier results recorded by the Standardised Achievement Tests [SATs] and the Southern and Eastern Africa Consortium for Monitoring Educational Quality [SACMEQ] III and IV. The post-test results saw an improvement in the scores of the learners from the experimental group. The t-tests calculated indicated that the experimental groups scored significantly better than the control groups at $\alpha = 0.05$ significance level.

The study, therefore, recommends an intervention for teachers to enable a democratic practice of a variety of computational strategies in the classrooms as opposed to rigidly adhering to the use of standard algorithms. The curriculum materials used by teachers and learners need to be revised to accord learners the freedom of choice with computational strategies. The study further recommends thorough emphasis of mental computation strategies in mathematics education programmes at teacher training institutions so that the graduating teachers will possess the mental computation strategies and boost up the development of numeracy in the schools.

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This work is complete thanks to many people. I hence hereby acknowledge a few and I apologise for any omissions – I may not finish the list.

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I am indebted to my colleagues at the University of Namibia; Dr Miranda, thank you for your thoughtful analysis in editing and proof-reading this thesis. Dr Naukushu for you have been always available to give quick tutorials. Rauna Haitembu, you have always been there when I needed you, thank you for walking this journey with me. Dr Kambeyo, thank you for proof reading sections of this work.

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Most vital, Thank you father God!

LIST OF ACRONYMS

BES	Basic Education Support
BETD	Basic Education Teachers' Diploma
CME	Critical Mathematics Education
CT	Critical Theory
DNEA	Directorate of National Examinations and Assessment
ETSIP	Education Training Sector Improvement Programme
HCF	Highest Common Factor
HoD	Head of Department
IIEP	International Institute of Educational Planning
IUM	International University of Management
LCM	Lowest Common Multiple
MoE	Ministry of Education
MoEAC	Ministry of Education, Arts and Culture
NIED	National Institute for Educational Development
NVCI	Naukushu's Vicious Cycle of Innumeracy
RME	Realistic Mathematics Education
SACMEQ	Southern and Eastern Africa Consortium for Monitoring Educational Quality
SATs	Standardised Achievement Tests
UNAM	University of Namibia

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CHAPTER ONE: INTRODUCTION

1.1 Introduction

This study was concerned with the classroom practices of mental arithmetic strategies in senior primary schools in northern Namibia. The context of the study is presented in this chapter beginning with a reflection on mathematics education in Namibia prior to and after independence. Also in chapter one, the researcher positions the numeracy level of learners by reflecting on results of major studies done on primary school learners and then discusses the mental arithmetic focus. The researcher then presents the motivation for the study followed by the problem statement, research questions and the research hypotheses. Towards the end of the chapter, the researcher rounds off with a discussion of the study limitations, delimitations, interpretation of key terms and then the outline of the thesis chapters.

1.2 Orientation to the study

Namibia is a vast and meagrely populated country situated along the South Atlantic coastal line of Africa, bordering Angola and Zambia in the north, Botswana in the east and South Africa in the south and south east.

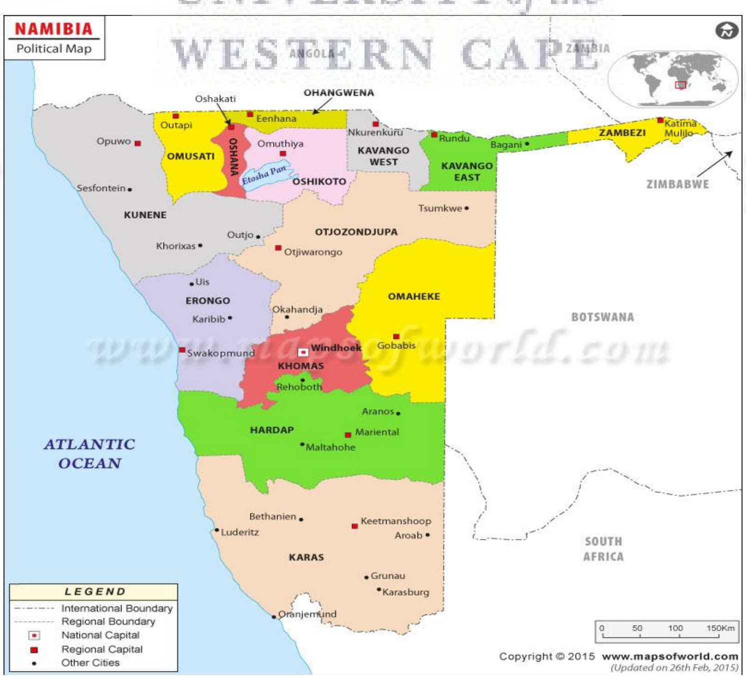


Figure 1.1. The map of Namibia and its 14 administrative regions (Google, 2018).

In Namibia, English is used as the official language. Over 11 indigenous languages are spoken in Namibia and used as medium of instruction in the junior primary grades (Grade 0 to 3). In most urban areas with a high mix of ethnics and their languages, Afrikaans is the lingua franca and it is used as the medium of instruction in the junior primary grades. From Grade 4 to 12, English is the medium of instruction in all public schools. Moreover, Namibia is demarcated into 14 political regions, which also serve as the educational regions (see Figure 1.1). One of these regions is the Oshana region where this study was conducted. Oshana region is in the northern part of the country with most of its schools in the rural areas and a few in the urban or sub-urban areas (Oshakati, Ongwediva and Ondangwa towns).

In the next four sections, the following aspects are discussed in order to further present the background orientation of the study. These are: mathematics education in Namibia pre-independence, mathematics education in Namibia post-independence, the Southern and Eastern Africa Consortium for Monitoring Educational Quality [SACMEQ] III results, and mental arithmetic.

1.2.1 Mathematics Education in Namibia pre-independence

Namibia gained independence in 1990 from South Africa. The education offered during the colonial period was designed to suit the aims of the then regime. Namibia was under the Germany colonial rule from 1885 to 1915 and then under the South African administration from 1920 to independence in 1990. Both Germany and South Africa did not have interest in developing the education of the indigenous people and thus the fight for better education was part of the ideologies behind the national liberation struggle (Peters, 2016).

During the Germany colonial era, the education of the natives was a responsibility of the missionaries who established schools in the rural areas and taught indigenous people how to read and write with the main purpose of spreading Christianity. The curriculum of the missionaries did not go beyond reading, writing and basic numeracy skills. The Germany administration funded some private schools which were meant for Germany children (Amkugo, 1993).

During the South African colonial era, the education system was regarded as highly disproportional due to apartheid – a Nationalist government policy of the early 1950s that targeted to separate whites and blacks. The Bantu education act of 1954 legalised the racially segregated educational facilities in order to provide inferior education for the black majority (Khuzwayo, 2005). During the apartheid era, three separate education systems existed in the country; education for the whites, education for the coloureds and education for the blacks (Amkugo, 1993). The mathematics curriculum, like for any other subject, differed across the three education systems. In the education for Blacks, it was highly perceived that the minds of the Blacks were not meant for mathematics (Amutenya, 2002) and thus the mathematics curriculum for the Blacks did not go beyond basic arithmetic. The education for Blacks placed prominence on achieving basic levels of understanding and encouraged rote learning of mathematics. Even the teachers for the three systems were trained at racially segregated institutions across the country (Naukushu, 2016).

Khuzwayo (2005, p. 317) presents the history of mathematics education in South Africa from the early 1950s when the apartheid education came into full effect. In his work, Khuzwayo highlights how the apartheid administration believed that blacks lag behind in the field of mathematics and could not even "perceive a three dimensional structure". This hypothesis was imparted into the teachers so that they believe Blacks and Whites were not equal and had to be taught different contents. Although Khuzwayo (2005)' s work was about the history of mathematics education in South Africa, the present study wishes to point out that both Namibia and South Africa were under the same white administration rule before 1990 as Namibia was regarded (by the then administration) as another province of South Africa. Hence, the same rules and oppressive strategies were applied in both South Africa and Namibia.

Naukushu (2016) asserts that the supposition that Blacks could not be proficient in mathematics made it difficult for many black learners of mathematics to attain greater levels of understanding mathematical concepts at that time. As argued by Freire (1993), the colonial oppressions used the education systems to maintain the culture of dominance and ensured that the oppressed never attained the liberation of the mind. Affirming Freire's words in the Namibian context are the words of the former Minister of Native Affairs in the pre-independent Namibia, Dr. H.F. Verwoerd, saying:

> When I have control of native education, I will reform it so that natives will be taught from childhood to realise that equality with Europeans is not for them. People who believe in equality are not desirable teachers for the natives. Education must train and teach people

> in accordance with their opportunities in life, in accordance to the sphere in which they live. (Amkugo, 1993, p. 57)

The education systems thus deliberately delivered a very shallow content to the natives to keep them oppressed. The main purpose was to make black people think in specific unyielding ways and deny them chances to develop and perceive the world differently. The target was to make the Blacks accept the oppression as normal and unchangeable (Khuzwayo, 2005). As Khuzwayo (2018) and Skovsmose (2005) indicate, mathematics education operates to both deliver and substantiate assured forms of inclusion and exclusion, thereby serving as a gatekeeper. Denying Blacks access to mathematics and other sciences was the approach to discriminate who develops consciousness and who does not (Khuzwayo, 2018). Many of the students who believed less in mathematics as a result of the system became mathematics teachers in Namibia and hence recycled the lack of mathematics in content and confidence (Naukushu, 2016). The majority of these teachers continued teaching in the post-independent Namibia.

1.2.2 Mathematics Education in Namibia post-independence

When Namibia got its independence in 1990, education had to go through major reforms. The curriculum became one for all Namibians under the reform policy known as *"Towards Education for All"* (Namibia. Ministry of Education & Culture [MEC], 1993). All learners in the same grade were to be taught the same mathematics across the country. A new teacher-training programme known as the Basic Education Teacher Diploma [BETD], which was earmarked to prepare teachers of Grade 1 to 10 with professional insights, pedagogical skills and content knowledge, was then introduced. The BETD was offered mainly by four teacher training colleges in the country, Ongwediva College of Education, Windhoek College of Education, Rundu College of Education and Caprivi College of Education. The BETD was also enrolled for by the in-service teachers who required further training.

Other than the BETD, the curriculum reform also rolled out the Basic Education Support [BES] in three phases known as BES I, BES II and BES III. The BES III began in 2005 and targeted to provide greater service to the Namibian schools by providing support to language and mathematics teachers (Haimbodi, 2012). The BES III worked in six remote regions of the country; Oshana, Ohangwena, Oshikoto, Omusati, Kavango and Caprivi (now Zambezi) as this was where about

70% of the Namibian learners lived. The six regions were also recorded with high levels of poverty. The BES III project activities included designing teaching resources, advising teachers on effective teaching strategies to improve writing, reading and numeracy skills. BES III also developed continuous assessment tools for teachers to measure learners' performance on an ongoing basis of professional growth.

The BETD was later found to be producing teachers who lacked subject content and hence abolished in 2011. The teacher training colleges were then merged with the University of Namibia's faculty of education in order to monitor the quality of subject content and pedagogical skills gained by the pre-service teachers.

Despite reform in teacher-training programmes and the school curriculum, the quality of mathematics education in Namibia did not improve. Learners' achievement in mathematics has been low across the grades over the years (Haimbodi, 2012). Ilukena (2009) conducted a study seeking to determine whether there was a need to implement a complementary course in mathematics education for in-service teachers in Namibia to improve the performance in the schools. Ilukena's study revealed that there were many mathematics teachers who were not qualified to teach mathematics and that some teachers had low subject content knowledge. He then recommended a complementary course to be implemented in order for Namibian teachers to upgrade their subject content and pedagogical skills. Additionally, Naukushu (2011 & 2016) is of the opinion that the Namibian education system could be trapped in a three vicious cycle of numerical deficiency which he termed the Naukushu's Vicious Cycle of Innumeracy [NVCI]. Figure 1.1 presents the NVCI.

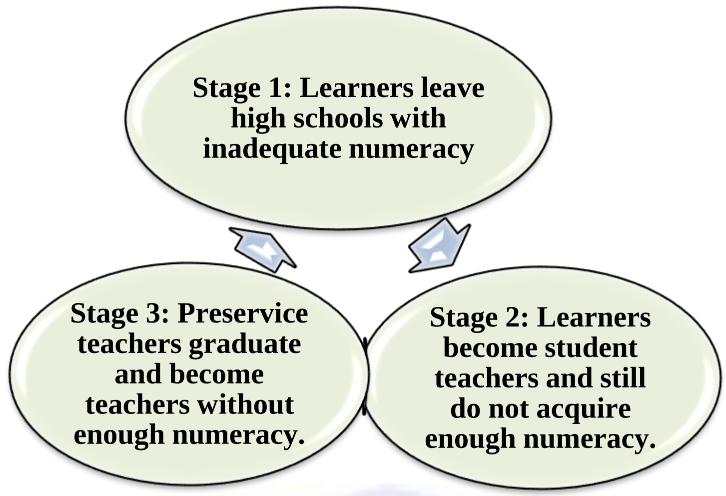


Figure 1.1.2. Naukushu's three stages of NVCI (Naukushu, 2011).

Naukushu (2011)'s three-stage cycle of numerical deficiency assumes that in stage one: learners leave school with numerical deficiencies; Stage two: learners become mathematics student teachers and still do not acquire enough numeracy probably due to a lack of required basics to grasp mathematics concepts; and Stage three: students graduate as teachers and go serve in the schools with insufficient numeracy and thereby passing on the same inadequate mathematical understanding to the learners. The NVCI could have revolved over the years as a result of the Bantu education system in which teachers for the indigenous learners were deliberately given shallow training.

Peters (2006) investigated the teaching strategies of Mathematics teachers in Windhoek schools. Her findings revealed that the teaching strategies of teachers were sub-standard and were negatively affecting the learners' performance in Mathematics. Peters (2006) thus recommends that Mathematics teachers design instruction that involves active learners' participation and the ones where learners can view mathematics as a subject that gives them power to solve problems in real life.

The reform policy required teachers to holistically develop learners into individuals who could identify, analyse and solve problems independently (MEC, 1993). The policy expected teachers to develop new visions, new instructional approaches and creativity in selecting teaching aids to allow learners to construct their own knowledge through the manipulation of available resources (MEC, 1993). In addition, the reform policy directed that the classroom instructional approach should be learner-centred – a constructivist teaching-learning approach that promotes learning through active learner participation. According to MEC (1993):

> Our children need to learn to think independently and critically. They must master strategies for identifying, analysing, and solving problems. Most important, they must develop self-confidence. Our teaching must be learner-centred: a methodology that promotes learning through understanding and practice directed towards autonomous mastery of living conditions. (MEC, 1993, p. 120)

In a learner-centred education, teachers need to view learners as active humans who should be engaged in constructing their new knowledge. The classroom activities should be designed to stimulate curiosity in learners to explore and gain knowledge and skills to master their surrounding world (Haimbodi, 2012). To date, the education system in Namibia calls for teachers to ensure that the learning process permits learners to communicate and interact with their fellows in pursuit of new knowledge. Teachers are expected to possess a sense of commitment, confidence, a reflective attitude, critical curiosity, problem solving skills and a sense of empowerment to enable them to employ active learning strategies in their classrooms (Amutenya, 2002). This indicates that the Namibian educational policy makers see 'empowerment' of learners as one of the goals of education that the teachers need to realise in their classrooms by developing learners who can think 'independently and critically'.

Education in Namibia is divided into four phases; junior primary phase from Grade 0 (pre-primary) to Grade 3, senior primary phase from Grade 4 to Grade 7, junior secondary phase from Grade 8 to Grade 9 and the senior secondary phase from Grade 10 to Grade 12. The senior primary mathematics was designed to provide a broad foundation for the future study of mathematics and other related disciplines as well as to enable learners to apply mathematics in everyday life (Namibia. Ministry of Education, Arts and Culture [MoEAC], 2016).

The Ministry of Education developed the Standardised Achievement Tests [SATs] under the Education and Training Sector Improvement Programme [ETSIP] to monitor the senior primary learners' acquisition of identified skills and competencies in key subject areas; Mathematics, English and Natural Science and Health Education (Mutuku, 2015). The SATs also aimed to set the baseline performance targets and monitor the progress of senior primary learners at individual schools.

The Mathematics SATs are psychometric tests which demonstrate candidates' abilities to work with numbers quickly and accurately. The tests contain items that assess knowledge of ratios, percentages, cost and sales analysis, rates, trends and currency conversions. Such computations are to be processed mentally as calculators are not allowed at the senior primary phase. The result of the SATs indicates that the senior primary school learners have performed below the required standards of numeracy over the years (Mutuku, 2015).

The Namibian government attaches great significance to the teaching of Mathematics in Namibian schools. For instance, "Mathematics is indispensable for the development of science, technology and commerce" (Namibia. Ministry of Education [MoE], 2010, p. 18). However, since the country's independence in 1990, there have been poor results in mathematics countrywide and across all school grades (National Institute for Educational Development [NIED], 2010). It is hence indispensable that scholars make interventions to remedy the situation.

1.2.3 The Southern and Eastern Africa Consortium for Monitoring Educational Quality [SACMEQ]

The Southern and Eastern Africa Consortium for Monitoring Educational Quality [SACMEQ] was created in 1995 to facilitate the expansion of quality education in Sub-Saharan Africa by providing necessary data to monitor educational quality, and by improving the research capacity and technical skills of educational planners (Spaull, 2011). By the year 2005, the consortium had 14 member countries participating. The SACMEQ data allow researchers to compare the quality of education amongst the participating countries.

Over the years since its inception, SACMEQ has developed research instruments, conducted studies in its member countries and collected useful data depicting the quality and standards of education. The SACMEQ runs three tests; a literacy test, a numeracy test and a test on HIV/AIDS. The Mathematics (numeracy) test measures the learners' capacity to understand and apply mathematical procedures as individuals and as members of a wider community (Spaull, 2011). The tests cover mathematical domains which are common in mathematics classrooms in Southern and Eastern Africa.

Spaull (2011) compared primary school performances among four southern African countries: Botswana, Mozambique, Namibia and South Africa using the SACMEQ III results which were collected in 2007. The study found a high prevalence of innumeracy in all the four countries, with Namibia being the worst of the four. Almost half of the Namibian learners (47.69% of the 6398 learners who participated in the SACMEQ III tests) were classified functionally innumerate as compared to 40% in South Africa, 33% in Mozambique and 22% in Botswana. Spaull (2011, p. 33) defined functional numeracy to indicate whether an individual has "acquired sufficient numeracy skills such that he or she is able to satisfactorily use those skills in everyday life". If a learner cannot relate basic arithmetic skills into real world situations such as interpreting common everyday units of measurement, then that learner was classified as innumerate.

Furthermore, the SACMEQ IV results came out as a reflection of the previous studies. Only 17.4 percent of the senior primary learners reached the competent numeracy level (Shigwedha, Nakashole, Auala, Amakutuwa, & Ailonga, 2017). These results indicate that the low level of numeracy has persisted over the years.

1.2.4 Mental arithmetic in Namibian senior primary school curriculum

In the independent Namibia, the teaching of mathematics has shifted from the era of teachers transmitting routine skills to an era of developing mathematical power. The Blacks, once deprived of learning sound mathematics are now at the fore of battling the innumeracy levels in schools and aiming to develop future citizens with required functional numeracy. A key element in the catalogue of developing numeracy is the ability to compute answers mentally. Learners should gain power over the mathematics they are taught in order to be able to use it.

Mental arithmetic in this study refers to the development and use of strategies for calculating mentally (Swan & Sparrow, 2001). It is more than the mere recalling of basic numerical facts and multiplication tables. It is mainly concerned with computation strategy building. This study embraces the position that learners develop a range of mental strategies by being exposed to rich situations requiring them to think, relate and build up a computational strategy.

The Namibian mathematics curriculum emphasises that upon completion of the senior primary phase, learners should be able to perform operations utilising mental arithmetic strategies to numbers in the range of 0 to 1 000 000 (Namibia. Ministry of Education [MoE], 2010). It is arguable that the development of mental calculations is regarded as one of the most important objectives at the senior primary phase. However, the concept mental arithmetic is not integrated in the syllabus contents as “nobody understands how it works” (MoEAC, 2016, p. 73). The argument by MoEAC (2016) that “nobody understand how it works”, referring to the mental arithmetic strategies, points to discrepancies within the senior primary curriculum in Namibian schools. The same curriculum calls for teachers to implement mental arithmetic strategies in the senior primary grades to develop learners’ autonomy, the same curriculum admits the mental arithmetic strategies are not integrated in the syllabus as nobody understands these mental arithmetic strategies and how they work. The contradictory statements in the curriculum seem to add urgency to a need for an exploratory study looking into the incorporation of mental arithmetic strategies in the senior primary schools.

Mental arithmetic is a fundamental element of children’s basic education and every adult’s daily life (Morgan, 1999). Basic arithmetic knowledge (i.e. being able to compute simple addition, subtraction, multiplication and division problems as well as making accurate approximations) is a prevalent necessity of everyday living and it provides the foundation for more advanced mathematical skills central to all modern scientific disciplines. This study holds a stand that computing mentally is a viable alternative to a calculator and the development of mental computation procedures contribute to the formation of powerful mathematical thinking strategies (Morgan, 1999). The mental arithmetic strategies therefore need to form part of senior primary classroom computations.

1.3 Motivation for the study

Mental arithmetic is an essential skill in every individual’s everyday life as numbers are part of human existence. Some authors (e.g. Morgan, 1999; Imbo, 2007 & Tabakamulamu, 2010) stress the importance of teaching mental arithmetic strategies in primary schools. This study explores current practices in the mathematics classrooms and determine how mental arithmetic strategies may be incorporated in the curriculum as a tool to address poor academic achievement at the senior primary phase.

The researcher is not aware of any studies that focused on mental arithmetic in Namibia. Hence this could be the first of its kind in Namibia, and the results may make an original contribution to research in developing numeracy teaching and learning at the senior primary level via mental arithmetic strategies. The results of this study point out aspects that need to be addressed in order to improve the numeracy level of senior primary learners. The results are also a contribution to pedagogical approaches in the training of mathematics teachers on mental arithmetic in the Namibian context.

It is thus credible to argue that incorporating mental arithmetic strategies in primary schools should start with developing mental arithmetic strategies in teachers. This intervention will be conducted with the anticipation that an improved understanding of mental arithmetic among primary school mathematics teachers could help boost academic achievement in primary school mathematics.

Moreover, the curriculum policy that mandates mental arithmetic simultaneously admitting the system have little ideas on the practicalities of its implementation (MoEAC, 2016). Such serves as a motivation for an exploratory study on how the implementation was happening in the primary schools.

The researcher of the current study served as a secondary school mathematics teacher (grades 8 - 12) for seven years. During the last two of the seven years he also served in the school management as a Head of Department [HoD]. Through these years he observed that learners entering the secondary phase lacked basic numeracy skills as they could not compute simple operations with whole numbers, without the help of a calculator.

For the two years the researcher served in the school management as a HoD, he conducted classroom observations and realised that teachers do not help learners acquire strategies for mental arithmetic as required in the curriculum. In Namibian primary school mathematics, learners are not allowed to use calculators, hence, the importance of great emphasis on mental arithmetic computation strategies. The researcher’s interest in a critical mathematics approach to research is partly a result of a concern with the manner mathematics is conventionally taught in Namibian schools – with standard algorithms strictly followed by learners. The role of mathematics educators should be to allow learners to become participative, active, competent and critical citizens.

1.4 Research Problem

Namibia has recorded unimpressive results in mathematics across all grades since independence (National Institute for Educational Development [NIED], 2010). Namibia recorded the highest proportion of functional innumeracy (47.69%) in a comparative study by Spaull (2011) in which he compared the SACMEQ III results of Botswana, Mozambique, Namibia and South Africa. The SACMEQ IV results indicated that only 17.4% of the Namibian learners reached the competent numeracy level (Shigwedha et al., 2017). The senior primary learners have been performing below average in the mathematics SATs (Mutuku, 2015; Shigwedha et al., 2017; Spaull, 2011). The failure to perform well in the SATs seems to indicate that learners cannot do computations mentally (i.e. without the help of a calculator). The low level of learners' numeracy persists despite numerous reforms that attempted to address the situation. As a result of the gloomy picture in Namibian education painted above, this study is therefore an attempt to address the following concerns: What mental arithmetic strategies are used by primary school mathematics teachers and learners in their classrooms? What are the views of mathematics teachers on the inclusion of mental arithmetic? How may mental arithmetic be developed in the primary schools?

1.5 Research Questions

The study sought to answer the following main research question: How is the senior primary mathematics curriculum incorporating mental arithmetic? In attempt to answer the main research question, the study specifically sought to address the following sub-questions:

1. What mental arithmetic strategies do primary school mathematics teachers and learners use in the mathematics classrooms?
2. From teachers' perspectives, why should the development of mental arithmetic strategies (or not) be emphasized in schools?
3. How can the development of mental arithmetic strategies be enhanced in primary schools?

1.6 Hypothesis

The second phase of data collection involved an intervention where senior primary learners were exposed to mental arithmetic strategies in a quasi-experimental design. In order to test the hypothesis that learners taught mental arithmetic strategies can perform significantly better than the learners who were not, the null and alternative hypotheses were formulated (Gay, Mills, & Airasian, 2013). The scores from the psychometric test were used to test the null hypothesis at confidence level, $\alpha = 0.05$, in order to answer the third research question.

H_o: There is no significant difference in the performance of the learners taught mental arithmetic strategies and those who were not.

H_1: There is a significant difference in the performance of the learners taught mental arithmetic strategies and those who were not.

1.7 Delimitation of the study area

This study was confined to the Oshana region, Namibia, where it explored the teaching strategies used in primary schools and sought to determine how mental arithmetic was incorporated in these strategies. In Namibia, primary education is divided into two phases, the junior primary phase with the Pre-primary (Grade 0) to Grade 3 and senior primary phase with Grade 4 to Grade 7. The study focused on the senior primary phase and included senior primary mathematics teachers and Grade 7 learners.

1.8 Interpretations of key terms

Arithmetic – is the branch of mathematics that deals with the properties and manipulation of numbers(Imbo & Vandierendonck, 2008). In this study, arithmetic will mean the manipulation of numbers based on their properties using the four basic operations.

Mental Arithmetic: Mental arithmetic in this study is used as defined by (Mclntosh, 2006) that it is an approach concerned with the development and use of strategies for calculating mentally. In this study, the term is used interchangeably with mental computation and they mean the same thing.

Mental Computation strategies: This refers to learner-deployed computation strategies, often untaught solution methods used by learners to solve a variety of mathematical problems, such as those involving adding and subtracting whole numbers (Van de Walle, Karp, & Bay-Williams, 2013). In this study, the terms *mental computation strategies* and *mental arithmetic strategies* are used interchangeably.

Teaching Strategies – refer to the methods used to enable learning of the desired course content (Imbo & Vandierendonck, 2008; McIntosh, 2005). In this study, the term teaching strategies is used to refer to methods identified by mathematics teachers to enable them to develop learners' numeracy skills.

Teachers' perceptions – perceptions refers to the ways in which someone understands a particular concept (Gay et al., 2013). In this study, teachers' perceptions refer to the ways in which senior primary mathematics teachers understand, regard and interpret the concept mental arithmetic and its implementation in the mathematics curriculum.

Critical Mathematics Education [CME] is a transformative worldview that advocates for consistent reflections on mathematical content and pedagogies to ensure learners gain critical consciousness. The theory questions the aspects often taken for granted within mathematics education and addresses such aspects to emancipate societies (Skovsmose, 1994).

Mathemacy is defined as a proficiency in handling mathematical procedures and appropriately applying these in a variety of contexts and later making critical reflections on these applications (Skovsmose, 1994).

Mathematics for social justice – refers to the guidance and opportunities provided to learners to make meaning of the world with mathematics and develop positive cultural and social identities (Gutstein, 2006).

Numeracy: The term numeracy in this study refers to the application of mathematical understanding in daily activities at school, at home, at work and elsewhere in the community, and be able to recognise the reasonableness of results (Morgan, 1999; McIntosh, 2006).

Performance – refers to academic achievement in mathematics (Mutuku, 2015). In this study the word performance refers to academic achievement of the senior primary learners in the mathematics psychometric tests.

1.9 Thesis outline

The thesis is divided into the following chapters:

Chapter One: Introduction – The first chapter gives the orientation of the study by giving a reflection on mathematics education in Namibia prior to and after independence. The chapter further discusses current problems faced by the education system in terms of learners' numeracy and academic performances building up to the problem statement of the study. The research questions, hypotheses, significance of the study as well as the operational definitions of key terms are discussed towards the end of the chapter.

Chapter two: Critical Mathematics Education as the Theoretical framework – This chapter discusses Critical Mathematics Education [CME] as theoretical framework of the study. It begins with the background of the CME and its applications in various settings. The chapter further discusses the critical theory as the parent theory to CME. Lastly, the chapter places the current study into the CME notions and makes a justification of the theoretical framework choice.

Chapter three: Literature Review – This chapter place the study in the Namibian context with particular references to mental arithmetic, mathematics education and numeracy development in Namibian primary schools. The chapter further reviews studies on mental arithmetic as applied in various settings, its benefits and links to computational estimation, numeracy development, number sense and mathematical reasoning. The chapter ends by presenting an adopted framework of a mental arithmetic curriculum.

Chapter four: Research design and methodology – This chapter discusses the mixed methods research design used to shape and collect data. The chapter discusses and justifies the choice of research design for the study. The sampling methods, data collection procedures, data analysis and research ethical considerations undertaken by the researcher are discussed in this chapter.

Chapter five: Presentation, interpretations and discussion of findings- This chapter presents and interprets the results of the study relating to the classroom practices in primary schools, teachers' views on mental arithmetic and its incorporation into the mathematics curriculum, the scores of the learners from the psychometric tests as well as the findings relating to the intervention programme. It then discusses these findings in pursuit to answer the research questions.

Chapter six: Summary, Conclusion and recommendations – This chapter summarises and draws together the findings of the study with the literature reviewed in chapter three. It further discusses possible implications the results have on the mathematics classroom practices and mental arithmetic, the contributions the study makes to the mathematics education community and then gives recommendations to address the shortfalls in the mathematics education arena. The chapter also points to aspects that may need further research.

1.10 Chapter summary

This chapter presented the context of the study. It made reflections on mathematics education in Namibia prior to and after independence, positioned the numeracy level of Namibian learners by reflecting on results of major studies done on primary school learners and then discusses the mental arithmetic focus thereby building up to the motivation for the study, statement of the problem, research questions and the research hypotheses. The next chapter discusses Critical Mathematics Education as the theoretical framework of the study.

CHAPTER TWO: CRITICAL MATHEMATICS EDUCATION AS THE THEORETICAL FRAMEWORK

2.1 Introduction

This chapter outlines the chronological perspective of Critical Mathematics Education [CME] and the suppositions of CME as the theoretical framework. It further discusses Critical Theory as a theory that informs CME. In the beginning of the chapter, the researcher discussed the Critical Mathematics Education theory, the suppositions underlying CME, and the applications of CME and Critical Theory to mathematics education. Towards the end of the chapter, the researcher presents the justification of CME as a theoretical framework for this study and how CME illuminates the approach in this study.

2.2 Critical Mathematics Education [CME]

The term '*critical*' in connection with mathematics education seems to have first been used by Frankenstein (1983), after she coined it from Freire's *critical consciousness* – an assertion that teachers must challenge their own selves and their learners' ways of thinking in order to understand their lives in new ways and consider a change to systems that may oppress in particular ways (Tutak, Bondy, & Adams, 2011). Frankenstein developed a *critical mathematics literacy* programme which sought to enhance the mathematical confidence of adult students through a collaborative approach where social issues are directly related with the learning of mathematics. She defined critical mathematics as "understanding mathematics in a way that will enable you to use that knowledge to cut through the assumptions which are taken for granted about how the society is structured and to act from more informed choices about those structures and processes" (p. 18). Research in critical mathematics pedagogy questions the 'taken-for-granted' structures in education and understand them in order to critically act on these as they routinely disadvantage humans (Tutak et al., 2011). Critical Mathematics Education aims at the implementation of equitable mathematics education to empower citizens and ensure a democratic society.

About 10 years after Frankeinstein coined the term *critical mathematics education*, Skovsmose published his book *Towards a philosophy of critical mathematics education*, in which he introduced the theoretical sparks for a critical mathematics education utilising the critical theory developed by the Frankfurt School (Skovsmose, 1994). Critical mathematics education is the education that addresses the conflicts in society by uncovering inequalities and oppression and sensitising individuals on how mathematics education may address or redress such inequalities. The critical role of mathematics in society entails an understanding of the risks and uncertainties that mathematics education conveys to society (Skovsmose, 1994).

The Critical Mathematics Education theory fits in well with the rationale of this study in that the researcher envisions to utilise the underpinning conventions of the CME to aid the development of mental computation strategies in the senior primary schools. An inquiry into the teachers' practices of mental computation strategies in their classrooms sheds light on how equitably the current mathematics curriculum is preparing learners for the future. Given the background of mathematics education in Namibia prior to and after independence, a critical mathematics education research approach may uncover taken-for-granted aspects in the curriculum.

The Critical Mathematics Education, through its notions, challenges the role of mathematical consciousness, the rationale of mathematics education and questions the curriculum implementation in terms of classroom practices (Naukushu, 2016). Consequently, in the context of this study it could be suggested that the development of mental computation strategies should be fully integrated in the senior primary mathematics curriculum to foster the growth of learners and total liberation of the minds.

The word *critical* in critical mathematics education embraces the general meaning of the expression of detailed analysis and assessment of mathematics and school pedagogies leading to a turning point where alternative actions are possible. The focus of critical mathematics education is to explore possibilities of teaching mathematics to enhance a critical attitude of individuals, society and culture to be an instrument in changing attitudes, convictions, and perspectives of learners (Skovsmose & Borba, 2004). The CME theory emerged from the restraint of being concerned primarily with how mathematics is *learned* and *taught*, and secondly, concerned with the mathematics embedded in *historical*, *cultural*, *social* and *political* contexts and the implications for classroom practices.

Scholarly works by Tutak et al. (2011) as well as Francois and Stathopoulou (2012) solely looked at the second aspect of how mathematics is embedded within historical, cultural and political contexts and the implications on education delivery. This study takes a different bearing to focus on the integration of the two aspects of critical mathematics education on how mathematics is *taught* and *learned* within the *historical*, *cultural*, *social* and *political* milieu, taking the notions of *mathemacy* and *social justice* into account.

2.2.1 Mathemacy

Skovsmose (1994) devised the concept *mathemacy* while working on CME and defined it as a proficiency in handling mathematical procedures and appropriately applying these in a variety of contexts and later making critical reflections on these applications. *Mathemacy* draws together three ways of knowing; mathematical, technical and reflective knowing, with reflective knowing being the potential catalyst for critical awareness of mathematics.

The *mathemacy* concept is crucial to the mathematics curriculum in terms of its content and pedagogical approaches in school. *Mathemacy* comprises reflections through *critical enquiries* (Cotton, 2012) on how mathematics is empowering or disempowering individual learners. These are enquiries on the mathematical processes and interactions between the teachers and learners in the schools aimed at determining how mathematics education is developing critical awareness in learners. It is these reflection type of critical enquiries that guide this study as it seeks to reflect on the mathematical processes and interactions between the teachers and learners in the senior primary classrooms. The study hence poses pedagogical questions through the curriculum offered to senior primary school learners in terms of content and teaching approaches. In what ways does the Namibian senior primary mathematics curriculum deal with issues which may allow teachers and learners to interpret the world they live in? How does classroom instruction support learners in developing an understanding of themselves within their settings through mathematics?

2.2.2 Social justice

Critical mathematics education includes a concern for addressing any form of suppression or exploitation through mathematics as a tool. Gutstein (2006) interprets social justice in mathematics education as the guidance and opportunities provided to learners to make meaning of the world with mathematics and develop positive cultural and social identities. Learners are guided to acquire socio-political consciousness of their immediate and wider contexts and also attain a logic of seeing themselves as humans capable of changing their world. The notion of *social justice* is that learners should be exposed to and learn rich mathematics so that they have opportunities to study, pursue eloquent lives, support their communities' growth, and be able to use mathematics to fight oppression and improve society. *Social justice* calls for a view in all learners that mathematics has real meaning in life and can specifically be used to make meaning of the world.

According to Gutstein, (2006) as well as Stinson, Bidwell and Powell (2012), reading the world with mathematics means to use mathematics to comprehend the relations of power, resource inequities, disparate opportunities and explicit discrimination among different social groups based on race, class, gender, language and other differences. Writing the world with mathematics means using mathematics to create meaning of the world (Gutstein, 2006). Developing positive cultural and social identities means to ground mathematics instruction in the learners' languages, cultures and communities while providing them with the mathematical knowledge needed to survive and thrive (Stinson et al., 2012).

The call by Critical Mathematics Education is for learners to be allowed to critically deconstruct the way in which mathematics designs reality, so that they can socially contribute as informed and critical citizens in the structure of a democratic, socially just society. Learners should not only be asked to solve general mathematical problems but contextualised mathematical problems potentially coming from all areas of everyday life. That will allow learners to critically analyse how mathematics is used to manipulate people's decisions and how they can use mathematics to interpret information, make informed decisions and transform their realities (Skovsmose & Borba, 2004). CME is concerned with providing access to mathematical ideas and their function in everyday life. The effective learning of mathematics may result in empowerment, citizenship and democratic participation or it may result in disempowerment, marginalisation and exclusion (Skovsmose, 2005a). Mathematics education should prepare learners "to investigate and critique injustice, and to challenge, in words and actions, oppressive structures and acts" (Eric Gutstein, 2012). This study is thus concerned with uncovering how classroom discourses deliberate on ideas and discuss issues leading to numerate citizenry.

2.3 Ideas fundamental to CME

Critical mathematics education is concerned with the social aspects of the learning of mathematics, the provision of access to mathematical ideas for everybody independent of skin colour, gender or economic class, and the use of mathematics in practice, being an advanced technological application in modern days (Skovsmose, 2005a). The use of CME within the classrooms is concerned with a democratic discourse where ideas are presented and discussed with the primary purpose of developing learners.

In this study the researcher therefore envisions to use the pivotal conventions of critical mathematics education to enhance the development of mental arithmetic in senior primary schools. Along these lines senior primary school mathematics teachers may be equipped with skills that they may use to develop their learners' *mathemacy*. Additionally, the researcher reckons that it is necessary to combine contemporary and innovative paradigms of mathematics education for emancipating the teachers and learners as well as the whole education system to develop critically conscious citizenry (Mutuku, 2015; Spaull, 2011).

CME in its setting, challenges the role of mathematical consciousness, the rationale of mathematics education and, sequentially, CME questions mathematics curricula along with classroom practices. Scholarly works (Ernest, 2001; Frankenstein, 2000; Knijnik, 2002; Skovsmose, 2005a) discuss education policies, curricula and teaching practices that position mathematics as an absolute, neutral, accepted body of knowledge that possesses pint-sized relationship with the socio-political settings we live in. Such curricular do less to develop students as participating, thinking citizens. This study therefore, holds a dissimilar view to such curricula, which is the belief that mathematics curriculum making is not a static dynamic exercise which is detached from the environment and cultures of the learners, thereby linking CME to ethnomathematics. More on the link between CME and ethnomathematics is discussed in a section on ethnomathematics later in this chapter.

2.4 Criticisms of CME

There are several critics of Critical Mathematics Education or the critical approach to research in education (Cohen, Manion, & Morrison, 2007). Firstly, critical mathematics education seems to have not been systematically applied in depth as many nations considered the approach too critical and resisted a change of the systems (Cohen et al., 2007; Tutak et al., 2011). The aspects considered to be of an oppressive nature were thus explored and discovered but change faced resistance as it would be costly.

Secondly, Morrison (1995) as cited in (Cohen et al., 2007) points out that the link between the ideology critique and emancipation of citizens is "neither clear nor proven, nor a logical necessity"

(p. 48). Morrison (1995) argues that the exercise of an ideology critique may not necessarily end up emancipating a society and that such needed empirical verifications. Moreover, the ideology critique may actually itself obstruct actions designed to bring about emancipation (Morrison, 1995).

Lastly, there exists a view that the critical mathematics education has a deliberate political agenda and has been viewed as such in many nations (Tutak et al., 2011). As argued by the Frankenstein, (1983) and Skovsmose (1994), mathematics exists within societies and it is not apolitical in its nature as even oppressors have used mathematics to dominate their subjects. The critical mathematics approach thus targets to use the subject in emancipating the citizens to gain critical consciousness and be able to re-write their world with mathematics.

The present study takes a stand that in order to support and further develop mathematics education, the education system needs an approach that does not treat teachers as robots to mindlessly implement pedagogies. Research should question and explore aspects in mathematics education to determine shortfalls, explore solutions and make recommendations to enhance numeracy development. Teachers are indispensable in the critical mathematics education pedagogy due to their roles as architects of learning and providers of prospects for critical thinking. Hence, critical mathematics education is essential to attaining change in the mathematics classrooms and to initiating critical consciousness among their learners (Tutak et al., 2011).

2.5 Applications of CME

Frankenstein (1983) used the CME-informed programme which sought to enhance the mathematical confidence of adult students through a collaborative approach where social issues are directly related with the learning of mathematics. She found that the adult college students learned much more out of critical mathematics education programmes than they would if they learned the standard, depoliticised mathematics curriculum. Drawing mainly on Freire’s ideas about critical literacy, Frankenstein (1983) integrates political consciousness-raising goals into the teaching of mathematics and her main focus is on the situation of the oppressed, exploited and historically marginalised people. Frankenstein argues that mathematics educators, in critical mathematics pedagogy, should respect learners, prior understandings and inspire learners to question the most aspects of mathematics taken for granted thereby expanding their interests and imagination.

Skovsmose (1994) coined the term critical mathematics education and worked significantly to develop the theory. He sees CME as a vehicle for examining the technical and frequently hidden use of mathematics by powerful decision makers and technocrats to oppress their subjects. Sokvsmose (1994) notes that to help learners understand social issues and the role of mathematics in a highly technological society, some conventional mathematics will need restructuring.

Postmodern theorists of Critical Mathematics Education, for example (Gutstein, 2003, 2006), and Stinson et al., (2012) expand on the work of Frankenstein (1983) and Skovsmose (1994) by looking at the reform of mathematics curriculum. Gutstein's (2003) work emphasises that critical mathematics learners should develop five tenets: 1. *Critical consciousness* – an understanding of the socio-political powers and establishments that shape their lives; 2. *Critical agency* – a sense that they can make a progressive transformation in the world and fight for social justice; 3. *Positive social and cultural identities* – by validating their language and culture and helping them unearth and appreciate their past; 4. *Changed dispositions towards mathematics* – coming to view themselves as capable of doing sophisticated mathematics and this as relevant to their lived situations; and 5. *Mathematical power* – confidently engaging in complex mathematics, exploring various strategies to solve problems and communicate results effectively as they engage in learning.

Drawing ideas from the Critical Mathematics Education, ethnomathematics and from data collected in lecture rooms and classroom settings in South Africa, Vithal (2002) coined the term *pedagogy of conflict and dialogue* as she searched for new ideas integrating theory and practice in a critical perspective. Vithal's (2002) pedagogy of conflict and dialogue is embedded in the South African settings as a new democratic country where the apartheid effects were still persisting.

Vithal (2002) offers a means of realising broader goals of critical citizenship, democracy, equity and social justice within every life of a mathematics classroom. She notes that becoming critical is not confined to pupils only but mathematics educators need to be 'reflective practitioners in the broad sense and to take an equally critical stance in relation to both mathematics and mathematics education' (p. 13). Although Vithal (2002) termed her work as *pedagogy of conflict and dialogue* and contextualized it in the South African settings, it appears embedded fully in the critical mathematics education theory (Skovsmose, 2014).

Gutstein (2003, 2006) and Stinson et al. (2012) have looked at critical mathematics education in the United States contexts while Vithal (2002) discussed CME in the South African contexts. All their work took place exclusively in elementary and middle school classrooms in under resourced urban areas with students of colour, as mathematics has been a determinant in their access to academic, professional, and economic opportunities (Brantlinger, 2013). Their work was hence driven mainly by socio-political factors and were more interested in the first three of Gutstein's (2003) tenets; the *critical consciousness,* the *critical agency,* and the *positive social and cultural identities*. In a Namibian context, the last two tenets of Gutstein (2003); changed *dispositions towards mathematics* and *mathematical power* are of most relevance given the need for our learners to engage in sophisticated mathematics with confidence and gain more power in mathematical domains.

This study hence applied CME differently in that it sought to explore classroom practices of the senior primary teachers and learners and the meanings such practices have in terms of developing mental computation strategies which may ultimately instil mathematical power amongst senior primary school learners.

2.6 Critical theory as the parent theory to CME

Critical mathematics education evolved from the Critical Theory, a theory which was developed in the 1800s in Germany but became more prominent in the 1920s under the quasi-Marxist theory of a group of interdisciplinary social theorists of the Frankfurt School. The Frankfurt School was established as an affiliate of the University of Frankfurt with its primary purpose of research and consisted a group of German philosophers, sociologists and economists. These scholars worked within the Marxist framework with an intention to develop a theory that could contribute to the struggle of social transformation against human oppression. The concern was on how the oppressors' exerted control by reinforcing beliefs about what is considered to be 'absolute truth' (Naukushu, 2016).

The theoretical perspectives of critical theory maintain socio-political critiques on social practices and inequalities by examining social interests, conflicts, and contradictions and addressing inequalities in education and society. Critical theorists contend that an analysis of the establishmentsof domination would bring about an awakening of consciousness and awareness of social prejudices, stirring self-empowerment and social revolution (Stinson, Bidwell, Jett, Powell, & Thurman, 2007).

Forester (2010) views Critical Theory (CT) as a crucial tool in empowering the oppressed individuals and further indicates that critical theorists attempt to seek out contradictions and social dissimilarities in a diversity of disciplines in order to empower those who are oppressed. Similarly, Ojelanki's (2010) main argument of the CT is that certain groups of people in society are oppressed and need to be empowered. All fundamental sorts of disciplines should be questioned to achieve emancipation of the oppressed. The human capacities of individuals must be developed and linked to democracy to improve society.

On the classroom practices, Freire (1970) pointed out that teachers tend to use a *banking pedagogy* in which they fill learners' minds, as containers, with the knowledge that someone has determined that learners need to know. From the perspective of a critical pedagogy, the banking pedagogy hinders the thinking of learners and allows for passive reception of knowledge, which in turn may be an effort to disempower both teachers and learners. Discussing the *banking pedagogy* concept, Khuzwayo (2018, p. 173) stresses that the banking concept of education encourages the form of teaching in which a "one-way dependence from the student to the teacher exists" which develops no critical consciousness. "The more students work at restoring the deposits entrusted to them, the less they develop the critical consciousness which would result from their intervention in the world as transformers of that world" (Gutstein, 2006 as cited in Khuzwayo, 2018, p. 173).

The notions of *banking pedagogy* was one of the main reasons for major curricula reforms engaged by the Namibian education ministry soon after independence (MEC, 1993). Then, education was regarded as a platform for teachers to transmit knowledge to learners who come to school with empty minds to be filled. To date, the Namibian education curriculum calls on teachers to practice interactive learning and help guide learners as they engage in the construction of their own knowledge. The practice is commonly known as learner-centred education [LCE]. In a learner-centred paradigm, learners adopt the role of an active member and should under no circumstance be viewed as an empty vessel to be filled with information (MoEAC, 2016). Learner centred pedagogy requires learners to actively participate, critique and contribute collaboratively to their learning and training of mental arithmetic strategies (Forester, 2010; Ojelanki, 2010).

Freire's *critical pedagogy* advocates for a critical learning through which teachers and learners are agents of curiosity, investigators, and subjects in an ongoing quest for the revelation of 'the why of facts and things' (Freire, 1993, p. 105). Through critical pedagogy, teachers and learners generate and examine problems from their own backgrounds and work collaboratively to construct solutions.

