UNIVERSITEIT VAN WES-KAAPLAND
UNIVERSITY OF THE WESTERN CAPE

Hierdie boek moet terugbesorg word voor of op
die laaste datum hieronder aangegee.

This book must be returned on or before the
last date shown below.

900001745

1995 -07- 07
1995 -11- 15
1995 -11- 18
1996 -01- 31
1996 -04- 04
1996 -04- 19
1996 -05- 09
1996 -05- 25
24 AUG 1996
1996 -11- -4
17 JAN 1996

RETURNED

UNIVERSITY of the
WESTERN CAPE

30001545791214

https://etd.uwc.ac.za/

AN INVESTIGATION INTO SOME CAUSES OF COGNITIVE DEFICIENCIES AMONG HIGH SCHOOL PUPILS IN THE LEARNING OF DIFFERENTIAL CALCULUS AND IMPLICATIONS FOR FURTHER STUDY

BY

SIMON MATSELA

UNIVERSITY of the
WESTERN CAPE

submitted in partial fulfilment of the requirements for the degree of Master of Education awarded by the Goldfields Science and Mathematics Resource Centre in the Faculty of Education at the University of the Western Cape.

1994

Supervisor : Dr. J.S. Rhodes

https://etd.uwc.ac.za/

047420 U

THE
UNIVERSITEIT VAN WES-KAAPLAND
BIBLIOTEEK
370.156 MAT
LIBRARY
UNIVERSITY OF THE WESTERN CAPE

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

Dedicated to my son Zachariah Matsela, my beloved wife Malineo and to my respected parents whose faith in my abilities has been a pillar of strength throughout my studies.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

ACKNOWLEDGEMENTS

The author would like to express his heartfelt gratitude and appreciation to the following people who contributed to the success of this study.

Dr. J.S. Rhodes, my supervisor for his meticulous guidance and encouragement in this study. Through him I learned the importance of inner strength and commitment.

Professor A.J.L. Sinclair for his optimism and demystification of engaging in this study and for being my teacher.

Mr. S. Adams, for offering transport during the field work, photo-copying and distributing (the questionnaires, finding an editor for this thesis and most of all for his willingness to help even if it meant spending most of his time assisting me. Thank you very much.

Dr. L. Molema for editing the first draft of this thesis.

Mrs. G. Enous for challenging the entire draft.

Mrs Shantall Renecke for her sacrifice in typing many drafts of this thesis. I appreciate her marvellous job.

And especially my appreciation to the students and teachers who were involved in the interviews and the completion of the questionnaires.

UNIVERSITY of the
WESTERN CAPE
https://etd.uwc.ac.za/

DECLARATION

I declare that *An Investigation into Some Causes of Cognitive Deficiencies among High School Pupils in the Learning of Differential Calculus and Implications for Further Study* is my own original work that it has not been submitted before for any degree or examination in any other University, and all the sources I have used have been indicated and fully acknowledged.

....04/94..........

SIMON MATSELA DATE

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

CONTENTS

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

PART 2

CHAPTER 3 : RESEARCH METHODOLOGY

CHAPTER 4 : PRESENTATION AND ANALYSIS OF THE FINDINGS OR RESULTS

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

CHAPTER 5 : IMPLICATION FOR FURTHER STUDY

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

SUMMARY

Little attention is usually given to the disadvantaged pupils' lack of problem solving skills and thinking skills needed for successful learning of differential calculus. Factors like what makes (disadvantaged) pupils not function (cognitively) very well are usually overlooked. The only factor considered by (some) teachers being only to comment on the impairments that already exists. That is for example labelling students as unintelligent "foolish" or "dull". In some cases the blame is placed fully on the system (Government and its hidden agenda). While others blame the environment in which the pupils live without finding what impact the situations have on the learners' cognition. There is a need to know where and what causes cognitive impairment in general. This study looks at the domain of differential calculus. This research therefore sets out to find what the causes of cognitive deficiencies are in the learning of differential calculus.

To find out some of the causes 67 High School pupils and 40 first year University students completed the questionnaires, 34 pupils wrote the differential calculus test, 10 final year matric teachers also contributed some information about some causes of cognitive deficiencies and how they could possibly be remedied. 4 first-year university mathematics students and 7 high school pupils were involved in the thinking - aloud interviews. The interviews were audio-taped and then subjected to a protocol analysis. Special references to this study were made to the works of Feuerstein, Perkins and Sternberg.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

Thus in general the research set out to answer the following questions:

1. Which problems do pupils think they have with regard to learning differential calculus?
2. Which cognitive deficiencies or difficulties do pupils have in the learning of differential calculus?
3. What role do motivation and affective processes and other non - cognitive processes have in the learning of differential calculus?
4. Is there any relationship between the pupils' cognitive deficiencies, motivation and affective processes as well as the problems they think they have with regard to learning differential calculus?
5. What perception do practising teaching have on the above?
6. Is there any change or changes for first year university mathematics students?
7. Which instructional strategies can be used to rectify such deficiencies?
8. What are the implications and recommendations for further study?

It was found that there are many cases of deficiencies among high school pupils. These can briefly be summarised as follows: lack of effective institutional processes, poor socio-economic and political climate for blacks, inadequate research (thinking aloud) among the pupils. Finally the list of 28 recommendations and implementation for further study were suggested. These 28 recommendations were further divided into 5 general headings that teachers could experiment with in the classroom and school.

UNIVERSITY of the
WESTERN CAPE
https://etd.uwc.ac.za/

AN OUTLINE OF THE THESIS

This thesis is divided into five Chapters divided as follows:

Chapter 1

This is the introduction. It provides the general background and outlines the rationale and problem of this study.

Chapter 2

This chapter is divided into two parts.

Part 1 surveys of the literature in the field of mathematics education (differential calculus learning in particular).

Part 2 reviews the literature related to differential calculus content. The two parts together discuss concepts that are relevant to the learning mathematics in general and differential calculus in particular.

UNIVERSITY of the
WESTERN CAPE

Chapter 3

This chapter outlines the methodology that was used in the research. This includes population, sampling, instrumentation and procedures for data collection.

Chapter 4

This Chapter analysis interprets the data that was collected, and presents the results.

Chapter 5

A brief summary of findings, recommendations and implications for further study are outlined in this chapter.

https://etd.uwc.ac.za/

CHAPTER 1 : INTRODUCTION

1.1 General background

The poor mathematics performance of Black students in South Africa is a result of the segregated education system which separated people along different racial lines, in accordance with apartheid laws. It is to this effect that Jansen (1991) quoted H.F. Verwoerd the architect of Apartheid who stated in parliament in 1953 when introducing the Bantu Education Bill: *"What is the use of teaching a Bantu child mathematics when it cannot use it in practice? ... that is absurd"* (Cited in Harrison, 1981). Verwoerd said, *"A Bantu pupil must obtain knowledge, skills and attitudes in the school, which will be useful and advantageous to him and at the same time benefit to his community ... The school must equip him to meet the demands which the economic life of South Africa will impose upon him ... The Bantu teacher must be integrated as an active agent in the process of the development of the Bantu community. He must learn not to feel above his community, with a consequent desire to become integrated into the life of the European community ..."* (cited in Behr, 1980).

Statements like this therefore show that poor education for blacks was (or is) deliberate. The assumption implicit in a formulation like this is that the position of Blacks in the social, economic and political life of South Africa is both fixed (therefore, their intelligent quotient not being dynamic) and natural (meaning that they were born like that). Therefore, the curriculum for Blacks had to be directed towards serving those predetermined ends (Nkomo, 1984). It was assumed that the intelligence quotient of blacks was very low and in particular

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

lower than that of "Coloureds", Indians and Whites, and hence it became a matter of policy for the whites to manipulate and control them "for their (Whites) own good". The assumption underlying these claims was that intelligence is a fixed entity and nothing could be done to boost it.

Whilst it is crucial to keep the above in mind, there are other views that closely examine the role that schools play in the academic performance of Black students with particular reference to mathematics (the context of this study). According to Olivier (1993), *"the recent statistics suggests that 25,8% of Black children enroled in grade 1 do not pass at the end of the year - a figure four times higher than that for white South African children"*. He goes further to state that, usually the blame is placed on the political situation with schools being intentionally set up to maintain the socio-political structures and at the same time reproduce and perpetuate racial differences in knowledge and culture. While he admits that indeed there is evidence for these claims, he, however, points out that there should be bridging programmes set up for the purpose of making children "ready" for school. He asserts (importantly) that it is usually the schools which are (basically) not ready for children. Applying this observation to black schools, it means that these schools are usually not ready for their own pupils and that their failure rate is not wholly to blame on race, culture and social class.

UNIVERSITY of the
WESTERN CAPE

Olivier's contribution to the debate is useful in that it focuses attention not only on the victim and his or her circumstances, but also on the role the school ought to and can play in alleviating poor mathematics performance in black schools.

https://etd.uwc.ac.za/

It is important at this stage to look at the work of Norman (1993) who states that there are two distinct approaches to the study of human cognition. The first approach is one upon which cognitive science is said to be founded, that is symbolic processing or simply the traditional approach. The second approach emphasizes the role of the environment, the context, the social and the cultural setting and the situations in which people live, this new or recent process or approach is called situated action or "situation cognition".

The traditional symbolic approach and the situated cognition approach do not seem contradictory on the face of it, but they seem to only emphasize different behaviours and different methods of study. Vera and Simon (1993) to this effect argue that the new approach can easily be incorporated within the old.

Looking at the two more closely, the traditional information studies of symbolic processing focus largely upon the processing structures of the brain and their symbolic representation in the mind. That is, it concentrates mainly on the internal processes. For this process all action takes place inside the head. Thus it regards the human brain as the computational engine of thought and focuses upon understanding the working of the brain and mental representations.

The situated action approach to cognition on the other hand focuses largely upon the structures of the world and how they constrain and guide behaviour. This approach stresses that the Human Knowledge and interaction cannot be separated or removed from the world. What really matters for this approach is the situation, and the part that people play. According to this

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

approach a person is not taken in isolation, instead he/she is taken to be part of his/her environment . To take a human being in isolation is to destroy the interaction and therefore, to eliminate the role of the situation upon cognition and actions. This, therefore, means that the proponents of situated action tend to emphasize the importance of historical influences, social interaction, culture and the environment (Norman, 1993).

It is important at this point to also look at the work of De Bono CoRT Program. The program is said to be simple and practical, to have utility across a wide range of ages, abilities and cultures (thus De Bono's CoRT appreciates that thinking processes are fundamental). CoRT propagates that thinking skills trained, should be the thinking skills required in real life, for CoRT program, training in thinking skills is not to be dependent on the prior acquisition of a knowledge base. In this program, instruction is based on an understanding of the information handling characteristics of the mind. For CoRT Program, the mind is pattern making and pattern using. It would be very interesting at this point to look at De Bono's NPI (Negative Positive and Interest). He states that in order for one to make an informed decision about something then the list of negative, positive and points of interest should be made. Thus the external factors are used to make the decision, which in turn could be internalized and used to make other decisions without making the list. In this case transference would take place and the whole mechanism would be internal, i.e the external would be informed by the internal.

Norman (1993) goes on to say that scientists have access to three forms of information about human cognition: observations of human

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

behaviour, neurophysiological observations and measurements and subjective impressions. The author states that all these are faulty as both the behavioral and neuro-physiological measurements are limited in accessibility and by lack of interpretability. It is not only difficult or hard to measure what is going on, but also the people's ability to interpret is limited.

Norman states that the brain computes by a wide variety of methods, the computational band with some 10^{12} neural, each with as many as 10^{4} synaptic connections, is enormous and neural firings are only part of the computation. The author further says that the low-level recordings do not tell of the higher-order encodings (Norman, 1993). To this it can be said that, it can take a huge endeavour with billions of individual symbolic representations to explain human behaviour and understanding, especially everyday action and "commonsense" reasoning as the world is ever changing and very unpredictable. Part of this problem is the limited nature of the information extracted through sensory systems in both quantity and accuracy, whereas the other parts are temporal and environmental.

In addition to Jansen's, Norman's and Olivier's findings, it needs to be acknowledged also that, very little research has been carried out to find out whether there are some cognitive deficiencies displayed by students when solving mathematical problems, and if so, what the deficiencies are and how they can be addressed. It is important, in this regard, to examine what is involved in the learning process of the child and to consider the views of teachers and students, the work done by students, as well as the analysis of thinking-aloud interviews done by

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

researchers. It is through such an endeavour that one approaches the answer to what the causes of cognitive deficiencies in children in the learning of mathematics (differential calculus in particular) are. Therefore finding the solution to what seems to be the problem in the learning and teaching of differential calculus.

It is important to note that an attempt to study a person's behaviour and actions to the exclusion of his/her environment is as detrimental to that person as trying to study the life of fish outside their natural habitat, that is water. Therefore the study of pupils or learners should be carried out in the context of the environment in which they live. It is to this effect, therefore that reference to Maslow's hierarchy of needs (Maslow quoted in Jones, (1982)) is useful. These needs are crucial because ultimately they determine how the child functions cognitively. For instance, physiological needs are very basic in the life of a child (or of an adult for that matter) and when these needs are not met the psychological impact becomes very apparent when the pupil lacks concentration in class and is very anxious due to hunger. A close observation of the lives of Black students reveals that most of the needs outlined by Maslow, such as safety needs, and love and belonging are not met and this has a tremendous impact on their academic performance.

This is shown by Hamacheck (1976: 74) who states that "*research shows that healthy people see themselves as liked, wanted, accepted, able and worthy. Not only do they feel that they are people of dignity and worth but they behave as though they were ... These factors distinguishes between the high and low achievers*".

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

It is important at this juncture to link this argument to the entity that is of utmost importance in learning. Namely, intelligence, for the reason that when students perform poorly in academic tasks, they are often labelled unintelligent, the underlying assumption being that intelligence is something one is born with or without and is independent of environmental influences. There are different perceptions about what intelligence is and also whether it is malleable or not.

According to Perkins (1989) intelligence can be divided into three categories namely, neural, experiential and reflective. Neural intelligence refers to the contribution of the neurological system to intelligent behaviour. Experiential intelligence refers to knowledge and skills acquired through formal education and through life experience. Reflective intelligence refers to knowledge gained through speculative reflection, as in philosophy for example.

UNIVERSITY of the
WESTERN CAPE

It is in the reflective intelligence category that most contemporary efforts to "teach thinking" fall (Perkins, 1989). Instructional intervention, therefore, plays a very vital role in this category. According to Perkins (1989) there is evidence that intervention can boost students' reflective intelligence quickly and hence Perkins calls reflective intelligence the "mindware". This "mindware" is a collective term for the tools that one's mind uses to organize and direct thought in useful ways, in the same way as "kitchenware" is a collective term for the tools that one uses for preparing food.

Reflective intelligence can further be broken up into three categories (i) the micro-level which involves brief and

https://etd.uwc.ac.za/

relatively simple kinds of thinking that might be called "operations". Operations are the "bricks" of thinking, thus they are the elements out of which more complex patterns or constructions of thought are made, (ii) the macro-level involves more than just "bricks", here there is a whole edifice or "building", for example good decision making involves a number of operations (iii) the meta-level, which is the "highest" level of the three, can be equated to the "architect" who oversees the construction of the entire building. This level of organization concerns metacognition or what is aptly characterized as "thinking about thinking". Meta-levels to keep themselves working, are self-monitoring. They plan in advance and reflect afterwards to see whether there are ways in which they might have managed their thinking better. It is at the level of reflective intelligence that researchers believe instructional intervention can enhance student performance.

It is important, in short, to find out whether the problem lies in the "brick", the "building" or in the plan as a whole. According to Feuerstein (1991) we would have to look first at the cognitive functions, which are the most elementary information processes (the "bricks"); then move on to the group of cognitive functions and operations (both of which comprise the "building"). Finally we would arrive at a cognitive map, the architectural plan or "meta-level" indicating the place of mediated learning in this scheme of things. This approach helps us to find out where exactly the problem lies, whether it is at micro, macro or at the meta level.

There are many factors that can retard the performance of the child. These can be detected by the qualitative differences in

UNIVERSITY of the
WESTERN CAPE
https://etd.uwc.ac.za/

the thinking processes of the pupils. This means that it is vital for teachers not only to concentrate on the content, but also on the thinking skills, and strategies for solving problems (Heuristic for solving problems should be taught) as these are the factors that bring about the differences in the thinking of pupils.

It is through clinical interviews that students have shown that mathematical deficiencies may arise during the elementary school years and persist (if unaddressed) through the middle and high school years (Cawler & Miller, 1989). In this regard teachers should provide diverse mathematical learning opportunities to such students and utilize different instructional interventions as part of sound mathematics programmes (Giordano, 1992).

It is against this background that this thesis sets out to find out some of the reasons for cognitive difficulties experienced by students in the learning of differential calculus at all levels (the micro, macro and/or meta level). This is done by means of thinking-aloud protocols, which are mainly concerned with the thinking that goes with solving a problem.

This thesis seeks to find out (in the context of high school and first-year University mathematics students' and practising teachers' perceptions of) what some of the causes of cognitive deficiencies are. A definition of the problem follows.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

1.2 DEFINING THE PROBLEM

It is interesting to note, on the one hand, how pupils say that differential calculus is easy but how, on the other hand they do not pass it well. The pass rate for matric mathematics indicate that mathematics is one of the subjects with the lowest percentage pass. One would like to know therefore what it is that hinders students' performance, and in particular, whether cognitive deficiencies or difficulties displayed by pupils in the learning of differential calculus are significant factors. Further, as a matric final-examination examiner of four years standing, the researcher's impression is that students do not perform very well in differential calculus even though Std 10 differential calculus is one of the relatively simpler topics.

Having taught matric students for a period of five years, the researcher is acquainted with students' statements about how differential calculus is easy merely to please the teacher. Conversely, teachers (due to the time factor), are often contented to take students' word for it because of the pressure to cover the syllabus. This might account for the gap between students' perception of their own ability and their actual performance in examinations. What the gap points to, in any case, is that concepts have not been fully understood by students. Feuerstein et al (1980), hit the nail on the head when they describe the disadvantaged (mathematics) student as *"one who shows a level of cognitive activity which appears retarded as measured by his present performance but which does not necessarily reflect his or her potential cognitive capacity" (See also Mehl, 1988).*

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

The present research, carried out in the Western Cape area, thus sets out to discover the empirical as well as the clinical factors that might be said to underlie the observations of Feuerstein and his colleagues. The sample of 141 high school and University first-year students, together with 11 "thinking-aloud" clinical interviews, contributed towards finding the solution the problem.

The following questions which sought to clarify the problem being investigated, were asked through questionnaires and/or interviews:

1. Which problems do pupils think they have with regard to learning differential calculus?
2. Which cognitive deficiencies or difficulties do pupils have in the learning of differential calculus?
3. What role do motivation and affective processes and other non-cognitive factors play in the learning of differential calculus?
4. Is there any relationship between pupils' cognitive deficiencies, motivation and affective processes and pupils' own perception of what the difficulties are in learning differential calculus?
5. How do practising teachers perceive the above four questions?
6. To what extent are these difficulties carried over into first-year University mathematics students?
7. Which instructional strategies can be used to address such deficiencies?
8. What are the implications and recommendations for further study?

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

1.3 PURPOSE AND MOTIVATION FOR THE STUDY

The purpose of this research was to identify some causes of cognitive deficiency among high school pupils in the RSA. This was done by finding out what their perceptions were or what they thought the causes of cognitive deficiency were that make them perform poorly in differential calculus. The purpose is to find which factors of cognitive deficiency need more emphasis in order to improve both the standard of pupils performance and their understanding of differential calculus .

The present research was carried out in the Western Cape area with the intention of finding out the causes of the disadvantaged mathematics students' (in the Feuerstein et al mould) distinguished qualities or behaviour patterns in the learning of differential calculus, this is done through questionnaires and the "thinking aloud" interviews.

UNIVERSITY of the WESTERN CAPE

The research seeks to test whether underlying factors contributing to cognitive deficiency in differential calculus learning can be identified using: (a) The findings about the effect of the environment on cognition;(b) The findings that neural and environmental emphases are not mutually exclusive in the study of cognition; (c)The importance of Perkins' micro-, macro-, and meta-levels; (d) The findings about the school not being ready for the children; and (e) the finding from the thinking aloud protocols. In addition, supposing these stages yield results, then it stands to reason that the research should take Perkins hints into consideration and make some suggestions to mathematics teachers concerning actual classroom practice. In general the work by Feuerstein et al on modifiability was

https://etd.uwc.ac.za/

considered critically by the study in trying to find ways of minimizing some of the causes of cognitive deficiencies in the learning of differential calculus.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

CHAPTER 2 : LITERATURE REVIEW

PART 1

2.1 Language, culture and mathematics education

2.1.1 Cultural impact of a second language on acquisition of academic knowledge

Most of the children's world views are shaped largely by the environment in which they live. Therefore to be able to discuss the cultural impact of a second language (in the case of this thesis it is the impact of both English language and mathematical language) on the acquisition of academic knowledge, one needs first to clarify the meaning of culture. According to Cao (1986: 45), a people's culture is the dynamic result of the interaction between man through his needs and capacities and the environment with its resources and constraints in which one lives and develops. It constitutes a fund of knowledge and technical skills, values, aspirations, beliefs, attitudes, patterns of behaviour and relationships with all that makes up the environment (kin, fellow-citizen, human beings, nature and other spiritual forces, images or representations). The culture of a people is a reflection of its genius and its art in the pursuit of progress and happiness, in accordance with its needs and aspirations, the problems, possibilities and constraints imposed by its environment, the perception and conception that it has of its place within the universe, its role and the meaning of its existence (Uno, 1974).

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

From the above observation Valdes (1988) maintains that the teacher needs to know or get to understand the student before making any conclusions. The teacher will also need to determine methods and techniques of presentation, concepts and areas to be stressed, areas requiring tact or extensive explication for certain ethnic groups regarding what to expand from the printed material and what to omit or compress and most vital of all, how to make it interesting and non-judgemental.

Bochner (1976) has pointed out that what the student must accomplish is a knowledge of the culture to understand behaviour, not necessarily to become part of it. Once the second language learner comes to understand the behaviour of the speaker of the target language (English language and mathematics language in this case) regardless of the original motivation for study, the task of adding the language becomes far simpler, both through acceptance of the speakers of the language and through increased knowledge of what the language means as well as what it says. It should therefore be noted that language, thought and culture are not the same thing but none can exist without the others (Ellis, 1987).

UNIVERSITY of the WESTERN CAPE

From what has been discussed thus far one can also say that the learning of the second language is often second culture learning. In learning the second language therefore one needs to understand the nature of acculturation, culture shock and social distance. Acculturation is the process of becoming adapted to a new culture. This means that the re-orientation of thinking, feeling and communication are necessary. Cultural shock on the other hand is a common experience for a person learning a second language in a second culture (Malavé & Duyguette, 1991 and

https://etd.uwc.ac.za/

Bishop, 1988). Culture shock is associated with a feeling in the learner of estrangement, anger, hostility, indecision, frustration, unhappiness, sadness, loneliness, home sickness and even physical illness.

It is only after understanding and appreciating (thus knowing) something of other cultures that one can realize the importance of providing cultural clues to assist the language learner in a new environment, and to recognize what values and behaviour patterns of the new culture the learner has or needs to know. This therefore means that one's being able to read and speak another language does not guarantee that understanding will take place. This is because words are too limited a dimension, meaning that the critical factor in understanding has to do with cultural aspects that exist beyond the lexical aspects that include the many dimensions of non-verbal communication. But it should also be noted that various cognitive levels of understanding and affective levels of appreciation matter a lot in the learning of any content (Valdes, 1988).

UNIVERSITY of the
WESTERN CAPE

The communication process is very much influenced by culture, as is the language used as its medium. In acquiring a modern European language, the students from the third world have to accept, in contrast to their own culture, the close bond between most of the people's interactions, above all intellectual moves and norms, rules, theories, ideas and opinions. Teachers should therefore relate language to culture if a co-ordinated system is to result from the learner's efforts, bearing in mind that each pupil is unique.

https://etd.uwc.ac.za/

This is further explained in the next section. Reading successfully implies the ability to interpret (the culture of the material being interpreted should be known and used) and integrate the text meaningfully, i.e to read with understanding (Sinclair, 1987: 82).

2.1.2 The Schema Theory

Successful learning depends on how successfully the reader interacts with the text and on the nature or culture of the text. According to the schema theory, the reading process is seen as the construction of meaning by the reader when what is in the head interacts with what is on the page. This theory expands reading comprehension theory from being exclusively text-based to becoming text and reader-based, but the text-based should be understood as the medium inclusive of the author in his/her own cultural base, otherwise the message could be misunderstood (Sinclair, 1987).

UNIVERSITY of the
WESTERN CAPE

Sinclair (1987) says that the metacognitive awareness refers to four factors encompassed in every act of reading (1) the characteristics of the reader, thus the culture of the reader; (2) the nature of the text (to this, one would ask whether the text-book culture is known to the reader); (3) the strategies used in learning (methods used); (4) the task used to indicate whether comprehension has occurred (how to evaluate).

From the above discussion one would come to the conclusion that the text on its own cannot carry meaning, but meaning has to be constituted by the reader (Fay, 1988). What the text does is just to provide the reader with cues as to how to retrieve or

https://etd.uwc.ac.za/

construct meaning from the text and in relation to his own previously acquired knowledge, meaning that comprehension will depend on how the reader interprets the text in this case mathematics text.

Using the schema theory one can say that pupils do not comprehend by osmosis but rather they bring histories of their pre-school, learning experiences that actually is responsible for preparing the way for a smooth transition (Sinclair, 1987: 3). It would be better now to relate what has been discussed so far to the mathematical language.

2.1.3 Mathematical Language

In addition to the above discussion mathematical language can be said to encompass two aspects, one involving terms or terminology associated with initial concepts, the other aspect involves the processing of language which the child needs to structure thinking patterns associated with learning of algorithms. In teaching infants and primary school children, a large number of mathematical concepts are introduced which represent a new language for the learner. Thus mathematics also has its own language and therefore its own culture. First pupils especially at primary school level are taught words like, one, two, three, four and the language associated with operations, more, less, add, take-away, groups, share, divide and so forth. Later verbal language would be replaced by symbolic language which in turn would reflect the terms in a more abstract form. Language (mathematical) is introduced throughout primary and later through secondary schools thus increasing mathematical vocabulary. The secondary school pupils are taught, bearing in mind that the

https://etd.uwc.ac.za/

children already possess some mathematical or general language ideas. It involves the gradual evolvement of new ideas based on the background of the learner, meaning that one would build on what is already there (Orton, 1987).

To this effect Bishop (1988) says that, *"it should be noted that mathematics is not just about words and language. Neither is it just a skill to be learnt on a training course. Particularly for a mathematical enculturator, it needs to involve values, perceptions and beliefs of a personal nature, the communication of which is conveyed strongly by behaviour towards the mathematical cultural group itself and towards the people with whom one is communicating" (p 167).*

It can, therefore be seen that (mathematics) language (impairment or lack of it) can be an obstacle in the path of children's learning or understanding of mathematics (calculus in particular) ideas. Even if the vocabulary is appropriate, there might be problems because children do not only interpret statements literally, but in some cases they also change the meaning into what they think the teacher intended to say.

UNIVERSITY of the WESTERN CAPE

Orton (1987) says that the special symbols of mathematics as an extension to the language of mathematics cause more problems. It has already been said that learning mathematics or mathematical language is the same thing as learning the second language and the second culture (ie learning the second language). The process of learning vocabulary or mathematical language ought to be increased gradually. This is because some words used in mathematics are used with a different specialized meaning in other subjects, like 'chord' (Otton, 1987). Many

https://etd.uwc.ac.za/

problems also arise in terms of mathematical symbols and their corresponding meaning or words, this is especially true for primary students (Buxton, 1981). "The fact that children can interpret what we say in a way that is different from what we expected is but one part of the relationship between language and learning" (Orton, 1987: 134).

To Vygotsky (1962) language plays an instrument of thought in the proper sense in seeking and planning the solution of a problem. Thus language is important not only for communication but also facilitates thinking. Most pupils in South Africa have a very restricted knowledge of English which is mainly used as the language for studying mathematics. Most third world countries use English because some of the home languages (which would be the best alternatives) are alleged to lack the technical vocabulary and symbolism.

Orr (1987) carried out a practical study in USA and found that black American english learners have trouble with understanding mathematics. He ascribes this to the fact that learners are themselves language-limited by the unfamiliarity with mathematically loaded terminology. In addition to this, another research carried out in algebra shows that disadvantaged students (especially mathematically disadvantaged) are not very skilful at creating higher order thinking skills such as "mathematizing". By "mathematizing" Resnick (1988) means constructing links between formal algebra expressions and the actual situations to which they refer (Resnick, 1988; Fynn, 1987).

In mathematics language interpretation tends to be difficult to teach. This in itself constitutes a difficulty for mathematics

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

learning by the disadvantaged students, this, therefore, means that teachers should know both the culture of their pupils and that of the text. This is explained more in the next section.

2.1.4 Mathematics and culture

D'Ambrosio (1984) argues that the social and the cultural reality of the learner is the major factor in determining what is learned and that each culture has its own mathematics that has developed out of various forces within the culture. He indicates that people who object to this view are those who view the teaching of mathematics as an enterprise involving the transmission of an immutable body of knowledge (static knowledge, knowledge that is not meaningful) to the student, without concern for the various psychological and sociological conditions that influence learning. It is further suggested that mathematics is just an activity inherent in the human beings' existence, spontaneously practised and determined by the socio-cultural context and the material reality in which the individual lives (D'Ambrosio, 1984; Taylor, 1993; Bishop, 1990).

UNIVERSITY of the
WESTERN CAPE

Mathematics, like other subjects that involve or are studied by human beings, can not be treated as knowledge that exists on its own, without a knower, and that is to be transmitted without due respect to the learner's reality. D'Ambrosio further suggested that culturally approved mathematical models have become too powerful in the society and have come to compete with naive mathematical models, a situation that often results in greater interference with learners' ability to develop further in mathematics (D'Ambrosio, 1984).

https://etd.uwc.ac.za/

Just like Vergnaud (1981) focused on the indissoluble link between the signified (the mental representation), and the signifier, (external representation system), Kaput (1987) also argues that language used by individuals is connected to their mental representations, that is, to the way one elaborates and organises knowledge. That is having a true or right picture in the mind, of the concept being discussed.

It should be noted that language exists because users have agreed on the symbols to be used and rules to be followed. That language is socially constructed rather than just being a logical or empirical constraint. Thus the users follow the rules of language system and if problems arise the users can agree to change the rules. Mathematical language, therefore, is a medium that influences (directly) our entire lives (Sinclair, 1992).

Cummins and Swain (1986) stated that the cognitive skills and the ability to structure knowledge and to approach learning tasks can easily be transferred to a second language. To this effect D'Ambrosio (1984) says that the use of native language is the best for study, because critical thinking skills and cognitive structuring are conditioned by linguistic and cultural knowledge and knowledge and experience that children usually obtain in the home and bring with them to school.

It is clear, therefore, that language can cause some cognitive problems or difficulties to the student learning mathematics (or calculus). For most students (in South Africa) English is used as the medium of instruction. They use Mathematics textbooks written in english and even write examinations in English. As a result of these, many students (especially disadvantaged

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

pupils) experience difficulties in understanding their teachers and the material used, and struggle to express themselves in English. This goes back to saying that learning in a foreign language can result in serious cognitive difficulties. In countries like South Africa, the textbook becomes one of the most important vehicles for learning mathematics available to students (Brodie, 1989). Therefore the language used in this textbook may have a serious negative impact on the child's learning, if not clearly understood.

Austin and Howson (1979: 176) noted that in the internationally accepted form mathematical symbolism is a brief way of writing, the bulk of which has been devised by speakers of a few closely related languages (Indo-European language group). The use of this symbolism can cause difficulties to those whose mother language has different structures. It is, therefore, important to check whether language is in place (Rhodes, 1992).

UNIVERSITY of the
WESTERN CAPE

2.2 Preferences, attitudes and affective motivational processes

2.2.1 (i) Attitude toward mathematics

The Assessment of Performance Unit (APU) Primary Survey (1982), found that the relationship between attitude and performance in mathematics that at age 15 boys, demonstrated greater confidence in their own mathematical ability than did girls. It should therefore be noted that this may be caused by many factors (for example teacher being male, therefore gender discrimination, allowing pupils to do what they want, or even turning a blind eye when he finds them smoking, for example). It should be noted that gains in attitude to mathematics (calculus) and increased awareness of the nature of the subject are not easily measured.

https://etd.uwc.ac.za/

According to Hudson (1966), a liking for mathematics stemmed from preferred styles of study. Mathematics is taught in some as a form of static knowledge or just as being there and being hammered into one. It is not a subject you can 'humanize' (Russel, 1983).

Russel (1983) in Orton (1987) goes on to show that pupils often perceive the mathematics classroom as being a place for competition, which is attractive to some and not to others. Such a situation can act as an incentive to some students but for less successful pupils or disadvantaged pupils, the outcome would be a negative attitude. It should be noted that people who decide to study mathematics do not necessarily have a positive attitude towards the subject. In the study conducted by Russell (1983), many sixth-form boys studying mathematics were found not to like the subject mathematics. They did or chose the subject because of its job opportunities. On the other hand girls were found not to be very interested in the subject mathematics as most were not interested in becoming engineers. So as a result of the above, boys perceived mathematics to be a high status subject.

UNIVERSITY of the WESTERN CAPE

The negative attitude of many girls to mathematics appeared to deteriorate steadily through the years of secondary schooling, alongside the growth of self-consciousness about errors and difficulties (Orton, 1987: 122). One of the negative attitudes towards mathematics is that certain topics in mathematics are considered irrelevant by pupils. That is, pupils need not be told only the positive side of things or topics but also the negative side should be shown. Thus the hidden agendas of mathematics should be revealed to the students.

https://etd.uwc.ac.za/

Dillon and Sternberg (1986) suggest that the mathematical acquisition is more likely to occur in low-anxiety situations, while conscious learning is likely to be associated with situations involving at least a moderate degree of arousal.

There is a consistent association between creativity and attitudes and also according to Nickerson, Perkins and Smith (1985) (found that) attitudes can or are directly connected to originality, (even though scientific confirmation is needed).

According to Hutchinson (1985) most students developed positive attitudes towards the materials after halting starts, some continued to believe that they could not manage the required activities. Thus pupils disbelieve in their own mental ability, buttressed by a history of academic failure, greatly militated against their willingness to give themselves a chance. Such pupils believe that they were born like that and nothing could change their mental ability and that they were or are not going to develop any longer. Thus Hutchinson (1985) calls attention to the importance of recognizing the affective, as well as the cognitive demands that are placed on students by the learning task. Hutchinson pointed out that a student with history of academic failures may find himself/herself and his/her self-view seriously threatened by the environment in which his academic difficulties or problems would be exposed to others (Nickerson et al, 1985). It is important therefore to look closely at the impact that Affective-motivational processes and anxiety can have in the learning of mathematics.

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

2.2.2 (ii) Affective-motivation anxiety and mathematics learning

Mathematics would be effective if the pupils' experiences are challenging enough to arouse one's interest and active participation without being so threatening as to cause an avoidance reaction. According to Cornelius, (1982) some degree of motivation is essential if learning, rather than conditioning, is to result and the conceptual difficulty of much mathematics learning demands a correspondingly high level of active thinking if real development is to take place. One of the keys to human motivation is that the mind is naturally curious and can be stimulated by mathematical materials, and problems or any other thing that would stimulate it in a particular area thus having enough space or room for exploration and experiment.

Learning would be more efficient when motivation is intrinsic. This therefore means that what must be learnt ought to be interesting and this should be accompanied by some form of extrinsic motivation. Irrespective of external pressures and the roots of motivation seeming to lie in an eager curiosity, Hopkinson maintains that above all most able children are motivated to use their talents (Hopkinson, 1978).

UNIVERSITY of the WESTERN CAPE

Many pupils fear mathematics and have great difficulty learning it. In cases like this, motivation is not likely to be intrinsic for such learners (slow learners). The teacher should motivate pupils to recognise or see their problem.

It can be seen then that the background of the learner is very important in learning, as we said earlier, human beings are what

https://etd.uwc.ac.za/

their experience made them, so the attitude of the parents (home), peers, teachers (or school) and or ecosystem in general do affect the learner. These factors will affect pupils differently as they come from different environments. The way students are taught and the curriculum as a whole should be changed so that it can benefit the pupils (Cornelius, 1982).

Piaget's references to the domain of emotions acknowledge the duality of the cognitive and the affective and he clearly believes that many of his cognitive findings are mirrored in the affective area. Affective, life-like, intellectual life, is a continual adaptation, and the two are interdependent, since feelings express the interest and the value given to actions of which intelligence provides the structure (Richard in Cornelius, 1982).

The calculus (differential) has to be made more interesting for its own sake, and the type of teacher approval also needs to become more 'mature' and incorporate responses that obviously value pupils' opinions and respect their views. It should be noted that the image of oneself as being poor at mathematics could well spread into a more general assessment of oneself as a failure, but though it does not help the image of mathematics, but also it should be noticed that there can be safe mechanisms to prevent this (Ellis, 1986).

The role played by anxiety can be summarized as follows, high anxiety produces unpleasant feelings, (some of which are physical), that can distract the performers' attention as a result of this the field of attention narrows considerably. This seems to simplify the environment for the learner but in most

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

cases it will be found to have oversimplified situations that require complex appreciation.

Problems of demotivation through anxiety may be created through unsuitable subject matter, unsympathetic teaching and a whole variety of environmental factors. Some children do appear to panic quite badly, and this is clearly not helpful in fostering learning. One does not know what such anxious children might achieve under different circumstances. It is not possible to completely separate the cognitive factors from the affective (where cognitive may be thought of as pertaining to the recall or recognition of knowledge and the development of intellectual abilities and skills and affective as pertaining to interests, attitudes, values and appreciations (Bloom; Engelhart; Furst; Hill and Hathwohl; 1956).

The pupils' motivation affects effort. That is the simple comments (made by the teacher) like saying, "let me check your answer, ... I think you did a good job here ... not, I want you to do ...", may encourage the student to be more efficient and to seek help from others for metacognitive processes such as monitoring and reinforcement and in turn should give a feedback which is student-oriented (Ellis, 1986).

The efforts by some teachers to run a highly organized and tightly structured classroom may inadvertently reduce students' opportunities to learn and use metacognitive skills of self-structuring and monitoring. Student self-concept may be affected by the instructional atmosphere.

