AN EXAMINATION OF A DIDACTICAL PROCEDURE TO ENGAGE FIRST YEAR UNIVERSITY STUDENTS IN MEANINGFUL MATHEMATICAL ACTIVITY

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ABSTRACT

The challenge to empower learners whose mathematical powers have been underdeveloped has lead the author to search for and implement teaching innovations designed to enhance students learning of mathematics. Undergraduate students receive much of their exposure to mathematics during lectures which are characterised by frontal teaching, a demonstration only by the lecturer. Students are passive recipients of the lecturer's knowledge and end up coming to lectures to gather information to learn at a later stage. The low level activity inherent in traditional lectures results in most students not developing the skills to grapple meaningfully with mathematical concepts and ideas. This has prompted the author to investigate changing the format of lectures in the quest to provide an environment in which students can engage more meaningfully with mathematical concepts.

Adopting the position of education theorists who advocate that mathematics as a meaningful activity is engendered by "doing" mathematics and that students must be allowed to enter the culture of the mathematical enterprise, the author has designed a teaching procedure called the "workshop-lecture". This mini-thesis reports on an examination of the design and implementation of the "workshop-lecture" which affords first year university students the opportunity to be involved in meaningful mathematical activity.

This examination provides evidence that the format of the "workshop-lecture" is conducive to more meaningful interaction amongst students and between lecturer

and students than would be the case in a traditional lecture, even with constraints such as venues with fixed, tiered seating and a relatively large class size of 56 students. It also highlights issues such as lecturer intervention, and the learning materials and aids that facilitate student interest and involvement in meaningful mathematical activity. A way of expanding the notion of the "workshop-lecture" to create opportunities for students to recognize more their own responsibility for their learning is proposed, as well as a strong recommendation for changing the curriculum to allow for meaningful involvement by students.

DECLARATION

I declare that AN EXAMINATION OF A DIDACTICAL PROCEDURE TO ENGAGE FIRST YEAR UNIVERSITY STUDENTS IN MEANINGFUL MATHEMATICAL ACTIVITY is my own work, that it has not been submitted before for any degree or examination in any other university, and that all the sources I have used or quoted have been indicated and acknowledged by complete references.

LARRY DICKSON KANNEMEYER DATE: MAY 1996.

SIGNED: ....

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CHAPTER 1

MOTIVATION

> Innovation can effect no more than to adapt education to a changing society, or at best it can try to anticipate the change.
>
> (Freudenthal, 1991: 156)

1.1 INITIAL DOUBTS

In 1993 I faced the challenge of developing and implementing an alternative first year mainstream mathematics course at the University of the Western Cape (UWC). This project formed part of an endeavour by the Mathematics Department of the university to tackle the problem of the continued poor performance of students in the first year courses. A report by the Department of Mathematics, "INTERVENTION AT FIRST YEAR LEVEL MATHEMATICS", discusses strategies employed by the Mathematics Department to improve the situation. In a section headed THE WAY AHEAD, the report (Department of Mathematics, 1991: 7) concluded that

> patch work will not work and the students need a longer time to mature in the concepts that they have missed in their high school mathematics make-up or have met in a very cursory manner.

One of the proposed strategies to deal more comprehensively with the problem was the development of a “Foundations Course” which would aim to “give students a solid grounding in basic mathematics; develop study skills and discipline; change students’ perception of mathematics” (Department of Mathematics, 1991: 7). Although dubbed as a “Foundations Course”, this course had to be an extended version of the regular first year mainstream mathematics course and was given an official university title of Mathematics 114/124. As is the case for the regular first year course, the "Foundations Course" consists of two modules, Mathematics 114 (M114) the first semester module, and Mathematics 124 (M124) the second semester module. Students identified to attend the course were those deemed by the Mathematics Department to be “at risk”, that is, those students who were judged to be underprepared for the regular first year course. Many of these students achieved a matric (school leaving) mathematics symbol of either at least a D (50% - 59%) on the Standard grade or an E (40 - 49%) on the Higher Grade. Students attending the course register for one fewer subject (three instead of four) than those students registered for the regular first year course, making it possible for them to have four extra lecture sessions for mathematics. Whereas the regular first year course has six lecture slots of 40 minutes each per week, the Foundations Course has ten lecture periods of 40 minutes each per week.

Initial ideas for the course were mainly linked to issues of content, time management and supplementary materials such as videos which could evoke student interest in mathematics. Backlog in content and weaknesses in algebraic manipulations were posited as major reasons in explaining the failure of students in coping with first year mathematics. In addition to the normal first year syllabus, the course had to have a big component related to revision of aspects of school algebra and an introduction to set theory. The extra lectures which these students had at their disposal were mostly devoted to these additional topics and no real attention was given to strategies to enhance the learning of mathematics by students. Pedagogical issues related to approaches to learning was, for me at that time, at best a secondary consideration and rarely entered the equation when I planned for the course. One of the problems was that there was just not enough time in the students' programme. It was packed with so much content that the overriding concern was to cover the syllabus. Twenty percent of the 64 students starting the course dropped out. Questions on tests and examinations never ventured beyond routine manipulations and even then at least 25 percent of those writing the final examination had to be given a second chance to obtain a passing grade. Since almost 50 percent of the students just about obtained a passing grade, I was not sure that the goal of giving students a solid grounding in basic first year mathematics was achieved. Students' perceptions of mathematics as compartmentalised and a series of isolated skills, with mechanical manipulation of symbols to obtain the right answer, were also not fundamentally altered.

By the end of 1993, I began questioning the major pedagogy underpinning the course. The transmission mode, with the lecturer occupying centre stage, was the modus operandi of lectures. These lectures were supported by small group tutorial sessions which presented students with the opportunity of at least gaining some space in coming to terms with some of the course work. That students were not gaining much insights in lectures became evident in these smaller more informal sessions.

1.2 SEEDS OF CHANGE, A FUNDAMENTAL SHIFT NECESSARY

The first seeds for making a fundamental shift in conceptualising the Foundations Course were laid in 1993 when a visiting lecturer, who observed some of the lectures, commented "Little attempt seem to be made to offer students heuristics, or ways of approaching the learning of mathematics, and alternative insights in the content material" (Powell, A.B. [1], 1993: 1). In his brief visit he introduced me to the possibilities of investigative activities around the use of a graphics calculator. He had developed "in-class laboratory sessions" which intended to provide students with "rich mathematical experiences" as well as to displace the lecturer "from the authorative center and distribute authority among the students" (Powell, A.B. [2], 1993: 3). We attempted two of the laboratory sessions Powell had developed for his students attending a precalculus course at the Rutgers University, Newark in the USA. Each of these sessions used up almost an entire week of lectures, causing further cramming of the rest of the syllabus. Although these laboratory sessions were not tailored to the conditions and courses within the South African context, the experience did provide me with glimpses of possibilities of shorter in-class laboratory sessions which would engage students more actively and meaningfully with fundamental mathematical concepts.

In 1994 my attempts at restructuring the Foundations Course turned around developing in-class experiences to involve students more actively during lectures. The "Powell experience" was pointing away from the traditional lecture, with the transmission mode of delivery as the chief conduit for exposing students to mathematical ideas, towards a teaching procedure which will promote more participatory and collaborative learning activity. I had taken on board the necessity to search for teaching strategies that would enhance my students learning of mathematics. To gain greater insight into the learning and teaching of mathematics I enrolled for a Masters degree in Mathematics Education. This experience afforded me the opportunity to be involved in a community of scholars grappling collaboratively with issues about the learning and teaching of mathematics. During these sessions I was exposed to the theories and practices of educators questioning authoritative teaching practices. It was also during these sessions that I gained exposure to the Realistic Mathematics Education school of thought with students' engagement in the mathematizing process as one of its major underpinnings where mathematizing is described as "an organising and structuring activity according to which acquired knowledge and skills are used to discover unknown regularities, relations and structures" (De Lange, 1987: 43). Besides stressing that learning mathematics is a "constructive activity" and that "learning of a mathematical concept or skill is a process" (Treffers, 1991: 24), proponents of the Realisitic Mathematics Education school of thought also maintain that learning is a social activity which is stimulated by "interaction" (Treffers, 1991: 25).

During the first semester of 1994, I received a draft copy of the text produced by the Harvard Calculus Reform Consortium which placed more emphasis on conceptual development of calculus. This text was produced in the context of producing a "lean and lively" calculus course. Impetus for changing the traditional calculus courses was similar to that which we faced in South Africa. These include the concern over access and dissatisfaction with students' performance (Huhges-Hallet,1994: 1546). The reforms which this group instituted, advocate presenting mathematical concepts numerically and graphically, as well as the traditional algebraic offerings. From this work I gleaned ideas which provided the scope to develop student discussion activities around mathematical concepts in lectures. Some of the activities included using technology which has given further impetus to restructuring the curriculum and the formulation of investigative activities for students to perform so as to understand some of the processes underlying mathematical ideas.

I view 1994 as "the year of eclectic implementation" of a number of different innovations. I made attempts throughout the year to implement strategies to enhance the learning of mathematics by my students. What crystallised for me during all these trials and experiments, was the necessity to involve students more actively in the learning process as it unfolds in the lecture room. At most universities, students are exposed to new mathematical ideas in the lecture room. It is thus necessary to take a look at the way students experience the learning of mathematics during traditional lectures.

1.3 TRADITIONAL LECTURES AND QUALITY OF STUDENT LEARNING

One of my past students, asked to comment on all lectures he attended, summed up his view by saying, "... it's where someone stands in front and explains to the best of his knowledge and hoping that the students will understand." This viewpoint resonates with my own experience as a student, a teacher and a lecturer. Having taught at high school and first year university level, I have gathered the sense that the traditional lecture consolidates the negative features of the students' school learning experience. Powell (J.P.,1981: 206) typifies traditional teaching as:

> a repeated demonstration of subject-matter knowledge and skills in such a manner that the student is left with little to do beyond the performance of routine intellectual and clerical tasks. The student becomes a victim of the teacher's role, a receptacle for highly-processed and over-simplified information which is quickly forgotten once it has served its purpose as examination fodder.

Traditional teaching is characterised by frontal delivery and is an articulation of the teachers' knowledge. It is how well students understand the teacher's way of knowing and the desire on the teacher's part to cover the syllabus that dominates classroom instruction. Passive reception by students is encouraged in this way at the expense of students critical engagement with the subject matter. Most first year university students suffer from years of conditioning about the role of the teacher and classroom activity. They often expect to be taught by the "template method", that is, they expect to be provided with detailed algorithms and worked-out solutions to sample problems, which they can, in turn, mimic on tests and examinations. Unlike at school though, the volume of knowledge and the pace at which it is delivered at university, does not afford students enough scope to keep pace with the work being presented during lectures. It is no wonder that students end up coming to lectures not to understand or learn new material but to gather information. Jenkins (1992: 66) echoes this sentiment when he states:

> The one-way flow of the conventional lecture places far too many potential demands on students - to listen to information, to note down what they consider to be important, to relate it to previous knowledge. It is not surprising that most can only attain low level recording of information.

From summaries and discussion of research evidence on the lecture method, Jenkins (1992: 64) cites two major problems with traditional lectures:

> One is that generally attention and student learning deteriorate markedly over the first 20 minutes or so. The second is that higher level educational goals which involve understanding, the application and evaluation of ideas, and so on, and which go beyond recall and description, cannot be easily achieved when students are largely passive, as they are during a conventional lecture.

These higher level educational goals require learners to be active participants in the meaning-making process. The lecturer-centred nature of traditional lectures does not encourage activities such as reflective discussion amongst students. Consequently, learning during traditional lectures is superficial.

Some researchers make the distinction between "surface approaches" and "deep approaches" to learning. A surface approach is one in which a student "reduces what is to be learnt to the status of unconnected facts to be memorised. The learning task is to reproduce the subject matter at a later date (eg in an exam)" (Gibbs, 1992: 2). With a deep approach, the student "attempts to make sense of what is learnt, which consists of ideas and concepts. This involves thinking, seeking integration between components and between tasks, and 'playing with ideas'" (Gibbs, 1992: 2). Traditional lectures are more conducive to students adopting "surface approaches" to the study of mathematics, consolidating rote learning as the most effective way of learning.

The quality of student learning is inextricably bound to the quality of educational provision. Included in the spectrum of issues determining such provision are the teaching methods employed and supporting sessions such as tutorials. It appears that the traditional lecture does not afford students the opportunity to be engaged in meaningful mathematical activity and in so doing develop "deep approaches" to their learning of mathematics.

1.4 SUMMARY

The quest to provide an environment in which students can engage more meaningfully with mathematical concepts has prompted me to investigate changing the format of traditional lectures. I have adopted the position of educational theorists, such as Alan Schoenfeld and Jean Lave, who advocate that meaningful mathematical activity is engendered by "doing" mathematics and involvement by students in what Powell (A.B.,1993 [1]: 3) describes as "the reality of mathematical activity" and "the possibilities that they can use their mental powers to develop the particular disciplined habits of the mind that mathematicians use". Towards this goal I have developed a procedural framework for my lectures which I call "workshop-lectures".

The aim of this mini-thesis is to reflect on the design and implementation of the "workshop-lecture" for engaging students at first year university level in meaningful mathematical activity. In Chapter Two I will unpack the major features of meaningful mathematical activity and how these impact on the pedagogy inherent in traditional lectures. The "workshop-lecture" is defined and explicated in Chapter Three which presents the research procedure and method. Chapters Four and Five provide an analysis and discussion of the data collected. A reflective discussion on research methodology and of the theoretical implications of the findings, as well as possible practical adaptations are presented in Chapter Six which closes with a brief summary of the main conclusions and recommendations that ensue from the discussions in the previous chapters.

