Mediation and a Problem Solving Approach to Junior Primary Mathematics

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by

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STATEMENT

I declare that *Mediation and a Problem Solving Approach to Junior Primary Mathematics* is my own work and that all the sources quoted have been indicated and acknowledged by means of complete references.

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June 1996

.... May 1996

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Key words

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Metacognition

Mathematical Problem Solving

Strategies and Skills

Peer Group Teaching

Autonomous Problem Solver

Abstract

This study argues that not all children in the Junior Primary phase benefit from the Problem Centred Approach in mathematics that was adapted by the Research Unit for Mathematics at the University of Stellenbosch (RUMEUS). One of the reasons could be that not all pupils can construct their own knowledge and methods. There are the highly capable pupils who cope well with this approach. These pupils are able to solve mathematical problems with little or no teacher interaction. Then there are the average and weaker pupils who cannot solve a mathematical problem on their own. These pupils need strategies and skills to solve problems and they need the teacher to mediate these strategies and skills to them, which will help these pupils to become autonomous problem solvers. Working in groups can, to some extent, supplement mediation or teacher interaction. Peer group teaching can be effective, whereby pupils are placed in groups so that the more capable pupils can teach concepts or make concepts clearer to the average or weaker pupils. There is, however, the possibility that when pupils of mixed abilities are placed in groups of four there might be one pupil who might refuse to work with the group. This pupil will work on her own and will not share ideas with the other members of the group. If this happens, mediation is necessary for those pupils who cannot solve a mathematical problem on their own.

The purpose of this study is to investigate how exposure to mediation can improve pupils' problem solving abilities. As directions for my research I've chosen the first six criteria of Feuerstein's Mediated Learning Experiences (MLE). The first three parameters: intentionality and reciprocity, mediation of transcendence and mediation of meaning are conditions for an interaction to qualify as MLE. Mediation of competence and regulation of behaviour are functions of specific experiences that combine with the first three to make an adult-child interaction one of mediated learning. Mediation of sharing behaviour can be added. Here the child and the mediator are engaged in a shared quest for structural change in the child. In addition to this, the five mechanisms of mediational teaching, i.e. process questioning; challenging or asking reasons; bridging; teaching about rules; and emphasising order, predictability, system, sequence and strategy are also used in the implementation of mediation as described by Haywood.

Two methods of investigation were chosen. The pupils' problem solving abilities were studied by means of eight word sums, of which the first four word sums were done in the pre-test and the other four word sums in the post-test. After the pre-test and before the post-test there was a period of mediational teaching for the experimental group. During this period and during the post-test the control group was denied mediation. After this research, mediation was also available for the control group. Two pupils from the experimental group were then chosen for further in-depth, think-aloud, person-to-person interviews. The aim of the interviews was to determine why these pupils could not solve the problem in the pre-test, but could successfully solve the post-test question.

The results of the word sums in the pre-test and the post-test were compared. The role of strategies and thinking skills is concentrated on in the results. Mediation was not equally successful in all of the four different types of problem sums. Questions one and five contained two or more numbers and here pupils tended to either plus or minus these numbers. Questions two and six also contained numbers, but this is a problem situated in a real life situation. Questions three and seven contained no numbers and questions four and eight compelled pupils to first work out a plan. Mediation was most successful in problem sums situated in a real life situation, followed by problem sums which compelled pupils to first work out a plan, and then by problem sums where there were no numbers. Mediation was least successful in problem sums that contained two or more numbers. Analysis of these results shows that with mediation there is an improvement in the pupils' problem solving abilities. Mediation can be viewed as S-H-O-H-R, in which the human mediator (H) is interposed between the stimulus (S) and the organism (O), and between the organism and the response (R). We can argue that the Problem Centred Approach without mediation can produce individuals who are little, if at all, affected by their encounter and interaction with new situations. Due to the lack of support in the Problem Centred Approach to Mathematics, it is the aim of this mini-thesis to propose mediation as an essential component in the Problem Centred Approach to Mathematics in the Junior Primary phase.

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Chapter 1

Introductory orientation

Introduction and Motivation

The challenge facing education has never been greater. Technology greatly influences today's school programmes. Today's technological society requires that schools help children prepare for a life of continuous learning. New jobs are appearing regularly, and the requirements for old ones are changing at an ever-increasing pace. Instruction should be directed towards helping students cope with a changing world to ensure that they can function properly once their formal education is completed.

The mathematics programme of the Junior Primary phase can be the starting point in helping to meet the challenge. It must provide children with knowledge, attitudes, and skills they need to be mathematically literate. This can only be done if mathematics is a joint discovery, involving teacher and learner. The researcher is of the opinion that children by themselves do not seem to be able to solve maths problems. Using the information that young pupils prefer to construct their own methods, Murray (1991:2) suggests that teachers in the problem centred approach are prohibited from showing pupils how to compute.

This approach to mathematics is based on the notion that a pupil must be given a problem and then be left alone to solve the problem on her[1] own or in a group. It is also suggested that the problem is solved without any interference from, or interaction with the teacher. The teacher thus gives no clues as to how the problem can or must be solved (Murray, 1991; Chantler, 1993).

This method is suitable for the highly capable learners who can construct their own knowledge and do not need the teacher to "show them how to perform the four basic operations", but what about the weaker pupil who cannot work on her own and who needs the teacher as mediator to guide her step by step through the problem to find a solution?

The ideal situation would be if pupils were to be given a "new" problem to solve on a regular basis, without any prior knowledge. They should attempt the unknown without input or interference from the teacher and without any assurance that they will solve the problem successfully. This work, however, insists that mediation plays an important role in mathematics. Not all the pupils have the ability to solve a "new" problem. They need the interaction with the teacher. Although many children come to school with informal mathematical knowledge, skills and vocabulary, many others are not as fortunate

[1]The feminine gender at all times includes the male gender.

and should therefore be provided with opportunities to experience and develop the knowledge, skills and vocabulary.

In mathematics we present pupils with word problems, but we, as teachers, need to go a step further. We should, through mediation, develop the child's ability to use various techniques and strategies for problem solving. Knowledge, skills, vocabulary and understanding are important elements of mathematical learning, but it is in problem solving that the pupil needs to synthesise these components in order to answer a question, make a decision or achieve a goal.

Definition of terms

Mediation

Mediational teaching is derived from Feuerstein's learning theories. It is an application of the principles of interaction between adult and child. Feuerstein's conception of the nature of intelligence stresses the role of mediated learning experience in the development of intelligence. Lack of essential mediation is what Feuerstein means by cultural deprivation; it deprives the child of the vital structuring that is needed for full cognitive development and understanding (Sternberg, 1985:224).

Mediational teaching refers to the way in which the stimuli emitted by the environment are transformed by a "mediator". The mediator selects and organises the world of stimuli for the child. The mediator selects stimuli that are most appropriate and then frames, filters and schedules them. Through this process of mediation the cognitive structures of the child are affected. The child acquires behaviour patterns and learning sets which, in turn, become important to the child's capacity to respond well to direct exposure to stimuli (Feuerstein, 1980:16). Virtually any experience can be a mediated learning experience if someone intervenes to make the sensory experience "transcend itself" (Fisher, 1990:10).

Why do some children learn more effectively than others? What can we do to improve learning? What makes a good thinker? These are some of the questions that mediated teaching attempts to answer. Mediated teaching describes a special kind of learner/teacher interaction which attempts to stimulate more effective learning and thinking. In this situation the teacher becomes a mediator or go-between whose task is that of interpreter and enricher rather than that of the ultimate source of all knowledge. Mediated interaction helps students to understand that persons, events and objects have a meaning that goes beyond the immediate circumstances of the classroom and contributes to the structure of the universe (Feuerstein, 1980).

With mediational teaching, teaching becomes a joint discovery, involving teacher and learner. It describes a special relationship between teacher and learner. The teacher's primary goal is to help students to acquire the fundamental thinking skills which underlie the ability to learn effectively and to adapt to a rapidly changing environment. So, ultimately the role of mediation is to broaden educational goals beyond the confines of the curriculum to the development of independently thinking and caring human beings. It is an approach which encourages sharing and also celebrates diversity. Feuerstein (1980:145) stresses that the more individuals are mediated, the greater becomes the capacity to learn.

Another line of research shaping current instructional views, centres on the teacher's mediational role (Dole, Duffy, Rochler & Pearson, 1991: 252). Just as pupils are cognitively active in comprehending instructions, effective teachers should be cognitively active in their efforts to develop students' understanding. The teacher interprets students' verbal and non-verbal actions during instruction and, on the basis of that interpretation, provides students with additional instructional information. Such teacher activity is designed to bring students' understanding in line with the intended curricular outcomes and has been referred to as alternative representations and as responsive elaboration. Under a drill-and-practice model, the teacher's role was essentially one of ensuring students' attention to the task. In the cognitive mediational paradigm, the teacher engages in what Shulman (1986:20) calls pedagogical representations and actions: There are ways of talking, showing, enacting or otherwise representing the ideas so that the unknown can come to be known; those without understanding can comprehend and discern; the unskilled can become adept.

For the purpose of this mini-thesis, "mediation" will thus refer to a teacher-pupil interaction in an attempt to make the unknown known. A teaching style is concerned not with what one teaches, but primarily with how one teaches. Every interaction between a pupil and teacher has some potential for being a mediated interaction. But whether or not a given interaction will be useful in promoting the cognitive development of children will depend upon the quality of that interaction.

A Problem

A problem arises when an individual faces a situation for which an immediate answer is unavailable. The person must also be committed to finding a solution before a problem can exist (Kennedy, 1980). Whether a problem is truly a problem or merely an exercise depends on the person faced with it. What is a problem for one person is not necessarily a problem for the next (Reys, Suydam & Lindquist, 1984). To the extent that it is possible, the mathematical problems children face should grow out of the real-world situation they face. Since there are some types of problem situations children do not usually encounter, vicarious situations from word sums and other sources can give them exposure to situations they would miss (Kennedy, 1980).

If a "problem" is so easy that children know how to obtain the answer, or know the answer automatically, there is really no problem at all. Many teachers are prone to select only problems that can be solved immediately or easily. This leads to children forming the idea that problems should be solved easily – so that a problem where the route to the solution is not immediately apparent, is viewed as "impossible". Finding the right level of challenge for a student is not easy, but it can be found by trying out a range of problems, providing the time, and then encouraging students to explore many ways around the obstacle initially posed (Reys et al., 1984).

Thus mathematical problem solving involves more than just giving an answer. Whenever children are faced with providing a solution to a task they have not mastered, they are solving a problem.

Problem solving and Mathematics

Mathematics, like problem solving, involves more than computation. (Reys et al., 1984: 1-2)

- **It is a study of patterns and relationships.** Children need to become aware of recurring ideas and of relationships between and among mathematical ideas. Children must come to see how one idea is like or unlike other ideas already learned.

- **It is a way of thinking.** It provides us with strategies for organising, analysing, and synthesising data, largely, but not exclusively, numerical. People who are comfortable with mathematics use it as they meet everyday problems.

- **It is an art,** characterised by order and internal consistency. Many children come to think of mathematics as a confusing set of discrete facts and skills that must be memorised. Because of a tendency to focus on developing the skills required to do mathematics, we forget that children need to be guided to realise and appreciate the underlying orderliness and consistency.

- **It is a language,** using carefully defined terms and symbols. This enhances our ability to communicate about science and other real-life situations.

- **It is a tool.** It is what mathematicians use, and it is also used by everyone during daily life. Thus, children can come to appreciate why they are learning facts, skills, and concepts that the school programme involves. They too can use mathematics to solve both abstract and practical problems, just as mathematicians do. It is useful in many vocations, serving as a "critical filter". Mathematics is a prerequisite to many occupations.

An additional characteristic can be added:

- ☐ **It must be fun.** Children must be encouraged to learn mathematics for its own sake, and not to excel or outdo their classmates in competition.

Aim of study

The aim of this study is to determine whether all pupils have the ability to solve a mathematical problem on their own; then to determine why some pupils find it difficult to solve a mathematical problem on their own; and, lastly, give an example of the role mediation can play in the Problem Centred Approach to mathematics.

This study will thus be twofold, focusing on both problem solving and mediation in mathematics. It sets out to investigate a possible exemplar to teaching problem solving in the Junior Primary phase with the incorporation of mediation in the Problem Solving Approach to mathematics. This study will thus focus on how to apply mediation in problem solving in an attempt to enhance the child's cognitive functions and thus make her an autonomous problem solver.

Method of research

This research departs from the assumption that pupils on their own do not seem to be able to solve word problems. The researcher will thus first try to identify difficulties and then recommend ways of improving the teaching of problem solving. The researcher will compare the performance of pupils who did problem solving without mediation, with those who received mediation.

Consequently the method of research will be as follows:

- ☐ A review of literature on the topic with the aim of keeping this research in harmony with research already carried out.

- ☐ An experimental investigation consisting of:
  - A pre-test without any teacher direction or interaction.
  - A period of mediational teaching whereby the experimental groups will be exposed to intensive mediation.
  - A post-test in which there is teacher direction and interaction for the experimental group, but not for the control group.

- ☐ Interviews with two pupils from the experimental group to determine their thought processes while solving a problem and to determine why they could not solve the problem in the pre-test, but successfully solved it in the post-test.

- ☐ The researcher will conclude with an analysis of the pre-test and post-test. It will consist of:

- An analysis of the results and a discussion of the word problems done in the pre-test.
- An analysis of the results and a discussion of the word problems done in the post-test.
- A comparison of the results of the post-test of the experimental and control groups.
- An analysis and discussion of results of the interviews.

Conclusion

The purpose of this study is to provide possible examples for the teaching of the Problem Solving Approach to Mathematics. It aims to investigate the need to apply "mediation" to the Problem Solving Approach in Mathematics. If we, as teachers, want every child to experience a feeling of competence while solving problems, we need to equip each child with the appropriate skills. This implies making them capable of gathering the data necessary for solving a problem and to offer them opportunities to confront certain situations which have to be mastered.

Chapter 2

Mediation

Introduction

In this chapter the development of the theory of Mediated Learning Experience (MLE), the criteria for MLE and their relationship to Feuerstein's Instrumental Enrichment (IE) will be discussed. The issue of MLE will concentrate on the theories of Reuven Feuerstein and Lev Vygotsky, two seminal writers on mediation. Piaget's understanding of psychological development and learning will also be discussed briefly, but for the purpose of this mini-thesis, greater concentration will be on Feuerstein.

The Development of the theory of Mediated Learning Experience (MLE)

The theory of MLE was developed over the period 1950–1963 during which time Feuerstein worked with large numbers of children who showed massive intellectual and academic disfunctioning. These encounters occurred concomitantly with Feuerstein's exposure to the Piagetian school of thought. He spent time in Geneva and in the camps of France where thousands of children were examined and prepared for their emigration to Israel (Feuerstein, Klein, Tannenbaum, 1991; Sharron, 1987).