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

There are two aspects to the role of feelings to be noted, first that they are an important and integral part of learning and secondly that children's feelings, like their thinking processes, pass through various stages of development. Emotionally, as well as intellectually, children develop from an ego-centric stage where basic feelings such as pleasure, success, fear or failure are predominant, to a formal operational stage associated with questions of values and judgements (Cornelius, 1982).

Slow learners need to be motivated more, that is they need to feel that they can be successful if they try and that someone cares about their success, they need to feel that the work they do in school is worthwhile to them and is considered to be worthwhile by their teachers, their parents and the community.

One of the questions relating to reasoning is how people come to believe what they do. A closely related question of considerable practical educational significance is how to increase the willingness and ability of people to base their beliefs on evidence rather than to seek and mould evidence to support their beliefs.

UNIVERSITY of the
WESTERN CAPE

2.2.3 (iii) "Beliefs" and mathematics

Due to their experiences in mathematics classrooms, pupils develop a set of beliefs about mathematics and mathematical problem-solving. Most students in both secondary and high school, believe that mathematics is mostly memorization, that is only one right way to solve every mathematics problem and that mathematics problems should be solved, if at all, in a few minutes or less. This statement as it can be seen reflects pupils' attitudes towards mathematics. Schoenfeld (1987) goes

https://etd.uwc.ac.za/

further to say that if students do not view themselves as mathematics thinkers but view themselves as recipients of the 'inert' mathematical knowledge that others possess (Whitehead, 1929), then mathematics education for thinking is going to be problematical because the agent would be missing (Whitehead, 1929; Schoenfeld, 1987).

What students pick up from lessons is highly dependent on what presumptions they bring into the classroom. For example if a student has the impression that mathematics consists solely of symbols pushing, then regardless of what the teacher says, the student may think about any new algorithm in only superficial ways. If a teacher tries to "make sense" of the procedure, pupils will see this as "mere justification". Similarly, if students believe that mathematics is understood only by a "genius" and is supposed to be memorized by ordinary people like themselves, then they will memorize instead of trying to understand (Schoenfeld, 1987: 101).

UNIVERSITY of the
WESTERN CAPE

Some students even believe that if one really understands the subject matter, then any assigned problem can be solved in relatively short order. But to this Schoenfeld stated that some problems may take the class a few days or even a few weeks to solve.

Schoenfeld (1985) argues that belief systems an individual holds, can dramatically influence the possibilities or the work or understanding of mathematics education. He states that "Students" abstract a "mathematical world view" both from their experiences with mathematical objects in the real world and from their classroom experiences with mathematics ... These

https://etd.uwc.ac.za/

perspectives affect the ways that students behave when confronted with a mathematical problem, both influencing what they perceive to be important in the problem and what sets of ideas, or cognitive resources they use" (Schoenfeld, 1985: 157).

Kagan (1990) called for a line of research designed to identify alienation processes within classroom and schools noting the need for "a theoretical foundation for this approach drawn from (a) labelling theory and (b) the evidence that cognitive activity is interwoven with the environmental context in which it occurs. Thus for Kagan in Cooper (1993) " the labels teachers assign to students affect how the teachers perceive classroom events and how they respond to students", and (b) "students do not necessarily use the cognitive skills they possess unless provided with appropriate motivating factors". This, therefore, means that pupils need some guidance or need to be mediated in the learning of mathematics.

2.3 <u>Mediated learning experience</u> (MLE)

UNIVERSITY of the
WESTERN CAPE

2.3.1 <u>Defining MLE</u>

Feuerstein believes that there are two modalities of learning (a) direct approach and (b) mediated approach. A divided approach is based on Piaget's formula (S - O - R) which stands for or can be explained as follows: the organism (O) or the individual learner interacts directly with the stimuli (S) of the surrounding world and Response (R) or responds to it. This type of learning (or interaction with environment) is incidental.

https://etd.uwc.ac.za/

Mediated Learning Experience (MLE) on the other hand is the vital approach that ensures effective learning. For Feuerstein the formula for this would be S - H - O - H - R where H is the human mediator. The human mediator interposes herself/himself between the learning organism and the world of stimuli to interpret, guide and give meaning to the stimuli (Feuerstein, 1991). This kind of learning or interaction with the environment is guided, therefore, it is intentional.

MLE helps the child to be more receptive to direct exposure and benefit more from it. Mediated learning interaction is between the child and the mediator and therefore both can be blamed if a problem arises (thus the process of reciprocity takes place). In this way mediation becomes an open and dynamic process.

Feuerstein (1980) stated that he was concerned with the improvement or modifiability of individuals or simply one can say that he was concerned with the ability to learn and solve problems, which is why this ability fails to develop in the absence, (during early childhood) of systematic learning mediated by a caring adult (in this case differential calculus teacher) and how, much later than generally thought possible identified cognitive deficits can be remedied by a formal instructional programme. This deficient function relates to and helps identify the prerequisite of thinking, meaning that they refer to deficiencies in those functions that underlie internalized representational operational thought.

Feuerstein (1980) believes that the low level of scholastic achievement and low level of general cognitive adaption of the retarded performer, especially among culturally disadvantaged

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

adolescents, are a product of or of inefficient use of those functions that are prerequisites to adequate thinking. There are many reasons or causes of these low cognitive performances, they range from generic to environmental factors.

Modifiability according to Feuerstein refers to structural change or to changes in the state of the organisms brought about by a deliberate program of intervention that will facilitate the generation of continuous growth by rendering the organism receptive and sensitive to internal and external sources of stimulation (Feuerstein, 1980). Thus it can be concluded that MLE is basically a multidirectional and highly complex process in which both the inner and outer environments are mediated to one individual by another, more initiated human being, capable of organizing the environment in a meaningful and efficient way (Rand, 1991).

The deficient cognitive functions are of four kinds (i) impairments in cognition at the input phase (ii) impairment of the elaborational phase (iii) impairment in cognition at the output phase and (iv) affective motivational factors. These are intrinsically linked to MLE. That is they result from lack of or insufficient MLE. The categorization serves basically as a didactic paradigm, helping to understand better both assessment procedures and interventional methods (Feuerstein, 1980).

These deficient functions are taken or conceived of as elements that are weak and vulnerable. The deficient functions are distinguished on the basis of phases of mental act only to be understood as an artificial allocation, since the dimensions cannot be regarded in isolation from one another. The

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

subdivision is important because it provides one with the opportunity or possibility of producing the desired changes in the cognitive functioning by focusing our intervention on the appropriate phase while taking into account the current responsibleness of the organism.

The first group of deficiencies include impaired cognitive functions that affect the input phase of information processing (thus the gathering of all the information needed). The second group of deficiencies in cognitive functions is that affecting the elaborational phase during information processing. The third group of deficiencies include impaired cognitive functions affecting the output phase of cognitive operation or information-processing behaviour. For a list of the deficiencies refer to Appendix A. This list is based upon clinical observations and analysis of low functioning individual (or pupils) under various conditions and within different psycho-education settings.

UNIVERSITY of the
WESTERN CAPE

Feuerstein originally did not itemize the details of affective-motivational factor as he did for others. He only stated that these factors can combine negatively to influence the attitudes of the retarded performer.

In later works (Rand and Tzuriel; 1991) it is shown how a motivational-affective condition can affect the individual (the learner). Tzuriel for example said that lack of curiosity, self-confidence, competence and self-determination, optimistic view of life and general feelings of well-being, vividness, basic trust and harmonious relations with parents would be conceived as negative factors. It is also stated that the effective mediation made by an adult can facilitate motivational and

https://etd.uwc.ac.za/

affective processes such as arousal of exploration, seeking of challenges, feeling of competency and feeling of warmth towards the mediation.

Tzuriel argued that the continuation of efforts to arouse exploration without taking into account the child's motivational-affective condition will act as a boomerang to create a negative reaction. A schematic transactional model of the relations among MLE, Affective-motivational factors and cognitive modifiability is shown in Appendix B, figure 2. In this diagram four components - MLE, cognitive modifiability, motivational processes and affective processes are connected by double arrows, representing the transactional nature of the relationship.

It is from the above that it can be seen that a child who is alert, explorative, shows basic trust in social interactions and feels competent, invests more effort in learning and can therefore change his/her performance more than will an apathetic, passive child or a child who is absorbed with his/her emotional problems.

UNIVERSITY of the
WESTERN CAPE

2.3.2 Cognitive map

Instrumental enrichment of Feuerstein as well as the didactics of their application is based on cognitive map that aids in the categorization and definition of the components of mental acts. The cognitive map is the basis for analysis of the cognitive behaviour. The specific parameters of the cognitive map or model that serve to analyze the various components of the programme are:

https://etd.uwc.ac.za/

1. **Content**

The mental act can be described according to subject matter and can be analyzed in terms of the content on which it is operating. Content is one of the areas of cognitive functioning in which people differ (or perform quite differently) greatly with differences determined directly by experimental background.

2. **Operation**

Which may be understood as internalized, organized, coordinated set of actions in terms of which we elaborate upon information derived from external and internal sources. The operation may range from simple recognition (which in turn is made up of small operations or functions) to complex activities such as categorization.

3. **Modality**

Refers to the fact that mental act may be expressed in a variety of languages, including figurative, pictorial, numerical symbolic, verbal or a combination of these or others.

UNIVERSITY of the
WESTERN CAPE

4. **Phase**

There are three phases as already discussed namely: input, elaborational and output phases (Ref Appendix A)

5. **Level of complexity**

May be understood as the quality and quantity of units of information necessary to produce a given mental act.

6. **Level of abstraction**

Refers to the distance between the given mental act and the object or event on which it operates.

https://etd.uwc.ac.za/

7. **Level of efficiency**

Refers to the rapidity with which a problem can be solved for a specified level of precision. Rapidity-precision complex or the amount of effort objectively and subjectively extended by the individual in his or the production of the particular act can be considered as criteria of efficiency.

2.3.3 Summary

MLE may be viewed as the means by which elementary cognitive sets and habits are transformed into the bases for effective thinking. Therefore, the earlier and the more the children are subjected to MLE, the greater will be their capacity to efficiently perceive, understand and respond to information and stimulation in and out of school.

In effective mediator helps the child focus on certain features and interpret various experiences. Lack of MLE can occur if the agent fails to mediate to the child (eg apathetic parent, a parent too busy with other things, parent changing from "bad" culture or when the child possess barriers to mediation, eg emotional disturbance or impairment in cognitive functions).

By inducing into the individual repertoire new behaviours and new methods of reaction as well as by reinforcing existent adequate functions and coping skills, MLE may act to enhance the capacity (CA) of that individual and hence to improve the level of functioning. MLE should also act on the existent or induced function-bound need (NE) system so as to increase and reinforce the individual's propensity to put into action the various

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

functions. MLE also can act to enrich the spectrum of available orientation (OR). For the diagram refer to Fig 1 in Appendix B.

The quality of MLE interaction is best described by a series of twelve parameters as follows: (1) Intentionality and reciprocity, (2) Transcendence, (3) Mediation of meaning, (4) Mediation of feeling of competence, (5) Mediation of regulation and control of behaviour, (6) Mediation of sharing, (7) Mediation of individuation and psychological differentiation, (8) Mediation of goal seeking, goal setting and goal achieving behaviour, (9) Mediation of challenge, the search for novelty and complexity, (10) Mediation of an awareness of human the being as a changing entity, (11) Mediation of the search for the optimistic alternative, and (12) Mediation of the feeling of belonging. Of all, the first three are the necessary conditions for MLE, thus these three are universal and found in all races, ethnic groups, cultural entities and socio-economic strata.

UNIVERSITY of the
WESTERN CAPE

2.4 AUTOMATIZATION, CRITICAL THINKING AND CREATIVITY

2.4.1 Triarchic theory

Sternberg on the other hand specifies that the internal mental mechanisms are responsible for intellectual behaviours. Thus his theory relates intelligence to the internal world of the individual, to what goes on inside a person's head when he thinks and behaves intellectually. Unlike the psychometric view, for Sternberg the important question would be "what are the underlying mental processes that contribute to individual difference in intelligence?" and not measuring what is inside the

https://etd.uwc.ac.za/

head that makes a person intelligent (Sternberg, 1986). So this means to him intelligence is not a static but a changing entity.

The part of Sternberg's theory being how kinds of processes interrelate, this part deals with the roles of dealing with novelty and automatizing mental processing in intelligence. The third part is the theory of development. This relates intelligence to the external world of the individual, specifying three kinds of acts, environmental adaptation, environmental selection and environmental shaping that characterize intelligent behaviour in the everyday world. Thus to Sternberg the theory of intelligence (cognitive operations approach) relates to the internal world of the individual, to the external world of the individual and to experience.

One considers the basic construct in Sternberg theory as the sub-component or the exercises (instruments) that Sternberg gives, then these sub-components combine to form the components.

UNIVERSITY of the WESTERN CAPE

According to Sternberg a component is an elementary information process that operates upon internal representation of object or symbol (Sternberg, 1977).

The components may translate a sensory input into a conceptual representation, transform one conceptual representation into another or translate a conceptual representation into a motor output. As already stated the component can be spilt into successively finer subcomponents (which are elementary at a convenient level of analysis). It is at this level that deficient function like lack of encoding skills (for example) can be detected (Sternberg, 1986). The components can be classified

https://etd.uwc.ac.za/

by functions and by level of generality. Components can serve (at least) three kinds of functions namely meta-components, performance components and knowledge-acquisition components.

Meta-components are higher order control processes that are used for executive planning and decision making in problem solving. Thus the executive processes are used to plan what one is going to do, to monitor it while it is being done and to evaluate it after it has been done. Performance components are processes that implement the plans and decisions of meta-components in carrying out actual tasks. While acquisition components are processes involved in learning (acquiring or gaining) new information. With this in mind it would be better to look at some of the factors that help in problem solving.

2.4.2 <u>Novelty and automatization</u>

The ability to deal with a novel task and situation is vital when dealing with intelligence. This is illustrated particularly by the processes of insight. For Sternberg (1986) novelty is one of the several ways to measure intelligence through assessment of insightful problem-solving. Sternberg (1985) divides insight and components into three kinds, selective encoding, selective combination and selective comparison. [Selective component (like in acquisition component) involves shifting out relevant from irrelevant information. Selective combination involves combining selectively encoded information in such a way as to form an integrated plausible whole, thus one must know how to combine the pieces of information into an internally connected whole. Selective comparison involves relating newly acquired or retrieved information to information acquired in the past.

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

It is also vital when dealing with intelligence and/or problem solving to have a close look at automatization. The good example of ability to automatize information processing would be that of reading. The performance of a task as complex as reading seem to be possible only because a substantial proportion of the operations required are automatized. This is done without conscious thought and thus requires minimal mental effort. That is it is being proposed that complex, mathematical and other tasks can feasibly be executed only because many of the operations involved in their performance have been automatized. Failure to automatize such operations may result in a breakdown of information processing and hence less intelligent task performance. This goes well with what Riley and Shapiro (1989) stated when pointing out that, a student's processing deficiencies ought to be understood first before correcting the problem; they further stated that more focus must be on the correction component.

Schneider, Steeg and Young (1982) found that consistency in information processing is a necessary condition for the development of automatization. That is, a consistent strategy should be used throughout the task that one needs or wished to practise in order to automatize. They also found that improvement in performance appears to be primarily a function of people "learning from their mistakes", but this process is not reversible.

The full level of automatization can take anywhere from 200 to 2000 (or even more) trials of performance on a task, automatization can begin to set in at as few as 10 trials, so long as those trials involve consistency in strategic

UNIVERSITY of the
WESTERN CAPE
https://etd.uwc.ac.za/

performance. There may also be some degree of "consolidation time" in our automatization of information processing. The material to be automatized should be learned under moderate speed sprees. Automatization can also be more rapid if one devotes full attentional resources to the task at hand. It should be noted that automatization can be influenced by the context in which that performance occurs.

It has already been discussed in this paper that motivation is important in learning. It is also believed that motivation is often more important to automatizing information processing than is any other single variable. Self-esteem also plays a role in the automatization of information (Sternberg, 1986: 253). It would, at this point be worth looking at the role played by creativity and critical thinking in the solving of problems.

2.4.3 Creativity and critical thinking

UNIVERSITY of the WESTERN CAPE

Some problems need to be solved in a creative way; thus some problems cannot be solved without a fair degree of inventiveness, meaning that the ability to look at things in new and unconventional ways is undoubtedly an important problem-solving skill. Creativity is, therefore, an important aspect of problem-solving.

According to Nickerson et al (1985) creativity is the collection of abilities and dispositions that leads to a person most regularly or frequently producing creative products. Critical thinking on the other hand seems to be a sufficient condition for creativity, necessary on psychological if not logical grounds, meaning that creativity requires critical thinking (Nickerson et al, 1985: 89).

https://etd.uwc.ac.za/

Creativity is a highly complex trait that involves several qualities in the creative person. Nickerson et al (1985) state that some of these qualities may be primary and others consequential. The four plausible components of creativity are abilities, cognitive style, attitudes and strategies.

Abilities

Creative ability should in itself make a person more inventive in diverse or different ways. There have been many creative abilities proposed by various researchers to account for human inventiveness. That is ideational fluency, remote associates and intuition are some of the examples.

Ideational fluency refers to the ability to produce large numbers of appropriate ideas quickly and easily. Thus, people who can think of more ideas are supposedly in a better position to invent and to do so in diversified field.

UNIVERSITY of the
WESTERN CAPE

Remote associates

Creative ability (often) reflects the retrieval of information remotely associated with the problem at hand (Mednick, 1962 in Nickerson, 1985).

Intuition

An appeal to intuition is the last resort of a person having difficulty explaining something. Intuition could also be understood as the ability to reach out for sound conclusions from minimal evidence.

https://etd.uwc.ac.za/

Various investigators have found consistent associations between creativity and attitudes. Strategies for aiding creative thinking have been suggested by a number of authors. A few of the more common of these strategies are: making analogies, "brainstorming" effecting imaginative transformations such as "magnifying or reversing" listing attributes, challenging assumptions, defining a problem, seeking a new entry point, and setting a quota of ideas. In order to be (more) creative a problem solver needs to consider more alternatives before making a final choice. Brainstorming is designed to avoid the inhibiting critical attitude that often pervades formal meetings, the basic strategy being to generate a list of options and then to select from among them. It should, therefore, be realized that all creative processes are non-algorithmic, but not all the non-algorithmic processes are creative processes (Nickerson et al, 1985).

As already mentioned every genuine creative discovery is accidental in the sense that it is a result of a lucky stumbling upon an appropriate object, but it is not accidental in the sense that both the stumbling upon are results of active, directed, purposefully-conscious, or directed post-conscious search actions. It can therefore be seen that creative and critical thinking are important components of problem solving.

UNIVERSITY of the WESTERN CAPE

2.4.4 Problem solving

In order to make sense of incoming information and to be able to apply it appropriately, learners need access to a wide range of knowledge about how to solve problems. Pupils frequently need to engage in extended problem-solving in a domain in order to

https://etd.uwc.ac.za/

assimilate new facts and ideas, the problem is that the students' knowledge of problem solving is limited or even counter productive (Perkins and Simmons, 1988).

Usually, as said earlier, students like memorizing mathematics thus they develop stock answers and respond in stereotyped ways instead of actively engaging in problem-solving or being involved in active knowledge (Ingram, 1988). Heuristics (problem-solving) can be used to regulate the problem-solving process. The Heuristic-oriented approach sees thinking skills as a matter of appropriate know-how. According to this view in order for one to be an effective thinker one would need a repertoire of heuristics that are likely to be effective in a variety of problem situations.

What is this "Heuristics"? According to Groner and Groner (1983: 153) it means "serving a discovery". Thus when one is solving a problem, there would be a bit of discovery that would be involved in "real" problem-solving, hence problem-solving involves heuristic processes.

UNIVERSITY of the WESTERN CAPE

Groner, Groner and Bishof (1983) go on further, to explain heuristics as the science of finding solutions to problems whenever there are algorithms. Where algorithm is a formula for solution, that is a plan for the sequence of steps required in order to find a solution with certainty, if one exists.

Heuristic process, like any other cognitive process, can proceed in a non-rule-governed way, but it can be modeled and controlled only on the basis of rules. Heuristics and algorithms, differ regarding resultivity or more precisely, the degree of

https://etd.uwc.ac.za/

resultivity (Landa et al, 1978). In solving problems with an undefined field of choice the solver does not have available a complete set of all the possible paths and often does not know what they might be and so has nothing from which to pick or choose. Before choosing possible ways, paths or alternatives ought to be determined (Landa, 1976: 117). It should be noted that problems raised in artificial intelligence are not radically different from the problems raised in other disciplines working on heuristics. Examining what people do in an attempt to solve problems, is central to much cognitive science on problem-solving. What people do when solving problems can be explained as heuristics.

According to interpretive educational theory and from constructive perspective human beings build on their interpretive frameworks for making sense of the world, and then see the world in terms of these frames. But it should be noted that what one sees may not correspond with what is "actually" there or correspond to "objective" reality (Schoenfeld, 1987: 22).

UNIVERSITY of the
WESTERN CAPE

According to Piaget to know something is to assimilate it to a scheme. When a scheme is used it may need to be changed in order to fit the particulars of a new situation. This change is called accommodation. Assimilation on one hand can be taken as the modification of observation to make those observations fit internal models (schemes). Whereas on the other hand accommodation is modification of internal models to make them fit observations. In accommodation for example when a new scheme is developed, it is not copied from somewhere, and the new scheme is not given in the old scheme or in new data. In this way one can say that scheme is created through equilibration (equili-

https://etd.uwc.ac.za/

bration being a factor which "notices" self contradictions within person's knowledge and which reduces this imbalances through construction of new schemes (Piaget, 1977). It should, therefore, be noted that in the process of seeking a solution to a difficult problem, external (with real objects) and internal (with ideal objects) operations constantly alternate, which ensures the appearance in one's mind of new images and concepts or recall of available one's or reconstruction of their connections from both inside and outside (Landa, 1976).

Whimbey and Lockhead (1984) emphasise that lack of accuracy and thoroughness in thinking are primary causes for errors on the types of questions that are commonly found in tests of reasoning abilities. Whimbey and Lockhead also encourage students while solving problems to work in pairs and/or in groups and to "think aloud" while solving the problems to make it easy for one's approach to be analyzed and criticised.

UNIVERSITY of the WESTERN CAPE

According to Whimbey and Lockhead, there are five characteristics for one to be a good problem solver: (a) Positive attitude (confidence that reasoning problems will yield to careful persistent analysis); (b) Concern for accuracy, this involves rechecking one's work and making sure that one understands the problem completely; breaking the problem into parts (small manageable steps); avoiding guessing (solving the problem); and Activeness in problem solving.

Hutchinson (1985) attempted to describe how the classroom can be made a safe place in which to make mistakes. To do this the teacher or facilitator in this case encouraged students to discuss and analyze the mistakes that he as the facilitator

https://etd.uwc.ac.za/

makes. The purpose of this being to show students that mistakes are not a sign of stupidity, but they can be used profitably to understand or correct the next person (Chipman et al, 1985). The heuristic approaches suggested by Whimbey helped students to break away from the rote learning to an active one so that they could become efficient and motivated problem-solvers.

According to Rubenstein (1980), the failure to use the information that is known and the introduction of unwanted or unnecessary (data) are constraints in problem-solving. To overcome this problem, memory can be helped by (external method) the use of paper and pencil in solving the problem.

Gagné (1977) has expressed the view that problem-solving is the highest form of learning. Descartes in Orton (1986) in relation to that, stated that each problem he solved became a rule which served afterwards to solve other problems in this way. This means finding a heuristic for deriving heuristics. Problem solving would then imply a process by which the learner combines previous knowledge rules, techniques, concepts and skills to provide a solution to a novel situation. One aspect of problem-solving in mathematics is that often the problems are divorced both from the mainstream subject matter and from the real world (Orton, 1986).

UNIVERSITY of the
WESTERN CAPE

Lurkin and Reif (1979) give the following instruments as the prerequisites to problem-solving: (i) an efficient strategy to decompose the problem into sub-problems; (ii) a knowledge base which contains a set of solvable problems that act as building blocks for the solution of more complex problem; and (iii) a

https://etd.uwc.ac.za/

carefully organized knowledge base which allows easily retrievable information in different contexts.

Polya (1957) said that research into human problem-solving has a well-earned reputation for being the most chaotic of all identifiable categories of human learning. Thus the essence of Polya's How to solve it was the elaboration and justification of the self-questioning technique to be carried out by the solver. This technique involves the following four stages: (1) understanding the problem, (2) devising a plan, (3) carry out a plan, and (4) looking back. The first stage being the most important of them all, that is it involves steps such as drawing a diagram and introducing suitable notation, in addition to considerations such as whether the information provided is sufficient (whether the solver has gathered all the necessary information or data and whether it incorporates any redundancy). On the other side Hadamard (1945) also suggested four stages of problem solving: (1) preparation, (2) incubation, (3) illumination, and (4) verification. It can be seen that the first stage and the last are similar to those of Polya.

Schoenfeld (1985) indicates how useful dialogues among mathematics problem-solvers can be in learning to think mathematically. In his earlier book Schoenfeld (1978) has shown that students' abilities to evaluate and integrate can be greatly enhanced if they are given or taught a coherent manner of approaching problems. Schoenfeld's research methodology includes (novice) the observation of experts to determine: (a) regularities, strategies or implicit rules experts may unconsciously use as they solve problems, (b) breaking down the problem-solving process into smaller "chunks" which can be taught to students.

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

2.5 The role of memory in problem solving and learning

It is important to note the important role the memory plays (thus retrieval of information stored in one's head). There is a transient store of sensory information (via, for example the echoic and iconic memories) which gives a detailed replication of stimuli from the external environment as these impinge upon the human information processing system. The processes associated with this state are feature extraction and pattern recognition. Short-term memory is also a transient source of information. For this information can be retained for up to some minutes by rehearsal. According to Miller (1956) information is processed serially up to a maximum of 7 $\pm$ 2 chunks (where a chunk is a related segment of information not necessarily one discrete piece). Short term memory gets information from the sensory information store (Simon, 1981)

Short term memory (STM) appears to have two components: (i) the echo box (in which information is only retained by rehearsal and it is here where information can be integrated into the long-term memory (LTM) by rehearsal. It appears to serve as the "receptacle" for information retrieved from the long-term memory (LTM) which is being processed and manipulated.

Long-term memory is essentially infinite in capacity. Before knowledge can be used it can be transferred to STM or the STM must contain a symbolic pointer to the location of the information. The long-term memory and STM both store information and also there must be means of getting or having access to the stored information. That is in this way the memory will be having both the text and the index.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

Information can be retrieved from LTM not only via the index but also by following paths of links from one mode to another through intermediate modes. Retrieval using the index is called recognition, while retrieval using sequences of links are called association. Information in long-term memory is stored in so-called "semantic networks" (Mehl, 1982: 34); Schoenfield, 1987: 36 and Simon, 1981).

Semantic memory consists of concepts and relations among concepts it is the memory receiver and stores information about temporality-dated episodes or events and temporal-spatial relations among these events. Whereas episodic information is stored in terms of its autobiographical reference to the already existing contents of the episodic memory store. Memory representations also consist of images. These presentations are based on sensory impressions of actual objects or events. Images are very important aspects of human memory, therefore, very helpful and useful in problem-solving. This can be deduced from the following Chinese quotations, "A picture is worth 1000 words" (Simon, 1981: 413).

UNIVERSITY of the
WESTERN CAPE

According to Reese (1977) the studies of imagery are based on three general characteristics of images, all of which can affect memory: (a) imagery or an image is highly memorable, (b) images are coded distinctly from verbal memory code and thus provide a second mode of access to store information, and (c) images provide an effective framework for organizing material to be remembered.

It should be borne in mind that the "information processing" consists of controlling flow of information into and out of

https://etd.uwc.ac.za/

working memory by processes such as receiving information from sensory buffer and retrieving information from LTM, recognizing work, comparing and manipulating symbols in working memory and storing information in LTM. If a person can easily and quickly retrieve required information (active), then such a person is usually called an intelligent person. The history of intelligence is intimately connected with the notion of heuristics or rather the notion of heuristic search. Intelligent behaviour can be characterized as search, the search for solutions to problems requiring intelligence is typically so large that any system must use heuristics to reduce it to a manageable size given the system's limited resources of time and space (Groner, 1983: 16).

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

PART 2

2.6 Differential calculus

2.6.1 Learning differential calculus

One of the reasons why students do not perform well in differential calculus is the lack of pre-knowledge. For example some students at high school level still have difficulty with addition and subtraction, especially of fractions. In order to overcome this problem Young and Savage (1983) suggested that students should practice with concrete material for example cars, use cents and toys. They said that subtraction in particular is a stumbling block, so when involved in this process the idea of "take away" meaningfully by physically removing concrete materials would make more meaning or sense.

Concrete materials make differential calculus meaningful for children. Handling and manipulating them help the pupils to understand differential calculus better. Once the student is sure of oneself, then the transference of motor activities to mental acts will take place and hence application of that concept will be easy (Young et al, 1983).

UNIVERSITY of the WESTERN CAPE

Cornelius (1982) states that any discussion of mathematics and mathematics teaching have to be based on conscious or unconscious assumptions about the nature of mathematics as already mentioned earlier. The basic method of mathematics is to construct a simplified model but paradoxically, the greater the degree of simplification the greater also the degree of abstraction. Lots of problems are created by oversimplifying the models, by so

https://etd.uwc.ac.za/

doing variables which are signified in real life are cast away. It should be noted that the success of the model depends entirely on using the correct focus. No pupil will truly learn mathematics unless it becomes a skill which is used in personal mathematics activity.

The learning of mathematics depends very much on the pupils' grasp of earlier basic concepts which according to Cornelius (1982) should not be inferred from the ability to perform routine tasks in a narrow and limited context. The development of children's concepts is dependent on mental activity that take place as the child experiences and interacts with his environment.

According to Piaget the four main stages of intellectual development through which children pass on their way to forming adult cognitive structures, have an influence in a way pupils grasp mathematical concepts. The main value of these stages being characterized by the ways of thinking associated with each stage. Piaget's four stages are sensori-motor (period before the appearance of language), second, the period from about two to seven years of age (the pre-operational stage), the third is the concrete operation stage (a period from seven to twelve) and the final stage is the formal operations or propositional operations.

This Piagetian model is useful to teachers as a broad guide to the way in which pupils develop and as such has implications for the way in which mathematics should be geared so that the complexity of the subject matter is matched to the conceptual abilities of the child.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

Children of all abilities tend to be interested in practical work and the teachers should avoid falling into a trap of thinking that able children do not need concrete materials and that they prefer to think in the abstract. It has been suggested that there are two aspects of differential calculus ability namely, reproductive, which relies upon memory and the productive which relies upon thought and imagination (Cornelius, 1982 and Avital, 1987). Some of the characteristics of the mathematically (therefore in differential calculus) able child are ability to generalize, to switch rapidly from one operation to another, to remember, to find the quickest and easiest way of solving a problem and a "never tiring" attitude towards differential calculus or mathematical activities.

2.6.2 Content

Calculus is only mentioned in the Standard 10 syllabus directly, but there are many latent opportunities for the calculus from Standard 6 upwards as shown below.

UNIVERSITY of the WESTERN CAPE

Standard	Topic
6	The area and the circumference of a circle, irrational numbers. Relative frequency of obtaining heads in coin tossing.
7.	Ratio and rate. Gradient of a straight line. Division by zero not defined .
8.	Gradient of secant to curves, average rate of change and function values, speed and rate problems. The hyperbola.

https://etd.uwc.ac.za/

9. Tangents to circle, gradient of secants and average rate of change, absolute value of function.

10. Analytical geometry, sequences and series Exponential function - growth.

Avital (1987) states that calculus as it is now being taught is based upon two different concepts. (1) The rate of change of a function (R ChI) over finite interval or area and (2) the limit when a certain variable approaches zero. The first concept is fully algebraic, that is when given a function f: x --> f (x) one will have to compute and simplify the operation

$$\frac{f(x+h) - f(x)}{h}$$

Pupils usually have difficulty with algebra needed for simplification of the RchI while without simplication the limit cannot be reached. A good example of this is when given

$$\lim_{x \to 2} \frac{(x^2 - 4)}{x - 2}$$

UNIVERSITY of the
WESTERN CAPE

In calculus derivative of $f(x)$ at x_o is usually defined to be

$$\lim_{h \to o} \frac{f(x_o + h) - f(x_o)}{h}$$

leaving the students with the impression that this is the only way to define $f'(x_o)$ and not telling pupils other options. They later get surprised when told that for $\frac{[f(x_o + h) - f(x_o)]}{h}$

small h is a very poor approximation to $f'(x_o)$. Thus usually teacher like $\lim_{h \to o} \frac{f(x_o + h) - f(x_o)}{h}$

(or simply writing $\lim_{h \to o} \frac{f(x + h) - f(x)}{h}$)

to $\lim_{h \to o} \frac{f(x + h) - f(x_o - h)}{2h}$

https://etd.uwc.ac.za/

Therefore using points symmetrically located on either side of $(x_o; f(x_o))$, even though the latter converges to $f'(x_o)$ faster. By relating each limit definition of $f'(x_o)$ to the underlying geometry of secant lines, the validity of expression are illustrated in concrete, visual terms.

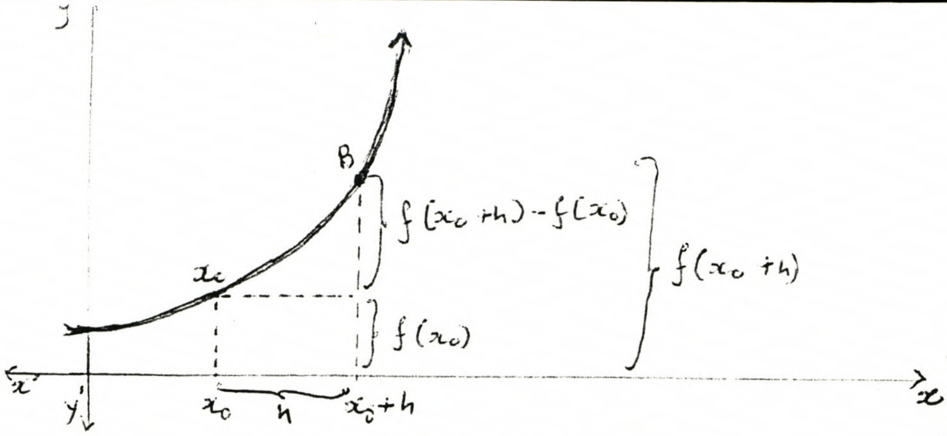
All beginner calculus students are shown that the target line to the graph of $y = f(x)$ at $(x_o; f(x_o))$ is the limiting position of the secant line using $(x_o; f(x_o))$ for one point and allowing the second point to approach the first point infinitely close. This phenomenon has the obvious result that the slope of the tangent line (denoted $f'(x_o)$ is the limiting position of the slope of the secant line through $(x_o; f(x_o))$ and $(x_o + h, f(x_o + h))$ or steepness of tangent line = lim (steepness of secant line through $(x_o, f(x_o)$ and $x_o + h, f(x_o + h))$.

$$f'(x_o) = \lim_{h \to o} \frac{f(x_o + h) - f(x_o)}{h}$$

This last expression can be/is used as the definition of the derivative at x_o. In a numerical analysis class, students learn that there is no need to use a secant line through $(x_o; f(x_o)$ in fact, it is found that the slope of the secant lines converge to the tangent line slope faster if one uses points symmetrically located on either side of $(x_o; f(x_o)$ (Avital, 1987).

More emphasis should be placed on diagrammatical or schematical representation. That is the average gradient of a curve between two points on the curve should be shown as the gradient of the line segment which joins the two points. Thus the average gradient of a curve f between the points $x = x_o$ and $x = x_o + h$ should be determined as follows:

UNIVERSITY of the
WESTERN CAPE
https://etd.uwc.ac.za/



It is only after this that one (teacher) should move on to notation which is used to indicate that the limit of an expression is to determine if the value of the variable in the expression tends towards (or approaches) a certain number or infinity. Thus in this way, gradual move towards differential calculus would be made.

According to Laridon P (1989) in order to develop correct concepts it would be valuable to spend some time on cases such as the ones illustrated below. If f is defined by the graph (starting with straight line, the quadratics and finally cubic graphs), evaluate, if possible.

(a) (i) $\lim_{x \to a} f(x)$ (ii) $f(1)$

where "a" is an element of Natural numbers. This being to check whether students really understand what limit means and how to apply it.

Next example would be if

$$f(x) = \begin{cases} x - 1 & x \geq 3 \\ 5 & x = 2 \\ -x + 1 & x \leq 1 \end{cases}$$

Evaluate the following, if they exist:

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

(a) $\lim_{x \to 0} f(x)$ (b) $f(0)$

(c) $\lim_{x \to a} f(x)$ (d) $f(a)$

The other example that needs attention is $\lim_{x \to 0} \frac{|x|}{x}$

Laridon (1989) states that in order for the above examples (problem of these nature) to be treated efficiently in Standard 10 it would be necessary to expose pupils to the manner of defining functions which they entail in Standard 8 and 9. The first and second examples lead one to the definition for continuity as: Definition 1.

Where f is said to be continuous at x = a if the following three conditions are met

(i) $f(a)$ is defined

(ii) $\lim_{x \to a} f(x)$ exists and

(iii) $\lim_{x \to a} f(x) = f(a)$

To this effect Laridon (1981) gave the following example to illustrate the above definition

(a) $\lim_{n \to x} \frac{(1 - (\frac{1}{2}))^n}{x}$ (b) $\lim_{h \to o} \frac{(x - h)^2 - x^2}{h}$

The difference between these would be that n is an element of natural numbers but h is an element of real numbers; n -->x but h --> o; n approaches x from below only but h approaching 0 from above and below.

Definition 2 $f'(x) = \lim_{h \to o} \frac{f(x + h) - f(x)}{h}$

provided the limit exists

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

Definition 3 $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

provided that this limit exists.

Laridon further suggests that the product rule should be introduced in high school syllabus, particularly for pupils who are likely to continue with mathematics after matriculation, that their original schemata in the calculus be sufficiently formed to be receptive to later developments. Proper equilibrium can only occur if there is closure of the concepts concerned. The present higher grade syllabus is said to cause difficulties in this regard (Lardon, 1989).

2.6.3 Other factors to be considered in teaching and learning of differential calculus

Mastery of differential calculus facts requires more things than just accurate performance. Rate of performance, or fluency, is an important aspect of skill mastery that is often overlooked as a target for instruction in the classroom by both students and the teachers. Conducting time trials can be an effective way for building fluency. Time trials also provide students with many opportunities to respond. The time trials can be easy to implement, and therefore could easily be enjoyed by students.

