THE INTERPRETATION AND CONSTRUCTION OF GLOBAL GRAPHS BY ERITREAN GRADE NINE STUDENTS (AN EXPLORATORY STUDY)

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Declaration

I declare that **THE INTERPRETATION AND CONSTRUCTION OF GLOBAL GRAPHS BY ERITREAN GRADE NINE STUDENTS: AN EXPLORATORY STUDY** is my own work, that has not been submitted before for any degree or examination in any university, and that all the sources I have used or quoted have been indicated and acknowledged by complete references.

MENGISTEAB TEKLEMICHAEL JUNE 2000

Signed: . . . .

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ABSTRACT

In this study the interpretation and construction of global graphs by grade nine Eritrean students is explored in a qualitative research paradigm called developmental research. The experience of these students in an environment in which they were encouraged to work independent of the teacher is analysed. A situation was set up in which students worked individually as well as in groups for implementing the task-based activities with the accompanying research method. Successive observations of students' work in this environment were conducted. Their written works in this environment were collected for analysis.

The result of the analysis revealed the fact that students at this grade level were to some extent using their prior knowledge to plot points in the Cartesian co-ordinate plane. Furthermore, the analysis confirms that students' answers were based on the appearance of the graphs on the co-ordinate plane. Their interpretations were also directed towards graphing straight lines. In general it was observed that students predominantly used a visual strategy for solving problems in the task-based activities.

This study shows that to make the topics of graphing simple and interesting so that the general public can communicate with it, the concept of global features of graphs should be introduced in all grades in the form of a spiral curriculum. Since this study was conducted in one cycle, to make it complete and fruitful, successive cycles should be undertaken in the future.

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Chapter One

Introduction

1.1 Back ground Information about Eritrea

Eritrea is located in the northeastern part of Africa; bounded by the Sudan in the west and north, by Ethiopia in the south, by Djibouti in the southeast, and by over 1000km long Red Sea in the east. As described in the paper published by the Ministry of Education of Eritrea (MOE) (1997), the name Eritrea was derived from the Greek and Roman ancient names of the Red Sea. *"The Greeks used to call the Red Sea 'Sinus Aeritreyus' while the Romans came to know it as 'Mare Erytreum' that is the sea of Eritrea."*

Its size is about 124300 sq. km and there has not been any survey or census conducted in the country. However, some rough estimates put the country's population at about 3.5 million. The population is culturally and linguistically diverse. There are nine ethnic groups each having their own language.

During the war for liberation, Eritrea suffered utter damage in all aspects. The Eritrean people have inherited a dilapidated infrastructure. Moreover the struggle has displaced its population specifically; it has caused a serious brain drain, as Eritrean specialists of all kind were obliged to leave their country. Even though the people inherited a dilapidated infrastructure, after liberation progress has been attained in reconstructing all the sectors.

1.2 The education sector

The existing education system as described by MOE is *"Five-Two-Four system."* The compulsory cycle, which is the primary level, is composed of five years of elementary and two years of middle level schooling. This is followed by four years of secondary level.

1.2.1 Goal of education at the secondary level

The declaration of policy on Education issued by the government of Eritrea (1991:2) as stated by Habtai (1994) is that education at the secondary level would be used to prepare *"productive citizens through practical oriented provision and to meet the demand for skilled manpower."*

1.2.2 Mathematics Education in Eritrea

Mathematics has a conspicuous place in the school curricula of many countries. In Eritrea, mathematics is one of the basic academic subjects at all levels. However, it is regarded by many as *"a system of abstract ideas and relationships"* (Mathematics Panel, 1993) and there are many deeply rooted misconceptions about mathematics along with the attached fears among students and the general public. The opinion is that it is separated from the real world and can be mastered by only a very few. The belief was further strengthened by the introduction of new mathematics in many parts of the world.

Unless school learning becomes more relevant and approachable to the majority, it will be impossible for schools to generate favourable attitudes towards mathematics.

The teaching of mathematics in our schools is a continuation of the old style. That is, the talk and chalk method. Having these difficulties in mind, the reshaping of the current syllabus, textbooks and methodology must work towards enabling students to gain a fuller and more lasting understanding of the basics of mathematics (Mathematics Panel, 1993).

The following are excerpts from the unpublished paper on the role of mathematics education in Eritrea

- *To develop mathematical skills among pupils which will enable them to function in all practical affairs of life.*

- *To develop the ability of pupils to discover mathematical concepts and ideas and also their ability to think logically.*

- *To develop in pupils positive attitude towards the subject and thus enjoy learning it* (Mathematics Panel, 1993).

1.3 Statement of the problem

I am a qualified mathematics teacher and have been teaching for the last seventeen years. During this time, I have taught at four secondary schools, starting from grade eight through grade twelve. The normal practice at schools where I taught, and in almost all schools in the country, was teacher-centred. The teacher did everything starting from explaining the topic up to solving the problems to consolidate the topic.

Whenever students were given exercises in the form of classwork or homework, very few students completed or attempted them. Others considered themselves as if they know nothing and expecting that everything to be written on the chalkboard by the teacher for the students to copy.

In relation to mathematics results in the tests and final examinations at the schools where I taught and in other schools were extremely poor. When some students were asked about their poor performance, they replied as follows. *"Even if I study hard, I could not understand mathematics."* Some others said: *"I hate the subject."*

Though most of the mathematics teachers in schools where I taught were depressed by what we saw, we were also motivated by the activities of handful of students who were struggling with mathematical skills and concepts. Despite my interest to help students, this was seldom possible because of the number of students who needed individual assistance and the pressure from the school administration to complete the syllabus in a fixed time limit.

Whether you are teaching in grade eight or in grade twelve it was not possible to leave some of the problems to students to try themselves during their recess time. Even the teacher accepts that everything must be written on the chalkboard by the teacher. Such belief was the source of weakness for the majority of students not to be active participants in the mathematics class. Sometimes in answering questions raised by students especially, when their solutions were to be evaluated during the mathematics class, it was necessary for me to go back to the basics of the topic. But this was difficult due to the following reasons:

1. the number of students in one class is large. For example, in a school where I was teaching there were minimum of 80 students per class;

2. the class itself is of mixed ability groups;

3. an overloaded syllabus with too much content;

4. because of the problem stated in number one, lack of guidance to independent work both inside and outside the class; and

5. frequent absenteeism by a number of students.

In the mathematics department it was part and parcel of our routine work to ask advice, exchange ideas, share experiences, sharing teaching aids and conducting classroom observations among each other while one of us would teach a section of the syllabus. This kind of sharing experiences gave me access to the teaching practices of each of the members of the mathematics department in those schools where I taught. From such interactions I did observe that we all were having almost similar teaching methods.

Teachers were complaining about the majority of the students being unable to do independent work. That is, students always required all the steps for solving problems which the teacher had already explained.

I realised that the teaching method we were using was a factor for students' poor performance in every activity and for being dependent on the teacher. To remedy the above difficulty, a method, which enhance students' independent work, should be investigated. Students' independent work could specifically be helpful in the following ways.

- they will be able to confidently evaluate their own work; and

- The teacher in turn will be able to provide the necessary assistance to those students who badly need it.

1.4 Motivation of the study

Now the time has come to work with the idea, which I had in my mind for a long period of time. I was seeking for some approach that would develop self-confidence in the students; and in the process, change the focus from teacher-centred to student participation in the learning process. The dream turned into being because of the motivating factors that were released by the University of Asmara (UOA) in the year 1998.

From a study done by the University of Asmara in 1998, on the item analysis of the 1995-1997 of the Eritrean Schools Education Certificate Examination (ESECE) *"of a particular concern is the very low average for the mathematics suggesting that this course seems to be very difficult."* (University of Asmara, 1998: 13) Furthermore, the report comments on the contents of Mathematics and other textbooks as:

There is too much knowledge given but not enough in the approaches of analysis or ways of thinking. Emphasis is not *given to the Eritrean situations and numbers of modern subjects relevant to the high school are not included* (University of Asmara, 1998).

With the problems suggested by the UOA and from my long experience as a mathematics teacher, I observed that many of the students in secondary schools were facing difficulties to challenge the problems given in the textbooks of the respective grades. With all these and other problems related to mathematics, I was not in a position to fix my research topic at a particular issue from the difficulties and problems that were faced by many students. The reason was clear. Students face so many difficulties and problems and it was impossible to treat all those difficulties and problems at the same time. After my arrival at the University of the Western Cape (UWC), I was impressed by the research on global graphs organised by Realistic Mathematics Education in South Africa (REMESA) project, which was one of the motivating factors initiating me to conduct my study in this area.

I believe that the system used in the above research could pave the way to make my dream a reality. Students' difficulties were not restricted to one specific area in the field of mathematics. So it was reasonable to restrict myself to a very specific issue which seems to be simple and trivial but a source of error in most students' work.

- How do students interpret and construct graphs?
- What are the basic sources of errors, while students' are interpreting and constructing graphs?
- Do students use their prior knowledge in interpreting and constructing graphs?

- How do we change the existing teacher-student relationship, which was a transmission of knowledge from the teacher to the student?
- How do we enable students to change their attitude towards mathematics?
- How do we make them active participants in the mathematics class?

These were some of the questions that were raised during our departmental meeting, though we were not able to find an everlasting solution. I hope my study will pave the way for teachers to run a study in different areas around the current difficulties of students. Because of these and other burning issues related to the mathematics curriculum, such as, shortage of qualified mathematics teachers, there was a workshop held in Asmara in August 1998. I was one of the participants. In that workshop the participants were discussing about what should be done in order to make the presentation in the text book as simple as possible so that students can grasp the ideas and concepts easily and apply it to solving problems related to mathematics and related subjects. After a long and hot discussion, all participants were coming to a single conclusion that every mathematics teacher in all parts of Eritrea must forward his/her comments to the mathematics panel of the curriculum unit of Eritrea. I believe that in order to have an understanding of mathematics, the mathematics itself should start from the pupils' day to day experiences. That is, it should start from what the pupils can understand. This is supported by De Lange (1993) as cited by Julie et al (1998:38) " . . . *mathematics education philosophy must start off in the real world and that mathematics education should search for its problems and* *questions in reality."*

From the researches done in different countries, I am motivated to undertake this research. For example, Lillian et al (1986), in the analysis of graphing errors, identified that *"many are a direct consequence of an inability to make connections between a graphical representation and the subject matter it represents."* In their study they observed two major difficulties. They are difficulty in connecting graphs to physical concepts and difficulty in connecting graphs to the real world.

Though there are a lot of problems in the present mathematics textbooks of Eritrea, I am interested to focus on one issue, that is pupils' difficulty in interpreting and constructing graphs. Such problems are common problems in different countries such as, South Africa. As cited by Julie et al (1998:37)

> *The present curriculum deals with specialised graphing techniques . . . However, there is a lack of emphasis on the understanding and interpretation of the global feature of the graphs like the general shape of the graph, intervals of rise and fall or of a maximum increase.*

The present Eritrean mathematics textbook is in its pilot stage, and it is open to constructive criticism. Hence, as an Eritrean citizen and as my duty and responsibility of being a mathematics teacher, I have decided to make my research on construction and interpretation of global graphs in the Eritrean context. I am hopeful that the result obtained from this research may contribute to update the text.

1.5 Limitation of the Study

This study has its own limitations due to the following basic reasons.

1 Students' participation was limited because of the coincidence of harvesting time and data collection time.

2 Limited prior research about Eritrean mathematics education.

1.6 Summary

In this first chapter I have tried to give an overview of the background information of Eritrea in relation to its geographical location, the estimated population and the education sector with a specific focus on mathematics education. Beside this, I also explicated the statement of the problem and the motivation for this study both from the existing condition in Eritrea and from the experience of other countries found in the literature. Furthermore I mentioned some of the limitations of this study.

The second chapter deals with the literature that underpins the theoretical constructs of this study. Chapter three deals with the methods of data collection and procedures followed by chapter four, which is the analysis part. Discussions, conclusions and recommendations are treated in chapter five.

Chapter Two

Literature Review and Theoretical Framework

In this chapter the theoretical constructs underlying this study are given.

2.1 Importance of global graphs in school mathematics

A lot has been said about the importance of graphs in school mathematics in general. Among these I would like to present some of them as a basis for my study. Julie et al., (1998) described the importance of graphs as:

> *Graphs are commonly used forms to represent information, which is closely related to the adage "a picture is worth a thousand words." Because, graphs are related with information, any citizen needs to have a firm grasp of construction and critical interpretation of information.*

Graphs occupy an important position in school mathematics. Examples of this can be seen from the prominence of graphs in many branches of mathematics. Giordano and Weir (1985) justify the importance and value of graphical representation as follows.

A graphical model has the important advantage of appealing to one visual intuition. It gives us a picture and a 'feel' for what is happening that often eludes us in more symbolic analysis. Graphs are very good for gaining qualitative information . . . Graphical analysis is also a useful prelude *to a more detailed analytical model often providing clues to which factors should be considered more thoroughly in subsequent analysis* (Giordano & Weir, 1985:3).

Leinhardt et al., (1990:2) has also described graphs and graphing as

> *. . . Graphs represent one of the earliest points in mathematics at which a student use one symbolic system to expand and understand another . . .e.g. algebraic functions and their graphs. Graphing can be seen as one of the critical moments in early mathematics.*

According to Shuard and Neill (1977:xii) "graphs are used in order to convey a simple pictorial and immediate way ideas which otherwise require many words, figures or symbols to portray." They also further describe about the importance of graphs in schools as:

> *. . . To extract as much as possible from graphical messages, pupils must learn to read the messages and to become fluent in the vocabulary of graphing language . . . All pupils whatever their ability, should learn to read as many of the messages contained in graphs as they can, for otherwise they will be deprived of communication of proven use* (Shuard and Neill, 1977: xii).

Julie et al., (1998:37) further explained about graphs as:

> *In addition to the usefulness in mathematics and applied mathematics graphs are probably second to numbers as a representational form through which information of a*

mathematical nature is conveyed to the general public (Julie et al 1998:37).

Most text approach graphing as information display, usually in the form of bar graphs, pictographs, circle graphs, and line graphs (Stein & Baxter, 1989).

Recent works in assessing what students know and do not know about graphing reflects a growing sensitivity to the importance of students' prior knowledge (Confrey, 1990, 1993; Smith et al.,1993) cited by (Mevarech and Cramarsky, 1997).

The literature on graphs can be considered from several perspectives. This study was conducted:

- *to consider an analysis of the tasks and their presentations;*

- *to consider the learner and the development of understanding about graphs* (Leinhardt et al., 1990: 4).

2.2 Classification of tasks related to graphs

Most of the tasks that relate to graphs and functions can be classified into two main categories: interpretation and construction. These are not mutually exclusive categories. Much of the existing studies look mainly at interpretation tasks. In this study interpretations and constructions of graphs are analysed in a qualitative dimension.

2.2.1 Interpretations of Graphs

At present it seems necessary to define what global graph mean. According to (Leinhardt et al., 1990:9) *global graph in its fullest sense means whole graph reading.*

According to Leinhardt et al., (1990), interpretation usually refers to students' ability to read a graph and make sense or gain meaning from it. Leinhardt et al., further describe that "*interpretation can be global and general or it can be local and specific.*" In relation to information processing related to graphs, Wainer (1992) identified three levels of information processing.

a) The elementary level involving data extraction.
b) The intermediate level involving trends seen in parts of the graph, and
c) The overall level involving an understanding of the data.