The considerable gap many of these children manifested in their performance on Piagetian and other cognitive tasks when compared with the children in Geneva, made Feuerstein consider two sources of contrast: cultural differences and cultural deprivation (Feuerstein, Rand & Hoffman, 1979; Feuerstein and Jensen, 1980). Rautenbach also mentioned other sources as a cause for these gaps: socio-economic problems, organic conditions of the individual, language differences, adaptation and emotional problems or even genetic or racial differences (Rautenbach, 1984). Even though cognitive performance was similar for both types of children, one important difference could be seen: whereas the culturally different individual showed a good capacity to become modified through direct exposure to stimuli, even without special investment, this was not the case in the culturally deprived individual (Feuerstein et al., 1991:4).

The culturally different grabbed at each new experience to which they were exposed and turned it into a source of new and more efficient strategies to learn. This contrasted strikingly with the culturally deprived who showed more or less severe difficulty in benefiting from new experiences to which they became exposed. They needed an investment on the part of the examiner that went far beyond what the examiner was accustomed to making. The difference in capacity to benefit from direct exposure type of learning required an explanation that could not be easily found in the classical conventional approaches to the testing and nurturing of intelligence (Feuerstein et al., 1991:4).

These observations led Feuerstein and co-workers to consider the culturally different child as an individual equipped with learning capacities which she acquired while being exposed to, and affected by, her own culture. In defining the problem, it led to the shaping of MLE (Feuerstein et al., 1980; 1991). What became evident is that the culturally deprived had to "learn to learn" via mediation.

Feuerstein's dissatisfaction with conventional methods of intelligence testing stemmed from efforts to use such tests in the planning of educational schemes for young immigrants to Israel in the early 1950s. These test results had utility, in his judgement, because they revealed what an individual had or had not learned, but not what he was capable of learning. Feuerstein believes that IQ is of limited usefulness to educators who are interested in producing cognitive change because it gives no clue to the process that determines the level of an individual within a normally distributed population. What is needed is an indication of the individual's potential for learning (Feuerstein et al., 1980:7; Rutherford, 1989:101).

Feuerstein recognised the inadequacy of existing psychometric instruments for the assessment of the disadvantaged, low functioning child. Changing the structure of the test instrument was necessary because conventional psychometric tests could not fulfil what was to be an essential task in testing the culturally deprived: that of assessing the examinee's potential for being modified by learning. It led to the development of the Learning Potential Assessment Device Battery (LPAD), (Feuerstein et al., 1979).

The LPAD is designed to assess learning potential by producing cognitive changes during the testing process. The idea is to assess the individual's ability to learn by observing his learning performance in a controlled situation. The fundamental notion underlying Feuerstein's approach to assessment is that what one wants to measure is not so much the current level of the individual's intellectual development, but her susceptibility to change (Feuerstein et al., 1979, Rutherford, 1989). Feuerstein (1979) elaborated an approach that de-emphasised genetic or person variables and focused on the person-environment interaction as determining intellectual potential. He calls "dynamic testing" the major feature which involves testing in the act of learning.

In this way Feuerstein determined the role of MLE in the development of the propensity to learn. Its source lies in the work with thousands of children, adolescents, and adults. In developing the theory, Feuerstein and co-workers relied on clinical, observational and empirical data (Feuerstein et al., 1980; 1991).

The LPAD approach is based on a model of cognition that Feuerstein refers to as the "cognitive map". The cognitive map includes seven parameters of which phase is one by which a mental act can be analysed. This model is a direct result of Feuerstein's work (Feuerstein et al., 1979; Nickerson, Perkins & Smith, 1985; Sharron, 1987). The mental act can be divided into three phases: The input phase, the elaboration phase and the output phase (Feuerstein, 1979).

Years of observation that followed allowed Feuerstein and co-workers to consolidate the theory of the role played by the human mediator in the development of the autoplasticity and flexibility of the efficient learner (Feuerstein et al., 1991:5).

The characteristics of MLE

Feuerstein et al.'s (1980) theory is largely based upon the concept of MLE. According to Feuerstein, the development of cognitive structure in an organism can be viewed as a product of two kinds of interaction between the organism and its environment: direct exposure and mediated learning (Sternberg, 1985:224).

Direct exposure to sources of stimuli

This first, and most universal, modality is the organism's direct exposure to sources of stimuli impinging on it from the earliest stage of its development, throughout its life span (Feuerstein et al., 1980:15).

Piaget (1964) suggests that essential intellectual development is that the child must be exposed to sources of stimuli. The stimulus environment must also match the developmental stage of the child.

Learning for Piaget is based on readiness that is actively promoted and not passively entered. Instruction should pace development, but not precede it. Piaget's theory of development and Skinner's (1972) theory are similar in their emphasis on the child's direct and active interaction with the environment as the key factor in her development.

In a Piagetian view, the role of the human factor is not clear, since, theoretically, development occurs without mediation and intervention and therefore, to the extent that the human intervenes in this process, she is conceived by the Piagetian theoretical framework as an object among others. The human is not conceived as a mediator of the world to the growing child, but rather as a direct source of stimuli among other sources (Feuerstein & Rand, 1974:15).

Vygotsky conceived of intellectual development as a social transaction that forms the basis of the child's education and he believed that social relations underlie all psychological functions. For Vygotsky all higher mental functions have their genesis in social exchange. These functions are external and experienced in connection with others before becoming internalised (Tenzer, 1990:46). Vygotsky (1981) claimed that the intellectual skills children acquire are directly related to how they interact with others in a specific problem solving environment.

The nature of social transactions is central to the concept of a zone of proximal development (ZPD). It suggests that we should think of the zone as a characteristic not solely of the child or of teaching, but rather of the child engaged in collaborative activity within specific social environments. The focus is on the social system within which we hope children learn, with the understanding that this social system is mutually and actively created by teacher and student. This interdependence of adult and child is central to the Vygotskian analysis of instruction (Moll, 1990:11).

In Vygotsky's developmental theory, the child achieves self-regulatory capacities by actively manipulating the environment by the use of signs. The developmental progression on the road to self-regulation can be seen as a social process. Firstly, from the very beginning, auxiliary signs are brought in and given to the child by her social environment in order to control, direct and regulate the child's behaviour. Secondly, the child actively begins to use signs in order to influence other people and act on others around her. Words with socially shared meaning are the most useful signs in children's first attempts to master their environment. Finally, basic processes are transformed, mostly as a function of children's use of speech as a tool for planning and guiding their activity: the same speech that mediates social interaction is used as the main mediator of cognitive activity (Moll, 1990:135). Feuerstein, like Piaget, believes that children can learn from interacting with the environment; but, like Vygotsky, he emphasises the importance of mediation in the child's learning from adults (Savell, Twohig & Rachford, 1986:385).

The difference in the amount and nature of change as a result of direct exposure that occurs between individuals and even within the same individual when she is confronted with different tasks, cannot be explained by appealing solely to the theory of learning by stimulus-response or even Piaget's stimulus-organism-response approach (Feuerstein et al., 1982: 199).

Feuerstein views the development of cognitive structure in the organism as a product of two modalities of interaction between the organism and its environment: direct exposure to a source of stimuli and mediated learning experiences (MLE). MLE refers to the way stimuli emitted by the environment are transformed by a "mediating" agent, usually a parent, or adult caregiver (Feuerstein, 1991:8). MLE thus involves interactional processes between the learner and the "teacher" who, by interposing herself between the child and external sources of stimulation, "mediates" the world to the child by framing, selecting, focusing, and feeding back environmental experiences in such a way as to produce in her appropriate learning sets and habits (Feuerstein et al., 1979:71).

The concept of MLE provides the theoretical basis for Feuerstein's belief in the reversibility of deficient cognitive processes. MLE as a quality of interaction is responsible for two major phenomena unique to human beings: modifiability and diversity. These two phenomena, which are strongly intertwined, namely the propensity of the human being to modify and diversify her cognitive structure throughout stages of development, are basic to the theory of structural cognitive modifiability. The theory postulates that one of the most important characteristics unique to humanity is its plasticity and flexibility, making the human capable of modifying and diversifying her cognitive structure in a radical way that will affect her capacity to learn to adapt to more complex and unfamiliar situations (Feuerstein et al., 1991:12).

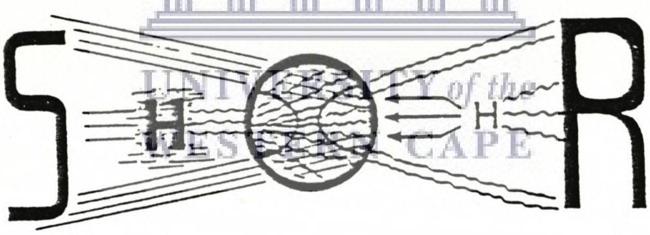
All children have potential, what Vygotsky called the "Zone of Proximal Development" (ZPD). He proposed that each child, in any domain, has an "actual development level" which can be assessed by testing him or her individually, and an immediate potential for development within that domain. He defined ZPD as "the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978:86). Vygotsky found that the traditional measures of attainment and intelligence lack one vital ingredient: they do not assess what a child might achieve given the right help and support by way of "mediation" (Fisher, 1990:136). Although there has been a recent upsurge of interest in Vygotsky's theory (Wertsch, 1985a), most of this interest has centred on the role that adults play in fostering children's development (Moll, 1990:55), and not on the mediational role of their peers.

Vygotsky's theory differs from Piaget's theory in that Vygotsky conceives "readiness" as the child's ability to learn through mediation. Change within the ZPD is characterised as individual change, that the child can do something independently that she could do only with assistance yesterday (Moll, 1990:12). Thus the notion of ZPD enables us to propound a new formula, namely that the only "good learning" is that which is in advance of development (Vygotsky, 1978:89).

The power of Vygotsky and Feuerstein's ideas is that they represent a theory of possibilities. Whereas Feuerstein (1980:2) conceives the human organism as an open system that is receptive to change and modification, Vygotsky (1978:86) emphasises functions that have not yet matured, but are in the process of maturation. Functions that will mature tomorrow but are currently in an embryonic state.

[For further discussions of Feuerstein's "connection" with Vygotsky see Adams and Adams (1991); Kozulin (1990); Moll (1985); Morphet (1985); Sharron (1987).]

If we were to describe the behaviourist direct exposure learning of Skinner and Rosseau as S-R, and Piaget's (1968) approach as S-O-R, then the MLE is conceived of as S-H-O-H-R (Feuerstein, 1982:55), in which the human mediator (H) is interposed between the stimulus and the child. Feuerstein argues that S-O-R without the H (Human) factor will produce individuals who are little, if at all, affected by their encounter and interaction with new situations. It is Feuerstein's contention that the more optimal the MLE, both quantitatively and qualitatively, an individual has received, the more readily and efficiently she is able to use direct exposure to stimuli to become modified in the direction of greater adaptability and greater mobility. Conversely, the lack of appropriate MLE leads to inadequate cognitive behaviour as a result of impairment and/or deficient cognitive functions that are the prerequisites for higher mental operations (Feuerstein et al., 1980; 1982; Kaniel & Feuerstein, 1989).



Human mediator (H) is interposed between the stimulus and the organism (Adapted from Feuerstein, Klein and Tannenbaum, 1991:7)

In direct exposure to sources of stimuli, the stimuli that impose on the organism flow into her system in a randomised, direct, non-mediated way. The stimuli appear and disappear in a totally non-systematic way with a double randomisation of their appearance: the one pertaining to the stimulus *per se* and the other determined by variations in the readiness of the organism to register them. This double randomisation makes the encounter with the stimuli in the direct exposure modality highly chance-like and probabilistic. The two modalities hypothesis, where both direct exposure and MLE co-exist, explains at once the high level of modifiability of the human being.

The relationship between MLE and direct exposure to stimuli

These two modalities for the development of cognitive structures can be set forth as follows: the more and earlier an individual is subjected to MLE, the greater will be her capacity to efficiently use, and be affected by direct exposure to sources of stimuli; the less MLE offered to the growing child, both quantitatively and qualitatively, the lower will be her capacity to become affected and modified by direct exposure to stimuli (Feuerstein et al., 1980:16).

Unlike Piaget, Feuerstein et al. (1980) view the cognitive development of a child as not solely the outcome of the process of maturation of the human organism itself and its autonomous independent interaction with the objective world. Rather, it is a combined result of direct exposure to the world and what has been termed the mediated experience by which cultures are transmitted.

MLE should never be considered the only effective influence on the development of the individual. Both direct exposure and MLE are necessary in order to create the prerequisites of thinking, the flexibility necessary for adaptation and the content base for operational functioning (Feuerstein et al., 1991:7-11).

Determinants of differential cognitive development

The cognitive development of the child (according to Feuerstein et al., 1980:16) should be viewed as this combination.

MLE can be considered as the ingredient that determines differential cognitive development in otherwise similarly endowed individuals even when they live under similar conditions of stimulation. By recognising the role played by Mediated Learning Experience in the course of cognitive development, a new perspective is cast on the etiology of cultural deprivation, as manifested in retarded intellectual performance. In terms of the theory of MLE, it is possible, and in fact necessary, to distinguish between distal and proximal determinants of differential cognitive development (Feuerstein et al., 1980; Feuerstein et al., 1979).

Distal determinants

Distal etiologies comprise those determinants that do not lead directly and invariably to certain outcomes but have rather a trigger effect. Distal determinants include such variables as genetic factors, organicity, level of environmental stimulation, emotional balance of the child or parent, and socio-economic status, etc. When these factors are deficient in some respect, they can, but do not necessarily, lead to inadequate cognitive development. Thus, the distal etiological factors are defined as determinants that neither directly nor inevitably cause retardation.

Proximal determinants

On the other hand, the proximal determinants are a lack of, or reduced exposure to MLE, the outcome of which is retarded performance. The proximal etiologies comprise those determinants which triggered by distal factors, lead directly and almost invariably to specific outcomes. When the distal factor triggers the proximal determinant, then the specific outcome appears. However, when for certain reasons, the distal determinant does not succeed in, or is prevented from triggering the proximal one, the expected end product may not necessarily appear. As already mentioned, any of the distal conditions may trigger the proximal etiology, so that, although the indirect distal determinants contributing to the retardation may vary, the result is nevertheless retarded performance.

The cause of a lack of MLE may be rooted in two broad sources:

- the nature of the individual's environment (exogenous factors) and
- the conditions of the individual at a given point in her development (endogenous factors) (Feuerstein et al., 1991: 37).

Both these potential areas of cause for a lack of MLE may become interrelated in such a way that the individual's condition affects her environment or vice versa. Exogenous factors are cultural transmission, poverty, ideology, parent-child relationship and pathological condition of parents. Endogenous factors are autism, constitutional factors and emotional balance of the child.

Criteria of MLE

MLE is perceived to be a universal phenomenon, for it is the process of mediated learning experience, rather than its content or even the language in which it is expressed, that becomes important in the construction of a cognitive repertoire and its elicitation. Not all mother-child interaction fulfils the requirements of mediated learning experience. Parental interactions oriented to the provision of stimuli, maintenance and control, and giving instructions, important though they may be, do not necessarily qualify as MLE (Feuerstein et al., 1982:56).