The teacher should get stop-watches (if she/he thinks it is necessary) and a chart and encourage each student in the class to beat his or her best scores, the score improve the speed of the pupils (Miller & Heward, 1992).

It is important that teachers should give the feedback as quickly as possible because the biggest problem with delayed feedback

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

during the acquisition stage is that it provides "an opportunity for children to practice mistake and learn poor habits that are often difficult to replace (Miller et al, 1992).

The assessment of mathematics abilities is vital for formulating an instructional program for students with special needs. Thus students show that mathematical deficiencies may arise during the elementary school years and persist through the middle and high school years (Cawler and Miller, 1989). Differential calculus teachers, therefore, need a deep understanding of subject matter, for without it one would not teach "the right stuff" and also teachers need an understanding of learning and thinking processes and skills (Schoenfeld, 1987).

It should be noted that what is important is not only the transmission of specific skills or abilities but the development of the prerequisite cognitive schemata to enable an individual to derive maximum benefit from direct exposure to sources of stimulation.

UNIVERSITY of the
WESTERN CAPE

Thus as Barnard and Strauss (1989) state, mathematics is indeed of paramount importance to identifying the obstacles that impede pupils' progress, the types of errors pupils make and the reasons why they make them. Barnard et al. go on further to state that many of the pupils did not know what the mathematical terms, expression and formulae mean. Thus pupils often gave correct answers but failed to understand the concepts and principles on which these were based. These also apply to differential calculus.

https://etd.uwc.ac.za/

To be able to teach for understanding we have to relate the new as much as we can, to what has already been learned before, which is previous knowledge, so that new knowledge will grow from and upon the old, thus for example students cannot be taught how to differentiate before first having done the limits. Thus interrelated ideas are fast to store in memory and harder to forget than the compartmentalized ones. Meaning that the interconnected ideas are easily retrieved from memory when needed rather than isolated ideas or knowledge (Avital, 1987).

There is indeed inadequate or no time for Standard 10' students to improve their skills in algebraic manipulating, but if there would be time then it would be very beneficial (Avital, 1987).

It should be noted that teachers should try as much as possible to create a visual image of concept that they try to introduce. Thus for example there are very interesting ways to visualize the concepts of the derivative namely via linear approximation. Thus a search for the derivative at a given point on a given function as the search for a line which passes through the point and will produce the 'best' linear approximation the values of the function in the vicinity of the given point. Pupils will generate the idea that such a 'best' linear approximation is a line whose slope is the same as the "slope of the function of the given point. This slope is the limit of RchI when the interval that starts at the given point gets smaller and smaller. This visualization is a good motivation for the construction and analytical description of the tangent to the function of the given point. To survive in mathematics courses, many students attempt to compensate for their lack of understanding and

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

visualization by memorizing mathematical procedures and formulas (Simon, 1985: 41).

Facilitating students' discovery of differential calculus concepts requires far more than letting the students do things for themselves, but rather needs the process or form of teaching that requires teachers to examine the cognitive structure of the concepts to be taught and then to create series of experiments or exercises that will offer students the opportunity to explore the domain and discover concepts (Simon M, 1986).

Students' discovery of differential calculus concepts can further be enhanced by having two or more students solving the problem (group or peer problem-solving). The students are first exposed to diverse thinking and problem-solving approaches. Secondly, they develop metacognitive skills-knowledge about their own cognitive processes and the ability to use them (Flavell, 1976 & Schoenfeld, 1985).

UNIVERSITY of the
WESTERN CAPE

Let us look at the following example by Volmink (1993), "there are 20 sheep and 6 goats. How old is the shepherd?" The author says that if experience elsewhere in the world is anything to go by, there is much greater chance that most students will attempt an answer to this "Problem" and that their answers would be remarkably similar. Thus the cumulative experience of students has lead them to adopt the view that mathematics is the necessary outcome of meaningless things. This is very common also in calculus word problems, whereby students just answer the problem for the sake of answering but without real understanding. This view is supported by the culture of learning and the dominant modes of teaching.

https://etd.uwc.ac.za/

2.7 POLITICAL DIMENSIONS OF MATHEMATICS EDUCATION

Political mobilization and democratic decision-making are said to be necessary but not sufficient elements to start an emancipatory learning process from concrete conditions. Thus through Praxis, the politics of mathematics need to develop consciousness of struggle in the process. It should therefore be noted that if differential calculus is produced in practices with specific properties then analysis of the production of those practices has profound implication for mathematics education (Noss, 1990: 10).

At the first meeting on July 1991, many South African mathematics education organizations (members) argued that the Mathematical Association of Southern Africa (MASA) indeed has been very political (even though it was said to be apolitical), supporting the status quo "white only" membership and that all mathematics education is political (Julie, 1991: V). This is one of the underlying assumptions of People's Mathematics, thus it attempted or addressed the following problem: "How mathematics is utilized to marginalize certain sectors of society". In order to address this problem it is shown that Mathematics is a panhuman activity and that "abstract mathematical concepts and reasoning had very concrete roots in practical and everyday problems of commercial and social value" (Julie, 1991: 4). As already discussed under "Culture" People's mathematics takes into consideration the culture of the students together with that of the material to be studied. It needs to be noted that social and/or political movements are dynamic projects, not finished products. This therefore means that studying early ideas and debates would help pupils or students even educators remember to keep developing new

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

ideas and debates. Slammert (1991) said in this regard that pupils need to learn from others and think for themselves.

Pupils should be taught not only one side of mathematics (that is computation) but they should be taught the other side of mathematics, meaning the side that will affect their lives and their future. The hidden agenda's of mathematics should be exposed so that pupils would know what is frustrating them (Mellin-Olsen, 1987). Fasheh (1982: 8) to this point argues that "*... teaching mathematics through cultural relevance and personal experiences helps the learner know more about reality, culture, society and themselves. That will in turn, help them become more aware, more critical, more self-confident. It will help them transform some existing structures and relationships.*"

Freire (1970) has stated that education either acts as an instrument to integrate students into the logic of the status quo or it serves as the means of enabling people to critically transform their world.

UNIVERSITY of the
WESTERN CAPE

The powerful insight into both the contemporary curriculum dilemma and the emerging problem of curriculum reconstruction can be gained by describing the curriculum as the product and expression of the political interests, values and knowledge of the dominant social group (Jansen, 1987). It should be noted that for many years white politicians and educationists in South Africa have recognized the vital connection between curriculum content and the ideological and material interests of a racially stratified political economy, and used it to their own benefit (Jansen, 1990).

https://etd.uwc.ac.za/

2.8 UNDERSTANDING

Understanding can roughly be explained as the ability of people to display a certain range of generative performance involving explanation. Perkins (1987) states that while the present analysis emphasizes the importance of explanation to understanding, its worth remarking that people, might be said to "understand" in certain senses of the word without being able to explain or putting it in words. This type of understanding is very important especially for second language learners and therefore mathematics learners , who find it hard to explain or express themselves using a foreign language.

A person who is said to understand should, mentally explore a rich conceptual network that is usually called an explanation structure (which is a rich network of explanatory relationships that are encoded mentally in any of the many ways in which the mind operates it can be through words, images, cases in point, anecdotes, formal principles and many others. It is extensible and revisable).

UNIVERSITY of the
WESTERN CAPE

To display understanding, thus building, extending and revising explanation structures one would need to be able to call upon several resources. For example, (1) knowledge, (2) representation, (3) retrieval mechanisms, and (4) construction mechanisms. This four dimensions are called Access Framework (Perkins and Crismond, 1990).

It should be noted that a person cannot understand or achieve understanding without content knowledge. The understanding of differential calculus for instance would call for knowledge of

https://etd.uwc.ac.za/

such concepts as variable, graphs, equation, solution and expression.

As said earlier the learner needs access to a wide range of knowledge about how to solve problems in order to make sense of the incoming information and to be able to apply it appropriately (cf Brown, Bransford, Ferrara and Campione, 1983).

Students need epistemic knowledge in order to be able to build cohesive and flexible explanation structures. Whereby epistemic means knowledge of the "rule of the game" for justification and explanation in a domain.

But it is worth noting that in order for knowledge and representations to function as tools of understanding they (knowledge and representations) should be retrieved from long-term memory as discussed earlier in this chapter). Thus understanding something fully means having active knowledge about that particular concept (Perkins, 1987). Therefore, just as pupils' physical access to good teachers, facilities and materials, there is a need of access (mental access) to a wide repertoire of higher-order knowledge, accessible representations and rich contexts that facilitate activation of relevant knowledge (Perkins & Crismond, 1990).

Each and everyone of us has a conceptual model of events in the world. Thus models serve as the instruments of understanding. These models become sophisticated over many years and can help in making sense of stimuli that impinge upon one's sense (Adams, 1990).

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

Cornu (1991) states that students often believe that they 'understand' the definition of a limit without truly acquiring all the implications of the formal concept. He said that there have been different investigations which have been carried out showing that the majority of students do not master the idea of limit, even at a more advanced stage of their studies.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

CHAPTER 3 : RESEARCH METHODOLOGY

3. INSTRUMENTS AND TECHNIQUES

3.1 INTRODUCTION

3.1.1 CLINICAL INTERVIEW TECHNIQUES

Researchers in both Science and Mathematics Education have become interested in the cognitive processes which students use as they "think about science" or "think about mathematics". To investigate these cognitive processes, researchers have made extensive use of techniques known as Clinical Interview (Cannell and Kahn 1968).

Clinical Interview is a popular investigative tool among Science and Mathematics Education researchers. It is highly interactive, deeply probing and at the same time producing a vigorous data. Clinical interview allows the interviewer to have access to the child's highly individual world and his/her special experiences, making it easy to diagnose the child and using an appropriate therapy as a result thereof.

Clinicians are expected to have certain talents, that is they have to know how to stay out of the child's way as s/he reveals, at his own pace and in his own "words", the character and contents of his experiential world. At the same time the talented skilled and/or trained clinicians will know when and how to step in and help the child communicate in an increasingly rich and personal manner. Due to their natural ingenuity, children will share their thoughts and feelings with the clinician using

https://etd.uwc.ac.za/

various communication channels. The clinicians' task is to be tuned into all these channels simultaneously and to be alert to every subtle difference in the meaning conveyed (DSM III, 1986).

A Clinician must, therefore, be a highly observant and should possess the necessary skills to do so, which includes seeing, hearing and various ways of sensing all the data the child provides. A fundamental requirement of being a good observer is to resist the common human tendency of jumping into conclusions on formulation. It is only when all the relevant and necessary information is in, that one should formulate a necessary diagnosis. It can, therefore, be concluded from this that the basic requirement of a good clinician is the ability to be observed in several dimensions simultaneously (Greenspan 1981)

There are other features that need to be looked at when conducting an interview. For example it should be noted that children put their messages across in the way they tend to look at or even avoid looking at the Clinician, both the depth and style of their personal relatedness, their mood, various emotions they manifest, the way they negotiate the space of the interview setting and the themes they develop in talk and play and other ways as well. This can be summed up by stating that a good observer must continuously monitor every communication channel, both verbal and non-verbal, that the child employs throughout the interview session. It is important to have a thorough understanding of the processes that are involved in solving a problem because this is absolutely necessary in the formulation of models of human cognition in addition to the obvious pedagogical implication (Davis 1975).

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

It is on the basis of this methodology that various kinds of analytical person-to -person interviews which I conducted in clinical research (techniques) will be discussed. Then an analysis of Eleven (seven high school pupils and four first year university students) person-to-person interview, will be made, determining the difficulties which students experience and display. Before analyzing this person-to-person interview it is worth looking at the various kinds of analytical person-to-person interviews.

3.2 <u>The various kinds of analytical Person-to-Person interviews</u>

Although there are numerous ways in which an interview can be conducted and analyzed, here only three types of problem-solving interviews and three levels of protocols analysis will be discussed. The three interview formats differ primarily in the kinds of comments (Probes) the interviewer is allowed to make.

UNIVERSITY of the WESTERN CAPE

They are (i) Thinking-aloud interview, (ii) Indepth interview and (iii) Tutorial interview.

(i) Thinking-aloud interview

In this type of interview the interviewee is given a problem to solve and is expected to verbalize his\her thoughts as he\she solves the problem. The interviewee is not expected to engage in self-analysis or to reflect on previous thoughts. Probing [if at all used] is just used to encourage interviewee to vocalize more and not to lead the subject [interviewee] to the answer (Konold and Well 1981)

https://etd.uwc.ac.za/

(ii) Indepth interview

This type of interview is also referred to as a Piagetian or clinical interview just like in ''thinking-aloud'' interview, the indepth interviews also involves presenting subjects with a specific problem and questions as they attempt to solve the problem. The probing technique, however, is much more flexible than with the thinking-aloud instructions. Unlike in thinking-aloud interview, here the interviewees are asked to reflect back on what they have just done and are sometimes offered subtle challenges on their thinking. However, the interviewer is not expected to (purposefully) give evaluative responses, nor provide hints in the form of questions, statements, gestures etc. (Konold C. et al 1981).

(iii) Tutorial interview

Here although the minimum necessary help is provided, the interviewer is interested in getting (or eliciting) a correct solution. So probes are used to guide the interviewee toward solution strategies they might otherwise not have considered.

The description of different types of interviews given above is very important. However, it is also crucial to discuss (briefly the levels of protocol analysis so that a complete picture of clinical interview techniques can be created.

For the levels of protocol analysis the following will be discussed: (i) coded analysis, (ii) Descriptive analysis, and (iii) interpretive analysis (Konold et al. 1981)

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

(i) Coded analysis

This involves identifying key elements of interest in the protocols and defining them in such a way that their presence or absence may be noted by raters. Elements may be defined as grossly as the use of certain problem-solving strategies, or more finely as the presence of key words, phrases and equations.

(ii) Descriptive analysis

The interviewer or researcher is interested in providing a clear restatement of what the subjects said and did during the interview, describing the data as they are, making no inference about underlying structures that may account for the data. The focus of this analysis is on the surface structure of the subjects verbalizations, that is on the meanings they are explicitly trying to communicate.

UNIVERSITY of the
WESTERN CAPE

(iii) Interpretative analysis

Here inferences are made about the deep structures of subjects' reasoning processes. What the interviewee says and does to make statements about the processes and knowledge structures (both explicit and tacit) the subjects are using to solve the problem is used by the researcher (Konold, 1981)

3.4 Discussion

It is worth noting that a single interview can proceed through different bases. It can begin with the interviewer using only the facilitatory probes characteristic of the thinking-aloud

https://etd.uwc.ac.za/

interview. Once a solution has been arrived at, more indepth probing may be used, followed (if required) by probes designed to lead the subject to a correct solution. This is what happened in all the interviews I carried out. That is all the different types of interviews and analysis were used: thus first leaving students on their own, then using probes just to facilitate, then moving on to indepth probing to find out where the problem lies.

Some investigators (myself inclusive) who seem to be primarily interested in descriptive or interpretative analysis also code parts of the interview, and it is often the case that entirely coded protocols are used to support descriptive or interpretative statements. If coded information was included in a report, then the attempt was made to account for all or most of the subjects' verbalizations and behaviour, it was categorized as a description of interpretive analysis. These studies often include large segments of protocols along with their analysis.

UNIVERSITY of the
WESTERN CAPE

Now having discussed types of interview and the levels of analysis one can proceed on to addressing the actual process of conducting the analysis. The simple analytic approach is to code the selected parts of the interview, transforming them into manageable, quantitative data. (Ericson and Simon 1980).

It should be taken into consideration that psychological research, by its nature is reflective, that is, human beings hold particular views about how and why humans behave as they do and if one takes these views seriously, they ought to be equally powerful in describing the behaviour of psychologists or clinical researchers.

https://etd.uwc.ac.za/

When analyzing the protocols it is critical that one should recognize the limits of one's objectivity. For example in problem solving, subjects are guided and limited by their existing knowledge and their prior knowledge. Thus prior knowledge serves as the means by which one (interviewer) comes to an understanding of the interpretation of the subjects' understanding. But it should be noted that the role of the researchers is to accommodate the protocols they collect to inform them, rather than simply confirm prior expectations.

Person-to-person interviews help in developing models of students' problem solving that are powerful enough to capture important individual differences, yet not so specific there would be as many models as the number of students.

In analyzing the protocols there will be frequent alternatives between indepth analysis of individual protocols in which the interviewer tries to understand what an interviewee thinks about a particular problem to a more general analysis in which interviewers ask what characteristics do all subjects, or a subgroup of subjects, share in thinking about the same problem. Thus this strategy provides interviewers or researchers with an interactive framework for hypothesis formulation and testing, meaning that the hypotheses formulated on the basis of a single protocol are "tested" on individual protocols. So in this way the interviewer would be avoiding the extremes of either nomothetic or idiographic approachers (Fredette, 1979).

There are important issues that also need to be addressed in the report of interview research. For example interviewee, characteristics, interviewer characteristics, material and instructions and Interview situation and characteristics

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

3.5 Description and selection of schools

As a full-time student (thus not having my own school pupils) I had to make arrangements with three schools in the Western Cape. One school was for pupils who are repeating subjects that they failed in standard 10. This is usually termed a "finishing school" and it is based in the Western Cape. The other school was situated in the area for Africans (location). I shall term this a "Black high school" and the last school is a so- called "Coloured" high school.

These schools have the same characteristics. The pupils attending these schools (selected for the interviews) have generally the same background (socio-economic and political) and mostly from the lower class level (working class) as compared to their white counterparts (who are of the first world status).

The schools were just selected at random (bearing in mind that as the full-time student I had to choose the schools close to the University campus). The ages of students both interviewed and the pupils given questionnaires ranged from seventeen years to thirty years.

3.6 Interviews

The interviews were conducted in the format described earlier in this Chapter. There were seven high school interviewees and four University first year mathematics students or interviewees. Of the seven high school pupils three were girls and four boys. Two girls and one boy were doing standard grade mathematics whereas the rest were doing higher grade mathematics. The university

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

students were two boys and two girls. (See Tables on Page 78 and 80).

The interviewer explained to each interviewee what the purpose of the interview was. The interviews were conducted in the office of the Head of Department of Science which the student seemed to be relaxed and at ease with.

This was done so as to rid the students (especially high school ones) of the misunderstanding that this was some kind of test. An audio-tape was used for the thinking aloud protocols and for the general discussion of the questionnaire with the pupils. This was done in full agreement with research participants.

3.7 Interview situation and procedure

Each participant was interviewed alone and each interview proceeded as follows, an interviewee was given a calculus problem to solve. It began with the interviewer using only the facilitator probes characteristic of the thinking-aloud protocols. Once the solution was reached, more indepth probings/probes were used of necessity. The probes were designed to lead the subject through a correct solution and then a correct answer would follow.

The interviewees were allowed to remain silent for at most two minutes before a probe was given. Unless the interviewee murmured, in which case at most three minutes were allowed to lapse.

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

3.8 Questionnaires

There were three different kinds of questionnaires that were given out. First a semi-structured questionnaire consisting of open-ended questions was handed out to eighty high school students from four different schools. Check the table below for details.

Table 1 : Number of Questionnaire respondents

Institution	No. of boys	No. of girls	Total
African High School	10	7	17
"Coloured" High School	16	7	23
Finishing school	11	13	24
Saturday school	1	2	3
TOTAL	38	29	67

UNIVERSITY of the WESTERN CAPE

Students were allowed to complete the questionnaire at the time they were handed out to them. This was done so as to maximize the return rate. Nevertheless thirteen questionnaires were discarded because they were not completed correctly. In addition to these, one hundred and sixty semi-structured questionnaires with open-ended question were handed to first-year mathematics students. They were given the questionnaires to complete at their leisure. They were given a time limit of a week. The return rate was 28,1% of which 3,1% had to be discarded because they were incomplete and are not reflected anywhere in the discussion. All in all sixty-seven questionnaires (excluding the discarded one) were analyzed.

https://etd.uwc.ac.za/

As a follow-up to both the interviews and the questionnaires forty matriculation students were given a short multiple-choice calculus test and all the scripts were returned. Finally ten mathematics teachers were given questionnaires that were intended to clarify and shed some light on the pupils' responses. Four of the ten teachers returned them while the other teachers were reluctant to fill in the questionnaire so the interviewer had to discuss the question (verbally).

3.9 Characteristics of the interviewer

It should be noted that it was not the first time for the interviewer had conducted the thinking-aloud interviews. These type of interviews had been used by the interviewer while doing his Honours degree and also for the M.Ed assignments, he had done a number of such interviews. The interviewer in addition to what had already been said had watched several audio-visual cassettes on how to conduct thinking-aloud interviews and he had read and studied a lot of literature on this subject. This, therefore, means that the interviewer has some experience (both theoretical and practical) on how to conduct the interviews.

UNIVERSITY of the
WESTERN CAPE

3.10 Interpretation and analysis of data

The interpretation and analysis of data are given in the next chapter. The purpose of these is to find out whether there is any pattern in the data. The first year university mathematics pupils' data was analyzed first. This was done by analyzing individual questions and making or drawing the relationships between question testing, finding, or seeking for the same information. After these interpretation and analysis of

https://etd.uwc.ac.za/

questionnaires for high school pupils were made (using the same method as the one used by first year mathematics students). After interpreting and analysis (high school pupils) the presentation and analysis of the interviews with teacher is given, showing the common mistakes or misconcepts made by pupils, and how they think differential calculus could be made interesting. Then the summary of analysis of the thinking aloud interviews is given. Finally the interpretation and analysis of multiple choice test given to high school pupils were given. Thus showing which questions were well done and those which were badly answered. (At the end of chapter 4 there is discussion of the findings which extends to the first pages of Chapter 5).

3.11 Characteristics of the participants in table forms

1. Thinking aloud interview for High school pupils

Table 2: Characteristics of high school thinking-aloud interviewee

UNIVERSITY of the
WESTERN CAPE

Characteristics	Gender		Maths grade SG	HG
(a) Students who scored high marks in maths.	1	1	1	1
(b) Students who scored low marks in maths.	2	1	1	2
(c) Students who scored average marks.	1	1	1	1

https://etd.uwc.ac.za/

Table 3: Thinking-aloud interviews for university students

Characteristics	Gender M	F	No of years in first year
(a) Students who usually score high in maths.	1		2
(b) Students who do not score very high but manage to pass.		1	1
(c) Students who always fail maths.		1	1
(d) Students who sometimes pass mathematics and other times fail.	1		1

3. <u>High school pupils</u>

(i) Black High School

Characteristics	Gender		Grade HG	Standard Grade
(a) Above average students	3	2	4	1
(b) Average students	3	3	5	1
(c) Below average students	4	2	2	4

UNIVERSITY of the
WESTERN CAPE

(iii) <u>"Coloured" High School</u>

Characteristics	Gender M	F	Grade HG	SG
(a) Above average students	4	2	6	-
(b) Average students	4	2	5	1
(c) Below average students	4	3	3	4

(iv) <u>Finishing High school (repeaters)</u>

Characteristics	
Number of boys	11
Number of girls	13

https://etd.uwc.ac.za/

It should be noted that in all the above tables only the number of pupils whose questionnaires were analyzed are indicated. Those who did not complete the questionnaire or those who just completed the questionnaire partly were excluded, as will be indicated in chapter four.

Of the first year university students there were fifteen boys and twenty five girls who completed the questionnaire. For the teacher the criteria was just anyone teaching standard ten mathematics or differential calculus to be more specific.

3.12 Limitations of the research

As a full-time student the researcher had a problem of interaction with the students interviewed. This means that he only met the pupils on the day of the interview. So pupils were not acquainted with him. This actually caused some problems as he was a complete stranger to the pupils. Problems such as being fully feeling at home with the interviewer, of not knowing school or pupils' discipline and many others.

UNIVERSITY of the WESTERN CAPE

Students were a bit uneasy at the beginning of the thinking aloud interviews. It should also be noted that getting standard ten teachers to complete the questionnaire was not an easy task. The researcher, therefore thinks that generalizing from the results obtained from 10 teachers is not without dangers.

It should also be noted that the interviews and handing out of questionnaires were done in October (1993) at the time when the Standard ten pupils were busy with their revisions for final matric examinations. So pupils' anxiety should be taken into

https://etd.uwc.ac.za/

consideration. The attendance of pupils is usually very poor at this time, as they prefer studying at home or any other place they deem appropriate. This really affected the sampling and the researcher had to use (or be helped by) those who were present. The interviews and handing out of questionnaires took place towards the end of the year as most teachers treat differential calculus at this time. It was the only appropriate time for handing out the questionnaires.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

CHAPTER 4 : PRESENTATION AND ANALYSIS OF THE FINDING OR RESULTS

4.1 Problem areas identified by first year University students/ Undergraduate

General information

4.1.1 (i) First year mathematics students

The ages of students ranged from 17 years to 30 years with most students at 18 years of age. The following table shows the age range of Forty first year university students (who completed the questionnaires).

Table 1: Age of University first year students

AGE	17	18	19	20	21	22	23	24	25	26	27	28	29	30
FREQ	2	9	5	5	6	2	2	2	-	3	1	1	-	2

UNIVERSITY of the WESTERN CAPE

It should be noted that One hundred and sixty questionnaires were given out but the return rate was only 25%. This return rate resulted from the timing of the interviews and distribution of questionnaires, which was close to the end-of-year examinations. However, it was the only time available for the interviews and also the appropriate time for administering the questionnaires.

There were four different streams for first-year mathematics students. One stream was for students doing commerce "A", the

https://etd.uwc.ac.za/

other stream was for students majoring in mathematics (BSc students) "B"; the next stream was for people taking mathematics just as an auxiliary, "C"; and the last stream was for students who did not pass mathematics well in matric, "D". The frequency of students who responded to the questionnaire is as per table 2 below.

Table 2 : Streams of Maths students

Stream	A	B	C	NOT SPECIFIED	D
Freq.	13	8	11	3	5

The students' parents or guardians were divided into two main branches. Namely "Educated" or "not educated". refer to Table 3.

Table 3 : Occupation of Parents

Parent or guardian	Mother	Father	Guardian	Mother	Father	Guardian
Freq.	5	10	--	29	24	--

UNIVERSITY of the WESTERN CAPE

The occupation section was further divided. Thus students were asked to state whether they had single parents or not, this was explained when handing out the questionnaires.

Table 4 : Number of children with single parents

Parents	Single parents	Not specific
Frequency	12	1

https://etd.uwc.ac.za/

It is important to note that all (except one) students who completed the questionnaires sat only once for their matric final examination. The symbols obtained in matric mathematics ranged from "A" to "F", with most frequency in C,D, and E. Only one student obtained an 'A', refer to table 5 below.

4.1.2 Preference for differential calculus

Table 5 : Symbols Frequency Table

Symbol of matric maths	A	B	C	D	E	F
Frequency	1	4	8	8	16	3

Of all the students, 70% sat for higher grade mathematics and 30% for standard grade. 50% of the students who completed the questionnaires said that they found differential calculus easy, while 15% found it moderate and the 17,5% of the pupils found differential calculus difficult. The other (17,5%) did not respond to this item (see Table Fy I in Appendix C).

UNIVERSITY of the
WESTERN CAPE

4.1.3 (a) <u>Preliminary topics</u>

60% of the first year students said that they were not sure of the topics that are the prerequisites for differential calculus, while 25% of the pupils stated that areas, functions and exponents were the prerequisites for differential calculus.
Those who thought that trigonometry should be taught before introducing differential calculus constituted 10% and 5% said that everything that they studied in earlier levels contributed or would contribute or help (see Table Fy 2 in Appendix C).

https://etd.uwc.ac.za/

4.1.3 (b) Blame on teachers

62,5% of the students did not blame the teachers for anything. They said that their teachers really helped them to make it to the university, in addition to this they said that they had good teachers, who "knew their stuff". 25% complained about their teachers, claiming that their methods of teaching were complicated for them, meaning that they did not understand their teachers because they could not explain the content as well as the pupils would have liked, but seemed to be in a hurry to cover the syllabus without caring whether they (pupils)understood or not. Students also complained of mathematics teachers being very hostile, as a result making it difficult for pupils who have problems to consult them. The last 12,5% did not comment on teachers, saying that they were not sure whether the teachers' approach was good or not (See Table Fy 3 in Appendix C).

4.1.3 (c) Calculus learning

UNIVERSITY of the
WESTERN CAPE

80% of the students said that they gained a lot from the calculus that they learnt in standard ten; that it was not like starting from scratch, like when studying integration which they found comparatively difficult. They said that high school calculus gave them the essential basis. 12,5% said that they did not benefit at all from the calculus they studied in Std 10. They claimed that it was like starting a new topic altogether, but this time with a better teacher. They said that when they were at high school differential calculus did not make meaning at all, it just involved standard derivatives and substituting things (numbers) in the formula or using the first principles which one just memorised. The other 7,5% of the students did not respond.

https://etd.uwc.ac.za/

When asked how or what needs to be done to make high school pupils pass differential calculus, 37.5% of the students said that high school pupils need to be taught how to solve problems. Thus students should be given problems and be taught the strategies of how to go about solving problems of that nature. 5% of those who responded said that students should be motivated by teachers so that they could have a positive attitude toward differential calculus. 12,5% indicated that students should be taught with the help of teaching aids like video cassettes. 5% said that students would perform better if teachers could simplify in their presentation of the content and their methods of teaching. 12,5% said that they thought that students should attend extra classes like Saturday classes in order for them to pass differential calculus. While 10% said that pupils should do a lot of practice, this should solve lots of differential calculus problems. 7,5% said it was up to the individual students what they wanted to do, as people are unique, what may work for one may not for the other. The last 10% did not reply or respond to this section.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

4.1.3 (d) **Some factors that may affect the learning of differential calculus**

(i) **Teachers**

As already mentioned some students said that the methods teachers use, have an impact on the learning of differential calculus. 75% of the students said that teachers (motivation, strategies and other factors) have a good impact on the learning of differential calculus whereas 25% said that they learnt on their own and teachers did not even motivate them, except talk a lot of non-academic stuff in class. Some students said that without their teachers they would not have "got the chance to come to the university". This was because their teachers were hardworking and encouraging, giving them a lot of exercises and positive feedback. (Refer to Table Fy 6 in Appendix C the first block.

(ii) **Syllabus**

UNIVERSITY of the
WESTERN CAPE

82,5% responded by saying that there was nothing wrong with the differential calculus syllabus. All what one needed to do was to practise and understand what was practised. On the other hand 17,5% said that the syllabus was for an inferior education system (DET) and not the same as that of their white counterparts. That for them it was difficult and also that it was included in the syllabus to make them fail mathematics. (Look at Table Fy 6 in Appendix C).

https://etd.uwc.ac.za/

(iii) <u>School</u>

90% of the students said that the school did not have any negative influence on their studies of differential calculus. By school the researcher is referring to school administration and the pupils' peers. Some students however, (10%) said that they had bad company (peers) and that the school in general had a very bad influence on their studies, like administering of corporal punishment when failing certain subjects. (Refer to Table Fy 6 in Appendix C).

(iv) <u>Home</u>

82,5% of the students said that they had no problem at their homes, meaning that their homes did not interfere with their studies. While 17,5% said that they had problems at home, such as lack of opportunity to read, parents always fighting either physically or verbally, no food or any other income.

UNIVERSITY of the
WESTERN CAPE

(v) <u>Politics</u>

80% of the students said that politics had no effect whatsoever on their studies. 2,5% of the students, even said that for them the political activities like stay-aways, helped them to have more time for studying. 20% of the students said that they were disturbed by politics as the classes were affected and the classes were worst affected when the teachers went on strikes for a long period of time (See Table Fy 6 in Appendix C).

https://etd.uwc.ac.za/

(vi) Sources of motivation

Most students (45%) said that they were mainly motivated by their teachers and they also had a goal in life which they knew they would achieve only if they would pass mathematics. 22,5% said that they were motivated by the political climate in South Africa, that is they wanted to represent the black nation, in technology, accounting and economics fields. 17,5% of the students were encouraged by their parents, and also their teachers. While the rest said that they were just interested in the competition between themselves and their friends (5%). The other students were encouraged and motivated by their brothers/sisters (10%) (See Appendix C, Table Fy 7).

(vii) Factors that make high school pupils fail

Item 11 in the questionnaire elicited the same information as Item 8, the responses of the two were found to be the same. For Item 12, 50% of the students said that most students do not pass differential calculus because of lack of interest either, in the subject or in the person offering the subject. 12.5% of the students said that some of their friends failed differential calculus because they did not at all like the subject saying that some students had a "phobia for figures". While the remaining percentage failed because of full involvement in political movements, lack of properly qualified teachers, examination fever, poor study habits and others who were not adequately "intelligent". Other students mentioned working together (team-work) they (15%) said that team-work motivates one to work when one is lazy. 2.5% said that some students "are born clever" that is why they pass; 10% said that some students fail because they

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

are too involved in non-academic matters. 10% did not answer this question (See Appendix C, Table Fy 8).

(viii) The task of solving differential calculus problems

22.5% of the students said that the mistakes that were commonly made when solving differential calculus problems were mainly to do with "signs", such as, changing the signs unnecessarily or using wrong formulae or equations, or forgetting the first principles/formulae. 12,5% said the other common mistakes were those concerned with simplification. 7,5% of the students indicated that differential calculus was usually done or learned by rote method (memorising without understanding), therefore, when the structure of the question was different from the one that the student had practised (the one familiar to students), even if they required the same thing, students would start to fumble. 37,5% of the students did not reply to this question. The other 20% said that students are usually in a hurry and as a result make careless mistakes due to being impulsive and also due to and lack content subject (See Appendix C, Table Fy 9).

UNIVERSITY of the
WESTERN CAPE

(ix) Impact on medium

77.5% of the pupils said that they had no problem with understanding differential calculus using English as a medium of instruction. 22,5% said that there is a great problem in mathematics of using English language as it is not their mother language. Items 14 and 16 gave similar responses (See Appendix C, Table Fy 10).

https://etd.uwc.ac.za/

4.1.3 (e) First year University differential calculus

57,5% of the students said that the first year differential calculus is very interesting as they now know what they are doing and what differential calculus means unlike when they were in standard 10. However, they still maintained that the matric calculus really helped them for better understanding of first year differential calculus. 35% on the other hand said that the calculus at the university is difficult as it involves lots of things like the product rule and other difficult types of differentiation. 7,5% did not answer this item (See Appendix C, Table Fy 11).

4.1.3 (f) Gender

It is really surprising to see that not even one student (0%) wanted to be taught by females only (no feminists). 15% of the students from both sexes said they wanted to be taught by males only as females are "moody and like gossiping". 85% said it does not matter whether it's a male or female as long as a person is confident and has the content and can "deliver" it properly, then that is all they require from a teacher (See Appendix C, Table Fy 12).

Item 19 and 20 can be summed up together as follows: pupils who consulted peers, teachers and senior students seemed to be in the majority. For example 40% of students said that when they had problems with calculus they usually go to their peers, senior students and teachers. While 47,5% in total also said that they consulted their teachers (15%), peers (27,5%), and peers and textbook (5%) for help. While 7,5% of the pupils said that they

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

usually help themselves by thinking hard and giving a problem a break and going back to it again. The other 5% did not complete the item.

4.1.3 (g) Differential calculus knowledge

The last two items actually test the content knowledge of differential calculus to check whether students learn by memorization without understanding (rote method or static knowledge). 75% of the students showed some knowledge gains, that is they knew what f'(x) $D_x f(x)$ and dy/dy stands for. 52,5% of the students also showed some understanding of what f'(x) = 0 means while 5% did not know at all what f'(x) = 0 stands for (meaning that they possibly managed to pass matric differentiation calculus using the rote method.) One could explain it by saying that it may be students who did not pass their matric mathematics (therefore obtained an 'E' or 'F' symbols for maths). NB. the sample of questionnaire for First year mathematics students is included in Appendix 2A).

4.2 Interpretation and Analysis of the Questionnaire for pupils. (High school pupils)

General information

4.2.1 High School mathematics pupils

It should be noted that the aim of this study is not to make a comparative analysis of African students, "Coloured" students and the African student repeaters, but to find some cause of cognitive difficulties that students encounter in the study of

https://etd.uwc.ac.za/

differential calculus. Therefore, the comparison made was just for finding the causes and not checking as to who were "smart" comparatively.

Of the pupils given the questionnaires 44,8% of the pupils were doing standard ten mathematics for the first time, the other 55,2% students were repeaters of whom three were doing standard ten mathematics for the third time. The ages of pupils ranged from seventeen years to thirty-one. (See Table 6 below shows the range of age).

Table 6 : Age for frequency of high school pupils

Age	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
Freq	17	6	6	13	6	2	4	5	3	3	-	-	-	-	2

The pupils, parents are categorized into two, the educated and those not educated this all determines the type of occupation they hold. Most parents are not highly educated. This can be detected by the types of job they are holding. The following table 7 gives an indication of what their frequencies look like.

Table 7 : Parents Status (Occupation)

Parent or Guardian	Mother	Father	Guardian	Mother	Father	Guardian
Freq.	7	15	--	40	32	3
Education	EDUCATED			NOT EDUCATED		

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

The reason why the number reflected in the above table appears to be bigger than the actual number of pupils is because some students wrote both the occupation of the mother and that of the father. For the pupils who completed the questionnaires 29 girls and 38 boys, responses were analyzed. The others were discarded as they had not been completed or had only answered a few questions.

Generally speaking almost all the students (91%) said that they liked mathematics, algebra and especially differential calculus. Some of the reasons for liking mathematics was because it gave job opportunities and most students said they liked mathematics because it made them think better. 26.9% said they liked differential calculus especially because that is where they score marks.

(a) <u>Some factors that retard pupils performance</u>

UNIVERSITY of the WESTERN CAPE

10,4% of the pupils said the teachers hindered them from passing (according to their abilities) because they were not properly trained and also because they could not teach them, so that they could understand what was being taught. 28,3% said that they did not pass differential calculus as well as they would have wished because they lacked enough basics and practice. 23,9% students said that differential calculus involved many words which are difficult to understand. 7,5% of the pupils complained of being very slow and forgetting concepts taught earlier very quickly. 9% of the learners said that the textbooks supplied by the government were not adequate and not good enough for them to understand. 14% on the other hand said they were not sure of what actually causes poor results in differential calculus (referring to word problems) (See Appendix D, Table P. 1).

https://etd.uwc.ac.za/

4.2.2 Some factors that may affect learning of differential calculus

(a) Teachers

19,4% blamed teachers for not being to be able to explain. Pupils stated that teachers like pupils who show some understanding and do not give slow learners attention at all. They blamed teachers for not giving them feedback and not marking the exercises given in class.

