Analysis of the ways of working of learners in the final grade 12 Mathematical Literacy examination papers: Focussing on questions related to Measurement

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A thesis submitted in partial fulfilment of the requirements for the degree of Magister Educationis in the Department of Education, University of the Western Cape.

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DECLARATION

I declare that *Analysis of the ways of working of learners in the final grade 12 Mathematical Literacy examination papers: Focussing on questions related to Measurement* is my own work, that it has not been submitted for any degree or examination in any other university, and that all the sources I have used or quoted have been indicated and acknowledged by complete references.

MARIUS DERICK SIMONS DATE: May 2012

SIGNED: ………………………………………………

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DEDICATION

To my family: I thank you for the patience and moral support you have availed to me when I have spent so much time away from home. May God reward you for your steadfastness in all things throughout your life, Amen.

In loving memory of my mother through whose inspiration I have, thus far, been able to succeed in my educational and academic pursuits.

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ABSTRACT

Mathematical Literacy has a dual meaning in South Africa. On the one hand it alludes to an understanding of the role of Mathematics in the real world. On the other hand it refers to a subject that is taken by students who generally do not do well in Mathematics and who do not wish to do a pure Mathematics course, This research focuses on the identification and investigation of errors, misconceptions and alternative ways of working in the responses of students in the final grade 12 Mathematical Literacy examination. The aim was to identify the errors, misconceptions and alternative ways of working and to discuss possible reasons for these errors and misconceptions. This aim was governed by the principle that feedback to students and teachers form a vital component in the teaching and learning process.

This analysis only focused questions pertaining measurement in Mathematical Literacy. An analytical framework was constructed based on common errors and misconceptions identified by various researchers' in the field of Mathematics. This analytical framework was used to classify and analyse the errors, misconceptions and alternative ways of working in Mathematical Literacy.

The analysis was done using document analysis on a randomly selected sample of Mathematical Literacy scripts taken from across all education districts in the Western Cape. Great care was taken to prevent bias and cross checking was done by peers to ensure that the categories of errors were agreed on.

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The results of the analysis revealed that the errors identified for Mathematics are common to those of Mathematical Literacy. The findings in this project suggest that the analysis and feedback of errors and misconceptions may help to improve teaching and learning in Mathematical Literacy.

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Chapter 1

Introduction

1.1 Assessment in Mathematics

Niss (1999) outlines the two primary areas of investigation in Mathematics education research as the teaching of Mathematics and the learning of Mathematics. The teaching of Mathematics focuses on matters pertaining to organised attempts to transmit or bring about mathematical knowledge, skills, insights, competencies, and so forth, to well-defined categories of recipients. The learning of Mathematics focuses on what happens in, around and with students who engage in acquiring such knowledge, skills, with particular regard to the processes and products of learning. A closely related area of investigation is the outcomes (results and consequences) of the teaching and the learning of Mathematics respectively. In this regard, assessment plays a very important role.

Assessment in mathematical education is has to do with the judging of the mathematical capability, performance and achievement of students whether as individuals or in groups. Assessment thus addresses the outcomes of mathematical teaching at student level (Niss, 1999). Desforges (1989) offers a simple but comprehensive overview of the purpose of assessment by stating that in general, assessment provides information to help people make decisions. He elaborates on this by stating that the decision-makers are pupils, teachers, local and national educational officers, and parents. He further suggested that pupils and teachers need diagnostic information; parents need evidence that schools provide sound bases for learning, and local and national educational officers need information on which to make monitoring judgments.

We can safely say that assessment outcomes or results are used to:

- Measure the effectiveness of the system that is being used
- Monitor curriculum delivery
- Improve teaching and learning
- Measure learner performance in high-stakes examinations.

Currently performance in Mathematical Literacy in grade 12 is primarily judged by a final examination or the National Senior Certificate Examination. When tests and examinations are considered to be ways of judging student performance, they are special forms of assessment and are thus subsumed under the assessment category. In essence this means that these examinations are major assessment tools in determining pupil's competence in Mathematical Literacy. According to Niss (1992) the roles, functions and effects of assessment in Mathematics education should no longer be neglected; rather, they should become objects of investigation and examination for several reasons.

Examinations are used by different countries around the world. These examinations are normally used to determine whether a learner can continue to the next phase or grade. In other cases examinations are used to ascertain whether or not teachers are delivering the curriculum requirements. In the Cockcroft Report, Cockcroft (1982) shares this view on examinations as follows:

> We believe that there are two fundamental principles which should govern any examination in Mathematics. The first is that the examination papers and other methods of assessment which are used should be such that they enable candidates to demonstrate what they do know rather than what they do not know. The second is that the examination should not undermine the confidence of those who attempt them. There is, however, nothing wrong with examinations being objective, because generally we test pupils to get to certain conclusions.

This is commonly accepted that students will make errors during the course of their learning. It is how these errors are used to inform teaching but more importantly, to inform learning, that should be a priority in assessment. Teachers should attempt to understand why students make the errors they do and to use this knowledge in their teaching in order to minimize the occurrence of the errors in future. This implies that feedback should be given to learners with regard to what has been found during assessment so that the learning of Mathematics and Mathematical Literacy can be improved.

In this thesis it is acknowledged that examinations, being one method of assessment, can be used as a measure to determine whether a student progresses to the next phase, but the emphasis would be on how examinations can be used to give feedback to teachers and students in order to improve the teaching and learning of Mathematical Literacy. The final examination scripts in Mathematical Literacy will be looked at to determine the types of errors that learners make during high-stakes examinations. The purpose is to analyze the errors and to discuss the possible reasons why students would make these errors. This information could then be used to give feedback to the broader Mathematics community and to inform the teaching of Mathematical Literacy.

1.2 Motivation

According to the National Curriculum Statement (NCS), assessment is a process of collecting, synthesizing and interpreting information to assist teachers, parents and other stakeholders in making decisions about the progress of learners. It involves gathering and organizing information (evidence of learners), in order to review what learners have achieved. It informs decision making with respect to teaching, and helps teachers to establish whether learners are making progress towards the required levels of performance (or standards), as outlined in the Assessment Standards of the NCS (National Curriculum Statement, 2005). The focus of the study is to gather evidence of the errors and misconceptions manifested in the written work of learners and to investigate how this evidence can be used in teaching and learning in order to minimize the recurrence of those errors.

Looking for the correct final answer is not always the most accurate way of identifying competent Mathematics students. An in-depth analysis of solution path is also required. Error analysis in Mathematics education has a long history. In the United States, Buswell and Judd (1925) cited more than 30 studies dealing with the diagnosis of arithmetical errors. In Europe aspects of Gestalt theory and ideas on pedagogical reformers have been influential by that time.

In the Soviet Union, the correction of the changed educational system and the curriculum were important factors. Recent years have seen an increased interest in error analysis that has not been limited to errors in arithmetic computation. (Radatz, 1980).

This study will emphasize which errors, misconceptions and alternative ways of working are evident in students' written answers. Currently, the feedback given to teachers in the form of a moderator's report is very general and does not provide a comprehensive overview of the errors and misconceptions. Also, it does not provide possible reasons for these errors, misconceptions and ways of working. This research aims to fill that gap. It hopes to fulfill one of the purposes of assessment as stated in the Assessment Guidelines for Mathematical Literacy (2005), that is, to "revisit or revise certain sections where learners seem to have difficulties". By doing an error-analysis which gives an in-depth analysis of the errors and misconceptions of students as evident in their written work, and implementing the findings of the study, teachers may give learners opportunities for a better understanding of mathematical concepts. This in turn will result in improved results in Mathematical Literacy. Secondly, the investigation into the possible reasons why students make the mistakes they do, may help teachers reflect on their teaching practices and whether certain teaching practices may cause some of the misconceptions that students have.

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1.3 Research questions

English (2008) is of the opinion that one of the major challenges facing Mathematics education is how to effectively assess learners' mathematical achievement. This can also be seen as the most difficult task facing teachers today. According to the NCTM (2000), assessment should support learning of important Mathematics and furnish useful information to both teachers and learners. Therefore, this study will focus on difficulties learners' encounter in high-stakes examinations.

The aim of the study is to:

- Identify errors and misconceptions in learners' responses in the final grade 12 Mathematical Literacy examination.
- Explain the errors and misconceptions that occur in the learners responses in the final grade 12 Mathematical Literacy Exam.

Thus, in order to fulfil the aims of the research, the following research questions have been formulated:

- What are the ways of working of learners when they are engaged in a high-stakes examination in Mathematical Literacy?
- What are the errors and misconceptions that are manifested in the written responses of learners?
- What possible explanations, from a mathematical perspective can be provided, for these misconceptions, errors and alternative ways of working?

To answer any research question and staying focused to the aims of the research study, one has to recognize the imperative role of language and the contextualization of certain terms. For this reason I briefly describe key terms for the benefit of the reader.

- **High-stakes examination –** examination with great consequences in terms of progressing to a next level of study or entry into a job category which requires success in the examination at a specific level.
- **Assessment** - judgement of mathematical capability, performance and achievement.
- **Feedback** – provides information about your learning.

The thesis consists of five chapters. In this chapter, the introduction, motivation and the research questions were presented. An overview of the thesis is also provided.

The second chapter provides an explanation of what is meant by errors, misconceptions and alternative ways of working.

An overview of the literature regarding common errors as well as how the different scholars have classified these errors is also discussed. A template for the classification of the errors is provided.

In chapter 3 the methodology used in this research is explained. A sample of grade 12 Mathematical Literacy examination papers was analyzed in terms of the errors made by students, using document analysis.

A comprehensive discussion of the errors found is provided in chapter 4. The errors are classified according to the template and possible reasons why students have made these errors are discussed.

Chapter 5 consists of the findings of the research. These are discussed with reference to the literature and relate the research questions to the findings. Suggestions are provided for teachers to use the knowledge gained from error analysis in their teaching in order to improve the learning of Mathematical Literacy. Suggestions for further research are also given.

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Chapter 2

Review of Literature

2.1 Introduction

Mathematical Literacy was introduced into South African as an alternative to mathematics in 2006. Although there existed controversy in the Mathematics education community regarding the theoretical concept of Mathematical Literacy, the South African National Education Department continued with the implementation of Mathematical Literacy as a new school subject. As part of a progressive agenda for transformation towards increased democracy and social justice, the department of education viewed its introduction as necessary and appropriate.

According to the South African National Department of Education, Mathematical Literacy provides learners with an awareness and understanding of the role that Mathematics plays in the modern world. In this regard it is stated that:

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> "the subject Mathematical Literacy should enable the learner to become a self-managing person, a contributing worker and a participating citizen in a developing democracy. The teaching and learning of Mathematical Literacy should thus provide opportunities to analyse problems and devise ways to work mathematically in solving such problems. Opportunities to engage mathematically in this way will also assist learners to become astute consumers of the Mathematics reflected in the media". (Curriculum Assessment Policy Statement, 2011, p. 10)

The Programme for International Student Assessment (PISA) (2003, p. 24) defines Mathematical Literacy as follows:

> Mathematical Literacy is an individual's capacity to formulate, employ and interpret Mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that Mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens.

Both definitions above define Mathematical Literacy in terms of the role Mathematics plays in the real world and its impact on citizenship. Furthermore PISA's domain for Mathematical Literacy is concerned with the capacity of students to analyse, reason and communicate ideas effectively as they formulate, solve and interpret solutions to mathematical problems.

This definition is also relevant to Mathematical Literacy in the South African context. However, it should be noted that in terms of subjects, Mathematical Literacy is distinguished from Mathematics in South Africa. Mathematical Literacy is a separate subject and it is often considered as a softer option for those students who are struggling with the rigour of pure Mathematics. Thus, the term Mathematical Literacy can refer to the role of Mathematics in the real world on the one hand, and to the subject Mathematical Literacy on the other.

As the rapidly changing society becomes more information- and goal-driven, it puts greater challenges on the learners. Learners are forced to become more mathematically literate. They need to broaden their thinking and learning, have a better understanding of the world around them and become more informed with the challenges which the increasing technological society offer them.

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The National Council of teachers of Mathematics (NCTM) (1989, p. 5) proposes five goals for mathematical literacy. They are:

- That learner learns to value Mathematics.
- They must become confident in their ability to do Mathematics.
- They need to be mathematical problem solvers.
- That they learn to communicate mathematically.
- They learn to reason mathematically.

The above-mentioned emphasise the necessity to develop the subject through research, for the improvement of learner achievement as well as developing our learners to become productive and functional citizens.

This chapter deals with Mathematical Literacy as part of Mathematics education by looking at issues of assessment and focussing on how learners respond to assessment tasks. It then discusses the importance of feedback on learners' responses to these tasks. According to Barnes, Clarke and Stephens (2000, p. 623), "assessment is increasingly viewed as an instrument of system-reform, monitoring or system management and is linked to powerful global discourses of performability, efficiency, quality assurance and accountability".

2.2 Assessment in Mathematics education

According to the Curriculum Assessment Policy Statement (2010, p. 101), "assessment is a continuous planned process of identifying, gathering and interpreting information about the performance of learners, using various forms of assessment. It involves four steps: generating and collecting evidence of achievement; evaluating this evidence; recording the findings and using this information to understand and thereby assist the learner's development in order to improve the process of learning and teaching". This research focused on the last three steps.

Dietel, Herman and Knuth (1991, p.1) state that assessment may be defined as "any method used to better understand the current knowledge that a student possesses". This implies that assessment can be as simple as a teacher's subjective judgment based on a single observation of student performance, or as complex as a five-hour standardized test. The idea of current knowledge implies that what a student knows is always changing and that we can make judgments about student achievement through comparisons over a period of time. Assessment may affect decisions about grades, advancement, placement, instructional needs, and curriculum.

Currently performance in Mathematical Literacy in grade 12 is judged by the National Senior Certificate examinations. The NCS is an examination for certifying students as having successfully completed the Further Education and Training (FET) phase. Mathematical Literacy is one of the subjects offered in the FET phase; therefore if a learner is successful in Mathematical Literacy it will provide access to further education and training.

Hence the purpose of NCS is in line with what was put forward by the previous authors as to how they define assessment. These examinations are considered to be ways of judging student performance. In essence this means that these examinations are major assessment tools in determining pupil's competence in Mathematical Literacy. Assessment thus provides a framework for sharing education objectives with students and for charting their progress. However, assessment can generate feedback that can be used by students and teachers to enhance learning and teaching and ultimately performances.

2.2.1 Attainment and achievement of assessment objectives

Attainment is at the level of students' outcomes, which is a result of what had taken place in the classroom. However, academic achievement and the student's belief measures capture part of this attainment (TIMSS International Curriculum Analysis (ICA), 1995). Travers and Westbury (1989) cited by Orton and Wain (1994), calls the attained curriculum as that which pupils acquire in terms of body of mathematical knowledge which includes concepts and skills, the processes involved in doing Mathematics and attitudes towards the subject. It will be ascertained whether pupils have attained certain objectives and to what benefit it is to Mathematics education through assessment.

To assess the extent to which pupils have attained certain objectives as laid out in the intended curriculum may not reveal all that is appropriate to know about pupil's overall mathematical achievement, or from where such achievement originates (Satterly, 1989).
Niss (1990) is of the opinion that profiling and graded certification both record a learner's achievement, but the supporting evidence of attainment is frequently omitted from profiling. He goes on by saying that a learner must demonstrate a particular level of attainment before a grade is awarded to such learner.

Nelson and Frobisher (1993) argue that the attainment of mastery by students on a particular occasion does not guarantee that the level of performance will be maintained. They further conclude that the main issue is not solely "how many answers are correct on one occasion" but "on how many testing occasions" (p. 123). This conclusion emphasizes the importance of diagnostic assessment to the field of Mathematics education which ultimately boils down to the overall mathematical ability of the learner. Therefore, it is imperative to get a sense of errors and misconceptions present in the responses of pupils. Similarly Brooks, Schraw and Crippen (2005) state that to improve student learning, the quality and quantity of performance-related feedback must also be improved.

To them the important aspect of this type of feedback is the link between the student's action and response. The marking process produces a wealth of information which could be used in the improvement of teaching and learning.