The quality of MLE interaction is best described by a series of twelve parameters as follows: (Feuerstein et al., 1991:15).

1. Intentionality and reciprocity
2. Mediation of transcendence
3. Mediation of meaning
4. Mediation of feeling of competence
5. Mediation of regulation and control of behaviour

6. Mediation of sharing behaviour
7. Mediation of individuation and psychological differentiation
8. Mediation of goal seeking, goal setting and goal achieving behaviour
9. Mediation of challenge; the search for novelty and complexity
10. Mediation of an awareness of the human being as a changing entity
11. Mediation of the search for an optimistic alternative
12. Mediation of the feeling of belonging

Whether or not a given interaction will be useful in promoting the cognitive development of children will depend upon the quality of that interaction (Haywood 1993:32). Of these twelve parameters of MLE, the first three parameters: intentionality and reciprocity, mediation of transcendence, and mediation of meaning are all necessary conditions for an interaction to qualify as MLE. Feuerstein insists that if these three are present, and only if these three are present is mediation sure to take place. These three are considered responsible for what all human beings have in common, viz. structural modifiability. The other parameters are neither uniformly necessary conditions, nor are they supposed to be present always in order for an interaction to be considered MLE. They are task dependent and strongly related to the culture to which one belongs. The first three parameters of MLE are of a universal nature and can be found in all races, ethnic groups, cultural entities, and socio-economic strata. Mediation for competence and/or regulation of behaviour are functions of specific experiences that combine with the first three to make the adult-child interaction one of mediated learning (Feuerstein et al., 1982:57). Mediation of sharing behaviour can be added to the above mentioned five criteria of MLE. Here the child and mediator are engaged in a shared quest for structural cognitive change in the child. The interaction, then, is not one-way and while it is directive it is neither authoritarian nor patronising (Haywood, 1993:33).

Whereas Feuerstein mentioned twelve parameters of MLE, Haywood (1993:32) concentrates on only six and Skuy et al. (1991:11) on ten parameters. According to Skuy, "Mediation is a dynamic and open process and should not be rigidly applied nor seen to be fixed at ten criteria." For the purpose of this mini-thesis, I will concentrate on the first six parameters of MLE, because these six parameters are the critical elements that should be present in mediation.

Intentionality and reciprocity

Intentionality and reciprocity are the main conditions of an MLE interaction. In MLE the specific content of the interaction – no matter how important – is shaped by the intention to mediate to the mediatee, not only the particular stimuli, activity, or relationship, but also to share this intention with the mediatee. The interaction initiated by the particular goal of the mediator, and reciprocity is a way to turn an implicit intention into an explicit volitional, and conscious act

(Feuerstein et al., 1991:17).

The interaction animated by an intention and an effect to create a relationship of reciprocity can be viewed as powerful and rich in behavioural, mental and emotional components. The intention transforms the three partners involved in the interaction – the stimuli, the mediator and the mediatee. The transformation of the stimuli to be mediated must be accompanied by efforts on the part of the mediator to transform the mental, emotional, and motivational state of the mediatee. Feuerstein et al. (1991:18; 1982) state that rendering the mediatee more accessible to experiencing the stimuli may demand an alteration of her state of mind to increase her level of vigilance and to create conditions of consciousness and awareness that will affect her mental process as well as the peripheral sensory motor system involved in the perception and execution of a given act.

Thus mediation of this parameter requires the establishment of an optimal interpersonal relationship between mediator and mediatee in such a way that the mediator is responsive to the learner's needs and characteristics, and the learner is receptive to the focus and goals provided by the mediator (Skuy, in press:7).

Mediation of transcendence

Here the mediator tries to produce structural cognitive changes in the child, i.e. changes that transcend the immediate situation. An event could be seen as only an isolated event, but a mediator will give such an event transcendent (generalised) meaning by attempting to relate the event to previous and even future events of a similar nature, and thus to extract a generality (Haywood, 1993:32).

Transcendence means going beyond the goal of interaction. Thus the criterion for MLE interaction is its transcendence from the immediate here-and-now situation in which it is elicited to others that are remote from time and space. A mediated interaction is not limited to satisfying immediate needs or coping with immediate problems, but aims at remote goals by producing in the child structural changes that will help her respond to new experiences and demands. It is the transcending characteristics of MLE that are responsible for establishing in the child the anticipation of events and search for horizons that go beyond the immediate. Transcendence produces a constant expansion of the individual's spatial and temporal life space. The constant expansion of the need system, so typical of the human, is responsible for the enormous amount of transformation or structural cognitive modifiability that characterises humanity. Feuerstein et al. (1991:21) conclude that the mediation of transcendence is considered among the most elementary forms to the most complex and advanced forms of human dialogue.

The intention to "transcend" is not necessarily dependent on the explicit awareness of the individual. The very simple, uneducated, and even "low functioning" mother may transcend the immediate goal of interaction. The need to transcend the "immediacy" of one's existence forms the operations and techniques deployed by each "culture" in its striving to transmit itself to the next generation and to ensure its perpetuity in this way. It is through transcendence that cultures go beyond the individual's immediate physical needs for survival and accede to the collective goals of existence. Indeed it is the intentional transcendental nature of the interaction that is the defining characteristic of a mediated interaction (Feuerstein et al., 1982; 1991; Haywood, 1993).

Mediation of meaning

The third attribute of MLE is the assignment of meaning by the mediator in a variety of ways to both the stimulus and the interaction. In the interaction the object or event presented to the child is not neutral but embodies affective, motivational and value-oriented meaning. In the process of acquiring meaning, objects and events gain significance in relationship to one another, in addition to their relationship to the individual. Feuerstein et al. (1982:56) further state that the meaning that is assigned in the mediated learning interaction cannot be grasped by the senses alone.

The implicit reasons for certain of our interactions do not become easily understood, nor do we always attempt to make them explicit and share them with the partner of our interaction. MLE animated by intention to transcend, mediates to the mediatee the meaning of the interaction, its significance, its "why" and its "what for", and it renders explicit reasons for the changes produced for, and the primary and secondary goals of the interaction (Feuerstein et al., 1991:24). With the growth of understanding the child moves from behaviour that is determined by the intrinsic properties of the stimulus to behaviour that involves non-intrinsic meaning. As with the other dimensions of mediation, this involves both a cognitive and affective domain (Skuy, in press:9).

The mediation of meaning of the adult mediator on the child gives rise to an ethical question: Who gives one the right to mediate, i.e. to impose one's own meaning and significance on the mediatee, determining by this not only immediate behaviour, but also future decisions? Feuerstein et al. (1991:25) debate this. My belief is that as a professional teacher, I am obliged to mediate.

Mediation of feeling of competence

Children's feelings about their own competence as learners are extremely important (Haywood, 1993:32). The mediator, both implicitly and explicitly, makes the child feel she is able. The child's feeling that she is capable of functioning independently is fostered by the mediator who organises opportunities for the child's success and interprets to her the reasons for her demonstrated competence. The affective and emotional elements that accompany mastery and competence give rise to the willingness of the child to cope with new and strange experiences (Feueurstein et al., 1982:57).

An impaired feeling of competence is one of the cognitive deficiencies that is most difficult to repair because once children conceive themselves as incompetent, not even the most brilliant successes can shift a poor self-image (Sharron, 1987:44).

The mediator must help the individual identify those aspects of her functioning that serve as strengths for her, and which can contribute to success. This can be done by rewarding appropriate responses, especially process-oriented responses. Haywood (1993:32) adds that the mediator must also make sure that children understand exactly what aspects of their behaviour were good and should be repeated.

Mediation of regulation and control of behaviour

This involves the heightening of the individual's responsibility for and control of his behaviour, both in learning and in social situations (Skuy, in press:11). Children, especially young children, often require some regulation of their behaviour in order to demonstrate their cognitive competence. Mediators must help children to inhibit their impulsive responding and thus to improve the quality of their responses. By creating an effectively safe and expectant environment, the mediator helps children to "unblock" available responses, to be willing to give answers (Haywood, 1993:33).

Feuerstein et al. (1991:36-40) maintain that the mediation of regulation and control of behaviour involves two main components. One has to do with the creation of functions to be used for gathering data necessary for decision-making, i.e. the input phase of the mental act. This component is concerned with the nature of the perceptual process involved in gathering data, the precision, the acuity and the exhaustiveness of the perceived. This is followed by the use of multiple sources of information that, once the data are given, gives rise to the establishment of a metacognitive orientation. The second component is the analytic establishment of a metacognition consisting of the combined evaluation of the gathered data and the evaluated competence of the individual. This is followed by an assessment of the meaning of the particular event, results in the decision whether or not to execute the particular response, in what form, where, how? The mediation of regulation of behaviour based on cognitive and metacomponents is vital today, more than ever, considering the nature of human adaptation required by modern technological ever-changing and faster-changing circumstances.

Mediation of sharing behaviour

This parameter involves the establishment of a climate and conditions for co-operative learning and living, and provides the opportunity for an emphasis on conceptual convergence, and other commonalties in knowledge and experiences, and among people (Skuy, in press:11).

Mediators convey the attitude that they and the children are engaged in a shared quest for structural cognitive change in the children. Each has an identifiable and separate role, but each is a participant and shares in a "we're in this together and you can count on me to do my part" manner. The interaction, then, is not only one-way, and while it is directive it is neither authoritarian nor patronising (Haywood, 1993:33).

Feuerstein et al. (1991: 40) state that sharing behaviour thus reflects the need of the individual to go out of his own self in the direction of participating with others and to make others participate with him. This is a strongly felt need mediated in cultures where cognitive, emotional, and magical participation is a main characteristic .

Where no sharing of behaviour occurs it can lead to a lonely, emotionally isolated child who will find it difficult to communicate with others or to make friends. This child will express feelings of rejection and abandonment.

It is, however, important to note the following characteristics of MLE and their relationship to other aspects of the learner and the contents to be learned (Feuerstein et al., 1991:50).

Summary of comments about the criteria of MLE

MLE should never be considered the only effective influence on the development of the individual. Both direct exposure and MLE are necessary in order to create the prerequisites of thinking, the flexibility necessary for adaptation, and the content base for operational functioning. The teacher should never function only as a mediator but should also act as the source of information and skill. In other words, the learner should never be exposed solely to MLE.

MLE should not be considered as having taken place unless the loop stimuli-mediator-mediatee has truly been closed. Whenever MLE has been limited for reasons associated with the mediator, the stimuli or the mediatee, the process cannot be registered, and no MLE experience can take place. The failure of expected effects to appear as they should, is accounted for by the failure of the MLE experience to take place as it should.

There is thus, no way to describe MLE as a fixed curriculum that can be applied automatically, mechanically and uniformly to all individuals. The "how" of MLE interaction depends on "who" mediates to "what" of a particular content of mediational intervention. The individual who masters the parameters of MLE will be endowed with a high level of freedom to influence the human interaction and to introduce the kind of change that is best suited.

Application of MLE

MLE is a basic, universal phenomenon, but despite the underlying commonalities, it appears in different forms in different situations. The primary modality of MLE is, of course, early childhood parental mediation. The second form of mediation is used in classroom-teaching, of which the teaching of Feuerstein's Instrumental Enrichment (FIE) is a special instance. The differences in method between these two forms of mediation are not accidental. They reflect the numerous situational differences between the classroom and the home. A third type of mediation, mentioned here only in passing, which is different from both parental mediation and school teaching, is used during assessment of the learning potential by Feuerstein's LPAD (Egozi, 1991:347).

Feuerstein's Instrumental Enrichment (FIE)

The concept of mediated learning experience provides the theoretical basis for Feuerstein's belief in the reversibility of the deficient cognitive processes. Among the best documented and most widely known approaches to enhancement of cognitive performance is the Instrumental Enrichment programme of Reuven Feuerstein (Egozi, 1991; Hadassah-Wiso-Canada-Research Institute, 1985; Nickerson et al., 1985; Sharron, 1987).

The materials and the didactics for their use are based on a cognitive map (Feuerstein et al., 1979:122-125) which constitutes a definition and categorisation of the components of the mental act or cognitive behaviour. The cognitive map involves seven parameters that guide the construction of the various instruments, help establish the sequencing, and contribute to an understanding of the dimensions of culture and environment that affects learning (Passow, 1980:396).

As Feuerstein puts it: "In our theoretical framework, the map, in conjunction with the inventory of deficient functions, explains cognitive behaviour by analysing its components, and locating and interpreting any weakness that may occur (Feuerstein, Rand, Hoffman & Miller, 1980:113). The cognitive map and the inventory of deficient functions make possible dynamic assessment of the individual's mental functions and, at the same time, provide the teacher with guidance in the selection of the enrichment exercises and procedures to be used with that individual.

Goals of Instrumental Enrichment (IE)

The prime goal of Instrumental Enrichment is "to increase the capacity of the human organism to become modified through direct exposure to stimuli and experiences provided by the encounters with life, events and with formal and informal learning opportunities" (Feuerstein et al., 1980:115). The major goal of IE is to enable individuals to function cognitively at a normal level. There are six subgoals that guide the construction of the exercises used and the application of the programme. These include:

1. Correction of the deficient functions.

2. Enrichment of the cognitive repertoire, including cognitive operations, strategies, concepts and relationships.
3. The production of intrinsic motivation through formation of appropriate habits.
4. Production of reflective, insightful thinking processes.
5. Creation of task-intrinsic motivation.
6. The development of attitudes in the learner that results in a self-perception of himself as a possible generator of information in addition to a user of information (Passow, 1980:397).

Thus, the goal of the FIE programme is one of modifying the cognitive behaviour of culturally deprived learners, and the subgoal contributes to that cognitive development. The instruments of the Instrumental Enrichment Programme consist of units, each of which emphasises a particular cognitive function. Each instrument deals with a number of cognitive deficiencies and is aimed at contributing to all of the subgoals (Fisher, 1990:143). The instruments, a series of paper-and-pencil exercises, are divided into fifteen "units". The materials are selected to match the learners' needs, the instruments are introduced by teachers, followed by discussions to prompt insight and application to other areas of learning (Rautenbach, 1985; Sharron, 1987).

By design, the instruments are content-free, in so far as possible. The major goal of FIE is to nurture acquisition of the prerequisites of thinking, to help the learner learn how to learn. Consequently, while the content has been selected because it is familiar to the learner and will hold her attention, the focus of the individual's orientation and efforts is on the modification of deficient cognitive functions, not the acquisition of subject matter. The FIE programme, by being content-free and non-curriculum bound, makes it possible to keep the child aware and conscious of the goal of the tasks without dissipation and diffusion of her efforts on aspects and content (Feuerstein et al., 1980:397).

The relationship between FIE and MLE

This relationship can be regarded as threefold:

MLE as the theoretical rationale for FIE

According to Feuerstein, MLE forms the theoretical foundation for our belief in the reversibility of deficient cognitive processes under specified conditions of intervention. The programme, FIE, was developed from the concept of mediated learning. It is a direct and focused attack on those cognitive functions diagnostically determined as being responsible, because of their weakness or non-existence, for poor intellectual performance (Feuerstein et al., 1980:17).