(b) School

6% of the pupils complained that there was a lot of noise and other disturbances during the study periods. 3% added that usually calculus is introduced too late when they already had "exam fever"

UNIVERSITY of the
WESTERN CAPE

(c) Home

14,9% of the pupils complained of the home environment, saying that they cannot study at home due to the large numbers of brothers and sisters at home, and also due to homes being very noisy. Others complained of parents coming home drunk, lack of study facilities (rooms) and even if they would like to study there would be no light as the paraffin used for lighting is unhealthy and too expensive for their parents, who in most cases do not work. Only 3% complained of the language that is used at school being different from the home language. (Reference of 'a', 'b' and 'c' is in Appendix D, Table P2).

https://etd.uwc.ac.za/

(d) Politics

68,7% of the pupils said that party politics affected their studies seriously. They complained of stay-aways, their involvement in such movements, therefore, as a result wasted a lot of time which would otherwise be utilized for studies. The other 31,3% said that politics does not affect their studies, but rather the stay-aways help them to study more, to have more time out of school for studying. At this point is should be noted that of the 68,7% who said they were disturbed by politics, 49,3% were responses from repeating pupils (reference Table P3, in Appendix D).

4.2.3 Problems with understanding differential calculus.

49,3% of the pupils said that they have no problems at all with understanding differential calculus. While 50,7% said that they had problems with differential calculus especially its application. 23,9% of the pupils said that they did not fare well in differential calculus because of their poor background knowledge of it. 6% did not respond to this problem (See Appendix D, Table P.4).

Pupils also complained of time, saying that usually the time set for solving the differential calculus problem is very limited. 64,2% of the students complained of the time factor, saying that it seemed that they were not tested on how much they know but on how fast they work. To this effect it should be noted that the intention of solving problems in mathematics is not only whether pupils can solve problems, but the time and accuracy efficiency of the students, thus how many problems could be solved

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

accurately within the stipulated time. 6% said that because they were always prepared for the exercise or test given they usually finish before the given time (Refer to Table P 5).

4.2.4 Language

64,2% in Item 7 whereas 38,8% in Item 14 complained of the language that is used saying that because it is not their mother language it creates problems; that they struggle with understanding the question, and then afterwards that they try to understand the content. Most of these pupils believed that if differential calculus (mathematics) would be taught using their first language they would pass or understand it better. Some pupils complained that the examples or the application of differential calculus is mainly related to physics and because of a negative attitude towards physics they ended up not liking mathematics or differential calculus due to the physics language used (See Appendix D, Table P. 6).

UNIVERSITY of the
WESTERN CAPE

64,2% of the pupils said that they liked using differential calculus rules or formulae because the rules or formulae can easily be memorized even if one does not understand what is going on (look at Table P7 in Appendix D). When asked what pupils should do in order to like or have a positive attitude towards differential calculus they suggested that students should work hard, making sure that they practice seriously and regularly. 71,6% of the pupils responded this way. To do this 7,5% pupils said that teachers should motivate them. 10,4% said that good attendance and being attentive could help pupils to pass differential calculus well (See Appendix D, Table P. 8).

https://etd.uwc.ac.za/

41,8% said that to make differential calculus interesting and easy, teachers should give pupils more work. Teaching them step-by-step so as not to skip important information; and telling, and showing pupils how differential calculus is applied in everyday life. Another 22,4% of the pupils said that in order to make differential calculus interesting and easy, teachers should give pupils more work and should stop insulting pupils or shouting at them when they make mistakes. 16,4% pupils said that teachers should learn to start with simple problems and gradually move towards demanding ones. 9% said because differential calculus is one of the simple topics in mathematics that teachers should start in the second term of Std 10. The other 9% of the pupils also said that differential calculus would be interesting if everyday life experiences would be used (See Table P. 10 in Appendix D).

4.2.5 Syllabus

Most pupils who responded to this question blamed things which are to some extent outside the syllabus, like having qualified teachers; while others suggested that teaching aids should be introduced or should be used so that they could have the real picture of the situation and therefore, proper mental models. 59,7% of the pupils said "nothing should to be done to the syllabus, it should just be left as it is".

4.2.6 Content knowledge

61,2% of the pupils said that there is a relationship between finding the gradient and finding the derivative. Most of them said that they mean the same thing, or can be found in the same

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

way, showing that at least they understand the work. 16,4% of the pupils said that there is no relationship whatsoever while 22,4% did not answer the question. (Look at Table P.12 in Appendix D)

For Item 14 it should be noted that students did not understand this question properly; they thought that the question is asking whether they should be taught in English or Afrikaans. So most pupils said they prefer English to Afrikaans. But for those who understood the item properly 38,8% said that English, indeed, affects their understanding of differential calculus, for reasons mentioned in Table P.14 in Appendix D. While 55,2% said that English does not affect their understanding of differential calculus. 6% of the students did not answer this question.

4.2.7 Prior topics

29,9% of the pupils said that "graphs" and limits" (algebra) are necessary topics for differential calculus. While 22,4% said that geometry and trigonometry are the prerequisites for differential calculus (See Table P. 14, Appendix D). The questionnaire for pupils is given in Appendix 2B.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

4.3 Interpretation and analysis of data from teachers

The questionnaire sample for teachers is given in Appendix 2C.The return from practising teachers was very poor, 40% of the ten standard ten teachers completed the questionnaire, the other 60% just responded to the questionnaire verbally (discussing the questions with the researcher). Their years of experience ranged from five years to twelve years teaching maths at high school.

4.3.1 What makes pupils like or hate differential calculus

All (100% of the responses of the) teachers complained that students usually have problems with drawing cubic graphs; while 80% complained of the pupils' negative attitude towards mathematics (in general). 80% also complained that pupils have very serious problems with "applications of differential calculus" saying that "students usually do not attempt such problems at all". Only 20% of the teachers believe that some pupils were not born for mathematics, blaming this on career guidance counsellors, for not advising pupils properly. 75% also agreed that pupils do not do well or pass mathematics well especially differential calculus, because of their laziness to study. (Refer to Table T.1 in Appendix E).

One teacher mentioned that to motivate or make pupils like mathematics especially differential calculus he starts teaching differential calculus from back to front. "*That is starting off with "how to differentiate, sketching the graph without some details, maxima and minima and then doing the more "abstract" work like limits etc*". Two teachers agreed on scripts that pupils should be taught differential calculus in such a way that

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

it can be interesting for instance using everyday life experience for application of differential calculus. The other teacher mentioned that in order to motivate pupils to make them like the subject *"pupils should be told about the job opportunities of pupils doing or studying differential calculus"*. Pupils should also be told that if they intend furthering their studies in colleges or universities then high school calculus would lay a good foundation for them. Of the teachers 50% also made mention of starting early with differential calculus.

It is important to note at this point, that in a general discussion with the teachers who completed the questionnaire teachers complained of pupils who are promoted from standard 8 through to standard 10 not having passed mathematics at all but scoring high marks in other subjects especially languages and as a result getting an overall pass mark. Teachers said that this is really a drawback as such pupils have a very negative attitude towards the mathematics teachers as a result of having an negative attitude towards mathematics. They said that it is usually such students who are full of misconcepts and have a mental blockage, therefore they do not understand mathematics.

UNIVERSITY of the
WESTERN CAPE

Three teachers suggested that pupils who seem to have problems with understanding differential calculus, should be given extra classes, which they said is very taxing on them. 20% of the teachers suggested peer group studies during or after school.

4.3.2 <u>Socio-Economic status</u>

80% of the teachers said that most pupils in their schools are affected by the same factors. The society in which they live is

https://etd.uwc.ac.za/

almost the same for all the pupils' so that does not affect pupils individually but is structural. Even though they all agreed that the background or experience of the pupils plays a major role in the learning of differential calculus (or mathematics in general). Structurally the socio-economic factor should be an encouraging factor as pupils should be ambitious to change the situation in which they live. It should be noted that if physiological needs are not met pupils cannot function well in schools.

All the teachers interviewed said that pupils cannot blame time or the syllabus as they (teachers) make sure that they finish the syllabus in time to give more time for revision. In all the schools visited, English is used as the medium of instruction.

20% of the teachers said that English as the medium of instruction has no effect on students' understanding, while the remaining percentage (80% said that usually they found themselves explaining many concepts using the pupils' first language, saying that it is through such language that the pupils culture can be understood and integrated into new culture (differential calculus). This they said promotes understanding as pupils understand their own languages and culture better. The problem with using their mother tongue is that pupils are not asked questions in their own languages, but in English and as a result using mother language for teaching may cause serious problems especially during examination time.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

4.3.3 Pre-topics and academic background

60% said limits should be taught, this is surprising since some (60%) teachers seemed to take limits as if it is not part of differential calculus even though one of the teachers suggested the working backward method. 80% of the students said that 'straight lines" 'parabolas" and quadratic equations should be the prerequisite of differential calculus.

Poor academic background mentioned earlier, always leads to failure and frustration. All the teachers agreed that it is very difficult to bring pupils with poor academic grounding to a high standard or level so that they could pass standard ten. Such pupils are usually "full" of very serious misconceptions.

4.3.4 Content knowledge

Item 9 is very interesting as it brings up some common misconceptions in differential calculus. Teachers said that for the first question "What would be the initial speed given that the distance, s metres that a body travels in t seconds is given by $s= {}^{1}/_{12} \; t^3 - t^2 + 4t$", students usually equate s to zero and try to solve the problem.

For part two "determine the time it will take the body to come to rest" students instead of writing $\frac{1}{4}t^2 - 2t + 4 = 0$ they simply write it like $\frac{1}{4}(0) - 2(0) + 4 = 0$ resulting with $4=0$ which they always do not mind (thus feeling content with). For the last part "What does $f'(x)=0$ mean?" most students (with misconcepts) would write it like this ${}^{1}/_{12} \; t^3 - t^2 + 4t = 0$, instead of writing $\frac{1}{4}t^2 - 2t + 4 = 0$. It should be noted that students, when given problems like these

https://etd.uwc.ac.za/

also make many other mistakes like sign mistakes, (mixing up signs or leaving them out), displaying the inability to differentiate behaviour or just substituting a number without differentiation.

4.3.5 Age and performance

80% of the teachers agree that the young students (who are motivated) work better than old pupils who are usually "full" of problems (either at home or at school). They added that the attitude of such pupils (young)toward mathematics can easily be changed.

4.3.6 In-service training

50% of the teachers who completed this questionnaires complained of lack of in-service training. Saying that even when conducted, they are usually for people with mathematics III. Meaning that teachers who only studied at the college will encounter problems during such a training session. One teacher to this effect even said that differential calculus was not in the syllabus when he was at college, so he is just learning it on his own.

UNIVERSITY of the WESTERN CAPE

4.3.7 Improvement

20% of the teachers suggested that teachers should teach backwards. 80% suggested that the misconceptions or mistakes that students make should be addressed immediately and to this effect 20% of the teachers even said that the best way would be to teach using misconceptions that are common to prevent pupils from doing them. Teachers also suggested that pupils should not

https://etd.uwc.ac.za/

be forced into doing mathematics and that those (pupils) who failed mathematics in standard 8 and standard 9 should not be allowed to do standard ten mathematics.

4.4. Summative analysis of the thinking aloud interviews.

Under this topic the following concepts will be discussed: the common mistakes made by pupils when solving the differential calculus problems, the problems caused by lack of understanding especially the semantic difficulties and also the summary of the cognitive deficiencies or difficulties displayed by pupils when solving the problem or during the process of solving the problems. (It should be noted that a detailed or full list of cognitive difficulties displayed by individual pupils for each question is given in the Appendix A. A sample of questions given for thinking-aloud are in Appendix 2E).

4.4.1 Common mistakes or misconcepts made by pupils

UNIVERSITY of the
WESTERN CAPE

Question i (i)

The results show that most pupils' (81,7%) problems or mistakes are due to the lack of the basic knowledge, for example (pupils) saying that zero divided by zero is zero, the other example would be that of substituting 2 for x in x -4 and 2 for x in x-2 without visually or mentally checking the answer or simply without factorizing x -4 to simplify the expression.

When solving the problem pupils leave off limit and just write the unconnected statement. The other problem that the pupils had is that of not fully understanding what the limit means, thus

https://etd.uwc.ac.za/

having many misconcepts surrounding the word, 'limit'. 57,1% of the high school pupils interviewed had difficulty with solving the problem

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

because they could not see how the expression x -4 could be factorized, this therefore, showed that it is not that the student found differential calculus difficult, but that the problem is the preliminary knowledge. It is interesting to note that 28,6% of the students interviewed said that they can solve problem 1 (i) by using differentiation. One very serious misconcept held by the pupils is that x^2 -4 is the square root (reference: analysis of interview 6 in appendix 1) so the answer for it will be x-2. The pupils' terminology also had great influence as they said "factorization" means the same thing as taking out the common factor.

Question 1 (ii)

UNIVERSITY of the
WESTERN CAPE

Item 1(ii) is quite a difficult question for standard tens, the reason for including such a question was just to find out how creative pupils can be or simply to test the ability of the pupils to deal with novelty. 28,6% of the students did not even attempt the problem (there was mental blockage and seemingly lack of knowledge) they said that the problem was too difficult for them to solve. For those who tried, the following mistakes or misconcepts were displayed.

Pupils said that x^2 in the numerator can cancel with x^2 in the denominator ending up with $\frac{x^2 - x - 1}{x^2 + 2}$.

https://etd.uwc.ac.za/

Some pupils said that the problem can be solved using the difference of the two squares. When asked what they meant by this they could not explain. Pupils had a wrong idea that whatever is in the denominator will cancel what is in the numerator as long as the cubic expression is factorized. 42,9% of the pupils said that this problem could be solved by using differentiation. It should be noted that 3 of the 5 (there 60%) students who attempted question 1 (ii) think that items in the denominator can cancel those in the numerator irrespective of the signs connecting them; for example $\frac{x(x^2 - x - 1)}{x(x + 2)}$.

2 pupils said that infinity is equal to zero. From this, one would come to the conclusion that such students have problems with pre-knowledge. The other problem encountered in question 1, is that pupils had problems of leaving out lim before writing the expression (5 out of 5 who answered or attempted problem 1 (ii) made this mistake).

UNIVERSITY of the
WESTERN CAPE

Problem 2

There were many mistakes made in this section, pupils said that

$\frac{f(2 + h) - f(2)}{h}$,

is the same expression as $\frac{(2 + h) - (2)}{h}$,

thus just removing f and then solving the problem. Pupils seemed unable to understand the question and seemed to be in a hurry to finish, the reason for saying this is because most pupils left out f(2) when substituting, therefore writing

$\frac{f(2 + h) - f(2)}{h}$ as $\frac{f(2 + h)}{h}$,

leaving out -f(2). Instead of writing -f(2) would write -2.

https://etd.uwc.ac.za/

It is clear that some pupils learn by rote or simply by memorising concepts without really understanding them, saying for example that "$\lim_{h \to o} \frac{f(2 + h) - f(2)}{h} = \frac{2 + h - 2 + 2x + 7}{h}$"

thus one would start to wonder as to where the "x" terms come from. This shows that the pupils have $\lim_{h \to o} \frac{f(x + h) - f(x)}{h}$

in their minds. It should be noted in the example given above that the second expression is without $\lim_{h \to o}$

whereas the first one is with it. This as already mentioned is one of the common mistakes made in solving the differential calculus problems.

Problem 3

For this problem pupils just substituted the numbers (-1 and 2) without first finding the derivative or differentiating $f(x) = x^2 (x-3)$. (85,7% of the students interviewed made this mistake).

UNIVERSITY of the
WESTERN CAPE

28,6% of the pupils 'Just' differentiated $(x^2)(x-3)$ like 2x not dealing with what are in the brackets first, that is expanding the (or multiplying out the) brackets first. 85,7% of the pupils said initially that $f'(-1) = f(-1)$ saying that f'(-1) is the opposite of f(-1) so to them f' means opposite of "f".

14,3% of the pupils said that when one says that the gradient is zero for a cubic equation one would be saying or means that the graph cuts the axis at the origin. The pupils (72%) said that or read f'(-1) as f prime into (-1) and f'(2) as f prime of or by 2. This reflects that pupils just knew what to do or how to

https://etd.uwc.ac.za/

solve the problem without really understanding what the problem means.

When asked what the gradient would be at the turning point 28,6% of the pupils said that it would be zero. When asked to sketch the graph all of them except one were not able to draw the cubic graph.

Problem 4

Instead of finding the gradient two pupils just substituted 2 for x in f(x), thus finding the y co-ordinate when x=2. This is enough evidence to show that pupils do not take some time to think about the problem, but just solve the problem impulsively.

28,6% of the pupils tried to find the gradient of f(x) at (2;0) using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$

and ultimately gave up as they could not find the other point to be used.

Question 5

Only two pupils attempted question 5 (well), while others did not have enough courage to solve the problem saying that it was difficult for them, saying also that they did not like word problems.

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

4.4.2 The semantic and cognitive difficulties displayed by pupils interviewed

(i) Semantic difficulties

As discussed in the literature review (Valdes, 1987 Sinclair 1992, Fynn 1990) language, especially the one used as a medium of instruction (which in the case of this research is English), has or is said to have a great impact on the pupils' learning and understanding of mathematical concepts. This was reflected by all the pupils. Thus they had very serious problems trying to explain the concepts (they were asked to explain). They were not clear at all, at some stages (the interviewer) could see that the pupils understood the concepts but lacked enough vocabulary to express themselves.

A good example of the above would be that when pupils were asked to explain clearly what the term limit means; 71,4% of the pupils lacked enough vocabulary to explain (refer to interviews Appendix 1). The other common mistake or difficulty that pupils had was that of reading, for example $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

42,9% of the pupils read this as "limit is equal to x^2 - 4 when x approaches two divided by x - 2" "others read it as" x^2 - 4 by x - 2" if x approaches two. When reading the second question pupils read f (2 + h) - f(2) as follows "f into (2 + h) minus f by or of 2". These examples show that pupils really have difficulty with the use of English. Reading an expression like this to some students seemed like a punishment.

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

(ii) Cognitive difficulties

It should be noted that there are many of cognitive difficulties displayed by pupils but here only those which are common to (interviewees) many pupils will be discussed.

I found that pupils have a very serious problem with the subject, especially the word problems, these could be seen from the way pupils had problems with gathering information necessary for answering the question as well as not being able to elaborate on such data. Pupils lacked the verbal tools for communicating, this was shown by their inability to communicate their responses precisely and accurately. Question 5 is a good example of students' reaction towards the word problems. Only two students attempted it while the rest just did not even give it a trial. There was a complete mental blockage.

When solving the problems especially problems 1 and 2, most pupils just substituted without really making an attempt to understand what the question needed. So in solving the problems pupils seemed to be in a hurry to finish, and did not plan their work. Such students were very impulsive and unsystematic in (exploratory) behaviour. Apart from being impulsive students made mistakes because of poor retrieval (from the memory) of things or concepts previously learned, for example saying that $^0/_0$ (zero divided by zero) is zero, or the other example would be that of finding the derivative of $f(x) = x^2$.

$(x - 3)$ some pupils just substituted (-1) in $f(x)$ when finding $f'(-1)$ while others did not get rid of the brackets but found the answer as $2x(1-0)$ which showed that pupils could actually

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

differentiate, but had problems with remembering things that were learned earlier. Therefore, most students used rote learning, that is learning without proper understanding. Most pupils had to be guided in order to solve the problem and it was found that for such pupils (42,9%) content matter was really a problem. They liked using trial-and-error responses, praying hard to get the answers correct. These, therefore, show narrowness of the mental field.

The imagery of pupils is actually very poor, when asked for example to draw a cubic graph so that they could explain themselves better, pupils could not draw the graph or if they did they drew the wrong one. This actually shows that pupils were just solving the problems without having a clear picture (in mind of what actually is taking place (refer to Rhodes 1992 as discussed in literature review). These results, therefore, show that pupils lack or have a limited interiorization of behaviour.

As it could be seen pupils were very inadequate in experiencing the existence of an actual problem and subsequently defining it and as a result of this they had lots of problems in expressing the solution to the problem or simply, they could not analyze the problem. Pupils also could not remember and keep in mind the various pieces of information needed to solve the problem. When solving problems some pupils could not think about different possibilities and figure out what would happen if she or he chose one or the other.

For example saying $\frac{f(2 + h) - f(2)}{h} = \frac{2 + h - 2 + 2x + 7}{h}$

https://etd.uwc.ac.za/

When answering especially question one pupils showed or demonstrated blurred and sweeping perception as they attempted to analyze the problem. Most pupils did not trust themselves enough that is why they could not solve the problems (no self-confidence).

4.4.3 <u>Common mistakes or misconcepts made by First year mathematics students</u>

<u>Question 1</u>

One student said that 0/0 is undefined or infinity meaning that to his/her infinity is equal to zero. When asked to explain what the limit is the student tried to solve the problem using the graph showing a bit of an understanding even though two students did not get it right. But at least, unlike in the case of high school pupils the students could picture what the limit is or could possibly be.

UNIVERSITY of the
WESTERN CAPE

<u>Question (ii)</u>

One student said that in order to find the limit one has to differentiate first. When simplifying the expression one student said $\lim_{h \to \infty} \dfrac{\frac{3x^2}{x^2} - \frac{2x}{x^2} - \frac{1}{x^2}}{\frac{2x}{x^2} + \frac{2}{x^2}} = 3$ other student wrote that

$$\lim_{x \to \infty} \frac{x^3 - x^2 - x}{x^2 + 2x} = \lim_{x \to \infty} \frac{3x^2 - 2x - 1}{2x + 2} = -\tfrac{1}{2}.$$

It should be noted that even though these first year students made mistakes to get $-\frac{1}{2}$ they were able to say that this is a special case as one will find $\frac{\infty}{\infty}$.

https://etd.uwc.ac.za/

Question 2

When reading f (2+h) students saying that f into (2+h) and reading f (2) as f into 2. The other mistake that the student made in this section is that of applying what he knows (the formula $\lim_{h \to o} \frac{f(x+h)-f(x)}{h}$) without real understanding of what the question requires (thus not using f(2+h)-f(2)) as a result ending up in complete confusion (with expression with x's and h's).

Question 3

Just like for the standard ten pupils reading f'(-1) was also a problem to the first year students. They read it as f prime into minus one. Drawing the cubic graph (fully) was also a problem even though they could explain what turning points are, but they could not draw the cubic graph.

UNIVERSITY of the
WESTERN CAPE

Question 4

One student said in order to determine whether y has a maximum or a minimum value one has to find y". Finding whether the turning point is maximum or minimum really created problems for students, as most students 85,7% ended up just using trial-and-error responses. One student even said that he had forgotten his high school mathematics completely.

Question 5

50% of the students could not solve problem 5 saying they hated word problems and that they had forgotten the standard ten

https://etd.uwc.ac.za/

methods of solving such problems. The other 50% who seemed to be having active knowledge, not learning by rote, and who were able to retrieve things learned earlier, were able to solve the problem without too many problems.

4.4.4 <u>**The semantic and cognitive difficulties displayed by first year's University mathematics students**</u>.

<u>**Semantic difficulties**</u>

When answering questions asked by the interviewer the students murmured (in Xhosa) saying things that were not clearly audible to the interviewer. This showed that students had problems expressing themselves using the English language. (For example(s) refer to interviewee B_1 when answering question 4 in Appendix 1).

<u>**Cognitive difficulties**</u>

UNIVERSITY of the
WESTERN CAPE

They tried very hard to hide the fact that their vocabulary, especially mathematics vocabulary was very limited. So they described concepts and events by calling them "things". Even though compared to high school pupils first-year students were better, still their imagery power was found to be strong. They still had misconceptions of what turning points are (for the cubic curves) so they had limited interiorization (of one's behaviour). As already mentioned, the students lacked clarity and precision in language. They also lacked or had impairment in remembering and keeping in mind various pieces of information that one needs to solve a problem. An example for this would be

https://etd.uwc.ac.za/

that of students not being able to solve problem 5 and of some having even problems with problem 4.

In most cases students were in a hurry to finish solving the problem as a result they did not plan their work and therefore became very impulsive and unsystematic (in exploratory behaviour) when trying to solve a problem. The other factor was the inability to break the problem into small parts or relating it to things that one knows in order to simplify the problem. As a result of these, students lacked a good picture in their mind of what the question required them to do or what they should look for.

The other important thing to consider is the time factor. All the first year university students were able to finish their work within a short time compared to the high school pupils.

4.5 Analysis and interpretation of multiple-choice test given to high school pupils.

UNIVERSITY of the
WESTERN CAPE

Out of thirty-four students who sat for the test 64,7% of the pupils got a mean percentage below fifty. This indicated that there was indeed a problem with solving differential calculus. Of the questions answered pupils seemed to fare well on question 10, which just required the students to solve the following problem "Given g (x) =x^2 + 2 to find the gradient of the graph at the point where x = 2". I actually expected all the students who had done differential calculus to at least get only this question correct but six students did not manage to get it right. One would say that such students had very serious problems with differential calculus or were not interested in the test, they

https://etd.uwc.ac.za/

just ticked (guessed) any letter. Question 11 was the second best. Seventeen or 50% of the students managed to get this one correct. All the other questions had a score of less than 50% whilst questions 6 and 7 had the lowest scores. Full results of the test are given in the Table 1 and 2 and the sample of the question paper is given in Appendix 2D.

Table 1 : Analysis of multiple choice test

Number of Question	1	2	3	4	5	6	7	8	9	10	11
Frequency of pupils who got it correct	12	14	9	10	8	7	7	12	11	28	17

Table 2 : Score Table

Score out of 11	2	3	4	5	6	7	8	9
Frequency of pupils who scored the mark	2	8	6	6	8	3	-	1

UNIVERSITY of the
WESTERN CAPE

4.6 Discussion of results

(i) MLE

The role played by the teacher in mediating the pupils is very important in the learning of differential calculus. Students both at University level (25%) and those at high schools (19,4%) said that the reason why they did or do not perform well in calculus was because they were not properly guided by teachers as to how to study, poor teaching methods, lack of content matter, teachers being hostile to learners, and also teachers not

https://etd.uwc.ac.za/

perform at a lower cognitive level (Shapiro, 1989). Students both at high school and university levels stated that it is through a lot of practice (automatization) that one can pass differential calculus.

(v) School

Most students (90%) said that school does not have a particularly negative impact on their learning, except that the school culture is a bit different from the home one. One student said, *"at school you mix with bad friends, friends who are not interested in education"*. Most high school pupils said that usually they are forced into doing the subjects. They are not given a chance to choose subjects for themselves. For example one student said, *"I had passed mathematics well in Standard 7 so when I moved to high school I was forced to do mathematics even though I am more interested in History, so now I do not pass it well"*. Teachers said that they tried as much as possible to make school a conducive place for learning.

UNIVERSITY of the
WESTERN CAPE

Students complained of text books being difficult and not being supplied in time. It is important to note that the school can frustrate the learner and consequently retard his/her cognitive functioning if it (school) is not ready for the learner. Olivier (1993) to this effect asserts that usually the schools are not ready for the children that is why most students fail. Even though Oliviers' debated that it is useful not to "blame the victim" but to blame the school. I think that similarly the school will have a negative cognitive impact on the pupil (or victim).

https://etd.uwc.ac.za/

marking pupils' work. As mentioned earlier, children who lack mediation are not easily or more receptive to direct exposure and as a result do not benefit much from it. The results, therefore, support the studies done by Feuerstein (1991), where it is stated that in mediated learning experience, learning or interaction with the environment is guided therefore learning is intentional. Thus Feuerstein stated that the teacher (who is the mediator) should interpose himself or herself between the students and the world of stimuli to interpose, guide and give meaning to the stimuli (Feuerstein, 1980, as discussed in Chapter two of this thesis).

In the above discussion of "the results" of the thinking aloud protocols whereby most students (92%) were found to have learned calculus "by heart" (memorising without real understanding). 50% of first year university mathematics students said that they used to learn high school calculus for exams but did not understand what was going on. This shows how detrimental lack of mediation can be, and how it can retard student learning and therefore have an impact on students' cognitive ability or performance. There were many similar mistakes made by pupils like the inability to express himself/herself. A caring teacher could find these and help pupils to overcome them.

UNIVERSITY of the
WESTERN CAPE

All of the cognitive difficulties displayed by students (which) are or could be caused by lack of mediated learning experience (for these refer to "The semantic and cognitive difficulties displayed by pupils interviewed").

Of the ten teachers who were given the questionnaires only four responded showing how unmotivated or committed 60% of the teachers are, not interested in finding the ways of detecting

https://etd.uwc.ac.za/

pupils' mistake or deficiencies. This 60% just discussed the questionnaire, said that it was better for them not to write their answers. It can therefore be concluded that lack of mediated learning experience indeed leads to some of pupils' cognitive difficulties as stated by Feuerstein (1980).

(ii) Language and culture

It should be noted that as stated earlier in this chapter and in the previous chapter, the students interviewed are from the so-called working class or rather those whose parents or guardians earn the lowest wages. Most of their parents may not have even reached standard ten level in their education. So such parents usually use their mother language (Xhosa in this case) for blacks and Afrikaans for "Coloureds"), which ultimately will have an impact on the pupils cognition when learning in English due to both the culture and the learning of mathematics.

UNIVERSITY of the WESTERN CAPE

22,5% of the first year mathematics students said that English language has a great impact on their learning of differential calculus while 38,8% of the high school pupils who completed the questionnaires also said that they had problems with English language being used as the medium of instruction emphasizing that they struggle to understand English, at the same time struggling to understand the content (differential calculus). Valdes (1988) as discussed in chapter two, says that it is the responsibility of the second language teachers, including mathematics teachers, to realise this trauma the pupils experience in learning differential calculus. Valdes goes on to say that teachers should assist (therefore mediate) and bring pupils through to the point where their values, technical skills, aspirations, beliefs,

https://etd.uwc.ac.za/

attitudes, patterns of behaviour and relationships can be realized and utilized efficiently.

(iii) <u>Syllabus</u>

Some students blamed the syllabus claiming that it is segregated. That their syllabus is different form the syllabus of their white counterparts. First year students to this effect also said that *"The syllabus for the blacks (DET) is an inferior education meant for labourer not to be the planner."* Some teachers (50%) agreed that something has to be done to the syllabus but said that this will take some time so in the meantime teachers themselves should teach their pupils (for their own (pupils') benefit). It should be noted that, as Laridon stated, calculus is only mentioned in standard 10 syllabus directly, but it actually starts from standard 6 upward. Looking at the Verwoerd statement, it is clear that what the pupils said about the syllabus holds.

(iv) <u>Content and calculus learning</u>

UNIVERSITY of the
WESTERN CAPE

As stated by students it is important to have teachers that are well conversant with the content. As Avital (1987) states pupils should be taught other options of solving the problem and not just one. It is important as some of the teachers wanted to know exactly how to introduce differential calculus as failure to do this may cause some cognitive problems.

Most first-year students (60%) said that usually they are not told the prerequisites (pre-knowledge) for the differential calculus in order that to do revision before the introduction of the new topic. They said that if one does not have a good

https://etd.uwc.ac.za/

foundation then no matter how hard s/he tries s/he will always run short of something which will have a serious impact on academic learning. They said that pupils should be taught not only in differential calculus but more concentration should be placed on how to solve problems. Therefore on the learning of heuristics and algorithms, they further said that a lot of practice is needed for differential calculus and this (like Simon (1981) stated) can help for the better storage and retrieval of information if practice takes place with full understanding of what is going on.

Most high school pupils (50,7%) said that they had problems with differential calculus, especially its application. They blamed their poor performance on their poor background, the time set for solving the problem and the underqualified teachers. Teachers on the other appeared to differ greatly on how to approach the differential calculus. Some teachers suggested the working backward method as the best. While others suggested checking the pupils' pre-knowledge first. As a result that the researcher found that in-service training is really needed for matric mathematics teachers as some teachers seemed not to be confident or at ease with differential calculus. When dealing with the learning of calculus it is important to look at factors such as problem-solving. Sternberg (1986) states that one should be able to learn the necessary skills to be able to deal with the novel situations. As novelty is one of the several ways to measure intelligence through assessment of insightful problem solving. The ability to automatize is also vital in the learning of differential calculus. The lack of ability to deal with novelty and lack of ability to automatize information may end up with the breakdown of information processing hence causing the student to

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

(vi) Political and socio-economic factors

A few (20%) first year university mathematics students complained of the political activities like, stay-aways and teachers strikes. They said that some of their colleagues leave school because their parents do not have money to pay the school fees. Most high school pupils especially (all) the repeaters said that political situation affects their learning of differential calculus because of stay-aways. Which they asserted are good to put pressure on the government for changes, but bad because it affects their performance and takes most of their time. Teachers said students need to know what is going on around them but at the same time should work very hard to pass. Concerning the socio-economic factors they stated that all their students come from almost the same (poor) backgrounds. Even though students mentioned that politics affect their studies, they (most of interviewees) are not aware of the Department of Education and training's hidden agendas. That is things like how 'mathematics' is utilized to marginalize certain sectors of society (Julie, 1991). Even though it was not explicit students did mention that mathematics should be taught in such a way that it should be related to everyday happenings instead of being divorced from reality. In relation to this, D'Ambrosio (1984) and Julie (1991) stated that mathematics should be taken as a panhuman activity. It is also important to note that students said that some of their colleagues leave school because of poor economic factors, some students continue schooling even though some of their physiological needs, their safety needs, love and belonging are not met. Failure to satisfy these needs has tremendous cognitive impact on their academic performance as stated in Jones (1980).

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

UNIVERSITY of the
WESTERN CAPE
https://etd.uwc.ac.za/

(vii) **<u>Preference, attitude, believes and affective motivational factors</u>**

First year university mathematics students, high school pupils and matric teachers all agreed that affective-motivational factor is the most important in the learning of differential (This is in agreement with Ellis (1986). Both the university and high school students said that one needs to have a good and positive attitude towards differential calculus in order to pass it well. They said this can be done successfully by teachers thus teachers should relate the content or simply differential calculus to everyday experience. They went on to say one has got to believe in oneself first before one can actually perform well in calculus. Students said that teachers should consider students' choices (or preferences) and not choose subjects for the pupils. As stated in the literature review, Orton (1987) to this effect stated that pupils need not be told only the positive side of things or topics but the negative should also be discussed.

UNIVERSITY of the
WESTERN CAPE

The pupils' views on the other hand are in agreement with what Hutchinson (1985) pointed out. He said that students with bad records, for instance those with history of academic failure are or may be seriously threatened by the environment in which their academic difficulties would be made known to others. It can therefore be seen that lack of all the above factors should be taken care of in order to enhance students' cognitive functioning, as Cornelius (1982) stated, mathematics or differential calculus in particular can be effective if pupils' experiences are challenging enough to arouse one's interest and active participation without (much) anxiety.

https://etd.uwc.ac.za/

CHAPTER 5 : IMPLICATION FOR FURHTER STUDY

5. SUMMARY, RECOMMENDATIONS AND CONCLUSION

5.1 A brief summary of findings

The objective of this study was to find out what the causes of cognitive difficulties were in the learning of differential calculus among High school pupils. These are discussed in the previous chapter. In summary one can say that the problem which pupils think they have regarding the learning of differential calculus are those of lack of motivation, their attitude towards the subject, inadequate practice, unqualified teachers, unstimulating teaching methods unmarked work, lack of feedback, homes not conducive for study and the socio-economic political conditions of blacks in South Africa. All these are thought of as having an impact on ones' cognition in the learning of differential calculus.

UNIVERSITY of the
WESTERN CAPE

On the other hand for those pupils who were interviewed (thinking aloud interviews) the following deficiencies were found (for full information refer to analysis of thinking aloud protocol in chapter 4). There were serious problems of expression as students could hardly express themselves (in English). Students were found to be impulsive and very unsystematic when solving the problem. They were found to have a very poor retrieval capacity as they were not able to adapt what they had learned in the previous lessons to the new situation so the transference ability for the students was lacking. They were found to be very poor in gathering the information necessary for solving the problem (therefore impairments during input phase manifested), the

https://etd.uwc.ac.za/

imagery ability or the mental models of the pupils were also found to be full of misconcepts. In addition to these, students were making many mistakes showing the misconceptions that they had been carrying along from the lower standards, for example $\frac{x^2 - 4}{x - 2}$ dividing x^2 by x and get x and dividing 4 by 2 to get 2 resulting with x-2 as the answer. The other difficulties are semantic in nature for instance, pupils having difficulty with the language (both mathematical and English).

It is already stated that high school pupils (71,6%) said that affective motivational processes influence in the learning of differential calculus while 96% of the university first-year mathematics students said if it was not for motivation they would not be at the university. This therefore shows that affective motivational factors are indeed variables that need teachers, students' and parents' real attention as without this pupils can develop cognitive dysfunctions.

Indeed it has been found that all the factors stated that pupils mentioned had an impact on the study of differential calculus (apart from the lack of content or basics). Teachers also mentioned that without motivation learning cannot be successful, (for more recommendations made by the teacher refer to chapter four). Therefore, for such pupils (from low social economic class) lack of motivation would result in the failure of differential calculus.

It is through this research and the researcher's experience as the teacher and as an external marker that the conclusion drawn is that something ought to be done to improve or modify the cognitive impairments that the children have and to alter some

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

of the variables that cause these impairments. The following are the suggestions for preventing or trying to minimize some of the causes of cognitive difficulties as found in this study.

5.2 Recommendations and implications for further study

5.2.1 Thinking Skills

(1) Pre-knowledge should always be tested before introducing a new topic (differential calculus in particular). Pupils found to be having problems with this should be given extra-classes or an audio-visual tape should be played for them showing them or teaching them the lacking skills or knowledge. Alternatively they can be allowed to attend lower level mathematics classes. Special arrangements should be made for them if they find it necessary.

(2) Further research should concentrate on the importance of the thinking or the logic that goes with solving the problem (i.e. heuristics) and not putting more emphasis on finding the correct answers which can just come if the (algorithms) methods and the thinking are in place. That is the algorithms should be considered as intrinsic to problem solving. Teachers should therefore concentrate more on heuristics for problem solving.

(3) Standard ten pupils should have tutorial sessions which should be attended by all the school mathematics teachers to help the Std 10 teachers with diagnosis and rectifying the cognitive difficulties shown by pupils during the mathematics (especially differential calculus) problem solving sessions. The time taken to solve the problem should also be taken note of. This should

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

be done as pupils and first year university students said that is only through (guided) practice that success can be attained.

