# A Critical Analysis of the Pre-Calculus Course at U.W.C.

**Namari Myburgh**

A thesis submitted in partial fulfillment of the requirements for the degree of Magister Scientiae in the Department of Mathematics, University of the Western Cape.



Supervisor: Prof. R.L. Fray

November 2002

# Table of Contents



**Contents**

**Chapter 4 Profile of Students in the Pre-Calculus Course**

## Appendices

## Abstract

Mathematical knowledge and skills have, over the last few decades, become very important in terms of study and work opportunities. Unfortunately, schools in South Africa have not met the expectations of their pupils or the country in terms of delivering the appropriate number of students mathematically equipped for scientific study at universities. To redress this imbalance many tertiary institutions offer foundation courses to accommodate students who did not attain a matriculation exemption to gain entry to tertiary study. The Pre-Calculus course at the University of the Western Cape forms part of the foundation course of the university.

This study analyzes the Pre-Calculus course for the identification of its attributes that would act as substitute for a matriculation exemption in terms of higher grade mathematics and for the empowering of its students to study other subjects in the science faculty. To this end questions from higher grade mathematics matriculation examination papers were analyzed to identify the mathematical thinking and algorithmic skills that are tested at this level. Subsequently it is shown that the Pre-Calculus course does indeed have the content that can facilitate the development of these skills.

The students in the Pre-Calculus course were given the opportunity via an extensive questionnaire to give their opinion of the course, the problems they had while studying Pre-Calculus, their motivation for studying mathematics and the effect the course had on them emotionally. They also had the opportunity to criticize and make recommendations about the course. The information gained from the questionnaire supplemented by the observations of the author gives good insight into the problems and ideals of these students.

Recommendations to improve the effectiveness of the Pre-Calculus course and recommendations for further research conclude this study.

## Declaration

I declare that

*A Critical Analysis of the Pre-Calculus Course at U.W.C.*

is my own work, that it has not been submitted for any degree or examination in any other university, and that all the sources I have used or quoted have been indicated and acknowledged by complete references.

Namari Myburgh November 2002

-------------------------------------

# Acknowledgements

I wish to express my gratitude and appreciation to the following people who have supported and encouraged me in completing this thesis.

To my supervisor, Prof. R Fray, for the stimulating and friendly atmosphere in which supervision was conducted.

To Larry Kannemeyer, for his valuable guidance in the writing process.

To Rika Potgieter, for encouraging me and believing in me.

To Barbara Adams, for the valuable proof reading and constant encouragement.

To my husband, Chris, and children who have been patient and understanding. Their support, encouragement and assistance are greatly appreciated.

To my Lord, for granting me the opportunity to study.

# List of Tables

# Chapter 1

# Introduction

## 1.1 The Importance of Mathematics in the World Today

In his statement before the Committee on Education and the Workforce in the U.S. House of Representatives in the year 2000, Alan Greenspan, Chairman of the Board of Governors of the Federal Reserve System, discussed improving Mathematics and Science education in the elementary and secondary schools. He said:

> In 1900, only one out of every ten workers was in a professional, technical, or managerial occupation. By 1970, that proportion had doubled, and today those types of jobs account for nearly one-third of our workforce.
>
> … this simple statistic undoubtedly understates the ongoing increase in the analytic content of work, because there also seems to have been a marked increase in the need for conceptual skills in jobs that a decade ago would have been easily characterized as fully manual labor.
>
> … the new jobs that are created by the surge in innovation require that the workers who fill them use more of their intellectual potential. This process of stretching toward our human intellectual capacity is not likely to end any time soon.
>
> Expanding the numbers of individuals prepared to use a greater proportion of their intellectual capacity means, among other things, that our … students must broaden their skills in mathematics and related sciences.

These extracts from Mr. Greenspan's statement clearly mark the road of progress of mathematics in becoming such an important subject as it is today. It also highlights the two aspects of Mathematics that make it an important subject, namely, knowledge, and the intellectual skills fostered by studying Mathematics.

Mr. Greenspan continues to say:

> Addressing this issue is crucial for the future of our nation. It is obviously just a matter of time before the bulk of our workforce will require a much higher level of problem-solving skills than is currently evident.

In South Africa the situation is no different. The effects of globalization bring about the same expectations, expectations of employers of their employees and of employees for themselves. Additionally, as a result of the history of South Africa, many learners at school were disadvantaged with regard to the opportunities of good teaching and a healthy learning environment and this has resulted in many of them not being adequately equipped to start university studies when they leave school.

In the Financial Mail of 17 October 2000, the cover story was:

Education: Asmal needs help with uphill push.

(Mr. Kadar Asmal is the current Minister of Education in South Africa.)
The following paragraph from this article, about the state of education in South Africa in the year 2000, indicates that this desired increase in mathematical skills is not yet happening in South Africa.

> The report confirms worst fears. Only 4%-5% of matrics pass higher grade mathematics and science; 8,6% pass science and 14% pass mathematics on standard grade. Over the past three years, there has been no improvement in the number of higher grade mathematics and science passes and though standard grade enrolments are increasing, pupil performance remains dismal. … "Second Chance" programs should also be improved to enable 2000 pupils to rewrite matric annually and improve their performance.

It is clear from the above that Mathematics as a subject has become very important in recent times.

## 1.2 Background to the Study

Universities and tertiary institutions worldwide offer bridging courses for Mathematics to address the problem Mr. Greenspan spoke about above. The problem Mr. Asmal addressed resulted in a dramatic drop in student registration at tertiary institutions. Universities had to explore new ways of getting students. The different ways in which a student can be admitted to a tertiary institution is discussed under the heading 'Matriculation Exemption' in paragraph 1.3.

R.L. Fray, L.D. Kannemeyer and M.G. Salasa of the Department of Mathematics and Applied Mathematics at the University of the Western Cape prepared a report for the SASEN 2001 (South African Science Education Network) conference. This report states: "The Mathematics requirement to register in the Science Faculty at the University of the Western Cape is a pass of at least 40% in the Higher Grade or 50% in the Standard Grade in the matriculation examination. This same requirement applies to the first year mathematics and applied mathematics courses offered by the Department of Mathematics and Applied Mathematics." However, the report also says: "Increasingly students wishing to enroll in the Science Faculty, do not meet this requirement."



The following table indicating the enrolment for the matriculation examinations in Mathematics Higher Grade and Standard Grade and Physical Science Higher Grade and Standard Grade in 1999 and 2000 paints a bleak picture of the future of mathematics and science in South Africa.

| | 1999 | 2000 | Change |
|---|---|---|---|
| Mathematics HG | 51 716 | 39 640 | -12 076 |
| Mathematics SG | 242 184 | 256 687 | 14 503 |
| Physical Science HG | 68 276 | 57 121 | -11 155 |
| Physical Science SG | 98 491 | 112 085 | 13 594 |

Table 1: Enrolment for matriculation examination in Mathematics and Physical Science in 1999 and 2000

The SASEN report further states: "As a response to the … situation, the University of the Western Cape granted admission to students who did not obtain a matriculation exemption into degree programs in 2000 on certain conditions. Because of the weak performance in mathematics in the matriculation examination, the Science Faculty decided to institute a non-credit-bearing bridging course in mathematics. This constituted the so-called "value-added" part (see par 1.3) of the requirement for the granting of university exemption to such students by the UWC Senate."

The Department of Mathematics and Applied Mathematics at the University of the Western Cape (UWC) anticipated this problem and acted pro-actively by instituting help for first year students in as far back as the early 1980s. This program went through many phases until, in the year 2000, the first students were enrolled in a Pre-Calculus course in the Science Faculty. In 2000 the first year courses for these students were Pre-Calculus, Computer Literacy and English for Educational Development. From 2001 onwards, the first year courses are Pre-Calculus, Science Foundation and a first year Science course of the student's own choice.

During the development phase of the Academic Development Program at UWC, it was felt that a profile of students in their first year would be helpful in structuring a course that would accommodate them optimally with regard to their study of mathematics, but also with regard to their adaptation to university.

The author has always been intrigued by the discrepancy in mathematical ability between different persons and dismayed by the fact that an inability to do mathematics can have such a profound effect on a person's future in terms of study and career possibilities. The Pre-Calculus course at UWC is one way of addressing this discrepancy. However, offering a course does not necessarily mean that the desired outcome is attained and for this reason the course still needed to be assessed to ascertain whether the course indeed does empower students with skills they lacked when enrolling for the course.

## 1.3 Key Concepts

The key concepts in this study are:

- Matriculation Exemption in South Africa at the time of the writing of this thesis, 2002.
- Higher grade mathematics
- Pre-Calculus

### 1.3.1 Matriculation Exemption

Matriculation exemption is granted in any one of the following ways:

1. The matriculation examination administered by the Independent Examinations Board. A pass means that a student is eligible to be considered for admission to degree studies.
2. Exemption (from the matriculation examination described above) on the basis of the combination of subjects and the level at which they are passed in the senior certificate. This means students do not have to write the Matriculation examination because their Senior Certificate is taken as the equivalent. Those passing grade 12 examinations are given exemption if they have the right subject choice and have taken an appropriate selection of them on the Higher Grade.
3. Conditional exemption awarded to students who registered for the senior certificate but failed in one respect to meet the requirements for full exemption. These students may be permitted to register for degree study,

but must meet the outstanding requirement for matriculation within three years, either by repeating the senior certificate subject and passing it at the required level, or by passing the same subject (or another from the same group of subjects in the matriculation regulations) at university.

4. Conditional exemption on the grounds of mature age is granted to prospective students 23 years or older, who have passed at least four subjects from the matriculation groups. They may apply for (and will be granted) full exemption as for conditional exemption.

5. Post-school qualifications can also lead to exemption. At present this route is restricted to people with a senior certificate or its equivalent and a three-year diploma acceptable to the matriculation board and obtained in South Africa at a recognized university, technicon, or teacher's training college.

6. Conditional exemption by virtue of a certificate issued by the Senate of a university. This flexible area was opened up in 1994, when the regulations for Matriculation exemption had the following paragraph added:

   31(1) The Committee of Principals [now the SA Universities Vice-Chancellors' Association] shall issue a certificate of conditional exemption to a person who, in the opinion of the Senate of a university, has demonstrated, in a selection process approved by that Senate, that he or she is suitable for admission to bachelor's degree studies, which certificate shall be valid for admission to that university only.

   The regulation under which UWC is gaining Matriculation Exemption for students who have not met the older requirements is thus widely accepted. One outstanding condition is that for the conditional exemption granted in terms of this regulation to be converted to full exemption there shall be "value added" over and above the credit-bearing parts of the course, this value addition taking the student beyond the school-leaving certificate Grade 12.

This last group of students is called the Senate Discretionary (SD) students and they are, as such, the subjects of this research.

There are two main reasons why a student would not have matriculation exemption on leaving school. The first would be the choice of subjects generally and the second, the choice of higher grade subjects the student offered for the matriculation examination. These problems regarding incorrect choices for the matriculation exam can be attributed to more than one factor. Inadequate vocational guidance compounded by the immaturity of the learners at the time they choose their matriculation subjects, namely 15 years, is one factor as at this age many learners at school do not yet have a set idea of their academic goals in life. Another factor is inadequate teaching/learning in the junior standards at school that lead to poor performance on higher grade resulting in a drop to standard grade.

The result however of not having matriculation exemption is far-reaching for someone who decides to study at a university. Not having a matriculation exemption means that a person is excluded from studying at university. In order to give such persons a "second chance" (Asmal, 2000), universities were permitted to admit students at the discretion of the senate of the university.

### 1.3.2 Higher grade

Higher grade mathematics in Grade 12 differ from standard grade mathematics in content and types of questions asked.

Higher grade mathematics include the topics: Absolute value functions, Linear programming, Quadratic inequalities, Graphs of trigonometric functions of which the domain exceed $(-360^0, 360^0)$, Converses of certain theorems in Euclidian geometry, Exponential and logarithmic functions and their graphs, Convergence of geometric series, Trigonometric identities for sum and difference of angles as well as double and half angles. If it is further noticed that, in the guidelines to the syllabus for mathematics for Grade 12, it is stated that questions integrating different sections of the syllabus may be asked, it is clear to see that the higher grade examination paper has more possible variations in integration because it has more variation in content.

The guidelines of the Department of Education for setting a mathematics paper give the following table to show the difference between questions on higher grade and standard grade:

| Cognitive Abilities | Approximate Mark Allocation | |
|---|---|---|
| | Higher Grade | Standard Grade |
| Knowledge and Skills | 40% | 50% |
| Comprehension | 40% | 40% |
| Application and Creative thought | 20% | 10% |

Table 2:Guidelines of the Department of Education for setting mathematics papers

The pass mark on standard grade is $33\frac{1}{3}\%$ (100/300). This means that a pupil could pass on standard grade with mere knowledge and skills. Getting a D (50%) on standard grade could also be achieved without much comprehension or ability to apply his/her knowledge. On the other hand, a pass mark on higher grade is 40%, which means that a pupil has to exhibit at least some form of comprehension or application of knowledge to attain a D (50%) on higher grade. This then is the tangible difference in the mark allocation for different kinds of questions on higher grade as opposed to standard grade.



That higher grade mathematics does indeed increase a student's chances of passing first year mathematics at university is corroborated by a report on a first year placement test at UWC (Amoah et al, 1997). Results for Mat 111 were compared to the results for the placement test. The following table gives the average for the years 1994-1996:

| Variable | Grade | N | Mean | SD |
|---|---|---|---|---|
| Placement Test | HG | 153 | 57.7 | 12.3 |
| | SG | 184 | 47.7 | 12.0 |
| Mat 111 | HG | 139 | 51.4 | 14.2 |
| | SG | 167 | 41.6 | 14.4 |

Table 3: Results for mathematics 111 and a placement test at U.W.C.

This indicates that the average student coming from higher grade mathematics at school had a better chance of passing the first year mathematics course. Significantly the standard grade students scored 10% lower in both cases and this was the 10% they needed to pass.

It is thus reasonable to accept that a student enrolling for a first year course in mathematics at a university will be better prepared for his/her studies if he/she has passed mathematics on higher grade in Grade 12 as opposed to having passed or even attained a higher symbol for standard grade mathematics.

### 1.3.3 Pre-Calculus Course

The Pre-Calculus course in mathematics at the University of the Western Cape is a mathematics course instituted to prepare students who are not eligible for enrolling in the Faculty of Science on grounds of inappropriate matriculation results. The Pre-Calculus course aims to broaden the students' mathematical knowledge while strengthening their understanding of the mathematics they learnt at school. This is done by the selection of appropriate topics and careful structuring of the curriculum. The actual content of the course is discussed in paragraph 3.6.

## 1.4 The Aims of the Study



The aim of this study is to facilitate the development of an effective mathematics course for students registered in the science faculty at the university of the Western Cape who require bridging into the mainstream courses by:

- Investigating the Mathematics offered in this course in order to evaluate it for its effectiveness in achieving the aim of preparing students adequately for further science studies.
- Doing this by comparing the school leaving skills a student is supposed to have for matriculation exemption to be granted, to those skills the Pre-Calculus course offers and hopes to develop in this respect.
- Investigating the thinking skills involved in studying science as it is manifested in the curriculum of the Pre-Calculus course.

- Assessing the content of the course for the possibility of developing these thinking skills.
- Drawing up a profile of the students so that their strengths can be utilized and their problems can be addressed.

It is clear from the statement of Mr. Greenspan that the cognitive skills learnt alongside Mathematics are of the utmost importance. These cognitive skills are also responsible for the discrepancies between standard grade and higher grade mathematics achievement of any individual. As these are the skills that students should gain from studying Mathematics, it is the aim of this study to identify these skills and to assess whether the Pre-Calculus course offers the students the opportunity to learn these skills.

The students in the Pre-Calculus Course are subjected to all the stress factors of normal first year university students. As this is widely known to be a year of adaptation to new circumstances in terms of living conditions, social interaction, study load etc., it became obvious that an assessment of the Pre-Calculus Course had to include the drawing up of a profile of the students in the course. In the specific case of the Pre-Calculus course at UWC, additional factors come into play when the first year students are profiled. More than 60% of the class does not speak English at home and this has a huge impact on their understanding of what is said in class during lectures, what is explained in textbooks and how to communicate with the lecturers or tutors. This aspect and the fact that they often had ineffective teaching at school needs to be addressed when drawing up a profile of the students in the Pre-Calculus course.

## 1.5 The Main Research Question

The main research question this study wants to answer is then:

Does the Pre-Calculus Course in Mathematics at UWC
indeed teach students the skills
they did not acquire during their school studies?

This question can only be answered by considering the two crucial factors, namely

- What is being learnt? and
- Who is learning?

Literature study identified the skills that should ideally be learnt through study in Mathematics. The content of the course was then tested against these skills to see whether the content is appropriate to develop these skills. Information about other universities and institutions that offer bridging programs was obtained from the Internet and was helpful in placing the topics and organization of the course in context against what these other institutions offer. The correlation between courses is discussed in chapter 3.

To draw up a profile of the students, those 'Who are learning', a questionnaire was filled in by the students, giving valuable information on their background and their own assessment of the course. This information was correlated by literature study to corroborate the claims made by the students with regard to their problems and views.

Finally the mathematics results and course results of the Pre-Calculus students (Appendix H) were correlated with their overall results and with the first year mathematics results and course results of students not having done the Pre-Calculus course. In this way an estimate could be made of the academic success of the course.

## 1.6 The Structure of the Thesis

**Chapter 1: Introduction**

Chapter 1 provides a general introduction to the thesis. It is aimed at orientating the reader with the thesis and explaining the key concepts. At the same time it is aimed at indicating how broad the study will be by posing the main and secondary questions to be answered. The chapter further outlines the aims and objectives of the study including the significance and rationale of the study. Finally, the structure of the thesis is also outlined.

**Chapter 2: Literature Review to identify Cognitive and Procedural Skills**

Chapter 2 provides a literature background against which the Pre-Calculus Mathematics course will be assessed. It highlights desirable aspects of the design of a mathematics course. It also identifies the cognitive skills taught in higher grade mathematics at school as well as the general algorithmic or procedural skills necessary for effective study in mathematics.

**Chapter 3: Analysis of the Pre-Calculus Course**

Different aspects of what the students are taught in the Pre-Calculus course are discussed in this chapter. In turn the following is analyzed and discussed: the actual content of the course, how the content correlates with that which other tertiary institutions offer in the same type of course, the design of the course, the cognitive skills addressed by the course and finally the procedural or algorithmic skills attended to during the teaching of the course.

**Chapter 4: Profile of the students**

To draw up a profile of the Pre-Calculus students, a questionnaire was drawn up to gain demographic information about them, to ask their opinion about the course and to give them opportunity to express their feelings about the course. Chapter 4 discusses the literature underlying certain issues in the questionnaire and also discusses the feedback from the students against the background of this literature

**Chapter 5: Research Findings and Methodology**

Chapter 5 deals with the methods used to collect and analyze data for the purpose of this study. This chapter describes the students constituting the sample that filled in the questionnaire. It also describes the measuring instruments and the process of the study. Finally it also notes the limitations of the data.

**Chapter 6: Suggestions and Recommendations**

From the analyses and discussions in chapter 4, suggestions on how the course can be improved upon are made in this chapter. Recommendations for further study are also noted.

# Chapter 2

# Literature review to identify Cognitive and Procedural Skills

## 2.1 Introduction

A comprehensive knowledge of Mathematics is the most important foundation for studies in the sciences. This importance lies in two aspects of the subject: the actual content of the subject on the one hand, and the thinking skills involved in learning and doing mathematics on the other. These are the reasons why mathematics is such an important subject for a matriculation certificate and why it is an integral part of obtaining a matriculation exemption. Universities generally require prospective science students to have at least a 40% pass on higher grade or 50% pass on standard grade for mathematics. The distinguishing factors between standard grade mathematics and higher grade mathematics at school are:

- firstly, the expanded content of the standard grade curriculum to form the higher grade curriculum and
- secondly the difference in the kind of questions posed in examinations.

Higher grade questions test not only mathematical knowledge but also a vast array of thinking skills that are not tested on standard grade. For a Pre-Calculus course in mathematics then to make the necessary difference to what a student brings from school in terms of mathematics, it needs both to supplement the content and foster the thinking skills the students are lacking.

In this chapter a literature review of the development of both thinking and algorithmic skills via both the design of the curriculum and the content of the course will be discussed. Where applicable, examples from past higher grade papers will be quoted and discussed to show that these skills are tested on higher grade and should thus ideally be taught and tested in the Pre-Calculus course.

## 2.2 Aspects of the design of a course

Piaget (In Howson et al, 1981:118) is quoted to have said:

> ...since everything in Mathematics is interconnected in an entirely deductive discipline, failure or lack of comprehension where any single link in the chain is concerned, entails an increasing difficulty in following the succeeding links, so that the pupil (student) who has failed to adapt at any point is unable to understand what follows and becomes increasingly doubtful of his ability.

As all the students in the Pre-Calculus class come from bad experiences with Mathematics, it is of great importance that the course should be structured and taught in a way that will first of all give them confidence and after that enhance their confidence to such a point that they will see themselves as students of Mathematics in the future. Their newly acquired confidence should be guarded fiercely, because they will revert back to their feelings of incapability at the slightest provocation. This could be any one of many factors like

- an overly accelerated pace,
- a newly introduced but not yet fully explained topic or
- problems using background knowledge they do not have readily available at the time of having to solve the problems.



In this light, the following aspects of the curriculum are deemed very important to be given the appropriate attention:

- Sequencing of topics
- Hierarchy of concepts
- Conceptual thinking
- Level of difficulty

### 2.2.1 Sequencing of the topics

J.S. Bruner (In Howson et al, 1981:112) expresses the need for "something approximating a spiral curriculum, in which ideas are presented in homologue form, returned to later with more precision and power, and further developed and expanded until in the end, the student has a sense of mastery over at least some body of knowledge."

Bruner (Howson et al, 1981:112) also states that the teacher should find the ideas that have been presented earlier and deliberately use them as much as possible for the teaching of new ideas. Bruner (Howson et al, 1981:112) also expects the teacher to look to the future and teach some concepts and understandings even if complete mastery cannot be expected. This leads us to the hierarchy of concepts.

### 2.2.2 Hierarchy of Concepts

"In mathematics, not only are the concepts far more abstract than those of everyday life, but the direction of learning is for the most part in the direction of still greater abstraction" (Skemp, 1979:26).

A hierarchical order of introducing concepts accommodates the statement made by Freudenthal (1991:94), in which he says: "learning processes are marked by a succession of changes of perspective which should be provoked and reinforced by those who are expected to guide them."



These statements are the reasons why it is advisable that course content should be carefully analyzed to follow a strong hierarchy of introducing concepts. It makes the communication of concepts simpler than when all concepts would just be "understood" from their definition (Skemp, 1979:32). It would further seem that, not only should topics be arranged in a hierarchical order, but the examples should also be chosen to expand concept forming gradually (Skemp, 1979:26). This can be accomplished by an order of presenting topics in such a way that some concepts of a topic are used in subsequent topics.

This hierarchical aspect of the design of a course also makes it easier to teach and subsequently easier to comprehend and study.

### 2.2.3 Conceptual Thinking

Skemp (1979:30) emphasizes the power of conceptual thinking, saying that the more abstract the concept, the greater its power to combine and relate many different experiences and classes of experience. The power of conceptual thinking also lies in the expansion of our short-term memory (Skemp, 1979:31). Mathematics, according to Skemp (1979:31), is the most abstract, and so the most powerful, of all theoretical systems. "It is, therefore, potentially the most useful system for conceptual thinking, and scientists in particular, but also economists and navigators, businessmen and communications engineers, find it an indispensable 'tool' (data-processing system) for their work" (Skemp, 1979:31).

A curriculum designed to foster conceptual thinking would then apply a single concept/calculation in all kinds of different situations, thus strengthening the concept by using its various attributes in appropriate situations.
This expanding of a concept encourages the vertical growth of the mathematical knowledge (Tall, 1991:83). The extent to which a curriculum incorporates structures to foster conceptual thinking should then be an indication of the success of the course in adding value to the mathematical knowledge that students bring to the course.



### 2.2.4 Level of Difficulty

The level of difficulty of a course has to be such that it:

- challenges the students to expand their mathematical boundaries and
- challenges them to improve to the required standard in terms of content, depth and thinking skills involved.

At the same time, the level of difficulty should not make the subject inaccessible otherwise it would act as a demotivating force.

The principal known as the Yerkes-Dodson law (Skemp, 1979:125) states that the optimal degree of motivation for a given task decreases with the complexity of the task. This has a profound impact on the choosing of examples in the teaching of the subject. Examples should ideally lead the student as unobtrusively as possible to a higher level of difficulty without the student losing interest or motivation.

## 2.3 Thinking skills

Greenspan (2000) stresses the importance of improving the intellectual skills of the workforce, saying that there is a "marked increase in the need for conceptual skills in jobs" and that "innovation requires that the workers ... use more of their intellectual potential". He also concludes that this transformation can only be brought about by broadening skills in mathematics and related sciences. In this regard, the following intellectual skills were identified as important for the aims of the Pre-Calculus course:

- Generalizing
- Synthesizing
- Abstraction
- Classifying
- Forming conceptual entities
- Generic generalization
- Coping with a disequilibrium
- Imagery
- Reflectivity
- Transition
- Reversibility, and
- Refined intuition.

### 2.3.1 Generalizing

"To generalize is to derive or induce from particulars, to identify commonalities, to expand domains of validity" (Tall, 1991:35). Skemp (1979:59) defines generalization more particularly as the detachment of "the method from any particular example of their use", for example: $\frac{ab}{ac} = \frac{b}{c}$ is the generalization of $\frac{14}{12} = \frac{7}{6}$.

Generalizing, as a component of abstraction to form conceptual entities, also has the effect that it saves on memory space in the brain (Tall, 1991:83). Quite simplistically, it is easier to remember how to multiply than to remember all the answers to all possible products. In the same way it is easier to remember how to transform a graph than to remember a different graph for each different function.

Generalizing, according to Tall (1991:35) and Skemp (1079:59), is also useful in learning new information based on what is known, e.g. using the existing knowledge of logarithms and its laws, and exponents and its laws, as general concepts of how $\log_e x$ and $e^x$ will behave.

In a 1993 higher grade mathematics paper (Prins et al, 1994:30), the following question draws on a pupils ability to generalize.

a)

Prove that $\dfrac{\cos 2R + \cos R}{\sin 2R - \sin R} = \dfrac{\cos R + 1}{\sin R}$

b)

Hence prove that $\dfrac{\cos 4x + \cos 2x}{\sin 4x - \sin 2x} = \cot x$

In (a) a certain result is to be derived. In (b) the identification should be made that the information of (a) can be generalized to fit the pattern of (b) and therefore that the result of (a) applies to (b) and need not be proved again. Thus:

a) $$\frac{\cos 2R + \cos R}{\sin 2R - \sin R} = \frac{2\cos^2 R - 1 + \cos R}{2\sin R\cos R - \sin R} = \frac{(2\cos R - 1)(\cos R + 1)}{\sin R(2\cos R - 1)} = \frac{\cos R + 1}{\sin R}$$

b) $$\frac{\cos 4x + \cos 2x}{\sin 4x - \sin 2x} = \underbrace{\frac{\cos 2(2x) + \cos(2x)}{\sin 2(2x) - \sin(2x)} = \frac{\cos 2x + 1}{\sin 2x}}_{\text{generalizing the result of a)}}$$

$$= \frac{2\cos^2 x - 1 + 1}{2\sin x\cos x} = \frac{2\cos^2 x}{2\sin x\cos x} = \frac{\cos x}{\sin x} = \cot x$$

According to Krutetski (1976:53), the generalization process consists of recognizing a problem's type and is related to discriminating its relevant features and abstracting them from the irrelevant features.

The following question was asked in a 1995 higher grade mathematics paper (Prins et al, 1996:18), illustrating the use of the thinking skill of generalizing in terms of Krutetski's definition:

The sum of 11 terms of $\log_{\frac{1}{2}} x + \log_{\frac{1}{2}} x^3 + \log_{\frac{1}{2}} x^5 + \ldots$ is 242.

Calculate the value of x.

This question gives mixed signals in terms of different topics addressed: indices, logs and series. It is necessary to generalize so that the "relevant features can be abstracted from the irrelevant," in the words of Krutetski (1976:53).

For the first step the kind of series is irrelevant, and the logs and their laws are relevant.

Applying the third log law, the expression becomes:

$$\log_{\frac{1}{2}} x + 3\log_{\frac{1}{2}} x + 5\log_{\frac{1}{2}} x + \ldots$$

Now the kind of series becomes relevant and the expression is identified according to the properties of series as an arithmetic series. The fact that the values of 'a' and 'd' are in terms of logs should be overlooked at this stage (regarded as irrelevant) and the logs should be regarded as 'expressions acting like numbers', so that the general formula for the sum of an arithmetic series: $S_n = \frac{n}{2}(2a + (n-1)d)$ can be applied. Eventually a stage is reached where the equation is reduced to: $2 = \log_{\frac{1}{2}} x$. This is then solved with the general rules that apply to logs, which have now become relevant again. Writing in the exponential form gives: $\left(\frac{1}{2}\right)^2 = x, \quad x = \frac{1}{4}$. From this question it is clear how the relevant and irrelevant features each have their time and place in the calculation.

## 2.3.2 Synthesizing

"Synthesizing means to combine or compose parts in such a way that they form a whole, an entity. This whole then often amounts to more than the sum of the parts," according to Tommy Dreyfuss (Tall, 1979:35). The calculations and decisions leading up to the drawing of a graph is an example of the kind of synthesis students on a Pre-Calculus level can achieve.

Dreyfuss (Tall, 1991:35) continues to say that the very fact that many previously unrelated facts could be merged into a single picture by the process of synthesizing means that mathematics is highly compressible. He also says that while this is the joy of mathematics, it is an irreversible process and as such has serious consequences for the teaching of mathematics. It places the burden of seeing the facts before synthesis from the student's point of view and adapting the teaching approach to accommodate the student's learning process.

Krutetski (1976:188) names a high level of development of the ability for synthesis as one of the criteria for mathematical ability. It would thus be prudent to include learning experiences in the curriculum that aim to teach or train students to synthesize to improve their mathematical ability.

The following question that calls for synthesizing comes from a higher grade mathematics paper of 1995 (Prins et al, 1996:16):

Consider a continuous function $f: y = f(x)$ with:

$$f(-3)=0,\ f(-1)=0,\ f(2)=0,\ f'(-2)=0,\ f'(1)=0,$$
$$f'(x)>0 \ if \ x<-2 \ or \ x>1,\ f'(x)<0 \ if \ -2<x<1.$$

Sketch the graph.

Given so much information that is completely in symbolic language, it is important to look at each piece separately to ascertain how it fits in with the other pieces. It is obvious that -3, -1 and 2 are the roots of the function. -2 and 1 can be identified as the x-values of the turning points. The rest of the information pertains to the shape of the graph: increasing (therefore ascending) for $x < -2$ and for $x > 1$ and decreasing (therefore descending) for $-2 < x < 1$. This culminates in the graph:

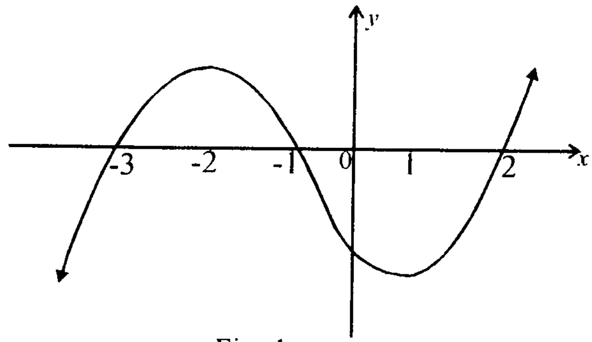


Fig. 1

Another example from a 1995 mathematics higher grade paper (Prins et al, 1996:17) where synthesis is the thinking skill tested is:

> In the figure 2 functions are represented graphically:
>
> $f(x) = ax^2 + bx + c$ and $h(x) = -x + 7$.
>
> Suppose the straight line is a tangent to $f$ in the point A(4;3).
>
> Derive the equation of the parabola using the given information, hence show that $a = -\frac{1}{4}$ and $b = 1$.



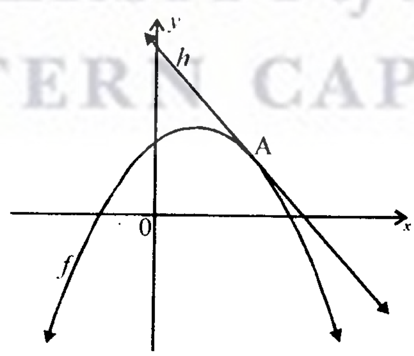


Fig. 2

To find the values of two variables, two equations are needed. The first equation is easily found by substituting the coordinates of A into *f(x)*. For the second equation, information must be combined – synthesized: The gradient of *f(x)* must equal the gradient of the straight line. The gradient of *f(x)* must then be found by differentiating *f(x)*. Only then the second equation which is needed in order to solve for $a$ and $b$ is achieved: At A: $2ax + b = -1$. Now another piece of information needs to be taken into account as well, therefore another synthesizing thought is needed. The coordinates of A must also be substituted into this equation, giving: $2a(4) + b = -1$. The stage is now set to solve for $a$ and $b$. This question compels the student to synthesize the information concerning the gradient gained from the derivative, from the tangent and from the point where the tangent touches the curve, truly a mini-exercise in synthesis.

These two skills, generalizing and synthesizing, form the background to acquiring the skill of abstraction, which will be discussed next.

### 2.3.3 Abstraction

Abstraction is the culmination of the two processes named above, namely generalizing and synthesizing, which are the prerequisite skills for the ability to abstract.

According to Skemp (1979:238), abstraction is to "take out just those parts of the problem which are relevant to the solution of the problem." This is essentially the mathematical activity when solving a word problem. An example of such a question from a 1995 higher grade mathematics paper (Prins et al, 1996:21) is:

> The manager of an apple farm must decide when the apples must be picked. If the apples are picked immediately the average harvest is 50 kg per tree. These apples can be sold for 96c per kg. Experience has shown that if harvesting is delayed, the harvest per tree will increase by 5 kg per week and the price will decrease by 6c per kg.