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2.2.2 Importance of feedback

In general the purpose of feedback is to motivate students by acknowledging good work and providing encouragement for approaching new assignments or tasks. It provides evidence of what was done well or poorly and enables students to adopt recommendations for future assessment. Students gain confidence as they learn from their errors and lastly, lower achieving students get the support by their teachers addressing the errors which lead them to find ways of improving their work. This helps them to overcome their feelings of inadequacy or failure.

An essential part of monitoring students' progress is through effective feedback. Webb (1993) describes learner's responses as what the student is expected to produce. According to him the appropriateness of a particular response depends on the purpose of the assessment and other factors such as the time available for scoring and the number of students tested. He emphasises that a crucial step in any assessment involves the consolidation of the information collected or observed as well as the comprehensive feedback given, so that meaning can be assigned in light of the identified purposes of the assessment.

Black & Wiliam (1998) report that in order to raise standards in schools, governments must focus on processes of teaching and learning, rather than on standards and goal setting, external testing, school planning and management and school inspections. This process of teaching and learning can be strengthened by the correct implementation of feedback. This would bring about students that are not so much concerned about what grades they achieve than the actual attainment of understanding. Therefore, feedback would assist students to fix the problems or clear up misconceptions. Boston (2002) notes that feedback on tests and homework can be very helpful when it provides comments about errors and specific suggestions for improvement and encourages students to focus their attention on the task rather than simply getting the answer right.

Similarly Marzano, Pickering and Pollock (2001) cited by Barry and Hickman (2008, p. 4) states that feedback needs to be "corrective in nature, timely, specific to criterion and should involve the student". Their research found that telling a student simply what answers they got wrong or right can have a negative effect on achievement.

2.2.3 Feedback on Mathematical Literacy from High-stakes examination

The Department of Basic Education (DBE) has introduced a form of feedback which focuses on the evaluation of the marking process and the analysis of learners' responses.
The evaluation of the marking process is geared towards the improvement of the marking process for future examinations. The analysis of learner responses allows chief markers and internal moderators to comment on questions that were poorly answered and to provide possible reasons for the poor responses.

The extract from an examiner's report presented below is based on the analysis of learner responses done by moderators, senior markers in consultation with some of the experienced educators at the marking centre. The purpose of the report is to point out certain areas in the subject that do not pose challenges to learners as well as those that are misunderstood by learners.

It aims to assist teachers and curriculum advisers in their preparations for the grade 12 class of 2011 and beyond. The report aims to give teachers an overall idea of which types of questions need more emphasis and in which questions learners need more exposure. The report also suggests how problematic areas could be dealt with. It should be noted that the report is based on a small sample of 100 randomly selected scripts.

QUESTION 1
1. General comment on the performance of learners. Was the question well answered or poorly answered?
This question was well answered by most learners. The total mark for the question is 26 and the highest recorded mark from 100 scripts analysed is 26 with 05 being the lowest mark. The average mark is 53.8%
Some of the learners struggled with the graph question on 1.2.2 and the writing of a formula from expressions in words in 1.2.1. Learners also struggled with question 1.2.4, that requires graph interpretation and analysis or use of a formula.
2. Why question was poorly answered: Also provide specific examples
Learners were not provided with a table (as this used to be the case) for the calculation of values to be plotted. (1.2.2) Learners show no understanding of what a formula is. Many of them simply wrote numbers related by an equal sign without any variable. For a mathematical statement to be a formula it must contain at least one variable. (1.2.1) The instruction on the question that learners should motivate their choice by a calculation confused learners in 1.2.4.

(Internal moderator's report, 2011)

The WCED has utilized an effective system of capturing the learners' marks per question and this has enabled the province to identify weaknesses of learners in specific domains. Therefore problems in performance are no longer generalized but the detailed analysis is able to pinpoint a domain-specific area of deficiency which needs to be remedied. In the case of other provinces, the moderator was tasked to collect data and generate an analytical report. This feedback to teachers will improve teaching and learning, as teachers focus on particular areas of weakness in their subject (Report on the National Senior Certificate results for 2010, p. 28).

Table 1 below presents an example of the question analysis given to teachers of all schools in the WCED as a report on the performance of learners in the respective Mathematical Literacy Final grade 12 examination papers of 2010.

Table 1: Question analysis of final Paper one and Two on Mathematical Literacy

MATHEMATICAL LITERACY								Paper average difference
Questions	1	2	3	4	5	6	7	
Paper 1	- 6.2	- 7.1	- 8.2	- 7.2	- 4.4	- 0.6	- 10.2	- 6.2
No of candidates	127	127	127	127	127	127	126	127
Paper 2	- 5.2	- 4.1	- 8.6	- 8.7	- 4.7			- 6.4
No. of Candidates	127	126	126	126	125			127

(Western Cape Education Department, 2010)

The question analysis gives a clear indication of the learners' performances in each question, in relation to the provincial average. The negative numbers indicates that the learners' performances in the different questions are below the provincial average, while a positive number indicates a performance above provincial average. In questions 2, 3, 4 and 5 of paper two, the number of candidates who wrote each question was less than 127 indicating the number of learners that did not answer those questions at all. Question 7 of paper one also shows that one learner left the question blank.

This form of feedback is called summative feedback. It sums up the final judgement of the quality of the learners' work. Feedback should be critical, but supportive to learning to encourage learners' scrutiny of their future work. However, this type of feedback does not show the learners' ways of working.

As stated above, one of the general purposes of assessment is to revisit or revise those sections where learners seem to have difficulties. The feedback in Table 1 and what was given in Appendix A does not show any significant way to the improvement of teaching and learning as mentioned in the National Curriculum Statement.

Neither does it emphasize which errors, misconceptions nor are alternative ways of working evident in the written answers of the learners. The report also does not attempt an in-depth explanation of the reasons why learners make these mistakes, but rather attempts to explain the mistakes in terms of how the structure of the question may have caused the students to answer the questions incorrectly.

Despite the quality of current strategies to improve performances, students continue to develop misconceptions, encounter difficulties in their learning and make errors in their Mathematics. I have been teaching Mathematical Literacy since its inception as a FET subject in 2006 and experienced that learners struggle with questions which involve measurement. This phenomenon seems like a broader problem. Preston & Thompson (2004, p. 437) refer to this by stating that "measurement scores, along with geometry, are consistently lower than other content areas on national and international assessments". According to them measurement is an important foundation for many real-world applications and this focus area of the curriculum needs greater emphasis.

2.3 Measurement

The above statement emphasises the reason for measurement as the topic of analysis for this study. Measurement is the process of determining the magnitude of a quantity such as length or mass, relative to a unit of measurement, such as a meter or a kilogram. In general, a unique number assigned to a line segment, plane region or a space region is respectively called the length, area or volume of these elements. Generally length, area or volume is called measures. The smallest length used by ancient civilisation of Egyptians 1700 BC was a cubit which refers to the length of a forearm or ulna. The ancient Greek and Roman 400 BC to 600 AD used a palm which was the similar to hands, to measure the height of horses and for long distance the use of the *pes* or foot. Many ancient cultures measured volumes of grain in basketsful.

The most striking Mathematics feature of these early measures is that although the units themselves may be inexact; the number of units is absolutely exact, because we are dealing with whole numbers.

The whole idea of using numbers to quantify amounts has two parts:

- First the mathematical idea of choosing a fixed unit that unit to match a given amount, which can then be assigned a certain number of units or quantity. This idea is abstract.

- Secondly the practical implication of this scheme by agreeing, how to realize the abstract idea of the fixed unit in practice and how to replicate the unit reliably and fairly.

The key idea behind all measurement is that once we choose a particular segment as our unit (u) of length any other segment (A) is measured exactly in principle by the number (x) of times that the segment (u) fits in to (A). It is this idea that is exact. Hence measurement units are independent of context. If the length of an object is measured in one place and that same object is measured in a different place or country, using the same unit, the measurement will be the same. In practice measurement introduces its own inexactness.

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For very long the acceptance of measurement systems that was applied throughout the Greek and Roman world had first been perfected in Egypt. What is universally referred to as the 'common Egyptian' foot of 300mm continued to be used in Rome in conjunction with the system that is acknowledged as 'Roman'. This Roman system is related to the Greek by a ratio of 24 to 25, and it is the measurements adopted by the Greeks that are the true basis of the Egyptian structure.
Another of the fundamental truths, which must be acknowledged to come to an understanding of the complexities of ancient meteorology, is the fact that the primary system adopted by the Greeks is the root from which all other systems develop.

Measurement started in trade and construction, long before Mathematics and Science emerged as disciplines. By that time the commercial, architectural, political and even moral necessities for abstract, exchangeable units of unchanging value were well recognized (Wright, 1997). Thus the definition of measurement is rooted in this historical background as an act of measuring or the process of being measured. It involves determining of quantity, capacity or dimension.

Several systems of measurement exist, each one comprising units whose amounts have been arbitrarily set and agreed upon by the International System of units which was adopted in 1960 by the 11th General Conference on Weights and Measures. Given the importance of measurement as a foundation for many real-world applications, this strand of the curriculum needs greater emphasis. According to the Curriculum Assessment Policy Statement (2011); conversion; measuring length, distance, mass, weight, volume and temperature; calculating perimeter, area and volume and time forms all part of the topic measurement. These focus areas will form part of the research analysis process. Preston and Thomson (2004) mentions that measurement is one of the most useful mathematic topics for the average citizen, national and international examinations point to measurement as a weakness for middle school students.

According to Preston and Thomson a problematic feature of measurement education is the lack of challenging and meaningful measurement experiences for students. This, however, puts the emphasis on mensuration, the part of Mathematics which deals with measurements of length of lines, areas of surfaces and volume of solids.

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2.3.1 Mensuration

Mensuration is based on the use of formulae and geometric calculations to provide measurement data regarding the width, depth and volume of a given object or group of objects. Thus measurement provides the 'numbers' with which mensuration is executed.

While the measurement results obtained by the use of mensuration are estimates rather than actual physical measurements, the calculations are usually considered very accurate within certain tolerable errors of measurement. Mensuration is thus the calculation of length, area and volume. In the broadest sense, mensuration is all about the process of measurement.

2.3.2 Difficulties and misconceptions regarding measurement

Most often, certain difficulties and misconceptions are attached to measurements, particularly by the learners.

According to Hapkiewicz (1992, p.14), some difficulties and misconceptions regarding measurement experienced by a number of learners are:

- That measurement is only linear.
- That you can only measure to the smallest unit the measuring device allows you.
- That an object must be touched to be measured.
- That time is measured with a clock or watch.
- That only the area of a rectangular shape can be measured in square units.
- That you cannot measure the volume of certain objects because they do not have regular lengths, widths or heights.

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This research aims to analyse learners' responses in the Mathematical literacy examination paper 2 that was written in 2010. The analysis will seek to look at the types of errors and misconceptions as well as alternative ways of working present in their answers when answering questions relating to measurement. What will follow is a discussion on errors, misconceptions and alternative ways of working.

2.4 Errors and Misconceptions

2.4.1 Errors

The Free dictionary (2011) defines an error as an act, assertion or belief that unintentionally deviates from what is correct, right or true. An error is also defined as the condition of having incorrect or false knowledge. Errors in Mathematics are the difference between a computation or conceptual measured value and a true or theoretically correct value.

During the 39th meeting of the International Commission for the study and Improvement of Mathematics teaching (CIEAEM) in 1987 in Sherbrooke - Canada, a definition for an error was agreed upon: "Error takes place when a person chooses the false as the truth". An error is caused by an "insufficient mastery of basic facts, concepts and skills. Such an error in learning something new is called normal" (Rouche, 1989).

Example: "8 + 4 = [??] + 5" with 12 or 17 as the answer.

The answers 12 or 17 can be thought of as an error. The faulty algorithm can be that learners only conducted the first step and got the definite value "12". Another faulty algorithm in this problem was to add all the numbers and get "17".

According to Falker, Levi and Carpenter (1999), such wrong answers clearly indicate that learners had a partial understanding of the equality relation and equal sign, hence a misconception. It has been acknowledged that errors can be a powerful tool to diagnose learning difficulties and consequently direct remediation (Borasi, 1986). However, not all errors that students make are the same. Some errors in procedures can be caused by faulty algorithms or "bugs" where others can have certain conceptual basis and can be termed as misconceptions.

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2.4.2 Misconceptions

Misconceptions are conceptual structures that interact with new concepts, and influence new learning, mostly in a negative way (Oliver, 1988). According to Davis (1984) misconceptions arise from information gained from experiences which interact with previous information experiences stored in the passive memory. It is these underlying beliefs and principles in the cognitive structure that are the cause of systematic conceptual errors, often referred to as misconceptions.

Benander and Clement, (1985) refers to the following example of a misconception:

I went to the shop and bought the same number of books as DVD’s. Books cost \$20 each and DVD’s \$60 each. I spent \$400 altogether.
Assuming that the equation $20B + 60R = 400$ is correct, what is incorrect, if anything, with the following reasoning?
$20B + 60R = 400$. Since $B = D$, we can write,

$20B + 60B = 400$, and therefore,

$80B = 400$

This last equation says 80 books cost \$400, so one book cost \$50.

In this problem the letter “B” is interpreted as “books”. The number of books is also the number of DVD’s as shown by the substitution “B = D” B stands for a variable which is an unknown number not for the word book, hence the misconception.

Rosnick (1981) and Mestre (1982) on the other hand, referred to a common case of misconception:

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At this university there are six times as many students as professors. Use “*S*” for the number of students and “*P*” for the number of professors. The correct answer is $S = 6P$. 35 % of the Anglo engineering students tested wrote $6S = P$ because they confuse the variables as labels where *S* stands for students and *P* stands for professor, rather than interpreting them correctly as variables that are containers of numerical values.

It is obvious from above example; the learners’ misconceptions of some mathematical concepts are particularly difficult to change even under reasonably “good” teaching over a long time. This is emphasised by Al-Khalifa’s (2006) presentation of the effect of learner misconceptions on learning in a novel approach suggesting that misconceptions are induced by teaching students erroneous materials and then correcting that knowledge to identify how this affects future learning. Al-Khalifa proposes that even if future knowledge is taught correctly, it is still affected by the previous erroneous materials. This notion merely shows how crucial it is to try and minimise learners’ misconceptions in order to curb errors that might result from them.

It is important to understand the learners' mathematical perceptions and ways of thinking when they engage with mathematical problems, and also to develop teachers' knowledge about different ways in which learners think and reason mathematically (Even and Wallach, 2005). Although learners might use their mathematical understanding of certain concepts in an improper or "non-standard method", it must not be excluded that learner's might use alternative ways to solve a mathematical problem and still get the correct answer.

2.4.3 Alternative ways of working

This is the use of a "wrong method" method to get to the correct answer.

For example:

$$\frac{1\not{6}}{\not{6}4} = \frac{1}{4}$$

The cancelation of the 6 in 16 and the 6 in 64 shows a wrong method of solving the problem, but can still give the correct answer in some instances (Davis, 1984).

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The next example by Neria and Amit, (2004, p. 225) shows the use of a "non-standard method" to get to the correct answer using the following example:

In Hanukah we light candles each day of the 8 day holiday. Every day we light one leading candle and additional candles, according to the day of the holiday.

On the first day we light the leading candle and one more candle,
On the second day we light the leading candle and two more candles,
On the third day we light the leading candle and three more candles and so on until the last day of celebration.

The questions to this were as follows:

How many candles do we light altogether in all eight days of the holiday?

If the holiday was 30 days long, how many candles would we have to light?

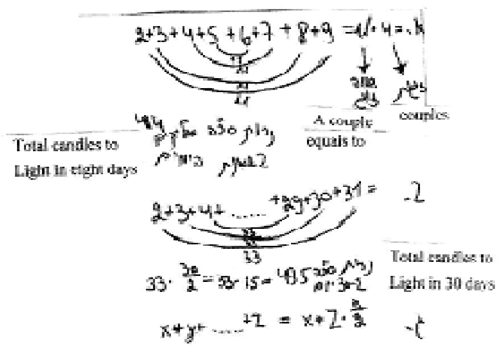
If the holiday was *n* days long, how many candles would we have to light?

The response to the questions above was as follows:

Figure1: Alternative way of solving a word problem, method one.