(4) Misconcepts that are carried by pupils are mainly blamed on pre-knowledge. To this one would suggest that projects like instrumental enrichment suggested by Feuerstein (1980) and the "Project intelligence" suggested in Venezuela (1979) and any other projects of this nature (like Debono's) be researched on and put into practice so that pupils' impairments or retardation can be detected and rectified. Thus more remedial teachers need to be trained for this purpose.

(5) If a given problem solving strategy (or heuristics) is intended to be generally useful, teachers should make sure that it should be encountered and practised in a variety of problem contexts. That is nothing that one says (teachers) in class is likely to have a lasting impact on the students' attitudes towards thinking as will those displayed by the teacher. This, therefore, means that research needs to be carried out on types of models or material that would be more triggering and interesting in the learning of differential calculus.

(6) Pupils should be made aware that mathematics is a deductive theory (a Claim by Vinner 1991) and as such it starts with primary notion and axioms and should note that by means of primary notions all other notions are or can be defined. Vinner (1991) said that all theorems which are not axioms are proved from axioms by means of certain rules of inference.

(7) More research should be directed towards teaching the pupils to draw the distinction between finding the limit and determining

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

the derivative as most students confuse these two terms. (This can serve to promote both Creative and Critical thinking).

(8) The stages (cognitive and physical) at which pupils are should be taken into consideration when dealing with pupils, but should never be exaggerated to their (pupils') disadvantage.

(9) In teaching any section of a syllabus, it is necessary to research or determine: which phase of the mental act (input, elaboration and output) are necessary for the impairment incurred or lack of understanding and/or applying the particular concept or content material in general (Feuerstein 1980).

5.2.2 Affective Motivational factor

(10) If the problem lies in the terminology (differential calculus) then pupils should not be told the topic at the beginning but rather just do it without specifying the field. In cases like those pupils should be told afterwards.

UNIVERSITY of the
WESTERN CAPE

(11) Exercises should be designed and selected so as to find out whether students experience the right level of challenge, which means that a sincere effort should usually be made through research to success and a sense of accomplishment.

When giving or providing feedback teachers should make sure that a feedback is informative as well as telling pupils how well they performed on a task. Thus telling pupils what they have done well (like writing the $\lim_{x \to 2} \frac{x - 4}{x - 2}$ then simplying it like

$$\lim_{x \to 2} \frac{(x - 2)(x + 2)}{(x - 2)}$$

https://etd.uwc.ac.za/

Then substituting 2 without first cancelling (x - 2) so in this case students should be told that the first two steps are correct but made a mistake when moving on to the third step. Thus telling pupils what they have done well and what they have done poorly and how they can do better. It is important to note that students should not be penalized when inventive approaches to problems yield unorthodox or unanticipated results.

(12) Teachers should try and find out whether telling students the objectives of the topic (differential calculus) before introducing differential calculus or any new topic would not make some improvements in the understanding of the topic. Making them few but specific.

(13) Teachers should feel free to do or approach the syllabus the way they deem appropriate, but the researchers' suggestion is that this can be done with the help of other standard 10 teachers. The new approach should be made interesting so that the pupils would like differential calculus.

UNIVERSITY of the
WESTERN CAPE

(14) Pupils should be taught South-African history of mathematics so that they can understand why most people (maybe even most of their teachers) take or consider mathematics as a status subject that can only be done by a "genius". This should be done so as to find out whether pupils' interest in mathematics will not improve.

(15) Pupils should be given real life examples of how people survive academic frustrations. This can help pupils to realize that academic life can sometimes have very serious problems, but

https://etd.uwc.ac.za/

pupils should keep on their level best so that they cannot give up (too) easily. Thus pupils should persevere.

(16) When giving exercises teachers should try as much as possible to select problems that can be intrinsically interesting to the students. It is vital to note that the programmes designed to enhance thinking must respect the students' desire to know why they should be interested in learning what they are being asked to learn. Thus in this case the differential calculus word problems should be interesting and they should be written in simple English. Differential calculus teachers to this effect should try as much as possible to make differential calculus a place where mistakes are made and corrected, as pupils should know that they should learn by their mistakes (Sternberg, 1986).

(17) Pupils should be told that "the essential condition for learning is the purposeful activity, the willingness to work hard to learn, (of the individual learner). Pupils should know that learning is not a gift any school can give, but it is the prize the learner himself/herself must pursue. If a pupil is unwilling or unable to make the effort required, he will learn little in even the best school". This does not mean that teachers are powerless and unwilling to motivate and make the effort that learning requires.

(18) Pupils should not be overloaded with work as this can affect their (memory) retrieval and even storage of information in memory. This, can, therefore aggravate pupils' attitude toward differential calculus saying that it is too demanding on

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

their work. For example, teachers should not give pupils exercises to do before marking the previous ones.

5.2.3 <u>Clinical Interviews</u>

(19) There should be in service courses to enrich teachers and for teachers to learn new topics like differential calculus. There should also be a place for training teachers to provide diverse mathematical learning opportunities and to be able to utilize different instructural interventions as part of sound mathematical programme (Giardano 1992). On top of these there should be subject advisers who should contact courses and discuss individual teachers' problems (mathematical) in this case differential calculus, as many teachers (especially those trained in the colleges in South Africa before 1988 did not do differential calculus). Such teachers should be spotted and given the proper training - this can be done through clinical interviews with teachers.

UNIVERSITY of the
WESTERN CAPE

(20) The other important factors which should be considered for further research are that of teachers conducting thinking-aloud-interviews in standard eight (mid-year) and then they will have to rectify and modify the weak points. Then in standard nine (mid year) they should do the same thing as in standard eight. By the time pupils reach standard ten they would have a better understanding of mathematical concepts. (It should be noted that the thinking-aloud protocols should be done using the areas related to differential calculus as the objective would be to rid the causes of cognitive difficulties in differential calculus among high school pupils.

https://etd.uwc.ac.za/

(21) There is need for teachers to be taught the importance of diagnosing cognitive difficulties among students.

5.2.4 Imagery

(22) Teachers should not recommend only one textbook for use in class. They should recommend other books for reference. This should serve to promote students to have a better picture of concepts being discussed.

(23) When introducing $\lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$ pupils should first know what it means before just using it to solve problem. Thus the previous knowledge ($m = \frac{\Delta Y}{\Delta X}$) should be linked to the new, there should be transference. This should be introduced in such a way that pupils will be able to see and have a picture of what the word 'approach' means, what the word "limit" means that to be able to see what happens as 'h' gets smaller because this could best be solved using the graphical representation. Which on the other hand helps for the mental model or help to promote or enhance the imagery power of the learner. The concept should be integrated (synthesized) into a whole, (refer to chapter four under "Differential Calculus"). It should also be shown how the rules of differentiations were derived.

(24) More graphs, diagrams, and even concrete objects should be used as much as possible in differential calculus teaching and learning. These also would help in forming the mental picture of concept being discussed.

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

(25) Standard ten pupils should know that to acquire a concept means to form a concept image for it. Thus learning of differential calculus should not just be memorization or learning it to pass standard ten (as one university student interviewed said) but students should have a mental model or picture of what exactly one is talking about. Therefore, pupils should know that learning by memorization the concept definition does not guarantee understanding of concepts. It is in this regard that the researcher suggests that reference should be made to Perkins (1986) where it is stated that students should be taught the four design questions so that they can easily check whether they have really understood the concept or not. (The four design questions are: (i) What is the purpose(s)? (ii) What is its structure? (iii) What are model cases? and (iv) what are explanatory and evaluative arguments?) In this way students can develop the skills or have an interest in reflecting on their own activities (Praxis) Skemp (1979) calls this the feature of all intelligent behaviour.

UNIVERSITY of the
WESTERN CAPE

5.2.5 Others

(26) More individual work on problems should be encouraged unless pupils want to ask their friends something. That is even if they work in groups (discussing), when it comes to real solving of the problems students should be allowed to solve them alone and then to compare their answers with those of other group members. If they differ then each can explain how s/he has come to the answer until they find who made a mistake and why. In this way students can learn to be independent and be able to deal with a novelt situation easily. Where novelty is a function of knowing what to do in a new kind of task or situation (Sternberg,

https://etd.uwc.ac.za/

1986) one of the vital mental skills for dealing with novelty is that of insight which involves a process whereby one should be able to take out the relevant from the unwanted information (selective encoding). It also involves a process whereby a simple combination of what might originally seem to be an isolated piece of information into a unified whole that may in turn look different or the same to its parts. Lastly it involves relating newly-acquired information to information acquired in the past. It is through a process like this that a teacher can easily find the misconceptions held by pupils and can, therefore, quickly rectify or modify them.

(27) For pupils with "home problems" that is those who cannot study at home (due to factors mentioned in Chapter four), special arrangements should be made for them to use the classrooms at night. This means that there is a need for research in this field to find out how these situations can be remedied.

UNIVERSITY of the
WESTERN CAPE

(28) There is a need for research to be done to find out whether those pupils who have been passing other subjects but failing mathematics (and promoted to the next class), as a result developing a very negative attitude towards the subject, should be allowed to do a new stream of mathematics. Whether this new stream should be recognized by the matric board (as mathematics for general knowledge) which should be different from that of students who want to major in mathematics. This would allow the weak student to have a better picture of what goes on in mathematics.

It is not very hard to attain at least twenty of the above mentioned suggestions for a really hard-working, devoted teacher,

https://etd.uwc.ac.za/

as most of them simply need looking very closely at one's own pupils (the ones he/she interacts with daily).

CONCLUSION

The above implications and recommendations for further study can be summarized under five headings, namely thinking skills, affective-motivation factors, the benefit of clinical interviews, imagery and 'others' (for example environment factors). Research on the improvement of learning and teaching of differential calculus should revolve around these factors.

Firstly, thinking skills, (items 1 - 9) can be summarized as follows, pre-knowledge should be tested prior to the teaching of differential calculus to ensure that students are more or less at the same level. It would be useful and helpful to teach students the logic that goes with solving differential calculus problem (heuristics). Furthermore individual work should be encouraged so that students will be able to work with novel situations and thus show a need for more tutorial sessions which will allow them more time to practice solving differential calculus problems. The impact of students' age of the learning of differential calculus should be taken into consideration.

UNIVERSITY of the WESTERN CAPE

Secondly, affective motivational factors (items 10 - 18) should be taken cognizance of. They are highlighted in the following discussion. Students should be made aware of essential conditions for learning of differential calculus. Students would most likely learn better and will be motivated if they know the objective of the studying of differential calculus. The exercises that are given in differential calculus should be at

https://etd.uwc.ac.za/

the pupils' cognitive level and should depict everyday life experiences; this is because, it is when students feel that what they are learning is meaningful to them that they are encouraged to learn. There should also be a dynamic approach to the teaching of differential calculus syllabus. Students need to be taught South-African mathematics history so that stereotypes in the learning of mathematics especially the differential calculus can be broken. It is also crucial that students be shown how to survive academic frustrations in the learning of mathematics. The classroom situation should be such that students can feel free to learn even by making mistakes. All these factors need to be tested and validated in the research on teaching and learning of differential calculus.

Thirdly, teachers should realize the benefit of clinical interviews (items 19 - 25), which are summarized below. This necessitates that those teachers who lack differential calculus skills should be given inservice training. Teachers should be taught ways of diagnosing students with cognitive difficulties in the learning of differential calculus. It is important that the deficiencies be diagnosed at an early stage so as to determine the phase of the mental act.

UNIVERSITY of the
WESTERN CAPE

Fourthly, imagery which includes items 22 - 25 summarized below is another concept that research on differential calculus learning and teaching should focus on. Forming concept image is crucial in the learning of differential calculus. There is a need to break the habit of learning by memorization because this does not guarantee an understanding of the concept. More textbooks should be prescribed so that students can have a clear picture of what goes on. Model diagrams and other teaching aids

https://etd.uwc.ac.za/

should be used in the learning and teaching of differential calculus. The four design questions formulated by Perkins (1986) should be used and if students can answer them satisfactorily it would mean that they have a clearer picture of the concept in discussion. The four design questions are as follows, (i) What is the purpose(s)? (ii) What is its structure? (iii) What are its model cases and lastly, (iv) What are its explanatory and evaluative arguments.

Finally we have "Others" (Items 26 - 28) The example in this case would be environmental factors that may have serious impact on cognition. Institutions such as school should be made ready for the pupils. They should be able to accommodate the needs of their own pupils. Peer group working and the attitudes and beliefs (that is culture) of the pupils should be taken into consideration. Thus in general the ecosystem of pupils should be looked at when deciding on the syllabus that should be followed for teaching the pupils.

UNIVERSITY of the WESTERN CAPE

In general the researcher suggested that not only the internal/endogenous and the endo-exogenous factors but also the exogenous or external factors (situated action) should be noted and taken care of as these factors can combine positively in mediated learning or negatively in lack of mediation resulting in inadequate cognitive development and cultural deprivation and reduced modifiability.

The endogenous factors include heredity or genetic factors, organicity and maturational level, the endo-exogenous factors include maturational level, emotional balance of the child and of parents. Exogenous on the other hand include the following

https://etd.uwc.ac.za/

factors: environmental stimuli, socio-economic status, educational level and the cultural difference.

Whilst these factors do not necessarily provide a blue-print on the learning of differential calculus, they however, should guide research in this field and they can be either validated or invalidated in the process.

There many problems which were stated by students as the causes of their differential calculus failures. Most of these were found to be mainly exogenous factors. It is true that other factors were found to be endogenous. This was done by means of the thinking aloud interviews. It is through the interviews that the lacking functions were identified and so could easily be located within a particular phase.

The study also shows that affective-motivational factors play a vital role in the learning of differential calculus. This was found through the questionnaires given to high school pupils, to university first year mathematics pupils and to the matric mathematics teachers. The research also shows that other non-cognitive factors can affect or retard the performance of the learners.

The pupils cognitive deficiencies, the motivation and affective processes and the pupils' perception of what the causes of their difficulties in learning differential calculus were found to be very close related. So closely related that it is not even very easy to separate one from the other. The teachers response on the other hand showed that most impairments that pupils have are not solely dependent on internal mechanism of the brain but that

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

these are mainly influenced by the factors which are outside, like the environment.

The first year students indicated that even though they had some problems with high school calculus they (most of them) think that it helped them a lot and therefore promoted their understanding of first year university calculus.

It can therefore be deducted from the results and findings that Perkins' stages are very important. That is not only that the micro and macro level that should be taken into consideration but the whole structure "architect" meta-level should also be considered as the cause(s) may lie on this and then affect others.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

BIBLIOGRAPHY

Achola, P. and Bless, C. (1988). Fundamentals of Social Research Methods: An African Perspective. Lusaka: Government Printing Departments

Adams, S. (1990). A study of Misconceptions in the Understanding of the Simple Electric Circuit in a Sample of South African and American School Pupils. Unpublished M.Phil Mini-Thesis, University of the Western Cape, South Africa.

American Psychiatric Association. (1986). Diagnostic and Statistical Manual of Mental Disorders (3rd edition). Washington D.C.: APA.

Armbruster, B.B., Anderson, T.H. and Meyer, J.L. (1991). Improving Content-area Reading Using Instructional Graphics. Reading Research Quarterly, XXVI/4, 393 - 416.

Assessment of Performance Unit. (1982). Mathematical Development: Secondary Survey. Report Number 3. London: HMSO.

Austin, J.L. and Howson, A.G. (1979). Language and Mathematical education. Educational Studies in Mathematics, 10, 161 - 167.

Avital, S. (1987). Teaching Calculus for Better Understanding. Spectrum, 25 (4), 27 - 29.

Barnard, J.J. and Strauss, J. (1989). Verband Tussen Basiese Begrippe en Wiskundeprestasie. South African Journal of Education, 9 (2), 228 - 233.

Bell, J. (1987). Doing Your Research Project. Philadelphia: Open University Press.

Behr, A.L. (1980). New Perspectives in South African Education. Durban: Butterworth and Company (S.A.) (PTY) LTD.

Best, J.W. (1977). Research in Education. New Jersey: Prentice-Hall, Inc.

Biggs, J.B. (1962). Anxiety, Motivation and Primary School Mathematics, London: National Foundation for Educational Research.

Bishop, A.J. (1990). Western Mathematics: The Secret Weapon of Cultural Imperialists. Race and Class, 32 (2), p 51 - 65.

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

Bishop, A.J. (1988). Mathematical Enculteration. London: Kluwer Academic Publishers.

Bloom, B.S., Engelhart, M.D., Furst, E.J., Hill, W.H. and Hathwohl, D.R. (eds.) (1956). Taxonomy of Educational Objectives: Handbook I-Cognitive Domain. New York: David Mckay.

Bochner, S. (1976). Problems in Culture Learning. In Bochner, S. and Wicks, P. (eds.), Overseas Students in Australia. Sydney: The New South Wales University Press of Hawaii.

Bozsahin, H.C. and Findler, N.V. (1992). Memory-Base hypothesis Formation: Heuristic Learning of Common Sense caused Relations from Text. Cognitive Science, 10, 431 - 454.

Brodie, K. (April, 1989). Education in a second language: Its effects on Mathematics Learning. Pythagoras, 19, 33 - 39.

Brown, A.L., Bransford, J.D., Ferrara, R.A. and Campiones, J.C. (1983). Learning, Remembering and Understanding. In: Mussen, P.H. (ed.), Handbook of Child Psychology. Vol III. New York: Wiley and Sons.

Buxton, L. (1981). Do You Panic About Maths? London: Heimann.

Cannell, C.F. and Kahn, R.L. (1968). Interviewing. In Lindzey, G and Aronson, A (eds.), The Handbook of Social Psychology, (2nd ed.), Vol 2, Research Methods. Reading MA: Addision-Wesley, pp 526 -595.

Cao, F. (1986). Strategies of Endogenous Development. New Delhi: Oxford and IBH Publishing Co. Pvt Ltd.

Cawler, J.F. and Miller, J. (1989). Cross Sectional Comparison of the Mathematical Performance of Children with Learning Disabilities: Are we on the Right Track Toward Comprehensive Programming? Journal of Learning Disabilities, 22, 250 - 254.

Cooper, E.J. (1993). A Comprehension and Cognitive Development Approach to School Reform. Journal of Negro Education, 62 (1), 91 - 99.

Corneluis, M. (1982). Teaching Mathematics. New York: Nicholis Publishing Company.

Cornu, B. (1991). Limits. In. Tall, D. Advanced Mathematical Thinking. Dordrecht: Kluwer Academic Publishers.

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

Cummins, J and Swain, M. (1986). Bilingualism in Education. London: Longman.

D'Ambrosio, U. (1984). Culture, Cognition and Science Learning. Proceedings of Science Education in the Americas. Panama: NSTA-OAS.

D'Ambrosio, U. (1985). Ethnomathematics and its Place in the History and Pedagogy of Mathematics. For the Learning of Mathematics. 5, No. 1: Montreal, Quebec, Canada: F.L.M. Publishing.

Davis, R.B. A Second Interview with Henry, Including Some Suggested Categories of Mathematical Behaviour. The Journal of Children's Mathematical Behaviour. 1, 36 - 62.

Davis, R.B. (1984). Learning Mathematics: The Cognitive Science Approach to Mathematics Education. London: Billing and Sons Ltd.

De Bono, E. (1985). The CoRT Thinking Programme. In Chipman, S.F., Segal, J.W. and Glaser, R. (eds.), Thinking and Learning Skills: Relating Instruction to Research. Vol. 1. Hillsdale, N.J: Lawrence Erlbaum Associates.

De Mey, M. The cognitive paradigm. Dordecht: D. Reidel Publishing Company.

Dillion, R.F. and Sternberg, R.J. (1986). Cognition and Instruction. London: Academic Press Inc.

Ellis, E.S. (1986). The role of motivation and pedagogy on the generalization of cognitive strategic training. Journal of Learning Disabilities. 19 (2), 66 - 69.

Ellis, R. (1987). Understanding Second Language Acquisition. London: Oxford University Press.

Ericson, K.A. and Simon, H.A. (1980). Verbal Reports as Data. Psychological Review. 87, 215 - 251.

Etuk, H.U. (1992). Predicting Educational Performance at Tertiary Level on the basis of Secondary Level Performance. International Journal of Mathematics Education and Science Technology, 24 (2), 283 - 288.

Fasheh, M. (1982). Mathematics, Culture and Authority. "For the Learning of Mathematics, 3 (2), 2 - 8.

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

Fay, B. (1988). Social Theory and Political Practice. London: George Allen and Unwin (Publishing) Ltd.

Feuerstein, R., Rand, Y. and Hoffman, M.B. (1979). The Dynamic Assessment of Retarded Performers: The learning Potential Assessment Device, Theory, Instruments, Techniques. Baltimore: University Press.

Feuerstein, R., Rand,Y. and Hoffman, M.B. and Miller, R. (1980). Instrumental Enrichment. Baltimore: University Press.

Feuerstein, R., Klein, P. and Tannenbaum, A.J. (eds.), (1991). Mediated Learning Experience. London: Freund Publishing House.

Flavell, J.H. (1976). The Development of Metacommunication. Paper presented at the twenty-first International Congress of Psychology, Paris.

Fredette, N.H. (1979). The Clinical Interview as a Tool for Investigating Cognitive Processes. Unpublished manuscripts. Amherst: University of Massachusetts.

Freire, P. (1970). Pedagogy of the Oppressed. New York: Seabury.

Fynn, C.A. (1990). The Interdependence Between Language and the Learning of Mathematics by Disadvantaged High School Mathematics Learners. Unpublished M.Phil Mini-thesis, University of the Western Cape.

UNIVERSITY of the
WESTERN CAPE

Gagné, R.M. (1985). The Conditions of Learning. New York: Holt Saunders.

Giordano, G. (1992). Heuristic Strategies: An aid for solving Verbal Mathematical Problems. Intervention in School and Clinic. 28 (2), 88 - 96.

Glencross, M. (1991). A New Mathematics Curriculum for a New South Africa. Pythagoras. 25, 8 - 11.

Gordon, A. (1993). Constructivism and the Politics of Pedagogy: The 'Social' in Constructivist Mathematics Programmes. Paper prepared for the conference, Political Dimensions of Mathematics Education: "Curriculum reconstruction for society in transition". Johannesburg, 1 - 9.

Greenspan, S.I. and Greenspan, N.T. (1981). The Clinical Interview of the Child. New York: R.R. Donnelley and Sons, Inc.

https://etd.uwc.ac.za/

Groner, R., Groner, M. and Bischof, W.F. (1983). Method of Heuristics. Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Hadamard, J. (1945). The Psychology of Invention in the Mathematical Field. Princeton: Princeton University Press.

Hamachek, D.E. (1976). Quoted In: Confield, J. and Wells, C.H. 100 Ways to Enhance Self-Concept in the Classroom: A Handbook for Teachers and Parents. New Jersey: Prentice Hall, Inc. Engelwood Cliffs.

Harrison, D. (1981). The White Tribe of Africa: South Africa in Perspective. Los Angelos: University of California Press.

Hendricks, M.A. (1991). The Nature and Value of Mathematics. Spectrum. 29 (2), 32 - 35.

Hudson, L. (1966). Contrary Imagination. Harmondsworth: Penguin Books.

Hutchinson, R.T. (1985). Teaching problem solving to Developmental adults: A Pilot study. In: Segal, J.W., Chipman, S.F. and Glaser, R. (eds.), Thinking and Learning Skills, Volume I: Relating Instruction to Research. Hillsdale, New Jersey: Erlbaum Associates.

Ingram, A.L. (1988). Problem solving. ECTJ, 36 (4), 211 - 230.

Jansen, J.D. (1987). Curriculum: Context, Conflict and Change in Black South African Education. Paper presented at the 31st Annual Conference of the Comparative and International Education Society, Washington D.C. March, 12 - 16.

Jansen, J.D. (1989). Curriculum as a political phenomena: Historical reflections on Black South African Education. Journal of Negro Education, 59 (2), 195 - 204.

Johnstone, A.H. and Al-Naeme, F.F. (1991). Room for Scientic Thought. International Journal of Science Education, 13 (2), 187 - 192.

Jones, V.J. (1980). Adolescents with Behaviour Problems: Strategies for Teaching, Counselling and Parents Involvement. London: Allyn and Bacon.

Julie, C. (ed.) (1991). People's Mathematics: Early Ideas and Debates. Bellville: University of the Western Cape.

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

Julie, C., Angelis, D. and Davis, Z. (eds.), (1993). Political Dimensions of Mathematics Education for Society in Transition. Pinelands: Maskew Miller Longman Ltd.

Konold, C. and Well, A. (1981). Analysis and Reporting of Interview Data. Amherst: University of Massachusetts.

Kagan, D.M. (1990). How do schools Define Students at Risk? Clearing House, 61, 320 - 324.

Landa, L.N. (1976). Instructional Regulation and Control: Cybernetics Algorithmization and Heuristics in Education. Englewood Cliffs, New Jersey: Educational Technology Publications.

Lamon, W.E. (1972). Learning and the Nature of Mathematics. California: Science Research Associates, Inc.

Laridon, P.E. (1988). An experiential approach to the Introduction of the Differential Calculus. Spectrum, 26 (2), 31 - 34.

Larkin, J. and Reif, F. (1979). Understanding and Teaching Problem Solving in Physics. European Journal of Science Education, 1, pp 191 - 203.

Macdonnel, J. (1989). What Pre-calculus skills do Students Retain? International Journal of Mathematical Education, 20, 174 - 177.

Malavé, L.M. and Duquette, G. (1991). Language, Culture and Cognition. London: Billing and Sons Ltd.

Maslow, A. (1982). quoted In Jones V.F. Adolescents with Behaviour Problems. Boston: Allyn and Bacon.

Means, B. and Knapp, M.S. (1991). Cognitive Approaches to Teaching Advanced SKills to Educationally Disadvantaged Students. Phi Delta Kappan, 282 - 289.

Mehl, M.C. (1985). The Cognitive Difficulties of Disadvantaged Students at the University of the Western Cape and Various Compensatory Programmes. Unpublished Doctoral Thesis. University of Cape Town.

Mellin-Olsen, S. (1987). The Politics of Mathematics Education. Dordrecht: C.D. Reidel Publishing.

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

Miller, A.D., Hall, S.W. and Heward, W.L. (1992). Effects of Sequential 1-Minute Time Trials, with and without Inter-Trial Feedback, on Regular and Special Education Students' Fluency with Maths Facts. Manuscript submitted for publication.

Miller, D. and Heward, W.L. (1992). Do your students Really Know Their Maths Facts. Intervention in School and Clinic, 28 (2), 98 - 104.

Montague, M. and Bos, C.S. (1986). The Effects of Cognitive Strategy Training on Verbal Maths Problem Solving Performance of Learning Disabled Adolescents. Journal of Learning Disabilities, 19 (1), 26 - 33.

Miller, G.A. (1956). The Magical Number Seven, Plus or Minus Two: Some Limits on Our Capacity for Processing Information. Psychological Review. 63, 81 - 97.

Nagy, P., Traub, R.E., Macrury, K. and Klaiman, R. (1991). High school calculus: Comparing the Content of Assignments and Tests. Journal for Research in Mathematics Education, 22 (1), 69 - 75.

Nickerson, R.S. (1984). Kinds of Thinking Taught in Current Programs. Educational Leadership,. September, 26 - 36.

Nkomo, O.M. (1984). Student Culture and Activism in Black South African Universities: The Roots of Resistance. Westport: Greenwood Press.

Nickerson, R.S., Perkins, D.N. and Smith, E.E. (1985). The Teaching of Thinking. Hillsdale, N.J.: Lawrance Erlbaum Associates.

UNIVERSITY of the
WESTERN CAPE

Norman, D.A. (1993). Cognition in the Head and in the World. Cognitive Science: A Multidisciplinary Journal of Artificial Intelligence, Linguistics, Neuroscience, Philosophy, Psychology, 17 (1), 1 - 6.

Noss, K. (ed.), (1990). Mathematics versus the National Curriculum. London: Falmer Press.

Olivier, A. (1989). Handling Pupils' Misconceptions. Pythagoras, 21, 10 - 19.

Orr, E.W. (1987). Twice as less: Black English and Performance of Black Students in Mathematics and Science. New York: W.W. Norton.

Orton, A. (1985). When Should We Teach Calculus? In: Mathematics in Schools. March: 11 - 5.

https://etd.uwc.ac.za/

Orton, A. (1987). Learning Mathematics: Issues, Theory and Classroom Practice. London: Cassell Educational Ltd.

Perkins, D.N. (1987). Thinking Frames: An Integrative Perspective on Teaching Cognitive Skills. In: Baron, J. and Sternberg, R. (eds.), Teaching Thinking Skills: Theory and Practice. New York:Freeman.

Perkins, D.N. and Simmons, T. (1988). The Cognitive Roots of Scientific and Mathematics Ability. Presented at the Conference on Academic Latent in Science, Mathematics and Technology. Duke University, March.

Perkins, D.N. (1989). Mindware: The new science of learnable intelligence. In: Proceedings of Paper Presented at the 4th International Conference on Thinking. San Juan, Puerto Rico, August.

Perkins, D. (1986). Knowledge as Design. Hillsdale, New Jersey: Lawrence Erlbaum.

Perkins, D. and Crismond, D (1990). Inside Understanding. Educational Technology Centre Havard Graduate School of Education.

Piaget, J. (1954). The Construction of Reality in the Child. (Margeret Codd, Trans.) New York: Basic Books.

Piaget, J. (1977). The Development of Thought: Equilibration of Cognitive Structures. New York: Viking Press.

Plug, C. (1990). Writing Scientific Articles: A Review and Guide for Authors. South African Journal Higher Education/SATHO, 4 (2), 7 - 25.

Polya, G. (1957). How to Solve It. 2nd edition. Princeton. New Jersey: Princeton University Press.

Polya, G (1981). Mathematical Discovery: On Understanding Learning and Teaching Problem-Solving. (2 vols: ed). New York: John Wiley and Sons.

Rand, Y. (1991). Deficient Cognitive Functions and Non-Cognitive Determinants. An Integrating Model: Assessment and Intervention. In Feuerstein, R. et al. (EDS), (1991). Mediated Learning Experience. London: Freund Publishing House.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

Reese, H.W. (1977). Imagery and Associate Memory. In Karl, R.V. and Hagen, J.W. (eds.), Perspectives on the Development of Memory and Cognition. Hillsdale, New Jersey: Lawrence and Erlbaum Associates.

Resnick, L.B. (1988). Treating Mathematics as an Ill-Structured Discipline. In Charles, R. and Silver, E.A. (eds.), Research Agenda for Mathematics Education: Teaching and Assessment of Mathematical Problem-Solving. Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Reynolds, P. (1989). Childhood at Crossroads: Cognition and Society in South Africa. Claremont: David Philip Publisher (Pty) Ltd.

Rhodes, J.S. (1993). Analogy and Imagery in Mathematical Learning. Fifteenth National Convention on Mathematics and Natural Science Education. Bloemfontein: MacMillan Boleswa Services Ltd.

Rhodes, J.S. (1992). Within the Subject Discipline of Mathematics in Differential Calculus to Investigate the Learning/Teaching of some Specific Learning Skills. Unpublished Doctoral Thesis, UWC.

Riley and Shapiro (1989). Diagnosis and Correction for Reading Problems as a Problem Solving. Clearing House, 62, 250 - 255.

Rivera, D.M. and Bryant, B.R. Mathematics Instruction for students with Special Needs. Intervention in school and clinic, 28 (2), 71 - 86.

Rubenstein, M.F. (1982). A Decade of Experience in Teaching, and Interdisciplinary Problem-Solving Course. In Tuma, D.R. and Reif, C (eds.) Problem Solving and Education: Issues in teaching and research. Hillsdale, N.J.: Lawrense Erlbaum Associates. 25 - 38.

Roos, R.C., Dekker, A.J., Visser, D.P. and Van Rensburg, J.N.J. Mathematics in Action: Standard 10. Wynberg: The Rustica Press Ltd.

Russel, S. (1983). Factors Influencing the Choice of Advanced Level Mathematics by Boys and Girls. University of Leeds, Centre for Studies in Science and Mathematics Education.

Schoenfield, A.H. (1985). Metacognitive and Epistemological Issues in Mathematical Understanding. In Silver, E.A. (ed.), Teaching and Multiple Research Perspectives, Hillsdale, New Jersey: Lawrence Erlbaum Associates.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

Schoenfield, A.H. (ed.) (1987). Cognitive Science and Mathematics Education: A Collection of Papers. California: University of California Berkeley.

Schoenfield, A.H. (1984). Learning Mathematics: The Cognitive Science Approach to Mathematics Education. Worcester, Billing and Sons.

Schoenfield, A.H. (1985). Students' Beliefs about Mathematics and Their Effects on Mathematical Performance: A questionnaire analysis: Paper presented at the Annual Meeting of the American Research Association, March 31 - April 4, Chicago, Illinois.

Schneider, M., Steeg, M. and Young, F.H. (1982). Linear Algebra: A Concrete Approach. New York: Macmillan.

Segal, J.W., Chipman, S.F. and Glasser, R. (eds.), (1985). Thinking and Learning Skills: Relating Instruction to Basic Research. Vol. 1. Hillsdale, New Jersey: Lawrence Erlbaum and Associates Publishers.

Silver, E.A. (1985). Teaching and Learning Mathematical Problem-Solving: Multiple Research Perspectives. Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Simon, H. (1981). Models of Thought. London: Yale University Press.

Simon, F.F.; Stierlin, H. and Wynne, L.C. (1985). The Language of Family Therapy: a Systematic Vocabulary and Sourcebook. New York: Falmer Press.

UNIVERSITY of the
WESTERN CAPE

Simon, M.A. (1986). The Teacher's Role in increasing Students' Understanding of Mathematics. Educational Leadership. 40 - 43.

Sinclair, A.J.L. (1987). On Reading and Comprehension. Part 1: Schema Theory. Unpublished Manuscript Research group on cognition. University of the Western Cape.

Sinclair, A.J.L. (1992). Reading and not Understanding: The Encoding Problems in English of Non-English First-year Students. Paper presented at the Annual SAALA Conference, University of Cape Town.

Skemp, R. (1977). The Psychology of Learning Mathematics. London: Penguin Book Publishers.

Skemp, R.R. (1979). Intelligence, Learning and Action. New York: John Wiley and Sons.

https://etd.uwc.ac.za/

Skovsmose, O. (1993). Towards a Critical Mathematics Education. In Julie, C., Angelis, D. and Davis, Z. (eds.), Political Dimension of Mathematics Education 2. Cape Town: Longman (Pty) Ltd.

Slammert, L. (1991). Mathematics and Politics. In Julie, C. (ed.), People's Mathematics: Early Ideas and Debates. Bellville: University of the Western Cape.

Sternberg, R.J. (1979). Intelligence, Information Processing and Analogical Reasoning: The Componential Analysis of Human Abilities. Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Sternberg, R.J. (ed). (1982). Handbook of Human Intelligence. New York: Cambridge University Press.

Sternberg, R.J. (1985). Instrumental and Componential Approaches to the Nature and Training of Intelligence. In Chipman, S.F., Segal, J.W. and Glasser, R. (eds.), Thinking and Learning skills. Vol 2: Research and Open Questions. Hillsdale. New Jersey: Lawrence Erlbaum Associates.

Sternberg, R.J. (1984). How Can We Teach Intelligence? Educational Leadership,. September, 38 - 42.

Sternberg, R.J. (1986). Intelligence Applied. New York: Harcourt, Brace, Jovanovich.

Sundberg, N.D. and Tyler, L.E. (1962). Clinical Psychology. New York: Meredith Publishing Company.

UNIVERSITY of the
WESTERN CAPE

Swanson, H.L. and Barbara, R.M.S. (1985). Strategy Transformations in Learning Disabled Children's Maths Performance: Clues to the Development of Expertise. Journal of Learning Disabilities, 18 (10, 596 - 603.

Tall, D. (1991). Advanced Mathematical Thinking. Dordrecht: Kluwer Academic Publishers.

Tall, D. and Razali, M.R. (1993). Diagnosing students' Difficulties in Learning Mathematics. International Journal of Mathematical Education and Science Technology, 24 (2), 209 - 222.

Taylor, N. (1993). Desire, Repression and Ethnomathematics. In Julie, C., Anglis, D. and Davis, Z. (eds). Political Dimensions of Mathematics Education 2: Curriculum Reconstruction for Society in Transition. Cape Town: Longman (Pty) Ltd.

https://etd.uwc.ac.za/

Toumasis, C. (1992). Explorations in school calculus: A case study. International Journal of Mathematical Education and Science Technology, 23 (6), 881 - 894.

Thompson, T.M. and Wiggins, K.L. (1990). The Treatment of Limits in the Calculus: Are We Becoming Careless? International Journal of Mathematical Education and Science Technology, 21 (2), 235 - 242.

Tzuriel, D. (1991). Cognitive Modifiability, Mediated Learning Experience and Affective Motivational Processes: A Transactional Approach. In Feuerstein, R. et al. (1991). (eds.), Mediated Learning Experience. London: Freund Publishing House.

UNO (1974). Res 3210 S -VID Item 7.

Valdes, M.J. (1988). Culture Bound: Bridging the Cultural Gap in Language Teaching. London: Press syndicate of the University of Cambridge.

Van Duyn, C. (1988). How about some Third Degree? Spectrum, 26 (1), 37 - 38.

Van Erp, I.W.M. and Heshusius, L. (1986). Action Psychology: Learning as the Interiorization of Action in Early Instruction of Mathematically Disabled Learners. Journal of Learning Disabilities, 19 (5), 274 - 279.

Vera, A.H. and Simon, H.A. (1993). Situated Action: A symbolic Interpretation. Cognitive science. Vol 17 (1). pp 7 - 48.

Volmik, J.D. (1993). A Different Mathematics Education for a Different South Africa? Pythagoras, 31, 32 - 37.

Von Glaserfeld, D. (1991). Radical Constructivism in Mathematics Education. Dordrecht: Kluwer Academic Publishers.

Vygotsky, L.S. (1962). Thought and Language. Cambridge, M.A.: MIT Press.

Walker, R. (1985). Doing Research: Handbook for Teachers. Cambridge: Cambridge University Press.

Wesso, I. (1992). An Intervention Mechanism for the Reading and Thinking Difficulties of Physiology Students at Second-Year University Level. Unpublished M.Phil Mini-Thesis, University of the Western Cape.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

Whimbey, A. and Lochhead, J. (1984). Beyond Problem Solving and comprehension. Hillsdale, New Jersey: Lawrence Erlbaum Associates Publications.

Whitehead, A.N. (1929). The aims of education. New York: Macmillan.