Write down an expression for the total income derived from one tree after x weeks. Use this expression to determine after how many weeks the apples should be picked to earn a maximum profit.

Here, leaving out all the words to concentrate on the essentials that make out the profit in the end brings one to the point where the variable number of kilograms and the variable cost are the essentials, with

kg = $(50 + 5x)$ and cost in cent = $(96 - 6x)$.

That makes the expression for total profit:

$$(50+5x)(96-6x) = 4800+180x-30x^2.$$

To find the maximum profit, this expression is identified as a quadratic expression and for the maximum profit $x = \frac{b}{2a} = -\frac{180}{2(-30)} = 3$. Thus, harvesting after 3 weeks will deliver the maximum profit.

In this way, abstraction, according to Skemp (1979:238), reduces the "noise" of additional information not essential to the computation of the solution.

An example of a higher grade question (Rossouw, 1997:12) that draws on a pupil's ability to think abstractly is:

The curve of the function $f(x) = 2x^3 - cx^2 + mx + 6$ has only one stationary point. Show that $c^2 = 6m$.

Without being able to draw the function or determine its roots to place it on the Cartesian plane, a calculation has to be done in the void and this calculation has to be trusted to deliver the appropriate answer. Bold knowledge of the processes involved in finding a stationary point is necessary. Then the calculations flow quite easily: $f'(x) = 6x^2 - 2cx + m = 0$ at the stationary point. This quadratic equation must have equal roots for the curve to have only one stationary point. Thus, $\Delta = 0$, which means that $(2c)^2 - 4(6)m = 0$ and this equation delivers the required result: $c^2 = 6m$. Thus, during the whole calculation, the value of x never became known and all calculations had to be done with an abstract (variable) value for x.

### 2.3.4 Classifying

The skill of classifying (Skemp, 1979:22) is fundamental to the study of mathematics. It means the collecting together of experiences on the basis of their similarities. This can happen in two different ways:

i. a single object in its own class among other classes
ii. a single object in many different classes.

The mathematical consequence of this fact is far-reaching (Skemp, 1979:80).

To illustrate the "single object in its own class among other classes," the following question from a 1995 higher grade mathematics paper (Prins et al, 1996:17) can be cited:

$$\text{If } h(x) = -x + 7,$$

which of the following sketches represents the graph of $-h^{-1}$?

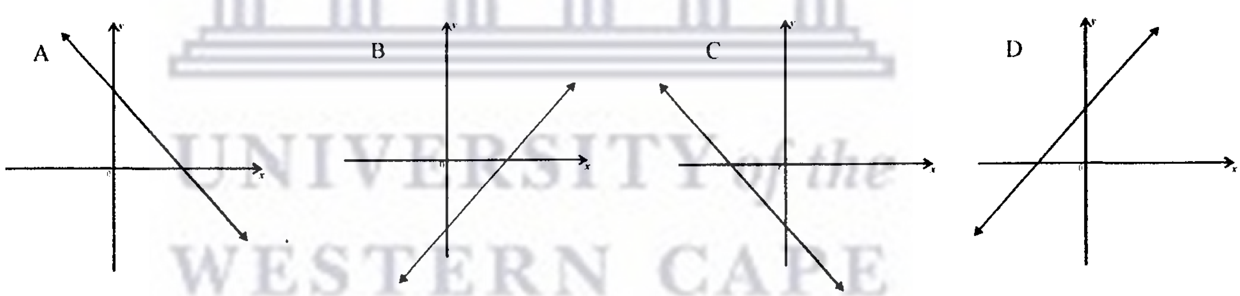


Fig. 3

This question calls for the classifying of each of the two minuses accompanying the $h$ to be classified, each into its own class. This will enable the student to do the computation in the correct order. The minus in $h^{-1}$ constitutes the inverse of $h$, therefore $h^{-1} = -x + 7$ and the minus in front simply means $-1$ times the value of $h^{-1}$ thus: $-h^{-1} = x - 7$ To be able to identify the matching graph the answer must now also be classified:

- the gradient must be positive,

- while the $y$-intercept must be negative, bringing the correct answer to B.

This constitutes a very simple example of classifying but serves to illustrate the concept of a single object in its own class among other classes.

To illustrate the "single object in many different classes" the following example from a 1995 higher grade exam paper (Prins et al, 1996:43) serves well:

A function is defined by $f(x) = -ax^3 - 4x^2 + 8x + b$.
The graph of $f$ has a turning point at (-2,-13).
Calculate the values of $a$ and $b$.

Because two variables have to be solved, two equations have to be set up. Substituting the given coordinate into the equation for $f(x)$ will only give one equation. The other equation comes from the fact that $f'(x) = 0$. However, only one coordinate was supplied. This means that the coordinate has to be used twice and thus be classified in two different classes:

- first as a non-critical point on the graph and then
- as a critical point, the turning point of the graph.

In this way the one coordinate fits into more than one class, depending on what has to be done with it.

It is clear to see how important this thinking skill is for mathematics. It is thus imperative that a mathematics curriculum provides the student with appropriate examples in which a specific piece of knowledge can be applied to as many different situations as is possible at the level of mathematical functioning of the student, and in doing so, teaches the student to classify abstract information.

## 2.3.5 Forming Conceptual Entities

Resnick & Ford(1981:101) states that the development of mathematics teaching from traditional computational approaches to conceptual approaches began in the late 1950s. It became necessary "to teach ... the basic concepts and principles that give coherence to the subject matter of mathematics (i.e. the structures of mathematics)."

Tall (1991:82) says that a conceptual entity is a body of knowledge that is permanently available as an object in the individual's mind. These entities have the result that they alleviate working memory or processing load, facilitate comprehension of complex concepts and assist with the focus of attention on the appropriate structure in problem solving.

Skemp (1979:37) calls these mental structures "schema". He states that each secondary concept is derived from other concepts and contributes to the forming of yet other concepts, thus forming schemas in the mind of the learner.

A good curriculum, that would give the student opportunities to form conceptual entities or schema, would provide teaching that is structure orientated. The content of the course should emphasize the commonalities and the differences between different situations to enable the student to make the right choice in applying a certain concept. It should also build onto existing concepts, say graphs, to form the student's knowledge of the topic into a conceptual entity. It is then to be hoped that at least the more successful, if not all, students will store this information, not as little unattached pieces of information, but as an entire body of knowledge under the key word 'graph' in their brain. This could then be 'opened', very much like a file in a computer, to make all the knowledge available when needed, while it does not clutter the memory when it is not needed. It thus consolidates the knowledge and sets the mind free to process or work with other concepts. The following is a schema on the gradient of a line. This combines the knowledge on this topic that a pupil should ideally acquire at school.

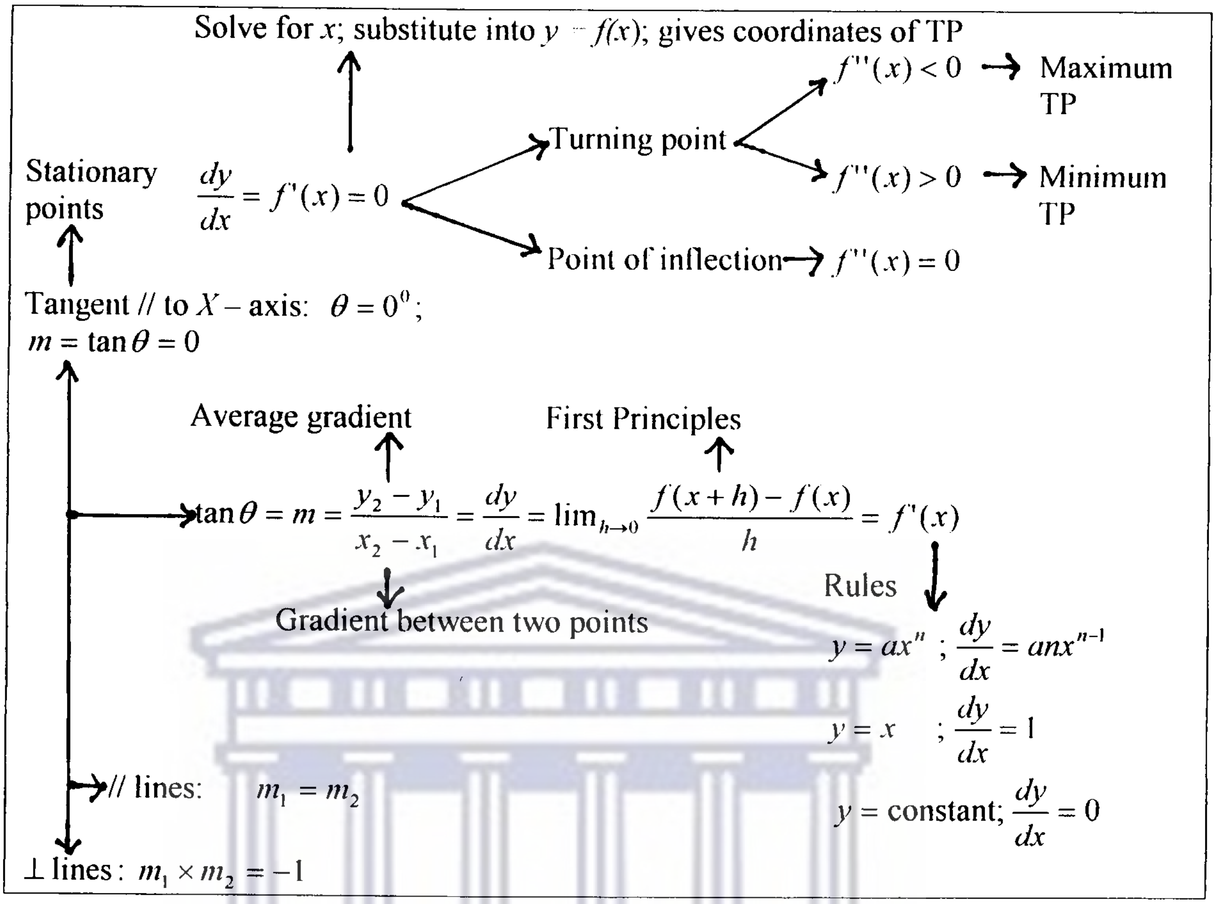


Table 4: A mental schema on the gradient of a line



## 2.3.6 Generic generalization

Ausubel (1968:515) sees generic generalization as the most significant aspect of the concept assimilation process. He states that it involves "relating, in non-arbitrary, substantive fashion, to relevant, established ideas in the learner's cognitive structure, the potentially meaningful generic content contained in the definition or contextual cues (its criterial attributes). The phenomenological emergence of the new generic meaning in the learner is a product of this interaction. It reflects both:

a) the actual content of the new concept's criterial attributes and of the anchoring ideas to which they are related, and

b) the kind of relationship (derivative, elaborative, qualifying, or super-ordinate) established between them."

Tall (1979:12) defines this thinking skill as the application of a well-known process in a broader context.

Generic generalization is achieved, for example, by applying the knowledge regarding the nature of the roots of a quadratic equation to find the turning points of the graph of a rational function. The formula used to determine the roots of a quadratic equation is recognized at school level for its qualifying of the nature of the roots of a parabola. Later, this attribute of the formula will be elaborated on to find the turning points of the graph of a rational function, thus transferring information from one concept to another. No longer does the formula fit only into the framework of a parabola, but can now be used in appropriate cases to find other values required.

In this way the use of the formula is superordinated to have more meaning than before and to be applicable to other situations than just the graph of a quadratic function. But, more specifically, students become aware of the multiplicity of a mathematical concept - its generic content. This awareness extends their mathematical insight and attunement.

### 2.3.7 Coping with a Disequilibrium

Teaching via the forming of a mental equilibrium and then establishing a disequilibrium to encourage further studying, is an integral part of how people learn. This means that the students realize the need for a different approach when previously used algorithms are inadequate to deal with an altered situation/question, and that this constitutes a challenge to them to try/find/learn a different algorithm that will solve the problem.

In this manner, the establishment of a mental disequilibrium, according to Piaget, as quoted by Wadsworth (1978:82), is a stimulus for intrinsic motivation for learning in the student, as people generally strive to a better mental equilibrium. Thus the student is motivated to accommodate the new knowledge in his cognitive structures and work with and on it until it is assimilated into his cognitive structures and equilibrium is once again attained, the new knowledge now forming part of previous knowledge and the student ready for the next experience of disequilibrium.

A question from a 1995 higher grade mathematics paper (Prins et al, 1997:16), illustrates the testing of this thinking skill:

If $a$ and $b$ are the roots of $x^2 - 5x - 1 = 0$,

determine the value(s) of

(i) $b^2 - 5b - 1$ and (ii) $a^2 - 5a + 1$

It is reasonably easy to see that (i) fits into the equation $x^2 - 5x - 1 = 0$, and as $b$ is a root, $b^2 - 5b - 1 = 0$. For (ii) the situation changes - disequilibrium. Now (ii) does not fit the equation $x^2 - 5x - 1 = 0$ and the equation needs to be altered in an appropriate way to produce the correct answer: It is true that $a^2 - 5a - 1 = 0$ because $a$ is a root of the equation. Therefore $a^2 - 5a - 1 + 2 = 0 + 2$, making $a^2 - 5a + 1 = 2$. It is the recognizing that (ii) is not the same but related to (i) that the forming of a disequilibrium is presented, and the challenge to overcome the disequilibrium is set.
A curriculum that enables students to study using the force of intrinsic motivation in this way will then be a curriculum that periodically challenges the students with content beyond their comfort zone.

### 2.3.8 Imagery

Skemp (1979:94) writes about mental imagery, saying "some people have strong visual imagery and others think mainly in words while some people have both kinds of mental imagery available." For studying mathematics, it is crucial to have both kinds of thought imagery available. It is therefore also crucial to develop both these kinds of imagery in mathematics students.

Imagery can, in terms of mathematics be separated into two parts: verbal imagery as in algebraic symbols and visual imagery as in graphs or diagrams. Verbal, especially symbolic thought plays a crucial part in mathematics (Skemp, 1979:94-96).

When doing linear programming, in the matriculation mathematics syllabus for higher grade, the profit/loss is determined by a line with constant gradient but with variable y-intercept. To visualize this line moving across the graph to obtain the maximum/minimum required is a case of mental imagery very often found in mathematics.

In higher grade mathematics at school, the following kind of question is often encountered (Rossouw, 1997:17):

> In the figure Q, T and R are points in the horizontal plane such that TQ = TR = y and TP represents a vertical pole positioned at T.

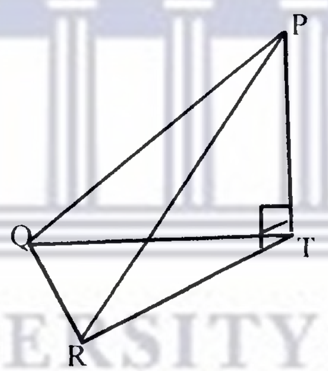


Fig. 4



Interpreting this three dimensional sketch proves to be an exercise in visual imagery based on the verbal imagery of the words of the question.

## 2.3.9 Reflectivity

One of the major differences between the SG and HG math syllabi at school is the way questions are asked in tests and exams. A SG paper would probably consist of 75% of the questions requiring only one or two steps to achieve the answer with the information presented reasonably straightforward. On the other hand, a HG paper would have the information obscured and the answer likewise not acquired in a simple step or two.

In November 1995 (Prins et al, 1996:17), the following question was asked in a HG mathematics exam paper:

In the figure two functions are represented graphically:

$$g(x) = \frac{k}{x} \qquad \text{and} \qquad h(x) = -x + 7$$

Suppose the straight line is a tangent to $g$. Determine the value of $k$.

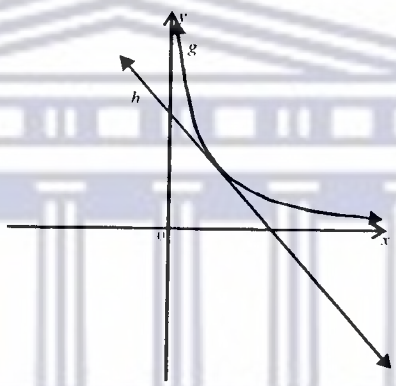


Fig. 5

The information given includes a hyperbola, a straight line and a tangent. The question is: which to use first? Therefore what information to 'receive' first for mental processing, and what information is to follow on that. This leads the HG student to have to think reflectively (Skemp, 1979: 58) where information is received into a person's mental schema (Skemp, 1979:37) on the topic processed, but not generating the ultimate answer, and is therefore received again into a different mental schema of the person, where it can then be solved, understood, redefined, until it can be formed into the desired answer. Skemp names this the reflective activity and illustrates it with the following diagram:

| Intervening Mental Processes | |
|---|---|
| ↑ | ↓ |
| Receptors | Effectors |
| ↑ | ↓ |
| Intervening Mental Processes | |
| ↑ | ↓ |
| Receptors | Effectors |
| ↑ | ↓ |
| External Environment | |

Table 5: Reflectivity

Applying this scheme to the question above, we find:

Information received from external environment:

Straight line tangent to hyperbola.

Intervening mental process: This means that the two graphs have only one common point, which is at the value of $x$ for which:

$$\frac{k}{x} = -x + 7$$

Information received: Quadratic equation with 'equal' roots.

Intervening mental process: Apply $\Delta = 0$

Information output to external environment: $k = 12\frac{1}{2}$

This kind of question is generally known as a question that requires 'insight'. It is clear to see how this name came about, as it actually requires the student to look in until he/she gets to a point from where he/she can begin to see out. A curriculum that aims to raise the level of mathematical thinking of the students in the course would offer content that requires reflective thinking and would definitely test students with questions requiring reflective thinking.

## 2.3.10 Transition

The process of transition from one mental state to another consists of the assimilation, taking in new ideas, and accommodation, the modifying of an individual's cognitive structure, according to Piaget's theory for learning mathematics as cited by Tall (1991:9).

"Skemp (1979) puts similar ideas in a different way by distinguishing between the case where the learning process causes a simple expansion of the individual's cognitive structure and the case where there is cognitive conflict, requiring a mental reconstruction" (Tall, 1991:9). This is a case where students have to overcome the obstacle of preconceived ideas about a topic to make the transition to be able to see other aspects of it.

A question from a 1995 mathematics higher grade paper (Prins et al, 1997:18) that calls on this kind of thinking skill can be the following:

Given $r = a^{\log 2}$ and $t = 5^{\log a}$, prove that $rt = a$.

Although the expressions for $r$ and $t$ are given as exponential expressions, the product $rt$ needs logs to simplify the resulting product of exponential expressions. Thus

$$\log r = \log a^{\log 2} = \log 2.\log a \quad \text{and} \quad \log t = \log 5^{\log a} = \log a.\log 5$$

Now the expression for $rt$ must be derived from the logs:

$$\log rt = \log r + \log t = \log 2.\log a + \log a.\log 5 = \log a(\log 2 + \log 5) = \log a.\log 10 = \log a$$

It is now clear to see that, as $\log rt = \log a$, $rt = a$.

So, although the question was stated in exponential form, the student had to make the transition to logs to be able to solve the problem.

### 2.3.11 Reversibility

Krutetski (1976:85) notes: "the ability to switch from a direct to a reverse train of thought characterizes the difference in mathematics ability in students." Tall (1991:105) describes it as the reversal of a process – add vs. subtract, draw a graph vs. find the equation that defines the graph, differentiate vs. integrate, etc. Piaget, as quoted by Wadsworth (1978:19), claims that "reversibility is necessary for operational solutions to problems" and that it is "the logical tool that permits logic to triumph over perception in thinking."

In practice it is the one tool that enables a student to check an answer to a question. It is also the backbone of reasoning in geometry. It is probably the reason why geometry is still included in the matriculation syllabus – to offer learners the opportunity to learn to think in reverse.

When a question is asked in a reverse order, it states the answer and requires the student to find the origin of the answer. In a 1995 mathematics higher grade paper (Prins et al, 1996:16), the following question is such an example:

Consider $\sqrt{12-2x} = 4x-3$.

Without solving the equation, show that $\frac{3}{4} \leq x \leq 6$.

Here the solution for $x$ is supplied. Normally the question would be (on standard grade): Solve for $x$.

Starting with the answer, as required in the quoted question, forces the student to think in reverse. Obviously $\frac{3}{4} = x$ come from $4x - 3 = 0$, but why should $x > \frac{3}{4}$? Thinking in reverse is thus not just a matter of linking steps of a calculation in a reverse order, but it also entails the stating of reasons why this order is correct. Thus, $x > \frac{3}{4}$ so that the answer of the square root can be positive: $x > \frac{3}{4}$ means that $4x - 3 > 0$. On the other hand, $x \geq 6$ clearly comes from $12x - 2 \geq 0$, according to the properties of surds. This question then illustrates the fact that reverse thinking is tested in higher grade mathematics at school.

## 2.3.12 Refined Intuition

Tall (1991:14) defines primary intuition as cognitive beliefs that develop themselves in human beings. Secondary intuitions are those developed as a result of systematic intellectual training. This kind of "trained" intuition is also called "refined" intuition or "second degree" intuition. He continues to say:

> Thus aspects of logic too can be honed to become more 'intuitive' to the mathematical mind. The development of this refined intuition should be one of the major aims of more advanced mathematical education.

Only by practicing mathematics does this refined intuition develop (Tall, 1991:14). Of all the topics in school mathematics, both on higher and on standard grade, the one that uses a student's refined intuition most is geometry. In the following example from a higher grade question paper (Rossouw, 1997:22) refined intuition is used to find the best starting point for the reasoning.

The diagonals PR and QS of square PSRQ, with sides 6 units long, intersect at K. T is the midpoint of SR and PT cuts QS at G. Calculate the length of PG, leaving your answer in simplest surd form.

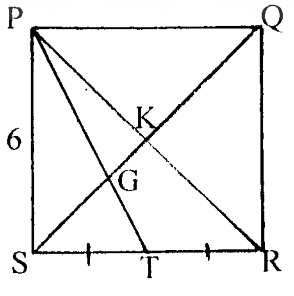


Fig.6

Refined intuition should tell one that starting with the given lengths here would not easily lead to the answer to the question. It is much more appropriate to start with the point G and determine what can be said about its place in the sketch. Lying on two medians, G is the centroid of $\Delta PSR$ and now it is known that $PG : GT = 2 : 1$. Applying the theorem of Pythagoras now, is a good choice to find the length of PT and hence the length of PG is easily found. Geometry questions excel at giving information that has to be synthesized. Where to start is discovered by doing many examples and so honing one's refined intuition.

## 2.4 Practical mathematical skills

Mathematics is a practical subject par excellence in the respect that it has syntax of its own. The way knowledge is communicated is reasonably rigid and competency in the writing of mathematics is indispensable to students of mathematics. A mathematics course that presumes to raise the level of mathematical competence of the students will emphasize the following practical mathematical skills:

- Adequate algorithmic skills
- Recording knowledge appropriately
- Correct representation of mathematical information
- Understanding and being able to use the terminology of the subject
- Checking of own work and evaluating the correctness.

### 2.4.1 Algorithmic activity

To eventually become creative mathematicians, students have to acquire the algorithmic skills needed in the proving and disproving of hypotheses made in problem solving (Skemp, 1979:88).

Most failures in mathematics stem from the inability of students to do routine manipulations automatically. At matriculation level, such automatic manipulation would include, e.g.

- drawing a parabola to ascertain which $x$-values will give positive or negative values for $y$;
- factorizing a trinomial;

- switching from logarithmic form to exponential form and back;
- solving simultaneous equations in two variables;
- drawing a straight line graph, finding the equation of a straight line graph, to name but a few.

Not being able to do these computations/manipulations automatically:

- will influence the ease with which students can follow lectures.

  Lecturers, when lecturing on more advanced mathematics generally assume these manipulative skills to have been acquired, and do not give time or attention to such basic detail. If the students, however, cannot perform the manipulations/computations with the required ease and speed, they get left behind in the subsequent explanations, and find it hard to follow the logic of the explained work.

- will also distract them from the main line of thinking/computation when working on problems.

  This makes it hard to see the bigger picture because they keep getting bogged down in detail rather than being accelerated by the automatic processing of that information.

- will also impact on their problem solving skills because they will not readily see the applicability of a computation/manipulation if they find it a tedious activity to do.

- will also impact on the time it takes them to work through and practice examples.

  This then leads to the process taking even longer to become automatic.

Being fluent in the algorithmic skills pertaining to the topic being studied is to the benefit of the student and any course aiming at raising the mathematical competence of the students will have to include time and opportunity for these skills to be practiced.

### 2.4.2 Recording Knowledge

Recording is a special case of communicating, since it is normally done with the intention that these records shall, in the near or distant future, be seen by others. (Skemp, 1979:73)

Skemp distinguishes 3 categories of this kind of communication – each having its place and time in mathematics instruction and study.

Firstly, there is communication with someone who doesn't know what we are talking about, but wants to know.

> This is essentially what happens in a lecture. The lecturer carries the responsibility of not alienating the student, but attracting him/her to the subject. Thus the lecturer bears the responsibility of choosing the best follow-up of concepts, the correct use of symbols, how much of the topic to reveal in the time of the lecture and to which depth of detail to progress.

Secondly, communication with those who do know what we are talking about, but only as a general background, within which we are trying to communicate some particular aspect.

> Such is the communication of a consultation or a revision class, where much can be taken for granted, and time saved if only the essentials are concentrated on – given the students are willing to 'go along' with this (and not get stuck on non-essentials, see 'Making routine manipulations automatic.', par 2.4.1)

Lastly, there is the situation in which we are communicating with those who do know what we are talking about, but want to fault it.

> This is the test situation, wherein the student has to convince the lecturer of the validity of the student's knowledge. This is then also the time when mathematical notation, symbols, logic, computation and representation are crucial skills the student should have acquired.

The aim of study is to become more knowledgeable, but having knowledge that one cannot share with other people in the same field of study has very little meaning. As students thus gain knowledge, they should also gain communication skills.

## 2.4.3 Representation

Students often have problems understanding the representation of concepts in Mathematics. The representations containing the expression $(a+b)$:

$$5(a+b),\ \log(a+b) \text{ and } f(a+b)$$

all look the same to an uninformed person, but should mean three vastly different things to a mathematician. This is often not the case among pupils at school and/or students at university. Apparently they have much difficulty in interpreting the symbolic language of mathematics.

According to Tall (1991:30), symbols involve relations between signs and meanings: they serve to make a person's implicit knowledge – the meaning – explicit in terms of symbols.

Piaget, as quoted by Wadsworth (1978:165), addresses representation of mathematical operations and states that before students can comprehend a representation of mathematical operations they must comprehend the operations themselves.

As abstract as the concepts of mathematics are, it is even more difficult to read and write the sign language accompanying it. Once again, this skill should be addressed in any mathematics course aiming at raising the level of mathematical ability in students.

### 2.4.4 Terminology

The terminology of any subject is very important. Without knowing the correct terminology, there can be no effective communication between lecturer and student or even between textbook and student and most importantly between student and lecturer during a test.

Questions that are stated in words, but need to be 'translated' into symbols to be calculated, rely heavily on the understanding of the terminology of the subject. In this regard, questions in geometry are excellent examples. Geometry as a topic has a vocabulary that seems to make it a completely different language. Pupils at school who have to learn words and expressions like: cyclic quadrilateral, tangent, opposite interior angle, subscribed by the same chord, etc. find it very difficult to make this language their own and this is one of the great stumbling blocks for many pupils in their study of geometry. It is thus clear that it is impossible to study a subject if the terminology of the subject is not well known to the student.

### 2.4.5 Checking

Checking as a mathematical process is described by Tall (1991:40) as "taking actions to convince oneself that a result indeed does answer the question that was asked, and does answer it correctly!"

Very few students check their answers. The reason for this is that they consider the answer to be a product to be judged for correctness by the lecturer and therefore they do not accept responsibility for ascertaining the correctness of their answers. Mathematics and other mathematics-based subjects are also unique in this respect. Subjects that have mainly word content, like Psychology, Sociology, Law, etc. do not offer the student any means of checking an answer and thus train the student to rely on external assessment of the worth of an answer.

Other reasons for not checking answers are found to be:

- Most importantly, the students do not know what the answer means and therefore do not know what to do with the answer to check it.
- Students are reluctant to accept that they can be wrong.
- In some cases they do not realize that answers can be checked.
- Often students are not able to do the calculations involved in checking the answer.

## 2.5 Summary

As shown in this chapter, the design of the curriculum and the thinking and practical skills developed by the content of a course can offer students a wide variety of learning experiences. Thus, apart from learning actual mathematical content, they can also be given opportunities to develop into mathematicians. In the next chapter the Pre-Calculus course will be compared to what other higher education institutions offer students in similar circumstances. The Pre-Calculus course will also be examined for the presence of the skills mentioned in this chapter. Lastly the content of the course will be analyzed to show how school mathematics is revised and enhanced by the Pre-Calculus curriculum.

# Chapter 3

## Analysis of the Pre-Calculus Course

### 3.1 Introduction

This chapter will show the level of correlation between the Pre-Calculus course offered at UWC and similar courses offered at other higher education institutions in South Africa and overseas. The first question to be answered in this chapter.is thus 'How does the UWC Pre-Calculus course compare to other higher education institutions locally and abroad?' This research was done via the Internet. After ascertaining what the other courses offer in comparison to what the Pre-Calculus course offers, the content of the Pre-Calculus course was analyzed to show which skills, identified in chapter 2, are supported by each topic and also to show how the school mathematics is used and expanded by the Pre-Calculus course.

The current chapter also takes each skill and matches it with subject matter in the course curriculum, to show that the course did indeed offer the students opportunities to add value to their matriculation results in mathematics. Firstly, the design of the curriculum will be discussed against the background of which skills it can foster. Secondly, the desired thinking skills will be matched up with the content and topics identified that could teach these skills. Lastly, the course will be analyzed to see if it offered learning experiences for the acquiring of the essential practical mathematical skills.

### 3.2 Correlation between the Pre-Calculus course at U.W.C. and other courses with similar aims

As to types of courses, two types can be identified. Maritz Snyder (University of Port Elizabeth) spoke at the Unify Indaba II Conference, held at the University of the North from 7-8 June 2002, and distinguished between a 1+3 and a 2+2 year course. A 1+3 year course he calls a bridging course. This type of course would typically be aimed at filling in the gaps left by inadequate learning at school by repeating the school content. 1+3 means a year of preparation for a degree followed by a 3 year degree. A 2+2 year course on the other hand is called a foundation course. This means that the 'first year' of a normal degree course is spread out over 2 years and then the 2nd and 3rd years of the degree follow, each in the allotted 1 year.

The UWC Pre-Calculus course constitutes a merger of the two options mentioned. Although the UWC Pre-Calculus course does not form part of a degree like the 2+2 case, it also does not only fill in the gaps like the 1+3 model. The UWC Pre-Calculus mathematics course is contained in 1 year but has all the attributes of a foundation course as listed by Snyder (2000).

In comparing the Pre-Calculus course to courses with similar aims at other higher education institutions, emphasis was placed on the content of the courses. With respect to duration of course, lecture and tutorial time allocated and entry requirements, the variation was such that no 'median' could be established for these variables.

The curricula of courses preparing students for first year mathematics courses at the following institutions were accessed via the internet: La Trobe University (2001), Australian National University (2001), Unisa (Singleton, 2000), Naenae-college (2001), Canadian Mennonite University (2002), University of Michigan (2002), Woodsworth College (University of Toronto) (2001), St Louis University (2002), University of California, Irvine (2002), and University of Southern California (2002).

From these curricula the following topics occurred in 50% of the curricula: (in order of frequency)

Exponential and logarithmic functions
Linear and quadratic equations
Trigonometry
Polynomials

Inequalities

Functions and graphs of functions

Analytic geometry

The UWC Pre-Calculus course offers all these topics except analytic geometry.

Other topics that are less generally taught are:

Rational functions, Systems of linear equations, Absolute Value, Complex numbers, Binomial theorem, Networks, Vectors, Series, Sets.

Of these, the UWC Pre-Calculus course includes Rational functions, Systems of linear equations, Absolute Value and Complex numbers.

It can thus safely be deduced that the content of the UWC Pre-Calculus course is in line with what other institutions deem necessary to teach in a preliminary mathematics course.

## 3.3 Aspects of the design of the course

Aspects of the curriculum that are deemed very important and should be given the appropriate attention were identified by the literature study as being:

- Sequencing of Topics,
- Hierarchy of Concepts,
- Conceptual Thinking, and
- Level of difficulty.

Each of these have been defined and analyzed in the literature review (chapter 2) and will be discussed here with reference to the content of the course.

### 3.3.1 Sequencing of the Topics

The spiral curriculum that J.S. Bruner (In Howson et al, 1981:112) deems necessary is "a curriculum in which ideas are presented in homologue form, returned to later with more precision and power, and further developed and expanded until in the end, the student has a sense of mastery over at least some body of knowledge."

In this regard the Pre-Calculus curriculum sequence introduces the graphs of a polynomial where the use of a sign table is taught. Later, when drawing the graph of a rational function, the sign table is revisited. Even later, when inequalities are solved, the sign table is again used. This constant revision of the concept of a sign table creates the opportunity to master the concept and to grasp the different situations in which it can be of help.

Another example of the appropriate sequencing of topics is the calculation of the turning point of the rational function graph. Initially the graph is drawn without emphasis on turning points. This is done so that the students can concentrate on the attributes of the function with regard to asymptotes and end point behavior. Later in the year, the drawing of the graph is revisited and then the turning points are calculated as well. In this way the concept of the graph of a rational function is expanded over time and the students given time to master the concept.

Bruner (In Howson et al, 1981:112) also states that the teacher should find the ideas that have been presented earlier and deliberately use them as much as possible for the teaching of new ideas. This is done explicitly with the factorizing of a polynomial, which is applied again in drawing the graphs of the polynomial, and also in solving inequalities. Another example of sequencing of topics has to do with radian measure. Radian measure is introduced and used in the section on trigonometry and again in the polar form of a complex number, thus giving it more weight in terms of applicability.