CHAPTER 2

THEORETICAL FRAMEWORK

> In the community of mathematics educators, the view of mathematics as a system of definitions, rules, principles, and procedures that must be taught as such is being exchanged for the concept of mathematics as a process in which the student must engage. (Gravemeijer, 1994: 443)

2.1 LECTURES AND TRADITIONAL TEACHING

Part of the experience which students bring with them to university is the perception that mathematics is situated within the realm of certain knowledge. These formalist notions of mathematical knowledge establishes for mathematics a neutrality and an existence "out there" not tainted by cultural and social considerations. Mathematics is presented by teachers as "a system of definitions, rules, principles, and procedures" (Gravemeijer, 1994: 443) giving the impression that mathematics has a strict hierarchical structure. An authoritarian relationship is induced, placing the "intellectual authority" (Lampert, 1990: 32) in the teacher and the textbook. Unquestionable acceptance of this authority by students is normative, perpetuating the alienation many students experience towards mathematics. Concepts are handed down by the teacher in a manner divorced from the processes that give them life. Knowing mathematics is signalled by quickly getting the right answer which is a function of remembering and applying the correct rule. Knowledge of mathematics is thus predicated by the remembrance of rules, so that doing mathematics means following the rules as explicated by the teacher. Students react against their estrangement towards mathematics by their total dependence on the authority figures and a distance from their own intuitions.

This experience by students of mathematics as disconnected pieces of information to be learnt and regurgitated in tests and examinations is consolidated and perpetuated at university. My observations has been that our first year university students are bombarded with a great deal of content relative to what they are accustomed to at school level. Lectures are accompanied by tutorial sessions which provide them with the opportunity to discuss problems related to lectures, but there is a disjuncture between the lecture presentation and the way in which students take up the problems. They attempt to ape the examples used in lectures or those set out in textbooks without much attention to the processes that produce the results. There does not seem to be any time given for students to explore ideas deeply, thus consolidating their understanding and building "confidence in their abilities to generate and codify mathematical knowledge as well as to do mathematics that has already been codified" (Powell, A. B., 1993 [1]: 1) Students are not exposed to the "reality of mathematical activity" which supersedes the superficial task of remembering definitions and statements of theorems.

Inherent in the traditional lecture is the absence of meaningful mathematical activity on the part of students. The completeness of presentation by the lecturer belies the hours the lecturer spent in preparing for the talk, especially if it is the first time round that she is presenting the course being offered. Included in her preparation are many "habits of the minds", which "characterize how mathematicians think about phenomena" (Powell, A. B., 1993 [1]: 3). It is via these habits that mathematicians make sense of mathematical ideas and develop a means of communicating with them. That students do not obtain this facility can be attributed to the practice of mathematics being taught "as received knowledge rather than as something that (a) should fit together meaningfully and (b) should be shared" (Schoenfeld, 1990: 6). In the next section I will explore what these "habits of the minds of mathematicians" are and in the process explicate what can be constructed as meaningful mathematical activity.

2.2 MEANINGFUL MATHEMATICAL ACTIVITY

In an exercise with the mathematics department at UWC, Powell (A.B., 1993 [1]: 4) engaged mathematicians about their habits of the mind as they went about their business as researchers. Out of the ensuing discussion, 15 activities were identified. Some of these activities could be listed under three habits of the mind of a mathematician which Powell had illustrated, namely: "thinking deductively, justifying conjectures, and posing 'what if' questions". A fourth activity that emerged strongly through this exercise was the social nature of the mathematicians enterprise. "Consulting others", either directly in the office, the tea room, and seminars, or indirectly via conference papers, journal articles, and texts, was a unanimously agreed upon habit which afforded mathematicians with insights that assisted their own research.

The exercise referred to above begins to shed light on what some mathematicians consider to be meaningful mathematical activity. Schoenfeld (1990: 9) captures this for me when, in summary, he asserts that:

> Mathematics is an inherently social activity, in which a community of trained practitioners (mathematical scientists) engages in the science of patterns - systematic attempts, based on observation, study, and experimentation, to determine the nature or principles of regularities in systems defined axiomatically or theoretically ('pure math') or models of systems abstracted from real world objects ('applied math').

Stressing mathematical activity as a social enterprise challenges the stamp of certainty given to mathematical knowledge by traditionalists. It would appear that mathematical knowledge, far from being accepted as absolute truths, reflect "our best current understanding of the quasi-empirical data out of which our understanding of mathematics is built" (Schoenfeld, 1990: 9). Mathematicians have to convince the mathematical community at large in order to assert the validity of their arguments. Truth in mathematics is thus socially defined and arrived at through collaboration with members of the mathematical community.

Meaningful mathematical activity also involves learning to think mathematically. This habit of the mind, according to Schoenfeld includes "mathematical sense-making" which he posits as "developing competence with the tools of the trade, and using those tools in the service of the goal of understanding structure" (Schoenfeld, 1990: 9). Schoenfeld is here drawing an analogy with a craftsperson actually using shop tools to construct an item and not just knowing how to use these tools. It is the former practice which actually makes him/her a craftsperson. The tools of mathematics are considered by Schoenfeld to be "abstraction, symbol representation, and symbolic manipulation" (Schoenfeld, 1990: 9). In order to be involved in mathematical sense-making, mathematicians need not only to know about the tools of mathematics, but they need to actively use these tools. It is in this actual "doing" of mathematics that mathematicians make sense of the particular dynamics that underlie the relationships between mathematical objects and are enabled to make contributions to the bank of mathematical knowledge.

Mathematical practice, that is the way mathematicians engage with mathematics, is thus a specific cultural practice within which the practitioners are mathematicians. To become a mathematician would mean to enter this practice and take on board the specific cultural traits that governs the practice.

2.3 MATHEMATICAL PRACTICE AND "COGNITIVE" APPRENTICESHIP

It has been argued above that mathematicians make sense of mathematical ideas by "doing" mathematics which , necessarily, entails their engagement with the "tools of their trade". What is suggested, then, for aspirant mathematicians to become members of the mathematical community, is that they need to learn how to do mathematics as practitioners would. This is best achieved in the cauldron of mathematical practice in which "on site learning" can take place as in the case of craft apprenticeship. The term "cognitive apprenticeship" (Brown, Collins, Duguid, 1989: 32) has been proposed for the skills usually associated with traditional teaching. Elements of such "cognitive apprenticeship" begin to make their appearance at the graduate level when students have to behave as practitioners to develop their conceptual understanding. These aspiring professionals engage in the "habits of the minds of mathematicians" which they learn through their apprenticeship with senior professionals and their exposure to the wider mathematical community via work presented at conferences and workshops, and in journals and other recognised texts. What is central to all forms of apprenticeship is that learning is achieved through participation in a community of practice, and that this participation takes place in a cultural setting which is determined by the collective wisdom and collaboration characteristic of the specific community.

In her study of situated activity, Lave (1989) stresses that learning does not take place "sui generis", but rather that learning is a function of the learners "changing participation in a culturally designed setting." Lave describes learning as "legitimate peripheral participation" in which the learner as a "newcomer" has "legitimate access to ongoing community practice" and develops "a changing understanding of practice over time from improved opportunities to participate peripherally in ongoing activities of the community" (Lave: 1989). It is the newcomers participation in the activities of the community which is an important condition for learning. Newcomers become full participants or "oldtimers" by their growing engagement with authentic activities, that is, recognized activities, of the community of practice.

Drawing on research conducted by Jordan (1989) on Yucatec Mayan midwives, Lave observes that it would be difficult to find evidence that suggests that direct teaching is the way in which midwives obtain know-how in order to practise the skill of midwifery. Inherent to such apprenticeships are resources by which newcomers develop the knowledgeable skill to become oldtimers not characteristic of teaching. Newcomers are aware at the outset of what is expected of them. They are people who have a broad overview of the practice's artifacts and goals and thus make a conscious decision to become newcomers and enter the community of practice. Newcomers are also accepted by the community as legitimate participants. This affords them the opportunity to develop their knowledge and skill in the process of taking on more responsibilities, doing more of what is expected of oldtimers.

To enable students of mathematics to learn mathematics as effectively as craft apprentices come to learn to use the tools of their trade means changing the participation structure of the traditional classroom. Intervention at the classroom level is necessary since it is predominantly the site of learning in which our students experience and develop their understanding of the nature of the mathematical enterprise. In particular, at UWC, first year students are for the first time in direct contact with "oldtimers", the research mathematicians who are their lecturers and tutors. The continuing experience of the lecturers as practitioners can more readily be made transparent to students in the lecture halls. A quasiapprentice relationship could be constructed in which students are accepted as "legitimate peripheral participants" involved, at their level, in mathematical sense-making.

However, unlike the newcomers cited in the midwifery example above and other similar apprenticeships, most first year students have not made the conscious decision to be newcomers. Their "bad" prior experience have consolidated an impaired and truncated view of mathematics and mathematical practice. It is thus not de facto, given the opportunity to be involved in meaningful mathematical activity, that our students can immediately accept their roles as newcomers who have a broad overview of the mathematical enterprise and who would want to develop the "habits of minds of mathematicians" as the most effective way of learning mathematics. Learning to participate in activities that lend themselves to more meaningful mathematical activity is one of the first hurdles that must be cleared. Educating students about what knowing mathematics entails, involves something which is different to that pertaining in traditional classrooms. As Lampert (1990: 58) remarks, it is "Like teaching someone to dance, it requires some telling, some showing, and some doing it with them along with regular rehearsals".

2.4 THE CLASSROOM AS A "LOCAL INTELLECTUAL COMMUNITY"

In craft apprenticeships, as is the case in most graduate classes at universities, there are sufficient numbers of "oldtimers" to induce "newcomers" to the culture and practice of the particular community of practice. Typically, young professional mathematicians work on a one-to-one basis with experienced mathematicians. In some cases the number of "oldtimers" even exceeds the number of "newcomers", with a graduate student enjoying the collective experience of more than one supervisor. The situation for "newcomers" is even more favourable at mathematics conferences. Graduates and young professionals find themselves surrounded by a whole host of "oldtimers" participating in the business of communicating their research. Undergraduate classes and the school classrooms present a different scenario. Besides being confronted by a large number of students who have not made the conscious decision to be "newcomers", it is not possible to teach students mathematics by having them interact with practitioners as graduate students would.

What is possible though, is affording students the opportunity to enter the culture of mathematical practice by allowing them to engage in authentic mathematical activity. Based on the notion of "intellectual community", Schoenfeld works at making his problem solving classes "microcosms of selected aspects of mathematical practice and culture" (Schoenfeld, 1990: 17). The emphasis is strongly on student participation in the classroom in ways which reflect some of "the values of the mathematical community at large" (Schoenfeld, 1990: 17).

The classroom and its participants, namely the students and the teacher, form an "artificial" community, or a "local intellectual community" in which the notion of localization is conceptualised as: "a contribution is significant if it helps the particular intellectual community advance its understanding in important ways" (Schoenfeld, 1990: 18). In this setting students generate "mathematics" and generalizable mathematical strategies which contributes to the bank of mathematical knowledge of their "local" community.

Schoenfeld is suggesting a pedagogy that allows students to enter the culture of mathematical practice at their level, that is, that students engage in meaningful mathematical activities "at a level commensurate with their knowledge and abilities" (Schoenfeld, 1990: 20). Underlying this pedagogy is the social nature of the mathematical enterprise and the way in which mathematicians contribute to the mathematical community's knowledge bank. In the exercise on the "habits of the minds of mathematicians" to which I referred in section 2.2 above, consultation was cited as a major activity of practitioners. Students interaction in their "local intellectual community" allows for "local consultation" and discussion in which some students, gaining in confidence and understanding, increasingly take on the mantle of "oldtimers".

2.5 SUMMARY

In this chapter I have laid the theoretical basis for a didactical procedure which I am proposing as an alternative to traditional lectures. I have called this procedure a "workshop-lecture" as it has evolved out of experiences which attempted to enhance the learning of students during lecture sessions as traditional lectures did not allow for meaningful mathematical activity on the part of students. In order to arrive at an alternative procedure it was necessary to explicate what meaningful mathematical activity entails and how possibly to induct such activity in the lecture room.

Meaningful mathematical activity finds its expression in the view that posits that mathematical practice is a specific cultural practice within which the practitioners are mathematicians. Mathematicians make sense of mathematical ideas and contribute to the bank of mathematical knowledge by “doing” mathematics. Aspirant mathematicians are inducted into the culture of mathematical practice by apprenticeship into the community of mathematicians. Usually this achieved through a relationship between an "oldtimer" (an expected, experienced knower of the community of practice) and a few "newcomers" who have made the conscious decision to become part of the community of practice (that is, "oldtimers").

Students should be afforded the opportunity to enter the culture of mathematical practice. This can be achieved (Schoenfeld) by transforming traditional classes into “local intellectual communities” in which students are involved at their level in “doing” mathematics. In chapter three I provide a description of a teaching procedure which attempts to simulate an environment that affords students the opportunity to engage in authentic mathematical practice. The rest of chapter three and the ensuing chapters detail the investigation of this simulation which is conducted in an educational setting characterised by relatively large student numbers and by students who have not made the conscious decision to be "newcomers" in the community of mathematicians.

CHAPTER THREE

RESEARCH METHOD AND PROCEDURE

> Soul-searching isn't everybody's business, but change through accident can be a source of search, just as search can lead to change. (Freudenthal, 1991: 158)

3.1 INTRODUCTION

In an attempt to capture some of the aspects of meaningful learning activity discussed in the previous chapter, and bearing in mind that the students need to be weaned from being passive receivers of knowledge into more active learners, I have developed a teaching procedure which I have called the "workshop-lecture". Section 3.2 provides a description of a typical "workshop-lecture" as I have developed it working within the constraints of the students' academic timetable and the venues provided by the university.

The "workshop-lecture" has been conceived within the paradigm which holds that mathematics is a social activity and that students should engage in meaningful mathematical activity in order to enhance their learning of mathematics. Thus, practical examination of the effectiveness of the "workshop-lecture" should also bring under scrutiny these theoretical underpinnings. My experience of trying out innovations in teaching also suggested development and modification of the actual syllabus constructed around the subject matter, thus the necessity for integrating instructional issues with considerations of curriculum changes. For the purpose of my thesis I have turned to the research methodology of developmental research as explicated by Freudenthal (1991) and Gravemeijer (1994). An outline of this research methodology is presented in section 3.3.