FIE as a substitute for mediation

According to Feuerstein, FIE is a substitute for early mediated learning experiences. It is a phase-specific method and technique for the alleviation of the ill effects resulting from the lack of MLE (Feuerstein et al., 1980:17). In other words, the FIE programme is intended to compensate for the lack of early childhood MLE, which caused reduced cognitive modifiability and created the need for remedial intervention.

The FIE teacher as mediator

The third strand of the relationship between early childhood parental mediation and the FIE programme is the use of a mediational teaching style by the FIE teacher. The FIE teacher uses a set of didactical methods to achieve the goals of the programme, i.e. the correction of the effects of the lack of early childhood MLE. An important member of this set of methods is the mediational teaching style. The use of the principles of mediation in teaching is, however, not a method specific to the FIE programme (Passow, 1980:325).

Feuerstein's work has in fact been around for a considerable time but it is only since 1984 that the Feuerstein package, including books, training programmes, instruments and personal promotion came to prominence in South Africa.

Critique of Feuerstein's FIE and MLE

Some criticism of Feuerstein's work has been mentioned in the development of this chapter. Here some further consolidated critique is presented. The issue of mediated learning that recently acquired certain prominence in Western psychology because of the innovative work of Feuerstein (1978:1980) has been a hotly debated subject. The notion of mediation, together with the notion of psychological tools, played the central role in Vygotsky's theory of human development (Vygotsky, 1978). When Piaget, Vygotsky and Feuerstein's theories are compared, it becomes evident that different agents mediate between a child and the world in the course of learning interaction. Since, for Piaget, the child's direct experimentation with the physical and social world is of primary importance, the role of a mediator is assigned to the inner mental schemes of the child. For Vygotsky, in contrast, a child's interaction with the world is mediated by symbolic tools provided by the given culture. Adults and more competent peers introduce symbolic tools to the child and teach her how to use them. Still, the role of the human mediator is not fully elaborated within this theoretical framework. This theoretical gap can be filled with the help of Feuerstein's (1991) theory of mediated learning, which assigns the major role to a human mediator (Kozulin, 1994:285). However, this is debatable. Feuerstein's definition of mediation may be one of many.

Feuerstein mostly concentrates on IE and the skills of the individual learner. It is his co-workers, following his advice, who research the notion of mediation applied to content. Feuerstein's IE activities are constructed so as to be outside the content range of the school curriculum. The instruments are free of academic content, but in the lesson designed for each part of the programme careful provision is made for a "bridging" discussion (Morphet, 1985:39). Bradley (1983) commented on the limitations of studies with IE in this respect. Because of the emphasis on the thinking process, content is relegated to a secondary role. This concentration on the thinking process enables systematic treatment of the obstructions to adequate cognitive functions that cannot be achieved when the main purpose is to teach a subject matter. On the other hand, the lack of content in IE implies that the programme is not intended to replace teaching of subject matter and has to be taught as a separate subject. Such an approach and alternatives, such as integrating the teaching of thinking into the regular school subjects, are in dispute among educators (Egozi, 1991:358).

For the successful implementation of the enrichment programme Feuerstein insists that teachers undergo special training at the Institute and that the programme be implemented to the laid down specifications. It is thus in every sense a closed system to be applied in full or not at all (Morphet, 1985:41).

According to Morphet (1985:44), mediated learning is not the same as Mediated Learning Experience. Mediated learning is a materialist concept used to describe the way in which a child enters the historically constructed culture of its social context. He refers to Vygotsky (1978) who conceives the child as an active learner using mediation of the adult to construct and interpret material reality in the terms made available by culture.

According to Vygotsky (1978:30) the path from object to child and from child to object passes through another person. This complex human structure is the product of a developmental process deeply rooted in the links between individual and social history. He also conceives that human learning presupposes a specific social nature and a process by which children grow into the intellectual life around them.

The materialist concept, as expressed by Vygotsky, sees mediation as the social process that operates at any given time and place under particular conditions of the historical development of any given culture and society. In this sense the human learning process is afloat in the currents of history. It is not subject to permanent and universal laws or conditions nor can it be precisely conceptualised and analysed except in terms of the particularity of a historical culture (Morphet, 1985:45).

By adding the term "experience" to mediated learning, Feuerstein changed the construct drastically. He takes along the concept of a process operating between adult and child but he truncates it severely by making it a one-way transmission – a bestowal by the adult of a fundamental structure of the intellect which, Feuerstein argues, remains the same no matter what the conditions and historical development of the culture (Morphet, 1985:45).

This conceptual shift is crucial because it validates Feuerstein's positivist approach. His task is first the discovery and analysis of the inherent quasi-natural law governing the formation of cognitive structures. Secondly, once the laws are in place, the diagnosis and remediation of deficiencies can follow (Morphet, 1985:45).

The theory of MLE serves Feuerstein's practical intervention goals perhaps better than it does his socio-cultural interpretation because it provides him with a justification for focusing his practical improvement strategies on the condition of lack of MLE itself – rather than on any of the other distal factors (Morphet, 1985:36).

Moll (1985:48) has a different point of view concerning the materialist notion of mediation. According to him, Feuerstein does not use a materialist conception of mediation. Morphet quotes Vygotsky on mediation, and then mistakenly assumes that Feuerstein is operating with the same theoretical notions, even though he does admit that MLE is a radically truncated conception of an adult-child interaction. There are radical differences between Feuerstein's enterprise and the Vygotskian conception of mediation.

Morphet also neglects to point out that the cognitive transformation of children can only be understood as being entailed within the overall transformation of social relations. Luria (1976:570) calls this the leap from the world of necessity to the world of freedom. The acquisition of skills is not the same thing as the transformation of cognitive structures, which depends on transformation of forms of adult-child discourse within overarching political structures. This is what the Vygotskian notion of mediation signifies.

Moll's critique against Feuerstein is that no account is taken in the intervention itself of concrete social activities or structures that determine the child's cognitive capacity. Mediation, as manifested in MLE, is nothing more than the identification of particular skill deficits by means of the LPAD, and their subsequent remediation by means of the FIE. Rather than being a materialist concept, mediation as used by Feuerstein is a concept describing a psychological intervention technique.

Both Morphet and Moll reject Feuerstein's package, both agree that it has dangerous outcomes which need to be avoided (1985:50), but Moll adds that mediation is necessary to tackle the kinds of cognitive developmental problems we increasingly face in South Africa during transition, and the LPAD and the FIE might well have something to offer in this regard. Engage Feuerstein we must (Moll, 1985:51) was a conclusion 10 years ago. More contemporary critiques exist (e.g. Blagg, 1991, Sternberg in Blagg, 1991), but for this research on balance the writer sees more advantages in using Feuerstein's framework than disadvantages.

Conclusion

It thus becomes evident that there are different approaches to a child's learning and this allows us to distinguish some pivotal points. The first is the child-centred character of the contemporary learning theory. A child's cognition has its own specific features, which should be constantly kept in mind if we wish to meaningfully improve the process of teaching and learning. The second pivotal point is the recognition that learning is not an individual process but a cultural-historical one, both in respect of symbolic tools internalised by the child as her mental processes and in respect of the socio-cultural goals of education established by society. The last pivotal point is the role of human mediators in learning. Specific contributions of the human mediator should be distinguished both from the content of learning and from the symbolic tools offered by the given culture (Kozulin, 1994:286). It is only by providing each child with adequate mediated learning experience that we can ensure her normal cognitive development and an opportunity for further learning.

This concludes my theoretical considerations of Mediation. In the next chapter "A problem centred approach to the teaching of mathematics" will be discussed.

Chapter 3

Problem centred approach

Introduction

To know mathematics is one thing, and to be able to teach it – to communicate it to those at a lower conceptual level – is quite another and it is this latter that is most lacking at the moment. As a result, many pupils acquire a lifelong dislike, even fear, of mathematics at school. It is good to report that widespread efforts are being made to remedy this. These efforts would, however, be of greater value if they were combined with greater awareness of the mental processes involved in the learning of mathematics (Skemp, 1971).

In this chapter the writer will discuss the different approaches to the teaching of mathematics and a pair peer Problem Solving Approach, and how these approaches are related to the Problem Centred Approach. For the purpose of this mini-thesis concentration will be more on the Problem Centred Approach. The focus will be on the role of metacognition in solving a problem and the importance of mediation in this approach.

Approaches to teaching mathematics

Traditional approach

Romberg and Carpenter in Desforges (1986:8) state that traditional teaching is based on a metaphor of production in which students are seen as "raw material" to be transformed by "skilled technicians". This view of learning and teaching upon which this metaphor is based is no longer in harmony with a constructivist perspective. In this approach the teacher is the bearer and transmitter of knowledge. The teacher's role is to show pupils how to fill their empty vessels. A distinct feature of this approach is "rote learning".

Investigation approach

The original goal of "investigations" was that pupils should know from experience what mathematics is like. The major part of the activity is the investigation by individual pupils of substantial open problems... formulating problems, solving them, extending and generalising them. This kind of work might be described as research type activity at the pupil's level.

Investigations are often seen as progressive ways of teaching mathematics in a non-authoritarian way. Frequently the emphasis has been on "freedom", on putting children in a situation and just telling them to "investigate", while not telling them anything else. This open-endedness might come from an unwillingness to impose on pupils. It was often noted, of early sources of investigation, that they told you where to start, but not where to go or to end.

Discovery approach

This approach involves a substantial reorganisation of thinking. It requires the most sensitive teaching. The teacher must be adroit at setting the stage, asking exactly the right questions at the right time, encouraging students to think, identifying what pupils mean when they respond, and rewarding creativity. One aspect of this approach is the "Eureka" syndrome, whereby the discoverer is strongly motivated by success. This encourages the pupil to make further, and often still deeper, explorations. This approach involves students in the learning process, that they embrace the content discovered, making it their own (Rising & Harkin, 1978:202).

Experimental approach

An experiment is a procedure for teaching a hypothesis, so this method starts with a hypothesis which is either stated by the teacher or the pupils. The teacher often knows that this hypothesis is either true or untrue, but the pupils have to investigate its truth by devising a proof that suits their level of mathematical development (Dean, 1982:75).

Activity based approach

This approach is clearly rooted in the real world. This approach has no underlying theory. It deliberately goes straight into an activity. These activities become the driver for the learning that must take place. This approach is linked with situated cognition – that from an activity comes thinking skills.

Problem Centred Approach

The word problem is derived from the Greek word "problema" which actually means something "thrown forward", whereas the standard definition is "a question raised for inquiry, consideration or solution, a source of perplexity". For the purpose of this mini-thesis problem solving in mathematics will refer to students' performance on mathematical tasks where the solution or goal is not immediately attainable, and there is no obvious algorithm for the students to use (McLeod, 1985:267).

Solving a problem means finding a way out of difficulty, a way around an obstacle, attaining an aim that was not immediately attainable. Solving problems is the specific achievement of intelligence and intelligence is the specific gift of mankind. It can be argued that learning to solve problems can be regarded as the most characteristically human activity (Polya, 1962:v). It is a complex cognitive skill – one of the most intelligent human activities.

According to the Problem Centred Approach, pupils construct their own knowledge through problem solving. They are active mathematical thinkers who try to construct the meaning of what they are doing. Children are confronted with problems and from this basic construct find ways of solving problems (Junior Primary Mathematics Handbook, 1994).

How do these approaches differ from each other? The investigation approach includes experimentation and discovery. Now let's take Investigations and Problem Solving. Is there a distinction? Cockroft (1982) makes a distinction but suggests that:

Problem Solving is a convergent activity where pupils have to reach a solution to a defined problem, whereas investigations are more divergent activities where pupils are encouraged to think of alternative strategies to consider what would happen if a particular line of action was pursued, or to see whether certain changes would make any difference to the outcome. More formally we would say.

Problem = an end without a beginning.

Investigation = a beginning without an end.

If there is a distinction, then it rapidly becomes "blurred". As mentioned above, the investigation approach includes discovery and experimentation. Most problems can become investigations and most investigations, no matter how simple the starting point, reach a stage where the pupil doesn't know what to do next – then she has a problem (Shabalala, 1992).

A new approach to the teaching of mathematics

Since the early 1980s, there has been a strong perception that the mathematics curriculum and/or method of instruction in South Africa was not meeting the current and future needs of the pupils, industry and society. There is a demand for a more rapid change of pace.

Industry expects from pupils:

- The ability to set up problems with the appropriate operations.
- Knowledge of a variety of techniques to approach and work on problems.
- Understanding of the underlying mathematical features of a problem.
- The ability to see the applicability of mathematical ideas to common and complex problems.
- Belief in the utility and value of mathematics.

(Junior Primary Mathematics Handbook, 1994:4)

Background

With the co-operation and support of the Cape Education Department, an alternative approach for teaching Junior Primary Mathematics was adapted by the Research Unit for Mathematics Education at the

University of Stellenbosch (RUMEUS) and introduced in 1989 in eight primary schools in the Western Cape on an experimental basis. The Superintendents of Education and the teachers involved did much to refine the approach and to try out and share appropriate classroom activities. Even within a few months, it became clear that the approach had a high degree of success, and the number of schools involved increased quickly. According to a decision from the Committee of Education Heads, the Education Departments of the other three provinces (as they were in 1989) had to initiate trial implementations in their own provinces as well (Murray, 1991:2).

The most recent development in mathematics education highlights the fact that pupils construct their own knowledge and do not need teachers to "show them how to perform the four basic operations" (Junior Primary Mathematics Handbook, 1994:6).

Piaget (1968), on whose theory the Problem Solving Approach is based argues that a stimulus is not a stimulus unless the child acts on it, that is, the interaction between the child and environment is important. This does not only mean a psychological interchange between the individual and the learning environment, but includes erroneous ideology expressed as constructivism - the view that the learner actively constructs knowledge.

Constructivism

The idea of "constructivism" – hardly mentioned a few years ago – nowadays attracts a lot of attention in the world of mathematics. What is happening now is that we are beginning to see the kind of research and instructional innovation that builds on new psychological and epistemological insight and is leading to new educational programmes. This new development is called constructivism (Slater, 1994:14).

Fundamental to this new research is what Von Glaserfeld (1984) calls "radical constructivism". To claim that one's theory of knowing is true, in the traditional sense of representing a state or feature of an experience-independent world, would be perjury for a radical constructivist. One of the central points of the theory is precisely that this kind of "truth" can never be claimed for the knowledge that is produced by human reasoning. Human understanding is always searching for new ways to explain events and is not converging towards "truth". Constructivism needs to be radical and must explain that one can indeed manage without the traditional notion of truth. The essence of constructivism is that "knowledge cannot simply be transferred ready-made from parent to child or from teacher to pupil but has to be actively built up by each learner in her own mind" (Von Glaserfeld, 1991).