Interpretation actions are subject to both over and under generalisation and to confusion or confounding with pictorial events of a similar nature (Leinhardt et al., 1990:8).
During interpretation action the kind of interpretation required of the students depends largely on what the graph represents (Julie et al., (1998); Leinhardt et al., (1990)). Leinhardt et al., further explicate that, the graph that are used in studies represent either a situation[1] or an abstract functional relationship (usually expressed by an equation and sometimes represented by a table of ordered pairs) or they are

[1] Situation encompasses two aspects: the surrounding setting and the context of the problem. The first aspect of a situation is the setting in which the task is presented, such as a mathematics lesson, a social studies class, or a science laboratory activity (McKenzie & Padilla, 1986).

considered entities in their own right. Depending on what the graph stands for the meaning gained by interpretation can either reside within the symbolic space of the graph or it can shift to a different space (the situation space or the algebraic space).

Interpretation that requires moving form one representation form to another is according to Janvier (1987d) and Caput (1987c) a type of translation (Leinhardt et al., 1990:8). Interpretation tasks tend to involve graphs that represent situations (Janvier 1980, 1981a, 1981b; Preece, 1988a). Given a specific graph representing a situation, there are a variety of questions asked; the interpretation can be a local process. (For example, one regarding point-by-point attention) or a more global one (e.g. trend detection).

There are many global features of a graph that can be interpreted. These include general shape of the graph, interval of increase or decease. And intervals of extreme increase or decrease (Julie et al., 1998; Leinhardt et al., 1990). It should be noted that one could attend to global features of a graph whether the graph represents a specific situation or whether the graph represents an abstract functional relationship. Another dimension along which interpretation tasks can be analysed is their progression from a quantitative to qualitative interpretation. According to Leinhardt et al., (1990:11)

> *Qualitative interpretation of a graph in its fullest sense requires looking at the entire graph (or part of it) and gaining meaning about the relationship between the two variables and in particular, their pattern of co-*

variation (Leinhardt et al., 1990:11).

Qualitative interpretation often is associated with global features. Though global features can be interpreted either quantitatively or qualitatively, it is not accustomed to interpret local features qualitatively, except for dramatic change of shape, rate or direction.

To summarise, interpretation actions can vary depending on where most of the action takes place entirely within the graph or telling the behaviour of the graph at a certain point or at infinity.

2.2.2 Construction of graphs

Construction, as to Leinhardt et al., (1990:12)

> *It refers to the act of generating something new, to building a graph or plotting points from data. (From a function rule or a table) or to building an algebraic function for a graph. In its fullest sense, construction involves going from raw data (or abstract function) through the process of selection and labelling of axes, selection of scale, identification of unit and plotting. In the past construction has been rather a tedious, point-by-point undertaking* (Leinhardt et al., 1990:12).

However with the advent of technology, part of these actions can be performed by a computer in a more global fashion and need not be the task of the student. It is important to note, however that "*releasing the student from constructing a graph from scratch with paper and pencil alters the task domain* *enormously"* (Leinhardt et al., 1990:). *"Construction can be fairly simple or quite difficult"* (Kerslake, 1981). For example, plotting points from a table of ordered pairs, once the axes and scales are set up is quite straightforward. Leinhardt et al., (1990:12) stressed that,

> *Construction is quite different from interpretation. Whereas interpretation relies on and requires reaction to a given piece of data (e.g., a graph, an equation, or a data set), construction requires generating new parts that are not given* (Leinhardt et al., 1990:12).

There are four typical tasks that involve interpretation and construction.

- *Prediction tasks which rely mostly on construction and address the issue of pattern.*

- *Classification tasks, which require interpretation and address the definition and special properties of functions.*

- *Translation tasks, which can be either construction or interpretation tasks.*

- *Scaling tasks.* (Leinhardt et al., 1990:4)

2.3 Defining some terms related to interpretation and construction of graphs.

After defining what interpretations and constructions of graphs are, it is better to come across the four kinds of tasks that involve interpretation and/or construction. They are prediction, classification, translation and scaling. For this study, I would like to focus on the literature on prediction and scaling tasks.

2.3.1 Prediction tasks

According to Leinhardt et al (1990:13)

> *Prediction refers to the action of guessing from a given part of a graph where other points (not explicitly given or plotted) of the graph should be located or how other parts of the graph should look.*

At the heart of most prediction tasks is an action of construction, which can be done either physically or mentally. Bell & Janvier (1981) explained that, some prediction tasks rely primarily on estimation and to some extent on measurement skills where others depend on pattern detection. The characteristic of many prediction tasks is that they can not be tested. In addition they cannot have one correct answer (Lienhardt et al, 1990: 14). *"Prediction tasks vary with respect to the extent of diversity of acceptable answers"* (Bell & Janvier, 1981).

2.3.2 Scaling Tasks

As to Lienhardt et al., (1990), scaling tasks are typical of the domain of functions and graphs (especially to graphing). Scaling tasks require particular attention to the axes and their scales and to the units that are measured. The issue of scale becomes more fundamental when using graphical techniques

(Goldenberg et al., 1988; Heid et al., 1988; Yerushalmy, 1988) as cited by (Leinhardt et al., 1990). Scaling tasks can encompass either interpretation or construction tasks. Leinhardt et al., (1990:19), further stressed that:

> *A graph cannot be interpreted fully, without taking into account its scales. A full understanding of a graph mean realising what visual features of the graph will not change under the change of scales and what features change when the scales are altered* (Leinhardt et al., 1990:19).

The change of scales is one of the main sources of graphical visual illusions (Goldenberg et al., 1988). According to Kerslake (1981) and Wavering (1985) scaling tasks can rely primarily on construction, such as setting up the axes to plot a graph from a scratch.

2.3.3 Conceptions

Students' conceptions are used to represent students' knowledge that is in accord with the accepted meaning. According to Leinhardt et al., (1990):

> *Conceptions are features of student's knowledge about specific and usually instructed piece of mathematics. They are meaningful ideas that student's develop that can serve as powerful workhorses in students continued efforts to reach deeper more integrative levels of understanding* (Leinhardt et al., 1990: 6).

Conceptions are contained in the domain in the sense that they consist of content that is specific to understanding in a particular domain. By nature conceptions are in translation or in the process of being fleshed out to their fullest realisation or capacity. This is why they can at times appear to be fragile, situated, bounded or misapplied. Nevertheless, they are well enough structured to do work for the student and explicit enough to be the object of communication with others.

In their ideal form conceptions are highly interactive, they cannot easily fit into the intuitive structure. Intuitions are features of student's knowledge that arise commonly from everyday experience. In general they are seen to exist prior to specific formal instruction. The most recent thinking in mathematics education views intuitions as positive and as occasions around which to build instruction and learning (Fischbein, 1973, 1978; Resnick, 1989) cited by (Leinhardt et al., 1990:5).

When those conceptions are deemed to be in conflict with the accepted meanings, various terms have been used in the literature including the following: alternative conceptions (e.g., Confrey, 1990; Mevarech et al., 1997), misconceptions (Julie et al., 1998; Leinhardt et al., 1990), systematic errors (Brown and Vanlehn, 1980). The different terms reflect different perspectives of students' knowledge. For this study, I use the term "misconception".

2.3.4 Misconceptions

As to Leinhardt et al., (1990:5):

Misconceptions are features of student's knowledge about *specific piece of mathematics knowledge that may or may not have been instructed. A misconception may develop as a result of over generalising an essentially correct conception, or may be due to interference from everyday knowledge. To qualify a misconception must have_ a reasonably well-formulated system of ideas not simply a justification for an error* (Leinhardt et al., 1990:5).

Some misconceptions can be traced logically to intuitions (student's tendency to interpret graphs iconically may be related to their intuitions regarding picture reading). Other misconceptions can be interpreted as a result of incomplete formal learning.

2.4 Difficulties related to attempts to construct and interpret graphs that represent situations

Although graphing has long been considered a fundamental part of mathematics and science curriculum, recent studies have indicated that students' understanding of graphs are limited. Many students have difficulties when asked to shift between the different modes of presentations (Barson & Rowe, 1993; Modvhkovich et al 1993; Yerushalmy 1991) as cited by (Mevarech and Kramarsky 1997:229), others cannot use graphs for either imparting or extracting information (Wainer 1992). Still others cannot apply what they have learned about graphs in mathematics classes to physics and/or other subject matter. (Mc Dermott, Rosenquist and Vanzee, 1987). Most secondary pupils are weak in the ability to interpret global graphical features so as to extract information about many everyday scientific situations (Bell and Janvier, 1981).

There is ample evidence that difficulties relating to graph interpretation are widespread, manifest at all levels of education including students attending an honour section of calculus-based university physics course. (Beichner 1989; Padilla et al., 1980; Schneider, 1993) cited by (Mavarech et al., 1997).

The advantage of many graphs is that their patterns and shapes highlights feature of the underlying situations that otherwise would be hard to detect.

Nevertheless when examining graphs of situations students often restrict their focus to an individual point or group of points as opposed to the more global features of the graph such as the general shape, interval of rise or fall, and so forth (Bell & Janvier, 1981). Janvier (1978, 1981a, 1981b) has suggested that this pointwise focus is not surprising, given traditional instruction in which students are asked to plot a graph from a table of ordered pairs and then are presented with a series of questions that can be answered by the table alone. Under these conditions, students typically use graphs in much the same way as they use tables to transmit specific piece of information. Little or no attention is devoted to elaborating the properties of the underlying situations.

The difficulties and misconceptions associated with students learning to focus more broadly on the overall shape of the graph or parts of the graph are classified under three categories.

1. **Interval/point confusion:** As they interpret graphs students often narrow their focus to a single point even

though a range of points (an interval) is more appropriate. This is most apt to occur when the wording of a question is ambiguous.

For example, with respect to-the graph referring to the age versus average weight of boys and girls on the same co-ordinate plane, Preece (1983b) found that; students often responded with a single point when asked the questions, *"when are girls heavier than boys?"* and *"when are girls growing faster than boys?"* According to Preece (1983b), students found the word "when" to be rather imprecise and *"took the easiest option and gave a single point as the answer."* Given the ambiguity of the word "when", it must be granted that technically these students were correct. Nevertheless their focus on overall tendency to interpret graphs pointwise.

2. **Slope/height confusion:** There are numerous examples in the literature of students' confusing gradients with the maximum or minimum values (Bell & Janvier, 1981; Janvier, 1978; McDermott et al., 1987; Preece, 1983b). Students have been found to confuse these two graphical features on both interpretation and construction tasks.

3. **Iconic Interpretation:** the most frequently cited students error with respect to interpreting and constructing graphs that represent situations is iconic interpretation. A host of findings support the notion that students some times interpret a graph of a situation as a literal picture of that situation (Janvier, 1978; Kerslake, 1977, 1981; McDermott et al., 1987; Stein and Leinhardt, 1989). A frequently cited finding in this regard is student interpretation of travel graphs as the path of the actual journey (Kerslake, 1981).

Clement (1989) has called attention to two kinds of errors that students often make when dealing with graphs of situations: features correspondence errors and global correspondence errors.

Several studies have pointed to students' tendency to tend toward linearity in a variety of situations. Markovits et al., (1986) found that when asked to generate examples of graphs of functions that would pass through two given points students produced mostly linear graphs. This tendency towards linearity also has been displayed on tasks that focus on more than two points (Dreyfus & Eisenberg, 1983).

Scaling issues also arise in the interpretation of graphs. The inclination and shape of a graph are, to a great extent, dependent on the coordinate system. Learners need to develop an understanding of which features of a graph are indigenous to the graph itself (e.g., the y-intercept) and which features are responsive to the system on which it is constructed. (For example, the slope of the graph). In this situation, Kerslake (1981) investigated the degree to which students understood the effect that changing the scale of the axes would have on the appearance of the graph. Yerushalmy (1989) analysed the type of explanations that students gave for their answers to provide insight into their relatively good performances. On average four times as many explanations were based on computations as opposed to visual considerations.

2.5 Summary

An overview of the importance of graphs in schools and about the interpretation and construction of global graphs was broadly given. Furthermore, what prediction, scaling, translation and classification tasks are discussed. In addition to these, varieties of misconceptions and difficulties revealed in different studies were presented as a reference for the ongoing study. Particular attentions are given to the literatures of Julie et al., 1998; Kerslake, 1981; Mevarech et al., 1997; Leinhardt et al., 1990, since the tasks for this study were developed based on the results from their studies.

This literature review helped me to focus on issues, which look straightforward but are very critical in relation to students' ability referring to interpretation and construction of global features of graphs.

Chapter Three

Research methodology

3.1 Arriving at a research strategy

The broad aim of this study is to investigate students' ability in interpreting and constructing global graphs accompanied by inculcating an investigative nature and self-confidence in learners. Furthermore, it aims at promoting students' independence from teacher assistance and thus to change the focus from teacher-centredness to student participation in the learning process.

Given the exploratory nature of this study, there was a need for a research methodology that would be open to change. For this purpose Developmental Research seems to be well suited as a research methodology to gather information about students ability and experience.

Although there are many differences regarding the details, nature and characteristics of developmental research, Gravemeijer (1994) explicates the following characteristic of developmental research:

- direct contribution for the improvement of practice;
- great emphasis on subject matter;
- importance of the relationship between theory and practice;
- diverse methods of data collection;
- co-operation and collaboration of the researcher and practitioner: and

- Cyclical process.

My duty and responsibility during data collection was to:

- develop materials which was to be used in the class;
- make observations and make audio recordings of students' conversations whilst they are involved with the tasks;
- keeping discussions focussed on the topics, which appear on the worksheets;
- take observational notes during the whole implementation process. Where possible observational notes were also taken by teachers for triangulation purposes.

3.2 Sample

The implementation period of the study coincided with the harvesting period in the country. That was from the end of November onwards. Consequently, almost all students from grade eight through grade eleven were on the harvesting campaign for three consecutive weeks. After their return from the campaign some students volunteered to participate in the study.

Those students participating in the study were of mixed ability. Some of them were scoring the highest marks in mathematics examinations in the previous grades. Some other students were average learners. Still there were other students who were scoring below average in mathematics examinations in the previous grades.

3.3 Data Collection

Data were collected in the following ways:

- Students were requested to do all their work on the worksheets provided. If necessary, they were provided with some other blank papers. The written works of the students were collected.

- They were asked to explain what they did using the local language "Tigrigna." This was audio recorded. The reason for doing so was that, English language is used as a medium of instruction in middle and senior secondary schools. Students as well as the teachers were exposed to this language only during the teaching and learning process inside the classroom. Hence, most of them were having a difficulty for expressing themselves in English. So audio recordings were used to overcome the difficulties created by language problems.

- Personal observational notes; and

- Observational notes of colleagues.

3.4 The Environment Constructed

Since students' ability for constructing and interpreting graphs is the focus of attention of this study, there had to be an environment which complements the research method and vice versa. With this in mind, an environment was constructed to promote student-student interaction, which was the basis of action within the environment. Great emphasis was given to the activities of students who were actively involved in the tasks delivered. These tasks were questions that would reflect students' understanding of interpretations and constructions of graphs. The tasks in the worksheets primarily included points in a co-ordinate plane, increasing, decreasing, constant and curvilinear functions, and also graphs of situations.