In figure1, solving question B the student used the strategy of selecting operations and calculating. The student spread out the days (right column), the candles to be lighted each day (middle column). The correct total sum, 495 candles, appears on the right.

Figure 2: Alternative way of solving a word problem, method two.



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In figure 2 the upper part is the answer to question A. The student wrote the number of candles to be lighted each day, and paired them (1^{st} and 8^{th} day, 2^{nd} and 7^{th} day and so on). The student found that there were four equal sums of 11 (11.4). The answer to question B is located in the centre. The student applied the pattern she found in question A to B and got a correct answer.

In the lower part is the answer to question C. The student marked the number of candles lighted on the first day as x (and not 2, as expected), y stood for the number of candles lighted on the second day and z stood for the number of candles on the last day. The student then proceeded and multiplied the sum of candles ($x+y+\ldots+z$) by half of the number of days (n), just as she had found in question A and B.

2.5 Conceptual Framework

This framework will demonstrate errors and misconceptions identified by various authors.

Orton (1983) identify the following types of errors, namely:

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Structural errors which are errors which arise from some failure to appreciate the relationships involved in the problem or to grasp some principle essential to solution.

For example: $3x^2 - 6x = 0$

$$3x\,(x-6)=0$$

The essential principle not grasped is an incorrect factorisation of a quadratic expression consisting of two terms.

In the 2010 Mathematical Literacy Paper the following question appeared:

A casual worker employed during the soccer world cup was paid an hourly rate of $R12{,}50$. Given the formula:

$$Daily\ \ payment = hourly\ \ rate \times number\ \ of\ \ hours\ worked$$

One casual worker worked for $8\frac{1}{2}$ hours. How much did she/he earn?

The learner's response: $12{,}50 \times 8{,}30 =$ 103, 75.

The principle not grasped by the learner is an incorrect conversion from a mixed fraction to a decimal number.

There is a direct translation from $\frac{1}{2}$ into 30 minutes. Instead of using a $\frac{1}{2}$ as 0.5 it is used as 0.30, hence an incorrect conversion from time into a decimal. (Final grade 12 examination Mathematical Literacy Paper one 2010)

Executive errors are errors involving the failure to carry out correct manipulations, although the principles involved may have been understood.

For example: Substitute a = 1; b = -2 and c = -3 into the quadratic formula

$$\frac{2 \pm \sqrt{4 - 4(1)(-3)}}{2(1)}$$

$$2 = \frac{\sqrt{4+12}}{2}$$

Here the error refers to the $\pm$ sign that is being replaced by the = sign.

Arbitrary errors which are errors, in which the subject behaves arbitrary and fails to take account of the constraints, laid down in what was given.

For example: Arbitrary assigning zeros to some variables like below:

$$x + y + z = 30$$

$$x = 30$$

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Newman (1983) identified the following errors based on the Newman's Model for error identification:

Reading errors

An error would be classified as reading if the child could not read a key word or symbol in the written problem to the extent that this prevented him/her from proceeding further along an appropriate problem-solving path.

For example: Billy's answer to the question: "What does fifty-six minus forty equal?" his answer was"96", when asked how he had obtained his answer, Billy replied: "Well, it says, "What does fifty-six minutes forty equal?" It didn't tell me what I had to do, so I added and got ninety-six, because it is more than sixty, so the answer must be in hours". Billy had made a reading error by reading minus as minutes (Clements, 1980, p.1).

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Comprehension error

This occurs when the student has been able to read all the words in the question, but has not grasped the overall meaning of the words and therefore, was unable to proceed further along an appropriate problem-solving path.

For example: Charles, read the following question perfectly, but wrote "15" for his answer.

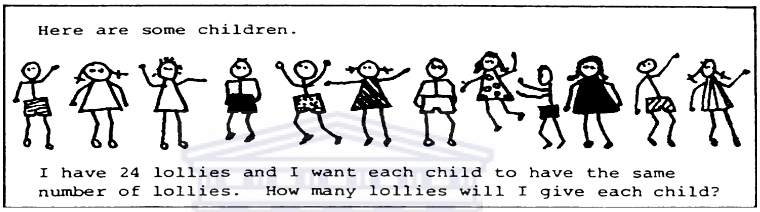
> Sam goes to bed at 10 minutes to 9. John goes to bed 15 minutes later than Sam. What time does John go to bed?

On being asked why he gave this answer he explained that "he says John goes to bed fifteen minutes later, so the answer must be "15".

Charles could read the words but had not grasped the meaning of all the information given in the question. He had not been able to proceed towards the solution of the problem because of a reading comprehension difficulty (Clements, 1980, p.2).

Transformation error

This occurs when the child has understood what the question wanted him/her to find out but was unable to identify the operation, or sequence of operations, needed to solve the problem. Consider the following example:



(Clements, 1980)

For the question shown above, John wrote “144” as his answer. He explained that “there are twelve children, and twenty-four “lollies”; 12 into 24 go 2, so we have two twelve, you multiply these two twelve’s, 12 times 12 are 144”. When asked what the answer to the question was, John said: “Each child gets 144 lollies”. John could read the question well, and knew that he had to find out how many lollies each child should get. He failed to solve the problem correctly because he did not formulate a correct sequence of mathematical steps. He did not transform from the written problem to an acceptable ordered set of mathematical procedures (Clement, 1980, p.2).

Encoding error

This occurs when the child correctly work out the solution to a problem, but cannot express this solution in an acceptable written form. Referring to the same question as above, another response was simply “93” This was an encoding error, because of a failure to present the answer in an acceptable written form (Clements, 1980, p.3).

Mayer (1984) found that students make four classes of errors when solving mathematical problems. These errors relate to knowledge requirements. The table below shows the relationship between error and knowledge type.

Type of error	**Type of knowledge**
Transformation and understanding	Linguistic and factual
Understanding and calling upon relevant knowledge	Schematic
Planning	Strategic
Execution	Algorithmic

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In an in-depth study of learners' work, Davis (1984) postulated a number of error types. These are:

Procedural errors: This involves a defective procedure that occurs when one or more steps do not flow from the underlying mathematical logic. For example, a learner solved the problem $2x^2 + 2x + 3 = 0$ as follows, $(x-1)(x+3) = 0$ (Step a)

When the answer is given as $x = 1$ or $x = 3$ for (step b), then step (a) was correctly performed but in step (b) the rule for the procedure for solving the second linear equation was incorrectly applied.

Random errors and careless errors: These are errors that have nothing to do with a wrong method and can easily be corrected with sufficient care.

The third error identified by Orton (1983) is arbitrary error which is called a random error by Davis (1984). These errors points to an illogical operation. Davis (1984) distinguishes between slips, errors and misconceptions.

Slips are wrong ways of processing mathematical concepts. They are not systematic, but are sporadically and carelessly made by both experts and novices. He also describes that these wrong ways of working are easily detected and are spontaneously corrected.

Errors are wrong ways of working which are systematic and applied regularly in the same circumstances.

Misconceptions as the mistakes or the symptoms of the underlying conceptual structures that are the cause of errors.

According to Davis (1984) some errors occur so commonly that he calls them classical errors. These are:

Binary reversion: This occurs when a well established frame learned in the classroom is extracted by a visually-similar cue.

For example $2^3 = 6$

The error shows a confusion with 2×3 instead of $2 \times 2 \times 2$

Language-induced Errors: These are errors caused by inaccuracies or ambiguities within the text. For example, when learners' are presented with the following problem:

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Write the following algebraic expression in descending order of x

$$3x^2y - 4xy^2 + 6x^3y - 7$$

A learner responds by writing $x^3 + x^2 - x$, because the question states "in terms of x the learner wrote down only the variable x". The learner wrote down only the x and its exponents.

Hodes (1998) identified errors during test taking, namely:

Test-taking errors: These are the mistakes made because of the specific way a test is taken, such as missing questions; not completing a problem; changing test answers from the correct ones to the incorrect ones; miscopying and leaving answers blank.

Other errors that are part of Hodes' study include the following:

Careless errors: These are the mistakes made and can be noted automatically upon reviewing one's works. Careless errors as well as errors during test taking, which Hodes (1998) refers to, form part of what Davis (1984) calls slips.

Concept errors: These are the mistakes made when learners' do not understand the properties or principles in the subject. For example: A casual worker earns an hourly rate of R12.50/hour. One casual worker work 8½ hours daily. How much did she/he earn?

One learner's response was: $R12.50 \times 8\frac{1}{2} = R12.50 \times 4 = R48.00$

Here the 8 is multiplied by a ½ to obtain the answer 4. The response shows a misunderstanding of the concept of the fraction that was use to indicate time.

Application errors: These are mistakes learners made when they know the concept but cannot apply it to a specific situation or question. This type of error is particularly important when it comes to solving word problems. Consider the following example:

Kadija (2010) presents her students with the following problem:

> Given a rectangle where the base (length) is twice the size than the height (breath). Find the dimensions of the rectangle if its perimeter is 120m. One student responds to this by writing 2x multiply by x is equal to 120. Then $2x^2 = 120$. The student then divide by 2 on both sides of the equation to get $x^2 = 60$, then complete the problem with x equal to the square root of 60.

Elbrink, (2007) identified three main errors learners make in their mathematical work. These include:

Calculation errors: These errors are often the result of carelessness or short attention span.

Example: $9x + 14x - 50x = 60 + 3 + 14 - 8$

$x = 69$

This solution shows that the simplification on the left hand side of the equation was totally ignored.

Procedural errors: Students do not have an understanding of how a procedure works; therefore, students do not recognize the importance of applying the procedure correctly.

Consider the following example:

$$= 120 + \left(\frac{45}{14}\right) \times 20$$

$$= 120 + \left(\frac{13}{14}\right) \times 20$$

$$= \left(\frac{133}{14}\right) \times 20$$

The learner shows lack of knowledge of the order in which operations (BODMAS) are applied, which is crucial to the solution of this problem.

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Symbolic errors: Students create their own meanings for mathematical symbols and signs rather than interpreting them according to actual meaning.

Example: When ask to solve $\sqrt{a^2 + b^2} = a + b$, the response shows incorrect interpretation with addition under the square root Elbrink, (2007).

The following extracts are from a grade 12 Mathematical Literacy examination administered by me in June 2011. It shows learners' ways of working with questions relating to measurement.

3.2 The use of fertilisers for crops such as mealies, sorghum, fruit and vegetables, can result in an increased harvest of these crops.

In South Africa farmers use an average of 0,65 kg of fertiliser per hectare (ha), while farmers in Egypt use an average of 4,32 kg of fertiliser per hectare, where $10\ 000\ m^2 = 1$ ha.

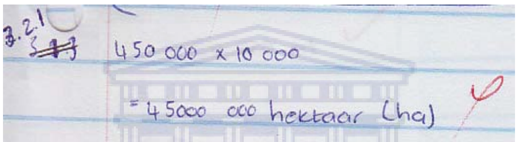
3.2.1 Convert 450 000 m^2 to hectares. (2)

3.2.2 Calculate the number of hectares that could be fertilised with 5 000 kg of fertiliser by a farmer in South Africa. Give the answer rounded off to the nearest hectare. (3)

3.2.3 Calculate the number of kilograms of fertiliser that would be needed in Egypt to fertilise 2 000 ha. (3)

3.2.4 Write the average amount of fertiliser used per hectare in South Africa as a percentage of the average amount of fertiliser used per hectare in Egypt. (2)
[19]

In question 3.2.1 it is expected of the learner to work with conversion of units. Converting m^2 to hectares.



Here the learner multiplies 450 000m^2 with 10 000 instead of dividing by 10 000.

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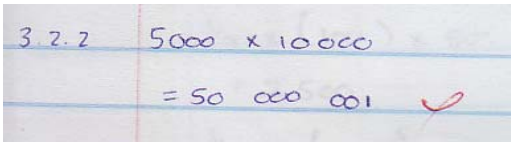
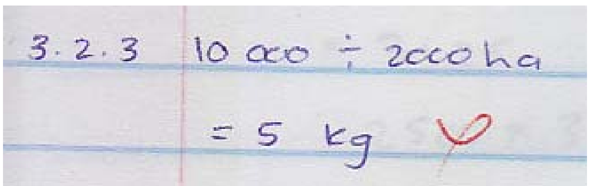
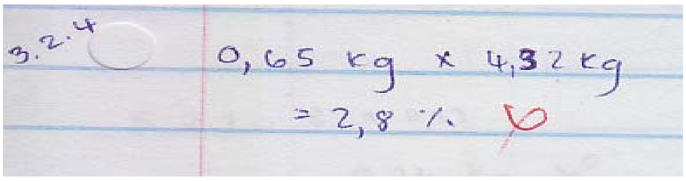


In question 3.2.2 the learner multiplies 5000 by 10 000 instead of dividing 5000 by 0.65.



In the response to 3.2.3 the learner divides, 10 000 by 20 000ha instead of multiplying 2000ha by 4.32kg.

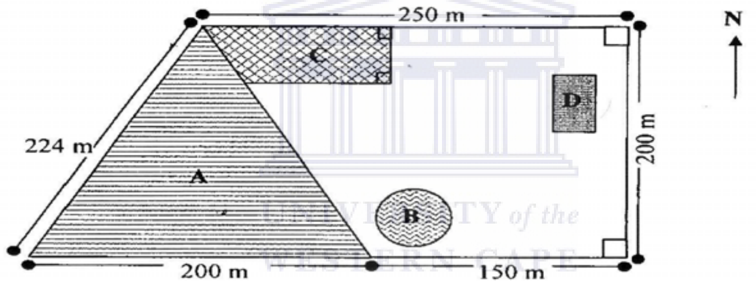
The response to 3.2.4 shows that the learner could not work out how one quantity can be expressed as a persentage of another quantity.



QUESTION 5

5.1 Mr J Khoso owns a plot, as shown in the diagram below (not drawn to scale). His house (D) is on the eastern side of the plot. Also on the plot is a cattle kraal (A), a circular water tank (B), and a vegetable garden (C).

Diagram of Mr J Khoso's plot



KEY		**DIMENSIONS**
A	Cattle kraal	Height = 200 m Base (south) = 200 m Slanting side = 224 m
B	Water tank	Radius = 10 m
C	Vegetable garden	Parallel sides = 100 m and 125 m Distance between parallel sides (height) = 50 m
D	House	Length = 25 m Breadth = 8 m

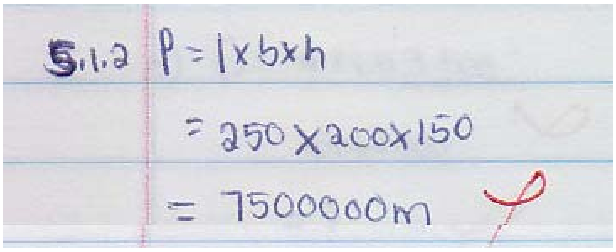
5.1.2 Determine the perimeter of Mr Khoso's plot.

5.1.5 Calculate the total area of Mr Khoso's plot.

Use the formula:

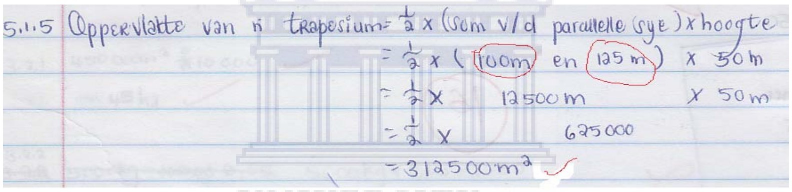
Area of a trapezium = $\frac{1}{2} \times$ **(sum of the parallel sides)** $\times$ **height**

The learner was asked to determine the perimeter of Mr Khoso’s plot and the response was as follows:



The learner calculated the area of the vegetable garden instead but made a mistake in the calculation because the learner multiplied the parallel sides instead of adding them.

Question 5.1.5 the learner was asked to calculate the total area of Mr Khoso's plot using the given formula and the response was as follows:



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The learner's response shows a wrong substitution of the parallel sides of the plot. The learner used the size of the parallel line of the garden.

