Perceived experiences that grade seven learners have in learning algebra.

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Declaration

I declare that *The perceived experiences that grade seven learners have in learning algebra* is my own work, that it has not been submitted for any degree or examination in any other university, and that all the sources I have used or quoted have been indicated and acknowledged by complete references.

Matjala Lydia Matsolo………………………..June 2006

Signed ……………………………………….

Special dedication

I would like to dedicate this thesis to my mother, Mamatsolo you gave me not only love but courage. My father, Lira for always believing in me, my brothers Matsolo, Morabeng, Masheane and my sisters Baba, Mapule and Semantoana most inspired and inspiring friends.

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Key words:

Arithmetic

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Introductory

Variables

Letters

Concepts

Misconcepts

Notation

Convention

Transition

Abstract

This study investigated grade seven learners' perceived experiences in learning algebra. Things that learners do and say during algebra lessons and about algebra as a topic were investigated.

An empirical investigation technique was used to carry out the study and some literature was reviewed. In the empirical part of the study, the grade seven learners' perceived experiences in learning algebra were investigated. The study was done at one of the previously disadvantaged schools in Cape Town, South Africa. The data were collected through observations, a questionnaire and interviews. A total of 90 copies of the questionnaire were distributed and the response rate was 100%. Observations were made from the day the topic was started in two grade seven classes. Two different teachers taught the two classes. Focus group interviews were conducted; two groups of the learners, ten learners from each of the two classes were interviewed.

Learners devised a number of strategies for solving problems related to sums and differences. The principal learning difficulties experienced by learners in algebra related to the transition from arithmetic conventions to those of algebra, the meaning of literal symbols and the recognition of structures. Learners frequently mentioned obstacles that arose from meanings of the equality sign, the conversion of a description into an equation or a system of equations, and the manipulation of symbolic forms.

Additionally, this study produced results from learners' work in the classroom which showed that, algebraic reasoning is more accessible to learners than algebraic symbolising. It became obvious then that developing algebraic thinking is not necessarily dependent upon algebraic notation and that the presence of algebraic notation says little about the level of problem solving. It was also revealed in learners' work that the unknown has a remarkable function in the informal solving of (systems of) equations. Letters and symbols help learners organise the information in a given problem, but the unknown does not play a meaningful role in the solution process.

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Chapter 1

Introductory Chapter

1.1 Introduction

The general purpose of mathematics as a learning area as outlined in the South African Government's policy (2002) is for learners being mathematically literate enabling them to contribute to and participate with confidence in society. Access to mathematics is, therefore, a human right in itself. Therefore, the teaching and learning of mathematics aims to develop the learner to become critical, confident, appreciative, curious and aware of how mathematical relationships are used in social, environmental, cultural and economic relations.

The policy document, Department of Education (DOE) (2000), advocates the learning of algebra from grade R to grade 9. According to the document, algebra is the language for investigating and communicating in most parts of mathematics. Algebra can be seen as generalised arithmetic and can be extended to the study of functions and other relationships between variables. "A central part of this outcome is for the learner to achieve efficient manipulative skills in the use of algebra" Revised National Curriculum Statement (RNCS), (2002:62).

The learning of mathematics has always posed a problem in South African Schools, especially the previously disadvantaged schools. The researcher is not aware of any research that has been done after the introduction of algebra in grade 7; therefore the researcher felt that some research has to be done on grade 7 learners' perceived experiences with algebra. The purpose of this research is to investigate the perceived experiences that grade 7 learners have in the learning of algebra.

1.2 The constructivist perspective of learning and teaching

The RNCS is underpinned by constructivism that is why in this research the constructivist perspective is being used. The focus of attention from this perspective is the role of the sociocultural environment in the reconstruction of the individual's cognition. What counts as ultimate truth, is determined by sociocultural consensus. The researcher is therefore following Simon's (1995) coordination of the two perspectives, cooperative and progressive/learner centered perspectives, in order to make sense of how learning takes place in everyday classrooms.

Constructivism can have different meanings for different people (Simon, 1995; Wheatly, 1991). There is example, the distinction between "radical constructivism" and "social constructivism". Radical constructivists, in keeping with the psychological or cognitive perspective, focus on the individual learners' cognitive constructs. From this perspective social interaction and consensus is of secondary importance; the primary concern is the reconstruction of the individual's cognition. Social constructivists, on the other hand, perceive cognitive processes as socially determined.

The research is in agreement with Wheatley's (1991) interpretation that two main pillars support the theory of constructivism:

- The principle that knowledge is not passively received, but is actively reconstructed by the cognizing receiver. This means that the leaner does not passively absorb knowledge in an intact form; instead the learner uses the received knowledge to construct his or her own meaning. In the attempt to convey our own meanings we are more likely to evoke meanings in others. Sometimes these evoked meanings may differ radically from the intended ones.

- The other pillar is formed by the principle that presupposes the nonexistence of an independent and objective reality. We construct our own reality based on our experiences. Our knowledge of the world is constructed from our own perceptions and experiences, which in turn are mediated through our sensory experience of the past and the present, no ultimate truth is attainable; at best we can only hope to construct viable explanations for our experiences (Simon, 1995; Wheatley, 1991). Without an ultimate truth to strive for, we must settle for what is viable.

As explained by Simon (1995), a concept works or is viable to the extent that it does what we need it to do: to make sense of our perceptions or data, to make an accurate prediction, to solve a problem, or to accomplish a personal goal. When our concepts are not viable by our own standards, our adaptive processes are triggered, and we are ready to learn. By reflecting on successful adaptive operations, we put ourselves in a position to modify our existing concepts or to build new ones.

From a constructivist perspective, knowledge evolves as the result of a learner's activity performed on mental constructs variously referred to as 'objects' (Wheatley, 1991), 'schemas' (Dubinsky, 1991) or 'frames' (Davis, 1986). Knowledge is always intimately related to the actions and experiences of the learner, always within the particular schematic context of the learner's experiential knowledge. The learner's activity is transformed into a mental 'object' when they are able to think it through, come up with a result, and take it as it is given (Wheatley, 1991). Once the actions have been transformed into objects, the learner is able to reflect on them; and through the process of 'reflective abstraction' learning can take place, (Dubinsky, 1991; Simon, 1995; Wheatley, 1991).

Constructivism provides us with a theoretical framework for discussing the adaptation of existing cognitive constructs and the creation of new ones in the minds of our learners. Simon (1995) sums this up and makes a point, which is often overlooked by both teachers and policy makers: "constructivism, as an epistemological theory, does not define a particular way of teaching. It describes knowledge development, whether or not there is a teacher present or teaching going on" (Simon, 1995:117).

While Parker (1995) reports that following changes in the discourse about the mathematics curriculum in South Africa:

- The child is no longer seen as an 'empty vessel', but as an active, mathematical thinker who enters school with powerful informal mathematical methods.

- A shift from the transmission of esoteric knowledge to the exploration of pupils' everyday knowledge for generalisable mathematical knowledge that can be recontexualised into school mathematics.

- The teacher is no longer seen as an external regulator and knowledge disseminator, but as a consultant and facilitator.

These changes in the discourse about mathematics education are indicative of the current trend towards a more constructivist based curriculum discourse in South Africa.

1.3 Motivation

The researcher is not aware of any study in South Africa concerned with determining the grade 7 learners' perceived experiences with algebra. It is hoped that this research will make an important contribution towards the gap between grade 7 and grade 8 learners in connection with the learning of algebra, current methods offered for teaching algebra.

Algebra has traditionally been regarded to be the domain of the gifted and they have always been considered the only ones who could do algebra and dare to further their studies in the subject. The 'ability to do algebra' is often regarded as a 'sure sign of intelligence'; especially by those who do not use it. Teachers discouraged by the high failure rate in algebra, often discard the idea of teaching algebra in grade 7 as an irrelevant jingle or too difficult a task to accomplish. Usiskin (1992) made the following points related to this idea:

- The whole of society needs algebra, the engineers, scientists, statisticians, etc; but also, the carpenters, plumbers, builders etc.

- There now exists technology that makes the graphing of functions and data, and even curve fitting and data analysis, accessible to all. One no longer needs to know huge amounts of mathematics to do all these things. Algebra has become more accessible and so also has elementary analysis.

- However, the available technology does not yet cover all of algebra, so there is still a need to know some algebra.

- The economic well being of a country depends upon having jobs for its people, and the creation of new jobs in the 21st century depends upon achievements in sectors such as biotechnology, telecommunications, computers and software, micro electronics, robotics and machine tools. Advancements in these areas demand considerable amounts of mathematical knowledge of which algebra is an indispensable part.

Usiskin (1992) also argues that the learning of algebra is like the learning of a language and that anyone who can learn to read, write and comprehend his or her native language should be able to learn how to read, write and comprehend algebraic symbolism. What one must bear in mind, however, is that a native language is learned in a particular context that gives it immediate meaning. It is my contention that, if algebra were to be taught in a context that gives it immediate meaning, it would be learned like a language and would indeed be accessible to all.

Researches into mathematics education e.g. Cortes, Vergnaud and Kavafian, (1990); Herscovics and Linchevski, (1994) have identified a cognitive gap between the arithmetic framework that is sufficiently wide for pupils to experience difficulties in trying to make a transition from arithmetic to algebra. They and other researchers have documented some of these difficulties e.g. Herscovics, (1989); Herscovics and Linchevski, (1994); Kieran, (1989); MacGregor and Stacey, (1993); Orton (1994); and Reggiani, (1994.) the researcher hopes to use their work as a theoretical background for her own explorations into the perceived experiences that grade 7 learners' have in algebra at a primary school.

This has led the researcher to work with one of the previously disadvantaged schools in Khayelitsha, where the problem seems to be particularly in algebra. This problem could result from various factors such as learners' attitudes towards mathematics, semi-qualified mathematics teachers and the ideologies (mathematics is a difficult subject that can only be done by brilliant learners) of the society about mathematics.

1.4 Scope

The research focuses on investigating the grade 7 learners' perceived experiences with algebra. In the study, the phrase 'perceived experiences' is used and defined as things said and done by the learners in their engagement with algebra.

1.5 Organization of the study

Chapter one opens with the contextualisation of the study. It serves to introduce the study by delineating the area of focus. It then proceeds with an articulation of the aims of the study, the rationale and significance.

Chapter two gives a detailed account of the literature around grade 7 learners' perceived experiences with algebra. This chapter is a review of the relevant international and national literature on the topic under investigation.

Chapter three delineates the methodological procedures, instrumentation and analytical processes applied to this study. Qualitative and quantitative research methods will be employed, with the use of a questionnaire; observations and interviews. The rationale behind the choice of this methodology is discussed in detail in this chapter. The school chosen for this study is located in Khayelitsha, Western Cape province in South Africa. Different instruments of data collection (questionnaire; interview and observation) are employed to gather the empirical data. The approaches used to decide on the choice of the schools and participants for this study are also clearly outlined in this chapter.

Chapter four reports on the analysis of the data, and discusses the findings. It focuses mainly on the aim of the study and some issues that surface from the data.

In chapter five, based on the analysis of data collected, interpreted and discussed, recommendations are made with particular emphasis on the gap between grade 7 learners, current methods offered for teaching of algebra and learners' perceived experiences, that should be taken into consideration.

Chapter 2

Literature Review

2.1 Introduction

This chapter deals with the literature consulted in connection with the perceived experiences grade 7 learners have in learning algebra. The research question in this study is: what are learners' perceived experiences in learning algebra. Therefore this chapter deals with definitions of school algebra according to different authors; other authors' views about algebra, for example, introductory algebra; research into learners dealing with school algebra, (the findings and their importance to the study); notation and convention in algebra; letters and variables; learners' understanding of arithmetic; the misunderstanding of arithmetical conventions; learners' use of informal methods and the views on teaching algebra.

2.2 Definition of school algebra

What is algebra? According to Usiskin (1999) there is no simple definition for algebra. Usiskin points out that what is taught as algebra in school is different from what is taught as algebra to mathematics majors at university. He says that in school, algebra deals with the rules for manipulating 'letters' standing for numbers; at university 'letters' are still used but may not stand for numbers anymore, for example, in abstract algebra letters may stand for structures such as vector spaces, groups or rings. These letters are known as 'variables' in mathematics. The word *algebra* is of Arabic origin (Joseph, 2000).

Reeuwijk (1997) observed that the belief underlying the algebra strand is that algebra is a tool for making sense of the world, which is to make predictions and to draw inferences. Reeuwijk (1995) says algebra is a subordinate of mathematics that seems to be hard to define. He further says that there is no universal outline of what should be classified as algebra. But for the purpose of this study and according to Reeuwijk (1995), algebra is defined as the domain in which learners learn and use a variety of algebraic representations such as relationships, patterns, symbols, tables, graphs, formulae (including functions), variables, expressions and equations to make sense of real world problem situations and to solve problems. Algebra (its structure and symbols) is not a goal on itself; rather it is a tool to solve problems (Reeuwijk, 1995). There are three strands in algebra that are strongly related. These are formulae, equations and graphs. Patterns and symbols are the starting point for both the formulae as the equations' ministrands (Reeuwijk, 1995).

Algebra is a branch of mathematics, which may be roughly characterized as a generalization, and extension of arithmetic; it also refers to a particular kind of abstract algebraic structure, the algebra over a field (Dirk, 1967). According to Hollingdale (1989), algebra is a branch of mathematics that substitutes letters for numbers. An algebraic equation represents a scale, on which what is done on one side of the scale is also done to the other side of the scale. The numbers are the constants.

Some authors like Sardar, Ravetz and Van Loon, (1999) describe algebra as a branch of mathematics. Originally (and still in high school context), it refers to the art of calculating with unknown quantities, represented by letters. Modern algebra has expanded this to manipulating symbols represented by letters, following certain rules which may differ from the ones applying to numbers, e.g. vector algebra, matrix algebra etc. Callinger (1995) describes it as a branch of mathematics, which deals with relationships and properties of quantities by means of letters and other symbols. It is applicable to those relationships that are true for every kind of magnitude.

According to Hill (1994) algebra is a form of advanced arithmetic in which letters of the alphabet represent unknown numbers. Children use simple algebra when they solve a problem such as 4 + ? = 7 (a problem they would rephrase as "4 + x = 7" when they get older and begin to study algebra) (Hill, 1994). Therefore, one could say that algebra is a branch of mathematics, which may be roughly characterized as a generalization, and extension of arithmetic, in which symbols are employed to denote operations, and letters to represent numbers and quantities; it also refers to a particular kind of abstract algebraic structure, the algebra over a field.

2.3 Some authors' views about algebra.

French (2002) has made an observation that a proper understanding of algebraic processes is inevitably very dependent on a corresponding understanding and facility with arithmetical operations. Therefore, the researcher believes that helping learners to do calculations mentally is particularly valuable in this respect because simple numbers are involved and underlying principles have to be secured for success. However, practice with mental methods constantly reinforces the distributive law through examples, like 7 x 17 calculated as (7 x 10) + (7 x 7) and 8 x 98 calculated as (8 x 100) – (8 x 2).