Another approach to constructivism, as put forward by Vygotsky (1978) is social constructivism. Like Piaget, Vygotsky views learners as active organisers of their experiences but, in contrast, he emphasises the social and cultural dimension of development. Vygotsky differs from Piaget in that he believes that development and learning take place in the child through mediation. Change within the Zone of Proximal Development is characterised as individual change, that the child can do something independently today that she could only do with assistance yesterday. Piaget, on the other hand, conceives the child as an object among others. The mediative value of human intervention is not regarded by Piaget (1964) as essential to the cognitive development of the child. He believes that the child will on her own, through her direct experimentation with the physical and social world, develop formal operational thought processes. The role of mediator is assigned to the inner mental schema of the child. This interaction with the tools provided by the given culture to develop her learning is why Vygotsky's theory is classified as social constructivism.

A programme conducted at RUMEUS, linked to the Madison programme, explicitly includes the "social" in its description of its programme. "The socio-constructivist didactical contract" (Gordon, 1993:5). Oliver (1989:11) writes: "Knowledge does not simply arise from experience. Rather, it arises from the interaction between experience and current knowledge structures." Constructivism can be described as essentially a theory about the limits of human knowledge, a belief that all knowledge is necessarily a product of our own cognitive acts. We construct our understanding through our experiences, and the character of our experience is influenced profoundly by our cognitive lenses (Confrey 1980:108).

When constructivism is applied to the issue of teaching, one must reject the assumption that one can simply pass on information to a set of learners and expect that understanding will result. Learning mathematics requires construction, not passive reception. It assumes that learners have to construct their own knowledge – individually and collectively. Each learner has a tool kit of conceptions and skills with which she must construct knowledge to solve problems presented by the environment. The role of the community, other learners and the teacher is to provide the setting, pose the challenges and offer support that will encourage mathematical construction (Maher, 1990:3).

A problem-centred approach based on RUMEUS (1991)

Using the information on constructivism described above that young pupils prefer to construct their own methods, teachers in this alternative approach are prohibited from showing pupils how to compute, yet they must create and sustain situations in which pupils will manage to develop computational skills.

The teacher must also ensure that each child realises that a new type of "contract" now exists between teacher and child. The teacher expects each child to tackle a problem in a way which best suits her understanding, that she will respect each child's effort and that children should take responsibility for each other's learning (Murray, 1991:6).

The aim of the Problem Centred Approach to Mathematics is focused on the versatile ability of pupils to solve problems. In this approach the solving of problems is the most important means to learning. According to this approach, pupils develop numerosities, an understanding of the basic computations and a variety of methods of computation through communication on how they solved problems (Teachers Guide for Mathematics, 1993).

This approach suggests that a child's understanding of numbers grows with experience, and her level of understanding can easily be identified if the method she uses to solve an addition problem is observed (Murray, 1991:6). The child's method of computation will thus reflect the maturity of the child's number sense. At level one, the child will use a "counting all" strategy, e.g. to solve 2+3 she will count "One, two, three, four, five". Pupils at this level are limited to direct modelling solutions using concrete objects. Problems that cannot be modelled easily cannot be solved at this level (Silver, 1985:29).

At level two the child uses a "count on" strategy, e.g. to solve 6+4, she says "six, seven, eight, nine, ten". According to Silver (1985), this is a transition period. At this level some pupils use both modelling and counting strategies.

At level three, the child has realised that a number can be "broken up" as convenient. The child is now able to decompose and recompose numbers for which she had already constructed numerosities. She is now able to think of 34 as 34 ones as a whole, as well as 34 as a 30 and a 4 (Carpenter, Fennema, Peterson, Chiang & Loef, 1989). Now the child will find it boring to solve 24+37 by counting on. The level three child makes life easier for herself by transforming the given numbers to 20+4 and 30+7 and transforming the problem to 20+30+7+4. Murray (1991) stresses that these levels can in no way be taught. The child develops (constructs) this understanding by just working with numbers in a natural way: counting, playing games with dice and money, and solving problems.

It is, however, expected of the teacher to create a learning situation and to match an activity to a child's level of development, and manage the classroom in such a way that children remain involved in worthwhile tasks and discussions. She may not suggest methods, but she has to supply that kind of knowledge that children need to communicate with others.

Murray (1991:3) suggests that most mathematical knowledge has to be constructed by the child herself and that it develops gradually over a period of time depending on what type of experiences the child has had. If the teacher shows the child how to solve a problem, the knowledge is not properly constructed from within, but is memorised in bits and pieces that are frequently assembled and which may be applied in the wrong order for a particular situation. When children use methods they have constructed themselves, they only choose those that they really understand – which accounts for the much higher success rate.

This Problem Centred Approach in mathematics rests on three pillars: realistic-real-life-world problems, number concept development and social interaction and communication (Junior Primary Mathematics Handbook, 1994:16). The pupils progress at their own pace and no limitation on number is prescribed should pupils cope. It is possible for a Grade 1 pupil to work with units of hundreds.

This approach thus suggests that the way pupils progress is by allowing them to construct their own understanding in their own time, and allowing them to adopt those methods they feel comfortable with at a given time.

Metacognition including mental management in problem solving

It seems as if the teacher in the Problem Centred Approach to mathematics views this approach as the method for solving mathematical problems. Teachers do not seem to realise that metacognition plays an essential role in problem solving. The child and teacher should be aware of the processes by which thinking takes place. The view that metacognitive decision-making plays a vital role in problem solving is becoming increasingly popular within the mathematics education community (Lesh, 1983; Lester, 1982; Schoenfeld, 1985 & Silver, 1982). Metacognition refers to an individual's awareness of her own cognitive processes and "to the active monitoring and consequent regulation and orchestration of these processes" (Flavell, 1976). Thus metacognition consists of a cognitive self-awareness component and a behaviour regulation component.

According to the Junior Primary Mathematics Handbook (1994:47) problems form the focal point of all mathematical activities in the Junior Primary phase.

They serve to embrace:

- acquisition of the concepts of the four basic operations
- refinement and extension of number development

- development of strategies and methods of computation
- reflection and communication about how problems are solved
- development of an awareness of the properties of number.

The essential aim of mathematical education should be to help pupils to discover how to describe, tackle, and ultimately solve problems which require the use of mathematical knowledge and techniques. Until recently, research into most problem solving instruction could be classified into four categories:

- instruction to develop thinking strategies (e.g. originality and creative training)
- instruction in the use of specific "tool skills" (e.g. making a table, organising data, writing an equation)
- instruction in the use of specific heuristics (looking for patterns, working backwards)
- instruction in the use of general heuristics (planning).

None of these four approaches has been shown to be clearly superior to the others; rather, good problem solving instruction probably involves some combination of instruction in the use of both specific and general heuristics, together with training to develop tool skills. Also experience in solving a wide variety of problems over an extended period of time is essential (Silver, 1985:45). Even an ideal combination of these approaches might fall far short of the mark unless attention is also given to the "guiding forces" of problem solving (i.e. the metacognitive aspects). I will discuss what I believe are the primary stumbling blocks for mathematics students when they solve problems.

Polya's four phases of problem solving

A discussion of the major stumbling blocks encountered by problem solvers, especially novices, can be organised around the four phases of problem solving specified by Polya: understanding, planning, carrying out the plan, and looking back (Souviney, 1981:21). Instruction that enables students to avoid or overcome these stumbling blocks should be very effective.

Understanding the problem

Understanding involves two things: First the problem solver must read the sentences of the text and extract information from them by grammatical and semantic analyses. Secondly, she must construct from newly extracted information a representation of the problem that is adequate for its solution. This representation must include the initial conditions of the problem, its goal, and the operators for reaching the goal from the initial state. Dillion & Sternberg (1986:47) advocate that problem solving involves building a representation of the words of the problem and finding the solution. A major difficulty in students' performance on word problems seems to involve representation of the problem, i.e. moving from the words in the problem to a coherent mental representation of the problem. One major sub-component in the representation process for word problems is the translation of each sentence. Research shows that learning to solve word problems also involves learning how to translate the sentence and construct it into a meaningful representation. For example, students have difficulty in translating relational propositions. Thus instruction could provide practice in phrasing sentences in one's own words, or in a drawing to represent the sentence.

Holmes (1985:38) suggests that teachers give guidance and support to children, especially those who experience difficulties as they solve problems. Children must know that they have a resource person to help them as they solve problems. Guidance could be in the form of comments that lead learners to discover essential meaning in the problems they are trying to solve. It could be hints about solution procedures. Procedural options are helpful to many learners. Teachers could suggest using concrete or graphic material to aid solving problems. A pupil who cannot comprehend instructions will not be able to solve a word problem, unless the teacher acts as mediator to the pupil and makes sure that the pupil understands the wording of the problem. It consists of selecting the right elements of the situation and putting them together in the right relation, with the right amount of weight of influence or force for each. The mind is assailed, as it were, by every word in the word sum. It must: select, repress, soften, emphasise and organise, all under the influence of the right mental set or purposes or demand (Stauffer, 1969:10).

Planning and carrying out plans

Results from my own experience with Junior Primary students indicate that children in the age range of 6-9 years can be taught several rather sophisticated strategies. A teacher can model the use of a strategy for students and then ask them to solve a different problem the solution to which can be obtained by using the modelled strategy. The students will do well, but this would not be problem solving. Problem solving takes place when there is uncertainty involved. The major obstacle to success at the planning and implementing stages lies with the difficulties in recognising the conditions under which a strategy can be used effectively and in selecting a particular strategy from among two or more alternatives. What is needed in instruction is guidelines to enable the problem solver:

- to recognise when a strategy may be useful
- to select an appropriate strategy when more than one can be used, and
- to execute the strategy correctly.

Unfortunately, most instruction has been restricted to the third step. There is also the use of metacognition at these stages, in particular the sort of managerial activities that help the problem solver decide if progress is being made.

During this phase in mathematical problem solving the pupil must devise a solution plan and include steps to reach the goal. A pupil who understands the problem will have no difficulty with this phase. But in order to devise a plan of attack, the problem solver needs to have some knowledge of problem-solving heuristics, e.g. strategy knowledge. The problem solver may decide to use a "count all" strategy for computing the answer e.g. 7 7 7 7x3 = ... 1, 2, 3, 4, 5, 6 ....21 or the child may decide to use the method of counting in sevens 7, 14, 21. According to Sternberg (1986:51) to solve the problem the problem solver must choose steps that are the right size for solving the problem. The first step must be an easy one and the right size for the pupil too and the pupils must consider alternative steps.

Although the semantic structure of a problem appears to be the major factor that determines how it will be solved, the wording of a problem can also significantly affect whether a child will be able to solve a given problem or not. Some wordings appear to make the semantic structure of the problem clearer than others. As a consequence, they are easier to model. Although traditional compare problems are generally relatively difficult for young children, almost all children could solve them if the language was made more transparent (Silver, 1985:24). Children in the first grade were given a compare question.

They were shown a picture of a group of birds and a group of worms. The question then was, "How many more birds than worms are there?" and an alternative "Suppose the birds all race over and each one tries to get a worm, will every bird get a worm?" Most children were unable to solve the standard problem but could solve the alternative version. In this alternative version pupils used the one-to-one correspondence. This suggests that much of the difficulty that children at this level experience with problems of this type is with the language of the problem, not with representing the problem structure.

Another way of making the problem more understandable for the Junior Primary pupil is to change the time frame to the present tense. For example "Ceril had 5 apples. Mark gave him some more apples. Now he has 13 apples. How many apples did Mark give Ceril?" This version could be changed to "Ceril has 5 apples. How many more apples does he have to pick to have 13 apples?" The difference between the two versions of the problem is the time frame in which the action is performed. The question is then framed in the context of "How many apples have to be added to get 13?" This seems to cue the child that the purpose of adding elements to the initial set is to find out how many have to be added to get 13. Therefore the elements added need to be kept track of so they can be counted. In the more difficult version the initial quantity is described in the past tense. Apples were added so that now there are 13. The question of how many apples were added comes later and is not clearly connected to the process of adding elements to the initial set. Since 13 apples exist in the present, it may be necessary to somehow transform the problem to model it. Problems are more difficult if they must be transformed and cannot be solved directly. It is clear that the difference in wording contributes to the difficulty of the problems, but it is not at all clear exactly how. We have a reasonably clear picture of how semantics affects children's solution processes, but beyond knowing that certain wording is more difficult, we have a much less precise picture of how differences in wording influence the children's solutions.

Looking back

This phase in problem solving requires that the problem solver be able to look back over the solution of the problem. This stage is beneficial for two reasons. Firstly, it provides an opportunity to evaluate and refine the results. Secondly, it brings the process of solution into clear focus. At no time are children in a better position to clearly understand and explain the process in which they are engaged than immediately following a successful problem solving experience. Not only does the child evaluate the validity of the results, but more important, she links the current solution with previously solved problems (Souviney, 1981:35).

If each solution is carefully reconsidered in the light of past experience and linked with prior results, then the ability to solve novel problems in future may be enhanced. If the goal is to improve general problem solving abilities, children must be strongly encouraged to re-examine their successes, refine their results, and imagine problems which have been or could be solved with similar techniques. Otherwise each problem is but a lonely star shining in the unknown (Souviney, 1981:35).

Pupils must be given a chance to explain how they solved the problem. They should tell others what procedures they undertook in solving the problem. This would enable other pupils to see the differences and the similarities in the strategies that they have used. Pupils need to learn and experience mathematics, that is read, write and speak mathematical ideas in an environment that not only encourages communication, but also gives opportunity to reflect, clarify, and communicate their thinking. The pupil must thus be willing to verbalise and share ideas.

This social interaction between and among pupils forms an important part of the learning process because it compels pupils to refine existing knowledge as well as to construct new knowledge (Junior Primary Mathematics Handbook, 1994:16).

In this final analysis of the looking back phase, the success of any instruction can be judged by the extent to which what has been taught can be applied to novel situations. The problem has been solved with "good understanding" only when the problem solver recognises the relation of the solution to some general principle. Transfer has taken place only when the problem solver recognises that task B has certain properties in common with a previously solved task A. It is likely that little transfer will take place unless direct attention is given in instruction to having students look back at what they have done with an eye towards answering the question. "What have I learned by solving this problem?" It is not enough to train pupils to "check your work" and to make sure the solution satisfies all conditions of the problem. It is as important for the problem solver to identify the key features of a solution effort that may prove to be useful in future problem solving. A step in the direction of making students better able to look at their efforts might be for teachers to focus more attention on solution attempts and less on correct answers. Post-problem solving sessions in which students share their attempts and discuss reasons for their actions might be one way to bring about this change in focus (Silver, 1985:47).

In this final phase of looking back, mediation plays an important role. There are pupils who are willing to communicate the process of how they went about solving the problem, but there are also some pupils who are very hesitant to communicate. Here the teacher must ask a leading question that will enhance communication, e.g. "Why did you draw four circles? Why are there the same amount of elements in every circle?" These pupils who are hesitant to talk may later communicate freely after repeated mediation.

The information processing approach

This approach is based on task analysis, which means that any type of mathematical problem can be broken down into information processing components, e.g. simple mental operations, skills and knowledge that are required for problem solution. Mathematical abilities are defined as all the cognitive operations, skills and knowledge that are components of a mathematical task (Sternberg, 1985:129). When a child is confronted with a mathematical word problem there are steps which she can follow to solve the problem and in her attempt to solve the problem she displays certain cognitive operations.