Brodie and Berger (2010) developed and classified a set of possible errors by focusing on the analysis of a multiple choice question paper. They pointed out systematic, persistent and pervasive patterns of mistakes made by learners. These are the errors identified by them that are relevant to this study:

Keyword trigger: A certain keyword or term in the distractor that may signal to the learner to apply a particular routine, even though the term or word does not serve such a purpose to the problem.

Example: Lisa wrote this formula to work out the total numbers of liters in 24 soft drink cans. $24x = y$. What does x represent?

A the numbers of liters in one can

B the number of liters in 24 cans

C the numbers of soft drink cans

D the number of cans per liter

If the answer (B) was chosen, it means that the use of x in the given equation triggered a routine. The routine had the following form: *x* is what was found. We have to find the number of liters in 24 cans. So *x* is the number of liters in 24 cans. (Brodie and Berger 2000, p. 174)

Errors of visual mediators

- **Inappropriate use of visual detail**: When a learner wrongly interprets visual detail such as photographs, diagram or graph by ignoring certain information.

When learners are asked to determine the size of (x) which is one of the interior angles of triangle ABC:

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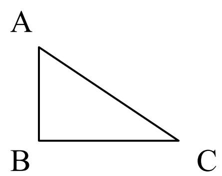
Example:

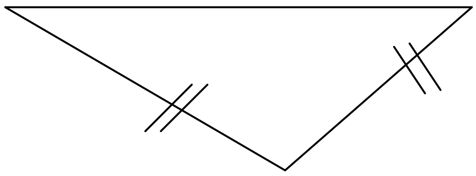


Angle A = 35°

Angle C = 65°

A response which will show a symbolic error will be $x = 90°$. This response will indicate that the learner was misled by the shape of the triangle instead of calculating the size of (x).

Difficulty with visual construction: Lack of visual mediation in mathematical communication or thinking. For example: All the faces of a solid are identical copies of this isosceles triangle.



How many faces does the solid have?

A 3
B 4
C 6
D 8

The answer 6 will show a lack of visual mediation (Brodie 2000, p. 178).

From what was mentioned above, one can identify differences and similarities between the identification and description of errors put forward by the different authors. Orton (1983) and Hodes (1998), have all identified errors related to a lack of knowledge of important principles.

Hodes (1998) calls them concept errors whereas Orton and Wain (1983) call them structural errors. On the other hand Orton (1983) mentioned executive errors which also refer to calculation errors mentioned by Elbrink (2008) and algebraic manipulation errors identified by Mason (2000). All these errors are connected with multiplying or dividing wrongly an expression or a number. Other important errors mentioned under this category that Orton (1983) calls executive errors, are symbolic misunderstanding which Mason (2000) refers to as symbols and notations and misinterpretations, (which Mason (2000) calls notations) which all points to a lack of association.

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Davis (1984) and Elbrink (2007) on the other hand, classified procedural errors which Davis referred to as systematic errors. Procedural errors are consistent over a range of similar problems which reflect important flaws in children's computation. Brodie (2010) calls these types of errors Halting signal errors. Another important error mentioned by Hodes (1998) is the application errors which refers to mistakes learners made when they know the concept but cannot apply it to a specific situation or question. These types of errors are particularly important when it comes to solving word problems.

Therefore, structural and executive errors mentioned by Orton (1983) as well as procedural error mentioned by Davis (1984) contain a variety of other errors mentioned by other different authors. A commonly encountered presence of word problems in the Mathematical Literacy question papers makes the mentioning of application errors very relevant. These coherent ideas and concepts which exemplify what was done by other researchers provide a platform and is ultimately the basis for this research study.

2.6 Building a conceptual framework

The conceptual framework is constructed from a set of ideas and theories from the literature, with a purpose to structure the research, give direction and guide the data collection and analysis. The main focus of the conceptual framework in this study is on the definitions of different errors identified for Mathematics. The analysis of the causes of errors and the application of the results from analysis in the process of planning of Mathematics teaching and learning can provide valuable insights for the teaching of Mathematics.

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When introducing new concepts or procedures, the knowledge about errors informs teachers what to focus on, what to clarify and, how to negotiate the comprehension of new terms in order to avoid a certain type of errors and how to positively use the identified errors as the basis for mathematical understanding (Olivier, 1989).

Mathematics has an internally coherent structure and its concepts are built on the basis of other concepts. Therefore, the facilitation of learning Mathematics needs to keep this in consideration. A seemingly small gap in comprehension or knowledge creates further errors that are built upon another, which after some time are revealed in an error avalanche. An unrevealed error is rooted in the mind of students, and therefore it is a major threat to the construction of students' mathematical knowledge. Hence, a revealed and clarified error may be extremely useful both for students and teachers (Krygowska and Booker 1988). The understanding of why students make mistakes can be interpreted in terms of learning theories. The behaviourist theory and constructivist theory are considered in this study.

2.7 Theoretical Considerations

Student's errors in Mathematics education as assumed by behaviourist are simply a result of ignorance and situational accidents. According to this theory, most student mistakes are due to uncertainty and carelessness, and all knowledge originates in experience.

Behaviourism therefore assumes that students learn what they are taught, because it is assumed that knowledge can be transferred intact from one person to another. Olivier (1989) cited Strike (1983) that according to a behaviourist perspective, errors and misconceptions is not important because it does not consider students' current concepts as relevant to learning. Rather, errors and misconceptions are seen as a faulty byte in a computer's memory; meaning that if we don't like what is there, it can simply be erased or written over, by telling the pupil the correct view of the matter (Strike, 1983).

This view is concisely put by Gagne (1983, p. 15) who states that: "the effects of incorrect rules of computation, as exhibited in faulty performance, can most readily be overcome by deliberate teaching of correct rules. This means that teachers would best ignore the incorrect performances, and set about as directly as possible teaching the rules for correct ones".

A constructivist perspective on learning assumes that concepts are not taken directly from experience, but that a person's ability to learn from and what she/he learns from an experience depends on the quality of the ideas that she/he is able to bring to that experience. Knowledge does not simply arise from experience; rather, it arises from the interaction between experience and our current knowledge structures. The student is therefore not seen as passively receiving knowledge from the environment, as it is impossible that knowledge can be transferred ready-made and intact from one person to another. Although instruction clearly affects what children learn, it does not determine it, because the child is an active participant in the construction of his own knowledge. Therefore, student errors are the result or the product of previous experience in the mathematics classroom (Piaget, 1970).

From a constructivist point of view, misconceptions are crucially important to learning and teaching because misconceptions form part of a pupil's conceptual structure that will interrelate with new concepts and influence new learning, mostly in a negative way. Therefore, if the general principles of cognitive functioning are understood from a constructivist perspective, it becomes obvious that, for the most part, students do not make mistakes because they are stupid. Rather, their mistakes are rational and meaningful efforts to cope with Mathematics; and these mistakes are derivations from what they have been taught.

Of course, these derivations are objectively illogical and wrong, but, psychologically, from the student's perspective, they make a lot of sense (Ginsburg, 1977).

Other related literature alerts us that students' errors are causally determined and can very often be systematic. It shows in the descriptions that it can be persistent, or they can be easily detected. The analysis reflects certain error techniques. It explains the possible causes for certain errors made by students while receiving and processing information in the mathematical learning process, or from effects of the interaction of variables acting on Mathematics education.

These analyses of errors illustrate individual difficulties; and shows that students failed to understand or grasp certain concepts, techniques and problems in a "scientific" manner. Student errors reveal faulty problem-solving processes and provide information relating to the understanding of and the attitudes toward mathematical problems. From this perspective, error analysis is important in two respects.

First, with regard to the requirements of academic practice, error analysis provides an opportunity to diagnose learning difficulties, in mathematical education, and as a means to create more awareness and support for the performance and understanding of individual students. Secondly, error analysis seems to be a starting point for research on the mathematical teaching and learning process. Error analysis must be considered a promising research strategy for clarifying some fundamental questions of Mathematics learning. In my reading of the literature and from what is written in the literature on error analysis, I have found that there are different categories of errors. Some errors come from misunderstanding and others are a result of wrong computation.

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This study will attempt to analyse students' mathematical errors and misconceptions by focusing on what was done by the authors as indicated in the conceptual framework, on error identification.

These notable errors summarised in Table 3 below are all a result of misconceptions.

Table 2: Framework for identification of errors

Error Type	**Description**
Structural Errors	Failure to grasp principles essential to solution
Executive Errors	Failure to carry out manipulation
Procedural Errors	Defective procedure, when one or more steps do not flow from the underlying mathematical logic.
Application Errors	Mistakes made when they know the concept but cannot apply it to a specific situation or question.

The analytical template above will be used, to analyse students' errors as exhibited in the high-stakes Mathematical Literacy final external examination.

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CHAPTER 3

Research Design

3.1 Introduction

This chapter discusses the research context, methodological approach and the data collection process as it was employed in this study. The chapter also touches on the reliability and validity issues which is a critical part of this research. A short overview of qualitative research and document analysis is presented. The second part describes the data collection process. It describes how the sample for data collection was constructed.

3.2 Research context

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It is necessary to see the environment and mention the circumstances under which learners completed the examination. It was in this context that learner responses were developed in which the errors, misconceptions and alternative ways of working occurred.

3.2.1 Examination rules and responsibilities set by the WCED

According to the 2007 Examination Procedure Manual, supplied by the Chief Directorate: Assessment and examinations in the WCED, the following arrangements must be adhere to:

- No two candidates must be seated together at one desk or table.
- Candidates must be seated one metre apart where possible.
- Candidates must enter the examination room 30 minutes before the official starting time.
- The first 15 minutes are used to settle the candidates in their seats and distribute answer books and question papers.
- During the remaining 10 minutes candidates must read through the question papers.

- No writing or making of notes is allowed during the 30 minute preparation period, for such an infringement will constitute an irregularity.

The above rules and responsibilities laid out by the WCED is what set the boundaries for what was allowed and not allowed during this high-stake assessment. It emphasises the strictness under which these high-stake examination took place.

3.3 Methodological Approach

To best answer the research questions put forward in this study a qualitative research approach was preferred because it seeks to appreciate and understand phenomena in context, such as: "real world setting, where the researcher does not attempt to manipulate the phenomenon of interest" (Patton, 2001, p.39). Qualitative research approach seeks clarification, understanding and extrapolation of findings (Hoepfl, 1997).

According to Jacob (1987), qualitative research methodology attempts to present the data from the perspective of learners. This is to ensure that researchers' cultural and intellectual biases do not distort the collection, interpretation or presentation of data. Seliger and Shohany (1989) point out that qualitative research is considered as synthetic or holistic and heuristic with little or no manipulation of the research environment.

3.3.1 Document Analysis under Qualitative Research Approach

Document analysis in qualitative research is a systematic procedure for reviewing or evaluating documents, both printed and electronic material. Documents exist prior to research, it reveal what people do or what they value. This study involved the analysis of documents in the form of examination scripts.

Like other analytical methods in qualitative research, document analysis requires that data be examined and interpreted in order to elicit meaning, gain understanding and develop empirical knowledge (Cordin and Strauss, 2008).

Although document analysis has served mostly as a complement to other research methods, it has also been used as a stand-alone method (Bowen, 2009). Furthermore Bowen (2009) explains that documents can serve a variety of purposes as part of a research undertaking, he specifies five uses of documents.

Firstly, documents can provide data on the context within which research participants operate. Such information and insight can help researchers understand the historical roots of specific issues and can indicate the conditions that impinge upon the phenomena under investigation.
Secondly, information in documents can suggest some questions that need to be asked and situations that need to be observed as part of research.

Thirdly, documents can provide primary or supplementary research data. The information and insight derive from documents can be valuable additions to a knowledge base.
Fourthly, documents provide a means of tracking changes and development.
Fifthly, documents can be analysed as a way to verify findings or corroborate evidence from other sources. From above purposes it is clear what strengthens the idea of documents and what one can get from it.

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3.3.2 Advantages of document analysis

Yin (1994, p. 80) cited by Bowen (2009), proposed the advantages of using the qualitative document analysis research method:

- It is less time consuming
- It requires data selection instead of data collection
- It can be reviewed repeatedly
- It is cost effective because data has already being gathered; and what remains is for the content to be evaluated.
- Documents are unaffected by the research process.

3.3.3 Limitations of documents

According to Yin (1994, p.80) cited by Bowen (2009), the following are some of the limitations associated to the use of documents:

- Documents are produced for some purposes other than research; consequently, they usually do not provide sufficient details to answer a research question.

- Documents are not always retrievable and that access to documents may be purposely blocked.

- Certain incomplete documents suggest biased selectivity.

Bowen (2009) is of the opinion that efficiency and cost-effectiveness outweigh limitations. Hence, I acknowledge that both the advantages and limitations played an essential role in the planning and performing of the documents analysis process which was employed in this study.

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3.4 Data Base

Data used in this study is a sample of the final Grade 12 Mathematical Literacy examination scripts of 2010 that was written in the WCED. The research question focuses on the responses of students which is ultimately what the question papers produce. The question papers were selected according to the centre numbers. No specific reference was made in terms of complete or incomplete question papers. It is expected of the grade 12 learners to write a Mathematical Literacy Paper 1 and 2. Both papers count 150 marks and were to be completed in 3 hours respectively on different days. These papers are written in their numerical order. The distribution of marks according to taxonomy levels for paper 1 and 2 are shown below.

Paper 1 examination includes questions at the different levels of the taxonomy according to the following mark distribution:

• 60% of the marks at Level 1 (knowing);

• 35% of the marks at Level 2 (applying routine procedures in familiar contexts);

• 5% of the marks at Level 3 (applying multi-step procedures in a variety of contexts);

Paper 2 examinations include questions at different levels of the taxonomy according to the following mark distribution:

• 25% of the marks at Level 2 (applying routine procedures in familiar contexts);

• 35% of the marks at Level 3 (applying multi-step procedures in a variety of contexts);

• 40% of the marks at Level 4 (reasoning and reflecting). (Curriculum Assessment Policy Statement, 2011, p. 113)

Over the years learners' performance in Paper 2 Examination was weaker than in Paper 1. The administration of the 2010 paper one to my 2011 grade 12 students indicated that they were doing sufficiently well and that possible errors that they might commit will surface again in paper two. The analysis in chapter 2 gives a further indication in this regard. Therefore paper two was the object of analysis.

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3.4.1 Sampling

Sampling is the act, process or technique of selecting a suitable sample or a representative part of a population for the purpose of determining parameters or characteristics of the whole population (http://socialresearchmethods.net/tutorial). Sampling is just as important in qualitative research as it is in quantitative research. One cannot study everyone everywhere doing everything. Therefore sampling requires decisions on setting boundaries and processes (Punch, 2009).

For the purpose of this research study the respondents needed to reflect the diversity of the population. Therefore all groups had to be divided based on their proportionality to the total population.

Mindful of the above; this study was confined to schools which had a total of 20 to 80 learners who wrote the examination. A stratified sample was compiled by selecting from the eight districts a population of 10% of the number of learners who wrote the examination. In Qualitative research the concern is with the issue of access. Once gaining access, the researcher is obliged to follow certain rules so as to maintain access (Potter, 1996). This is important in relative private settings such as the Western Cape Education Department where access to the data is not observable by the general public. The examination papers are kept at the Western Cape Education Department, and after following all ethical procedures, authorization was given by the department to access the scripts needed for the research.

Deciding on the sample size was influenced by all the preceding concerns. A random selection of 9.2% based on proportion was done from all eight districts. These districts are grouped in terms of their geographical setting within the Western Cape. The table below show all eight districts in no specific order. This 9.2% allowed for the unforeseen restrictions and necessary limitations that occurred on the side of the WCED. This 9.2% was proportionately divided between the eight districts. Table 3 shows a breakdown of learners in all eight districts, who entered the examination and the total learners who wrote the examination as well as the total number of scripts selected in each ex-departments, Independent and WCED schools within each district.

The ex-department refers to the schools as it was known in apartheid South Africa. Education during this time was segregated according to race groups. Different government departments were responsible for the administration of schools for different race. The establishment of Tri-cameral Parliament which was introduced in 1983 saw the establishment of Coloured Affairs which administered coloured schools under the department of House of Representative (HOR), Indian schools was administered by the House of Delegates (HOD) and whites schools was managed by the Cape Provincial Administration (CED) which is now the Western Cape.