Awkward looking examples like 7 x 3.5 + 3 x 3.5 and 7 x 237 + 3 x 237 also reinforce the distributive law and give appropriate meanings to an algebraic simplification like $7a + 3a = 10a$. Short and simple algebra questions with key information written on the board could be much better. Asking learners to perform simple calculations with a minimum of written work and then, most importantly to engage in discussion about the methods used best develops mental arithmetic skills (French, 2002). "Short, orally given and quickly a variety of types of calculations and of discussing those areas where difficulties that continue to arise" (French, 2002:47). The researcher is in agreement with French's observation; teachers tend to believe that giving learners a lot of work with long procedures is good, yet it makes them reluctant to try to do the work. Giving them a little can encourage them to try to work on it.

In the process of reviewing and practicing algebraic skills using the same technique as with mental arithmetic, the emphasis should be on identifying errors and discussing them as they arise. Learners then have immediate feedback, something that they do not get with a timed test where attention tends to focus on right answers and the total marks obtained. French (2002) also agrees and says that questions should be short and simple because fluency and understanding are best developed and reinforced when attention is focused on the key idea involved with no distraction from awkward numbers and that the skills which have already been developed are constantly viewed and enforced in this way.

Kieran (1990) supports the above authors and states that solving problems in arithmetic is primarily directed at finding numerical solutions in specific situations. The objective of algebra, on the one hand, is usually to discover and express generality of method, looking beyond specifications. Generalization requires the learner to recognize common factors on the one hand and unique characteristics on the other. For example, equation solving is not useful if each new problem requires a new approach. The strength of equation solving is in its general applicability: define the unknown, describe the relationship between the quantities, and solve the problem with algebraic means. Algebra also constitutes the reduction of uniformity; in contemporary mathematics this is done with symbolic language (Kieran, 1989).

According to Reeuwijk (1997), the algebra in the Mathematics in Context (MiC) curriculum is developed through the process of progressive formalization. Carefully chosen, realistic problem situations and starting with learners' informal knowledge to motivate learners to move from informal to formal strategies. He has observed that in MiC, as learners move from fifth to eighth grade, the focus changes from informal to more formal algebra. The movement from informal to more formal is not a rigid linear sequence, learners can move back and forth between the levels as they feel the need. Time is taken to develop algebraic language and concepts slowly (Reeuwijk, 1997).

From the project, Application Reform in Secondary Education (ARISE), the strategy is partly underpinned by the realistic mathematics education philosophy, that, mathematics starts off in the real world and that mathematics education should search for its problems and questions in reality (de Lange, 1993).

Algebra is an introductory topic in grade 7, the researcher is not aware of much work that has been written about it, especially in South African context. The research is hoped to make teachers and relevant people aware of some of the experiences learners have when dealing with algebra, and may help them to look at how they can deal with such experiences.

In this research, the researcher is going to look at notation and convention in algebra; learners and arithmetic; learners' use of informal methods and letters and variables. In arithmetic, letters stand for a certain thing, for example, 'c' stands for centimeters and 'm' stands for meters, while in algebra letters are said to be variables and stand for any number, this means any number can be represented by any letter. This is something which learners need some kind of transition and teachers have to be careful when they are dealing with this part in order to avoid any confusion that may arise.

2.3.1 Introductory algebra

This theme starts in grade 7. Equations like problems are presented in pictures and stories. Learners are encouraged to develop their own problem solving strategies. According to French (2002), linear equations and systems of linear equations with more than one variable are introduced. He has further made an observation that no formal algorithms are introduced as the focus is on pre-formal methods such as trial and improve, combining and reasoning and exchanging. It is only at the end of grade 7 that a more formal notation could be introduced. By presenting the learners with complex situations in which there is a 'natural' need for more efficient and 'sophisticated' strategies to solve the problems, learners reinvent the algebraic language of writing (and solving) equations with letters for the unknowns and the formalization only partly takes place in the unit comparing quantities; the concepts *variable* (or better, *unknown*) and equation are further formalized and used in the units decision-making (French, 2002).

In agreement with French, Kindt (1995:47) says that " in grade 7**,** only pre-formal methods such as trial and improve, combining and reasoning and exchanging should be introduced". Linear equations and systems of linear equations with more than one variable are introduced; no formal algorithms are introduced.

Wijers (1995) agrees with French and Kindt as she shows that in progressive mathematization, the teacher needs to allow learners to develop the mathematical concepts at their own pace. She further says that the role of the teacher becomes that of a facilitator and guide in the learning process of the learners. The teacher facilitates the learners' reinvention of mathematics by means of leading class discussions, knowing how to deal with different answers, and so on. According to Wijers (1995), this new role is not easy, but the experiences show that more learners are learning and enjoying mathematics. She has discovered that learning the structure and language of algebra is not a goal on itself, but exploring structure and language contributes to the development of a mathematical attitude.

One does not need to strive for perfect mastery of algebraic skills, especially not for learners at the middle grades. Conceptual understanding of the algebra – and of mathematics in general – is more important. Kindt and Drijvers (1995) have observed that learners need to develop a mathematical attitude so that they at least will start to do something with an equation even when the equation is new to them. They suggest that a possible method to develop such a mathematical attitude is to practice a lot, so learners can rely on their routine.

2.4 Research into learners' dealing with school algebra

In this study when the researcher talks about learners' perceived experiences she is referring to the things that learners do and say during the algebra lesson about algebra as a topic. Things and activities that we can see and hear them say. According to French (2002:99), the following are the difficulties that learners experience as soon as equations are represented symbolically and formal methods of solution are introduced.

- Difficulty in translating a 'word problem' into an algebraic form.
- Reliance on informal methods that work well in simple situations, but are less suited to equations whose solutions necessitates a more formal procedure.
- Lack of fluency when operating with negative numbers and fractions.
- Choosing the wrong inverse operation.
- Difficulty in deciding the order in which to carry out operations.
- Confusion over what is seen as a complicated written procedure.

The most difficult part of solving any problem in symbolic form is interpretation of the problem. It is easy to neglect this aspect of solving problems and concentrate learners' energies solely on learning techniques for solutions. French (2002) says that this has two bad effects: firstly, learners fail to develop an ability to carry out this vital initial stage in solving problems and the second is that the motivation is weakened because the reason for learning to solve equations is lost.

There is a need to jump back and forth between the concrete level of the realistic problem situation and the global general level that is more abstract (Kemme, 1990). The symbols and operations acquire the meanings in realistic problem situations; but it can still be efficient to 'forget' this reality during the execution of the problem solving strategy and think within the 'world of algebra', where the meaning of the object is related to the algebraic rules and properties. Descartes personified this distinction by considering the connection between symbolizations that were detached from their origin (Gravemeijer et al, 2000). As Malle (1993) indicates, the separation of meaning and form of semantics and syntax is one of the difficulties of algebra.

One way of trying to find out learners' perceived experiences in algebra is to identify the kinds of things learners commonly do in algebra and investigate the reasons for these actions. Despite differences in age and experiences in algebra, similar errors appear at each grade level (Coxford, 1988). According to Booth (1984), interviews with algebra learners showed that many of their experiences could be traced to their ideas of aspects such as: the focus of algebraic activity and the nature of 'answers'; the use of notation and convention in algebra; the meaning of letters and variables and the kinds of relationships and the methods used in arithmetic.

Development through the ages shows that algebra had long been practiced as 'advanced arithmetic' and that arithmetic questions can serve as an introduction to algebraic reasoning. Moreover history reveals that knowledge of symbolizing and algebraic jargon are not prerequisites for algebraic reasoning, so with little preparation, learners can start and in this way come up with their own abbreviations (Bednarz et al., 1996).

According to Freudenthal (1968), table 1 below shows the characteristics of arithmetic as opposed to those of algebra. It is important to this study because in order for learners to do algebra they have to transit from arithmetic to algebra and knowing those characteristics could help learners to compare and differentiate between the two. Learners should be able to tell when they are dealing with arithmetic and when they are dealing with algebra. (Refer to section 2.6.1.)

Table1: Characteristics of arithmetic and algebra (Fruedenthal, 1968).

Arithmetic	**Algebra**
General aim: To find a numerical solu	General aim: To generalize and symbolize met problem solving.
Generalization of specific number situ	Generalizations of relationships between n reduction to uniformity.
Table as a calculational tool	Table as a problem solving tool
Manipulation of fixed numbers	Manipulation of variables
Letters are measurement labe abbreviations of an object.	Letters are variables or unknowns.
Symbolic expressions represent proce	Symbolic expressions are seen as produc processes
Operations refer to actions	Operations are autonomic objects
Equal-sign announces a result	Equal-sign represents equivalence
Reasoning with known quantities	Reasoning with unknown
Unknown as end-point	Unknown as starting point
Linear problems in one unknown	Problems with multiple unknowns: syste equations

2.5 Notation and convention in algebra

According to Kieran (1992), learners' learning difficulties in algebra are centered on the meaning of letters, the change from arithmetical to algebraic conventions, and the recognition and use of structure. Some of these problems are amplified by teaching approaches: often the structural character of school algebra is emphasized, whilst procedural interpretations would be more accessible for children (Kieran, 1990; Sfard and Linchevski, 1994 in Kieran (1992)). Operational (or procedural) and structural (relational) modes of thinking, and problem solving are general difficulties of algebra (Sfard and Linchevski, 1994). They suggest that problems encountered in learning algebra can be partly ascribed to the nature of algebraic concepts. Sfard (1991) also agrees and says that there are two fundamentally different ways to conceive mathematical notations: operationally and structurally. "Learners struggle to acquire a structural conception of algebra, which is fundamentally different from an arithmetical perspective" (Sfard, 1991:231).

Part of the problem in learners' attempt to simplify expressions such as $2a + 5b$, concerns their interpretation of the operation symbol. In arithmetic, symbols such as + and = are typically interpreted in terms of actions to be performed, so that + means to actually perform the operation and = means to write down the answer (Behr, Erlwanger, and Nichols 1980; Ginsburg, 1977). The student may not readily appreciate the idea that the addition symbol may signal the result of addition as well as the action, or that the equal sign can be viewed as indicating an equivalence relation rather than a 'write down the answer' signal, although both notions are necessary to algebraic understanding. The restricted way of reading the operation symbol also underlies the 'name-process dilemma' (Davis, 1975), which is the situation whereby a learner confuses the interpretation of the question.

2.5.1 The need for notational precision

According to Coxford (1988), another area that is more critical in algebra than in arithmetic is the need for exact precision in the recording of statements. Such precision is, also important in arithmetic, but inadequacies in this regard can be of smaller consequence if the student knows what is intended and performs the correct operation regardless of what is written. In arithmetic, whether the student writes $12 \div 3$ or $3 \div 12$ makes little difference if the subsequent computation is correct. However, in algebra the distinction between $p \div q$ and $q \div p$ is crucial. This apparent lack of rigor may reflect a lack of attention in mathematics classrooms to the correct and precise verbal statements of ideas in mathematics (French, 2000).

More notably, the free interchange of expressions such as '12 ÷ 3' and '3 ÷12' can often be traced to the learners' earlier experiences with arithmetic. Some learners think that division, like addition, is commutative. Others see no need to distinguish the two forms, believing that the larger number is always divided by the smaller. This appears to have been derived from well-intentioned advice given by the arithmetic teacher when learners first started learning division and from learners' own experience, in which all division problems met in elementary arithmetic, did in fact require the larger number to be divided by the smaller one (Coxford, 1988).

2.6 Letters and variables

2.6.1 Letters in algebra

According to Booth (1984), one of the most obvious differences between arithmetic and algebra, is in the latter's use of letters to represent values. Letters also appear in arithmetic, but in quite a different way. The letters 'm' and 'c' for example, may be used in arithmetic to represent 'meters' and 'cents,' rather than representing the number of meters or the number of cents, as in algebra. Confusion over this change in usage may result in a 'lack of numerical referent' problem in learners' interpretation of the meaning of letters in algebra.

The use of letters to represent a range of values is far more neglected in the teaching of pre-algebra (introductory) than their use as unknowns (Kieran, 1991). Nevertheless, literal symbols may be used to generalize patterns, to stand for unknowns or constants, and to represent the parameters of situation (Speer *et. al*., 1997). As learners have little experience in using algebraic symbolism as a tool with which to think about and to express general relations, they encounter difficulty with these uses of letters.

English and Warren (1989) point out that patterning activities offer a meaningful introduction to early algebraic ideas. However, they added that these activities could present difficulties for learners who lack the requisite skills and knowledge of processes. They further say that flexible, articulate thinking and an understanding of equivalence are particularly important in the learners' success with this approach. Some studies reveal that a great proportion of learners ignore letters used in algebra, they replace them by numerical values, or regard them as 'shorthand' of names or units of measurement. In Kuchemann's (1980) study, it was observed that most learners were unable to cope with items that required interpreting letters as generalized numbers or specific unknowns.

2.6.2 The notion of variable

Perhaps one of the most important aspects of algebra is the idea of the 'variable' itself. Even when learners do interpret letters as representing numbers, Kuchemann (1981) states that there is a strong tendency for the letters to be regarded as standing for specific unique values, as in '$x + y = 8$,' rather than as numbers in general or variables, as in '$x + y = y + x$' or '$a = l \text{ x } w$'. In arithmetic, symbols representing quantities always do signify unique values. One problem arising from this view of letters is that learners often assume that different letters must therefore stand for different numerical values. Consequently, many learners consider that '$x + y + z$' can never be equal to '$x + p + z$,' (Booth, 1984).

A number of research studies have shown that the interpretation of algebraic expressions, particularly of letters in the algebraic code, is not an easy matter for many learners (Kuchemann, 1980; Oliver, 1989). These studies, from a range of perspectives, have found that most learners were unable to cope with items that required interpreting letters as generalized numbers or specific unknowns.

2.7 Learners and arithmetic

2.7.1 Learners' understanding of arithmetic

The above experiences have been discussed from the perspective of the differences between arithmetic and algebra. However, algebra is not separate from arithmetic; it is in many respects 'generalized arithmetic.' To appreciate the generalization of arithmetical relationships and procedures requires first that those relationships and procedures be apprehended within the arithmetical context. If they are not recognized or learners have misconceptions concerning them, then this may well affect the learners' performance in algebra.

Learning algebra is a little like learning another language. In fact, algebra is a simple language, used to create mathematical models of real-world situations and to handle problems that we can't solve using just arithmetic. Rather than using words, algebra uses symbols to make statements about things. In algebra, we often use letters to represent numbers. Since algebra uses the same symbols as arithmetic for adding, subtracting, multiplying and dividing, one is already familiar with the basic vocabulary. (Refer to table 1 under 2.4). The first step in learning to "speak algebra" is learning the definitions of the most commonly used words (Swan, 2000).

According to French (2002) many learners fail to make much sense of algebra and see it as lacking in both meaning and purpose. Motivation is clearly influenced for the worse when learners find ideas difficult to understand and the only apparent purpose for the subject is to do the questions in the next test. Arcavi (1994) notes that even those learners who manage to handle the algebraic techniques successfully, often fail to see algebra as a tool for understanding, expressing and communicating generalizations, for revealing structure, and for establishing connections and formulating mathematical arguments.