Problem translation: The first step in representing a problem is to be able to translate each proposition from the problem into an internal representation (Dillon et al., 1986:147). In order to translate each proposition in a word problem the pupil needs some knowledge of language (linguistic knowledge) and some knowledge about the world (factual knowledge). For example: Tom has 50 cents and Peter has 2 times more than Tom. How many rand does Peter have? Linguistic knowledge is required to parse the sentence into variables, or what various words mean (e.g. 2 times more than) and factual knowledge is required to represent "cents" as 100 cents is equal to one rand. The child needs to know the units of measure.

Sternberg (1986:62) suggests that an important part of many kinds of problem solving is the way that information is represented mentally. It might be in a form of a pictorial image especially for younger children. In solving a mathematical problem, the pupil may find it helpful to draw a diagram that represents the terms of the problem.

Problem integration: The second step in mathematical problem solving is to put the propositions of a word problem into a coherent representation. In order to integrate and understand the problem, the pupil needs to have some knowledge of problem types (schemata knowledge). The pupil may note that the problem is a "comparison problem" because two sets are compared to one another, e.g. Tom has 50c and Peter has two times 50c (50c x 2).

Feuerstein et al., (1979:59) refer to this stage as the input phase. Here the child gathers all the data as she attempts the solution of the problem. Fisher (1990:142) refers to this stage as the child's capacity to gather and organise information. Children often fail to solve problems because they have only partially collected the necessary data. They need certain verbal tools with which they can extend their conceptual grasp of things. These tools help the child to receive and differentiate data. Here the teacher must act as mediator. To enable pupils to become aware of the interactive steps the teacher must first establish with the pupils what they are asked to do, as the starting point is often the major stumbling block. At this stage it may be that pupils cannot comprehend the instruction. The teacher must now read the problem to the pupils and make sure the pupils know the meaning of the words in the sentence. If not, the teacher must, with the help of the pupils, restructure the question and use words that the pupils understand.

Solution planning: The third step in mathematical problem solving is to devise a solution plan and include steps to reach the goal. But in order to devise a plan of attack, the problem solver needs to have some knowledge of problem-solving heuristics, for example the problem solver may decide to use the "count all" strategy for computing the answer, e.g. 2 x 50c ... 1,2,3,4,5 ....100 or use the method of counting in tens 10, 20, 30, 40, 50, (60, 70, 80, 90, 100). Feuerstein et al. (1979:59) refer to this stage as the Elaboration stage. Here the information in the problem is being processed. It is the "thinking things through" phase; here the child confronts the elements of the problem.

Solution execution: The fourth step in mathematical problem solving requires that the problem solver be able to carry out the operations, such as computations. In order to execute problem solutions the pupil needs some knowledge of solution procedure, that is algorithmic knowledge. For example the problem solver needs to know that 2 x 50c = R1,00. Pupils may differ in their ability to carry out operations, and these differences may be related to algorithmic knowledge

(Sternberg, 1985:140). Computational algorithms depend on the availability of subskills in problem solving. In particular, early computational algorithms tend to make use of students' experience in counting, e.g. "count all" 2 times 50 cents is 1, 2, 3, 4,.........100 or "use known facts" 2 x 50 = 100 cents. Once the students have acquired some experience, they move from less sophisticated counting procedures to a more sophisticated counting procedure.

Feuerstein refers to this as the output phase where pupils communicate their thinking and try out their solutions. Sometimes the need is there but the expression or output may not meet that need. One cause of failure in solving the problem is vagueness and lack of accuracy and precision.

Pair peer problem solving

Whimbey and Lochhead identify and describe methods that good problem solvers should use. They also describe a pair peer problem solving method (Nickerson, Perkins & Smith, 1985:206). The pair problem solving method involves two people working together on a problem, with each person having a specific role as problem solver or listener. The problem solver reads the problem out loud, then continues to talk aloud, describing what she is thinking while solving the problem. The listener listens carefully to the problem solver and asks for clarification whenever the problem solver leaves out information or makes a step that the listener does not understand. The listener should not solve the problem independently of the problem solver, but should make sure that she understands how the problem solver is thinking about the problems. Errors should be pointed out only after the problem solver has had ample opportunity to discover them herself. When they are located they should be studied in detail to find the reason why errors were made, so that there is less chance of making the same mistakes again (Whimbey & Lochhead, 1984:2). This method involves mediation. Here the listener acts as mediator, she asks for clarification, points out errors and through communication the listener and problem solver find the reason why the error occurred.

Whimbey & Lochhead advocate working in pairs and "thinking aloud" while trying to solve problems, for two reasons. Firstly, by listening to other people solving problems one may learn something about the techniques that work and those that do not and secondly, by exposing one's own thought processes verbally both to others and to oneself makes it possible for one's approach to be analysed and criticised (Nickerson et al., 1985:106). Here the pupils' communication leads to an adjustment to their mathematical vocabulary.

The process of pair problem solving has been shown to have many important advantages in the development of good analytical reasoning abilities. First, thinking aloud slows down the thinking process, making it both more explicit and accurate. Learning to become a good listener allows you to learn from other people's reasoning skills and to pay more careful attention to your own method of thought (Whimbey & Lochhead, 1984:2).

In addition to thinking aloud, another way of examining your thoughts is to write them down. A thought process protocol takes more effort and self-discipline than thinking aloud, and it can be frustrating because it slows the thought process dramatically. But people who have been persistent enough to master the process find it exceedingly helpful, particularly with difficult problems (Whimbey & Lochhead, 1984:8).

Teachers who use pair work and group work in this way give their pupils the opportunity of thinking aloud. This makes pupils' thinking visible and through the interactive play between peers and teachers misconceptions can be corrected.

Managerial strategy

Schoenfeld (1985), a mathematician interested in the nature of expert problem solving and how to teach it, suggests that:

- Heuristics helps students to solve problems when the students know and apply heuristics
- Students lack a good set of heuristics
- Students do not reliably pick up heuristics spontaneously from examples; but heuristics have to be taught explicitly

- Students do not reliably apply heuristics they know, some sort of guidance or prompting is necessary
- A "managerial strategy" for approaching problems, taken together with heuristics, can lead to substantially improved problem solving performance in mathematics.

Krulik and Rudrick (1980:19) describe heuristics as a general strategy that individuals can follow to help them approach a problem, understand the problem, and arrive at a solution to that problem. Simons (1979:152) uses heuristics to denote any principle or device that contributes to speed up the search for a solution. Landa (1976:106) understands heuristics as specific rules of instruction governing actions and not as the actions themselves or any process influencing the solution of a problem involving creativity. Heuristics must thus be understood to mean the rules of heuristic activities, or as instructions on how to perform these activities.

Through experience and experiments Schoenfeld (1985:198) developed a general managerial strategy and a series of heuristics for mathematical problem solving – he has since reviewed his approach – whereby instruction concentrated on the process of problem solving. The importance of self-monitoring was stressed, in keeping the design phase of the managerial strategy. He stressed that it is not enough to have heuristics, management is also needed. The management of the mental processes is thus metacognition.

In conclusion, cognitive and metacognitive strategies enhance learning. Adept learners invoke cognitive and metacognitive strategies to monitor that progress. The teacher thus has an important role through her teaching approach to enhance individuation. The approach should be process based and should encourage autonomous learning.

Language and problem solving

To read with understanding plays an important role in a problem centred mathematics approach. There is a connection between reading and thinking. The notion of reading being akin to thinking has a long history. Earlier researchers in this area characterised reading as "thought-getting" and viewed its purpose to stimulate readers' thinking powers (Stauffer 1969:8). Reading is not a simple mechanical skill; nor is it a narrow scholastic tool. Properly cultivated, it is essentially a thought process. However, to say it is a "thought getting" process is to give it too restricted a description. It should be developed as a complex organisation of patterns of higher mental processes. It can and should embrace all types of thinking, evaluating, judging, reasoning, and problem solving. Indeed it is believed that reading is one of the best media for cultivating many techniques of thinking and imagining (Stauffer, 1969:9).

- realistic thinking, or reasoning
- imaginative thinking
- personalised thinking

Stauffer (1969) also likens purposeful reading to problem solving in that both involve three phases: confrontation by a problem, reading to find a solution, and finding the solution or failing to find it. Reading is narrowly associated with reasoning in solving a problem in mathematics. It consists of selecting the right elements of the situation and putting them together in the right relation. The mind is assailed, as it were, by every word in the paragraph. It must select, soften, emphasise, correlate, and organise, all under the influence of the right mental set or purpose or demand. The reading process is considered to be an interactive social event that involves internal regulation, and it is influenced by context and utilises higher order thinking processes (Bloome, 1991). Examples of such processes are analysis (to clarify information by examining the component parts), synthesis (to combine relevant parts into a coherent whole), and evaluation (to set up standards to judge the reasonableness of ideas). But these cognitive processes are not the end products of a hierarchy of separate skills, they are part of an interactive process that uses several levels of thought simultaneously.

According to researchers on critical reading, being a critical reader demands that students correctly identify valid (versus invalid) instances of argument, reasoning, or presentation of evidence in written materials, make reliable observations, produce sound inference, and form reasonable hypotheses (Flynn, 1989). The above-mentioned abilities are also characteristics of good problem solvers.

The kinship between reading and thinking has motivated several authors to advocate that reading may be one of the best ways of cultivating techniques of thinking. Thus, according to Whimbey (1982:121), although intelligence manifests itself in many forms, there is one expression of it that is especially important in scholastic pursuits: the ability to read comprehendingly.

To solve a problem, we use previously acquired knowledge, often in creative ways, to restructure a situation so that it is comprehensible and manageable. The cognitive approach to learning is often referred to as a Problem Solving Approach. Learning occurs in response to disequilibrium. It involves the construction of new knowledge. This is problem solving in a broad sense. Problem solving reorganises what is known in terms of the requirements of a problem situation (Holmes, 1985:35).

Across all levels of age and ability, readers use their existing knowledge as a filter to interpret and construct the meaning of a given text. They use this knowledge to determine importance, to draw inference, to elaborate on the text and to monitor comprehension. But although students' existing knowledge is crucial to comprehension, the relationship between that knowledge and text comprehension is not a simple, orthogonal one. Sometimes the knowledge is inert and therefore not brought to bear in the comprehension process. Other times the knowledge is incomplete, fragmented, naive, or even misleading. And when students possess knowledge that conflicts with the information encountered in the text, their existing knowledge can, and often does, prevail over textual information. Students are not likely to change their existing knowledge unless they recognise and are dissatisfied with the fact that it no longer provides an adequate account of everyday experiences. Thus, for better or for worse, knowledge plays a critical role in cognitively based views of reading and problem solving (Dole, Duffy, Roehler & Pearson, 1991:24).

Critique

The Problem Centred Approach conducted at Stellenbosch University, linked to the Madison programme, is based on a Piagetian model of thinking. Here the child's learning is seen as an individual, not a social activity, reflecting the value placed on individualism in these classrooms. Hence, successful teaching practice entails each teacher taking cognisance of and engaging with each child's unique problem solving strategies. Junior classrooms in North America tend to have less than 20 children per teacher making these pedagogical practices feasible. But the over-crowded and under-resourced classrooms in most South African schools make such exigencies impossible (Gordon, 1993:5).

Where the "social" in social constructivism is used we find a distinct difference in the meaning of social in Piaget's (1968) theory and Vygotsky's (1978) theory. Piaget's theory, implicitly conceptualises children as "individual" active scientists, whereas Vygotsky views children as social beings who construct their understanding from social interactions within specific social-cultural settings. The Problem Solving Approach is based on a Piagetian theory. This approach is thus preaching a Vygotskian theory, but is built on Piagetian theory.

South Africa aims to reconstruct the mathematics curriculum so as to make it accessible to all sectors of the community in order to empower people in the context of their everyday activities (Gordon, 1993:2). It is rather irresponsible of a teacher to think that a child can interact with a written problem on her own and produce her own solution without mediation.

It is argued that the Problem Centred Approach method of teaching is only suitable for the highly capable learners who can construct their own knowledge and do not need the teacher to show them how to perform the "four basic operations". But what about the weaker pupils who cannot work on their own and need the teacher as mediator to guide them step by step through the problem that must be solved? Mediation plays an important role in mathematics. Not all pupils have the ability to solve a "new problem". They need the interaction with the teacher. Many children come to school with informal mathematical knowledge and mathematical vocabulary. However, many children are not as fortunate and therefore they need mediation.

The Problem Centred Approach applied by RUMEUS (1991) may easily lead to dangerous and uncertain practices in problem solving. This approach says "do not mediate". What actually happens now is that the teacher thinks she can just pose the problem. The pupils then have to sort it out themselves. This means that some teachers may take the easy way out and not do any teaching.

Even if the teachers fully appreciate what lies behind the Problem Centred Approach and the weakness in the Problem Centred Approach, they may give wrong mediation, or no mediation, because they have no guidelines. Therefore, teachers must be given guidelines on how to mediate. Teachers only attend workshops on how to implement the Problem Centred Approach. They do not receive any formal training on how to apply this approach in the classroom. This situation can lead to the deterioration in the teacher/pupil environment, because the pupil does not know what she is supposed to do or what is expected of her and neither does the teacher.

The policy of RUMEUS (1991) is that the Problem Centred Approach is not compulsory, but it is the prescribed approach in the Mathematics Syllabus. We must take into consideration that learners are individuals who learn in different ways at different times. If we only adopt one approach all the time there will always be those who will not interact at the time.

Why must the Problem Centred Approach be used all the time? Why do we not rather use a variety of approaches? Taking into account that children learn in different ways, why not choose an approach that will suit that child's needs by giving the child an experience of the other methods? Do not place too much emphasis on problem solving as if the other approaches are inferior.

Another major area of difficulty is in the non-use of algorithms. The lack of teaching algorithms can disable pupils. They need to be taught some standard algorithms because these algorithms are tried and tested. They have been tested over a number of years and they will always give an answer. When a child is left alone to make up her own algorithms, her algorithms may be weak or they may break down in different cases.

Evidence from research findings does indicate that children do not necessarily compute in the ways they are taught (Junior Primary Mathematics Handbook, 1994:6). But algorithms have a place in the curriculum. They represent known ways of solving problems. At the same time they need not be the only ways and none of a pupil's own adventurous inclinations need to be suppressed. On the other hand, the open-ended underpinning of the "new maths" methodology might represent sound psychology but it also contains a fundamentally erroneous premise, viz., that knowledge should be discovered anew, instead of being experienced anew. The discovery process is exclusive, whereas the experienced one is inclusive. A map analogy will suffice: I can at the hand of basic input leave it to an individual to find a particular place (sic). It may take her days (of frustration) to discover a way of finding the place. Or, I can make use of a map and show the person a way of finding a place. There may still be more than one route to follow or the person may even wish to devise her own way of finding it, but in the event of getting lost there is at least a known possibility to fall back on. This is a component which is lacking in the New Maths Methodology (Sinclair, 1995).