Section, 3.4, sets out the general implementation plan for collecting data and includes the data gathering techniques I used.

3.2 THE "WORKSHOP-LECTURE"

A "workshop-lecture" is conceived as a *hybrid of lecturing and workshopping activity.* It is conducted during the normal period allocations for the subject. Sessions are thus, in UWC's context, either single periods of 40 minutes or double periods of 90 minutes with a five to ten minute break within the session. Essentially five overlapping types of activities occur in a "workshop-lecture". These are

(1) Whole group interaction: The lecturer engages the whole class interactively.

(2) Small group interaction: Students engage in small groups with lecturer circulation and/or lecturer interaction.

(3) Lecturer circulation: The lecturer moves amongst students while they are

interacting in small groups, possibly taking part in their discussions.

(4) Lecturer intervention: While circulating the lecturer interacts with the whole class based on observations made in a small group.

(5) Plenary: Either students from various groups present the results of their discussion with conclusions drawn for the class by the lecturer, or a discussion in lecture format is given on the concepts discussed in the small groups, or a combination of these.

The time given to these activities vary dependent on the topic under study during a session.

In order to facilitate the "doing of mathematics" during the "workshop-lecture" particular types of learning materials are made available. An essential characteristic of these materials is that it should engage students in discussion, debate and reflection of the relevant mathematical ideas. For the purposes of this study, integral parts of the learning materials were worksheets and graphic calculators.

- The worksheets:

  A worksheet determines the framework of the lecture. The questions posed or the investigations suggested on the worksheets drive the discussions of the lecture. The completed worksheets of each lecture together constitute the core notes for any particular section of the work.

- Technology:

  Beside having an impact on what we teach, the advance in technology has increased the store of teaching and learning tools that can be employed for more active engagement with mathematical concepts. With their ability to easily perform complex computations, sketch graphs and manipulate symbolic information, technology such as calculators and computers allow for the connection to be made among the numerical, graphical and symbolic points of view. In some of the "workshop-lecture" sessions students use a graphics calculator, (the Texas Instrument 85: TI-85), in their investigation of mathematical concepts as proposed by the worksheets. The graphics calculator (GC) has been employed since it is easily held and used in one's hand, and can be transported to any venue.

Table 3.1 describes a typical "workshop-lecture" session over a double period dealing with the topic "Using the derivative". The worksheet for the session appears in Appendix 1. The "workshop-lecture" described here took place in a lecture hall, which has fixed, tiered seating. Students form groups of two or three where they are seated so that collaboration and discussion amongst them can take place. Although the graphic calculator is not mentioned on the worksheet for this particular session, a set of graphic calculators was available in the venue. Students would make use of the graphic calculator if they deem it necessary for their investigation.

Table 3.1: Typical "workshop-lecture" session of 90 minutes.

Stage	Activity	Time
1	Whole group interaction	8 minutes
2	Small group interaction and lecturer circulation	7 minutes
3	Plenary 1	5 minutes
4	Small group interaction and lecturer intervention	10 minutes
5	Small group interaction and lecturer circulation	10 minutes
	Break	5 minutes
6	Whole group interaction	5 minutes
7	Small group interaction and lecturer intervention	10 minutes
8	Plenary 2	10 minutes
9	Whole group interaction	10 minutes
10	Small group interaction and lecturer circulation	8 minutes
11	Plenary 3	2 minutes

3.3 DEVELOPMENTAL RESEARCH

Essential to developmental research is that curriculum development is not divorced from the goal of changing educational practice, which is improvement of practice. So, from his position that the purpose of educational research is "change", Freudenthal (1991) stresses that in educational practice, research and curriculum development should not be separated. Freudenthal's conception of educational development thus embeds curriculum development in a more holistic framework "which embraces all the developmental activities and interventions between the initial idea and an actual change in educational practice" (Gravemeijer, 1994: 444).

Central to Freudenthal's concept of developmental research is that, in educational practice, research and development should be interwoven by their cyclical alternation. Gravemeijer (1994: 449) describes a cycle as follows:

> What is invented behind the desk is immediately put into practice; what happens in the classroom is consequently analyzed, and the result of this analysis is used to continue the developmental work.

The cyclical alternation of research and development in this way provides a greater synthesis between what Freudenthal (1991: 160) refers to as "development ensuing from research" and "research as fall-out of development". Also, this cyclic process is "more efficient when the cycle is shorter" (Gravemeijer, 1994: 449). To ensure that research and development are not separated, educational development must take place within the educational environment. Consequently, developmental research deals with "Research *in*, rather than *on* Mathematics Education, not in order to exclude the latter, but to emphasise the former" (Freudenthal, 1991: 158).

Each cycle in developmental research is a consequence of and preceded by the developer engaging in a thought-experiment conceived by a question or wonderment arising from the need for change in the classroom. Instructional design, "What is developed behind the desk", is formulated via the developer's deliberation with respect to lesson content and delivery style. Evidence which supports or negates hypotheses or ideas arising as a consequence of the thought-experiment are then sought in a teaching experiment, that is, in practice. Feedback from the teaching experiment results in new thought experiments inducing the iterative character of development research.

An overarching concern of developmental research is the development of a global theory, which on the one hand provides a general framework for local instruction theories, and which on the other hand develops during the process of research and development. This theory "guides developmental work" and "functions as a basis for a learning process by the developer" (Gravemeijer, 1994: 449). During the cyclic alternation of thought experiment and teaching experiment the local theories are explicated. The basic tenets of the global theory are elaborated and refined in the analyses of these local theories. This "reconstruction" of the global theory in practice ensures "theoretical progress" (Gravemeijer, 1994) by allowing adaptation of the general framework to the changing conditions in which it develops.

Developmental research is situated within the realm of qualitative research. Importance is placed on the interpretation of the events as they unfold in the teaching experiment so as to deepen our understanding of the nature of the activities involved in the process. An important step in the analyses and interpretation of the mainly qualitative data is, according to Gravemeijer (quoting from Smaling), "the construction of categories of data and the construction of concepts" (1994: 454).

Another essential feature of developmental research is the issue of dissemination - what is transmitted so that an innovation can be implemented and further developed. Since the credibility of the educational project is at stake, dissemination cannot be divorced from the developmental process. Towards this end, knowledge of the processes that give life to an innovation is essential. One must be clear about how the product of research activity, manifested as a teaching innovation, came about as such a product can be interpreted in different ways. This knowledge is crucial if an innovation in education development is to be sensibly used. Gravemeijer (1994: 452) elucidates this further when he states that:

> An educational experiment cannot be repeated in the same manner, under the same conditions. Therefore new knowledge will have to be legitimized by the process by which this new knowledge was gained.

The researcher thus needs to report on as much of the developmental process as possible which demands that the researcher be in a constant state of reflection. As thought experiments are the cradle of developmental research, reporting on them is also fundamental to explicating the processes that produces the final innovation.

Developmental research is thus an integrated approach to curriculum development with change or improvement of practice as its major goal. By adopting the developmental research approach, I intend to distil and examine the ways in which the "workshop-lecture" affords students the opportunity to engage in authentic mathematical practice as a means of enhancing their learning of mathematics.

3.4 IMPLEMENTATION AND DATA COLLECTION

3.4.1 GENERAL PLAN

The participants in this study were the 56 students who registered in 1995 to do the "Foundations Course" referred to in chapter one and myself as the lecturer. There were 16 female students (28,57%) and 40 male students (71,43%). Eleven of these students had exposure to first year mathematics at UWC. The duration of this exposure ranged from 2 weeks to a whole academic year. Table 3.2 provides a profile of the students in terms of their matric symbol for mathematics.

Table 3.2: Students profile (matric symbol for mathematics)

Symbol	Number of students
Standard Grade D	28
Standard Grade C	14
Standard Grade B	7
Standard Grade A	1
Higher Grade E	6

For the purposes of collecting data I decided to implement two cycles in the manner of developmental research. Given that the academic semester at UWC is only twelve and half weeks, and that my 1994 experience indicated that I needed to establish the tenor of the course right at the outset, I decided to expose my students very early on in the semester to the way I perceived the "workshop-lecture" to operate. The first cycle was thus planned for the first three weeks of the semester. The second cycle was planned for implementation 2 weeks after, also over a three week period. This was to allow time to reflect on the first cycle, engage in a thought experimentation, and plan the instructional setting for the second cycle. The short duration of the semester, together with the students' intense academic programme, did not allow time to focus on a single mathematical topic over the two cycles. However, I did not consider this as a problem as the purpose of this thesis is to test the central hypothesis of the effectiveness of "workshop-lecture" as a teaching strategy which afforded students the opportunity to engage in meaningful learning. The second cycle would thus present the opportunity to test the accumulative effect of the two cycles on the way students worked and performed. In particular, cycle two is intended to provide sharper focus of some of the pertinent issues raised in cycle one.

3.4.2 DATA COLLECTION TECHNIQUES

I made use of the following data collection techniques to obtain the data for analyses and interpretation.

- Observational notes

  These include my own observations of each session during the course of the experiment, as well as those of two colleagues who agreed to observe some of the sessions. My own observational notes were in the form of a diary, so that they included "observations, feelings, reactions, interpretations, reflections, hunches, hypothesis, and explanations" (Elliott, 1981: 10). I entered these diary notes immediately after each session.

- Questionnaires

  A short questionnaire, appendix 2, to obtain the views of the students on mathematics and the learning of mathematics based of their previous experience of mathematics classes.

- Document analyses

  Students' written assignments and tests, and the "worksheets" used during the "workshop-lecture" sessions, were also collected for analyses.

- Interviews

  Semi-structured interviews with at least 30 percent of the class. These were structured around pre-set questions, but freedom was allowed for the students to digress and raise their own topics as the interview progresses.

- Photographs

  Two sessions were photographed by an observer to visually capture the operation of these sessions as a source of triangulation.

- Triangulation

  This is more of a method than a technique which is used to compare and contrast evidence obtained from different sources so as to establish a greater degree of reliability in the analysis of the data. It also allows for "competitive argumentation" around points of disagreement between two observers of the same activity.

3.5 SUMMARY

In this chapter a description of the "workshop-lecture" as a teaching procedure to engage students in meaningful mathematical activity is given. An example of a typical "workshop-lecture" session and the types of activities that occur, is provided. The research methodology of developmental research is adopted for the purposes of investigating the implementation of the "workshop-lecture" in the setting of a first year mathematics class at UWC. In the manner of developmental research, the investigation is planned over two cycles, employing data collecting techniques appropriate for gathering mainly qualitative data. Cycle one is an initial reconnaissance stage providing the first sweep of issues related to implementing the "workshop-lecture". The data to be collected in cycle two will provide further evidence to tease out these issues. Chapters four and five report on and sets out the analysis of the data collected for cycles one and two, respectively.

CHAPTER FOUR

CYCLE ONE

> Secondly, students cannot know without training *how* to do something. It is not giving a person autonomy to throw him into water without teaching him how to swim. Such an approach grants him neither success nor satisfaction. Skills are most rapidly learned where most conscientiously taught, and where structures are set up to allow for their practice. (Potts, 1981: 111)

4.1 DESCRIPTION OF CYCLE ONE

Cycle one was implemented over the first three weeks of the first semester. The students were given time in the first session to complete a questionnaire. A copy of the questionnaire appears in appendix 2. The intention of the questionnaire was to gather information to gain some insight of the expectations students might have of lectures in mathematics.

To drive the sessions of the cycle, I produced a series of "worksheets" on the first chapter of the course which included the function concept, graphs and features of functions (increasing/decreasing, local/global minima and maxima, concavity and inflection points, and continuity). The intention was to give the students an opportunity to develop an intuitive feel for some of these concepts which would be given more rigour later on in the course.

I had planned for two colleagues to observe some of the sessions, as well to video record two of the sessions. A colleague, Jansie Niehaus (hereafter referred to as JN), who also served as research assistant to the mathematics department at UWC, and who observed and participated in my lectures in 1994, attended at least two sessions per week. This enabled me to immediately share my reflections with her and she provided me with the observational notes she had made over the period of the cycle. See appendix 3, Academic Development Journal (3-3-95), for her overall remarks, as well as her observations as a participant-observer in one of the sessions of cycle one. Unfortunately, the second colleague could not attend this early in the semester, as well as the video crew let me down at the last moment. After each session I made observational notes which included any discussion I had with students immediately after lectures or when students consulted me in my office. These notes I have cross-referenced with those of my colleague's, and they serve as the major source of analyses and discussion for the first cycle. Given the lack of data for more effective triangulation, I have included anecdotes from my observational notes to serve as "evidence" for one or two tentative assertions.

4.2 STUDENTS' INITIAL VIEW OF MATHEMATICS AND THE LEARNING OF MATHEMATICS

In this section an attempt is made to establish the effect students' previous experience of mathematics classes might have on the implementation of the "workshop-lecture". This would affect the pace at which the innovation would be implemented.

Tables 4.1 to 4.5 below are constructed from the students' responses to the questionnaire given in appendix 2.

Table 4.1: Students' responses to the question "what is mathematics?" Number of student responses = 47.

Category	%	Representative quotes from questionnaire
A. Mathematics is problem solving.	27,66	Mathematics is the study of finding methods to solve problems. Getting solutions for problems.
B. Mathematics is the manipulations of numbers.	21,28	Mathematics is the study or the science that deals with numbers. Mathematics is a means of measuring and of manipulating numbers to get a satisfactory answer.
C. Mathematics is a way of thinking.	21,28	It is an alternate way of broadening your mind. It is a subject which asks you to think logically.
D. Mathematics is a language.	12,76	Mathematics is a language that uses numbers, quantities and so on in order to communicate information.
E. Mathematics is interesting and a challenge.	8,51	It is a fun and exciting subject, a bit challenging, but if you play your part it will be of your second nature.
F. Mathematics is relevant for under- standing the world.	8,51	A powerful tool, a necessity if one wants to become a good physicist, chemist or whatever filed of science one wants to become involved with.