Furthermore, Critical Theory assumes that knowledge is not static but dynamic. Horkheimer (1937) as cited in Naukushu (2016) in an attempt to discuss the dynamic nature of Critical Theory explains the concept of mind as:

> The mind is liberal. It tolerates no external coercion, no revamping of its results to suit the will of one or other power. But on the other hand it is not cut loose from the life of society; it does not hang or suspend over it. Truth is therefore liable to change but under no circumstances is it an illusion. The abstract reservation that one day a justified critique of one's own epistemic situation will be put into play, that it is open to correction expresses itself among materialists not in a tolerance for contradictory opinions or even in a sceptical indecision, but in watchfulness against one's own error and in the mobility of thought a later correction does not imply that an earlier truth was an earlier untruth. (p. 25)

In addition to the debate on the dynamic nature of knowledge, Higgs and Smith (2002) as cited in Hamunyela (2008) argue that Critical Theory discards classical philosophy in that it uses some form of independent, objective, critical thinking to reflect on society:

> According to critical theory, reason is immanent, not transcendent. For, according to critical theorists, certain rational principles (e.g. the law of deduction, formal logic and mathematical reasoning) are not timelessly true. They are only "true" because we human beings at this point in human history believe that this principle achieves certain things. The idea that 1+ 1 = 2 is simply an arrangement of symbols that are pronounced to be a truth in this era. In deed the whole mathematics system which is seemingly unassailable has a history. The fact that the year 2000 is more important to the west means nothing to the Chinese or the Indians, neither of whom rely on the Christian dating system. (p. 10)

The idea that knowledge is not static aided the researcher when conducting the mental arithmetic workshops; to view senior primary mathematics teachers to be in possession of some prior knowledge of mental arithmetic strategies which could then be used as a foundation to extend what they already know. Therefore, the researcher envisions using instructional paradigms that value the existence of prior leaning and experience such as the learner centred method which is the core of the mental arithmetic workshops as it allows individualised flexibility, democracy and freedom as required by the assumptions of Critical Theory.

Critical theory thus raises concerns about 'state of affairs' in the education system and how humans may be educated differently (Tutak et al., 2011). Being critical is not synonymous with being negative towards the education system, it is rather a commitment towards democratic principles of equality and equity in pursuit of raising learners with a critical eye to examine social justice in their world. Therefore, Critical Theory in education is concerned with liberating, enlightening, emancipating, and empowering learners (Tutak et al., 2011).

This researcher considered that the senior primary school mathematics teachers are from a previously disadvantaged section of the Namibian society where education was a tool of domination by the apartheid regime (Naukushu, 2016). The whites' domination accounted for the inferior resourcing for the Bantu education system prior to the Namibian independence. Schools for the blacks had a shallow mathematics content and some schools had no mathematics (Vatileni, 2015). It can hence be concluded that the domination caused underdevelopment of numerical competences as well as other mathematical competences in Namibia, hence the need to 'liberate', 'enlighten', 'emancipate' and 'empower' the teachers. Both critical theory and the critical mathematics education theory concurs on the enlightening and empowerment of individuals. The researcher therefore envisages to utilise the underlying assumptions carried by these theories, critical theory and CME theory, to enhance the practice of mental arithmetic strategies in the primary schools thereby boosting up the development of learners' numeracy skills. The next section discusses realistic mathematics education [RME] as a theory with close links to critical mathematics education.

2.7 Mathematics education theories allied to CME

Although Critical Mathematics Education philosophy guided this study in questioning the practices in mathematics education, the theory seems to lack practical guidance on how classroom instruction in mathematics should be designed in order to fully emancipate learners. This section, therefore, discusses transformative philosophies in mathematics education which may complement the CME theory in developing critical teachers. On the one hand, the realistic mathematics education [RME] pleas for the teaching and learning of mathematics to be contextualized in the learners' world for them to realise the realistic nature of mathematics and how it is connected to their daily activities (Freudenthal, 1991). On the other hand, Ethnomathematics advocates that the teaching and learning of mathematics should be embedded in learners' cultural experiences in order to realise the true meaning of mathematics (D'Ambrosio, 1985).

2.7.1 Realistic mathematics education [RME]

Realistic mathematics education (RME) is a teaching and learning theory in mathematics education that was first introduced and developed by Hans Freudenthal of the Freudenthal institute in the Netherlands. The theory has two main views; one: mathematics must be connected to reality and two: mathematics is a human activity. Freudenthal (1977) argues that mathematics must stay close to learners and should be relevant to society within which it is learned. The connection between mathematics and reality as reasoned by Freudenthal (1977) aims at making mathematics to be of human value. As a human activity, Freudenthal deduced that the teachers have a duty to play a facilitative role to give students a guided opportunity to re-invent mathematics through practicing it (Freudenthal, 1991). Taking into account that the recent CME theorists (Alrø, Ravn, & Valero, 2010; Francois & Stathopoulou, 2012; Skovsmose, 2014) also call for new and innovative methods of teaching and learning mathematics, where the teacher plays the role of facilitator and the learners are active participants guided towards unveiling mathematical ideas, the researcher therefore deemed it necessary to infer a connection between CME and RME.

The perspective underpinning RME is that learners should cultivate their mathematical comprehension by working from settings that make sense to them (Dickinson & Hough, 2012). Mathematics learners should be allowed to devise their own intuitive methods for solving problems using carefully chosen steps before appropriate interventions by the teachers to guide them towards the development of a more formal understanding of mathematics. RME treats learners as active participants in the learning process, which in the light of this study comes back to the whole notion of empowerment as one of the appeals for the CME.

The RME approach to mathematics education is hailed by studies (for example Dickinson & Hough, 2012; Peters, 2016) to be a good instructional pedagogy as its applications have resulted in several significant benefits. Firstly, the use of realistic situations as a means of letting learners develop their mathematics in their contexts has improved the practice of mathematics education. Secondly, the realistic mathematics developed numeracy via placing less emphasis on standard algorithms and more efforts on making sense and gradual refinement of informal procedures. Lastly, RME's focus on gradual development of learners' ideas resulted in increased learner participation in mathematics compared to classrooms where teachers drilled learners on mathematical problems which were isolated from their settings.

The activity principle further states that in RME mathematics is best learned by doing mathematics, which is strongly reflected in Freudenthal's interpretation of mathematics as a human activity, as well as in Freudenthal's and Treffers' idea of mathematisation (Heuvel-Panhuizen, 2003). It also emphasises that RME is a product of its time and cannot be isolated from the worldwide reform movement in mathematics education that occurred in the last decades Freudenthal (1991). This study thus holds a strong view that mental arithmetic can be best developed if the primary school mathematics learners are afforded an opportunity to explore mental arithmetic strategies working on activities designed within the learners' realistic contexts as opposed to the traditional approach of teaching mathematics which is often disconnected from the learners' realities. In view of the foregoing characteristics of RME, this study used the RME notions as it sought to develop primary school mathematics teachers' mental arithmetic strategies so that teachers could become critical mathematics educators equipped to teach mathematics and produce learners that are able to cope with the demands posed by the societies' expectations of schooling. The next sub-section discusses ethnomathematics as another theory closely linked to critical mathematics education.

2.7.2 Ethnomathematics

Coined by D'Ambrosio (1985), ethnomathematics as a theory was necessitated by the desire to accommodate cultural and environmental aspects in mathematics education. The tussles against

Eurocentrism in mathematics, viewing mathematics from whole European perspectives and inclusion of cultural groups in mathematics education led to the ideas of ethnomathematics and it is a form critical mathematics education pedagogy in the classrooms. Ethnomathematics strives to reconstruct mathematics and mathematics education to empower cultures by using authentic mathematics from their lives (Tutak et al., 2011).

The theory brings to the fore the mathematics practised by cultural groups such as urban and rural communities, labour groups, indigenous societies and many other groups in order to improve the cultural dignity of human beings. D'Ambrosio (1990) analyses the concepts of ethnomathematics by dividing it into three constitutive parts as and defines it as:

> I call *mathema* the actions of explaining and understanding in order to survive. Throughout all our own life histories and throughout the history of mankind, *technés* (of *tics*) of *mathemacy* have been developed in very different and diversified cultural environments, i.e. in the diverse ethos. So, in order to satisfy the drives towards survival and transcendence, human beings have developed and continue to develop, in very new experience and in diverse cultural environments, their *ethno-mathema-tics.* (p. 369)

In his analysis, D'Ambrosio recognises the traditional knowledge as a way of elucidation and understanding in order to live. Secondly, he talks about the techniques and applications of such in handling and practicing mathematics. Thirdly, D'Ambrosio refers to a critical part as he talks about the existence of the diverse ways constituting applications of diverse mathematical ideas. This is his reflective part, the notion of ethnos that refers to the diverse nature of mathematics (Francois & Stathopoulou, 2012).

Ethonomathematics calls for the inclusion of the cultural, social and environmental elements in mathematics education for the teaching of mathematics to make sense to learners. The main aim of ethnomathematics is hence to bring the content of mathematics to the learners' settings for learners to benefit from the teaching of mathematics.

Additionally, Rosa and Orey (2003; 2007; 2009) all cited in Naukushu (2016, p. 41) viewed ethnomathematics as follows:

> The *ethno* part of ethnomathematics would imply the members of a cluster within a cultural milieu, whose brand is their cultural customs, codes, symbols, mythology, taboos and unambiguous ways used in such cultural milieu to reason and deduce. *Mathema* on the other hand could imply to put in plain words and identify with the human race in order to surpass, deal with and muddle through with certainty so that the members of cultural groups can carry on and thrive, and *tics* refers to techniques such as counting, ordering, sorting, measuring, weighing, ciphering, classifying, inferring, and modelling.

D'Ambrosio (1983) affirms that the onus of the ethnomathematics in education is to enlighten educators that there exists a variety of ways to exploit mathematics by taking into account the inclusion of mathematical knowledge nurture by different sectors of society as well as by considering different approaches in which diverse cultures discuss their mathematical practices. Ethnomathematics is hence based on a notion that everyday life is soaked in the knowledge and practices of a culture where people are comparing, classifying, quantifying, measuring, explaining, generalising, inferring, and evaluating using intellectual instruments and devices that belong to their culture. It's for these reasons this researcher chose to use the notions of ethnomathematics together with the notions of the CME in the hope to develop primary school teachers' mental arithmetic strategies.

The notions of Ethnomathematics have similarities with the notions of the Critical Mathematics Education. For some studies (see Francois & Stathopoulou, 2012; Skovsmose & Borba, 2004; Vithal & Skovsmose, 1997) ethnomathematics is regarded to be embedded within the critical mathematics education theory. Critical Mathematics Education advocates for a move away from teacher-directed top-down instructions towards engaging learners in meaningful problem-based instructions, something that requires thorough consideration of aspects of learners' lively experiences and contexts. That encompasses the notions of ethnomathematics. The study hence takes a stand that by using the CME theory, and engraving notions of ethnomathematics within the objectives of *mathemacy* may enhance numeracy development among learners. This study hence focused on exploring the mental arithmetic strategies as practiced in senior primary schools, where cultural and political factors influences the learners' daily lives and thus ground mathematics instruction within learners' contexts and cultural experiences.

Although the CME theory allows us to question the taken-for-granted aspects in mathematics education, it does not seem to be specific on how the mathematics content should be structured to fully develop numerate citizenry. RME and Ethnomathematics seem to give specific guidance on instructional designs. These theories are thus discussed here as they complement CME in a call for a transformative action in mathematics education for the realisation of genuine emancipation of the society. The concepts of human emancipation and social empowerment via mathematics education is common in Critical Mathematics Education, Realistic Mathematics Education and to Ethnomathematics as all these theories call for learning of mathematics to be relevant and meaningful connecting school content to learners' contextual realities hence deepening learners' understanding and arousing critical consciousness (Lekoko, Suping, & Pitso, 2018). The present study takes a stand that the transformative principles of RME and ethnomathematics when well encompassed in CME pedagogies may boost up the development of numeracy (Ernest, 2001; Francois & Stathopoulou, 2012; Skovsmose & Borba, 2004; Vithal & Skovsmose, 1997).

2.8 CME and research methodology

Guba and Lincoln (1994) developed the notion of resonance in order to conceptualise the possibility of fit between the CME theoretical perspective and methodology of a research approach. They characterised the critical paradigm as:

> The aim of inquiry is the critique and the transformation of the social, political, cultural, economic, ethnic, and gender structures that constrain and exploit humankind, by engagement in confrontation, even conflict. The criterion for progress is that over time, restitution and emancipation should occur and persists. Advocacy and activism are key concepts. The inquirer is cast in the role of instigator and facilitator, implying that the inquirer understands a priori what transformations are needed. (p. 113)

In accordance, the research approach this study took is in line with the description of Guba and Lincoln (1994). The notion of inquirer should understand *a priori* what transformations are needed and researcher playing instigator roles are advocated for in this study. This researcher believed that critique and transformation include formulated intervention (*a priori* goals) informed by the base-line needs situation study (often filled with uncertainties and doubts). This study thus sought to clear the uncertainties and doubts surrounding the incorporation of mental arithmetic computation strategies in primary schools.

The study focused on the interactions in the classroom settings and how mathematics operations are done, i.e. the mental strategies practiced by teachers and learners during the mathematics lessons. The learners' and teachers' expressions were registered and body language noted. The researcher concentrated on bringing about empirical materials from the classroom situations and addressing what has taken place.

Doing research in the Critical Mathematics Education theory should not only consider what is taking place but also what could have taken place and what could be imagined as possible alternatives to what is taking place. A critical task for a critical research is to explore alternatives and opportunities in such detail that they confront what might be conceived as given (Sriraman, 2007).

2.9 Justification of CME

School systems need to change their approach in teaching mathematics so that mathematics education develops humans for society. This researcher used critical mathematics education due to a concern with the way mathematics is traditionally taught in primary schools in Namibia: as a subject disconnected from learners' reality. From the CME perspective, the teaching of mathematics disconnected from learners' realities is likened to Freire's *banking pedagogy* in which learners' minds are filled with the knowledge perceived to be appropriate (Tutak et al., 2011).

The activities in mathematics should be developed to allow learners to uncover, and understand the role of mathematics in different social situations. The Namibian senior primary mathematics curriculum explicitly stipulates that mathematics has the purpose to develop learners' functional numeracy and enable them to apply mathematics in everyday life (MoEAC, 2016). The mathematics taught in schools should therefore bring to light the social dimensions of mathematics, making sure that the activities are relevant to learners' social realities and allow a critical analysis of the mathematics behind the modelling of a specific social activity. As a critical mathematics educator, the researcher took to reflect on the classroom practices in the senior primary schools.

Research in critical mathematics education should draw attention to problematic issues related to mathematics education; the organising of work in mathematics classrooms, the reliability of mathematics practice and the ability to practice the mathematics outside classrooms (Skovsmose, 2002). CME allows this researcher to *reflect through* the mathematics curriculum and its pedagogies in the senior primary schools.

2.10 Theoretical lens of the Study – *Mathemacy* and *Social justice*

From the CME theory discussed, the researcher hoped that the development of mental arithmetic strategies of teachers could be best achieved by adopting the *mathemacy* and *social justice* as theoretical lenses to guide the enquiry. The *mathemacy* and *social justice* concepts together aim for developing mathematical power via deducing mathematical generalisations, constructing creative solution strategies to problems, and perceiving mathematics as a tool for socio-political critique (Stinson et al., 2012). The use of these two theoretical lenses together can be used in developing mental arithmetic strategies of teachers that may enhance numeracy of their learners. The researcher is of the view that a *mathemacy* and *social justice* driven approach to mathematics education may enlighten teachers that mathematics is not a series of detached, rote rules to be memorised and regurgitated, but as a powerful and significant analytical tool for understanding complex and real world aspects. Such an understanding may enhance thorough development of mental computation strategies.

2.11 Summary

Critical mathematics education requires more than a discussion on the implication that the cultural, historical, social and political aspects have on mathematics education. CME requires an in depth application by critical teachers who consistently critique self and their learners in an ongoing process of critical consciousness. The *mathemacy* and *social justice* notions require consistent reflections on the mathematics curriculum to restructure content and pedagogies for a better yield in numerate citizenry. The study thus took a stance in reflecting on the mathematics delivered in the senior primary schools and critique in terms of value and its ability to develop learners into critical beings aware of and able to apply mathematics in their daily lives. The CME shifts the focus on approaching mathematics and its teaching from a critical viewpoint (Ernest, 2001). Figure 2.1 displays the exploratory approach

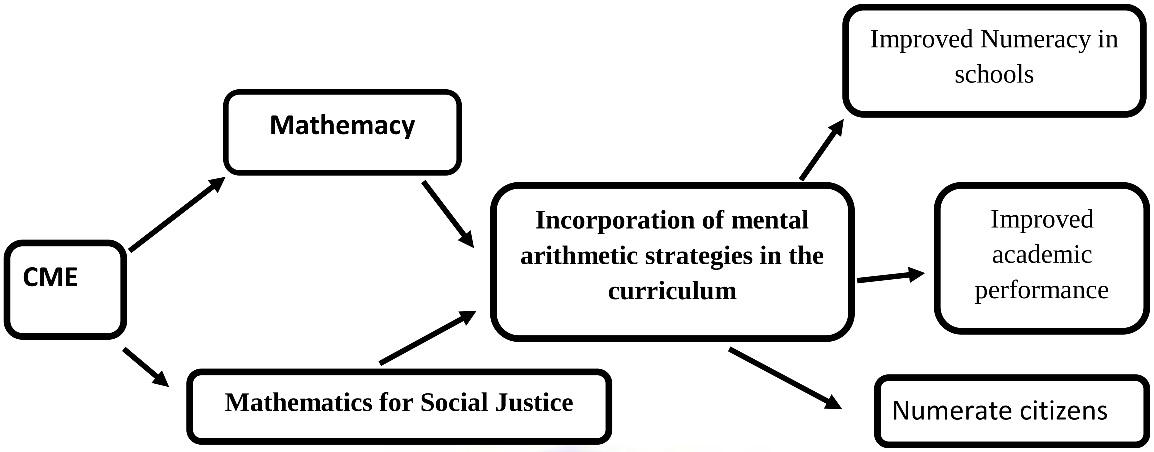


Figure 2.1 The CME approach to exploring mental arithmetic in primary schools.

Intrinsically, the fundamental purpose for critical mathematics education is to stimulate critical thinking allied to mathematics amongst learners so that they could use mathematics in their lives to empower themselves both personally and as functional citizens in the society. The researcher thus hoped to use CME to acquire a critical view of the status of mental arithmetic in classrooms and make a critical intervention to emancipate teachers and learners. Figure 2.2 shows how the CME notions link to research questions and the methodological approach in this study.

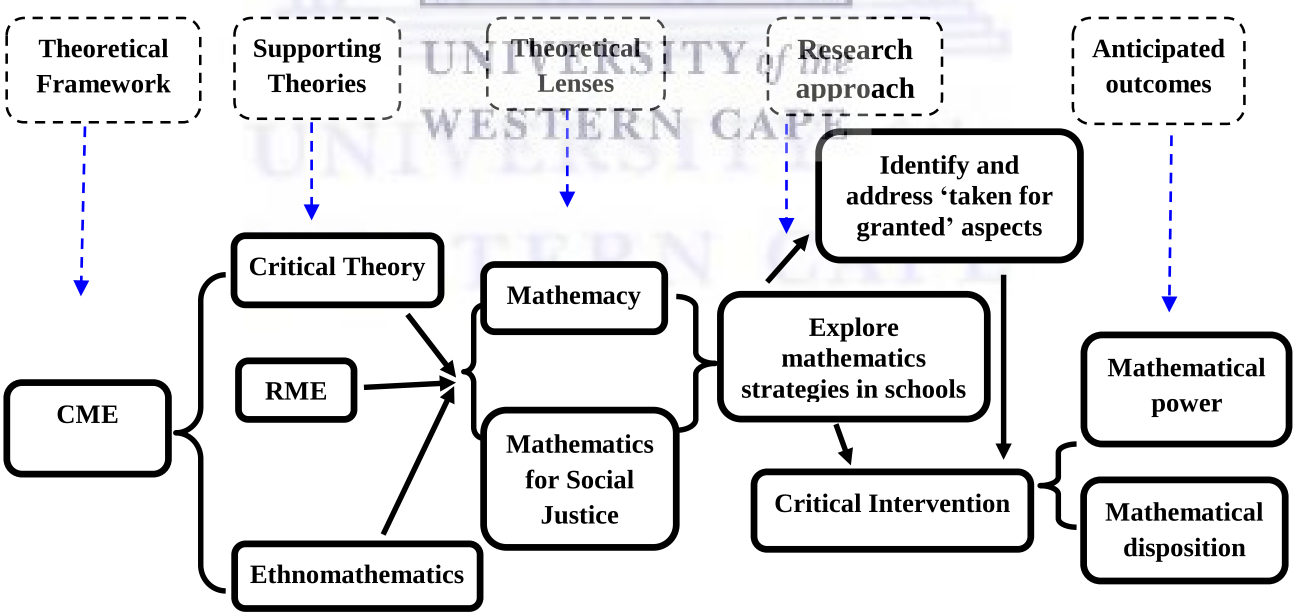


Figure 2.2 Summary of the theoretical underpinnings of the study

The mathematics teachers in senior primary schools need to be sensitised to recognise that they must challenge their own and their learners' thinking that frequently limit their potentials. It is this kind of critical reflections that may lead the notion of critical consciousness which enables humans to understand their lives in different perspectives and consider a change as they liberate themselves from possible oppressions. The next chapter presents a review of literatures related to this study.

CHAPTER THREE: LITERATURE REVIEW

3.1 Introduction

This chapter places the current study in the Namibian education context with specific reference to mental arithmetic and the development of learners' numeracy skills. Informed by the Critical Mathematics Education [CME] theory, the study sought to explore the state of mental arithmetic computation in the senior primary mathematics classrooms. Mental arithmetic, although a very old concept in mathematics education globally, seems to be a relatively new topic in the Namibian mathematics education. This chapter starts a discussion on mental arithmetic and the teaching strategies for mental computations, the research studies on mental arithmetic and concludes with the state of numeracy education in Namibia.

3.2 Mental Arithmetic

Mental arithmetic has been defined, on the one hand, as "the speedy recall of numerical facts and multiplication tables" (Rogers, 2004, p. 193). According to studies (Caney, 2004; Rogers, 2004; Reys, 1984), the education system which practised mental arithmetic from this perspective consisted of ten or fifteen questions given at the beginning of a lesson. These questions were circumscribed to basic facts of the basic operations and speed and accuracy were emphasised. Education systems in many countries around the world, (McIntosh, 2005), have a view that mental arithmetic mainly means a recall of simple addition, subtraction or multiplication facts. Such is still the practice in Namibia and South Africa, (MoEAC, 2016; South Africa Department of Basic Education [DBE], 2011). There seemed to be less emphasis on the experiences that encourage discussion and learning and more on activities that focus on testing in primary schools.

On the other hand, studies (for example Imbo & Vandierendonck, 2008; McIntosh, 2005; Morgan, 1999) define mental arithmetic as the computational strategies selected by learners in performing quick calculations mentally. The development of mental procedures is viewed as a powerful formation of mathematical thinking and reasoning. The emphasis needs to be placed upon the mental processes involved in working out mathematical problems.

This study hence defines mental arithmetic, as defined by McIntosh (2005) that it is an approach with two parts: *Recalling* and *computation.*

Recalling

The *recalling* part is concerned with the quick remembering of numerical facts and multiplication tables which relies mainly upon the skill of a child's memory. The *recalling* practice has received criticism that the testing sessions take the emphasis of learners' comprehending how they are finding answers and hardly provide them with the skills to further develop their understanding (Caney, 2004; Rogers, 2004). The gifted learners are made to feel good about themselves while the rest of the class develops anxiety (Rogers, 2004).

The fact recalling part of mental arithmetic has received criticism that it promotes rote learning, increases number anxiety, and make children more neurotic about numbers instead of enhancing the development of their numeracy skills (Caney, 2004; Rogers, 2004). Rogers (2004) further argues that mental arithmetic makes children with strong memories feel good about themselves and leave the rest of the class feeling disappointed. A study by Biggs (1967) cited in McIntosh (2005) revealed that the *fact recalling* practice was of limited value. Biggs (1967) studied 69 classrooms where mental arithmetic was practised with emphasis on speed of response and accuracy. Among the conclusions of Biggs' (1967) study were: (a) learners developed number anxiety, (b) the daily speed and accuracy tests did not make the learners noticeably more competent, but did make learners slightly neurotic about numbers. For these reasons, education systems need be cautious in implementing the mental arithmetic as to avoid developing anxiety in learners.

Computation

The *mental computation part* is concerned with the development and use of strategies for computing mentally. These skills are unique in that once learnt, learners will not need to store all arithmetic facts in long-term memory as problems can be calculated in simple series of steps (Imbo & Vandierendonck, 2008). McIntosh (2005) clarifies that mental arithmetic is not solely based on the quick and accurate recall of number facts which relies mainly upon the skills of children's minds, but includes developing children's understanding and promotion of metacognition. McIntosh (2005) further argues that mental arithmetic has a much wider relevance in education of individuals, as adults, in their everyday lives, use mental computation for over three quarters of all their calculation.

According to Imbo and Vandierendonck (2008, p. 533), arithmetic strategy uses entail two components: “strategy selection (occurring before a particular strategy is executed) and strategy execution (occurring when a particular strategy is used to solve the arithmetic problem)”. While computing mentally, an individual always chooses the best strategy to use based on the nature of numerals and operation in the problem, and then executes the strategy. The integration of mental arithmetic strategies into the senior primary school curriculum thus need the view of Imbo and Vandierendonck (2008) as the departure point to ensure learners are well exposed to a variety of computation strategies so that they have a choice of methods before executing the computation process.

According to MoEAC (2016, p. 1), senior primary teachers are expected to place emphasis on the “*mental arithmetic strategies*” to develop the learners’ awareness of numbers and number sense. This study was thus concerned mainly with the second aspect of mental arithmetic, that of *computation* and strategy building. Mental arithmetic requires regular and systematic practice for learners to master the computation strategies (Morgan, 1999). However, it seems no study was done to determine how primary school mathematics teachers make use of mental arithmetic strategies in their classrooms to enhance numeracy - a task which has presented itself challenging in the Namibian education system.

3.3 Features of Mental strategies

The mental arithmetic strategies require understanding, and often provide early approximations of correct answers (Morgan, 1999). The range of contexts for which a strategy is appropriate are limited. The mental strategies are *fleeting, variable, flexible, active, holistic, constructive and iconic* (Plunkett, 1979, p. 3). These characteristics are in contrast to those of the traditional algorithms which are considered to be *standardised, contracted, efficient, automatics, symbolic, general* and *analytic.* With mental strategies, learners can always explain how they arrived to answers unlike with standard methods.

Plunkett (1979) and Morgan (1999) discuss paper and pencil usage as belonging to traditional algorithms and not mental strategies. The same views are held by the Namibian senior primary mathematics curriculum developers where they are listed as part of the objectives that “learners are expected to use mental arithmetic and paper-and-pencil methods appropriately" (MoEAC, 2016, p. 4). This study, in contrast, considers that the assessments done at senior primary school are mainly in written form thus mental arithmetic strategies should not be detached from paper-and-pencil. The researcher, thus takes the stance of Rogers (2004) and merges mental arithmetic procedures with paper and pencil usage. The development of mental computation strategies can be better enhanced by allowing learners to have aids and jot down a few steps as they decompose and recompose numbers.

The mental arithmetic strategies revolve around the four basic operations; addition, subtraction, multiplication and division. Under each operation are a variety of strategies learners may conveniently employ to solve a mathematical task at hand.

3.4 Mental arithmetic in mathematics curricula

Mental arithmetic strategies gained place in the formal mathematics curriculum around the mid-nineteenth century in order to address the perceived slowness with which learners carried out written calculations (Morgan, 1999). Today, instead of quick recalling of simple facts and multiplication tables, learners are allowed to use pencil and paper as aids in quick computations through selected strategies.

Encouraging learners to develop eccentric cognitive methods for carrying out computations is compatible with the constructivist approach to learning, which asserts that humans acquire knowledge by constructing it from within instead of adopting it from the environment (Kamii, 1990). According to Kamii, an emphasis on mental arithmetic strategies contributes to the development of: a deeper understanding of the number structure and its properties; creative and independent thinking; ingenious ways of number manipulation; and strategies and skills associated with problem solving and computational estimation.

There are similarities among the skills listed by studies (see Morgan, 1999; Kamii, 1990) and the aims of the senior primary mathematics curriculum in Namibia. It is believed that during the mental computation, learners construct their own mathematical understanding that not only enhances learning but also encourages them to view mathematics as a meaningful subject (Morgan, 1999). The need is now drawn to focus on computational tools as well as a decreased emphasis on the traditional algorithms. Learning has now focused attention on learner autonomy and the use of learner-invented strategies as methods for developing basic facts and computational procedures. Hence, there is a need to study how to effectively incorporate mental arithmetic strategies when teaching primary school mathematics.

3.5 Mental Arithmetic and number sense development

The purpose of mental arithmetic is to develop meaningful strategies for learners to aid in the problem solving process and help develop number sense (Kim, 2016). Kim defines number sense as a persons' general intuitive understanding of number and operations and the ability to work effectively with them. It includes using numbers in flexible ways to make mathematical judgements and develop useful strategies for handling numbers and operations. Similarly, Markovits and Sowder (1994) define number sense as "a well organised conceptual network of number information that enables one to relate numbers and operations to solve problems in flexible and creative ways" (p. 23). Morgan (1999) extends that number sense includes the ability to (a) perform mental computations with nonstandard algorithms that take advantage of the ability to decompose and recompose numbers, (b) use numbers flexibly to estimate numerical answers to computations and to realise when an estimate is appropriate, and (c) judge the reasonableness of solutions obtained. Learners with number sense tend to analyse the whole problem first rather than immediately applying a traditional algorithm. They look for relationships among the numbers, and with the operations and contexts involved, and then invent computational strategies based on the observed relationships.

Number sense is deemed important in the senior primary phase (MoEAC, 2016). A learner classified as possessing number sense should be able to represent numbers in multiple forms, recognising relative and absolute magnitude of numbers, decomposing and recomposing numbers, understanding the relative effects of operation with numbers, and flexibly performing mental computation and estimation (Naukushu, 2016). Although the development of number sense has possibly occurred in many classrooms, the pivotal role that it plays in the ability of individuals to respond flexibly and creatively to number situations requires its development to be viewed as the main goal of primary school mathematics.

Graven, Venkat, Westaway and Tshesane (2013) argue that number sense includes developing an understanding of the meaning of numbers, the relative size of numbers, the relationships between numbers, knowledge of representing a number in various ways and the effects of operating numbers. Number sense especially in terms of relationships between numbers and operations in the form of mental strategies, is also the critical basis for the development of algebraic reasoning (Greeno, 2001). Without fluency and flexible knowledge of the commutative and distributive properties of numbers for example, one cannot efficiently compute answers to sums like $3 + 49$ as $(49 + 3)$ or to products like 14×101 as $(14 \times 100 + 14)$ (Graven et al., 2013).

Griffin (2003) reviewed research evidence pointing at a strong correlation between basic-facts fluency and achievement in mathematics. This suggests that developing learners' number sense results in improved academic achievement. Veloo (2010) argues that mental computation builds number sense by boosting learners' ability to reason in their own ways and build increasingly higher levels of thinking that are rooted in their own knowledge. Mental computations for single and multi-digit number operations teach learners how numbers work, how to decide on strategies, and how to create different methods to solve mathematics problems. Mental arithmetic encourages learners to reflect about the process and to think about what numbers mean in relation to the problem. Mental computation is a subset of number sense (Heirdsfield & Cooper, 2004), hence, developing mental computation skills in learners directly develops learners' number sense.

3.6 Mental arithmetic and Numeracy development

Numeracy is acknowledged as an indispensable form of elementary mathematical skills needed for use in all facets of life. It would be difficult for an individual to live a normal life without making use of mathematics of some kind (Lekoko et al., 2018). In this study numeracy is used as defined by (Westerford, 2008) that it is the ability to use mathematical ideas efficiently to make sense of the world. It involves understanding notations and techniques as well as being able to draw knowledge of particular contexts and circumstances in deciding when to use mathematics, choosing the mathematics to use and critically evaluating its use. It is thus imperative that the teaching of mathematics be done with caution to ensure a better link between "what it takes to live in today's societies and what learners learn" (Lekoko et al., 2018, p. 206). Lekoko et al. link the development of numeracy to the social justice notions, that of teaching mathematics which develops learners into conscious individuals functional in their societies (Gutstein, 2006; Stinson et al., 2012). It is hence arguable that mathematics for social justice and mathemacy lenses may enhance the development of numeracy in primary schools.

The development of numeracy or mathematical literacy is mainly held as key purpose for studying mathematics in school. Numeracy is hence the main objective which the Namibian senior primary curriculum of mathematics hopes to develop in learners via mental arithmetic strategies (MoEAC, 2016). School mathematics should equip learners with skills and understandings necessary for efficaciously dealing with other aspects of their lives (Morgan, 1999). Morgan further adds that schools should not only equip learners to develop competence and confidence in computational skills, they should also develop a much broader competency, one that includes an ability to apply understanding of numbers, space and measurement in realistic situations, to be able to use a wide range of technological aids and to recognise the reasonableness of such results.

3.7 Mental arithmetic and mathematical reasoning

Mathematical reasoning is defined as the process of reaching a decision by using critical, innovative and reasonable thinking (Erdem & Gürbüz, 2015). In mathematics, truths are arrived at through reasoning and not by experiment or observation as in the sciences and other fields. Mathematical concepts and operations are associated via reasoning. The reasoning includes abilities like following and assessing chains of arguments, knowing what a proof is and how it differs from other kinds of reasoning, uncovering the basic ideas in a given line of argument, and devising formal and informal arguments (Niss, 2003 as cited in Erdem & Gürbüz, 2015). Thus, individuals who possess sound mathematical reasoning skills about a particular case have enough knowledge about that case. They can thoroughly analyse newly encountered information and associate it with previous information, make reasonable estimates and assumptions, justify their thinking, reach some conclusions and explain and defend them (Umay, 2003 as cited in Erdem & Gürbüz, 2016). People who reason and think analytically tend to note patterns, structure, or regularities in both real-world situations and figurative matters; they ask if those patterns are fortuitous or if they occur for a reason; and they conjecture and prove (Erdem & Gürbüz, 2015).

Mental computation and mathematical reasoning are two vital skills that are crucial for nurturing higher order thinking because they help learners realize relationships among numbers while increasing their number sense (Umay, 2003). Thinking strategies involve and overlap with other widely discussed concepts, including mental computation, estimation, and number sense (McIntosh, 2005). Thus, mental computation training can be used as a vehicle for stimulating thinking, conceptual understanding on numbers and operations, and developing number sense

(McIntosh, 2005). Supporting the above, Heirdsfield (2011) found evidence that the teaching on mental computation strategies in early mathematics was not only helping children develop mental computation strategies, but also helping them to reserve energy to develop higher order reasoning, evaluating, and making sense of numbers and operations.

Mental computation strategies are needed not only at school but also in daily life since there is evidence that daily mathematics in which four basic operations take part mainly is suggested to be a part of human's natural ingenuity and reasoning (Erdem & Gürbüz, 2015). Devices such as calculators, computers and other calculating devices may not always be available to make calculations in everyday life. In such cases, making estimates and using mental computation skills can help lessen the problems. Hence, logical thinking is needed for mental computation to be carried out correctly. This requires mathematical reasoning, which is defined as the process of reaching a sensible deduction by thinking through all possible aspects (Umay, 2003 as cited in Erdem & Gürbüz, 2015). Varol and Farran (2007) argues that the steps taken by individuals to solve a problem give a clue on whether his or her mathematical reasoning has developed. In other words, once a learner develops practical strategies and uses such while carrying out mental computation, then it suggests that such a learner's mathematical reasoning is at a good level. When learners begin to use terminologies such as "If … then …", "because …" when computing, then it can be said that mathematical reasoning is being applied (Umay, 2003).

3.8 Mental arithmetic and computational estimation

The distinction between mental computation and computational estimation derives from the degree of precision required in arriving at appropriate solutions (Morgan, 1999). Computational estimation is defined by Lemaire and Lecacheur (2002) as the process of converting from exact to approximate numbers and mentally computing with those numbers to obtain an answer reasonably close to the result of an exact computation.

Whereas an approximate answer can be obtained for every arithmetical problem through computational estimation, the calculation of an exact answer is limited to a subset of problems. The degree of arithmetical complexity which can be mentally processed is dependent upon such factors as an individual's (a) knowledge of mental computation strategies, (b) recall a large number of numerical equivalence, and (c) ability to remember partial answers at various stages of a computation (Morgan, 1999). The first step in arriving at an approximate answer is to convert from exact to approximate numbers so that an estimate can be easily computed. It follows that the range of numbers that learners deal with when doing computational estimations are subsets of those that may be required to manipulate when computing exact answers.

Computational estimation

Computational estimation is defined as the process of finding an approximate (but satisfactory) answer to arithmetic problems using some set of mental calculation rules or procedures without actually (or before) computing the exact answer (Lemaire & Lecacheur, 2002). Computational estimation is an important mathematical cognition as it provides information about the general understanding of mathematical concepts, relationships and strategies. The computational estimation skills are useful in everyday situations in which a rough answer provides a contextually appropriate degree of precision (Tsao & Pan, 2012) . The widespread use and availability of hand calculators place additional importance on computational estimation skills to allow the user to evaluate whether the values displayed on the calculator are reasonable. Apart from the benefits of computational estimation as an everyday life skill, it allows learners to check the reasonableness of their answers and helps them to develop a better understanding of place value, mathematical operations and general number sense (Courtney-Clarke, 2012).

Sowder and Wheeler (1989) scrutinised the components involved in computational estimation and identified three types of conceptual knowledge. These are (a) understanding of the role of approximate numbers in estimation, (b) understanding that estimation could involve multiple processes and have multiple answers, and (c) understanding that the appropriateness of an estimated answer depends on the context. The related concepts and skills require knowledge of place value, basic facts and properties of operations as well as an ability to compare numbers by magnitude and mental computation.

Kilpatrick, Swafford, and Findell (2001) argue that computational estimation is a multifaceted activity that should incorporate all elements of mathematical proficiency. It entails a flexibility of computations that places emphasis on adaptive reasoning and strategic competence, guided by learners' conceptual understanding of both the problem context and the mathematics underlying the calculation as well as the proficiency with mental computational strategies. A disposition to judge the context in order to produce a sound answer instead of wild guesses is needed for good estimation (Courtney-Clarke, 2012).

Mental arithmetic computation and computational estimation share a common background characterised by factors essential to a well-developed sense of number. These include: (a) an understanding of place value concepts related to whole numbers and decimals, (b) an ability to operate with multiples and powers of ten, (c) an ability to use properties of operations, and (d) an understanding of the symbol systems used to represent numbers (Case & Sowder, 1990).

Computational estimation process

Efficient computational process requires three key cognitive processes: (a) *Reformulation,* (b) *Compensation,* and (c) *Translation* (Morgan, 1999).

Reformulation is dependent on the learners' ability to compare numbers so that they can convert exact to approximate numbers. In most school mathematics curricula, the development of computational estimation skills has been confined primarily to approximating numbers using the rounding conversion, a *reformulating* strategy (Reys, 1984). *Reformulation* is defined by Morgan (1999) as the process of altering numerical data to produce a more mentally manageable form without altering the structure of the problem. Its characterised by two forms; the *front-end strategies* and *substitution* by more acceptable forms. The most often practiced method of approximating numbers is *rounding,* usually to the nearest multiple of 10.

Translation entails modifying the mathematical structure of a problem to a form which is more easily managed mentally (Reys, 1984). Sowder and Wheeler (1989) characterised *averaging* as the main translation strategy. *Averaging* is mainly useful for operations in which the numbers cluster around a common value. The estimate is calculated by first multiplying the common values in the cluster. For example, 6164 + 7122 + 5112, an appropriate answer can be calculated by first selecting the average of the three numbers, 6000 and multiplying by 3. The strategy is translated to a form which an individual can comfortably compute mentally yet arriving at an acceptable estimate. The process of *Translation* is more flexible than *Reformulation* as learners have a wider view of the problem and are less restricted by the numbers involved (Morgan, 1999).

The third key cognitive process in computational estimation process is *Compensation,* which entails a comparison of numbers involving tolerance of errors in estimation (Reys, 1984). It is computational estimation process whereby the user first *reformulates* or *translates* the operations and then makes adjustments to compensate for errors.

Computational estimation has been researched and studies (for example Alajmi & Reys, 2007; Dowker, 2002; Hanson & Hogan, 2000; Sowder & Wheeler, 1989) revealed that many children and even adults lack the basic skills of estimation, a deficit attributed to limited exposure to estimation in school curriculum by Courtney-Clarke (2012). Hanson and Hogan (2000) found that college students were better at estimates of addition and subtraction operation than estimates of multiplication and division operations and attributed this to lack of deep understanding of multiplicative reasoning.

The nominal treatment given to estimation in the mathematics curriculum makes it one of the most neglected skills in mathematics as estimation strategies covered are insufficient to build any appreciable skills (Tsao & Pan, 2012). In the Namibian senior primary mathematics curriculum, estimation is included under the topic of Numbers, where learners are expected to: "demonstrate an understanding of numbers and be able to use mental and paper-and-pencil methods sensibly and appropriately. Learners are expected to use a variety of processes, e.g. comparison, classification, problem solving, abstraction and generalization, approximation and estimation" (MoEAC, 2016, p. 4).

The approximation and estimation part of the curriculum is further broken down into two objectives in Grade 7, which are to: (a) apply approximations in real life situations and (b) estimate answers to calculations with reasonable accuracy. This appears to be too shallow given the importance attached to computational estimation in real life. Moreover, there are no strategies mentioned which teachers should make use of during classroom instructions to aid learners when making approximations.

3.9 Measuring mental arithmetic fluency

The mental arithmetic strategies are not designed for recording as they require an understanding of the mathematical relationships (Plunkett, 1979). The use of mental arithmetic strategies facilitates a focus on quantities and not on digits and thus allows individuals to make meaningful alterations to the problems encountered and to work with quantities that can be manipulated more easily.

Mental arithmetic fluency has been measured via daily oral, speed and accuracy tests. This was mainly done with the view that mental arithmetic is about memorizing and recalling simple facts (Rogers, 2004). The daily oral, speed and accuracy sessions are criticized by McIntosh (2005) and Caney (2004) for making learners with strong memory feel good about themselves and left the rest of the class disappointed. These daily oral tests did not make learners noticeably more competent, but it did make them more neurotic about numbers.

Mental computation can be simply and effectively assessed by a written class test (McIntosh, 2005). McIntosh explained that if learners are asked to solve a problem and write down steps of the procedures and strategies they used to solve it, teachers can easily assess the depth of their mental computation knowledge.

Mental computation can be associated with the development of number sense. At times, number sense is measured by determining how competent a learner is in mental strategies. Hence, a teacher administering a number sense test is simultaneously measuring mental strategies fluency. Tests such as the ones administered by the Ministry of Education, the SATs, are good measures of mental strategy fluency.

3.10 Research on mental arithmetic

The concept mental arithmetic has been researched and experimented with in several countries such as Australia and Zambia (see Morgan, 1999; Tabakamulamu, 2010). This section positions the current study within the existing discourses and highlights the gaps in the existing knowledge that the study sought to fill.

Tabakamulamu (2010) conducted a study to assess the extent to which teachers in early primary mathematics in Zambia could adopt the use of strategies for mental calculations for the addition and subtraction of double-digit numbers in Grade 2 classes. His study was informed by the constructivist paradigm. The results show a significantly better performance of the children who were taught mental arithmetic strategies than those who were in control groups. The pupils from experimental groups were also found to have developed more positive attitudes towards learning mathematics. Tabakamulamu therefore argues that instruction in mathematics classrooms should take a mental arithmetic approach.

Furthermore, Tabakamulamu also found that the teachers who were involved in his study had inadequate understanding of the mathematics they were supposed to be teaching in their classrooms. He further recommended an in-service or bridging programme to supplement the teachers' knowledge. Given the common geographical features shared between Namibia and Zambia, it was thus deemed necessary to explore how the senior primary teachers in Namibia integrate mental arithmetic strategies in their classrooms.

Morgan (1999) conducted a study to explore aspects of mental computation within primary school mathematics curricula in Queensland, Australia, and as benefits of mental arithmetic cited; deeper understanding of the structure of numbers and their properties, ingenious ways of manipulating numbers, and development of skills associated with problem solving and computational estimation. Morgan's study has recommended that in order to test the success of an intervention a model should be designed to suit the context and subjects being studied. Although the model used by Morgan was successful in Queensland, Namibia is a far a different contextual setting with different experiences. It was thus found necessary to determine the reactions and outcomes of an intervention model on mental arithmetic strategies in Namibian primary schools. Furthermore, this study appears a replica of the works of Tabakamulamu and Morgan but with major differences in theoretical approaches, methods and contextual settings.

3.11 Mathematics education in Namibia

Namibia, like many other African countries as highlighted in section 1.2.1, has been colonised and subjected to a segregated education system. Before 1990, the country has been ruled by the apartheid government of South Africa and shared a racial educational policy background with the natives of South Africa; Blacks in both countries were under the Bantu education system. The apartheid government which ruled the country ensured that black people received the education that only enabled them to serve the whites better. Subjects like mathematics were taught with very shallow content.

Khuzwayo (2005) points that South African apartheid government in 1954 legalised the Bantu Education Act which formalised racially segregated educational facilities. According to Khuzwayo

(2005), the Bantu education act was engineered to provide inferior education for the blacks in order to sustain their position of social, political and economic suppression. The black learners had different schools with a different and substandard curriculum which would prepare them with skills to serve as labourers under white control (Peters, 2016). The teachers for the blacks were also trained in different institutions.

Khuzwayo (2005, p. 310) as well as Amkugo (1993, p. 57) both cited the words of the then minister of native affairs in the South African apartheid government, Dr HF Verwoerd, in a speech delivered to parliament on 17 September 1953 as:

> When I have control over native education I will reform it so that the natives will be taught from childhood to realise that equality with Europeans is not for them. People who believe in equality are not desirable teachers for natives… what is the use of teaching the Bantu child mathematics when it cannot use it in practice?

The educational policies were then formulated to keep blacks from taking up careers in mathematics. Many of the schools for blacks did not offer mathematics at the senior secondary level (Khuzwayo, 2005).

Namibia gained her independence in 1990 from South Africa and the then new Namibian government through the Ministry of Education and Culture, (MEC, 1993), embarked on reforming the education system in order to train its citizens into economic productive individuals. A numerate nation can competently participate in the increasing global technological world of work and information. Mathematics is needed in order to access to and succeed in economic activities.

The identified skills shortage in the professional, technical and trade careers (e.g. engineers, accountants, technicians, electricians, etc) require appropriate numeracy skills or a higher level of mathematical proficiency (Courtney-Clarke, 2012). The mathematics curriculum is thus targeting to develop learners into mathematically literate citizens who may take up careers in these fields and address the shortage.

Namibia is experiencing challenges in mathematics education. Studies (Courtney-Clarke, 2012; NIED, 2010; Naukushu, 2016; Peters, 2016) pointed out aspects such as poor teachers' content knowledge, learners' social backgrounds, teaching and learning materials among others as the predicaments to mathematics education.

3.12 Mathematics at primary schools

As Peters (2016) puts it, if the performance of learners in mathematics is to improve, the main focus should be placed at the primary school level. Namibia has recently introduced another education reform implemented in 2015. The reformed primary school mathematics curriculum aims to impart knowledge and skills on learners in order to be able to solve everyday tasks, and to take care of personal interests and duties. It further targets to develop logical and scientific thinking and provide pleasure and satisfaction when learners solve problems (MoEAC, 2016).

The Namibian primary education structure consists of two phases; the Junior Primary phase (grades 0 to 3) and the Senior Primary phase (grades 4 to 7). At the end of the Senior Primary phase, i.e. Grade 7, learners write national examinations in a transition to the Secondary education phase (grades 8 to 12). The main interest of this study is in the Senior Primary phase as it's the one that directly prepares learners for the Secondary phase.

At the Senior Primary phase, the education system aims to develop learners' mathematical skills and numeracy, and the following are listed as aims (MoEAC, 2016):

- ✓ *to develop functional numeracy and mathematical thinking;*
- ✓ *to develop positive attitudes towards mathematics;*
- ✓ *to enable learners to acquire basic number concepts and numerical notation;*
- ✓ *to enable learners to understand and master the basic mathematical concepts and skills;*
- ✓ *to enable learners to apply mathematics in everyday life.*
- ✓ *to prepare learners for present and future studies in mathematics and other related subjects*

The core themes covered in the senior primary mathematics curriculum extends from the basic competencies in computation with whole numbers to computations with common and decimal fractions. Measurement, time, and money and finance relate to the learner's everyday situation. Geometry is the mathematical understanding of space and shapes. The themes of problem solving, number patterns and data handling are ways of working with, understanding and communicating about and through Mathematics.