Conclusion

Learners are to be thought of as "active mathematical thinkers" and we as teachers need to "develop their thinking", by using a wide variety of styles and approaches which include activity-based learning, discussion between pupils as well as between teacher and pupil, problem solving, open-investigation and discovery. Southwood (1995) asks the question: "Does this sound like the maths we know and teach?"

This concludes my theoretical consideration of the Problem Centred Approach. In the next chapter the experimental investigation will be discussed.

Chapter 4

Experimental Investigation

Introduction

The main goal is to prove that teacher mediation improves problem solving performances of children in the Junior Primary phase. In particular, the emphasis was on improving children's abilities to use certain heuristics, that is to break a task into steps, think aloud, use strategies, and other problem solving tactics to solve a wide range of types of Junior Primary mathematics problems.

The investigation proceeded in four stages. The first stage involved a pre-test. The second stage consisted of a teacher-mediated interaction. Here I made use of the five mechanisms of mediational teaching and the first six criteria of MLE. I also made use of certain aspects "helping the child" to foster cognitive development. The third stage was the post-test and here the pupils applied the Mediated Learning Experiences that are thought to be essential for the successful solving of mathematical problems. In the fourth stage individual interviews were conducted with two subjects from the experimental group. This experimental investigation was done with Afrikaans medium pupils in the second grade (Substandard B, 7 – 9 years).

Procedure

The investigation constituted a pre-test and post-test experiment. Pupils were deprived of mediation on a random basis. They thus constituted a control (C) group (N = 10) while the other pupils (N = 10) constituted the experimental (E) mediation group. The E-C group division did not constitute a difference in motivation between the E and C groups as neither group had a choice in their participation in this research. The C group was broadly the same as the E group in terms of age, gender, intellectual and academic ability.

Pre-test

During the first stage the pupils in the C group and E group were given a pre-test that consisted of four word problems described below. The pupils were placed four in a group, each pupil working on a different word problem. Within easy reach were pencils, paper, crayons, erasers, counters and sticks of different sizes. The use of these materials was neither encouraged nor discouraged. The reason for four different word problems was to allow all pupils to work on their own on a different word problem and also to prevent them from copying other pupils answers. Thus, over a period of four days every child completed the four word problems. The problems were presented as "word problems" written out on their worksheets. During each session the problems were read aloud to the group of 20 pupils. No questions concerning the problem statement were answered by the teacher. While the pupils worked on the problem the teacher was not available to answer questions. No hints were given and pupils were left on their own to solve the problem.

The purpose of this first stage was to engage the entire 20 pupils in problem solving that would involve no teacher direction or interaction. Secondary purposes were to give an example of what sort of teacher mediation would be most appropriate given the goals of the investigation.

The word problems in the pre-test

1. There are 36 sweets. Paul has the same amount of sweets as Mark. Cole has 5 sweets more than Paul and Mark. How many sweets does each have?
2. A chocolate has 4 blocks on the short side and 7 blocks on the long side. Cyril gave half of the chocolate to his friend. How many blocks are left over?
3. Paul is shorter than Mark, but taller than Kevin, Kevin is shorter than Paul but taller than Cole. Who is the tallest and who is the shortest?
4. On a plate is 26 biscuits. Mary ate half of the biscuits and Joan ate 4 less than Mary. How many biscuits are left over for Anne?

Questions 5–8 in the post-test were comparable to these word problems in the pre-test. They only differ in numbers, names, objects and situations.

The questions were divided into four categories.

Questions 1 and 5 contained two and more numbers. Pupils tend to either add or subtract when they see numbers. The aim of mediation is to help pupils acquire strategies to solve these problems.

Questions 2 and 6 also contained numbers but it is a real life situation, which should make it easier for pupils to represent this problem.

Questions 3 and 7 contained no numbers and pupils are compelled to invent a visual manner of representing the problem.

Questions 4 and 8 contained two numbers but mentions three children. To find a solution the pupils must work out a plan. This compels them to stop and think and not to act impulsively.

Implementation of mediational teaching

During the second stage, pupils in the E group were exposed to intensive mediation for about one hour a day during the mathematics lesson over a period of one month. This stage consisted of teaching pupils strategies on how to solve a problem. I also used Feuerstein's (1991) mechanisms of mediational teaching and the first six criteria of MLE to help develop the E- group's problem solving abilities.

Of the twelve parameters of MLE (see pp. 30/31), I have only chosen the first six criteria of MLE in the implementation of mediated teaching. The first three parameters: intentionality and reciprocity, mediation of transcendence, and mediation of meaning are all necessary conditions for an interaction to qualify as MLE. Mediation of competence and/or regulation of behaviour are functions of specific experiences that combine with the first three to make the adult-child interaction one of mediated learning. Mediation of sharing behaviour was added to the above mentioned five criteria of MLE, because here the child and the mediator are engaged in a shared quest for structural cognitive change in the child.

No problem can be solved without first understanding thoroughly what the problem is about. The statement of a problem often includes descriptions of events or situations that, in their verbal form, are difficult to interrelate. In these cases, it is helpful to visualise the verbal description and learn how to represent the underlying event or situation. Many times this is all it takes to solve the problem because the solution can simply be read out of the representation (Grignetti, 1986; Polya, 1962).

In the 1980s an experiment on a large scale was conducted in Venezuela to try and improve intelligence on the whole in school children. This was Project Intelligence and resulted in pupils' and teachers' books, marketed in USA under the title of Project Odyssey. To teach the E-group strategies, I made use of some of Odyssey's techniques (Grignetti, 1986:1-46). First I introduced a strategy called Represent and showed how to use it in solving problems that require the construction of a one-dimensional figure to obtain a solution. The wording of the problems is simple and direct. The representation can be constructed by following the order of the words in which information is given in a problem statement, without having to leave any part pending, until further reading clarifies the meaning.

My aim in teaching this strategy was to enable pupils to experience what it is to see a problem. I posed the word problem "Peter has 5 apples, Mary gave him three more apples. How many apples does Peter have?" Most of the pupils impulsively answered 8 apples without actually thinking. Then I posed another word problem "Peter has 5 baskets, in each basket are 3 apples. How many apples are there?" Again the answer was 8 apples. Pupils were asked to close their eyes and the question was repeated aloud. Pupils were then asked to draw, what they saw when their eyes were closed. Here the teacher acted as mediator. She guided the pupil, she asked questions and answered the pupils' questions. She allowed them to discuss what they were doing with their friends and the outcome was that most of the pupils could draw a representation of the problem and gave the same answer 15 apples. The second word problem was "Peter has 5 baskets, in each basket are 3 apples, he saw 4 rotten apples. Mother took the 4 apples. How many apples does Peter have?" While watching them attempting to solve the problem I saw some of them counting 5+3+4+4. Pupils were again encouraged to use the strategy of representation. But now they were also encouraged to postpone representing a part of the problem until, by proceeding with the statement, its meaning became clear.

My aim here was to teach the pupils how to use the Postpone strategy in order to avoid getting stuck. I emphasised that when it is too difficult to solve a problem, break it up into parts that are manageable and when it is too difficult to solve a part of a problem, put it aside and tackle another part of the problem. Through pupil-teacher interaction, pupils were encouraged to draw what they saw in the first part of the question and then to tackle the second part of the question.

The third problem posed was "Peter has 5 baskets, in each basket are 3 apples. While counting the apples, he saw 4 rotten apples. Mother took the 4 apples and gave him 6 fresh apples. How many apples are there now?" By now some pupils were aware that part of this problem was the same as the one they had done before. They could successfully solve the first part of the problem. Pupils were now guided to see that this problem consisted of three parts and through solving the problem part by part, it was easier to reach a solution.

This routine of problem solving was applied every day. Pupils were first encouraged to try and solve the problem mentally, that is to say without drawing a picture. Afterwards, they were asked to solve the problem with the help of representation. Here the teacher stressed how much easier it becomes to solve the problem once the representation has been drawn. Pupils were also reminded that there was usually more than one way to represent a problem. The only requirements for a good presentation were that the symbols used should not be confusing, and that the differences or similarities that existed between the properties of the represented dimensions had to be clearly noted. Every problem solution was also discussed one by one before moving on to the next problem.

Pupils could not recognise the meaning of words and phrases contained in some of the problems. Here the pupils were encouraged to discuss the difficulties they came across either with their classmates or with the teacher. The teacher stressed that social interaction between and among pupils forms an important part of the learning process because it compels pupils to reflect on their work. Reflective thinking leads to the refining of existing knowledge as well as the construction of new knowledge (Grignetti, 1986).

There is no substitute for thought. Strategies are therefore presented not as a means for accomplishing particular tasks, but as tools for making their accomplishment easier or more manageable. The goal is to develop in pupils a repertoire of widely applicable strategies and the necessary attitudes and skills for flexibly using that repertoire at any appropriate opportunity (Grignetti, 1986:vi).

Mechanisms of mediational teaching

With Feuerstein's mediational techniques, I incorporated Haywoods five mechanisms of mediational teaching to help develop the E-group's problem solving abilities. These are:

- Process questioning
- Challenging or asking reason
- Bridging
- Teaching about rules
- Emphasising order, predictability, system, sequence and strategy.

Process questioning

While the pupils attempted to solve the word problem the teacher asked many questions of a process nature. She asked "how" questions. In the class discussion she asked, "Yes, but how did you know?" This mechanism is extremely important as a metacognitive tool, i.e. it helps to focus children's attention on their own thinking processes and encourages them to engage in similar "small conversations" with themselves (Haywood, 1993:35). By asking the pupils to close their eyes and to focus their attention on what is formed in their minds, the pupils' attention is focused on their own thinking process. The learners then have "small conversations" with themselves to work out the solution of the problem. Children who can focus their attention on their own thinking processes become good problem solvers. They learn the ability to internally work out a plan of action.

Challenging or asking reason

Here the teacher established a habit of challenging both correct answers and incorrect answers. She accepted as much as possible of the children's responses. She challenged the children. She asked, "Yes, but why —?" By challenging a correct answer, the child is conditioned against the expectation that a challenge by the teacher means that her answer is wrong (Feuerstein et al., 1980).

Bridging

Here the teacher applied bridging to the thinking skills or cognitive functions rather than to content alone. The teacher encouraged the pupils to think of different applications of the thinking process and strategies they had been discussing. In the class discussion pupils were encouraged to first form a picture of the problem and to "think aloud". These pupils were able to apply this strategy in a situation where no numbers were given in a word problem. At times a pupil was unable to solve a problem even when she formed a picture in her mind or tried to draw a picture. When she used some of her friends to help her represent the problem, she successfully solved the problem. Here she used the strategy she had learned in the class discussions and applied it to a real, concrete situation. The learning of a concept or principle of thought does not take place merely in the verbalising of the principle but in the children's successive attempts to apply the principle in a variety of different situations and applications (Feuerstein, 1980; Haywood, 1993).

Teaching about rules

The idea here is to make pupils realise if one can make an applicable rule one will know what to do in future similar situations. Pupils were asked to generalise "Can we make a rule about how to do this kind of problem?" Rules of this sort are to emphasise what to do when confronted with a mathematical problem that must be solved (Landa, 1976:107):

- Begin not with what is given but with what is to be proved.
- Break up the goals into subgoals.
- Find the difference between that which is to be proved and that which is given. Transform that which is to be proved into something nearer the given. (Subgoals are introduced in such a way as to reduce these differences.)

In the class discussions, pupils were made aware that they must apply these rules. They were encouraged to generalise and ask themselves:

- Can we make a rule about this kind of problem?
- Does it apply here as well?

- Would it help to have a rule here?
- How could we make one?
- When do we need to know rules, to know what to do?

(Haywood, 1993:36)

Emphasising order, predictability, sequence and strategies

Pupils were made aware that there is order in the universe, that events are predictable if one understands the rules and has enough information, that objects, events and even persons are arranged in a system and that the order in which one looks at or does things is important and that problem solving is often primarily a question of finding an effective strategy. This was done by planning the days' activities with the pupils and ending with a discussion. Both served to emphasise the orderliness and predictability of the universe as well as the satisfaction of planning events in agreeable sequence (Feuerstein et al., 1980; Haywood, 1993).

In the maths class discussion, working out a plan of action and breaking the problem into manageable parts helps to solve a problem which at first seems impossible. Pupils learn that even in problem solving there is order and that a situation is predictable if rules are carried out. They are made aware that there are strategies that they can apply and that events happen in sequence. To solve a novel problem you must first: (Polya, 1962).

- Understand the problem

- Make a plan
- Carry out the plan
- Look back on how you solved the problem.

Pupils are made aware that the looking back phase can only be last, because now they have the opportunity to evaluate and refine results and it brings the process of solution into clear focus.

Criteria of Mediated Learning Experience

Feuerstein has suggested a number of criteria of mediated interactions. From this list I applied the first six criteria and suggest that they were the most important during my investigation (refer to Mediation Chapter) (Feuerstein et al., 1980; Haywood, 1993)

- Intentionality and reciprocity
- Transcendence
- Mediation of meaning
- Mediation of feeling of competence
- Mediation of regulation and control of behaviour
- Mediation of sharing behaviour

Every interaction can be examined to determine to what extent it reflects each of the six criteria. Awareness of these criteria will encourage teachers to try consciously and deliberately to construct their interaction with pupils during problem solving in such a way as to reflect these criteria (Haywood, 1993:32).

Intentionality and reciprocity

Given the theoretical discussions of Feuerstein, Haywood and Skuy, this is what I did in practice.

Classroom procedures

- The teacher orientates the learner towards a goal. She elicits the pupils' interest and motivation in the word problems and thus elicits responsiveness from the pupils.
- The pupils listen and react to the teacher in an atmosphere that is conducive to learning.
- The teacher shows interest in the pupils while they solve problems and shows pleasure and gives praise when the pupils are successful in solving the problem.
- The teacher is prepared to make the necessary adjustment when pupils do not understand the word problem. She takes time to ask questions, she elicits rather than give answers.
- The teacher and pupils are "equal partners" in the learning process. Each takes responsibility in and for the learning process.

Transcendence

Transcendence means to go beyond the goals of interaction. The goals of mediating transcendence are the expansion of need and the application of certain principles, processes and/or behaviours beyond the situation in which they are encountered (Skuy, in press:9). The learner learns to think reflectively and to discover relationships between events. The mediator tries to produce structural cognitive changes in the child, i e. transcend the immediate situation. An event could be seen as only an isolated event, but a mediational teacher will give such an event transcendent meaning by attempting to relate the event to previous and even future events of a "similar nature", and thus to extract a generality (explanatory rule) (Haywood, 1993:33).

According to Skuy (in Press) mediation of transcendence lets children develop:

- a better conception of the world
- a perception of how things are interrelated
- an interest to investigate and to discover relationships
- a desire to know more and to look for explanations
- a constant expansion of their spatial and temporal life space.

Classroom Procedures

- The teacher uses the topic of the lesson and relates it to previous and even future events.
- The teacher selects subject material according to the pupils' ability.
- The teacher prefers WHY and HOW questions, instead of WHO and WHAT questions.
- The teacher applies bridging to the thinking skills or cognitive

functions rather than to the content only.