2.7.2 The misunderstanding of arithmetical conventions

One area where learners' ideas on arithmetic can influence their performance in algebra is in the use of parentheses. Learners typically do not use parentheses (Kieran, 1979) because they believe that the written sequence of operations determines the order in which the computation should be performed. In addition, many learners think the value of an expression remains unchanged even if the order of calculation is varied. A further view is that the context to which the written expression relates will determine the order of occupation regardless of how the expression is written.

French (2002) claims that it is too easy for algebra to be associated in the learners' mind with a set of procedures such as simplification, substitution and finding factors, and a set of topics, such as simultaneous equations, graphs, quadratic equations and algebraic fractions, each of which appears to sit in a separate compartment. Apart from some common language and symbols, the procedures and topics often seem to have little connection either with each other or with other areas of mathematics. Labels for processes and topics have a powerful effect in compartmentalizing knowledge and tend to set up barriers in the student's mind if explicit links between ideas are not made (Swan, 2000).

2.8 Learners' use of informal methods

There is considerable evidence that learners at the elementary school level use informal problem solving methods (Ginsburg, 1975; Carpenter and Moser, 1981) and a similar observation has also been made at secondary school level (Booth, 1981 and Petito, 1979) where the solution of an equation is concerned, for example, the availability of only informal procedures can have a marked effect on learners' facility with seemingly similar items.

The use of informal methods in arithmetic can also have implications for learners' ability to produce (or understand) general statements in algebra. For example, if a student typically does not find the total number of elements in two sets of, say, 35 and 19 elements by using the notion of addition as represented by 35 + 19 but rather solves the problem by a 'counting on' procedure, then the chances are slight that the total number of elements in two sets of x and y elements will be readily represented by $x + y$. Here the difficulty is not so much one of generalizing from the arithmetic example as it is of having an appropriate procedure, in arithmetic from which to generalize in the first place. This has been discussed by Case, (1974); Booth, (1981) and Booth and Hart, (1983).

According to Tall and Thomas (1991), the learning of algebra is not easy. Along the line of the difficulties already discussed, there still exists more: that is the learners cannot relate to informal and meaningful approaches. Kindt (2000) says that the learners' first approach to algebraic problems is often a natural and informal one. In education, however, we often focus on developing more formal routine methods that are supposed to be automatised. Formal routine methods are efficient and, once mastered, free the learners from renewing the solution at each instance. According to Kindt (2000) the transition from informal to formal algebra is a level jump that learners find difficult to make. The lack of time spent on the informal phase and on the learners' schematization might be responsible for this. Too soon, strategies are shortened and condensed into compact algebraic forms (Kind, 2000).

Wijers (1995), in the philosophy of Real Mathematics Education (RME), says that school mathematics is a subject in which mathematical topics are integrated. She further says that there are no separate courses such as, algebra, geometry, statistics, and calculus and so on, because instruction is not hierarchically organized, all learners get the opportunity to learn about all topics of mathematics. She continues and says that in traditional instruction, especially in the middle grades instruction often takes a linear approach, and its beliefs in repetition: a topic is introduced, taught and practiced, all in a short period of time in one unit of instruction; learners are expected to 'master' this topic, and the topic is not revisited until probably the next year in the same chapter of the book.

It is from the latter idea that if the learning of algebra instructions take a linear approach and the belief in repetition where the topic is introduced, taught and practiced, all in a short period of time in one unit of instruction; learners are expected to 'master' this topic, and the topic is not revisited until probably the next year in the same chapter of the book. Then learners would be able to cope with algebra very well and it would be easy for the teachers to clear learners' misunderstanding. In RME, mathematics is seen as an integrated subject that is developed as a whole and the connections between the different sub-domains are constantly made (Wijers, 1995). "Algebraic skills can be reviewed and practiced by using the same technique as with mental arithmetic – short and simple algebra questions with key information written on the board" (French, 2002:47).

As indicated earlier on, there is no simple definition for algebra. What is being taught, as algebra in schools is different from what is taught as algebra to mathematics majors at university (French, 2002; Wijers, 1995 and Kind, 2000). As mentioned earlier on, Usiskin (1999) has made an observation that at school algebra deals with the rules for manipulating 'letters' representing numbers, and at university 'letters' are still used but may not stand for numbers anymore, e.g. in abstract algebra letters may stand for structures such as vector spaces, groups or rings. These letters are known as 'variables' in mathematics. Research shows that learners' experiences could be traced from their ideas of aspects such as: the focus of algebraic activity and the nature of 'answers'; the use of notation and convention in algebra; the meaning of letters and variables; the kinds of relationships and the methods used in arithmetic.

2.9 Views on teaching algebra

According to Julie *et. al.* (1998), any design activity is normally based on some philosophy of mathematics education and the design process itself. Thus, it is found that the primary philosophical orientation is that school mathematics is something with which learners must have “automaticity” when certain clues are presented, and then a Corrective Feedback Paradigm (CFP) (Siegel and Misselt, 1983) might be the major design strategy. Essentially this strategy requires that, the content and the skills be conveyed to learners efficiently and effectively. After the presentation of the content and the skills, sets of carefully designed practice activities are presented to the learners. Careful notice is taken of the items (questions) where the learners do not attain mastery in; feed back on the assumed misconceptions and the mistakes learners make are provided for, and learners redo items until mastery has been attained (Julie, *et al*., 1998). If this could be incorporated in the teaching and learning of algebra, it could be of the utmost help as it can help learners to recognize their strengths and weaknesses. Teachers may also be able to assist learners with the encountered problems.

Swedosh (1997) provided a method of remedial intervention. The reported method was highly effective in eliminating hitherto persistent misconceptions about algebraic processes, ‘easy and effective method’, which shows simple numerical examples, by substitution, in which the misconception (which is a falsely generalized process of computation, or a misuse of pronumerals and algebraic operations) leads to ridiculous conclusions. This establishes a cognitive conflict for the student: the student’s reliance on (faulty) computation clashes with obviously incorrect results of substituting numbers for pronumerals and doing simple arithmetic. This then makes it possible to (re-) teach the correct method or concept. During this remedial teaching stage, different examples (different numbers, pronumerals and values) are used, thus emphasizing the general concepts, not the particular instances.

According to Bednarz et al. (1996), ‘The future of the teaching and learning of algebra’ raised issues like ‘why algebra?’ ‘approaches to algebra’, ‘language aspects of algebra’, early algebra education’, and such like. They said that it has become clear that there is no agreement on what algebra is or what it should be, and each classification has its strong and weak points. Therefore the researcher has tried to give some recommendations and a conclusion that will consider algebra in terms of its role in different areas of application based on the research findings.

French (2002) has shown that there is a strong prevailing tradition in algebra teaching whereby the teacher introduces learners to a new topic by demonstrating 'worked examples' and then seeks to reinforce the procedures involved through extensive practice exercises. This has been a case in this study as most of the time the teacher would come in, teach and give learners an exercise to do.

Reeuwijk (1995) suggests revisiting the topic slowly but thoroughly developing mathematics. The step towards formal algebraic manipulations is made slowly. At first, the context supports the manipulations. Even when the step towards a higher level of abstraction and formalization is made, the student can always 'fall back' on the context. This approach has implications for instruction. The teacher needs to allow learners to develop the mathematical concepts at their own pace. The role of the teacher becomes that of a facilitator and guide in the learning process of the learners. The teacher facilitates the learners' reinvention of mathematics by means of leading class discussions, knowing how to deal with different answers, and so on. This new role is not easy, but the experiences so far show that more learners are learning and enjoying mathematics (Wijers, 1995).

Freudenthal (1991) discovered that reflection and discussion are critical factors for the teaching and learning process. In Realistic Mathematics Education (RME), interaction between learners and learners, teachers and learners and teachers and teachers are all-important ingredients. Interaction can take place in many ways, one of which is cooperative learning. It is up to the teacher to decide what the most appropriate form of instruction is for the learners and for the situation in the class. Interaction is important because it leads to crucial activities of problem solving: cooperation, discussion, sharing and reflection.

2.10 Summary

From this chapter the researcher has given a definition of school algebra; looked into the research into learners' dealing with school algebra and its importance to this study, much concentration was on the research findings; the notation and convention in algebra; letters and variables; learners understanding and misunderstanding of arithmetical conventions; learners' use of informal methods and views on teaching algebra. In conclusion, the literature review helps the researcher to have a broad understanding of the issue under investigation, which was used to depict the source of relevant information to answer the research question.

Chapter 3

Research Methodology

3.1 Introduction

This chapter discusses the study's research methodology. Research methodology is about the whole range of questions pertaining to appropriate ways of going about social research (Hitchcock and Hughes, 1995). Silverman (2000) defines methodology as a general approach to studying topics and the choice of methods (tools) of the overall research strategies. Likewise, Wellington (2000) describes methodology as a procedure or a mechanism used to decide, reflect upon, ascertain and validate tools employed to address research questions. In short, methodology refers to the ways in which the general scientific statements or procedures of disciplines or perspectives are acted out in research situations (Hitchcock and Hughes, 1995).

The research methodology describes the research design, data collection procedures and instruments, the data analysis framework and the ethical issues relevant to investigating the question(s) under study.

3.2 Research design

A research design is a logical sequence that connects the empirical data to the research questions of the study and, ultimately to its conclusion (Yin, 1997). It provides the conceptual framework for the procedures used in collecting the data. According to Ogunniyi (1992), a research design presupposes the kind of methods to be used and the type of instruments to be developed and used in collecting appropriate data. In other words, the research design helps the researcher to spell out clearly what they do with participants (subjects) and the procedure to be followed during the course of the investigation. For the researcher, the first methodological challenge is to select a suitable design for the study. The design chosen for a study depends on the nature of phenomena being investigated (Wellington, 2000).

This study used both qualitative and quantitative research methodologies. A combination of the two methods was envisaged to be appropriate because of the need to get in-depth information on the topic under investigation. A qualitative approach allows the interaction between the researcher and the informants. This interaction makes the approach particularly appropriate to elicit adequate information from both the teachers and the learners, since learning does not happen in isolation from teaching. A quantitative approach gives a broader understanding of the phenomena and a qualitative approach gives a deeper understanding of the phenomena.

Quantitative methods and quantitative instruments have got the advantage of studying large numbers of participants and easy data collection. However it is generally believed that qualitative approaches can achieve greater validity than quantitative methodologies (Cohen *et. al*., 2000). According to Cohen and Manion (1994), a qualitative approach has the disadvantage that the analysis of responses can be both time-consuming and difficult. Another disadvantage that they state is that reliability tends to be poor as it is determined by too many variables, for example the nature of the research and the researcher's experience.

To explore grade 7 learners' perceived experiences in algebra, three data collection strategies were used. These are: semi-structured and open-ended interviews for learners; a learner questionnaire and classroom observations. Triangulation was used to corroborate and validate information from the three data sources. Cohen and Manion (1989) assert that multiple data sources increase the validity and reliability of research findings. The different methods of data collection can complement each other's shortcomings. Furthermore, the use of the questionnaires and the interviews in this research helped the researcher to obtain in-depth information about the learner's perceptions. Observation alone could not fully provide this insight.

The questions used in both the questionnaire and during the interviews were the same. Learners and their teacher were asked questions about the topic (algebra) that they had just done. Figure 1 below summarizes the methodological framework used in this study (Cresswell, 2003).

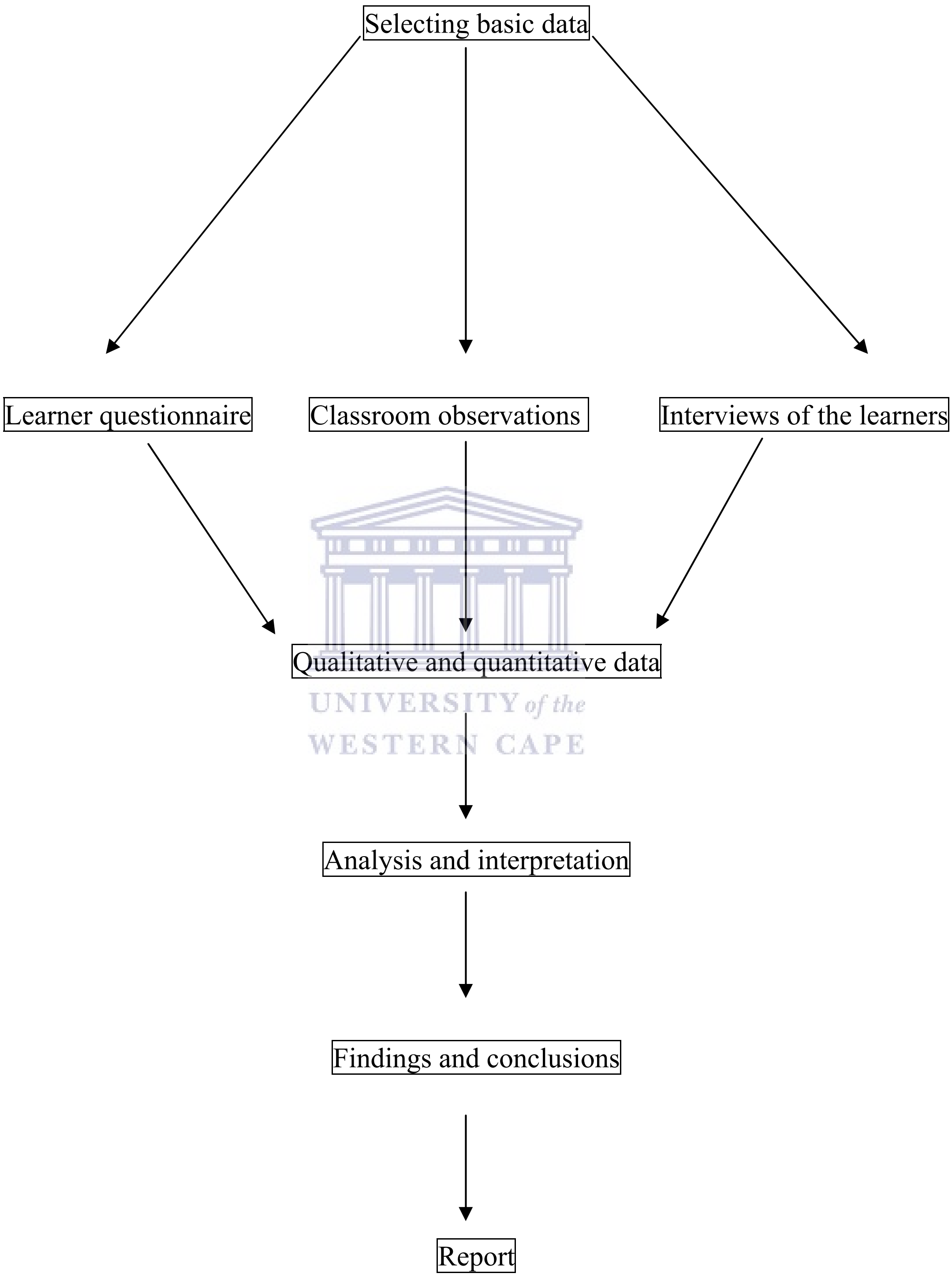

Figure 1: Methodological framework

3.3 Procedure

The chosen school was visited to make arrangements and see the teachers' work plans. A meeting was then held with the grade 7 mathematics teachers to find out when they were going to start with algebra. Although a time frame was given, the researcher was told to keep calling to find out when the teachers were to start with the said topic. During the beginning of the topic, observations were done. Notes were taken with the guide of an observation schedule that was developed by the researcher.