Mathematics teachers at the Senior Primary level are tasked to help learners develop these skills in effective and meaningful ways. In the entire Primary phase (grades 0 to 7), learners are not allowed to use calculators and teachers are expected to teach their learners various strategies of mathematical computation that do not require a calculator. Learners are required to mentally compute exact answers and make appropriate approximations. Senior primary school teachers of mathematics are thus tasked to place an emphasis on mental arithmetic strategies to enhance numeracy and number sense among learners.

The approach to teaching and learning in the Namibian education system is based on a paradigm of learner-centred education (LCE) described in ministerial policy documents and the LCE conceptual framework (see Haimbodi, Kasanda, & Kapenda, 2015). The starting point for teaching and learning is the fact that the learner brings to the school a wealth of knowledge and social experience gained continually from the family, the community, and through the interactions with the environment. Learning in school must therefore involve, build on, extend and challenge the learner's prior knowledge and experience. Consequently, teaching strategies must be varied but flexible within well-structured sequences of lessons. Co-operative and collaborative learning should be encouraged wherever possible to promote discourses in learner-centred settings.

Studies have revealed a point of concern in the performance of senior primary school learners in mathematics (see e.g. Mutuku, 2015; National Institute for Educational Development [NIED], 2010; Spaull, 2011). The points of concern emerging from this will be discussed under the following sections: SACMEQ; the SATs; and teacher training and the development of mental arithmetic strategies.

3.12.1 The Southern and Eastern Africa Consortium of Monitoring Educational Quality [SACMEQ]

The Southern and Eastern Africa Consortium of Monitoring Educational Quality [SACMEQ] evolved from a small research project that was initiated by the International Institute of Educational Planning [IIEP] in Zimbabwe during 1991-1992 into a very important web of 14 Ministries of Education (Botswana, Kenya, Lesotho, Malawi, Mauritius, Mozambique, Namibia, Seychelles, South Africa, Swaziland, Tanzania, Uganda, Zambia, and Zimbabwe) (SACMEQ, 2010).

The purpose of SACMEQ is to undertake research and training activities that aims to expand opportunities for Educational Ministries in the participating countries to gain the technical skills required to monitor and evaluate the general conditions of schooling and quality of their own basic education systems. It also aimed at undertaking research that generates data to be used in improving the quality of education.

The first educational research by SACMEQ was done 1995 – 1998, and the second project (SACMEQ II) was conducted in the years 1998 – 2004. SACMEQ III project was conducted from 2005 – 2010. At the end of each of these three projects, each ministry prepared a report with set agendas for action in order to improve the quality of the education system.

SACMEQ III ran three tests; a literacy test, a numeracy test and a test on HIV/AIDS knowledge. The Mathematics (numeracy) test measured the learners' capacity to understand and apply procedures, knowledge and mathematical understanding as individuals and as members of a wider community (Spaull, 2011). The tests covered mathematical domains which were common in mathematics classrooms in Southern and Eastern Africa.

Spaull (2011) compared primary school performances in mathematics among four southern African countries: Botswana, Mozambique, Namibia and South Africa. The study found a high prevalence of innumeracy in all the four countries, most outstandingly Namibia. Almost half of the Namibian learners (47.69% of the 6398 learners who participated) were classified functionally innumerate as compared to 40% in South Africa, 33% in Mozambique and 22% in Botswana.

Spaull (2011, p. 33) defined functional numeracy as an indication of whether or not an individual has "acquired sufficient numeracy skills such that he or she is able to satisfactorily use those skills in everyday life". If a learner could not relate basic arithmetic skills into real world situations such as interpreting common everyday units of measurement, then that learner was classified as innumerate.

The Spaull's (2011) analysis of the SACMEQ III report revealed that a large proportion of learners in the four countries had not mastered the basic numeracy skills. In Namibia, the ministry of education developed and implemented strategies to improve the teaching of mathematics in primary schools to help remedy the situation (Shigwedha et al., 2017). The report by (Shigwedha et al., 2017) presented the results of SACMEQ IV and claims that an intervention was done in Namibian schools between SACMEQ III and SACMEQ IV to enhance the development of numeracy skills among primary school learners in Namibia.

SACMEQ IV was conducted in 2013 to further determine the quality and standards of literacy and numeracy of primary school learners in the member states. The results indicated that only 17.4% of the Namibian learners reached the level of competent numeracy (Shigwedha et al., 2017). The competent numeracy level is defined by (Shigwedha et al., 2017) as the level where learners are able to solve problems in order of arithmetic operations, compute fractions, decimals and make estimations. These results show that the low levels of functional innumeracy persist in Namibia and suggests that the intervention strategies by the ministry of education which were implemented between SACMEQ III and SACMEQ IV did not improve numeracy in primary schools. The report by Shigwedha et al. (2017, p. 86) recommends that the Ministry of Education, Arts and Culture [MoEAC] should ensure that "mathematics teaching strategies in place are used optimally in order to see a positive change in SACMEQ V". It remains unclear what these strategies referred to in the report are and whether the teachers in the primary schools are aware of and are implementing such, and hence, necessitates a study on classroom practices and perhaps a model which may enhance numeracy.

3.12.2 The Standardised Achievement Tests [SATs]

In order for the education sector to play its key role in the realisation of the Namibian national vision, *Vision 2030*, (a national vision that by the year 2030 Namibia should join the ranks of high income countries and afford all its citizens a quality of life that is comparable to that of a developed world), it embarked on pursuing equitable social development. In 2006, the Ministry of Education, thus, developed the Education and Training Sector Improvement Programme [ETSIP]. The key purpose of ETSIP is to "substantially enhance the sector's contribution to the attainment of strategic national development goals, and to facilitate the transition to a knowledge based economy" (Namibia. Ministry of Education [MoE], 2005, p. 35). At the Senior Primary level, ETSIP focused, among other items, on establishing assessment benchmarks of international standards to monitor the quality of education and performance in Literacy, Science and Numeracy.

The Standardized Achievement Tests [SATs] were thus developed to monitor the learners' acquisition of identified skills and competencies in key subject areas; Mathematic, English and Natural Science & Health Education (Mutuku, 2015). The SATs also aimed to set the baseline performance targets and monitor the progress of learners at individual schools.

The SATs are psychometric in that they demonstrate candidates' abilities to deal with numbers quickly and accurately. The tests contain items that assess knowledge of ratios, percentages, cost and sales analysis, rates, trends and currency conversions. Such computations are to be processed mentally as calculators are not allowed. The results of the SATs indicate that the senior primary school learners have performed below average in mathematics over the years (Mutuku, 2015).

Both the SACMEQ and SATs consisted of items measuring learners' functional numeracy. The reports indicate that learners could not engage in computation strategies that quickly and efficiently helps to solve the presented items. Then one wonders how the senior primary teachers are preparing learners to function as numerate individuals. Moreover, how are the mental arithmetic strategies integrated in the daily mathematics classrooms? These are some of the basic questions that this study tried to address.

3.12.3 Teacher training and the development of mental arithmetic strategies

Before 1990, the Namibian education system existed in three segregated categories; education for Whites, Coloureds and for Blacks. The apartheid South African government believed that mathematics was not meant for Blacks and hence the separate curricula. The teachers for these three races were also trained at segregated institutions across the country (Amutenya, 2002; Naukushu, 2011).

Khuzwayo (2000) in his study that explored how the South African apartheid ideology and the political practice influenced how mathematics was taught and learnt revealed how the then education system was engineered to deliver substandard mathematics education to the blacks. While not all schools offered mathematics, those that offered mathematics did not go beyond abstract meanings and emphasised memorisation of simple facts. The blacks were also made to believe that their brains were not wired for mathematics (Naukushu, 2016).

The assumption that black learners could not be competent in mathematics made it difficult for many of them to attain higher order understanding of mathematical concepts at that time. As a result of this perception and the oppressive education system, blacks acquired substandard numerical and mathematical competencies.

Given that mental arithmetic seems not explored in the Namibian mathematical discourse, it is highly likely that the senior primary school teachers have been mainly exposed to standard (traditional) algorithms during their school years and during their teacher training programmes. The reader should be mindful of the fact that most of the Namibian teachers are directly or transitively affected by the apartheid education system which targeted to keep the natives mentally unliberated. It is therefore logical to assume that these teachers were not exposed to methods which holistically develop number sense and numeracy and that the "scars" of oppression still exists to date (Naukushu, 2016, p. 2). This is likely to influence the teachers' classroom practice.

Allowing learners to do operations in standard algorithms presents a risk of practicing mathematics which is "divorced from reality" and hence the need for RME perspectives in mathematics education (Morgan, 1999, p. 51). The efficient standard algorithm requires that the digits be dealt with separately without reference to their meaning or their relationship to real world or representational models. Traditional algorithms do not correspond to the ways in which people tend to think about numbers; for example, in the context of the use of the most conventional algorithms, the digit '6' in the number 5683 is treated as '6' and not 600. The standard algorithms thus seem to contradict the core purpose of mathematics education which is to develop numeracy via understanding of mathematical concepts, procedures and operations.

The traditional algorithms do not solicit thinking in learners, leading to a loss of ownership of ideas. Thinking is not the focus, the focus is put on quick and reliable answers. Learners therefore tend to accept answers and resort to calculations without thinking. An example from the findings by Northcote and McIntosh (1999) of learners asked to work out $100 - $ 95 and resorted to standard algorithms.

Another issue is of the relevance of the standard algorithms in everyday life. It has become increasingly unusual for standard written algorithms to be used anywhere except in the classrooms. The practical world has calculators and computers to carry out big number computations. The classroom should therefore focus on developing numeracy skills. It should enhance development of number sense and the minds that can think and reason mathematically.

Today the Namibian senior primary school teachers are trained by the University of Namibia [UNAM] via its satellite campuses distributed across the country, and by the International

University of Management [IUM]. The teaching programme runs for four years and upon graduation, teachers are expected to have acquired the necessary skills to transform the education sector of the country. How are the graduating teachers equipped to develop number sense and numeracy? How likely are these teachers to revert to using the traditional algorithms which they used during their school years (Naukushu, 2016)?

3.13 Mental arithmetic in Namibian senior primary school curriculum

The Namibian Mathematics Curriculum for the senior primary school phase supports the acquisition of mental computation skills through the development of thinking strategies across the senior primary phase. The curriculum states that “Learners are expected to use mental methods and paper-and-pencil methods sensibly and appropriately” (MoEAC 2016, p. 2).

MoEAC (2016) further emphasises that senior primary school learners are not allowed to use calculators and that teachers must place emphasis on mental arithmetic strategies that develop the learners’ awareness of number and number sense. Learners are required to compute exact answers and make appropriate approximations mentally. Before learners exit the senior primary phase, they should possess a variety of strategies of mental computation. It is important for teachers to recognize these strategies, develop and improve over time with regular practice. It is thus peculiar that teachers incorporate mental computation strategies in their lessons daily.

3.14 Developing mental strategies

Mental arithmetic strategies are methods that learners deploy in performing calculations and are selected to suit the numbers involved (Threlfall, 2000). In selecting a strategy, learners examine the features of a given calculation, determine an appropriate approach and then execute a method for the calculation. This approach is opposed to standard algorithms where learners have no choice over methods e.g. division has to be done using the long division approach.

There is a need to teach and expose learners to a variety of strategies for mental computations to help make them more efficient and to allow them to have access to different strategies when solving problems (Thompson, 2001). It has been observed by Murphy (2004) that children who employed a range of mental computation strategies attained higher scores and the below average children often relied on the taught traditional algorithms. Therefore, teachers should teach such strategies to enhance the performance of below average learners.

Some researchers even argue that students should not be taught algorithms, but should invent their own methods instead. Of course, this would need to be carefully managed by teachers, so that students do not practise wrong methods or inadequate methods that only work in special cases (Kamii, 1997). Furthermore, Murphy (2004), Morgan (1999) and Kamii (1997) have cautioned that although learners in the senior primary grades can become proficient with mental computation in everyday situations, such will not occur unless learners are provided with definite mental arithmetic experiences in school. In addition to that, Threlfall (2000) affirms that careful interventions, modelling of the mental computation strategies by teachers and working jointly with learners on strategies is effective in raising the level of learners' performance.

The teachers need to have knowledge of mental strategies in order to offer fitting responses to their learners. These responses may be in asking learners for clarifications (Swan & Sparrow, 2001). Teachers need to be armed with the right knowledge in order to determine the exact help to give their learners as they work on mathematical problems. The classroom focus should be on the discussions of the devised mental strategies with peers and teachers so that learners may think about alternative strategies as they listen to the ideas of others (Morgan, 1999).

This study proposes that the approach to developing mental computation strategies should involve teaching one strategy on a particular operation at a time as to not cause confusion. The teacher will then give a variety of problems and encourage learners to solve choosing any of the discussed strategies. That way teachers may not run the risk of losing flexibility with strategies (Swan & Sparrow, 2001). Flexibility is the key to the development of mental computation skills. The aim of any session designed to develop mental strategies should be to develop flexibility in learners' thinking and gain an insight into the structure and properties of number. Instructions should be designed in such a way that it elicits from learners the use of a variety of methods to give them a democratic voice and expressions on their learning. This will allow learners to explore and discuss a variety of strategies and adopt those that are suited to solving the particular problem at hand.

Markovits and Sowder (2004) concur that when learners with number sense are given a mathematical task, they are expected to have in mind that there is not only one way, that there are always several strategies to solving the task, that mathematics is related to real life, and that decisions and judgements are expected.

Developing awareness of mental computation strategies is an important skill in making sensible selection of computation strategies. To build mental computation skills, each strategy should be practised in isolation until learners can give correct solutions in a given time frame. Learners should understand the logic of the strategy, recognise when it is appropriate to use it and be able to explain how to use the strategy. The activities for reinforcement should be structured to ensure thorough participation and should not focus on the correct answers but on how to get to the correct answers.

After the teacher is confident that the learners have internalised the strategy, the learning should shift to integrating the strategy with other strategies developed earlier. This can be done by providing activities that includes a mix of number expressions, for which this strategy and others would apply (Imbo, 2007). Learners should then practice by choosing the most suitable strategy and discussing their choices. The goal for teaching mental computation should be to show learners a range of various mental strategies, provide opportunities where each strategy can be used, and encourage learners to use mental strategies regularly to improve their skills.

In the Namibian senior primary school mathematics curriculum, mental arithmetic is listed as a topic with objectives distributed across the grades 4 to 7. This topic is mostly taught at the beginning of each phase. The key assumption is that the learners acquire computation strategies at the onset and employ these strategies throughout the course. How the teachers integrate mental strategies in their lessons and in all other mathematics topics throughout the academic year was among the interests of this study. Threlfall (2000) advises that in order to develop mental strategies, teachers need to avoid mere rehearsals of strategies as routines and shape teaching to develop strategies rather than acquiring them. Teaching for the development of mental arithmetic strategies should involve offering continuous opportunities to compute in a supported sequence of challenges to expand the learners' awareness of inferential possibilities in numbers.

The Namibian senior primary school curriculum has listed a few of mental arithmetic strategies that teachers may practice with their learners. These are labelled as: (a) Preserve the first number, split the second number, (b) Split both numbers into base ten and units, (c) Changing the order of addends to form multiples of 10, (c) Compensatory strategy, (d) Bridging the decades, (e) Mentally multiply up to four digit numbers by 10 and 100, and (f) The relationship between multiplication facts (Morgan, 1999).

Table 3.1: The mental arithmetic strategy model

Addition	1. Left – Right Approach 2. Compensation 3. Bridging a decade 4. Breaking and Bridging
Subtraction	1. Left –Right Approach 2. Compensation 3. Breaking and Bridging 4. Constant Difference
Multiplication	1. Distributive Principle 2. Compensation 3. Compatible Factors 4. Half and Double 5. Aliquot parts
Division	1. Additive distribution 2. Subtractive distribution 3. General Factoring

Sequential framework for senior primary mental strategies. Adapted from Morgan (1999).

These strategies seem to be mainly on addition and subtraction. They also seem not to give a wide range of various strategies for learners to choose from. This study tested the effects of an intervention based on the framework in Table 3.1. The framework comprises of various mental computation strategies within the four basic operations.

3.15 Summary of the chapter

This chapter discussed literature on mental arithmetic and its place in mathematics education. It adopted the definition of mental arithmetic as defined as the development, selection and execution of computation strategies. These strategies may be developed with the aids of pencil and paper. Namibia is faced with poor results in primary school mathematics. Studies pointed to instructional approaches of the teachers, the teachers' content knowledge, teaching resources and learners' social and economic backgrounds as possible contributing factor to low performance. Mental arithmetic strategies seem to enhance academic performance of learners. How are the primary school teachers in the Oshana region integrating mental strategies in their classrooms? The study explored to determine answers to that question.

A number of studies and reports cited low performance at primary schools and highlighted possible reasons for poor performance in Mathematics. Reasons for poor mathematics performance as confirmed by these reports are: teachers' competencies in mastering the curriculum content, under-qualified teachers as a result of shortage of qualified mathematic teachers, availability of teaching materials, methods of presentation, learning environment, lesson preparation, gender, and motivation to learn. Namibian learners were recorded to have performed the lowest in mathematics comparing to the performance of the same group of learners in the southern African countries. This study, hence, sought to explore how teachers incorporate mental arithmetic strategies in primary school mathematics and determine the impacts these strategies may have on learners' academic achievement. The next chapter presents the research design and methodological approaches of the study.

CHAPTER FOUR: RESEARCH DESIGN AND METHODOLOGY

4.1 Introduction

This chapter discusses the research design, data collection procedures and methods of data analysis for this study. It begins with a discussion of the research paradigm, followed by the sampling techniques, data collection procedures, research instruments as well as the data analysis procedures. Towards the end, the chapter discusses the pilot study that was conducted and concludes with a discussion on the steps taken to upkeep research ethics.

4.2 Research Design

The study used a mixed methods paradigm as it sought to explore the incorporation of mental arithmetic strategies in primary schools in the Northern Namibia. A pragmatic mixed methods paradigm is an inquiry involving collecting both qualitative and quantitative data, and integrating the two forms of data to generate new meanings (Creswell, 2014). As Gay et al. (2013) put it, utilising a mixed paradigm allows the two data to complement each other and form a strong basis of findings, rather than confining ones' self to a single paradigm. It is on this ground that the researcher used a mixed methods design in pursuit of exploring the incorporation of mental arithmetic in senior primary school mathematics.

Informed by the Critical Mathematics Education theory, the researcher took into account that objects in both quantitative and qualitative research designs are shared by any social group and would yield better meaning towards the objectives of the study. The core assumption of the mixed methods form of inquiry is that the combination of both qualitative and quantitative approaches provides a more complete understanding of a research problem than either approach alone (Creswell, 2014). A mixed paradigm for this study was thus taken for a collective yield of data to answer the research questions.

According to Creswell (2014), the field of mixed methods is relatively new as it stems from the late 1980s to the early 1990s. The early thoughts about the value of mixed methods resided in the idea that both qualitative and quantitative methods had bias and weaknesses, and the collection of both qualitative and quantitative data neutralised the weaknesses of both form of data. The mixed methods designs were developed to further guide studies. This study assumed an exploratory sequential mixed methods approach (Creswell, 2014; Cohen, Manion, & Morrison, 2007) where the researcher first conducted qualitative research and explored the views of the participants, the researcher then formulated a hypothesis for the quantitative phase and went on to collect quantitative data.

The study used the QUAL-Quan model, also known as the *exploratory mixed methods design* where qualitative data were collected first and were more weighted than the quantitative data (Gays et al., 2012). As Cohen et al. (2007) put it, the qualitative study comes first in the QUAL-Quan model and it is typically an exploratory study preceded by classroom observations and open-ended interviews or group interviews and then concepts or potential hypotheses are identified, and then, in the second phase, the hypotheses are verified with quantitative techniques. The concept QUAL-Quan was derived from taking the first four letters of *qualitative* and *quantitative*, the *QUAL* is written in capital letters to indicate that qualitative paradigm is more weighted than the *Quan* for quantitative paradigm. Moreover, the QUAL is written first to indicate that the study first collected qualitative data.

4.2.1 The QUAL

In this study, the qualitative methods weighed more as the study was of an exploratory nature. Most of the data collected on the types of mental arithmetic strategies practiced by the senior primary teachers and learners in their classrooms were of a qualitative nature. The qualitative paradigm is the collection, analysis and interpretation of comprehensive narrative and visual data to gain insights into a particular phenomenon of interest (Gays et al., 2012; Cohen et al., 2007). From the qualitative paradigm perspectives, all meaning is situated in a particular perspective or context, and because different people and groups often have different perspectives and contexts, the world has many meanings, none of which is necessarily more valid or true than the others. The efforts to understand the mental arithmetic practices and perspectives of the senior primary teachers and learners required that the researcher interacted extensively with these participants, using time-intensive data collection instruments such as classroom observations and interviews.

4.2.2 The Quan

After the qualitative data were gathered, the researcher developed a hypothesis in seeking to establish the effects that the mental arithmetic strategies may have on the learners' mathematical understanding. The quantitative data focus on information that can be represented by means of numerical values such as frequencies, percentages, proportions and averages (Creswell, 2014).

This study gathered numerical data on the performance of learners in psychometric tests and hence used descriptive statistics like mean, median, standard deviation and also included inferential statistics like the *t*-tests.

4.3 Population

The population of the study comprised of senior primary mathematics teachers and senior primary (Grade 4 – 7) learners at 10 primary schools in the Oshana region in 2017. Sited in the northern part of Namibia, Oshana region has a wide range of 91 schools that offer the senior primary education phase. These schools are evenly located in urban, sub-urban and rural areas of the region and hence giving a collective information on the practices of mental arithmetic across all economic-social settings. The researcher chose this region conveniently as most schools were easily accessible to him.

4.4 Sample and sampling techniques

A sample is defined by Gay et al. (2013) as a group of individuals, items, or events that represents the features of the population from which the sample is drawn through the sampling process.

In this study, the sample consisted of 10 primary schools in the Oshana region. The 10 schools, all in the radius of 20km from the Ondangwa town, were purposively sampled based on accessibility to the school as this required minimal resources in terms of time and transport costs. *Purposive sampling* is a sampling procedure whereby the researcher handpicks participants, research sites, documents or visual materials, which are most likely to help in providing more information to understand the problem being studied (Creswell, 2014). In this case, the researcher wanted to explore, understand and gain insight into the classroom practices of mental arithmetic at primary schools. The researcher thus purposively sampled the experimental schools whose teachers had attended mental arithmetic workshop as these teachers had an idea on the type of strategies to use. The participants in the *mental arithmetic strategies workshop* were sampled based on their proximity to town and willingness to attend the workshop.

Ten senior primary school mathematics teachers and 300 senior primary learners at these schools were involved in the study. At each of the 10 sampled schools, one senior primary mathematics teacher was observed while teaching and interviewed after the lesson observations. Most of the schools (seven out of the 10 schools) in the sample schools had only one mathematics teacher for the senior primary phase.

After the initial phase of data collection, the researcher drew up a manual for a workshop on mental arithmetic strategies. Five out of the ten schools were represented in the training. Although the researcher intended to have more teachers attend the workshop, only five teachers made it for the workshop sessions. The rest of the teachers registered concerns that they could not make it for the workshop. The five teachers who took part in the mental arithmetic strategies workshop sat for the focus group discussions.

In the second phase of data collection, two schools were randomly sampled out of the five schools which were represented by teachers during the mental arithmetic strategies workshop to be the experimental schools. Two other schools, out of the five schools which were not represented by teachers at the workshop were randomly sampled as the control schools.

4.5 Research Instruments

The study made use of data collection methods of observations, interviews, focused group discussions and tests in order to collect data from the sampled participants. This section presents the different data collection instruments that were used in drawing the data from the interactions of participants in the sub-sections as highlighted in Table 4.1.

Table 4.1. The research data collection plan

Research question: How is mental arithmetic being incorporated in the senior primary mathematics curriculum?				
SUB - QUESTIONS	**METHOD**	**INSTRUMENT**	**RESPONDENTS**	**ANALYSIS TECHNIQUES**
1. What mental arithmetic strategies do primary school mathematics teachers and learners use in the mathematics classrooms?	Observations	Observation protocol	Teachers and learners	Interpretive analysis
2. From teachers' perspectives, why should the development of mental arithmetic strategies be (or not) emphasized in schools?	Individual teacher interviews	Interview schedule	Teachers	Thematic analysis using Atlast.ti
3. How can the development of mental arithmetic strategies be enhanced in primary schools?	Intervention for teachers – mental arithmetic strategies workshops	a) Mental arithmetic strategies manual b) Focus group discussion guide	Teachers	Thematic analysis using Atlast.ti
Includes hypothesis testing: H_o: There is no significant difference in the performance of the learners taught mental arithmetic strategies and those who were not.	Intervention for learners – quasi experimental to test the hypothesis	a) Mental arithmetic strategies manual b) Pre-test c) Post-test	Learners	Descriptive and inferential statistics using SPSS

Additionally, brief accounts on measures taken by the researcher to ensure trustworthy, validity and reliability of the instruments are made in this sub-sections too. However, later in this chapter, a detailed section discussing validity and reliability issues is presented.

4.5.1 Observations

Observations consist of detailed notation of behaviours, events, patterns, interactions and their settings (Simpson & Tuson, 2003). In this study, observations were conducted to generate data to answer the first research sub-question 1: What mental arithmetic strategies do primary school mathematics teachers and learners use in the mathematics classrooms?

Observations as a method of data collection has strengths in that the researcher has direct access to events or interactions which are the focus of the research (Cohen et al., 2007; Simpson & Tuson, 2003). This enabled the researcher to understand the context of the programmes, to be open-ended and inductive, to see things that might otherwise be unconsciously missed, to discover things the participants might not freely disclose in interviews and to move beyond perception based data as opposed to interviews. This method was used in this study so that the researcher, in pursuit to explore the incorporation of mental arithmetic strategies in the senior primary schools, could get access to interactional settings to understand situations in their contexts. The observations were unstructured and sought determine the types of mental arithmetic strategies practiced in senior primary schools and to further generate the hypothesis later tested in the quantitative phase of the study. The researcher took note of the teaching strategies of the teachers and how mental arithmetic was incorporated in solving problems. The learners' responses to verbal questions and to problems on the chalkboard as well as in their note books were recorded. The researcher took video clips and snapshots of learners' written work. This gave the researcher the picture of the state of mental arithmetic integration in the mathematics classrooms at the senior primary schools.

To enhance validity of the observed data, the researcher started the preliminary data analysis parallel to observations. The coding and grouping of data was done during observations and frequencies and consistencies recorded and the bases of the observed behaviour were reflected upon during teacher interviews for triangulation purposes (Simpson & Tuson, 2003).

4.5.2 Interviews

An interview is an interchange of views between two or more people on a topic in order to construct knowledge (Cohen et al., 2007). The interviewer and interviewee(s) interact discussing their interpretations of the world in which they live and express personal perceptions of life aspects. The individual teachers' interviews were conducted to answer the second research sub-question: From teachers' perspectives, why should the development of mental arithmetic strategies be (or not) emphasized in schools?

The researcher conducted oral interviews right after observing each teacher. All the 10 observed teachers agreed to be interviewed. The interviews gave insights into the perspicacity and views of teachers with regard to the mental arithmetic concepts. The interviews were audio recorded and transcribed at a later stage. Although some of the teachers looked nervous prior to interview, the researcher gained their trust by re-assuring them that the information disclosed in the interview sessions will be treated with utmost confidentiality and their names were not going to be identified with the provided information. The researcher also assured them that the tape recordings will not contain any names. The interview items sought to determine how teachers understand the concept mental arithmetic, their views on how mental arithmetic strategies are being incorporated in classroom computations and whether such strategies should be part of the curriculum or not (see Appendix P). The interviews further sought for teachers' views on how mental arithmetic strategies were to be developed in the senior primary schools.

Lee (1995) as cited in Cohen et al. (2007) warns that researchers need to be careful during interviews to avoid seeking for answers that support their preconceived notions as this poses a threat to the validity of collected information. In the present study, the researcher took care to ensure neutrality as he explored to uncover classroom practices. Teachers were assured of confidentiality and displayed confidence that the information they were providing was to be used for the purpose of the study only, and that they would not be identified with such (see Appendix H). Moreover, the researcher being from an outside institution should have provided confidence in teachers as he (researcher) was not in an authority position capable of imposing sanctions on the interviewees (Cohen et al., 2007). In addition, the data collected via interviews was triangulated with data from observations, focused group interviews and psychometric tests.

4.5.3 Focus Group Discussions

A focus group discussion is a dialogue whereby several participants are interviewed at the same time and place by the moderator or interviewer, who uses the group and its interactions as a way to gain information about a specific or focused issue (Patton, 2002). Similarly, Masadeh (2012) defines focus group as a group of interacting people with some collective interest or characteristics, brought together by a mediator, who uses the group and its deliberations as a way to gain information about a particular concern of focus.

Focus group discussions were conducted by the teachers who went through the *mental arithmetic strategies workshop* in order to generate answers to the third research sub-question: How can the development of mental arithmetic strategies be enhanced in primary schools? The items on the focus group discussion guide helped determine what the participants learned from the workshop regarding mental arithmetic strategies that new knowledge informed the participants' views on how mental arithmetic strategies are to be developed in the senior primary schools. The participants were also asked to evaluate the effectiveness of the workshop (see Appendix O).

It was found necessary to use the focus group discussions in order to create room for interactions between the participants and open up a platform for generation of ideas. The researcher considered that the participants were fulltime teachers with rich classroom experiences and a lot could be learnt from focused group with them. The interactions of participants is the crucial feature of focus group discussions because the interaction between participants reviews their perceptions of the world and their values about a situation (Masadeh, 2012; Morgan, 2008). The main purpose of focus group discussions in research is to draw upon respondents' attitudes, feelings, beliefs, experiences and reactions in a way which it would not be feasible using other methods, for example observation, individual interviews or questionnaire (White & Thomson, 2009). These attitudes, feelings and beliefs may be partially independent of a social setting, but are more likely to emerge during social gatherings and multiple explanations of these may occur during the interactions.

4.5.4 Psychometric tests

Two psychometric tests (pre-test and post-test) were written by the sampled Grade 7 learners. The tests covered the basic competencies on; Number concepts, Measurements and Money. The three topics were chosen as these are the topics in which mental methods plays the main role. The test items required learners to demonstrate their abilities to deal with numbers and have free manipulation via strategies of their choice and then arrive to answers (see Appendices Q and R).

The intervention for learners and subsequent psychometric tests was conducted to determine the learners' response to a mental arithmetic strategies treatment. The generated data was used to test the null hypothesis:

H_o: There is no significant difference in the performance of the learners taught mental arithmetic strategies and those who were not.

The null hypothesis was tested using t-tests calculated using the SPSS software. An example of a test item falling in each of the three competencies covered are presented next.

Number concepts

Question 15. (Pre-test)

Question 15. Work out 13 × 3016

The question sought to determine, firstly, the type of computation strategy learners would choose to use and secondly how correctly they compute the multiplication problem.

Measurement

Question 21. (Post-test)

Question 21.

How many **hours old** will the baby be when she lived for **36 000 seconds**?[2]

The question sought to measure whether learners could convert seconds to hours. Solving the problem would involve simplifying using highest common factors [HCF] and dividing.

Money

Question 15. (Post-test)

Question 15. Helvi bought a phone at N$ 1800 and sold it to her friend making a profit of 20%. Work out her actual profit.

The question measured the *percentage profit* section in the *Money* topic. It sought to determine whether learners could work out the actual profit. The computation steps involve the use of HCF to simplify, multiplications and division.

The tests were acclimatised standardised achievement tests. The items were adapted with open access permission from the National Curriculum Tests (Key stages 2 & 3) used in the United

Kingdom and ‘contains public sector information licensed under the Open Government License v.3.0’. The researcher takes confidence in the reliability of the standardised test items at 0.9 (Bew, 2011). For construct and face validity, three experts in the mathematics education field were engaged to verify the quality of the items advised on the acclimatisation of such items to the Namibian contexts.

4.6 Data Collection Procedures

Data collection was done in two phases. The first phase comprised collection of mainly qualitative data and the second phase comprised of the quantitative data collection. The first phase consisted of observations and teacher interviews. The second phase, preceded by a workshop for teachers on mental arithmetic strategies, consisted of the focus group discussions and the pre-test and post-test for learners.

4.6.1 Data collection Phase 1

4.6.1.1 Observations

After securing permissions to enter schools, the teachers were observed while teaching in order to determine the types of mental arithmetic strategies used by the senior primary teachers and learners in their classrooms. Every teacher was observed for one school day while teaching in the senior primary classes (grades 4 to 7).

Researcher as a participant observer

In this exploratory study, the researcher began observations from an explicit position of uncertainties with regard to the type of computation strategies used by the teachers and learners in the senior primary mathematics classrooms. The researcher thus assumed a position that allowed him to gain insights into how teachers and learners chose and executed the computation strategies within their classroom settings. In the process of generating meaning to what was observed, the researcher generated categories based on the nature and/or frequency of events (Simpson & Tuson, 2003). Initially, the researcher had planned to be a non-participant observer, however, he later discovered that the classroom settings are human settings impossible to alienate oneself. The learners at points asked me if their work was correct, and I had to respond and seek clarity as to how they arrived to some answers I could not figure out their steps. Such interactions with the learners gave me further insight into learners’ understanding with executing algorithms. The interactions with the learners was kept as low as possible so that the researchers' presence could not influence the direction of the class progress.

4.6.1.2 Interviews

The researcher conducted oral interviews right after observing each teacher in order to determine the perspectives of teachers on why mental arithmetic strategies should (or should not) be integrated in the entire senior primary mathematics curriculum. This helped to get the accuracy of the impressions picked up from observations (Neuman, 2003). An audio recorder was used to make sure every phrase the teachers uttered was recorded.

4.6.2 Data collection phase II

4.6.2.1 Intervention

This study is informed by the critical mathematics education theory, hence, studying alternatives to actual practices forms part of the study. Skovsmose and Borba (2004) provide a summary of what a critical mathematics education entails:

> The principal feature of a critical research approach in mathematics education is the change of action or collaboration between researchers and the researched. The concern is to improve a situation through active intervention and in collaboration with the parties involved. (p. 211)

After the first phase of data collection and subsequent preliminary analysis of the data, the researcher prepared mental arithmetic strategies manual and had two interventions, one with the teachers and the other with the learners.

4.6.2.2 Teachers' intervention

The ten teachers who were earlier observed and interviewed were invited to partake in the mental arithmetic strategies workshop to which they all agreed, however some teachers withdrew their participation due to some personal problems and/or other commitments. Only five of the ten teachers attended the workshop. The workshop was run for four weeks and took place on the Saturdays to avoid interfering with school activities. The workshop introduced the teachers to mental arithmetic strategies and gave teachers a chance to practice with these methods. A training manual was developed comprising of mental arithmetic strategies on the four basic operations and contained examples on how problems were to be approached (see appendix S).

This researcher developed an *Intervention Mental Arithmetic Workshop Manual* (see appendix S) based on the framework adopted from Morgan (1999) in Table 4.1. The basic operations are discussed in the order: addition, subtraction, multiplication and division.

Table 4.2. Extracts from the mental arithmetic strategies manual.

Addition	1. Left – Right Approach – *this strategy involves adding the highest place values and then adding the sums of the next place value(s).* *Example: 450 + 380: 400 + 300 is 700, and 50 and 80 is 130 and 700 plus 130 is 830.* 2. Compensation - *this strategy involves changing one number to a ten or hundred; carrying out the addition and then adjusting the answer to compensate for the original change.* *Example: For 4500 plus 1900: 4500 + 2000 is 6500 but I added 100 too many; so, I subtract 100 from 6500 to get 6400.* 3. Bridging a decade – These facts all have one addend of 8 or 9. The strategy for these facts is to build onto the 8 or 9 up to 10 and then add on the rest. *9 + 6 = 9 + 1 + 5 = 10 + 5 = 15* 4. Breaking and Bridging – *this strategy involves starting with the first number and adding the values in the place values, starting with the largest of the second number*. *Example: 5300 plus 2 400: 5300 and 2000 (from the 2400) is 7300 and 7300 plus 400 (from the rest of 2400) is 7700.*
Subtraction	1. *Left –Right Approach* – learners do the computation starting at the front end. Example: For 3700 subtract 2400, simply record, starting at the front end, 1300. 2. *Compensation* - involves changing one number to a ten, hundred or thousand; carrying out the subtraction and then adjusting the answer to compensate for the original change.

	Example: 3660 – 996 = 3660 – 1000 + 4 = 2 664. 3. *Breaking and Bridging* - involves starting with the first number and subtracting the values in the place values, starting with the highest, of the second number. Example: 8369 - 204: 8369 subtract 200 (from the 204) is 8169 and 8169 minus 4 is 8165. 4. *Constant Difference* - involves *adding* ***or*** *subtracting* the same amount from both the Subtrahend and the minuend to get a ten, hundred or thousand in order to make the subtraction easier. This works because the two numbers are still the same distance apart. Example: a) 345 – 198: Add 2 to both numbers to get 347 – 200; so the answer is 147. b) 567 – 203: Subtract 3 from both numbers to get 564 -200; so the answer is 364.
Multiplication	1. *Distributive Principle* - involves finding the product of the single-digit factor and the digit in the highest place value of the second factor, and adding to this product a second sub-product i.e. $a(b + c) = ab + ac$. Example: 2 × 716 = 2 (700 + 10 + 6) = 1400 + 20 + 12 = 1426. 2. *Compensation* - involves changing one of the factors to a ten, hundred or thousand; carrying out the multiplication; and then adjusting the answer to compensate for the change that was made. This strategy could be used when one of the factors is near ten, hundred or thousand. Example: 7 × 198 = 7 × (200 – 2) = 1400 – 14 = 1386. 3. *Compatible Factors* – this strategy involves looking for pairs of factors whose product is a power of ten and re-associating the factors to make the overall calculation easier (associative property of multiplication). Example: 25 × 63 × 4 = 25 × 4 × 63 = 100 × 63 = 6300. 4. *Half and Double* – this strategy involves halving one factor and doubling the other. At least one factor must be even. Examples:

	1. $18 \times 4 = 9 \times 8$ (or $36 \times 2) = 72$ 2. $3\frac{1}{2} \times 12 = 7 \times 6 = 42$ 5. *Aliquot parts* – this strategy involves multiplying by an equivalent fraction of the given factor. Example: $48 \times 5 = 48\ (10 \div 2) = 480 \div 2 = 240$
Division	*1. Compatible numbers (The distributive property;* $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$*)* Examples**:** $145 \div 5$ 1.1. *Additive distribution* – look for two numbers divisible by 5 whose sum is 145. $\frac{100}{5} + \frac{45}{5} = 20 + 9 = 29$ 1.2. *Subtractive distribution* – look for two numbers divisible by 5 whose difference is 145. $\frac{150}{5} + \frac{5}{5} = 30 - 1 = 29$ 2. *General Factoring* – this strategy works by breaking down the divisor into its factors. Example: $\frac{240}{16} = \frac{240}{2\times 8} = \frac{30}{2} = 15$

On the last day of the workshop, teachers sat down with the researcher for about 45min to conduct a focused group interview. The researcher clarified to the interviewees that the purpose of the interview was to collect data on how senior primary teachers were integrating mental arithmetic strategies in their classrooms as well as other perceptions teachers may have with regard to the incorporation of mental arithmetic in the school curriculum. Each teacher was interviewed for about 15 minutes and the interview was audio recorded.

4.6.2.3 Learners' intervention

At each of the 10 schools in the study, one Grade 7 class sat for the psychometric test. The test was used to determine the numeracy level of the learners. Two of the five schools which were represented by teachers during the mental arithmetic strategies workshops were sampled out for the learners' intervention and served as the experimental schools. Two schools whose teachers did not partake in the workshops were sampled to be in the control group. For the control group, the researcher picked schools which were not represented during the workshops because the workshop participants agreed to introduce mental arithmetic computations strategies in their classrooms. The researcher wanted to compare to rather neutral classrooms.

The psychometric test taken during the first phase of the study was used as the pre-test. In the events where schools had more than one Grade 7 classes, one class was randomly selected to participate in the test. Before administering the test, the researcher obtained written consent of the learners' parents/guardians as well as learners' signed assent.

Control Group

Two schools were in the control group. The learners and their parents were asked for consent to partake in the study through signed assent forms and consent forms respectively. One of the two schools in the control group is situated in the urban district and the other school was in the rural areas of the Oshana region. The learners continued with their classes as normal. During study sessions while the other schools where receiving intervention, the control schools received the same activities to practice but via their usual strategies. The researcher asked the mathematics teachers at the control schools to facilitate the handing over of activities and practical sessions.

Experimental Group

The two schools which formed part of the experimental group were conveniently sampled based on the teachers' willingness and availability to work extra and conduct the treatment. Like the case in the control group, one of these two schools was in the urban area and the other was in the rural areas of the Oshana region.

The researcher worked with the teachers who exposed the learners in the experimental group to a series of eight lessons on solving problems using the mental arithmetic strategies. The lessons were conducted in the afternoons during the study session for a period of two weeks. The afternoon study sessions lasted for an hour, four days in a week, and hence the learners' intervention was about 8 hours long.

Post-test

After the treatment sessions given to the experimental group, the post-test was written by both the experimental and control groups. In all cases of the pre and post-tests, the papers were written at the same time in all the schools and a life skills teacher at each school was tasked to administer the tests. The pre-test and post-test results were then compared and the hypothesis tested as found in chapter 5 ahead. A summary of the data collection process is illustrated in Figure 4.1.

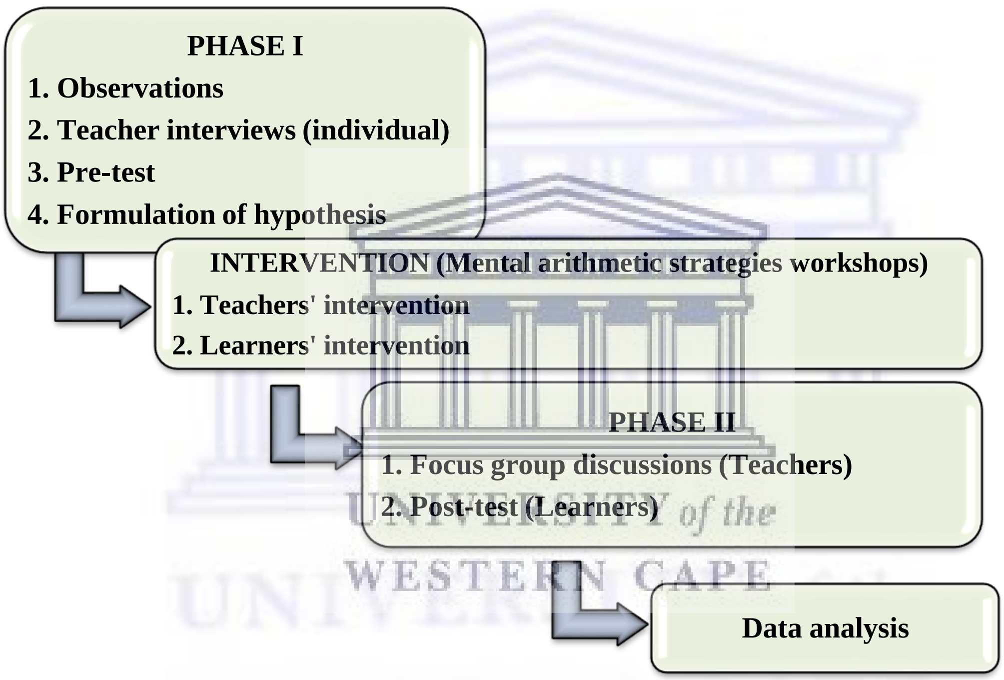


Figure 4.1: An illustration of the data collection procedures.

4.7 Pilot Study

The pilot study was conducted to gather information on the appropriateness of the research instruments and determine faults in the administrative logistics and reveal deficiencies in the design of the study (Lancaster, Dodd, & Williamson, 2004). The pilot study was done at one primary school in the Kavango East region. A senior primary mathematics teacher was observed for a day and then later interviewed. Grade 7 learners at that school wrote the psychometric tests. Half of the class (16 learners) wrote the pre-test and the other half sat for the post-test. Analysis of the gathered data was then done to detect discrepancies in the instruments. Not unexpectedly, several inconsistencies were picked up during the pilot study.

The researcher earlier allocated 40min as duration for the test. It was discovered during the pilot study that the time was insufficient. The test duration was then changed to an hour. The test time was then scheduled for the afternoons during the study sessions as to not interfere with normal class time and to also avail one-hour time for the learners to write the test.

During the interview with the pilot teacher, it was revealed that the teacher changed the topic she earlier intended to teach as she felt there was less for the observer. The researcher thus noted and planned to inform all the teachers in the main study to keep to their routine so that the observations would be done under normal circumstances. The researcher also noted to request teachers to add some practical component to their lessons so that the researcher could observe learners at work.

4.8 Reliability and Validity

Validity and reliability do not carry the same connotations in qualitative research as they do in quantitative research (Creswell, 2014). Given that the present study used a mixed methods approach, both threats to validity and reliability in the separate qualitative and quantitative designs are discussed in this section.

Qualitative validity and Qualitative reliability

Qualitative validity is concerned with verifications that the researcher does to ensure accuracy of the findings by employing necessary measures throughout the data collection, analysis and reporting (Cohen et al., 2007; Creswell, 2012, 2014). Validity is one of the strengths of qualitative research and is based on determining whether the findings are accurate from the viewpoint of the researcher, the participants and the readers of an account. In a qualitative design, validity may be addressed through the "honesty, depth, richness and scope of the data achieved, the participants approached, the extend of triangulations and the objectivity of the researcher" (Cohen et al., 2007, p. 105). In the present study, qualitative validity was enhanced via triangulation of the data from observations, interviews and focused group interviews. Triangulation is defined as the use of two or more methods of data collection in a study where findings are attributable to similarities of methods (Cohen et al., 2007; Gay et al., 2013).

Qualitative reliability on the other hand, is concerned with the consistence in the researcher's approach across different researchers and different projects (Creswell, 2014). To ensure qualitative reliability, researchers should document the procedures of their case studies so that it is possible for others to follow the same procedures and replicate the results. The transcriptions in the present study were revised for obvious mistakes and shifts in the drifts of the codes as advised by (Cohen et al., 2007).

Quantitative validity and Quantitative reliability

The study took on account the threats to quantitative validity and reliability of the research instruments in order to attenuate the effects of these threats. On one hand, the quantitative validity, concerns the extent to which the test tests what it is supposed to test. This devolves on content, construct, face, criterion-related, and concurrent validity (Cohen et al., 2007) as it is these threats that menaces the researcher's ability to draw correct inferences from the data about the population in an experiment. At the design stage of the data collection instruments, threats to validity were minimised by selecting appropriate methodologies to answer research questions, taking an appropriate time scale, using an appropriate sample which is representative of the population and keeping the time frame between pre-test and post-test a month apart. The face validity of the research instruments was determined by three experts in the mathematics education field. The instruments were also reviewed by the supervisor to ensure these were in line with, and would obtain data to answer the research questions of the study. For content validity, the existing literatures on numeracy and mental computations were used to make crosscheck and ensure the instruments tested the content required.

On the other hand, the quantitative reliability of instruments concerns the degree of confidence that can be placed in the results and the data, often a matter of statistical calculations and subsequent test redesigning (Cohen et al., 2007; Creswell, 2014). The reliability measures for the consistency and replicability over time, over instruments and over groups of respondents. For research to be reliable, it must demonstrate that if it were to be carried out on similar group of respondents in a similar context, then similar results would be found. In the case of this study, instruments were piloted and then subjected to a test-retest technique to assure their reliability.

4.9 Field constraints

This study had to be completed within a given time set and hence the magnitude of the study was reduced. A longer detailed mental arithmetic workshop for a wider community could have had a more significant impact to teachers' views on the classroom practices and on learners' performance.

Given the financial and material resources, only 10 primary schools formed part of the study and from each school, only one teacher and one Grade 7 class were involved in the study. The researcher took care in choosing the participating schools by choosing the schools with analogous features with other schools in the Oshana region so that it was possible to generalize the findings of the study beyond immediate research sites.

Another constraint faced during data collection was the teachers' availability and willingness to partake in the mental arithmetic workshops. The workshop sessions were run on weekends to avoid interfering with school activities. Teachers had difficulties with accepting this arrangement as during the weekends most of them were engaged with personal chores. However, the researcher managed to re-arrange workshop time schedules to times convenient to participants.