Bruner also expects the teacher to look to the future and teach some concepts and understandings even if complete mastery cannot be expected. In this regard, a good example is the drawing of the graphs of polynomial functions in which the turning points are not determined at this stage as this requires calculus, which does not form part of the Pre-Calculus curriculum. Nevertheless, the general attributes of a polynomial were studied and the graph drawn, showing where turning points will approximately be but not calculating them.

## 3.3.2 Hierarchy of Concepts

The course content was carefully analyzed and proved to follow a strong hierarchy of introducing concepts. This makes the communication of concepts (Skemp, 1979:24) simpler than when all concepts would just be 'understood' from their definition.

Graphs of functions form the central concept of the course. They are presented in the following sequence:

Polynomials, Rational Functions, Piecewise Graphs, Absolute Values, Transformation of Graphs, Exponential Graphs and Logarithmic Graphs and ultimately Trigonometric graphs.

This order of presenting the topics is such that some concepts of each topic are used in subsequent topics. For example, in the following table, the follow through of concepts is clearly illustrated.

| **Polynomial function** | **Rational function** | **Piecewise graphs** | **Absolute value function** | **Transformation of graphs** | **Trigonometric functions** | **Exponential and Log functions** |
|---|---|---|---|---|---|---|
| Long division to find factors | Long division to find horizontal and oblique asymptotes | | | | | |
| | Domain subdivided by asymptotes | Domain subdivided by definition of each piece | Domain subdivided by calculation | | Domains of graphs of tan $x$ and the inverse trigonometric functions | Restrictions on domains |
| | | Revision of basic graphs done at school | Revision of the straight line graphs | Revision of basic graphs | | |
| | End point behaviour | | | | | End point behaviour |
| | | | Restrictions on Range | | Restrictions on Range | Restrictions on Range |

Table 6: Hierarchy of concepts in the Pre-Calculus course

Not only are the topics in the curriculum arranged in a hierarchical order, but the examples are also chosen to expand concept forming gradually.

Such was the case in the factorizing of polynomials that preceded the drawing up of the sign table to predict the shape of the graph. This is taught in a concept expanding hierarchy (Skemp, 1979:31) as is illustrated by the factorizing of polynomials that successively:

- fully factorized into linear factors:

  Example: $3x^4 + 8x^3 - 15x^2 - 32x + 12 = (x+2)(x-2)(x+3)(3x-1)$

- factorized into linear factors and a quadratic factor with irrational factors. Here the formula for solving a quadratic equation was used to find the irrational factors:

  Example:

  $$\begin{aligned} 3x^4 + 20x^3 + 17x^2 - 44x + 12 &= (x^2 + 4x - 4)(x+3)(3x-1) \\ &= (x + 2 - 2\sqrt{2})(x + 2 + 2\sqrt{2})(x+3)(3x-1) \end{aligned}$$

- factorized into linear factors and a quadratic factor that has no real roots:

  Example:

  $$\begin{aligned} 3x^4 + 20x^3 + 44x^2 + 28x - 15 &= (x^2 + 4x + 5)(x+3)(3x-1) \\ &= ((x+2)^2 + 1)(x+3)(3x-1) \end{aligned}$$

  Here completing of the square was used to find the sign of the quadratic factor so it could be represented on the sign table, or, if the factor was of the kind $x^2+a$, the sign was obviously positive.

In this way, while still basically factorizing a polynomial, other concepts are confronted, discussed, used, and eventually taken up into the students' mental schema (Skemp, 1979:40) on factorizing, and the effect that factors have on the sign of a function. This hierarchical aspect of the design of the course makes it both easier for the lecturer to teach and for the student to study.

### 3.3.3 Conceptual Thinking

The Pre-Calculus mathematics course proved to be an example of a curriculum put together to foster conceptual thinking. This means that it provided learning experiences where a single concept is applied in different situations, so strengthening the concept by using its various attributes in appropriate situations. For example, the formula for solving a quadratic equation is applied in all kinds of different situations

- to find irrational factors of a polynomial;
- to find the turning points of a rational function;
- to emphasize the fact that e.g. $\sqrt{2}$ is an exact number, whereas its decimal 1.41... is not, (this is shown in the calculation of the turning point of a rational function, where 1.41... does not give equal roots, but $\sqrt{2}$ does);
- to find the complex factors of a polynomial;
- to illustrate the fact that the two roots produced by the formula are conjugates of one another.

This expanded the concept of the quadratic formula and encouraged the vertical growth of the mathematical knowledge (Tall, 1991:83) of the students. Fostering conceptual thinking does indeed add value to the level of mathematical thinking that the students brought to university.

### 3.3.4 Level of Difficulty

The level of difficulty of the Pre-Calculus course has been found to be such that it:

- challenged the students to expand their mathematical boundaries and
- challenged them to improve to the standard of HG school mathematics in terms of content, depth and thinking skills involved.

At the same time, the number of passes in the course showed that the level of difficulty kept the subject accessible.

Quite frequently the subject matter in the Pre-Calculus course starts with a new basic concept and these times should be experienced by the students as 'easier' than when the concept is being developed into more depth and detail.

Synthetic division is an example of such a topic. At first it is easy to follow the algorithm. Later, however, when deductions have to be made from the outcome of the division, the level of difficulty escalates. This motivates students to work harder at mastering the algorithm in order to consistently get a correct answer in order to make the accompanying deductions.

Another topic in the Pre-Calculus course that started out simple is the concept of complex numbers. Initially it is just an easy revision of basic algebra. Eventually, however, it escalates in difficulty to find the nth root of a complex number.

An increasing level of difficulty in a mathematics topic is so characteristic of the subject that the Pre-Calculus curriculum quite naturally attended to the fact that the subject matter should always be approachable.

## 3.4 Thinking skills developed by the Pre-Calculus course

### 3.4.1 Generalizing

In the Pre-Calculus course, the method of generalizing is particularly applied to the transformation of graphs. The transformations of the graph of $y = x^2$ to be the graphs of $y = (x \pm a)^2$ or $y = x^2 \pm a$ or $y = ax^2$ and combinations of these are discovered by working out tables of values to see how each alteration in the equation of the function transforms the graph of the function. Once this was done to the students' understanding and satisfaction, and they had the general idea, they were expected to generalize this information to encompass the transformations of any other given basic graph. These basic graphs include graphs of logarithmic, exponential and trigonometric functions, so the concept is fairly widely generalized. In terms of the definition of generalizing, the different graphs the transformations are applied to thus form the discriminating features, and the transformation itself, the commonalities in the algorithms. This also connects to the description of generalizing that it is easier to remember an algorithm than to remember the answers to all calculations that could be done with the algorithm.

Another example of generalization that is used extensively in the course is the factorizing of a quadratic trinomial. In its most simple form, this is: $ax^2 + bx + c$, say, $x^2 + 7x + 12$ with factors $(x+3)(x+4)$. The general method for factorizing a quadratic trinomial was then expanded and applied to expressions like: $5^{2x} + 7.5^x + 12$; $25^x + 7.5^x + 12$; $\tan^2 x + 7\tan x + 12$; $x + 7\sqrt{x} + 12$; $(\log x)^2 + \log x^7 + 12$; etc. The fact that each of these expressions fit into the form $x^2 + 7x + 12$ means that the factorization could be done quite automatically (commonality) and attention could then be focused on the subsequent answers (discriminating factors), instead of the factorizing taking up most of the attention.

In conclusion, the Pre-Calculus course offers the students many opportunities for and much practice in acquiring the skill of recognizing the general form in any altered state and then applying the generalization of the method instead of having to memorize different methods for each given alternative.

### 3.4.2 Synthesizing

In the Pre-Calculus course, a mild form of synthesis is achieved in the drawing of the graph of a rational function. The calculations leading up to the drawing of the graph generates a variety of information:

- the roots and y-intercept as used in all previous functions: roots at $y = 0$, $y$-intercept at $x = 0$.
- asymptotes – formerly only used in the graph of $f(x) = \tan x$. This has now expanded to:
- Vertical asymptotes at the values of $x$ for which the denominator = 0.

- Horizontal and oblique asymptotes: Choosing between these asymptotes by inspecting the degree of the numerator in comparison to the degree of the denominator.
    - Deg(numerator) = deg(denominator) + 1 : oblique asymptote
    - Deg(numerator) = deg(denominator) : horizontal asymptote
    - Deg(numerator) < deg(denominator) : vertical asymptote only
    - Applying algebraic long division to determine the oblique/horizontal asymptotes.
- Inspecting endpoint behavior when $x \to -\infty$ and $x \to +\infty$
- Factorizing the function and drawing up a sign table to determine the behavior of the graph between the endpoints.
- Calculating the turning points. This is done without the advantage of calculus and thus the quadratic equation and the fact that for equal roots, $\Delta = 0$, is used. (This in itself is an exercise in accuracy, incorporating surds, substitution and resubstitution, providing an attention detractor for a less competent student.)

Incorporating all of the above information to eventually draw the graph of the rational function constitutes an elementary exercise in synthesis. The resultant graph is then also indeed more than just the representation of the calculations. It also reflects the relationship between x and y for the parts of the domain not directly involved in the calculations.

Drawing both the graphs of the polynomial and the rational function give the students valuable exercise in the thinking skill of synthesis.

### <u>3.4.3 Abstraction</u> (Tall, 1991:37)

Many topics in the Pre-Calculus course stand out for the use of abstracting. Among these synthetic division was experienced by the students as extremely abstract. It was also done early in the year, challenging them quite early in the course to think abstractly.

Synthetic division uses the same kind of abstraction as matrices (using only the coefficients). The result of the computation does not, however, give the direct answer. After each division the result must be analyzed and evaluated to extract the information, as the last row of entries is just an array of numbers. This array carries information about the roots and factors of the polynomial in the following ways:

- The signs of the last row of entries indicates a lower or upper bound of the roots of the polynomial, directing one in one's choice of subsequent divisors. The fact that the lower and upper bounds of the roots of the polynomial do not feature in the drawing of the graph made this deduction in itself very abstract. Many students struggled with this concept.
- A zero as last entry indicates that the divisor is a root or a factor (depending on the type of computation that was done).

When none of these descriptions fits the last row of entries, a next attempt at division must be made to find a different last row of entries and hopefully a factor.

The Pre-Calculus course offers the students several opportunities to practice the skill of abstract thinking in the teaching of topics like: solving systems of linear equations using matrices, drawing and transforming graphs and working with complex numbers.



### 3.4.4 Classifying

In the Pre-Calculus course the skill of classifying is widely used. At least three examples spring to mind:

- <u>Classifying the asymptotes of a rational function</u>

  The kind of asymptotes the graph will have depends on the relationship between the power of the function in the numerator and the power of the function in the denominator, leading to the classification:

  - power (numerator) < power (denominator) indicating only a vertical asymptote at the value of $x$ for which the denominator = 0, etc.

leading coefficient and the constant term.

- Finding the factors by synthetic division causes a further classification to be made when the upper and lower bounds for roots are established, based on the signs in the last row of the division.

Clearly this is a case of classifying answers as the computation progresses.

It could thus be accepted that the Pre-Calculus curriculum gives the students ample opportunity to practice the skill of classifying.

### 3.4.5 Forming Conceptual Entities

One area the Pre-Calculus curriculum focused on to form a conceptual entity in the students' minds is the drawing of a graph. This emphasized the commonalities (roots, y-intercept), and the differences (asymptotes in rational functions, periodicity in trigonometry graphs). These concepts were reinforced by the transformation of the basic graphs, namely:

$y = x, \quad y = x^2, \quad y = x^3, \quad y = |x|, \quad y = \sqrt{x}, \quad y = a^x, \quad y = \log x.$

The conceptual entity of the quadratic trinomial is seen as an aid to focus on the essentials of a quadratic expression. It is to be recognized in expressions like $5^{2x} - 5^x - 2$ or $(\log x)^2 - \log x^2 - 3$ or $\tan^2\theta - 2\tan\theta - 3$, etc. that they are essentially still ordinary quadratic trinomials. Expressions like these are often used in the Pre-Calculus course, encouraging the forming of a conceptual entity with regard to quadratic trinomials.

### 3.4.6 Generic generalization

Generic generalization is achieved in the Pre-Calculus curriculum by applying the knowledge regarding the nature of the roots of a quadratic equation to find the turning points of the graph of a rational function. The formula used to determine the roots of a quadratic equation was recognized at school level for its qualifying of the nature of the roots of a parabola. In the Pre-Calculus course this attribute of

$$\therefore x^2 - 10x + 25 = 0$$

$$\therefore (x-5)^2 = 0$$

This indicates that for $x = 5$ there is a turning point at $\left(5, -\frac{8}{9}\right)$.

Normally the turning point would be determined by solving $x$ from the equation: derivative = 0.
As Calculus does not form part of the Pre-Calculus course, the derivative is not used, but instead, known algebraic manipulations are expanded in meaning to find the turning points.

In this way the curriculum makes provision for the use of the formula to have more meaning than before and to be applicable to other situations than just the graph of a quadratic function. It also affords the students the opportunity to become aware of the multiplicity of a mathematical concept and this awareness causes their mathematical insight and attunement to expand.

### 3.4.7 Coping with a Disequilibrium

Teaching via the forming of a mental equilibrium and then establishing a disequilibrium to encourage further studying is an integral part of the curriculum of the Pre-Calculus mathematics course (Wordsworth, 1978:78-85).

An example from the Pre-Calculus curriculum where this process was observable is in the drawing of the graph of a polynomial, which included the drawing up and interpreting of a sign table. This follows on the factorizing of the polynomial and is thus relatively easily accomplished and understood, thus forming a mental equilibrium.

disequilibrium. This kind of factorizing is done when the question allows it and then the opportunity to work with surds – another disequilibrium in this context - is also utilized.

In accordance with the spiral design of the course, all topics are developed along the lines of alternating equilibrium and disequilibrium, affording the students excellent learning opportunities.

### 3.4.8 Imagery

Quoting Skemp (1979:96) again for context - imagery can, in terms of mathematics be separated into two parts: verbal imagery as in algebraic symbols and visual symbols as in graphs or diagrams. Verbal, especially symbolic thought plays a crucial part in mathematics.

One 'sentence' of symbolic language that stands out in the Pre-Calculus course is the description of the end point behavior of the graph of a rational function where:

$$\text{If} \quad x \to +\infty, \quad \text{then} \quad \tfrac{1}{x} \to 0^{+} \quad \text{and} \quad y \to (x+2)^{-}$$

means that if $x$ becomes very large positive, then $\frac{1}{x}$ will become very small, but stay positive and $y$ will approach the asymptote $y = x + 2$ from beneath (below) the asymptote, as $y$ will always be less than $x + 2$, when $x$ becomes very large positive. This last part $y \to (x+2)^{-}$ would be defined by the rational function in the determining of the horizontal or oblique asymptotes. 'Seeing' these values approach infinity in the case of $x$ and approaching a line from above or below in the case of $y$, was a real challenge to the students.

The result of the 'sentence' is, of course, a visual thought, prescribing how the graph behaves in the case of $x$ becoming very large positive. In this way the curriculum is found to cater for the transcription of verbal thought into visual thought.

The Pre-Calculus curriculum gives visual thought attention in the drawing of graphs, but the real value of the Pre-Calculus course in regard to imagery lies in the interaction between the two kinds of thought imagery, where the interaction between these two is given extensive attention. As graphs are the ongoing topic, the description of a graph by calculation and symbolic exposition constitutes the verbal thought, while the sign table and actual graph represents the visual thought.

Trigonometric graphs combined with the transformation of the graphs are especially valuable in this respect to foster the interaction between these two kinds of thought. The trigonometric graph is a representation of the change in the ratio of two sides of a triangle as the angle of reference changes. Developing the graph from the unit circle increases the amount of visual thought used. Drawing the graph of $y = \pm a\sin(x \pm b) \pm c$ from the basic graph of $y = \sin x$ extensively uses the verbal thought interpretation of the transformation and then necessitates it to be converted to visual thoughts to draw the graph.

Thus, this very important development of thought was given much attention and exercise in the Pre-Calculus course.

### 3.4.9 Reflectivity

In the November exam for the Pre-Calculus course, the following question was asked:

Given the graph of one period of a function of the form:

$$y = a\sin k(x-b) \qquad \text{or} \qquad y = a\cos k(x-b)$$

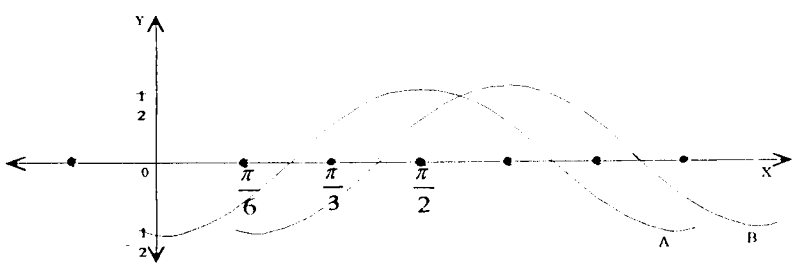


Fig. 7

Determine *a, b* and *k* if graph A is moved to the position of graph B.

- Receptor: The first information to be received here is that a choice should be made between $\sin x$ and $\cos x$.
- Mental intervention: This choice is not so simple, because the given graph is inverted, so the choice is really between $-\sin x$ and $-\cos x$.
- Receptor: At the same time the graph is condensed - both horizontally and vertically, making the information even more obscure.
- Mental intervention: After this, the student had to access a completely different set of information - the information on the transformation of graphs, and apply it to the identified trig graph.

Thus the student had to think through more than one 'layer' of information to uncover the most basic facts and then reconstruct the information to produce an answer.

This question then indeed had multiple levels of reflectivity. Thus, when a question was asked incorporating the transformation of graphs applied to trigonometric graphs, the student had to process the information through more than one layer of knowledge, acting reflectively in doing so.



### 3.4.10 Transition

Students in the Pre-Calculus course are forced to do a mental reconstruction on the topic of graphs, a transition of thought. They come into the course with the belief that all graphs of functions have a fixed form: straight line, parabola, circle, hyperbola and even the graphs of the trigonometric functions. The drawing of the graph of a polynomial and even more, the drawing of the graph of the rational function, bring them to acknowledge that, at least for these two kinds of functions, graphs do not have fixed forms and that quite a number of calculations must be done to predict the form of the graph of a polynomial or a rational function. Now roots, $y$-intercept and turning point - crucial information for the graph of a quadratic function - only constitute general information. Determining where the function is positive or negative and in the case of the rational function, also where it does not exist and what its end point behavior is, turns out to be the crucial information.

In this way they have to overcome the obstacle of preconceived ideas about a graph to make the transition and so to be able to draw a graph for any given function.

### 3.4.11 Reversibility

Reversibility of thought is given much attention in the Pre-Calculus course. As graphs are the central theme of the course, drawing a graph after doing the appropriate calculations is the direct way of dealing with the topic.. However, once the students are acquainted with the direct line of thought, the curriculum also requires them to be able to make deductions from a drawn graph. Graphs of trigonometric functions are especially well suited for this purpose. The graph of a trigonometric function can be altered in all the ways a graph can be transformed: a vertical shift changing the $y$-intercept of the graph, a horizontal shift causing a phase shift, vertical stretch or shrink affecting the amplitude of the graph, horizontal stretch or shrink affecting the period of the graph, reflection in the $X$ or $Y$-axis affecting the sign of the angle or the function and then to top it all the graph needs to be fitted to only one of the six functions available, namely $\sin x$, $\cos x$, $\tan x$, $\cos ecx$, $\sec x$ or $\cot x$. Finding the equation of a trigonometric graph that has been changed in any of these ways or a combination of them, constitutes an exercise in reverse thinking and is catered for by the Pre-Calculus curriculum.

### 3.4.12 Refined Intuition

Refined intuition is a secondary intuition that can be developed as a result of systematic intellectual training, according to Tall (1991:14). He continues to say that aspects of logic can also become more 'intuitive' to the mathematical mind and that the development of this refined intuition should be one of the major aims of more advanced mathematical education.

Refined intuition is especially useful in the study of indices, surds and logarithms, topics that are important for subjects like Physics, Chemistry, Statistics, Economics, etc. However, these topics also excellently serve to teach and learn the process of identification of a question with the relative law or process it matches best. Working with indices is closely related to the refined intuition (Tall. 1991:14) that helps a student 'hit' on the most appropriate solution to a question. Thus these topics best developed this kind of intuition. On the level of the Pre-Calculus students, they could not always discern the difference between $5^x + 25^x$ and $x^5 + x^{25}$ and therefore could not determine what the appropriate approach would be in each case. They then had difficulty understanding the factors $5^x(1+5^x)$ and $x^5(1+x^{20})$ and why the one 25 turned into 5, while the other became 20. This is the result of not seeing the fine difference between the two original questions that contained the same numbers, variables and operation, but needed to be interpreted differently. Although this is an extremely simple example, it is nevertheless a real example of what students had difficulty with. If a student has a problem with this concept, it is to be understood that more advanced examples will pose bigger problems.

Incorrect intuition brought from school re indices had a most detrimental effect on their performance on this topic. To unlearn these proved to be much more difficult than to acquire the new knowledge on such topics as matrices, polynomials and rational functions.

Another example, from surds, illustrates the confusion that may arise if two questions are started in the same way, but continue differently. A student needs to be aware of the subtle difference to make the right decision all along.

| Simplify: | |
|---|---|
| $\sqrt{12}.\sqrt{75}$<br>$= 2\sqrt{3}.5\sqrt{3}$<br>$= 10.3$<br>$= 30$ | $\sqrt{12} + \sqrt{75}$<br>$= 2\sqrt{3} + 5\sqrt{3}$<br>$= 7\sqrt{3}$ |
| Here the surd disappears. | Here the surd does not disappear, but is treated as a variable. |

Table 8.1: Refined intuition

Another example of where refined intuition is necessary, even on Pre-Calculus level, is when the following calculations are to be performed. The same symbols and signs are used, but in different combinations and therefore the calculations for simplifying differ:

| For $x = 5$, find the values of: | | |
|---|---|---|
| $(-x)^2$<br>$= (-5)^2$<br>$= +25$ | $-x^2$<br>$= -5^2$<br>$= -25$ | $x^{-2}$<br>$= 5^{-2}$<br>$= \frac{1}{5^2}$<br>$= \frac{1}{25}$ |

Table 8.2: Refined intuition

Again these examples are simple but they best illustrate the problem students have in distinguishing between various calculations befitting various ways of combining the same symbols.

Only by practicing mathematics does this refined intuition develop (Tall, 1991:14) and students in the Pre-Calculus course had many opportunities to practice mathematics, on their own as well as under supervision, and so develop this refined intuition in at least some topics.

## 3.5 Practical mathematical skills developed by the Pre-Calculus course

### 3.5.1 Algorithmic activity

As most failures in mathematics stem from the inability of students to do routine manipulations automatically, it is important to note that the curriculum of the Pre-Calculus course includes practice opportunities in the most basic algorithmic activities, such as:

- drawing a parabola;
- factorizing a trinomial;
- switching from logarithmic form to exponential form and back;
- solving simultaneous linear equations in two variables;
- drawing a straight line graph;
- finding the equation of a straight line graph;
- working with fractions, both numerical and algebraic, to name but a few.



The Pre-Calculus course gives the students many opportunities, both to see the value of these manipulations being automatic, and to see which manipulations fall into this category. As they progress in their vertical growth of mathematical knowledge, this list will obviously become longer, but hopefully their expertise in these manipulations will improve as well.

### 3.5.2 Recording Knowledge

The three categories of communication that Skemp (1979:73) distinguishes are:

1. Communication with somebody who doesn't know what we are talking about, but wants to know. This is essentially what happens in a lecture. As the common method of teaching in the Pre-Calculus course is by lecture,

the students are given good practice in recording knowledge by listening, taking down notes, questioning the lecturer, and generally ensuring that they get the information, all or as much as possible, and in the correct symbolic notation.

2. Communication with those who do know what we are talking about, but only as a general background, within which we are trying to communicate some particular aspect. Such is the communication of a consultation or a revision class. Pre-Calculus lecturers are freely available for consultation and extra classes are frequently offered. These are opportunities the students can use to acquaint themselves with this kind of communication. This kind of communication also serves as a barometer of what the lecturer deems 'background' knowledge to be and motivates the student to acquire this knowledge.

3. Lastly, there is the situation in which "we are communicating with those who do know what we are talking about, but want to fault it." This is the test situation in which the student records his/her own knowledge. Weekly tutorial tests and two class tests per term in the Pre-Calculus course, afford the students ample opportunity to express themselves mathematically and to have these efforts evaluated so they know where improvement is necessary.

Communicating in writing implies that the recording of mathematics is to be done in the correct way. This aspect of communication is given due attention in the course. Students are made aware of the need for clear stating of ideas and correct use of mathematical structure and symbols. One of the ways in which students are assisted in this respect is by correct and clear memoranda being available after tests so they can judge and better their own efforts at communicating mathematics.

### 3.5.3 Representation

According to Tall (1991:30), symbols involve relations between signs and meanings: they serve to make a person's implicit knowledge – the meaning – explicit in terms of symbols. This kind of implicit knowledge being made explicit by symbols is best illustrated in the Pre-Calculus course by the description of the end point behavior of a rational function.

E.g. The rational function: $y = \frac{(x-1)^2}{(x-2)(x+1)}$, has a horizontal asymptote at $y = 1$ and the equation $y = 1 - \frac{x-3}{x^2-x-2}$ predicts the endpoint behavior of the graph.

As $y = 1 - \dfrac{\frac{1}{x} - \frac{3}{x^2}}{1 - \frac{1}{x} - \frac{2}{x^2}}$, the endpoint behavior is described in symbols as:

If $x \to +\infty$ then $\frac{1}{x} \to 0^+$ and $y \to 1^-$

If $x \to -\infty$ then $\frac{1}{x} \to 0^-$ and $y \to 1^+$

This reads as follows:

If $x$ becomes very large positive, then $\frac{1}{x}$ becomes very small, but stays positive, and $y$ approaches the line $y = 1$ from below the line. (Less than $1 = 1^-$)

If $x$ becomes very large negative, then $\frac{1}{x}$ becomes very small, but stays negative, and $y$ approaches the line $y = 1$ from above the line. (More than $1 = 1^+$)

In the Pre-Calculus course, students are required to represent their deductions regarding the endpoint behavior of the rational function, in symbolic language, giving them practice in expressing the meaning of their deductions symbolically.

Piaget (Wadsworth, 1978:165) stresses the need for students to comprehend a mathematical operation before they can comprehend a representation of the operation. In the Pre-Calculus course synthetic division is based on algebraic long division. As algebraic long division is no longer taught at school, the Pre-Calculus course makes provision for the teaching of algebraic long division so students can understand the abstraction to synthetic division, where they are only working with a symbolic skeleton of the algorithm. This is a very challenging transcribing but gives good exercise in the symbolic representation skill.

### 3.5.4 Terminology

The Pre-Calculus course in mathematics affords the students the opportunity to get acquainted with the correct terminology of the subject. As most of them are not studying in their first language, the terminology of the subject is a very important form of communicating content to them and for them to communicate with the lecturers, in either asking or answering questions.

Early on in the course the students encountered the polynomial function. They had to discriminate between statements like:

- the possible rational roots;
- the possible number of positive/negative roots;
- upper and lower bounds for the roots; and
- rational, irrational and imaginary roots.

Each of these 'statements' regarding the roots of the polynomial function needs to be understood very well as they indicate vastly different aspects of the roots and deal with hypothetical information. It is a valuable part of the curriculum in as much as it stresses the importance of the correct word to carry the implied meaning.



Students were thus made aware of the importance of using the correct terminology right from the outset of the course.

### 3.5.5 Checking

In the Pre-Calculus course it is stressed throughout that most mathematical operations have an inverse operation, e.g. factorize ↔ multiply; divide ↔ multiply; and that these can be used to check an answer. Many instances of working with the inverse are included in the curriculum:

- logarithmic form vs. exponential form,
- the equation of the inverse of a function and the relationship between the two graphs of the functions,

- the solving of a system of linear equations by using matrices and the subsequent substitution of the answers into the original equations to assess the validity of the answers, etc.

It is clear that the Pre-Calculus course is geared to letting students experience the possibility and value of checking answers, by deploying the information in a way that encourages them to see the connection between two inverse operations.

## 3.6 Content of the Pre-Calculus Course

This paragraph lists the actual topics covered by the Pre-Calculus course, the algorithmic and cognitive skills taught in these topics and the way in which each topic uses and expands the school mathematics standard grade curriculum.

**1. Polynomial Functions**

| | | |
|---|---|---|
| 1.1 | Leading coefficient and constant | |
| 1.1.1 | Possible roots | – conjecture |
| 1.1.2 | Number of positive roots | - delimitation of calculation |
| 1.2 | Synthetic division | - abstracting |
| 1.2.1 | To determine factors | - interpretation of roots, factors, quotients, remainder |
| 1.2.2 | To determine upper and lower bounds | - classification |
| 1.3 | Factors | - algebraic manipulation |
| 1.3.1 | To determine roots | - algebraic manipulation |
| 1.3.2 | To draw up the sign table | - predicting the shape of the graph<br>- interpreting each factor's sign behaviour |
| 1.4 | Sign table | - working with abstracted information<br>- interpreting abstract information |
| 1.5 | Drawing the graph | - synthesis of abstract information |
| 1.6 | School Mathematics used and expanded | |

- Concept of roots
- Factorizing a quadratic trinomial
- Completing the square - expansion of SG mathematics
- Drawing up a sign table - expansion of SG mathematics

**2. Rational Functions**

2.1 Intercepts - algebraic manipulation

2.2 Asymptotes - classifying

2.2.1 Vertical asymptotes based on the fact that division by 0 is not permitted

2.2.2 Horizontal and oblique asymptotes by algebraic long division and using the algorithm: $R(x) = Q(x) + r(x)/d(x)$

- algebraic manipulation

2.3 Behaviour of the graph - abstract reasoning
- working with perceived values

2.4 Turning Points - algebraic manipulation

2.5 Sign Table - abstracting

2.6 Drawing the graph - synthesis of information from calculations

2.7 School mathematics used and expanded:

- behaviour of 0 in a fraction
- expansion of the concept of an asymptote encountered at school when drawing the graph of $y = a\tan bx$
- algebraic long division
- graph of a straight line
- changing the subject of the formula
- quadratic formula
- nature of the roots
- sign table

**3. Absolute Value Functions - Linear and Quadratic**

3.1 Concept of a piecewise function - analysis

3.2 Domain and range, Sets - reflectivity

3.3 Solving inequalities - algebraic manipulation

3.4 School mathematics used and expanded:

- absolute value - expansion of SG mathematics
- representation on a number line
- inequalities – linear and quadratic - expansion of SG mathematics

- sign table - expansion of SG mathematics
- factorizing
- graph of a straight line
- correspondence to the equation of a parabola

**4. Functions**

4.1 Function of a function - algebraic manipulation

4.2 Inverse functions - imagery, algebraic manipulation

4.3 Translations of functions - classification

4.4 School mathematics used and expanded:

- function of a function - expansion of SG mathematics
- inverse functions - expansion of SG mathematics
- graphs of basic functions: straight line, parabola, hyperbola, trig functions

**5. Solutions of Systems of Linear Equations**

5.1 Setting up a matrix from the equations - abstraction
- terminology

5.2 Operations on matrices - terminology
- numeric manipulation
- representation

5.3 Solving the system of linear equations - synthesizing
- communication

5.4 Concepts of unique, infinite many and no solution - imagery

5.5 School mathematics used and expanded:

- manipulations on numbers especially fractions
- solving simultaneous equations

**6. Partial Fractions**

6.1 Type - classification

6.2 Separation into partial fractions - reverse thinking

6.3 Simultaneous equations to find coefficients - algebraic manipulation

6.4 School mathematics used and expanded

- algebraic fractions
- finding a LCM
- solving simultaneous equations
- simplifying algebraic expressions

**7. Indices, Surds and Logarithms**

7.1 Exponential function - concept expansion

7.2 Natural log - concept expansion

7.3 School mathematics used and expanded

- all work done at school on these topics were revised

**8. Trigonometry**

8.1 Radian measure - concept expansion

8.2 School mathematics used and expanded

- graphs of trigonometric functions - expansion of SG mathematics
- revision of all the trigonometry done at school

**9. Complex Numbers**

9.1 Concept of a complex number - abstract thinking

9.2 Operations on complex numbers - algebraic manipulation

9.3 School mathematics used and expanded:

- basic algebra
- non-real roots of a parabola
- conjugate of an expression

## 3.7 Conclusion

As shown in this chapter, the design of the curriculum and the thinking and practical skills developed by the content of the Pre-Calculus course, offer students a wide variety of learning experiences. Thus, apart from learning actual mathematical content, they are also given opportunities to develop into mathematicians.

In the next chapter a literature review will attempt to form a framework for the profile of students that would possibly be in a mathematics course like the Pre-Calculus course under discussion.

# Chapter 4

## Profile of Students in the Pre-Calculus Course

### 4.1 Introduction

In chapter 3, the question 'What is being learnt?' was addressed and in this chapter attention will be given to the question 'Who is learning?' The questionnaire that students filled out contained questions on a wide variety of topics. The reasons for including these topics spring from the experience of faculty at the university. Some questions, those on biographical details, are obvious choices for a questionnaire of this kind. Other questions spring from universal problems with studying and in particular with studying mathematics. This questionnaire covered the following aspects:

- general information concerning first language, age and gender;
- place of residence while studying;
- English as second language;
- matriculation exemption;
- reasons for doing mathematics at university;
- influence of circumstances at school on mathematics marks in the matriculation examination;
- the motivational aspect of the course;
- study habits in mathematics at university vs. at school;
- the emotional aspect of studying mathematics;
- experience of success;
- use of study resources; and
- general comments made by the students.