Respondents completed the questionnaire in their classrooms and altogether 90 questionnaires were successfully completed. In the first place learners were given questionnaires to complete at home. Not all were brought back; those that were brought back had problems, as learners could not answer all the questions clearly. The same questionnaire therefore had to be redone, and administered in class by the researcher. Learners were given a chance to ask the researcher wherever they did not understand the questions.

Preliminary meetings were held with the principal and the grade 7 mathematics teachers of the selected school to solicit for their support and to arrange for an appropriate time for the questionnaire to be administered to the learners. The learners were assured that anonymity and confidentiality would be maintained within the boundaries of the research process. Respondents were encouraged to express their views without discussing them with their peers and to use any language of their choice (Sotho or English). The aspect of anonymity and confidentiality was emphasised. Interviews were done and learners signed a consent form. See appendix 3 for the details on the form.

3.4 Sampling

Sampling is the process of selecting the site and number of individuals for the study in such a way that the number represents a large group from which they were selected (Behr, 1988). Babbie and Mouton (2002) say that a sample is a special subset of a population observed in order to make inferences about the nature of the total population itself. According to Gay (1981), the population is the group of interest to the researcher, the group to which they would like the results of the research to be generalized. The individuals selected comprise a sample and the larger group is referred to as a population. The purpose of sampling is to gain information about the population.

The concept of sampling involves taking a portion of the population, making a study of this smaller group and generalising the findings to the large population. Generalising is a necessary scientific procedure, since it is rarely possible to study all members of a defined population (Behr, 1988). According to McMillan and Schumacher (2001), in random sampling, participants are selected at random from the population so that all members of therein have the same probability of being chosen.

3.4.1 Sampling learners for the interviews

Random sampling was used to select learners for the interviews; this was adopted from McMillan and Schumacher, (2001). Each learner had to pick a number of their choice and those whose numbers had the digit five in it were to form a group of the interviewees, only ten numbers had the number five for each table. In order to avoid missing or left over numbers, two tables were used. One table had 46 numbers as one of the classes had 46 learners and the other table had 48 numbers as the other grade had 48 learners. (See appendix 1 for table 1 and table 2 for random sampling).

3.4.2 Choosing the school

There are seven education districts in the Western Cape Province, and the study was only limited to one education district, in the Metropole East. This Primary School is a dual medium school, catering for both Sotho and Xhosa speakers. Many of the pupils come from sub-economic areas where poverty and other related social problems are endemic. The school has a lack of adequate teaching and learning resources. These are just some of the features that, in the researcher's own opinion, contribute to some of the unpleasant experiences learners have. There were quite a number of learners in the school (858) even though there were not enough buildings. There were also a number of pupils in each of the two classes and according to the principal it is due to a general lack of buildings. As a result learners are overcrowded in their classrooms. There are 48 learners in one grade 7 class and 46 in the other. The researcher decided to observe both classrooms, give questionnaires to all the learners in both classrooms and interview only twenty learners, ten from each grade. The twenty learners interviewed were chosen randomly. The researcher found it convenient and manageable to interview only 20 learners.

The reasons for selecting the school that was used in the research were firstly, that the school was accessible and convenient to the researcher during her stay in Cape Town. The school was also convenient in that the researcher could use her mother tongue language given that it is a Sotho speaking school and it was therefore easy to communicate some issues in Sotho with the learners. Secondly, it is in this school that the researcher had made some classroom observations in 2002 and the researcher was therefore familiar with some teachers and the principal. This helped the researcher to communicate easily with the respondents and the researcher was also able to get a fair level of cooperation and support from the teachers and the principal. This included getting access to the premises, calling teachers for the meeting in order to arrange for the observation in classes, and providing me with a separate and suitable office to conduct the interviews. The school as a sample was convenient and opportunistic and as indicated before the learners were randomly selected for the interview.

A purposive sampling was used to select this school in its area. In the first place the two classrooms were observed, and ninety-four learners were each given a questionnaire to complete. Twenty of the learners were interviewed, ten from one class and the other ten from the other class.

3.4.3 The learners

Two classes were observed and participated in the quantitative data collection part. This means that 94 learners were requested to complete the questionnaire. Most of the pupils came from low socio-economic status families. The age and gender distribution of the learners are presented in table 1 below.

Table 2: Age and gender distribution of learners

Age	**No. of respondents**	**Females**	**Males**	**Responses %**
12-13 yrs	10	6	4	11%
14-15 yrs	54	33	21	60%
16-17 yrs	25	13	12	28%
18-19 yrs	1	-	1	1%
Total	**90**	**52**	**38**	**100%**

3.5 Research instruments

3.5.1 Classroom observations

Besides the use of questionnaires and the interviews, observation was another research technique employed in this study. According to Mouton and Babbie (2002), when doing participant observation, one is faced with the difficulty of simultaneously being one of the members of the group, and also observing everyone else from the researcher's point of view. This can become a dilemma at times. However, I decided to be a participant observer. I believed that being a participant observer helped learners not to change their behaviour as I was part of them, but when knowing that they are being observed their behaviour might have changed and I might not have got what I really needed to know about their perceived experiences in algebra. Denzin (1989b) defines participant observation as a field strategy that simultaneously combines document analysis, interviewing of respondents and informants, direct participation and observation and introspection.

Mouton and Babbie (2002) are in support of the idea of being a participant observer however they indicate that the researcher observes from a member's perspective but also influences what is observed owing to his/her participation. An observation has also been made that "it is a logic process of inquiry that is open ended, flexible, opportunistic, and requires constant redefinition of what is problematic, based on facts gathered in concrete settings of human existence" (Jorgensen, 1989:34)

Like any other method of data collection, participant observation also has its problems in conducting it. One problem according to Flick (1999) is how to delimit or select observational situations in which the problem under study becomes really 'visible'. According to Spradley (1980), social situations may generally be described along the physical place, the people involved, a set of related acts people do, a set of related activities that people carry out, the sequencing that takes place over time, the things people are trying to accomplish and the emotions felt and expressed for observational purposes.

In addition, the other problem with this method is that not all phenomena can be observed in situations. Biographical processes are difficult to observe and not easily understood, but also comprehensive knowledge processes are not accessible to observation. Events or practices that seldom occur, although they may be crucial to the research question, can be captured only with luck or if at all by a very careful selection of situations of observations. As a way of discovering these problems, often additional interviews of participants are integrated into the research programme that allow the reconstruction of biographical processes or stocks of knowledge which are the background of practices that can be observed. Therefore, the researchers' knowledge in participant observation is based only in part on the observation of actions. A large part is grounded in participants' verbal statements about certain relations and facts (Flick, 1999).

As a participant observer, the researcher attended classes and became part of the class, the researcher had with her the pen and notebook ready at all times and took down some notes. It was not easy for the researcher to note everything since she was not only an observer but she was also the participant as well. The idea of being a participant observer was adopted so that the learners could not change their behaviour in the class. During the observation the researcher was looking for the learners' perceived experiences, things they did and said in class, their interaction with their teacher and with their peers in class.During the observation period, the researcher took notes on incidents and critical issues that arose. Observation notes were typed and returned to the concerned teachers to check if it captured a correct record of the things that had happened in their classrooms during my observations, and thereafter were analysed. For this study the observations were done from the beginning of each mathematics lesson until the end of the lesson. The researcher observed the classes for all the mathematics periods over a three-week period. Some days

Mathematics lessons were double periods and others single periods. Each period lasted for 55 minutes.

3.5.2 Interview and interview questions

Besides the questionnaires and observations, interviews were another research instrument used in this study. An interview, according to Cohen and Manion (1989), is a purposeful conversation between the researcher and the subject to gather descriptive data in the subject's own words so that the researcher can develop an insight into how participants interpret some phenomenon. In this research an interview format was developed for the learners in order to obtain more information about their perceived experiences in the learning of algebra. Cohen and Manion (1989) describe the interviews as used for gathering information that provides access to what is inside a person's mind. Through interviews, it is possible to collect information about what a person knows (knowledge information), and what a person thinks (attitudes and beliefs). The main purpose of interviewing is to determine those feelings, thoughts and intentions that cannot be directly observed by the researcher, allowing entrance into another person's perspective (Behr, 1988).

Tape recording is one of the most convenient techniques of recording the interview. This technique allows the researcher to maintain eye contact with the interviewee. It is more appropriate for recording emotional conversations (for example, if someone is crying or shouting) and verbal conversations.

A structured interview is one of the most frequently used methods of eliciting information in social and educational research (Cohen and Manion, 1989). However Behr (1988) states that a structured interview is one in which the procedure to be followed is determined in advance. An interview schedule is prepared in such a way that the pattern to be followed, the wording to the questions, instructions and methods of the coding and categorising the answers are detailed. Ogunniyi (1992) argues that a structured interview may prevent the tendency to wander off the subject matter but it might also limit the level or the latitude of response obtained. Behr (1988) also agrees that in the unstructured interview, the interviewer is permitted to use their discretion and to depart from the questions prepared beforehand, which makes it difficult to compare the data obtained from the various respondents so as to arrive at reliable generalizations.

According to Cohen and Manion (1989) there are three kinds of items in the construction of schedules in a structured interview, namely; “fixed-alternative items, open ended items, and scale items.” In this research, the interview is called a semi-structured interview (Behr, 1988). Its interview questions are flexible; allow the interviewer to probe so that she/he may go into more depth if she/he chooses, or clear up any misunderstanding. They enabled the interviewer to test the limits of the respondent’s knowledge. They encourage cooperation; help establish rapport and they allow the interviewer to make a true assessment of what the respondent really believes (Cohen and Manion, 1989). (See appendix 4 for the interview questions and appendix 3 for the consent form).

From the researcher’ experience as a teacher, the researcher was aware that interviews, particularly tape-recorded interviews, are a threatening experience for many learners. Nevertheless, effort was made to establish a rapport with all the learners who were the participants of the study. The researcher played with the learners some of the indigenous games that they were playing in school at lunch during the observation period. All the interviews were conducted at the school as it suited the preference of the participants. This allowed us to communicate effectively and to elicit more information. This effort was helpful in increasing the reliability of the study and establishing rapport with the respondents.

A convenient time was arranged for interviews so that it would not cause any disruption in the normal functioning of the school. A mini cassette recorder was used to capture the data and all interviews were conducted in the special needs classroom and sometimes the principal’s office. Dalen (1979) has acknowledged this aspect, suggesting that most interviews have to be conducted in a private setting with one person at a time so that the participants feel free to express themselves fully and truthfully. The interviews were transcribed. The researcher had to replay the cassettes and write down the answers and regroup the answers according to the questions, and the same answers were discussed and put together as one point.

For this study the normal duration for the interviews was 35-40 minutes, including the first five minutes reserved and used for informal conversation to introduce the interview process and create a culture of trust, respect and honesty. This further promotes the interpersonal relationship. Mouton (1996) alludes that the advantage of strong interpersonal relationship between researcher and participants is that it neutralises initial distrust. The interviews were done over a week. The interview questions were used to elicit learners' views and experiences in algebra.

3.5.3 Questionnaire

The questionnaire is a cheap and quick instrument to obtain a lot of information covering a large area within a relatively short time. By the same token, a large percentage of questionnaires are never returned even after several dispatches to the participants. The lack of personal, face-to-face contact increases the objectivity of responses, but also increases the problems of ambiguity. Since the researcher is not around to clarify difficulties that may be encountered by the participants, several items may not be responded to, thus reducing the reliability of the response. Among the many problems common to questionnaires the most obvious relates to ambiguity and poor return rate (Ogunniyi, 1992).

According to Bell (1987), when a self-administered questionnaire is compared with a personal interview, each has its own advantages over the other in certain respects. A questionnaire tends to be more reliable, because it guarantees confidentiality; it helps to avoid fear and embarrassment, which may result from direct contact. It allows respondents to be free to answer in their own time and at their own pace and it enables the researcher to collect large quantities of data from a considerable number of people over a relatively short period of time. On the other hand, in a questionnaire there might be a low percentage return rate, lack of clarification for the respondents if there is ambiguity among the items or if only open-ended items are used and respondents who are unwilling to write their answers for one reason or another.

In this research, a self-administered questionnaire was purposefully used to extract learners' perceived experiences with regard to the learning of algebra and also to investigate how they feel about the topic as a whole. The questionnaire was hoped to access and measure respondents' preferences, their attitudes and their beliefs about algebra. (Tuckman, 1978). The main advantage of the questionnaire is that it is a useful means of obtaining information about sensitive issues. It would therefore, be easier for the learners to disclose their feelings with regard to the learning of algebra without the fear of being censored. Sensitive topics can therefore be explored more accurately.

The questionnaire comprised of two parts. The first part was composed of questions that were aimed at unearthing the biographical details of learners. The second part was composed of questions focused on the learning of algebra and the experiences learners have in learning algebra. The items in the questionnaire were open-ended and there were also some closed ones. The questionnaire was developed to find learners perceptions about algebra. An example of the questions is: was the topic easy to understand? Why? (See appendix 2 for the learner questionnaire).

Different approaches can be used to fill in the questionnaire. For this research, learners were given questionnaires in their different classrooms during the time when the purpose of the research was explained. Forces and Richer (1973) recommended this procedure as it helps to avoid some of the negative consequences of a self-administered questionnaire. The following are noted advantages according to Forces and Richer (1973), it creates some pressure for the learners to participate; an opportunity to clarify unclear instructions, and it will be easier to collect the completed questionnaires and will be easy to follow up incomplete or unclear answers.

3.6 Validity and reliability

To ensure that this study and the instruments used measured what the study intended to measure, and that the study and the instruments used would give comparable results when used again to collect the same kind of data, several corroboration techniques were used. The following precautions were undertaken to ensure that the research instruments used in the study have attained high validity and reliability.

The first questionnaire was done and tested on the same class involved in the study. Out of 90 questionnaires only four came back. The reason for this according to participants turned out to be that the questionnaire was too long and difficult to answer. Steps taken in the process included a thorough scrutiny of the instrument by a panel of four teachers who gave their comments about their suitability for gathering the required data. The panel was chosen randomly based on their availability. The same instrument was given to this panel of four people. The panel was given a brief summary of the participants' histories and that of their school as explained earlier on in this chapter. Thereafter they were asked to assess the questionnaire based on a given a criterion. They were also allowed to go beyond this criterion if they found it necessary. This panel looked at the questionnaire at different times and independently of each other, and the researcher approached them as individuals.

The panel was specifically required to assess whether or not the questions were presented in a comprehensible manner for the learners. Whether or not the level of language used was appropriate for the target learners. If the question in both the interview and questionnaire would capture the desired data. Whether the questions were presented in a logical manner and also whether the questions were clear and could identify the overlapping and poorly structured questions. They were to look at the flow and linkage of the questions and whether the questions were adequate enough to answer the research question. They were asked to give their comments and recommendations, which would then be used to refine the final versions of the instruments. All the instruments were trial tested in a class comparable to the one involved in the actual study. Particular attention was paid to the English language used in the instruments and the objectives of the instrument.