4.10 Data Analysis

Through the critical exploratory mixed methods approach, the study generated both qualitative and quantitative data. The data analysis began during data collection as the researcher needed to make sense of the information being supplied by the participants and judge the worth such information was contributing towards the research objectives. Data analysis consists of taking the data apart to determine the value in individual responses or pieces of data and then re-assembling the data in order to summarize it and make conclusive meanings (Creswell, 2012). The next sub-section discusses how each kind of data was analysed.

4.10.1 Analysis of qualitative data

The analysis of qualitative data from the observation field notes and interviews started during data collection and this ensured that the data was recorded as accurate as observed (Gay et al., 2013; Taylor-Powell & Renner, 2003). The interviews audio recordings and field notes were first transcribed to written form and then entered into the Atlast.ti software for qualitative data analysis.

In order to understand the data well and generate meanings, the analysis comprised of breaking up data into themes to enable inspection of relations between concepts, constructs, contrasts and whether there are any patterns, and trends that can be grouped for better meaning (Cohen et al., 2007; Creswell, 2014; Gay et al., 2013). The researcher used Atlast.ti to create codes and groups and later develop themes for discussions to allow drawing meanings from the responses in the themes. The qualitative results are presented in chapter five. The following paragraphs discusses how data gathered by each instrument was analysed.

4.10.1.1 Observations

After several times of reading through the transcriptions of the field notes and studying pictures from the classrooms, the researcher decided to present the observed data according to the schools from which such data was obtained as was the approach used by Tabakamulamu (2010). The data collected comprised of teachers and learners' classroom practices captured in pictures during mathematics lessons and is presented in chapter five under the theme *Strategies used by mathematics teachers and learners in senior primary schools*. This theme comprised of all aspects related to classroom practices of mathematics teachers and learners. The presentation and discussion of such data gave ideas building up to help answer the first research sub-question which sought to uncover the types of mental arithemtic strategies used by teachers and learners in the mathematics classrooms. After presenting the observed data, further analysis was conducted to determine how the current implementation of the mathematics curriculum integrates mental arithmetic strategies in senior primary classrooms.

4.10.1.2 Teachers' interviews

The transcriptions of the teachers' interviews were coded using the Atlas.ti software and the coding and recoding guided the researcher in coming up with two themes under which the qualitative data from interview transcripts is presented and discussed (Creswell, 2014; Taylor-Powell & Renner, 2003). The first theme is *Teachers' perspectives on the integration of mental arithmetic strategies in the curriculum* which entails sub-themes as: teachers' interpretation of mental arithmetic and teachers' views on why mental arithmetic should be (or not) part of the curriculum. Analysis of the data from interviews helped in generating ideas towards answering the second research sub-question which sought to determine the teachers' perspectives on the inclusion of mental arithmetic in the senior primary school curriculum. The second theme from the transcriptions of the teachers' interviews is *Curriculum concerns* and encompassed the data provided by the teachers pointing to shortcomings in the current Namibian senior primary school mathematics curriculum which are possibly hindering effective teaching and learning of mathematics in general and mental arithmetic in particular.

4.10.1.3 Focus group discussions

The focus group discussions were conducted to determine possible ways to enhance the practice of mental arithmetic strategies in the primary schools and enhance the development of mathemacy. The data obtained is presented, analysed and discussed under one main theme titled the *Development of mental arithmetic strategies*. The transcriptions of the focus group discussions were analysed and coded using the Atlas.ti software. The sub-themes were formulated as: teachers' suggestions on how mental arithmetic should be developed; the data pointing to 'a need for a critical mental arithmetic intervention'; and the teachers' views on mental arithmetic after a critical mental arithmetic intervention. The analysis, presentation and discussion according to these sub-themes helped the researcher generate answers to the last research sub-question which sought to determine teachers' perceptions on how mental arithmetic may be developed in primary schools.

4.10.2 Analysis of quantitative data

The quantitative data was collected using the psychometric tests – pre-test and post-test – and was analysed by means of descriptive and inferential statistics using the Statistical Package for Social Sciences (SPSS). The descriptive statistics comprised of the calculations of mean, median, standard deviations and quartiles of the learners' scores (Gay et al., 2013). Data were presented in bar charts and box and whisker plots to help give a pictorial representation of the distribution of the learners' scores. An analysis of these statistical values and pictorials suggested the mathemacy value of the senior primary mathematics learners in the Oshana region.

For inferential statistics, on the other hand, a parametric *t*-test was used to test for statistical significance in the difference between the mean scores of the pre-test and post-test between and within the control and experimental groups (Cohen et al., 2007; Creswell, 2014; Naukushu, 2016; Tabakamulamu, 2010). The tests were standardised and hence the use of parametric methods. The t-test for statistical significance was tested at 95% confidence interval articulating to $\alpha = 0.05$ on the hypothesis that tested for differences between the mean scores of the pre-test and post-test within and between the experimental and control groups. These results are also presented and discussed in chapter five.

4.11 Ethics statement

In this study, the researcher took into account ethical care pertaining to research which involves human subjects. The study sought for and received approval of methodologies and ethical clearance from the Humanities and Social Science Research Ethics Committee of the University of the Western Cape (see Appendix C). Secondly, the researcher sought for and obtained permission from the office of the permanent secretary in the Ministry of Education, Arts and Culture, in Namibia to conduct a study of this nature in the Oshana education region (see Appendices A, B, D and E).

To observe further research ethics, the names of the participants were not revealed as the participants were given codes and their views will only be used for the purpose of this study. The researcher ensured that participants were fully informed about the purpose and intention of the study before asking them to decide whether or not to participate in the study. The participants were informed of their right to refuse to participate or to withdraw from participating anytime they feel like doing so without any penalty against them (see Appendices; G, H, I, J, K, L, M, and N).

Furthermore, the learners were allocated codes as pseudonyms and did not reveal their real names on the answer scripts. That helped to make sure learners would never ne be identified with theirs scripts in order to protect the learners' identity. The test scripts were handled with utmost confidentiality as the scripts were marked by the researcher himself, marks recorded and scripts locked away to prevent access by third parties. The learners (and their parents) were assured that the results from the tests were to be used for the sole purpose of the study. No possible chances of learner victimisation based on responses to items of the tests occurred. The pictures captured during observation sessions did not record learners' faces and or items they could be identified with. The researcher obtained consent from the learners and their parents/guardians via signed letters of assent and consent respectively.

4.12 Chapter summary

This chapter presented the exploratory sequential QUAL-Quan mixed methods design and the methodology used in this study. The chapter justified the option of the mixed method research paradigm, presented the population, sample and research instruments. The chapter also presented the data collection procedures in the two phases of the QUAL-Quan design where qualitative data was first collected and the preliminary analysis helped design the hypothesis which was tested by the quantitative data collected during the second phase of data collection. The study included interventions for teachers and learners. The teachers' intervention was a series of workshop sessions that introduced teachers to mental arithmetic strategies. The learners' intervention involved a quasi-experimental control group design and a post-test was administered at the end of the intervention. Data was analysed using the software Atlas.ti and SPSS. The reliability and validity concerns were discussed towards the end of the chapter. The next chapter presents the findings, the interpretations and discussions thereof.

CHAPTER 5: PRESENTATION, INTEPRETATIONS AND DISCUSSION OF FINDINGS

5.1 Introduction

This chapter presents the findings, interpretations thereof, and then a discussion of these findings. The study sought to answer the main research question: How is the senior primary mathematics curriculum incorporating mental arithmetic? To answer the main question, sub-questions were derived to guide the study.

The Sub-questions were:

1. What strategies do primary school mathematics teachers and learners use in the mathematics classrooms?
2. From teachers' perspectives, why should the development of mental arithmetic strategies (or not) be emphasized in schools?
3. How can the development of mental arithmetic strategies be enhanced in primary schools?

In the quest to answer the preceding questions, this chapter presents the data in four main sections; it begins with the presentation of the biographical information of participants, followed by the presentation of qualitative results, the presentation of quantitative results and lastly the discussion of the findings. The last three sections sought to generate answers to the research sub-questions which guided the study.

5.2 BIOGRAPHICAL INFORMATION OF PARTICIPANTS

This section describes the biographical information of the participants. The participants were 300 learners and 10 teachers from the senior primary schools in the Oshana region. The biographical information of the learners and teachers are presented in the following sub-sections.

5.2.1 Learners' demographics

The senior primary school learners from ten schools in the Oshana region were observed during lessons. Five of the schools were in rural areas and the other five in urban areas. The learners' age ranged from 10 – 13 years. The researcher sat in the classrooms and observed the interactions between the teachers and learners during the teaching learning process. Where the need arose, the researcher engaged with learners to seek clarity on responses on how they worked out the mathematical problems on tasks given by the teachers.

5.2.2 Teachers' demographics

Ten teachers were involved in the study. Teachers warmly welcomed the researcher to their classrooms and most of them hoped to learn something from participating in the study. The next sub-section presents the gender, distribution per circuit and teaching experiences of the participants.

5.2.2.1 ***Gender***

As in Table 5.1, only two of the teachers who participated in the study were female. Most of the schools had only one mathematics teacher for the senior primary grades who happened to be a male. The sampling process could thus not balance the gender of the participants.

Table 5.1. The Gender of the teacher-participants.

	Gender	Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Male	8	80.0	80.0	80.0
	Female	2	20.0	20.0	100.0
	Total	10	100.0	100.0	

The scope of this study did not allow the researcher to explore reasons for a skewed distribution of the senior primary mathematics teachers with respect to gender.

5.2.2.2 ***Education circuit***

The study covered three education circuits out of a total of five circuits in the Oshana region. As highlighted earlier in chapter four, 10 schools were purposively sampled based on accessibility to the schools. The circuits were evenly represented in the study, with Onamutai 30%, Eheke 30% and then Oluno 40% (see Table 5.2).

Table 5.2. The distribution of teacher-participants per circuit.

	Circuit	Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Onamutai	3	30.0	30.0	30.0
	Eheke	3	30.0	30.0	60.0
	Oluno	4	40.0	40.0	100.0
	Total	10	100.0	100.0	

*5.2.2.3 **Teaching experience***

Three of the teachers had less than five years of teaching experience. This figure includes Teacher B, a Grade 12 graduate, who was employed on a one-year contract basis and hence possessed no teaching experience. The rest of the teachers had five and more years of teaching experience (see Table 5.3). This suggests that the data obtained from these teachers and their classrooms might have prevailed in senior primary mathematics classrooms for a long period of time.

Table 5.3. The teaching experiences of the participants.

Experience (years)		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	< 1 - 5	3	30.0	30.0	30.0
	6 - 10	4	40.0	40.0	70.0
	10+	3	30.0	30.0	100.0
	Total	10	100.0	100.0	

The last three teachers with over 10 years of teaching experience includes a male Principal, a male Head of Department and one of the female teachers, Teacher A. Teacher A indicated that she trained as a language teacher but had to take over mathematics since the school had no mathematics teacher and could not advertise the post due to over staffing.

5.3 PRESENTATION OF THE QUALITATIVE RESULTS

The study was informed by the critical mathematics education theory and used a mixed method approach as the utilisation of a mixed pragmatic paradigm allows the both qualitative and quantitative data to complement each other thereby forming a strong basis of findings (Gay et al., 2013). The qualitative data collected are presented in this section in sub-themes. This section begins with the presentation of strategies used by teachers and learners in senior primary classrooms which were observed in the schools. The section then presents the teachers' perspectives on the integration of mental arithmetic strategies in the schools; the teachers' concerns about mental arithmetic in the senior primary syllabus, and then, the development of mental arithmetic strategies in the schools. Lastly, the section presents the intervention and the effects thereof, which was done as a part of this study.

5.3.1 Strategies used by teachers and learners in senior primary schools

This section presents the classroom strategies employed by the teachers and learners in the senior primary classes. This study, in its critical nature sought to explore the existing practices of learners and teachers in the senior primary classrooms and determine how these practices incorporate mental arithmetic strategies. This section presents the findings sought to answer the first sub-research questions.

> Sub-question 1: What strategies do primary school mathematics teachers and learners use in the mathematics classrooms?

The researcher utilised observation methods. Ten schools were sampled for the study. Ten senior primary mathematics teachers in the Oshana education region were observed. The researcher sat mostly behind the classroom and took record of events that caught his interest. The observations made at each school are presented and interpreted below. The schools are labelled School A to School J to protect the identities of the schools and the participants. The sub-sections below presents the observations made at the schools.

***<u>School</u>* <u>A</u>**

School A is a rural school in the Oshana region. There was only one mathematics teacher for the senior primary grades. The teacher indicated that she was not trained to teach mathematics but asked by the school management to help teach the subject as nobody could do it. She was a trained language teacher. The school has no trained teacher for senior primary mathematics and could not advertise a vacancy due to overstaffing. However, the teacher was very friendly and hoped to learn something by participating in the study.

The lessons observed were in a Grade 7 class. The learners were quiet throughout the lesson and only spoke when the teacher asked questions. The lesson was on the topic *'Percentage profit'*. The teacher wrote the topic on the chalkboard and told learners that profit was the money left after subtracting the cost price from the selling price. She then gave the example below:

1. *The owner of a shop buys a dress for N$248 and adds N$40 for her profit. Calculate the percentage profit the shop owner wants to make.*

The teacher started to solve the problem on the board. The teacher used a teacher centred-approach to teaching. She worked out the steps as follows:

$$\% \text{ profit} = \frac{\text{profit}}{\text{cost price}} \times 100 = \frac{40}{248} \times 100$$

$$= \frac{4000}{248}$$

Teacher (to learners): "*You can now use repeated addition to divide 4000 by 248*"

Perhaps the teacher meant learners should use the long division method and not repeated addition method because after a short period of time, about a minute long, the teacher went to the board and worked out $\frac{4000}{248}$ to get 16.1 % as follows:

$$\begin{array}{ccc|ccccl}
 & & & 1 & 6. & 1 & & & \\
\cline{4-7}
2 & 4 & 8 & 4 & 0 & 0 & 0 & & \\
 & & & 2 & 4 & 8 & & & \\
\cline{4-7}
 & & & 1 & 5 & 2 & 0 & & \textit{0 is less than 8. borrow} \\
 & & & 1 & 4 & 8 & 8 & & \\
\cline{4-7}
 & & & & & 3 & 2 & 0 & \textit{Add 0 to last column} \\
 & & & & & 2 & 4 & 8 & \\
\cline{5-7}
 & & & & & & 7 & 2 &
\end{array}$$

Looking at the step where the teacher had to divide 1520 by 248 to get 6, the teacher did not explain how she got the answer. It was also not clear how the long division method made the division operation easier.

The teacher cleared the chalkboard and wrote the second example.

2. *A car dealer sells a second hand car for N$ 45 600. He makes a loss of N$4400 on the car. Express the loss as a percentage.*

Loss of 4400 at a selling price of 45 600 implies value of the car is 50 000. The teacher took on to solve the problem as follows:

$$\% \text{ loss} = \frac{loss}{cost\ price} \times 100 = \frac{4400}{50000} \times 100$$

$$= \frac{4400}{500}$$

Teacher shifted to the one side of the board to do long division and got the answer 8.8 %. The learners were copying notes as the teacher solved the problem on the chalkboard. It appears long division was commonly practiced to arrive at answers. The lessons were teacher-centred and the learners were quietly following as the teacher solved mathematical problems. No alternative methods were explored to find percentage profit/loss or in dividing the expressions $\frac{4000}{248}$ and $\frac{4400}{500}$. At this stage the concept of simplifying using common factors seems forgotten.

School B

The school is situated in a rural area. It offered classes from Grade 0 to Grade 7. A male teacher was appointed on a one-year contract to teach mathematics in the senior primary grades (grades 4 to 7). He was a Grade 12 graduate with no teaching experience. The teacher was very friendly and the classes were active. The researcher sat at the back of the class. Since the teacher had given homework the previous day, he collected learners' homework books and started to give solutions to the homework problems on the chalkboard. The class was working on *percentage discount*. The first problem was as follows:

1. *Calculate 10% discount on a bicycle that is marked N$2050. What is the new price?*

The teachers' lessons appeared to be satisfactorily learner-centred. The teacher wrote the problem on the chalkboard and called a learner to the chalkboard to solve the problem. A snapshot of her working has been presented in Figure 5.1.

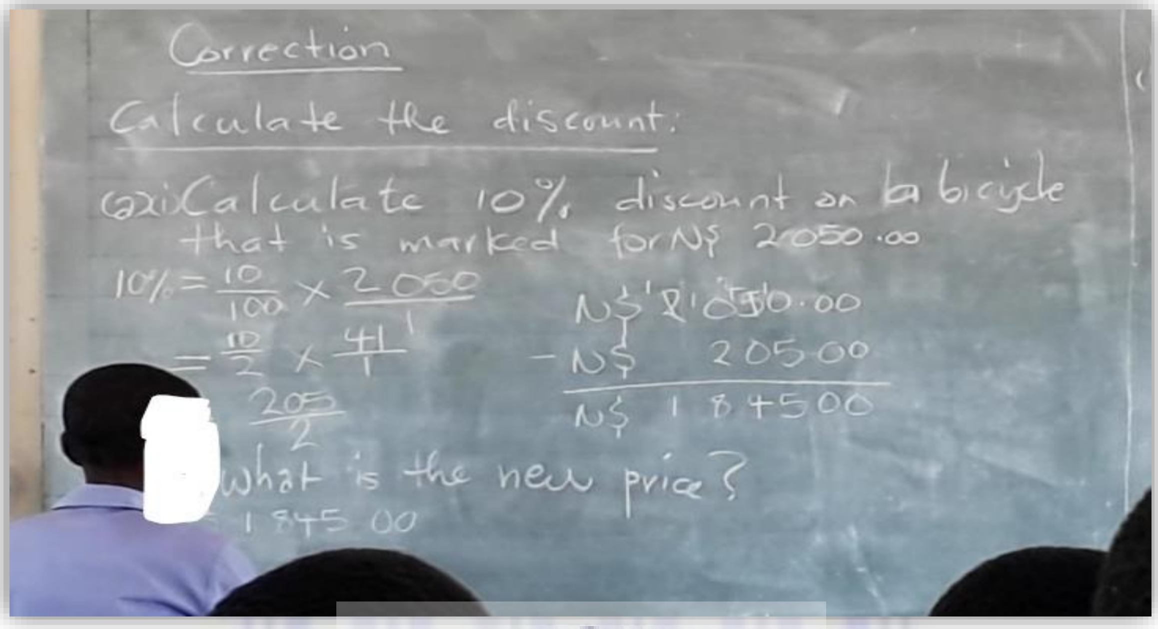


Figure 5.1. A learner working out 10% of N$2050.

<u>Learner's working:</u>

$$10\,\% = \frac{10}{2\,\cancel{100}} \times \cancel{2050}^{41}$$

$$= \frac{10}{2} \times \frac{41}{1}$$

$$= \frac{205}{2}$$

The teacher stepped in and told the learners that the answer is not $\frac{205}{2}$ but N$205. The teacher did not point out where the learner went wrong or how the correct answer was to be obtained. Instead, the teacher went on to work out the new price as follows:

$$\begin{array}{rrrr} \cancel{2}^{1} & 0^{10} & \cancel{5}^{4} & 0^{10} \\ - & 2 & 0 & 5 \\ \hline = 1 & 8 & 4 & 5 \end{array}$$

The teacher approached the problem using a standard algorithm of subtracting in columns with digits aligned according to place value. He further advised learners to always 'borrow' when the digit above is less than the digit below. The class did not explore alternatives to obtaining the difference 2050 – 205. The teacher wrote down the second problem on the chalkboard.

2. *Calculate 12% discount on a CD that is marked for N$ 128.00. What is the new price?*

The second problem was also based on percentage discount. The teacher called up a learner to solve it.

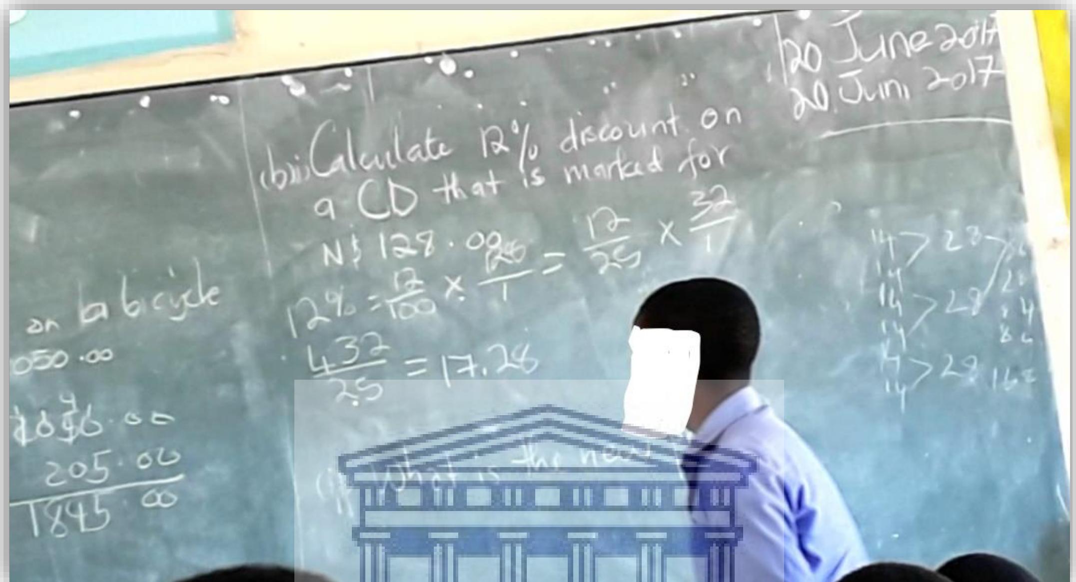


Figure 5.2. A learner working out a division problem on the chalkboard.

The learner wrote: $\frac{12}{100} \times 128 = \frac{12}{\cancel{100}_{25}} \times \cancel{128}^{32} = \frac{12}{25} \times 32 = \frac{432}{25}$

The learner simplified correctly but wrongly worked out the product $12 \times 32 = 432$ as in Figure 5.3. The learner did not show his workings and the teacher did not ask how he arrived to a wrong product. Instead, the teacher told the class that the product of 12 × 32 is 384 and not 432. This is the second time a learner gets a wrong product in this class. The same was recorded in the first example.

The teacher then asked the learner (who was still standing by the chalkboard) to use long division and divide 384 by 25. The learner got $384 \div 25 = 15$ and looked at the teacher. The following conversation arose:

Teacher: You are not done. Does 25 go exactly 15 times into 384?

Learner: No. There is remainder 9.

Teacher:	What do you do to nine when dividing in long division?
Learner:	9 divided by 25 is impossible.
Teacher:	Class! What do we do?
Class:	(goes quite)
Teacher:	What if we add a 0 to nine?
Learner:	Oh, we get 90!

The learner then continued with long division and got a value of the discount as N$ 15.36. The teacher did not explain why a zero should be added to 9 to give 90.

What is the new price?

The teacher called up another learner to the chalkboard who solves in a rectangular array format (a standard algorithm) as follows:

$$\begin{array}{lcccccc} & 1 & 2 & \cancel{8}^{7} . & \cancel{0}^{9} & 0^{10} \\ & & 1 & 5\,. & 3 & 6 \\ \hline = & 1 & 1 & 2\,. & 6 & 4 \end{array}$$

The standard algorithm involving trading was used. It appears the practice of standard algorithms was the regular approach to solving problems in this class. It was also noted how the teacher did not explain certain steps in the standard algorithms for example, adding a zero next to nine during long division thereby the teacher failing to link the lesson to the place value concept.

School C

The school was situated in an urban area of the Oshana region. There was one mathematics teacher for the senior primary phase. The teacher was a young male and he welcomed the researcher to his classes where he taught *division of decimals*. The classroom interactions between the teacher and his learners were very good, allowing the researcher to record the types of computation strategies used in these classrooms and how these strategies were executed. The following examples on strategy execution were recorded from classes:

The teacher gave three examples on dividing decimals to a Grade 7 class:

1. Work out: $0.65 \div 0.5$
2. Work out: $1.74 \div 0.3$
3. Work out: $24.42 \div 0.11$

The figure below shows the workings of the teacher for examples 1 to 3.

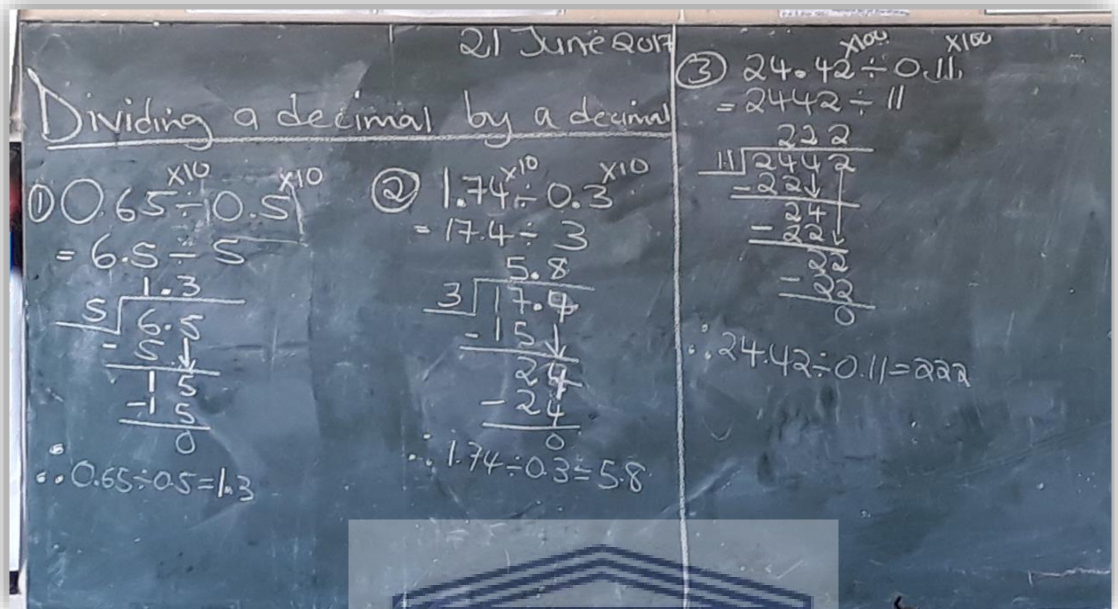


Figure 5.3.Teacher C's solutions to division of decimals.

For all the three examples the teachers used long division (a standard algorithm) as shown in Figure 5.3. The first approach was moving the decimal points a required number of steps until the divisor became a whole number. The same number of steps were also moved on the dividend to keep the ratio the same. In all three examples, there was no alternative to long division discussed.

The teacher then gave a class work. In one of the problems, the teacher required learners to work out: 2.136 ÷ 1.2. The learners started moving the decimal point before putting the numbers in the long division format. Figure 5.4 contains the snapshots of the learners' workings.

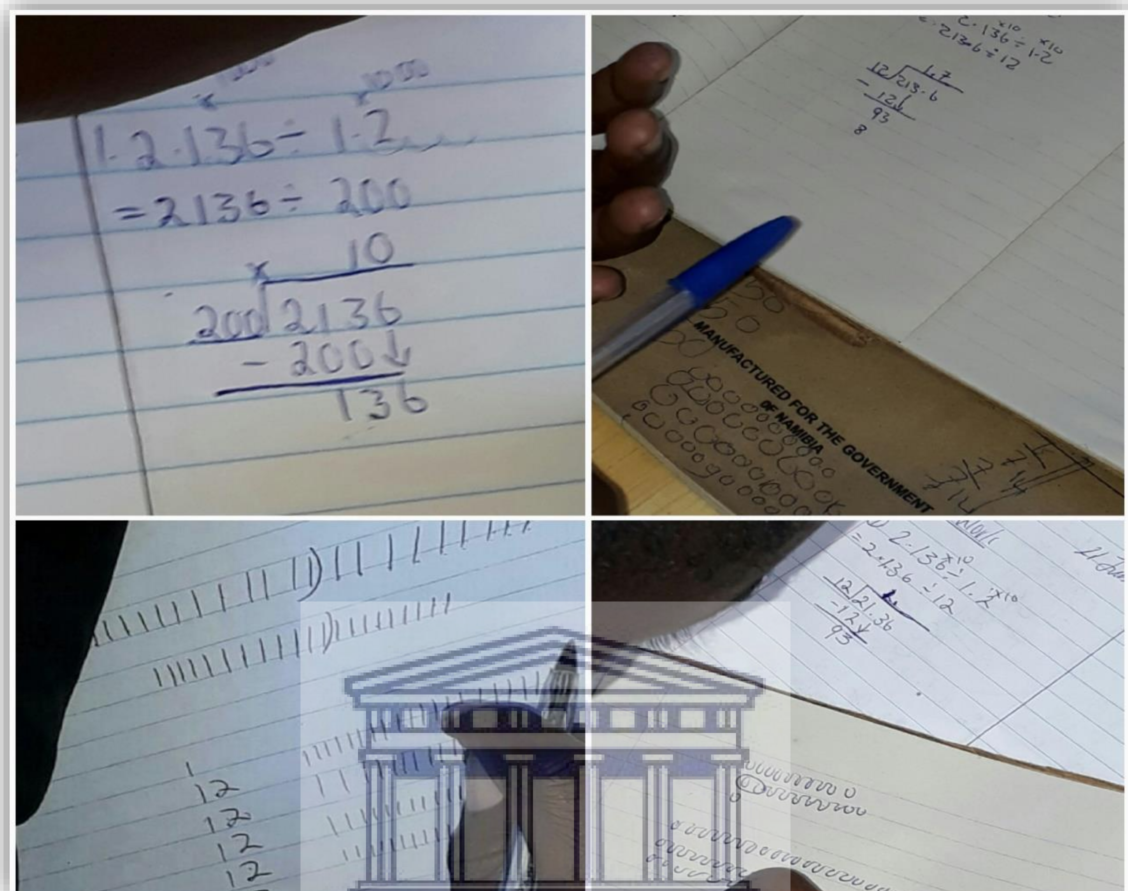


Figure 5.4. Learners' calculations.

From the images, it can be spotted how the 'moving decimal point' concept was missed by a learner. The rest of the images in the collage shows how learners resorted to using drawn sticks and circles as well as fingers to help carry out long division. This seems to suggest that learners would prefer an alternative to long division in order to work out the division problems.

The bell rang and Teacher C had to go to a Grade 6 class. The following are observations from the said class. Teacher began by giving solutions to a homework which was given the previous day. The home work was as follows:

Work out:

1. $13.24 + 21.875 + 6.7$
2. $65 + 0.173 + 0.08$

As was the case with the Grade 7 class, the teacher arranged numbers in place value columns/rectangular array and added/subtracted.

Figure 5.5 shows the worked out solutions.

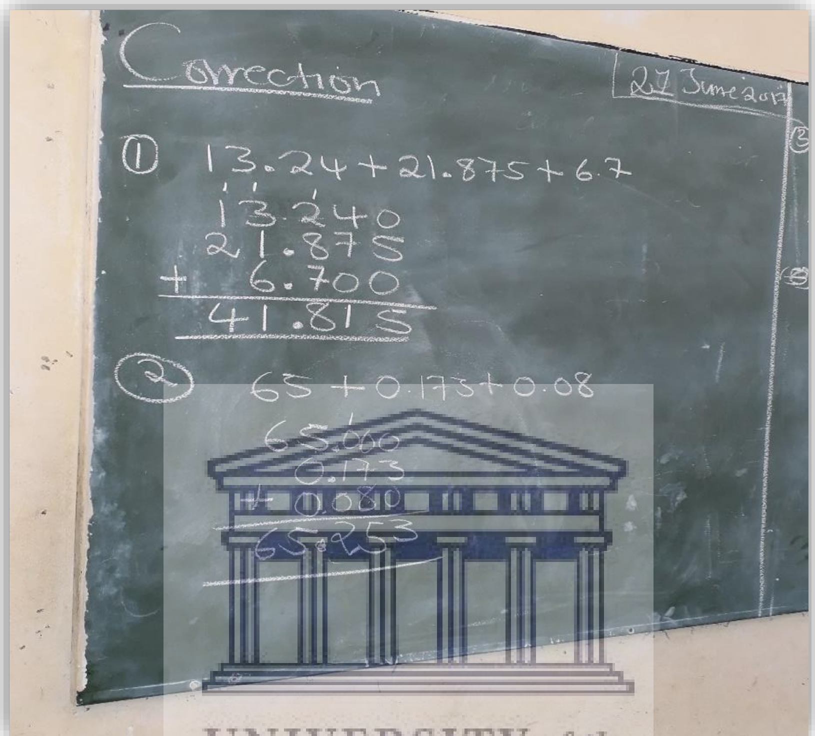


Figure 5.5. Teacher's solutions to addition of decimals.

The teacher then went on to a new section: Multiplying decimals by whole numbers. He linked how an operation 12.1×3 is closely worked out as computing 121×3. Teacher C did not point out to the learners that $12.1 \times 3 = \frac{121}{10} \times 3$, which could have been an alternative strategy to standard algorithms of multiplying decimals.

Teacher C then gave an examples on the chalkboard as in Figure 5.6.

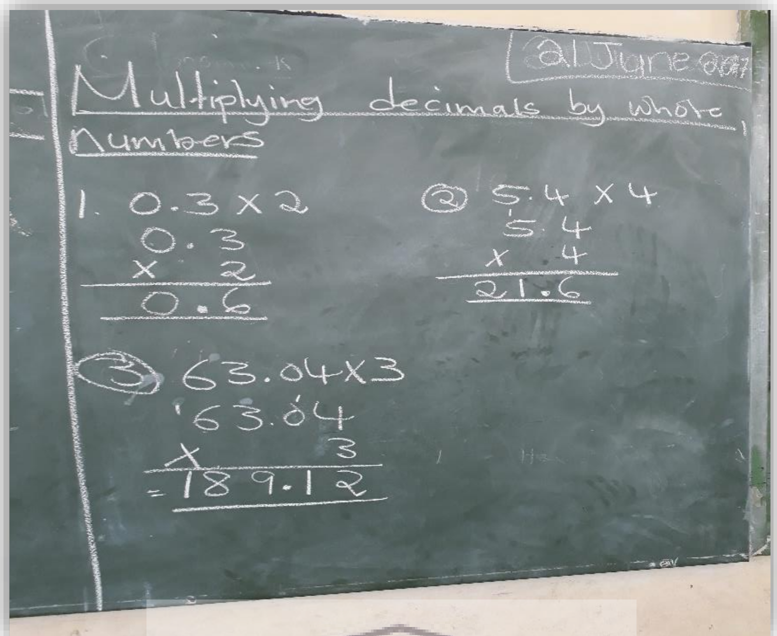


Figure 5.6. Teacher C's solutions to multiplication of decimals.

The classroom practices were mainly based on standard algorithms. There was not a case where a teacher referred learners to mental computational methods.

The following conversation ensued between the teacher and a learner who brought her book to be marked. The given problem was; "work out 0.8×11". The learner got an answer 14.8.

Teacher: *How did you do your multiplication?*

Learner: (Keeps quiet)

Teacher: You should arrange your numbers in columns like this (writes on paper)

$$\begin{array}{cc} 0 & .8 \\ \underline{1} & \underline{1} \end{array}$$

It was worth noting how the teacher immediately directed learners to the use of tabular-multiplication, a standard algorithm.

School D

School D is situated in an urban area of the Oshana region. The researcher was welcomed to the school and linked to the senior primary mathematics teacher. Although the teacher agreed to participate in the study as Teacher D, he looked uneasy from the start as he was hesitant about when the researcher should start observing lessons and trembled as he asked whether the researcher wanted a lesson plan designed in specific ways. The researcher took some time to re-assure the teacher that the data to be collected will be used for the sole purpose of the study only and that his name, names of his learners and that of his school were not going to be revealed and no information that may identify with him will be revealed. Only then the teacher looked more calm.

The lessons observed were on *percentage of a given quantity*. The teacher started the lesson by telling learners that they should not forget that the percentages are always out of a hundred and indicated that the lesson was going to be on writing given quantities as percentages. The teacher then wrote the first example on the chalkboard and asked learners to work out the problem:

1. *Express 40 min as a percentage of 1 hour.*

Most of the learners could work out the following steps:

$$\frac{40}{60} \times 100 \quad = \quad \frac{4}{6} \times 100 \quad = \quad \frac{400}{6}$$

From there, $\frac{400}{6}$, learners move to the long division symbol and had difficulties dividing 400 by 6. Learners, like the case at previous schools, resorted to drawing small marks on pieces of papers to aid counting, a practice which so far seemed to have not bothered the teachers. It seemed that learners resort to their own methods when confronted with the challenges of correctly applying the standard algorithms they are taught.

After some minutes of learners working out the 400 ÷ 6, the teacher stepped in to solve the problem on the chalkboard.

Teacher (writes): $\frac{40}{60} \times 100$,

Teacher: Find a number that divides both 60 and hundred to simplify.

Learners: It is 20!

Teacher (simplifies):	$\frac{40}{3\cancel{60}} \times \cancel{100}^{5} = \frac{40\times5}{3} = \frac{200}{3}$
Teacher:	Now, what is $\frac{200}{3}$?
Learners:	O, Sir!
Teacher:	I told you that you can always use long division.
Learners:	(start the long division)

Once again, it was observed as a teacher advised learners to use long division, a standard algorithm. It appeared as if division is only possible via the long division algorithm. After sometimes the teacher had to give feedback. He wrote:

$$3\,\overline{)\,200}$$

He then asks the learners the same question he asked before "What is 200 divided by 3?" This seems to suggest that with or without the long division sign the learners had to determine the number of times 3 divides into 200. The long division did not seem to make the problem any easier. The second observation was noted when a learner was working out 50% of 1 Kg. The learner's work was recorded in the Figure 5.7.

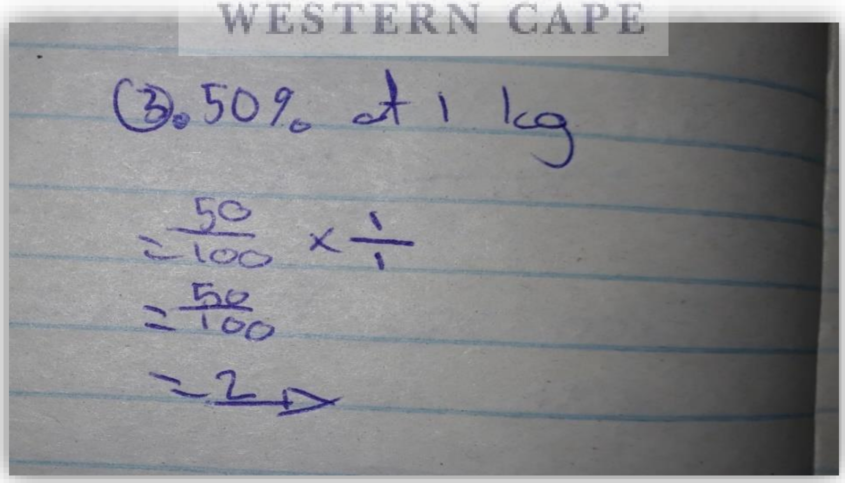


Figure 5.7. Learners' solution to 50/100.

The learners seem to be doing mathematics without reasonableness. The learners divided 100 by 50 to get answer 2 although the 100 was in the denominator.

School E

This school is situated in an urban area of the Oshana region. One male teacher, Teacher E, was responsible for mathematics in the senior primary phase. Teacher E had 15 years of teaching experience and was the school principal. He displayed keen interest in the study and narrated how the development of numeracy was facing many challenges in the primary schools. He welcomed the researcher to observe. In a Grade 7 class, Teacher E taught on *division of decimals*. The learners quietly listened and recorded notes. The teacher told class to pay attention as the division of decimals was done slightly different from the division of whole numbers. He wrote down the first example on the chalkboard as follows:

1. Work out: $6.4 \div 0.4$.

The teacher told learners to 'move the decimal point' until the divisor is a whole number. The same number of 'moves' were also to be performed on the dividend to keep the figures in the same ratio. That gave $6.4 \div 0.4 = \frac{64}{4}$. Then the teacher worked out the problem as in Figure 5.8.

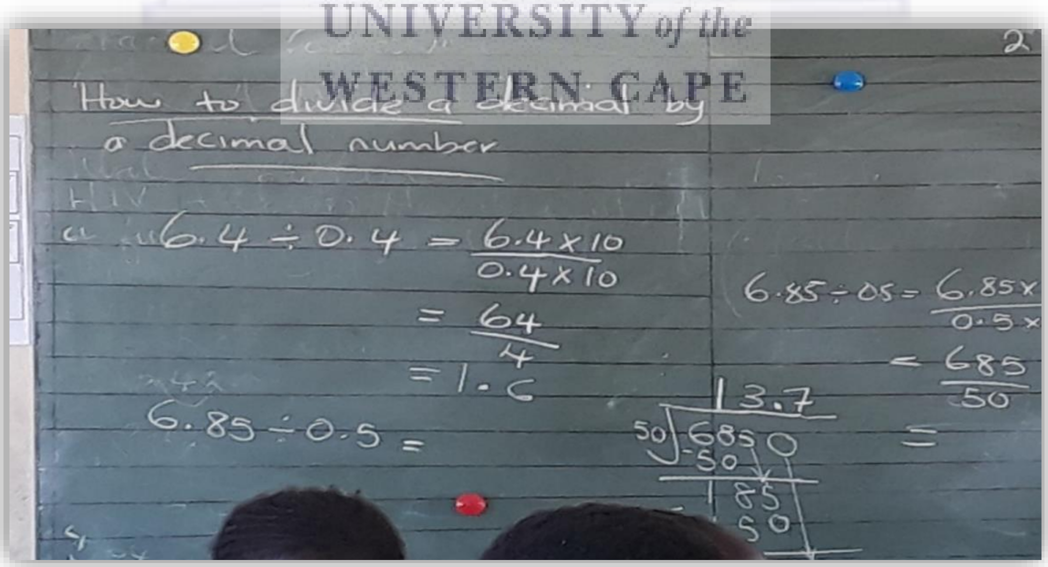


Figure 5.8. Teacher E's solutions to 6.4 ÷ 0.4

Teacher E divided 64 by 4 and got 16. Then he told learners that since the original values where $6.4 \div 0.4$, the answer should have a decimal point 'between the two digits'. The answer 16 was correct as a final answer. Standard algorithms seem to be performed by both teachers and learners as series of steps with incorrect mathematical reasoning.

The second example discussed was as follows:

2. Work out $3.0 \div 1.2$.

Like in the previous example, learners had to 'move the decimal point' so that the divisor becomes a whole number. The problem then became $30 \div 12$. The teachers asked what 30 divided by 12 equals to. The learners were quiet and the teacher, once more, asked the learners to use long division. The learners wrote down their operation as:

$$12\,\overline{)\,30}$$

The following interactions were recorded:

Teacher: What is 3 ÷ 12?

Learners: 3 is less than 12!

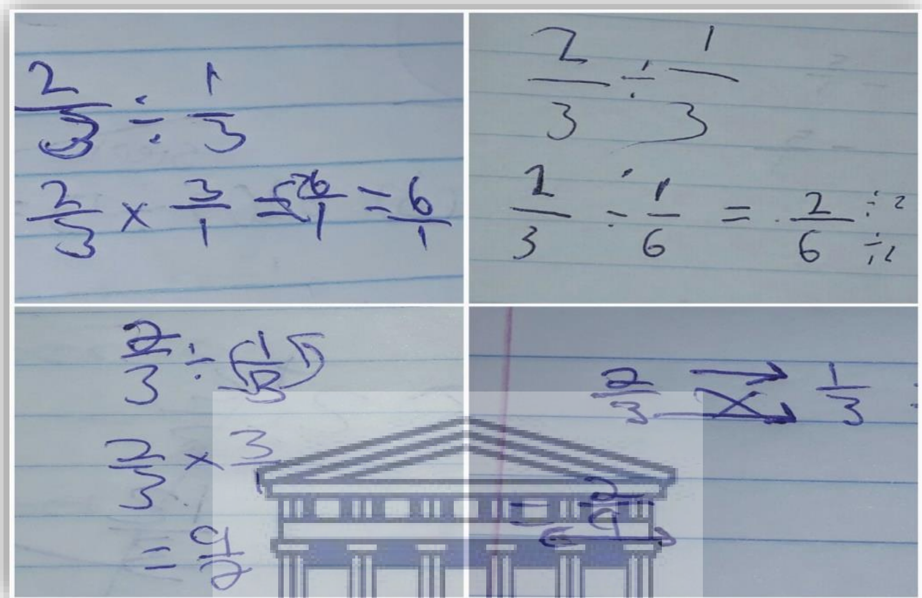
Teacher: Good. Then we have to consider the whole 30. What is 30 ÷ 12?

Learners grabbed pencils and marked tallies/counters on their notes to work out 30 ÷ 12. The process of solving this problem, although made use of the long division symbol, ended up dividing 30 by 12 via a practice of tallies/counters. The similar observations were recorded at school D when learners were dividing 200 by 3. It seemed the teacher was not aware of learner-invented strategies.

School F

School F is sited in an urban area. There were two mathematics teachers responsible for the senior primary phase and they were all male. The principal assigned the researcher to one of the teachers. The teacher welcomed the researcher to his lessons and agreed to participate in the study as Teacher F. The learners sat in rows and were actively participating in the lessons.

The class was working on *Fractions*. The methods employed to manipulate fractions were mainly standard algorithms. Figure 5.10 shows several answers of learners to a multiplication problem $\frac{2}{3} \times \frac{1}{3}$



.

Figure 5.9. Learners' solutions to 2/3 by 1/3.

The learners' workings suggest lack of conceptual and procedural understanding of fractions. It appears that learners do not know when to change the ÷ to × and take a reciprocal of the second fraction. The practice of 'cross multiply' was also wrongly applied. It appears that the numeracy at the senior primary level is low. Learners rarely look for common factors to simplify fractions before operations.

The researcher asked the learner whose work is on the picture at the top right corner of the collage to explain how she got the 6 in the second step.

> Researcher: Please tell me how you got the six?
>
> Learner: 'It's the LCM of 3 and 3'.

The learner does not understand when the lowest common multiple [LCM] is used during operations.

School G

School G is in the rural areas of the Oshana region. The principal welcomed the researcher to the school and encouraged the senior primary mathematics teacher at the school to give his very best to help the researcher. The senior primary mathematics teacher, who also served as the school Head of Department [HoD], agreed to participate in the study as Teacher G. The school looked very old as most of the ceilings in the classrooms were worn out.

The researcher followed the senior primary teacher to his class. The researcher noted information he considered to be interconnected to low pace of numeracy development in the primary schools. The class worked on *Percentage of a quantity*. The teacher came up with class examples out of his own head as he seemed to have no lesson plan with him. He told the class that they were going to express quantities as fractions in percentages. The class was quiet most of the times and only spoke when the teacher asked a question. Teacher G wrote the first example on the chalkboard as:

1. *Your father has N$70 in his wallet. Suppose you want N$42 dollars to go to Oshakati (from Ondangwa), how much should your father give you in percentages?*

It is worth noting that the question is not well framed. The learners seemed to have a problem figuring out the answer. Teacher G encouraged learners to 'visualise' the problem.

The following conversation ensued between the teacher and a learner.

Teacher:	The answer? Anybody?
Learner:	60%
Teacher:	(walks up to the learner) how did you get 60%?
Learner:	$\frac{60}{100} \times 70 = N\$42.00$'
Teacher:	But how did you choose to start with 60%?
Learner:	(goes quiet)

Teacher G walks to the board and says: "problems like this should be solved via trial and error. Take several numbers and find the one that gives you enough amount."

Teacher works it out as: $\frac{60}{100} \times 70$

$= N\$42.00$

Answer: 60%

Just as the learner did, the teacher's working started with the required answer. The topic of the day was *expressing one quantity as percentage of another* and hence learners were supposed to express the N$42 as a percentage of N$70 as $\frac{42}{70} \times 100 \quad = \frac{\cancel{42}^{6}}{\cancel{70}} \times \cancel{100}^{10}$ = 60%. The teachers' advice that learners should solve problems of this nature via 'trial and error' is incorrect.

School H

School H is located in the rural areas of the Oshana region and offered grades 0 to 10. The researcher was welcomed to the school. The only senior primary phase mathematics teacher at the school, Teacher H, a male, agreed to participate in the study. The teacher did not carry a lesson plan. He came up with the examples and questions from the head.

The first observation was in a Grade 4 class. Teacher H told the learners they were continuing with the topic *Time* and asked the following questions orally:

1. *How many hours are between 12 – 3?*

The framing of the question was not correct as there was no indication as to whether am or pm. However, Learners got on their fingers and gave 9 as the answer, to which the teacher agreed.