In this chapter, the feedback from the students will be discussed against the background of literature research in the above areas. In this way hopefully, an approximate profile of the students registered in the Pre-Calculus course will emerge.

The students' numerous and expansive comments will be used to assess the course from their point of view. In this regard, it must be pointed out here that words that are written in italics, are actual comments made by the students in answering the questionnaire. These comments were also reproduced as they were written by the students without being edited, and thus give an indication of their competence in English.

Certain aspects of the questionnaire draw on the differing experiences of the students. E.g. a student passing with an A will respond differently to the question on success than the students who is failing. For this reason the class group will roughly be divided into three groups where applicable. The division into groups was entirely objective taking the students' marks for the Pre-Calculus course as the basis for the selection of the groups:

Group 1: students who achieved above 75% average for the course,
Group 2: students with marks below 75% but still passing and
Group 3: students who did not pass the course.

The characteristics of each group can be summarized as follows:

Group 1:
The students in this group were almost misplaced and could possibly have coped with 1st year Mathematics. In the Pre-Calculus course, they would score their marks on understanding the questions and would generally lose their marks on the calculations, with which they had become bored. They more often asked questions on the reasons for the calculation(s) and what the result means, than on the actual calculation, as they could grasp the algorithms easily enough on their own.

Group 2:

These are the students who benefited most by doing the Pre-Calculus course. They would frequently ask questions. Unfortunately these questions were mostly about the calculations and very seldom about the insight needed to understand the question. They passed the course on the marks they acquired by mastering the algorithms. They gained valuable knowledge about Mathematics, but are still not really better equipped to cope with innovative problem solving in Mathematics.

Group 3:

The students in group three had very little interest in or motivation for Mathematics. They would be inattentive in class or even disruptive, if they at all turned up for class. In this group, 2 subgroups could be identified:

- One group hardly worked at all and passed/failed tests on what they could learn in the 4 periods on Wednesdays between the last lecture and the tutorial test. They seldom asked questions and, if they did ask, it was mostly about the organization of the lectures and tutorials, times, dates, and such.
- The other group apparently worked very hard and often consulted the lecturers, but never reached the point where they took responsibility for their own success and thus failed/passed on the incidental knowledge they gained from consulting the lecturers.

The respondents to the questionnaire formed groups representing 11%, 64% and 25% of the class for groups 1, 2 and 3 respectively. As the answers to the questionnaire can often also be divided into the answers of these 3 groups, it makes the interpretation of the answers more meaningful.

## 4.2 General Information regarding age, gender and matriculation mathematics status

### 4.2.1 Age

The age distribution of the respondents (age at the end of 2001) shows that the majority (modal age = 19) finished school at the normal age of 18. The average age of 19.7 years on the other hand shows that there is a large contingent of older students. These were students who had been working in the years between finishing school and coming to university, e.g. Plaatjies (20), Mndwangu (25), Tsabalala (23). The age distribution is as follows for the class group's age at the end of 2001, therefore at the end of their Pre-Calculus year:

| Age | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 37 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Number | 1 | 2 | 24 | 49 | 28 | 9 | 1 | 3 | 2 | 4 | 1 | 1 |

Average age: 19.7 years Modal age: 19 years Range: 16 – 37 years.

Table 9: Ages of Pre-Calculus students 2001

The wide range of ages did not seriously affect the class situation. It did however, affect the individuals at the extreme ends of the spectrum. The student, Ramncwana, who was 37 years old, had been a nurse before coming to university and found the pace of acquiring new knowledge very daunting. It had been a long time since she did any Mathematics and she had virtually no background to draw on and not enough time in her course to make up for the deficit. She subsequently dropped out of the course during the fourth term. At the other end of the spectrum, the students who were younger than the average were: Fakude (17), Kubukeli (17), Randall (16). They presumably came through school faster than other students because they were quite clever. Their average mark for the Pre-Calculus course were: Fakude (72.5%), Kubukeli (74%) and Randall (76%). These marks give the impression that they did not find the course too hard and that this young age was not a hindrance to their studies.

## 4.2.2 Gender

The gender distribution of the respondents is included but, as there was no gender discrimination in the class, it is quoted here only for the sake of completeness of information.

| | | | |
|---|---|---|---|
| Male | 48.8% | Female | 51.2% |

Table 10: Gender of Pre-Calculus students 2001

## 4.2.3 Matriculation Mathematics Status

The students' qualification in matriculation mathematics prior to coming to university is the single most important reason why they could not be accepted into the Science Faculty of UWC or any other South African university. The following table summarizes their matriculation mathematics background in terms of the symbol they achieved for mathematics and the grade on which this was achieved.

| Grade of Mathematics in matriculation examination | Symbol obtained | Number of students |
|---|---|---|
| Higher Grade | E | 2 |
| | F | 1 |
| | G | 1 |
| | GG | 1 |
| | H | 1 |
| Standard Grade | B | 1 |
| | C | 12 |
| | D | 12 |
| | E | 26 |
| | F | 17 |
| | GG | 2 |
| Lower Grade | F | 2 |
| No Mathematics for matriculation | | 1 |

Table 11: Matriculation mathematics status of Pre-Calculus students 2001

It is clear from this summary that students came to the Pre-Calculus course with a very shaky background in mathematics. This caused many problems during the course when they were expected to use some of the knowledge they should have acquired at school. In chapter 2 the literature study regarding the thinking and algorithmic skills highlights the skills they lacked on coming to the Pre-Calculus course. In this chapter the measures taken to teach them these skills will be discussed.

## 4.3 Place of Residence while studying

### 4.3.1 On campus

While teaching the Pre-Calculus class of 2000, Prof. Fray found that some students were very tired and could not easily get through a day's lectures. He subsequently found, by interviewing them, that they spend many hours commuting to university each day. This had a real impact on their achievement in mathematics and it was decided that this study should also investigate this matter. To the question 'Do you live on campus?' the students responded as follows:

| Yes, I do live on campus | No, I do not live on campus |
|---|---|
| 34.1% | 65.9% |

Table 12: Place of residence while studying: On/off campus

### 4.3.2 Place of Residence

#### 4.3.2.1 Geographical

Students from all over South Africa came to UWC to attend the Pre-Calculus course. The university has a hostel policy that gives preference to students who live more than 50 km from the university. In light of this fact, it is interesting to see that 73% of the Pre-Calculus students live near enough to the university to commute to campus.

In 2001, the distribution of the students' origins was as follows:

| | | | Number of students |
|---|---|---|---|
| Northern Province | | | 6 |
| North Western Province | | | 1 |
| Eastern Province | | | 23 |
| Cape Province | Karoo | | 1 |
| | Southern Cape | | 1 |
| | West Coast | | 2 |
| | Cape Peninsula and vicinity | Helderberg | 3 |
| | | Oostenberg | 9 |
| | | Tygerberg | 29 |
| | | Cape Town | 39 |

Table 13: Place of residence while studying: Geographical

#### 4.3.2.2 Kind of accommodation

The following table summarizes the feedback of the Pre-Calculus class of 2001. According to this table most students who live off campus are still living with their parents. This means they still have the same support they had while at school. Presumably this would in general be a good support system as these students completed school to such a degree of academic achievement that they were at least eligible for one of the alternative ways of gaining matriculation exemption. Most adolescents living at home go through a phase of fighting the authority of their parents and this can have an impact on their studies. On the other hand, students who have to adapt to living in a hostel during their first year at university also have problems that impact on their studies and those living at home are thus not more compromised than others.

| | |
|---|---|
| 52.7% | At home with parents |
| 36.4% | Living with family |
| 5.5% | Commune with other young people |
| 3.6% | Lodger in someone's house |
| 1.8% | Rented room |

Table 14: Kind of accommodation at place of residence

#### 4.3.2.3 Impact of current place of residence on studies

Two sets of feedback to the question 'How does this (place of residence off campus) affect your studies?' were obtained. The responses of the class of 2001 is represented in table 15:

| | |
|---|---|
| 61.8% | I am tired when I get home |
| 36.4% | There is no one at home to help me in my studies |
| 29.1% | There isn't silence |
| 29.1% | It does not affect my studies at all |
| 25.5% | I have house chores to do when I get home |
| 25.5% | I am often late for class in the morning |
| 10.9% | I don't feel comfortable with the people in the house |
| 10.9% | Time spent in lectures and tutorials leave little time for studying on campus during the day |
| 10.9% | There is no library nearby |
| 3.6% | There is nowhere I can sit down and study |

Percentages relate to feedback from the Pre-Calculus class of 2001.

Table 15: Impact of current place of residence on studies

The Pre-Calculus class of 2002 was asked open-ended questions on how living off campus influenced their studies and their comments (Appendices B1, B2 and B3) were analyzed to give a better insight into the problem of living off campus.

The responses to the questions ‘How do the people you live with affect you?’ and ‘What do they do that interferes with your studies?’ are found in Appendix B1 and indicate that:

- 58% of the students are affected in a negative way,
- 37% are affected in a positive way, and
- 5% are apparently not affected at all.

Of the students that indicated that they are affected in a negative way, 74% mention noise as a problem they have where they stay. During the day students can study in the study hall or library, yet even these places have noisy conditions making it difficult to study there. Only 3 students indicated that they study in vacant venues during the day. However, all students, whether they stay on or off campus, are affected by the lack of silence for studying. In answer to the question ‘Would you like to live on campus?’ (Appendix B2) noise is cited as one of the reasons why students would prefer *not* to live on campus.

Being held to their cultural roles of doing house chores is a problem for 26% of the respondents. This affects their time and energy. They spend an approximate 1.7 hours on traveling per day (Table 17) and arrive home being tired (Table 15). They complain that they are not granted enough time to study, not even when they are writing tests or examinations are they exempted from house chores. Other matters that are mentioned are

- the jealousy of their housemates regarding their student status,
- a lack of privacy when they have to share a room, and
- financial problems, especially regarding travel expenses.

The complaints students have about their place of residence and the people they have to live with can be summarized as a lack of understanding of the demands studying at university have on a student. People are not considerate in keeping quiet, granting them privacy or time to study.

Of the 38% of respondents who indicated that they are affected in a positive way, 86% apparently still live at home with their parents, a deduction made from the comments in Appendix B1. They declare that they are not subjected to any interference and that they are supported in positive ways conducive to studying.

Reasons for wanting to live on campus (Appendix B2) are:

- Advantages of living on campus:
  - more study time,
  - the availability of resources like the library and computers,
  - being able to have group discussions with peers.
- Disadvantages of living off campus:
  - travel conditions that are unsafe,
  - fatigue due to traveling,
  - problems regarding money for traveling, and
  - a lack of privacy where they live.

Interestingly, 36% of the respondents declared that they would not like to live on campus. Of these students, 45% live at home, being satisfied with the comfort there. They have their own room, silence when they study and are not interfered with. The residences are perceived to be expensive and students have qualms about their safety in the residences. According to one respondent, much self-discipline is needed to overcome the temptations in the residences and to be a diligent student, making a university residence not necessarily a better place to stay than any other place.

In answer to the question calling for general remarks about the fact that they live off campus (Appendix B3), the above sentiments were reiterated with the emphasis again on transport, its cost and its time consuming effect. One student bemoaned the fact that he/she had no access to the facilities of the university and that *things have to "stand still" until I get back to campus the next day* because of not being able to use the computers or library. Another student raised the matter of not having a discussion group and ending up *losing interest*.

## 4.4 Modes of transport

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Train | 59.3% | Taxi | 18.5% | Bus | 9.3% | Car | 20.4% |

Table 16: Modes of transport

Their mode of travel has a profound effect on the emotional state the students are in on arriving on campus. Trains have, in South Africa, become a dangerous form of transport. Passengers are constantly in danger of being mugged or robbed or even raped. It is therefore even more dangerous late in the afternoon when it is dark. Students are thus anxious to leave campus as early as possible. This limits the time a commuting student can spend on campus to work in the library and/or in the computer laboratories on projects. Attending a science course, with all the tutorials and practical classes that are part of it, takes up most of the hours available during the day. These students have to be very well disciplined to be able to utilize the opportunities available to them on campus.

Students commuting by car have different problems. They frequently belong to a lift club and find that not all the members of the lift club are equally serious about being on time for lectures. They complain that they are often late for the first lecture of the day and this leads to them missing the strand of reasoning in the class. This, especially in science subjects, is very debilitating for understanding. In the afternoons, members of the lift club who do not have practical subjects are eager to get home, and put pressure on the science students to finish off their tutorial and/or practical as early as possible. This caused a lot of friction in the Pre-Calculus course as each tutorial included a tutorial test at the end of the session and students would leave to catch their lift without finishing their test. Understandably this had a detrimental effect on their marks as the tutorial tests form part of the continuous assessment of the course.

Students commuting by bus and taxi have very much the same problems as those traveling by train as both these modes of transport harbor the same dangers as the trains with the added dangers of road accidents and shooting in taxi disputes.

## 4.5 Time spent on Travelling

Students were asked to estimate the number of hours they spend on traveling and to include the time they spend waiting for the train/taxi/bus/car. Their responses were:

| 1 hour | 1½hours | 2 hours | 2½hours | 3 hours | 3½hours | 4 hours | More |
|---|---|---|---|---|---|---|---|
| 22 | 14 | 5 | 7 | 2 | 0 | 0 | 3 |

Table 17: Time spent on traveling

The average number of hours spent on traveling is thus 1.7 hours. The Pre-Calculus science course consists of the following subjects with their contact hours in brackets:

Pre-Calculus (6hours = 4x1hour lectures + 1x2 hour tutorial),

Science Foundation (5½ hours = 4x1 hour lectures + 1x1½ hour practical) and one first year subject (7 hours = 4x1 hour lectures + 1x3 hour practical).

This means that out of an approximate total of 35 hours a week (7 hours/day) that commuting students normally can spend on campus, 16 hours (approx. 3 hours/day) are available to make use of the resources like consulting with the lecturer, using the computers, using the library and participating in group discussions. However, as a result of the fatigue that the students all mention as a major disadvantage of living off campus, and the fact that they do not like to leave campus late in the afternoon or come to campus during peak time in the morning, they spend considerably less time on campus than the approximate 35 hours. Referring back to paragraph 4.3.2.3, it is clear that many students are not granted enough time at home to study. This means they have to do much of their studying during the day while on campus. In light of their timetable schedule it is clear that they also do not have much time during the day to study. As Prof. Fray suspected, these time constraints impacted negatively on their studies.

## 4.6 English as Second Language

The following language distribution of the respondents shows clearly that 82.9% of the students in the Pre-Calculus course were studying in their second language.

| Home language | |
|---|---|
| Xhosa | 62.2% |
| English | 17.1% |
| Afrikaans | 8.5% |
| Swati | 2.4% |
| Tswana | 2.4% |
| South Sotho | 1.2% |
| Venda | 1.2% |
| Tsonga | 1.2% |

Table 18: Home language

English as second language for studying is a worldwide problem that is given much attention globally. In their article 'Reforming Mathematics instruction for ESL (English Second Language) Literacy students', K Buchanan and M. Helman (1993) state that:

> In the mathematics classes, they (the ESL students) must learn in a linguistically and culturally unfamiliar environment, constructing understanding without the background knowledge that their classmates employ to make assumptions and process new information.

To the question 'Is your home language English?' the students responded as follows:

| | |
|---|---|
| Yes, my home language is English | 17.1% |
| No, my home language is not English | 82.9% |

Table 19: English as a second language

Buchanan & Helman's (1993) statement that students learn in a linguistically and culturally unfamiliar environment is especially true for the Pre-Calculus class in which 82.9% of the respondents were not English speaking. The students face huge problems in understanding a subject that many people find difficult to understand even in their own language.

On reading the comments of students in appendices B and D to G, the students' general level of English usage can clearly be detected. Many students express themselves in sentences that have poor structure, little reasoning and often no verb.

The fact that many students study in their second language makes it imperative that measures will be taken to accommodate their slow/inadequate/poor understanding of the language in which the work is explained. The following set of statements, that applies to the problems students experience with English as a second language, was voiced by students during informal discussions with the lecturer. These discussions were summarized by one of the students, ms. Mpofu. They said they

- have difficulty getting used to new words;
- find the explanations difficult to comprehend;
- find it difficult to follow fast speech;
- have difficulty understanding the language of the textbook;
- need more time to study;
- find it difficult to explain what they mean in English;
- often find words that nobody knows the meaning of; and
- have trouble understanding the terminology of the subject.

The following set of statements were made and students indicated which problems were the most prevalent:

| | |
|---|---|
| 31.7% | I find it difficult to follow fast speech |
| 27.0% | It takes more time to study |
| 22.2% | I have difficulty getting used to new words |
| 20.6% | I have trouble understanding the terminology of the subject |
| 20.6% | It does not affect me at all |
| 19.0% | I find it difficult to explain what I mean in English |
| 11.1% | I find the explanations difficult to comprehend |
| 11.1% | I have difficulty understanding the language of the textbook |
| 1.6% | I often find words that nobody knows the meaning of |

Table 20: Problems with English as second language

*Not understanding fast speech* is the most common complaint. This is true for everybody listening to speech in a 'foreign' language. However, understanding an ordinary conversation is not dependent on understanding everything that is said. Very often the context of the conversation can give a person a clue to what is being said. Unfortunately, mathematics does not allow one to make one's own deductions of what is being said, making it very important that every single word can be heard and given time to be understood.

Not only were *explanations difficult to follow* but the *questions were also difficult to interpret*. In one of the tests the words 'evaluate' and 'verify' left students not knowing what to do. They were thus robbed of the opportunity to gain marks, for although they knew how to work out the problems if the question had stated 'find the value of' and 'prove that', they could not use this knowledge for they knew not what the words meant.

Students often found it *difficult to explain what they don't understand/want to ask/think the answer is.* Accommodating this problem in class is highly time consuming, and limits the time to impart knowledge. In this way, the student is disadvantaged in one way or another, losing out on optimal knowledge input or losing out on the opportunity to express their questions/ideas because of time-constraints. Amongst the 68 respondents whose first language is not English, 14 were identified as having passed English First Language on HG in the matriculation examination. It was observed that these students each had a little group of students that would consult them and to whom they would then explain the work in their own first language. This practice was very successful, with many students becoming more and more independent of these translations as the year wore on.

As 66% of the respondents indicated that they are studying mathematics because they *need it for their other subjects* (See Appendix A, question 4.2), expressing themselves clearly in English constitutes a large part of their success at university. They will be translating their other subject matter, not only into English, but also into Mathematics and it is thus imperative that they are given opportunities to develop the skills to speak and write mathematically, in English.

The variety of English and Math proficiency in the class made it very difficult for this matter to be addressed in a large class of 100+ students. Many students are also shy to speak in front of so many strangers, as is to be seen from their comments on the questionnaire (See Appendix A, question 6.2). Buchanan & Helman (1993) suggests that smaller groups will afford better opportunities to speak and hear and write math in a less threatening environment. Unfortunately even the tutorials had 50+ students, and could thus not be reckoned as a small group. Smaller groups were formed, when the optional extra classes were introduced. Unfortunately, not everyone attended these classes and the benefits were thus very limited.

According to Buchanan & Helman (1993) "math students benefit from a variety of instructional settings in the classroom... individual, small group and whole group activities." All these options were available to the Pre-Calculus students: individual attention in consultation with the lecturer, small group in the optional extra classes and whole group in the classes as per timetable. In this way the students were given ample opportunities to address the problems they had with English as a second language.

Other options whereby this language matter could be addressed were put to the students, and their responses were: (See Appendix A, question 2.2)

What can be done at university to help you overcome these problems?

| | |
|---|---|
| 63.5% | Typed notes being handed out |
| 39.7% | List of reference books and these books on reserve |
| 34.9% | Lists of the terms used in each topic |
| 33.3% | Notes in simplified English |

Table 21: Options to address the language matter

Typed notes, or handouts, were only available for the work on matrices and students thus had an idea of the advantages of studying with handouts. In chapter 6, under suggestions and recommendations, the advantages and disadvantages of handouts will be discussed as well as the other options mentioned above.

Accommodating the language problem in class is highly time consuming, and limits the time to impart knowledge. In this way, the student is disadvantaged in one way or another, losing out on optimal knowledge input or losing out on the opportunity to express their questions/ideas because of time-constraints.

As mathematics is an auxiliary subject to most students studying mathematics in the first year of university studies, expressing themselves clearly in English is a large part of their success at university. They will be translating their other subject matter, not only into English, but also into Mathematics and it is thus important that they become skilled in speaking and writing mathematically in English. As almost all students in the Pre-Calculus course study in their 'second' language it means that mathematics becomes yet another language with its own grammar rules and its own unique set of 'shorthand' symbols. What complicates matters even more is that there is no 'creative' writing in the true sense of the word, as all writing must lead to the same correct conclusion, following one of a set number of paths. If these reasoning paths are not known to the student there can be no comprehensible writing, and thus no satisfactory evaluation (in terms of marks or otherwise) of the application of the 'language'. It is not always only a lack of mathematical knowledge but rather the students' inability to write down knowledge in an acknowledged form for evaluating that is often the reason why a student fails.

The impact made by studying in one's second language can be gauged if the other side of the matter is inspected. 21% of the 68 students who indicated that English is their second language also indicated that it did *not affect their studies at all.* (See Appendix A, question 2.1). These 14 students passed English on higher grade in the matriculation examination. Of these, 2 speak Afrikaans at home and 12 speak an African language. It is interesting to note that some of the best students in the class were amongst these 12:

| Student | Average percentage for Pre-Calculus course |
|---|---|
| Heyana | 80% |
| Majakajaka | 70% |
| Makhubalo | 60% |
| Malotana | 77% |
| Monakali | 63% |
| Ndindwa | 84% |
| Sigalelana | 78% |

This is an indication that their understanding of mathematics as a subject was supported by the fact that they understood the language of instruction. Indeed, these students confirmed that this is so during a subsequent discussion between them and the author.

According to the article of Buchanan & Helman (1993), the NCTM (National Council of Teachers of Mathematics) established 5 general goals for mathematical literacy and these goals could as well be applied to any school course in mathematics as well as to the Pre-Calculus course. These goals have a hierarchy of mathematical activity corresponding to the hierarchy of proficiency in English as language in which the mathematics is communicated.

The 5 goals are:

1. That the students learn to value Mathematics.
2. That the students become confident in their ability to do Mathematics.
3. That the students become mathematical problem solvers.
4. That the students learn to communicate mathematically.
5. That the students learn to reason mathematically.

The first goal that students learn to value mathematics posed no problem in the Pre-Calculus course. Students' reasons why they do mathematics as a subject at university (See Appendix A, question 4.2) indicated unequivocally that they are aware of the value of mathematics. They value it for affording better career opportunities and supporting other scientific studies, according to their answers to question 4.2 (Appendix A).

Becoming confident in their ability to do mathematics is the second goal according to Buchanan & Helman (1993). In this respect it is interesting to note that students in the second group (par. 4.1) tended to concentrate only on the calculations and procedures that would be asked as straightforward questions in tests in exams. These questions would have plain familiar wording like 'find the roots of' or 'show graphically…'. It would be the kind of standard grade school mathematics questions testing knowledge and application. The students banked on it that the examination would have such questions and that these questions would be worth enough marks to get them through the test/examination. Unfortunately this was not the case as part of the reason for offering the Pre-Calculus course is for it to act as a substitute for higher grade mathematics as in the matriculation examination.

This means that questions must also be posed that tests the students' ability to interpret and analyze information. In this way students are offered the opportunity to achieve the third goal, in understanding both English and Mathematics, namely to become problem solvers. These questions were very difficult for most students.

As to achieving the fourth goal of being able to communicate mathematically, the Pre-Calculus students fell short. Especially students in the second group (par. 4.1) would, for instance, do a calculation to prove that a certain number is a root of, e.g., a polynomial and then never write down in words what conclusion they have come to. Although this is very common practice with students who are weak at mathematics, it is also a by-product of having difficulty with the language. In this way they are omitting to communicate their conclusion to the examiner for evaluation and so lose marks as a result of ineffective communication. This is a special case of co-dependence of the two subjects, Mathematics and English.

As was discussed in the literature study, the fifth goal is to be able to reason mathematically. This is a field where the mathematics is so intertwined with the language of the question that it is difficult to determine whether someone who cannot answer the question in the examination/test has a problem with mathematics or with the language. It should however be obvious that a problem with the language of a question would prevent the understanding and execution thereof. This caused many students to lose marks in tests and examinations, marks that they could probably have achieved if the language was more accessible to them.

In conclusion it is clear that English as second language for studying affects the students' performance in mathematics adversely. Ways in which this problem was addressed in the Pre-Calculus course were discussed above and in the concluding chapter additional ways will be recommended and discussed.

## 4.7 Reasons for doing mathematics at university

In answer to the question

Why did you decide to do Maths as a subject at university?

students answered as follows:

| | |
|---|---|
| 65.9% | It affords better career opportunities. |
| 65.9% | I need it for my other subjects. |
| 39.0% | I am interested in studying Maths as a subject. |

(You may choose more than one option.)

Table 22: Reasons for doing mathematics at university

The reasons the majority indicated for doing mathematics at university are self-explanatory. There is no dispute about the usefulness of mathematics in terms of finding employment or of its usefulness, indeed its necessity, in other subjects. In this regard the speech of Mr. Greenspan (2001), wherein he explains the escalation of the percentage of jobs that need the knowledge and thinking skills of mathematics, comes to mind.

However, the third option that students could choose as a reason for doing mathematics at university had nothing to do with its usefulness but concerned their subjective feelings about mathematics:

'I am interested in studying Mathematics as a subject.'

This refers to the characteristics of mathematics that makes it an interesting subject. To understand the lure of mathematics for these students, their answers to the question 'What about Mathematics makes you happy?' (Appendix A, question 11.1) were studied.

Literature research to try to fit their responses to these two questions into a character description of a kind, found that it most closely matched the attributes of an inventor as described by David Perkins in his Six-trait Snowflake Model of Creativity (Kirby & Kuykendall, 1991). He lists these traits an inventor has as:

1. A strong commitment to personal aesthetic.
2. The ability to find problems.
3. Mental mobility.
4. A willingness to take risks.
5. Judging own ideas and also seeking criticism.
6. Inner motivation.

Students aspiring to or showing these traits are well on their way to enjoy and ultimately excel at mathematics. All these traits are examples of positive emotions, and when connected to mathematics, they form powerful inner motivators.

In answer to question 11 (Appendix A) on 'What about mathematics makes you happy?' students were very forthcoming with their comments (Appendix F) and here these are matched to the attributes of an inventor as described by Perkins in his Six-trait Snowflake Model of Creativity (Kirby & Kuykendall, 1991). These traits are:

1. A strong commitment to personal aesthetic. Students' comments supporting the fact that they find studying mathematics a pleasant experience were:

- *When I understand it, I feel happy;*
- *When I am doing Maths, I feel so relaxed and confident;*
- *I feel very happy about Maths because I am having a clear understanding of everything;*
- *I am good at it;*
- *If I understand it, it makes me happy and I am much more interested in it;*
- *Maths is an interesting subject, its enjoyable especially when you dedicate yourself to it and practice makes perfect.*

From these comments it is clear that each of these students are fulfilled by what the subject does for them on an emotional scale, making them feel well and confident.

2. The ability to find problems. This is very hard for the average Pre-Calculus student as they are still struggling with the basics of mathematics. They do however declare that on their level, they:

- *enjoy calculating sums,*

which indicates that they would also seek and find problems to solve, once they have acquired more mathematical skills.

3. Mental mobility. The students like this effect of Mathematics. They say:

- *It makes my brain count (calculate) things quickly;*
- *It helps me think quicker, tricky and effectively;*
- *It teaches me to be logical.*

These comments speak for themselves and reflect the invigorating effect mathematics have on these students.

4. A willingness to take risks. Students own up to this trait when they say:

- *To me it's a challenge to sit and struggle and solve a problem;*
- *I like maths because it is challenging;*
- *I feel challenged when I do maths;*
- *I am sure I can do very well in maths, it needs more practice.*

In the light of the fact that most people are too scared to do mathematics, these comments indicate the willingness to take the risk of being challenged by mathematics. Coming from relative failure in mathematics at school level, enrolling for a mathematics course at university, by itself, spells out the willingness to take a risk and this is what the Pre-Calculus students did, they took the risk of studying a subject that had before challenged them and beat them. In this way they show a remarkable amount of resilience in the face of possible failure.

5. Judging own ideas and also seeking criticism. One student echoed this statement in his/her comment:

*When you do something wrong, you practice it again and again until you get it right.*

This is indeed the cyclical learning pattern of mathematics: doing something, judging it yourself, correcting it yourself several times and eventually seeking approval/criticism from another mathematician. Only people with a strong commitment to what they do, can endure this process, and in the Pre-Calculus class there were many students committed to this process.

6. Inner motivation.

The students were exuberant in their comments about their inner motivation:

- *I feel proud of myself (when an answer is correct);*
- *Knowing that I know something about the subject that other people don't know;*
- *Wow, maths is my favourite subject;*
- *When I am doing maths I feel so relaxed and confident.*

These comments speak of powerful inner motivation gained from studying mathematics.



## 4.8 How school influenced mathematics marks in the matriculation examination



In the original questionnaire, the students gave answers to the question about circumstances at school that were deemed by the author to be inconsistent with their matriculation results. It was then decided to give a few of them a more detailed questionnaire (Appendix B1), covering only this topic. Only 11 students responded, but the answers still give a good review of what they experienced at school and the written comments appear in Appendix C3.

In order to verify the information from the 2001 class, the 2002 class were given the same questionnaire and the two sets' answers are reflected together to give a profile of how they experienced their respective schools. In each section the opinions that form the top 70+% are given. Percentages refer to the number of respondents who were of that opinion.

| | | |
|---|---|---|
| 1. | My teacher was absent sometimes and it bothered me a little. | 41% |
| | My teacher was not at all absent and it did not bother me at all. | 32% |
| 2. | I did not often have a new teacher and it affected me not at all. | 47% |
| | I sometimes had a new teacher and it bothered me a little. | 25% |
| 3. | I had a textbook and it did not bother me at all. | 60% |
| | I did not have a textbook and it affected me very much. | 23% |
| 4. | My teacher understood my questions very well and it did not bother me at all. | 47% |
| | My teacher understood my questions reasonably well and it | |
| | - bothered me not at all. | 19% |
| | - bothered me a little. | 19% |
| 5. | My teacher could explain the work very well and it affected me not at all. | 50% |
| | My teacher could explain the work reasonably well and it bothered me a little. | 22% |
| 6. | Not all topics were taught and it bothered me a lot. | 41% |
| | All topics were taught and it bothered me not at all. | 37% |
| 7. | Classes were noisy sometimes and it bothered me a lot. | 29% |
| | Classes were noisy sometimes and it bothered me a little. | 27% |

8. Teaching was exam-directed sometimes and I liked it a little. 24%
Teaching was exam-directed sometimes and I liked it a lot. 20%
Teaching was exam-directed sometimes and I liked it not at all. 17%
Teaching was exam-directed very often and I liked it a lot. 17%

9. My teacher gave me no personal attention and it bothered me a lot. 24%
My teacher sometimes gave me personal attention and it bothered me not at all. 24%
My teacher very often gave me personal attention and it bothered me not at all. 17%
My teacher sometimes gave me personal attention and it bothered me a little. 14%

10. I never had a tutor to ask if I had a problem and it bothered me a lot. 55%
I never had a tutor to ask if I had a problem and it bothered me not at all. 18%

From this the following deductions can be made:

Positive:

- Teacher absenteeism does not adversely affect the pupils.
- Change of teachers is not significant.
- Textbooks are readily available.
- Pupils are generally satisfied with the understanding teachers have of their questions.
- Teachers explain the work to the pupils' satisfaction.
- Most pupils received personal attention from their teachers.

Negative:

- Not all topics are taught and this poses a problem.
- Class noise (and therefore discipline) is a problem.

- Teaching seems to be exam-directed more often than not.
- No pupil had a tutor to ask for help.

In terms of the outcome of their schooling it seems as if students are generally satisfied with what happened at school on an emotional level but have great problems with the handling of the content.

Their response to the question on class size was very interesting. The categories and average class size for this category are:

| | |
|---|---|
| It does not affect me at all | 34 |
| It bothered me a little | 47 |
| It bothered me a lot | 40 |

One would have thought that there would have been a greater difference in the number of pupils in a class and especially that the number that leads to a little bother (47) would be less than the number leading to a lot of bother (40). This shows that different persons will experience the same circumstances differently and that this makes it very difficult to find the 'ideal' circumstances for learning.

All the above paints a not too dark picture of schools in South Africa. If, however, we look at the comments the 11 students of the 2001 class wrote (Appendix B2) a different picture emerges.

- 4 of the 11 students were not given a fair opportunity to do mathematics on higher grade,
- The teachers caused problems in that they:
  - Were too busy with other activities
  - Were not mathematics teachers
  - Did not use an appropriate teaching method
  - Were rushing through the work, a consequence of inadequate planning
  - Did not finish the syllabus

- Favoured some pupils above others

- Physical circumstances like class size and small classrooms where the teacher could not reach the pupils are detrimental to the learning process.
- Textbooks were inadequate to learn from.

These comments are more in line with the general idea of the circumstances at schools in South Africa. It also shows that these students are in need of opportunities to recover those they lost at school.

It is very encouraging to note that even while at school, some of these students would find ways of helping themselves with group study and with going outside school to the Institute of Race Relations.

## 4.9 Motivational Aspects

It is a popular view that part of the lecturer's responsibility is to motivate the students to learn, and furthermore that their failure to learn means that the lecturer could not motivate the students. Ausubel (1968:365-366) has an alternative view. He states:

> The causal relationship between motivation and learning is typically reciprocal rather than unidirectional. Both for this reason and because motivation is not an indispensable condition of learning, it is unnecessary to postpone learning activities until appropriate interests and motivations have been developed. Frequently, the best way of teaching an unmotivated student is to ignore his motivational state for the time being, and to concentrate on teaching him as effectively as possible. Some degree of learning will ensue in any case, despite the lack of motivation; and from the initial satisfaction of learning he will, hopefully, develop the motivation to learn more. In some circumstances, therefore, the most appropriate way of arousing motivation to learn is to focus on the cognitive rather than on the motivational aspects of learning, and to rely on the motivation that is

developed from successful educational achievement to energize further learning.