The results of the panel were looked into and analysed. The panel recommended that the researcher should leave out certain questions as they were too general and would not give specific answers to the research question. They also thought that questions should be shortened and that the researcher should avoid combining two questions as one question as that would confuse the participants. They recommended that the level of English be reduced to lower standard as they felt that it was somewhat complicated for the intended participants.

A systematic development of questionnaires was done. The instrument was then structured in such a way that it had two sections. Section one was done on its own day and section two on its own day as well. This was done to avoid learners having to answer a lot of questions at once. The researcher also thought that many questions might leads to the point whereby they just answer the questions without giving a thorough thought, or coming to the point where by respondents just want to get rid of either the questionnaire and or the researcher.

The first part of the questionnaire included questions about the topic (algebra, which in this case they had just done); the learner and the teacher. The instrument was then given to the participants at two different times on two different days. Participants were able to answer the questions and it was evident that the instruments were up to their standard and appeared adequate.

According to Hammersley (1992) in Silverman (2001), reliability refers to the degree of consistency with which instances are assigned to the same category by different observers or by the same observer on different occasions. Huberman and Miles (1994) define reliability as a measure of whether the process of the study is consistent, reasonably stable over time and across researchers and methods, they speak of quality control and whether things have been done with reasonable care. In this study, a reliability test was not done but a number of steps were followed to ensure dependability/authenticity. Silverman (2001) says that 'authenticity' rather than reliability is often the issue in qualitative research. The aim is usually to gather an authentic understanding of people's experiences and it's believed that open-ended questions are the most effective route towards this end.

3.7 Data presentation and analysis

Analysis is the process of bringing order to the data, organising what is there into patterns, categories and basic descriptive units (Patton, 1987). The data obtained from the qualitative research was analysed. The data from the questionnaires was subjected to descriptive analysis. Descriptive analysis is a set of concepts and methods used in organising, summarising, tabulating, depicting and describing collections of data (Cohen and Manion, 1989).

Data obtained from observations was summarised through content analysis. According to Patton (1987), content analysis involves identifying coherent and important examples, themes, and patterns in the data. The analyst looks for quotations or observations that go together, that are examples of the same underlying idea, issue, or concept. Sometimes this involves pulling together all the data that addresses a particular evaluation question (Patton, 1987).

In the qualitative approach, the data collected by the interview method were presented by writing up the raw data as themes and categories. These themes and categories were then analysed using conceptualisations. Finally, the three methods of data collection were triangulated to examine the possible trends of the responses, and relationship among concepts.

3.8 Ethical issues

Permission was obtained from the University of the Western Cape, and the identified school to conduct the research. All persons involved in the research were informed of the purpose of the study. Participation in the research was voluntary and thus the respondents were invited to cooperate in the study and they were made to feel as relaxed as possible. Guarantees of confidentiality were given to the sample participants. They were told that information from the individuals was to remain confidential and that individuals would not be identified. It was assured that information given was treated with confidentiality and discretion. The results of the research would be made available to the school upon request.

3.9 Conclusion

In conclusion, this chapter elucidated the concepts and ideas pertaining to the methods that were used for data collection. In doing this, much emphasis was placed on the key research technique of the method used to collect data. To this end, the questionnaires, observations and interviews were the key instruments in this study. According to Cohen and Manion (1989), the inclusion of multiple sources of data collection in a research project is likely to increase the reliability of the observation. The use of both methods in this study made triangulation possible. Therefore the questionnaire, observation and interview were found more acceptable and helpful. The use of the three instruments helped the researcher to elicit the grade 7 learners' perceived experiences in learning algebra. The results of the investigation are reported in the next chapter.

Chapter 4

Results and Discussion

4.1 Introduction

In this chapter the results of the study are presented, analysed and discussed. The first part of this chapter presents and discusses results from the questionnaire and the interviews. This is followed by presentation and discussion of the findings from classroom observations of the two teachers and their learners. As mentioned in chapter three, the researcher conducted the study using a questionnaire, interviews and classroom observations. The responses of the learners from the questionnaire and the interviews were grouped according to the themes that emerged from the analysis. Results from the questionnaire and interviews are presented together since similar questions were asked to learners in both the questionnaire and interviews. This was seen as convenient because it allowed triangulation and validation of the collected data. Notes from the lesson observations were analysed and interpreted with the view of capturing the nature of the learners' experiences in algebra.

4.2. Results from questionnaires and interviews

In this section, results are presented and discussed on: learners' feelings and views about algebra; the usefulness of the topic in life, the clarity of the teacher, the level of attention they need from her, and how best they think they could learn algebra.

4.2.1 Learners' views about algebra

Learners gave a variety of answers to the question: How do you feel about the topic (algebra) that you have just done? This question elicited learners' views about learning algebra. Learner responses to this question are summarised or shown in Figure 1 below.

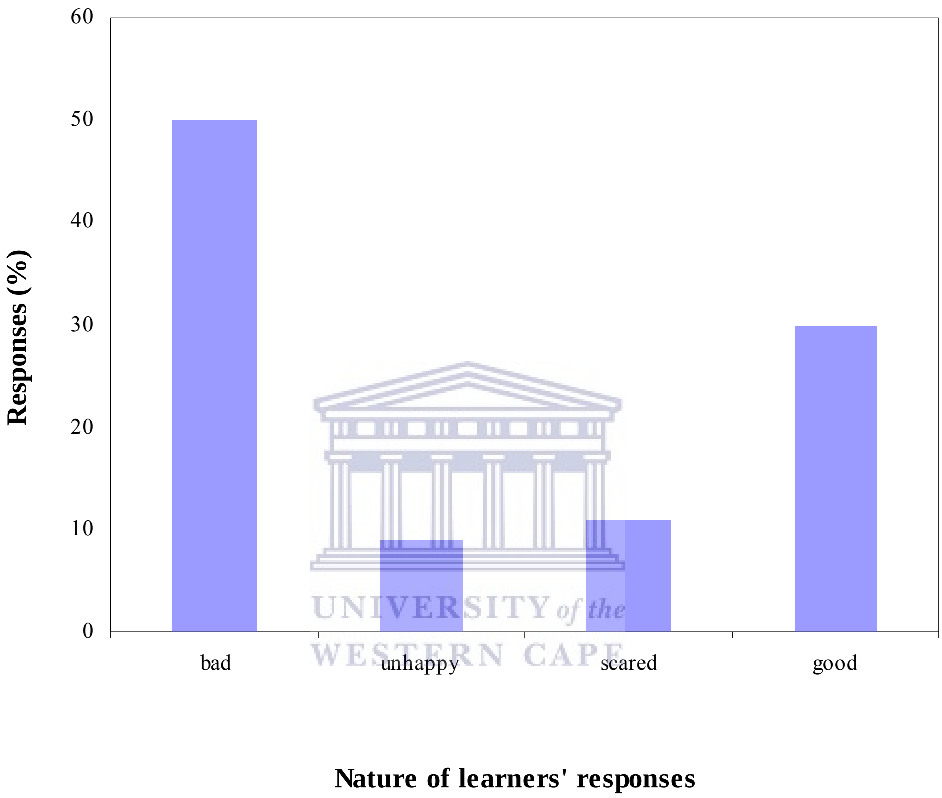

Figure 1: learners' views/feelings about algebra

Learners' had shown that they felt bad, unhappy and scared because they did not understand the topic and they were concluding that they were not going to do it the following year. During the interview some of the learners said that they did not understand the topic because they did not ask questions while the teacher was teaching. Others said that they could not understand the teacher herself and others were saying that they never asked questions because their class was noisy. As a result they did not understand what the teacher taught.

One of the learners commented:

> I felt so scared that I am going to fail the test and unhappy. Because I did not even understand that topic and I did not even try to ask questions.

From the questionnaire responses, 70% of the learners said that the topic itself was difficult to understand. The topic had so many steps to do. Some said that they were doing the work that they cannot even explain or follow. They just did the work because the teacher gave them work to do. The learners said that they also found problems when changing signs. They did not know when and why to change the signs. They continued and said that they did not understand what they were doing and their teacher was fast, though they were doing the topic for the first time. During interviews 16 of the 20 learners said that they did not feel good about the topic. One of them said:

> I am not happy at all because I did not understand anything. The topic was difficult.

Another learner said:

> I am unhappy because I didn't understand the questions asked, because I did not understand what was taught.

From the questionnaires 30% of the learners who felt good about the topic said that the topic was enjoyable and easy to understand. During the interviews, 4 of the interviewees said that they felt good because they understood the topic. Some reasons they gave during the interviews included that they answered the exercise given correctly and were able to help those who did not understand. A number of them showed that they enjoyed and liked the topic.

One of them said:

> I felt good and enjoyed the topic because it was not difficult to do; and I learned something from it.

Another learner who was in agreement said:

I was so happy because the topic was easy for me, I could simplify and add. I concentrated, shut my mouth and listened to the teacher. Now I know mathematics better than before, and wish I could go to the next level.

While they were being interviewed, they said that although there were many steps, they believed that the steps were of help as they could see when and where they had gone wrong. They said that they understood as their teacher explained and the teacher had used a lot of examples. They even felt that the topic allowed them to show their mathematics knowledge like adding and subtracting. They concluded by saying that they liked it when their teacher asked them to help others who did not understand. They added during the interviews that helping others helped them improve their own knowledge and also gave them practice in what they did. They mentioned that they discovered new methods with the help of those whom they are helping.

4.2.2 Learners' response to questions about algebra

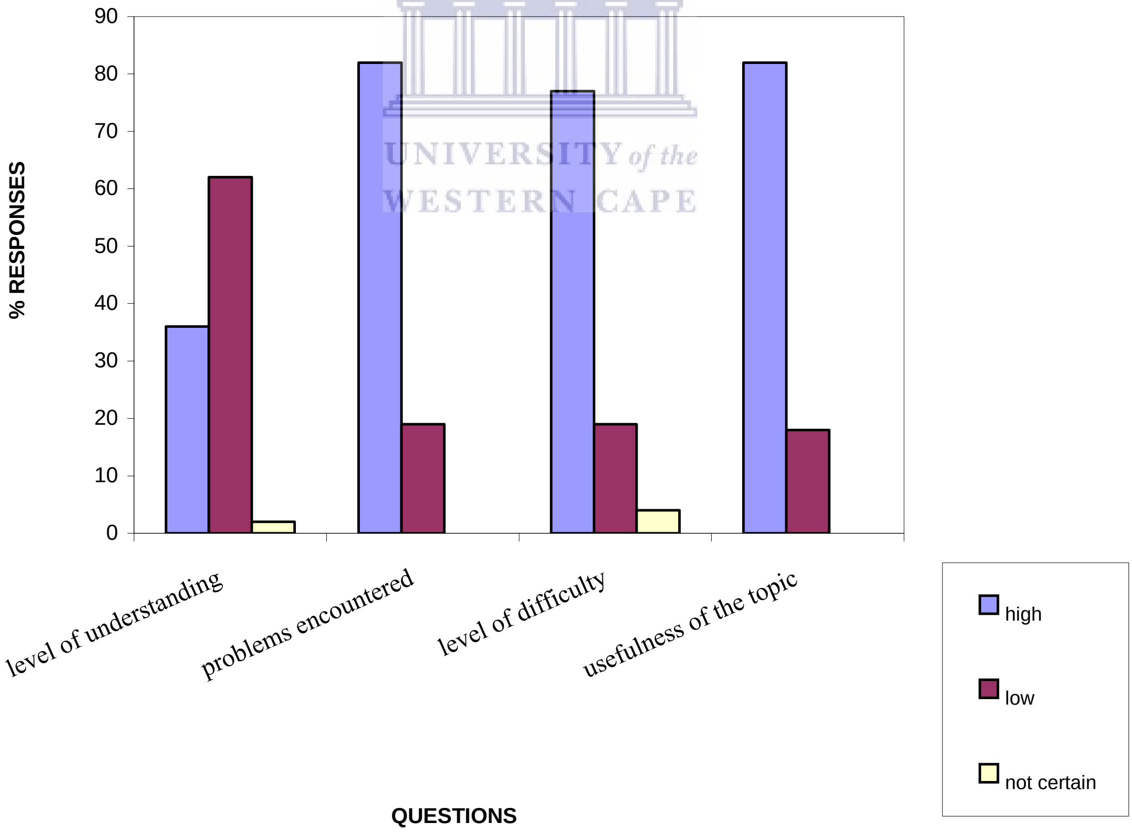

Figure 2: **Learners' views on difficulty and usefulness of algebra.**

Figure 2, summarises the questions below:

- Was the topic easy to understand? Why?
- Did you find any problems with the topic? Where?
- Did you find the topic difficult? Why?
- Other than learning it at school, do you think the topic can help you anywhere in life? Where?

Answers found from the questionnaires were yes, no and uncertain, and learners had the chance to give reasons for their answers. In the interviews, learners said the topic was easy because their teacher explained everything until they understood and when they were given work to do, they did not experience any problems. They said that they paid attention to their teacher and asked questions where they did not understand. The statement below captures what the learners said.

> Because our teacher does teach, then ask us to come to the board and work out some of the sums, ask questions to the classmates and explain how we came across the answer and give them chance also to ask questions. We listened when she was teaching and asked her questions where we did not understand.

Those that answered "no" said that the topic itself was difficult and they did not understand their teacher. When they were interviewed they said that the topic had too many steps to do. The teacher's explanation was not clear and some learners were playing and talking, which disturbed them. Some of the learners said that it was their first time to do the topic, and yet they did not receive enough time to grasp the topic and the topic had a lot of things to deal with. Some learners said that they believed that the topic was not difficult; it was just that it was their first time to do the topic.

One of them said:

> May be next year after doing the topic, we shall be able to tell if the topic is easy or difficult.

During the interviews learners who mentioned that they had problems said that it was because they did not understand what they were asked to do. They had problems with addition and/or subtraction, especially when variables were not the same. Some had problems changing signs even if the variables were the same as in $2x - 4 + 3x = 7$. Others had problems even if the variables were the same, but mixed with numbers. They had indicated even on their questionnaires that the more variables involved, and the more steps they had to deal with, the more the topic became difficult.

Some learners had indicated during interviews that they had problems with like and unlike terms. They mentioned that they could not say when it is a like term and when it is not a like term. Learners mentioned during the interviews that they also got confused when they were to tell the coefficient of a variable in a term. One of the learners said:

> I did not understand the terms, and then I was asked to tell the coefficient, that became even more difficult for me.

From 20 learners interviewed, 16 had shown that they did not encounter a lot of problems with algebraic expressions. They said they believed it was because their teachers had worked through a lot of examples, which had helped them to understand better. The statement below states what they had to say.

> We had fewer problems with algebraic expressions as our teacher explained using a lot of examples.

Above all these issues, they indicated that they had a problem with the number of steps involved in the exercise they had to work out. From the questionnaire responses, 70% of the learners said that the topic itself was difficult to understand as it had so many steps to do. Some said they could not "explain or follow" the work. Those who said this thought the teacher just gave them work to do without explaining. In particular, the learners reported that they had difficulties with algebra problems that required them to change signs as they did not know when and why the signs should be changed (for example changing from plus to minus). Some felt that the teacher was fast as they were doing the topic for the first time. During interviews 16 of the 20 sampled learners said that they did not feel good about the topic, especially with the number of steps that they were to do when adding and subtracting from the equation. One of them said:

> I am not happy at all because I did not understand anything; the topic was difficult.