2. *How many hours are between 11am and 2pm same day?*

Most learners insisted that the answer is 9. The use of fingers in counting was popular in this class. The learners worked out the problem as: 11 – 2 = 9. The teacher gave the correction as '3 hours' and did not explain how or why the answer is 3 hours and not 9 hours. The question and answer method prevailed in this class. Learners gave answers verbally.

The next lesson observed at the same school was in a Grade 5 class. The class worked on converting cents to dollars and vice-versa. After greeting learners, the teacher wrote down the questions on the chalkboard and asked learners to raise their hands so they could give the answer.

1. *Convert 72 cents to dollars.*

Three learners gave the answer, verbally, as:

Learner1:	N$702
Teacher:	No
Learner2:	N$72
Teacher:	No
Learner3:	N$0.72
Teacher:	Yes. The correct answer is N$0.72.

The teacher did not discuss how learners were supposed to get the correct answer. The lesson seemed to have required mental computations as learners were not required to show their working.

While the class proceeded, teacher invites to the chalkboard a boy who sat in the corner and looked rather withdrawn as he had no note book on his desk like the rest of the learners:

Teacher:	Convert 5000 cents to N$.
Learner:	(writes N$5. 000)
Teacher:	No, do not put three digits after the comma. Only 2 digits.
Learner:	(erases N$5.000 and writes N$50.00)
Teacher:	Ok. Now write 500 cents in N$.
Learner:	(writes N$5.00)
Teacher:	Good. But it seems you still do not understand.

After that the teacher gave a topic task. The activity sheets did not have space for learners to show their work, only the final answers. The learners worked on rough papers and recorded the final answer on the worksheets.

The last lesson observed at this school was in the Grade 7 class. The class worked on the topic *money and finance – Percentage profit/loss*. The teacher wrote the topic on the chalkboard and explained the meaning of profit and loss in the business contexts. He then gave an example from his mind as he did not carry any book or lesson plan. The first example was as follows:

1. *Sam bought shoes at N$120. He wore them once and realised that the shoes did not fit him properly. He sold them to a friend at N$96. Work out the percentage loss.*

He called out one learner to solve the problem on the chalkboard. The learner proceeded as follows:

$120 - 96 = 24$

$\frac{24}{120} \times 100 \qquad = \frac{24 \div 2}{12 \div 2} \times 10$

$= \frac{12}{6} \times 10 \qquad = 2 \times 10 = 20\%$

The learner thought well to simplify the $\frac{24}{120}$ although she did not use the highest common factors [HCF] to simplify fractions. The teacher seems to not have realised there was a problem as no efforts were made to help learners simplify using the HCF. The teacher then wrote the second problem on the chalkboard as:

2. *Tom bought a toy for N$ 80 and sold it making 30% profit. Work out his actual profit.*

The teacher solved the problem himself. He advised learners there are two methods to getting the answer and presented these methods as shown below:

Method 1

$\frac{13\cancel{0}}{10\cancel{0}} \times 8\cancel{0}$

$= 13 \times 8$ (Asks learners for the answer. Learners said 164. Teacher wrote it down)

$= 164$

The teacher left the above as correct. This, according to Teacher H, means 30% of N$80 is N$164. He then went on to give an alternative method;

Method 2 (teacher continues)

$\frac{30}{100} \times 80$

$= 3 \times 8$

$= 24$

Actual profit = 80 + 24 = 104

He then asks: '*The answers must always be the same. Where did we make a mistake*?' He went back to Method1 and realised, via column multiplication that $13 \times 8 \neq 164$. After that he said:

> *'You gave me a wrong answer. I trusted you too much'.*

The actual profit was also wrong. The N$24 obtained via method 2 is the final answer but the teacher added N$24 to N$80 to get N$104. This seems to suggest that the teacher was following the algorithms without judging the reasonableness of the answers.

It was also noted how in all the examples the teacher came up with referred to a male subject, Sam and Tom. The same was earlier recorded from Teacher G's classes. Both teachers had no lesson plans.

School I

The school is in the rural area. The principal was not at the school on that day, however, the teacher acting welcomed the researcher to conduct the study. The senior primary mathematics teacher agreed to participate in the study as Teacher I. There was only one mathematics teacher for the senior primary phase at the school, a male.

After the self-introductions and brief orientation to the school, we headed to the Grade 7 class. The lesson was on *money and finance – profit/loss*. The teacher told learners that they will be working out the loss or profit and express it as a percentage of the cost price. The teacher then gave out the following problem on the chalkboard:

1. *Cost price = N$360, Profit = N$18, what is the percentage profit?*

Worth noting here is how the structure of the question lacks sufficient context.

A learner was invited to the chalkboard and solved the problem as follows:

$$= \frac{18}{\cancel{360}_{18}} \times \cancel{100}^{5}$$
$$= \frac{54}{18}$$
$$= 3\%$$

The teacher told the class that the answer is incorrect. He did not point out why the learner was incorrect and/or where the learner went wrong. Instead, the teacher worked out the problem as in Figure 5.11:

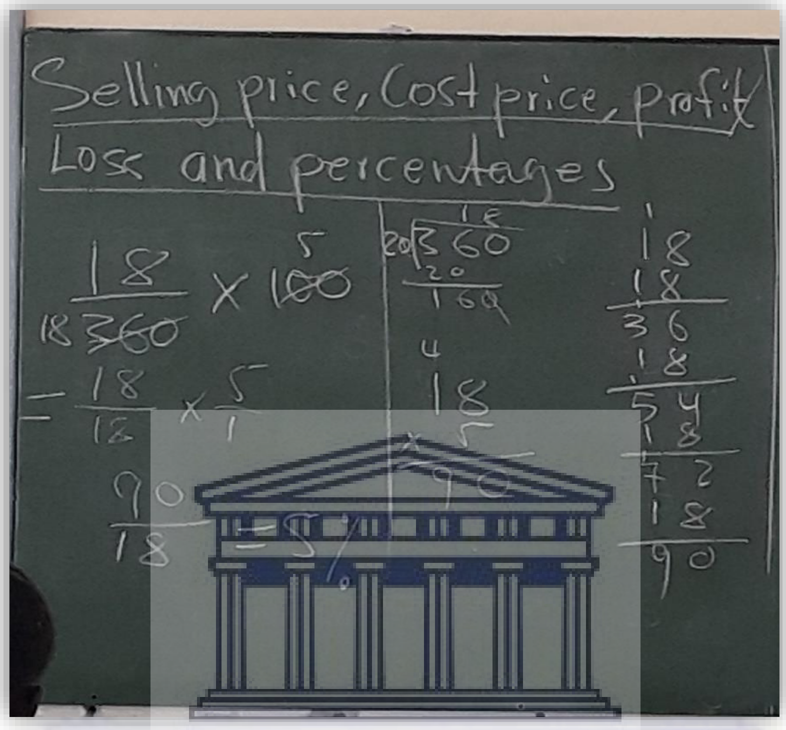


Figure 5.10. Teacher I's solutions to the problem on % profit.

The teacher simplified to get $\frac{18}{18} \times 5$ which is readily equal to 5 percent but, instead, the teacher went on to multiply the numerators. The whole process became longer as it involved long division to simplify $\frac{18}{360}$ and then multiply 18 × 5 via the repeated addition.

The teacher also chose the repeated addition method which involved many steps. Other methods such as the distributive principle would have been faster as it involved less to compute 18 × 5. For Teacher I too, it seemed, he performed the series of algorithm rules without judging the reasonableness of the steps.

School J

The last school was a private school in an urban area. The researcher had to wait for two days to obtain permission to engage the teachers and learners in the study from the 'owners' of the school as the principal could not grant such. When permission was granted, the principal welcomed the researcher and introduced him to the mathematics teacher. There was one female teacher responsible for mathematics in the senior primary phase. The teacher, referred to as Teacher J, agreed to participate in the study.

Observations were done in a Grade 5 class. The learners were very active and participated well in the lessons. The class discussed *decimals*. The addition of decimals was done in columns. The teacher emphasised the importance of aligning the digits according to place value. e.g. 8.203 + 2.07, the learners were told to make sure that the hundredths, tenths, and units were vertically aligned before adding. The same practice of directing learners to use standard algorithms was also observed in the other schools.

It was observed that during the multiplication of decimals such as 16.9 × 1.02, the teacher gave these standard algorithm rules to learners:

1. Remove decimal point so that: 16.9 × 1.02 becomes 169 × 102
2. Multiply as whole numbers: 17 238
3. Add the number of "decimal places in the factors" to get the number of decimal places the product will have. i.e. 1 + 2 = 3 decimal places in the product.

Answer: 17.238

Learners followed these rules. The problem came in when learners had to multiply 169 × 102. They used the standard algorithm and most could not get to the correct answer. No alternative was explored. It seems learners were left to feel that the only way to getting correct answers was via the standard algorithms. An alternative to column multiplication would have been the distributive principle as 169 × 102 = $169\ (100 + 2) = 16\ 900 + 338\ = 17\ 238 \ldots$ and final answer as 17.238.

Summary of the observations

The researcher observed senior primary mathematics classrooms. The interactions between the teachers and learners were noted for further analysis. Mathematics is mostly taught by male teachers. Eighty percent of the observed teachers were male. Most of the schools had only one mathematics teacher for the senior primary phase. One of the teacher-participants was a trained language teacher. At school B, the teacher was an unqualified young matric graduate employed on a one year contract. These two teachers are expected to develop numeracy in the senior primary schools despite their qualification backgrounds.

Teachers on two occasions did not carry lesson plans and had to come up with class activities and examples from their own heads and most of these cases the teachers' questions had language and structural flaws. For instance, the first example by Teacher G which required learners to express N$42 as a percentage of N$70 was not well phrased. The structure of the question influences how learners understand the problem and hence their solution methods. In most cases, the questions also seemed to be gender biased as the characters used were all males. An example of such was the questions posed by Teacher G on percentage of a quantity and by Teacher H on actual profit.

The classes mainly practiced standard algorithms. The teachers' examples on the chalkboard were solved using standard algorithms and no other strategies. The teachers recommended these methods to learners and often gave a series of steps to be performed during computations. Teachers did not explore alternatives to solving problems. The mental arithmetic strategies were absent. There were many occasions where teachers did not attempt to correct learners work by explaining to them why their responses were incorrect. Neither did the teachers ask learners to analyse the sources of error.

Learners seemed to lack procedural fluency with the standard algorithms. They got stuck following the provided series of steps of the standard algorithms. Most of the times learners resorted to the use of drawing allies on rough papers. The fact that learners resorted to manipulatives seemed to suggest they were not fully prepared or did not understand how to follow the taught standard algorithms and would have preferred other alternatives.

There were cases where teachers and learners gave unreasonable answers. An example is when a teacher represented 30% of 80 as equal to 164. Percentages are introduced in primary schools as fractions which are 'part of a whole'. In that case the teacher meant a portion of 80 was equal to 164. The classes seemed to perform standard algorithms without reasoning. In these lessons, standard algorithms seemed to be focused on the final product and not wholly on procedural fluency.

The mental arithmetic strategies were lacking in the observed lessons. There were no recorded incidences where teachers made use of mental computation strategies to ease the computation process. Moreover, teachers did not encourage learners to explore different strategies to computing operations. Contrarily, on many occasions, teachers insisted that learners use standard algorithms such as long division and column multiplication.

5.3.2 Teachers' perspectives on the integration of mental arithmetic strategies in the curriculum

This section presents the teachers' views of mental arithmetic and its importance in the senior primary mathematics curriculum. The study sought to determine the teachers' clear-sightedness of mental arithmetic, the teachers' views on the relevance of mental arithmetic to numeracy development and further determine the teachers' views on the incorporation of mental arithmetic strategies in the senior primary classrooms.

5.3.2.1 Teachers' interpretation of mental arithmetic

To determine the teachers' clear-sightedness of the mental arithmetic concept, the researcher, during the interviews, asked teachers to define mental arithmetic. From the definitions given by the teachers, the researcher developed sub-themes based on the frequencies of phrases such as 'recalling of facts', 'use of own methods' etcetera. The summarised teachers' responses are recorded in Table 5.4.

Table 5.4. The teachers' definition of mental arithmetic.

Teacher	'recalling of facts'	'development of computation strategies'	'working without any aid (no pen and pencil)'	'learners use own methods'	Other utterances
A				X	Do it mentally or on their own.

B				X	Own methods only.
C			X		No calculators.
D			X		I am not sure about this.
E				X	Knows place value and expanded notation.
F	X				Calculate long ones too with paper and pencil.
G				X	No calculators. Done mentally and with pen and paper.
H			X	X	Learners are not using. They are doing, mentally calculating.
I	X	X			They use it to memorise.
J		X			Mental power to carry operations.

Key: An X indicates that the teachers mentioned the heading of the column in the definition.

Table 5.4 indicates the frequencies of the phrases; mental arithmetic as a recalling of facts i.e. memorising, mental arithmetic as development of strategies, mental arithmetic as working without aids (such as paper and pencil) and mental arithmetic as the use of learners' own methods.

The teachers' definitions of mental arithmetic vary widely. Some teachers seem to have no idea as to what the mental arithmetic concept entails. For example, Teacher D indicated to be unaware of what the concept is about. Most other teachers defined mental arithmetic as '*working mentally*'. Teacher I, on the other hand believes that mental arithmetic is about learners memorising numbers. It appears teachers are not aware that mental arithmetic focuses on developing learners' computation strategies and encompasses both pen and paper as well as oral procedures.

5.3.2.2 Teachers' views on why mental arithmetic strategies should (or not) be emphasised in schools

The second research question of the study sought to determine teachers' views on the inclusion of mental arithmetic in the primary school curriculum. The participating teachers were asked about their views on the inclusion of mental arithmetic in the school curriculum during individual interviews. Their responses are as presented in Table 5.5.

Table 5.5. Teachers' view on inclusion of mental arithmetic in the curriculum

Teacher	Mental arithmetic should be in the curriculum	Mental arithmetic NOT needed	Reasons
A	×		• Exposes learners to various strategies. • Learners develop and think more critically. • Teachers need practice more of mental arithmetic.
B	×		• Need be increased more in schools.
C	×		• It should be done across curriculum in order to develop learners. But teachers need workshop to help integrate.
D	×		• They reduce pressure on learners. • It should be in the schools, now it lacks.
E	×		• Enhances academic growth in learners. • Teachers are however struggling with these strategies. Training for teachers is needed.
F	×		• It is necessary as it develops numeracy. We need more of it. • Some teachers may not well understand the concept and that influences our pedagogies.
G	×		• It is in the curriculum. It is needed.
H		×	• I am not supporting it. Not at all.

I	×		• Needed to provide a variety of approaches for learners. • It can allow us as teachers to cater for all our learners in ranges of abilities.
J	×		• Very much needed. It will develop learners more.

All the participants, but Teacher H, are of the view that mental arithmetic strategies are needed in the senior primary mathematics curriculum. Various reasons such as mental arithmetic provides a variety of approaches for learners towards problem solving, develops learners' academic strength and develops learners' numeracy were provided. Most of the teachers also pointed out a concern on the readiness of integrating mental arithmetic in the curriculum as some or many teachers may be facing problems with these strategies themselves and will need training via workshops to enhance their teaching. Teacher J had more to say about mental arithmetic strategies:

> I see that these mental arithmetic strategies would develop our learners' mathematics skills and if we as mathematic teachers can start now, developing this in learners, their mental arithmetic skills, I am very sure that it will yield better numeracy.

Teacher J believed that the thorough emphasis of mental arithmetic strategies in schools would promote better understanding and performance in mathematics. An exception was Teacher H who did not support the inclusion of mental arithmetic in the primary school curriculum. The researcher engaged the teacher in a longer discussion as follows:

> Researcher: You said you do not support the inclusion of mental arithmetic in the primary school curriculum, will you please share more into your views?
>
> Teacher H: So, well while people are running away from that mind colonising approach to computers and IT devices, it's a contradiction now, me I was even having the idea of, for learners to start using calculators while they are at primary grades and not mental arithmetic. If you look at it, nowadays people are trying to incorporate in the ICT. No need, yeah. The devices are solving the problems and therefore learners should not be…, we should no longer drill on the methods because then people have access to devices they are using in society. When they finish school they will use computers and other IT at work places. A calculator means the person is going to pass, now and if the person is doing it with a correct method, correct way when using a calculator, that's the best way because one thing mental arithmetic sometimes

people who set up like questions they don't even have it clear, they don't even give enough time for learners to finish, but someone who did not give answer correct does not mean they do not understand it, it might be according to the time that is given.

And about the properties brought in, I just wanted to know, honestly, what is really the use of those properties for them to be incorporated in lessons? Now we need to give clear reasons why we want them to use that method, where are they going to use it in real life situations. Unless good valid reasons are going to be given, to me it's making zero sense. If it's only for test and examination to me it's not valid. Where are we going to use it and why should our learners do long calculations? I don't see the point of us going for a long method, is like for impressions but necessarily what is important is not the way how to do it but the correct answer.

Teacher H expressed his views on mental arithmetic explicitly. His main point is that modernisation has people working with IT devices and hence feels learners do not need to be taught computation strategies but rather be trained on how to use calculators, computers and other devices used by the society in calculations. Teacher H might be one of a few or more mathematics teachers serving in the primary schools who do not see the need of developing learners' numeracy skills.

Another aspect brought to light by Teacher H is the question of 'where are they going to use it in real life situations?' His views are that the curriculum seems to be presenting topics which are divorced from learners' lives. He said "If reasons can be brought on the table like ok, after school or the community we are doing distributive property, or commutative property you are going to apply it here".

Teacher H feels that classroom applications of mathematics should be linked to real life outside classrooms and learners need the mathematics which will help them function in the society as opposed to performing algorithms for the sake of examinations. Another teacher who seemed to have similar sentiments to those of Teacher H was Teacher G who said:

We want learners to appreciate topics, for example, I am teaching them percentages for life, this topic should not be taught considering only the examination but we want at least this to be part and parcel of their life and they (learners) will able to say, this topic that will help me to solve some of the problems at home. Then we can be able to teach mathematics

> with meaning, without doing that one, our mathematics will remain in the dark and leaners will look at this subject as a foreign subject.

The two teachers are of the view that school mathematics should be taught in ways that awaken the desire to learn among learners and that this can be well achieved if learners know the real life applications of such mathematics.

On the other hand, Teacher J, who supported the inclusion of mental arithmetic in the school had more to say:

> In my opinion, the reason kids are struggling with mathematics is because their mental arithmetic is not well developed. We need mental arithmetic to be relooked at in the syllabus. It appears in there but it is not clear. That is a challenge because the way the syllabus is drawn up its like once you have administered that topic and you are done with it, you are out of it so when you proceed to other topic it's, we barely integrate that. Something is missing.

Teacher J gave a detailed description of where she believes the problem that constitutes low numeracy in primary school originates. She points to a curriculum concern. More comments gave a view that teachers appear to have a concern on the way mental arithmetic is offered in the senior primary mathematics curriculum. The researcher asked teachers to clarify their concerns on the current curriculum and these are presented in the next section.

5.3.2.3 **Curriculum concerns**

In section 5.3.2.2, it was pointed out that teachers brought to light the concerns rising from the way the concept mental arithmetic is taught in the senior primary mathematics curriculum. The researcher asked teachers to give their views on how mental arithmetic is covered in the senior primary school curriculum. The teachers indicated several concerns related to the senior primary mathematics curriculum. The teachers also made suggestions on how these issues may be addressed. The concerns indicated are as presented next.

First, mental arithmetic is taught as a topic at the beginning of each year and appears forgotten for the rest of the year. The teachers feel this is due to the manner in which mental arithmetic appears in the syllabus – mainly as a topic on its own, usually done in the first trimester. Teachers do not practice these computation strategies when they move on to other topics but switch to the use of standard algorithms as they are more used to these. The examples in the prescribed books are also applications of the standard algorithms. The researcher quotes verbatim of teachers as:

> Teacher A: I think if it (mental arithmetic strategies) was distributed in every topic it could help the learners develop own strategies.
>
> Teacher C: Our curriculum also sometimes it's the way the basic competencies are also outlined. Sometimes not really that clear and a bit confusing,
>
> Teacher E: Yeah, um, now the current practice is that mental arithmetic is taught in the beginning of the year, and then from there one moves on to the other lessons… now in decimal fractions, so we no longer use them though it could may be help.
>
> Teacher F: The syllabus only focuses on that, especially on the whole numbers where there is that specific topic where learners are supposed to do that, but in terms of now moving on towards other topics it, it's rarely appearing.
>
> Teacher G: The syllabus needs to link back to these strategies, sometimes you (as in teacher) and the learner might end up forgetting the strategies.
>
> Teacher I: Um, I think it would help if syllabus was modified so mental arithmetic strategies are integrated throughout the year, so that learners cannot forget them. Because if you just learn something at the beginning of the year and then all of a sudden we leave it there, they may not know the importance of learning about those methods.
>
> Teacher J: We barely integrate that into other topics. It's needed throughout from the beginning to the end. That will help out.

Secondly the teachers feel that the emphasis on mental computation strategies takes up a lot of time. Due to a lot of objectives teachers are bound to cover, they opt not to drill much on the mental arithmetic strategies. It appears as teachers believe that standard algorithms are performed faster than the mental arithmetic strategies and hence they switch to standard algorithms.

> Teacher F: You know the competencies are a lot and if you did not really learn it thoroughly considering what you have taught in the first term obviously you will just leave it there.
>
> Teacher G: The syllabus is overloaded and we have very little time to spend on the competencies.

Teacher H: Completing the syllabus is one of the concerns. Otherwise learners can be given calculators and they will work faster.

Teacher J: Also, when planning you are bound to the syllabus and its number of topics that you have to cover within a term, you just do away with it. The applications of these strategies in other topics will be difficult as we have to complete the syllabus in a limited time.

Thirdly, teachers indicated that the competencies under mental arithmetic concept in the syllabus appears not clear and confusing and teachers find it hard to make sense out of these competencies. This is a curriculum concern. It also has to do with how teachers understand and interpret the mathematics syllabus. Teacher C said:

Our curriculum also sometimes it's the way the basic competencies are also outlined. Sometimes not really that clear and a bit confusing, one really has to go into details and try to make sense out of what the competencies are referring to.

Teacher G later echoed the words of Teacher C:

At some point I might also blame the, how the curriculum is set up, like the syllabus, the syllabus is overloaded with unclear competencies and we have very little time to spend on understanding what is required with regard to computations.

Additionally, the teachers believe that the mental arithmetic strategies should be thoroughly practiced in the lower primary phase to enhance early development of numeracy rather than waiting to introduce such at the senior primary phase as it's the current practice. This concern was raised in the teachers' remarks below:

Teacher C: And then the emphasis should also be put on the lower grades for the learners to develop that mental arithmetic.

Teacher D: So I think the computations at lower primary could be the major cause, they were not taught properly there while early. When they get to the other (senior) grades they will never struggle if their mental arithmetic was fully developed.

Teacher E: At Grade 3 they are doing it but not to such extend to say they really understand it; they are just saying it but without understanding. It needs emphasis down there.

> Teacher J: It should start from lower primary grades because if you make learners come to upper primary and their mental strategies are not well developed it becomes very difficult, so for me I argue that it should start from the pre-primary.

Lastly, the teachers might be unaware of how to develop mental computation strategies as they themselves were never taught such strategies in school as indicated by Teacher G:

> Sometimes you cannot blame teachers because they went through the same process… I mean the way their teachers, when they were learners in their days, their teachers did not emphasise mental arithmetic strategies and thus they do not know these strategies.

Teacher G's concerns seem to point at teachers' readiness to incorporate mental arithmetic strategies in the curriculum and linked to teachers' pedagogical knowledge. The pedagogical knowledge and more concerns within the curriculum may be some of the aspects that need to be looked at in the development of mental arithmetic strategies in senior primary schools. The next section looks at the development of mental arithmetic in senior primary classrooms.

5.3.3 Development of mental arithmetic strategies

The study also sought to explore ways of enhancing the development and practice of mental arithmetic in senior primary schools. During the individual teacher-interviews, teachers were asked to give suggestions on how the development of mental computation strategies may be enhanced in schools. The research further considered an intervention, a mental arithmetic strategies workshop, for teachers. This section presents the qualitative findings.

5.3.3.1 **Teachers' suggestions on developing mental arithmetic**

Teachers were asked for their views on how mental arithmetic development may be enhanced and these are presented next in sub-themes which emerged:

Mental arithmetic in the syllabus

The teachers mostly indicated that the development of mental arithmetic strategies in senior primary schools could be better enhanced if the strategies were distributed across all topics in the syllabus. For example, Teacher A said:

> So I think that could be more, if it (mental arithmetic) was distributed in every topic I think that could help maybe the learners to develop their strategies to get to the required answer.

Teachers indicated that as per the current senior primary mathematics curriculum, mental arithmetic is discussed as a topic at the beginning of the year, assessed via a test or topic task and considered done. Such mental arithmetic strategies are not applied to the rest of the topics in the syllabus. Next are more quotes of the teachers relating to the distribution of mental arithmetic in the senior primary mathematics syllabus:

> Teacher C: Incorporate mental arithmetic in the topic of fractions, decimal numbers and so on.

> Teacher E: So I think it need to be integrated in every topic, it's really needed just like now in decimal fractions, learners they need to know how to re-arrange the numbers.

> Teacher G: To develop their mind in terms of mathematics, I think we need to emphasise that strategy to be part and parcel of t every topic else, our learners they won't develop the sense of numbers.

> Teacher J: In the syllabus learners are to perform the mental strategies only when it comes to calculations like the addition, subtraction, multiplication and division but when it comes to other area where by all his basic operation are also integrated but it's like it's not incorporated .so when we are teaching mathematics like when, I teach mathematics at some point you just let learners use their system, use their strategies as long as they reach the correct answer.

Teaching of strategies

The second point raised by the teacher on how to develop mental arithmetic strategies in the senior primary schools is that the learners should be well exposed to these strategies. For example, Teacher B and Teacher F said:

> Teacher B: They (learners) need to be taught mental arithmetic strategies. They should not be left alone.

> Teacher F: We have to help our learners to know how to calculate using steps.

It however appears that the development of mental arithmetic strategies does not receive attention in the senior primary classrooms as Teacher C said:

Teacher C: We are only focusing on the traditional algorithms in schools and if learners don't understand we just repeat and repeat or leave them like that as time wouldn't allow.

Teacher I: But for me I think that it need to be stressed. We need to stress more no mental arithmetic's as for now they do more of standard algorithms only.

Training of teachers

The third suggestion brought up by the teachers in order to enhance the development of mental arithmetic strategies in the school is to offer workshops or training to 'refresh' teachers' memories for them to be able to deliver as per the expectations of the curriculum. Teacher C and Teacher J said:

Teacher C: I had a feeling that workshop was supposed to be done...

Teacher J: Yes, because even if I the teacher, because this is something I have realised I have realised it and I try to incorporate it in every topic, now what about the other teachers who are bound to the syllabus?

Teacher J believes there are senior primary mathematics teachers who are bound to the syllabus and do not explore a variety of ways to help learners in computing. Her remark seems to insinuate that something needs to be done to help teachers to be able to do more than what is in the syllabus. The next section presents findings related to mental arithmetic interventions which were conducted.

5.3.3.2 Interventions – A Mental Arithmetic Strategies Workshop

After the first phase of data collection and subsequent preliminary analysis of the data, the researcher drew up an intervention schedule for the senior primary teachers. The intervention was a workshop on mental arithmetic strategies. The next subsection presents the views of the teachers which made the researcher conduct such interventions.

5.3.3.2.1 A need for an intervention

During the interviews with the teachers, a high number of them pointed to a need for a refresher workshop - a training or intervention to equip teachers with the necessary skills to integrate mental arithmetic strategies in the curriculum. Despites the direct request for a training, a remarkable utterances and strategies employed by the participating teachers gave an indication that an intervention was necessary. Next are verbatim quotes of the teachers' views on their readiness to integrate mental arithmetic in their classrooms.

> Teacher A: Hmmm, well I think, I think teachers should also be, maybe holding workshops and…that will make us to be comfortable and be able to also teach these methods.

The word 'comfortable' as used by Teacher A, draws the researcher closer to the possible feelings of low self-efficacy in teachers when it comes to developing the mental arithmetic skills in the primary schools. As noted in the early sections of this chapter, Teacher A was trained as a language teacher but had to teach mathematics as there was no one to teach it. The school was overstaffed and hence could not advertise a post for a mathematics teacher. The word 'comfortable' from her perspectives may mean a lot. Other teachers had this to say:

> Teacher B: Iyaa, we need be given that quiet enough exposure in terms of number sense, in terms of, all this (mental arithmetic strategies) as opposed to just these traditional ways.
>
> Teacher C: I have never attended any workshop; I want to undergo mental arithmetic workshop to refresh my mind.
>
> Teacher D: I am not sure about this (when asked to define mental arithmetic).
>
> Teacher E: The emphasis should be here that more training on that is needed. So, because you find even in some teachers they are still struggling. Training is needed.
>
> Teacher F: We need to be refreshed on that one, we need to be refreshed on how to go about, especially when it comes to decimal fractions, be given that hint say workshops (on) how to integrate.
>
> Teacher G: If somebody could organise a workshop on mental arithmetic where we can also try to check if it can be able to incorporate in other topics. But the workshop should not focus on one topic only on whole number, we want at least somebody who come with other examples how can be used in.

> Teacher H: Maybe you also need to bring it again to my attention how do you understand mental arithmetic, you mean learners are not using calculators?

> Teacher J: I have actually attended workshops but never ever heard mention of mental strategies, perhaps, maybe I don't know how it works or they don't know it should be arithmetic, mental strategy but the facilitator does not go to that point as to say, this is how you teach your learners to develop their mental arithmetic.

The teachers quoted above all indicate the need for a refreshing workshop or intervention training focusing on the mental arithmetic strategies and how such strategies may be incorporated into the other topics of the senior primary syllabus. As for Teacher D who could not define the concept mental arithmetic but yet expected to develop mental arithmetic strategies in the senior primary classrooms, a critical intervention may be necessary.

Teacher G, who serves as the Mathematics Head of Department and hence responsible for monitoring the implementation of the mathematics curriculum at the school believes the problems leading to teachers not being 'comfortable' at developing mental arithmetic strategies may be deep rooted as he said:

> Sometimes you cannot blame teachers because they went in the same process, we only need as teachers, maybe we need to attend workshops where we need to learn and understand… teachers went through the same process in such a way that when they were learners, their teachers didn't emphasise on numeracy development and mental arithmetic was not known.

To that, Teacher J added:

> Perhaps teachers are the one who didn't know how and what exactly to teach. I think I also have a role to play myself, how to work out my methods and what to give to learners.

During the interview with Teacher I, the researcher asked the question relating to the way the teacher worked out the example as in Figure 5.10.

> Researcher: How often do you link to mental strategies in your class room, like today in your class, leaners had to work on 18 as a percentage of 360, when you were working on, calculating profit percentage and then they came to a stage whereby you had $\frac{18}{18} \times 5$, but did not use common factors to simplify.

Instead, you had to multiply 18 by 5 and they used the multiplication as a repeated addition approach, where you had to add 18+18 they get 36, add 18 again, they get 54. How about the use of the distributive property where;

$18\times5 = 5 \times (10+8) = 50 + 40 = 90$, instead of the repeated addition?

Do you find some challenges with incorporating the mental arithmetic strategies across the syllabus?

Teacher I: *Yeah*, the distributive properties and use of common factors, so I did not use it because learners we are already used to this repeated addition and multiplying across, so that is the methods they know better. Just to avoid confusions, I used what we are used to.

The teacher replied that he did not use the mental arithmetic approach as they are already used to addition. This seems to suggest that the classes get accustomed to certain methods of computing and do not explore several other ways in order to avoid confusion. It also appears that the teacher did not simplify $\frac{18}{18}\times 5$ in order to avoid confusing learners as simplifying would have been an unusual way of doing the mathematics.

5.3.3.2.2 The intervention sessions – mental arithmetic strategies workshop

Five teachers attended the workshop which were run for four weeks and introduced the teachers to mental arithmetic strategies. The instruction during the workshop were based on a training manual (see appendix S) with examples on how to solve problems using mental computation approaches. On the first Saturday, the session was based on mental addition strategies and teachers practised the left-to-right, compensation, bridging a decade and breaking-and-bridging approaches.

Left-to-right addition

The left to right approach strategy involves adding the highest place values and then adding the sums of the next place value(s). An example of the left-to-right strategy discussed during the workshop was:

Mrs Ndara owns a goat farm. She sells 3500 goats on auction. She is now left with 2300 goats on her farm. How many goats did she own before the auction?

The teachers were to first write out the highest place values of the two addends which are 3000 + 2000. The next place values have 500 + 300. The whole computation line was:

3000 + 2000 + 500 + 300 = 5000 + 800 = 5800.

Compensation addition

The compensation strategy of addition involves changing one number to a ten or hundred (multiples of 10), carrying out the addition and then adjusting the answer to compensate for the initial change. One of the examples that teachers practiced with was:

A truck with the weight of 3150 kg was carrying timbers of mass 2800 kg. Find the gross vehicle mass.

The compensation involved adding 50kg to 3150kg to become 3200kg. The addition yielded:

3200kg + 2800 = 6000kg and then subtracting the earlier 50kg,

6000kg – 50 kg = 5950kg

This strategy seems to make addition easier by changing the addend to an easily manageable value.

Breaking and bridging addition

The breaking & bridging strategy for addition involves starting with the first number and adding the values in the place values, starting with the largest place value of the second number. An example discussed during the workshop was:

A driver covered 233km on a tarred road and 190km on a gravel road to deliver chairs. What is his total distance?

The solution involved 233 km + 100 km (from 190km) = 333 km.

Now, 333 km + 90 km (which remained after the 100km added earlier) = 423 km.

Bridging a decade

The bridging a decade strategy is applicable to cases where the last digit is a 7, 8 or 9 (or products of these digits with powers of 10) and involves building on to the last digit to obtain a decade (10 or its multiple). The multiples of ten are relatively easier to add. An example practiced by the teachers during the workshop was:

The headman had 137 goats in 2016. During January 2017, he received 15 goats as taxes from the villagers. Work out the total number of goats in his kraal by end of January 2017.

The solution involved 137 goats + 15 goats = 137 + 3 (from 15 now 12 remains)

= 140 goats + 12 (which remained)

= 152 goats

These addition strategies are different from the addition standard algorithm were numbers are aligned in columns and addition is based on digits and not place value.

The mental computation strategies for the subtraction operation are closely linked to the addition strategies. On the second Saturday, the participants discussed the subtraction strategies as left-to-right, compensation, breaking & bridging, and constant difference. Examples for each of these strategies are as follows:

Left-to-right subtraction

The left-to-right approach during the subtraction operation involves learners doing the computation starting at the front end. An example was:

Namene has a bank balance of N$6720 and Mary has a bank balance of N$5845. How much more does Namene have?

The solution is: 6720 – 5845 = (6000 – 5000) - (800 – 700) - (40 – 20) - 5 = 875

Compensation for subtraction

The compensation strategy for subtraction involves changing one number to a ten, hundred or thousand (i.e. a power of 10), carrying out the subtraction and the adjusting the answer to compensate for the original change. An example of this approach was:

Lisa received a discount of N$996 on a laptop which worth N$3360. Work out how much she paid.

The solution involved changing N$996 to N$1000, subtracting and then adjusting to compensate for N$4.

That is; N$3660 – N$1000 = N$ 2660 + N$ 4 (adding 4 to since we subtracted 4 more)

= N$2664.

Breaking & Bridging for subtraction

This strategy for subtraction involves starting with the first number and subtracting the values in the place values, starting with the highest, of the second number. An example worked out by participants during the workshop was:

A primary school has 901 people altogether. Learners are 880, workers are 5 and the rest are teachers. Find the number of teachers in the school.

Solution via the breaking & bridging method:

901 – (800+80+5) = 901 – 800 = 101 – 80 = 21 – 5 = 16 teachers.

Constant difference

This strategy for subtraction involves adding or subtracting the same amount from both the subtrahend and the minuend to get a ten, hundred or thousand (multiples of powers of ten) in order to make the subtraction easier. This works because the two numbers are still the same distance apart. The following example was one of the examples done by teachers during the workshop on mental computation strategies for subtraction.

Leo bought a CD at N$398 and sold it for N$ 534. Work out his profit.

To work out the solution, teachers wrote: 534 – 398

= 536 – 400 (2 is added to both amounts)

= N$136.

On the third Saturday, the workshop was on mental multiplication strategies. These are the distributive principle, compensation for multiplication, compatible factors, half & double, and aliquot by parts. These strategies are discussed next and an example of each is given.

Distributive principle

The distributive principle involves finding the product of the single-digit factor and the digit in the highest place values of the second factor, and adding to this product a second sub-product. The computation is based on the general principle that: $a(b + c) = ab + ac$.

An example of the problems the teachers worked on during the workshops is:

The school starts daily at 07:10. There are 4 periods of 45 minutes each before break. Calculate the time the first break starts.

Solution: $4 \times 45 = 4\,(40 + 5) = 160 + 20 = 180 \text{ min} = 3h00\ min$

Now, we get; $07{:}\,10 + 3h00 \text{ min} = 10{:}\,10$.

Compensation for multiplication

This strategy for multiplication involves changing one of the factors to a multiple of a power of ten, carrying out the multiplication, and then adjusting the product to compensate for the change that was made. The strategy may be convenient when one of the factors is a multiple of a power of ten. Two of the examples worked out by teachers was as follows:

Example 1. A farmer planted carrots in 11 rows. Each row has 250 carrots. How many carrots did the farmer plant?

Solution: 250 × 10 = 2500 then we take 2500 + 250 = 2750 carrots. This example contains adjusting via addition. The next example contains adjusting via subtraction.

Example 2. The length of the rectangular garden is 149cm and its width is 5cm. Find the area of the garden.

Solution: 149 × 5 can be worked out as; 150 × 5 = 750 and then subtracting a 5 = 745 cm^2.

Compatible factors

This strategy for multiplication involves looking for pairs of factors whose product is a power of ten (or multiple of a power of ten) and re-associating the factors to make the overall calculation easier (makes applications of the associative property of multiplication). Among other examples, teachers worked on:

A farmer has 25 goats. Each goat requires 63 litres of water per week. How much water does the farm need to sustain the goats for a month?

Solution: Multiply $25 \times 63 \times 4 = 63 \times 25 \times 4$(25 and 4 are compatible factors as they give 100)

$= 63 \times 100 = 6300$ litres of water.

Half and Double

The half & double strategy involves halving one factor and doubling the other. At least one of the factors must be even for this strategy to work. One of the examples done by teachers during the workshop was:

Work out the area of a rectangular swimming pool which is $3\frac{1}{2}$ m wide and 12m long.

Solution: Multiply $3\frac{1}{2} \times 12 = 7 \times 6 = 42$ cm^2.

Aliquot by parts

The aliquot by parts strategy involves multiplying an equivalent fraction of the given factor to make computation easier (Morgan, 1999). The next example was discussed by teachers during the workshops:

A dealer sells a radio at N$280 making a profit of 15%. Calculate the profit.

Solution: 15 % of N$ 280 = $\frac{15}{100} \times 280 = 1.5 \times 28 = \frac{3}{2} \times 28$

$= 3 \times 14 =$ N$ 42

On the fourth Saturday, which was the last day of the workshops, the teachers discussed the mental computation strategies for division. These are the distributive property (both additive and subtractive distribution) and factoring division. These strategies are executed as shown in the examples that follows.

Distributive strategies

These are additive and subtractive distribution of division based on the principle: $\frac{\boldsymbol{a+b}}{\boldsymbol{c}} = \frac{\boldsymbol{a}}{\boldsymbol{c}} + \frac{\boldsymbol{b}}{\boldsymbol{c}}$.

An example that was solved by teachers is:

A kindergarten will receive a donation of N$145 000 in equal portions over five months. Work out the amount the kindergarten will receive monthly.

Method 1. (Additive distribution): $\frac{100\,000}{5} + \frac{45\,000}{5} = 20\,000 + 9000 = \text{N\$}29\,000$ per month.

Method 2. (Subtractive distribution): $\frac{150\,000}{5} - \frac{5000}{5} = 3000 - 1000 = \text{N\$}29\,000$ per month.

Factoring division

The factoring strategy for division works by breaking down the divisor into factors for easier computation. An example of this strategy as was worked out by the participants is:

Three business partners, Etuna, Helvi and Vickie shared a profit of N$40 000 in the ratio 1:2:5. How much was Vickie's share?

Solution: 1 + 2 + 5 = 8, Vickie's share: $\frac{5}{8} \times 40\,000 = \frac{5}{2\times 4} \times 40\,000$

$= \frac{5}{2} \times 10\,000$

$= 5 \times 5000 = \text{N\$}25\,000.$

These are the type of mental computation strategies practiced by teachers during the workshops. At the end of the workshop on mental division computation strategies, teachers set for the focus group discussions. The researcher guided the discussions to generate a discourse on issues pertaining mental computations and reflections on the workshops (see appendix O). The next section presents the views of teachers on mental arithmetic after the intervention.

5.3.3.2.3 Teachers' views on mental arithmetic after the intervention

After the researcher realised the need for an intervention, a workshop on mental arithmetic strategies was organised. Five teachers from the ten interviewed teachers participated in the training. After the intervention sessions, the teachers who participated were asked to share their views on mental arithmetic after the workshop. This was done during the focus group discussions. The following revelations were made during the discussions:

New computation strategies learnt from the workshops

In this section the teachers were asked to mention anything new they had learnt from the intervention sessions. The five participants all indicated that they learned a lot of new aspects from the workshop. In particular, Participant 1 said:

> From the sessions I have realized that all the approaches like left to right, compensation, compatible numbers and breaking and bridging, and most of these can be used in all the operations.

To add on, Participant 2 said:

> I have learnt lots of methods to give to my learners as an alternative to these standard algorithms where they do the calculations and spending more time.

The participants agreed that too much time is spent on the standard algorithms. They indicated to having acquired different approaches which may come in and make computing faster and easier. In addition, Participant 3 said that they had learned that there are various ways of doing mathematical operations, indicating that has not been the practice in the schools. He said:

> It is quite clear that, in fact while we have been pushing just same strategies, there are lots of strategies that can easily be used to carry out any calculation and help our learners understand mathematics.

To which Participant 5 added:

> Rather than just using the standard or traditional methods that our schools are using, the compensation method and compatible numbers method, um, are very good methods that one can use in the classrooms as varieties of computational approaches.

Participant 4 agreed with the other participants that they had discovered that there exist various strategies of computing that learners can be exposed to instead of being limited/reduced to a specific standard algorithm like it is mostly the practice in senior primary schools. The participants in the focus group discussions indicated what they learnt from the mental arithmetic strategies workshop. They also indicated how these strategies would be helping learners to carry out operations as alternatives to standard algorithms. The next subsection presents the teachers views on how the intervention workshop helped change their earlier perceptions of mental arithmetic.

Effects of the intervention workshop on the participants' perception earlier held about mental arithmetic

The researcher asked participants whether the workshop has had caused any change in how they earlier perceived mental arithmetic. During the individual interviews, teachers were asked of their perceptions towards the mental arithmetic concept. These perceptions and beliefs about mental mathematics suggested that some teachers did not really understand the concept before the workshop.

After the intervention, the participants had developed different views of mental arithmetic. They admitted that they did not fully understand what mental arithmetic is about and that they have not been practicing the mental arithmetic strategies in their classrooms. Some of the verbatim quotes contrary to what has been presented earlier are as follows:

> Participant 1: I often don't use the mental arithmetic strategies, really, in the classroom. Yeah, we did not explore other ways than the standard algorithms. I just became more aware after attending the workshops.

> Participant 2: I agree. We have not tried like various problems and solve them using various approaches with learners mostly coming up with their own methods. But I see we were struggling. Seeing these strategies presented here today has changed me. I will introduce these strategies in my classes. The time will not need to be as much as we allocate now, they will work faster.

> Participant 3: Most definitely it has changed me. I now know what mental arithmetic is, um, entails. I will not only, specifically focus on the traditional or standard algorithms where learners have to be so specific in following a method sometimes without really understanding what they are doing. Learners got much of problem when they align numbers. Perception is changed in such a way that actually one can now go and free-style so to say. Not only insisting that learners solve mathematical problems by aligning numbers in standard ways.

The teachers agreed that there are many times where learners follow the steps of standard algorithms yet lack understanding of the number concept and operations thereof. This confirms cases observed earlier by the researcher where learners' answers indicated lack of reasoning. The column-aligning of numbers was again mentioned as a problem faced by learners during standard algorithms. Participants also nodded that mental arithmetic strategies will enable learners to complete tasks faster than when using standard algorithms.

> Participant 4: um, now from here, I can say we have some light on the different methods that we should use in the classrooms. We have been just using those foreign books but then in these books the things are not well explained to us as well. But now we got exposed to those methods that we can use in the classrooms and these are very much good.
>
> Researcher: Tell me, the prescribed books at the senior primary phase, how are the examples solved in there, are various methods explored?
>
> Participant 4: Its mostly standard algorithms. And yeah, a topic on its own discussing pencil and paper methods and then expanded notations.
>
> Participant 5: That's correct, um, mostly weighted in the standard algorithms. Thus as teachers we use the same examples. It is also how we were trained both in schools and at university. Yeah, I have captured and, um, probably what the clue that I got here is that, I will definitely not just stick to the old methods.
>
> Participant 2: I will change. We have been forcing learners to work by explaining to them over and over, just repetition. Repeating exactly same methods taught already. With mental strategies shown here today, I now have a view of what mental arithmetic is about.

The participants pointed out how they have been practicing only standard algorithms in their classrooms. From the focus group discussions, it is emerging that the teachers seemed to have had no awareness of the existence of various mental strategies which may ease computation. The prescribed books also seem to be too much on the standard algorithms and hence schools practiced the examples using the algorithms found in the books. The teachers also indicated that they were taught using standard algorithms and were trained to use standard algorithms and thus not well exposed to mental arithmetic strategies.

New expectations from the participants' future mathematics lessons

The participants were asked to mention the new expectations the intervention created in them regarding their future mathematics lessons. The participants indicated that they learned a variety of strategies that they will be introducing in their classrooms. The participants verbatim were as follows:

> Participant 1: I will definitely incorporate these. We have some very fast learners, to them getting especially these methods they will consider fun and will follow with excitement. Now it will be quick you know, already, when I am to think all the way forward.

> Participant 2: No, with me I had a feeling that this workshop was supposed to be done earlier. This topic, most of the strategies discussed here, even though I am going to integrate them in all the other topics, you know, um, where computation was done there, if you could have gotten all this information earlier then I would have helped my learners. When a learner doesn't understand I can introduce him/her to another approach. I will try to change from one method to another. At least to give them a chance to choose a method they find easy. Yeah.

> Participant 3: Uh, as mentioned earlier that we are only focusing on the traditional algorithms in schools and if learners don't understand we just repeat and repeat or leave them like that as time wouldn't allow. It would have helped a lot. These strategies are going to be the solution to a problem that persisted for long now and improve the performance of the learners.

> Participant 4: You know I, I was someone who believed in that traditional method and believed mental arithmetic is to be done with whole numbers only and did not link with other topics say for example Measures. But from today I will be able to integrate these methods into other topics, which I couldn't do earlier.

> Participant 5: I am going to change the entire approach from here. The planning and teaching will be more practical and using the mental strategies learnt here. I am grateful to have attended these and the exposure is rewarding.

The participants believe the practice of mental arithmetic strategies in their classes will make learning fun and quick. It was observed how the participants were keen to make the learning of mathematics fun. The participants also felt the training was long overdue as it would have helped them to exercise a variety of computation approaches to improve the learning of mathematics in senior primary schools and enhance learners' performance. They indicated readiness to integrate mental arithmetic in their teaching. The participants pledged to change their approach in the teaching of mathematics to develop mental arithmetic strategies and enhance learners' numeracy.

Critical reflection on the workshop manual – content relevance

The researcher asked participants to reflect on the manual (see appendix S) used in the intervention sessions relative to content relevance. The participants made the following reflections:

> Participant 1: The work out problems one would say really, you know, they are usable. The examples, the explanation is quite clear, you know. One can actually present these to learners as they are here.
>
> Participant 2: The information on the manual is ready to digest, it's actually at the very level of a learners. It's quite clear and self-explanatory.
>
> Participant 3: I think, its spot on. One could see that all the information that are here are the ones that we are dealing with in the classroom situations and you know, um, the other calculations shown here are well simplified and ready for use in the classrooms. I think it is quite clear.
>
> Participant 4: Yeah, just to add on the slides, these were very good slides, very nice methods we have, topics are there, and examples that can be used in the classrooms.
>
> Participant 5: As said by everyone, the guide or manual is very rich. We just have to integrate these methods in the curriculum topics.