3.4.1 Tasks and the purposes of each task

Mathematical tasks are central to students' learning because "*tasks convey messages about what mathematics is and what doing mathematics entails*"(NCTM, 1991:24). "When students 'do mathematics' in the classroom, the activity has most likely not occurred in a vacuum" (Henningsen and Stein, 1997: 526).

The tasks given to students were divided into five sections, for the following reasons:

- The first section starts with assessing students' ability in using their prior knowledge in finding solutions to the problems given in the task.
- The second section was designed to investigate students' ability in interpreting graphs.
- The third section was referring to the investigation of students' ability in constructing graphs.
- The fourth section was focussing on both interpretation and construction of graphs.
- The last section was to assess students' ability in relating the concept of graphing with other subjects.

3.4.2 Procedure

Students were asked to work in groups of five or six, in some cases they were told to work individually. They were also free to suggest other possible options.

When the number of students was over populated (unmanageable class size), they were conducting their work in different sessions.

There was no restriction in using other materials, which could assist them in performing the task delivered. For that matter, some of them were usually using textbooks and exercise books of the previous grades as reference materials.

At the start of the program the students were urged to read the worksheets prior to any activity, and then continue with the problem. In such circumstances my duty was to register the time elapsed in reading the task and also to register the time taken during discussion and the time taken to complete the task either individually or in group.

One task was delivered in each session. After the completion of the task for that day, most of the time they were asked to explain their work using the local language "Tigrigna" to check the reliability of their work.

After the procedure described above, students' were given the chance to ask any question related to the problems, which they deemed to be difficult. In such cases I was acting as a teacher, otherwise I deemed myself as a mere facilitator and observer.

3.5 Developmental Research

The core concept of developmental research is that curriculum development is not divorced from the goal of changing educational practice, which is the improvement of practice. So, from Freudenthal's position that the purpose of educational research is "*change*". Freudenthal (1991) stresses that in educational practice, research and curriculum development should not be separated. Freudenthal's conception of educational development thus embeds curriculum development in a more holistic framework "*which embraces all the development activities and interventions between the initial idea and an actual change in educational practice*" (Gravemeijer, 1994:444).

Fundamental to Freudenthal's concept of developmental research is that in educational practice; research and development should be interwoven by their cyclical alternation. Gravemeijer (1994: 449) describes a cycle as follows:

> *What is invented behind the desk is immediately put into practice; what happens in the classroom is consequently analysed, and the result of this analysis is used to continue the development work.*

The cyclic alternation of research and development in this way provides a greater synthesis between what Kannemeyer (1996:27) quoting from Freudenthal (1991:160) refers to as "*development ensuing from research*" and "*research as fall-out of development.*" To secure that research and development are not separated, educational development must take place within the educational environment. Developmental research deals with *"research in, rather than on mathematics education, not in order to exclude the latter, but to emphasise the former"* (Freudenthal, 1991: 158).

Gravemeijer (1994:449) quoting Freudenthal (1988) *"that thought experiments are important in educational development"* stresses the importance of thought experiment in the *"behind the desk"* phase. The developer will formulate instructional design with respect to the teaching-learning processes and afterwards he/she will try to find evidence in a teaching experiment that shows whether the expectations were right or wrong. Feedback from the teaching experiment results in new thought experiments bringing about the iterative character of developmental research. According to Freudenthal (1991) *"the cyclic alternation of thought experiment and practical experiment is interpreted as theory development: which can be seen as sediments of a local instructional theory."* Which on the one hand provides a general framework for local instruction theories, and on the other hand develops during the process of research and development. According to Gravemeijer (1994:449) *"This theory 'guides developmental work's functions as a basis for a learning process by the developer."*

In developmental research, the contribution coming from a student's own construction and interpretation do have a great role in developing global theories. Similarly, explicit negotiation, intervention, discussion, co-operation and evaluation are essential elements in which the student's informal methods are used as a lever to attain the formal ones.

Developmental research is situated within the sphere of qualitative research. The experimental experiences are subject of an interpretative process. An important step in the analysis and interpretation of the mainly qualitative data is according to Gravemeijer quoting Smalling (1990), *"the construction of categories of data and the construction of concepts"* (1994:454).

Another essential feature of developmental research is the issue of dissemination. Dissemination cannot be divorced from the developmental process. Towards this end, knowledge of the processes that give life to an innovation is essential. According to Gravemeijer (1994:452):

> *An educational experiment can not be repeated in the same manner under the same conditions. Therefore new knowledge will have to be legitimised by the process by which, this knowledge was gained.*

Developmental research is thus an integrated approach to curriculum development with change or improvement of practice as its major goal. By taking the developmental research approach, I intended to extract and explore the ways in which the interpretation and construction of global graphs affords students the opportunity to engage in authentic mathematical practice as a means of enhancing their learning of mathematics.

3.6 Summary

In this chapter, I discussed with what developmental research is and some characteristics of developmental research, which are helpful for organising and analysing data. Furthermore, the difficulties faced during data collection time were mentioned.

As Gravemeijer (1994:444) stated, curriculum development is embedded in a holistic framework, defined as *"educational development"*, which holds all the developmental activities and interventions between the initial idea and an actual change in educational practice.

Chapter Four

Data Analysis

4.1 Introduction

In this study, the data were collected from students of grade nine in one secondary school. After explaining the aim of the research to the class containing 71 students, eighteen students volunteered to participate, out of which five were female. Students' age varied from 15 through 19. Male students are represented by the letter Mi and female students are represented by the letter Fi, where i is an identification number.

Collection of data was carried in the afternoon shift in the science laboratory. Two teachers were assisting me in observing some activities at different times. After the completion of some of the tasks, students were asked to clarify their work.

At times there were no other teachers assisting me and the number of students became so large it was difficult to manage, I was then obliged to give the task in two different sessions, one after the other. Some of the tasks were given as group work and some of them given as individual work. The discussions held by one group was audio recorded. All the tasks were given in English; and students were communicating dominantly in the local language "Tigrigna."

4.2 Tasks given to students, presentation and analysis of students' work

4.2.1 A task on investigating students prior knowledge

Task One

The first task delivered to students was the following.

A Plot the points (1,3), (3,7), (4, 9), and (2, 5).

B Draw the line joining these points. Find some other points on the line and write them down.

C Plot the points (2 ½, 6), and (4.6, 10.2). Are there any points on the line between the points (1, 3), and (3, 7)?

Available materials for task 1

Available equipment were pencils, rulers, and duplicating paper. Graph paper were not provided, because, I was eager to see how they were using the available materials to do the task.

Purpose of the task

With this task, I investigated students' understanding of constructing and interpreting graphs based on their prior knowledge. By prior knowledge, I was referring to:

- Plotting the co-ordinate axes and naming each axis.
- Writing the co-ordinates of points in ordered pairs according to the general convention. That is in the form of (x, y).
- Determining the slope of the line.
- Finding the equation of a line passing through two distinct points.
- Finally, drawing the graph, referring to the task delivered, using the above mentioned ideas (concepts).

The following were the topics treated by students in their grade eight mathematics in relation to graphs of linear functions.

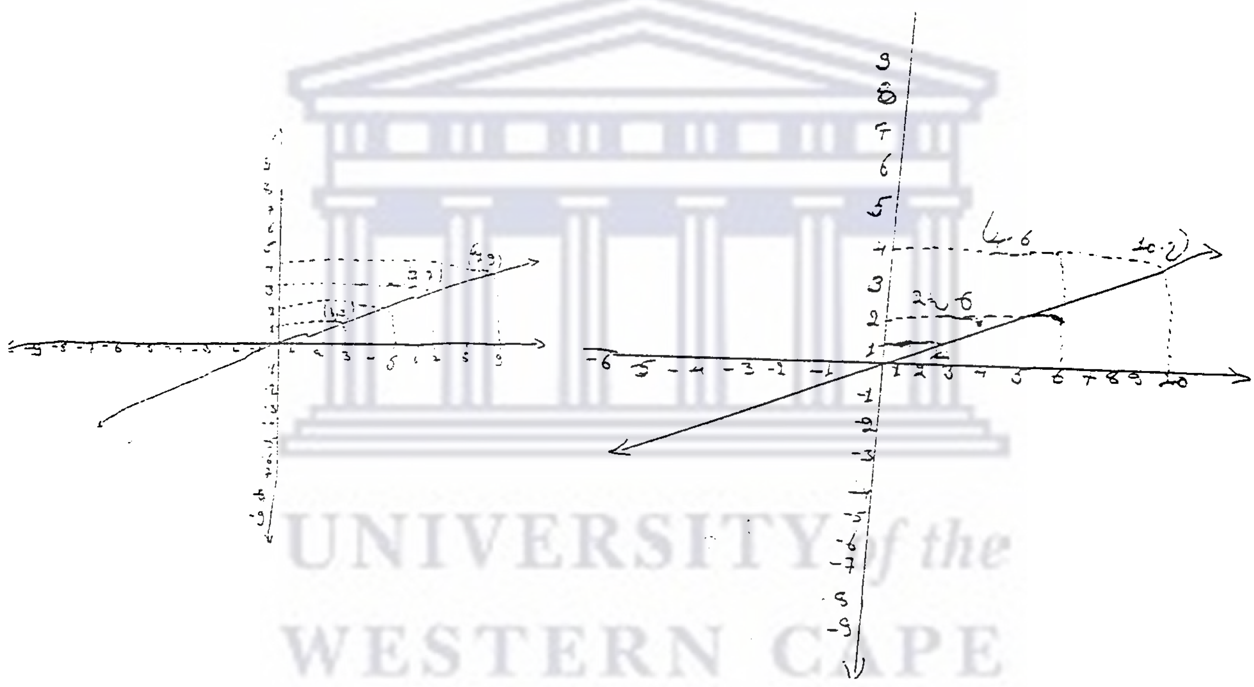
- Graphs of linear equations
- Graphs of linear equations using intercepts.
- Slope of a line.
- The equation of a straight line (slope-intercept form).
- Describing and Plotting Points on the Co-ordinate Axes (In grade seven and grade eight).

Presentation of Data

The first task delivered was to be done in three groups, but students favoured individual work. So, they did it according to their wishes. The following were observations of the tasks did by each of the students.

Respondent1 (Female, aged 16, F1)

The first female student (F1) of age 16 was responding to the first task as: She drew the co-ordinate axes, but not naming any of the axes. In plotting the given ordered pairs she was reversing the entries, and then she drew a line passing through the reversed entries. That line was passing through the origin. She did the same thing in part C of the task, and she plotted the points (2 ½, 6) and (4.6, 10.2) in reversed order. That is 10.2 and 6 along the x-axis and 2 ½ and 4.6 along the y-axis, and then she drew the line through the origin. (Refer to figure 1)



(Figure 1)

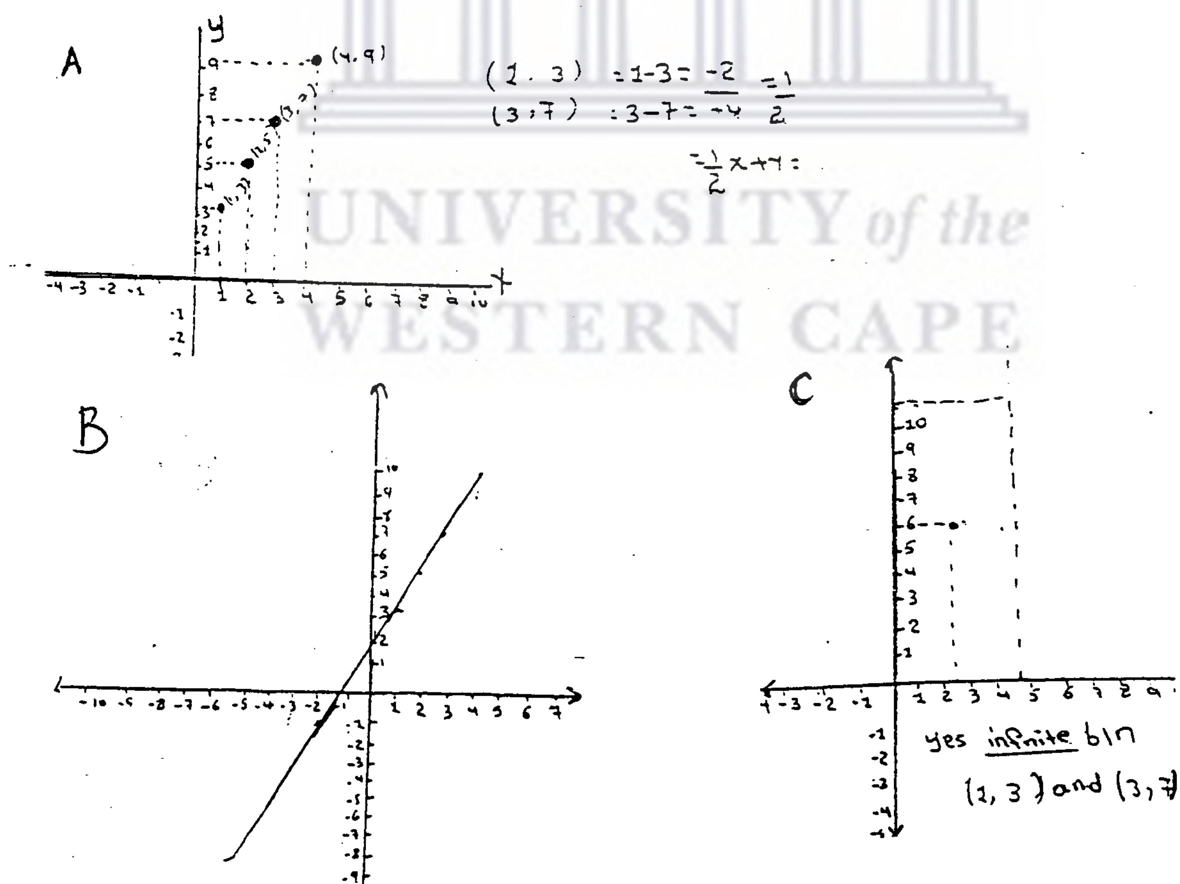
Respondent2 (Female, aged 17, F2)

After drawing the co-ordinate axes, she marked the co-ordinate axes. She did not plot any of the points given in part A of this task. Further more she did not give any other points which lie on the line. She was drawing one line, which passes through the point (0, 1) by looking it from her neighbour.

Respondent3 (Female, aged 16, F3)

She plotted the co-ordinate axes and marked it, after which she drew the line passing through all these points and containing the point (0, 2), which was not a point on the line. To give some other points which lie on the line, this student used the graph she has drawn and then tried to guess the point the line passes through by simple inspection, rather than finding a relationship between values of the given ordered pairs or by finding the equation of the line. (Refer to figure 2)

Her response to part C of the problem was "Yes there are infinitely many points between the given points."