It is therefore reasonable, according to Ausubel, that students, given time to acquaint themselves with the content of a course and for the intrinsic goal-orientated motivation to work, would experience the ensuing success in the subject that would carry the motivation on, and that no great campaign of the lecturer to motivate them would be necessary.

In his article on math abused students, Greg Fiore (1999) writes about math anxiety as follows: "One effective technique that helps reduce math anxiety focuses on teaching the content of mathematics. Teach so that students understand, because the more students understand, the less math anxiety the students will have."

In his book, Educational Psychology – A Cognitive View, Ausubel (1968:393) writes about increasing classroom motivation. This can be accomplished by making "the objective in a given learning task as explicit as possible" and by setting "tasks that are appropriate to each learner's ability level." These quotations refer to the method of teaching and explaining as motivational factors.

The study matter also has inherent motivation. "Internal motivation comes from being in a state of disequilibrium and desiring to return to equilibrium", according to Wadsworth (1978:81) in his book Piaget for the Classroom Teacher. In the same vein, W.G. Perry (1970:9) states: "it has been observed for two millennia that in any learning situation the learner requires the support of some elements that are recognizable and familiar. Then, if the experience is to be anything more than drill, the second requirement is a degree of challenge." Referring to the students' comments in Appendix D1 it is this kind of challenge that motivates them. Having approachable lecturers is an important motivating factor for students. This fact is corroborated by Fiore (1999) when he states: "the belief that an individual can learn mathematics, linked to the feeling that the instructor cares about the student's learning, will push a student harder to learn."

Another motivating factor is the understanding of math topics previously not understood. Concepts not understood in mathematics are bound to come up in subsequent study and have a cumulative negative impact on each successive topic in which these concepts feature. In his discussion of the levels of mathematics, Usiskin (2000) describes level 2 as the level of a student who is "diligent, who does daily homework". In South Africa this would apply to a learner at school who could, under ideal learning conditions, achieve an A, B or C on standard grade in the matriculation examination. Mastery of level 2 is essential for access to level 3, the level of graduate study in mathematics (Usiskin, 2000). As the Pre-Calculus course was instituted to eliminated some of the problems the students had with concepts in school mathematics, and thus with level 2 activities, it is hoped that students find the course motivating.

Finally it is clear that there are many factors influencing the motivation of the students. These factors include the conducting of the course, the course content, the lecturers teaching the course and the positive attitude of the students and all contribute to the motivation to study and to want to be successful.

To establish the motivational aspects of the Pre-Calculus course as experienced by the students, they were given the opportunity to write about it. They were asked to:

- Name 3 ways in which the Pre-Calculus course motivates you.
- Name 3 ways in which the Pre-Calculus course does not motivate you.

The motivational factors of the Pre-Calculus course as identified by the students can be matched in many instances to the theories on motivation that are in existence. Some of these theories will be used to highlight the answers the students gave and to show the correlation between what they experienced and what the theories expect them to experience. Ausubel (1968:365-366) states that the very fact that one is learning something leads to the motivation to learn more. He also says that one should not wait for motivation to set in before one starts to teach, but that one should accept that some motivation would flow from the very act of learning and "from successful educational achievement to energize further learning." Ausubel also feels that "the best way of teaching an unmotivated student is to ignore his motivational state for the time being, and to concentrate on teaching him as effectively as possible."

It is a fact that the Pre-Calculus students came into the Pre-Calculus course from a bad experience with math at school. It could thus be expected that the motivation stemming from the content of the subject was very low. As they were enrolling for studies in science though, their motivation for coping with math was very high as can be deduced from their answers to the question: 'Why did you decide to do Mathematics at university?' (Appendix A, question 4.2). This question had an answering percentage of 171%, as most students gave more than one reason for doing mathematics at university. So, although they might have lacked seeing the content of the curriculum as motivational, they definitely saw the accomplishment of their ideals through successful mathematics studies as motivational. It is therefore reasonable that, given time to acquaint themselves with the content of the course and for the intrinsic goal-orientated motivation to work, the ensuing success in the subject would carry the motivation on, and that no great campaign to motivate them would be necessary. This was proven by the answers to question 6.1 (Appendix A): Name 3 ways in which the Pre-Calculus course motivates you.



### 4.9.1 Motivational factors identified by the Pre-Calculus students

At the end of the year when the students filled out the questionnaire, 67% of the respondents indicated that they were motivated by the course. The reasons they supplied also gave a very good indication of their assessment of the emotional value of the course.

In his article on math-abused students, Greg Fiore (1999) writes about math anxiety that focusing on the teaching of the content helps to reduce the stress. If the content is taught in such a way that students understand, they will be less anxious. This correlates with the responses of 32% of the students in which the words *understand/understandable/clear* were used in describing the motivational aspects of the course. This indicates their great need to understand mathematics and their relief at finally being able to understand it. It also indicates a feeling of safety in terms of Maslow's hierarchy of needs (Beck, 1983:380) from which they can venture out to further study in the subject.

The main aim of the Pre-Calculus math course is to *qualify students for entry into B.Sc.* courses at university. 11% of the responses named this aspect as a motivational factor. This was substantiated by their answers to question 4.2 (Appendix A) on the reasons for enrolling in the course. It also correlates with the self-actualization level of Maslow's hierarchy of needs (Beck, 1983:380), and follows on from the previous level of safety above.

In his book, Educational Psychology – A Cognitive View, Ausubel (1968:393) writes about increasing classroom motivation. This can be accomplished by stating the aim of a certain learning experience as explicitly as possible and by setting tasks that take each learner's ability level into account. This was apparently accomplished in class as the students also named the *method of teaching and explaining* as motivational factors. Fiore (1999) says: "success for many students is related to how we make them feel in class." The fact that students named the *assistance lecturers and tutors lent* as a motivating factor implies that they felt they could rely on these people for help and were thus motivated by the fact that they were not struggling alone. They felt assisted and therefore cherished in class. This hangs together with their statement: *having approachable lecturers* is an important motivating factor, and indeed Fiore (1999) also says that a student will be pushed harder to learn if the instructor cares about his learning. This will motivate the student to think that he/she CAN learn mathematics. In this respect, the students mentioned the *attitude, attentiveness, good mood, willingness and patience of the lecturers.* It seems something they were not used to at school and now appreciate even more at university. The students' using of the words *new* and *more* *knowledge* spells out the motivation that the study matter inherently has. Piaget's theory (Wadsworth, 1978:81) on the learning in general and of learning mathematics particularly, is that learning takes place through the successive confrontations a students has with learning matter that puts the student's mind in a state of disequilibrium.

Exploring this phenomenon, the marks for each group for two topics at a time were statistically analyzed. The topics are Transformation of graphs vs. Exponents and logarithms and Matrices vs. Trigonometry. No significant differences were found. Known topics, exponents and trigonometry, did not lead to a better mark than unknown topics, transformation of graphs and matrices. Thus it was also established that the new topics motivated students to such an extent that they did not underachieve on them but rather found it important to list them as motivating factors.

Another motivating factor the students mentioned is *the understanding of math topics they did not understand at school*. The Pre-Calculus students did not achieve high symbols for mathematics at school – ideal learning conditions not existing for everybody. Many of them were however working as is described by Usiskin (2000) for the attaining of level 2 mastery of mathematics. Many students were working diligently and daily, according to their answers to question 9.1 (Appendix A) where they listed the actions they took to ensure success. As their diligent work did not give them access to university through matriculation exemption, they would naturally be motivated by the fact that they could finally understand the work that was a stumbling block to them as recently as a year before. As mastery of level 2 is essential for access to level 3, the level of graduate study in mathematics (Usiskin, 2000), and the Pre-Calculus course eliminated some of the problems the students had with school mathematics, and thus with level 2, it is understandable that they found this aspect of the course motivating.

More particularly they commented on the content that it was *nicely outlined, not too much work, easy/simple/not very complex/excellent*. They also appreciated the *semester and module courses* as it made it easier to study for tests. These motivating factors correlate again with Maslow's level of feeling safe, as mentioned by Beck (1983:380), knowing what is going to happen and being prepared for it.

Based on the theory of mastery learning (Block, 1971:2-11), the students were given opportunities to *rewrite tests*. Many appreciated these opportunities to better their marks and enthusiastically attended the *extra classes* held to help them.

*Homework* and *frequent tests* were also cited as motivating factors. This correlates with Ausubel's (1968:393) discussion of increasing classroom motivation in which his advice is to "help students set realistic goals and to evaluate their progress toward these goals by providing tasks that test the limits of their ability."

The motivational impact of *practice* and *consistent working* were also mentioned and ties in with the statement made by Cooley (1918:89) that "if a man is working zealously at a task worthy in itself and not unsuited to his capacity, he has commonly the feeling of success". This feeling of success that comes from being committed to doing Mathematics is the motivating factor the students write about.

Comments expressing their feelings were:

- *I feel attracted to mathematics,*
- *I look forward to a challenge,*
- *I am overcoming my negative attitude* and
- *math is challenging and fun.*

In terms of Maslow's hierarchy of needs (Beck, 1983:380), these comments indicate the level of affection and affiliation needs that are strong motivators on the road to self-actualization, the ultimate goal of study.

It is thus clear that the conducting of the course, the course content, the lecturers teaching the course and the positive attitude of the students all contributed to the motivation of the students to study and to want to be successful.

### 4.9.2 Demotivating factors of the Pre-Calculus course

Demotivating factors cited by the respondents and relating to the Pre-Calculus course concern the following aspects:

- the organization of the class,
- the students themselves,
- the content,
- the lecturer,
- the time it takes and
- English as instructional language.

Although the comments of the students concern the topics above, their comments can also be categorized in four categories, namely

- Lack of self-discipline;
- Externalizing the problem;
- Problems they experienced regarding the course content; and
- Problems they experienced regarding the course organization.

#### 4.9.2.1 Lack of self-discipline

Psychological reasons for students being demotivated mainly have to do with their lack of self-discipline. Students expect to be told how to handle the *freedom at university*. They especially had difficulty with the *freedom to attend class or not*. They were made aware of the importance of attending every lecture but peer pressure plays an important role in this matter. Coming from school where the teacher is responsible for your success, to university where you are the responsible person is a big step for many students and one many students cannot take. They still seem to be caught up in the social group and to adhere to its rules. Those that did manage to break free of the group, for study purposes, were seen still to be part of the group for other activities and have gained recognition for their study diligence. This is the result of a maturation process that each student achieves in his/her own time. None of the students in Achievement Groups 1 and 2 complained about this matter, and it is one of the main reasons why some students found themselves in Group 3 – erratic attendance of classes and a lack of self-discipline in their studies.

Noticeably some students had no regard for lecture times and came and went, as they liked, to their own detriment and the disturbance of others. Fortunately this practice was eradicated by private discussion with the students concerned.

#### 4.9.2.2 Externalizing the problem

A lack of responsibility on the part of the students was a stumbling block to many students. They like to hold the lecturer responsible for their success, asking that *the lecturer mark homework so the student can become more serious.* They complain of *a lack of strictness on homework and exercises*, again ridding themselves of the responsibility for their own actions.

#### 4.9.2.3 Problems with the course content

The content of the course elicited two different responses. Students found the content *difficult to cope with* which made them *lose faith* in their abilities and interest in the subject. This again relates to the need for safety in Maslow's hierarchy of needs (Beck, 1983:380). The fact that they experience the content as too difficult impacts negatively on their emotional readiness to work at it. This causes failure that in its turn causes them to lose faith in their ability to do the mathematics. Some students felt that the content was *not preparing them for Mat 111* and that *some of the work is unnecessary.* This might be true for certain sections of the work that were done in class but not directly examined. It takes a fair amount of maturity to realize that exam oriented or mark-generating information is not the only information in a subject. Some topics are taught as background to the topics in the actual curriculum, and should not be regarded as unnecessary just because they are not directly tested. As to the Pre-Calculus course not preparing them for Mat 111, it would be interesting to know what they expect to study in Mat 111, for the Pre-Calculus course indeed is not structured to prepare them for the Mat 111 course, but to help them attain matriculation exemption.

The *inability of some students to understand* the work done in the Pre-Calculus course was understandably demotivational to them. They complain that they *cannot do it on their own.* The students know that the subject *needs more attention*, but feel that they will not benefit by giving it more *untutored attention.* These students, mostly of group 3, did not avail themselves of the many hours of consultation time that were available with the lecturers and were put to good use by the vast majority of the students. If a student felt that he/she had to go it alone, it was definitely a situation that they themselves caused. This could be because they are too shy to approach the lecturer, but several students overcame this stumbling block by sending a friend, or inviting a friend along to the lecturer. Extra classes were also offered on a regular basis, which these students seldom attended.

*Questions asked in the tests are far more difficult than those done in class* is a very obvious demotivational factor stressed by the students and they had every right to be upset by it. The fact is that some of the Pre-Calculus students will not achieve exemption because they are not intellectually capable of university studies. If they then complain about the *difficulty* of a course, the reason might be this inability on their part. On the other hand, one student from Group 1 complained of being *spoonfed* and that his *responsibility and maturity was never tested.* This variation in expectations of the students makes the lecturers' handling of the course a very difficult task.

Some students rejected topics done in school and *repeated at university.* This was most probably because of the familiarity of the content and that they knew they had struggled with it before. These topics, exponents and logs and trigonometry, are the topics at school that learners find most confusing and therefore most difficult. This is one of the reasons they are included in the curriculum of the Pre-Calculus course. Another reason is that they are most applicable to Mat 111 Calculus and other scientific subjects and therefore it is very important for students to have a fair competency in these topics.

#### 4.9.2.4 Problems with the organization of the course

The organization of the class was a serious concern expressed by the respondents. In terms of Maslow's hierarchy of needs (Beck, 1983:380), this meant they did not feel safe in the class environment. They felt the class was *too big*. They were *too shy/scared to ask questions in such a big class*. *Noisiness in class* made it difficult for some students to concentrate and learn. In this respect class discipline was difficult to maintain as often students had to translate to each other what was being said and this, together with other talking, made the class very noisy at times.

While many students regarded *frequent testing* and even *retesting* as motivational (Groups 1 and 2), others (Group 3) experienced it as demotivational. They reason that having to write a test over and still not passing does not motivate them. This is indeed so, and the aim of mastery learning - the strategy followed in this case – is to get the student to the point where he/she does indeed pass. Not passing after the retesting can have several reasons – inability of the student to pass the course on grounds of insufficient intellectual ability, insufficient preparation for the tests, not taking advantage of the consultation times available with the lecturers, etc. Students felt that the *explanations were not done as well as at school* and *that some students received more attention in class than others*. These complaints can be a very personal view of the students concerned and could be explained by the interaction of personalities between the students and the lecturer regarding the content.

Economics is the reason for the *textbook not being available for leasing*. In order to have the book on the shelves for students to lease means that the Department of Mathematics must purchase a total of 130 books, an expense that the department cannot carry. However, with each successive year the course is offered, textbooks become more available for students to buy, as more second hand books become available from the students of the previous years.

Several students named the fact that it was a *year course instead of a semester course* and that it *did not gain them any credits* as a demotivating factor. It should be understood by the students that this course could not give them any credits, as it is not yet a university course but a preparatory course for university. As it is not yet part of the degree courses for mathematics, it cannot carry any credits. As regards the length of the course, the course is especially introduced to give students another opportunity to gain matriculation exemption, and in this light it is surely not reasonable to expect this to be done in a mere semester of 16 weeks' tuition.

Lastly, English as instructional language posed a problem to students. This was discussed under the heading English as Second Language in paragraph 4.6 of this chapter.

#### 4.9.2.5 Problems concerning tutorial classes

Apart from the students' comments on demotivational factors, it was noticeable how reluctant some of them were in attending tutorial classes. They obviously needed help, but came to class with hopelessness in their facial expression and in their body language. They did not mind being taught, but resented having to do any Mathematics themselves. The following observations by Krutetski (1976:310) throw a new light on the possibilities and realities of a Mathematics tutorial.

Krutetski (1976:310) conducted tests to determine the fatigue that sets in when pupils do Mathematics. Gifted pupils were found to be able to work well for up to 3 hours while the same pupils showed early fatigue studying other subjects.

In conjunction with this, there was noticeably increased fatigue of mathematically less capable pupils as they study Mathematics in comparison to their fatigue in studying other school subjects. These pupils do not tire during Mathematics lessons when they are not laboring (presumably just listening), or when they are cut off from the lesson (as often happens when their minds wander to think of other things). But, if they are working (like in a tutorial), they tire much more easily than other pupils, since establishing bonds in the world of mathematical objects is always a strenuous effort for them.

This phenomenon was easily detected during the tutorial periods of the Pre-Calculus course. These tutorials could last up to 3 hours and contrary to the belief that more time spent on Mathematics would help the average/weak student, quite the opposite happened. The weak students came to class with a resistance against the tutorial, no doubt because they resented the tiredness that would be the physical result of the tutorial. On the other hand, the top students were still interested and still asking questions long after the others had left and class time had expired, putting weight to Krutetski's (1976:310) findings. This situation has two negative effects feeding off each other. The student who needs more exercise is also the one who cannot endure spending time on Mathematics for long spells at a time and thus, in stead of the tutorial acting as a motivating factor, it turns out to be demotivating.

In conclusion, some factors like problems students have with the content and organization of the course could be addressed by the lecturer. Some factors, those stemming from the students' own perceptions and attitudes and abilities, are facts of life and will occur in any lecture room in any subject.

## 4.10 Study habits at university compared to those at school

The Saint Louis University (SLU), in their handout on Success in Mathematics, discusses the difference between College (university) Mathematics and High School Mathematics, and states that students at university "probably need more time studying per week" and they would "do more of the learning outside the class than in high school."

Two factors have an influence on the time it takes to study Pre-Calculus. Firstly, these are the problems experienced by students studying in their second language and thus spending more time reading and rereading study matter to comprehend and clarify concepts, facts, algorithms, etc. Secondly, the depth of mathematical context and the compactness of the concepts at university require more time to analyze and contextualize. This second factor is also the reason why more of the learning is done outside the classroom at university than at high school.

Analyzing study matter to make sense of it is a highly individual matter in the sense that each person has to do it for him/herself. Bringing this newly acquired knowledge into context with what is already known is likewise an individual endeavor. Stemming from the individual nature of these activities, they will be done outside of the classroom.

This leads us to conclude that it is indeed necessary that study habits at university should be different from those at school.

The volume and level of difficulty of subjects at university make it imperative that students adapt their study routine to differ from what they had at school, especially if these study methods were not successful as in the case of the Pre-Calculus students. To ascertain to what extent and in what way students adapted their study habits they were asked to answer the following five questions: (Appendix A)

- Do your study habits for Mathematics at university differ from your study habits for Mathematics at school?
- If YES, in which respects?
- Why did your study habits change?
- Do you study more regularly at university than what you did at school?
- If YES, what are the reason(s)?

The outcome of their responses to the questions above was that:

Study habits the students have at university differed from those they had at school for an overwhelming 76,8% of the respondents.

The reasons they gave were:

- they spend more time on mathematics problems than at school (52,4%),
- they work with fellow students (41,3%), and
- they spend more time reading the study material (27%).

This just means that the Pre-Calculus students found out for themselves what the Saint Louis University (SLU)(2001) tells its students in their handout on Success in Mathematics. In this handout they discuss the difference between College (university) Mathematics and High School Mathematics, and state:

- You probably need more time studying per week – 52.4% of respondents agree,
- You do more of the learning outside the class than in high school – this must be the reason 27% of the students claim to spend more time reading the study material. Spending more time reading can also reflect on the difficulty many students had studying in English which is not their home language, as 27% replied to question 2.1 (Appendix A) that it takes more time to study as a result of English not being their first language.

The students at SLU are also advised to form study groups. This was a natural step for 41.3% of the Pre-Calculus students and they benefited in more ways than one by doing so. The main objective was of course to get help with the mathematics, but for many it was also a means of having the mathematics discussed and explained in their mother tongue. Some students are not agreeable to the practice of studying with peers and will have to learn along their journey through mathematics that collaboration is a means of enriching your own and other mathematicians' understanding of the subject topic under scrutiny.

76,8% of the respondents indicated that they study more regularly at university than at school. Their reasons, in the order of importance they placed them, were:

- There is more work to study,
- The work is much harder to understand, and
- There is no revision in class.

It could be perceived that there was more work to study, but it was not always entirely new work. The whole section on graphs of polynomial and rational functions, for instance, was indeed a cleverly disguised revision of school algebra while combining it in new structures to be able to make new conclusions. Their inability to think laterally with the school mathematics knowledge made the Pre-Calculus seem more work than it indeed was.

That the work was harder to understand can be proved correct and incorrect. What plays a great role here is that it was important to know more about the work than at school as, at university, a pass mark is 50%, while 34% (standard grade) and 40% (higher grade) was sufficient to pass at school. This must have made the work seem harder. In some cases the work could seem harder because of its level of abstractness. Students placed the study of matrices and the transformation of graphs in this category.

Students who said that there was no revision in class, were either not in class or not paying attention while in class. The curriculum was drawn up and executed in such a way as to repeat and utilize previous knowledge cumulatively. So, although no formal revision was done, incidental revision was part of each lecture.

So, in conclusion, one can deduce that the students did change their study habits when they came to university.

## 4.11 Success as experienced by the Pre-Calculus students

A theory of success from the book Social Process by Charles Horton Cooley (1918: 88-98) forms the background for the discussion of the students' feeling of success.



Although Cooley (1918:88) discusses success in relation to what a person can do for the community, he also lists the personality traits and physical or organizational attributes a successful person will have.

Success hinges on "the idea of personal self-realization" and the feeling of success is "the fullest consciousness of personal existence and efficacy", according to Cooley (1918:88). Cooley (1918:89) further states:

> If a man is working zealously at a task worthy in itself and not wholly unsuited to his capacity, he has commonly the feeling of success.

The personality traits that successful people exhibit are, according to Cooley (1918:90):

> being energetic, having initiative, being self-reliant, tenacious and adaptable and having courage, resolution, faith and composure.

According to Cooley (1918:92), to be successful, "we need all the opportunity that society can give us, but it will do us little good without our own personal force, intelligence and persistence."

Questions 9.1 and 9.2 of the questionnaire (Appendix A) address the subject of success. The feedback of the students to these questions will be discussed against the background of Cooley's theory of success. (1918:88-98). Cooley (1918:90) lists the personality traits and physical or organizational attributes a successful person will have as:

> energy, initiative, self-reliance, tenacity, adaptability, courage, resolution, faith and composure.

Cooley's (1981:88) statements that success hinges on "the idea of personal self-realization" and the feeling of success is "the fullest consciousness of personal existence and efficacy", are verified by the answers to question 11.2 (Appendix A) where students were asked to select the words applicable to their feelings when taking a Mathematics test.

They named the following positive feelings:

> *challenged (51%), confident(32%), capable(30%), in control(15%), strong(14%), energetic(11%) and clever(3%).*

This gives a total of 156%, which means that many students experienced more than one of these positive feelings of self-realization (*challenged, capable*) and of personal existence (*confident, energetic*) and of efficacy (*in control, strong, clever*).

Cooley (1981:89) also says that if a man is working enthusiastically at a task that is worthy in itself and not totally outside his ability, he will commonly have a feeling of success. In students' comments in answer to question 9.1 (Appendix A): What did you do to ensure your success? the phrases *work hard, practice a lot, study, do exercises* come up in 53% of the responses, showing a consciousness of the enthusiasm needed to be successful at Mathematics. On the other hand only 33% of the students, who did not see themselves as successful, contributed their failure to lack of hard work, indicating that these students are not likewise aware that it takes hard work to be successful at Mathematics.

That the task at hand, Pre-Calculus, was worthy in itself, was overwhelmingly positively answered in question 4.2 (Appendix A) where 65,9% of the students stated that Mathematics offers better career opportunities and 65,9% declared its usefulness in accessing other subjects. The students were therefore obviously conscious of the worthiness of the task. At the same time, a passing rate of 69% showed that the task was suited to their capacity.

Students' answers to the question: What did you do to ensure your success? can be classified under the personality traits Cooley (1981:90) listed. These are:

self-reliance:-

> *redo class work on my own; ask questions; I tried and managed to pass; I made sure I don't leave class without understanding the work done; I practice a lot on my own; I studied for myself, not because I was forced to.*

tenacity:-

> *I begin to get my strength; manage to work hard; I study and I don't understand some stuff easily; I tried my best in passing my tests.*

adaptability:-

> *It's better than at High School because I now ask questions I do not understand and don't wait until the exam date is closer; I have learnt more things I did not expect to know and understand.*

resolution:-

*I listen in class; I did my best not to make the same mistakes as last year; I attend class regularly; now working twice as hard; I've done more work; because I want to go to the first year; I paid attention; I attend extra classes.*

faith:-

*I have improved a lot; I did not fail; I know I can make it; I passed the 1st semester and now this semester I try my utmost best; I am confident enough to pass.*

composure:-

*I understand the work these days; I passed all my modules and this shows some kind of success; I am doing very good in my Maths; I understand my teachers and they understand me.*

According to Cooley (1981:88), everybody needs to exert their own personal force, intelligence and persistence in order to utilize the opportunities that society give us.

The Pre-Calculus class was established to give students who were not academically ready to enter into B.Sc. studies an opportunity to become ready. Based on their matriculation results, they were all credited with enough intelligence to study in science. What made the difference between the successful and the unsuccessful students was their personal force and persistence. Their personal force is noticeable in comments like: *because the work is interesting; I have always enjoyed Maths; I enjoy Maths,* and their persistence in the many instances that *hard work* is named (53%).

The students that did not see themselves as successful, noted the following (in their own words):

Their feelings:-

*did not feel confident; lost interest; know I don't deserve to pass.*

Their actions:-

*did not use the library; had no textbook; became lazy; did not attend class; hardly sat with Maths; did not revise Maths; did not work hard; did not ask for help; did not prepare for tests; did not pay attention when in class.*

This leads to the conclusion that they were not actively involved in their studies, nor were they feeling positive about the subject/activity of studying mathematics. Such negative and demoralizing feelings do not set a person up for success.

In question 9 (Appendix A), the students expressed their views on their possible success. Many of the virtues of a successful person that Cooley (1918:90) lists can be verified in the comments of the students, and his theory thus forms a good background to the analysis of their comments.

## 4.12 Use of study resources

Students in the Pre-Calculus class do not know the importance of having good/excellent study resources. The textbook in use is Precalculus by J. Stewart, L. Redlin & S. Watson. 50% of them did not even acquire a textbook. There are non-academic reasons for this fact, such as not being able to afford the textbook at R270, and also not having it available for leasing from the university. Fortunately, several copies were placed on reserve and 55% of the students indicated that the textbook (whether it was their own or the library's) helped them in their studying.

They used the textbook in the following ways:

1. *Do the exercises* (41%) – the most common use of a mathematics textbook.
2. *Reread class work* (33%) – a commendable act. Even if the work was only reread for the language's sake, the incidental benefit was that the mathematics was also reread.

3.1 *Read ahead* (11%) – this is the way a really interested student would use a textbook and it is encouraging to know that some students are already fostering good study habits in their Pre-Calculus year, their first year at university.

3.2 *Read the interesting facts about the topics I study* (11%) – similar to the above about reading ahead, this activity is highly commendable and to be encouraged so as to 'spur' on students' interest in the subject.

3.3 *Try doing some of the other topics* (11%) – a sign of the inquisitiveness that will carry a mathematics student far into undergraduate study and hopefully into post-graduate studies too, eventually.

4. Only 5% found the *histories of the mathematicians* interesting,

5. Only 4% found the *pictures* interesting, which is a pity as the textbook has some interesting applications of the work covered in the course.

It is thus clear that the textbook greatly enhanced their study experience, if it did not necessarily help them achieve higher marks it at least gave the majority of the students a positive disposition towards the subject.

Concerning handouts, which they only received for the work on matrices, 77% indicated that they were sufficient and 87% regarded these as a study resource. Notes written from the board were meant to augment the textbook and were appreciated by 77% of the respondents as being sufficient, while 80,5% used them as a study resource. However, the author's experience of seeing students' notes when they came for consultation belies the fact that they could serve as a study resource. Their notes were inaccurate, incomplete and often without the proper structure. That students did indeed pass studying from their notes is to be marveled at.

Only 36,6% of the respondents indicated that they ever consulted other mathematics books in the library. This is highly understandable as there are only a few books dealing with the Pre-Calculus work available on the shelves.

All in all, it seems as if the students were satisfied with the way the information was made available.

Secondly, mathematics anxiety is popularly cast as a factor prohibiting the learning of mathematics. On mathematics anxiety Steen (1991) says that observations dispel the belief that "for most adults the emotional baggage of mathematics is an overwhelming burden" (Steen, 1991). Mathematics anxiety originates in the practice that students have of imposing the lecturer's measure of correctness to the students' answers instead of testing and verifying their answers for themselves and thus become emancipated from emotional dependence on teachers or lecturers for approval.

Perry (1968:glossary) describes this in his forms of development as the form of dualism. In this form a student sees everything as absolutely right or wrong, sees learning as an exchange of information, knowledge as quantitative and honors authority with the ultimate say in whether an answer is right or wrong. These are ultimately the ways a first year student at university, therefore also the Pre-Calculus students, can be expected to see the acquisition of knowledge.

However, at UWC the annual report 2000 for the Institute for Counseling for the University of the Western Cape rates the occurrence of mathematics anxiety much lower than other fears and problems. In this study, mathematics anxiety ranks 31.5 out of 41 in the rank order of problems and concerns presented by students. This throws new light on the matter of mathematics anxiety in that it is probably not such an important factor in the learning of the subject as what is commonly expected.

Taking Krutetski's (1976:74) schema for readiness for an activity as background, the two topics

- What about Mathematics make the students happy, and
- Mathematics anxiety in the Pre-Calculus course

will be discussed under this heading.

Krutetski (1976:74) identifies the following general psychological components of readiness for an activity:

- A positive attitude towards the activity
- Certain character traits
- An appropriate mental state
- Knowledge, skills, habits applicable to the activity.

Students' comments to the question on what about mathematics makes them happy can be categorized to indicate that the components for readiness for an activity were indeed present in them and available to be used in the activity of learning mathematics.

The students' positive attitude is reflected in their comments:

- *When I have an answer to a problem and it is correct, I feel proud of myself.*
- *I love it (mathematics)*
- *What I like most about Maths is that it needs more practice.*
- *It's enjoyable, especially when you dedicate yourself to practice.*
- It is their *favourite subject.*
- They like *having to constantly work hard to get satisfactory results.*

Character traits identified by the students are:

- They most of all like to win. They measure this in their *test results that they like to be high and to be a good pass.*
- They *like facing challenges* and Mathematics presents them with such challenges.
- They know themselves, they *like numbers better than words*, and the fact that they *do not have to read in order to pass.* (This last statement should be understood as that Mathematics takes comparatively little reading when compared to subjects like Botany, History, etc., not really that there is no reading in Mathematics.)

- They like feeling special. This happens when they *know something other people don't know*. It boosts their self-image and makes them *feel proud* of themselves.
- They like being part of the social life on campus. Studying Mathematics allows them to be social beings as *they can work with other students* and they *can even study if they are listening to the radio.*
- They like being independent. It enhances their independence if they work with problems of which the outcomes *(answers) are known* or can *be checked by them themselves.*
- They like being singular. One student stated that what makes him happy is the fact that *everybody hates it* (mathematics). This must be a student that likes being independent of the opinion of others and this is a good character trait for a scientist to have, under certain conditions.

Students also feel happy about Mathematics because of their mental state:

- They are mentally active when they *like using their brain to reason out problems.*
- They are positively orientated when they *feel relaxed and confident when doing Maths.*
- They experience orderliness when the topics are *taught in an order from easy to difficult*. (This was not really so, they just found subsequent topics easier than the first ones, because they were more knowledgeable by the time they got to these topics.)
- They have a positive feeling of *support from the lecturers*, both in *the way the lectures are presented* and in the *extra time and opportunities* that they are afforded by the lecturers.
- They don't feel isolated by the fact that they are studying Mathematics as they experience that they can even study when not on their own and even listening to the radio.

The students affirmed that *being able to solve problems* also makes them happy. This has to do with their knowledge and skills and habits. They readily express their enjoyment of working Mathematics problems and this would not have been so if they did not have the skills to do this with positive outcomes.

It is thus clear that the general psychological conditions needed for successful performance of the Mathematical activity, as stipulated by Krutetski (1976:74), are present in the students who declared that they are happy doing Mathematics.

One other statement regarding the positive emotions Mathematics elicits is that it is an empowering subject. It opens doors to study fields not accessible without Mathematics, and it creates job possibilities (Appendix A, question 4.2). It is easy to understand why these attributes of the subject would make students happy.

With regard to Mathematics Anxiety, it is amazing that students who were not achieving in mathematics at school, still enroll in mathematics courses and seem to enjoy and excel at it. Question 11.2 (Appendix A) dealt with the possible presence of mathematics test anxiety. There are two sources that show that only a low level of mathematics test anxiety existed in the Pre-Calculus class.

The first are the answers to question 11.2 (Appendix A) where the responses indicated that only 9% felt sweaty, 9% short of breath and 7% cold. These are normal physical indications of anxiety. However they were not present in any significant way in the answers supplied by the students. These observations dispel the belief that "for most adults the emotional baggage of mathematics is an overwhelming burden: (Steen, 1991). The fact that the Pre-Calculus students did not experience mathematics test anxiety to this extent is actually quite surprising. After all, being in the Pre-Calculus class presupposes that they were not really very successful at mathematics while at school, a situation that could easily lead to mathematics anxiety.

Mathematics anxiety originates in the practice that teachers/lecturers have of imposing their measure of correctness to students' answers instead of leaving them to test and verify their answers for themselves and thus become emancipated from emotional dependence on teachers or lecturers for approval. Students in the Pre-Calculus course were given the memoranda to all tests and their textbook also had answers to selected questions. It was thus quite possible for them to become independent of the lecturer's approval of the correctness of their work.