The teachers' reflections on the content relevance of the training manual used during the intervention workshops suggests the manual contained informative strategies and examples ready to use in the senior primary classrooms. It was also noted that the participants preferred contextualised problems within the senior primary curriculum.

Critical reflection on the time duration spent on the workshop

The participants were asked to reflect on the time spent on the intervention sessions.

> Participant 1: I think the time was adequate. For all the information we shared here we had amble time to look at examples.

> Participant 2: I think time was ok for me. Information were well planned accordingly and presented in that time.

> Participant 3: The duration itself was ok, for this, um, presentation, um, however, one may say that we need more of these types of trainings. There are teachers out there that needs these. Every teacher needs these.

> Participant 4: Yeah, in actual fact as to, to me as a teacher the time spent here is ample. Its actually after all it's a workshop and we should acquire the glimpse of doing these and yeah, to me as a teacher the time is ample.

> Participant 5: The time was well distributed and enough insight has been given from all the perspectives. Yeah, it is quite enough time.

The participants said the time spent on the workshop was sufficient. This information may be useful in recommending the time required if mental arithmetic workshops were to be conducted for more senior primary mathematics teachers in the country.

Suggested improvements for the workshops

The participants were asked to suggest improvements to the workshops. They said:

> Participant 1: uh, on mental arithmetic and uh, to improve on the presented strategies here, I mean, really? I am even like; I am struggling to find one area needing improvement on these slides.

> Participant 2: The workshops were fine. As teachers we will come in with insight, sometimes we have got the insight on these, only that either we are too ignorant or as we usually cover up for ourselves and blame it on the 'time' available … (laughs). But otherwise, I mean, really, really, in all honesty, there is little if any to be improved about this, in itself, it is quite enough just the way it is.

Participant 3: We are already saying the time was enough, soo I don't think there is anything to be improved about the workshop. The slides are clear, we got handouts, and there is nothing to improve here.

Participant 4: Yeah, no, the workshop or may be the strategies that were for this workshop I think they are all clear and one could really, cannot find areas needing improvement as the other colleagues said.

Participant 5: Well, if you could perhaps, try to give us um, ways on how to integrate these in all the other topics as well like for instance fractions and percentages and all the other topics within the syllabus. Just add that information on that one as our books are all standard algorithms.

The only suggestion made on improving the workshops was inclusion of how mental arithmetic strategies might be integrated into other topics of the syllabus. This is a good point to consider so that teachers are given examples where mental computation strategies are integrated in other topics as it appears the examples in the prescribed text books are based on standard algorithms alone.

Possible barriers to integrating mental arithmetic strategies in mathematics classrooms

The participants were asked for the possible barriers that may hinder them from integrating the mental computation strategies as practiced in the workshops. The views of the teachers on possible barriers were:

Participant 1: I hold resources as the only thing that is going to be the main challenge. If one is to incorporate these methods in all the other topics yet the prescribed books use different strategies, it's going to be teacher does it differently and book differently.

Participant 2: The books has different methods and those are the examples we give to our learners. To replace the book examples with these strategies will perhaps mean we divert. Or perhaps we will first teach the standard algorithm and then later the mental strategies or vice versa. That will be a challenge to teachers, I foresee that.

The participants all agreed that there will be a mismatch between how they solve the problems using mental arithmetic strategies and how the problems are solved in the books. They had a view that the books in schools need to be flexible with strategies and thus suggested the books be edited to include mental arithmetic strategies.

The participants continued to list possible barriers:

> Participant 2: The problem as teachers we fail to see the mathematics topics as linked. We don't really, uh, link the topics to each other, really, there is no flow between the topics. Just because of that holding topics as separate, otherwise if we link the connections it was going to be easy.
>
> Participant 3: I agree with my colleague. There is a need to incorporate these strategies in all the other topics and link the topics. The current arrangement of the curriculum where mental strategies are taught at the beginning of the year only is not helping. That will be the main challenge.
>
> Researcher: I want us to look at the incorporation of mental arithmetic strategies this way, after discussing these strategies as a topic in trimester one, as you move on to other topics you keep solving the problems using these strategies instead of resorting to standard algorithms. Example, multiplication may easily be understood and solved using the distributive principle rather than the standard algorithm. How about you always use the distributive principle?
>
> And whenever you have division, say for example $30 \div 12$, instead of the standard algorithm (long division approach), you as a teacher may choose to rewrite that as $\frac{24+6}{12} = \frac{24}{12} + \frac{6}{12} = 2 + \frac{1}{2} = 2\frac{1}{2}$. This should be done without necessarily repeating all the other methods. You should be using one method at a time only.

After the researchers' guidance on how mental arithmetic strategies are supposed to be integrated, the participants nodded in agreement. They discussions continued as follows:

> Participant 3: um, a challenge will then be obviously calculations carry marks for the methods not only the answers. During examinations, lots of marks are currently allocated for long standard algorithm procedures and the mark allocation will be tricky with the mental methods as part of the procedures. And, um, most at times the marking scheme is set by someone else and the markings allocated in that order.

Participant 1: uh, examinations! Suppose we teach the learners how to approach questions using those mental strategies and somebody else has to set the examination question paper with a marking scheme, usually the marking scheme restricts teachers on how to award marks. So now, I remember at a certain workshop I attended, I told them how long division has been a problem to me and to my learners, you know the moment they see that long division sign, they tend to ignore the problem as they find it very difficult. So, if you bring another alternative they mostly to your learners and in the marking scheme one is directed to follow the long division method like my colleague has mentioned, you will have a problem with mark allocation. So, even though the learners got the correct answer there using another method, uh, they will not get all marks.

Participant 4: Adding on, perhaps another fear is on the learners not showing all the steps involved and claim they did the work mentally.

Researcher: well, that sounds highly likely, how about we always urge them to put down the series of steps involved as they compute with these new strategies? Remember colleagues that the mental arithmetic strategies we discussed here comprises pencil and paper usage in recording the steps.

Participant 4: Perhaps that will then work, um, yes.

The possible challenge raised at this stage is that the examinations are set at the regional education office. These papers are sent with the memorandum which guides teachers as to where they should allocate the marks. Such memorandums solve problems via standard algorithms. It seems the marking is not based on the correctness and applicability of any method used but based on standard algorithms.

The discussions continued:

Participant 5: The additional challenge I am thinking of is how to get these to the colleagues out there. They need to be trained on how to use these methods. Within in our minds are only standard algorithms, and the examples in the books mostly use the standard algorithms. The syllabus has indicated some of these methods, but only as a topic at the beginning of the year and teachers are not helped on how to integrate these into the other topics.

> Participant 2: May be what I will recommend is the, uh, teachers should at least be trained if these mental arithmetic are to effectively be part of the curriculum, even the advisors need to know these methods. That way we will all follow the same, um, and allow learners to use a variety of methods as they may like. Learners should not be limited to standard algorithms as the current practice.

Participant 5 and Participant 2 raised that a challenge to integration of the learnt mental arithmetic strategies will be when only the five of them who attended the workshops will be the ones integrating the mental arithmetic approaches in their classrooms. They feel the workshop should be rolled out to more teachers in order to make a difference as far as the development of mental arithmetic is concerned.

The participants had a discussion on time, marks allocation, other teachers lacking exposure to mental computation strategies and a lack of teaching resources such as books that has examples solved using mental strategy techniques. In order to enhance the development of mental arithmetic strategies in the schools, the participants recommended a review of the senior primary books and the intervention workshops for all senior primary teachers.

General comments on the intervention – mental arithmetic strategies workshops

At the very end of the focus group discussions, the researcher asked participants for any remarks, comments or addition they had about the mental arithmetic strategy workshops. They participants responded as follows:

> Participant 1: I would recommend that all teachers be exposed to these methods including examiners at circuit, regional and national levels so that when they are setting examinations they know and will allow learners to explore various ways of computing operations freely. One should always bear in mind that there is a high possibility that these learners may use different methods apart from this that you are only thinking about as an examiner.

> Participant 2: Yes, indeed the strategies discussed here need be known by everyone in the teaching arena. There will be contradictions or disagreements if this is not well disseminated to teachers out there. Always good if we are on the same boat.

Participant 3: Yeah, I am also saying it must be extended to other teachers. It doesn't help at all if only one teacher knows about these methods at school, so, uh, the learners in the senior primary phases are mainly passed on to a different teacher as they progress from one grade to the next. So, we don't want to have a gap in the understanding. At least we must have, as teachers, the ability to make use of a variety of methods which our learners may choose from. This must be definitely extended to teachers out there.

Participant 4: I want to add that when we as teachers meet learners informally like in the Maths Club we should always encourage them to use different approaches to enhance their number sense. Learners, uh, would ask me if they can use a specific method and get all score in a test and they indicate they fear that their teacher might not accept the method. They got that fear you know.

Participant 5: Yeah, definitely that fear. They fear teachers might penalize them. They would always ask "is this method safe to use anywhere"? So, it is thus a good idea for this workshop to be extended to other teachers to cut across so we all do the same things. And learners be allowed to exercise the freedom of picking any correct strategy.

Participant 3: I want to add on the part of the Maths clubs. In fact, the main purpose of the maths clubs is to expose learners to lots of activities with problems and methods they may have not met in the mainstream classrooms. The mathematics out there which rarely comes in the classroom is explored in the maths club. So, the items you share with us are going to be very useful to us. Sometimes we are using those learners to help teach other, peer learning, those that struggle in the class are taught by the fast learners, mostly we select the best ones or 'A learners' to help teach the other ones as learners at times learn better from each other.

It appears that teachers and examiners restrict learners to standard algorithms. The teachers feel the mental arithmetic strategies will allow learners to exercise freedom of choice among several methods and promote learning with understanding. The participants recommended that senior primary teachers should be exposed to the mental arithmetic strategies via workshops.

The participants indicated that they have learned a lot from the workshops. They recommended the workshops be extended to all senior primary mathematics teachers and examiners. The possible barriers to effective implementation are the lack of resources e.g. text books with examples solved via mental strategies. Teachers allege that the current books in the schools only contain standard algorithms.

5.3.4 Summary of the qualitative data

This section presented the findings and gave an interpretation thereof. The study was of an exploratory design which embraced the critical mathematics education theory to determine how mental arithmetic is being incorporated in senior primary schools in the Oshana region, northern Namibia. Ten primary schools participated in the study. A mathematics teacher at each school was observed and interviewed. Five of the interviewed teachers underwent an intervention – a training on mental arithmetic strategies, after which the researcher had a focused group discussion.

The observations revealed that mental arithmetic strategies are not practiced in the senior primary schools. Remarkably, teachers have different views of what mental arithmetic is and what it entails. Learners are restricted to standard algorithms and various computation approaches are not exercised. Teachers and learners gave unreasonable answers at occasions. During the interviews, most teachers indicated a need for mental arithmetic development.

An intervention in forms of workshop on mental arithmetic strategies was conducted with five teachers. After the training, the teachers indicated having acquired a different perception of mental arithmetic and that they learned new computation strategies which may be helpful in their classrooms. Teachers indicated that there is a need for a bigger intervention to include all senior primary teachers in the country so that the entire senior primary phase is sensitised on how to develop mental arithmetic in the schools. The participants also had a strong view that mental arithmetic strategies need to be thoroughly practiced at the junior primary phase.

5.4 PRESENTATION OF THE QUANTITATIVE RESULTS

The study administered a psychometric test to Grade 7 learners at each of the ten schools. The results were used to determine the level of the learners' numeracy based on the performance.

5.4.1 Psychometric test

The psychometric tests were administered to a total of 300 Grade 7 learners from the ten schools during the first phase of data collection. The results showed that seven out of the ten schools scored an average below 50% (see Figure 5.11). This is a low level of performance by the Grade 7 learners in a numeracy test.

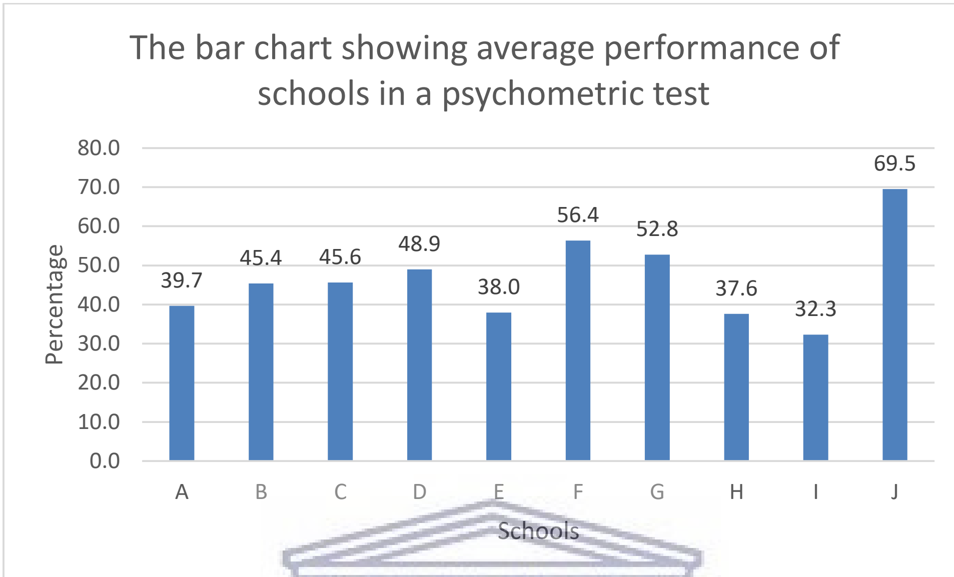


Figure 5.11. The bar chart showing the average psychometric test scores of the schools.

The mean scores from the schools are recorded in table 5.6. The average mean for all the schools is 46.62 with standard deviation of 21.017. These figures indicate a low numeracy score by the senior primary learners in the Oshana region.

Table 5.6. The psychometric test mean scores of the 10 schools.

School	A	B	C	D	E	F	G	H	I	J	Total Average
Mean	39.7	45.4	45.6	48.9	38.0	56.4	52.8	37.6	32.3	69.5	46.62
n	12	32	30	38	46	34	28	27	27	26	30
Std. Deviation	29.712	19.769	19.753	18.133	18.371	17.180	27.223	17.374	21.453	21.199	21.017

After writing the test, five teachers from the sampled schools attended a teachers' intervention – a mental arithmetic strategy workshop - and thereafter the, researcher purposively sampled out two out of the five schools which were represented in the workshops to form two experimental schools. The researcher further purposively sampled out two control schools from the other five schools which were not represented at the workshop. The researcher worked closely with two teachers in the experimental schools and provided assistance in integrating mental arithmetic strategies into classroom activities which teachers carried with to their lessons. The collected data presented in this section aims to test the null hypothesis, $H_{o,}$ at $\alpha = 0.05$ significance level.

H_o: There is no significant difference in the performance of the learners taught mental arithmetic strategies and those who were not.

H_1: There is a significant difference in the performance of the learners taught mental arithmetic strategies and those who were not.

To test the null hypothesis, the researcher had to compare the performances within and between the experimental and control group. The next sub-sections present an analysis of these results.

5.4.2 Pre-test scores – control versus experimental group

There were four schools purposively sampled to take part in the quasi-experimental treatment. School A and School C were the control groups while School B and School I were the experimental groups. For the experiments, the results from the psychometric test were used as the pre-test scores for the four schools. The four schools scored an average below 50 % (see Figure 5.12).

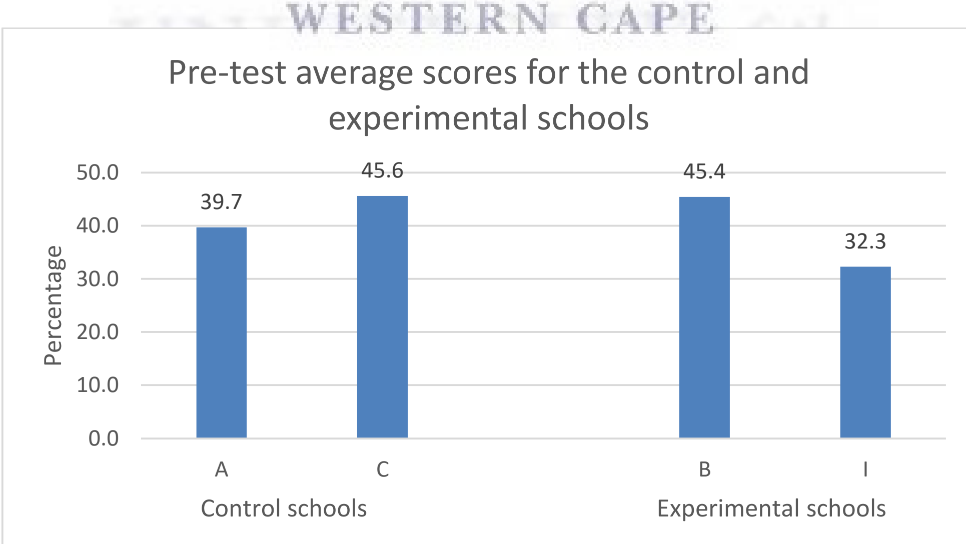


Figure 5.12. The bar chart showing the pre-test scores of the control and experimental schools.

To closely compare the performances between the experimental schools and the control schools prior to the interventions, a box and whiskers plot was used.

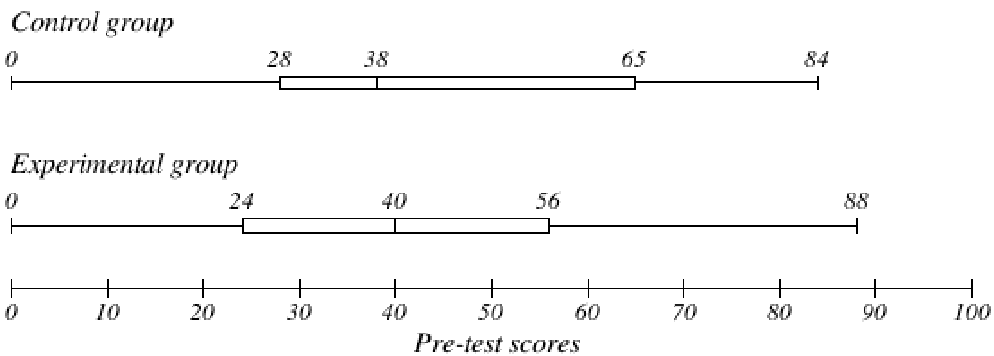


Figure 5.13. The box and whisker plot of the pre-test scores.

Figure 5.13 shows that half of the learners in both control and experimental group scored below 40% in the psychometric test. For the experimental group, three quarters of the learners scored below 56% (upper quartile) as compared to 65% score in the control group. Although the box and whisker plots gives a picture of how the learners' scores were distributed, it does not tell whether the differences between the control and experimental groups were statistically significant. Hence, the t-tests were conducted.

An independent t-test was calculated to test for significance in the differences between the mean scores of the control and experimental groups. The sub-hypothesis tested is:

> H_o : There is no significant difference between the mean scores of the control and experimental groups prior to intervention.
>
> $H_{1:}$ There is a significant difference between the mean scores of the control and experimental groups prior to intervention.

Table 5.7 contains the independent t-test scores of the control and experimental groups from the pre-test.

Table 5.7. The independent t-test scores of the control and experimental groups in the pre-test.

T-test for equality of means at 95% Confidence interval of the difference

Group	n	Mean	Std. Deviation	Std. Error Mean	$t_{critical}$	df	$t_{calculated}$
Experimental	28	43.1429	19.05492	3.60104	2.704	57	0.32
Control	31	42.9677	22.79106	4.09339			

The calculated t-test value, $t_{calculated} = 0.32 < t_{critical} = 2.704$ indicating that the test *fails to reject* the null hypothesis, H_o, that there was no significant difference between the pre-test means of the control and experimental group. This indicates that the control and experimental groups performed at the same level in the pre-test. Another indication of the similarity in the pre-test performance of the groups can be deduced from the mean scores which were both approximately 43%.

5.4.3 Learners' intervention – The mental arithmetic sessions

Two of the five schools which were represented by teachers during the mental arithmetic strategies workshops were sampled out for the learners' intervention as the experimental group. Two schools were purposively sampled from the other five schools which were not represented at the workshop to prevent contamination threats as teachers who attended workshops could have influenced a different approach to computations of mathematical problems. The psychometric test taken during the first phase of the study was used as the pre-test. The four classes that participated in the study all wrote the pre-test.

Control Group

Two schools were in the control group. The learners and their parents were asked for consent to partake in the study through signed assent forms and consent forms respectively. One of the two schools in the control group is sited in the urban district and the other school was in the rural areas of the Oshana region. The learners continued with their classes as normal – they were not exposed to any treatment but were given the same activities given to the experimental group to practice using their usual methods.

Experimental Group

The two schools which formed part of the experimental group were purposively sampled based on the teachers' willingness and availability to work extra and conduct the treatment. Like the case in the control group, one of these two schools was in the urban area and the other was in the rural areas of the Oshana region. The researcher worked with the teachers who exposed the learners in the experimental group to a series of lessons on solving problems using the mental arithmetic strategies. The lessons were conducted in the afternoons during the study session for a period of two weeks. The learners practised mathematics problems using mental computation approaches (see Table 5.8). For specific examples done by learners, see Appendix S.

Table 5.8. Computations strategies framework

Addition	1. Left – Right Approach 2. Compensation 3. Bridging a decade 4. Breaking and Bridging
Subtraction	1. Left –Right Approach 2. Compensation 3. Breaking and Bridging 4. Constant Difference
Multiplication	1. Distributive Principle 2. Compensation 3. Compatible Factors 4. Half and Double 5. Aliquot by parts
Division	1. Additive distribution 2. Subtractive distribution 3. General Factoring

Sequential framework for senior primary mental strategies. Adopted from Morgan (1999).

Learners worked on examples and were encouraged to explore alternatives of computation strategies. At the end of the two weeks, a post-test was administered. The post-test was written by both the experimental and control groups. In all cases of the pre and post-tests, the papers were written at the same time in all the schools and a life skills teacher at each school was tasked to administer the tests. The pre-test and post-test results were then compared and the hypothesis tested as explained in chapter 5, section 5.4.

5.4.4 Post-test

After the intervention with the experimental schools, the post-test was administered to all the four schools. The results of the post-test were compared within and between the control and experimental groups. Figure 5.14 shows these comparisons.

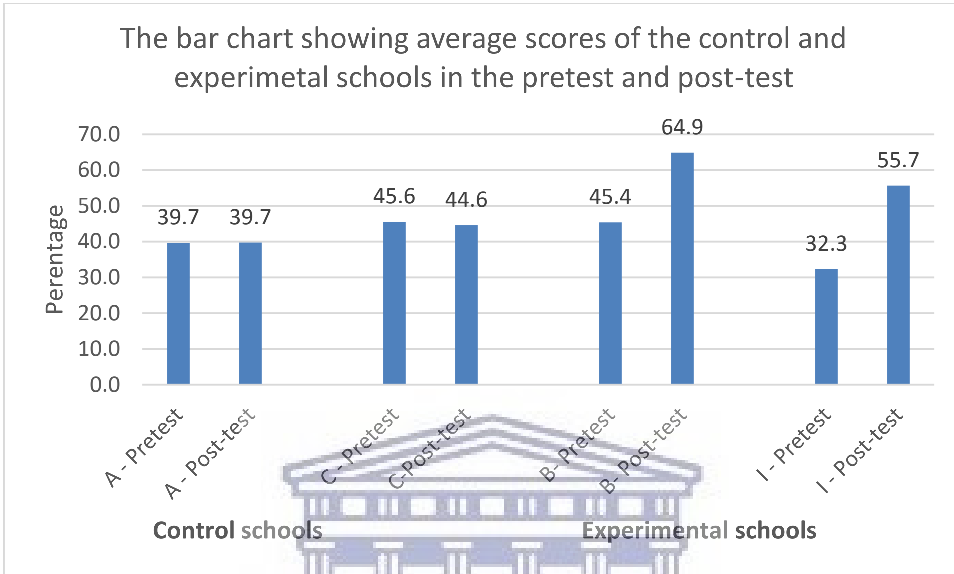


Figure 5.14. The bar chart showing the pre-test and post-test average scores in the control and experimental schools.

Figure 5.14 shows an improvement in the post-test performances of the experimental schools. The control schools performed nearly the same in both pre-test and post-test. These results are further closely compared in the next sub-section.

5.4.5 Control Group – pre-test versus post-test scores

The scores of the control group in the pre-test and post-test are compared to determine any differences. A box and whisker plot gives a comparison of how the scores were spread in the pre-test and post-test. Figure 5.15 contains the analyses.

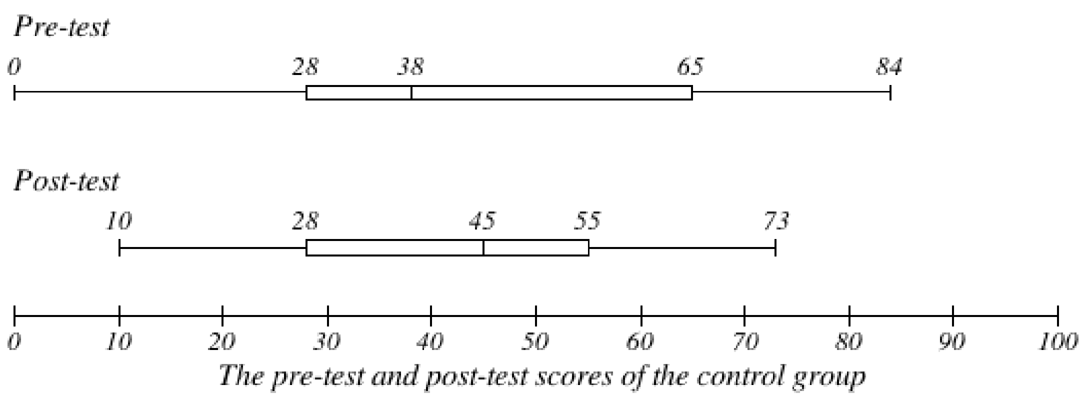


Figure 5.15. Box and whisker plot illustrating the pre-test and post-test distributions of the control group.

The pre-test and post-test scores of the control group were centred around the same values although there were no outliers in the post-test as was the case with the pre-test. The pre-test scores showed that 50% of the control group scored below 38%. In the post test, half of the class scored below 45% (the median value). To test whether these differences were statistically significant, a t-test was conducted.

A dependent t-test was calculated to compare and test for significance in the difference between the pre-test and post-scores of the control groups. The hypotheses tested to compare these means is:

H_o: There is no significant difference between the pre-test and post-test mean scores of the control group.

H_1: There is a significant difference between the pre-test and post-test mean scores of the control group.

The t-test results are recorded in Table 5.9.

Table 5.9. The dependent t-test results for the pre-test and post-test scores of the control group.

Control group	n	Mean	Std. Deviation	df	$t_{critical}$	$t_{calculated}$
Pre-test	31	42.9677	22.79106	30	2.750	0.616
Post test	31	44.2258	16.98766			

The dependent t-test calculated, $t_{calculated} = 0.616 < t_{critical} = 2.750$. The t-test results *failed to reject* the null hypothesis, an indication that the difference between the mean scores of the control group in the pre-test and post-test is not significant. This implies that the Grade 7 learners in the control group performed relatively the same in the two tests.

5.4.6 Experimental Groups - pre-test versus post-test scores

The scores of the experimental group from the pre-test and post-test are compared in this sub-section. Figure 5.16 illustrates the distribution of the scores using box and whisker plots.

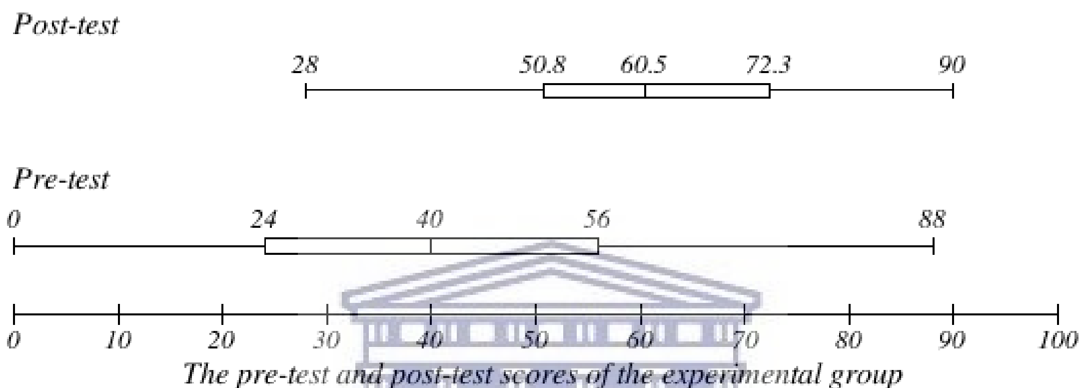


Figure 5.16. Box and whisker plot illustrating the pre-test and post-test distributions of the experimental group.

The post-test lower quartile = 50.8 compared to the pre-test lower quartile = 24. The median score of the post-test (60.5%) is also higher than the median score of the pre-test. Put differently, half of the experimental group managed to score above 60.5% in the post-test while in the pre-test, half of the class was below 40%. To test whether such differences were statistically significant, a t-test for independent groups was calculated.

A dependent t-test was calculated to compare and test for significance in the difference between the pre-test and post-scores of the experimental groups using the hypotheses:

H_o: There is no significant difference between the pre-test and post-test mean scores of the experimental group.

H_1: There is a significant difference between the pre-test and post-test mean scores of the experimental group.

The t-test results are recorded in Table 5.10.

Table 5.10. The dependent t-test results of the pre-test and post-test results of the experimental group.

Experimental l group	n	Mean	Std. Deviation	df	$t_{critical}$	$t_{calculated}$
Pre-test	28	43.1429	19.05492	27	2.771	5.466
Post test	28	59.8214	14.80003			

The calculated t-test, $t_{calculated} = 5.466 > t_{critical} = 2.771$. These results indicate that we *reject* the null hypothesis and conclude that the difference between the mean scores of the experimental group in the pre-test and post-test is statistically significant. The mean score of the experimental group in the post-test (mean = 59.8214) is significantly higher than the mean score of the same group in the pre-test (mean = 43.1429). Such differences in the mean scores is attributable to the treatment that the learners were exposed to.

5.4.7 Post-test scores of control versus experimental group

The post-test results of the control and experimental groups was also compared. The experimental groups scored higher than the control groups in the post-tests (see Figure 5.17).

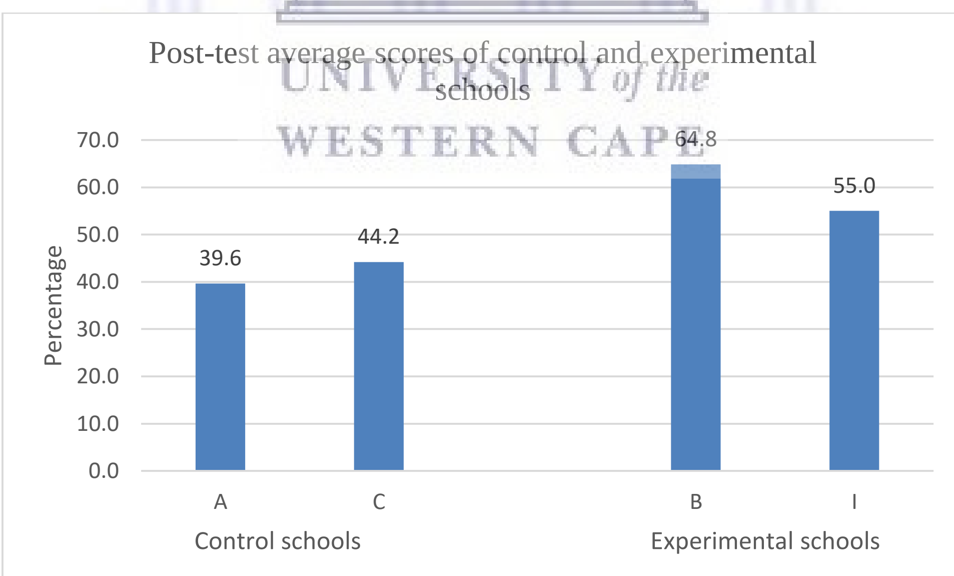


Figure 5.17. The bar chart showing the post-test mean scores of the control and experimental schools.

To further compare the scores of the experimental and control groups in the post-test, box and whisker plots are used.

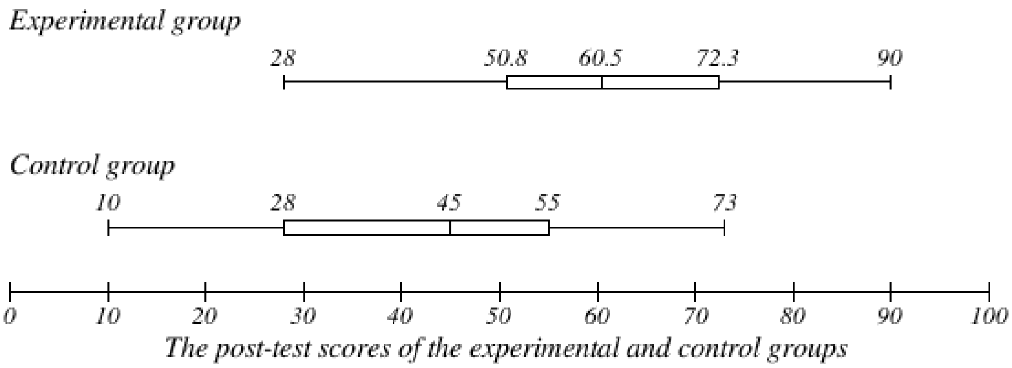


Figure 5.18. The box and whisker plot showing the post-test scores of the control and experimental groups.

As it can be seen in Figure 5.18, the post-test scores of the experimental group are distributed above the scale compared to the control group post-test scores. Half of the control group scored below 45% (median) compared to three quarters of the experimental group who scored above 50.8% (lower quartile). To test whether such differences were statistically different, the following sub-hypotheses were formulated:

H_o: There is no significant difference between the post-test mean scores of the control and experimental group.

H_1: There is a significant difference between the post-test mean scores of the control and experimental group

The independent t-test was calculated to test for significance in the differences between the mean scores of the control groups and the experimental groups and results are recorded in Table 5.11.

Table 5.11. The independent t-test results regarding the post-test scores for the control and experimental groups.

Group	n	Mean	Std. Deviation	Std. Error Mean	df	$t_{critical}$	$t_{calculated}$
Experimental	28	59.8214	14.80003	2.79694	57	2.704	3.741
Control	31	44.2258	16.98766	3.05107			

The independent t-test result shows the calculated value, $t_{calculated} = 3.741 > t_{critical} = 2.704$, *rejecting* the null hypothesis. This indicates that the difference between the mean scores of the control and experimental group in the post-test are statistically significant, with the experimental group having performed better than the control group.

5.5 DISCUSSION OF FINDINGS

This section discusses the findings of the study in its quest to generate meanings to answer the research questions. The section begins with a discussion of qualitative data presented in section 5.3, and then the discussion of the quantitative data presented in section 5.4.

5.5.1 Discussion of the qualitative results

This section discusses the qualitative data presented in section 5.3. It begins with a discussion of strategies used by teachers and learners in the senior primary schools, followed by a discussion of the teachers' perceptions on the integration of mental arithmetic in the mathematics curriculum and concludes with a discussion on the need for development of mental arithmetic in primary schools in Namibia.

5.5.1.1 **Strategies used by teachers and learners in senior primary schools**

The study, in its critical exploratory nature, sought to uncover the strategies used by teachers and learners to solve problems in the senior primary schools in the Oshana region – Namibia. The findings discussed in this sub-section were obtained from ten schools via classroom observations and provides answers to the first research sub-question of this study:

1. What strategies do primary school mathematics teachers and learners use in the mathematics classrooms?

5.5.1.1.1 Teachers' practices

The senior primary mathematics teachers mainly practiced standard algorithms. The teachers' examples were solved using the standard algorithms and the teachers recommended these methods to their learners. The teaching approaches seemed to be transmitting mathematical skills as enshrined in curriculum guides without considering learners' ideas of preferred methods of learning. As Khuzwayo (2018) would say, this absence of flexibility in the classrooms "denies any creative work" with the learners (p. 178). It is thus arguable that such classrooms could have limited the development of learners' consciousness.

When learners in Teacher A's class failed to get the percentage loss as they could not divide $\frac{4400}{500}$ using long division, the teacher seemed to have concluded that the learners have a problem with *percentage profit/loss* topic. However, learners failed because they could not perform the long division algorithm. Had the teacher exercised alternative approaches such as simplifying using the highest common factor [HCF] to divide through numerator and denominator, the problem could have been alternatively solved as: $\frac{4400}{500} = \frac{44}{5} = \frac{40}{5} + \frac{4}{5} = 8\frac{4}{5}\%$. The concept of factors, equivalent fractions and simplifying need be upheld. Varol and Farran (2007) argues that teachers should minimise the time spent on standard algorithms and begin explicit exploration of mental algorithms to enhance understanding of mathematical applications.

The teachers insisted that learners make use of the standard algorithms even when different approaches would have yielded results much easier and faster. These approaches used are described by Tabakamulamu (2010, p. 231) as being "inflexible and undemocratic" in that the teachers seemingly forced learners to follow precisely other people's mathematical knowledge as found in the textbooks.

Sometimes teachers gave unreasonable answers to examples on the chalkboard. For example, in Figure 5.8 one can see solutions of a teacher after dividing 64 by 4 and getting the answer 1.6. It appears that the teacher did not evaluate the numerical sense in his answer but performed the problem following the 'rules' without reasoning. An alternative approach to dividing 64 by 4 is the 'distribution of division over addition' as:

$$\frac{64}{4} = \frac{40}{4} + \frac{20}{4} + \frac{4}{4} = 10 + 5 + 1 = 16 .$$

A second example of an unreasonable answer was given by Teacher H. He was working out the actual profit given that the cost price = N$80 and profit was 30%. Teacher H worked out the problem as:

$$\frac{13\cancel{0}}{10\cancel{0}} \times 8\cancel{0} = 13 \times 8 = 164.$$

The teacher here reasons that 164 is 30% of 80. This unreasonable answer from the teacher leaves questions about the number sense level of primary school teachers (Courtney-Clarke, 2012; Naukushu, 2016). The critical mathematics education theory moves for an exploration of aspects within education which may have been taken for granted (Heuvel-Panhuizen, 2010). The education system seems to have taken it for granted that the number sense level of teachers is sufficient, an aspect that needs to be addressed in order to develop learners into emancipated critical thinkers.

5.5.1.1.2 Teachers' preparations

Some teachers did not have lesson plans as they carried only a chalk and duster to the classroom. These teachers came up with examples instantly, without prior investigation. Most of these examples had language and structural flaws. For example, one of the problems given by Teacher G read as follows:

> Your father has N$70 in his wallet. Suppose you want N$42 dollars to go to Oshakati (from Ondangwa), how much should your father give you in percentages?

The structure of the question is not correct and may mislead learners. Learners failed to get the solution to the given problem which may be attributed to the language and question structure. The teacher seemed not prepared as he approached to solve the question:

> 'Problems like this should be solved via trial and error. Take several numbers and find the one that gives you enough amount.'

He then solved the problem as: $\frac{60}{100} \times 70 = N\$42.00$

Answer: 60%

The teacher started with the required answer, 60%. He said that was 'trial and error', which is not correct as he did not do 'trial and error'. Given that the needed amount is N$42, learners should work out N$42 as a percentage of N$70. That is: $\frac{42}{70} \times 100 = \frac{\cancel{42}^{6}}{\cancel{70}} \times \cancel{100}^{10} = 60\%$.

As Erdem and Gürbüz (2016) put it, the standard algorithms lead to an inability to employ mathematical reasoning, which involves high-level thinking while performing a mathematical calculation. Therefore, it is necessary that teachers and learners practice mental computations to make sure they can practice mathematics sufficiently.

5.5.1.1.3 Learners' practices

Learners seem to lack procedural fluency with the standard algorithms. They got stuck following the provided series of steps of the standard algorithms. Most of the times learners resorted to the use of fingers, drawing tallies on rough papers as it could be seen in Figure 5.4. Similar results were found by Heirdsfield and Cooper (2004) as well as Siegler and Campbell, (1989) all cited in Varol and Farran (2007) where learners always resorted to the use of fingers as they could not perform the taught standard algorithms. Resorting to the use of fingers and tallies seems to suggest that learners are not comfortable performing the standard algorithms and would prefer a different approach to computing problems. The CME pedagogy would thus recommend that teachers expose learners to various computation strategies so that learners have choices to approach mathematical problems from.

Learners gave unreasonable answers on several occasions. For example, Figure 5.7, presented the work of a learner who divided 50 by 100 and got answer of 2. It seems the learners memorise steps of standard algorithms and follow these without logic. This weakness had also been observed in other studies (for example Varol & Farran, 2007; Erdem & Gürbüz, 2016) on mental computations in elementary schools where it was found that standard algorithms encourage learners to follow different steps without thinking about what they are doing. Erdem and Gürbüz (2016) reports that regular work with mental computation contributes to the development of learners' strategies, learners' reasoning, critical skills, number sense and operation sense. Teachers, therefore, need to practice mental computation strategies and provide learners with various computation alternatives that stimulates thinking prior to attempting a problem.

Learners were not asked to explain how they arrived at answers. For example, the learner whose work is in Figure 5.2, who multiplied 12×32 and got a product of 432 was not asked to explain how he arrived to this product. Instead, the teacher only said "the correct answer is not 432 but 384, now go on to your long division". It hence leads to a question of whether the strategies used in the mathematics classrooms are developing numeracy and whether teachers paid attention on the procedures or just to the end products. Varol and Farran (2007) argue that the steps taken by an individual to solve a problem give a clue on whether his or her mathematical reasoning has developed. It can hence be argued here that the senior primary learners lack mathematical reasoning.

Studies (for example Heirdsfield & Lamb, 2005; Morgan, 2008; Varol & Farran, 2007) revealed that several mental arithmetic strategies such as ‘breaking and bridging’ and ‘decomposing’ were commonly practiced by learners in primary school. The findings of this study are different as it appeared that mental arithmetic strategies were not practiced in senior primary schools as learners were restricted to standard algorithms. It hence suggests that there exist gaps in the mathematical practices of Namibian primary school learners and their age mates in the rest of the world. These gaps may be contributing factors to the fact that Namibia limbs behind on numeracy levels (Shigwedha et al., 2017).

There were no efforts by teachers to expose learners to several methods, other than standard algorithms, which may give learners a variety of options when it comes to computations. Although, the Namibian mathematics curriculum expects senior primary teachers to place emphasis on the mental arithmetic strategies to develop the learners’ awareness of numbers and number sense (MoEAC, 2016), the teachers felt that the curriculum is less prescriptive on the need to apply these strategies across all topics in the curriculum. The development of mental arithmetic strategy is necessary as learners will be flexible at operations well aware there exists more than one way to arrive at the solution to a problem (Swan & Sparrow, 2001). The results also illustrate how learners faced problems with long division among other standard algorithms. In these cases, a teacher is likely to conclude that learners have problems with ‘percentages/money topics’ while they are failing due to a standard algorithm strategy they have not mastered well (Graven et al., 2013). The results of this study agree with the recommendations by Graven et al. (2013) that in order to enhance numeracy, instructions and assessment should focus on the development of strategies and not just end results. The next section discusses perceptions of teachers with regard to mental arithmetic.

5.5.1.2 Teachers’ perspectives on the integration of mental arithmetic strategies in the schools

This subsection discusses the views of teachers on mental arithmetic and its importance in the senior primary mathematics curriculum. The sub-section provides answers to the second research sub-question of the study;

2. From teachers’ perspectives, why should the development of mental arithmetic strategies be (or not) emphasized in schools?

5.5.1.2.1 Teachers' interpretation of mental arithmetic

During individual interviews with the teachers, the researcher asked teachers to define the concept mental arithmetic in order to determine how well teachers understood the concept. The teachers' definitions of the mental arithmetic concept are summarised in Table 5.4 by listing the frequent phrases uttered by teachers. This sections discusses these definitions by looking at the frequent phrases.

Mental arithmetic as recalling of facts

Two of the ten teachers stated that mental arithmetic is concerned with the recalling of facts or memorising short facts. The other utterance added by one of the two teachers was '*the methods used to memorise'*. That suggests these teachers do not understand the concept mental arithmetic strategies yet expected to develop these strategies in their classrooms. McIntosh (2005) advocates for an emphasis on good knowledge on number facts and strategies for efficient mental computation as opposed to demands on short-term memories. Hence primary school teachers need to be aware that mental arithmetic strategies do not necessarily entail memorisation of facts. Erdem and Gürbüz (2016) advices that mental arithmetic does not amount to operations 'in the mind' but doing operation 'with the mind'. Hence the focus should shift from the recalling of short facts to strategies employed in generating the answers. Learners making use of mental strategies are working mathematically and thinking about numbers rather than remembering procedures and facts (Swan & Sparrow, 2001). The CME pedagogy would thus recommend that senior primary school teachers should move away from the notion of recalling basic facts to the development of the strategies of computing with the mind.

Mental arithmetic as development of strategies

The second part of mental arithmetic has main focus on the development of strategies of computing. Only two teachers mentioned this feature of mental arithmetic. Teacher I went on further to mention that mental arithmetic is about '*mental power to carry operations*'. This should be the focus of mental arithmetic in the schools (Swan & Sparrow, 2001). Teachers need to make sure that learners are exposed to rich and various mathematical operations in order to develop flexible ways to compute operations. However, only two teachers view mental arithmetic from the development of strategies perspective. It therefore leaves a question as to how teachers focus on developing strategies when they seem not to be aware of this important main component of mental arithmetic.

Mental arithmetic as working without paper and pencil

There were two teachers who stated that the learners do not use pencil and paper during mental arithmetic. This view is contrary to the practice of mental arithmetic with focus on developing strategies. Learners should be allowed to use pencil and paper to record a few steps as they construct own ways of solving problems (Callingham, 2005). Teacher C went on to further say that mental arithmetic means learners are not using calculators while Teacher D said he was not sure as to what the concept entailed. Teacher C was right to eliminate the calculator usage as calculators remove the computation process from learners and for that reason Namibian learners in primary schools do not use calculators. Teacher D indicated that he was not sure as to what mental arithmetic entails. Although it was only Teacher D who indicated not sure as to what the mental arithmetic concept entails, the definitions given by other teachers also suggests the concept is not fully understood by teachers. It is these sorts of aspects in the education system that a critical mathematics education approach to research intended to uncover (Heuvel-Panhuizen, 2010). If teachers are to develop mental arithmetic strategies with the aim of developing numeracy, then they need to be clear about of the concept.

Own methods

Three teachers mentioned that mental arithmetic has to do with learners using own methods. These teachers, however, did not mention the part of enhancing learners' development of own strategies as the main focus of mental arithmetic. This takes us to the question as to who should develop learners' mental arithmetic which was posed by Swan and Sparrow (2001). Are learners supposed to be left alone to explore and develop these strategies or are teachers supposed to expose learners to rich activities and guide the development of mental computation strategies? Swan and Sparrow (2001) as well as McIntosh (2005) assert that teachers should teach mental computation with an emphasis on how answers are obtained. The teachers need to first learn about the strategies that learners use, develop these strategies and enhance the use of mental strategies appropriately.

Teacher E further mentioned that learners 'work on their own, know of place value and expanded notation'. This teacher understands mental arithmetic from the same perspectives as McIntosh

(2005) who argues that learners work out strategies from their own understanding of place values creating various ways of solving problems. Classroom instruction should therefore encourage sharing of methods via collaborations among learners.

5.5.1.2.2 Teachers' views on why mental arithmetic strategies should (or not) be emphasised in schools

All the participants, but Teacher H, are of the view that mental arithmetic, if well-developed, provides various problem solving approaches, enhances learners' academic strength and develops numeracy. Despite the high practice of standard algorithms, teachers seem aware that learners need mental development and an alternative to standard algorithms may help achieve that. Teachers also pointed out possibilities that many teachers may not be aware of what mental arithmetic concept fully entails. This has an effect on the curriculum implementation as teachers won't comfortably develop the concept they do not understand. Furthermore, the teachers indicated that the low performance of learners is attributable to poor development of mental arithmetic strategies in the schools as the curriculum has focus on high drills of standard algorithms. These revelations seem to suggest that a change in the teaching approaches is needed if schools are to realise an improved level of numeracy.