(Figure 2)

Respondent4 (Female, aged 18, F4)

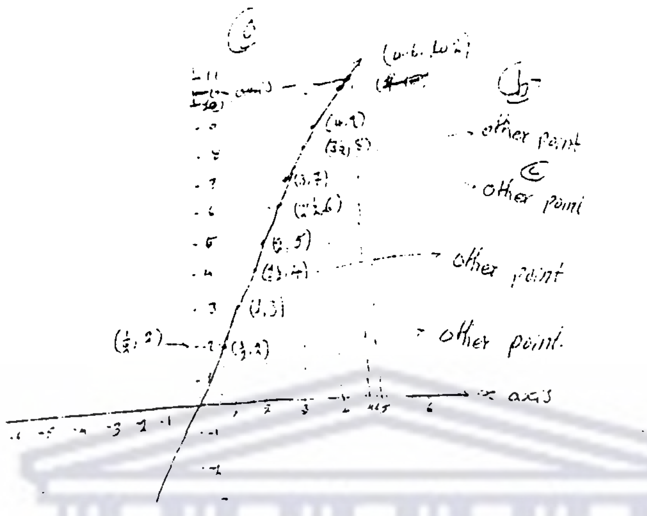
She was among the groups who were participating in all the tasks. She plotted the co-ordinate axes and marked it, without naming any of the co-ordinate axes. She also plotted all the given points in part A of the problem, and then joined them with a line. She has also suggested some other points that lie on the line, but some of those suggested points do not lie on that line.
Example: (-2, -2), (2, 4.3).

Responent5 (Female, aged 16, F5)

She plotted the co-ordinate axes without naming it. Next to that, it was difficult to identify which number corresponds to which part along the co-ordinate axes. She did all the work in a very small area. After plotting each point she was writing the ordered pair corresponding to the given point, beside that point. But she forgot to put the pairs of numbers in brackets. For the question in part B, she did not respond anything. For part C of the problem, her response was yes, and there are eight points.

Respondent6 (Male, aged 19, M1)

He plotted the co-ordinate axes. Name and label the co-ordinate axes and plot the given points. Beside each point, he wrote the corresponding ordered pair. And finally he has drawn a line through all these points. He pointed out four points on the line with values (1/2, 2), (2 ½, 4), (3 ½, 8) and(2 ½, 6). The line he drew was passing through the origin. (Refer to figure 3)



(Figure 3)

Respondent7 (Male, aged 16, M2)

In his response to the task, he plotted the co-ordinate axes, marked it and plotted the given points in parts A and C. Finally, drew a line through all these points. In answering part B Problem he has suggested that, "(1, 2), and (2, 4) are points on the line." For part C Problem he replied as: "*there are so many points lying on the line containing the given points.*"

Respondent8 (Male, aged 17, M3)

He has drawn the co-ordinate axes, name each of the axes. He had also, plotted the points given in part A of the problem, and then drew the line through these points. He also answered part B of the problem. In part C, he has plotted the two points and drew the line through the origin.

Respondent9 (Male, aged 17, M4)

After drawing the x-y co-ordinate axes, he marked it. Then he plotted the four points given in part A of this task. He then drew a line passing through these points, containing the point (0,2) which was a point not on the line. He did this from the nature of the graph he has already drawn. Basically from the scale he used on the co-ordinate axes (refer to figure 4). To answer part C of the problem, he wrote the ordered pairs (1, 3) and (3,7) as {1,3} and {3,7}. Then he proceeded with determining the slope of the line using the above points: As slope = (horizontal increase)/(vertical increase). Therefore, as to his work, Slope = (1-3)/ (3-7) = 2. Using this value as the value of the slope, he proceeded with finding the equation of that line.

(Figure 4)

Respondent10 (Male, aged 15, M5)

He drew the co-ordinate axes without naming any of the axes. He marked the axes and then plot the points given. Though he used a ruler, the way he located the points was depending on inspection. So from what he did, we can see that, the point (1,3) is a bit shifted to the right side, similarly, other points are also shifted with the exception of the y-intercept, which was (0,1). The line drawn in part A of this student work is different from the line drawn in part C. that is the line in A passes through (0, 1). Where as the line in C passes through (0, 2). In giving other points, which lie on the line, he sighted five points, out of which, one is correct. The other points sighted were (-2.4, 0), (0, 2), (3, 5), and (-3, -1).

Respondent11 (Male, aged 17, M6)

After drawing the co-ordinate axes, he marked it, without naming any of the axes. Then he plotted the given points and drew the line through these points. He also sighted other points, which lie on the line. In answering the last question in this task, his response was, there is no point between (1,3) and (3,7).

Respondent12 (Male, aged 16, M7)

After drawing the co-ordinate axes, he marked it. Without naming any of the axes. He then drew a line through all the points given in part A of the problem. In his work, he sighted other points on the line as (5, 12) and (6, 16) by simple inspection. He was guided by what he saw on the graph he has drawn. His answer to part C of the problem was, "*there are two* *points. They are (2, 5) and (4, 9)."*

Respondent13 (Male, aged 16, M8)

He plotted the co-ordinate axes and marked it. He also plotted all the points given in part A and C on the same x-y co-ordinate axes and then drew the line joining these points. In naming each of the co-ordinate axes he used the term x-intercept for the x-axis and the term y-intercept for the y-axis. He also gave other points by simple inspection of what he drew. He sighted the following points (-2, -2), (-1, -1), (6, 10), and (7, 11) as points lying on the line. His response for the second part of part C of the problem was *"yes there are infinite points."* Later on he cancelled out the term infinite and replaced it with the number 16.

Respondent14 (Male, aged 17, M9)

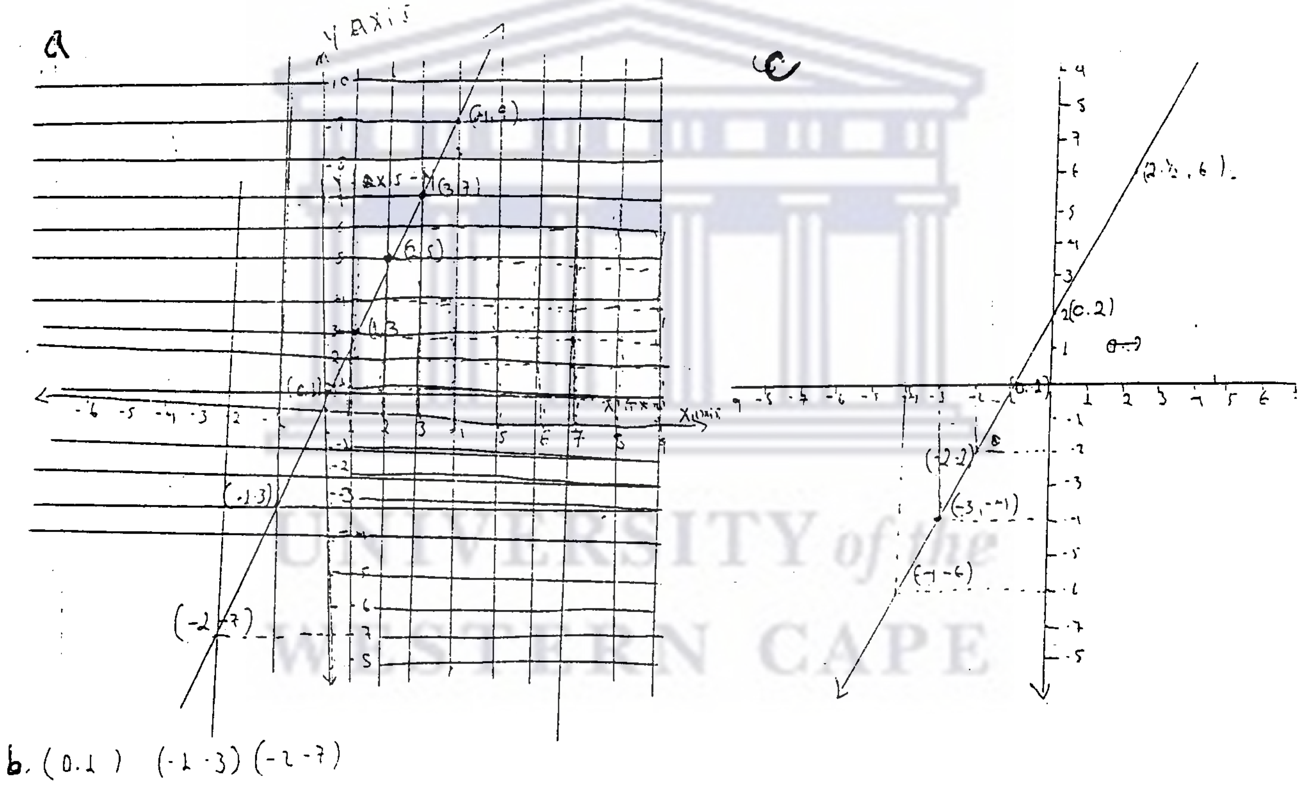
Draw the x-y co-ordinate axes, marked it and name the two axes properly. He also plotted the given points. He did not use a ruler. From what he suggested, some of the points such as (0,1.5), and (-1, 0) are the points not on the line. In part C of the problem to say, there are infinitely many points. He used the phrase "*many many.*"

Respondent15 (Male, aged 15, M10)

He did part A and B of the problem correctly. For the part C of the problem, his response was *"one and that is the point (2,5)."*

Respondent16 (Male, aged 17, M11)

After drawing the x-y co-ordinate axes and naming it, he formulated vertical and horizontal lines to help him locate or plot the co-ordinates of points. The distance between the lines was not the same. He plotted the points and joined them with a line, since the distance between those parallel lines was not the same, the line passing through the given points also passes through (-1, -3) and (-2, -7) according to his drawing. (Refer to figure 5)



(Figure 5)

Respondent17 (Male, aged 16, M12)

After plotting the co-ordinate axes, he marked it, and then he plotted all the points in part A and C. Finally, he drew the line containing all these points. He has mentioned other points such as (0, 1) and (-1,0). But, (-1, 0) is not a point on the line. He was suggesting these points by inspection rather than formulating a relationship between the given values. For the last problem of part C,- his reasoning was similar to that of the student (M10).

Respondent18 (Male, aged 16, M13)

For part A of the problem, he has drawn the co-ordinate axes, marked it. He named each of the co-ordinate axes. Plot the points given and then drew a line through them. For part B of the problem, he has drawn a line passing through the points (1, 0) and (0, 1). His answer for part C of the problem was, "there are nine points."

4.2.1.1 Analysis of the first task

Although students did not receive any special instruction on how to construct the graphs, when analysing students' graphs, I classified them into the following categories.

1) Plotting, naming and marking the co-ordinate axes, using proper scaling to mark points on the co-ordinate plane.

- All students drew the co-ordinate axes. With the exception of three students, the rest used rulers for drawing the co-ordinate axes.

- All students marked the axes. Only six students name each of the axes as x-axis and y-axis.

- One student used the term x-intercept and y-intercept to name the x-axis and y-axis respectively.

- Most of the students' did not use proper scaling for marking points on the co-ordinate plane.

As was described in the available materials for this task, most of the students were using rulers for drawing the co-ordinate axes. But, they did not use them in locating co-ordinates of points or in marking the points on the co-ordinate plane. Since, they were provided with duplicating papers, only one student used his ruler to draw vertical and horizontal lines on the co-ordinate plane, in order to help him plot the points. Though he was drawing these lines, he did not keep the distance between successive lines the same. In marking points on each of the axes, most of the students did not use proper scaling. Which again was a source of mistake for the majority of the students in guessing points from the graph to answer the questions in part B and C of the problem.

2) Plotting points on the co-ordinate plane.

In plotting the points in part A and C of the problem, one student assigned the values of x to y, and the values of y to x. i.e., reversing the entries on the co-ordinate plane.

All other students plotted the points given in part A of the problem. With the exception of one student, which did not plot any of the points. In joining the points with a line some of the students drew the line freehand like F1, M1, M4, and M12. The rest of the students used rulers. In extending the line far up or far down, students were using two styles.

Two Students drew lines, through the origin. Two other students drew the lines through (0,1.5) and (0,2) respectively. The rest of the students drew the lines through the point (0, 1).

On the whole, students were using their prior knowledge to plot points. However, three errors were detected. These were:

- reversing of the co-ordinates in plotting them on the co-ordinate plane;
- drawing the line through the origin;
- drawing the line through the points other than the given ones, which were not lying, on the line.

When we examine students' work on this task it leads us to the fact that students were having different interpretation of the given problem, such things were observed especially in part C of the task. The majority of the students did not give much importance to other points, whether they lie on the line or not. As a result, they were committing mistakes. They were also unable to formulate a mathematical relationship between the values of x and y in the given ordered pairs. Rather, they were dependent on their current work and were thus locally driven.

3) *Describing points on the line other than the given points in the task.*

In suggesting some other points, which lie on the line, there were three different answers suggested. They were:

Three students suggested points entirely on the line. From the suggested points by five students, some were on the line, and some were not. Four students suggested points, which were not on the line.

In suggesting points, which lie on the line, most of the students were using inspection and guessing.

From my experience in teaching whenever students were asked to draw a line through two or more points, most of them were not conscious of other points on the line. That is, they were joining the given points only. They did not pay any attention to the points of intersections with the x-axis or with the y-axis. In other words they did not consider intercepts as fundamental parts of graphing. Further more, most of them did not come across the link between parts of a problem as it was seen in this task. Hence, I do believe that students were not in a position to relate what they have learned in their previous grades with the existing problem. As Leinhardt et al., (1990:3), Quoting Greeno(1988a, 1988b)

> *Just because learners know something in one way does not mean that they can make immediate use of it from a different perspective or in a different situation.*

The difficulties exhibited in this task in plotting points other than the given ones were, because of lack of the use of proper scaling. That is, the distance between successive numbers was not a constant. Which again was a source of mistake for the majority of the students in guessing points from the graph to answer the questions in part B and C. Leinhardt et al., (1990: 19) explained about the significance of scaling in graphing as: *"a graph cannot be interpreted fully without taking into account its scales."*

The difficulties experienced by students in this task were also difficulties in similar researches done by different researchers such as: *"Many pupils found difficulty with the idea that there are any more points on the line other than those they had plotted."* (Kerslake, 1981: 122)

4. Describing the number of points between two given points.

To answer part C of the problem, half of the students' were suggesting that there were infinitely many points passing through those two points. Other students suggested there was one point (M10, M12), eight points (F5), nine points (M13), sixteen (M8), and no points at all (M6).

Those students who suggested the points between (1, 3) and (3,7) as one point only, were interpreting the problem as: *"how many points are there between these points from those given in A?"* Such students were using their prior knowledge in working with discrete points.

The following table describes students' response for the number of points between the two given points (1, 3) and (3, 7).

Infinite points.	Finite number of points.					No point at all
	One	Two	Eight	Nine	Sixteen	
F1,F2, F3,F4, M1,M2,M3,M4,M5,M9,M11,M13	M10, M12	M7	F5	M13	M8	M6

(Table 1)

In general, almost all students were using a visual strategy in telling the points that were on the line. The exactness of points was completely dependent on the use of scales and they used no other mechanism for checking whether the suggested points were on the line or not.