Table 4.2: Students' responses to "why people cannot do mathematics". Number of student responses = 47.

Category	%	Representative quotes from questionnaire
A. Undermining of ability.	31,92	Some people think mathematics is not for them because they are not as clever as others. They believe that they cannot do it. They are not confident enough.
B. Mathematics is difficult/complex /complicated.	27,66	Mostly they believe that mathematics is difficult. Most people see mathematics as a complex subject which requires some serious reasoning.
C. "Lazy" attitude.	19,15	People don't like to struggle. They want everything smoothly. They are lazy and do not want to think.
D: Discouraged by teachers.	8,51	The way mathematics is presented makes many people feel turned off by it.
E. Mathematics is not relevant.	8,51	They don't see the point of doing it.
F. Wrong approach.	4,25	Because they like memorising and you will never learn maths by memorising, you'll have to understand it.

Table 3: Students' response to " What does it mean to understand mathematics". Number of student responses = 47.

Category	%	Representative quotes from questionnaire
A. "Know" what you are doing.	27,66	To understand means not merely solving problems as such, but trying to understand the mechanics of mathematics. Being able to explain how to get to an answer even if the answer is wrong, i.e. understand each and every step.
B. To be able to solve problems.	21,28	You must know what a "problem" is about and you must be able to solve it. To look at a mathematical problem and be able to get a solution.
C. Understand the basic principles.	21,28	To understand maths is to know the basic principles on which all the concepts are built. To know the concepts and rules of maths.
D. Apply in everyday life.	17,02	I believe understanding maths means able to apply the maths concepts into our everyday living. How exactly I don't know. Understanding mathematics means able to relate the theoretical side of maths like solving equations, to daily use in the work-field.
E. Know formulas.	6,38	To know what formula to use in certain problems.
F. Your "approach".	6,38	You have to know how to confront it and not to let it get the better of you.

Table 4: Students' response to "How do you feel about mathematics".
Number of student responses = 47.

Category	%
A. Positive	87,23
B. Negative	12,77

Further analysis of Table 4:

Eleven of the students who had indicated a positive feeling towards mathematics gave negative reactions in explaining their experiences and notions of mathematics. These students would describe mathematics as difficult, causing them headaches, and did not enjoy mathematics periods at school.

Of the positive contingent, 19 expressed determination to "work hard at it" as underlying their present feeling towards mathematics.

Twenty students attributed their positive feeling in terms of their "enjoyment" of mathematics or relished the "challenge" of it.

Of the 6 negative feelings, 3 saw no relevancy in doing mathematics and 3 cited their own incapabilities for their feelings.

Table 4.5: Students' reasons for "enjoyable" classroom experience. Number of student responses = 39.

Category	%
A. "When able to solve a problem"	46,15
B. Role of the teacher	33,33
C. "Maths is a practical course"	12,82
D. "Working together"	7,69

4.3 INITIAL RECONNAISSANCE - CYCLE ONE

The analysis for this section is constructed from my own observational notes which I have crosschecked with those of a colleague, JN, provided in appendix 3.

In this initial reconnaissance I have discerned three interrelated issues which I have identified as:

- The effect of students' previous experience of "doing maths".
- What can be afforded as the result of the mechanism of the "workshop-lecture".
- The learning materials and aids used to get students involved in meaningful mathematical activity.

What stands out as a central point of departure is how the "workshop-lecture" as a teaching procedure allows for the integrated consideration of issues related to constructing an environment which affords students the opportunity to engage in meaningful mathematical activity. Although, not the "magic method", it provides a framework to introduce elements of authentic mathematical practice in the classroom. It is in this spirit that I take on board JN's remarks when she states in her journal that:

> LK is also practicing action research very actively! His thorough planning is upset every lecture, as he realises what is necessary or lacking every step of the way. ... So one can't say the "magic method" is workshops, or calculators, or writing, action research, or whatever - it is the integrated use of these things that we have to evaluate. (Appendix 3: 88)

4.3.1 THE EFFECT OF STUDENTS' PREVIOUS EXPERIENCE OF "DOING MATHS"

The in-class activity that students were accustomed to at school was the working out of a set number of problems at the end of a section or chapter from the text the teacher was employing for the course. Discussing mathematical concepts or ideas which are building blocks for the definition of concepts seem to be a foreign exercise. This appeared evident in the first two sessions, both double lectures, when most of the groups took some time to get started with the discussions related to the function concept that did not involve manipulation and arriving at a numerical or symbolic answer.

In the second lecture session a student in the class presented an interesting example of a functional relationship, and without prompting by me used symbols other than x and y: Amount of oxygen per km^2 as a function of the height of a mountain in km, which he wrote down as O=f(H). It was only after my acceptance of the legitimacy of the example that discussion ensued and many more examples were offered by the class. The discussion of this example in the class afforded me with the opportunity to break the ice with respect to students participation and to encourage them that their input is as valuable as mine. There was still a bit of hesitancy though, when I called for more examples. This once again could be ascribed to students not feeling comfortable in participating in a bigger discussion. They would have to be eased into this feeling. One is fighting against years of giving the teacher "exactly what she wants".

Three students consulted me the afternoon after the second lecture session about some of the questions they had attempted. Wanting to encourage student participation in the whole class scenario (plenary), I used these questions in the next session for the whole class to discuss. The first part of the session was dealt with in an interactive way between me and the students. I asked students to agree or disagree with answers given and to give their reasons. This session witnessed a much greater eagerness on the part of students to participate in the discussion. The second part of this lecture was devoted to students discussing their solutions of the rest of the questions in the problem section within their groups. I managed to get around to most of the groups and got the impression that the students were coping. Their discussions were lively with a collegial respect quite noticeable.

4.3.2 WHAT THE MECHANISM OF THE "WORKSHOP-LECTURE" ALLOWS

A feeling of community began developing amongst students with the concomitant development of confidence in their peers and themselves. The more relaxed and non-threatening experience of the "workshop-lecture" began having a positive affect on the students' participation in the class. This could be why "The students are spontaneous about asking, seem less shy than last year" as remarked by JN (appendix 3: 89) at the end of the cycle.

The “workshop-lecture” format also provides a basis for breaking down “barriers” between course activities used in providing for students' learning. The different activities become more complimentary rather supplemental appendages of one another, and hence not perceived as integrated by students. In the traditional setting, lectures are for gathering information, tutorials for aping class examples, assignments as easy/non-examination way of getting marks, etc. It is in this sense that "Tuts and pracs are very definitely workshops, with students using worksheets and graphics calculators to do fairly independent exploration, in groups" (JN, appendix 3: 88).

The “workshop-lecture” format allows the scope to immediately glean instances of students' weaknesses which would have a major bearing on their future comprehension. I had taken for granted that students' had a graphical understanding of the sign of a function. That they were confusing this with the negative or positive growth of a function (increasing and decreasing) was revealed in the sixth lecture session of the cycle. I could in this session deal with this misconception - not adequately, though, see later, but still it provided a signpost for clarity as the course proceeded. Not tied rigidly to a lecture format it was thus possible to "troubleshoot", instead of discovering these conceptual blocks at a much later stage in the course. The flexibility afforded by the format allows for the "crux" of the course as "creating the opportunity for trouble shooting", stated by JN in her journal (appendix 3: 90).

4.3.3 THE MATERIALS AND TEACHING AIDS USED TO GET STUDENTS INVOLVED IN "DOING" MATHS

Integral to the way the "workshop-lecture" operates are the materials which are used to drive the process. "Worksheets" need to be adapted to allow for student comprehension and not what the lecturer assumes to be a logical course of presentation.

The seventh lecture session involved a discussion on the composition of functions. I perceived that many of the students were actually struggling to comprehend this work, and the session degenerated into a traditional lecture in which I spoke for most of the time. The students were quietly staring on, not offering any comment or raising any query. I sensed already the feeling of rejection to this one way transmission. Judging by the look of almost disbelief on the faces of the students, I could feel a negative reaction to the proceedings. Prompted to do a great deal of explaining with not much participation by the students, I did not get a good feel of the problems confronting students with the concept of composition of functions. The worksheet for this section seem to be wanting, as it did not allow for effective "workshopping" on the part of the students. A copy of the worksheet appears in appendix 4.

The graphics calculator (GC) played a significant role in the way some of the sessions proceeded. In the final session of cycle one, students were engaged in discussions related to some of the major concepts introduced in the previous sessions. In their investigation to establish whether the statement, $\sqrt{x+3} = \sqrt{x} + \sqrt{3}$, was true or not, it was suggested that they enter all their variations on the GC and graph them together to see if they are the same. JN remarks that "This lead to great amazement and curiosity" (appendix 3: 108) and that

> In this whole debate the GC was crucial; without it we would have had to laboriously draw the graphs in question, with students uncertain about our very methods of sketching, and losing the point of the argument somewhere along the line (Appendix 3: 89).

4.4 REFLECTION ON CYCLE ONE

Reflecting on cycle one, it emerges that students are not familiar with an open-ended approach to the learning of mathematics and, that it takes time for them to adopt to a classroom procedure which is structured around their participation. A transition is necessary in which students learn to participate in activities that demand deeper engagement on their part. This transition is a process which is dependent on the students' acceptance of what is expected of them in the "workshop-lecture" setting. The lecturer's interactions with the students in the whole group setting plays an important part, not only in conveying enthusiasm for mathematics, but also in demonstrating to students the role they need to assume in relation to themselves and one another. Grappling with mathematical ideas in the form of discussion and argumentation is an important aspect of "doing" mathematics. An atmosphere must be developed in which students feel free to offer a viewpoint and in which it is acceptable to make mistakes. Using the input from students as much as possible and not impatiently imposing a viewpoint seems to be a vital skill the lecturer needs to adopt.

The large class size also means the lecturer having to play a management role in facilitating students' involvement in discussions. Having other colleagues and even tutors circulating and intervening during "workshop-lecture" sessions could offset the large class size. However, this implies greater overhead in terms of teaching personnel. On the other hand, the "workshop-lecture" format does allow for student collaboration and their sharing of ideas in smaller groups. This local consultation and deliberations gives students the opportunity to develop the confidence and competence to become "local oldtimers", that is, to play the role of more experienced knowers within their subcommunity. The number of authority figures is increased and a feeling of community amongst students is engendered which builds their confidence to participate in a wider audience. It appears that providing students with more opportunity to interact in this way would enhance the attempts to instil confidence in students and that which they would have for their peers.

The single, 40 minute, lecture periods, which in most cases were reduced to 35 minutes or even less, were not conducive to small group interaction amongst students. Given the large number of students and that they were not used to the "workshop-lecture" procedure, these single periods did not allow time for adequate student interaction as well as plenaries. A possibility for these sessions would be to have a whole class interaction, but with the smaller groups still operating at moments for conferring on ideas and possible answers to the questions either posed by the worksheet for the session or which arise as the session develops.

Learning materials and aids play a vital role in affording students the opportunity to engage in meaningful mathematical activity. This is especially the case with students who have not developed the confidence and competence to grapple with mathematical ideas to make sense of them. In this regard careful attention needs to be given to the conceptualising of worksheets and the questions that could guide students in discussions and investigations. What might be clear and logical to the lecturer can be quite daunting and unreasonable for students.

4.5 SUMMARY

It is evident that students are not conscious “newcomers” to the practice of “doing” mathematics. They have experienced the traditional way of learning mathematics for a greater part of their 12 years at school and expected to continue in this way. To clarify the role students need to adopt in their engagement with authentic mathematical practice requires a great deal of demonstration by the lecturer as “oldtimer”. Consequently, a more gradual induction of students into the procedure of the "workshop-lecture" is necessary. The large class size also requires putting into place mechanisms which would speed up the process by developing more students into "local oldtimers", that is, students confident enough to take on the role of more experienced knowers. One way of achieving this is to expand the notion of the "workshop-lecture" to include more intimate settings in which students can develop the confidence and competence to become "local oldtimers".

Cycle two needs to provide a closer scrutiny of issues related to the implementation of the "workshop-lecture" such as the students' involvement and interaction, the lecturers' participation and intervention, class size and venue, and the learning materials which allow for meaningful mathematical activity. Included in this investigation is the notion of the "expanded workshop-lecture" and the effects it has in enabling students to better recognize their role as "newcomers" on the way to becoming "oldtimers" in the cultural practice of "doing" mathematics.

CHAPTER FIVE

CYCLE TWO

> Of course photographs have the same potential to create the basis for manipulation, distortion and the exercise of authority, but for Berger's notion of the private photograph, which relocates ownership and authority in the process of interpretation, which confuses and complicates the relation between the subject and the object, which multiplies the sources of information brought to bear on interpretation and which threatens to mock any attempts that are made to impose singular views. (Walker, 1993: 84)

5.1 DESCRIPTION OF CYCLE TWO

I had planned to video record and make observations on sessions dealing with the first few sections on the derivative concept. These sections are an attempt to build on the concepts raised intuitively in the first chapter and seemed to me to be a cyclical development of the topics which included graphs and features of functions (slopes and rates of change, increasing/decreasing, local/global minima and maxima, concavity and inflection points, and continuity). Circumstances (disruptions in the lecture programme) dictated otherwise and I had to shift cycle two back by a month. This upset my original plans of having a colleague in the department other than JN to observe my planned lectures and a delay which resulted in cycle two organised around a different section of the planned semester syllabus. I also felt quite pressurised to get on with the work of the syllabus, given that the course had to "cover" a set number of topics. Quite fortuitously, cycle two then coincided with the visit by researchers of the Freudenthal Institute in the Netherlands. On their previous visit, in 1994, they proved to be valuable colleagues both in terms of ideas for materials production and as seasoned observers. They assisted me with the development of materials related to the concept of limit and agreed to observe a few sessions. I used their ideas together with my own, which I had garnered through experience, to develop a series of worksheets to drive the "workshop-lectures" for cycle two. One of the members of the Freudenthal Institute took a series of photographs during one of the sessions, as well as one of the "lab" sessions that took place during the same time period. Unfortunately, due to their tight schedule, the colleagues from the Freudenthal Institute could not observe as many sessions as I would have liked, nor was there much time for reflective discussions. I did request them to provide me with some general comments as well their comments related to the photographs. Appendix 5 contains the observation of one their number, Aad Goddjin (hereafter referred to as AG), who used the photographs he had taken as the basis for his comments.