Winkel, B.J. (1990). First Year Students as In-Class Consultants. International Journal of Mathematical Education and Science Technology, 21 (3), 363 - 368.

Young, M.R. and Savage, H. (1983). Better Learning: How to Help Students of all ages overcome Learning Problems and Learning Disabilities. London: Prentice-Hall International, Inc.

Zimbardo, P. (1979). Psychology and life. Glenview Illinois: Scott Foresman.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

APPENDIX 1

TRANSCRIPTIONS AND ANALYSES OF CHAPTER 4 INTERVIEWS

INTERVIEW 1

Problem 1 (i)

1 I: Read this problem for me and then explain what you think the problem requires you to do.

2 S: (Reads the question) Determine the following limits, Lim is equal to $x^2 - 4$ when x approaches two divided by $x - 2$.

3 I: I don't see any equal sign on this problem but ...

4 S: No, it is not equal to.

5 I: OK. Can you read it again (problem)?

6 S: lim when x approaches 2 of $x^2 - 4$ divided by $x - 2$.

7 I: Ok.

8 S: First thing to do is to factorize $x^2 - 4$ (he factorizes murmuring and gets $(x - 2)(x + 2)$.

9 I: Why is it that you did not just substitute for x but simplified the expression first?

10 S: Because to solve the problem you must first factorize so that you can simplify.

11 I: Ok.

12 S: (continues) $x - 2$ cancels $x - 2$ leaving $x + 2$ in the numerator.

UNIVERSITY of the
WESTERN CAPE

13 I: Ok.

14 S: Now you take x as if it is equal to two.

15 I: Ok.

16 S: Then you say lim as x approaches two is two so 2 + 2 will give you 4.

17 I: Ok. so you just add 2 + 2 to give you 4 or what will be the answer?

18 S: The answer will be four.

19 I: Ok. Can you explain to me what the term limit means?

20 S: (Silence 1 min) What limit means!

21 I: Yes, what do you understand 'limit' to mean?

22 S: According to this sum I can put it as if you are told that a certain number is approaching two.

https://etd.uwc.ac.za/

23 I: Ok. How?

24 S: Certain number is approaching 2 then that number is replacing x (sic).

25 I: Can you give me examples of the numbers that are approaching two?

26 S: Ok, like 1,9.

27 I: Is that all?

28 S: The other ones may be 1.96, 1.97, 1.98 etc.

29 I: Is it only those below two like 1.9, 1.99 etc that approach 2 or are there other numbers approaching 2?

30 S: That approach 2! no.

31 I: Ok. What about 5, 4, 3, 2.1, 2.001 etc.

32 S: Oh! They are also approaching two but in the negative direction.

33 I: In the negative direction?

34 S: Yes, because they are decreasing.

35 I: Ok. Now how do you link the two?

36 S: They mean that two can be approached from both sides.

37 I: Ok.

38 S: You know what the problem is, I can calculate this limits but I don't understand them quite well.

UNIVERSITY of the
WESTERN CAPE

39 I: Ok. What is it that you don't understand well?

40 S: Why, Mmm but it's ok.

41 I: Why what? just ask don't be afraid.

42 S: Where in real life can you use this limits? You see Sir this 'limits' and other parts of maths are just done to make students fail mathematics. Ok, maybe we can use them in finding derivatives using the first principles. Is it not so?

43 I: Eh, I think so.

44 S: But the "application problems" (of differential calculus) are very difficult.

45 I: Ok, thank you.

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 1

Problem 1 (i)

Error factor

1. Saying that the number that approaches 2 are only those approaching it from the bottom and there are not other numbers approaching two (ref #28).

Semantic difficulties

1. When reading the question saying "limit is equal to $x^2 - 4$, when x approaches two divided by $x - 2$". Whereas there is not "equal sign" anywhere in the questions.

2. Not being able to explain clearly what the term limit means.

Cognitive difficulties

1. Not trusting oneself.

2. Lack of or impairment in remembering and keeping in mind various pieces of information one needs especially in explaining the steps taken.

UNIVERSITY of the
WESTERN CAPE

3. Not seeing or showing relationship between concepts previously learnt, for example in the work sheet no relation is shown from one step to the other.

4. Not very clear in expressing the solution due to language problems.

https://etd.uwc.ac.za/

INTERVIEW 2

Problem 1 (i)

1 I: What you have to do is to read this problem and explain what it requires you to do. As you solve the problem you should tell me what you are doing. You should remember that this is not a test and will not be used against you in any way.

2 S: (Silence = 1 min) I write 2 substitute 2 in the place of x because the problem states that x is approaching 2.

3 I: Ok.

4 S: Therefore 2^2 makes 4 minus 4 over 2 minus 2.

5 I: Mmm.

6 S: That will be zero.

7 I: So the answer will be zero?

8 S: Yes.

9 I: Here you said $2^2 - 4$ over $2 - 2$. So you mean here (pointing at the answer) you will just get zero?

10 S: It will be 4 - 4 so thy will cancel and the same applies on the bottom.

11 I: Ok, meaning that we will have zero at the top and zero at the bottom? Meaning we will have zero divided by zero? which is ...

12 S: Zero.

13 I: Ok. Do you have a calculator?

14 S: Yes.

15 I: Can you divide zero by zero and find out what will be the answer?

16 S: (presses the calculator) It's undefined.

17 I: Ok it's undefined does it mean that $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ is defined?

18 S: Yes. It's undefined. Can I try another option?

19 I: Yes, please.

20 S: What about one? (silence 1 min) Substituting 1 for x?

21 I: Ok try it?

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

22 S: (student works out the problem and ends up with 3 as the answer).

23 I: Where does this "1" come from?

24 S: I have tried using one instead of x.

25 I: Mmm?

26 S: One squared will give me one minus four which will give me minus three. Therefore I will take the sign of the bigger number which is minus 4 divided by minus one (sic)

27 I: Why substitute one?

28 S: Because one is also approaching two, its not really at two but its approaching two.

29 I: Ok. So one is approaching two?

30 S: Yes.

31 I: So you decided to take the number approaching two from below. Why not 1,999.

32 S: Because it is not easy to work with.

33 I: Ok. For one you said the limit will be three. What about if it approaches two from above?

34 S: You mean using four or three?

35 I: Yes.

36 S: Can I do ...

UNIVERSITY of the WESTERN CAPE

37 I: No, I am just asking. Does it mean $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ will keep on changing?

38 S: Yes it will keep on changing.

39 I: Can you factorize $x^2 - 4$ for me?

40 S: (silence 1 min then solves the problem correctly.) (look at the paper then multiply the factors).

41 I: No I just want the factors in this form $(x - 2)(x + 2)$.

42 S: Ok, this are the factors that you need.

43 I: Yes. Is $(x - 2)(x + 2)$ the same as $x^2 - 4$?

44 S: Yes, (continues solving the problem) So the answer will be four.

45 I: So it will no longer be three?

https://etd.uwc.ac.za/

46 S: Yes. Because I factorized and cancelled the denominator by the numerator.

47 I: So ...

48 S: Yes. So 4 is the correct answer.

49 I: Why is it that you did not factorize first .

50 S: Yeah! I forgot to factorize. I could have applied BODMAS.

51 I: What is BODMAS?

52 S: It means removing brackets first then factorizing before solving the problem.

53 I: Ok, is that all what it means? (silence 1 min) Ok, do you feel satisfied now?

54 S: Yes.

55 I: I would like us to go back to zero divided by zero. When you get zero divided by zero in a problem like this what is it that you should actually do?

56 S: I should ... should factorize the numerator.

57 I: What about if the problem lies in the denominator.

58 S: Factorize the denominator.

59 I: Then what will be the next step?

60 S: Simplify or cancel.

61 I: Ok, thanks.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 2

Problem 1 (i)

Error Factor (or Problems with Content)

1. Just substituting 2 for x in $x^2 - 4$ and 2 for x in $x - 2$ without first visually or mentally checking the answer. (ref # 2).

2. Saying that $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ is undefined (ref #16)

3. Saying that zero divided by zero is zero shows lack of pre-knowledge (ref #12).

4. Substituting one in the place of x (Look at answer sheet and ref #20).

5. Saying that the $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

   will keep on changing (ref #38).

6. That the numerator should always have the factors that should cancel the denominator.

7. Leaves out limit throughout problem solving (look at answer sheet).

8. Lack of confidence. Running away from answering question (ref # 18).

Semantic difficulties (others)

1. Could not read the problem correctly omitting a lot of words. For example the interviewee reading the question as follows, "Determine the following limits $x^2 - 4$ by $x - 2$ if x approaches two".

2. To the interviewee factorization and simplification are synonymous.

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

Cognitive difficulties

1. The student demonstrates blurred and sweeping perception as he attempts to analyze the problem.

2. Student seemed to be in the hurry to finish or solve the problem as a result interviewee displayed or showed impulsivity, unplanned and unsystematic exploratory behaviour in trying to solve the problem.

3. Interviewee lacked a good picture in mind of what he was looking for or what was to be done (i.e. lack of or poor interiorization).

4. Interviewee was not able to remember and keep in mind various pieces of information needed. For example the interviewee should have factorized the expression before substituting. So this reflects restricted mental field of the interviewee.

5. During the output phase the interviewee could not express himself as well or much as he would have liked to. This was reflected by the interviewee expressing himself in his first language for explanation.

6. Lack of or impairment in remembering and keeping in mind various pieces of information one needs to solve the problem.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW 3

Problem 1 (i)

1 I: Can you solve this problem for me please? And as you solve it you must tel me exactly what you are doing. Why you are doing it the way you would be solving it and also it is important for you to explain whether you understand the question or not

2 S: Ok, Sir

3 I: Can you read question one for me please?

4 S: (Reads the question) Determine the following limits. Limits x approaches 2 x^2 - 4 x - 2. So here you have to calculate the limit of this (Points at $\frac{x^2 - 4}{x - 2}$).

Then after you calculate it you substitute the value of x by 2.

5 I: Can you solve it?

6 S: (Silence 1 min).

7 I: What does limit mean? What do you understand by the word limit?

8 S: Suppose x is approaching 2. The value of x such that ... given that the constant in this case ... Given that the limit of x is equal to 2 from the bottom. So which simply means that we must put 2 instead of putting x because that 2 indicates that the limit of x are mmh can just be solved by it (sic) (he then writes $\lim_{x \to 2} \frac{2^2 - 4}{2 - 2}$.

9 I: Can you explain it for me again?

10 S: Put the value of x as equal to 2. Then $\frac{2^2 - 4}{2 - 2} = 0$

Therefore the answer is zero.

11 I: So you mean zero divided by zero is zero.

12 S: Yes.

13 I: Do you have a calculator?

14 S: (Taking out the calculator giving it to Interviewer).

15 I: No, press it yourself and let's see what will be your answer.

https://etd.uwc.ac.za/

16 S: (student presses the calculator. Then get puzzled by E. Indicating Error) I don't know now. The limit is undefined.

17 I: Ok, in the case like this what should be done? (Silence for 2 min) Can you factorize $x^2 - a^2$ for me?

18 S: I don't think problems like this has got factors

19 I: (factorize $(x^2 - a^2)$) the factor of this therefore would be $(x - a)(x + a)$. Now what would be the factors of $x^2 - 4$?

20 S: $x^2 - 4$? eh (silence for abut 2 min)

21 I: Ok. Can't you see any similarity between $(x^2 - a^2)$ and $x^2 - 4$

22 S: mmh. No Sir.

23 I: What about between $(x^2 - 2^2)$ and $(x^2 - a^2)$?

24 S: They both have x^2 and minus.

25 I: Ok, factorize $x^2 - 2^2$ for me then.

26 S: (Look at it for sometime then started chewing the pen).

27 I: Ok. What does it mean to say that the limit is undefined? Or is undefined the number?

28 S: No, undefined is not the number.

29 I: So what is it that should be done in order not to get the answer as undefined.

30 S: Calculate it by using a ... law of differentiation, that is like multiplying the deficient of x (sic).

31 I: Go on ...

32 S: I did not do this problem in the past so I cannot go on.

33 I: Ok. Let us go back to factors of $(x^2 - a^2)$ and $x^2 - 2^2$ Look at the two expression carefully and see whether they don't look alike.

34 S: Eh. You said that the factors of $(x^2 - a^2)$ are $(x - a)(x + a)$. So .... so ... (silence for about 1 minute) so the factor $(x^2 - 2^2)$ are $(x - 2)(x + 2)$. No I am not sure, but I think they are ok. What about if I can say they are $(x - 2)$ and $(x + 2)$?

35 I: Ok. I think you would be correct. So now ...

36 S: You mean now I can write $(x + 2)(x - 2)$ instead of $x^2 - 4$?

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

37 I: Try it and let's see what will happen.

38 S: (write the problem and tries to answer it)

$$\frac{x^2 - 4}{x - 2} \qquad \frac{(x-2)(x+2)}{(x-2)} \qquad \begin{aligned} &= x + 2 \\ &= 2 + 2 \\ &= 4. \end{aligned}$$

39 I: Ok. Therefore two plus two is equal to four?

40 S: Yes, the answer will be four.

41 I: What will happen to the undefined now?

42 S: It's no longer undefined.

43 I: Why?

44 S: Because this answer is correct the first one was not.

45 I: Why is the first one not correct?

46 S: Because it gives zero divided by zero which gives undefined answer.

47 I: Ok. What must be done when one ends up with zero divided by zero.

48 S: I have to simplify the expressions.

49 I: Ok. Thank you.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 3

Problem 1 (i)

Error Factor Problems with Content

1. Substituting for x before factorizing (look at answer sheet).

2. Saying zero divided by zero is zero (ref #10).

3. Saying the limit is undefined (ref #16) the reason being zero when divided by zero using the calculator gives "E".

4. Saying that to get rid of undefined one has to use differentiation (ref #30).

5. Works out the problem leaving out limit throughout i.e. solving meaningless expression with equal sign correlation between them (look at answer sheet).

Semantic difficulties

1. Inability to express oneself. Student found himself explaining in his first language (African language) most of the time (ref. #30).

Cognitive difficulties

UNIVERSITY of the
WESTERN CAPE

1. Not planning before hand as to what to do so as a result interview becoming impulsive and unsystematic in problem solving.

2. Cannot transfer things or concepts learnt earlier to a new situation. That is not being able to see that $(x^2 - 4)$ can be factorized.

3. Not having a good picture in mind of what he must do or is looking for.

4. Lack of verbal tools for communicating adequately elaborated responses.

5. Not being creative, therefore not being able to solve or attempt problems not familiar with.

6. Lack of or impairment in remembering and keeping in mind, the various pieces of information one needs to solve the problem.

https://etd.uwc.ac.za/

INTERVIEW 4

Problem 1 (i)

1 I: As you solve the problem you must tell me what is going on in your mind, that is telling me why you are using a certain method for solving the problem and also what you understand the problem to mean.

2 S: So do I have to ask you when I have problems?

3 I: Yes, you can ask. Can you start now. Please read question number one and tell me what it requires you to do.

4 S: (reads the question) I think you have to find the common factor.

5 I: Common factor? How?

6 S: (factorizes the problem) I will have (x - 2)(x + 2) divided by (x - 2). (then she solves the problem quietly).

7 I: Tell me what you are doing as you solve the problem.

8 S: I am having limit 2 with the two in brackets then the answer is 4 (silence).

9 I: Is that all?

10 S: Yes. I just want to prove it (silence 1 minute).

11 I: Why are you so quiet?

UNIVERSITY of the WESTERN CAPE

12 S: What about if I find the derivative?

13 I: Derivative? but is that what the question needs?

14 S: No.

15 I: So. We want the limit isn't it? And you said the limit is 4. Is four not the answer?

16 S: It is.

17 I: Have you solved the problems like this before?

18 S: Yes and we used to solve them this way (point at what he has written).

19 I: Why is it that you leave out lim x --> 2 as you moved on to the other steps?

20 S: Because ... (speaks in Xhosa).

21 I: Can you translate for me please as my Xhosa proficiency is not that good?

22 S: No. I don't know.

https://etd.uwc.ac.za/

23 I: Ok! To conclude what would you say the word limit means?

24 S: Substituting something for x.

25 I: Ok! go on.

26 S: That is all.

27 I: Thank you.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 4

Problem 1 (i)

Error Factor

1. Factorizing $x^2 - 4$ correctly but not able to explain what she was doing.

2. Saying factorization is taking out the common factor (ref. #4).

3. Writing statements without any connection between them.

Semantic difficulties

1. Very serious language (English) problem could not express herself well in English. When asked to explain using mother tongue also complained that she could not translate English terms into the mother-language (ref. #20).

Cognitive difficulties

1. Not knowing or understanding what one is doing (rote learning) i.e. Narrowness of mental field.

2. Not using a plan so as not to skip or miss important factors like she did leaving out "limit". The displaying of unsystematic impulsive exploratory behaviour.

3. Not having a good picture in mind of what exactly the question need. For example (reference #13) shows that she did not know what she was doing she was just guessing.

4. The interviewee not clear and precise in language when expressing the solution of the problem.

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW 5

Problem 1 (i)

1 I: Please try to do the first problem. How do you describe it?

2 S: (reads the question) We used to solve it like first we used to say f of x like you give me $x^2 - 4$ then we use f(x) neh! so must I derive it? (sic).

3 I: What does the question say?

4 S: (Reads the question again. Determine the following limits? Limit as x approaches 2. First I have to make this thing (pointing at $\frac{x^2 - 4}{x - 2}$

like I want to have $x^2 - 4$ (sic).

5 I: Just solve it the way you feel its right, anyway you feel its right.

6 S: Ok (tries to solve the problem) I say lim x - 2 then would say x substituted by 2 would give x ---> 2. Therefore two minus two would give zero.

7 I: Where does x - 2 come from?

8 S: $x^2 - 4$ divided by x - 2 x cancel x^2 leaving x and 2 into 4 goes 2 times so you are left with x - 2 then you have to put 2 in the place of x and you say 2 - 2 and you get zero. (He tries again this time writing lim x -> 2 $x^2 - 2$ and no longer x - 2 then substitutes 2 for x then cancel the power 2 then have the original expression which still gives her zero).

9 I: Do you mean when you divide $x^2 - 4$ by x - 2 you get? Ok.

10 S: No I don't mean that. Did I make a mistake? Oh !

11 S: I have made a mistake. It's x - 2 then I have x then.

12 S I substitute x by 2 then I get 2.

13 I: You substitute x by 2?

14 S: Yes.

15 I: So what happens to this two here? (pointing at $x^2 - 4$).

16 S: I don't do it with the denominator we used to do it without denominator.

17 I: What do you mean?

18 S: (whispers) I am used to do it with $x^2 - 4$ without x - 2 (sic).

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

19 I: Ok. Just solve this one for me please (Interviewer writes $\lim_{x \to 2} x^2 - 4$.

20 S: Solves the problem $\lim_{x \to 2} x^2 - 4$ $\lim 2^2 - 4$

$4 - 4 = 0$) So the answer will be naught?

21 I: So the answer for this one is zero?

22 S: Yes.

23 I: So then what do you think $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ would give? Why is it that you can not solve this one?

24 S: It is not that I cannot solve it but I don't know whether I must divide first or I don't know whether it is right to find the limit for the top or for the bottom?

25 I: Ok.

26 S: It is something like this $\frac{x^2 - 4}{x - 2}$

then I should substitute for x.

27 I: Ok just substitute.

28 S: (Laughs and solves the problem) lim

$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$, $\lim_{x \to 2} \frac{2^2 - 4}{2 - 2}$

so our answer will be naught.

UNIVERSITY of the
WESTERN CAPE

29 I: I heard you say $2^2 - 4$ is zero.

30 S: Yes.

31 I: So, what about $2 - 2$.

32 S: It is also naught.

33 I: So what will be the answer?

34 S: Naught divided by naught (sic).

35 I: You mean zero divided by zero.

36 S: (write zero over zero).

37 I: What is zero divided by zero.

38 S: It's naught (meaning zero).

39 I: (gives the interviewee the calculator to divide zero by zero) What does that "E" stand for or mean?

https://etd.uwc.ac.za/

40 S: Its an error (keeps quiet for some time).

41 I: Can you factorize this one for me. Factorize $x^2 - 4$? Can you find out what the factors are?

42 S: (whispers then writes $(x - 2)(x + 2)$ oh I forget I should have used $(x - 2)(x + 2)$ and $x - 2$ will cancel $x - 2$ then I will be left with $x + 2$. Then I would substitute.

43 I: Ok.

44 S: (continues solving the problem)

$$\lim_{x \to 2} \frac{(x - 2)(x + 2)}{x - 2}, \quad \lim_{x \to 2} x + 2$$

$2 + 2 = 4$.

45 I: So, what happened to the limit. Does it just disappear or what.

46 S: I substitute 2 so I get 4.

47 I: Ok now will the answer be 4 or zero?

48 S: No e-eh it will be four.

49 I: Why not zero?

50 S: The first method was not right. I forget that we have to factorize first.

51 I: Ok so tell me when you have a problem whereby you end up with zero divided by zero what do you think should be done?

52 S: Factorize first.

53 I: Thank you, or is there anything that you would like to ask?

54 S: No.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 5

Problem 1 (i)

Error Factor

1. Assuming that $\frac{x^2 - 4}{x - 2} = x - 2$ (ref. #6) reasons being that x in the denominator cancels x^2 in the numerator, leaving x. That 2 also goes two times into two thus

$$\frac{x^2 - 4}{x - 2} = x - 2 \quad \text{(ref \# 8).}$$

2. Saying that zero divided by zero is zero. (ref # 20)

3. Impulsively substituting 2 for x without first simplifying.

Semantic difficulties

1. Not able to read the expression $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ (ref #4)

2. Uses a lot of words to explain a simple thing or concept. This shows lack of vocabulary and usage of language (ref #8).

3. Always calling zero naught (ref # 20, 32, 34).

4. In most cases whispering using mother language.

UNIVERSITY of the WESTERN CAPE

Cognitive difficulties

1. Unplanned, impulsive and unsystematic exploratory behaviour (look at answer sheet). Interviewee just solves the problem without much thinking being put into the problem.

2. Lack of precision and not being able to explain what is being done.

3. Not being able to use or transfer things previously learnt to a new situation.

4. Not clear and precise in language when expressing the solution to a problem.

5. Giving out trial-and-error responses.

6. Lack of or impairment in remembering and keeping in mind, the various pieces of information one needs to solve the problem.

https://etd.uwc.ac.za/

INTERVIEW 6

Problem 1 (i)

1 I: Can you read through the first problem and solve it for me. As you solve the problem you must tell me what you are doing.

2 S: (Reads the questions) Determine the following limits lim x approaching two. x^2 - 4 over x - 2.

3 I: Ok, so what does the question ask you to do?

4 S: (murmurs) It must be x approaching 2.

5 I: Pardon!

6 S: Find the limit as x approaches two.

7 I: Ok.

8 S: Since x^2 - 4 is the square root I have to break it into two brackets.

9 I: Why do you have to break it into two brackets?

10 S: To make it smaller and because z - 2 is ... (stops for 1 minute) .. because x^2 s one of the brackets as you can see x is the square root of x^2 and 2 is the square root of 4 so x - 2 and x + 2 (he writes them down).

11 I: What do you call this process of breaking x^2 - 4 into (x - 2)(x + 2)?

UNIVERSITY of the
WESTERN CAPE

12 S: I forget.

13 I: Is it not called factorization?

14 S: Yes, it is called factorization.

15 I: Ok go on.

16 S: So x - 2 and (x - 2) cancel each other and we are left with x + 2. So x approaching 2. We substitute x by 2.

17 I: Ok. You mean you substitute x by 2 or for 2?

18 S: Yeah.

19 I: Ok. So what will your answer be?

20 S: It will be 4.

21 I: Do you think this is the correct answer?

22 S: Yes.

23 I: Why?

https://etd.uwc.ac.za/

24 S: Because I substituted correctly.

25 I: Ok. Can you tell me what the limit means to you?

26 S: It means substituting x by a given number.

27 I: Ok. Generally what does the word limit mean?

28 S: It means something approaching something.

29 I: Can you make yourself a bit clearer.

30 S: The number that x is approaching is substituted.

31 I: Is that all?

32 S: Yes.

33 I: Ok. Thank you.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 6

Problem 1 (i)

Error Factor

1. Saying $x^2 - 4$ is the square root and therefore should be factorized (be "broken") (ref # 8).

2. Calling factorization taking out common factor".

3. Saying that x is the square root of x^2 and 2 square root of 4 so that is why x^2 will have factor as $(x - 2)(x + 2)$ (ref # 10).

Semantic difficulties

1. Inability to express oneself properly, insists on murmuring in mother language.

Cognitive difficulties

1. Using wrong term but solving the problem correctly.

2. Just solving the problem impulsively without clear understanding of what she is doing (rote learning).

3. Lack of transferring thing that she previously learnt.

4. Not clear in expressing the solution due to language problems.

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW 7

Problem 1 (i)

1. S: (Reads the problem thrice and then keeps quite for some time) I find this one complicated writes.

$$f'(x) = \frac{x + h) - f(x))}{h.}$$

2. I: Is it the first time for you to see a problem like this are?

3. S: No I tried to solve a problem like this yesterday.

4. I: Ok.

5. S: But I find them very complicated.

6. I: Ok, can you find $\lim_{x \to 2.} x$

7. S: Even this one I find it complicated.

8. I: Can we move on to the next question if this one is complicated for you.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 7

Problem 1 (i)

Cognitive difficulties

1. Lack of content showing poor interest in mathematics could not even attempt the simplest problem $\lim_{x \to 2} x$ (ref # 6).

2. Complete mental blockage.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW 1

Problem 1 (ii)

1. S: (Reads the question, then kept quiet for 2 minutes).

2. I: What are you doing now?

3. S: Factorizing $x^3 - x^2 - x$ to get $x(x^2 - x - 1)$

then $\lim_{x \to \infty} \frac{x^2 - x - 1}{x^2 + 2x}$

Ok, in the first place I have take out the common factor so even in the denominator I have to take out the common factor.

So $\lim_{x \to \infty} \frac{(x^2 - x - 1)}{x(x + 2)}$

$\lim_{x \to \infty} \frac{x^2 - x - 1}{x + 2}$.

4. I: Ok, so you mean x in the denominator will cancel x in the numerator (pointing at what interviewee wrote .

$\lim_{x \to \infty} \frac{x(x^2 - x - 1)}{x(x + 2)}$

(I repeat the question again).

5. S: Mmm, ok then (silence 1 min).

6. I: What will be the limit of $\frac{x^2}{x}$. when x approaches infinity

Thus $\lim_{x \to \infty} \frac{x^2}{x}$.

7. S: The limit will be undefined.

8. I: Why undefined.

9. S: Because we have infinity. No, ok $\frac{x^2}{x}$ will give me ∞.

So $\lim_{x \to \infty} x$

substituting x by infinity then we get x as the answer.

10. I: Can you apply what you have done here to the problem?

11. S: It is not easy.

12. I: Thanks.

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 1

Problem 1 (ii)

Error Factor

1. Cancelling x^2 in the denominator with x^2 in the numerator therefore ending up with $\frac{x^2 - x - 1}{x^2 + 2}$.

Cognitive difficulties

1. Lack of enough content to solve the problem.

2. Not having a good picture in mind of what one is looking for or of what must be done.

3. Unable to remember and keep in mind the various pieces of information needed to solve the problem.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW 2

Problem 1 (ii)

1. I: Can you read part 2 for me?

2. S: (Reads the question) The limit of x approaching ∞ if $x^3 - x^2 - x$ over $x^2 + 2x$ (sic).

3. I: So what does the question requires you to do?

4. S: To determine the limit.

5. I: Ok. So now what is the next step?

6. S: If x is approaching infinity. The first thing to do is to find the factors of cubic. To find difference of two squares.

7. I: Difference of two squares?

8. S: I think so.

9. I: I don't understand what you mean. Can you explain a little more.

10. S: Take out factors, just factorize.

11. I: Ok, factorize.

12. S: (He writes $(x^2 + 2x)(x - x - 3x)$ I know that its going to cancel the bottom (continues solving the problem). Can I now prove this.

13. I: Ok, just tell me what you are doing.

14. S: (Continues solving the problem) No its wrong.

15. I: What's wrong?

16. S: This does not cancel with this (pointing at the long expression, the denominator and the numerator).

17. I: How did you get $(x^2 + 2x)$ in the numerator.

18. S: I know that if the above is going to factorize there must be one that must cancel this one (pointing at the denominator.

19. I: So you have denominator as $x^2 + 2x$ now you must make the numerator have two factors the other one must or ought to be the same as the denominator. Am I interpreting you correctly?

20. S: Yes.

21. I: What about if the numerator does not have factors that are the same as those in the denominator?

22. S: I will just use the differentiation.

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

23. I: Just us the differentiation?

24. S: Yes.

25. I: So how can you solve a problem like this one?

26. S: By differentiation.

27. I: Will you be solving the problem.

28. S: No, it says determine the limit.

29. I: Yes, so!

30. S: I don't know whether this one will cancel with this one.

31. I: You mean x^2 in the denominator cancelling with x^2 in the numerator and x cancelling 2x?

32. S: Yes, but it doesn't work.

33. I: Can you solve this one

$$\lim_{x \to \infty} \frac{x^2 - 4x}{x} \; ?$$

34. S: (He writes $x^2 - 4x - x$).

35. I: Why does this expression come from?

36. S: When you take the denominator to the top it changes the sign. So we will have $x^2 - 4x - \underline{x}$.

37. I: Ok, what about taking out the common factor x in the numerator.

38. S: Ok. Yes $\frac{x^2 - 4x}{x}$ will be $\lim_{x \to \infty} \frac{x(x-4)}{x}$

the x's will cancel leaving $\lim_{x \to \infty} x - 4$.

39. I: What will be the answer then?

40. S: (No reply).

41. I: What is infinity minus four?

42. S: (silent for 2 minutes).

43. I: What does infinity mean?

44. S: It is not a number. It is something very big like million or trillion and so on.

45. I: Ok, what is x - 4 then?

46. S: It is a big number. I think four will not affect the answer. Ok, so it is ∞.

47. I: Ok, thank you.

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 2

Problem 1 (ii)

Error factor

1. Saying that whenever x --> ∞ the first thing for one to do is to find the factors of the cubic equation (ref # 6).

2. Not very sure what the difference of two squares are (ref # 6 to 10).

3. Saying that whatever is the dominator wil cancel what is in the numerator as long as the cubic equation is factorized (ref 11 - 22).

4. Finding the factors of $x^3 - x^2 - x$ as $(x^2 + 2x)(x - x - 3x)$.

5. Not sure when to differentiate or not even sure what derivative means (ref #21 and ref # 26).

6. Thinking that items in the denominator can cancel those in the numerator irrespective of the sign connecting them.

Semantic difficulties

1. Not having a good command of the language.

UNIVERSITY of the
WESTERN CAPE

Cognitive difficulties

1. Student does not have a clear and complete information about how to solve a problem.

2. Unplanned, impulsive and unsystematic exploratory behaviour.

3. Lack of, or deficient need for precision and accuracy in data gathering.

4. Inadequacy in experiencing the existence of an actual problem and subsequently defining it.

5. Lack of, or limited interiorization of one's behaviour.

6. In most cases the learner uses trial-and-error responses.

7. Lack of verbal tool for communicating adequately elaborated responses.

https://etd.uwc.ac.za/

INTERVIEW 3

Problem 1 (ii)

1. S: (Reads the question) the first think I must do is to factorize $x^3 - x^2 - x$ like $(x^2\ )(x\ )$. I also have to solve the problem in my head to find out what I will get if it is $^{\infty}/_{\infty}$ then I will have to do it again. Thus if I get undefined then I will have to do it again.

2. I: Ok, then what will be the answer?

3. S: If I substitute infinity for x.

4. I: Yes?

5. I: Do it in your head without writing it down. You said you can do it in your head like you did for the previous one.

6. S: Actually I would get the same answer undefined. I would substitute and get

$$\lim_{x \to \infty} \frac{(\infty)^3 - (\infty)^2 - \infty}{\infty^2 + 2\,\infty}$$

7. I: So what will be the answer?

8. S: It's going to be a big number.

9. I: Ok, what about in the denominator here?

10. S: But I don't understand this problem, because it says you substitute ∞ for x.

11. I: Ok, just try it.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 3

Problem 1 (ii)

Shows some knowledge of how to solve the problem even though it became too difficult for him to solve. Just leaving the answer as $(x^2\ )(x\ \ )$.

UNIVERSITY of the
WESTERN CAPE

N.B. For Interviewees 4 and 5 there was complete mental blockage. Pupils did not even make an attempt to answer problem 1(ii).

https://etd.uwc.ac.za/

INTERVIEW 6

Problem 1 (ii)

1. I: What I am coming to ask you to do is to read through this question and explain how you would work out a problem like this just as you did for the first part. I am actually trying to determine how students think about these things, the sorts of ideas that come into your mind as you work through the problem.

2. S: Ok, $x^3 - x^2 - x$ divided by $x^2 + 2x$ when x is approaching infinity.

3. I: Ok, what is it that you are actually required to do here?

4. S: I am required to find the limit when x is approaching infinity.

5. I: Ok.

6. S: In order to solve it I think I must first ... nie! ok, yes ... simplify this equation then substitute x with zero.

7. I: Why zero?

8. S: Because x is approaching infinity ...

9. I: So is infinity equal to zero.

10. S: Infinity is about 0,007; 0,000027 it is so small that I take it to be zero.

11. I: Infinity equal to zero?

12. S: Yeah, something that is close to zero so I take it to be zero.

13. I: Ok, can you just solve it for me?

14. S: I have to use the factor theorem.

15. I: The factor theorem?

16. S: Yeah, to factorize this.

17. I: Just do it I want to see how you would apply the factor theorem.

18. S: Eh ... Let me try this first $\lim_{x \to \infty} 2^3 - 2^2 - 2 =$

$8 - 4 - 2.$

19. I: Is this factor theorem?

20. S: No.

21. I: What is it that you are doing now?

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

22. S: Finding out the common factor.

23. I: The common factor?

24. S: Yes, so

$$\lim_{x \to \infty} \frac{x(x^2 - x - 1)}{x + 2} \qquad \lim_{x \to \infty} \frac{x^2 - x - 1}{(x + 2)}.$$

(the interviewee then writes the last step again).

25. I: What is the next step now?

26. S: Substituting x by zero.

27. I: Ok, tell me what do you usually called the number that is increasingly big?

28. S: Very big?

29. I: Yes.

30. S: You mean infinite number.

31. I: Ok, so if the number which is very big is called infinite number, what does infinity mean now?

32. S: Big number.

33. I: Why is it that you first said infinity is equal to zero?

34. S: Anyway we don't know whether the number is negative or positive and zero is neutral. It is between them.

35. I: So you mean zero is between the negative infinity and the positive infinity?

36. S: Yeah, so it is better to take zero because you don't know in which direction the number is on ...

37. I: But here if I have 4. Is it positive or negative?

38. S: It is positive.

39. I: So here is infinity negative or positive?

40. S: (Laughs) I must be positive.

41. I: What does it mean then?

42. S: It means a positive big number.

43. I: Ok, you can now continue solving the problem.

44. S: (Silence 2 min).

45. I: Do you have any problem?

46. S: I can't go on.

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

47. I: Why.

48. S: It is difficult for me.

49. I: Ok.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 6

Problem 1 (ii)

Error Factor

1. Assuming that infinity is equal to zero (ref # 8 and 10).

2. Saying that to solve this problem he will have to use factor theorem.

3. Substituting 2 for x (ref # 18) and saying that by substituting 2 he is finding out the common factor. Then leaves it and moves on to the original equation.

4. Saying that the last step would be to substitute x by zero.

5. Taking infinity to be zero because it is not stated whether infinity is negative or positive.

Cognitive difficulties

1. Very impulsive, does not take some time to check whether he really understand the question.

UNIVERSITY of the WESTERN CAPE

2. Does not plan his work and very unsystematic in solving the problem.

3. Lack of or impairment of gathering or relevant data.

4. Lack of, or impaired verbal tools for communicating adequately elaborated responses.

5. Trial-and-error responses.

N.B. The interviewee 7 was not able to answer question 1(ii) but only said it is too difficult.

https://etd.uwc.ac.za/

INTERVIEW 1

Problem 2

1. S: (Reads the question then writes) $f(x) = 5 + 7x - x^2$

then $f \dfrac{\underline{2 + h) - f(2)}}{\dfrac{(2 + h) - (2)}{h.}}$

2. I: What does $f'(2)$ mean? Is it just two. Getting rid of f and just getting two?

3. S: No.

4. I: Can you find for me what $f'(2)$ is?

5. S: I didn't practice this things.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 1

Problem 2

Error Factor

1. Saying that $\frac{f(2 + h) - f(2)}{h}$ is the same expression as

$$\frac{(2 + h) - (2)}{2}.$$

Cognitive difficulties

1. Mental blockage, he tried trial-and-error response and ultimately gave up.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW 2

Problem 2

1. S: (Reads the question) Therefore f(x) is $5 + 7x - x^2$ given the formula

$$\frac{f(2 + h) - f(2)}{h}.$$

2. I: Ok.

3. S: (solves the problem quietly) Ok, substituting for f (2 + h) will give $x^2 - 7x - 5 + h$ and then just write f(2) as f(2).

That's $\lim_{h \to 0} \frac{x^2 - 7x - 5 + h - f(2)}{h}$.

4. I: Just tell me what you are doing.

5. S: (Murmurs as she solves the problem).

6. I: Ok, can you tell me how many questions are there in this section.

7. S: There are two.

8. I: Which one are you answering now.

9. S: (Interviewee reads the question again) I am determining

$$\frac{f(2 + h) - f(2)}{h}$$

So I have to substitute this value (pointing at $x^2 - 7x - 5 + h$) in the place of x and I have to take f(2) as it is so ...

10. I: What does f 2 mean? What will be the answer if I give you f(2)?

11. S: Two is the value of x.

12. I: So what will be the answer of this?

13. S: It will be $5 + 7(2) - 2^2$ (interviewee substitutes correctly and get the answer as 15).

14. I: So our answer will be 15?

15. S: Yes f(2) = 15 (writes it down).

16. I: Ok, so what will be f(2 + h).

17. S: It will be 15 + h (writes f (2 + h) = 15 + h down).

18. I: You said for f(2) you substituted 2 wherever you have x.

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

19. I: So now what about f(2 + h)?

20. S: Substitute it in the place of x.

21. I: Ok.

22. S: [Substitutes (x + h) correctly in the place of x until reaching the answer.

$\frac{3h - h^2}{h}$ then at the end just writing $\frac{f\ (2 + h)}{h}$

ie 19 + 7h - 4 - 4h - h^2 then just divided by h without subtracting f(2)].