Summary about the use of prior Knowledge

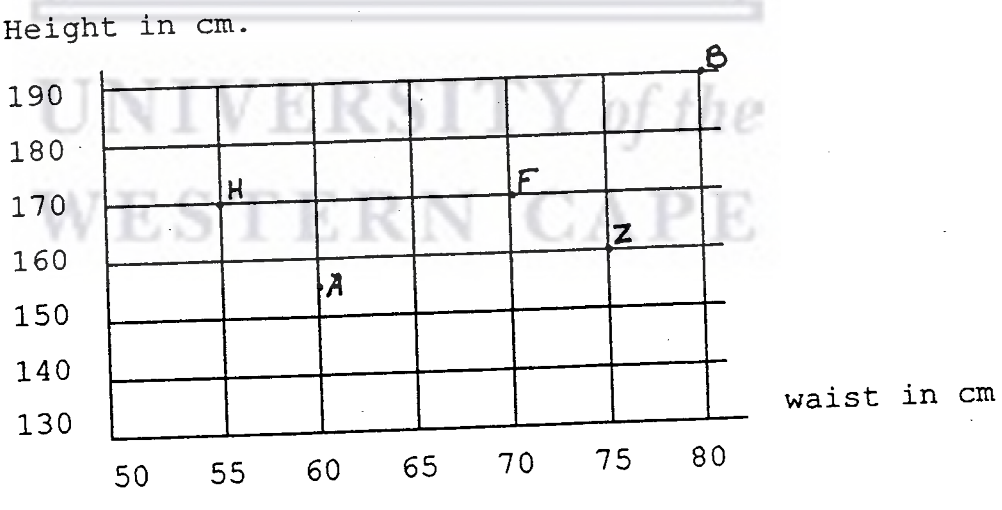
Though, the number of students participating in this task was low, the data collected from these students could give an indication about the problems in using students' prior knowledge for drawing graphs in the school and in the country as a whole. Almost all of the students were from similar living standard and the way they were taught were also in a similar fashion. That is, it was teacher centred. From the analysis of their work, it is possible to generalise that, students at this grade level were to some extent using their prior knowledge to plot points on the Cartesian co-ordinate plane.

It was observed that most of the students' activities were locally driven. That is to say they were influenced by what they drew in order to give other points on the line. It was also noted that, they were negligent about points other than the given ones. There was also a problem of using proper scaling in locating co-ordinates on the number plane, as a result, exposing themselves to committing mistakes. The data collected in this task also gave us insight on students' ability to interpret the given set of ordered pairs in forming pictorial representations. So this task paved the way to the next tasks which were focusing on interpretation of graphs.

4.2.2 Tasks on investigating students' ability to interpret graphs

The aim of the following task was to investigate students' understanding of discrete points, continuity (in the case of part D of the problem), and their ability to interpret whether points are plotted between the ones plotted or given.

Task 2



Haile drew a diagram to show the height and waist measurements of himself (H), Ali (A), Bereket (B), Sofia (S), and Freweini (F), Zubeda (Z).

What is Haile's height?

What is Bereket's waist?

Mark in Yonas (Y) whose height is 180cm and whose waist is 80cm.

Should we join up the points on the diagram?

Why do you think this?

What can you say about the height of a child whose waist is 65 cm?

Presentation of students' work

In the first three problems students read and gave the correct information concerning the individuals. These responses differ for the rest of the problems.

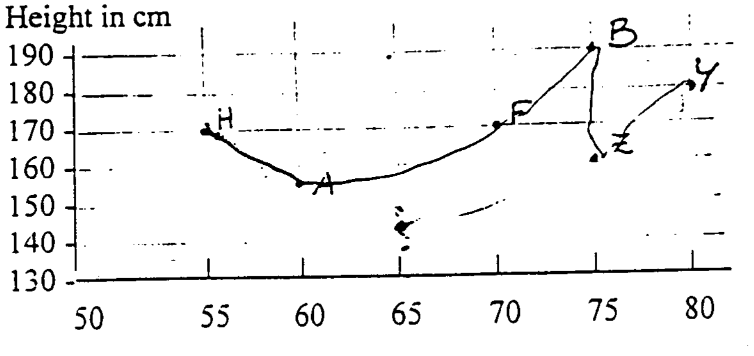
Those students responding for part D of the problem as YES were eight in number. The same number of students responded as NO, and two students did not say anything about this problem. They were forwarding different reasons to support their answer for part D of the problem. Among those suggested ideas were the following.

Students who said NO responded as:

The response of one student was: "*The diagram is a wrong diagram. Because, there is no equation, which fits this diagram.*" Another student responded as "*they are not on the same straight line or curve.*" A third student responded as: "*height and waist are not related. There should have to be a fixed multiplying constant (scale factor).*" In other words, there is no ratio of similitude. Another student was responding as: "*if we join them, we can not get a straight* *line."* Which was the same idea as the second person. In fact most of the students were suggesting similar ideas as these two students.

Among those students who said YES, they were suggesting the following. One of the students' responses was: *"short people are not fatter than long people."* The other student responded *"as far as you can take any comparison, you can join and you can show."* In relation to part F of the problem, there were different answers suggested. Such as, 160cm, 164cm, 157.9cm, 148.25cm, -170cm, around 148cm, *"There is no student or child whose height was 65cm."* *"We can not say anything about this child."* Two students suggested that, *"it could be any positive number less than 190cm."* After submitting their work, some of them were asked to explain how they did it. In order to make them free in explaining their work, they were told to use any language. For that matter, they were explaining their work using local language Tigrigna. The following were the points taken from the transcript.

Student (F4) was explaining as " . . . *by looking at the positions of the points I was joining them with two different curves, because, the points plotted on the co-ordinate plane could form two curves in the way I drew."* (Refer to this student's work below).

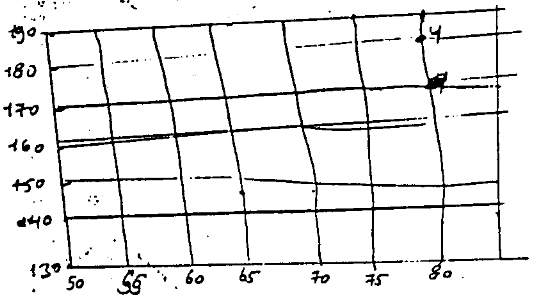


Answer

a) Haile's height is 170cm

b, Bereket's waist is 75cm

c



d) we can't join the points

(E) as I think we can't join with one curve But in to two curves they are joined.

f His height must be 145cm because when we put here, it can join (form curve) with the other points specially with Yonas & Zubeda

Question: Does your answer contradict with what you have already written in the paper?

"Yes, but, as to my understanding, I was joining the points under normal condition. Later on I have observed that, the points could lie in two different curves. So, I did like that. At this time, I am in favour of my previous idea. That is they should have to be joined."

Question: What do you mean by under normal condition?

"What I mean is, if points are given on the co-ordinate plane, then necessarily you need to join them."

Student M3: *"At first, I saw the points plotted on the co-ordinate plane. I was trying to join all the points, if it was possible to have a straight line; but then I was observing that, they were not on the same straight line. So, I said; the points must not join up."*

Question: If they were collinear, then would you join up them?
His reply was "yes, absolutely."

Student M5: "Since, they were not on the same straight line, they shouldn't be joined up."

4.2.2.1 Analysis of task 2

In this task it was possible to classify students' work into: Students were biased by the appearance of the graph.

Associating each of the points either with a line or with a curve.

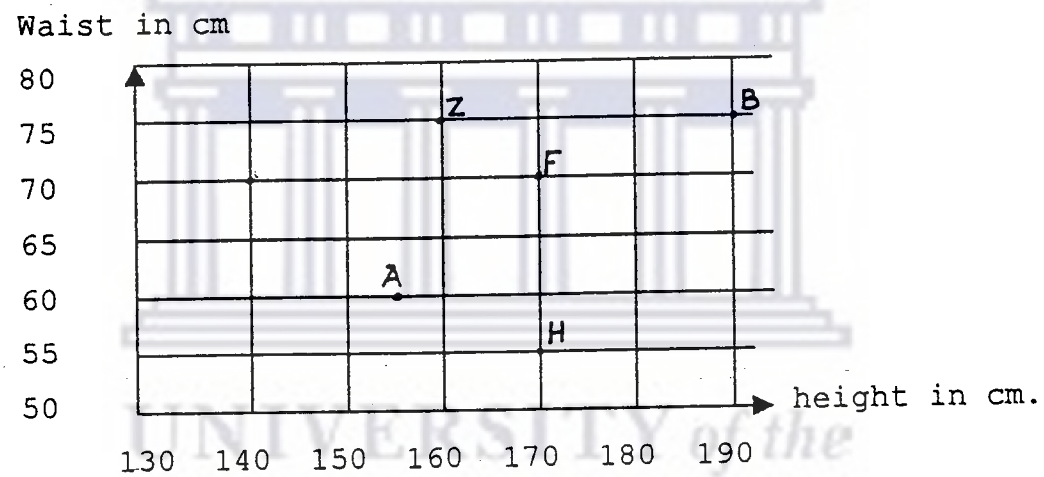
If a graph exists then necessarily there should have to be an equation corresponding to that graph. According to some of the students' interpretation, the equation will be either a linear equation or a quadratic equation. It was also noted that, students' responses to the question referring to an object or a thing, which was not labelled on the graph was that, "the problem is wrong." According to these students: in order to give a response other than the one given, then the thing should be objectively present.

Using the sum of the values of x and y in a given ordered pair and/or using the idea of ratio and proportion from the topic of their eighth grade portion to answer the question in part F of this task.

Generally speaking, students' answers were based on the appearance of the graph on the co-ordinate plane. The other thing that we can observe from their work was, their interpretation was directed towards graphing a straight line. If all the points were collinear, definitely all students would have drawn straight lines. Two basic factors can be sighted for doing so. First and foremost they were dealing with linear graphs in their previous grades. The first task was also having impact in students' response on this task.

Task 3

The aim of the following task was to investigate students' interpretation ability to describe graphs.



Describe the appearance of a person whose height and waist are plotted.

Nature of the class in this task

Five of the students did this task in a group. The rest of the students were doing their work in groups of two or individually. There were other students from the same grade but in other sections, who were constantly asking me to participate in the task. So, at this moment I was extending my working time and delivered them with the task. Hence, their work is included here. At the same time, some of the students who were involved with previous two tasks were not present in performing this task. Altogether there were twenty two students participating in this task.

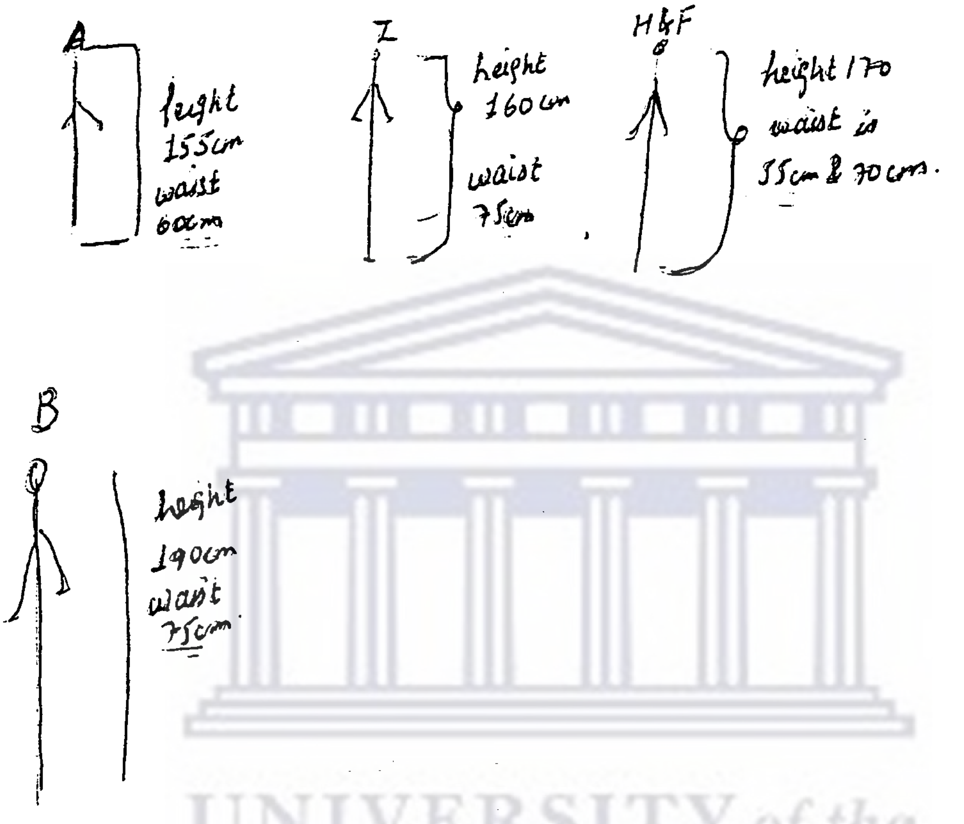
At first, I was giving much attention to the work of the students in the group consisting of five students. In the mean time, I was also observing the activities of the rest of the students now and then.

Presentation of students' work

The first group gave their answer in the form of ordered pair where the first entry of the ordered pair referred to the height of the person; and the second entry referred to the waist of the same person. Since, they were discussing in groups; the medium of communication was Tigrigna. Its translation was the following.

> *We observe that people with the same height are along the same vertical line; and people with the same waist are along the same horizontal line on the given co-ordinate plane. It was also possible to show the shortest and tallest person. Similarly, we can show the fattest and thinnest persons.*

From those students working individually (refer to figure 6), three of them used pictorial representation to describe the appearance of a person. Further more, one of these students used one pictorial representation of a person to represent two people H and F having the same height 170cm, and with different waists of 55cm and 70cm.



(Figure 6)

Another student was arranging the persons (people) in table form with increasing order of their height. He was also describing the tallest and shortest persons, fattest and thinnest persons.

One of the students' responses was completely different from other students. That is, he was interpreting the graph, as one person moving from one point to the other, his height and waist varies to the values corresponding to his/her destination point. He wrote it as:

When the person is at A, his height is 155cm and waist is 60cm. Then at Z his height increases by 5cm and his waist by 15cm. Then at the level of H, his height increased to 170cm but his waist decreased to 55cm. At this stage he gets thinner. But then at the level of F his height remains the same as H, but his waist increase to 70cm. Finally at B, his height increased by 20cm to 190cm and his waist is 75cm. Therefore through many level of growth, the person's height didn't decrease, whether it remains constant when he gets thicker or increase, but his waist some times increase some times decrease. Now, he is 190cm tall and 75cm thick.

4.2.2.2 Analysis of task 3

In this task two exceptional descriptions were detected.

1. Describing two different individuals having the same height but different waists by one pictorial representation.

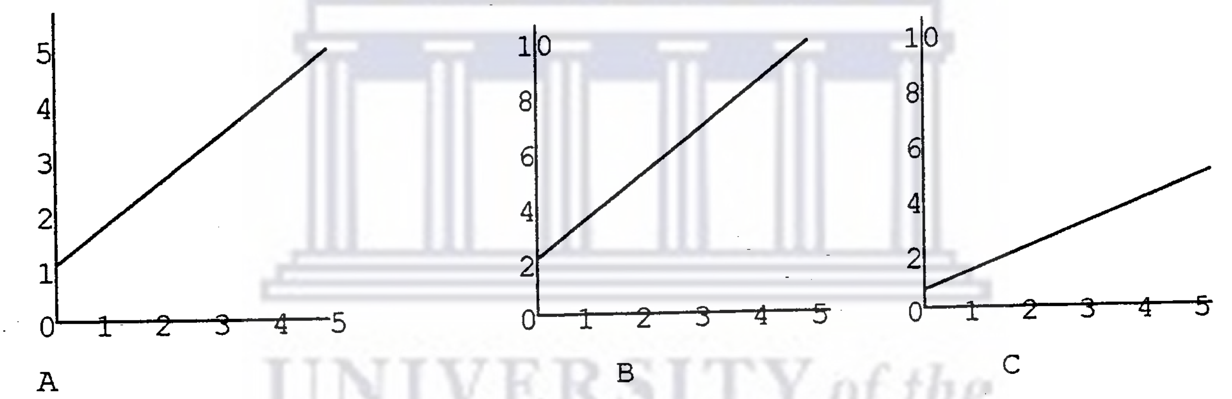
2. Considering the points as points corresponding to one person. Which means, the person's height and waist varies as he/she moves from one point to the other. The waist of a person may vary slightly as s/he moves from place to place, but in relation to the height, there will not be any difference in shifting from place to place.