In addition to my own observations and those of AG, JN has provided me with her observations, appendix 6, in which she also uses the photographs as backdrop to her comments. For cycle two, I also developed a few "lab" sessions as a result of my reflections of cycle one which is reported on in section 5.2. Photographs were taken of one of these sessions for which I obtained the observations of JN, in appendix 6, as well as one of the tutors for the course who facilitated one of the "lab" sessions, and who had keen interest in teaching (appendix 7). A colleague from the Academic Development centre of the university had also indicated his willingness to observe some of the sessions in cycle two. Due to the delay and his own commitments he was only able to attend one session on which he briefly commented (appendix 8). The students views are gleaned from interviews I conducted during the last week of lectures for the semester with 18 members of the class. Although, most of the students in my class of 56 at this stage of the semester were at ease when speaking and relating to me, my own inexperience in conducting interviews of this nature is exposed. Reading through the transcribed interviews, it was evident that in some cases I had reacted to students brief comments by prompting them along rather than allowing them to develop their responses. Notwithstanding these shortcomings, the interviews did reveal some of the students feelings delivered in a sincere manner. The photographs, which were spread on open on the table during the interview, were used at the beginning of each interview to illicit students' responses of their experience of the "workshop-lecture". Not being too happy about the interviews, I have included students free writing responses to a few a questions as indicated in appendix 9. This was done at the beginning of the second semester. Consequently, students had their first semester results as one yardstick in the construction of their appraisals.

5.2 DIDACTICAL REFLECTION AND DELIBERATION

Although cycle one provided some insight into some of the issues related to implementing the "workshop-lecture", it was necessary to gather more observational evidence to tease out these issues further and in the event open up the process to more scrutiny. Included here would be observations on students' involvement and how the lecturer's intervention facilitates their "doing" of mathematics, that is, the relationship between the "newcomers" and the "oldtimer" within the context of the "workshop-lecture". Also to be considered are the role that the learning materials play, and the effect the class size and venue have on students participation in the "workshop-lecture". For these observations the double period lecture sessions would be organised in similar fashion to the way it was done in cycle one.

In the reflection on cycle one, I suggested that the single, 40 minute sessions should be conducted as whole group discussions with the small groups functioning as forums for quick conferring on questions posed during the session. This was in response to the these sessions not being conducive to adequate small group interaction amongst students. To this end, I planned that some of the 40 minute sessions would take the form of a plenary to discuss conclusions or generalisations of investigations which occurred in previous sessions. The "generalising" session for the investigation on the two-sided limit is an example of such a plenary.

Just as traditional lectures do not stand on their own (it seems to be necessary to supplement them with tutorial sessions), the "workshop-lecture" must be coupled with work outside of the formal lecture times. This work must engage them, not merely in the traditional tutorial sense (attempting problems following a section of work), but in a proactive way, as an integrated part of the development of the concepts. In this way it is hoped that students take on board the responsibility they have for their own learning and develop the confidence to become more experienced knowers. To this end, I thought about converting some of the whole group discussion lectures into smaller group "lab sessions". These would take place in a smaller venue for more intermit collaboration by students. The work that would be organised for these sessions would be integrated into the work plan, and would serve either as prerequisite work for subsequent lectures or as completion of sections of work not to be dealt with in lectures.

5.3 CLOSER SCRUTINY - PHOTOGRAPHS AND ALL!

In this section, my own observations are triangulated with those of the photographs presented on pages 55 to 59 (workshop-lecture) and 63 to 64 (math-lab), the comments of students interviewed, AG (appendix 5) and JN (appendix 6), who used the photographs as a major backdrop for their observations. Reference is also made to observations by a tutor (appendix 7) and a colleague (appendix 8). The worksheet for the session which was photographed appears in appendix 10 (IN-CLASS LAB ON LIMITS).

Based on the reflections of cycle one on issues pertaining to the "workshop-lecture" as a mechanism affording students the opportunity to engage in meaningful mathematical activity raised in cycle one, I have grouped the observations into the following categories:

- Students' involvement and interaction

- Formation of groups, and considerations of class size and venue

- Lecturer's participation and intervention

- Learning materials and aids

- The math-lab: notion of expanded "workshop-lecture"

5.3.1 STUDENTS' INVOLVEMENT AND INTERACTION

In all the interviews, the first comments by the students were about the "group work" taking place in photograph 1. All, except one student noted the concentration and involvement of all students or at least that "Everybody looking busy and interested in the work". This "looking busy" was more critically observed by this student as " people at the back not working". This is not something that I as the instructor can easily pick up in the course of the session, especially if I am involved in discussion with a particular group, as photograph 1 depicts.

WORKSHOP-LECTURE 1

WORKSHOP-LECTURE 2

WORKSHOP-LECTURE 3

WORKSHOP-LECTURE 4

WORKSHOP-LECTURE 5

WORKSHOP-LECTURE 6

WORKSHOP -LECTURE 7

WORKSHOP-LECTURE 8

WORKSHOP-LECTURE 9

WORKSHOP-LECTURE 10

Photographs 2 and 3 home in on the middle group in the first row of photograph 1, indicating, maybe, that the female student is not part of the discussion. She seems to be on the periphery of the discussion involving the other two members of her group and could very well be showing her consternation by this in photograph 3. JN on the other hand observes that this student is only "slightly distracted ... momentarily, probably by the camera" and that "her body, hand and pen are still directed towards the centre of attention of 'her' group" (Appendix 6: 98). The situation in this group can be contrasted with that of the group in photograph 6. AG refers to this scenario as "shared concentration" in which the eyes of each member of the group is focused on the same object, the graphics calculator in this instance (Appendix 5: 95).

5.3.2 FORMATION OF GROUPS, AND CONSIDERATIONS OF CLASS SIZE AND VENUES

Based on photograph 1, AG comments "It is beautiful new hall, but the architecture is in complete contradiction with your teaching-aims!" and goes on to say that "The better situation is free moveable tables of course, even if the group is this large" (Appendix 5: 95). JN concurs with AG with her observation that "the seating arrangement is fixed in such a way that group work is actually discouraged" (Appendix 6: 97). I agree that a flat-floored room with movable tables would be preferable, but a tiered, fixed seating arrangement is not insurmountable. One problem is that the instructor is not able to reach all groups equally easily. I have attempted to overcome this problem to a certain extent by targeting particular groups at each session. Then the question of how to handle an even bigger class size arises. One way forward is to have team-teaching with more than one "oldtimer" circulating and intervening. Closer collaboration amongst lecturers is then also facilitated. Limiting group sizes to a maximum of three persons seems also to be another consideration for a tiered lecture theatre.

Groups formed spontaneously. Students start working together with those sitting next to them. In photograph 1, there is a student in the second row, with a zig-zag patterned jersey, sitting on his own. By photograph 5 he has formed a group with the student in the same row, wearing a black top. This could be, according to JN, a function of the student sensing "that he/she is missing out, deprived of the interaction in the small group" (Appendix 6: 97). The rows of the lecture theatre does tend to allow for students to quite easily turn away from a group and "hive off" on their own to complete their own worksheet. AG, referring to photograph 9, points out that "Some people feel secure with that kind of work, are more busy with cleanness and drawing accuracy than with math itself" (Appendix 5: 96). In a subsequent session, I handed out only one worksheet per group, which resulted in less groups "breaking up" for long periods of time.

5.3.3 LECTURER'S PARTICIPATION AND INTERVENTION

The lecturer interacting with the small groups gives students a chance, in the lecturer's presence, to show their understanding of concepts. They are less inhibited in the their smaller group and might not have contributed in the context of the whole class. JN points to this lack of self-consciousness which is evident in photograph 3 (Appendix 6: 98).

By circulating and interacting with the smaller groups, the lecturer picks up on issues arising in a particular group discussion and shares this with the rest of the class. By intervening from the discussion of a local group, the lecturer is deflecting exclusive authority from himself. AG picks up on this and says that he took photograph 4

> because it expressed well the way you talk to the group. Not as a guru, but as someone who want to keep contact with the group and at the same moment is excited about the subject itself (Appendix 5: 95).

5.3.4 LEARNING MATERIALS AND AIDS

As observed in cycle one, the worksheets that drive the "workshop-lecture" are integral to the way students interact. A "mix of text and tasks to do" (AG, Appendix 5: 94), characteristic of the worksheets, also seems to provide students with a sense of security that they are getting down some notes.

Beside the graphic calculator, which together with worksheets form the focus of discussion in the small groups, it is possible to incorporate other teaching aids. AG, referring to photograph 10, that it is possible to employ "Kindergartenmethods like paperwork, folding etc on university level also" (Appendix 5: 96).

MATH-LAB 1

MATH-LAB 2

MATH-LAB 3

MATH-LAB 4

5.3.5 THE MATH-LAB

References are made to the photographs, MATH-LAB 1 to 4, on pages 63 and 64.

All the photographs show intense interaction amongst the students. JN sees this as "due partly to the seating arrangements (around tables and partly to the absence of a lecturer" (Appendix 6: 99). I had withdrawn myself from the room for long periods of time. In this particular session, students were not disturbed by my entrance or that of the photographer.

The seating arrangements in the math-lab are more conducive to group work. Students are able to huddle closer together, and interact more effectively. Photograph 4 shows that the arrangement allows for better eye-contact than the tiered lecture theatre and subsequently improves communication.

With the absence of the lecturer, students take more responsibility for their own learning and the role of more experienced knowers. This is observed by a teacher training student as "Students develop an attitude of helping each other, that means they see themselves as source of knowledge" (Appendix 7: 100) .

5.4 FURTHER OBSERVATIONS

One of the double period sessions was based on an investigation of the two-sided limit. As the session was built around a series of 6 worksheets (we managed to discuss 5 out of the 6), it was quite easy to have "logical plenaries", with each new example indicating increasing confidence and what seemed to me at the time greater insight. Appendix 11 provides an example of one of the worksheets.

The follow-up lecture to the one on the investigation of the two-sided limit, a single period of 40 minutes, was an attempt at drawing out some general conclusions based on the previous sessions investigations. The session did not proceed as I had anticipated. I, initially, had still to do a great deal of coaxing. It appears, on the part of the students, that the deliberations of the previous sessions were not adequate enough to draw meaningful conclusions, or that I had not allowed sufficient time for the students to reflect on their investigations. Arguably, the concept under consideration, that of the limit of a function at a point, is one of the most difficult concepts to comprehend at this level. One of the issues here, in my opinion, is the greater time students need to reflect on the data they obtained from the worksheets, with more leading questions to force more awareness on the part of students.

5.5 STUDENTS' EVALUATION OF THE "WORKSHOP-LECTURE"

The analysis presented in this section is based on the students' free writing responses to the questions which appear in appendix 9. My categorization of these responses is shown in table 6.

Table 6: Students appraisal of the "workshop-lecture"
Number of student responses = 40, some students in more than one category.

Category	%
A. It helped "understanding" of mathematics.	50,0
B. Enjoyable/exciting, because of the participation.	47,5
C. Appreciation of the "relevance" of mathematics.	27,5
D. Learnt to work in groups.	25,0
E. Developed confidence to ask questions and give opinions.	15,0
F. Indifferent or not beneficial.	5,0

The following is a list of representative comments by students. The category of each comment is indicated by the letter preceding the comment.

(A) Its like you are given a chance to discover what you know rather receiving from the lecturer all the time. You are challenged with questions and that leads to more understanding of the work.

(B) I think it is a good way to learn mathematics as one is constantly involved in the approach.

(B) If a problem in the work is encountered the solution is not only from the lecturer's side but more of an argument between students and lecturer.

(C) I have come to see mathematics as something more wide.

(D) Personally, I feel that this method of teaching/lecturing creates an atmosphere of equality amongst students.

(E) Also everyone gets to know each other and that makes you fee free to say anything and not worry if people are going to laugh at you.

(F) It was not to my satisfaction, because I was unable to relate to the work. And as the work progressed I fell behind.

(F) I do not prefer to study/ work in a group. There's nothing wrong with the method, though.

The predominant student reaction to the "workshop-lecture" is very positive. 2 students, although indicating that the "workshop-lecture" was generally a good learning experience, did each raise a point of criticism. One student's comments related to the newness of the approach:

> It was not as effective due to time constraints and because its a new concept that some students feel sceptical about.

The other student's comments relates to the possible negative effects of the approach:

> The workshop approach makes you feel as if the work is done for you, so it is possible to be a little lazy as in my case.

5.6 REFLECTIONS ON CYCLE TWO AND SUMMARY

There appears to be a greater acceptance by students of their role as "newcomers". This is evident in their recognition of the need to be more actively involved in the learning of mathematics: The "workshop-lecture" approach kept them "constantly involved" and impressed on them that the "solution is not only from the lecturer's side", but that they can gain understanding from "given the chance to discover what you know rather than receiving from the lecturer all the time", when "challenged with questions" and through "argument between students and lecturer". It is within the social organisation of the "workshop-lecture" and the interaction that it engenders that the benefits of collaboration and group-work as vital to "doing" mathematics is understood.

The students' recognition of the value of active engagement and collaborative learning was further demonstrated in the "math-lab" sessions. The expansion of the "workshop-lecture" to encompass more intimate settings for student collaboration seem to give greater impetus to students "becoming independent learners, and 'owning' their work" with "even more freedom to form non-verbal agreements, and for students to control their own interaction" (JN, Appendix 6: 118).