Teacher H, on the other hand believes that the world is now at an advanced stage where calculations are carried out with devices and therefore sees no purpose of drilling on mental computation strategies. He argues that classroom strategies do not link to real life after school and hence curriculum needs to be changed to allow learners to use calculators from early school grades. The Namibian senior primary mathematics curriculum call on teachers to put an emphasis on nurturing leaners' mental arithmetic strategies in order to develop their awareness of number and number sense (Namibia. Ministry of Education [MoE], 2010). The aim of developing mental computation strategies is to develop flexible thinkers (Swan & Sparrow, 2001). An approach to teaching should embrace the development of learners as critical thinkers prepared to contribute to the growth of society. Learners need flexible approaches to solving problems and an insight into the properties of number systems. This level of development may not be achieved if learners are left to use calculators as suggested by Teacher H. Nevertheless, the aim of this CME study was to uncover perceptions of this nature.

5.5.1.2.3 Curriculum concerns

Curriculum concerns were raised in both individual interviews and the focused group interviews. The first concern raised is that the mental arithmetic appears as a topic taught in the first trimester of the year and thereafter no reference to these methods is made. The development of *mathemacy* occurs overtime with each mathematics lesson prepared focusing on mental arithmetic strategies as a way of teaching and not as a topic to be taught (Kuldas, Sinnakaudan, Hashim, & Ghazali, 2017). Moreover, the examples in the prescribed text books are solved using standard algorithms which seems to limit the teachers' practices.

Although the Namibian senior primary mathematics syllabus calls on teachers to develop mental arithmetic strategies in their classrooms, it appears the syllabus content is not clear and teachers are not always aware of how to bring in the mental arithmetic strategies. The syllabus has prescribed that by the end of the senior primary phase learners should be able to "demonstrate an understanding of numbers and be able to use mental arithmetic and paper-and-pencil methods sensibly and appropriately" (MoEAC, 2016, p. 4). However, the mental arithmetic strategies are taught as a section under the topic on Numbers and as teachers move on to other topics, they practice the "formal algorithms" (p. 23). The layout of the syllabus and the description of standard algorithms as 'formal' may thus be confusing to teachers. The teachers might thus take it that the 'formal algorithms' are more important and hence pay less attention on mental arithmetic strategies.

Similar problems with regard to curriculum layout were found in Malaysia where curriculum instructions and textbook activities were heavily geared towards the memorising of specific algorithms rather than the recognition of why and how the strategies work (Kuldas et al., 2017). The schools fail to develop *mathemacy* when teachers utilise the syllabus and textbooks overemphasising standard algorithm rules and only give positive feedback to learners who have strictly adhered to these rules. Kuldas et al. (2017) thus recommended that the teachers should use textbooks with care not to limit learners' thinking and practices but encourage them to explore alternatives of solving problems.

A second concern on the curriculum structure was the amount of content to cover in limited time frame. The participants indicated having experienced difficulties with completing the syllabus on time and hence they do not exercise a variety of strategies in their classes. Teachers also believed that mental arithmetic strategies take a lot of time. This points to another misconception that teachers have about mental arithmetic strategies. Contrary to this belief, mental arithmetic strategies allow learners to effectively compute and arrive to reasonable answers faster compared to when using standard algorithms.

A third concern on the senior primary school mathematics curriculum structure is that the competencies in the syllabus appears confusing to teachers. The participants expressed difficulties with interpreting the objectives suggesting that the teachers are unaware as to how to incorporate mental arithmetic strategies across all topics of the syllabus. It appears the syllabus is silent on how the mental arithmetic approaches may be applied across the entire curriculum. The teachers also indicated lack of resources such as textbooks with clear examples on mental arithmetic strategies. The remark on text books concurs with Volmink (1994) cited by Khuzwayo (2018) that most textbooks are written in a style which emphasises drill and practice on routine un-contextualised exercises. These books are thus likely to deny teachers and learners chances of flexible practice of alternative computation strategies.

Additionally, the participants indicated that mental arithmetic strategies needs to be practiced as early as the junior primary school phase. They believed it's too late to start with these methods in the senior primary phase. The same call was made by Kuldas et al. (2017) that the teaching and learning of mental arithmetic strategies should be developed early to foster their number sense and enable learners to take on advanced mathematics without difficulties.

Lastly, the teachers indicated lack of teaching skills on how to teach mental arithmetic strategies as the concept was not taught when they were learners and it also did not feature in the teacher training programmes. This indication by teachers gives weight to the phrase by (MoEAC, 2016, p. 37) that "nobody understands how it works". They hence lack awareness of what the concept entails and hence little is done with regard to the development of mental arithmetic strategies. The same point was already mentioned in this section and this is what was termed the Naukushu's Vicious Cycle of Innumeracy [NVCI] (Naukushu, 2011).

Given the raised curriculum concerns, the redesigning of the senior primary mathematics curriculum in order to improve the clarity of objectives, make less reference to standard algorithms and integrate mental arithmetic strategies across topics of the syllabus can be a way to develop learners' *mathemacy*. If designed properly, textbooks may be a source of knowledge for learners and hence schools need to be using books which exercises a freedom of methods and not only standard algorithms.

5.5.1.3 **Development of mental arithmetic strategies**

This section discusses findings on how to enhance the development of mental arithmetic strategies in the primary schools. The discussion in this section provides answers to the third research sub-question;

3. How can the development of mental arithmetic strategies be enhanced in primary schools?

5.5.1.3.1 Teachers' suggestions on developing mental arithmetic

The teachers suggested that the syllabus be re-structured so that the mental arithmetic strategies are distributed across the entire syllabus. This will be against the current structure where mental arithmetic is taught as a topic and teachers do not make reference to the use of these strategies as they move on to other topics in the syllabus. That a suggestion to restructure the curriculum to accommodate the thorough practice of mental arithmetic came from the teachers fits in well with the transformative worldview ideology of the critical mathematics education theory (Cohen et al., 2007) as the ideas of how to change the critical aspects are supposed to come from the society whose change is meant for.

Secondly, participants suggested that to develop mental arithmetic strategies, the teachers need to thoroughly teach and practice these strategies in their classrooms. Mental computation strategies play a significant role on teaching learners how numbers work, how to make decisions about procedures, and how to create different strategies to solve mathematics problems (Varol & Farran, 2007). Thus senior primary school mathematics teachers need to develop mental arithmetic strategies as instructions on mental computations can lead to both increased understanding of number and flexibility in working with numbers (Hiebert & Wearne, 1996 cited in Varol & Farran, 2007).

Lastly, the participants suggested that a training on mental arithmetic strategies be conducted for teachers to sensitise them in order for them to deliver as per expectations of the curriculum. When linking this call to how teachers interpreted the concept mental arithmetic as discussed earlier, it gives an indication that an intervention needs to be carried out as it appears some teachers are not aware of what mental arithmetic entails.

5.5.1.3.2 Need for an intervention

The lack of mental arithmetic strategies in senior primary schools point to a need of an intervention to enlighten the teachers and enhance the development of mental strategies. The teachers gave some answers with incorrect mathematical reasoning. This points to a need for teachers to be introduced to mental arithmetic strategies so that they can engage thoroughly rather than following series of rules without reasoning as it seems to be the case with standard algorithms. Additionally, the teachers indicated that their teaching approaches were being informed by how they were taught during their times as school learners and during teacher training.

Naukushu (2016, p. 6) believes that the Namibian education system seems to be trapped in a three stage cycle of numerical deficiency which he termed "Naukushu's Vicious Cycle of Innumeracy (NVCI)". It is against these grounds that an intervention into developing teachers' mental arithmetic skills are needed with hopes that the teachers may be empowered to develop mathemacy in the primary schools. To be an empowered teacher implies the ability to use mathematics to address both personal challenges and those that affect the learners. It further means a teacher possesses pedagogical knowledge, mathematical content knowledge and self-confidence in facilitating teaching and learning (Lekoko et al., 2018).

5.5.1.3.3 Teachers' views on mental arithmetic after intervention

The teacher participants indicated that they have discovered the existence of various mental arithmetic strategies that learners can make use of in everyday computations to develop *mathemacy*. They indicated that they learned how mental computation strategies may help ease the classroom practices and help improve a democratic exercise in the schools as opposed to the current practice where learners are restricted to the use of specific standard algorithms which learners seems to be experiencing difficulties with. Participants agreed they were not aware of faster mental arithmetic strategies. The participants also discussed how standard algorithms are performed by both teachers and learners as a series of steps with incorrect reasoning and acknowledged how the practice of mental arithmetic strategies may improve the situation as in agreement with (Erdem & Gürbüz, 2016).

As a threat to effective implementation of mental arithmetic strategies in their class rooms, the participants indicated that teaching is mainly assessment driven and they will face problems with the examiners. The same sentiments were raised by Tutak et al. (2011) that resistance of change as advocated by CME stands a high chance as change of structures may be regarded by systems as merely critical of the status quo. The senior primary examinations are regionally set and examiners restrict learners to the use of standard algorithms. The marking schemes often indicate how marks should be allocated for each of the steps along standard algorithms which influenced the focus of classroom practices.

All the teachers who participated in the intervention programme indicated that they will integrate mental arithmetic strategies in their mathematics classrooms instructions. This is evident in their own expressions that the intervention workshop should have taken place earlier than it did.

The teacher participants said that their classes will now be more democratic and exercise a variety of strategies of computations. This is what Stinson, Bidwell, Jett, Powell, & Thurman (2006) refers to as a move away from traditions of transmitting knowledge to critical mathematics pedagogies.

5.5.2 Discussion of the quantitative data

This section discusses the quantitative results collected through psychometric tests written by the Grade 7 learners in the schools.

5.5.2.1 **Psychometric tests (Pre-test and post-test)**

The psychometric test items required learners to demonstrate their abilities to deal with numbers and have free manipulation via strategies of their choice and then arrive to answers. The results from the test show that seven out of the ten schools scored an average mean below 50%. This is a low level of performance by the Grade 7 learners. The results concur with results of previous studies (Mutuku, 2015; Shigwedha et al., 2017; Spaull, 2011) that Namibia has a low level of functional numeracy. In Namibia, Grade 7 is the highest grade in the senior primary phase and learners are expected to possess mathematical skills to allow them to take on secondary school level mathematics. Results seem to suggest that learners exit the senior primary phase without the required level of numeracy to take on mathematics in the secondary school phase.

5.5.2.2 Pre-test scores - control versus experimental groups

The pre-test average mean score for the control group was 42.9677% and the pre-test average mean score for the experimental group was 43.1429%. There was a difference of 0.1752%. This indicates there was a very small difference between these mean scores of the control and experimental group in the pre-test. An independent t-test was calculated to determine if the difference was statistically significant. The t-test calculations yielded $t_{calculated} = 0.32 < t_{critical} = 2.704$. The test fails to reject the null hypothesis, H_o, and concluded that there was no significant difference between the mean scores of the control group and the experimental group in the pre-test. This suggests that the groups were at the same level of numeracy prior to the intervention.

5.5.2.3 Control group – pre-test versus post-test scores

The pre-test mean scores and post-test mean scores within the control group was compared and t-test calculated to determine whether a significant difference existed. The non-independent t-test results showed $t_{calculated} = 0.616 < t_{critical} = 2.750$, failing to reject the null hypothesis and suggesting that there was no significant difference in the pre-test and post-test mean scores of the control group. The control group was not exposed to any treatment and hence, no change in performance was expected since the group was not exposed to any treatment. The box and whisker plots (see Figure 5.17) illustrated how close the control group scores were in the two tests.

5.5.2.4 Experimental group – pre-test versus post-test

The pre-test and post-test scores of the experimental group were compared using the box and whisker plots (see Figure 5.17) which illustrated a better performance of the experimental group in the post-test. The pre-test mean score was 43.1429 and the post-test mean score was 59.8214. The pre-test mean score and post-test mean score were compared using non-independent t-test to determine if the intervention had any effect on the performance of the experimental group. The $t_{calculated} = 5.466 > t_{critical} = 2.771$, rejecting the null hypothesis and concluded that there exists a significant difference between the pre-test and post-test mean scores within the experimental group. This suggests that the treatment (teaching through mental arithmetic strategies) given to the experimental group resulted in a statistically significant improved performance of the group.

5.5.2.5 Post-test scores of control versus experimental group

The post-test scores of the control and experimental groups were also compared to determine differences. Figure 5.18 illustrates the differences in performance using a bar chart. More comparisons were shown using box and whisker plots (see Figure 5.19) where the experimental group performed higher than the control group in the post-test scores. Half of the control group scored below 45% in the post-test compared to three quarters of the experimental group who scored above 50.8%.

The post-test mean scores of the control and experimental groups were compared to using the independent t-test to determine whether the differences were statistically significant. The $t_{calculated} = 3.741 > t_{critical} = 2.704$, rejecting the null hypothesis. This indicates that there existed a significant difference between the post-test mean scores of the control and experimental group with the learners in the experimental group having performed significantly better than their counterparts in the control group. This suggests that the critical intervention informed by the *mathemacy* concept seem to have improved performance in mathematics.

5.5.2.6 Results of the main hypothesis

The results presented in the subsections builds up as the test results to the main hypothesis as follows:

H_o: There is no significant difference in the performance of the learners taught mental arithmetic strategies and those who were not.

H_1: There is a significant difference in the performance of the learners taught mental arithmetic strategies and those who were not.

The results in Table 5.5, Table 5.6, Table 5.7 and Table 5.8 indicates that the study *rejected* the null hypothesis and concluded that there is a significant difference between performance of the learners taught using mental arithmetic strategies and those who were not. This indicates that at $\alpha = 0.05$ and degree of freedom, df = 57, the experimental group scored significantly higher than the control group. Since the two groups were at the same level before intervention, the differences in performance during the post-test may be attributed to the treatment given to the experimental group. This results concurs with the findings by other studies (e.g.Yang, Hsu & Huang, 2004;

Yang & Li, 2008; Ho & Lowrie, 2014 all cited in Kuldas et al., 2017) where it was found in Taiwan and Singapore that the learners who followed mathematics activities and instructions for mental computation strategies demonstrated better performance than those who followed the standard algorithms through teaching with little emphasis on mental development.

5.5.3 Triangulation of data

The triangulation section attempts to map out the richness of the data sourced by different techniques via the observations, interviews, focused group interviews and tests as instruments used in the present study. Triangulation is a powerful way of demonstrating validity in a study (Cohen et al., 2007; Creswell, 2012; Gay et al., 2013) as it distorts the possibility of collected data to be a result of artefacts of one specific method of data collection. Defending the use of triangulation of data, Boring (1953) as cited in Cohen et al. (2007) argues that:

> As long as a new construct has only the single operational definition that it received at birth, it is just a construct. When it gets two alternatives operational definitions, it is beginning to be validated. When the defining operations, because of proven correlations, are many, then it becomes reified. (p. 130)

The above statement resonates with the arguments by Naukushu (2016, p. 122) where he defended his use of triangulation arguing that "hearing a statement from just one source is an anecdote; from two, a coincidence and hearing it from three makes it a trend" and its these trends that should guide the researcher to drawing valid conclusions.

The results from the observations suggests the absence of mental arithmetic strategies in the senior primary grades in the Oshana region. Teachers practiced standard algorithms and guided learners towards the same algorithms. The data obtained from individual teacher interviews suggests that most teachers do not understand what mental arithmetic strategies entail, which appears to support why teachers do not practice these strategies in their mathematics classrooms. Teachers also revealed, during both individual and focus group discussions that the lack of mental arithmetic strategies in the classroom is attributable to teachers' lack of awareness of such strategies. These revelations seem to agree with MoEAC (2016) which argues that the lack of mental arithmetic practice in the senior primary mathematics curriculum is attributed to lack of awareness as to what the mental arithmetic concept entails. These revelations, together with observed unreasonableness in the answers to problems worked out by teachers during classroom observations gave weight to a need for a critical intervention for teachers. The findings of this study are summarised in Table 5.12.

Table 5.12. Summary of the findings of the study

Research question: How is mental arithmetic being incorporated in the senior primary mathematics curriculum?		
SUB - QUESTIONS	**METHOD / INSTRUMENT**	**Results**
1. What mental arithmetic strategies do primary school mathematics teachers and learners use in the mathematics classrooms?	Classroom observations/ Field notes	• Mental arithmetic strategies missing • Teachers and learners mainly practice standard algorithms • Teachers seemed to restrict learners to the usage of standard algorithms. Undemocratic exercises of computation methods. • Some flaws in calculations suggest lack of reasoning in both learners and teachers
2. From teachers' perspectives, why should the development of mental arithmetic strategies be (or not) emphasized in schools?	Individual teacher interviews /Interview schedule	• Some teachers not aware of what mental arithmetic entails. • Most of the teachers feel mental computation strategies need to be emphasised more. • Some teachers do not support integration of mental computation strategies as they believe such methods have no applications to real life situations. • Most teachers pointed out several concerns with current curriculum as main impediments in the development of mental computation strategies. Such are lack of resources, unclear syllabus objectives, and the preparedness of teachers.
3. How can the development of mental arithmetic strategies be enhanced in primary schools? *Includes hypothesis testing:* H_o: There is no significant difference in the performance of the learners taught mental	Intervention for teachers – mental arithmetic strategies workshops Mental arithmetic strategies manual Focus group discussion guide	• A need for 'refresher workshops' for teachers. • A need to restructure syllabus to make objective clear and to incorporate mental computation strategies thoroughly. • A need for classroom interactions to be more democratic and allow learners' voices. • The intervention exposed teachers to new computation strategies and changed their perceptions of what mental arithmetic strategies entails. • The participants in the intervention indicated they were going to change their classroom instructions and incorporate democratic practice of various computational strategies.
	Intervention for learners – quasi	• Pre-test yielded low scores with average 46.6%.

arithmetic strategies and those who were not.	experimental to test the hypothesis Mental arithmetic strategies manual Pre-test & Post-test	• The t-test results rejected the null hypothesis and concluded that the experimental groups performed significantly better than the control groups. • The statistically significant difference between the experimental and control groups was attributed to the treatment.

5.5.4 Summary of discussions

The study uncovered that the teachers and learners only practice standard algorithms in their classrooms. In most of the cases teachers restricted learners to the use of standard algorithms and no alternative methods were allowed. Although, on some occasions such algorithms resulted in unreasonable answers, the teachers did not seem to not have noticed it.

Most of the teachers seemed to lack awareness of what the concept mental arithmetic entails. These teachers are nonetheless serving on a fulltime basis and are expected to effectively implement the curriculum to ensure good development of numeracy. There is a need for an intervention to train teachers on metal arithmetic strategies to enhance the development of mental arithmetic strategies at the senior primary school phase. Teachers pointed to curriculum concerns such as lack of clear text books with examples solved via mental arithmetic strategies, unclear syllabus objectives, too much content to cover in a limited time space and examination-driven teaching as main issues hindering the development of mental arithmetic. Other challenges to the development of mental arithmetic were teachers' readiness as some teachers have never been exposed to mental arithmetic strategies in their schooling years and/or during teacher training programmes.

The intervention exposed teachers to new computation strategies and changed their perceptions of what mental arithmetic strategies entails. The participants in the intervention indicated they were going to change their classroom instructions and incorporate democratic practice of various computational strategies. The participants also suggested that the intervention be rolled out to more teachers to help develop numeracy. This seem to be a positive outcome of the intervention.

An intervention for learners found that the learners who were exposed to mental arithmetic strategies performed significantly better than the learners who continued to do mathematics in the traditional algorithms. It is thus recommendable that a critical mathematics approach to mental arithmetic in schools may enhance *mathemacy*. The next chapter presents the summary, conclusion and recommendations of the study.

CHAPTER SIX: SUMMARY, CONCLUSION AND RECOMMENDATIONS

6.1 Introduction

This chapter presents the summary, conclusion and recommendations of this study. It begins with a summary of the chapters and then presents the conclusion by discussing the answers to the research questions of the study. The recommendations were then drawn from the conclusion, research gaps of this study identified and avenues for further research pointed out.

6.2 Summary of the study

This section presents the summary of the study:

Introduction: This study explored the incorporation of mental arithmetic in primary schools by looking at the computation strategies used by teachers and learners during classroom mathematics sessions at senior primary grades in the Oshana region – northern Namibia. Namibia has recorded unimpressive results in mathematics across all grades since independence despites numerous educational reforms to improve the learners' performance (National Institute for Educational Development [NIED], 2010). The senior primary learners have been performing below average in the mathematics SATs (Mutuku, 2015; Shigwedha et al., 2017; Spaull, 2011). The low level of learners' numeracy persists despites numeral reforms that attempted to address the situation. As a result of the glooming picture in Namibian education painted above, this study therefore an attempted to address the following concerns: What mental arithmetic strategies are used by primary school mathematics teachers and learners in their classrooms? What are the views of mathematics teachers on the inclusion of mental arithmetic? How may mental arithmetic be developed in the primary schools?

Theoretical framework: The study was informed by the Critical Mathematics Education philosophy as the theoretical framework. Critical mathematics education is the education that addresses the conflicts in society by uncovering inequalities and oppression of whatever kind (Skovsmose, 1994) having in mind that the learning of mathematics can empower or disempower the lives of the individual learners. Addressing the critical role of mathematics in society entails an understanding of the risks and uncertainties that mathematics and societal progress conveys. Learners should be presented with situations in which mathematics may format the way they understand and behave in reality.

The critical mathematics education, through its notions, challenges the role of mathematical consciousness, the rationale of mathematics education and questions the curriculum implementation in terms of classroom practices (Naukushu, 2016). Consequently, in the context of this study it could be suggested that the development of mental computation strategies should be fully integrated in the senior primary mathematics curriculum to foster the growth of learners. CME requires an in depth application by critical teachers who consistently critique self and their learners in an ongoing process of critical consciousness.

The researcher hoped that the development of mental arithmetic strategies of teachers could be best achieved by adopting the *mathemacy* and *social justice* as theoretical lenses to guide the enquiry. The *mathemacy* and *social justice* concepts together aim for developing mathematical power via deducing mathematical generalisations, constructing creative solution strategies to problems, and perceiving mathematics as a tool for socio-political critique (Stinson et al., 2012). The use of these two theoretical lenses together can be used in developing mental arithmetic strategies of teachers so that they may enhance numeracy of their learners. The researcher is of the view that a *mathemacy* and *social justice* driven approach, complemented by the Realistic Mathematics Education and ethnomathematics theories, may enlighten teachers that mathematics is not a series of detached, rote rules to be memorised and regurgitated, but as a powerful and significant analytical tool for understanding complex and real world aspects. Such an understanding may enhance thorough development of mental computation strategies and ultimately numeracy in the primary schools.

Literature review: The literatures reviewed defined mental arithmetic as the development, selection and execution of computation strategies. Mental arithmetic strategies seems to enhance academic performance of learners. These strategies may be developed with the aids of pencil and paper. Furthermore, a number of studies and reports cited low performance at primary schools and highlighted possible reasons for poor performance in Mathematics. Reasons for poor mathematics performance as confirmed by these reports are: teachers' competencies in mastering the curriculum content, unqualified teachers as a result of shortage of qualified mathematic teachers, availability of teaching materials, methods of presentation, learning environment, lesson preparation, gender, and motivation to learn. Namibian learners were recorded to have performed the lowest in mathematics comparing to the performance of the same group of learners in the southern African countries. This study, hence, sought to explore how teachers incorporate mental arithmetic strategies in primary school mathematics and determine the impacts these strategies will have on learners' academic achievement.

Design and methodology: Embedded in the critical mathematics education theory, the study used a pragmatic paradigm in its efforts to explore and understand the classroom practices of the senior primary teachers and learners. The qualitative paradigm involved the collection, analysis and interpretation of comprehensive narrative and visual data to gain insights into a particular phenomenon of interest (Gay et al., 2013). From the pragmatic paradigm perspectives, all meaning is situated in a particular perspective or context, and because different people and groups often have different perspectives and contexts, the world has many meanings, none of which is necessarily more valid or true than the other. The efforts to understand the mental arithmetic practices and perspectives of the senior primary teachers and learners required that the researcher interacted extensively with these participants, using time-intensive data collection instruments such as classroom observations.

The study conveniently sampled 10 schools in the Oshana region in northern Namibia. Senior primary school teachers and learners at these schools were observed during mathematics lessons. The computation procedures used by both teachers and learners were recorded. The study utilised a mixed methods approach in two phases. The first phase consisted of classroom observations, individual teacher interviews and a psychometric mathematics test (pre-test) for the Grade 7 learners. The second phase consisted of a critical intervention for teachers and learners. The intervention involved series of workshops that trained teachers on mental computation strategies. The workshops were followed by a focused group interview for teachers. Out of the 10 schools, four were sampled (two experimental and two control) and the learners in the experimental schools were exposed to mental arithmetic strategies. A mathematics post-test was then administered to all the four schools. The study evaluated the effectiveness of the mental arithmetic approaches on learners' performance.

Results and Discussions: The teachers and learners mostly practiced standard algorithms even when learners displayed difficulties in comprehending these strategies. In most of the cases, the teachers restricted learners to the use of standard algorithms. It was observed on several occasions that teachers and learners practiced standard algorithms without logic or reasoning. Learners, while adhering to teachers' instruction of using standard algorithms, made use of tallies/counters marked on pieces of papers to aid computation. These findings suggest that an alternative to standard algorithms is needed. The individual interviews with teachers sought to determine how teachers understood the concept mental arithmetic and their views on its incorporation in the senior primary school curriculum. Teachers had different views of what mental arithmetic is and what it entails. Some teachers believed that mental arithmetic is about memorizing numerical facts and multiplication tables. It emerged that teachers do not practice mental computation strategies as most of them are not aware of these strategies. The prescribed textbooks rigidly made use of standard algorithms. The critical mathematics education pedagogy targets to question the education systems via critical analysis of their practices in order to determine the 'taken for granted' aspects in mathematics education and address such. The study thus identified teachers' shallow understanding of mental computation strategies and hence the lack of mental arithmetic strategies in classroom practices as the aspects the education systems has not taken note of, which may have an influence on the numeracy development and ultimately the performance of learners. Hence, the study identified the need to run a critical intervention – a series of workshops to introduce teachers to mental computation strategies.

A quasi-experimental control group design was conducted to test the effects of the mental computation strategies on learners' understanding. At the end of the intervention, a focused group discussion was conducted with the teachers. The teachers who participated in the intervention expressed hope that the mental computation strategies they got exposed to during the critical intervention can be helpful to develop numeracy in the senior primary schools. The teachers pointed to discrepancies in the Namibian education system where they believe there are contents, such as mental computation strategies, which they were never taught, not in their school years and not during teacher training, and yet they are expected to teach such content in the current senior primary curriculum.

The findings from the psychometric tests suggest low level of numeracy among learners. Seven out of the 10 schools scored an average score below 50%. These results concur with earlier results recorded by the Standardised Achievement Tests [SATs] and the Southern and Eastern Africa Consortium for Monitoring Educational Quality [SACMEQ] III and IV. The post-test results saw an improvement in the scores of the learners from the experimental group. The t-tests calculated indicated that the experimental groups scored significantly better than the control groups at $\alpha = 0.05$ significance level. The next section presents the conclusion of the study.

6.3 Conclusion

The study to address the main critical question: '*How is mental arithmetic being incorporated in the senior primary mathematics curriculum?*' The following sub-questions guided the study:

1. What mental arithmetic strategies do primary school mathematics teachers and learners use in the mathematics classrooms?
2. From teachers' perspectives, why should the development of mental arithmetic strategies be (or not) emphasized in schools?
3. How can the development of mental arithmetic strategies be enhanced in primary schools?

The study further sought to determine how a critical intervention may enhance the development of mental arithmetic in the senior primary schools. A hypothesis was formulated and tested:

H_o: There is no significant difference in the performance of the learners taught mental arithmetic strategies and those who were not.

H_1: There is a significant difference in the performance of the learners taught mental arithmetic strategies and those who were not.

The summary of the main findings is presented in the following sections.

6.3.1 The strategies practiced in senior primary classrooms

The study, in its critical exploratory nature, sought to uncover the strategies used by teachers and learners to solve problems in the senior primary schools in the Oshana region – Namibia. The findings in this section were obtained from ten schools via classroom observations and provides answers to the first research sub-question of this study:

1. What strategies do primary school mathematics teachers and learners use in the mathematics classrooms?

The teachers and learners made use of standard algorithms such as column multiplications and long division. There were many incidences where teachers' and learners' solutions lacked conceptual understanding, suggesting that standard algorithms seem to be performed in the schools without logic. On many instances when learners were presented with problems, they resorted to the use of counters/tally marks drawn on rough papers to aide computation. Learners resorting to tally marks to compute suggests that they are experiencing difficulties with the standard algorithms. This is what the critical mathematics education defines as 'the taken for granted' which research in mathematics education should question and understand to make sure the education system does not in any way oppress the learners (Tutak et al., 2011).

There were no efforts by teachers to expose learners to several other methods which may have exposed them to a variety of options when it comes to computation strategies. However, the Namibian mathematics curriculum expects senior primary teachers to place emphasis on the mental arithmetic strategies to develop the learners' awareness of numbers and number sense (MoEAC, 2016). The development of mental arithmetic strategy is necessary to make sure that learners are flexible at operations and are well aware there exists more than one way to arrive at the solution to a problem (Swan & Sparrow, 2001). The results of this study agrees with the recommendations by (Graven et al., 2013) that to enhance numeracy, instructions and assessment should focus on the development of strategies and not just end results. The next section presents main findings on the perceptions of teachers with regard to mental arithmetic.

6.3.2 Teachers' perspectives on mental arithmetic

This section presents a summary of the main findings of the views of teachers on mental arithmetic and its importance in the senior primary mathematics curriculum. The section provides answers to the second research sub-question of the study;

2. From teachers' perspectives, why should the development of mental arithmetic strategies be (or not) emphasized in schools?

During individual interviews with the teachers, the researcher asked teachers to define the concept mental arithmetic in order to determine how well teachers understand the concept. The teachers' definitions of the mental arithmetic were diverse. Some teachers, for instance, defined mental arithmetic as the 'recalling of facts or memorising short facts'. McIntosh (2005) advocates for an emphasis on good knowledge on number facts and strategies for efficient mental computation as opposed demands on short-term memories. Erdem & Gürbüz (2016) advices that mental arithmetic is not doing operations 'in the head' but doing operation 'with the head'. Learners making use of mental strategies are working mathematically and thinking about numbers rather than remembering procedures and facts (Swan & Sparrow, 2001). That suggests that teachers should move away from the notion of recalling basic facts to the development of the strategies of computing with the mind. A few teachers see mental arithmetic as a focus on the development of strategies of computing. One of the teachers said that mental arithmetic is about '*mental power to carry operations*'. This should be the focus of mental arithmetic in the schools (Swan & Sparrow, 2001). Teachers need to make sure that learners develop flexible ways to compute operations. However, only a few teachers view mental arithmetic from the development of strategies perspective. It therefore leaves a question as to how teachers focus on developing strategies when they seem not aware of this important main component of mental arithmetic. There were teachers who stated that the learners do not use pencil and paper during mental arithmetic. This view is contrary to the practice of mental arithmetic with focus on developing strategies. Learners should be allowed to use pencil and paper to record a few steps as they construct own ways of solving problems (Callingham, 2005). The definitions given by many teachers suggest that the concept is not fully understood by teachers.

Most of the participants were of the view that mental arithmetic, if well-developed, may provide various problem solving approaches, enhance learners' academic strength and develop numeracy. Despites the high practice of standard algorithms, teachers seem aware that learners need mental development and an alternative to standard algorithms may help achieve that. Teachers also pointed out at possibilities that many teachers may not be aware of what mental arithmetic concept fully entails. This has an effect on the curriculum implementation. The teachers further indicated that learners' low performance is attributable to poor development of mental arithmetic strategies and high drills on standard algorithms. These revelations seem to suggest that a change in the teaching approaches is needed if schools are to realise an improved level of numeracy. The Namibian senior primary mathematics curriculum call on teachers to put an emphasis on nurturing learners' mental arithmetic strategies in order to develop their awareness of number and number sense (Namibia. Ministry of Education [MoE], 2010). The aim of developing mental computation strategies is to develop flexible thinkers (Swan & Sparrow, 2001). An approach to teaching should embrace the development of learners as critical thinkers prepared to contribute to the growth of society. Learners need flexible approaches to solving problems and an insight into the properties of number systems. The aim of this CME study was to uncover perceptions of this nature.

Curriculum concerns were raised in both individual interviews and the focused group interviews. The first concern raised is that the mental arithmetic appears as a topic taught in the first trimester of the year and thereafter no reference to these methods is made. The development of *mathemacy* occurs overtime with each mathematics lesson prepared focusing on mental arithmetic strategies as a way of teaching and not as a topic to be taught (Kuldas et al., 2017). Moreover, the examples in the prescribed text books are solved using standard algorithms which seem to limit the teachers' practices. Although the Namibian senior primary syllabus calls on teachers to develop mental arithmetic strategies in their classrooms, it appears the syllabus content is not clear and teachers are not always aware of how to bring in the mental arithmetic strategies. The syllabus has listed that by the end of the senior primary phase learners should be able to "demonstrate an understanding of numbers and be able to use mental arithmetic and paper-and-pencil methods sensibly and appropriately" (MoEAC, 2016, p. 4). However, the mental arithmetic strategies are taught as a section under the topic on Numbers and as teachers move on to other topics, they practice the "formal algorithms" (p. 23). The layout of the syllabus and the description of standard algorithms as "formal" may thus be confusing to teachers. The teachers might thus take it that the 'formal algorithms' are more important.

Similar problems with regard to curriculum layout were found in Malaysia where curriculum instructions and textbook activities were heavily geared towards the memorising of specific algorithms rather than the recognition of why and how to strategies work (Kuldas et al., 2017). The schools fail to develop *mathemacy* when teachers utilise the syllabus and textbooks overemphasising standard algorithm rules and only give positive feedback to learners who have strictly adhered to these rules. Kuldas et al. (2017) thus recommended that the teachers should use textbooks but with care not to limit learners' thinking and practices but encourage them to explore alternatives of solving problems.

A second concern on the senior primary school mathematics curriculum structure was the amount of content to cover within a limited time frame. The participants indicated having experienced difficulties with completing the syllabus on time and hence they do not exercise a variety of strategies in their classes. Teachers also believed that mental arithmetic strategies take a lot of time. This points to another misconception that teachers have about mental arithmetic strategies.

Contrary to this belief, mental arithmetic strategies allow learners to effectively compute and arrive to reasonable answers faster compared to when using standard algorithms.

A third concern on the curriculum structure is that the competencies in the syllabus appears confusing to teachers. The participants expressed difficulties with interpreting the objectives suggesting that the teachers are unaware as to how to incorporate mental arithmetic strategies across all topics of the syllabus. They also indicated a lack of resources such as textbooks with clear examples on mental arithmetic strategies. Additionally, the participants indicated that mental arithmetic strategies need to be drilled as early as the junior primary school phase. They believed it's too late to start with these methods in the senior primary phase. The same call was made by Kuldas et al. (2017) that the teaching and learning of mental arithmetic strategies should be developed early to foster their number sense and enable learners to take on advanced mathematics without difficulties.

Lastly, the teachers indicated lack of teaching skills on how to teach mental arithmetic strategies as the concept was not taught when they were learners and it also did not feature in the teacher training programmes. They hence lack awareness of what the concept entails and hence little is done with regard to the development of mental arithmetic strategies.

Given the raised curriculum concerns, the redesigning of the senior primary mathematics curriculum in order to improve the clarity of objectives, make less reference to standard algorithms and integrate mental arithmetic strategies across topics of the syllabus can be a way to develop learners' *mathemacy*. The textbooks designed properly may be a source of knowledge for learners and hence schools need to be using books which exercises a freedom of methods and not only standard algorithms.

6.3.3 Development of mental arithmetic

This section presents a summary of the main findings on how to enhance the development of mental arithmetic strategies in the primary schools. The discussion in this section provides answers to the third research sub-question;

3. How can the development of mental arithmetic strategies be enhanced in primary schools?

The teachers suggested that the syllabus be re-structured so that the mental arithmetic strategies are distributed across the entire syllabus. This will be against the current structure where mental arithmetic is taught as a topic and teachers do not make reference to the use of these strategies as they move on to other topics in the syllabus. Secondly, participants suggested that to develop mental arithmetic strategies, the teachers need to thoroughly teach and practice these strategies in their classrooms. Mental computation strategies plays a significant role on teaching learners how numbers work, how to make decisions about procedures, and how to create different strategies to solve mathematics problems (Varol & Farran, 2007). Thus senior primary mathematics teachers need to develop mental arithmetic strategies as instructions on mental computations can lead to both increased understanding of number and flexibility in working with numbers (Hiebert & Wearne, 1996 cited in Varol & Farran, 2007). Lastly, the participants suggested that a training on mental arithmetic strategies be conducted for teachers to sensitise them in order for them to deliver as per expectations of the curriculum. When linking this call to how teachers interpreted the concept mental arithmetic as discussed earlier, it gives an indication that an intervention needs to be carried out as it appears some teachers are not aware of what the mental arithmetic concept entails.

The lack of mental arithmetic strategies in the senior primary schools points to a need of an intervention to enlighten the teachers and enhance the development of mental strategies. The teachers gave some unreasonable answers. This points to a need for teachers to be introduced to mental arithmetic strategies so that they can engage thoroughly rather than following series of rules without reasoning as the case with standard algorithms. The teachers indicated that their teaching approaches were being informed by how they were taught during their times as school learners and during teacher training. Naukushu (2016, p. 6) believes that the Namibian education system seems to be trapped in a three stage cycle of numerical deficiency which he termed “Naukushu’s Vicious Cycle of Innumeracy (NVCI)”. His three stage cycle of numerical deficiency assumes that in stage one: learners leave school with numerical deficiencies; stage two: learners become mathematics student teachers and still not acquire enough numeracy; and stage three: students graduate as teachers and go serve in the schools with insufficient numeracy. It’s against these grounds that an intervention into developing teachers’ mental arithmetic strategies is needed.

All the teachers who participated in the intervention programme indicated that they would change classroom instruction to incorporate mental arithmetic computation strategies. The teachers indicated that their classes were now going to be more democratic and exercise a variety of strategies of computations a 'move away from traditions to critical mathematics pedagogies' (Stinson, Bidwell, Jett, Powell, & Thurman, 2006). The teachers recommended that a broader intervention be rolled out to include more senior primary teachers to enhance the development of mental arithmetic strategies. The intervention provoked the teachers to realise that mental arithmetic is 'a way of functioning' and not 'a list of strategies to be covered', which is critical in the teachers new learning.

6.3.4 Mental arithmetic intervention for learners

The results presented in this sections builds up as the answer to the main hypothesis tested:

H_o: There is no significant difference in the performance of the learners taught mental arithmetic strategies and those who were not.

H_1: There is a significant difference in the performance of the learners taught mental arithmetic strategies and those who were not.

The results of the t-test calculations suggested that the null hypothesis be rejected and conclude that there is a significant difference between performance of the learners taught using mental arithmetic strategies and those who were not. This indicates that at $\alpha = 0.05$ and degree of freedom, df = 57, the experimental group scored significantly higher than the control group. Since the two groups were at the same level before critical intervention, the differences in performance during the post-test may be attributed to the treatment given to the experimental group. This results concurs with the findings by other studies (Yang, Hsu & Huang, 2004; Yang & Li, 2008; Ho & Lowrie, 2014 all cited in Kuldas et al., 2017) where it was found in Taiwan and Singapore that the learners who followed mathematics activities and instructions for mental computation strategies demonstrated better performance than those who followed the standard algorithms through teaching with little emphasis on mental development.

6.4 Contributions of this study to the body of knowledge

Mental arithmetic is a relatively new concept in the Namibian mathematics education and literature points to a lack of studies done on the concept within the mathematics education in Namibia. This study thus contributes as a basis enquiry into the classroom practices of teachers and learners and how these practices may influence numeracy development. The critical approach engaged by the study brought to light that computation strategies practiced in the senior classrooms may be a taken-for-granted aspect and the education system needs to act on these as it may be disadvantaging learners (Tutak et al., 2011). Thus, in the practical realm the study can be taken as a viable diagnostic and agency resource for curricular improvement across subjects in Namibia, and a tool for agitating activation by all stakeholders. Such a tool was not really in existence; the study has put it onto the table for use by the system wishing to exploit it for improvement.

The intervention for teachers and learners had an impact on the learners' performance. The workshop manual may be used to train teachers on the mental arithmetic strategies and enhance the development of mental arithmetic strategies in the schools ultimately resulting in an improved level of functional numeracy. The study thus contributes towards the knowledge on continuous professional development and/or in-service models that are change-based and internally inspired and driven.

While the study by Tabakamulamu (2010) in Zambia used the constructivism approach to observe the effects of an intervention to teachers' perceptions on mental computations and on learners' performance. This study took a different route by using the CME to explore the classroom practices and perceptions of teachers with regard to mental arithmetic. The constructivist approach is based on claims that individuals generate knowledge by interacting with the environment. This study is an illustration of how the application of the CME framework illuminates some aspects of mathematics learning and instruction that have remained hidden in other studies (e.g. aspects of the emancipation effects on the researched participants).

6.5 Implications of this study for mathematics education community

The critical mathematics education theorist Skovsmose (2005b) states that mathematics education could mean empowerment of learners via inclusion of all learners but could also mean suppression via exclusion and discrimination. Skovsmose therefore argues that critical reflections on mathematics education are necessary to identify aspects within mathematics education which poses as threats to empowerment of learners. In this section, the researcher discusses three aspects from the findings of the present study which may mean a form of exclusion in the mathematics classrooms. These three are: lack of student voices in the mathematics classrooms; undemocratic choice of methods; and the reasonableness of answers.

The classroom practices lack student voices. The classroom discourses did not go beyond what Skovsmose (1998, p. 200) terms a "ritualised communication" which flowed as: the teacher asks a question; learners raise their hands; teachers points out one learner; learner gives the answer; teacher corrects the learner if necessary and asks a new question. There were also some case where the teachers did not correct the learners but instead said 'wrong, anybody else to give us the correct answer?' According to Skovsmose (1998), such practices by teachers are brought forth by the notion that school mathematics should be organised around activities with exactly one answer and that the teachers' job is to eliminate mistakes. Such classrooms often tend to regard mathematics as absolute truths and facts to be received by the learners with teachers acting as the transmitter of knowledge thereby taking us to Freire's banking pedagogy.

To avoid the banking pedagogy, teachers should thus evoke classroom deliberation following learners' responses and try to understand how learners construct responses. During classroom deliberations, teachers should challenge learners' thinking and reasoning and encourage dialogue within the classrooms in order to nurture critical consciousness.

The second aspect under this section is the reasonableness of answers given by teachers and learners. The answers often lacked critical, creative and logical thinking and hence creating an impression that mathematics seems to be practiced as series of calculations without logic. Critical reflections on the lack of logic in answers leads to questions whether mathematics education is exposing learners to learn rich mathematics so that they have opportunities to study, pursue eloquent lives, support their communities' growth, and be able to use mathematics to fight oppression and improve society as in the notions of *social justice*.

Erdem and Gürbüz (2016) established that there exist a significant positive relationship between mental arithmetic and mathematical reasoning and that standard algorithms where inefficient when it comes to developing mathematical reasoning. The reasons for this are that mental computation necessitates calculating operations using different strategies thereby engaging the mind in thinking about strategy options, as compared to following established steps in standard algorithms. It is hence easier to reason after having thought through an operation.

The last aspect this researcher wishes to discuss in this section is the perception within the mathematics education that some computational strategies are 'formal' and answers should be strictly presented via such methods. These methods are marked as formal methods in the mathematics curriculum and it's the same methods used in textbooks and teachers' guides. The examiners draw up marking schemes dictating how solutions should be presented for learners to earn marks.

Simultaneously, the education systems denounce undemocratic practices in classrooms and calls for open presentations of ideas of learners to enhance learning with understanding. Khuzwayo and Bansilal (2012) alludes that in order for schools to prepare learners for participation and contribution to a democratic society, the learners should be given an opportunity to experience democratic processes and realise that democracy grants all individuals opportunities to voice their views. It is, hence, imperative that mathematics teachers do not restrict learners to present their written answers in certain methods only but allow a democratic exercise so that learners follow computational strategies of their choice.

The undemocratic notion in mathematics education that some computation strategies are formal and learners' written answers should be presented via such strategies is comparable to what Khuzwayo (2005, p. 323) terms the "occupation of our minds". The belief is controlling the minds of the teachers and limiting options in their imaginations as it has blocked alternatives leaving only rigid ways of doing mathematics. The peril of the 'occupation of minds' is when the teachers are made to accept their situation as normal and unchangeable as it appears the case where teachers are made to mark learners' scripts strictly adhering to marking schemes. Critical mathematics education speaks against absolutism in mathematics education and calls for critical reflections on mathematics to identify and redress disparities.

6.6 Recommendations

The results of this study present the following recommendations hoping that these findings will be shared with the division of professional planning and development and all divisions responsible for teachers' professional development and monitoring of the mathematics curriculum implementation in the schools. The senior primary mathematics teachers are also the target audience of these findings as they serve to implement the mathematics curriculum and bears the responsibility of developing mental arithmetic strategies in the schools. The researcher also hopes that the findings are made available to teacher training institutions in Namibia and the world at large so that the teacher educators are sensitised of these prevalent issues pertaining to mathematics education in the primary schools. The CME driven study revealed the taken for granted aspects of the mathematics education curriculum which should be addressed to make sure rationale of developing a numerate citizenry is achieved. The findings of the study thus point to the following recommendations:

Recommendations to in-service senior primary mathematics teachers

The senior primary mathematics teachers should exercise a variety of mental computation strategies in their classrooms to ensure classrooms are democratic and learners are provided opportunities to learn mathematics with understanding. The teachers should seek to understand the mental arithmetic strategies themselves in order to effectively convey such strategies to learners. The findings provide evidence for the need for continuing professional development to improve teacher content knowledge on mental arithmetic computation strategies along with instruction specifically focussed on the significance of this challenging mental arithmetic strand of the syllabus. Teachers should delay standard algorithms as much as necessary until numeracy is well developed. And when introducing the standard algorithms, complement with learners invented or mental arithmetic strategies to help provide a variety of options for their learners.

Recommendations to curriculum developers

The curriculum developers should consider a critical intervention for the in-service senior primary mathematics teachers to develop their mental computation strategies. The results pointed to a lack of clarity amongst teachers as to what the concept mental arithmetic entails and also to a lack of mental arithmetic strategies in the classrooms as teachers persistently practiced standard algorithms and dictated that learners perform calculations via the same algorithms. Learners' perspectives are discouraged and silenced by teachers' tendency to resorting to the use of standard algorithmic procedures. Learners' imaginations are discouraged in contrast to a critical mathematics education perspective.

The curriculum developers should further consider restructuring the mathematics syllabus for the senior primary phase. The current structure consists of mental arithmetic taught as a topic at the beginning of a school year and thereafter, no reference is made to use of mental arithmetic strategies where teachers practice the "formal algorithms" (MoEAC, 2016, p. 4). The reference by the syllabus to standard algorithms as formal algorithms seem to be posing a confusion to teachers as it may be interpreted that standard algorithms are superior to mental arithmetic strategies. Furthermore, the curriculum developers should consider prescribing the textbooks which exercise both mental computation strategies and standard algorithms. Teachers revealed that the strategies in the textbooks were standard algorithms.

Recommendations to teacher training institutions

The teacher training institutes should make mental arithmetic an integral component of senior primary mathematics teacher education programmes. This may help break the NVCI and enhance the development of mental arithmetic in the schools once the trained teachers join the mainstream education.

6.7 Further research avenues

This study explored the incorporation of mental arithmetic in senior primary mathematics classrooms. It was underpinned by the critical mathematics education theory and determined classroom practices of senior primary mathematics teachers and learners. The study may be used to inform other studies on mental arithmetic and numeracy development in primary schools.

Although this study focused at the senior primary grades, functional numeracy is key in every adults' livelihood. Further research may hence look at determining the mental arithmetic skills of high school learners. How much mathematical problems high school learners are able to solve without the aid of a calculator is a measure of their own level of functional numeracy.

The study suggests that further research may be conducted to determine how well the pre-service teachers at the teacher training institutions in the country, University of Namibia (UNAM) and the International University of Management (IUM), are prepared to develop mental arithmetic strategies in senior primary classrooms. That way research may help to inform restructuring of the content used in preparing pre-service teachers for the teaching profession and equip them with the necessary content and pedagogical skills needed to develop numeracy.