So, mathematics anxiety did not really influence the feeling of success of the Pre-Calculus students, partly because of the high motivation they experienced and partly because of class and organizational activities, such as the mastery program, that worked at eliminating it.

## 4.14 Students' general comments regarding the Pre-Calculus course

Finally, in the last question but one (Appendix A), the students could write any additional comments they still wanted to make. They accepted this opportunity and these comments were made voluntarily and should therefore carry sufficient weight. They can be divided into categories as comments on:

- The content
- The emotions elicited by the course
- The lecturers
- The students' own actions, and
- Adaptation to university.

Then there are also criticisms expressed and suggestions made by the students for improving the course. These will be discussed in chapter 6.

### 4.14.1 Content

Regarding the content, the students singled out some topics as more difficult than others. These topics, *polynomials, absolute values, piece wise functions, determinants and matrices*, are in fact all the new topics taught, those they did not study at school. These topics also have a high degree of abstraction at this time of their studies and that could be the reason they experienced them as difficult.

In comparison to school mathematics, the students found the Pre-Calculus *course less difficult* or at least *on par* with school mathematics. They were appreciative of the fact that *basic concepts they did not understand at school became clear during the prelim course*, and that *they gained knowledge that enabled them to solve problems they could not solve before.*

They found the content had a high motivational value in their comments that it was *exciting, interesting, cool, enjoyable, nice.* They were also motivated by the belief that it was going to *help them in their B.Sc. 1 maths.*

As to the level of difficulty, they were not united in their opinions. They found it *understandable and not too much difficult.* The fact *that not all the work was new made it easier.* It also became easier as the year went on – no doubt because of their intense involvement – much higher than normally at school. Some, however, found it *challenging* but were fortunately motivated by this challenge.

### 4.14.2 Emotions elicited by the course

Feelings of motivation were expressed quite adamantly. The motivation stemmed from their *enjoyment of the course*, that *it prepared them for further studies*, that *it explained basic concepts* and that it made them *feel confident*, but also that it *started a career* for them – something they could not have envisaged, given their matriculation results.

Some students were outspokenly positive: *the maths is fine, good, useful, enjoyable and they love this maths, have no problems and are doing satisfactorily.* They *like the way they were taught and appreciate the opportunities they were given* as part of the course. It enhanced their self-esteem when they state: *Really I'm a new person now and I know in the first year I will make it.*

Negative feelings about the course came from students who *were failing, struggling a lot, felt unhappy because they wanted to pass but did not find it easy or were disappointed by their marks.*

Feelings of despair were expressed by students who *live off campus and had no one to help them once they were home*, while students living on campus were perceived to have peer group support.

### 4.14.3 Lecturers

The students appreciated what was done for them by the lecturers, that they were *treated fairly, given extra help, taught very well, given retest opportunities.* They found the lecturers always *willing to help and very encouraging.* They express their thanks towards the lecturers and *feel proud to have been taught by them.*

### 4.14.4 The students' own actions

Students realize from their experience in the Pre-Calculus course that *it needs extra effort and harder work* to pass mathematics at university. They name *commitment, dedication and love for the subject* as important traits for studying mathematics. They acknowledge that *the concepts of mathematics are not difficult if they study and practice every day.*

### 4.14.5 Adaptation to university

Adapting to university posed problems in that some students were in small Higher Grade classes at school and *found the large prelim class (100+) very overwhelming.* As English is their second language but the language of instruction, they were obviously initially reluctant to ask questions. This surely had a negative impact on their progress, but the majority did overcome it and were successful.

### 4.14.6 Criticism

Students also criticized the course, saying that there *was not enough attention to all students.* This is unfounded criticism as during lectures not only the lecturer assisted them but the author as well. At all other times also, both lecturers were readily available as is apparent from the many other comments in the questionnaire.

Other criticism was of *too much spoon feeding* and the course *not really being a challenge for higher grade students.* These comments came from students who had good mathematics marks in higher grade mathematics until the final matriculation exam, in which they inexplicably fared very badly. They were thus, to a certain extent, incorrectly placed in the Pre-Calculus course.

Quite severe criticism was expressed about *the handling of tutorials and tests.* They deemed it *unfair to be tested in a tutorial on a lecture they only attended that morning.* They explain that they *are not quick to understand* – the reason why were in the Pre-Calculus group in the first instance – and that they *need time to acquaint themselves with the content of the lecture before being tested on it.* This is indeed valid criticism coming from students who dearly wanted to do well in this mathematics course.

## 4.15 Conclusion

In this chapter the responses to the questionnaire were analyzed and discussed against a background of applicable literature. In chapter 6 the suggestions will draw on the responses of the students, which were discussed in this chapter.

In the next chapter the research methodology is discussed.

# Chapter 5

## Research Methodology

### 5.1. Introduction

Many factors have to be taken into account when evaluating a course for its validity in terms of what it offers the students and what students need. For this study the influence of the specific lecturer was omitted, as this is a variable influence, varying with each new lecturer appointed to the course. The physical and logistical parameters within which the course is conducted are practically the same for all courses at this university and as such practically a given constant.

The two main influences that were assessed in this study are the design and content of the course on the one hand and the students in the class on the other hand.

### 5.2 Hypothesis, key concepts and variables

Hypothesis: The Pre-Calculus Mathematics Course at the University of the Western Cape provides its students with adequate mathematical knowledge and skills to compensate for the fact that they did not pass the matriculation examination with an exemption.

Key Concepts: SD students; Matriculation Exemption; Profile of the students;

Variables: The students in the class; the lecturer.

### 5.3 Instruments

Four instruments were used in the gathering of information for this research project.

- A framework developed from literature review formed the basis for the identifying of the conceptual and procedural skills students should ideally learn in the Pre-Calculus course.

A framework developed from literature was also used as background to the discussion of the feedback on the questionnaire.

- A questionnaire (Appendix A) gathered information about the students so that a profile of the Pre-Calculus class could be established.
- Observation by the author, who, as observer in the class during lectures and tutorials and as administrator of the course, contributed her observations to substantiate certain conclusions she came to.

### 5.3.1 Framework for Analyzing Pre-Calculus Course

The design and content of the course are measured against criteria found in literature. These criteria describe the intellectual or thinking skills and the procedural or algorithmic skills that a good mathematics course should teach the students. These skills were matched to specific examples of questions, testing these skills, from matriculation higher grade mathematics examination papers.

To determine whether the Pre-Calculus course serves this purpose, these skills were identified in the design and content of the Pre-Calculus course. Examples from the content of the Pre-Calculus course that show the implementation of these skills have been cited in this regard.

### 5.3.2 Questionnaire

In order to draw up a profile of the students in the Pre-Calculus course, they filled out a questionnaire (Appendix A). This questionnaire determined general biographical information regarding age, gender, place of descent, and current place of residence.

The questionnaire (Appendix A) also identified the problems students have regarding certain circumstances like living off campus (Appendix A, question 1). In this regard, the choices through which the students could express their views were drawn up during interviews with four selected students. They gave their input on what it means to live off campus and have to spend time traveling and what it entails to live at home where they are expected to fulfill certain duties at the end of the day, imposing yet again on the little time they have for studying. The Pre-Calculus class of 2002 also answered this question as an open-ended question and their numerous responses gave a much clearer picture of what living off campus entails.

English, as a second language, is addressed in question 2 (Appendix A). The same four students helped to draw up the list of problems a student could have with English as the language in which you study if English is your second language. They also offered advice on how these problems could be addressed.

In question 3 (Appendix A) of the questionnaire students were questioned on whether they have matriculation exemption or not and what the reasons are for not having gained matriculation exemption while at school.

Question 4 (Appendix A) determined whether the students are doing Mathematics 1 and why they made this choice. This question was meant for the Pre-Calculus students of the year 2000 and was to be filled in during 2001. However, they could not be contacted and the Pre-Calculus students of 2001 subsequently answered this question, of their own accord, as if they were indeed already taking mathematics 1. Their answers gave valuable insight into the reasons for taking mathematics at university.

Circumstances at school and how it influenced them and possibly inhibited their learning of mathematics, is the topic of question 5 (Appendix A). The list of possible answers was compiled during an interview with Mrs. Benji Fray, the then lecturer of the Pre-Calculus course, who has extensive experience of teaching in schools similar to those many of the Pre-Calculus students originate from. However, the students misinterpreted the statements and the method of recording their preferences and this question was then subsequently altered and put to the

Pre-Calculus class of 2002. In the analysis and discussion of this question in chapter 3, the answers of the class of 2002 were used.
In question 6 (Appendix A) the students were given the opportunity to express their own ideas on the motivational aspects of the Pre-Calculus course. This was an open-ended question and the students freely gave their opinions, making the answers interpretable as if it was acquired during an interview.

A structure of options probes the students' study habits at school compared to their study habits at university in question 7 (Appendix A). In this respect, question 8 (Appendix A) hangs together with question 7 in finding information on the more subtle aspects of study methods, e.g. studying with friends, reworking examples to get them right, etc.

Question 9 (Appendix A) was an open-ended question giving the students the opportunity to write about their perception of their success, their own role in it and the role of the university in helping them to be successful.

Question 10 (Appendix A) probes the use of printed material and of the notes taken in class. It also determines whether the available printed matter in the Pre-Calculus course was sufficient and/or useful. The list of options which textbooks are used for was compiled in an interview with Prof. R. L. Fray.

Question 11 (Appendix A) addresses the emotional impact the Pre-Calculus course has on the students. Here they were given an opportunity to talk freely about their feelings of happiness and were given prompts to express their feelings about a mathematics test and failure in a mathematics test. The prompts were derived from an article on Mathematics Anxiety by Steen (1991).

Question 12 (Appendix A) attempted to find a hierarchy of difficulties and interest regarding the topics taught in the Pre-Calculus course. Students answered this question in a very unsatisfactory way, listing topics they did not cover, and it was decided to omit the answers to this question in the discussion of the questionnaire.

Students were literally "interviewed" on paper in question 13 (Appendix A). Their general comments on the course gave rise to many suggestions that are discussed in chapter 6.

For purposes of effective advertisement of the course and recruitment of students for the course, the ways in which students were informed of the course were determined in question 14 (Appendix A).

The questionnaire was thus in a way an interview as well as an opinion research.

### 5.3.3 Observation

In the year 2001, the Pre-Calculus course was taught by ms B. Fray. The author was an auxiliary lecturer in the sense that she was present in the lecture room, helping students. The author was also responsible for the marking of tutorial tests, class tests and exams. Furthermore, she took one of the two tutorial groups, taught extra classes and was available for consultation during the week for about 15 hours per week. She was thus closely involved with the Pre-Calculus students throughout the year and has a fair knowledge of their problems and triumphs.

## 5.4 Sample

### 5.4.1 The Curriculum

For the discussion of the curriculum the whole curriculum was scrutinized for examples to verify the presence of opportunities to teach the intellectual and procedural skills deemed necessary. Questions from matriculation examination papers for mathematics higher grade were taken as an affirmation that these skills were tested adequately at school level and should thus also be tested in any course presuming to substitute matriculation higher grade mathematics.

### 5.4.2 The Students

The Pre-Calculus class consisted of 121 students who were all given questionnaires to fill out. Eighty-four (84) questionnaires were returned, constituting an answering percentage of 69.4%. This was considered an adequate sample of the class population to warrant the deductions made in the study to be valid. For the discussion of the responses to the questionnaire, the respondents were grouped in three separate groups based on their academic performance. This was done because the response to a question like 'Do you see yourself as successful at mathematics?' would in all probability be answered differently by a student failing and a student achieving an A symbol. By separating the class in groups, a better interpretation of the responses was possible.

## 5.5 Processes

As regards the conceptual and procedural skills taught by the design and content of the course, the curriculum was the basis for the research. Questions from the higher grade matriculation mathematics paper matching these skills were easily obtained from past matriculation papers. Unfortunately it was not possible to gauge a student's progress in acquiring these skills since no copies of their written test answers were available, tests having been handed back to them for study purposes. Their progress could however be judged on the marks they obtained for successive tests and this had to suffice. It does, however, not adequately measure a student's progress and much of the progress the Pre-Calculus students made in their first year at university was in the field of adaptation to the university and to the study habits they necessarily had to develop.



The questionnaire was filled out during the first week of November 2001. This was at the end of the teaching year so students were able to give a summary of their experiences in the course. It was envisaged that the Pre-Calculus students of the year 2000 would also complete this questionnaire, but, as they have already dispersed into other courses, it was only possible to get two completed questionnaires from this group. This was deemed not a representative sample and was thus not incorporated into the findings of this study. As is already stated, the class of 2002 supplied the answers to questions 1 and 5 that were used in the analysis and discussion of the profile of the students.

## 5.6 Limitations of the Data.

Taking the intellectual and procedural skills that were identified to be taught by the Pre-Calculus course as data, it is certainly true that these are not the only skills that can be taught in a course aimed at elevating the level of knowledge and of insight into mathematics. `However, the skills listed were the skills that could be identified in the course. This meant that the opportunities to acquire these skills were available to the students of the Pre-Calculus course. Subsequently an attempt could be made to gauge the degree of successful use of these opportunities.

The limitations of the questionnaire lie in the fact that the lists given to the students to select from in the questions might have omitted some matters that would consequently not be addressed by this study. However, the opportunity for comments at the end of the questionnaire was deemed to have given them the opportunity to address any matter not raised in the contents of the questionnaire.

## 5.7 Summary

This chapter introduced and explained the methodology used in this particular study. In the following chapter suggestions and recommendations regarding the course and further study will be made.

# Chapter 6

# Recommendations

## 6.1 Introduction

In chapter 2 it was established which attributes a mathematics course should ideally have if it is to be a substitute for higher grade matriculation mathematics and to prepare students for study at university. In chapter 3 it was shown that the Pre-Calculus course at UWC does indeed possess these attributes. The choice of topics for the Pre-Calculus course did not form part of this research and therefore recommendations regarding the selection of topics for the curriculum will not be given.

However, the responses to the questionnaire (Appendix A), which were discussed in chapter 4 raised matters that need to be addressed. In this chapter these matters will be analyzed and recommendations made on how they can be addressed. The matters that will receive attention are grouped under the following headings:

- Language
- Handouts
- The Lecturer
- The Students
- Classes
- Tutorials
- Content
- Other Media

This chapter will conclude with a list of recommendations.

## 6.2 Language

To lend importance to the matter of English as second language, it is appropriate to examine again the correlation between mathematical thinking and language skills. According to the article of Buchanan & Helman (1993), the NCTM (National Council of Teachers of Mathematics) established 5 general goals for mathematical literacy. On close inspection it becomes clear that these goals run parallel to the acquisition and use of a language, in this case English. These goals are that:

- the students learn to value Mathematics;
- the students become confident in their ability to do Mathematics;
- the students become mathematical problem solvers;
- the students learn to communicate mathematically;
- the students learn to reason mathematically.

These goals have a hierarchy of mathematical activity that runs parallel to the hierarchy of proficiency in English as language in which the mathematics is communicated.

The first goal poses no problem because the students in the Pre-Calculus course indicated in their responses to the questionnaire (Appendix A, question 6.2) that they are already aware of the value and importance of mathematics. 65.9% of the respondents to the questionnaire feels that mathematics affords them better career opportunities and again 65.9% know that they need mathematics to study their other subjects. Similarly they are aware of the fact that being able to communicate in English is advantageous to their academic career.

As for the second goal, students attaining this goal will be concentrating only on the calculations and procedures that would be asked as a straightforward question, like 'draw the graph of…', 'factorize fully' and 'solve the system of equations'. This means that to be an able student of mathematics on this level, a finite English vocabulary will be adequate, as long as this vocabulary covers most words used in straightforward questions that can be answered by applying algorithms.

Students attaining the third goal will be able to answer questions that ask for the interpretation of information and instructions. These questions will draw on the reasoning power of the student in English. Words like 'but, therefore, because, if, then' and the sentence and meaning constructions that go with their use will constitute the vocabulary of a student functioning mathematically on this level.

Being able to communicate a conclusion of a calculation or proof would constitute the reaching of the fourth goal. The same language reasoning skills as for the third goal will apply to the fourth. Here the mathematics will probably begin to weigh more than the language, as a student understanding this kind of question and being able to calculate and interpret the answer will at this time be proficient enough in English for the language to stop being a barrier to his/her learning. Language at this receptive/productive level is also addressed by Bohlmann (2001). She suggests "that students find it more difficult to recall and express correct mathematical terminology than to respond to it. In an everyday sense most of us can respond to a wider vocabulary than we can actually use." This correlates with the language ability of 'understanding, speaking or reading' when one has to declare one's proficiency in a foreign language. Understanding what one hears come long before being able to read and even longer before writing and speaking. It is disconcerting that Bohlmann (2001) says: "research has shown that ESL learners may acquire BICS (basic interpersonal communication skills) fairly quickly, but the acquisition of academic language skills takes an average 5 years." Accommodating the students' problems concerning language in class should thus ideally start in the Pre-Calculus year, as it will then seem to be of the utmost importance.

As for the fifth goal, reasoning mathematically, the influence of the language is difficult to assess, as this is a field of mathematical thought that many people find difficult to access and poor results in this field could as well be because of the mathematics as of the language. It is however especially in this area of reasoning that the students' inability to express themselves clearly counts against them. This stage of reasoning, that Piaget as quoted by Tall, (1991:8) calls the "if ...then" stage, is also called divergent thinking. This entails the "setting up of hypotheses, followed by the elimination of the less tenable of the hypotheses by convergent thinking" (Tall, 1991:8) These thoughts are the most difficult to express in a language in which one is not fluent, and Mathematics consists for a large part of this kind of reasoning.

Finally it then seems as if a student's proficiency in English has a great effect on which level of mathematical activity the student can achieve.

When the students were asked in the questionnaire about what can be done at university to help them overcome the problems caused by English as second language, they were given four options to choose from and they arranged them in the following order:

| | |
|---|---|
| 63.5% | Typed notes being handed out |
| 39.7% | List of reference books and these books on reserve |
| 34.9% | Lists of the terms used in each topic |
| 33.3% | Notes in simplified English |

Table 20.2 (see p. 89)

A third of the respondents approved of a list of reference books and having these books on reserve in the university library as a way of addressing the problems they have with English as a second language as well as the subject matter of mathematics. This correlates with the 36.6% of respondents who indicated in question 10.5 (Appendix A) that they used other books from the library apart from the textbook. These students are: 1 from group 1, 25 from group 2 and 4 from group 3. Looking at the percentages, 11% of group 1, 50% of group 2 and 20% of group 3. It is interesting to note that it is mostly the students in group 2 who actually go out of their way to access help for their studies. It also shows that this is indeed a means of making the study material accessible.

This option, namely a list of reference books placed on reserve in the library, has the added value that it will foster a scientific approach to study in general and to mathematics specifically, while addressing the language issue in a way that makes the student independent in his/her study with regard to English.

Buchanan & Helman (1993), writing about the teaching of ESL students, makes the statement that "(it) ... requires the teacher to attend to teaching English in the content area, which includes both the language specific to math and additional English skills." The author has observed that students appreciate the fact that:

- words are explained in class. It reduces the stress in accepting new words if the choice of these words is explained. For example: Commutative – most students know that someone traveling by train or other transport to and from work every day is called a commuter. If they can carry the meaning of this word over into Mathematics, it then makes sense to talk of the commutative law, understanding that it means to be able to perform the operation in both ways and achieving the same result.

- words that are used as terminology are separated by explanation from their ordinary English use. Examples of words having a different meaning in mathematics as compared with their meaning in ordinary language are: 'leading' 'entry' of a row in a matrix, 'period' of a graph in trigonometry, row 'operations' on matrices. Students should be made aware of these words that have a different function in Mathematics compared with their function in other subjects or in ordinary speech. A list of the terminology of the subject per topic could benefit the students tremendously.

Thus this matter of the interaction between mathematics and English is very important for the understanding of mathematics. Research in this field that will include the drawing up of learning material to address this matter will be to the benefit of the students involved and to the ultimate benefit of the department of mathematics. Such learning materials should ideally include texts concerning Mathematics and Logic to help students attain the different goals of acquisition of language and mathematics cited at the beginning of the paragraph.

## 6.3 Handouts

Of the options named above, *typed notes* were requested by 63.5% of the students. At this point it is noteworthy to look at the benefits Newble (1991:16) found handouts to have. Students who selected typed notes as the most valued help in their studies did not know what Newble (1991:16) found, but conveyed their conviction that typed notes would be a great help. Newble (1991:16) found that handouts have the effect that:

- students get higher test scores from lectures accompanied by handouts;
- handouts are appreciated by students;
- the design of the handouts influences the way students take notes in class; and
- handouts used during a lecture are most useful if they provide space for the student to write in.

It seems as if the students' convictions on what kind of help will be most helpful coincides with what Newble (1991:16) found.

Getting higher test scores as a result of having handouts during the course could not be measured as the students in the Pre-Calculus course only received a handout on a single topic, matrices. Their comments did however reflect that they appreciated handouts (par. 4.12) and that they see handouts assisting them in their problem with English (par. 4.6).

The other effect of handouts, namely that it assists students in their note taking in class, is very important for the Pre-Calculus students. As mentioned in paragraph 4.6, the author often had the opportunity to judge the students' competence in taking notes. It was clear that to most of them, mathematics consisted only of the numbers and symbols they could write from the board. Their notes contained very little explanation, if any at all. It is thus clear that they need assistance and direction in taking notes, and handouts could be of immense value in this respect.

Handouts providing space for the student to fill in while in class are used with great success in the first year mathematics course at UWC by Mr. Kannemeyer. They offer the students the ordered approach to mathematics and also structures the way they set out answers. Marking the tests of these first year students the author found a remarkably high standard of recording mathematics in an orderly way.

Another important aspect Newble (1991:16) addressed regarding handouts is the guidance it gives students. It can spell out the structure of the course in general and the lecture in particular and guide further study by providing more study material and/or references for additional reading or studying. Handouts thus have an important role to play in instruction in as much as they afford the lecturer the opportunity to place the emphasis where he/she wants it to be in the topic under discussion and also to delimit the work to be studied.

Mathematics is a subject about order. Creating order in the minds of the students can only be beneficial to their studies in mathematics and science. In this respect, handouts have a very important role to play and the lecturer should use this very valuable tool to the benefit of the students and the subject.

The handout can support individualized learning by supplying worksheets and diagnostic instruments and it can also, for instance, incorporate the use of computers. (Ellington, 1985:36). This is likewise a very important kind of handout. It was found that the Pre-Calculus students were often satisfied with only being able to do the problems set out in the textbook. They could thus not develop a broader scope on questions on the same topics and could thus not develop lateral thinking in any specific topic. This is a great shortcoming of a lecture-only, textbook-based course.

Handouts can enhance the textbook in setting up lists of the terminology used. It will help the student identify the words that are used as terminology compared with the words used as part of the explanation. This will also address the language problems with understanding the terminology and getting used to new words (par. 4.6).

Handouts should be in simplified English. By definition notes are explanatory, and explaining something in a language style that cannot be understood is counterproductive. It thus follows that notes should be written in simplified English to bridge the gap between the information gained in the lecture and the information in the textbook. This will also address the problem students have with not understanding the textbook (par. 4.6).

It must be acknowledged that notes can also have a negative effect. They may lead the students to think that acquiring a textbook is not necessary. This should not be the case, as typed notes can never replace the richness of explanation and enhancing information that a textbook offers. Nevertheless, the pros vastly outweigh the cons in the matter of using handouts.

## 6.4 The Lecturer

The Pre-Calculus students mostly come from the group of students generally known as 'first generation' students. This means that they are the first members of their family to go to university. In the current context of South Africa, they might even have been the first members of their family who matriculated. They thus have no frame of reference as to how a university functions, what its standards are, its code of conduct, etc. It is part of the first year lecturer's responsibility to 'initiate' them into the world of the university.

The fact that they are first generation students is corroborated by the following quotations from the comments students made in answer to the questionnaire (Appendix G).

- *In 011(Pre-Calculus) I was lost because of the word university.*

- *(The university) Did everything; but more advice has to be given to students in terms of subject choice.*
- *We are fresh students from high school; the university should have a class where it explains everything to 1st year students because some things we don't know like the importance of attending a tut and lectures.*
- *The campus must work together to student that have the poor background like me.*
- *The freedom to come or not to come to class.*

Students would greatly benefit by a well-defined Code of Conduct for the Pre-Calculus course. In this document the expectations of the lecturer regarding the students' behavior in class, attendance, conduct in consultation, etc. could be spelled out. In this way they will more quickly become accustomed to the ways of the university.

The responses the students gave about living off campus (Appendices B1, B2, B3) also indicate that many of them live among people who have little understanding of what they need and experience. Also in this respect, Marshall, speaking about adult perceptions and attitudes to mathematics at the Australian Indigenous Education Conference (2001), found that students "have no real understanding of university culture and its demands" and that "university bridging programs must not only address gaps in students' mathematics education, but also provide strategies for student support and empowerment." All these factors place a large responsibility on the shoulders of any lecturer teaching this group of students and therefore also on the shoulders of the lecturer of the Pre-Calculus course.

At the Unify Indaba held during June 2002 at the University of the North, much of the success of the Unify program (University of the North Foundation Year program) was attributed to the support the students get from faculty. Faculty are responsible to educate the total student, academically, socially, culturally, emotionally, etc. In their circumstances it might be easier for faculty to be involved with the students than at UWC as almost all their students live on campus and the faculty in the town of Polokwane nearby. At UWC the students in the 'foundation year', therefore the students of the Pre-Calculus class, mostly live off campus and the faculty in the greater Cape Town. However, this does not mean that the students at UWC do not need the leadership of their faculty.

In this regard it is recommended that there should be a person on campus who will be available to the Pre-Calculus students on a full time basis. As the course is assigned two lecturers, at least one of them, or an appropriate substitute, should be available at any given time. Students often need encouragement more than the actual explanation they come to ask for, and if this is the reason they are looking for a lecturer, it is an opportunity to support the student in ways other than only with a mathematical problem and, in doing so, to strengthen the students' studying composure.

*Daily lectures* were requested by students to help them not lose interest. Unfortunately this is not possible because of timetable constraints. However, the availability of a lecturer as proposed above would help to combat this phenomenon, of losing interest, on days not offering mathematics lectures.

The lecturer is also responsible for the planning of the course. Students complained that inadequate planning demotivated them:

- *There is no course overview (were not given); test dates were a hassle to fit into schedule;* (Appendix D2)
- *It is very unfair to attend a lecture class on Wednesday morning and write a test afternoon.* (Appendix G)

A handout at the beginning of each term with the fixed dates for tests and the delimitation of the work to be prepared for each tutorial will significantly help the students to plan their own study programs to accommodate all subjects for all-round success.

## 6.5 The Students

Students in the Pre-Calculus course have all the problems first year students have.

Peer pressure plays an important role. Coming from school where the teacher is responsible for your success, to university where you are the responsible person, is a big step for many students and one many students cannot take. They still seem to be caught up in the social group and to adhere to their rules.

In this regard it might be feasible to extend the peer facilitator program that runs in the orientation week at the beginning of the academic year, into the rest of the year. This will give first year students a peer to consult on matters pertaining to all aspects of his/her life, especially matters that they would rather not discuss with a lecturer.

Referring to paragraph 4.1, where the Pre-Calculus students are divided into groups according to their results for the course, it is clear to see that a more stringent selection of students will be appropriate. At the same time many students were placed in the first year mainstream mathematics course that should have been advised to do the Pre-Calculus course. Students do write a placement test for the faculty and on these grounds they are assigned to one of the two courses. It seems however, as if the mathematics department would be wise to run its own test, testing the skills it requires and recommending a course to each individual student on grounds of these results.

The Australian National University (1997) runs bridging courses in the summer vacation for students planning to enroll in undergraduate programs at a tertiary institution and therefore also at ANU. These courses are offered on two levels: a general background for those who only need a strengthening of their mathematical background and a higher level course required for entry to some undergraduate courses. Criticism against a placement test is that it is written at the beginning of the year when students – especially those who speak English as a second language – are coming to the test 'cold' and are thus penalized for not understanding the

English. Summer vacation classes can remedy this as it can accustom the students to English before they write the placement test. In this way, the placement test would more accurately discriminate between those prospective students who would benefit by being in the Pre-Calculus class and those who could go straight into the mainstream first year mathematics.

Another way of making the placement test more reliable is to set up a list of topics on which students will be tested at arrival at the university. Students can then prepare for the test, the test's reliability will be greatly increased and students will be better placed according to their level of mathematical functioning. Better placing of students will also lead to better learning as the class groups will be more homogeneous and therefore easier to teach.

Students who do not attain a matriculation exemption after writing their matriculation examination because of the incorrect choice of subjects need more vocational guidance in the lower standards, grades 8 and 9. The mathematics department would be well advised to do this vocational guidance themselves and not to leave it to people who do not have anything to gain by doing it well. Especially in the proposed new school setup where learners will be streamed into academic or non-academic schools at the end of grade 9, the university should state its case to learners before they are committed to a non-academic future. Enthusiastic, well-motivated, Pre-Calculus students who are high achievers can be approached to help in this recruitment drive.

## 6.6 Classes

When addressing the problem ESL students have in coping with the language issue, Buchanan & Helman (1993) states: "math students benefit from a variety of instructional settings in the classroom…individual, small group and whole group activities."

The variety of English and Mathematics proficiency in the Pre-Calculus class makes it very difficult for this matter to be addressed in a large class of 100+

students. Many students are also shy to speak in front of so many strangers, as is to be seen from their comments on the questionnaire (Appendix A, question 6.2). As is already said, smaller groups will afford better opportunities to speak and hear and write mathematics in a less threatening environment. Unfortunately even the tutorials had 50+ students, and could thus not be reckoned as a small group. Smaller groups were formed when the optional extra classes were introduced. Unfortunately, not everyone attended these classes and the benefits were thus very limited. Another way to address this matter is for tutorial time to be extended from 1.5 hours to 3 hours, as this process of learning a language and a subject at the same time is time consuming, as is already stated.

Adapting to university posed problems in that some students were in small Higher Grade classes at school and *found the large prelim class (100+) very overwhelming*. They also said: *At school in our maths class we were only seven, I'm not used to the big class as I didn't ask questions during lectures.* As English is their second language but also the language of instruction, they were obviously initially reluctant to ask questions. This surely had a negative impact on their progress, although the majority did overcome it and were successful.

Students also criticized the course, saying that there was *not enough attention to all students*. This is unfounded criticism as both lecturers were readily available as is apparent from the many other comments in the questionnaire.

Groupwork as spelled out in the Calculus Students Guide of the University of Michigan (2002) can change the students' view that because of "the extreme emphasis on solo problem-solving that often is part of the traditional approach … it is somehow against the "rules of mathematics" to sit down with others and brainstorm". Working in groups also ties in with the current emphasis on collaborative learning in South African schools. It is also a studying method used by other departments at U.W.C., for example in the departments of Management and Psychology.

Buchanan & Helman (1993), in their statement: "Small-group work allows students to use language to talk about the math tasks at hand while they work to solve non routine problems", also address the problems students have with getting used to English as studying medium. They also advise that students should be encouraged to talk to one another as they work in pairs on math problems, that math language must be used in order for it to be built up and that the use of English must be encouraged, but that the use of students' first language should not be discouraged.

It is thus clear that there are many options for enhancing students' learning environment and these options should be explored to the students' benefit.

## 6.7 Tutorials

A student commented on the Pre-Calculus course in the following way (Appendix G):

*Class first it was big but I think if it could better easy for us to have 2 tutorial classes a week, meaning the one which was on Wednesday and the other one to be on Friday afternoon because so that when students are going to the weekend they will know all the work which they did during the week. I think that will help a lot of students like me who are not fast on understanding problems.*

Although these comments are very valuable, breaking up the tutorial into shorter time spans is not a solution. Two reasons support this statement:

- Firstly, students must eventually acquire the mental fitness (like endurance) to sit through the exam and it was interesting to notice that the weaker students were always the first to leave the test or exam room. Commonly this is ascribed to a lack of knowledge to answer the question paper substantially but, in the light of Krutetski's (1976:310) findings, it might also just be a case of mental unfitness to keep at a mathematical task for so long.
- Secondly, the weak students especially need time spent on a single concept before it crystallizes out for them. One student, ms. Firfirey, who struggled all year and often came for consultations, still always had a unhopeful attitude towards

Mathematics. However, she reported on her preparation for the supplementary exam that she had studied right through the night and that only then, at 3h00 in the morning, did she suddenly realize 'how matrices work'. This realization could have come to her much earlier in the year, had she been prepared to spend the energy needed persevering until the moment of discovery in her Mathematics studies. This is one example of why many of the students failed to be successful, a lack of appropriate perseverance.

Suggestions about tutorials are that there should be more tutors: *I think you should add more tutors – I know you had tutors, but I don't know what happened to them.* Having more tutors imply that tutorial groups can be smaller. A student from the 2000 class who studied mathematics 1 in 2001 commented on the advantages of being in a small tutor group such as she experienced in 2001 but not in the Pre-Calculus course. She appreciated the fact that she could have her questions heard and answered in a 'safe' environment. A shortage of suitable tutors is a problem experienced at many universities as first year mathematics students often choose other study directions in their senior years and are then not available for tutoring mathematics in their senior years. The McMaster University (2002) run an intensive training course for tutors and have reaped the benefits of this action. The author recommends that an officer or lecturer be made responsible for the tutorials of the whole group. This can free the lecturer from the administrative duties regarding tutorials and enlist one more person to assist the students.

## 6.8 Content

Powell (1993b) states that one of the principles that guides his thinking about the mathematics 114 bridging course at UWC at the time is that "less is more". By this Powell (1993b) means "that it is better to delve into depth and consolidate understanding of a few topics than to cover massive amounts of material superficially". He suggests that students should instead be encouraged and helped "to make connections with and between topics" and be guided "to develop mathematical insight". As can be seen in Table 6 the Pre-Calculus curriculum offers many opportunities for these processes to happen. The onus rests on the lecturer to make the students aware of the connections and guide them to see similar connections between subsequent topics for themselves. Students in their first year are still in the dualistic phase of their development (Perry, 1968: chart of development) and are as such dependent on the lecturer to help them grow academically. Not assuming that they can make all connections by themselves will help them, not only in the Pre-Calculus course, but also in their subsequent studies.