Learners were worried that they did not understand what they were taught. Arcavi (1994) notes that: 'even those learners who manage to handle the algebraic techniques successfully, often fail to see algebra as a tool for understanding, expressing and communicating generalisations; for revealing structure; and for establishing connections and formulating mathematical arguments'. It is therefore not something new.

In agreement with the above statement, for many learners, the whole subject of algebra is frequently associated at a very early stage with tasks, which appear to lack any meaning or serve any obvious useful purpose. This naturally leads to failure and aversion. French (2002) also agrees and says that even many of those who provide 'right answers' to routine exercises, commonly come to feel that the subject is obscure and unrelated to anything that is real or of interest to them.

About 4 learners indicated during the interviews that they were not in class when the topic was introduced. These four learners also said that the topic itself was difficult. Only 19% of the learners said that the topic was not difficult. The same four learners said that they had worked hard with their teacher and peers but still could not understand what they were doing. The 4% uncertain learners said that they understood some of the things, but just got lost and never followed thereafter. They said they felt confused, that was why they could not say it was either easy or difficult. While in the interview, one of the learners said:

> I used to be confused because today I would understand, but the next day I cannot remember what I understood yesterday.

The learners, who said that they were uncertain about the topic, said that on some days they would understand what has been said though they would soon forget what they did. Therefore they were not sure if the topic was really difficult. 80% of the learners showed that in life the topic will be important. During the interviews they were able to mention some careers to which they thought algebra might be applied. This is what some of them said:

- When I pass my grade 12, I want to do engineering at the university; therefore I want to understand maths.
- I want to be a mathematics teacher.
- I want to have my own building company, so I will do carpentry and building at the university.

4.2.3 Learners' views about their mathematics teacher

Learners said that it is the teacher's responsibility to see that they understand and the teacher must explain to make the topic easier. They believed that if the teacher explains thoroughly that could help them understand the topic better. Some learners complained that the teacher at times became angry and shouted at them when they asked questions or said that they did not understand. They said that she would even lose her temper and often used strong language. As a result they became scared of her. Some learners said that the teacher should have used both English and Sotho to explain so that they could understand better. One of the things that make learning enjoyable is a good learning environment. According to the learners' answers on the questionnaire and during the interviews, the environment was not conducive for learning.

Looking at what has been said by Julie *et. al.* (1998) as stated in chapter 2 of this study, learners' mother tongue plays a major role in learners' understanding of a concept. I therefore agree with the learners when they request the use of both Sotho and English during the class for better understanding.

4.2.4 Teacher's attendance to learners

From the question: how much attendance do you want from your teacher? Almost all the learners wanted the same thing from their teacher. They wanted her to give them enough attention and assistance in order to understand what she was teaching. They wanted her to answer their questions in full and clearly. They needed the attention from the beginning to the end of the lesson. Some of the learners felt that the teacher should be there everyday in class. While others felt that they needed about 30 minutes to an hour of extra attention on weekends or after school.

4.2.5 Learners' ideas about how they can best learn algebra

Learners said that if they were to do the same topic again, they would want their teacher to explain clearly and thoroughly. The teacher should help those learners that experience problems and repeat the work at least twice or three times a week. During the interviews they said that their teacher should deal with one thing at a time and for a number of periods. For example, if they are dealing with like terms, at least they should do them for two or three days until they all understand. This is what one of them said during the interviews:

> We can understand better if next year our teacher can use a lot of examples when we do the same topic. She must come to class everyday and we should do the work for at lease three periods.

Some learners felt that learners needed to be less noisy in class and listen to the teacher. They also indicated that they wanted to work in groups and wanted the teacher to attend to their groups. Learners have shown, during the interviews, that the noise level in their classes impacts on their learning, as they are not able to concentrate when it is noisy. Therefore they do not want to be given work to do on their own. Some learners felt that it would be good if they and their teacher worked together and discussed the topic before hand. They should also be given a chance to say how they would like to do the topic.

One of them said:

> The teacher must do the work with all of us in class, then after that work with those who have problems.

From the questionnaires and during the interviews, learners indicated that their teacher shouts at them. So they felt distracted and therefore they would want that kind of attitude to stop as it makes them uneasy, and scared to ask questions. They wanted their teacher to use different examples when teaching before she gives them work to do. When teaching they wanted her to demonstrate each step slowly and make sure all learners understood.

Learners were asked if they would like it if their teacher were to leave the work for them to do on their own and were asked to give their reasons for their choices. Figure 3 below shows their responses.

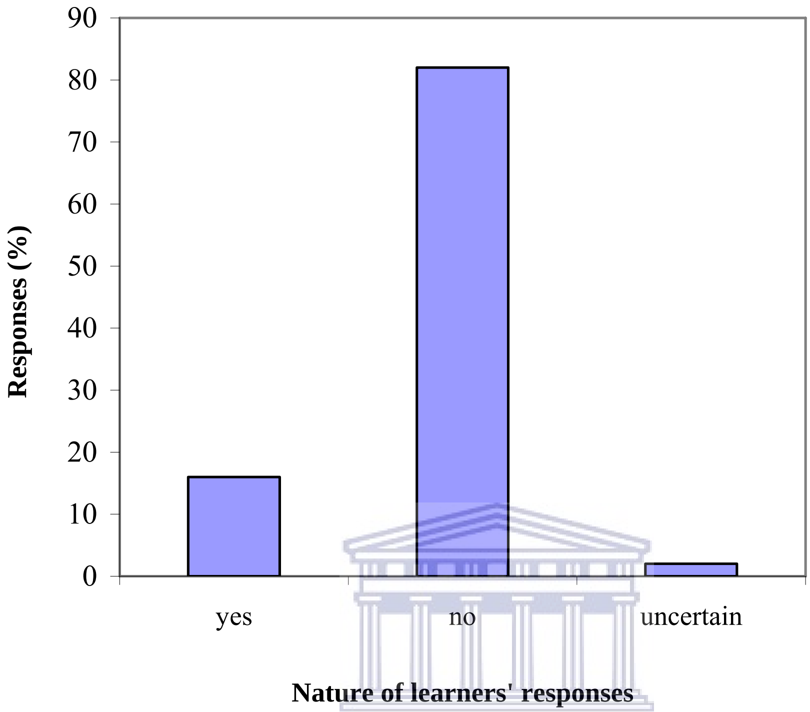

Figure 3: Learners' ideas about how they can best learn algebra.

Sixteen percent of the learners said that they like it when they are given work to do on their own, because this helps them to see if they understood what has been done in class. They also thought that this would help to minimize noise, as people would be busy working. During interviews they said that they believed that they would have understood well than when the teacher was teaching. Other learners believed that this would help their teacher to see if she explained well and be able to identify their problems.

Eighty two percent of the learners said no, because they needed to ask her questions while they did the work. They believed that they would need her assistance because in her absence the class became chaotic and work ends up not being done. So they did not want her to leave the work for them and ask them to do it on their own. They indicated that they needed the teacher to be in class at all the times.

When they were asked if they like it when she teaches first and then gives them work later. This is what they said. First presented on a graph then their reasons for their answers.

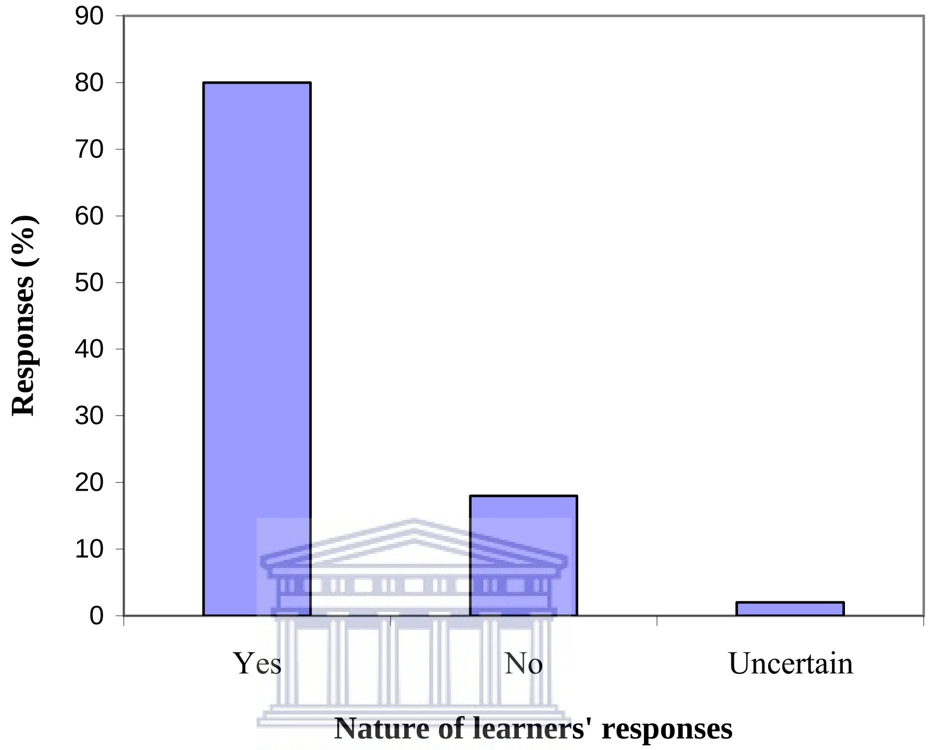

Figure 4: **Learners' views/feeling about algebra.**

Eighteen percent of the learners said that it would help if they discussed the topic first with the teacher. They said that this would assist them to have a better understanding. They also said that it would help them to understand what they had to do before doing an exercise. During the interview, learners indicated that, doing the topic on their own does not help because their classes are noisy and uncontrollable. They also added that it gives them problems because they sometimes do not agree on correct answers. And at home there is no one to help them. Therefore they wanted their teacher to teach first and give them work later.

From the questionnaire, 18% of the learners said no because they will forget what she said and they were afraid no one would ever explain the work to them. Therefore they wanted their teacher to teach first and give them work later. 2% of the learners who said that they were not certain, said that it was because they already did not understand and know mathematics, so whatever she did was fine. While interviewing the learners, one learner said that there are times when he felt that he did not like attending mathematics lessons because he never understood anything that was said and believed that mathematics was difficult.

4.3 Presentation and discussion of the classroom observations

In this section the results from classroom observations of the nature of experiences in both teachers' classes are presented and discussed. The two teachers are designated as T1 and T2.

In an incident whereby the teacher (T1) was dealing with writing algebraic expressions from the given sentence, the teacher told learners the symbolic meaning of the following words.

"More than" means that something has to be added; "sum" means total number; "and" means add ($a + b$); "product" means answer to multiplication, ($a \times b = ab$); "decrease" means subtract ($4 - 2$) and that "difference" also means subtract ($a - e$). An example was done in class, 5 more than the sum of a and b. $c = (a + b) + 5$.

In most of the lessons observed, there was not much interaction between the teacher (T1) and the learners during the lesson, learners were not much involved and not much activity was planned for them. The teacher did not communicate their views about the subtopic. She came in and told them how to write algebraic expressions from the given sentences.

A lot of questions came up as learners were asked to work out an exercise in class. Learners asked the teacher to explain further using their mother tongue. At times the teacher was not patient enough because she would shout at them saying that they never listen when someone is teaching and end up saying they did not understand. The teacher proved her impatience by stopping learners from asking questions, telling them to sit down and stop disturbing her class. She never attended to learners' question even after class. Sometimes her facial expression, especially the look she would give to her learners. She would look at them and they keep quite and stop asking any more questions.

The other teacher (T2) did the same thing. She came in and just taught by working out the examples. It was only when she gave learners the work to do that learners got involved as they wanted some clarity to the given exercise. It was from these questions that one could tell that the lesson included them in the activities. It helped them remember some of the things they did in some mathematics topics other than algebra, for example, addition, subtraction and multiplication. Learners had a chance to ask questions where they did not understand the teacher. When learners from the same class were interviewed, 6 of them said that they liked the way their teacher taught. This showed that the teacher makes them enjoy mathematics and even if something is difficult to understand they always wanted to try as they do it themselves. The other four said that even though their teacher sometimes let them do things on their own, they still encountered difficulties in some exercises.

It is not surprising to see teachers introducing the topic by working out examples as French (2002) has made an observation that confirms this incident. He observed that there is a strong prevailing tradition in algebra teaching whereby the teacher introduces learners to a new topic by demonstrating 'worked examples', and then seeks to reinforce the procedures involved through extensive practice exercises. These exercises are usually isolated from activities that could give meaning and purpose to the algebraic ideas and operations. He further says that, although there has been an increasing emphasis on looking at numerical patterns and on using graphical calculators or plotting software to investigate the properties of functions and graphs, it does not necessarily have a significant effect on approaches to learning skills or to using algebra in solving problems and explaining numerical properties.

Some learners came up with the same answer but instead of the letter/variable they substituted with a number. When they were asked where they got the number they said they assumed the number so that they could be able to work out the answer. It was evident that learners did not understand the role played by the letters in algebra. That is why they had to assume a number. In an exercise that they were given, learners were asked to give the product of 4 and 8. They said that they added 8 because they thought 'product' meant addition, and the teacher had to tell them that it means the total from two multiplied numbers. The teacher (T2) told them that they do not have to assume any unknown number; instead they should represent it with any letter. During the interviews one of the learners said:

> I did not understand the whole thing from the beginning; it became a problem when I was given some work to do.

Learners were left to do the work on their own. They went up and down to those whom they thought had understood. Those who had understood were helping others but the noise level was uncontrollable. Some learners who got correct answers had problems of explaining to their peers, and others could just explain half way and could not reach to the answer.

On some days teachers would leave some group work for the learners and ask them to prepare for class presentations. It is during these presentations that groups that were not presenting would ask those that were presenting some questions. Groups used to have the same answers to the questions but with different ways of working them out. The way they would explain their steps of working out the answers would also differ yet the answer would still be the same.

It is from learners' answers and methods used that the teacher (T1) would ask the whole class to choose the simple method that they understood. Learners were confused on when to use a number and when to use a letter to represent a certain number. They were also confused by the use of small letters and not a capital letters.

From the questionnaire learners indicated that they did not know when to use a number and or a letter to represent the unknown number. This is what one of them had to say:

> I had used a number for the unknown number but my teacher said I was supposed to use the number given. I was not sure of when to use the number and when to use the letter to represent the number.

The other one said:

> I did not know why we should not use the capital letter, because we were just told not to use it. So I found it confusing whether the problem was using the capital letter or I used the letter wrongly.

Looking back on chapter 2 from the ARISE project, in the task whereby learners had to do write ups, those who did them in their mother tongue produced more detailed and well explained documents though it was difficult to identify formal mathematical language. The same situation applied here**.**

The other group had the same understanding of the given problem (The product of a certain number and 8 decreased by 12.) though they put it differently. That is a x 8 – 12, a (8) –12 and 7 x 8 – 12. The teacher asked them to explain why they had multiplied, they said they knew that when they are asked for a product there should be two numbers that are multiplied and then the answer will be called the product that was why they multiplied by 8. The teacher (T2) showed other learners different ways in which this group had presented their answers and they discussed them.