It can be seen that, allowing students to work freely yields different interpretations, which in fact is having a great contribution to have a proper understanding of the problem.

Although, there were some misinterpretations on co-ordinates of points, their ability to read information from a co-ordinate plane was promising. It enables me to proceed further deep into the objective of the research. That is, to set other tasks related to interpretation and construction of global features of a graph.

Task 4

Which of the following three straight-line graphs show the same information? Why?



Presentation of students work

The number of students who realised that, the first two graphs represent the same information was three.
The response of thirteen students was the first and the last represent the same information. Five students responded that the second and the last represent the same information and one did not respond.

Graphs	A and B represent the same information	A and C represent the same information	B and C represent the same information
Number of Students	3	13	5

(Table 2)

Thirteen students forwarded their reasons because of having the same slope and having the same area. Some of these students were using the co-ordinates to determine the equations of the lines; in order to help them answer the question based on the equation they have found. One student was sighting co-ordinates of points from the graphs in A and C to justify his answer. The co-ordinates were (0,1) and (5, 5) from that of the graph in A, and (0, 1) and (5, 5) from that of the graph in C to show that the lines were having two points in common and hence they represent the same information.

Those who answered as A and B, gave the following reasons.

- Both start from the same point but different numbers. That is the distance from 0 to 1 in A, was the same as the distance from 0 to 2 in B.

- The slope of C is less than the other two. In case of A, the line starts from 1 until 5 and in case of B, the line starts from 2 until 10. The difference is only on scale but they give the same information.

Even those students, who answered the last alternative, reason out as, because of having the same slope B and C were having the same information. Two students respond as B and C represents the same information. Because, both the co-ordinates axes in B and C, were labelled in the same manner. That is, *"from 0 to 5 along the X-axis and from 0 through 10 along the y-axis."* In other words: the scales used in B and C are the same.

4.2.2.3 Analysis of task 4

In this task, most of the students' responses were A and C which of course was a correct result, but certain errors were detected. They were:

- Having the understanding that, the points of intersections of the graphs with the y-axis were different: neglecting this difference their responses were dependent on the distance. That is, if the distance between (0, 0) and (0, 1) was the same as that of the distance between (0, 0) and (0, 2) on the co-ordinate axes as it was in graphs A and B, then, irrespective what the slope of the lines were, they were suggesting the graphs represent the same information.

- By disregarding the scales used on the co-ordinate plane, students' responses were based on the marks on the co-ordinate axes. For example: as in the case of B and C, both were marked from 0 through 5 along the x-axis, and from 0 through 10 along the y-axis. Hence, their responses were "they represent the same information." The following is one of the students' works.

1) (a) B & C

(b) B I S their ~~...~~ straight line have the same scale that means their X & Y axis have the same scale that is 0 – 5 X axis, 2 – 10 Y axis. SO, They can show the same information.

- Answering based on the shape of the graphs: as in the case of the graphs in A and B.

- Though, determining the slope of the line was not the objective of this task; it was noted from one student's work that, the slope of the line in C was less than the other just from the appearance of the graph using simple inspection. Use of visual strategy was clearly manifested in all students' work.

Kerslake, (1981:129) investigated the degree to which students understood the effect that changing the scale of the axes would have on the appearance of a graph as:

> *Those students who are particularly strong visualises, that is they seem to think in visual terms - found extra difficulty - where a graph can be visually misleading. They found difficulty where the axes and scales of a*

graph were altered and the appearance therefore changed.

By and large, students were able to identify, the two graphs representing the same information. As in the case of the other tasks, in this task too, most of them were - using visual strategy. In giving a response to this task some of them were using additional reference materials such as, the grade eight-mathematics textbook and their grade eight mathematics exercise books.

4.2.3 Tasks on investigating students' ability to construct graphs

Task 5

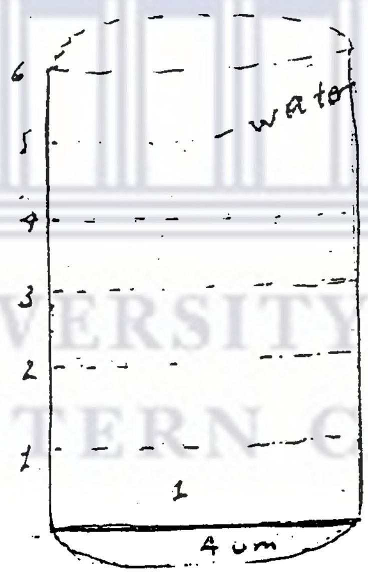
You are asked to fill an empty barrel, which is in the form of a right circular cylinder that contains six buckets of water from the nearby tap water. Draw the graph how the average height of water changes when you fill the barrel.

Aim of the task: To investigate students' ability to construct graphs.

There were ten students participating in this task. Among them one was female. The time taken to complete the task ranges from forty to fifty five minutes. Students were grouped into two different groups each consisting of five pupils. After discussing the task in groups, each of the students were submitting their own paper. This was to check whether they were fully participating in the discussion or not. The discussions in the first group were audio recorded. In relation to the second group, they were explaining their work after completing their task. This was also audio recorded.

Presentation of Students' Work

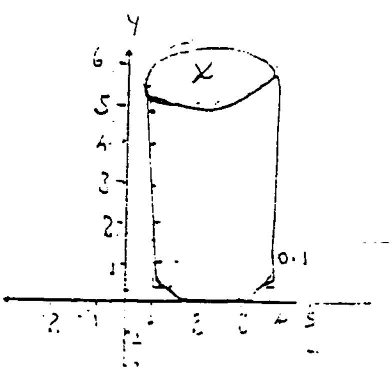
One student in the first group drew a barrel and divided the barrel in to six equal parts and describes, as the average height was 1 bucket/cm3. Another student in the same group drew the barrel by dividing it in to six equal parts and wrote the average height increase was 1 litre/ cm. The work of other students in this group was similar. That is, they drew the barrel and divide it into six equal parts to show that one bucket of water was equivalent to one partition of the barrel (cylinder).

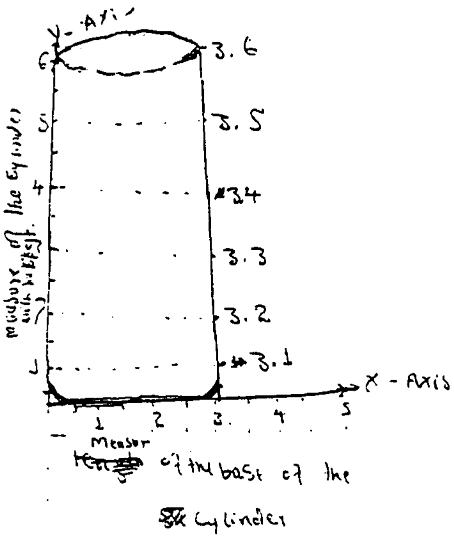


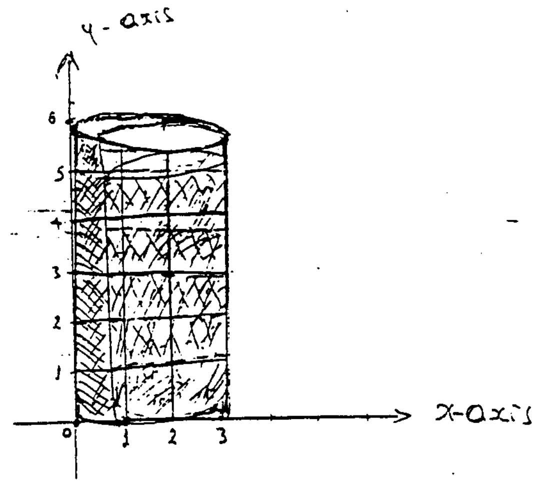
In the other group five of them drew the co-ordinate axes. They marked the axes using cm as the unit of length. They also drew the barrel by making the base of the barrel along the x-axis and the height of the barrel along the y-axis.

One student among this group divided the barrel into six equal parts and she fix the base radius to be 2cm. On the vertical line passing through (4, 0), she has plotted the co-ordinates with ordered pairs (4, 0), (4, 1), (4, 2), . . . (4, 6). Below the diagram drawn, she has described the base diameter or length of the base to be a constant 4cm, but in her explanation: *"as we pour water into the barrel, the height increases from 0 through 6cm."* Another student in the group did the same thing as that of this female student, but there was a slight difference in drawing. That is, he was drawing the base radius of the barrel to be 1cm. He plotted the co-ordinates (2, 0), (2, 1), (2, 2), . . . (2, 6) along the vertical line passing through (2, 0). Another member of the group took the base radius to be 3/2 cm.
Average = (3*6)/3 = 6 buckets/cm. He also described as "*x is a constant but y varies after or with the increase or rise of water.*"

The fourth member of this group drew the barrel with the base radius 2cm. Otherwise, the drawings were the same. Below the figure he wrote, "*when you apply one bucket, the height of the barrel raises in certain centimetre. Anyway, the height of the barrel changes constantly.*"







4.2.3.1 Analysis of task five

In this task, it was observed that:

1. All the students were not in a position to relate what they were taught in other subjects with mathematics.

2. They were all interpreting the statement as if it was asking to draw a barrel. Hence, there was a problem in understanding what was actually asked in the statement.

From their explanation and from what they did it was possible to come to a generalisation that, as Leinhardt et al., (1990:14) describes it "*the way a student constructs a graph depends heavily on how s/he thinks a graph should look.*"

Task six

a) Alem claimed that the more he studies, the better his grades are. Please, construct a graph that represents Alem's claim.

b) Sara argued that no matter how long she studies, she always get the same grades; Please, construct a graph that represents Sara's claim.

c) Solomon, however, said that, when he studies up to four hours, the longer he studies the better his grades are; but beyond four hours he becomes tired and his grades are lower. Please, construct a graph that represents Solomon's claim.

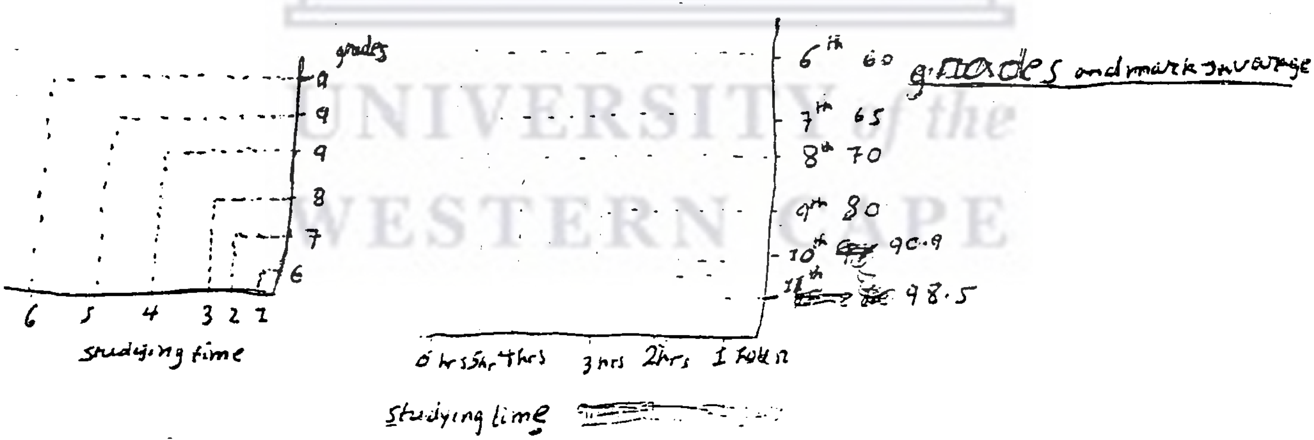
d) Henok confessed that, generally when he studies more, his grades decrease. Please, construct a graph that represents Henok's claim.

The response of students in this task enables me to classify them in different categories. According to their work, they were classified in to seven categories. Unlike to other tasks, the number of students participating in this task was much more in number

Category one: Students' who were drawing the co-ordinate plane by using the negative x-axis and the positive y-axis

In this category, there were two students. For part A and part B of the problem, these students responded by setting table of co-ordinates, which in-fact were not having any relationship with the problem, since the task was a construction task. It was their response for the rest of the problems, which enabled me to group them in one category.

One of the two was responding for part C of the problem in the following manner (refer to figure 9). First, he drew the co-ordinate axes as described above, and then he marked the horizontal axis (in this case the negative x-axis) by positive numbers in increasing order as you move from right to left. The vertical axis was also marked in increasing order; but after a certain interval there was a repetition of numbers. For example: 6, 7, 8, 9, 9, 9. Which were representing the grades attained by the individual in the question. For each of the whole numbers corresponding to the study time the respondent plotted points on the co-ordinate plane. The respondent's response for part D of the problem was not having any difference in plotting the co-ordinate plane from that of part C. The difference appeared in marking the vertical axis. He did it in decreasing order as you go up (e.g. 11th, 98.5; 10th, 90.9; 9th, 80; . . . 6th, 60) for the grades of the students.

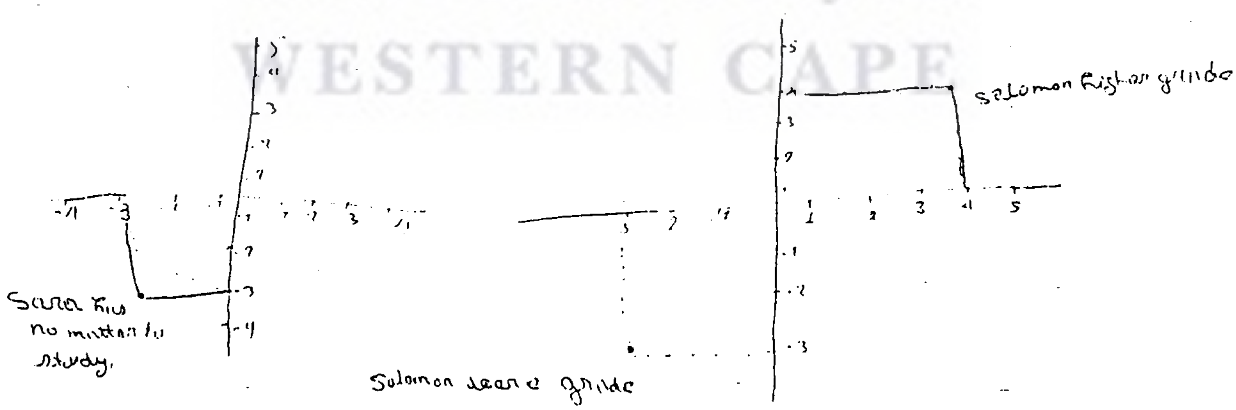


(Figure 9)

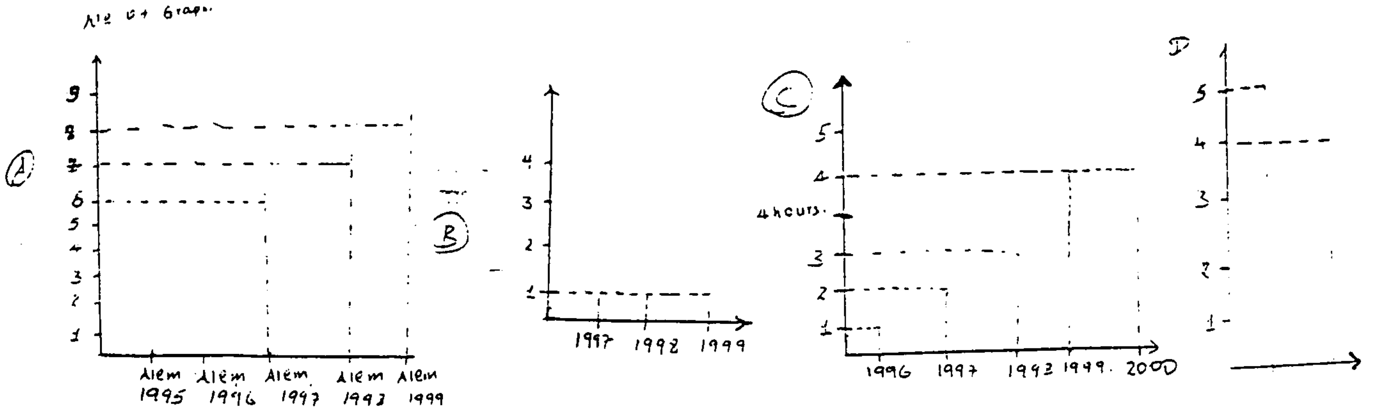
The other respondent did part C of the problem, in the following manner. Plotting the co-ordinate axes was similar to the first student. In marking the vertical axis, this respondent used descending order. Along the horizontal axis, numbers representing the time of study in hours were written in increasing order as we move from right to left on the axis. In relation to part D of the problem, points were plotted (located) to represent the time of study corresponding to the grades he scored.