Bantu education was replaced by the Department of Education and Training (DET). This was done to further segregate the blacks from the rest of South Africa. The Independent Education Department refers to private schools and Western Cape Education Department are schools which were established in the post-apartheid era.

Table 3: Summary of sample selection

Summary of sample selection							
	Ex-departments						
Districts	CED	DET	HOD	HOR	IND	WCED	TOTAL
Cape Winelands	94	7		305			406
Eden and Central Karoo	134	2		154			290
Metropole Central	97	70		191	49		407
Metropole East	82	91		173		24	370
Metropole North	119			286		63	468
Metropole South	55	61		259		47	422
Overberg	40			91			131
Westcoast	66			84			150
Total	687	231		1542	49	134	2644

3.4.2 Data selection

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After the sampling process was finalised, the data selection commenced. The data selection process involved a review of the question papers, in order to determine which questions fall under measurement. The questions identified are as follows: Q1, (1.1) which deals with the calculation of the circumference; Q2, (2.1.3b and 2.1.4) which deals with the calculation of actual distance and time; Q4, (4.1 to 4.3.2) test the learner's ability to do calculations on area and mass.

QUESTION 1

1.1 Ma Ndlovu makes circular place mats and circular tablecloths out of material (fabric) edged with beads and sells them. The place mats have a diameter of 30 cm.

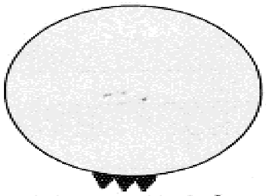
The radius of the tablecloth is FOUR times the radius of a place mat.

The following formula may be used:

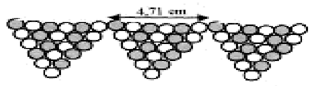
Circumference = $2\pi \times$ radius, and using $\pi = 3{,}14$

1.1.1 (a) Calculate the circumference of the tablecloth. (4)

(b) She uses a beaded edging consisting of triangular segments to decorate the edge of each tablecloth, as shown in the diagrams below. Each segment of the beaded edging is 4,71 cm long.



Circular tablecloth



Enlargement of beaded triangular segments

Calculate the number of beaded segments that she will need for each tablecloth. (2)

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QUESTION 2

Freedom High School's soccer team is taking part in a football tournament at iMbali in the iMbali Soccer Stadium.

2.1 On his way to iMbali, while travelling in a north-easterly direction, the driver of the school bus stopped in Selby Msimang Road (refer to the map on ANNEXURE B) to consult his map for directions to the iMbali Soccer Stadium.

(b) Hence, use a ruler to measure (in millimetres) the approximate distance of this shortest route on the map, and then calculate the actual distance, in kilometres, using the given scale.

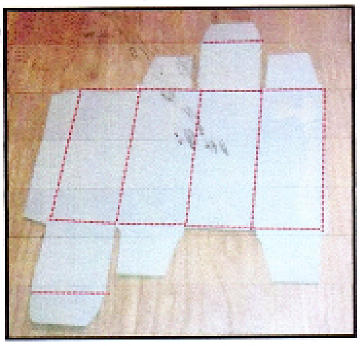
2.1.4 At 09:15, after looking at the map, the bus driver was ready to start driving again. He contacted the tournament coordinator to inform her that they would be at the stadium at 09:20. If the bus travelled at an average speed of 40 km/h, verify by means of relevant calculations whether the bus driver's estimated time of arrival was correct.

The following formula may be used:

$$\textbf{Average speed} = \frac{\textbf{distance}}{\textbf{time}}$$

4.3 To ensure that the box is strong enough, the cardboard used for the box has a mass of 240 grams per m^2 (g/m^2).

The layout of the opened cardboard box is shown below.



Picture of opened box

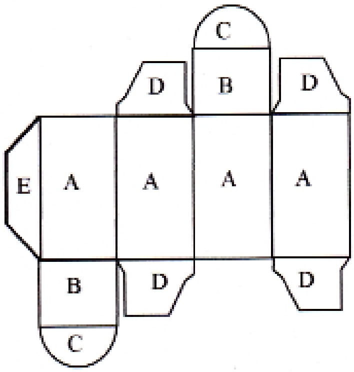


Diagram of layout of opened box

- Section C is semicircular.
- The area of each section D = 1 832 mm^2.
- The area of section E = 2 855 mm^2.

4.3.1 Calculate the total mass of the cardboard needed for one box, to the nearest gram. (11)

4.3.2 The total cost of the cough syrup includes the cost of the cardboard box.

Use the following formula to calculate the cost of a boxed bottle of cough syrup:

Total cost = R16,00 + (mass of cardboard box) × R20,00 per kg (3)

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3.5 Reliability and Validity

According to Patton (2001), validity and reliability are two factors which any qualitative researcher should be concerned about while designing a study, analysing results and judging the quality of the study. Credibility depends less on sample size than on the richness of the information gathered and the analytical ability of the researcher (Patton, 1990). Reliability points to the consistency of measurement. According to Golafsnani (2003) examination of trustworthiness is crucial to ensure reliability and validity in qualitative research. He states that if validity or trustworthiness can be maximized or tested then more credible and defensible results to a high quality qualitative research can be achieved.

Reliability can be enhanced through triangulation of data. Patton (1990) identified four types of triangulation: method triangulation; data triangulation; triangulation through multiple analysts and theory triangulation. From the four types of triangulation mention by Patton (1990), data triangulation and multiple analysts triangulation was part of this study. By triangulating data, one attempted to provide a confluence of evidence that strengthen credibility (Eisner, 1991, p.110). According to Patton (1990), triangulation helps the researcher to guard against the accusation that the findings of a single study are simply an artefact of a single method, a single source or a single investigator's bias. Some researchers have found that the use of more than one analyst can improve the consistency and the reliability of analysis (Daly, McDonald, and Willis, 1992). In terms of reliability, I acknowledge that the interpretation of the data is subject to human prejudice. Therefore, to further enhance the reliability and credibility, analyst triangulation was used. The analysts who were part of the triangulation process were my peers. They were asked to assist in the consistency of the interpretation of the analytical framework. This method was employed to reduce the potential bias coming from a single person doing all the data collection.

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3.6 Data analysis

Bogdan and Biklen (1982, p.145), define qualitative data analysis as "working with data, organising it, breaking it into manageable units, synthesising it, searching for patterns, discovering what is important and what is to be learned and deciding what you will tell others". Patton (1990) pointed out that qualitative data analysis requires some creativity. To Patton (1990) the challenge is to place raw data into logical, meaningful categories; to examine them in a holistic fashion and to find a way to communicate this interpretation to others.

3.6.1 Identification of errors

At first the data was organised in terms of the questions that were identified for analysis. These questions were organised in numerical order and kept under their respective centres.

During open coding the researchers identified and tentatively named the conceptual categories. This case was done by giving names to the error that occurred in the written responses of the learners. These names given to errors were guided by the analytical template.

This was done through an iterative and progressive process. During the identification process the researchers independently identified the errors from the relevant questions. They then produced outcomes in terms of the analytical template. These outcomes of error identification were then compared and through a process of inter-rater agreement an agreed outcome was reached. This inter-rater agreement helped to strengthen the reliability and emphasise the degree of agreement amongst raters. Among the raters were two masters' students who assisted in the analysis process. This whole process of noticing of new errors while in the coding process made this process recursive because the disagreement or differences was resolved by going back to the error and through discussion agreements were reached. Thus the holographic process of noticing errors and thinking about errors was the foundation of the coding process.

After the coding process was done the process of examining the data started. During this process an error was grouped in terms of their similarities and types. The aim of this process was to assemble or reconstruct the data in a meaningful and comprehensible way. This enabled the researcher to make some sense out of the collection; look for patterns and relationships both within and across the collection and ultimately make general discoveries about learners' response on high-stakes Mathematical Literacy examinations. Along the process of identification of errors, other realities were revealed which was not part of the primary objective of this study. In these cases these notable information were documented for future research.

This chapter gave a description of several considerations and decisions regarding qualitative research methodology with specific reference to document analysis. The discussion in this chapter placed the research in a specific context, which deals with high-stakes examinations.
This chapter placed emphasis on the validity and reliability in qualitative research as well as in the sampling and data collection process. What will follow in chapter four is an in-depth discussion on the data analysis.

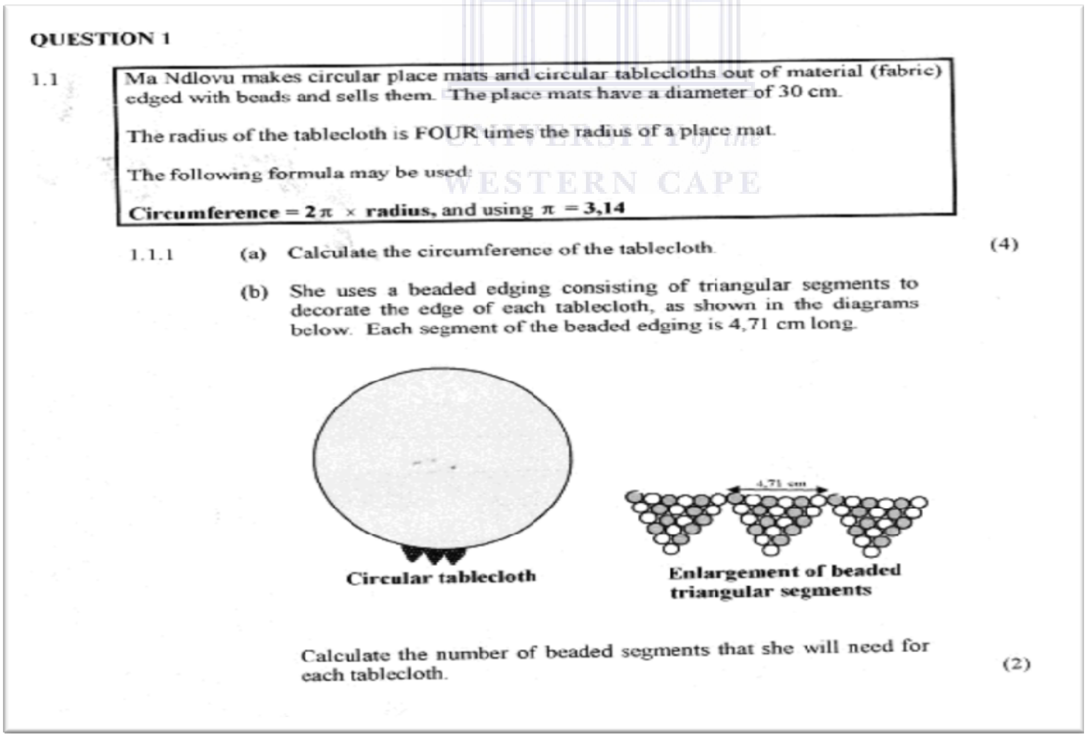
Chapter 4

Data analysis

4.1 Introduction

The research methods and data collection procedures were discussed in the previous chapter. This chapter describes how the data were analysed and made sense of. The analysis involved looking at a sample of 1807 grade 12 students' scripts repeatedly to identify errors and misconceptions in the students' ways of working. Only the parts pertaining measurement of questions 1, 2 and 4 of paper 2 were analysed. The questions chosen for analysis as they appeared in the examination question paper are as follows:

Questions from Mathematics Literacy examination 2010, paper 2


QUESTION 1

1.1 Ma Ndlovu makes circular place mats and circular tablecloths out of material (fabric) edged with beads and sells them. The place mats have a diameter of 30 cm.

The radius of the tablecloth is FOUR times the radius of a place mat.

The following formula may be used:

Circumference $= 2\pi \times$ **radius,** and using $\pi = 3{,}14$

1.1.1 (a) Calculate the circumference of the tablecloth. (4)

(b) She uses a beaded edging consisting of triangular segments to decorate the edge of each tablecloth, as shown in the diagrams below. Each segment of the beaded edging is 4,71 cm long.

Calculate the number of beaded segments that she will need for each tablecloth. (2)

Figure 4.1: Part of question 1.

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QUESTION 2

Freedom High School's soccer team is taking part in a football tournament at iMbali in the iMbali Soccer Stadium.

2.1 On his way to iMbali, while travelling in a north-easterly direction, the driver of the school bus stopped in Selby Msimang Road (refer to the map on ANNEXURE B) to consult his map for directions to the iMbali Soccer Stadium.

(b) Hence, use a ruler to measure (in millimetres) the approximate distance of this shortest route on the map, and then calculate the actual distance, in kilometres, using the given scale.

2.1.4 At 09:15, after looking at the map, the bus driver was ready to start driving again. He contacted the tournament coordinator to inform her that they would be at the stadium at 09:20. If the bus travelled at an average speed of 40 km/h, verify by means of relevant calculations whether the bus driver's estimated time of arrival was correct.

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The following formula may be used:

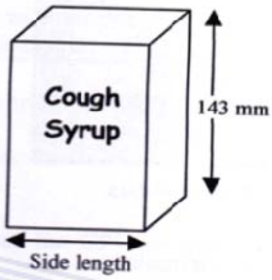
$$\textbf{Average speed} = \frac{\textbf{distance}}{\textbf{time}}$$

Figure 4.2: Part of question 2.

QUESTION 4

Triggers Enterprises was awarded the tender for making rectangular cardboard boxes to package bottles of cough syrup. Each bottle is packed in a cardboard box with a square base, as shown below.

- The diameter of the base of the bottle is 58 mm and the height of the box is 143 mm.
- The length of the side of the base of the box must be approximately 105% of the diameter of the base of the bottle.
- The height of the box is approximately 102% of the height of the bottle.



The following formulae may be used:

Area of circle = $\pi \times (\text{radius})^2$, and using $\pi = 3{,}14$
Area of square = $(\text{side length})^2$
Area of rectangle = length × breadth
Area of opened cardboard box = $4(A + D) + 2(B + C) + E$
(See design of open cardboard box in QUESTION 4.3)

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The following conversions may be useful:

$1 \text{ cm}^2 = 100 \text{ mm}^2$
$1 \text{ m}^2 = 10\ 000 \text{ cm}^2$

4.1 Calculate the height of the bottle to the nearest millimetre. (3)

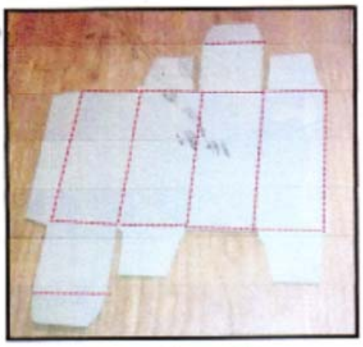
4.2 In order to minimise the cost of cardboard required for the box, the following guideline is used:

The difference between the areas of the base of the cardboard box and the base of the bottle should not be more than 11 cm².

Determine whether the dimensions of this cardboard box satisfy the above guideline. Show ALL appropriate calculations. (11)

4.3 To ensure that the box is strong enough, the cardboard used for the box has a mass of 240 grams per m^2 (g/m^2).

The layout of the opened cardboard box is shown below.



Picture of opened box

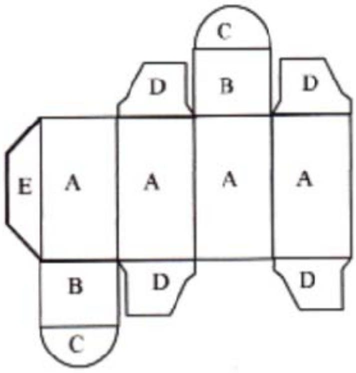


Diagram of layout of opened box

- Section C is semicircular.
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4.3.1 Calculate the total mass of the cardboard needed for one box, to the nearest gram. (11)

4.3.2 The total cost of the cough syrup includes the cost of the cardboard box.

Use the following formula to calculate the cost of a boxed bottle of cough syrup:

Total cost = R16,00 + (mass of cardboard box) × R20,00 per kg (3)

[28]

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Figure 4.3: Question 4.

The errors and misconceptions that were identified were classified according to the template designed in the literature review chapter. Errors and misconceptions that could not be placed in a category named in the template were given a different classification. These new categories were the "no responses" and the answers that show a clear breakdown in mathematical principles which is called the "break down" category.