Another area for further research may be on how mental arithmetic strategies might improve computational estimation skills. Estimation, just like basic arithmetic skills, is part of everyday living and schools should foster to develop these skills to enhance functional numeracy.

Further research may be conducted to determine the impacts of merging standard algorithms with mental arithmetic strategies. Teaching standard algorithms alongside mental arithmetic strategies and allowing these to complement each other may yield results the schools intent to achieve.

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APPENDICES

APPENDIX A: Letter to Permanent Secretary – Ministry of Education, Arts & Culture

UNIVERSITY of the WESTERN CAPE
RESPICE PROSPICE

FACULTY OF EDUCATION
Private Bag X17, Bellville 7535, South Africa Telegraph: UNIBELL
Contact Centre: +27 21 959 3888
www.uwc.ac.za

SCHOOL OF SCIENCE AND MATHEMATICS EDUCATION

7th February 2017

To: The Permanent Secretary
Ministry of Education, Arts & Culture
Private Bag 13186, Windhoek

Dear Ms Steenkamp

REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN OSHANA REGION ON THE TOPIC: **"INCORPORATING MENTAL ARITHMETIC IN PRIMARY SCHOOLS"**

I am a registered student for a Doctor in Philosophy degree in Mathematics Education at the University of the Western Cape, South Africa. I am kindly requesting for permission to do research on the topic stated above. My research will focus: '*to incorporate mental arithmetic strategies in primary schools and determine the impact it will have on learners' academic achievement. It will further explore current teachers' perceptions and practice of mental arithmetic.*

The primary school learners have been performing below average in the Standardised Achievement Tests (SATs) for over the years, an indication that they lack quick mental computation skills to solve the test items correctly. The Mathematics SATs which has been validated are psychometric tests which demonstrates candidates' abilities to deal with numbers quickly and accurately. Such computations are to be processed mentally as calculators are not allowed.

I kindly request your good office to allow me to use Oshana region as my research site. I intend to sample out 10 schools from the region and engage the upper primary mathematics teachers from these schools. I hope to complete the data collection exercise before the end of July 2017. The school and participants will be assured of confidentiality and anonymity in the final research report. I assure you the process will not interfere with the normal class teaching time at the schools. Teachers will be engaged after classes. Attached please find a copy of my research proposal. I would like to initially conduct a pilot study for a week in order to test for the validity and discrepancies in the research instruments before the main study.

Should you require further clarifications, please contact me at +264 81 148 8183/fhaimbodi@unam.na or my supervisor Professor Bekhumusa Khuzwayo at [bkhuzwayo@uwc.ca.za or +27 21 959 2798].

Yours Sincerely,

Frans Ndemupondaka Haimbodi

PhD Fellow: UWC

APPENDIX B: Permission letter from Permanent Secretary – Ministry of Education, Arts & Culture

REPUBLIC OF NAMIBIA

MINISTRY OF EDUCATION, ARTS AND CULTURE

Tel: +264 61 -2933200/02
Fax: +264 61- 2933922
Enquiries: C. Muchila/ G. Munene
Email: Cavin.Muchila@moe.gov.na/gm12munene@yahoo.co.uk

Luther Street, Govt. Office Park
Private Bag 13186
Windhoek
Namibia

File no: 11/1/1

Mr. Frans Ndemupondaka Haimbodi
Cell: +264 81148 8183
Email: fhaimbodi@unam.na

Dear Mr. Haimbodi

SUBJECT: PERMISSION TO CONDUCT RESEARCH IN KAVANGO EAST AND OSHANA REGIONS

Kindly be informed that permission to conduct research for your Doctor of Philosophy Degree in Kavango East and Oshana regions on the topic: "*Incorporating Mental Arithmetic in Primary Schools*" is herewith granted. You are further requested to present the letter of approval to the Regional Directors to ensure that research ethics are adhered to and disruption of curriculum delivery is avoided.

Furthermore, we humbly request you to share your research findings with the ministry. You may contact Mr C. Muchila/ Mr. G. Munene at the Directorate: Programmes and Quality Assurance (PQA) for provision of summary of your research findings.

I wish you the best in conducting your research and I look forward to hearing from you soon.

Sincerely yours

Ministry of Education, Arts and Culture
Office of the Permanent Secretary
2017-06-07
13186
Windhoek, Namibia

SANET L. STEENKAMP
PERMANENT SECRETARY

7/6/17.
Date

All official correspondences must be addressed to the Permanent Secretary

APPENDIX C: ETHICAL CLEARANCE

UNIVERSITY of the WESTERN CAPE

OFFICE OF THE DIRECTOR: RESEARCH
RESEARCH AND INNOVATION DIVISION

Private Bag X17, Bellville 7535
South Africa
T: +27 21 959 2988/2948
F: +27 21 959 3170
E: research-ethics@uwc.ac.za
www.uwc.ac.za

24 May 2017

Mr FN Haimbodi
Faculty of Education

Ethics Reference Number: HS17/2/9

Project Title: Incorporating mental arithmetic into primary school mathematics: A Case of Oshana Region, Namibia.

Approval Period: 22 May 2017 – 22 May 2018

I hereby certify that the Humanities and Social Science Research Ethics Committee of the University of the Western Cape approved the methodology and ethics of the above mentioned research project.

Any amendments, extension or other modifications to the protocol must be submitted to the Ethics Committee for approval. Please remember to submit a progress report in good time for annual renewal.

The Committee must be informed of any serious adverse event and/or termination of the study.

Ms Patricia Josias
Research Ethics Committee Officer
University of the Western Cape

PROVISIONAL REC NUMBER - 130416-049

FROM HOPE TO ACTION THROUGH KNOWLEDGE

APPENDIX D: LETTER TO EDUCATION DIRECTOR – Oshana region

UNIVERSITY of the WESTERN CAPE

FACULTY OF EDUCATION
Private Bag X17, Bellville 7535, South Africa Telegraph: UNIBELL
Contact Centre: +27 21 959 3888
www.uwc.ac.za

SCHOOL OF SCIENCE AND MATHEMATICS EDUCATION

8th June 2017

The Director: Education

Oshana Regional Council

Private Bag 5518, OSHAKATI

Dear Mrs. Hamukana

REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN OSHANA REGION ON THE TOPIC:

"INCORPORATING MENTAL ARITHMETIC IN PRIMARY SCHOOLS"

I am a registered student for a Doctor in Philosophy degree in Mathematics Education at the University of the Western Cape, South Africa. I am kindly requesting permission to do research on the topic stated above. My research will focus: '*to explore the incorporation of mental arithmetic strategies in primary schools. It will further explore the teachers' perceptions and practices of mental arithmetic.*

The primary school learners have been performing below average in the Standardised Achievement Tests (SATs) for over the years, an indication that they lack quick mental computation skills to solve the test items correctly. The Mathematics SATs which has been validated are psychometric test which demonstrates candidates' abilities to deal with numbers quickly and accurately. Such computations are to be processed mentally as calculators are not allowed.

Attached please find a copy of the permission letter from the office of the Permanent Secretary of the Ministry of Education.

I kindly request your good office to allow me to use Oshana region as my research site. I intend to sample out 10 schools from the region and engage the upper primary mathematics teachers from these schools. I hope to complete the data collection exercise before the end of August 2017. The school and participants will be assured of confidentiality and anonymity in the final research report. I assure you the process will not interfere the normal class teaching time at the school. Teachers will be engaged after classes.

Should you require further clarifications, please contact me at +264 81 148 8183/fhaimbodi@unam.na or my supervisor Professor Bekhumusa Khuzwayo at [bkhuzwayo@uwc.ca.za or +27 21 959 2798].

Yours Sincerely,

Frans Ndemupondaka Haimbodi

PhD Fellow: UWC

FROM HOPE TO ACTION THROUGH KNOWLEDGE

APPENDIX E: Permission letter from Education Director

REPUBLIC OF NAMIBIA

OSHANA REGIONAL COUNCIL
DIRECTORATE OF EDUCATION, ARTS AND CULTURE
Aspiring to excellence in Education for All

Tel: 065 229800
Fax: 065 229833
Enquiries: Levi Vries
E-mail: levi.vries@gmail.com
Ref no: 11/1/1

906 Sam Nuyoma Road
Private Bag 5518
Oshakati, Namibia

Mr Frans Haimbodi
Cell: 0811488183
fhaimbodi@unam.na

Dear Mr Haimbodi

RE: PERMISSION TO CONDUCT RESEARCH IN OSHANA REGION

Your letter on the subject has been received.

Permission to conduct research for your doctoral degree on **"Incorporating Mental Arithmetic in Primary school"** in schools in Oshana Region is granted. We are happy to note that you understand and undertake to do your research outside school hours. It is important that teaching and learning happens with minimum interruption for improved outcomes.

Kindly present this letter of approval to the principals of schools or the teachers you wish to interview. We further wish you all the best in you research.

Yours Sincerely

16/06/2017

HILENI M. AMUKANA
REGIONAL DIRECTOR

OSHANA REGIONAL COUNCIL
DIRECTORATE OF EDUCATION, ARTS & CULTURE
OFFICE OF THE DIRECTOR
16 JUN 2017
Private Bag 5518
OSHAKATI
REPUBLIC OF NAMIBIA

All correspondence should be addressed to the Director of Education, Arts & Culture

UNIVERSITY of the WESTERN CAPE

FACULTY OF EDUCATION
Private Bag X17, Bellville 7535, South Africa Telegraph: UNIBELL
Contact Centre: +27 21 959 3888
www.uwc.ac.za

APPENDIX F: LETTER TO PRINCIPALS

SCHOOL OF SCIENCE AND MATHEMATICS EDUCATION

7th February 2017

To: The Principal

………… Primary School

Oshana Regional Council

REQUEST FOR PERMISSION TO CONDUCT RESEARCH ON THE TOPIC:

"INCORPORATING MENTAL ARITHMETIC IN PRIMARY SCHOOLS"

I am a registered student for a Doctor in Philosophy degree in Mathematics Education at the University of the Western Cape, South Africa. I am kindly asking for permission to do research on the topic stated above. My research will focus: '*to incorporate mental arithmetic strategies in primary schools and determine the impact it will have on learners' academic achievement. It will further explore current teachers' perceptions and practice of mental arithmetic.*

Attached please find a copy of the permission letters from the office of the Permanent Secretary of the Ministry of Education and the office of the Director of Education, Oshana Region.

I hope to complete the data collection exercise before the end of July 2017. The school and participants will be assured of confidentiality and anonymity in the final research report. I assure you the process will not interfere the normal class teaching time at the school as teachers will be engaged after classes.

Should you require further clarifications, please contact me at +264 81 148 8183 /fhaimbodi@unam.na or my supervisor Professor Bekhumusa Khuzwayo at [bkhuzwayo@uwc.ca.za or +27 21 959 2798].

Yours Sincerely,

Frans Ndemupondaka Haimbodi

PhD Fellow: UWC

APPENDIX G: INFORMATION SHEET – TEACHERS

UNIVERSITY of the WESTERN CAPE

RESPICE PROSPICE

FACULTY OF EDUCATION
Private Bag X17, Bellville 7535, South Africa Telegraph: UNIBELL
Contact Centre: +27 21 959 3888
www.uwc.ac.za

My name is *Frans N. Haimbodi*, a PhD fellow at the University of the Western Cape. I am conducting a study on *"Incorporating mental arithmetic in primary schools*". My research will focus: '*to incorporate mental arithmetic strategies in primary schools and determine the impact it will have on learners' academic achievement. It will further explore current teachers' perceptions and practice of mental arithmetic.*

The primary school learners have been performing below average in the Standardised Achievement Tests (SATs) for over the years, an indication that they lack quick mental computation skills to solve the test items correctly. The Mathematics SATs which has been validated are psychometric tests which demonstrate candidates' abilities to deal with numbers quickly and accurately. The study, hence, focuses to explore possible ways to improve mental computation strategies among senior primary learners. I assure you the process will not interfere with the normal class teaching time at the schools and hence your classes will not be disturbed in anyway.

1. I kindly request you to volunteer take part in this research because I would like to learn more on the integration of mental arithmetic in your mathematics classrooms.
2. If you agree to participate in this study, I will ask to:
    - Observe your classes,
    - Have oral interviews with you,
    - Ask you to complete a questionnaire, and
    - Administer mathematics psychometric tests to your classes.

3. I do not foresee any potential risks associated with taking part in this study for you or your learners.
4. There will be no remuneration for partaking in the study.

NB: CONFIDENTIALITY

5. Any information that is obtained in connection with this study will be treated with utmost confidentiality and will not be identified with you. The interviews will be audio taped and all participants will have the rights to review tapes. Anonymity will be maintained during the study and all the possible ways of identification will be removed by means of using codes/pseudonyms.

Kindly grant me permission by signing the attached consent form.

If you choose to participate and later change your mind, you can stop participating anytime. No actions will be taken against you. If you have questions regarding your rights as a research subject or about the researcher, contact Professor H.B. Khuzwayo [bkhuzwayo@uwc.ac.za or +27 21 959 2798].

Yours Sincerely,

Frans Ndemupondaka Haimbodi

PhD Fellow: UWC

APPENDIX H: CONSENT FORM – TEACHERS

UNIVERSITY *of the* WESTERN CAPE

FACULTY OF EDUCATION
Private Bag X17, Bellville 7535, South Africa Telegraph: UNIBELL
Contact Centre: +27 21 959 3888
www.uwc.ac.za

APPENDIX J: TEACHERS' CONSENT FORM

UNDERTAKING.

I understand that:

- I am under no obligation to participate, and may withdraw from the study at any point prior to the publication or presentation of research results, without any penalty.
- Anonymity will be maintained through the use of pseudonyms. My name, name of my school or those of my learners will not be reported.
- The research will be used for academic and professional presentations and publications.

NB: Signing your name below means you agree to be in this study. You will get a copy of this form.

NAME OF TEACHER

____________________	____________________	____________
Print name	Signature	Date

FROM HOPE TO ACTION THROUGH KNOWLEDGE

APPENDIX I: INFORMATION SHEET – PARENTS

UNIVERSITY *of the* WESTERN CAPE

FACULTY OF EDUCATION
Private Bag X17, Bellville 7535, South Africa Telegraph: UNIBELL
Contact Centre: +27 21 959 3888
www.uwc.ac.za

APPENDIX: INFORMATION SHEET - PARENTS/GUARDIANS

7th February 2017

To: The Parents of ___________________________

Oshana Region

REQUEST FOR CONSENT TO INVOLVE YOUR CHILD IN A STUDY: *"Incorporating mental arithmetic in primary school mathematics".*

I am *Frans N. Haimbodi,* a student at the University of the Western Cape, South Africa. I am conducting a study focusing: '*to incorporate mental arithmetic strategies in primary schools and determine the impact it will have on learners' academic achievement.*

The primary school learners have been performing below average in the Standardised Achievement Tests (SATs) for over the years, an indication that they lack quick mental computation skills to solve the test items correctly. The Mathematics SATs which has been validated are psychometric tests which demonstrate candidates' abilities to deal with numbers quickly and accurately. The study, hence, focuses to explore possible ways to improve mental computation strategies among senior primary learners. I assure you the process will not interfere with the normal class teaching time at the schools and hence your child will not be disturbed in anyway.

FROM HOPE TO ACTION THROUGH KNOWLEDGE

1. I kindly request you to allow your child _________________________________ to take part in this study by sitting for two psychometric tests.
2. I do not foresee any potential risks associated with taking part in this study for your child.
3. There will be no remuneration for partaking in the study.
4. Any information that will be obtained during this study will be treated with utmost confidentiality and will not be identified with your child.

Kindly grant me permission to involve your child by signing the accompanying consent form.

Attached please find a copy of the permission letters from the office of the Permanent Secretary of the Ministry of Education and the office of the Director of Education, Oshana Region.

Should you require further clarifications, please contact me at +264 81 148 8183 /fhaimbodi@unam.na or my supervisor Professor Bekhumusa Khuzwayo at [bkhuzwayo@uwc.ca.za or +27 21 959 2798].

Yours Sincerely,

Frans Ndemupondaka Haimbodi

PhD Fellow: UWC

APPENDIX J: CONSENT FORM – PARENTS/GUARDIANS

UNIVERSITY of the WESTERN CAPE

FACULTY OF EDUCATION
Private Bag X17, Bellville 7535, South Africa Telegraph: UNIBELL
Contact Centre: +27 21 959 3888
www.uwc.ac.za

APPENDIX J: PARENTS/GUARDIANS' CONSENT FORM

UNDERTAKING.

I understand that:

- My child is under no obligation to participate, and may withdraw from the study at any point prior to the publication or presentation of research results, without any penalty.
- Anonymity will be maintained through the use of pseudonyms. The name of my child will not be reported.
- The research will be used for academic and professional presentations and publications.

NB: If you have questions regarding your child's rights as a research subject or about the researcher, contact me at 0812895357 or Professor H.B. Khuzwayo [bkhuzwayo@uwc.ac.za or +27 21 959 2798].

NB: Signing your name below means you agree your child to be in this study. You will get a copy of this form.

NAME OF PARENT

______________________	______________________	____________
Print name	Signature	Date

FROM HOPE TO ACTION THROUGH KNOWLEDGE

APPENDIX K: INFORMATION SHEET – LEARNERS

UNIVERSITY of the WESTERN CAPE

FACULTY OF EDUCATION
Private Bag X17, Bellville 7535, South Africa Telegraph: UNIBELL
Contact Centre: +27 21 959 3888
www.uwc.ac.za

APPENDIX K: LEARNERS' INFORMATION SHEET

My name is *Frans N. Haimbodi*, a student at the University of the Western Cape. I am conducting a study on Mental Arithmetic in Namibian Primary schools.

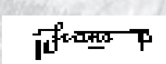
1. I kindly request you to take part in this research because I would like to learn more on the integration of mental arithmetic in your mathematics classrooms.
2. If you agree to participate in this study, we will ask you to write two mathematics psychometric tests. The tests will be two months apart.
3. I do not foresee any potential risks associated with taking part in this study.
4. There will be no remuneration for partaking in the study.
5. Any information that will be obtained during this study will be treated with utmost confidentiality and will not be identified with you.

Kindly discuss this over with your parents before you decide whether or not to participate. However, even if your parents say yes and you do not want to be in this study, you don't have to participate. If you choose to participate and later change your mind, you can stop participating anytime. No actions will be taken against you.

Kindly grant me permission to involve your child by signing the accompanying consent form

Should you require further clarifications, please contact me at +264 81 148 8183 /fhaimbodi@unam.na or my supervisor Professor Bekhumusa Khuzwayo at [bkhuzwayo@uwc.ca.za or +27 21 959 2798].

Yours Sincerely,



Frans Ndemupondaka Haimbodi

PhD Fellow: UWC

FROM HOPE TO ACTION THROUGH KNOWLEDGE

APPENDIX L: ASSENT FORMS – LEARNERS

UNIVERSITY of the WESTERN CAPE

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APPENDIX L: LEARNERS' ASSENT FORM

UNDERTAKING

I understand that:

- I am under no obligation to participate, and I may withdraw from the study at any point prior to the publication or presentation of research results, without any penalty.
- Anonymity will be maintained through the use of pseudonyms. My name will not be reported.
- The research will be used for academic and professional presentations and publications.

NB: Signing your name below means you agree to be in this study. You and your parents will get a copy of this form.

NAME OF LEARNER

Print name	Signature of learner	Date

SIGNATURE OF PERSON OBTAINING ASSENT

In my judgment the participant is voluntarily and knowingly agreeing to participate in this research study.

Name of Person Obtaining Assent	Contact Phone Number
Signature of Person Obtaining Assent	Date

FROM HOPE TO ACTION THROUGH KNOWLEDGE

APPENDIX M: INFORMATION SHEET – FOCUS GROUP

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Private Bag X17, Bellville 7535, South Africa Telegraph: UNIBELL
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APPENDIX M: FOCUS GROUP - INFORMATION SHEET

My name is *Frans N. Haimbodi*, a PhD fellow at the University of the Western Cape. I am conducting a study on *"Incorporating mental arithmetic in primary schools*". My research will focus: '*to incorporate mental arithmetic strategies in primary schools and determine the impact it will have on learners' academic achievement. It will further explore current teachers' perceptions and practice of mental arithmetic.*

The primary school learners have been performing below average in the Standardised Achievement Tests (SATs) for over the years, an indication that they lack quick mental computation skills to solve the test items correctly. The Mathematics SATs which has been validated are psychometric tests which demonstrate candidates' abilities to deal with numbers quickly and accurately. The study, hence, focuses to explore possible ways to improve mental computation strategies among senior primary learners. I assure you the process will not interfere with the normal class teaching time at the schools and hence your child will not be disturbed in anyway.

6. I kindly request you to volunteer take part in this research because I would like to learn more on the integration of mental arithmetic in your mathematics classrooms.
7. If you agree to participate in this study, I will ask you to:

- Ask to have four (4) meetings with you over four weeks to undergo a Mental Arithmetic Training Workshop and,
- Observe your classes,
- Contact focus group discussions

8. I do not foresee any potential risks associated with taking part in this study for you or your learners.
9. There will be no remuneration for partaking in the study.

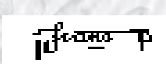
NB: CONFIDENTIALITY

10. Any information that is obtained in connection with this study will be treated with utmost confidentiality and will not be identified with you. The interviews will be audio taped and all participants will have the rights to review tapes. Anonymity will be maintained during the study and all the possible ways of identification will be removed by means of using codes/pseudonyms.
11. You are hence requested to keep any information shared by other participants during the focus group discussions strictly confidential.

Kindly grant me permission by signing the attached consent form.

If you choose to participate and later change your mind, you can stop participating anytime. No actions will be taken against you. If you have questions regarding your rights as a research subject or about the researcher, contact Professor H.B. Khuzwayo [bkhuzwayo@uwc.ac.za or +27 21 959 2798].

Yours Sincerely,



Frans Ndemupondaka Haimbodi

PhD Fellow: UWC and, Lecturer: Mathematics University of Namibia

APPENDIX N: CONSENT FORM – FOCUS GROUP

UNIVERSITY of the WESTERN CAPE
RESPICE PROSPICE

FACULTY OF EDUCATION
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Contact Centre: +27 21 959 3888
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APPENDIX N: FOCUS GROUP - CONSENT FORM

UNDERTAKING.

I understand that:

- I am under no obligation to participate, and may withdraw from the study at any point prior to the publication or presentation of research results, without any penalty.
- Agree to have four (4) meetings with the focus group for over four weeks to undergo a Mental Arithmetic Training Workshop.
- Anonymity will be maintained through the use of pseudonyms. My name, names of my learners and that of my school will not be reported.
- I will keep what other participants say in the focus group discussions strictly confidential.
- The research will be used for academic and professional presentations and publications.

NB: Signing your name below means you agree to be in this study. You will get a copy of this form.

NAME OF TEACHER

_______________	_______________	_______________
Print name	Signature	Date

FROM HOPE TO ACTION THROUGH KNOWLEDGE

APPENDIX O: FOCUS GROUP DISCUSSION GUIDE

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APPENDIX: FOCUS GROUP ITEMS

At the end of the intervention sessions, the researcher will ask the teachers from the experimental group some questions to gain the overall reflection on the training programme. The interview will be tape recorded.

NB: CONFIDENTIALITY

Any information that is obtained in connection with this study will be treated with utmost confidentiality and will not be identified with. The interviews will be audio taped and all participants will have the rights to review tapes. Anonymity will be maintained during the study and all the possible ways of identification will be removed by means of using codes/pseudonyms.

In the beginning the researcher will thank participants for their time invested in the workshop and for participating until at that stage of the interview. The researcher will then give guidelines on how the focus group discussions will proceed.

- ✓ One person speaks at a time.
- ✓ Avoiding mini-meetings.
- ✓ Giving fair chances to everyone to participate.
- ✓ Minimise phone interruptions.

1. Kindly mention the new computation strategies you think you have gained from this workshop e.g. what can you do better now than then?
2. What impacts will the workshop have on your future mathematics lessons?
   Hint: lesson planning, lesson facilitation.
3. Critically reflect on the workshop manual. Was it knowledgeable and helpful? Will it be useful? Did it make sense?
4. Critically reflect on the time duration spent on the workshop.
5. How should the workshop be improved if it was to be repeated?
6. What do you foresee as the main barriers to integrating mental arithmetic strategies in every senior primary mathematics class session?
7. Should this workshop be extended to more teachers in the region, country? Motivate.
8. Other reflections, comments, recommendations?

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APPENDIX P: INDIVIDUAL INTERVIEW ITEMS

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APPENDIX: INDIVIDUAL INTERVIEW ITEMS

This interview will be conducted after the first round of observations. It will be done right after observing each teacher to give an opportunity to talk about the aspects observed. The interview will be tape recorded.

NB: CONFIDENTIALITY

Any information that is obtained in connection with this study will be treated with utmost confidentiality and will not be identified with. The interviews will be audio taped and all participants will have the rights to review tapes. Anonymity will be maintained during the study and all the possible ways of identification will be removed by means of using codes/pseudonyms.

At the start the researcher will thank the participant for availing time to sit with him and agreeing to participate in the research study.

1. Kindly reflect on your lesson by looking at what you liked the most in today's lesson.
2. In your own words, what are mental arithmetic computation strategies?
3. To what extend did your learners employ the mental computation strategies?
4. Why should (or not) mental arithmetic strategies be incorporated in mathematics classrooms?
5. In terms of learners' prior knowledge, how often do your learners suggest different ways of computing problems? Any specific case you recall?
6. In your views, how should learners acquire mental computation strategies?

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APPENDIX Q: PRE-TEST

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MENTAL ARITHMETIC

Pre-Test

Candidate code: ....

School:

....

INSTRUCTIONS:

- **DO NOT write your name on anywhere on this paper.**
- **Attempt all questions.**

Question 1. Work out: 979 + 100

Answer = 1 mark

Question 2. Work out: 123 × 2

Answer = 1 mark

Question 3. Express 8kg as a percentage of 32kg.

Answer = 1 mark

Question 4. Calculate the mean of: 30; 40; 20; 50

Answer = 1 mark

Question 5. Work out: $48 \div 6$

Answer = 1 mark

Question 6. Work out the difference: $472 - 9$

Answer = 1 mark

Question 7. Find the product: $5 \times 4 \times 7$

Answer = 1 mark

Question 8. Work out: $630 \div 9$

Answer = 1 mark

Question 9. Calculate $\frac{3}{20}$ of 100.

Answer = 1 mark

Question 10. Work out: 1440 ÷ 12

Answer = 1 mark

Question 11. What is 20 % of 1500?

Answer = 1 mark

Question 12. Work out the sum: 5756 + 8643

Answer = 1 mark

Question 13. Work out: $7505 \div 5$

Answer = 2 mark

Question 14. Find the product of: 54×23

Answer = 1 mark

Question 15. Work out: 13×3016

Answer = 2 mark

Question 16. What is 95 % of 240?

Answer = 1 mark

Question 17. Work out 234 897 - 45 996

Answer = 2 marks

Question 18. Willem bought a belt marked N$200 and he got 10% discount. Calculate the discount received.

Answer = 2 mark

Question 19. Calculate the area of a triangle with base 20cm and perpendicular height 7cm.

Answer = 2 mark

Question 20. The percentage equivalent to $\frac{33}{50}$ is?

Answer = 2 mark

// 25//

The End

APPENDIX R: POST-TEST

UNIVERSITY of the WESTERN CAPE

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Contact Centre: +27 21 959 3888
www.uwc.ac.za

MENTAL ARITHMETIC

Post-Test

Candidate code: ……………….

School:

……………………………………………………………

INSTRUCTIONS:

- **DO NOT write your name on anywhere on this paper.**
- **Attempt to all questions.**

FROM HOPE TO ACTION THROUGH KNOWLEDGE

Question 1. Complete the sentence below: 125 + 38 = ... 125

Answer = 1 mark

Question 2. Work out: 486 - 9

Answer = 2 marks

Question 3. Work out: 936 + 285

Answer = 2 marks

Question 4. There are 9 parking lots at the school. Each parking lot takes 2 cars. How many cars can be parked at the school?

Answer = 2 marks

Question 5. Work out: 89 994 + 7643

Answer = 2 marks

Question 6. A class wrote a test out of 20 marks. Calculate the marks for a learner who scored 75% in the test.

Answer = 1 mark

Question 7. Work out: 879×3

Answer = 1 mark

Question 8. Work out: $30 \div 12$

Answer = 2 marks

Question 9. Etuna bought a book N$ 75.25 and a pencil at N$ 8.75. How much in total did she pay?

Answer = 2 marks

Question 10. A primary school has 901 people altogether. Learners are 880 and 5 are workers. The rest are teachers. Find the number of teachers at the school.

Answer = 2 marks

Question 11. Convert 780 mm to centimetres.

Answer = 2 marks

Question 12. Work out 122 456 - 11 999

Answer = 2 marks

Question 13. Complete the number sentence: $24 \div 6 = 6 - ....$

Answer = 1 mark

Question 14. Find the amount that is made up of two N$100 notes, three N$50 notes, and seven N$5 coins.

Answer = 3 marks

Question 15. Helvi bought a phone at N$ 1800 and sold it to her friend making a profit of 20 %. Work out her actual profit.

Answer = 3 marks

Question 16. Nangula bought a t-shirt costing N$39.50. She paid with a N$100 note. How much was her change?

Answer = 3 marks

Question 17. Convert 340ml to litres.

Answer = 1 mark

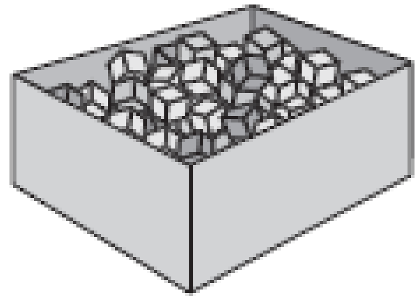
Question 18. Peter watched a movie for 1 hour 45 minutes. The movie started at 17:20, what time did it end?

Answer = 2 marks

Question 19. Etuna has a box of 123 cubes. She uses some of the cubes to build a tower.

77 cubes are leftover.

How many cubes has she used?



Answer = 2 marks

Question 20. Calculate: 750m + 6km + 350 m

Answer = 2 marks

Question 21. How many **hours old** will the baby be when she lived

for 36 **000 seconds** ?

Answer = 2 marks

THE END

APPENDIX S: MENTAL ARITHMETIC COMPUTATION STRATEGIES MANUAL

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UNIVERSITY of the WESTERN CAPE

MENTAL ARITHMETIC COMPUTATION STRATEGIES MANUAL

Workshop Instructional Guide

Frans N. HAIMBODI

FROM HOPE TO ACTION THROUGH KNOWLEDGE

Preface

Mental computation strategies are defined as any invented strategy that can be done mentally (McIntosh, 2005). Mental arithmetic in this manual is used as defined by authors above that it is an approach concerned with the development and use of strategies for computing mentally. However, the mental computation and mental arithmetic will be used interchangeably. Mental arithmetic is a process that involves the use of paper and pencil to write down steps when performing calculations. In many Namibian schools, learners are introduced to multiplication concepts in earlier grades, and are required to memorise the multiplication facts without applying a variety of strategies for example multiplying using fingers, area model, and Cartesian product approach to enhance their understanding of basic multiplication facts. In support of the assertion above the Namibian Mathematics Curriculum for senior primary supports the acquisition of mental computation skills through the development of thinking strategies across the phase. “Learners are expected to use mental methods and paper-and-pencil computation without a calculator sensibly and appropriately” (MoEAC, 2016, p. 4). This entails that those learners at the senior primary phase from grades 4 – 7 are not allowed to use calculators and mathematics teachers should teach various mental arithmetic strategies to develop learners’ awareness of number and number sense. The emphases should be that learners should develop and adapt strategies for mental computation with whole numbers, fractions, decimals and percentages. Strategies to be employed will require flexibility in moving from one representation to another that will be useful in deepening learners’ understanding of rational numbers and assist them in thinking flexibly about numbers (McIntosh, 2005).

This Manual incorporates mental mathematics strategies such as compensation, bridging a decade, distributive principle, aliquout by parts, compatible factors, as well as half and double consistently as part of instructions in computation across the senior primary grades before existing their phase. The main focus of manual will ensure that learners acquires a variety of strategies in computing mentally. Moreover, mathematics teachers should note that these strategies will not be developed over a short period of time as it requires regular practice. It is thus peculiar that teachers incorporate mental computation strategies in their lessons daily.

Senior Primary Mathematics Curriculum

The senor primary syllabus allows the application of the four basic operations and mental arithmetic strategies to numbers from 0 to 1000 000 (MoEAC, 2016, p. 48). It is arguably that the development of mental calculations is regarding as one of the most important objectives in mathematics (MoEAC, 2016) surprisingly the concept is missing in the syllabus as “nobody understands how it works” (p. 73). No wonder learners face challenges in mental computations. Therefore, the teaching of mental computation should be based on the constructivism approach or a paradigm shift from behaviouristic approach.

The approach to teaching and learning should be based on the paradigm shift of learner-centred education (LCE). The main focus should be considering the spontaneous knowledge acquired from social experience gained continually from the family, the community, and through interaction with the environment to be systematised at school arena. Therefore, learning in school must involve, build on, extend and challenge the learner’s prior knowledge and experience. Teaching strategies must therefore be varied but flexible within well-structured sequences of lessons. Co-operative and collaborative learning should be encouraged at all times.

The syllabus extends the basic competencies in computation with whole numbers to computation with common and decimal fractions. Measurement, time, and money and finance relate to the learner’s everyday situation. Geometry is the mathematical understanding of space and shapes. The themes of problem solving, number patterns and data handling are ways of working with, understanding and communicating about and through Mathematics.

Senior Primary Mental Arithmetic Objectives

The mathematics syllabus (MoEAC, 2016) has the following objectives upon completion of the senior primary phase that all learners are expected to be able display in relation to mental arithmetic:

- Perform calculations using all four operations on whole numbers and estimate answers.
- Add, subtract, multiply and divide using any paper and pencil algorithm or mental strategy.
- Apply the commutative, associative and distributive properties to aid calculations.

Values and attitudes: Appreciate and understand the place of Mathematics in everyday life and its widespread application to other subjects. Have an interest in and a positive attitude towards Mathematics

Manual focus:

- This Manual incorporates mental mathematics strategies consistently as part of instruction in computation across the senior primary grades.
- Before learners exit senior primary phase, they should possess a variety of strategies to compute mentally.
- It is important to recognize these strategies develop and improve over time with regular practice.
- It is thus peculiar that teachers incorporate mental computation strategies in their lessons daily.

NB: MA strategies comprises of mental and pen-and-pencil methods of computations.

Table 1: The manual content

Addition	5. Left – Right Approach 6. Compensation 7. Bridging a decade 8. Breaking and Bridging
Subtraction	5. Left –Right Approach 6. Compensation 7. Breaking and Bridging 8. Constant Difference
Multiplication	6. Distributive Principle 7. Compensation 8. Compatible Factors 9. Half and Double 10. Aliquot by parts
Division	4. Additive distribution 5. Subtractive distribution 6. General Factoring

Sequential framework for senior primary mental strategies. Adopted from Morgan (1999).

ADDITION STRATEGIES

I. LEFT TO RIGHT APPROACH

This strategy involves adding the highest place values and then adding the sums of the next place value(s).

Example: 450 + 380: 400 + 300 is 700, and 50 and 80 is 130 and 700 plus 130 is 830.

Exercises A.1 Work out the sums below:

a) 340 + 220 =   b) 470 + 360 =   c) 607 + 304 =

d) Mr Ndara owns a goat farm. He sells 3500 goats on auction. He is now left with 2300 goats on his farm. How many goats did he own before the auction?

e) Tom flew a distance of 1478 km from Cape Town to Windhoek. He then took a bus from Windoek to Rundu, a distance of 715km. Workout the total distance travelled by Tom.

II. COMPENSATION

This strategy for addition involves changing one number to a ten or hundred; carrying out the addition and then adjusting the answer to compensate for the original change.

Example: For 4500 plus 1900: 4500 + 2000 is 6500 but I added 100 too many; so, I subtract 100 from 6500 to get 6400.

Exercise A.2. Work out the following sums:

a) 1300 + 800 = b) 5400 + 2900 = c) 6421 + 1900 =

d) A truck weighs 3450 kg was carrying timbers of mass 2800 kg. Find the gross vehicle mass.

e) A driver covered 233km on a tarred road and 190km on a gravel road to deliver chairs. What is his total distance?

III. BREAKING & BRIDGING

This strategy for addition involves starting with the first number and adding the values in the place values, starting with the largest of the second number.

Examples:

1. Given 5300 + 2 400, think: 5300 and 2000 (from the 2400) = 7300 and 7300 + 400 = 7700.
2. Given 3.6 + 5.3, think: 3.6 and 5(from the 5.3) is 8.6 and 8.6 plus 0.3 = 8.9.

Exercise A.3.

a) 37 + 42 = b) 72 + 21 = c) 88 + 16 =

d) 7300 + 1400 = e) 2800 + 6100 = f) 3300 + 3400 =

IV. BRIDGING A DECADE (MAKE 10, 100 OR 1000)

These facts all have one addend of 8 or 9. The strategy for these facts is to build onto the 8 or 9 up to 10 and then add on the rest.

Examples

a) Given 9 + 6, think of it as: 9 +1(from the 6) is 10, and 10 + 5 (the other part of the 6) is 15 or 9 + 6 = (9 + 1) + 5 = 10 + 5 = 15.

The "make 10" strategy can be extended to facts involving 7.

b) For 7 + 4, think: 7 and 3 (from the 4) is 10, and 10 + 1 (the other part of the 4) is 11or 7 + 4 = (7 + 3) + 1 = 10 + 1 = 11.

Exercise A.4.

a) 58 + 6 = b) 5 + 49 = c) 680 + 78 =

d) 490 + 18 = e) 170 + 40 = f) 570 + 41 =

g) 450 + 62 = h) 630 + 73 = i) 5900 + 660 =

Exercise A.5. Use Breaking & Bridging or the Bridging a Decade approach to solve these problems

1. Mr Ndara owns a goat farm. He sells 3500 goats on auction. He is now left with 2300 goats on his farm. How many goats did he own before the auction?
2. Tom flew a distance of 1478 km from Cape Town to Windhoek. He then took a bus from Windoek to Rundu, a distance of 715km. Workout the total distance travelled by Tom.
3. A truck weighs 3450 kg was carrying timbers of mass 2800 kg. Find the gross vehicle mass.
4. A driver covered 233km on a tarred road and 190km on a gravel road to deliver chairs. What is his total distance?

B. SUBTRACTION STRATEGIES

I. **Left-to-right approach** - learners do the computation starting at the front end.

Example, given 674 – 358, simply record, starting at the front end:

674 – 358 = 274 – 42 = 316

Exercise B.1. Work out:

1. 725 – 358
2. N$65.85 – N$32.55
3. Namene has a bank balance of N$6720 and Mary has a bank balance of N$5845. How much more does Namene have?

II. COMPENSATION

This strategy for subtraction involves changing one number to a ten, hundred or thousand; carrying out the subtraction and then adjusting the answer to compensate for the original change.

Lisa received a discount of N$996 on a laptop which worth N$3360. Work out how much she paid.

Solution: 3 660 – 996 = 3 660 – 1000 + 4 = 2 664.

Exercise B.2 Work out:

Numbers in the 10s:	Numbers in the 100s:	Numbers in the 1000s:
a) 15 – 8 = b) 17 – 9 = c) 23 – 8 =	a) 673 – 99 = b) 854 – 399 = c) 775 – 198 =	a) 8 620 – 998 = b) 4 100 – 994 = c) 5 700 – 397 =

III. BREAKING & BRIDGING

This strategy for subtraction involves starting with the first number and subtracting the values in the place values, starting with the highest, of the second number.

Example

1. 8 369 - 204: 8 369 subtract 200 (from the 204) is 8 169 and 8 169 minus 4 is 8 16
2. For 745 – 203, think: 745 subtract 200 (from the 203) is 545 and 545 minus 3 is 542.
3. Given 8 369 - 204, we take: 8 369 subtract 200 = 8 169 and 8 169 – 4 = 8 165
4. A primary school has 901 people altogether. Learners are 880, workers are 5 and the rest are teachers. Find the number of teachers in the school.

901 – 800 = 101 – 80 = 21 – 5 = 16

Exercise B.3 Work out:

Numbers in the 10s:	Numbers in the 100s:	Numbers in the 1000s:
a) 79 – 37 = **b)** 93 – 72 = **c)** 98 – 22 = **d)** 79 – 41=	**a)** 736 – 301 = **b)** 848 – 207 = **d)** 3586 – 302 = **e)** 9564 – 303 =	**a)** 9275 -8100 = **b)** 6350 – 4200 = **c)** 38 500 -10 400 = **d)** 137 400 – 6100 =

IV. CONSTANT DIFFERENCE

This strategy for subtraction *involves adding* ***or*** *subtracting* the same amount from both the Subtrahend and the minuend to get a ten, hundred or thousand in order to make the subtraction easier. This works because the two numbers are still the same distance apart.

NB: Learners *may need to record at least the first changed number to keep track*.

Examples:

c) 345 – 198: Add 2 to both numbers to get 347 – 200; so the answer is 147.

d) 567 – 203: Subtract 3 from both numbers to get 564 -200; so the answer is 364.

Exercise B. 4 Practice Items

Numbers in the 10s:	Numbers in the 100s:
In these items you add to balance. a) 85 – 18 = b) 42 - 17 =	In these items you add to balance. i) 649 - 299 = j) 563 – 397 =

c) 36 – 19 =	k) 823 – 298 =
d) 78 – 19 =	l) 912 – 797 =
In these items you subtract to balance:	In these items you subtract to balance:
e) 83 – 21 =	m) 486 – 201 =
f) 75 – 12 =	n) 829 – 503 =
g) 68 – 33 =	o) 659 – 204 =
h) 95 – 42 =	p) 382 – 202 =

Exercise B.5 Solve these using any of the four discussed methods

1. A car of value N$ 75 316 was sold at N$ 9900 loss. Find how much the dealer received.
2. Leo bought a CD at N$398 and sold it for N$ 534. Work out his profit.
3. Rauna's gross salary is N$ 3660. Her total deductions are N$ 996. Work out her net salary.
4. Ndumba is driving from Windhoek to Rundu, a distance of 715km. After driving for 318km, he stopped to rest and fuel up. How far was his resting point from Rundu?

C. MENTAL MULTIPLICATION STRATEGIES

I. THE DISTRIBUTIVE PRINCIPLE

Distributive Principle - involves finding the product of the single-digit factor and the digit in the highest place value of the second factor, and adding to this product a second sub-product i.e. ***a(b + c) = ab + ac.***

Example: 2 × 716 = 2 (700 + 10 + 6) = 1400 + 20 + 12 = 1426.

Exercise C.1. Work out:

Items in the 100s	Items in the 1000s
a) 3 × 503 =	a) 3 × 4200 =
b) 209 × 9 =	b) 4 × 2100 =
c) 703 × 8 =	c) 6 × 3100 =
d) 503 × 2 =	d) 5 × 5100 =
e) 8 04 × 6 =	e) 2 × 4300 =
f) 309 × 7 =	f) 3 × 3200 =
g) 122 × 4 =	g) 2 × 4300 =
h) 320 × 3 =	h) 7 × 2100 =
i) 410 × 5 =	i) 4 × 4200 =

j) A school starts at 07:10. There are 4 periods of 35 minutes each before break. Calculate the time the first break starts.

II. COMPENSATION

Compensation - involves changing one of the factors to a ten, hundred or thousand; carrying out the multiplication; and then adjusting the answer to compensate for the change that was made. This strategy could be used when one of the factors is near ten, hundred or thousand.

Examples:

1. Given 7 × 198 = 7 × (200 – 2) = 1400 – 14 = 1386.
2. Given 6 × 39 =, we get 6 times 40 is 240, but this is six more than it should be because 1 more was put into each of the six groups; therefore, 240 subtract 6 is 234.
3. A farmer planted carrots in 11 rows. Each row has 250 carrots. How many carrots did the farmer plant?

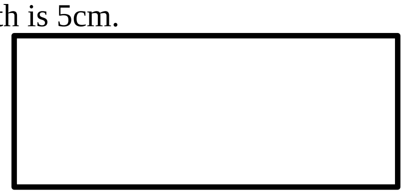
   250 × 10 = 2500 then we take 2500 + 250 = 2750 carrots.

Exercise C.2. Practice Items

In tens:	**In hundreds:**
a) 6 × 39 = **b)** 8 × 29 = **c)** 29 × 50 = **d)** 39 × 40 =	**a)** 5 × 399 = **b)** 3 × 199 = **c)** 4 × 198 = **d)** 9 × 198 =

g. The length of the rectangular garden is 149cm and its width is 5cm.

(i) Find the area of the garden.



III. COMPATIBLE FACTORS

Compatible Factors – this strategy involves looking for pairs of factors whose product is a power of ten and re-associating the factors to make the overall calculation easier (associative property of multiplication).

Examples

1. A farmer has 25 goats. Each goat requires 63 litres of water per week. How much water does the farm need to sustain the goats for a month?
2. Given 2 × 78 × 500, think: 2 times 500 is 1000, and 1000 times 78 is 78 000.
3. For 5 × 450 × 2, think: 2 times 5 is 10, and 10 times 450 is 4500.

Exercise C.3. Work out

a) 5 × 19 × 2 =

b) 2 × 43 × 50 =

c) 4 × 38 × 25 =

d) 500 × 86 × 2 =

e) 250 × 56 × 4 =

f) 40 × 25 × 33 =

IV. HALF & DOUBLE

Half and Double – this strategy involves halving one factor and doubling the other. At least one factor must be even.

Examples:

2. 18 × 4 = 9 × 8 (or 36 × 2) = 72
3. $3\frac{1}{2} \times 12 = 7 \times 6 = 42$

Exercise C.4 Work out:

1. $18 \times 4 = 9 \times 8$	4. $34 \times 5 =$
2. $15 \times 18 =$	5. $3\frac{1}{2} \times 12 =$
3. $4 \times 16 =$	6. $4.5 \times 8 =$

NB: At least one of the factor should be even.

V. ALIQUOT PARTS

The aliquot by parts strategy involves multiplying by an equivalent fraction of the given factor to make computation easier.

Examples:

1. $48 \times 5 = 48\ (10 \div 2) = 480 \div 2 = 240$
2. $12 \times 4.5 = 12 \times (9 \div 2) = 6 \times 9 = 54$
3. The cost price of a radio is N$280. A profit of 15% is made. Calculate the profit.

   15 % of N$ 280 = $\frac{15}{100} \times 280 = 1.5 \times 28 = 3 \times 28 \div 2 = 3 \times 14 =$ N$ 42

Exercise C.5. Work out

1. 340×5
2. 40×15
3. 500×19

D. DIVISION STRATEGIES

The distributive property; $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

Examples: 145 ÷ 5

I. *Additive distribution* – look for two numbers divisible by 5 whose sum is 145.

$\frac{100}{5} + \frac{45}{5} = 20 + 9 = 29$

II. *Subtractive distribution* – look for two numbers divisible by 5 whose difference is 145.

$\frac{150}{5} - \frac{5}{5} = 30 - 1 = 29$

Exercise D.1. Work out

1. 69 ÷ 3	4. 2442 ÷ 22
2. 391 ÷ 3	5. 670 ÷ 6
3. 30 ÷ 12	6. 105 ÷ 3

III. Factoring

General Factoring – this strategy works by breaking down the divisor into its factors.

Example: $\frac{240}{16} = \frac{240}{2\times 8} = \frac{30}{2} = 15$

Exercise D.2

1. 342 ÷18 =
2. 225 ÷15 =

3. N$4000 is divided between three schools in the ratio 1:2:5. How much was the largest share?

EXERCISE E

1. How much commission will James earn if he sells a car for N$47 000 and commission of 3% is paid to him?
2. Carrol buys bikes for N$1400 each and sells them for N$1720. What is her percentage profit?
3. A sum of N$1200 was divided between 3 people. Angela received $\frac{1}{2}$ of the money, Tyson received $\frac{1}{3}$ of the money and Charlie received the remainder.

   (a) How much did Angela receive?

   (b) How much did Tyson receive?

   (c) What fraction did Charlie receive?

4. As a special offer the mass of 300g of washing powder is increased by 15%.

   Calculate the increased mass of the washing powder.

5. Calculate 12% discount on a CD that is marked for N$128.
6. Find the amount that is made up of two N$100 notes, three N$50 notes and seven N$5 coins.
7. Etuna bought a doll costing N$79.50 and paid with a N$200 note. How much was her change?

The End of the Mental Arithmetic Strategies Guide