The teacher asked the learners to tell if they could see any differences among the answers, then they told her that a x 8 – 12, and a (8) – 12 are the same though written in different ways asked to comment on the third alternative, 7 x 8 – 12 one said that it was not correct because the 7 was not given, therefore they could not put any number for the number that is not given instead they should have used the letters.

It can be concluded that certain learners picked up that there is a difference between the use of a letter and a number where the number is unknown. They could then tell only the first two answers from the previous group were correct as they represented the unknown with the letter. On the other hand one could tell that there were some learners who still could not understand why a letter was used instead of the number.

Learners were introduced to the subtopic in which they dealt with terms, like terms and unlike terms. The teacher told them a story of a street vendor who had 5 boxes of avocados and 3 boxes of peaches, who then bought 15 boxes of peaches and 12 boxes of avocados. Then the teacher told them that algebraically the man had $18p + 17a$ which means 18 boxes of peaches and 17 boxes of avocados.

Learners asked the teacher to explain further. So she showed them that in the first place it was $5a + 3p$ then he bought $15p$ and $12a$ so then they have to add and they put the like terms together:

$$\begin{array}{r} 3p + 5\text{a} \\ \underline{+\ 15p + 12\text{a}} \\ \underline{18p + 17\text{a}} \end{array}$$

The teacher (T2) told them that the expression $18p$ + 17a has got two terms. She told learners that terms, in an expression are separated by the + and – signs. She continued to say that if brackets are used then everything in a bracket is considered to be one term.

From the exercises that learners were given, some of them had some problems identifying like terms while others were able to do so without any problems. During the interviews some of the learners said:

> It was not easy for me to choose the like terms from a group of terms. I used to be confused especially with the numbers that had about two or three letter and a number. Then I used to get confused.

The other one said:

> It was only in the beginning that I used to be confused, but as we went on with the like terms I found it easy to put them together. I used to remember the example of the apple and pear from the street vendor that the teacher once made in class.

The expression $20p + 15a$ was used to explain what a coefficient is. The teacher told learners that a coefficient is whichever number used to multiply the variable. For example in $20p + 15a$, the coefficient of p is 20 and that of a is 15. Then the teacher continued to say that in a case whereby there are two variables, and are asked to give the coefficient of one of those two, then they should take the number multiplying the variable. For example: in $2xy$ - the coefficient of x is $2y$, and that of y would be $2x$. Some exercises were given and most learners did them well. Only a few learners were not doing the work and said they did not know what to do. It can be concluded that most learners understood what they had been taught.

The teacher (T1) showed learners how to solve for the unknown when given other numbers. Examples done in class were as follows:

a) $5x + 9 = 15$
b) $2x + 8 = 12x$

a) $5x + 9 = 15$
$\quad -9 \quad -9$
$\frac{5x}{5} = \frac{6}{5}$
$X = 1\frac{1}{5}$

b) $2x + 8 = 12x$
$\quad -2x \quad -2x$
$\frac{8}{10} = \frac{10x}{10}$
$\frac{4}{5} = x$

Learners told the teacher that they did not understand what she had been doing. They asked the teacher to re-do the examples and they asked the following questions:

- Why do we subtract other numbers?
- When do we subtract and when do we divide?
- Why do we do what we did?

The teacher answered their questions as follows:

- Subtraction and or addition are done in order to put the like terms together.
- In a case whereby the expression is like in example (a) above, nine is removed to the side of 15 so as to leave 5x on its own and have the numbers on their own, then (collecting like terms) then since 5 is multiplying x we have to divide by it (5) so as to leave x on its own and get its value. (She repeated the statement, on learners' request). It is from this point that one can conclude that learners were listening and following what was done as they could stop the teacher whenever they did not understand.
- Again in example a) if 9 was a negative 9, then they would add it. She added by saying, the positive-ness and the negative-ness of the number is the one that determines whether we have to subtract or add.

The teacher concluded by saying that, if the equation has got the equal sign that is when we do the subtraction, addition, multiplication and or division of the numbers. In this case whatever is done on the left side of the equal sign of the equation should be done on the right side as well.

It was evident that this work was complicated for most of the learners, they tried to do the work, but in the middle of it they kept calling the teacher for assistance and explanation. Some learners were able to put together the like terms, but finishing off became a problem for some though a few could go on well from the beginning to the end.

French (2002) further says that, although there has been an increasing emphasis on looking at numerical patterns and on using graphical calculators or plotting software to investigate the properties of functions and graphs, though it does not necessarily have a significant effect on approaches to learning skills or to using algebra in solving problems and explaining numerical properties. I am in agreement with these observations that French has made. During classroom observations it was observed that most of the time the teacher would come in, introduce the topic by teaching and showing learners how to go about the topic and then leave an exercise for learners to do.

As in the case above, learners only wanted to do the exercise in the same manner as the teacher had shown them in class. They did not want to do something different from what the teacher had shown them, they were not aware that they could be able to come up with different methods of solving the same problem. As a result their thinking and discovery skills are not developed. It is from the exercises that the teacher gave learners that she could be able to tell whether they understood the concept. This might have not been effective because most learners did not bring their work to be marked. Some of the learners who got correct answers were asked to help those who did not understand. They did not find it easy to make sense of what they had written. They found it difficult to do the explanations because they could not follow what they had written.

Costello (1991) agrees with the above issue and says that many learners fail to make much sense of algebra and see it as lacking in both meaning and purpose. Learners do not get motivated when they find ideas difficult to understand and they think that the only apparent purpose for the subject is to do the questions in the next test. During observations, there was not much interaction between the teacher and learners and amongst the learners themselves. It is believed that it was because of the method of teaching which the teachers used, and this supports French's (2002) idea of the use of traditional method which teachers use. Learners were not made aware of their capabilities and the fact that they could be able to come up with something good without the teacher's guidance.

Arcavi (1994) has also observed that it may also apply to those who appear to be successful: 'even those learners who manage to handle the algebraic techniques successfully, often fail to see algebra as a tool for understanding, expressing and communicating generalisations, for revealing structure, and for establishing connections and formulating mathematical arguments'.

It seems that many difficulties with algebra arose because learners did not have sufficient understanding of essential numerical ideas of fluency in using them and this obviously applies equally to equation solving. It would seem appropriate therefore, as far as possible in the early stages, to establish principles and procedures, using only those types of numbers, which the student can use with fluency. Problems involving fractions and negative numbers should be delayed until confidence has been established with using these more demanding types of numbers and with the algebraic procedures through using simpler numbers. On the other hand, large whole numbers and decimals may be manageable when a calculator is used to help the solution process.

Insistence on a complicated written form of the solution, like that shown on figure 5 below where all the thinking steps are written down, makes a simple process look complicated and can create confusion rather than help in understanding. According to Drijvers's (1995), observations, the form shown on the right should be sufficient as a written solution, but there should be an accompanying discussion to highlight the process of deciding on the two undoing steps. Drijvers notes that adding 2 and then dividing by 3 on both sides of the equal sign. A flow diagram is a valuable way of aiding that thinking process and informally recording the steps that are involved.

$3x - 2 = 7$	
$3x - 2 + 2 = 7 + 2$	$3x - 2 = 7$
$3x = 9$	$3x = 9$
$\frac{3x}{3} = \frac{9}{3}$	$x = 3$
$x = 3$	

Figure 5 "complicating a simple procedure" (French, 2002:101).

The use of flow diagrams helps to establish the important principle of carrying out the same operation to both sides of an equation to maintain the equality and to make clear the order in which to carry out those operations. However, a flow diagram is not immediately applicable when the unknown appears in more than one place in an equation. The additional steps required to achieve that are not difficult, once learners have become fluent in solving a variety of linear equations where there is a single appearance of the unknown.

Both teachers tried to use real life examples in their teaching. The purpose was to show learners the importance of algebra in life. Some learners did not understand the examples made and they got even more confused. Those who understood the given examples were able to use them in the process of dealing with algebra, especially when they were dealing with like terms and unlike terms. For example, the teacher used a street vendor as an example. A man had 6 boxes of apples and 8 boxes of pears. Then the algebraic expression for that would be 6a and 8p (6a + 8p). They did not need to add the boxes of fruits to come up with one number because those are two different types of fruits and that there is never a fruit called applepear, so they could not add them together.

Interaction between learners is of great importance in their learning as it develops a lot of skills, like working together, listening and understanding others. It also helps learners who may not have heard the teacher to ask their peers to explain to them and may understand them better than their teacher.

The second teacher did this most of the time as she used different teaching methods. For example, she grouped learners and gave them the same task, then asked them to present. As the learners presented each member of the group was responsible for answering questions that were asked by those who were not members of the group including the teacher. It was from this that learners could see that they had different ways of working out the problem and yet the answer would be the same. It was also discovered that during presentations, some learners did not understand the methods used by others and could not make sense out of what others had done, they only understood their own method.

French (2002) agrees with the idea that some learners would only understand the way they have written their answers. They would not understand if somebody has the same answer but has used a different method. For example, in a case where the teacher asked two learners to tell if they saw any differences between the two answers, one told her that a x 8 – 12, and a (8) – 12 is the same though written in different ways. The other one said that they are two different things all together. When she asked them to say something about 7 x 8 – 12, and 7 (8) – 12, they still maintained their answers.

Some of the obstacles learners encounter in the transition from arithmetic to algebra are the uses of schemes and other forms of representation to support mathematical reasoning, symbolizing the unknown, misunderstandings of (unknown) quantities and the relations between them, and a flexible way of thinking of literal symbols.

One of the teachers used learners' mother tongue most of the time when she was teaching. This helped learners very much as they could understand quickly, but the problem arose when they were given an exercise to do. The exercises would come in a different language, which in this case was English.

T1 used to shout at her learners if they asked questions or said they did not understand, or they did not get the work correct. This made learners to become withdrawn and would not ask anymore question. Others did not do the given exercise, as they were scared that if they get it wrong their teacher would shout at them. One of the learners said:

> I never asked my teacher questions when I did not understand because she shouts at us when we say we did not understand.

As mentioned earlier, not much activity was done to stimulate learners to think of their own strategies. Though some activities about sum and difference brought forth pre-algebraic methods and notation, which facilitated the transition from an informal to a more general approach. As far as learners' reactions are concerned, 'brilliant' learners were able to evaluate the historic problems more positively than the 'weak' learners. The 'weaker' ones were distracted or discouraged by what was to them unusual question formulations, and their teacher's attitude also contributed to this.

A number of observed peculiarities in student work suggested that some arithmetic conventions caused misunderstanding, of relationships between quantities. This was because learners are used to calculating with established quantities, they experienced problems reasoning about unknowns. Primary school learners in particular have little grasp of the requirement of equality when using the equal sign.

Learners' work revealed that the unknown has a remarkable function in the formal solving of equations. Letters and symbols help learners organize the information in a problem, but the unknown does not play a meaningful role in the solution process. Apart from including the unknown in calculations, descriptions with words and abbreviations constitutes the part most done in introductory algebra and linear equations with one unknown can be solved using ratios. Learners tried to devise strategies for solving problems of sum and difference and the following were reflected:

- The gap between arithmetic and algebra can be bridged partially with the aid of some pre-algebraic strategies and informal ways of symbolizing, but not by all learners. Learners may also skip the pre-algebraic phase altogether, sometimes they consciously cling to an informal approach, especially in situations where it is equally as effective as a more advanced approach. Thus, pre-algebraic aptitude does not automatically lead to more formalization. Learners who could not overcome the arithmetic level in terms of reasoning often got stuck at a lower level of notation.

- Some of the obstacles learners encounter in the transition from arithmetic to algebra is the use of schemes and other forms of representation to support mathematical reasoning, the recognition of isomorphic problems, symbolizing the unknown, misunderstanding the quantities and the relationships between them, and a flexible way of thinking about literal symbols.

The observation that symbolizing and reasoning more or less developed in learners as independent skills is in agreement with Krutestkii's (1976) cited in Drivjers' (2001) observation, which showed that some learners are visually inclined while others primarily use mental processes. Moreover, he verified that high achieving learners are better in reasoning than in symbolising. At the same time, various researchers argue for mathematical activities that combine both aspects precisely because symbols receive meaning when they are employed in problem solving.

4.4 Conclusion

This chapter has dealt with data presentation and discussion of the study. The research question was used as a theme to discuss the collected data. The main topics were learners' feelings and views about algebra; the usefulness of the topic in life, the clarity of the teacher, the level of attention they need from her, and how best they think they could learn algebra and is also included activities that happened in class during the observations. Research results revealed that few grade 7 learners were successful in manipulating symbols to a certain extent, and in dealing with substitutions and simplification of symbolic forms. Most of the learners got confused and could neither manipulate symbols, deal with substitutions nor simplify the symbolic forms.

Chapter 5

Conclusions and recommendations

5.1 Introduction

The researcher has had a feeling that some research has to be done on grade 7 learners' perceived experiences with algebra. The purpose of this research was to investigate the perceived experiences that grade 7 learners have in the learning of algebra.

This chapter presents the study's conclusions and recommendations. Conclusions are drawn from both the literature study and empirical research. Recommendations emanating from the study are then made into a formal proposal that can be useful in laying a foundation for the teaching and learning of algebra as an introductory mathematics topic in grade seven and develop love and interest for the topic. Some limitations of the study are given. Recommended topics for future research are also suggested.

5.2 Conclusions

From the learners' responses to the questionnaire and interview questions, one can tell that the algebra that was done was more than introductory algebra. Looking at a number of steps involved and the changing of signs they did during observation period. Learners could not cope with all this work; they needed to do a lot of introductory algebraic lessons before solving equations. They needed to understand what algebra was all about before going into problem solving. Algebra ideas can often be conveyed by a visual approach that emphasizes understanding and meaning rather than symbol manipulation (Speer et al, 1997). "Teaching for understanding requires that the continuity of mathematical content be demonstrated to the student during, and prior to, the introduction of mathematical form" (Byers (1992) cited in Herscovics and Kieran 1980:579). This in this case did not happen.

Many learners failed to make sense of algebra and saw it as lacking in meaning and purpose. Most learners were not motivated and had no idea why they were doing algebra; during interviews some learners said that they thought they were doing it like any other subject, just for writing tests to promote them to the next grade for the following year. For many learners, the whole subject of algebra frequently became associated at a very early stage with tasks, which appear to lack any meaning or serve any obvious useful purpose. This naturally leads to failure and distaste. French (2002) says that even many of those who achieve some success in the sense that they are able to produce 'right answers' to routine exercises, commonly come to feel that the subject is obscure and unrelated to anything that is real or of interest to them.

Learners believed that the teacher's thorough explanations could help them understand the topic better. From the classroom observations, it seemed that their teacher liked to shout at those who did not understand. Some learners said that they felt threatened and thus stopped asking questions even when they could not understand, as she would loose her temper and shout at them and would even use strong language.