The following errors are named in the template:

- Structural errors: Failure to grasp principles essential to solution.
- Executive errors: Failure to carry our correct manipulations.
- Procedural errors: Defective procedure, when one or more steps do not flow from the underlying mathematical logic.
- Application errors: Mistakes made when the concept is known but cannot be applied to a specific question or situation.

The classification of errors in learners' responses was done by three independent analysts who are all Mathematical Literacy teachers as well as the researcher. Each of the analysts independently classified all of the responses in the sample. Not all the errors were agreed upon; hence the inter- rater agreement method was adopted. A total of 85% of the errors was agreed upon the 15% discussed and through this discussion an agreement was reached. Each analyst coded the errors as follows: S for structural errors, E for executive errors, P for procedural errors and A for application errors. The classification of errors was thereafter discussed by the team of analysts and the researcher. After scrutinising all the errors made by students in the selected sample, it was agreed upon to subdivide the above categories of errors into more specific errors. This was decided because of the high frequency of certain common errors. Not all errors were placed in subcategories.

An error that was identified as a structural error occurred when students chose the wrong operation to do a problem, for example, when multiplication instead of division was used in order to solve a problem. This type of structural error was noted frequently in the sample and to give prominence to this mistake it was thus decided to name it an "operation selection error". If students chose the wrong operation, it demonstrates a lack of knowledge of the mathematical principles which was necessary to solve the problem. Therefore, these errors were placed labelled as structural errors.

Executive errors occur when the wrong manipulation is carried out. For instance, if students knew that they had to multiply, but the execution thereof was done incorrectly. In Figure 4.2 question 2.1 the students' were asked to determine the actual distance in kilometres.

It was found that students' knew that they had to divide but did it incorrectly using the incorrect values. Errors occurred frequently when students converted the distance measured in millimetres to an actual distance in kilometres and therefore they were named conversion errors. Conversion errors were thus regarded as executive errors.

Similarly, another common error identified in the sample was when students substituted wrong values into correct formulas. These were named "substitution errors" and were classified as executive errors. The different types of substitution errors will be highlighted when specific examples of substitution errors are discussed.

Procedural errors occur when there is a defective procedure. During the analysis it was noted that a common procedural error, in Figure 4.1 question 1, was when students failed to transfer a value from one part of a problem to the next part where it should have been used in the subsequent calculation. This error was so common and it described the meaning of the notion of transfer so aptly that it was decided to give it the classification "transfer errors". Transfer errors would occur when students failed to transfer the answer obtained from one part of a question to the next part of the question, even if that answer was calculated incorrectly. Therefore, all transfer errors would be procedural errors since it shows a breakdown in the mathematical procedure.

Application errors happen when, although concepts are known, the application could not be done correctly. For example in question 2.1 (b) students were instructed to measure the shortest route between two places with a ruler in mm. What was discovered during the analysis was that students committed errors by either measuring a straight line between the two points or they measured the distance in centimetres or any other unit instead millimetres.

These types of errors are classified as application errors because they could not carry out the instruction. Again, these errors which derived from "measuring errors" were all classified as application errors.

The following table shows a summary of the main errors with its sub-categories. Executive errors have two sub-categories whereas the rest of the main errors in the table consist of one sub-category.

Table 4.1: Summary of types of errors

Error	Analysis Description of errors
Structural errors: Failure to grasp principles essential to solution	**Operation selection errors:** When wrong operation is selected in problem or using inappropriate mathematical method.
Executive errors: Failure to carry out correct manipulation	**Conversion error:** When one unit is incorrectly converted into another unit of measurement. **Substitution errors:** When wrong substitutions are made
Procedural errors: Defective procedure	**Transfer errors:** When answer from one part of problem is not transferred to the next part of the problem.
Application errors: Mistakes made when the concept is known but cannot be applied to a specific question or situation.	**Measuring error:** When physical measurement is done incorrectly.

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Errors encountered in the sample for which provision in the template was not made are:

- Errors occurring as a result of language difficulties encountered by learners. These errors occurred as a result of the manner in which the learner interpreted the question.
  The question in Figure 4.4 contains the words “distance” and “shortest route” in the same sentence. There was apparent confusion in the interpretation, whether it is a straight line or as the road leads.

  Errors occurred as a result of using a construct which have a different meaning in Mathematics as in other subjects.

For example, in question 2.1.b most students measured the shortest route by measuring the straight line between two points, instead of the route that the bus would travel by road. According to Davis (1984) these types of errors are caused by inaccuracies or ambiguities within the text. He called these errors language–induced errors. It was decided to adopt Davis's (1984) categorisation.

> Hence, use a ruler to measure (in millimetres) the approximate distance of this shortest route on the map, and then calculate the actual distance, in kilometres, using the given scale.

Figure 4.4: Language – induced error Davis (1984).

- Another observation made when analysing the scripts was that many questions were not answered at all and were left blank in the answer books. Hodes (1998) called the no response or leaving answers blank "test taking errors". These are named "no responses" in this thesis.

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In the next section the errors and misconceptions that were encountered in the sample of scripts are presented. Each error type is discussed and examples of where this type of error occurred are presented. A type of error may occur across all three questions; therefore it was preferred to discuss the type of error rather than analysing a question and then identifying the errors made in that particular question.

4.2 Analysis of errors and misconceptions

4.2.1 Structural errors

Structural errors are those errors which show lack of conceptual understanding of the mathematical principles involved in obtaining a solution.

When a mathematically inappropriate method is used in order to solve a problem or when a method does not make sense mathematically, it is classified as structural errors. The incorrect selection of an operation also illustrates this lack of understanding.

Errors caused by incorrect operation selection errors

In question 1, the students were required to calculate the number of triangular beaded segments that would be needed to fit around the table cloth. In order to find the solution, they had to take the answer in (a) and divide it by 4,71cm. Although the student transferred the correct value as obtained in 1.1.1(a) as showed in figure 4.5, the correct operation was not chosen and thus the correct answer could not be obtained. The student multiplied with 4,71cm instead of dividing by 4, 71. This was classified as an operation selection error.

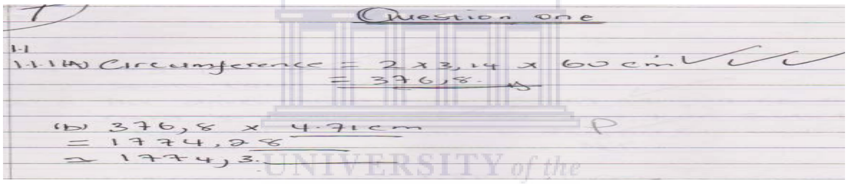

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Figure 4.5: Incorrect selection of operation.

Question 4 .1 required of the students to calculate the height the bottle. The question stated that the height of the box is 102 % higher than the bottle and that the box is 143 mm high. In order to obtain the answer, the students had to divide 143 mm by 1, 02 (or multiply $\frac{143}{102}\times 100mm$) to obtain an answer of 140,196… and then write the answer to the nearest millimetres which then would be 140mm. The operation selection error illustrated in figure 4.6 below is that the student did not divide 143 by 1, 02, but rather subtracted 102 from 143 and got 40 mm for the height of the bottle which shows an incorrect operation.

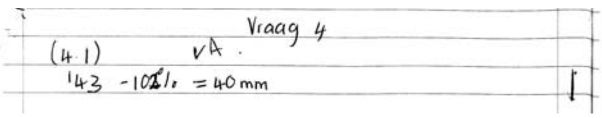

Figure 4.6: Incorrect selection of operation.

This error could have been made because the student did not understand the percentage or chose to ignore the percentage, thus using the 102 as an absolute measurement. Other structural errors which were classified as operation selection errors were those errors where a completely inappropriate method was used in order to solve the problem. These errors demonstrate a lack of conceptual understanding.

Figure 4.7 is an example of an operation selection error where the student used the volume of a right prism to determine the height of the bottle. This indicated that a wrong concept was used for the problem. Not only was the incorrect formula used but the student substituted percentages (105% and 102%) for the breadth and height, indicating a substitution error as well.

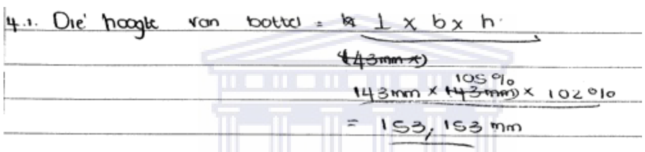

Figure 4.7 Operation selection errors (inappropriate method).

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The student's effort to calculate whether the dimensions of the cardboard box satisfy the difference of $11cm^2$ between the area of the base of the cardboard box and the base of the bottle indicates that the student used a mathematically inappropriate method in the attempt to solve the problem. Further, this same student substituted values that showed no relation to the problem in the given formula $4(A+D)+2(B+C)+E$.

Similar errors are illustrated by Figure 4.8 below. In question 4 the students were also asked to use the diagram of the layout of the opened box and calculate the total mass of the cardboard box needed, to the nearest gram. Figure 4.8 shows a student's attempt to calculate the total mass. The method used in the attempt was mathematically inappropriate. In fact, it made no mathematical sense. The number of times each letter was used in the diagram was used as an exponent in the calculations. The letter A^4 was used 4 times, so the student wrote A^4, D^4 were written because D was used 4 times.

The student wrote C^2 and B^2 because C and B appeared twice and E was used only once in the diagram. All this needed to be multiplied with the number of times it appeared on the diagram.

The student added A and D and then multiplied the answer with 4. The same method had been used for 2(B+C) where the answer of B+C in brackets had to be multiplied by 2.

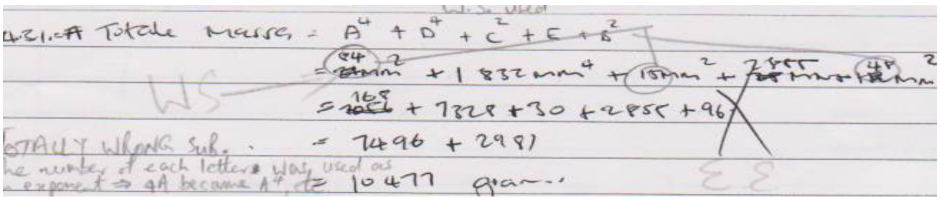

Figure 4.8: Operation selection error (inappropriate method).

Another example of these types of structural errors is illustrated by Figure 4.9 where the student had to determine the height of the bottle. Figure 4.9 shows that the student subtracted 102% from 105% and got an answer of $2.9cm^2$

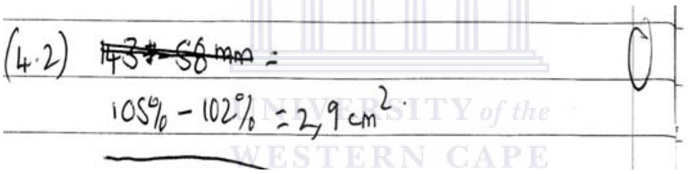

Figure 4.9: Operation selection error (inappropriate method).

4.2.2 Executive errors

Errors caused by incorrect conversion

Question 2.1 (b) required of the students to convert the scale distance to an actual distance. The scale given was 1:20000. The answer obtained through measurement had to be multiplied by 20000 and then the answer obtained thus had to be converted into kilometres. The acceptable answer range given in the memorandum was 2.6 km to 3.0 km.

Conversion errors were identified when the student used the incorrect operations as well as the incorrect values to convert. The students had to carry out the correct manipulations which are required to convert from millimetres to kilometres or centimetres to kilometres.

Figure 4.10 show that the student wrote “It 10 millimetres” which gives an indication that that was his measured distance. The 10 mm was then multiplied by 1000 and 10 000 kilometres was obtained as an answer. This indicated that the student could not convert the measured distance to the actual distance.

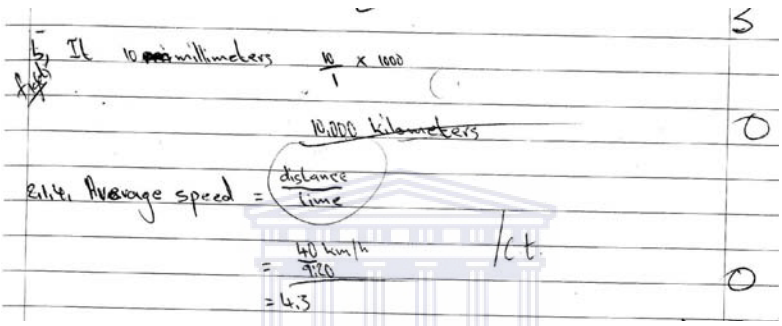

Figure 4.10: Failure to understand how to convert to kilometres.

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Figure 4.11 shows that the student obtained 13.2 mm as the measured distance. According to the addition of distances the student understood what “shortest route” means but the measurements indicates that the student measured in centimetres and wrote it in millimetres.

However, the student correctly multiplied it with 20 000 but wrote the answer as 264 000 km. It shows that he/she omitted to write the answer in millimetres. The student failed to convert the answer obtained from the multiplication of $(13{,}2 \times 20000)$ to kilometres, indicating that the student had a lack of understanding how to convert from mm to km.

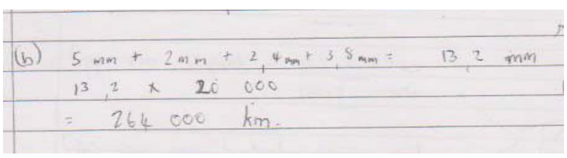

Figure 4.11: Incorrect conversion to actual distance.

Figure 4.12 indicates that the student got the correct measurement of 147 mm as indicated in the memorandum and divided the measured distance with 100 000. The student failed to multiply by 20 000, which makes this also a conversion error.

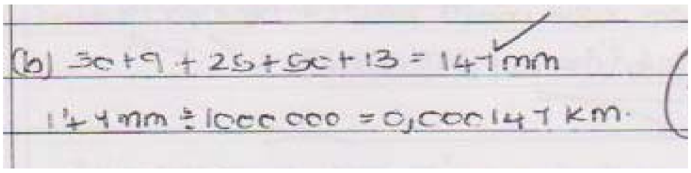

Figure 4.12: Omitting essential mathematical principles.

Errors caused by incorrect substitution

Question 1 required of the students to substitute values into the formula $2\pi \times r$. In question 2 students had to substitute the correct values into the formula $speed = \frac{distance}{time}$ and in question 4 they had to substitute into the formula *4(A+D) + 2(B+C) +E*. The various substitution errors identified were:

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(1) Students substituted incorrect values into a given formula. For example, in question 1 students substituted the diameter of the table cloth (30 cm) for the radius instead of 60 cm.

(2) Students substituted incorrect values which were not representative of the speed, distance or time. For example, in question 2 students substitute the actual time of 9:20 or 9:15 into the formula instead of 5 minutes (or 5 minutes which they converted to hours). Also, students substituted 40km/h in place of the distance in the formula $speed = \frac{distance}{time}$.

(3) Students substituted the incorrect value for *E* in the total surface area formula in question 4. The value of *E* was given in the question.

(4) Students substituted the percentages 105% and 102% directly into the given formula.

In question 1, in order to find the diameter or radius, students had to multiply the diameter 30cm with 4 and then divide the answer (in this case 120) by 2 to get the radius of 60 cm. The other option was to divide the 30 cm by 2 and then multiply the answer (15 cm) by 4 to get 60 cm. The 60 cm had to be substituted into the formula 2πr to determine the circumference of the table cloth.

In figure 4.13, the student substituted 140 in place of the radius. The student obtained 1380, 34 as the circumference. It is not clear how the student got to 140.

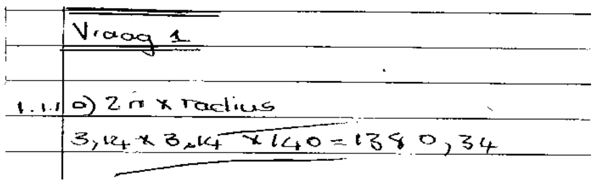

Figure 4.13: Substituting a non-related value in place of the radius.

Figure 4.14 indicates that the student multiplied the diameter of the place mat (30cm) by four and then got 120. The students substituted the 120 cm in place of the radius without dividing it by 2 to get the actual radius of the table cloth.