The process of inducting students to authentic mathematical activity is enhanced by having smaller groups of students and venues more conducive to small group interaction such as a level-floored room with movable desks. A situation in which this is not always possible and in which students have not made the conscious decision to be "newcomers" necessitates a more gradual induction of students to activities with which they are not accustomed. Also, the lecturer as "oldtimer", is mentor to a large number of students and has to constantly demonstrate how mathematicians use the tools of their trade and that this is partly done through interaction with one's peers. The intention would be to develop "local oldtimers" amongst the students and thus increase the number of students confident and willing to be "sources of knowledge". By circulating amongst the small groups during a "workshop-lecture" and intervening in the whole class group based on the discussion of a particular group impresses on students that a vital aspect in "doing" mathematics is sharing their ideas.

CHAPTER SIX

SUMMARY AND CONCLUSIONS

> My students were indeed learning mathematics, but learning is an ambiguous term. It is both the activity of acquiring knowledge and the knowledge that is acquired. What I have described here is the activity. The problem of defining what knowledge they have acquired remains. (Lampert, 1990: 59)

6.1 INTRODUCTION

In this mini-thesis I examined the design and implementation of an instructional procedure called the "workshop-lecture". The "workshop-lecture" attempts to simulate an environment which enables students to engage in meaningful mathematical activity. Meaningful mathematical activity is explicated within the theoretical framework which posits that mathematics is a specific cultural practice characterised by its practioners, the mathematicians, "doing" mathematics. Adopting the research methodology of developmental research, the examination was conducted during the first semester of the academic year (1995) in the mathematics department at UWC. The participants were the 56 students registered for the first year mathematics course at UWC developed for students considered to be "at risk", and myself as the lecturer. Data for analysis and reflection were collected over two cycles and reported on as candidly as possible.

This chapter concludes the mini-thesis with reflections on the implementation of the research (research methodology) and the results of the implementation (theoretical considerations). The main conclusions and recommendations for future research are given.

6.2 REFLECTIONS ON RESEARCH METHODOLOGY

This section considers deficiencies and limitations in the research design of the investigation for this mini-thesis.

One of the deficiencies in the research design is the insufficient evidence for cycle one. I had planned to video record some the sessions in cycle one. This did not materialise due to a misunderstanding between myself and the "video team". As I had not made contingency arrangements, there was little time to organise for an alternative arrangement. Reflecting on the research design, I feel that there is also a lack of evidence, during the investigation period, of how students encountered the "workshop-lecture", that is, the learner perspective on an on-going basis. I could have invited some students to make daily or weekly diary entries which could have been coupled with short student interviews at the end of each cycle.

My inexperience at interviewing proved to be a limitation. I could either have piloted the interviewing process to gain the necessary experience or brought in a colleague experienced in conducting interviews.

Factors hampering the implementation of the investigation were the instability of the university at the time and the flux of students in the first two weeks of the semester. Most of the students entering UWC are first generation university students and do not receive sufficient counselling on their career choices. Lecture periods in the first two weeks were continually disrupted, with students either changing their course of study or being allowed to register after closure of the registration date well into the academic year. Class disruptions because of students' grievances have been an integral part of UWC over the past few years. These are at times unplanned and can occur any time during the academic year. Putting research into practice at UWC cannot be divorced from this context, and in this way the planned second cycle for the investigation was affected and had to be postponed. Given the short duration of the academic semester at UWC (twelve and half weeks) and the adaptations I had to make to the syllabus for the semester, my plans for collecting data for the second cycle relied partly on the fortuitous visit by colleagues of the Freudenthal Institute in the Netherlands.

6.3 THEORETICAL REFLECTIONS

The underlying position for the "workshop-lecture" is that mathematicians make sense of mathematical ideas by "doing" mathematics which involves them using the "tools of their trade". This "doing" of mathematics or meaningful mathematical activity is inherently social in character establishing a community of practice of mathematicians. Mathematicians develop competence with the "tools" of mathematics through consultation, collaboration and discussion, and the sharing of ideas. Aspirant mathematicians or "newcomers" are afforded the opportunity of entering this culture of mathematicians by their exposure to the more experienced professional mathematicians or "oldtimers". A "cognitive apprenticeship" relationship is set up in which the "newcomers" are accepted as legitimate participants in the community of practice of mathematicians.

Enabling students as learners of mathematics is enhanced by allowing them to enter the culture of practice of mathematicians. This can be achieved by transforming the traditional classroom into a "local intellectual community" in which students are involved at their level in "doing" mathematics. However, two major constraints need to be overcome when simulating the "cognitive apprenticeship" for students in undergraduate classes. These are the large number of students in these classes and, especially for first year, students have not made the conscious decision to be "newcomers" (aspire to be mathematicians). In addition there are the constraints of unsuitable venues and time slots as students are simultaneously studying other subjects. By structuring lectures around group work activity, the mechanism of the "workshop-lecture" allows for the social interaction of students. But the lecturer, as "oldtimer", has to play a much larger role than just being a mentor. The extended role of the "oldimer", not only includes facilitating and managing the lectures so that students can work constructively together, but also clarifying to students what is expected of them as "newcomers". A more gradual induction is necessary in which some of the students become more confident and competent to take on the role of "local oldtimers", that is, they see themselves as more experienced knowers within their local group. Collaboration amongst students in their classroom community is localised to their individual groups. The facilitator has to ensue cross fertilization of the groups and in the a process a gradual gradation of the whole class of students to a "local intellectual community".

The development of "local oldtimers" amongst the students appears to be one way in which the class as a whole can operate as a "local intellectual community". This process can be accelerated by facilitating other experiences in which students can develop the confidence and competence to engage in authentic mathematical activity and become more experienced knowers. Expansion of the "workshop-lecture" could include activities such as group projects or small group presentations in the class. I have expanded the notion of the "workshop-lecture" by converting at least two lectures per week to "math-lab" sessions. For the "math-lab" sessions, the class is broken up into groups of approximately 15 students who meet in a more intimate setting to work almost independently on an "investigation" which serves as prerequisite to or an extension of a topic dealt with in lectures. These forums are less inhibiting and the students have more freedom in controlling their interactions.

6.4 CONCLUSIONS AND RECOMMENDATIONS

The mechanism of the "workshop-lecture" allows for the interchange of ideas between lecturer and students, and between students, which is limited during traditional lectures. Evidence indicates a greater involvement and enjoyment on the part of students in meaningful mathematical activity. Although, a flat-floored room with movable tables is more conducive to the activities of the "workshop-lecture", a tiered, lecture theatre does not prove to be a major obstacle in the implementation of the "workshop-lecture".

Students prior experience has predispositioned them to a passive acceptance of information and a reluctance to engage in more exploratory ways of working or in group-discussion around mathematical concepts. It appears that the bigger the size of the class the more gradual the induction of students to the different way of working needs to be. During this induction period students have to continuously be shown by the facilitator what it implies to be engaged in meaningful mathematical activity. Also, creating opportunities for students to recognize more their own responsibility for their learning, deepens their involvement in class discussion and collaborative learning. There is evidence that the notion of the "expanded workshop-lecture" discussed in the previous section does provide for such an opportunity.

From the lecturer's point of view, the "workshop-lecture" entails the honing of particular skills. Besides demonstrating to students how to grapple with mathematical ideas and helping them to work constructively together, the lecturer needs to become skilful at setting tasks that allows for meaningful mathematical activity. Included here is not only the production of materials such as worksheets, but also judging how long students need to work on them. This is a big task which is eased through collaboration and sharing of ideas with other "oldtimers", and via practice and developmental research.

Essential ingredients for the "workshop-lecture" to facilitate student interest and involvement in meaningful mathematical activity are the worksheets and technologies that are employed. Worksheets that are a mixture of text and tasks to do, and not merely notes with gaps to be filled in by students as the lecture proceeds, allows for students to engage in fairly independent discussion and exploration. These explorations are enhanced and the discussions heightened by the use of technology like the graphics calculator.

During "workshop-lecture" sessions more active learning is encouraged by the "uncovering" of concepts by students as opposed to the "covering" of the syllabus that characterises the pace set in traditional lectures. Students are working more intensely at understanding concepts and hence the "workshop-lecture" format tends to take more time for the development of key concepts and requires more lecture time to complete the syllabus resulting in students being overloaded with more mathematics lectures per week. One proposal is that a course adopting a "workshop-lecture" format is given over a longer period of time (three or four semesters instead of two). Another proposal, which is based on the principle that "less is more", explores a change in the curriculum constructed for undergraduate mathematics classes which would allow students to engage in meaningful mathematical activity. Powell (A.B. [1], 1993: 2) elucidates the underlying principle for this proposal by explaining that

> it is better to delve into depth and consolidate understanding of a few topics than to cover massive amounts of material superficially. Instead, encourage and help students to make connections with and between topics, guide them to develop mathematical insight. This would provide students with awareness of mathematical processes and opportunity to augment their facility into important areas of mathematics, and importantly, to development of confidence and evidence that they are capable of mathematical thinking.

Changing the curriculum in this way does not mean to make less rigorous. Rather, change implies that content of courses and concomitant pedagogical considerations must "evolve to a different standard of rigour than what may exist elsewhere" (Powell, A.B. [1], 1993: 2).

This mini-thesis is a report of a first examination of the design and implementation of the "workshop-lecture" as a procedure to engage students in meaningful mathematical activity. Besides the necessity to refine and expand on the work begun with the work reported, a further challenge is to illuminate what mathematical knowledge and how students access this mathematics while they engage in the activities of the "workshop-lecture". As Lampert (1990: 59) states: "[the] knowledge they have acquired still remains".

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APPENDIX 1

USING THE DERIVATIVE

A: LOCAL MAXIMA AND MINIMA

Recall what the first and second derivatives tell us about functions!

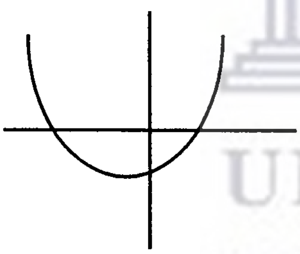
Critical Points

For any function f, a point p in the domain of f is called a **critical point of f** if • $f'(p) = 0$, or • $f'(p)$ is undefined.

The point (p,f(p)) on the graph of f is also called a critical point and f(p) is called a **critical value of f**.

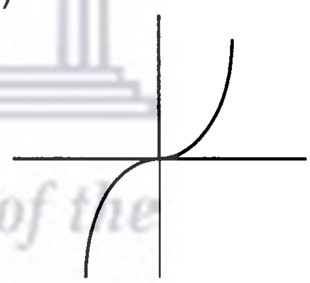
Indicate the critical points on the following graphs:

(a)

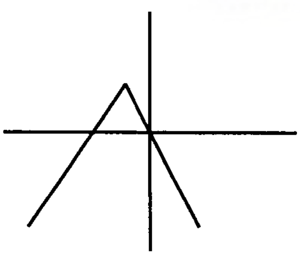


(b)

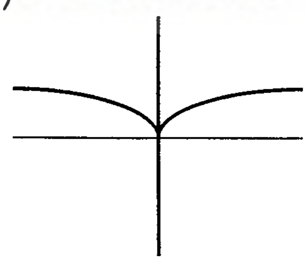
(c)



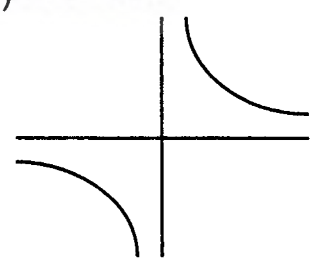
(d)



(e)



(f)



What do critical points tell us?

Stage

Find the critical points for the following functions and determine the intervals of x values in which the functions are increasing and decreasing.

(1) $f(x) = 2x^3 - 9x^2 + 12x - 4$ 4

(2) $g(x) = 3x^4 - 4x^3 - 12x^2$ 5

Stage

Local Maxima and Minima

Recall what we mean when we say that a function f has

- a local maximum at $x = p$:

- a local minimum at $x = p$:

The First Derivative Test for Local Maxima and Minima:

If p is a critical point in the domain of f, and if f' changes sign at p, then f has either a local maximum or a local minimum at p.

- If $f' > 0$ to the left of p and $f' < 0$ to the right of p, then f has a local   at p.
- If $f' < 0$ to the left of p and $f' > 0$ to the right of p, then f has a local   at p.

6

Give a pictorial version of the first derivative test:

Local maximum at p	Local minimum at p

Referring to examples (1) and (2) above, can you identify the local maxima and minima?

(1)

(2)

7

Does a function have to have a local maximum or minimum at every critical point? Consider $f(x) = x^3$ and $g(x) = 1/x^2$.

8

stage

Can we use the second derivative to identify local maxima and minima? Illustrate with sketches.

Local maximum	Local minimum

The Second Derivative Test for Local Maxima and Minima:

- If $f'(p) = 0$ and $f''(p) < 0$ then f has a local ____ at p.
- If $f'(p) = 0$ and $f''(p) > 0$ then f has a local ____ at p.

Use the second derivative test to identify the local maxima and minima of examples (1) and (2) above.

(1)

9

(2) $g(x) = 3x^4 - 4x^3 - 12x^2$

10

10

What can we tell if both $f'(p) = 0$ and $f''(p) = 0$?
Consider $f(x) = x^3$ and $h(x) = x^4$.

11

End of session

B: INFLECTION POINTS

Recall what we mean when we say that a point $x = p$ [or (p,f(p))] is an inflection point of a function f:

Illustrate with sketches:

If $f''(p) > 0$ then f is concave __ and if $f''(p) < 0$ then f is concave ____ .

Do you agree that at an inflection point, f" is zero or undefined?

APPENDIX 2

MATHEMATICS 114: QUESTIONNAIRE 1

DATE: ___________

This questionnaire is not a test. It is merely used to obtain a better insight to your feelings about mathematics.