Category two: Responding by using discrete graphs

Five students were grouped in this category. One student was interpreting each of the claims in the task by one point graph (refer to figure 10). Sara's, Solomon's, and Henok's claims were represented by single points in each of the co-ordinate planes corresponding to each problem. The rest of the students' were constructing point graphs for each of the problems in the task by plotting points corresponding to the whole numbers only. Three out of five students interpret the term "grade" as successive academic years in which the student was attending.



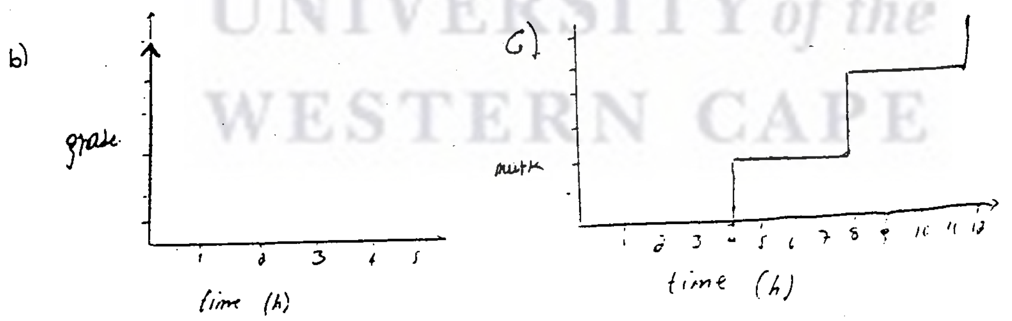
(Figure 10)



(Figure 10)

Category three: Describing by using histograms

Eleven students used histograms to describe the information graphically. Three students used line graphs to represent part A and B of the task. One other student drew a graph in the form of steps to represent the problem in part C of the problem. The same student drew a ray directed upward on the y-axis as a response to part B of the problem. (Refer to figure 11)



(Figure 11)

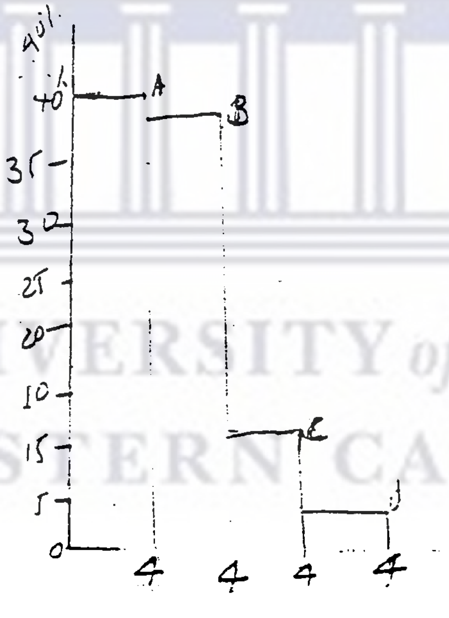
Common errors exhibited in this category were the following.

- Considering the problem as a problem representing a discrete graph.
- Unable to understand one block in the histogram represents a discrete nature.
- When they were asked to clarify their work, they were describing the real condition, which was completely different from their written work. That is to say, they did understand the statement and their reply was to the point. In other words: what they talked about and what they wrote were completely different.
- Some students were drawing parts of the histogram they drew below the x-axis to show that the individuals in part C and part D were decreasing their grades as they study more. They were associating the term decreasing with values below zero.
- There was a respondent considering the time as negative. In his drawing he was interested to show that for what so ever time he studies his grades never change. The unlimited time misled the respondent to consider the time to include negative numbers too.
- There was one other student who did the first two problems using constant histograms where the modal value is the same for all. For the remaining two using histograms in a negatively skewed form.
- Some of the students were giving their own interpretation for the task given. That is they were clarifying it and suggests some other ideas as a conclusion for their interpretation.

Category Four: Plotting all the graphs corresponding to the problems in the task on the same x-y co-ordinate plane (the horizontal axis represents the time and the vertical axis represents the corresponding grade).

One student was classified in this category. Refer to figure 12.

The respondent was marking along the time axis four points and four of them were assigned a constant value 4.i.e. (4, 4, 4, 4). Along the vertical axis there were marks given in intervals of five starting from 0 through 40.

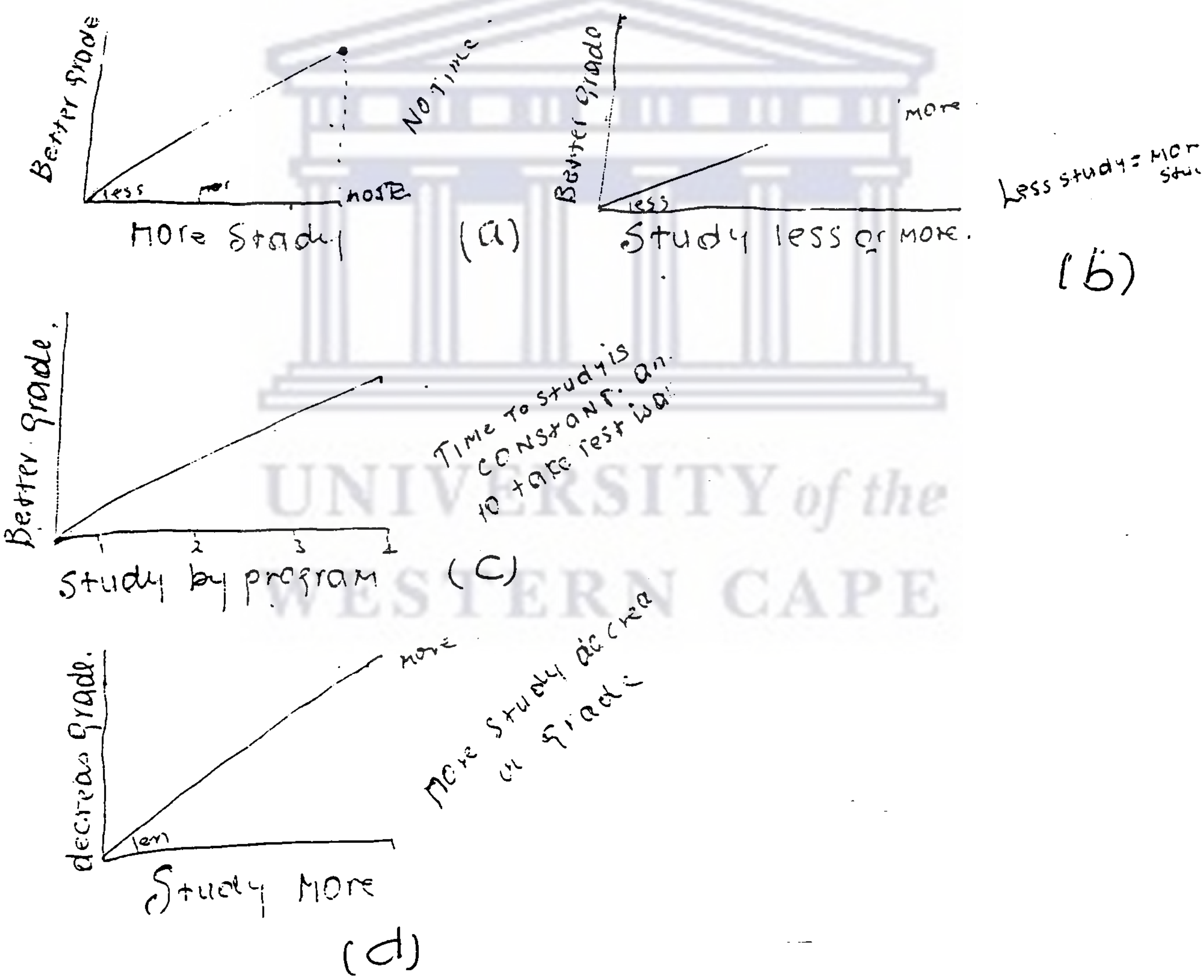


(Figure 12)

Category Five: Describing the task by graphs of increasing functions

In this category one student was classified. In marking the co-ordinate axes: for part A of the problem, he drew an arrow directing to the right side along the horizontal axis and wrote "more study" and along the vertical axis the arrow was directed upward and wrote "better grades." For part C of the problem, the marking was done in a similar manner, but the terms used were "study by program" on the horizontal axis and "better grades" on the vertical axis. Refer to figure 13.

For part D of the problem, it was a bit different from others. That is, beside the arrow which was directed upward along the vertical axis the term "decrease grade" was written.



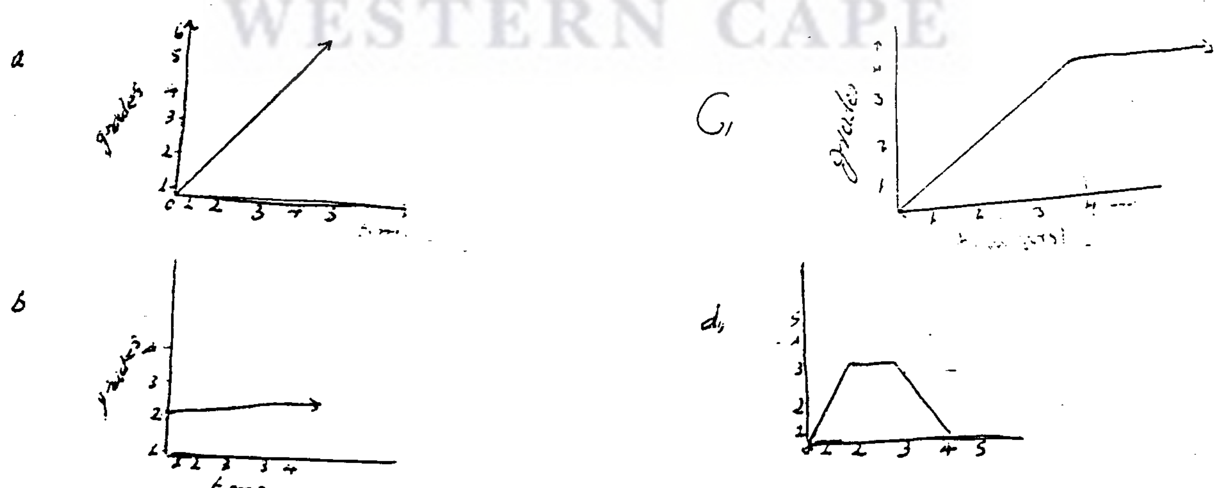
(Figure 13)

(Figure 13)

Category Six: Describing the claims using continuous line graphs (Refer to figure 14).

There were five respondents doing so. Two of them did it correctly. Two other students did the first three correctly, but there was a slight misunderstanding in their response to part D of the problem. One other student did the first and the second part correctly but his response for the rest of the problems was partly correct. The graph drawn by the later person for part C of the problem can be read as " up to four hours, the longer he studies the better his grades are, but beyond four hours his grades will never change no matter he studies."

Another respondent drew the graph for part D of the task as an increasing graph for the first two hours and then for the next one hour the graph remains constant attaining its maximum grade. Finally, for the last one hour by a decreasing graph. Other respondent for part D problem drew the graph starting from the origin and then after attaining a certain maximum grade which was 2, then it shows the decreasing behaviour.



(Figure 14)

Category Seven

Drawing the co-ordinate plane without plotting any thing inside it. There was one respondent who did like this.
The classification which I did was also supported by Mevarech et al., (1997) by classifying students according to their work into three major categories. They were:

I. Constructing an entire graph as one point.

II. Constructing a series of graphs, each representing one factor from the relevant data.

III. Conserving the form of an increasing function under all conditions

Though I classified students work further into pieces, generally speaking; all the seven categories can be condensed into three as described above. Category I includes three categories 1 and 2. Category II was also more simplified to engulf category 3 and 4. Category III includes the rest of the categories.

Remark

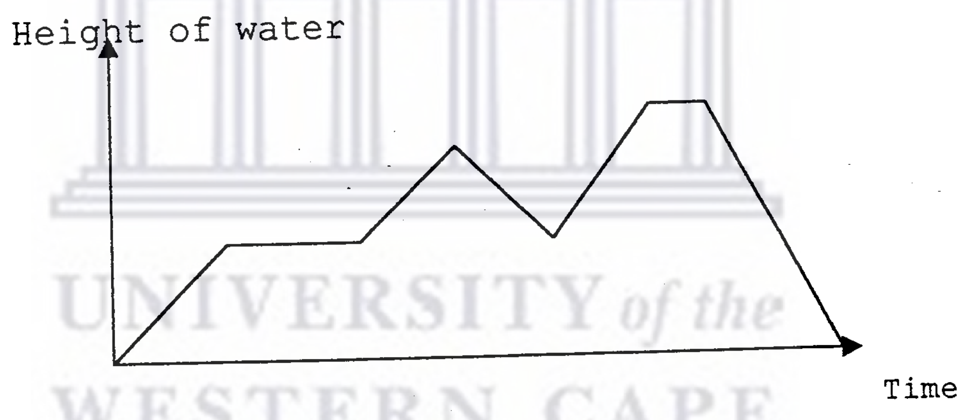
1) One of the respondents was suggesting the following for part A of this task. According to this respondent, "*Alem did not say clearly in what grade she was so it was difficult to construct the graph.*" This respondent was using terms that were different from other students. That is: for part C of the problem he was using the term "Better" for the interval from one through four; and the term "Bad" for the interval to the right of five and including five itself. In part C and part D he was plotting

points corresponding to whole numbers only.

2. In some cases it was difficult to identify students work. Because they were not writing the letter corresponding to the question beside graph they have already plotted.

Task seven

Aster followed the usual procedure for washing clothes. That is, she first prepared vessels, a barrel of water, bucket and the dirt clothes. Here is the graph that shows the first cycle of the washing process starting from the beginning of filling the vessel with water till the end where she spilled the dirt water.