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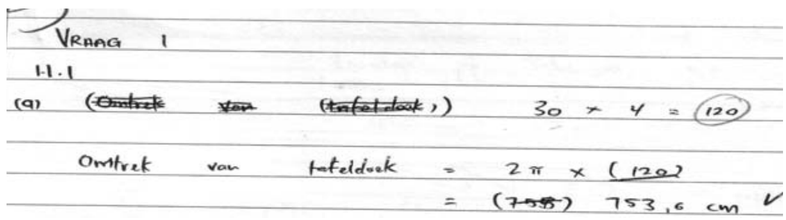

Figure 4.14: Substituting an incorrect value in place of the radius.

Figure 4.15 shows that the student substituted the value *4* into the formula which was taken out of the context of the question. The *4* in the question was used to indicate that the radius of the table cloth is four times the radius of the place mat.

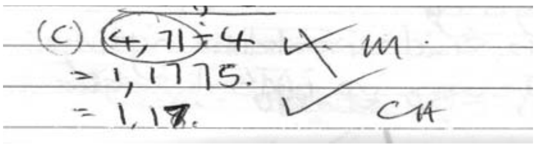

Figure 4.15: Substitute an incorrect value.

In figure 4.16 the student took the value 15 cm, which was half of the diameter of the place mats, i.e. the radius of the place mat, and substituted it in place of the radius in the formula to determine the circumference of the table cloth. The 15 cm had to be multiplied by 4 before substitution into the formula.

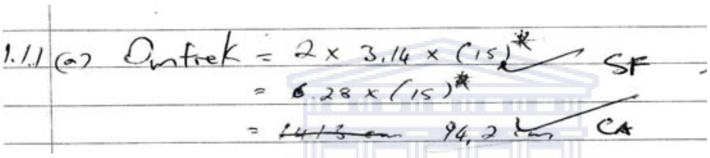

Figure 4.16: Substitute an incorrect value in place of the radius.

In question 2, the student had the choice to verify the speed to approximate 40 km/h by substituting the distance obtained in 2.1.3 as well as the difference in time. This difference in time was 5 minutes that needed to be converted to hours.

Figure 4.17 gives an illustration of a student that substituted the chronological time into the formula instead of the elapsed time that was the difference between 9:15 and 9:20. The figure indicates that the student divided 40km/h by the chronological time of 09:20, which was the time of departure.

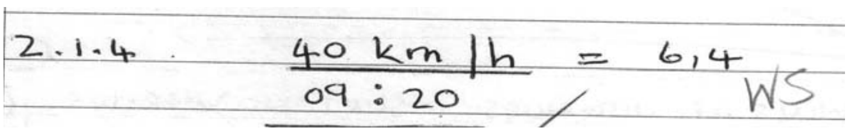

Figure 4.17: Substituting the chronological time as a single value.

In most of the cases, in question 4, students substituted the speed in place of the distance. This is also a substitution error because the formula that is given in the question is $speed = \frac{dis\tan ce}{time}$

Figure 4.18 below illustrates that the student copied the formula correctly but substituted the speed 40km/h in place of distance which shows an error in substitution.

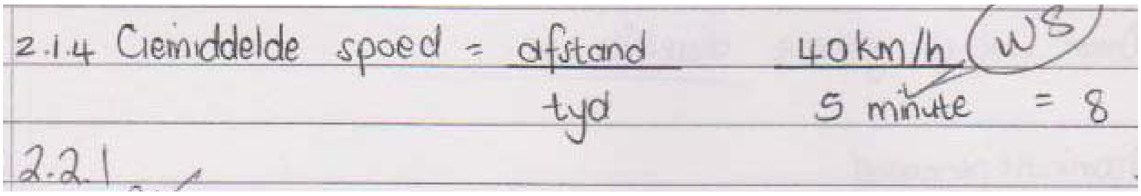

Figure 4.18: Wrong substitution into a formula.

In question 4 the student was presented with the value of E. There is only one segment that represents E in the diagram layout of the opened box. They had to take $2855mm^2$ and substitute it directly into the formula. Figure 4.19 show that the student used the height of the box, which is 143mm, for the value of E.

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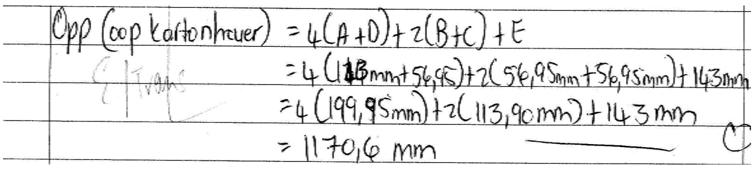

Figure 4.19: Substitute a wrong value into the place of a variable.

4.2.3 Procedural errors

Errors caused by transfer incorrect values

Students were required to use answers obtained from one part of a question in a follow-up part of that question. This is usually a question which consists of more than one part , for instance, a question 1(a) followed by a question 1(b) where 1(b) is a follow-up question on 1(a) .

The error arises when a student does not transfer the answer obtained in (a) to (b). For instance in question 1.1.1 (a) it was required that the circumference had to be found. Thereafter, the solution found in 1.1.1 (a) had to be transferred to 1.1.1(b) to determine the number of segments to be used around the table cloth.

The students had to divide the answer obtained in 1.1.1 (a) by 4,71cm. Students calculated the result in (a) but then substituted the value of the radius or any other value instead of the obtained result in (a), in 1.1.1(b). Figure 4.20 illustrates an example of transferring the incorrect value. This is classified as a transfer error since the value 120 was transferred and not the answer 376, 8 as obtained from the calculation in 1.1(a).

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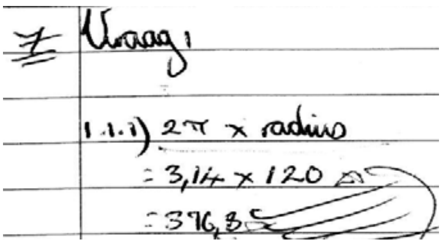
(a)



(b)

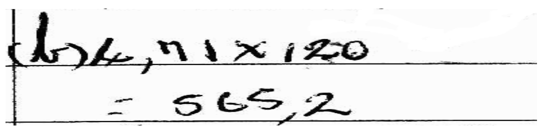


Figure 4.20: Transferring the incorrect value from problem (a) to problem (b) of the same question.

Another example of a transfer error was found in question 2. In question 2.1.3(b) the student needed to transfer the actual distance (0.02km) obtained in the first part of the question and use it in the formula to verify the time or distance in question 2.1.4.

Figure 4.21 below shows that the student determined the actual distance multiplying 40 km with 0.0005 and got 0.02 km for an answer. The student then transferred the 40 km in the place of the distance. Although this represented a distance, the student was supposed to transfer the answer obtained from the calculation. This is an indication that the wrong value was transferred.

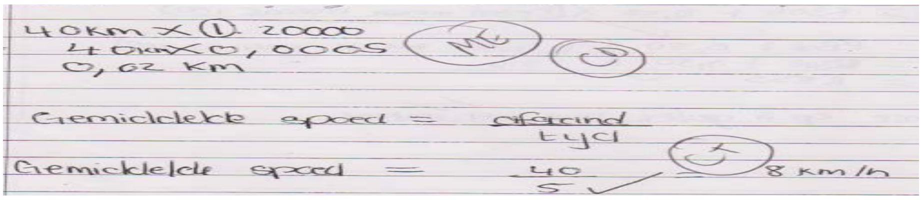


Figure 4.21: Transferring incorrect values.

4.2.4 Application errors

Errors caused by incorrect measurement

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In question 2.1.3 (b) it was required of the students to measure the shortest route from where the bus stopped to the sports field.

This measurement had to be done with the use of a ruler and it had to be measured in mm. The answer obtained from this measurement could be any distance from 130 mm to 150 mm. The measuring error identified was the following:

- The students did not measure the shortest route via the road, but rather used a straight line as for the shortest distance between two points and got the answer of 45mm.

In figure 4.22, the student measured a distance of 45 mm. This is an indication that a straight line and not a route along the road was used to measure, because this answer is far less than the approximate measurements as indicated in the memorandum. The manner in which the question was posed may result in some errors. It might be that the student's misconception resulted from an incorrect language interpretation, or a poorly phrased question. However it still resulted in an error related to measuring.

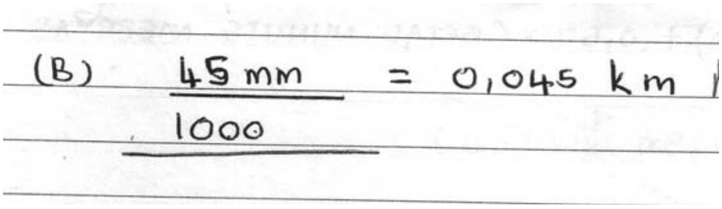

Figure 4.22: Incorrect measurement.

The table that follows gives an indication of the number of errors found for the sample during the analysis. The table provides the total number of errors found for each sub-category. The totals are written according to the main categories as per table 3.

Table 4.2: Total number of errors per sub-category

Error causes	Main categories				
	Structural error	Executive error	Procedural error	Application error	
1) Operation selection error	1480				1480
2) Conversion error		2738			2738
3) Substitution error		2291			2291
4) Transfer error			789		789
5) Measuring error				362	362
Total number of errors					7760

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4.3 No responses

This study focuses on the students' way of working. Therefore, it was necessary to investigate instances where there were no responses by students since this is seen as a way of working. No conclusive reasons can be put forward as to why students leave questions unanswered. One can only speculate the reasons for them not answering questions:

- The students might not have had enough time to complete the examination.
- The students might not have known the answer to a question.
- The students might have decided to come back to the question but ran out of time.

According to the analysis there were different degrees of blank errors:

- When no attempt was made to answer the question.
- When no attempt was made to answer a particular section or a sub-section of a question.
- When no attempt was made to answer only one subsection of a question.

Examples of cases where students had no responses

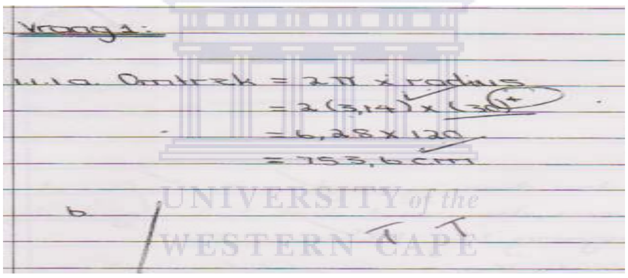
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Figure 4.23: Leaving a single sub-question blank.

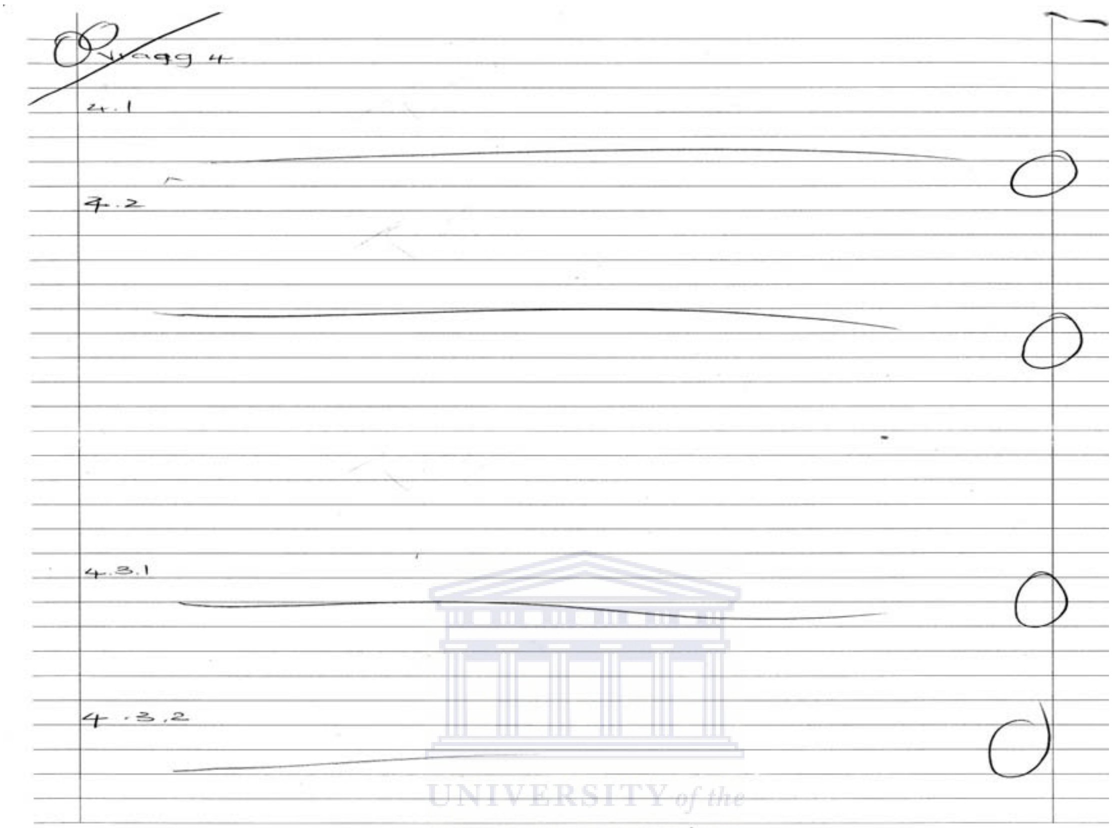

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Figure 4.24: No attempt to answer the whole question.

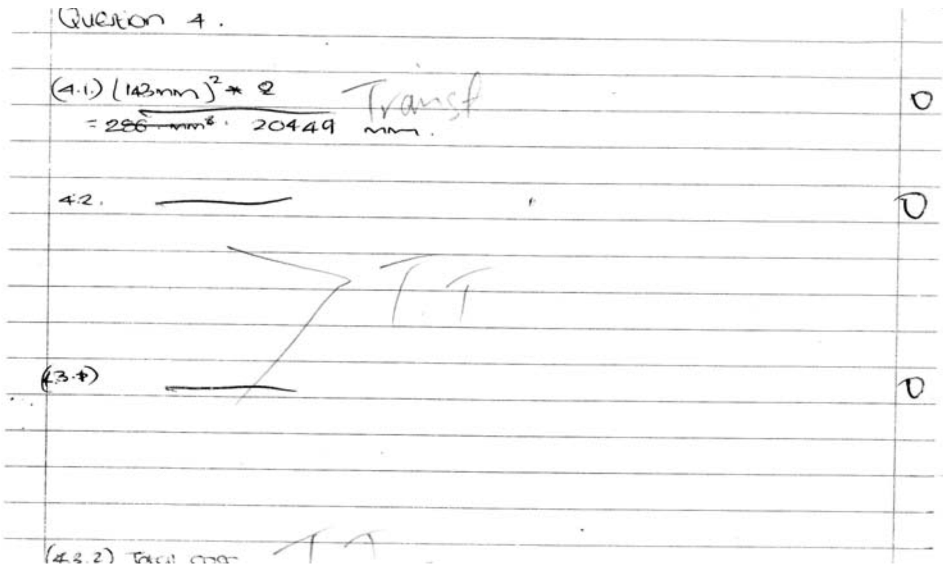

Figure 4.25: No attempt made to answer part of a question

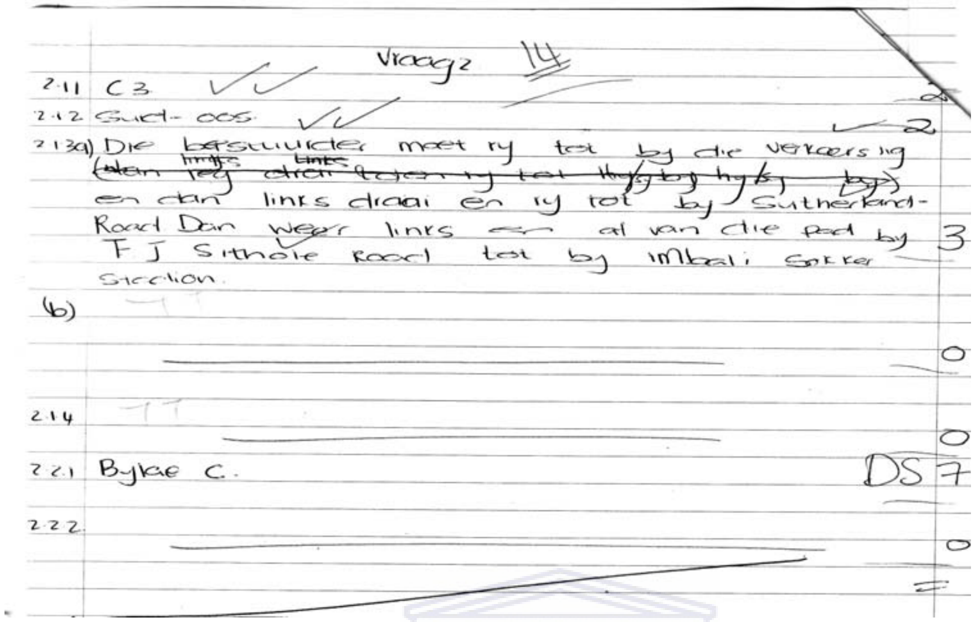

Figure 4.26: No attempt was made to answer a particular part of a question.