1. What is the first thing that comes into your mind when you hear the word "mathematics"?

____
____
____
____
____

2. In your opinion, what is mathematics?

____
____
____
____
____

3. Why do you think most people believe they cannot "do" mathematics?

____
____
____
____
____

4. What does it mean to "understand" mathematics?

____
____
____
____

5. How do you feel about mathematics?
a. very positive b. positive
c. negative c. very negative?

____

6. Explain your answer to no. 5.

____
____
____
____
____

7. Which topics of school mathematics did you enjoy most?

____
____
____
____
____

8. What did you enjoy about mathematics periods at school?

____
____
____
____
____

9. How would you describe a mathematician?

____
____
____
____
____

THANKYOU VERY MUCH FOR YOUR COOPERATION!

APPENDIX 3

UWC MATHS DEPT

ACADEMIC DEVELOPMENT

JOURNAL

Course M114/124

1995: SEMESTER 1

3-3-95

This year the course seems more consolidated, more consistent, LK seems to be clearer about what he wants, the direction to move in. The course now makes use of all (almost) the innovations tried before, but they are integrated very sensibly. Tuts and pracs are very definitely workshops, with students using worksheets and graphics calculators to do fairly independent exploration, in groups, and there is a regular, compulsory lab period per week, when students have to work on their own, on set problems. They then have to write a lab report for marks. In all this discussion is very important, and the students seem to take to it naturally and seem really positive and productively engaged. The GC has also come to play a central role, and I think it is being used as it should be now. LK is also practicing action research very actively! His thorough planning is upset every lecture, as he realises what is necessary or lacking every step of the way. (However he is not phased by this at all, just spends much time redrafting worksheets.) So one can't say the "magic method" is workshops, or calculators, or writing, action research, or whatever - it is the integrated use of these things that we have to evaluate.

This afternoon's prac was a case in point. The worksheet was labled "IN-CLASS LAB 1 (FRIDAY, 24 FEB) ARISING FROM WORKSHEETS", i.e. issues from the previous worksheet were used to structure the new worksheet. (action research) After some lecturing on the pitfalls of using the GC, students had to explore the question whether inflection pnt means change in concavity and vv. (group work) They had to look at their previous worksheet for this. Then they had to investigate 3 other functions (variations on functions they've seen) using the GC. (technology again)

There was plenty of opportunity for filling in gaps in students' knowledge and clarifying their understanding of concepts, but it does take fairly intensive work on our part, and the more tutors available for this purpose, the better. I move from group to group, ask whether they need help, or respond to a request, ask them a question to provoke further thinking (especially when they areon the wrong track). LK does something similar. The students are spontaneous about asking, seem less shy than last year.

Discussions within and with the small groups:
-What is meant by concave up/ concave down? one person thought that this was related to how steep the graph appeared, i.e. whether it was clearly moving "up and down", or more side-to-side.
-Interval notation: some people were still clarifying, and tentatively practicing their interval notation, and various forms of it. It struck me that writing things correctly, according to convension, is important, even if a student does know how to convey his intention somehow. I am impressed that students do think this important too. LK's lecture was about this this morning, so that may contribute to the students' keenness..
-What is a pnt of inflection? I thought that it was a point of change in the graph, not just a change in concavity, but LK disagrees with me (as i discovered after the prac). However this discussion was valuable for the students as well, with someone e.g. confusing the words concavity and inflection, and having to make a clear distinction for himself. Also, if a graph is not defined everywhere, is it an inflection point where it ends/begins?
-What is the correct way to enter square root of (x+3) in the GC? Does it matter? This was fascinating, informative for us about students' misconceptions, a fruitful learning experience for them, and a good illustration of the potential of the GC. One student had entered square root x + square root 3. Another: square root x + 3; another bracket square root x + 3 bracket; and yet another had drawn a graph with y-intercept = 3. To all groups battling with this question, I suggested entering all variations on the GC and graphing them together, to see if they are the same. This lead to great amazement and curiosity. They were surprised that all expressions did not lead to the same graph. One student remained adament that root x + 3 is correct, and that introducing brackets after the root sign could not be correct, until the last moment (then he seemed a bit embarrassed! - hopefully that does not make a student shy away from exploring, but makes him remember the lesson learned!) In this whole debate the GC was crucial; without it we would have had to laboriously draw the graphs in question, with students uncertain about our very methods of sketching, and losing the point of the argument somewhere along the line.

-Finding an x-intercept: to indicate the interval of increase for square root (x + 3), we had to indicate the point where the graph "begins". This happened to be on the x-axis. As the GC does not automatically lable this point, two students were unclear about how to find this x-value. I suggested using TRACE. Once they traced the graph to its beginning, it seemed that the value was (-2.8) or thereabout, but the GC did not give the exact point where y = 0. So I suggested checking algebraically. There was a little hesitation, but then rusty methods were recovered... the one student asked, as he was writing: "I've been wanting to ask this for a long long time: is it possible to square both sides of the equation?" I explained, and then pointed out that in this case it was very logical, because you cannot have a square root of 0 without having the "thing inside the root" = 0. (This required careful listening and repeating but I am sure it "clicked"!). After this it was easy for them to see that the point was x = -3 (and at the same time discover another shortcoming of the GC, which had lead in this case to a learning experience in itself!)

-Global max/min: can a sharp-point-shape be a global max, or only a smooth curved shape?

I remarked to LK that this is for me the crux of his course: creating the opportunity for "trouble shooting". If it is true, his course is now enormously successful.

APPENDIX 4

3. Composition of functions

Consider the following situation:
Oil is spilled from a tanker. The area of the oil slick will grow with time. Suppose that the oil slick is always a circle. (In practice, this does not happen because of winds, tides and the location of the coastline.)

Express the area, A, as a function, f, of its radius, r:

Find:

$f(0) =$ $\qquad$ $f(1) =$

$f(a) =$ $\qquad$ $f(\sqrt{a}) =$

$f(\frac{1}{\sqrt{\pi}}) =$

The radius is also a function of time, because the radius increases as more oil spills! Lets say that this relation is given by $r = 1 + t$.

Express this relation as a function called g:

Find:

$g(-1) =$

$g(\frac{1}{2}) =$

$g(\pi) =$

So, we have A as a function of r, and r as a function of t.
We wish to link A to t, explicitly, i.e. write A as a function of t!
We see that A and t are linked via r:

A ---------> r ------------> t

g(1) = = [ ], so f([ ]) =

g(a) = [ ], so f([ ]) =

Now, g(t) is also a number!

f(g(1)) = f( ) =

f(g(a)) = f( ) =

f(g(t)) = f( ) =

But, is'nt A = f(something)!
Can you now write A as a function of t?:

Find A at t = 0.

First calculation:

Second calculation:

We obtained A by the process of taking a "**function of a function**", called the **composition of functions**.
A is called a **composite function**, which has f as the "**outside function**" and g as the "**inside function**".

The composition fog of the functions f and g is defined by (fog)(x) = f(g(x)). The domain of fog consists of those x's for which g(x) is in the domain of f.

Let's change the order of calculations.

How would we write this down in terms of f and g?:
Does this still give us A as a function of t?:
If $f(x) = 2x + 3$ and $g(x) = x^3$, form the following composite functions:

(a) $f(g(-4)) =$; $g(f(-4)) =$

(b) $f(g(0)) =$; $g(f(0)) =$

(c) $f(g(x)) =$; $g(f(x)) =$

So, in general f(g(x)) g(f(x)) !

Let's **decompose a function**, i.e. write it as a composition of functions:

$$h(x) = (x^7 + 5)^{10}$$

outer function $f(y) =$
inner function $g(x) =$
composite function $f(g(x)) = f(\quad) =$

Do the same for $p(x) = \sqrt{1 - x^2}$,i.e find functions f and g such that $p(x) = f(g(x))$:

We can call our composite function f and then use other names for the inner and outer functions. We can also use variable names other than x.

Express the function $f(t) = (1+t^3)^{27}$ as a composite function:

APPENDIX 5

Hallo Larry,

Here are my reactions to your fax: the description of how you want to teach and the photographs.

I like the idea of it. I think that I use in my teaching to universitystudents here the same idea. I will tell something about my experiences.
I cooperate with one of the 'real' mathematicians of the Mathe matical Institute.
The kind of notes I make for the students are a mix of text and tasks to do. That gives some security, the content (at least what I think the content is) is available on paper and during the lessons you have complete freedom.
I started what polyeders and I know that later in the course their will be curves, Gaussian curveture, surfaces etc.
I started with the problem of finding a formula for the sum of the inner angles of a closed polygon. It can be non-convex, overlapping etc.
Finding something and proving something toke those students, (3 years) almost 3 quarters of an hour. The discussion was lively, enough contradictions, opposing approaches were present.
I did not expected that much time to take for an introductory problem, but it came out that it is good to invest time in working to an /mathematical attitude'.(They are also used to beeing lectured to and have in most case separate lab-sessions.) Im glad I started that way, because now it is quite more acceptabel for them that mathematics is also activity and discussion. If you have direct interaction with the students, it is also possible to talk abou math in a more informal way, where you can stress core-issues. In general those core issues are very basic and independent of the 'notated form' of math.
Also their is ample oppurtinity in this situation to bring in ideas about how mathematics is done. I have only superficially read Polya and Lakatos, but that is inspiring enough to use their ideas. And how it is learned.

Those students have syllabusse about analysis and linear algebra wher more in is than I have seen in my whole life. They finished exams and those subjects. Now I ask them for a formula relating the radius of the circle through the point of an regular n-gon with edge of length a. One student comes up with sqrt(a^2 - cos(360/n)) (something like that.) I am surprised that she did NOT learn the basic mathematical idea of testing your formula in known cases, in thinking about the general behavior of a formule (linearity in this case). That is what is possible what learning to much in an non-intrinsic way.

In my course we do the second half of the course in a different way. Students have to present something (related to the subjects of the first half). That is also a method to make them more participating! The evaluate each-other also about content, way of presentation and so on. In two weeks to of them present about geometry in 4D, using among other things a film of Thomas Banchoff about the hypercube.

I am telling al those stories, because I believe in the ideas you expressed in your notes whit the photographs.

I think the situation with your students is different, in preparation of the students before mainly, but the general idea about teaching are the same. I think it is important you do this specially with the students who are considered underprepared, because I observed also myself that the rote-learnig approach is implanted quite deeply in them. It is quite a fight you have with them , in the good sense!

You stress more than I do the element of cooperation among each other. I only stress the idea that math is not a body of truths, but a battlefield of discussion where people stimulate each other to better and better arguments. Basically that's the idea of Lakatos' Proofs and Refutations, (which I suppose You know and if not and you can't find a copy - it is probably out of print now - I send you one.) Cooperation among students is quite normal, some like to work alone, some work always together. If I fear that one cannot handele his/her presentation-subject alone, I will stress cooperation. and and You stress more 'the intelectual community' more expicitly. May be that's also related to the SA situation af the moment wher I find specially at UWC this idea of rebuilding together, etc.

I hope that , by comparing your ideas with what I try to do myself, I expressed how I feel about your teaching method.

Some more remarks related to the photographs.

I take them in general not with one particualry welldefined purpose.
I like picturing people in a situation where that is in some allowed by the work-situation I am in, not when I have no relation at all with them, like beeing on holidays (I have almost no photograps from SA with people outside the projectwork!).
Picture 4 I toke because it expressed well the way you talk to the group. Not as a guru, but as someone who want to keep contact with the group and at the some moment is excited about the subject itself. I think the photograph is rather good in that respect, because you made that possible. Your teachingmethod needs somebody who is inspired to do it that way, and we have to realise that not everybody is able to be that lively and radiant. Teaching style is personal.

Several of the phot's I toke because I liked the involvment of the students. You can easily see the cooperation on number 6, I like waht you can call 'shared concentration'. Photographically their should be an object on (of out of) the picure wher evrybody's eyes are aimed at. It's the calculator there.

I mad one picture with compass and calculatot combined in th e situation. That's because that was an important element in one of the task George and I came up. (The 60-degree limit I brought In).

Picture 10: It is beautiful new hall, but the architecture is in complete contradiction with your teaching-aims! On number 3 you see how you handel that problem. I found it for myself like beeing late in the cinema and the only free seat is in the middel of a long row. On e more aspect of this: in groupwork in this kind of situation, there is always one in the middele and 2 at the sides and matbe more at the side-sides. The better situation is free moveable tables of course, even if the group is this large.

I remeber one photograph (maybe your number 9, but the colorpicture will be better) expressing how some students work. There were this boxes in the middle of the sheet and the students had the go on wirh the halving etc. On 9 you se how accurate the boxes are copied. Some people feel secure with that kind of work, are more busy with cleanness and drawing accuracy than with math itself. I tried in a booklet for VWO-B (16 yera old) that they also learned to work sketchy. It is difficult. This morining I saw that text beeing used in class. A girl (this happens more with girls than boys) was time consumed useless accurate with the first 'sketchy' task!

Is picture 8 with the pieces of paper? I made such pictures because they show you can use Kindergartenmethods like paperwork, folding etc. on universitylevel also. Some teachers se that older students don't like it. That's simply not true, but you have to take the thing serious as a teacher, els it will not work.

Dear Larry, I can't come up with much more on the moment, so I mail you this now.

I hope me notes also express clear enough that I enjoied very much beeing present in your class and hope to see you working with your students in the same exciting way again sometimes!

Don't hesitate to correspond more with me about al this.
Last weeks wher very stressy in preparing things, so I didnt react fast. May be (but I als make planning mistakes) I am more relaxed from the beginning of november.

Greetings

Aad

APPENDIX 6

M114/124

COMMENTS ON PHOTOGRAPHS

By Jansie, Dec. 1995

The photos show something of the way the class is organised, how the students understand their class time, and the role of the lecturer. There are hints about how a small working group of students is 'formed' in the first place, what effect the lecturer's intervention has, and most of the photos give us a glimpse of what happens in small group interaction.

As can be seen in the photos, the seating arrangement in a lecture theatre is fixed in such a way that group work is actually discouraged. The students are forced by the seating to work in groups of no more than 3, and the lecturer has to move rather awkwardly between the benches when he visits each group.