Powell (1993b) also promotes teaching the heuristics of learning mathematics. Students in the Pre-Calculus class come from bad experiences with mathematics and helping them to form a successful study approach will be to their long-term benefit.

As seen in chapter 3, mathematics topics carry not only the knowledge of the mathematics in the topic, but also the potential for teaching skills and revising other aspects of mathematics. Such an example is the topic of complex numbers as taught in the Pre-Calculus course. The studying of complex numbers revises basic algebra (like terms, simplifying algebraic expressions, removing brackets), surds, radians, trigonometry (special angles, trigonometric ratios, quadrants, periodicity) and all this is done while the student is acquiring new knowledge. Topics should thus not only be selected for their intrinsic knowledge, but also for their possibilities of teaching beyond the knowledge.



Students' understanding of the content is tested in three ways: tutorial tests, term tests and examinations at the end of the course. Students complained that *questions asked in the tests are far more difficult than those done in class* and listed this as a demotivational factor. The lecturer should address this matter, ensuring either that class examples are in the same vein as test questions, or that exercises given as homework, are in line with ultimate test questions. Handouts and worksheets can help in defining the difference between questions done for the acquiring of knowledge and questions testing the use of the knowledge. In this way, the students will eventually be able to discriminate between the two kinds for themselves.

The fact is that some of these students will not achieve exemption because they are not intellectually capable of university studies. Usiskin (2000) identifies certain levels of mathematical activity that can be attained by students. He identifies level 2 as the level that about 15% of students in the U.S. can reach. Level 2 is roughly equal to a good pass on standard grade in grade 12 in South Africa. Level 3 – the level of "the terrific student" – is only reached by the top 1-2% of all students. It is thus understandable that not all students enrolled for the Pre-Calculus course will pass with a high percentage. If they then complain about the *difficulty* of a course, the reason might be intellectual inability on their part. On the other hand, one student from Group 1 complained of being *spoonfed* and that his *responsibility and maturity was never tested.* This variation in expectations of the students makes the lecturers' handling of the course an affair that needs careful planning and structured execution. It will however pay its dividends in more students' needs being met and thus more productive learning achieved.

The content elicited two different responses. They found the content *difficult to cope* with which made them *lose faith* in their abilities and interest in the subject. This again relates to the need for safety in Maslow's hierarchy of needs (Beck, 1983:380). The fact that they experience the content as too difficult impacts negatively on their emotional readiness to work at it (par. 4.13). This causes failure that in its turn causes them to lose faith in their ability to do the mathematics. This is a vicious circle of events and one that the lecturer should try to prevent by calling the students concerned for consultation, before the situation gets out of hand.

Students want to know: *Why can't you start with the difficult ones first and finish with the easy ones?* (Appendix G). In determining the hierarchy of presenting the topics the level of difficulty of each topic should receive intensive attention to at least try and alternate 'difficult' and 'easy' topics, keeping up the motivation for study in the students.

Likewise new and revision-of-school topics should be alternated. Students stated that they *find it (Pre-Calculus) easy because some of the work is not new.* This balance in content is a strong motivational outcome of good planning.

## 6.9 Other media

Regarding the content, the students singled out some topics as more difficult than others. These topics, *polynomials, absolute values, piece wise functions, determinants and matrices*, are in fact all the new topics taught, those they did not study at school. These topics also have a high degree of abstraction at this time of their studies and that could be the reason they experienced them as difficult. In this respect the Maple Program in the computer laboratory is a powerful tool to help ease the students into working with abstract concepts. An investigation of the effect of constants on the transformation of a graph can easily be done in a Maple worksheet. This will be of double benefit to the students, making them computer-able and independent learners.

Graphs of all kinds can also be drawn and thus also be checked with the Maple program. As graphs are the pivotal concept in the Pre-Calculus curriculum, it becomes imperative that the Maple program should be used for teaching and learning.

Rader-Konofalski (2000) quotes a mathematics student: "Thank … for the math lab! It saved me several times and I've overheard other students saying the same." The Pre-Calculus students should have the same opportunities to be saved. The Mathematics Department at U.W.C. has a well-equipped computer laboratory and it should be used to the maximum to help all students.

Powell (1993b) describes mathematics as the science of patterns and relationships. Computer-assisted learning is the ideal way to help students discover mathematical patterns. Instead of laboriously having to make the calculations to establish how a graph reacts to certain changes in its equation and drawing the graphs with a much too long time span in between examples to discover the pattern, many graphs can effortlessly be drawn with the Maple program and the pattern easily discovered. The quick follow up of the results that leads to the establishing of the pattern will also lead to the calculations having more meaning eventually.

However, for this component of the teaching program of the Pre-Calculus course, ideally someone other than the lecturer should be responsible. It takes a great amount of organization to ensure that all students have equal opportunities in the laboratory and that the material they are given is well developed and adequate. Assigning these activities to the lecturer will encroach on the availability of the lecturer to the students and will impact negatively on the lecturer's time for preparation and setting and marking of tests and examinations. This component should also carry its own weight in marks contributing to the Pre-Calculus course mark.

Homework is another instructional opportunity. Students ask that *exercises should not be more than ten because of time*. In another comment, they ask that *homework be marked by the lecturer*. Homework should be carefully planned because this is a very powerful way of setting the standard of the work. Homework can have different functions:

- Acquainting the students with the topic being studied at the time. This kind of homework assignment will not yet include test-like questions, but will include references to other textbooks for the benefit of the students who do not understand their own textbook or just need to read it in a different way. This is what students asked for in the questionnaire when 36% of them required textbooks on reserve in the library. This kind of homework assignment can also help to bridge the gap between the English and the mathematics.

- Acquainting students with questions of test standard. These questions can be set by the lecturer from the prescribed textbook and/or other available textbooks. The latter can help to widen the student's perspective on the topic and deepen his/her understanding of the topic.

That *homework should be marked by the lecturer*, as requested by the students, is not feasible. Two reasons exist for this statement. Firstly it does not make students independent of an outside opinion and thus does not lead them out of the dualistic phase (Perry, 1968: chart of development) of their development. Secondly it takes up too much of the lecturer's time. These are not reasons to prevent the giving of homework and although it is a fact that many students only work on assignments if the assignments 'count' for marks, it is also true that many students do work hard and would appreciate homework assignments even if they have to mark them themselves.

Worksheets can also enhance the learning of mathematics.

- They can serve as a teaching tool – especially in the case of background knowledge that has to be revised before a certain new topic is started.
- They can serve as an investigation – establishing patterns or relationships or practicing algorithms.
- They can also be used as a summary and revision exercise at the end of a topic.



The author does not support the practice that marks are awarded for worksheets and would rather see that, after the worksheet is done and marked by the students themselves, a similar worksheet be worked out by each student under test conditions. This is indeed the practice at some other universities in South Africa as was related by their faculty at the Unify Indaba (June 2002, University of the North).

## 6.10 Topics for further study

The Pre-Calculus group is a very important group at the university. The greater the percentage of students that pass, the greater the potential number of students who will continue their studies at UWC. It is therefore in the interest of the department and of the university to assure the optimum number of passes each year. Of the 109 students in the Pre-Calculus class of 2000, 62 students went on to study in the faculty of science, 17 repeated Pre-Calculus, 2 went on to study in other faculties and 28 did not return to university. Offering the foundation program of which the Pre-Calculus course forms a part thus delivered 81 students to the university in the year 2001. This indicates that the foundation program fulfills an important recruiting function and it also emphasizes the importance of assuring success to as many Pre-Calculus students as possible. To this end the author would like to recommend that regular research projects be run on this group. This will assure the intensive involvement of one or more faculty of the department of mathematics with this group and it will enhance the department's research portfolio. Such research can be done in cycles of two or more years depending on the kind of program researched. Topics for such research could be:

- Group work as done at Michigan University (2001);
- Training teaching assistants to be tutors (McMaster University, 20020);
- Concept mapping as done by Baralos (2002);
- Gateway testing as done at Michigan University (2002);
- Mastery learning (Davis, D. & Sorrell, J., 1995);
- Different ways of testing, achievement vs. diagnostic (Ausubel, 1968:213);
- English and Mathematics interaction, as well as any other current research on teaching or learning.

In this way the department can also collaborate with other universities doing the same research or lead them in research that the department finds appropriate.

## 6.11 Conclusions

Analyzing the responses to the questionnaire (Appendix A), it became clear that there are many issues that can and should be addressed to enhance the success of the students in the Pre-Calculus course. In this chapter the following recommendations on ways to deal with these matters were made:

- The language issue should receive extensive attention and ways to address the matter should be developed within the department of mathematics to assure an integrated approach.
- Handouts are a necessary aid for the Pre-Calculus students. Handouts can address many of the problems the students have with the organization of the course, the delimitation of work to be studied, the language and note taking in class. Handouts also support individualized learning.
- The lecturer should plan to address the many problems the students have regarding their studies. The lecturer should accept responsibility for the adaptation of the students to tertiary study in all its facets. Availability of the lecturer and his/her empathy with students would greatly benefit the students.
- The students could benefit by an extension of the peer facilitator program. A placement test can facilitate better selection for more effective teaching and learning. The reliability of the placement test can be improved by summer vacation classes or by informing students beforehand what they will be tested on. Recruitment of students should start as early as eighth and ninth grade at school. Successful Pre-Calculus students can assist with the recruitment.
- The negative effects of large class groups can be addressed by group work and smaller tutorial groups.
- Tutorial groups can be smaller and tutors can be better trained.
- The topics of the Pre-Calculus course should be presented in an alternating order of new and old, and easy and difficult. Students must be aware of the different standards of questions: homework, test, or examination. This will help keep up the motivation of the students.
- Other media, especially the computer laboratory and worksheets for exploration and practice, must be fully utilized to ensure maximum learning efficiency.

# Appendix A

Questionnaire to be put to the Prelim Maths students of the years 2000 and 2001 to determine how this course influenced them.

General Information:

| | | |
|---|---|---|
| Surname: | Student Number: | |
| Name: | Age: | Male/Female |
| Where do you come from? Where did you live before you came to UWC? | | |
| Qualification in Maths: | Year of studying Prelim Maths: 2000 2001 | |
| Home language: Eng Afr Xhosa …………… | | |

1. Do you live on campus? Yes / No

1.1 If NO: How do you travel to the university?

| | | | |
|---|---|---|---|
| By train | By taxi | By bus | By car |

1.2 How much time per day do you spend travelling? (Include time waiting for a train/taxi/bus.)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 hour | 1½hours | 2 hours | 2½hours | 3 hours | 3½hours | 4 hours | More |

1.3 Where do you live off campus? Suburb:……………………..

1.4 Kind of accommodation:

| | |
|---|---|
| | At home with parents |
| | Living with family |
| | Lodger in someone's house |
| | Rented room |
| | Commune with other young people |

1.5 How does this affect your studies?

| | |
|---|---|
| | It does not affect my studies at all |
| | I am tired when I get home |
| | I have house chores to do when I get home |
| | I don't feel comfortable with the people in the house |
| | There isn't silence |
| | Time spent in lectures and tutorials leave little time for studying on campus during the day |
| | I am often late for class in the morning |
| | There is nowhere I can sit down and study |
| | There is no library nearby |
| | There is no one at home to help me in my studies |

2. Is your home language English? Yes / No

2.1 If NO: What effect does this have on your studies in Mathematics?

| | |
|---|---|
| | It does not affect me at all |
| | I have difficulty getting used to new words |
| | I find the explanations difficult to comprehend |
| | I find it difficult to follow fast speech |
| | I have difficulty understanding the language of the textbook |
| | It takes more time to study |
| | I find it difficult to explain what I mean in English |
| | I often find words that nobody knows the meaning of |

| | |
|---|---|
| | I have trouble understanding the terminology of the subject |

2.2 What can be done at university to help you overcome these problems?

| | |
|---|---|
| | List of reference books and these books on reserve |
| | Typed notes being handed out |
| | Lists of the terms used in each topic |
| | Notes in simplified English |

3. Do you have Matric Exemption? Yes/No

3.1 If NO: Which of the following is the reason?

| | |
|---|---|
| | Not enough subjects on Higher Grade |
| | Not the right subjects on Higher Grade |
| | Marks in Higher Grade subjects not good enough |

4. Did you do/ Are you doing Maths 1 in 2001? Yes/No

4.1 If YES: Which course? Mat 111-141 Mat 115-125

4.2 Why did you decide to do Maths as a subject at university?
(You may choose more than one option.)

| | |
|---|---|
| | It affords better career opportunities. |
| | I need it for my other subjects. |
| | I am interested in studying Maths as a subject. |

5. In which degree did the following have an influence on your Maths at school?
1 = not at all 2 = a little 3 = quite a lot 4 = very much
(Responses are given in percentage of the total number of questionnaires received.)

| | | | | | |
|---|---|---|---|---|---|
| 1 | Teachers were often absent | 1 | 2 | 3 | 4 |
| 2 | Often having a new teacher | 1 | 2 | 3 | 4 |
| 3 | Not having a textbook | 1 | 2 | 3 | 4 |
| 4 | Teacher could not understand my questions | 1 | 2 | 3 | 4 |
| 5 | Teacher could not explain the work | 1 | 2 | 3 | 4 |
| 6 | Some topics were never taught | 1 | 2 | 3 | 4 |
| 7 | Classes were very large | 1 | 2 | 3 | 4 |
| 8 | Classes were noisy making it difficult to work | 1 | 2 | 3 | 4 |
| 9 | Teaching was only exam-directed | 1 | 2 | 3 | 4 |
| 10 | Teacher did not give personal attention | 1 | 2 | 3 | 4 |
| 11 | No 'tutor' to ask if you have a problem | 1 | 2 | 3 | 4 |

6.1 Name 3 ways in which the prelim Maths course motivates you.

| |
|---|
| |
| |
| |

6.2 Name 3 ways in which the prelim Maths course does not motivate you.

| |
|---|
| |
| |
| |

7.1 Do your study habits for Maths at university differ from your study habits for Maths at school? Yes / No

7.2 If YES, in which respects?

| | |
|---|---|
| | Work with fellow students |
| | I spend more time reading the study material |
| | Spend more time on Maths problems |
| Other, specify: | |

7.2 Why did your study habits change?

| | |
|---|---|
| | I have less time, so have to study differently |
| | I have problems understanding the language |
| | I have to read more to understand |

7.3 Do you study more regularly at university than what you did at school? Yes / No

7.4 If YES, what are the reason(s)?

| | |
|---|---|
| | There is more work to study |
| | There is no revision in class |
| | The work is much harder to understand |

Please rate the following on a 5 point scale for their applicability to you:

1= not at all; 2= only a little; 3= quite a lot; 4= very much

| | | | | | |
|---|---|---|---|---|---|
| 1 | I enjoyed Maths at school | 1 | 2 | 3 | 4 |
| 2 | I enjoy Maths at university | 1 | 2 | 3 | 4 |
| 3 | Mat 011-041 helps me in my 1st year studies | 1 | 2 | 3 | 4 |
| 4 | I am interested in Maths | 1 | 2 | 3 | 4 |
| 5 | I regularly attend class | 1 | 2 | 3 | 4 |
| 6 | I do my homework regularly | 1 | 2 | 3 | 4 |
| 7 | I study with friends | 1 | 2 | 3 | 4 |
| 8 | Passing Mat 011-041 has changed my self-image for the better | 1 | 2 | 3 | 4 |
| 9 | I like reworking examples to get them right | 1 | 2 | 3 | 4 |
| 10 | I like having an answer to work towards | 1 | 2 | 3 | 4 |
| 11 | I like studying new topics rather than revise previously learnt work | 1 | 2 | 3 | 4 |

Do you see yourself as successful at Mat 011-041? Yes / No

Reasons for this:

9.1 What did you do/not do?

| |
|---|
| |
| |

9.2 What did the university do/not do?

| |
|---|
| |
| |

Do you have a textbook? Yes / No

10.1 Does it help you in your studying? Yes / No

10.2 How do you use it?

| | |
|---|---|
| | Read ahead |
| | Reread class work |
| | Do the exercises |
| | I read the interesting histories of the mathematicians |
| | I find the pictures interesting |
| | I read the interesting facts about the topics I study |
| | I try doing some of the other topics |

10.3.1 Are your class notes (handouts) sufficient? Yes/No

10.3.2 Do you see your class notes as a study resource? Yes/No

10.4.1 Are the notes you write from the board in class sufficient? Yes/No

10.4.2 Do you see these notes as a resource? Yes/No

10.5 Do you ever use books from the library (apart from the textbook that is on reserve)? Yes/No

11.1 What about Maths makes you happy? (+/- 20 words)

| |
|---|
| |
| |
| |
| |

11.2 When taking a Maths test, which of the following happen to you? You feel: (Please underline)

| | | | | |
|---|---|---|---|---|
| Cold | Sweaty | Short of breath | Confident | Challenged |
| Strong | Capable | Energetic | Clever | In control |

11.3 When you fail / don't achieve satisfactory results for a Maths test, which of the following apply to you? (Please underline) I feel:

| | | | | |
|---|---|---|---|---|
| Hurt | Scared | Dumb | Embarrassed | Humiliated |
| Self-doubt | Defeated | Alone | Helpless | Confused |
| Discouraged | Anxious | Motivated | Angry | Victimized |

12. You were instructed in the following topics:

a. Polynomial Functions
b. Rational functions
c. Absolute value
d. Solutions of systems of linear equations (Matrices and determinants)
e. Trigonometry
f. Transformation of graphs
g. Exponential and logarithmic functions
h. Partial fractions (only in 2000)
i. Word problems (only in 2000)
j. Complex numbers (only in 2000)

12.1 Please arrange these topics in order of difficulty for you, starting with the easiest one:

12.2 Please arrange these topics in order of interest to you, starting with the most interesting one:

123 Which of these topics did you find useful in your first year of study?

13. Any additional comments on the Mat 011-041course?



14. How did you get to know about this prelim course offered at UWC?

Thank you very much for filling out this questionnaire.

# Appendix B1

Class of 2002: Students' Comments on living Off Campus.

How do the people you live with affect you?

What do they do that interferes with your studies?

They affect me negatively:

- They make lots of noise.
- I am staying with my family, brothers and sister. Every time they disappoint me to all things that I am doing. They are jelous of me because of my learning. When I take my books they make noise. Sometimes they told me a bad thing.
- They affect me badly. My mother is in Eastern Cape and now I am staying with my Aunt. She sometimes has discrimination towards me. Her children are so old by not any child of hers has passed standard 10. So she does not like that so she chooses to discriminate. I only study for sometimes 2 and 3 hours because she wants me to work every time.
- I am working too much. Such that they don't give me time for looking at my books.
- I first prepare for supper and clean after supper and prepare for tomorrow before study and sleep by 11h00.
- They take in alcohol and loud music.
- I live in the flat, we are 4, so I don't much pay much attention with my book. Because the only time I can concentrate is when I am alone in small room which is I don't have that so I'm struggling.
- They don't seem to understand that University demands a lot of money as a result they come in terms of not giving me the money to buy things which I will need here.
- My sister makes sometimes a noise.
- We live in a sheck which is a 4 room sheck, in most cases they all come and watch TV so sometimes its hard to study.

- I have to cook every day even if I have a big examination coming and do other hours chores as well. The loud music that they listen to in my neighbourhood is disturbing me a lot because when you complain they say you must go live in the suburbs.
- They make a lot of noise making it hard to concentrate.
- I have 2 small children at a home ( a brother and sister) who are both very active and loud. It's not easy to study with them around.
- I have a toddler niece and a 1 month old niece living with me. Very noisy conditions.
- They disturb me, because they are not studying, they are working. They make a lot of noise.
- Yes they interfer me a lot because I stay at new flats which are the hostels; there are a lot of people, you don't get a chance to study because its always a noise and at night they say the light switch should be off at 9pm, so you can't study after 9pm.
- They make a lot of noise, they don't want to learn, they tell me I must go and study at a library. I study sometimes in my room when the library is closed, because at the study hall people make noise we can't concentrate in the study hall.
- They affect me so much living next the juke box music, they don't care about education at all.
- No affection, its only one of the tickets to come to school, because my mother is a unworker although I'm stay with him.
- I have to cook everyday, even though sometimes I come late from school.
- Sometimes its good to stay with people and sometimes its not. We are from different backgrounds. Room mates put their interest first. Sometimes they don't care if they are disturbing me or not. I think it can be better if I can stay alone, so that I can have enough time and space to study my books.
- Sometimes there is a noise made from my small niece and nephew.
- Noise. They make noise.

- House chores.
- Yes, noise.
- Noise sometimes.
- Sister: sleep in same room. Brother: at home (make a noise). Mother: hairdressing.
- Put on loud music.
- Ask me to do chores.
- Bother me, give me unnecessary work.
- Sometimes I do get a problem with the noise level. But that's the only problem that occurs from time to time.
- They are very boring and sometimes they are okay the next they are bad.
- Make noise, bang the door, and very interrupting. I can't study in my room when my roommate is around, I can't study in the study hall.
- They disturb me, by shouting when talking. Turning the TV volume very loud as well as the radio volume. Asking me to go to places they want me to go to.

They affect me positively:

- We study together therefore they don't really interfere with what I'm doing during the day.
- Very understanding, encouraging, silent during study times.
- There's no interference.
- In a positive way. No one interferes because its only me and my father during the week. My mother comes home on the weekends from work.
- No, sometimes they smothered me that's all!
- They are very considerate and respect my study time.
- They affect me positively.
- They don't affect me and I do things in my own time, no one is pushing me when I'm studying.

- They do not interfer.
- They don't do nothing cause I am living with my sister of which she is also a student in Pentech so there is no interference.
- No people affect me.
- I am renting a single room in a commune so I do not have any interference from the other house mates.
- They don't affect me in any way because we're students and we all study.
- Not at all affecting me.
- They do nothing, but help to keep quiet.
- They don't affect me at all.
- They encourage me to do my best and learn. Give me nothing to do that affects my studies.
- My people are understanding, no interference when studying.
- They are okay because they do not interfere with my studies.
- They don't interfere.
- They don't affect me.
- No, they don't interfere with my studies.

Neutral comments:

- Sometimes they make me angry, when they stay up late playing games (TV games). Otherwise they don't bother me.
- Nothing.
- There is not much of an effect.

## Appendix B2

Class of 2002: Students' Comments on living Off Campus:

Would you rather live ON campus?

Please give reasons for your answer.

Yes:

- Yes! Because is better than outside, because you get too much time to study, and is less expensive.
- Yes, because of my problem.
- Yes, because here I can study my book every day and everytime and pass mathematic as I used to.
- Yes, because I would have had enough time for my books.
- Yes, only if I could get a bursary that would be able to pay for my studies, because without that I rather stay home.
- Yes, I would have saved money because I travel to campus and the library for study purposes would also be very useful.
- YES, YES, YES, I could pay more attention to my studies.
- Yes, I can get lot of chance to study.
- Yes, you have the private of your room to sit in silence to study and are near to the library.
- Yes! To be more close to the library, for more information and to be able to read with other discussing the problems we are having in that particular subject.
- Yes, because I'm traveling by a train, it makes me so tired because to get home I have to walk home.
- Yes for traveling when we wrote the test late and it is convenient.
- Yes! As I have said that I can't study well at home and its much easier on campus because I have friends that are in my class so we could study together as a study group.
- Yes please I would really like to live on campus for the sake of studying in a quiet place and studying without doing house chores first.
- Yes.

- Yes to be near computers. Library.
- Yes, its nearby and I can get more work done here that at home. I come to late at home.
- Yes, because it isn't nice to live with other people.
- Yes, availability of library and study hall. Peer tutas and easy access to my lecturer.
- Yes, because its to dangerous to travel by train.
- Yes.1. To be close to school. 2. so that I can study anytime I want to study even at night. 3. to be close to library.
- Yes, because most of the equipment like computers is here at school.
- Yes, because I am staying too far. There is no place to stay.
- Yes, because it'll help me to have more time with my book, because I sometimes have to wait for a bus that will come late, hence I don't get enough time to study due to the fact that I have to cook.
- Yes, if I can, there'll be more time for me to study than going up and down with a bus. I can sometimes go to the library and the study hall.
- Yes, peace.
- Yes, more freedom and I don't have to travel.
- Yes, too much people to discuss with (group work). Library.
- Yes, because then I don't need to travel and it will help with the finance problems. I'll get early to class and have more time to study.
- Yes, because there are may quiet places to study and the people to discuss with when you having problems, with studies.
- Yes, less walking.
- Yes, because traveling from campus to home takes about 2 hours of my time.
- Yes, then at least I wouldn't have to think about the train and new long it'll take me to home. And I could get a lot of time to study.
- Yes, where I would have much time to learn. It is where I can make a perfect timetable for what to do during each day without being disturbed.
- Yes, of course because it is very difficult for me to do some work during

the day and I may get hungry sometimes, that would me to leave the classes.

No:

- No, because I feel more comfortable at home.
- Not actually.
- No; home cooked food (I'll miss that).
- No, don't wanna be confined to one spot, like to travel.
- No, I like being in a comfortable environment where I know I wouldn't be interfered with.
- No. 1. safety. 2. comfort. 3. non-shalant attitude in res.
- No, because of not having more money.
- No, I would be reckless. Do my own thing. Parties!!! It would be boring, I don't have transport of my own.
- No.
- No, I'll miss my family too much.
- No – I like where I stay.
- No I prefer to study in my room which I don't have to share unlike in campus where rooms have to be shared.
- No.
- No , because they said there is no space.
- No, I am much better at home because I don't mind traveling.
- No, because I love my home.
- No, cause I'm comfortable at home.
- No, its too expensive.
- No, too many disruptive things are happening.
- No, I love my house and family too much.

Undecided:

- Maybe. For convience yes. But yet again no because of the noise in residence and having to share a room with someone.

# Appendix B3

Class of 2002: Students' Comments on living Off Campus:
Any other comments concerning your studies that you think is connected to the fact that you do not live on campus?

- No comments
- I think I can try to make better than last semester.
- Yes, because I don't have enough time of studying my books because of too much work that my mother is giving to me everyday because now she is my new mother.
- Also it cost me some money to pay a bus to go to school and sometimes when I don't have money for ticket I don't go to school.
- The transport is affecting me.
- None.
- No comments.
- It's a lot of responsibilities and a lot of influences.
- Yes, I would like to very much stay on campus because it would allow me to study hard.
- No access to a computer but that's all.
- Traveling and going home late and the fact that I don't have something to study with is killing me and frustrating me.
- Yes, because there are no other people who are doing the course I'm doing so that I can study with them.
- Sometimes you don't have enough time to study you come home late and you are tired, etc.
- Another comment is the fee for the transport and the buses don't arrive on time so I miss out on the first few minutes of class.
- I'm staying with my Aunt and I sometimes find it very hard to tell her that I can't do house chores because I'm writing a test. Reason number 2 is that she has a 4 month baby and we are staying in a one roomed house and I can't even get enough sleep with a baby crying all night long.

- Sometimes I need to use the computer hence this forces me to come to school or campus.
- I think its best off-campus because I study at my own convenient times.
- Finance.
- Don't have computers. I'm traveling, that is a problem.
- There are lots of people on the Langa hostels, most of them are not educated, they tell to do something else while you study.
- I live on campus because I have no other place to stay.
- It's only the studying about money.
- The group, studying with some people who are doing the same course. Even the text books, sometimes you don't have it but someone on campus has it and the access to the library and computer.
- The fact that I'm failing my subject. I don't think it depends on it. But it depends on the time that I make for myself and my studies.
- Access to all facilities – because I don't have them at home – things have to "stand still" until I get back to campus the next day. Eg. Computer, library.
- Study methods.
- A matter of not having a discussion group is a very big problem, because you end up losing interest.
- Finance problem. Mother doesn't work.,
- The time taken to get here.
- Yes, traveling makes it very hard for me to make up time for studying and doing other school work.

# Appendix C1

Addition to original Questionnaire: Filled in by Prelim Class of 2002

Please indicate on the following grid your perception of what circumstances was like at school for you. Please circle the statements that apply to you. This information will be used to assess the prelim maths course you are doing this year and will help to determine why it was necessary for you to do prelim instead of going straight into the first year of B.Sc. studies. Thank you very much. N.Myburgh.

| | |
|---|---|
| 1 | My teachers were absent<br>not at all sometimes very often<br>How did it affect you?<br>not at all it bothered me: a little / a lot |
| 2 | I often had a new teacher<br>not at all sometimes very often<br>How did it affect you?<br>not at all it bothered me: a little / a lot |
| 3 | I had a textbook Yes No<br>If No: How did it affect you?<br>not at all a little quite a lot very much |
| 4 | My teacher understood my questions<br>not at all reasonably well very well<br>How did it affect you?<br>not at all it bothered me: a little / a lot |
| 5 | My teacher could explain the work<br>not at all reasonably well very well<br>How did it affect you?<br>not at all it bothered me: a little / a lot |
| 6 | All topics were taught<br>Yes No<br>How did it affect you?<br>not at all it bothered me: a little / a lot |
| 7 | Classes were very large<br>Number in Maths class in matric: ....................<br>How did it affect you?<br>not at all it bothered me: a little / a lot |
| 8 | Classes were noisy, making it difficult to work<br>not at all sometimes very often<br>How did it affect you?<br>not at all it bothered me: a little / a lot |
| 9 | Teaching was very exam-directed<br>not at all sometimes very much<br>How did it affect you?<br>not at all I liked it: a little / a lot |
| 10 | My teacher gave me personal attention<br>not at all sometimes very often<br>How did it affect you?<br>not at all it bothered me: a little / a lot |
| 11 | I had a 'tutor' to ask if I had a problem<br>not at all sometimes very often<br>How did it affect you?<br>not at all it bothered me: a little / a lot |

# Appendix C2

Students' Comments in answer to supplementary question:
Why did you not achieve Mathematics marks that was
good enough for B.Sc. studies?
How were circumstances at school responsible for it to happen?

- The teaching method at school was the main problem. There was nobody who attempted higher grade and our class was very big (30). That's why my maths marks were so bad.
- Since the classroom was small and crowded with a large group (40), we couldn't all get personal attention. My teacher was a very busy man that year he was attending mathematics workshop after school and sometimes dance rehearsal, so we could not sit with him after school.
- My maths teacher wasn't a maths teacher. He was a physics teacher but the principal employed him as a maths teacher. As from grade 11, during the first semester the teacher left and he was substituted by Mr. Gysman another new teacher. When I was in grade 12 we had a new teacher who wasn't even a maths teacher but a physics teacher. We wanted to do higher grade maths but because of the lack of higher grade work we had to do standard grade. They decided that for us but not for ourselves. We used old textbooks. In order for me to pass my mathematics grade 12 I had to go to The Institute of Race Relations, they picked up the pieces and I had to borrow the Study and Master textbook.
- Firstly I was not determined enough. I always saw maths as a subject I was not good at, and that I could only work to a certain standard. That restricted me a lot. I suppose the fact that the teacher's attitude towards people like me was bad – people not putting in enough effort, did not help me very much. It was really disappointing that I did not make it even though I've learnt a lot in my year of B.Sc. Prelim.
- I did not understand the work. I did not receive individual attention. I was the only HG student in class and did not receive attention. I was afraid of tests and exams.

- The reason I didn't get good enough marks for Bsc 1, was because my matric teacher was rushing very fast with the syllabus. And he was not really teaching, he was more like lecturing of which that was a biggest problem. He was concentrating on the higher grades too much. And there were no consultation times for individual attention. Also the textbook that we used was not very detailed enough.
- First of all our teacher did not give us special/individual attention when we did not understand something. I did not have a tutor, I worked alone or sometimes with a group but you would find that sometimes we each knew various things which we helped each other with, but there were other things we had difficulties with. And lastly our school syllabus did not cover everything, something/many thing, we did not do at all.
- Our teacher focus a lot on standard grade mathematics so we struggle a lot as the students of higher grade. She was too fast though she did not finish the syllabus and I did not practise enough.
- My teacher had a problem in that he favoured a few pupils in the class. When ever it came to me answering questions, he would laugh even though I was right. He would be insulting to quite a few other pupils in the class and would never encourage us. He thought he was too good.
- We had a lecturer that took care of us individually and that made the difference between my matric and the prelim.

## Appendix D 1

Students' Comments in answer to question 6.1:

Name three ways in which the prelim Maths course motivates you

Group 1:

- Working on my own; it gives fundamentals of mat111; gives me more understanding of problem solving (not related to maths).
- I see new interesting methods; It is taught effectively and there are tuts; It's understandable.
- Tutors help a little; it is less work than matric maths; it is easier to understand.
- Knowing the basics for Maths 1 better than someone coming from school. I know many ways of solving problems now. I like extra classes.
- Tutors help a little; It's less work than matric; It is easier to understand.
- It motivates me because I understand it; The teachers are good at explaining it; And the testing is frequently.
- I understand maths better; I've learnt to work with my peers; I've learnt to work consistently.

Group 2:

- It's not that difficult; it is not too much work; you can always get help.
- The way the lectures are conducted; tutorial tests every Wednesday; the second chance we are given for a retest (tuts).
- In what I must expect of maths; to be able to prepare for computer science; to improve my maths in understanding it better.
- Know how to solve problems; not to give up; believe in myself.
- By the strategy they use in teaching Maths; By the way they take care of learners.
- I've got more information I needed to know; Everything seemed to be clear now; Some things I understand well in this course, the things I did not understand at High School.
- I was introduced to new chapters; my interest on maths has improved; I'm no longer studying maths to pass but to apply it in real world.
- The work is not very difficult; it gives me a good background.
- It is giving me a chance to accomplish my goals.