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The no-response type errors were very noticeable during the analysis process. It was a phenomenon that occurred throughout all the sampled districts. What is presented in the table is a summary of no-response errors found during the analysis. This summary presents no-response errors in terms of the preceding discussion. The summary is based on the questions that were part of the analysis.

Table 4.3: Summary no-responses

Types of no responses	Total no responses
Whole question	90
Part of question	357
Single question	557

The analysis of the errors made by students endorsed that the errors could be categorised as structural, executive, procedural or application errors.

However, certain common errors occurred frequently and this warranted a causal classification of the main errors. Examples of these errors were given to illustrate the type of errors made by students. The discussion of the errors will attempt to focus on possible reasons why certain errors were made and also give suggestions as to how to adjust teaching and learning methodologies that could minimise the occurrence of these errors. The following chapter will attempt to give possible reasons and recommendations to what was found during the analysis. This discussion will emphasise the importance of assessment and feedback through an effective and efficient error analysis.

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Chapter 5

Discussion, Recommendations and Conclusion

5.1 Introduction

The purpose of the study was the identification of students' errors and misconceptions when responding to questions in the final grade 12 Mathematical Literacy examination of 2010. This was done in order to gain insight into the types of errors learners make in their written responses in high-stakes examinations. This research specifically focuses on measurement. Measurement is found in the everyday experience of students. Measurement comprises 34% of the total marks of paper 2 and is also a section of the Mathematical Literacy examination that learners generally do badly in. If learners do not have a solid understanding of measurement, then it may result in them scoring low marks in the examination.

The research questions that lead this research are:

- What are the ways of working of learners when they are engaged in a high-stakes examination in Mathematical Literacy?
- What are the errors and misconceptions that are manifested in the written responses learners' work show?
- What possible explanations, from a mathematical perspective can be provided, for these misconceptions, errors and alternative ways of working?

A conceptual framework was used to set up a coherent set of ideas or categories of errors which would assist in the identification and analysis of errors of students' work. Miles and Huberman (1994) assert that the conceptual framework of a study is the system of concepts, assumptions, expectations, beliefs and theories that support and inform one's research.

The conceptual framework presented in this study shows consistent and systematic ideas and patterns that have been used by various authors to identify errors and misconceptions in mathematics. These systematic ideas and patterns made it easy to employ a qualitative document analysis method.

Document analysis is a method commonly used to identify the causes of students' errors and misconceptions when they make consistent mistakes. It is a process of reviewing students' written work by looking for patterns of misunderstanding. Any qualitative data reduction and sense-making effort that takes a volume of qualitative material and attempts to identify core consistencies and meaning is a defining feature of document analysis (Patton, 2002, p.453).

A template, with the different types of errors, was compiled from the conceptual framework and was used as a guide to identify the students' errors and misconceptions. After analyzing the scripts, this template was modified to include sub-categories. This modified template was used to categorise the different types of errors.

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5.2 Discussion of findings

This section discusses the main findings of the research based on the research questions generated. This includes possible mathematical explanations for the errors and misconceptions that were identified. Errors and misconceptions are normally part of the process of constructing knowledge and in fact may be a necessary corrective step when it comes to teaching and learning. The following errors and misconceptions were found during the analysis process and examples thereof were highlighted.

5.2.1 Structural errors

Errors caused by operation selection

What has been observed is that many students chose the incorrect operation and this was especially evident in question 1. Students had to divide the circumference of the table cloth by 4.71. Instead many students chose to multiply which shows that they did not grasp the mathematical principles presented in the question.

Calculate the number of beaded segments that she will need for each tablecloth.

Figure 5.1: Question 1.1b

It can however be argued that the students could read the question because they knew they had to perform an operation, but did not understand what they read or what the mathematical context was. After final examinations the Department of Basic Education issues reports for all subjects that highlights how students have performed in the different questions in the respective examinations. This report also highlights the errors of students and provides recommendation to teachers for remedial teaching. One of the suggestions given by the Department of education in the Report on the National Senior Certificate examination 2011 reads as follows: “Reading to comprehend should be encouraged”

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The question as stated in Figure 1 asked of the students to calculate the number of beaded segments that was needed for each table cloth. The phrase presented in the question “calculate the number of beaded segments’’ could have been an indication to the students that there had to be an increase in the beaded segments and therefore they should multiply instead of divide. The students in this case were led by how the figures and phrases were indicated in the question instead of reading the question in its totality.

Newman (1983) explains this phenomenon which occurs when the learner has understood what the question wanted him/her to find out but was unable to identify the operation, or sequence of operations, needed to solve the problem, as a transformation error.

On the other hand Davis (1984) mentions that errors can be caused by inaccuracies or ambiguities within the text. He called these errors language-induced errors. In question 1 the student were supplied with an enlargement of the beaded triangular segment to add some visual assistance.

However, the diagrams in Figure 5.2 below could have been interpreted by the learner as indicating that the number of beaded segments needed to increase; hence the possible reason for learners to multiply.

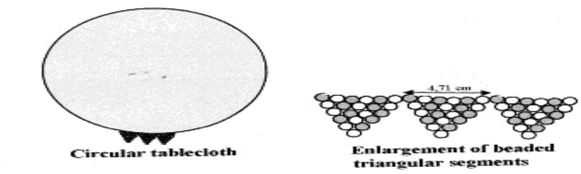

Figure 5.2: Diagram in question 1.1b

Brodie and Berger (2010) developed and classified a set of possible errors by focussing on multiple choice questions. They pointed out that of the systematic, persistent and pervasive patterns of mistakes made by students are when they wrongly interpret visual detail such as photographs, diagrams or graphs by ignoring certain information. In question 1, students might have looked at the sketch without taking the given information or the question into consideration and understood the instruction as finding the circumference of the tablecloth.

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Another occurrence that was common during the analysis was that students could not grasp certain mathematical principles which were expected of them. They showed a lack of conceptual knowledge in problems where a percentage had to be used. They did not know how to apply the concept of percentage in a problem. The meaning that Orton (1983) gives to these errors in relation to conceptual understanding of mathematical principles is the same that has been used in this research.

According to Ginsburg (1987) this conceptual misunderstanding is what he called “bugs”. This is when students systematically use an inaccurate or inefficient strategy. Ginsburg (1987) explained this type of error pattern as misunderstanding of an important mathematical concept. These are some of the reasons why students may have decided to multiply.

5.2.2 Executive errors

Errors caused by incorrect substituting

The first commonly made error in terms of substitution was that many made a mistake when replacing the representative alphabetical letters of a given formula with its given or determined values. This showed that, although they understood what the question required of them, they substituted a wrong value which pointed to the lack of understanding of how to use the values referred to in the question. Orton (1983) referred to errors like substitution errors as executive errors. These are errors that arose from failure to carry out a manipulation even though the principles involved may have been understood.

Secondly, substitution of time was also an error that was picked up during the analysis. Figure 5.3 below shows part of the errors made by students as stipulated in the Report on the National Senior Certificate Mathematical Literacy Final Examination of 2011, which indicates that the problem persisted. The reasons for this could be that students do not get enough practice in the algorithm or method of substitution or they don't get enough exposure to these types of questions.

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- Question 3.1.2 - Candidates made the error of not converting the time given in hours and minutes into a time given in hours only (i.e. total minutes divided by 60 leads to a decimal fraction of an hour). The answer can then be presented in hours or km. Some substituted the actual time (8:15 or 14:30);

Figure 5.3: National diagnostics report on learner performance

Errors caused by incorrect conversion

The incorrect conversion from measured distance to actual distance was a common occurrence found during the analysis. Students encountered difficulties when working with scale.

Their answers showed that they did not understand how to work with a given scale. Their knowledge about ratios is poor. The concept of ratio is dealt with in the lower grades. It means that there is a need for students to get an understanding of what ratios mean and how they are read and applied in the different situations. It is also important that subject integration is organized to achieve optimum understanding through a holistic approach because this concept is used in other subjects as well.

5.2.3 Procedural errors

Errors caused by transferring values incorrectly

This error was labelled a procedural error because it showed a breakdown in the step that link one part of a question to the second part of the same question. Students' responses to these types of questions showed that they struggled to follow procedure and failed to understand the problem as a whole. In question 1.1 (a) and 1.1 (b) students had to keep track with what the question expected of them. They however failed to follow the question from 1.1(a) to 1.1(b). One can argue that students did not understand that question 1.1(b) was a follow-up question to the first part. A possible reason for this might be that teachers tend to focus more on single problems rather than on problems where one part flows into the other.

5.2.4 Application errors

Errors caused by incorrect measurement

Here the error was committed when students measured with a ruler but could not read the correct measurement. Here one could argue that the students who committed this error were more focused on what the question expected of them instead on how to get the correct reading from the ruler. They then wrote the answer directly into kilometres. It was clear in their answers that they did not understand how to read and apply the given scale of 1: 20 000. Another possible

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reason for the incorrect measuring can be that students used the centimetres side of the rule and read it as millimetres. This could be due to them not knowing the difference between centimetres and millimetres. This can be an indication that the practical real life application of mathematical skills in this instance using measuring tools is neglected in Mathematical Literacy classrooms.

5.3 Recommendations

5.3.1 Implications for teaching and learning

The areas of focus of this research are assessment, errors, misconceptions and feedback. Therefore the implications for teaching and learning will incorporate these focus areas. Error analysis is commonly used to identify the cause of students' errors when they make consistent mistakes. By pinpointing students' errors, the teacher can provide instruction targeted to the students' area of need. What is recommended is that after assessment has been done, whether formal or informal, teachers should assess the tasks or tests as soon as possible in order to give feedback to the students. The process of feedback should be explained to the students beforehand and this should become part of the learning process and thus become part of the culture of Mathematics teaching and learning. This has to be done preferably with them having their answer scripts in front of them. The rationale behind this immediate feedback is that the content of the formal or informal task will still be fresh in the memory of the students. The errors and misconceptions should be highlighted, discussed and resolved and in this way the correct conceptual understanding will be inculcated.

Students should have the opportunity to give feedback and insight as to why they made the errors. This will give the teachers an opportunity to find the root of the problem or misconception. It would give the teachers understanding as to how to prepare better, and what teaching methodology will be the most effective in ensuring the minimisation of common errors and misconceptions. Students would gain understanding into why their methods were incorrect and strive to limit the number of errors when solving problems. Ideally, students should be given the time and opportunity to grasp the correct mathematical principles themselves through the intervention of teaching strategies instead of just being told what the errors had been and what the correct method or solution should have been.

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For this process to work students should attempt problems and allow themselves to make errors so that they can learn from them instead of leaving answers blank. In the analysis it was noted that many questions were left unanswered. Students should be encouraged to attempt solutions based on sound mathematical principles instead of not attempting a solution at all.

Through their errors they may deepen their mathematical understanding. Possible explanations for these errors and misconceptions would help to give teachers detailed feedback that could inform the teaching and learning in the classroom.

In the case of the error caused during conversion it is suggested that the report released by the department of education on the grade 12 results for Mathematical Literacy should encourage teachers to implement subject integration when teaching Mathematical Literacy as the subject lends itself to the embedding of different contextual situations. Teachers are recommended to , "make use of the contexts used in Geography when teaching the measurement and calculation of scale. This allows the learner to be aware of the integration of subjects as well as developing the students' life skills". Secondly another suggestion is that, "teachers need to ensure that when teaching students to determine the scale, they should insist that the distances be shown. Students need to develop the practice of writing the values and showing the addition."(WCED, 2012)

A recommendation to teachers would be to initiate cluster groups, or join existing groups, with the express purpose of error analysis. I these groups the errors of students are analysed and the possible reasons for these errors are unpacked. Some of these errors may be as results of how these concepts are taught to students. Thus teachers need to scrutinise their teaching practices in these groups as well.

The overall aim is for this information to be used to improve students' performance in Mathematical Literacy. The information on errors and misconceptions gleaned from this study might be useful for Mathematics as well.

5.3.2 Suggestions for further research

An area for further research could be to analyse how different methods of teaching can have an effect on how students perform and learn Mathematics or Mathematical Literacy.

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For example if students are only exposed to doing mathematics by doing algorithms and methods, how do errors and misconceptions manifest themselves?

This research only focused on those responses that contained errors and misconceptions in the ways of working in high-stakes examinations. It did not investigate the ways of working when correct answers were given. An idea for further research could be to focus on the ways of working when problems are correctly done as well. This can be done in order to gain insight into the level of questions that the students have no difficulty with. Are students able to handle questions in Mathematical Literacy or Mathematics based on all levels of Bloom's taxonomy or are students mostly able to do the lower-order questions? The questions analysed was level 3 and level 4 questions which according to the Curriculum Assessment Policy Statement has to do with applying multi-step procedures in a variety of contexts and (level 3) and reasoning and reflecting (CAPS, 2011)

Another suggestion for further research would be how a teaching methodology based on error analysis would impact on conceptual understanding. For example, it is recommended that a case study be conducted on what the impact of feedback on errors would be on students' performance, where not only written responses are analysed but discussions with students regarding their errors are held where students give motivations for their algorithms and ways of working.

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The idea is not to tell the students the mistake and then give methods of how to work out the problems correctly, but rather to listen to their reasoning, explain the errors in their reasoning and then from this explanation the students should be able to recognise their errors and then rectify them.

What may be currently happening in Mathematics or Mathematical Literacy classrooms nowadays is that teachers may realise that there are gaps in the basic knowledge of students, and yet they continue with the syllabus because they have to complete it in a certain time and then ignore to close the gaps in learners' legitimate knowledge.

The suggestion for further research should be not only focus at grade 12 level but on all grades of schooling

5.4 Conclusion

This research was done to identify the errors and misconception evident in students' responses when engaged in high-stakes Mathematical Literacy examinations. The objective was to identify the types of errors students commit and give possible explanations for it. The high-stakes examination brought certain limitations to the study. Firstly, only the written responses were concentrated on and no opportunity was possible to conduct in-depth interviews with students to understand their explanations for these errors and misconceptions. Secondly, high-stakes examinations have time restrictions. , Students have to complete the examination paper within a given period of time. In the case of the Mathematical Literacy second paper, the time period is 3 hours.

Thirdly, the anxiety that goes with high-stakes examinations may have had an impact on the errors that students made. It should also be noted that these students chose to do Mathematical Literacy instead of Mathematics. This generally implies that the students either do not have the aptitude for Mathematics or they do not particularly like Mathematics. The errors made should be seen in this context.

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This research showed that error analysis can provide an effective method for locating specific problems students have with Mathematical Literacy computation. The consistency of certain errors showed that students do not have an accurate working knowledge of mathematical concepts. Students' lack of knowledge could be a major reason why they cannot solve certain problems consistently (Hudson & Miller, 2006). Students' procedural effectiveness provided information on their lack of basic conceptual understanding.

What was a real worrying factor was the number of errors committed by students and the simplicity of the mistakes made. Students have to be taught how to structure an answer in problem solving types of problems. They need to be able to read and identify the correct choice of operations in sentences. After completing this research it was found that students lack in procedural, factual and conceptual understanding.

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