Make a story that will explain the graph.

Nine students were participating in this task. At first there was a difficulty to understand some of the terms used in this task. To overcome the difficulties, I was providing them with Oxford Learners Dictionaries.

They were forming two groups consisting of five and four students respectively. After discussing in groups, they were told to write their own paper using their own way of explanation.

The difficulty in expressing their ideas in English language was among those difficulties observed in the tasks prior to this task. It was reaching climax in this task. As a result, one student was describing the graph by using a local language "Tigrigna." From my observation of their group discussion, I came across the improvement in interaction with each other. One thing observed in this task was students' were discussing freely which implies that they were in the process of developing self-confidence.

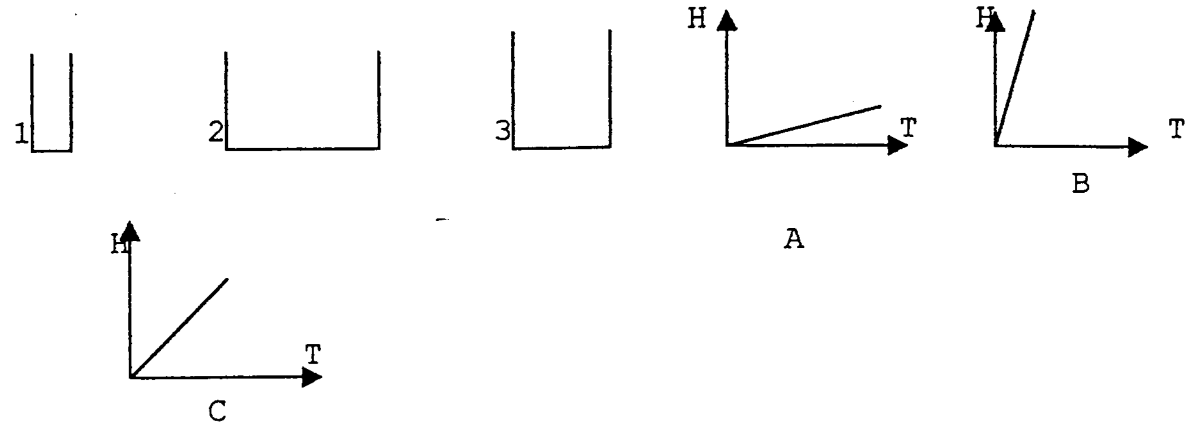
Almost all of them were having different understanding about the graph delivered to each of them. After a long debate they came to an agreement whose contents were described by each of the students.

Task eight

You are asked to fill the following three vases with water. Each vase was held under a tap dripping slowly at a constant rate.

- Match the given vases below (1, 2, or 3) with a graph (A, B or C) of your choice.

- Explain your choice to your group, then write it down.



Eight students were participating in this task. Participants were at first told to do their individual work and then to do the same task in groups. I did like this to check the reliability of their work. The time taken to read the task ranges from five to ten minutes. With in twenty five minutes they have finished their individual work. After submitting their paper, they were proceeding with their group work.

The following were the results of their individual work. Out of eight students doing this task five of them did the task correctly. Two students matched one correctly. One student did not respond for matching the vases with the available graphs except for one graph. Errors committed by those three students were the following.

- Two students were confused in identifying the time elapsed from the graph

- The other student was unable to identify the graphs corresponding to the largest and medium vases. While these students were asked to clarify their work, their reasons were quite correct and to the point. For example one of the students suggest the following.

The time required to fill the largest vase was much more than that of the time taken to fill the smallest vase. For the question: "how do you describe them in graphs?" His responses were clear and correct. Similarly the other two were also agreeing in his idea. This explanation of their work was done after completing their group task.
The difficulty actually exists in interpreting the graphs.

Results of the group work

Both groups were discussing using the local language "Tigrigna." Members of both groups were associating

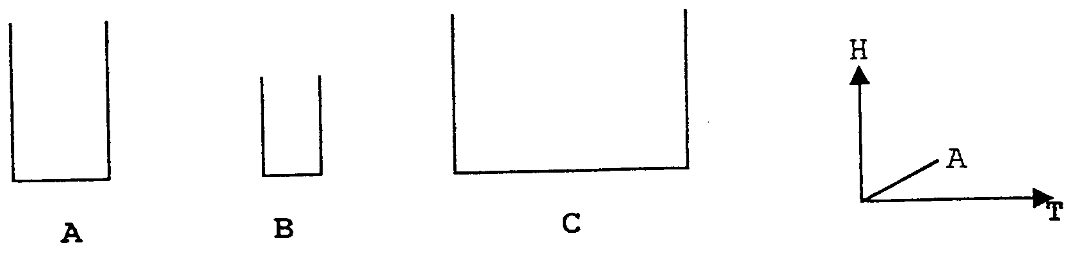
- Vase 1 with the graph in B
- Vase 2 with the graph in A, and
- Vase 3 with the graph in C

The difficulty of English was manifested in this group work. They were writing the reason for matching each vase with the corresponding graph correctly using "Tigrigna".
The result obtained from group work enables the three students to correct their previous understanding of the problem.

4.2.4 A task for investigating interpretation and construction of graphs.

Task nine

Kerosene siphons from a trough in to three different beakers (A, B or C) with the same rate. The graph given shows how the height of the kerosene changes in beaker A. on the same set of axes draw the graphs showing the time-height relationship for beakers B and C. Label your graph clearly.

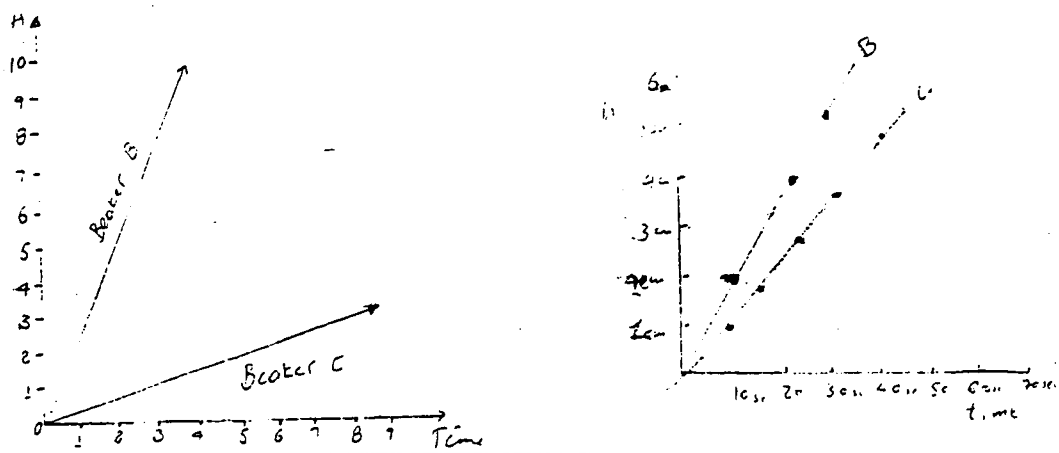


Analysis of students' work

Two students drew the graphs for B and C on one time-height axes. Five students drew the graphs for each of the two beakers on separate planes. One student drew beakers filled with kerosene.

Since the aim of the task was to investigate students' ability to construct and interpret graphs by drawing each of the graphs on the same set of axes all students fail to do so because of failure in understanding the instruction. Though, this was the case, the work of two students show correct drawing. (Refer to figure 15). From those students who drew separate graphs, one of them was responding as *"B is thin and has more height"* and *"C is wide so it has small height."* Another student also describe his graphs as *"B is narrow, so it full fast."* And *"C is wide so it full slowly."* Despite the separate drawings the reasons given by the above two students were also correct.

1 because the size of beaker is different from all and it depends on size.



* No relationship b/n B & C.

(Figure 15)

Generally speaking, these students' who were participating in all the tasks show confidence in challenging each of the tasks. The more they involved in the task the better they interact.

4.3 Summary on the analysis of data

The tasks were divided into five major parts, which enabled me to analyse students work in accordance with the aim of the study.

- A task on investigating students prior knowledge

- Tasks on investigating students' ability to interpret graphs

- Tasks on investigating students' ability to construct graphs
- Tasks on investigating students' ability to construct

graphs

- A task for investigating interpretation and construction of graphs.

In analysing students' work, I do come across the following points.

- Most of the students were to some extent using their prior knowledge to plot points on the Cartesian co-ordinate system.

- It was also noted that, for some questions referring to points which were not originally given, their replies were: *"the question is wrong"*, *"there is no point of this type"*, and the like. Which strengthened to the fact that they were locally driven.

- Some of the students were unable to formulate a mathematical relationship between the given ordered pairs; instead they depend on their current work and were thus locally driven.

- It was observed that most of the students did not make a link between parts of the problem, rather they considered them as separate entities.

- Although they were provided with materials to facilitate their work, they preferred doing freehand, which again paved the way to commit mistakes because of lack of proper scaling.

- Lack of proper scaling drives them to give the answers to the given task based on guessing which implies that, their work was basically dependent on the visual strategy associated with the shape, size, and nature of the graph.

- Since almost all of the tasks were related directly or indirectly with linear graphs, there were influences on the tasks to be performed later on.

- Understanding of graphs by most of the students participating in the implementation of the tasks was associated with continuous graphs. In other words, "if there is a graph, then it must be continuous."

Chapter Five

Discussions, recommendations and conclusions

5.1 Discussion

5.1.1 Introduction

In this mini-thesis I examined the interpretation and construction of global features of graphs using task-based activities in a qualitative research paradigm. The task based activities attempted to create an environment, which enable students to engage in meaningful mathematical activity. Adopting the research methodology of developmental research, the study was conducted during the months of December and January in the year 1999, in Adi Ugri Senior Secondary School in Eritrea. Data for analysis were collected in one cycle as candidly as possible.

5.1.2 Discussion on the implementation of tasks

Many of the students in this study did not hold a clear conception of graphs. Their conceptions were similar to students' conceptions as described in the literature (Kerslake, 1981). The students had specific expectations about graphs and their behaviours. For example, "*graphs are represented by straight lines.*" They also had no clear understanding of what continuous and discrete graphs are. There were responses of the type "*If a graph should exist then it must be continuous.*" "*If a graph exists, then there should have to be an equation or a formula corresponding to it*", and the like. There were instances in which students' rejected discrete graphs from being a graph.

After the implementations of successive tasks, the students' conceptions of graphs were directed towards graphs of straight lines. An interesting finding of this study indicates that given a set of points lying on the same straight line, when asked to give the number of points between the given points, students replies were of the type: *"there is no point", "there are finite points between the given points."* Some of the students' gave the actual number of points that they had plotted, others counted the number of locations where the line crossed the grid, and still others gave the number of whole number points as in the literature (Leinhardt et al., 1990:232).

Students were influenced by the nature and shape of the graph. Although, they were provided with materials for facilitating their work, they preferred doing freehand drawings, which was one of the basic factors for committing uneven scaling which again was the source of mistake.

Though some students do their work correctly according to the nature of the questions in the tasks, it was observed that, they were not paying much attention to other points on the graph as a result, exposing themselves to make conjectures. In some of the tasks students were applying fitting strategies which were developed basically from the visual features of the graphs.

As to interpretation of global features of graphs, students did three different tasks. The findings indicate that; their interpretations were mostly locally driven. Though there were a few students who were following some proper reasoning strategy for what they did, but the vast majority used to give the solutions not by relating to the reality and the corresponding mathematical formulae or equations but based on inspection. In almost all the tasks the visual strategy was the dominant means of giving solutions. From this study it has been noted that students interpretation of global features of the graphs by far depend on the visual features of the graphs.

5.2 Reflection on the research methodology

This section considers deficiencies and limitations in the research design of the investigation for this mini-thesis. One of the deficiencies in this research design was the insufficient evidence for cycle one, which was the composition of the nine tasks, delivered to students during the implementation period.

I feel that there is also a lack of evidence during the investigation period of how students encountered these task-based activities. I could have invited some other non-teaching staff for the process of triangulation, but I do believe from the very beginning that an observer for triangulation must be a practitioner of the subject matter. Due to this reason, there is a gap in having adequate information about the feelings of people towards the nature of the study.

My aim in this study was to conduct it with 30 to 35 students who were willing to participate with their full interest and motivation. I succeeded in getting about 18 students, which was by far less than my expectation.

During the implementation process, though much of it was done smoothly, it does not mean that, there were no problems. Among the problems I faced were:

- The coincidence of harvesting time and data collection time, this could not be avoided for a very practical reasons. I studied in South Africa and the data collection period had to coincide with summer vacation in the South, which is the harvesting season in the North.

- My expectation of getting teachers who can assist me during implementation of the tasks was not functional due to teachers work overload in the school.

5.3 Conclusion and Recommendations

This study has explored the students' ability to interpret and construct graphs, which would foster the following:

- promote an investigative nature and self confidence;
- promote students' independence of the teacher;
- change the focus from teacher-centred approach to an approach where students are actively involved with learning.

From the evidence which have been gathered it appears that task-based activities could create a conducive learning environment that provide a mechanism for a classroom environment to promote the features listed above.

The learning environment in which the students worked, allowed them to reach a consensus among themselves regarding the solutions to the problems posed in the tasks. In this interaction the students appeared to have accepted the group work as a reliable medium to supply information. In group work most of the students become more confident and competent that they see themselves as more experienced and develop their know-how within the group. Davidson & Kroll (1991:362) described the importance of group work or co-operative learning as:

> *Co-operative learning is generally understood to be learning that takes place in an environment where students in small groups share ideas and work collaboratively to complete academic tasks.*

While students were working with tasks and accompanying materials delivered, there were instances where students could assist each other in making conjectures based on local features of the tasks.

From this study, a lot of things on global features of graphs need to be revised in relation to curriculum development of Eritrea. In the present Eritrean mathematics curriculum, the concept of graphing is introduced in all junior and senior secondary schools. But as to that of global features of a graph, very few problems are included in grade eight. In the other higher grades though there are certain problems referring to global features of a graph. Their presentation however, is dealing with abstractions rather than problems of the reality. I believe that in order to learn mathematics the mathematics should start from the reality. In other words it must start with things or situations with which students are familiar and then, it can proceed to abstractions. The present mathematics curriculum is not functioning in this sense. To make the topics on graphing simple and interesting, so that the general public can communicate with it, the concept of global features of graphs should be introduced in all grades in the form of a spiral curriculum.

The other very important aspect of this study was the introduction of new approach to the teaching-learning method. The functional approach at present is more teacher-centred. If effective teaching-learning process as are to be fostered, an approach which enables students from being too dependent on the teacher should be introduced. This can be effective if task-based activities with the accompanying approach are introduced.

In this study, I was able to conduct cycle one of the study. To make the study complete and fruitful successive cycles should be undertaken in the future.

Although throughout the text reference is made to teaching methodologies, they are of secondary importance. The study focuses on the interpretation and construction of global graphs but for particularly readers in Eritrea the pointing to alternative teaching methodology is made to raise awareness of what the teaching context of the classroom experience was. This might contribute to further focussed studies on classroom methodologies.

Finally, I can say that the experience I gained from the activities and the developmental research approach has enriched my own knowledge and convinced me that to change the learning environment in the mathematics classroom. As Dawood (1995:107) quoting Waits (1988:334) stated we need "to change the way we teach mathematics and more important the way students learn mathematics.

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