- Prelim maths motivates me to solve problems like using sign table to know graphs.
- It helps me to cope with the university maths next year.
- Show me that maths is easy; VERY interesting if you know what maths is used for; you get second chances at tests.
- To pass for the next year; can help me to find a job; I also help the std 10's for the exam.
- I can achieve my goals; I'm able to cope with maths; I look forward to a challenge.
- It encourages me to continue with maths.
- At this semester I don't feel afraid to ask questions as the teachers are there if you want to consult them.
- It improved my knowledge of maths; it gives me courage that next year I can do maths; it gives me a little light about maths.
- Motivates you that you have hope of starting B.Sc. 1 ; hope of starting your career
- Make me understand maths better; to know some basic concepts of maths.
- By giving me opportunities to do even better; makes me attached to my work; Its really excellent; Writing tut tests every Wednesday.
- The lecturers are patient with the students; they can clearly explain the work; the care they have for the students.
- Lecturers are never bored, always in a good mood; lecturers give us homework for practice more often; if you practice it, you will pass it.
- It teaches me new and interesting concepts; at times it is challenging; it will help me in any further calculus course I intend to take.
- Seeing that I did not do well in my matric maths; it is giving me a chance to improve my results; It is quite understandable and sometimes challenging.
- I didn't do well my matric maths in school; It gives me a chance to improve my results; It also gives me qualifications for year 2002.
- It provide me with some skills.
- The fact that it gives us the opportunity to do a B.Sc. degree course next year and helping me to understand maths more; it is the key to mat111 because I want to do computer science.
- Retests; chances, eg. Still writing tests for the first semester while you are doing the

second one.

- It helps me understand better; it improves my thinking and my knowledge; it allows me to work alone.
- By explaining the things that I never understood at school; topics that were never taught at school were introduced; given more opportunities to pass maths.
- The work is nicely outlined; it is challenging and fun.
- The prelim course can be an encourage; I was learnt at 1999 matric so I forgot the most staff.
- I have improved at least than last year; The way the lectures teach is understandable.
- Gave me one more time to find more options; made me understand maths better; helped me not to fear maths.
- Prepares me for next year; there are things that I did not understand in school, but now it is clear.
- In groups incorporate; In practising my potential in calculations.
- I think it will give me a chance to study Maths next year.
- I did very well in my past modules, so far maths prelim motivates me.
- By providing with new topics I never learnt at school and it is a fair course by giving the opportunities in order to pass.
- Helps me to pass well in maths; I can apply the maths skills now; basically I have the better understanding.
- Gives me a simplified overview of tertiary mathematics; teachers/lecturers/tutors are more attentive; semester/modules make it easier to study for tests.
- Maths motivates me because I did not forget much about my work I have done at school.
- To give me a foundation in Maths; teach me about some of the stuff I missed at school; give me a strategy to the maths at university.
- Teachers explain very well in all chapters; there is a lot of tutors to assist in our work; they have a good mood anytime (attitude).
- To continue to do maths; understand it.
- Helping students to deal with challenging problems; analysing difficult or

confusing topics in a simple way; homework and individual attention.

- It helps with Maths 111 (makes it easier); better understanding of the work.

Group 3:

- It motivates me for knowing more about maths; I get more knowledge about maths; I get more knowledge about shift of graphs.
- I learned new things about Maths; Practice plays a big role in Maths; It motivates me to learn and understand what I am reading.
- It is like a window of hope for me for next year.
- It gives me background of mathematics because I arrived here with poor background of mathematics.
- It consists some things I've heard about in matric.
- To give more explanation for me to understand; To give me extra classes; Typed notes for the course.
- The lecturers are always available for any assistance; using textbook a lot and giving us homework; giving us a chance to prepare for the tut test.
- Shows me direction to a specific field.
- It motivates me because the lecturer make sure that the students understand the work; the method in which she teaches its easy and understandable at some point; the fact that there are 2 people who are willing to help during lectures.
- It's simple; it's not very, very complex; the lecturers explain well and take time for us.
- It gives me confident and skills; It foundation me to deal with hard math111 course
- Retesting gives us a second chance; Basics are taught to me - sometimes better than school; Special help available.
- To get a better understanding of maths for math111.
- Well I've always had a negative attitude towards maths and am learning to overcome it.
- No motivation at all.
- It gives me the background of mathematics; it also empowers me to reach my BSc; it gives me the light to enter the university.

# Appendix D 2

Students' Comments in answer to question 6.2:

Name 3 ways in which the prelim Maths course does not motivate you.

Group 1:

- There is nothing in the prelim course that does not motivate me.
- Tests were written too often.
- I don't like semester courses, I prefer terms; homework should be marked by lecturer to make us more serious; books are not leased.
- Redoing the std 10 work; everything is in English; not having lectures daily.
- We are spoonfed; responsibility and maturity not really tested; repeating high school maths.

Group 2:

- Lack of strictness, especially on homework and exercises.
- There is no course overview (were not given); test dates were a hassle to fit into schedule; class was too big.
- Sometimes I cannot understand the work, when I am doing it on my own.
- They did not let us lease the book.
- The prelim maths course can be discouraging. Because they took a whole year, at least a half year is better.
- The lessons before the tut tests.
- No break; marks allocation, sometimes it became unfair.
- The class is too big. Some of us are not used to that.
- The lecturer sometimes concentrate on one student; the noise that is caused by students; the students that walk in and out during lecture hours.
- I can't understand the exponent even if I try harder; in word problems.
- We don't do things to prepare us for mat111; we do things that are not necessary; because other students take it very lightly.
- It did not motivate me at all, instead I think it made me to realise that I must take care of my work; it's a waste of time because to have 2 years for 1st year as it does not have any credits.

- In the first semester I was not free to ask because the teacher was concentrating on the students at the front.
- Sometimes difficult; time consuming; complicating.
- Classes too big; I don't like asking questions in big classes; sometimes the English makes it difficult for me.
- Tells me it is only a small portion of much more, maths is not easy.

<u>Group 3:</u>

- Difficult to cope; makes me lose faith; makes me lazy.
- I struggle with the work; we get work in tests that's far more difficult than the work that we do in class.
- I wouldn't say that the course did not benefit me, it is just that I had a choice and that it makes it difficult for me. To choose whether to go to class or not.
- By studying about matrices; by not understanding the language correctly.
- No special tutoring.
- It is good for us to pass, but the fact that we have to write more than 3 times it doesn't motivate me instead it makes me lose confidence.
- Does not explain enough as my past teacher did to me in std 10; does not give notes.
- I understand it in class, even when I'm working alone, but when it's a test time I fail even though I thought I knew my work; it needs more attention.
- There is no overview at this stage.
- The freedom to come or not to come to class.

## Appendix E 1

Students' Comments in answer to question 9.2

Do you see yourself as successful at Mat 011-041?

YES, I see myself as successful:

Group 1:

- Because my grades have all been in the range os 60%; I always did my homework; I didn't have someone (tutor) to consult in any problems That I encount.
- I do a lot of exercises.
- Practice makes you perfect; I passed the first semester and now this semester I try my almost best.
- Attended classes regularly.
- I understand my teachers and they understand me; practise a lot; It's bette than at high school; I practised and asked questions I do not understand; I did not wait till the exam date is closer.
- I am confident enough to pass; I do not need a lot; I like doing more exercise.
- It motivates me because I understand it; the teachers are good at explaining it; And the testing frequently
- I understood most of the work.

Group 2:

- Work hard; redo class work at home on own.
- I have improved a lot.
- I practice a lot and try to understand each and every section.
- I practise maths very often and I also ask the lecturers if I don't understand.
- Because of what I been doing, I beginning to get my strength because of these tutorial class and groups which I have formed.
- I worked harder.
- Yes I see myself as successful but it is only the last chapter that sinks me (trigonometry).
- Passed!

- Because I pass it every time; I practise when I have time; I listen in class.
- Because I did my best this year not to make the mistake that I made last year.
- At school I got only D's and E's at least math 011 I tried and managed to get B's even though my year mark dropped down.
- Maths I was passed at school for standard grade.
- I tried to practise often.
- I do my work well and I always pass it with a high percentage; I always attend classes regularly and study and revise my work almost every time.
- Because it is more interesting; I study and I don't understand some stuff easy.
- I made sure I don't leave class without understanding work done so I can prctise on my own.
- I have always enjoyed maths; maths has always been a subject I've been good at.
- Because I understand the work this days.
- I enjoy maths and attend classes; I'm not regularly studying with friends.
- I studied for myself, not because I'm forced to.
- Each and every test I write I pass; each module and understand the work better; practised a lot which I was not used to do and since some of the work is not new so things were a little bit easier.
- I did all my exercises and homework.
- Go over the work taught.
- Work hard/study; do exercises.
- I passed all my prėvious modules and this shows some kind of success.
- Because I want to go to 1st year; new thing that I want to study at university.
- Try to understand my work and pass my tests.
- Because I am working much harder and I spend most of the time doing maths.
- I must work very hard to help myself.
- I study maths and like working with new ideas.
- Payed attention.
- I enjoy mat011-041 and I do well in my exams and tests.
- Tried my best in passing my tests.
- I attend extra classes; study with friends.

- I am doing very good in my maths; try and ask if I don't understand something.
- I practised and did some of the exercises that were done in class and the ones in a textbook.
- I give too much effort in it and most of work I did at high school.
- I have learnt more things I did not expect to know and understand.
- I practise and did some exercises.
- I passed the first semester with a better symbol but now my mark are not that good.
- I worked harder than at high school.

Group 3:

- I work hard; did not fail; manage to work hard and pass.
- I was not focused on my studies, but now working twice as hard.
- I improved on my maths since I came to university and studied maths 011-041.
- I know I can make it if I keep on practising; I passed 1st semester's test, I will make a point of it to pass this semester by practising my work.
- Maths is not that successful and constantly working hard to pass; I put more effort into my work.
- I've done more work; put a lot in maths.
- I see myself successful because the tut tests are boosting my results.

NO, I do not see myself as successful:

Group 1:

No students in group 1 saw themselves as not successful.

Group 2:

- My course marks does not reflect what I am; I am very poor in performance.
- Failed first semester, but now I think I'm doing much better now.
- Do not see where practical applications apply.
- Because I don't know my work but want to be successful and I will try to be.

- I tried to understand maths but it is difficult; I tried to spend more time.
- I tried to understand maths better especially graphs but I still don't get it.
- I failed my first semester.

<u>Group 3:</u>

- Did not ask for help.
- I tried to study my notes but they are not enough.
- Because I did not work hard enough to deserve what I should; I did not attend class regularly; had no textbook; almost never listened.
- Because when I practise maths stuff, I told myself that I will do it in the exam but unfortunately that doesn't happen and I end up on failing.
- I'm failing; lost interest in work; became lazy.
- Didn't work hard enough.
- I did come to class but I hardly sat with my maths, revising it; all I need to do is pay attention.
- I didn't attend as I was supposed to and needed a textbook to read on cause some exercises are done out of it and a little reminder.
- Did not buy a textbook.
- I did not make use of the library at first for the textbook to work out examples and exercises.
- I failed some of the tut tests because of not being prepared before writing; I didn't study well.
- I am not confident of myself.

# Appendix E 2

Students' Comments in answer to question 9.2

Do you see yourself as successful at Mat 011-041?

What did the university do? (This would presumably be positive things.)

Group 1:

- Help from my lecturers is always there so the university has been fair to me.
- Revised std 10 work.
- Given me extra help and time for understanding.
- Explained everything clearly to me.
- Lecturers helped a lot – made my task easier.

Group 2:

- They tried to make me understand.
- Helped us with the tutors and lecturers; give us the tut periods.
- With each aspect in maths show students differing applications.
- Gave me a chance to write over; extra classes and extra help.
- The university provides every skill we need to comprehend our studies.
- Taught me a lot about maths such as matrices and graphs (piecewise) the easy way to do it.
- Help us with extra classes; more tut test and exercise.
- It played an important role in helping; it provided extra classes to help.
- The tutorials and the tutors and also the tut tests were very useful and they helped a lot especially the re-tests.
- Supply with study material.
- Provide us with good lecturers.
- They provide us with tutors and extra classes.
- Give more opportunities such as writing a retest and having a tutor and extra classes.
- Was willing to help.
- University help to change my self image for the better; I understand the work more easily.

- It gave me lot of exercises and examples and there is also a logical explanation from the lecturers.
- Provide me with such good and motivating lectures.
- The university provided tutorial classes to sharpen my skills.
- Help me to understand the topic.
- Taught everything (went over all the work).
- There were extra classes for students who don't understand the work at lectures.
- Help in understanding the work.
- It introduced extra tutorials which makes me know what I must expect in the exam.
- We were offered tutorials where we discuss things and get more information from others.
- It has given us an opportunity to qualify for a degree without an exemption.
- Gave me a chance to improve my maths status.
- The university taught me a lot.
- Provides books/textbook at the library; we have good lectures
- Introduce two new lecturers who did their work better this year so that anyone understand.
- The university helped me esp. the help of mrs mayburg after classes and the extra classes that we used to have.
- It helped me to improve.
- Self-discipline and self-confidence at all times; the library have good books.
- It offered tutors to help; lecturers were available any time I needed them.
- Gave me another chance to prove to myself that I can do it.
- It gave a better system for maths.
- Give me enough info and people to consult when I don't understand.

Group 3:

- University taught me a lot at give me extra chances to pass.
- It changed my life, I mean I learnt new ways of studying.
- Give me too much freedom.
- Have extra classes.

- Gave me an opportunity to go further with my studies.
- Helped me in understanding maths better than the way I did at school
- Lecturers put in more effort only if you put in more effort.
- It allows you to give us some notes of which I didn't expect that from the university.

What did the university not do?

Group 1:

- Did not let us lease textbooks.
- Did not have an open day for maths.

Group 2:

- More consultation and helps but no revision.
- Did everything; but more advice has to be given to students in terms of subject choice.
- This made me feel more responsible because there's no-one to push me and tell me to do my work.
- The university do a lot of work for us to succeed, the problem is in me not feel confidence in English.

Group 3:

- We are fresh students from high school; the university should have a class where it explains everything to 1st year students because some things we don't know like the importance of attending a tut and lectures.
- The campus must work together to student that have the poor background like me.
- Must stick to things/work that we can't did at high school level to motivate students in this course.
- At university you must do things on your own.
- Self motivate me.
- Not much.

# Appendix F

Students' Comments in answer to question 11.1:

What about Maths makes you happy?

Group 1:

- When it encounters physics problems like amplitudes, sine curves, I'm prepared for the coming year. Problem solving in word sums. Ability to make calculation for other BSc subjects.
- You always give us chances to do our best.
- I enjoy calculating sums. When I have an answer to a question and it is correct, I feel proud of myself.
- It makes me happy because I enjoy doing it and I like numbers more than words. It helps me think quicker, tricky and effectively.
- I enjoy maths but when it comes to low pass mark I feel very bad and doubtful about my ability.
- The fact that I have to constantly work hard to get satisfactory results.
- I find it interesting and challenging. It stimulates the mind.
- It improves your thinking and there is a bright future for anyone interested in maths.

Group 2:

- I enjoy maths because you can solve problems applying skill of maths.
- Using one's brain to reason.
- No, I know that I can't do maths. It depresses me. I would love to know maths, but it seems as if it is not my subject.
- Passing a test.
- When I can conquer it. Actually maths stresses me out majorly, but it is challenging.
- I like maths because every subject I do need it, so it is important to me.
- Wow, maths is my favourite subject, and I am good at it so that is quite sure that I can do very well in maths that is if I study more of it.
- It is the topics or chapters which are taught and for the fact that they are taught in

a sequence of fram ease chapter to the difficult one.

- I like maths because it is challenging and motivating, and when you do something wrong, you practise it again and again until you get it right.
- When I understand it I feel happy.
- We do lot of exercises that help us understand maths. We are always tested which make us know where we are.
- Being able to work out problems on your own without assistance.
- Maths always make me feel happy if I understand the topic but if I don't it makes me angry.
- The fact that it opens doors to different avenues of study, also I am able to think more logically.
- No. (It does not make me happy.)
- I feel very happy about maths because I am having a clear understanding of everything except the graphs.
- Maths makes me happy because I enjoy solving problems and working with others. Also almost every test I write, I pass.
- The fact that I can prove things and the fact that I can handle problems.
- Maths makes me happy because I like maths from school so I passed my past modules. I like maths.
- It I understand the work we are busy with, it makes me happy and I am much more interested in it.
- I enjoyed very well and the lecturers were so active meaning they tried to explain everything so that everyone could understand.
- Maths is interesting to me. I feel challenged when doing maths. What I like most about maths is that it needs more practise, you do not have to read in order to pass.
- I understand work – need it for what I would like to do – pharmacy.
- It relaxes my mind because I prefer calculations than theory work, it doesn't bore me but rather excites me.
- It real makes me happy, whenever I understand how to deal with challenging problems – enough to reap the benefits of study.
- It makes me happy when I am doing well.

- I pass most test. I understand the work. The way the work is explained.
- It is the thing that I understand it better now than ever and it is not difficult for someone who want to pass to do it.
- Understanding the work, able to answer any problem that is given to you. Getting good maths results.
- I very happy for my result of the maths. Because always I passed this maths.
- The lecture explains thoroughly, there is extra classes, there are retests for somebody to fail is her risk.
- I often find it fascinating and has best interest.
- Our lecture always try (make sure) to make things easier for us like explain each chapter before doing the next chapter.
- It teaches me to be logical. It is an area where I can apply my skills.
- Exercises and examples are interesting to do.
- Knowing that I know something about the subject that other people don't know. Knowledge is power – sooner or later everyone will find out how important maths is – some will never learn.
- You don't need to go to the quiet place to study it. You can study it even if you are listening to the radio.
- When mathematics is made simpler, I understand it much better.
- It is a very tricky subject which makes you think and play around with numbers.
- I know how to calculate and to solve problems with maths. Now I can solve any problems like the business.
- It does not make me happy at all because as I am not good in it I usually pass it but here I fail it every time.
- It is an enjoyable course but its difficult.

Group 3:

- Maths is an interesting subject, it's enjoyable especially when you dedicate yourself to it and practice, makes perfect.
- Mathematics can open doors for me to go anywhere I which to go.
- When I am doing maths I feel so relaxed and confident. Mathematics keeps me

smiling all day long. I love it.

- I am happy because you always gives us chances to improve our mathematics even if we didn't understand, we end up understanding.
- Maths is logical thinking. When I get maths write or know what is happening I enjoy it because that tells me I have logic and can sort and deal with problems.
- At university – doing the work we've done of school making it more easier/happier to do.
- There is more than one way of doing/solving a problem.
- When I pass it!!!
- Nothing.
- Nothing, I am only happy when I get high marks which I never get.
- Maths makes me happy because I do have the ability to work but there's not enough practise and attendance/ing.
- You can use it in your physical life. I am able to work out problems. I personally don't have the love for maths.
- It makes me happy because I can get more opportunities to get a job and it make my brain count thing quickly.
- If I understand the topic, I became happy, and even if I know how to calculate. I feel enjoy to be in class to study. And also when lecture explain well I feel enjoy.
- I like mathematics even in lower standard but here at university I have terrible performance I'm still don't know why? but I'll talk to myself that I will never give up until I die.

## Appendix G

General comments made by the students at the end of the questionnaire.

- There's not enough attention to all students.
- No, keep it up doing the good work otherwise your concepts are not difficult if we as the students study our books and practise our maths.
- There is no more hope for people like me.
- Handouts starting from the 1st semester's chapters.
- I think it is the first start in my career that I looking to do in coming years. It is the foundation for the maths that I am going to do. It helps me in understanding of maths. It upgrade me from standard to higher grade level since I was doing SG in high school level.
- It is good.
- I think you should add more tutors. You had tutors but I don't know what happened to them. Why can't you start with the difficult ones first and finish with the easy ones?
- I find it easy because some of the work is not new.
- It was enjoyable.
- Maths 011-041 is good because it try to give us some motivation in university content and some revision of maths from school. And our lecture are fair to us try give us a retest when we fail the test and mrs Myburgh spend her time on Friday afternoon help us in some work.
- No. The maths is fine.
- It is a really exciting and interesting course.
- Too much spoonfeeding. Not really a challenge for higher grade students.
- It is very unfair to attend a lecture class on Wednesday morning and write a test afternoon. At least some of us are not quick to understand. At least the tut test may be written on Friday or make another day.
- This course was understood the same of term but the last work did not understand the determinants. Always I fail the test I didn't pass at all options.
- It was an interesting course although, commitment, dedication and the love of maths is important for the course. It helped me solve problems that I didn't know

in the past (maths problems). I think the reason I got an E for maths on my matric was a lack of information on how to calculate the problems. The maths class opened my eyes especially the help of mrs Myburg. At school in our maths class we were only seven, I'm not used to the big class as I didn't ask questions during lectures, thank you mrs Myburg.

- It was difficult to write a test on work when we started with it on the same day as the test.
- Exercises should not be more than ten because of time. No lessons before the test. Notices should not be late.
- I think that if mrs Fray was the only lecture we had I could have lower pass percentage because sometimes I real can't understand her explanation. I do believe that the help from mrs Myburgh has played a great role on me to pass this course.
- Start with easiest topic of the year and finish up with the tough ones for example you should have done trigonometry last semester and put more effort on those so called transformation of graphs.
- In 011 I was lost because of the word university and it was the first time to me to taught by the white person. In 041 it is better because I no the lectures.
- 'Bekumnandi': It was enjoyable.
- It is going to help me with first year maths because we learn the BSc1 maths basics. It has given me more knowledge, the things that I did not clear understand at school are more clear now.
- Maths 011-041 was never difficult, the only thing I did not understand until the end of the year were piecewise functions.
- Class first it was big but I think if it could better easy for us to have 2 tutorial classes a week, meaning the one which was on Wednesday and the other one to bo on Friday afternoon because so that when students are going to the weekend they will know all the work which they did during the week. I think that will help a lot of students like me who are not fast on understanding problems.
- Maths 011-041 helped me a lot because there are some basics which I didn't know but now I can see clearly. Really I'm a new person now and I know in 1st year I

will make it.

- I like the way we are taught. The lecturers made sure that everyone understands. If someone fails he/she is given a second chance or even a third time to improve and is given a chance to pass as others.
- I don't have any comments on the math 011-041, but I appreciate very much of what the lecture have actually done to us. The lectures has been very fair to us given us the second chance after the tests to improve or to better our marks. So I am very happy of what be have done, because we will not have the same opportunity we have next year, unless we will be having the same lectures.
- This course helped me a lot throughout the year. There are a number of topics that are much clearer to me now than when I was at high school last year. Although I've experienced some problems especially in the matrices and complex numbers, but with the help of mrs Mybutgh at least I can deal with them. Lastly I would like to give my sincere thanks to mrs Myburgh and mrs Fray for their immense job. Thank you.
- The teachers were always willing to help and that was encouraging.
- I really enjoyed this course, it made me motivated.
- It's very interesting because it becomes much easier as we are having opportunities in order to get pass. I can say it is useful.
- I love this maths. I find it easy that the maths I did at school. Which is maths HG. I ended up failing it in HG at the end of last year and passing it with a SG C. I was very disappointed with my marks but now I see I can do better.
- It was quite challenging, in a good way though, cause now we know that we have to work harder because we still have to do maths 111. So it was like a ladder because some of the things we did are done in maths 111, so when we go to BSc1 we will, at least, be familiar we some of the work.
- It is understandable and not too much difficult but it needs extra effort.
- So far I have no problem and I am doing satisfactorily. Lectures teach very well.
- Everything I have done in this course is really exciting and cool. I'm quite sure that it will help a lot in the next course that I will be doing in 2002. I really am proud of the lecturers who taught maths, they are my heroes because some of the

polynomials and absolute values I didn't understand what was really going on and they gave me more knowledge than I needed. Thank you very much.

- Maths 011-041 is just as maths I did in my school, but it makes me feel unhappy because I want to pass it, but it is not easy. I did practise every day.
- I think it will surely assist me in my first year cource and I am thankfull for the stepping-stone that it's become.
- Well 011-041 course was enjoyable, interesting and so nice. But at my side I was struggling a lot. But I near have someone nice who know maths too much to help me here were I'm staying. I think I can come with good symbol in maths like other student. Everything in earth is possible.

# Bibliography

1. Adey, P. & Sayer, M. (1994). *Really Raising Standards.* London: Routledge.
2. Amoah, V.K., Kannemeyer, L.D. & Nonxga, G.L. (1997). *First year Mathematics Placement Test Report 1994, 1995 & 1996.* Department of Mathematics and Applied Mathematics, University of the Western Cape, Bellville, June 1997. Unpublished paper.
3. Anutech Education Centre. (2001). *Mathematics Bridging Course.* Australia. [Online]. Available http://www.anutech.com.au/elc/mathsbridging.htm
4. Australian Indigenous Education Conference. (2001). *Indigenous Australian adults' perceptions and attitudes to mathematics and on-line learning of mathematics.* Perth: Edith Cowan University. [Online]. Available http://www.ecu.edu.au/ses/kk/aiec/papers/lmarshal01.htm
5. Australian National University. (1997). *Maths Bridging Courses 1997.* Canberra: Department of Mathematics, ANU. [Online]. Available http://www.users.bigpond.com/s-d.garth/teaching/anub/anub.html
6. Ausubel, D.P. (1968). *Educational Psychology A Cognitive View.* New York: Holt, Rinehart and Winston, Inc.
7. Baralos, G. (2002). *Concept Mapping as Evaluation Tool in Mathematics.* Centre for Educational Research, Greece. [Online]. Available http://www.math.uoc.gr/~ictm2/Proceedings/pap451.pdf
8. Beck, R.C. (1983). *Motivation.* New Jersey: Prentice-Hall.
9. Bell, A.W., Costello, J. & Küchemann, D. (1985). *A Review of Research in Mathematical Education.* Windsor: NFER_NELSON.
10. Biggs, J.B. & Collis, K.F. (1982). *Evaluating the Quality of Learning.* London: Academic Press, Inc.
11. Bisseker, C. (2000). Asmal needs Help with Uphill Push. *Financial Mail,* 27 October 2000.
12. Bohlmann, C. (2001). *Reading skills and mathematics.* SASEN 2001 Conference Proceedings, University of the Western Cape, Bellville, 7-9 March. Unpublished paper.

13. Brown, G. & Atkins, M. (1988). *Effective Teaching in Higher Education.* London: Methuen.

14. Buchanan, K. & Helman, M. (1993). *Reforming Mathematics Instruction for ESL Literacy Students.* NCBE Program Information Guide Series, 15 Fall 1993. [Online]. Available. http://www.ncbe.gwu.edu/ncbepubs/pigs/pig15.htm

15. Cooley, C.H. (1918). *Social Process.* New York: Time-Warner.

16. Crawford, D.B. & Snider, V.E. (2000). Effective Mathematics Instruction the Importance of Curriculum. *Education and treatment of Children,* 23 (2): 122-143.

17. Davis, D. & Sorrell, J. (1995). *Mastery Learning in Public Schools.* Valdosta, GA: Valdosta State University, December 1995. [Online]. Available. http://chiron.valdosta.edu/whuitt/files/mastlear.html

18. Davis, R.B. (1984). *Learning mathematics: The cognitive science approach to mathematics education.* London: Croom Helm.

19. De Lange, J. (1987). *Mathematics, Insight and Meaning.* Utrecht: OW&OC.

20. Department of Education. (2001). *Guideline Documents National Senior Certificate Examination 2001 Mathematics Paper I, II HG & SG.*

21. De Wet, J.J., Monteith, J.L.de K, Steyn, H.S. & Venter, P.A. (1981). *Navorsingsmetodes in die Opvoedkunde.* Pretoria: Butterworth an Kie (SA) (EDMS) BPK

22. Dörfler, W. (1993). *Educational studies in Mathematics.* Dordrecht: Kluwer Academic Press.

23. Ellington, H. (1985). *Producing Teaching Materials.* London: Kogan Page Limited.

24. Ellington, H. & Race, P. (1993). *Producing Teaching Materials.* London: Kogan Page Limited.

25. Fray, R.L., Kannemeyer, L.D. & Salasa, M.G. (2001) *Learning Materials Development.* SASEN 2001 Conference Proceedings, University of the Western Cape, Bellville, 7-9 March. Unpublished paper.

26. Fremont, H. (1969). *How to Teach Mathematics in Secondary Schools.* Philadelphia: W. B. Saunders Company.

27. Freudenthal, H. (1973). *Mathematics as an Educational Task.* Dordrecht: D. Reidel Publishing Company.

28. Freudenthal, H. (1991). *Revisiting Mathematics Education: China Lectures.* Dordrecht: Kluwer Academic Publishers.

29. Freudenthal, H. (1980). *Weeding and Sowing.* Dordrecht: D. Reidel Publishing Company.

30. Harris, N.D.C. (1979). *Preparing Educational Materials.* London: Croom Helm.

31. Howson, G., Keitel, C., Kirkpatrick, J. (1981). *Curriculum development in Mathematics.* Cambridge: Cambridge University Press.

32. Kannemeyer,L. (1996). *Report on the Mathematics 114/124 Course 1993 to 1996 (August).* Department of Mathematics and Applied Mathematics, University of the Western Cape. Unpublished Paper.

33. Kieran, C. (1998). Complexity and insight. *Journal for Research in Mathematics Education,* 29 (5) : 595-601.

34. Kirby, D. & Kuykendall, C. (1991). *Mind Matters.* Portsmouth: Boynton/Cook Publishers, Inc.

35. Krutetskii, V.A. (1976). *The Psychology of Mathematical Abilities in Schoolchildren.* Chicago: The University of Chicago Press.

36. La Trobe University. (1999). *Mathematics/Science Bridging Course - BEUGBR.* Bendogo: La Trobe University. [Online]. Available http://www.bendigo.latrobe.edu.au/mte/courses/msbp.html

37. La Trobe University. (2001). *Bridging Mathematics.* Bendigo. [Online]. Available http://www.bendigo.lathrobe.edu.au/mte/maths/subjects/Bridging_Mathematics.htm

38. Malan, S.P.T. & Du Toit, P.H. (1991). *Suksesvolle Onderrig.* Pretoria: Academica.

39. Naenae-college. (2001). *Mathematics bridging course.* New Zealand. [Online]. Available. http://www.naenae-college.school.nz/courses/senior/senmatbr.htm

40. Newble, D. & Cannon, R. (1991). *A Handbook for Teachers in Universities and Colleges*. London: Kogan Page Limited.

41. Polya, G. (1965). *Mathematical Discovery*. New York: John Wiley & Sons, Inc. Press.

42. Powell, A.B. (1993a). *Forcing Awareness of Mathematics: Self, Mind and Content in Dialogue*. Lecture delivered at the Department of Mathematics, University of the Western Cape, May. Unpublished paper.

43. Powell, A.B. (1993b). *Observations and Comments on Maths 114 with Implications for the Curriculum of the Mathematics Department at the University of the Western Cape*. Lecture delivered at the Department of Mathematics, University of the Western Cape. Unpublished paper.

44. Prins, E., van Kradenburg, E. & Vermeulen, N. (1996). *Pythagoras '94 HG*. Stellenbosch: Fisichem.

45. Prins, E., van Kradenburg, E. & Vermeulen, N. (1997). *Pythagoras '94 HG*. Stellenbosch: Fisichem.

46. Rader-Konofalski, W. (1991) Confessions of a Math Avoider. *Clearing House*, 64 (4):269-271.

47. Resnick, L.B. & Ford, W.W. (1981). *The Psychology of Mathematics for Instruction*. Hillsdale: Lawrence Erlbaum Associates, Publishers.

48. Rossouw, H. (1997). *Proverto Study Guides Mathematics St 10 (1997)*. Wierda Park: Proverto Educational Publishers.

49. Saffer, N. (1999). Math and your career. *Occupational Outlook Quarterly*, 43 (2) 31-35.

50. Skemp, R.R. (1979). *The Psychology of learning Mathematics*. Aylesbury: Hazel,Watson & Viney Ltd.

51. Steen, L.A. (2002). *Foreword to Math Panic*. St Olaf College Northfield, Minnesota. [Online]. Available. http://www.stolaf.edu/people/steen/Papers/panic.html

52. Strydom, A.H. & Helm, C.A.G. (1981). *Die Suksesvolle Dosent*. Goodwood: Nasou Beperk.

53. Tall, D. (1991). *Advanced Mathematical Thinking*. Dordrecht: Kluwer Academic

54. The University of Queensland. (2001). *Bridging Course.* Brisbane: The University of Queensland. [Online]. Available http://www.sph.uq.edu.au/acithn/students/cw/BridgCrs.html
55. Underhill, R.G., Uprichard, A.E. & Heddens, J.W. (1980). *Diagnosing Mathematical Difficulties.* Columbus: Charles E. Merrill Publishing Company.
56. University College London. (2001). *Mathematics Bridging Course.* London: Department of Mathematics, UCL. [Online]. Available http://www.ucl.ac.uk/Mathematics/Bridging.html
57. University of Michigan. (2001). *Calculus Student Guide.* Department of Mathematics. [Online]. Available http://www.math.lsa.umich.edu/programs/calculus/student.html
58. University of Michigan. (2002). *Gateway Testing in Mathematics Courses at the University of Michigan.* Department of Mathematics. [Online]. Available http://www.math.lsa.umich.edu/programs/calculus/gwydesc.html
59. University of the Western Cape. (1999). *Interim Admissions Policy.* Bellville.
60. University of the Western Cape. (2000). *Institute for Counseling: Annual Report.* Bellville.
61. University of York. (2002). *Teaching Methods.* Department of Mathematics. [Online]. Available. http://www.york.ac.uk/depts/maths/ugrad/methods.htm
62. Usiskin, Z. (2000). The Development into the Mathematically Talented. *Journal of Secondary Gifted Education,* 11(3):152-162.
63. Van Hiele, P.M. (1973). *Begrip en inzicht.* Muusses Purmerend.
64. Wadsworth, B.J. (1978). *Piaget for the Classroom Teacher.* New York: Longman Inc.
65. Wiseman, S. & Pidgeon, D. ((1970). *Curriculum Evaluation.* London: King, Thorne & Stace Ltd.