23. I: Ok. What is it that you are dividing by h?

24. S: f (2 + h).

25. I: Are you expected to divide only f (2 + h) by h or ...

26. S: Oh I am forgetting 15 which is f(2). Ok, so now I will have

$\frac{15 + 3h - h^2 - 15}{h}$ 15 will cancel 15 leaving

$\frac{h\ (3h - h^2)}{h}$.

Taking out h in the numerator we will have $\frac{h\ (3 - h)}{h}$ the h's will cancel leaving 3 - h as the answer. So this answer part one.

27. I: Ok. You can move on to part 2. What does it require you to do?

28. S: find the limit if h --> 0.

29. I: Ok, can you find it for me please.

30. S: 3 - h = 3. So the answer is three.

31. I: Can you prove it.

32. S: I think I can do it by differentiating [then writes 5 + 7x - x^2, 0 + 7 - 2x, 7 - 2x, 7 - 2(2)].

Now replace x with 2.

33. I: Which two? (student points out at f(2) Ok.

34. S: Therefore 7 - 2(2), 7 - 4 will give 3 as the answer.

35. I: Thank you.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 2

Problem 2

Error Factor

1. Saying $\lim_{h \to 0} \frac{f(2 + h) - f(2)}{h}$ will give the answer

   for the first part of the interview (ref # 1).

2. When substituting for f (2 + h) not substituting for f(2) (ref #3).

3. For f (2 + h) taking it to be -f(x) + h therefore writing the whole expression as $x^2 - 7x - 5 + h$.

4. Leaving out f (2) when substituting therefore writing $\frac{f(2 + h)}{h}$.

UNIVERSITY of the
WESTERN CAPE

Cognitive difficulties

1. Makes very limited analysis of the problem initially.

2. Had to be lead step-by-step in order to get to the answer. So meaning that he is not in good grip with the content.

3. Has problem of finding the relationship between f(2 + h) and f(2).

3. Is impulsive, does not plan and is also very unsystematic in behaviour.

4. Has problem expressing the oneself.

5. Uses trial-and-error responses.

https://etd.uwc.ac.za/

INTERVIEW 3

Problem 2

1. S: (Read the question correctly) Therefore the question says determine

$$\frac{f\,(2 + h) - f(2)}{h.}$$

2. I: Yeah.

3. S: Can I first calculate it on my own and then I will tell you?

4. I: Ok. (interviewee takes 3 minutes trying to solve the problem).

5. I: Can you tell me now what you have done?

6. S: (Reads the question again) Instead of putting the value of x in this case you put 2 + h.

7. I: Mmm.

8. S: Therefore $-x^2 + 7x + 5$ here we have

$$\frac{-(2 + h)^2 - 7(2 + h) - 14 - 7h + 5}{h}$$

$$= \frac{(4 + 2h + h^2) - 14 - 7h + 5}{h}$$

multiply by the negative number

$$= \frac{-4 - 2h - h^2 - 14 - 7h + 5}{h.}$$

9. I: Is this not only for $\frac{f(2 + h)}{h}$?

What happened to $-f(2)$?.

10. S: Oh, I did not subtract this one. It is minus 2 (He then writes minus two next to the first expression).

11. I: Why two for f (2 + h) I see what you have done substituting x by 2 + h. So what about $f(2)$?

12. S: We put 2 because in this case we don't have x.

13. I: Why 2? This two not substituting two by x whereas in the previous case we had (2 + h) in place of x?

14. S: Because in this case we don't have x. We haven't got x (sic).

15. I: Ok, are you through with this one?

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

16. S: (simplifies what is in the bracket, then writes -13 as the answer).

17. I: So what is your answer?

18. S: Minus 13.

19. I: Ok.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 3

Problem 2

Error factor

1. Making mistake with signs writing -x - 7x + 5 when substituting (ref # 8) instead of +7x.

2. Writing the expression $\frac{f(2 + h) - f(2)}{h}$

   only as $\frac{f(2 + h)}{h}$

   leaving out $-f(2)$.

3. Saying $-f(2)$ is -2.

Cognitive difficulties

1. Unplanned, impulsive and unsystematic behaviour (ref looking at the answer sheet, the learner wrote.

   $\lim_{h \to o} \quad -x^2 + 7x + 5$ calculating the first part.

2. Lack of, or limited interiorization of one's behaviour.

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW 4

Problem 2

1. S: (Reads the question) The answer is $7x - x^2$ because the "context" term is zero.

2. I: What is the "context" term? Oh you mean constant term?

3. S: Yes constant term.

4. I: Ok, so what about constant term?

5. S: Is equal to zero. Ok now I don't know whether I should do the derivative, which will be 2x + 7.

6. I: 2x + 7 is the derivative of what?

7. S: I think I must find the value, ok

$$\lim_{h \to 0} \frac{f(2 + h) - f(2)}{h}$$

$$= \frac{2 + h - 2 + 2x + 7}{h}$$

$$= \underline{2x + h}.$$

8. I: I can't follow what you are doing, so you just take of *f* and be left with 2 + h and remove *f* to get 2. Where does 2x + 7 come from?

9. S: The derivative.

UNIVERSITY of the
WESTERN CAPE

10. I: Ok.

11. S: Then the answer will be 2x + h. No it will be

$$\frac{f\ (x + h) - f(x)}{h}$$

$$= \frac{f\ (2 + h) - f(2)}{h}$$

$$= \frac{2x + 7 - 2x}{h} \qquad = \frac{2 + h - 2 + 2x + 7 - 2x}{h}$$

$$= 7.$$

12. I: I can't really follow what you are doing.

13. S: This is the way we do it in class it is called using the formula (sic).

14. I: So is seven the answer for the first part or the second part?

15. S: For the first part the answer will be 2x +7 but for the second part it will be 7?

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 4

Problem 2

Error Factor

1. She write the $\lim_{h \to o} \frac{f(2+h) - f(2)}{h}$

$$= \frac{(2 + h - 2 + 2x + 7)}{h} \qquad = \qquad 2x + h$$

Thus just taking out f from $f(2 + h)$ and removing f from $f(2)$ and get 2 + h - 2 then add the derivative of $f(x)$. So getting 2 + h - 2 + 2x + 7 all over h (ref # 7).

2. Dividing or cancelling the h in the numerator with h in the denominator ignoring the connectives.

3. Writing $\lim_{h \to o} = \frac{f(2+h) - f(2)}{h}$

showing that she does not understand what she is doing.

4. Saying that the answer will be 2x + h.

5. Writing $\frac{f(x+h) - f(x)}{h} = \frac{f(2+h) - f(2)}{h} = \frac{2x + 7 - 2x}{h}$

then saying that this is the way they were taught to solve the problem.

Semantic difficulties

1. Avoid using long explanation but keeps on saying this is like this (writing down something).

UNIVERSITY of the
WESTERN CAPE

Cognitive difficulties

1. Student unable to keep in mind or remember various pieces of information that would be needed to solve the problem.

2. Lack of or impairment in projecting relationships.

3. Not able to gather all the information needed to solve the problem.

4. Student not having a good picture in her mind of what she is looking for or what must be done.

5. Not thinking about different possibilities and figuring out what would happen if she choose one or another (for example saying that

$$\frac{f(2+h) - f(x)}{h} = \frac{2 + h - 2 + 2x + 7}{h}.$$

https://etd.uwc.ac.za/

INTERVIEW 6

Problem 2

1. I: Read the question or problem and then try to solve it. Also like in previous ones tell me what you are doing.

2. S: I am required to calculate the limit as x approaches zero.

3. I: Is that all?

4. S: (Read the question) You must first determine.

$$\frac{f(2 + h) - f(2)}{h}$$

and then calculate the limit if h --> o.

5. I: So ...?

6. S: So I must first solve the first problem.

7. I: Meaning you have more than one?

8. S: Yes, there are two of them.

9. I: Ok.

10. S: (Solves the problem by first finding $f(2)$ then $f(2 + h)$ then used the formula

$$\frac{f(2 + h) - f(2)}{h}$$

and found 3 - h as the answer). Therefore 3 - h will be the answer for the first part, now for the second part we have

$$\lim_{h \to o} \frac{f(2 + h) - f(2)}{h} = \frac{15 + 3h - h^2 - 15}{h}$$

$$\lim_{h \to o} \frac{3h - h^2}{h} \quad \lim_{h \to o} \frac{h(3 - h)}{h} \quad \lim_{h \to o} 3 - h = 3.$$

11. I: Why is it that you start with $\frac{f(2 + h) - f(2)}{h}$ and then substitute whereas you have already solved it in the first part?

12. S: For more marks.

13. I: So you think you will get more mark if you start from the beginning?

14. S: Yeah, it will also depend on the marks.

https://etd.uwc.ac.za/

15. I: Ok, tell me how can you check that 3 is the correct answer (silence 1 minute).

16. I: What does $\lim_{h \to o} x^2$ mean?

17. S: The value of x is tending toward zero. It is the same thing as finding the derivative.

18. I: Derivative! What do you mean by derivative?

19. S: It is change in y over change in x.

20. I: Which gives you ...

21. S: The gradient.

22. I: So now what does derivative give you?

23. S: The gradient.

24. I: So you mean three is the gradient.

25. S: Yes.

26. I: Of what?

27. S: Of the curve $5 + 7x - x^2$ at the point where $x = 2$.

28. I: Very good. Thank you.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 6

Problem 2

1. Students seem to be clear and show good understanding of the problem.

Error factor

1. When solving for part 2 repeat the steps that have already been covered. Saying that he is doing that for more marks (ref # 11 to 12).

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW 7

Problem 2

1. S: (Reads the question two times wrongly, saying "into" all the time for example reading $f(2 + h)$ as f into $(2 + h)$), I tried to solve problems like this yesterday.

2. I: Yes, so what happened? Did you get them right?

3. S: Ye ... Yes.

4. I: Ok, go on.

5. S: $\lim_{h \to o} \frac{f(2 + h) - f(2)}{} = \lim_{h \to o} f \frac{2 + h - 2}{h}$

   So the h's will cancel.

6. I: What is $f'(x)$ given $f(x) = x^2 + 2$.

7. S: It is 2x.

8. I: Ok then what will be $f(2 + h)$ of $f(x)$.

9. S: $\lim_{h \to o} \frac{x^2 + 2xh + h^2 - f(x)}{h.}$

10. I: So meaning that $f(2 + h)$ is equal to what?

11. S: You see x + h will be equal to x that's why I and saying x + h will be equal to x^2 (sic).

12. I: So $x^2 + 2xh + h^2$ is equal to $f(2 + h)$.

13. S: Yes (then substitute 2 for $f(x)$).

14. I: So you mean $f(x) = 2$.

15. S: Yes.

16. I: Ok, if I have $f(x) = x^2 + 2$ what will be $f'(x)$.

17. S: 2x.

18. I: Ok, what will be $f\ (x + h)$?

19. S: It will be $x^2 + 2xh + h^2$ or 2 + h because $f(x) = 2$.

20. I: So which one is the right one.

21. S: This $f(x + h)$ is always confusing me.

22. I: Is it that whatever is written in the bracket gives the value of x or should be substituted for x?

23. S: Mmm?

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

24. I: I mean if $f(x) = x^2 + 2$, $f(x + h)$ will be $(x + h)^2 + 2$ giving $x^2 + 2xh + h^2$.

25. S: Yes. That is what I said even though my $f(x)$ ... even though I made $f(x) = 2$. Ok now I remember I have to do it that way, yes (sic).

26. I: Ok, just try or $f(x) = 5 + 7x - x^2$ now.

27. S: $f(x) = 5 + 7x - x^2 \quad = \quad f'(x) \quad = \dfrac{f(2 + h) - f(x)}{h}$

$$\lim_{h \to o} \quad = f \dfrac{2 + h)^2 - f\ (5 + 14 - 4)}{h}$$

$= \ 4$

$$\lim_{h \to 0.} \quad (2 + h)^2$$

28. I: I can't follow what you are writing, can you please explain?

29. S: (The interviewee cancels the second step and keeps quite.

30. I: You mean you can't continue?

31. S: Yes.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 7

Problem 2

Error Factor

1. Writing $\lim_{h \to o} \frac{f(2 + h) - f(2)}{h}$ as $\lim_{h \to o} f \frac{2 + h - 2}{h.}$

2. Saying that for $\frac{2 + h - 2}{h.}$

   the h in the numerator will cancel the one in the denominator.

3. Saying that x + h is equal to x^2 thus can be easily substituted by x^2.

4. Writing f(x) as f (5 + 14 − 4).

Semantic difficulties

1. Poor command of the language so could not express himself as well as he would like to.

Cognitive difficulties

1. Unplanned, impulsive and unsystematic explanatory behaviour.

2. Not remembering and keeping in mind the various pieces of information needed to solve the problem.

3. Not having a very good picture in mind of what one is looking for or what must be done.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW 1

Problem 3

1. S: (Reads the question twice) $f'(-1) = (-1)^2 (-1 -3)$.

2. I: Is there a difference between $f'(-1)$ and $f(-1)$?

3. S: There is a difference because for f' you have to find the limit.

4. I: The limit?

5. S: Em .. but there is a difference between the two.

6. I: Tell me what does $\lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$ means?

7. S: I use it when I am finding the limit.

8. I: When?

9. S: When I find the derivative of the gradient (sic).

10. I: Is it the derivative of the gradient or just gradient?

11. S: The derivative or the gradient.

12. I: Ok, can you find $f'(2)$?

13. S: (Substitute two in the $f(x)$ equation without first differentiating). Ok, the answer will be $(2)^2 (2 - 3)$.

14. I: Is f' and f the same thing?

15. S: I have not actually revised this one.

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 1

Problem 3

Error Factor

1. Writing $f'-1$ as if it is $f(-1)$ (ref # 1).

2. Saying that he uses $\frac{f(x + h) - f(x)}{h}$ when finding the derivative of the gradient (ref # 9).

Cognitive difficulties

1. Unplanned, impulsive and unsystematic behaviour.

2. Mental blockage.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW 2

Problem 3

1. S: (Reads the problem, and read f^1 (-1) as *f* prime minus one and f (2) as *f* prime 2). So I must first calculate the value of prime one. If I can put prime minus one (he substitutes (-1) for x in $x^3 - 3x^2$ and find -4 as the answer). So f^1 (-1) = -4.

2. I: Why?

3. S: Because they say that x^2 multiplied by x and minus three multiplied by x^2 (ie x^2 (x - 3)) is the general formula.

4. I: Mmm.

5. S: So when substituting (-1) for x you get -4.

6. I: Ok.

7. S: So now I must calculate prime two.

8. I: Ok before finding the derivative when x = 2, f'(-1) the same thing as f (-1)? (silence) What will f(-1) give you?

9. S: Oh I see here I must put one because it is the opposite of f(x).

10. I: Opposite of f(x)? What do you mean?

11. S: Yes. If there is something like f'(x) and f(x) then one is the opposite of the other.

12. I: Ok.

13. S: I don't know whether to calculate the gradient here.

14. I: That is why I am asking whether this two things are the same (pointing at f^1(x) and f(x)).

15. S: Ok, I have used this one f^1(x).

16. I: So now what will be the answer for f (-1).

17. S: Solves the problem and changes -1 to one for f(-1) so the answer is -2.

18. I: Ok what does this f slash here (pointing at it) mean?

19. S: Opposite of something.

20. I: Opposite of something like what?

21. S: Like opposite of one is minus one.

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

22. I: Does this derivatives $Dxf(x)$, $\frac{dy}{dx}$ mean the same thing as f(x) .

23. S: Yes.

24. I: What is f(x) given $f'(x) = x^2$.

25. S: It is 2x.

26. I: So now what will be f(x) if $f(x) = x^2 (x - 3)$.

27. S: It will be $3x^2 - 6x$.

28. I: So now what will be $f'(-1)$.

29. S: Oh, Ok now it means substituting (-1) where there is x in the equation $3x^2 - 6x$ ok. I see I think its right I now remember yes.

30. I: Try it let's see what you will get.

31. S: (the interviewee solves the problem correctly) so now $f'(-1)$ will be 9.

32. I: Ok, you may move on to the next one.

33. S: Now $f'(2)$ will give $3x^2 - 6x$ substituting 2 3(2) - 6(2) = 12 - 2 giving zero. Now answering the last question, "What does the value of $f^1(2)$ indicates? I think it indicates the gradient. The gradient is zero. It means that the derivative is equal to zero.

34. I: Why equal to zero.

35. S: There is something which say this size must be equal to that side and equal to zero (looking at $3x^2 - 6x = 0$). It indicates that the gradient line lie horizontally.

36. I: What does it mean to say the that gradient line lie horizontally.

37. S: That means the gradient is zero.

38. I: Can you draw a sketch of the cubic graph and show me what you mean?

39. S: (draws the graph) Ok, the horizontal line is the x-axis.

40. I: So you mean the x- axis will give the gradient as zero? What is the relationship between x- axis and this problem?

41. S: X- axis is parallel to line passing at the point on the graph when x = 2.

42. I: Where will that be?

43. S: At the turning point.

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

44. I: Yes so what does $f(2) = 0$ give you.

45. S: It will give me the turning point with $x = 2$ as one of the co-ordinates.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 2

Problem 3

Error factor

1. Says that $f'(-1) = f(-1) = (-1)^3 - 3(-1)^2$ then goes on to say that $f'(-1)$ is the opposite of $f(-1)$ (ref #9).

Semantic difficulties

1. Unable to explain oneself clearly (ref #35).

Cognitive difficulties

1. The student does not take some time to make sure that she really understand the question (ref: from 9 to 10).

2. Does not also think through before answering the questions instead she is impulsive and very unsystematic in solving the problem.

3. Student not having a good picture in mind of what one is looking for.

4. Verbal tool for communicating adequately elaborated responses is also impaired or lacking.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW 3

Problem 3

1. S: (Read the questions) So must I calculate?

2. I: Just explain what you understand by the question.

3. S: Question says that I must find the derivative $f'(x)$ instead of putting x. I am supposed to put minus one.

4. I: Ok.

5. S: Ok, $f(x) = x^2 (x - 3)$

$$f'(-1) = (-1)^2 (-1-3)$$
$$= 1 (-4)$$
$$= -4$$

So the answer will be minus four.

6. I: So now the answer will be minus four. Tell me what is $f(-1)$?

7. S: Must I calculate it?

8. I: Yes, please.

9. S: (writes $x^2(x -3) - (-1)$) Oh I was suppose to differentiate here.

10. I: Where?

11. S: Here (pointing at $x^3 - 3x^2$). Multiply this x here by coefficient of x here (sic).

12. I: Can you do it? I just want to see what you are talking about?

13. S: $x^2 (x - 3)$ when differential will give $2x (1 - 0)$ zero because the differential of a constant is zero.

$$f(-1) = 2(-1) (1)$$
$$= -2$$

14. I: Ok, can you use the first principle?

15. S: To solve this problem?

16. I: Yes.

17. S: (Tries to use the first principle, but does not get the formula right).

18. I: (Helps interviewee to solve the problem using the first principle).

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

19. I: So what is it that must be done to make this answers the same?

20. S: So maybe I must multiply them.

21. I: Multiply them! ok.

22. S: And this becomes $x^3 - 3x^2$.

23. I: Ok.

24. S: When I differentiate this it will give me $3x^2 - 6x$ now I substitute the value minus one. The $f'(-1) = 3x^2 - 6x = 3(-1)^2 - 6(-1) = 3 - 6 = 3$.

25. I: So you mean -6(-1) is equal to -6.

26. S: Where?

27. I: Here (pointing at -6(-1).

28. S: -6x(-1) is 6.

29. I: Ok, so the answer will be three plus six giving you nine?

30. S: Yes, can I move on to the other one?

31. I: Go on to the next one, $f'(2)$.

32. S:

$$f'(2) = x^3 - 3x^2$$
$$= 3x^2 - 6x$$
$$= 3(2)^2 - 6(2)$$
$$= 12 - 12 = 0$$

(interviewee solves the problem quietly).

33. I: What does the value of $f'(2)$ indicates?

34. S: (Reads it).

35. I: What does it mean to say $f'(2) = 0$.

36. S: No I cannot explain it.

37. I: You can't explain it?

38. S: Yes.

39. I: What does derivate mean?

40. S: Like derive .. means derive.

41. I: Ok, what I would like you to tell me is what you mean by derivative and not how to differentiate. What does

$$\lim_{h \to o} \frac{f(x + h) - f(x)}{h}$$ give you?

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

42. S: It gives the gradient.

43. I: So meaning that derivative means finding the gradient?

44. S: Yes, it means the gradient.

45. I: So what does $f'(2) = 0$ mean?

46. S: Zero is the derivative of $x^3 - 3x^2$ at 2

47. I: So that means zero is the ...

48. S: The gradient.

49. I: Ok, what does it mean to say that the gradient is equal to zero, for a cubic equation like this one?

50. S: It means the graph is going to cut the axis at zero at the origin.

51. I: What type of a line has the gradient zero?

52. S: Horizontal line.

53. I: Horizontal how? Can you draw it for me?

54. S: Like the x-axis?

55. I: You mean its just like the x-axis when our gradient is equal to zero?

56. S: What we are actually calculating here is the gradient of the number at two.

57. I: What about if we have a line, parallel to the x-axis but not at the x-axis?

58. S: So the gradient will only be zero at the x-axis.

59. S: Yes, but not always, we do get the gradient equal to zero at the y-axis.

60. I: On the y-axis? Why?

61. S: Yeah.

62. I: Ah, ok thank you.

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 3

Problem 3

Error factor

1. Substituting (-1) for x when trying to find $f'(-1)$, therefore $(-1)^2(-1-3)$ and not differentiate $f(x)$.

2. Not multiply x^2 by $(x - 3)$ before differentiating the expression.

3. Saying that -6(-1) is equal to -6 (ref # 25).

4. Saying that gradient equal to zero for a cubic equation means that the graph is going to cut the axis at zero or at the origin.

5. Saying that the gradient at the y-axis can be equal to zero

Semantic difficulties

1. Student is not able to express himself clearly (ref # 11).

UNIVERSITY of the
WESTERN CAPE

Cognitive difficulties

1. Student is always in a hurry to finish, so is very impulsive and very unsystematic in behaviour.

2. Lack of, or, impaired, verbal took for communication adequately elaborated responses.

https://etd.uwc.ac.za/

INTERVIEW 4

Problem 3

1. S: (Reads the question) I put -1 in the place of x.

2. I: What does this dash or slash mean to you?

3. S: To do derivative.

4. I: Ok, continue. Is it difficult for you?

5. S: (Silent for a minute) No not so difficult but the way to solve it.

6. I: You cannot find a way of solving it?

7. S: Yes, here I can say $f'(-1)$.

8. I: Can you tell me what is $f'(x)$ if $f(x) = x^2$?

9. S: It is zero. Oh of x^2 it is $2x^2 - 1 = 2x$.

10. I: Is 2x, ok?

11. I: So what will be $f'(-1)$.

12. S: For calculus any number will be zero.

13. I: Meaning?

14. S: Meaning that the answer will be zero for $f'(-1)$ and $f'(2)$.

15. I: Ok, why equal to zero?

16. S: It is from the derivative formula that any positive number is zero.

17. I: So what about $f'(-1)$.

18. S: (no answer).

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 4

Problem 3

Error factor

1. Assuming that $f'(-1)$ is the same thing as $f(-1)$ (ref # 1).

2. Saying that $f'(x)$ of $f(x) = x^2$ is equal to zero (ref #8).

3. Thinking that $f'(-1)$ will be zero because (-1) is a constant term (ref # 12).

4. Saying it is from the derivative formula that any positive number is zero (ref #16).

Cognitive difficulties

1. The student uses rote learning, thus just memorizes things (wrongly) without understanding them.

2. Not seeing the difference between $f'(-1)$ and $f(-1)$.

3. No logic at all in answering the questions.

4. Lack of, or limited, interiorization of one's behaviour.

5. Lack of, or impaired, planning behaviour.

6. She uses trial-and-error responses and looks at the interviewer to see the reaction.

7. She is very impulsive for example (ref # 9).

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW 6

Problem 3

1. S: (Reads the question).

2. I: What does it actually want you to do?

3. S: It wants me to calculate the values of $f'(-1)$ and $f'(2)$ and to say what the value of $f'(2)$ indicates.

4. I: Can you read it again?

5. S: If f of x is equal to x^2 into (x - 3), calculate the values of f dash at minus and f prime of minus 2. What does the value of f prime two indicates?

6. I: How do you read $f'(-1)$?

7. S: f prime of minus one.

8. I: What about x^2 (x - 3)?

9. S: It is x^2 multiplied by (x - 3).

10. I: Is it x^2 into or by (x - 3)?

11. S: By not into.

12. I: Ok ...

13. S: First I must multiply x^2 by (x - 3) then get $x^3 - 3x^2$ then differentiate it with respect to x and get $3x^2 - 6x$.

14. I: Why do you have to differentiate?

15. S: Because f' means differentiation.

16. I: Ok.

17. S: So then put -1 in place of x (She substitute correctly then moves on to $f'(2)$ and get zero as the answer).

18. I: Ok, what does the value of $f'(2)$ indicate?

19. S: It indicates that the graph passes at zero.

20. I: Which zero?

21. S: (Draws the wrong graph).

22. I: Ok (interviewer draw the graph of cubic equation). Show me the point where it will pass.

23. S: Points at the origin.

24. I: So it will pass through the origin? What does $f'(x)$ mean?

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

25. S: The gradient.

26. I: So what does it mean to say $f'(2) = 0$?

27. S: The gradient at 2 is zero.

28. I: What do you mean?

29. S: I don't know now, maybe it passes through (2; 0).

30. I: What is the gradient of the cubic graph at the turning points?

31. S: At the critical value?

32. I: Yeah!

33. S: It is zero because it is the highest or lowest point.

34. I: Why do you say this?

35. S: Our teacher told us that the gradient is zero at the turning points.

36. I: Ok, what is the gradient of the line passing through the x-axis?

37. S: $\frac{Dy}{Dx}$ There is no change in y therefore $\frac{Dy}{Dx} = 0$

the gradient will be zero.

38. I: So what can you say about the gradient of the line parallel to the x-axis and the line passing through the turning point?

39. S: (silence 2 minutes).

40. I: Ok, (draws the line passing through the turning point, still no reply from the interviewee).

41. I: Ok, thank you.

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 6

Problem 3

Error factor

1. Reads x^2 (x - 3) as x^2 into (x - 3) (ref # 5).

2. Says that $f'(2) = 0$ indicates that the graph passes through zero (ref # 19) therefore passes through the origin (ref # 23).

3. Saying that the gradient at the turning point is zero because his teacher told him so. No reason behind.

Cognitive difficulties

1. Using trial-and-error response and does not show full or clear understanding of why the gradient at the turning points is zero.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW 7

Problem 3

1. S: (Reads the question) I have to write this as one expression. Therefore as $x^3 - 3x^2$, then $f'(x) =$

$$\frac{f(x + h) - f(x)}{h}.$$

2. I: Which one are you solving? Is it the first part or the second one?

3. S: The first one.

4. I: Ok, then where is $f'(-1)$ and $f'(2)$?

5. S: (Silence 2 minutes).

6. I: So you mean $3x^2 - 6x$ is the answer for both $f'(-1)$ and $f'(2)$?

7. S: Yes.

8. I: Ok, what is $f'(1)$ if $f(x) = 2x^2 - x$?

9. S: (Writes $f'(1)$ then stops).

10. I: Ok, what does $f'(2)$ mean?

11. S: It means where there is x you put 2.

12. I: Ok, what does "f'" mean?

13. S: It means the derivative.

14. I: Ok, then what will be $f'(-1)$ given $f(x) = x^3 - 3x^2$?

15. S: (Silence 1 min).

16. I: Can you solve this problem for me?

17. S: Reads the question again.

18. I: Can you solve it?

19. S: No.

20. I: Do you understand the question?

21. S: No.

22. I: What is it that you don't understand?

23. S: (murmur something the interviewer did not understand).

24. I: Ok, can you find the derivative of $x^3 - 3x^2$?

25. S: I can solve it using the first principles?

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

26. I: Can you solve it without the use of the first principles?

27. S: Yes.

28. I: Ok, solve it then.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 7

Problem 3

Error factor

1. Saying that he is solving the first one while writing the formula

$$\frac{f(x + h) - f(x)}{h}.$$

2. Saying that $f'(-1)$ and $f'(2)$ have the same answer and it is $3x^2 - 6x$.

Cognitive difficulties

1. Not able to define a problem that is what he has been asked to do and what he should figure out, i.e. lack of or impaired analysis.

2. Does not think things through before answering but instead gives immediate answers.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW 1

Problem 4

1\. S: Reads the question, murmur then writes
$y = 2x^2 - 5x^2 - 4x + 12)$

First I will find the gradient $m = \frac{y - y_1}{x - x_1}$

Then substitute for y_1 and x_1 $m = \frac{y - 0}{x - 2}$.

2\. I: You are given a curve in this case isn't it? So will the gradient of the curve be the same as the gradient of the straight line? (Silence) Ok, can you draw the graph of a cubic graph?

3\. S: Yes.

4\. I: Ok, what does it look like?

5\. S: (draws the curve).

6\. I: Ok, now if I have this two points here (pointing at the turning points) will the gradient of the graph be given by the line passing through this two points?

7\. S: No, ok this is the gradient of the straight line.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 1

Problem 4

Error factor

1. Saying that the gradient of $f(x)$ at (2; 0) can be calculated using the formula

$$m = \frac{y - y_1}{x - x_1}.$$

Cognitive difficulties

1. Lack of content and therefore the result is mental blockage.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW 2

Problem 4

1. S: (Reads the question) I must first find the derivative of $y = 2x^3 - 5x^2 - 4x + 12$ which will be $f(y) = 6x^6 - 10x - 4$.

2. I: Ok.

3. S: Now I must find the gradient at the turning point.

4. I: Why do you say at the turning point?

5. S: At the point that is given (ie at (2; 0)).

6. I: Ok.

7. S: Determine now the critical value.

8. I: Mm.

9. S: If I want to find out the critical value I must say $6x^2 - 10x - 4 = 0$. So I must divide by 2 which will give me $3x^2 - 5x - 2 = 0$. Factorizing this I get $(x - 2)(3x + 1) = 0$.

10. I: How did you get the factors?

11. S: Using cross trial-and-error method (shows it on the scrap paper). Then the answer will be $x = 2$ or $x = -{}^1/_3$. Now I must check the answer by substituting the values of x in the original equation.

12. I: Why?

13. S: I want to find out the gradient. (interviewee substitutes 2 in the original equation then get zero as the answer. Then substitutes $-{}^1/_3$ ...).

14. I: Ok. What really does the question ask you to do?

15. S: To determine whether y has a maximum or minimum value.

16. I: Have you answered the first part already?

17. S: Finding the gradient. I think so.

18. I: What is the gradient of $2x^3 - 5x^2 - 4x + 12$ at the point (2; 0).

19. S: Mmm (Then draws a table).

20. I: What is this table for?

21. S: Determining whether y has a maximum or a minimum value at (2; 0).

22. I: Ok, but what is the gradient at (2; 0)?

https://etd.uwc.ac.za/

23. S: It is zero.

24. I: Zero? How did you get the zero because the question ask whether y has a maximum or minimum value at (2; 0). So it must be the turning point?

25. I: Hmm. What about if it was not stated?

26. S: I would substitute 2 for x.

27. I: Where?

28. S: In the gradient equation.

29. I: Why is it that you did not do that?

30. S: Oh, Oh I see. Ok, I made a mistake. I didn't think properly, but I solved problem like this yesterday.

31. I: Ok.

32. I: So will it be maximum or minimum at the turning point?

33. S: (looks back at the table drawn) It will be maximum.

34. I: Ok, how did you find out?

35. S: The table directed me.

36. I: Can you explain how the table directed you?

37. S: (keeps quiet) I can't explain it, it is the way we do it.

38. I: Ok. Thank you.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 2

Problem 4

Error Factor

1. Wasting time finding the co-ordinators of the turning-points again.

Cognitive difficulties

1. Lack of or deficient need for precision and accuracy in data gathering.

2. Inadequacy in experiencing the existence of an actual problem and subsequently defining it.

3. Lack of or limited interiorization of one's behaviour.

4. Has lots of problems in expressing the solution to the problem.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW 3

Problem 4

1. S: (Reads the question aloud).

2. I: (S reads the problem again) Tell me what is it that you are expected to do.

3. S: The question requires me to calculate the gradient of the curve and given that x = 2 when y is equal to zero.

4. I: Can you try it?

5. S: $2x^3 - 5x^2 - 4x + 12$ Then substitutes two for x

   $2(2)^3 - 5(2)^2 - 4(2) + 12$ (reads the question again).

6. I: Ok, you can read the question as many times as you want.

7. I: Do you understand the question? The questions says you must determine the gradient of the curve of

   $y = 2x^3 - 5x^2 - 4x + 12$ and not find what will be the y- coordinate at the point where x = 2.

8. S: Ok, I am not suppose to do it like this.

9. I: Ok, then you are suppose to do it like what?

10. S: First differentiate this $6x^2 - 10x - 4$.

11. I: Why do you have to differentiate it?

12. S: Continues to solve the problem $6x^2 - 10x - 4$
    So substituting 2 for x to get y- value
    $= 6(2)^2 - 10(2) - 4$
    $= 24 - 20 - 4$
    $= 0$.

13. I: Ok, now that you have found zero as your answer, determine whether y has a minimum or maximum value at this point.

14. S: I can say y has a maximum value.

15. I: Why?

16. S: Because it is not negative, because once it is negative y- value has a minimum value.

17. I: (Draws a graph for him and ask him).

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 3

Problem 4

Error Factor

1. Instead of finding the gradient at (2; 0) student find what the ordinate will be when the abscissa is 2 i.e. x = 2.

Cognitive difficulties

1. Does not answer interviewer's question (ref # 11). Just goes on with the work.

2. Uses guess work. Just saying y has a maximum value without any working. Therefore student show that he does not plan his work.

3. Uses trial-and-error responses (ref # 14 to 16).

4. Could not answer the last part due to lack of content.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW 4

Problem 4

1. S: (Reads the question) I can start to solve x (sic).

2. I: Ok, just solve it.

3. S: In the place of x I put 2.

4. I: Eh.

5. S: Thus $y = 2x^3 - 5x^2 - 4x + 12$

$= 2(2)^3 - 5(2)^2 - 4(2) + 12$

So now removing the brackets I will get

$= 16 - 20 - 8 + 12$

So the answer is zero.

6. I: Ok, so what does it mean?

7. S: (no answer).

8. I: Are you answering the question?

9. S: No, sir. Now I must find the gradient. Now I must find the gradient. (She then writes $y_2 - y$, then cancels it.

UNIVERSITY of the
WESTERN CAPE

10. I: Why do you cancel this one?

11. S: If I had two points then I would use it but because I have one so I can not use it.

12. I: Ok.

13. S: Substitute zero (work out the problem using zero and gets 12).

14. I: So what is this 12?

15. S: It is the y- value. So our co-ordinates will be (0; 12). (uses this as the second point for calculating the gradient therefore

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{12 - 0}{0 - 2} = \frac{12}{-2} = -6$$

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 4

Problem 4

Error Factor

1. Finding out the y- co-ordinate when x is 2, which is already given as zero (ref # 5).

2. Thinking that she can find the gradient by using

   $\frac{y_2 - y_1}{x_2 - x_1}$ (ref answer sheet).

3. Finding the other point thinking that she can you this point together with (2; 0) to find the gradient at (2; 0).

Cognitive difficulties

1. Unplanned, impulsive and unsystematic exploratory behaviour.

2. Very poor in content matter.

3. Narrowness of the mental field.

4. Trial-and error responses.

5. Lack of, or impaired, verbal tools for communicating adequately elaborated responses.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW 2

Problem 5

1. S: (Read the question twice).

2. I: What is actually needed here?

3. S: (Interviewee reads the question again) They want a length of a side of the square that must be cut out so that the volume is a maximum. Therefore area is t xb.

4. I: Ok, go on.

5. S: I cannot solve this type of problem.

6. I: Why?

7. S: They gives me a head-ache.

8. I: Just try this one maybe it will not give you headache this time.

9. S: Really, Sir I cannot solve it.

10. I: Ok, thanks.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 2

Problem 5

Cognitive difficulties

1. Mental blockage.

2. Has a very negative attitude towards words problems.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW 3

Problem 5

1 S: (Reads the problem) I cannot do this one

2 I: Just try it

3 S: I have got problems with word problems. I do not understand them

4 I: Just try this one

5 S: I have forgotten all my standard ten maths for solving this problem

6 I: Do you understand what the question wants?

7 S: Yes, but it gives one a headache really trying had to solve

8 I: Ok, thank you

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW 3

Problem 5

Cognitive Difficulties

1. Student believes that because he did not get the correct answer for the similar problem the previous day he cannot solve this one. Therefore lack of self-confidence is manifested.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW B_1 (University first year student)

Problem 1 (i)

1. I: Read the question for me and solve the problem.

2. S: (Reads the question. Determine the following Limits the limit as x approaches 2 and then we have $x^2 - 4$ divide by x - 2).

3. I: Mmm.

4. S: So to solve the problem, one cannot substitute 2 here because the moment you substitute 2 the result will be zero over zero which is infinite, undefined.

5. I: Hmmm.

6. S: So first what must be done is to, you must say the limit as x approaches two and try to factorize the numerator and then can say (x + 2)(x - 2) all over x - 2. Then cancel, and get

   $\lim_{x \to 2} x + 2$ and then substitute our value 2 then

   we will get 2 + 2 and get 4 which is our answer.

7. I: Ok. What does the word limit means to you?

8. S: The word limit according to my explanation is whereby one distinguishes whether the function is continuous and then the limit is whereby you approach a certain number. For example the certain number here is 2 therefore you can go to the left or the right.

9. I: What do you mean by going to the right or to the left?

10. S: Ok, let me say from the left for this one will be (points at the number line). They will give us the same answer from the left or from the right. We will get the same answer you can see this from this graph (he draws a graph).

11. I: Ok. What I am asking is for you maybe to give an example when it is being approached from the left?

12. S: When it approaches 2 from the left? For example maybe it will be difficult but I will try (Try to explain using the graph murmuring and giving good example).

13. I: So what you are saying is that when it is approaching from the left we may say 1; 1.9, 1; 99, 1,999 ... Ultimately will it reach two?