In the early stages of learning algebra, coming to terms with the meaning of letters and the conventions associated with symbolic representation was a major hurdle. Learners experienced some complications, which took attention away from that major focus, these needed to be avoided by the teacher but it did not happen in this study. Learners had problems working with the symbols and or letters. They could not understand their meaning, as it was not clearly stated by the teacher from the beginning. The symbolism, when it is introduced, needs to have an immediate meaning and a local purpose. In other words, expressions involving letters need to be used in a helpful way at a time, not in a clumsy or unnecessary way. It is sensible to base a lot of work on small whole numbers with occasional reference to large numbers to illustrate the power of the ideas.

The principal learning difficulties experienced by learners in algebra related to the transition from arithmetic conventions to those of algebra, the meaning of literal symbols and the recognition of structure. Learners frequently mentioned obstacles that arose from meanings of the equality sign, the conversion of a description into an equation or a system of equations, and the manipulation of symbolic forms.

Development through the ages shows that algebra had long been practiced as 'advanced arithmetic' and that arithmetic questions can serve as an introduction to algebraic reasoning. Moreover history reveals that knowledge of symbolizing and algebraic jargon are not prerequisites for algebraic reasoning, so with little preparation, learners can start and in this way come up with their own abbreviations (Bednarz et al., 1996).

Both teachers did not spend much time on the subtopic introduced to learners. For example, T1 did algebraic expressions for one period and that was it. But there was some confusion amongst the learners. It was a few of them that understood what they had to do. Although they also showed that they needed more time to do what ever they were doing. Introducing negative numbers before learners are completely fluent in their use can make algebra seem hard when the difficulty actually stems from problems in handling these types of numbers correctly and fluently. For example, some of the problems associated with learning about simultaneous equations, could be avoided in the initial stages if work was limited to equations involving addition only and with solutions that are restricted to positive whole numbers.

According to French (2000), failure to develop an understanding and fluency with negative numbers and fractions is often a major barrier to later success with algebra, although there is much that can and should be done before the need for them becomes crucial. Since negative numbers have an immediate relevance to simplification, they must come next with some discussion of fractions towards the end of the topic.

Algebraic ideas are a direct extension of numerical operations and relationships and are therefore totally dependent on understanding and fluency with number. Skill with mental arithmetic, and the understanding and fluency that should accompany it is thus a key requisite for success with algebra. Beyond the initial stages, where the positive integers provide a sufficient numerical background, negative numbers and fractions become increasingly important. Failure to understand them adequately and to acquire sufficient mastery of operations involving them commonly acts as a barrier to success in algebra (Sfard, 1991).

The teachers did not motivate learners. There were a few activities planned for the learners to be engaged and discover things on their own. There were no varieties of teaching methods introduced to learners and therefore learners were obliged to the teacher's method of working out answers, even if they did not understand properly how to go about the method, they had no option. A variety of methods on how algebra can be taught have been discussed. Learners needed to acquire an initial feel for what is involved in solving equations and that was not provided. Consequently it seems that part of teaching was unattainable for the average learner.

In a case of symbolising, learners who had difficulties arriving at an algebraic way of thinking (from an arithmetic way of thinking) mostly seemed to get stuck in an arithmetic level of notation. Pre-algebraic and algebraic notation might not be necessary for algebraic reasoning, but appeared to support the solving of pre-algebraic and algebraic problems. When it comes to understanding connections, mathematical notions of numbers and connections between numbers impeded the development of algebraic understanding.

Learners worked in the classroom, demonstrated that algebraic reasoning is more accessible to learners than algebraic symbolising. It appeared to follow then that developing algebraic thinking is not necessarily dependent upon algebraic notation and that the presence of algebraic notation says little about the level of problem solving. It was also revealed in learners' work that the unknown has a remarkable function in the informal solving of (systems of) equations. Letters and symbols help learners organise the information in a problem, but the unknown does not play a meaningful role in the solution process.

Additionally learners devised a number of strategies for solving problems of sum and difference. This was seen when learners did their presentations. The analysis of learners' work and learning processes in the class yielded the following:

- The gap between arithmetic and algebra can be bridged partially with the aid of some pre-algebraic strategies and informal ways of symbolising, though not by all learners. Sometimes learners consciously cling to an informal approach, especially in situations where it is equally as effective as a more advanced approach. This may mean that pre-algebraic aptitude does not automatically lead to more formalisation.

- Some of the obstacles that learners encounter in the transition from arithmetic to algebra are the use of schemes and other forms of representation to support mathematical reasoning, the problem of symbolising the unknown, misunderstandings about the (unknown) quantities and the relationships between them and a flexible way of thinking about literal symbols.

5.3 Recommendations

- It is important to be aware of the difficulties that learners are having in trying to understand the different uses of letters/variables in algebra.

- Teachers need to be aware that pupils who appear to perform adequately on a limited range of tasks may be misled into relying on a strategy which is naive or inadequate or illogical and which may quickly break down in more complicated situations (Costello, 1991).

- Teachers need to help learners appreciate that algebra is a special language that has its own conventions and uses familiar symbolism in new ways. In order to meet the challenges of dealing with the prior knowledge that learners bring to their study of algebra, as suggested by Stacey and MacGregor (1997), teachers need to use algebraic notation more often, emphasise that letters in algebraic expressions stand for numbers, not for names of things and appreciate that learners come to algebra with rich prior experiences of symbol systems.

Developing learners' capacity to perform standard algebraic manipulations with fluency requires more than frequent practice with routine tasks. To be effective, the developing of skills needs to be presented in meaningful contexts that enable learners to see the underlying purpose in what they are doing. Learners should increasingly come to appreciate:

- The meaning of algebraic symbols and expressions

- The importance of arithmetical skills and understanding, particularly with reference to negative numbers and fractions.

- How algebraic results relate to numerical patterns and calculations
- Ways of checking for errors
- The importance of fluency with mental algebra using simple examples as a basis for understanding more complex written algebraic manipulations
- The role of algebra as a powerful tool for explaining and solving problems

It may be entirely appropriate to organize the curriculum and teaching in an essentially linear way, but we must never delude ourselves into thinking that learning will follow the same path. The uncertainty and expectedness about the paths that learning can take is what makes teaching such a frustrating and demanding task, but it is also one reason why it can be so stimulating and endlessly fascinating.

Symbolizing should receive more attention in primary school, especially as a support for mental processes. According to Drijvers (2001), as far as shortened notation is concerned, it is advisable to build upon knowledge of arithmetic; formal algebraic notation conflicts too much with arithmetic conventions. He further says that emphasis should be placed on dynamic action language, more activities that draw upon skills such as structuring and schematizing, so that tables and other schemes can evolve into aids in mathematical reasoning.

Learners should have more practice reflecting upon their own ways of working so as to increase their drive to improve. Teachers should not allow learning difficulties associated with algebra to fall by the wayside; instead they need to seize them as discussion opportunities, and make learners aware of the ongoing conflicts.

In-service training for teachers could be essential. It can also help teachers raise issues that they cannot solve on their own. For some teachers, the last courses they attended would have been when they left college or university, therefore would be lacking new ideas to handle the current curriculum effectively.

According to Sfard (2000), the way that algebra is taught is influenced profoundly by teachers' beliefs about the nature of the subject and how learners should learn it and also by a whole host of external constraints. There cannot be a definitive answer to the question of how learners should learn algebra, but one can point to the nature of the difficulties by looking critically at current research and examining practices in the light of the evidence it provides. An alternative is to start with problems, which will often be linked to specific topics, and learn necessary ideas and skills through working on the problems, so that mathematics can be seen as something to think about and make sense of, rather than as a set of rules to be remembered. This has a strong influence on learners' perceptions and performance.

According to Drijvers (2003), algebraic ideas are a direct extension of numerical operations and relationships and are therefore totally dependent on understanding and fluency with numbers. Skill with mental arithmetic, and the understanding and fluency that should accompany it, is thus a key requisite for success with algebra. Beyond the initial stages, where the positive integers provide a sufficient numerical background, negative numbers and fractions become increasingly important.

5.4 Limitations to the study

The research comprised a study of particular phenomenon at a particular school in Khayelitsha. Limitations in this case study are evident. Some of which are briefly discussed below.

The results cannot be generalized to other schools. However, the process developed at this particular school in Khayelitsha can be used as a basis for further development in other contexts. To test the reliability of the research, the findings will therefore have to be put into practice at another school.

Like other educational researches, the study was fraught with a host of unpredictable circumstances. Some learners could not bring back the questionnaires given to them at first and a second batch of questionnaires had to be done and administered in order to avoid disappointment. Teachers were given copies of the observations, a week before schools closed for the winter break in June 2005 but could not look at them. Three weeks after the schools had reopened and the researcher had made insistent follow-ups then only did the teachers come up and say they had lost their copies and again other copies had to be made.

I have realised that it could have been of good help to give learners the same controlled test on algebra before and after they had done the topic. This could have helped to draw firm conclusions as the controlled test could have helped me to see if they had any idea concerning algebra. For the researcher who might do the same study in future, it would be good to do so. One other weakness that the research noted was that the study was limited to one school of its own type and can therefore not be generalised for all the South African schools or even the Western Cape Schools.

5.5 Recommendations for further studies

The researcher may not conclude that teachers' practices were not up to standard as this was not part of the research, but it is recommended that for further studies the nature of teaching should be investigated to see how it influences learner experiences. Understanding of variables is fundamental to success in algebra. Graham and Thomas (2000) say that graphic calculators and computers play a major role in improving learners' understanding of algebraic variables, these could also be of utmost help if given attention for further study.

5.5.1 Suggested topics for further research:

- What is required of the teacher for the proposed experimental teaching sequence? (This will ultimately lead to improved expertise in the area of solving equations formally).
- Problems encountered by teachers when teaching introductory algebra. (This will give a picture of the kind of assistance that may be needed by teachers).

5.6 Concluding remarks

The study has focused on perceived experiences that grade 7 learners have in learning algebra. It is hoped that this study will help teachers to realize learner's experiences and concerns about the topic in discussion. The research has attempted to describe the experiences that the learners are having and the way they react towards these experiences in learning algebra.

The research has identified some teaching methods that can be applied in classrooms to ensure active and motivated learning of algebra. The research has also found that learners had problems with identifying *like* and *unlike* terms, writing algebraic expressions and solving some equations.

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Appendix 1

Random Sampling Tables

Table 1and 2:Randomly assorted numbers McMillan and Schumacher (2001).

Table 1

466	200	179	142
162	052	541	233
915	624	053	110
628	626	201	780
100	964	590	421
473	734	118	894
456	710	536	107
504	533	717	284
892	321	839	373
234	419	332	263
598	818	187	031
987	878	006	860

Table 2

466	200	179	142
162	052	541	233
915	624	053	110
628	626	201	780
100	964	590	421
473	734	118	894
456	710	536	107
504	533	717	284
892	321	839	373
234	419	332	601
598	818	187	
987	878	006	

Appendix 2

Questionnaire

Learners' questionnaire

This questionnaire is to be completed by you. It intends to elicit information about your experiences in learning of algebra. I believe that the information collected from you will be relevant and more realistic. Please give an answer for every question. Moreover I assure you that filling in this questionnaire will not affect your personal, social safety, your integrity and or the relationship between you, your peers and your teacher. All information received through this questionnaire will be treated as confidential and no individual's name or name of the school is needed or will be mentioned either in the questionnaire or in the report.

Biographical data

Could you please provide the following information

Gender: male ____________

Female ___________

Age: ____________

Grade:______________

1. How do you feel about the topic that you have just done?

2. Why?

3. Was the topic easy to understand?

4. Why?

5. Did you find the topic difficult?

6. Why?

7. Did you find any problems with the topic?

8. Where?

9. Other than learning it in school, do you think the topic can help you anywhere in life? Where give example.

10. How would you want to learn the topic in the next grade, or if you were to do it again this year?

11. What do you think could have been done by you and your teacher to make you feel different or even better than the way you felt?

12. What would you want your teacher to do next time when you do the same topic?

13. What would you want her not to do next time you deal with the same topic?

14. How much attendance/assistance do you want from your teacher?

15. Do you like it when your teacher leaves the work for you and asks you to do it on your own? Why?

16. Do you like it when she teaches first and then gives you work later? Why?

Appendix 30

Consent Form

Description of project and research ethics statement

Description of project

The purpose of this research is to investigate the grade 7 learners' perceived experiences in learning algebra.

Ethics statement

In order to gain access to the informants, the WCED, the principal at the site of research and the learners as the key informants will be approached and asked for permission and their participation in the interviews.

The aims of the research will be explained to participants in the language of their preference, (either Sotho or English). Informed consent of the participants will be obtained for this research. This will pursue with the aid of a consent form to be completed by each participant. The research participants will therefore have the right to the concealment of their names on all the information in the course of the research and the right to choose not to be tape recorded during the interviews. The participants also have the right to be informed of the results of the research in accessible form. This includes having access to the report of the research and the transcript of the tape recordings that transpired during the interview.

Consent Form

The grade 7 learners' perceived experiences in learning algebra

- The interviewees agree to participate voluntarily in this project, which means participating in a conversation with the researcher for not more than an hour, focusing on the perceived experiences they have in learning algebra.

- The interviewees will be protected through anonymity. Meaning that their names will not be revealed on any public documentation or to any one not even the teacher, not unless the interviewees themselves wish so.

- The interviews will be tape recorded, unless otherwise specified by the interviewees or the interviewer before or during the interview.

- The interviewee has the right to withdraw from the study anytime without any fear of penalty, including having their records from the study. The interviewees may choose at anytime not to answer a particular question or set of questions.

- To ensure that views of all concerned are accurately documented and shared, the interviewer will provider the interviewee with any notes and/or transcriptions from their interview for editing purposes, allowing for a maximum period of three months for response. The researcher will also provide the interviewees with copy (ies) of any publications emerging from this research.

- The interviewees are made aware that the findings of the research will be shared with the relevant communities, through different kinds of publications, the interviewees therefore agree to the findings to be published within the context of the aims identified.

- The interviewer and the interviewee commit themselves to mutual respect of one another throughout the interviewing process. This respect includes fulfilling the various aspects of this agreement.

I have read and understood the above agreement. Therefore I (interviewee) am willing to participate in this project and to have the findings used in the ways described above.

Interviewee's signature

………………………………………… Date …………………………………..

Interviewer's signature

……………………………………….. Date …………………………………..

Appendix 4

Interview Questions

1. How do you feel about the topic that you have just done?
2. How did you find the topic? Why?
3. What do you think could have been done by your teacher?
4. What should you have done?
5. During the lessons of the topic what did you like from the teacher/ something she did that you think helped you understand the topic better?
6. During the lessons of the topic what did you not like from the teacher/ something she did or did not do that you think could have helped you understand the topic better?
7. Why?
8. Did you find any problems with the topic?
9. Where?
10. Did you ever find some part of the topic that was easy to understand and do?
11. Which one? / Why?
12. Other than learning it in school, do you think the topic can help you anywhere in life?
13. Where, can you give an example?
14. How would you want to learn the topic in the next grade, or if you were to do it again this year?
15. What do you think you are going to do next time when doing the same topic so that you can enjoy it?
16. What would you want your teacher to do next time when you do the same topic?
17.
18. What would you want her not to do next time you deal with the same topic?
19. How much attendance/assistance do you want from your teacher?
20. Do you like it when your teacher leaves the work for you and asks you to do it on your own?
21. Why?
22. Do you like it when she teaches first and then gives you work later?
23. Why?