How a group is formed:
Photos 1-5 of the 'workshop-lectures' are pictures of the same group, one woman and two men, presumably taken chronologically. In (1) the woman is working alone, one man is looking at what the other is doing. Larry is talking to the next group (of 3 students). In photo (2) the attention of the woman is also focussed on what the man is explaining, in fact one could say that a group has now been socially 'defined' by the attention of all three focussed on the same object. The man doing the explaining is pointing and his head is moving a bit as he is talking, the eyes and bodies of the other two are directed at what he is pointing out on his paper. By photo (4) the woman, too, is actively involved, discussing with the man next to her.

In my experience, groups form quite spontaneously in this manner: When the lecturer poses a well-formulated problem, somewhat challenging and interesting, and he then allows the students the 'space' (time and opportunity) to accept the challenge of the problem, they start working together with those sitting next to them. Although students sharing the same first language tend to sit and work together, it doesn't generally matter that their backgrounds differ. Here and there a student prefers to work on his/her own, but often senses that he/she is missing out, deprived of the interaction in the small groups.

Looking at the row of students behind the group of 3 focussed on above: There is a student with a jersey with a zig-zag pattern directly behind them, who in photo (1) is sitting alone. To the left of him, (seen from our perspective) 2 seats away, is a student wearing a black top. By photo (2) these 2 students are now sitting together - the man with the black top having moved to the right of the student with the jersey. Cf especially photo (5).

Lecturer's intervention:
Photo's (3) and (4) show us how Larry 'teaches'. In (3) he talks to the group of 3 shown in photo's (1) and (2). There is an animated discussion going on - one man is in the process of gesturing or pointing, the other is about to say something or in the middle of saying it - the 2 of them obviously focussed on the object of their discussion. The woman is slightly distracted, but I think only momentarily, probably by the camera - (2 rows back there is also a woman starting to smile at the camera) - but her body, hand and pen are still directed towards the centre of attention of 'her' group. The focus has shifted from previously, but is not directed exclusively at Larry.

In my experience, the lecturer/tutor plays these roles in interaction with the small groups: urging students to get to work or try a problem, often this involves clarifying what the question is asking; once the students are working, to actively give them the opportunity to ask for assistance; if they don't ask anything, to watch them working or listen to them speaking, and intervening when an error/problem is encountered. Interacting with small groups rather than the whole class, gives the students a chance to respond - to ask, reformulate in an attempt to clarify, explain to show the lecturer they understand, discuss - with much less inhibition than if they had to talk in the contect of the whole class. This lack of self-consciousness is evident in photo (3). The art of this facilitation is have a good relationship with the students, and also not to cause the group's focus to be directed entirely at the facilitator - the focus should remain on the problem itself if the interaction is to be productive.

In photo (4) Larry is addressing the whole class. The group of 3 is focussing of their work. It looks like Larry is calling the class's attention to somthing discussed with this group, because the group apparently does not need to listen at this moment. Four students behind the group can be seen listening attentively to him. Larry is gesturing with both hands, and his gaze is directed somewhere towards the centre of the class (the group of 3 is sitting towards the right of centre of the room - cf photo (1)).

Over time, the students get used to Larry's body language. He has developed consistent ways of signalling whether it is time to address the class as a whole, or whether they should get on with their work and he will move around to their groups. He does not need to demand the rapt attention of every student in class before he speaks to them. When he speaks towards the centre of class, loudly enough, gesturing, they generally stop working for a moment and listen. When he looks down and talks to the people in front of him, they get on with their work in groups (photo (1)). His intervention in this way is helpful to the students when it is brief so that their attention to their group work is not entirely broken. The signals require consistency so that they become part of classroom culture.

Group work:
The rest of the photos show us how groups function.

Sometimes group members appear to work individually, but their body language shows that at any moment they may again interact with one another: e.g. photo (5). The group members remain sitting slightly turned towards the centre, as far as this is possible with the awkward seating. Sometimes group members sit and write or focus on their own work, but comment for another to hear, as they are doing so. I assume that is what is happening in photo (7). Students also look at what group members are doing without speaking: photo (8).

It is necessary to allow the students to interact spontaneously according to social, unwritten codes of interaction. The groups need time and opportunity for informal implicit (usually unspoken) 'agreements' like: "the 3 of us will work together", "we can look at one another's work without it being interpreted as plagiarism"; "all group members will participate in working on the problem (not just watch what the others are doing)", "no member will dominate"; or "no group member will distract others constantly from the work", etc. These are not things that can be enforced. They work best when mutually and informally agreed upon, and most adults are very capable of interacting productively in this way. For all this, the students need to be allowed the 'space' - if a lecturer dominates the class most of the time, it will not happen, group work will fail.

The graphic calculator can serve as a focal point to get students to interact and discuss mathematics, particularly as there are not enough for each student to have one. In photo (6) all attention is directed at the calculator. Perhaps it makes it easier for shy students to be drawn into this interaction situation, where they do not need to take the initiative to seek interaction with their class mates (?).

Math-lab:
In the photographs of the mathematics laboratory, the group work is even more visible, due partly to the seating arrangements (around tables) and partly to the absence of a lecturer. the students are clearly interacting with one another. They are also obviously actively involved with the work. The way students are looking at each other in photo (4) is typical of the math-lab situation: two students are looking intently at someone, listening, thinking. The arrangement allows for eye-contact, which improves communication.

The math-lab is an interesting innovation, encouraging students to become independent learners, and 'owning' their work. It goes one step further than the 'workshop-lecture'. Here is even more freedom to form non-verbal agreements, and for students to control their own interaction, within the framework of the expectation of the course/lecturer that they work on mathematics and specific tasks.

APPENDIX 7

MATHEMATICS 114 YEAR: 1995

NAZO Nyameka

Solving problems algebraicly is good, quick and sometimes easy but it does not develop in students the ability to analyse thing, think critically and with originality.

Lectures, tutorials, lab sessions in this course are conducted in such a way that they do not focus only on the subject matter but to develop in students a sense of responsibity. This is noticed when some of the students leave behind to ask questions even if the tutorial session has come to an end.

The approach used in this course encourages students not to accept things without questioning. Students in this course see themselves part and responsible for their learning. Students develop an attitude of helping each other that means they see themselves as source of knowledge.

Resources like video tapes, graphic calculators used in the course make it easy for students to relate their learning to everyday life. Creative tutorial activities stimulates the students interest for the course. Maximum participation always prevails in class and during the tutorial sessions.

APPENDIX 8

Mathematics 114. 5/5/95.

The students seems to perform the tasks when given. They are very alert (most of them) – Corrected you when you misstated to say the function is continuous at 1 when you should have said 0.

What I notice that it is the same student which manipulate the calculator throughout the lecture. My concern here is around the following: if the grouping remain the same, and the same students manipulate the calculator: – is the other students in the group getting the full benefit? But on the other hand, is it not the students responsibility to see that other contribute to the working of the group? I think it is.

All the students in the first four rows were very attentive. In the fifth & sixth rows the students sitting in the centre were paying attention. The group (3 students) in the fifth row sitting on the fringes seemed to be fairly inattentive and also the group (4 students) in sixth row sitting to the far right were fairly inattentive.

After the break it appeared as if all the students were attentive again.

APPENDIX 9

MATHEMATICS 114: QUESTIONNAIRE 2

Think back to the first semester of this year and provide honest answers to the following questions. Your cooperation is deeply appreciated!

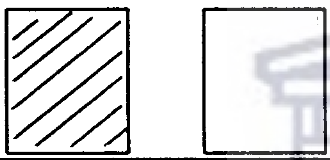
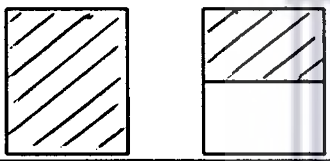
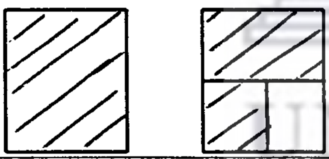
1. What did you think of the "workshop" approach to lectures?
2. Before you started studying at university, how did you expect to be taught?
3. Would you prefer the traditional way of lecturing?
4. Were the "workbook-notes" useful?
5. Was working with graphics calculator helpful?
6. How would you describe your feeling towards mathematics now?
7. What is your opinion of the course?
8. How difficult/easy was it to pass the course?
9. How do you think the course can be improved?
10. Anything else you wish to say about the course?

APPENDIX 10

LAB ON LIMITS

A: THE SKY IS THE LIMIT

Take two equal pieces of paper and carry out the process as illustrated in the first column. The sums (series) formed at each step by colouring in half of the remainder is noted in column 2, and the third column forms a sequence of the remaining uncoloured piece of paper. Continue the process for at least 10 steps, then answer the questions that follow.

	1	1
	$1 + \frac{1}{2}$	$\frac{1}{2}$
	$1 + \frac{1}{2} + \frac{1}{4}$	$\frac{1}{4}$

Questions:

1. Does this process stop?

2. Is the sequence in column 3 approaching any number? If so, what is it?

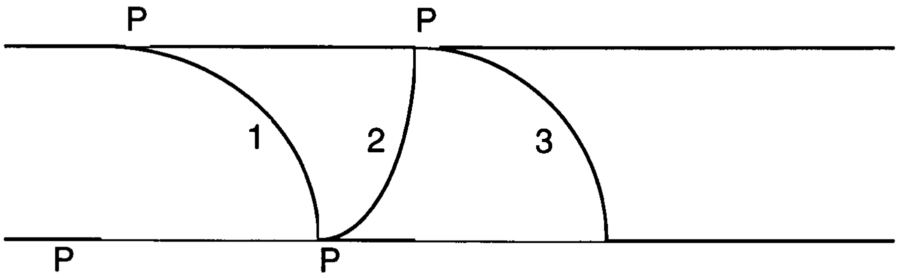
3. Referring to column 2: You can never reach 2 in this way! But ...

4. Make an estimation of the number of steps you need to have a sum of at least 1,999. ($2^{10} > 10^3$, $2^{10} = 1024$)

5. In how many steps can you, for sure, obtain a sum of at least 1,99999?

B: GROWING REGULARITY

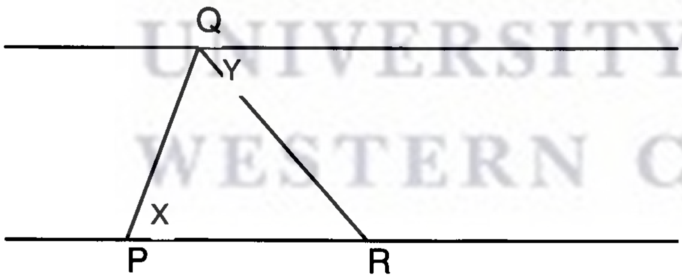
Look at the pattern in the diagramme below.



There are two starting points, P_1 and P_2, and circular arcs, drawn after each other: With your compass centred at P_1 and radius P_1P_2, draw arc 1 to obtain point P_3. Then with your compass centred at P2 and the same radius draw arc 2 to obtain P_4. And so on. The lines are parrallel.

1. Try the above process for yourself and see whether any regularity emerges if you go on for quite a while. Does this regularity depend on the starting points?

2. Here is one step in the process:



R is the last point found. Why is PQ = PR ?
If $x = 80°$, what is the value of y ?

3. Now take your TI-85 calculator. Make sure it is in degree mode.
Enter: 80
Enter: 90 - 0.5 * ANS
The ANS means: put the last answer there.

4. If you press [ENTER] again, the calculator will apply the same formula. Try it, say 10 times, and see what happens. Then go on until you seem to reach a limit.

   Does your observations agree with what you found in question 1?

5. We need the graph of the function

$$y = 90 - 0.5\,x \qquad (1)$$

   It tells you how the **next angle (y)** is found from the **current angle (x)**.

   Now in the following step, this next angle (y) becomes the current angle (x), so that it will also be necessary to have the graph of

$$y = x \qquad (2)$$

   Draw both graphs on the axis provided.

6. Mark a starting point on the x-axis for your first x and draw a line to graph (1) to obtain the first y. Now use graph (2) to find the next x. With this x find the next y. Repeat this process a few times.

   Does this process "home in" on any particular angle?

7. Which equation do you have to solve to obtain this limiting value? Does it give your the same answer?

8. Draw the graphs on the TI-85 using range of [0,90] by [0,180]. Then use the TRACE facility to find the value in 6.

If time allows, answer the questions on the next page.

9. Which answer below is correct:

(A) If your [ANS] is not 60, 90 - 0.5 * ANS is also not 60. You will never reach 60.

(B) If you press ENTER often enough you will reach 60 after some time.

10. We try to make (A) above more accurate:

Suppose your angle x is different from 60, say

$$x = 60 + d$$

then y will also differ from 60, say

$$y = 60 + e$$

Find a relation between e and d, by substituting these values for x and y in the formula linking x and y.

11. Conclusions:
If d is positive, e is __________ .
If d is negative, e is __________ .

But, always, $|e| = |d|$ * ____ .

12. Formulate your conclusions about:
(a) What happens with the angles **in the long run**!

(b) How the pattern looks **in the long run**!

APPENDIX 11

UWC, M114 (1995)

LIMIT AT A POINT OF A FUNCTION: A(8)

$$f(x) = \frac{x}{(x-1)^2}$$

1. Graph f on the GC using [-10, 10] by [-5, 5] and make a copy of the sketch in the space below:

2. Complete the two tables below:

x	f(x)
0.9	
0.99	
0.999	
....	
0.99999	
....	
0.9999999999	

x	f(x)
1.1	
1.01	
1.001	
....	
1.00001	
....	
1.0000000001	

3. Complete the following statements:

$$\lim_{x\to 1^+} \frac{x}{(x-1)^2} = \qquad\qquad \lim_{x\to 1^-} \frac{x}{(x-1)^2} =$$

$$\therefore \quad \lim_{x\to 1} \frac{x}{(x-1)^2}$$

4. Is $\lim_{x\to 1} \frac{x}{(x-1)^2} = f(1)$? Is f continuous at $x = 1$?