14. S: No its will just go like that without touching two but will be very close to two.

https://etd.uwc.ac.za/

15. I: Just for interest sake do you know what is meant by asymptote.

16. S: Yeah, Yeah, Yeah, asymptote, yes.

17. I: What does it mean?

18. S: The line getting closer and closer to the asymptote without touching it.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW B_1

Problem 1 (i)

Error factor

1. To him infinity and undefined means one and the same thing (ref # 4).

2. The diagram drawn not good for explaining what a limit is.

Semantic difficulties

1. Could not link $\lim_{x \to 2}$ to the expression $\frac{x^2 - 4}{x - 2}$. (ref # 2)

2. Unable to express oneself properly (ref # 6) and also in line (ref 8) he is unable to explain what a limit is even though one could see that he at least has a picture of what it might be.

Cognitive difficulties

1. Not very sure of describing things and events in terms of their names and where and when it occurs.

2. Lack of, or limited, interiorization of one's behaviour.

3. Unable to look for the relationship by which separate objects, events and experiences can be tied together (Projecting relationships).

4. Not clear and precise in language therefore egocentric communicational modalities and the lack of or impaired, need for precision and accuracy in communicating one's responses.

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

INTERVIEW B_1

Question 1 (ii)

1. I: Read part (ii) for me please

2. S: (Reads the question) Before doing this one also like the first one it will be difficult because we will have ∞/∞. So maybe we will have to simplify or factorize the numerator.

3. I: Ok.

4. S: The limit, eh, it is important that you always write the limit because the moment the lim is not reflected the answer will be totally wrong.

5. I: Ok.

6. S: You can take x because it is common in the numerator and then we have

$$\lim_{x \to \infty} \frac{x(x^2 - x - 1)}{x(x + 2)}.$$

7. I: Ok.

8. S: Now you can try the x then will be left with

$$\lim_{x \to \infty} \frac{x^2 - x - 1}{x + 2}.$$

9. I: Ok.

10. S: Then try to factorize the numerator. Oh it is not easy to factorize it - then substitute x --> x and then we get 0.0001 and then ultimately we will get answer as $-\frac{1}{2}$.

11. I: How did you get $-\frac{1}{2}$?

12. S: You see infinite can be substituted by something like 0.0001 which is almost zero.

13. I: Ok.

14. S: Yes.

15. I: What about if somebody just divided each and every term by x say

$$\frac{\frac{x^2}{x} - \frac{x}{x} - \frac{1}{x}}{\frac{x}{x} + \frac{2}{x}}.$$

16. S: Ok. Yeah its ok.

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

17. I: You mean it will still give the same answer or will the answer be different?

18. S: Ok. Let me try. (he solves the problem) Yes the answer will be different. So the answer in this case will be minus one.

19. I: Why minus one?

20. S: (he is ignoring the x assuming that x is almost zero so the answer becomes minus one).

21. I: Remember you said x is a big number, so ...

22. S: So, let me see (keeps quiet for about two minutes).

23. I: Therefore what will be the answer?

24. S: The answer will be ∞.

25. I: Why ∞?

26. S: Because x --> ∞ then we get 00 because $^1/_x$ will give zero.

27. I: Ok.

28. S: So ∞ - 1 divided by 1 give infinity

29. I: Ok

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

ANALYSIS OF INTERVIEW B_1

Problem 1 (ii)

Error factor

1. Saying that infinity is the same thing as 0.0001 there substituting it for x which will be the same thing as substituting zero therefore getting the answer as $-\frac{1}{2}$ (ref # 6 to 14).

2. After dividing each term by x and getting

$$\frac{x - 1 - \frac{1}{x}}{1 + \frac{2}{x}}$$

says

the answer will be minus 1. Thus still considering x to be zero.

Semantic difficulties

1. Expressing oneself is still a problem.

Cognitive difficulties

1. The student seemed to be in a hurry to finish or solve the problem as a result interviewee displayed or showed impulsivity, unplanned and unsystematic exploratory behaviour in trying to solve the problem.

2. Lack of a good picture in mind of what the question requires or what he is looking for.

Interview B_1 (2) to Interview B1 (5) are not included as there were no cognitive deficiencies displayed by the student. The same applies to Interview B_2 (i) to Interview B_2 Problem 5.

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

INTERVIEW B3

Problem 2

1. S: (Reads the question).

$$\frac{f\ (2 + h) - f\ (2)}{h}$$

can be determined as follows

$$\begin{aligned} f\ (2 + h) &= 5 + 7\ (2 + h) - (2 + h)^2 \\ &= 5 + 14 + 7x - 4 - 4h - h^2 \\ &= 15 + 7x - 4h - h^2. \end{aligned}$$

2 I: Where does 7x come from?

3 S: From the formula $f\ \frac{(x + h) - f\ (x)}{h}$.

4 I: Does the formula appear anywhere in the question?

5 S: No.

6 I: So?

7 S: Wait ... I have made a mistake here. You know this are the mistakes that make one fail maths. It is 7h not 7x.

UNIVERSITY of the
WESTERN CAPE

8 I: Ok, continue.

9 S: So $f\ (2 + h) = 15 + 3h - h^2$. Now moving on to $f\ (2)$

$f\ (2) = 5 + 7(2) - (2)^2 = 15$.

Now $\frac{f\ (2 + h) - f\ (2)}{h} = \frac{15 + 3h - h^2 - 15}{h}$

$= \frac{3h - h^2}{h} = 3 - h$

So 3 - h is the answer for part one.

10 I: Ok.

11 S: So for the second part as h approaches zero the answer will be 3.

12 I: Ok, are you satisfied?

13 S: Yes.

https://etd.uwc.ac.za/

INTERVIEW B3

Problem 3

1 S: If f at x is equal to x^2 by (x - 3). Calculate the values of f prime into minus one and f prime of two. What does the value of f prime 2 indicates?

2 I: How do you read $f'(-1)$?

3 S: It is the derivative at minus one.

4 I: Okey.

5 S: (Writes) $f'(x) = 2x\ (1)$.

6 I: Is this the way you are taught to differentiate?

7 S: Yeah. No I have to use the product rule or first simplify. Ok now I will have $f\ (x) = x^3 - 3x^2$ then $f'(x) = 3x^2 - 6x$.

8 I: Ok.

9 S: So $f'\ (-1) = 3\ (-1)^2 - 6\ (-1) = 9$ and $f'\ (2) = 3\ (2)^2 - 6\ (2) = 0$.

10 I: Ok.

11 S: Now the next question is what does the value of $f'\ (2)$ indicates?

12 I: Yes, what does it indicate?

13 S: That the derivative is equal to zero.

14 I: What does derivative equal to zero means?

15 S: ee ..h the gradient is zero.

16 I: That is right, but tell me what does gradient equal to zero mean for $f'\ (2)$?

17 S: It means (draws) the curve is equal to zero, I mean the gradient of the curve at turning point is zero.

18 I: Ok.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW B3

Problem 4

1 I: Read the question for me please and do as it requires you to do.

2 S: (Reads the question).

3 I: Which y do you have to determine?

4 S: $y = 2x^3 - 5x^2 - 4x + 12$.

5 I: Ok.

6 S: $y' = 6x^2 - 10x - 4$

So at the point where x = 2 we have

$$f'(2) = 6(2)^2 - 10(2) - 4$$
$$= 24 - 20 - 4$$
$$= 0$$

So the gradient is equal to zero.

7 I: Ok, now ...

8 S: Now for the second part I will have y as minimum because the gradient is zero.

9 I: So you mean whenever the gradient is zero one will have a maximum value at the turning point?

10 S: Yes.

11 I: Ok.

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

INTERVIEW B4

Problem 1 (i)

1 S: (Reads the question correctly) Now the first thing for me is to cancel.

So $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ then $\lim_{x \to 2} x - 2$ gives the answer

2 I: so you mean the answer is $\lim_{x \to 2} x - 2$

3 S: Ok, let me see. No it is not like this. The method is first factorize, therefore the factors of $x^2 - 4$ are $(x - 2)(x + 2)$

Then $\lim_{x \to 2} \frac{(x - 2)(x + 2)}{(x - 2)}$

$= \lim_{x \to 2} x + 2$ which gives the answer as 4.

4 I: So is 4 your answer?

5 S: Yes.

6 I: Tell me what does the word limit means to you.

7 S: It means something being approached from both sides (draws the diagram). That is the keep on moving so close to the point that one would think that there is no space at all between the two points.

8 I: Ok, thank you. Can we move on to the next question?

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW B4

Problem 1 (ii)

S: (Reads the question) First take out the common factor. Therefore

$$\lim_{x \to \infty} \frac{x^3 - x^2 - x}{x^2 + 2x} = \lim_{x \to \infty} \frac{x(x^2 - x - 1)}{x(x + 2)}$$

$$= \lim_{x \to \infty} \frac{x^2 - x - 1}{x + 2}$$

Now the answer is $-\frac{1}{2}$.

I: Why $-\frac{1}{2}$.

S: Because there is -1 in the numerator and 2 in the denominator.

I: So what happened to the x's?

S: x's? I don't know. Maybe infinity will be neutral so equal to zero. That is finding the average

I: What is infinity?

S: A very big number that is increasingly big and a very small number

I: So it means both big and small numbers?

S: Yes, but it is not small as such it is big, but negatively so

I: Ok, you mean there is a negative and a positive infinity?

S: Yes, so when you take the average the answer will be zero

I: Why do you find the average?

S: (Silent for 2 minutes)

I: What is ∞? Is it positive or negative?

S: It is neutral?

I: Is it? Is 2 neutral?

S: Ok, I see it is positive because for positive we leave signs. Ok, but no I cannot solve this one

I: Thanks

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

INTERVIEW B4

Problem 2

S: (Reads the question)

$$\frac{f\ (x + h) - f\ (x)}{h}$$

Ok, now $f\ (2 + h) = 5 + 7\ (2 + h) - (2 + h)^2$

wherever there is x you substitute it with (2 + h)

So $f\ (2 + h) = 5 + 14 + 7h - 4 - 4h - h^2$

$= 15 + 3h - h^2$ $f(2) = 5 + 7(2) - (2)^2 = 15.$

I: Ok.

S: Then $\frac{f(2 + h) - f\ (2)}{h}$

$$= \frac{5 + 14 + 7h - 4 - 4h - h^2 - (5 + 14 - 4)}{h}$$

$$= \frac{15 + 3h - h^2 - 12}{h} = \frac{0 + 3h - h^2}{h}$$

$= 3 - h.$

I: Ok.

S: Then calculating the limit of h -> o.

$\lim_{h \to o} 3 - h$ This will give 3 as the answer.

I: Ok, thanks.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW B4

Problem 3

I: Read the question for me please.

S: IF f at x is equal to x^2 into x - 3 calculate the values of f slash -1 and the derivative at 2. What does the value of f' dash 2 indicate?

I: Can you read $f(x) = x^2 (x - 3)$ for me again?

S: It is the function of x is given by x^2 by (x - 3).

I: Ok, go on

S: (solves the problem correctly) Now the value $f'(2)$ indicate that the gradient is zero at the critical point.

I: Ok, what is the critical point?

S: The point where the values are maximum or minimum.

I: Thanks.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

INTERVIEW B4

Problem 4

S: (Reads the question) If $y = 2x^3 - 5x^2 - 4x + 12$ at the point (2; 0). The derivative will be given by

$f'(x) = 6x^2 - 10x - 4$.

Then $f(2) = 6(2)^2 - 10(2) - 4 = 24 - 20 - 4 = 0$.

Then because it is zero it wil be minimum.

I: Minimum?

S: Yes, zero is the minimum number. Anyway I have forgotten how we used to solve this at high school.

I: Ok.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

APPENDIX 2A

QUESTIONNAIRE FOR THE FIRST YEAR B.Sc MATHEMATICS STUDENTS

NAME OF STUDENT :......................................

AGE OF STUDENT :......................................

MATHEMATICS STREAM :......................................

OCCUPATION OF PARENTS

UNIVERSITY of the
WESTERN CAPE

FATHER :......................................

MOTHER :......................................

GUARDIAN :......................................

YOUR HONESTY WILL BE HIGHLY APPRECIATED PLEASE ANSWER ALL QUESTIONS

https://etd.uwc.ac.za/

1. For how many times did you sit for matric mathematics?

....
....
....
....
....

2. What symbol did you get for matric mathematics?

....
....
....
....

3. Did you register for mathematics higher grade or standard grade?

....
....
....
....
....

4. How did you find matric differential calculus?

....
....
....
....
....

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

5. Which topics do you think need to be studied before dealing with differential calculus? Why?

....

....

....

....

....

6. What do you think (now) about high school teacher's approach to differential calculus?

....

....

....

....

....

UNIVERSITY of the
WESTERN CAPE

7. Do you think the calculus you learnt at high schools is of any help? Give reasons.

....

....

....

....

....

8. What do you think needs to be done to make high school pupils pass differential calculus? Give reasons.

https://etd.uwc.ac.za/

....

....

....

....

....

9. What impact do you think your (i) teacher (ii) syllabus (iii) school (iv) home (v) politics had on your studies in differential calculus?

(i) Teacher

....

....

....

....

....

(ii) Syllabus

....

....

....

....

....

(iii) School

....

....

....

....

....

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

(iv) Home

....

....

....

....

....

(v) Politics

....

....

....

....

....

UNIVERSITY of the WESTERN CAPE

10. Who and what actually motivated or helped you to have a positive attitude towards mathematics?

....

....

....

....

....

11. If you were asked to improve the teaching and learning of differential calculus what would you do? Why?

....

....

....

....

....

https://etd.uwc.ac.za/

12. What do you think made your other high school classmates to fail differential calculus?

....

....

....

....

....

13. What do you think made other students to pass differential calculus?

....

....

....

....

....

UNIVERSITY of the
WESTERN CAPE

14. What are the misconcepts or mistakes that were commonly made when solving differential calculus problems? Please give examples.

....

....

....

....

....

15. What can you say about the medium of instruction (English/ Afrikaans) in the learning of differential calculus?

https://etd.uwc.ac.za/

....

....

....

....

....

16. Are there any other problems that you encountered in the learning of differential calculus, please state them.

....

....

....

....

....

17. How do you find differential calculus now? Give reasons.

....

....

....

....

....

....

....

18. Do you like being taught mathematics by male, female teachers or does it not really matter who?

....

....

....

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

....

....

19. What do you usually do when you experience problems with mathematics (say after mathematics period) or at home during your own study time?

....

....

....

....

....

20. When having problems with solving differential calculus who do you often consult?

....

....

....

....

UNIVERSITY of the
WESTERN CAPE

21. What is the relationship between (i) f'(x); Dxf(x) and dy/dx?

....

....

....

....

....

https://etd.uwc.ac.za/

22. What does f'(x) = 0 mean? Explain.

....

....

....

....

....

THANK YOU VERY MUCH

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

APPENDIX 2B

QUESTIONNAIRE FOR PUPILS

NAME OF PUPIL :..

NUMBER OF YEARS DOING STD 10 MATHEMATICS:......................

AGE OF PUPIL (Year) :..

OCCUPATION OF MOTHER:..

UNIVERSITY of the
WESTERN CAPE

OCCUPATION OF FATHER:..

OCCUPATION OF GUARDIAN:..

PLEASE ANSWER ALL QUESTIONS ASKED OVERLEAF GIVING AS MANY REASONS AS POSSIBLE.

YOUR HONESTY WILL BE HIGHLY APPRECIATED.

https://etd.uwc.ac.za/

1. Do you like the subject mathematics?

....

....

....

....

....

Give reasons

....

....

....

What about algebra?

....

....

....

....

....

Give reasons

....

....

....

....

....

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

What about differential calculus?

....

....

....

....

....

Give reasons

....

....

....

2. What in your own opinion do you think makes you not to pass differential calculus as you think you would? Elaborate.

UNIVERSITY of the
WESTERN CAPE

....

....

....

....

....

3. Do you think some of the problems (if any) that you have with regard to understanding differential calculus problems are because of problems at (i) home (ii) at school (iii) with peers or any other body? Give reasons for your answer.

....

....

....

https://etd.uwc.ac.za/

....

....

4. Do you think politics (partly) (can) affect your studies? Why and how?

....

....

....

....

....

5. Do you have problems of understanding the differential calculus problems? Give and example of such problems.

....

....

....

....

....

6. When solving the problem do you always finish within stipulated time? Why?

....

....

....

....

....

UNIVERSITY of the WESTERN CAPE

https://etd.uwc.ac.za/

7. What do you think about the language use din differential calculus? Do you think it is difficult or not? Why?

....

....

....

....

....

8. Are you able to use the differential calculus rule(s) when confronted with the problem? Please give reason for your answer.

....

....

....

....

....

UNIVERSITY of the
WESTERN CAPE

9. What do you think students should do in order to like and pass differential calculus well?

....

....

....

....

....

10. What do you think is the main reason for student not to understand or like mathematics especially differential calculus?

https://etd.uwc.ac.za/

....

....

....

....

....

11. What do you think teachers should do in order to make differential calculus interesting and easy?

....

....

....

....

....

12. Do you think anything needs to be done to the syllabus of differential calculus?

UNIVERSITY of the
WESTERN CAPE

....

....

....

....

....

13. Is there a relationship between the gradient and finding the derivative? What is it (if there is any)?

....

....

....

....

https://etd.uwc.ac.za/

14. Does English language affect your understanding of differential calculus? Why?

....

....

....

....

....

15. Which topics do you think must be dealt with before dealing with differential calculus?

....

....

....

....

....

UNIVERSITY of the
WESTERN CAPE

THANK YOU VERY MUCH

https://etd.uwc.ac.za/

APPENDIX 2C

QUESTIONNAIRE FOR TEACHERS

NAME OF SCHOOL : ....

NUMBER OF YEARS TEACHING STANDARD 10: ....

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

1. What in you own opinion causes students to either pass or fail differential calculus in particular or mathematics in general? Elaborate.

....

....

....

....

....

2. What do you do to motivate or make students like maths especially differential calculus?

....

....

....

....

....

UNIVERSITY of the
WESTERN CAPE

3. What should be done to students who do not understand differential calculus?

....

....

....

....

....

https://etd.uwc.ac.za/

4. Do you think the background (socio-economic) has any effect on the pupils learning or performance in mathematics? Elaborate.

....
....
....
....
....

5. Do you usually finish Standard Ten mathematics syllabus in time? What do you think are the reasons for your answer?

....
....
....
....
....

UNIVERSITY of the
WESTERN CAPE

6. Do you think the use of English as a medium of instruction has any effect on understanding of differential calculus?

....
....
....
....
....

https://etd.uwc.ac.za/

7. Which topics do you think must be dealt with before introducing differential calculus? Give reasons.

....

....

....

....

....

8. What do you think about the academic background of the student? Does it have any effect on the pupils?

....

....

....

....

....

UNIVERSITY of the
WESTERN CAPE

9. If confronted with the following problem how do you think the students would solve them. Step by Step (Please show the misconceptions that are usually displayed by students).

Problem:

The distance, S metres that a body travels in t seconds is given by

$$S = \frac{1}{12} t^3 - t^2 + 4t.$$

https://etd.uwc.ac.za/

Determine (i) the initial speed (ii) the time it will take the body to come to rest (iii) what does $f'(x) = 0$ mean?

....
....
....
....
....

10. Is there a relationship between age of students and their performances?

....
....
....
....
....

UNIVERSITY of the
WESTERN CAPE

11. What do you think about the in-service training in mathematics especially differential calculus?

....
....
....
....
....

https://etd.uwc.ac.za/

12. What do you think should be done in order for students' understanding of differential calculus to be improved? Which instructional method should be used?

....

....

....

....

....

13. What other factors do you think need to be considered in order to improve the teaching and learning of differential calculus?

....

....

....

....

....

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

APPENDIX 2D

TICK THE CORRECT ANSWER

1. If $f(x) = x^2$ then $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ is

   A. $\frac{x^2}{h}$ B. 0 C. 2x D. $(x^2 + h) - f(x^2)$ E. 4

2. The gradient of $f(x) = x^3$ is given by

   A. $\frac{y_2 - y_1}{x_2 - x_1}$ B. $3x^4$ C. 3 D. $3x^2$ E. $\frac{x^2}{3}$

3. The equation of the tangent to $f(x) = 3x^2 + 6x + 7$ at the point (2: 31) is

   A. y = X + 7 B. y = 6X + 6 C. y = 7 D. Y = 3x + 6

   E. y = 18x - 5

The area A of an expanding paint blot (in cm^2), t seconds after it has been spilled, is given by the formula

$$A = 6 + 4t - t^2$$

INFORMATION FOR QUESTIONS 4 - 7

4. Initial area of the blot is

   A. $4cm^2$ B. $6cm^2$ C. $8cm^2$ D. $10cm^2$ E. 0

5. Initial rate of increase in area is given by

   A. $6cm^2.S^1$ B. $4cm^2.S^1$ C. 0 D. 6 + 4t E. t^2

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

6. The time at which the blot stops spreading is given by

A. 6 sec B. 2 sec C. 10 sec D. 9 sec E. 0

7. The area at the time the blot stops spreading is given by

A. $1cm^2$ B. $10cm^2$ C. $100cm^2$ D. t^2cm^2 E. $2cm^2$

8. $f(x) = x^2 + 3x^2 - 6$. The average gradient of f(x) between x = 2 and x = 3 is given by

A. 1 B. 6 C. 20 D. 10 E. 3

9. $\lim_{x \to 2} \frac{X^2 - 4}{X - 2}$ is

A. 2 B. 4 C. 6 D. 8 E. 10

10. $g(x) = x^2$

What is the gradient of the above mentioned graph at the point where x = 2

A. 2 B. 4 C. 6 D. 8 E. 10

11. Given that g(x) = x + 2

g(5 + h) - g (2) is given by

A. 3 + h B. 2 C. 5 D. 0 E. -2

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

APPENDIX 2 E

1. Determine the following limits

(i) $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

(ii) $\lim_{x \to \infty} \frac{x^3 - x^2 - x}{x^2 + 2x}$

2. If $f(x) = x^2 (x - 3)$

Calculate the values of $f'(-1)$ and $f'(2)$

What does the value of $f'(2)$ indicate?

3. $F(x) = 5 + 7x - x^2$

Determine $\frac{f(2 + h) - f(2)}{h}$

and hence calculate the limit if $h \to o$

UNIVERSITY of the
WESTERN CAPE

4. Determine the gradient of the curve of

$y = 2x^3 - 5x^2 - 4x + 12$ at the point (2; 0)

and determine whether y has a maximum or a minimum value at this point

5. A box which is open on top is made of a rectangular piece of cardboard of 50 cm by 30 cm.

The box is constructed by cutting out equal squares at the corners and bending up the sides.

Calculate the length of a side of the square that must be cut out so that the volume is a maximum.

https://etd.uwc.ac.za/

APPENDIX A

The deficient functions are presented in four categories.

A. Impaired cognitive functions affecting the input phase include all impairments concerning the quantity and quality of data gathered for problem solving.

- Blurred and sweeping perception
- Unplanned, impulsive, and unsystematic exploratory behaviour
- Lack of, or impaired, receptive verbal tools and concepts which affect discrimination
- Lack of, or impaired, spatial orientation, including the lack of stable systems of reference which impair the organization of space
- Lack of, or impaired, temporal orientation
- Lack of, or impaired, conservation of constancies (i.e. in size, shape, quantity, orientation) across variations in certain dimensions of the perceived object
- Lack of, or deficient need for, precision and accuracy in data gathering
- Lack of, or impaired, capacity for considering two sources of information at once, reflected in dealing with data in a piecemeal fashion rather than as a unit of organized facts.

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

B. Impaired cognitive functions affecting the elaborational phase include those factors that impede the individual in making efficient use of data available

- Inadequacy in experiencing the existence of an actual problem and subsequently defining it
- Inability to select relevant, as opposed to irrelevant, cues in defining a problem
- Lack of spontaneous comparative behaviour or limitation of its appearance to a restricted field of needs
- Narrowness of the mental field
- Lack of, or impaired, need for summative behaviour
- Difficulties in projecting virtual relationships
- Lack of orientation toward the need for logical evidence as an interactional modality with one's objectal and social environment
- Lack of, or limited, interiorization of one's behaviour
- Lack of, or restricted, inferential-hypothetical thinking
- Lack of, or impaired, strategies for hypothesis testing
- Lack of, or impaired, planning behaviour
- Non-elaboration of certain cognitive categories because the necessary labels either are not part of the individual's verbal inventory on the receptive level or are not mobilized at the expressive level
- Episodic grasp of reality

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

C. Impaired cognitive functions affecting the output phase include those factors that lead to an inadequate communication of the outcome of elaborative processes

- Egocentric communicational modalities
- Blocking
- Trial-and-Error responses
- Lack of, or impaired, verbal tools for communicating adequately elaborated responses
- Deficiency of visual transport
- Lack of, or impaired, need for precision and accuracy in communicating one's response
- Impulsive acting-out behaviour, affecting the nature of the communication process

D. Affective-Motivational factors affecting the cognitive processes can combine negatively in such a way as to influence the attitudes of the disadvantaged. (For more recent information on this factor refer to Tzuriel (1991).

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

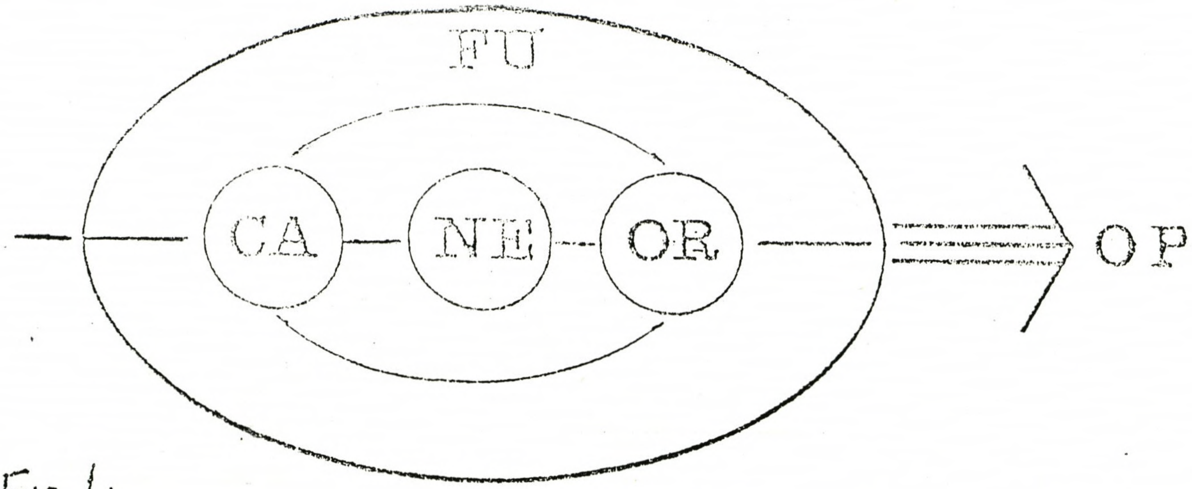


Fig 1.

Cognitive function (FU): An integrative model including capacity (CA), need (NE), orientation (OR), and operation (OP)

UNIVERSITY of the
WESTERN CAPE

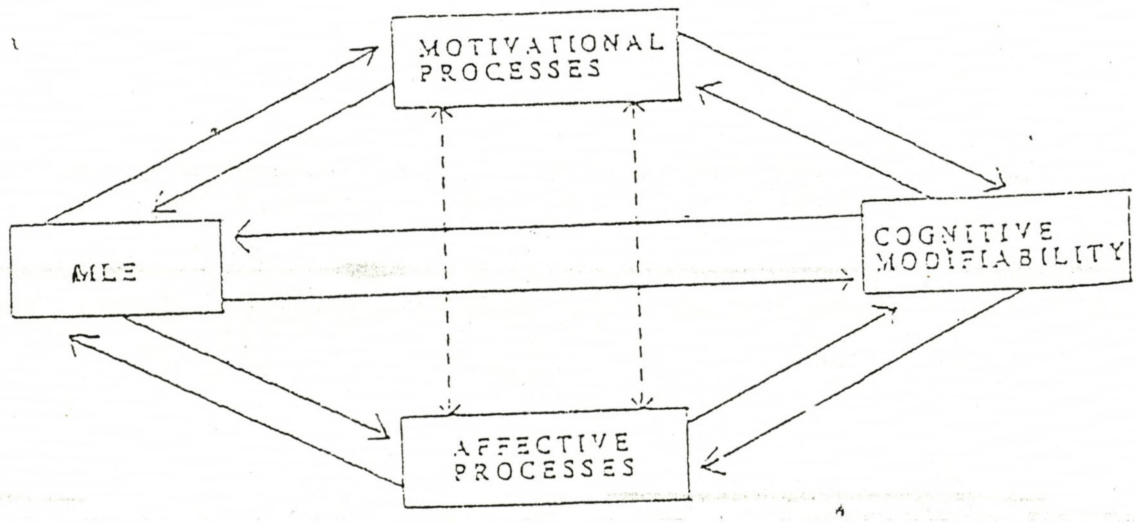


Figure 2

https://etd.uwc.ac.za/

APPENDIX C

First year differential calculus questionnaire summary

Table Fy 1 How did you find differential calculus?

	(Very) Difficult	Moderate	Easy	No response
Number of students	7	6	20	7
Percentage	17,5%	15%	50%	17,5%

Table Fy 2 Preliminary topics

Topics	Area, Functions and exponents	Trigonometry	Everything done earlier	Not Sure
Number of students	10	4	2	24
Percentages	25%	10%	5%	60%

UNIVERSITY of the WESTERN CAPE

Table Fy 3 Blame on teachers

High schools teachers approach	**Good**	**Bad**	**No Comment**
Number of students who responded	**25**	**10**	**5**
Percentages	**62,5%**	**25%**	**12,5%**

Table Fy 4 High School calculus

Is high school calculus of any help	Yes	No	No comment
Number of students	32	5	3
Percentages	80%	12,5%	7,5%

https://etd.uwc.ac.za/

Table Fy 5 What need to be done to improve high school calculus

What need to be done to improve high school calculus	Student should taught how to solve problems	Students should be motivated by teacher	Teaching aids should be used	Lowering the teaching standard to the level of students	Students should attend extra clas-sess	Up to each student	Practice and more exercise	No reply
Number of students	15	2	5	2	5	3	4	4
%	37,5%	5%	12,5%	5%	12,5%	7,5%	10%	10%

Table Fy 6 Some Factors affecting the learning of differential calculus

Factors affecting the learning of differen-tial calculus	Teachers Bad/Good	Syllabus Bad/Good	School Bad/Good	Home Bad/Good	Politics Bad/Good	No re-ply
Number of students	10 30	7 33	4 36	7 33	8 32	-
Percentages	25% 75%	17,5% 82,5%	10% 90%	17,5%82,5%	20% 80%	-

UNIVERSITY of the WESTERN CAPE

Table Fy 7 Some sources of motivation

Sources of motive for have positive attitude toward differential calculus	Teachers only	Political climate in RSA	Parents and teachers	Interested in competition	By friends and the brothers and other people
Number of students	18	9	7	2	4
Percentages	45%	22,5%	17,5%	5%	10%

https://etd.uwc.ac.za/

Table Fy 8 Factor that make pupils fails differential calculus

Things that make high school pupils fail differential calculus	Teachers should have interest in what they teach	Students not liking the subject mathematics	Lack of team work	Students are born clever or stupid so nothing can be done	Students to active in other matter than academic ones	No reply
Number of students	20	5	6	1	4	4
Percentages	50%	12,5%	15%	2,5%	10%	10%

Table Fy 9 Misconcepts

Misconcepts made ->	Signs	Not simplifying	Learning by rote method	No reply	Impulsive or not knowing the basics (lack of content)
Number of students	9	5	3	15	8
Percentages	22,5%	12,5%	7,5%	37,5%	20%

Table Fy 10 Impact of medium of instruction

UNIVERSITY of the WESTERN CAPE

Impact of the medium of instruction	English does not have nay bad impact on learning calculus	Have impact as it is not the students first language
Number of students	31	9
Percentages	77,5%	22,5%

Table Fy 11 First year university differential calculus

First university differential calculus	Interesting and challenging	Difficult	No Reply
Frequency of students	23	14	3
Percentages	57,5%	35%	7,5%

https://etd.uwc.ac.za/

Table Fy 12 Gender

Gender Preference	Male only	Female only	Any Sex (does not matter)
Frequency of students	6	0	34
Percentages	15%	0%	85%

Table Fy 13 Mathematical problems

What is usually done when confronted with mathematics problem	Consult Friends	Teachers	Teachers senior students and peer class-mates	Peers and text-books	No reply	Helping oneself
Number of Students	11	6	16	2	2	3
Percentages	27,5%	15%	40%	5%	5%	7,5%

Table Fy 14 Definitions

Relationship between $f(x)$: D_x and Dy/d'x	Different	No answer	Same answer
Number of students	30	4	6
Percentages	75%	10%	15%

UNIVERSITY of the WESTERN CAPE

Table Fy 15 Content

What does $f'(x) = o$ mean?	Gradient/ slope	Derivative equal to zero	Wrong answer	No answer
Number of students	21	12	2	5
Percentages	52,5%	30%	5%	12,5%

https://etd.uwc.ac.za/

APPENDIX D

High School differential calculus questionnaire summary

Table P.1: Some Factors that make students not to pass differential calculus well

What makes students not to pass differential calculus as they think they would	Teachers	Nothing I don't know	Lack of enough practice	Slow writers and poor memory	The terms or vocabulary for differential calculus word problem too difficult to understand	Text books not good
Frequency	7	14	19	5	16	6
Percentage of students who responded	10,4%	20,9%	28,3%	7,5%	23,9%	9,0%

UNIVERSITY of the WESTERN CAPE

Table P.2: Some factors affecting the learning of differential calculus

Factor affecting the learning of differential calculus	Politics said that hidden agendas, stay-aways affect their students	Teachers not explaining to slow learners	School Calculus being introduced late (3%) and lot of noise at school (6%)	Home Not able to study due to condition at home (lack of facilities)
Number of pupils	41	13	6	10
Percentages	61,2%	19,4%	9%	14,9%

NB. It should be noted in table P.2 that the number of pupils represents those who responded in the items.

https://etd.uwc.ac.za/

Table P.3

Do party politics affect differential calculus study	Yes	No
Number of pupils	46	21
Percentage	68,7%	31,3%

Table P.4: Problem with understanding differential calculus

Problems with understanding differential calculus	No problem	Problems	<u>Some Reasons</u> Lack of practice	<u>Some Reasons</u> Poor back-ground	Student not serious	No reply
Number of pupils	33	34	39	16	8	4
Percentages	49,3%	50,7%	58,2%	23,9%	11,9%	6%

Table P. 5: Time Factor

Do you always finish within stipulated time	No	Yes	Prepared	Not fast / too slow	Writing a lot of steps (unneces-sary)
Number of pupils	43	24	4	38	25
Percentage	64,2%	35,8%	6%	56,7%	37,3%

Table P.6: Language Impact

Impact of the language used	Negative therefore difficult	Not difficult	REASON: English not mother language	REASON: English being better than Afrikaans
Number of Pupils	43	24	31	18
Percentages	64,2%	35,8%	46,3%	26,9%

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

Table P.7: Differential calculus rules

Can standard rules of differential calculus be applied	Yes	No
Number of pupils	43	24
Percentages	64,2%	35,8%

Table P.8: Factors for pass differential calculus

What should pupils do in order to like and pass differential calculus well	Being serious, working hard and practising a lot	Teachers must motivate pupils	Should know the basics of differential calculus	Listen carefully ask teachers should make it enjoyable	No attempt
Number of pupils	48	5	4	7	3
Percentages	71,6%	7,5%	6%	10,4%	4,5%

Table P.9: Hate of differential calculus

What do you think make students hate differential calculus	Basics not explained well	Poor methods of teaching employed by the teacher	Being told how differential calculus is	Too theoretical	Not regularly attending too many people in the classroom	Lazy to Practice
Number of pupils	20	8	11	1	5	22
Percentages	29,9%	11,9%	16,4%	1,5%	7,5%	32,8%

https://etd.uwc.ac.za/

Table P. 10: Point of interest

Making differential calculus interesting and easy	Teacher should give more work to pupils, teaching and showing the important steps important for solving the problem	Teacher should not be hostile and insult students	Use everyday life examples and use teaching aids	Discord method	Should introduce differential calculus at the beginning of the year	Starting with easy problems then moving on towards the demanding giving examples
Number of Pupils	28	15	6	1	6	11
Percentages	41,8%	22,4%	9%	1,5%	9%	16,4%

Table P. 11: Syllabus

Do you think anything should be done to the syllabus?	Yes	No
Number of Pupils	27	40
Percentage	40,3%	59,7%

Table P. 12: Content Knowledge

Relationship between the gradient and finding derivatives	Exists	Does not exist	No response
Frequency	41	11	15
Percentages	61,2%	16,4%	22,4

UNIVERSITY of the WESTERN CAPE
https://etd.uwc.ac.za/

Table P.13: Language understanding

Does English language affect understanding of differential calculus?	Yes	No	No answer
Frequency	26	37	4
Percentages	38,8%	55,2	6%

Table P.14: Prior-topics

Which topics do you think must be dealt with before dealing with differential calculus	Don't Know	Limits Graphs and Algebra	Geometry trigonometry	Mathematics done in previous years	Physics (Mechanics)
Frequency	10	20	15	19	3
Percentages	14,9%	29,9%	22,4%	28,3%	4,5%

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

APPENDIX E

Table T. 1: Reasons for students failure

Why students fail differential calculus	Problems with drawing cubic curves	Negative attitude	Problems with application therefore with word problems	Not born for maths	Laziness and reluctance to learn	Not starting it early enough for students to have time for grouping it	No peer spirit or working
Percentage response	100%	80%	75%	20%	80%	50%	80%

Table T. 2: Language impact

English language impact on mathematics learning	Effect (negative)	Positive effect	REASONS: English not pupils first language	Culture of student unknown
Percentage response	80%	20%	80%	50%

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/

APPENDIX F

Analysis of differential calculus test

Table F. 1

Question number	1	2	3	4	5	6	7	8	9	10	11
Frequency of correct answers	12	14	Not well structured	10	8	7	8	12	11	28	17
% Pass question	35,3%	41,2%		29,4%	22,9%	20,6%	20,6%	35,3%	32,3%	82,4%	50%

Table F.2

Marks out of 11	2	3	4	5	6	7	8	9
Number of pupils who scored it	2	8	6	6	8	3	-	1
Marks in percentage	18,2%	27,3%	36,4%	45,5%	54,5%	63,3%	72,7%	81,8

UNIVERSITY of the
WESTERN CAPE

https://etd.uwc.ac.za/