- The teacher encourages pupils to apply bridging from what they are doing to other areas.
- The teacher also provides bridging from a certain area to other subjects.
- The teacher provides problem posing and problem solving activities.
- The teacher expects pupils to explain their answers.
- Children now learn to communicate with one another and respect each other's views.

Mediation of meaning

It is important to let the pupil know why she is doing any particular activity, i.e. to produce structural cognitive changes in the children (Haywood, 1993:2). The mediation of meaning deals mainly with the energetic dimension of the interaction; it answers the questions of WHY, WHAT FOR, and other questions related to the casual relationship reason for something to happen or to be done.

My classroom procedure for mediation of meaning

- The value, significance and/or reference of a task or event is conveyed.
- The teacher communicates both the immediate (content) meaning of events and their generalised relationship to other events.

- The teacher conveys to pupils why things are done a certain way. When you have a plan, you will know what to do as you go along.
- The teacher "conveys" the stimulus events, relationship, and concepts which are the purpose of the interaction.

Mediation of a feeling of competence

Children's feelings about their own competence are extremely important. The child must be equipped with appropriate skills of competence. This implies making her able to gather the data necessary for solving a problem and to offer the child opportunities to confront certain situations that she has to master. The child must also be encouraged to work independently (Sharron, 1987; Feuerstein et al., 1991).

Classroom procedures

- The teacher, both implicitly and explicitly, makes the child feel that she is able.
- The teacher makes sure that the stimulus is appropriate to the pupil's level of ability.
- The teacher rewards appropriate responses, especially process-oriented responses, with acceptance, acknowledgement and praise.
- The teacher makes pupils aware of what aspects of their behaviour were good and should be repeated.

- The teacher makes the children aware that they are capable of functioning independently.
- The teacher is well organised and makes sure that the children understand the process that can lead to successful behaviour.
- The teacher challenges both correct and incorrect answers and work. She gives positive feedback.
- The teacher makes pupils aware of their progress and encourages those who progress at a slower pace.

Mediation of regulation and control of behaviour

The mediation of self-regulation involves the heightening of the individual's responsibility for and control of her behaviour, both in learning and social situations (Skuy, in press:11). Pupils are thus made aware that certain situations require certain behaviour. The emphasis on regulation of behaviour is on the creation of the prerequisites, cognitive elements that, supported by the metacognitive self-reflective orientation, create the propensity for a reasoned way of behaving adaptively. Self-control over certain activities becomes possible through the use of cognitive components related to the particular activity.

Classroom procedure

- The teacher encourages pupils to inhibit their impulsive responding and thus to improve the quality of their responses.

- The teacher helps pupils to "unblock" available responses for the answering of questions by creating an effectively safe and expectant environment.
- The pupils are given the opportunity to solve problems in an environment that is warm, non-threatening, supportive, stimulating, challenging.
- The pupils are allowed to answer questions fully before the teacher reacts to the pupils' answers.
- Pupils are encouraged to collect the necessary data for discussion beforehand.
- The teacher gives pupils the chance to experience "hands on". She allows pupils to make mistakes – pupils learn through their mistakes.

Sharing behaviour

Sharing behaviour reflects the need of the individual to go out of her own self in the direction of participating with others and to make others participate with her. The mediator conveys the attitude that she and the child are engaged in a shared quest for structural cognitive change in the child. Each has an identifiable and separate role, but each is a participant and shares in a "we're in this together and you can count on me to do my part" manner. Where no sharing of behaviour occurs, it will lead to an emotionally isolated child (Feueurstein et al., 1980; Haywood, 1993).

Classroom procedure

- The teacher creates an atmosphere that fosters mutual respect and co-operation between children, and children and teacher so as to encourage sharing and learning from one another.
- By encouraging children to work co-operatively, negative competition is discouraged and the view that knowledge is and should be exclusive, is broken down.
- The teacher encourages pupils to help one another and to listen to one another.
- The teacher provides suitable challenging activities that must be done as a group.
- The teacher allows pupils to choose who must be part of the group activity. (Maths must be fun.)
- The teacher encourages social interaction between and among pupils as it forms an important part of the learning process, because it compels pupils to reflect on their work.

Underpinning the whole process of problem solving is the desire to encourage mathematical thinking, activity, experimentation and most importantly, discussion and communication so that children should be allowed to share their ideas with others.

Helping the child

Feuerstein has identified certain crucial ways in which the adults in a child's life foster cognitive development. If any of these links is missing then the child's cognitive ability will be impaired (Fisher, 1990:137). The aim of the teacher/mediator is thus to identify those missing links and then through mediation to improve the child's cognitive development. This is done through mediation of the following five aspects:

- Helping the child to see
- Helping the child to select
- Helping the child to plan
- Helping to develop self-control
- Helping to develop care and precision

Helping the child to see

A child's natural style of seeing is to scan the environment around her. This is also what happens when the child is confronted with a maths problem. The child needs help in learning to focus perception and attention. An absence of this mediation in helping children to focus attention on single objects can have profound effects. It can mean that the child will develop poor perceptual skills including "blurred" or "sweeping" perception. Perceptual skills are necessary for recognising shapes, patterns, and for comparing and differentiating objects. Children who lack effective mediation find it hard to attend to an object longer than it takes to register its existence. They lack the ability to discriminate and select. They find it hard to perform the necessary prerequisite of logical/critical thinking, putting objects into mental categories (Fisher, 1990).

Through mediation the teacher encourages the child to focus her attention on the word problem. Even when the child is confronted with a novel problem, she must select the necessary data. She must use all her senses to gather clear and complete information and she must restrain impulsive behaviour. The pupil is encouraged to write down what she "sees" in her mind. The child will find it helpful to draw a diagram that represents the terms of the problem. By representing the problem the child is able to translate each proposition from the problem into an internal representation. It might be in a form of a pictorial image especially for younger pupils in the Junior Primary phase. Here the child makes use of external representation (Sternberg, 1986).

Helping the child to select

Children are bombarded with visual and other sensory stimuli.They cannot concentrate on every stimulus so they learn to shut off and rely on random unthinking responses. They look but do not see. Their responses become haphazard and unfocused, unable to select. Skills in selection can be taught through mediation.The child is encouraged to think and concentrate for himself (Fisher, 1990:138).

During the mediation phase the child is encouraged to select only the information needed to solve the problem. The child must approach the data in an organised manner that will help her to select cues and their specific attributes that are relevant to solve problems.

Helping the child to plan

Research on children with learning difficulties shows that their most common characteristic is a lack of planning behaviour (Fisher, 1990:138). When such children are told to make a plan, their performance often improves. They rarely engage in planning spontaneously. Learning to plan begins as a mediated experience. When confronted with a maths problem the child is encouraged to work out a plan. After the child has made an external representation of the maths problem and selected the necessary data, she is encouraged to make a plan on how she will go about solving the problem. The pupil then defines what steps she will take to solve the problem. She chooses steps that she can manage.

Helping to develop self-control

Another prime reason for learning failure is impulsiveness. Children need to invest more time and energy in recognising and defining problems, to inhibit their first thoughts and impulses and to foster a more reflective and cautious approach. The slogan "Take time to think" is one that could prove useful in home and school (Feuerstein et al., 1980).

Without mediation of self-control the child's life becomes ruled by impulse, grabbing whatever is wanted - uncontrolled behaviour. The way self-control is mediated can help to enhance cognitive development. The teacher must make it clear to the child why she must do things a certain way, i.e. "Do not do that because if you do, X will happen" or "X and/or Y followed by Z." These sequences will engage the child in a chain of reasoned argument. It encourages the child to stop and think, and thus inhibit impulsive behaviour (Fisher, 1990).

Helping the child to develop care and precision

"Look, listen, think carefully," are the sort of verbal cues to help children take care and develop precision in their thinking and perceiving. One common problem with perception, is the difficulty many children have in using more than one source of information at any one time. They tend to rely on the dominant feature, looking for the one crucial clue, the one obviously right answer. Again impulsiveness can inhibit precise processing.

Questions asked to help pupils get a clearer conceptual grip on the elements of any given situation are:

- What else do you see?
- Is that all there is?
- What is the same? What is different?
  (What other features/aspects are there?)

We must remember that children only see what they want to see. They can be "field dependent". When in a forest children may see the trees, but do they see the wood? Can they relate the wood to other geographical features, to the nature of forests? Perception is a matter of seeing within and relating without. It involves looking into a field of perception with care and precision and connecting what is seen to a wider field or context (Fisher, 1990:140).

The adequacy of a given response depends not only on the cues of the representation stage, but in many instances on the individual's felt need to respond to a problem in a precise, accurate way. A child who is encouraged to perceive a maths problem with care and precision will not act impulsively when confronted with a problem. This child will define the problem carefully. When she sees the numbers 3 and 7 she will not assume it must be 3 + 7, 3 x 7, or 7 - 3, but with care and precision she will select the relevant cues that apply to the problem.

Mediated interactions differ demonstrably from non-mediated interactions. Firstly, of course, mediated interactions are characterised by the criteria discussed in the preceding paragraphs. Secondly, mediated interactions are strongly process-oriented, that is the dialogue between child and adult is focused on generalisable processes of thinking and learning rather than on answers to immediate problems in immediate situations. Thirdly, mediated interactions display the mediator's confidence in the children's ability to learn and apply appropriate thinking modes and strategies. In contrast to many kinds of non-mediated interactions, mediated ones constitute true dialogue, that is a "two-way street" in which information is sought by each participant from the other, and the unique role of each participant is recognised (Haywood, 1993:33).

Helping to develop a systematic approach to problem solving

During this stage, it will be determined what role mediation plays in the problem solving approach to Junior Primary mathematics. So what do I expect from the pupils in the experimental group, as they attempt to solve a problem? It will be desirable for them to develop a systematic approach to problem solving. It is quite common for some pupils to rush into a solution without proper consideration of the situation as a whole and although the more capable pupils can generally cope quite well without guidance, there are certain procedures which may well be followed by all pupils as they seek the solution to a mathematical problem (Mayer, 1983).

In a systematic approach to problem solving, I would expect the experimental group to apply the following useful steps to each of the word sums in the post-test:

1. Read the entire problem carefully to get a mental picture of the whole situation and to understand what is required.
2. Analyse the problem situation to determine the steps needed to solve it.
3. Set out the solution to the problem.
4. Check the reasonableness and the accuracy of the solution.
5. Lastly, seek help when stuck.

The first step is well defined and has immediate links with the language aspect and with the understanding of the problem in general. The other steps cannot be restricted to a definite procedure for all pupils. The teacher should pay particular attention to the individual differences of the pupils.

The control group

During the month of mediational teaching and during the post-test the control group was denied the full mediation described above. During this month they received normal mathematical lessons which consisted of the counting activities and doing mathematical exercises. They were also given "traditional" easy word sums to do on their own; traditional sums in the sense that no concerted cognitive effort and planning is required. After the experimental investigation was completed, they also received mediational teaching.

Post-test

The third stage of the investigation consisted of the post-test. This also involved four word problems, described below. The composition of the groups remained the same as in the pre-rest except that the C-group remained seated at the back of the classroom, whereas the E-group was allowed to work on the mat in front of the classroom. Again materials to help with problem solving were within reach of every pupil. Now the E-group was encouraged to make use of these materials, whereas the use of the material for C-group was neither encouraged nor discouraged. The C-group was again left on its own without any mediation, while mediation was available to the E-group. The problem was posed to the C-group and they were expected to solve the problem without any teacher interaction or intervention.

Mediation in the post-test was in the form of guidance to the E-group. The teacher acted as mediator, she asked leading questions. She made sure every one of the E-group understood the problem. She answered questions about problem comprehension, through questioning. She focused the pupils' attention on relevant information. She provided hints to help pupils make a start. She encouraged pupils who seemed confused and who felt like giving up. She also allowed the pupils to communicate with other pupils while attempting to solve the problems. Never during this stage did the teacher help pupils to attain the correct solution to the problem. This the pupils did entirely on their own.

Word problems in post-test

5. There are 42 blocks. Antonio has 5 blocks more than Eileen and 7 blocks less than Marco. How many blocks does each one have?
6. The roof of the dog's kennel has 16 tiles on the long side and 5 tiles on the short side. 9 of the tiles are cracked. How many whole tiles are there?
7. Ceril is older than Cole. Cole is younger than Zelda, but Zelda is older than Ceril. Who is the eldest and who is the youngest?

8. Mary has 28 sweets. She keeps half of the sweets for herself and she gives 4 to Sally. How many sweets are left over for Mark?

The questions of the pre-test and post-test were made more or less the same, they only differed in numbers, names, objects and situations.

Interviews

During the fourth stage two interviews were conducted with pupils from the E-group. My aim during the interviews was to gain understanding of their thought processes. I selected one boy and one girl. The interviewees were chosen deliberately because I wanted to choose pupils who had problems in solving the pre-test problem, but could successfully solve the post-test problem. Permission for the interviews was granted by the principal and staff. Each of the interviews was scheduled to last for at least 30 minutes. These conversations were recorded on audio-tape. The pupils were made aware of the fact that a recorder was being used even though it was hidden under a cloth. I told them beforehand why I was doing the interviews, what I would use and what I was about to do. The interviews were conducted on an individual basis at school in our classroom after the rest of the pupils had been dismissed. The reason for this was that I wanted the pupils to be relaxed and comfortable in their own environment. I wanted a free-flow of conversation that I felt could only be possible on the pupils' own territory.

The format of the interviews proceeded through different phases. It was "thinking aloud" because I asked the interviewees to verbalise their thoughts as they attempted to solve the problem. It was also an in-depth interview because I used probing. Because I asked leading questions to determine their thought processes, it could be classified as a tutorial interview.

Differences in problem-solving processes were determined by asking pupils to describe their thought processes while they solved a problem in the post-test. They were encouraged to say what was going on in their minds as they attempted to solve the problem. They were actually "thinking aloud". The aim here was to determine the thought processes of the interviewees while they solved the problems in the pre-test and the post-test, i.e. why they couldn't successfully solve the pre-test problem, but could solve the post-test problem successfully.

Limitations of research

The effect of the E-group receiving mediation and the C-group not receiving mediation was the subject of qualitative research. (Refer to Implication and Suggestions.)

The E-group received mediation during the mediational teaching sessions and also during the post-test (in the spirit of the LPAD). The fact that they received mediation during the post-test could be the reason why they scored a much higher mark than the C-group. The choice was made deliberately over denying mediation to both groups during the post-test.

The Junior Primary teacher should know when to mediate and when to encourage children who she knows do not need mediation, to work on their own. The teacher must, however, guard against the possibility that a pupil can develop a dependency. The teacher thus applies mediation where and when it is deemed necessary.

Conclusion

Evaluation of the programme described above does not seem to contradict Feuerstein's premise that with mediation we should be able to increase a child's power of attention, judgement, reasoning and memory, and thereby improve her performance in problem solving.

Children especially are in a permanent state of developmental growth and where that process will end, who can say? If the adult/teacher is there by the child's side as mediator, we can be sure it will enhance the child's cognitive function and thus make her an independent being who can reflect on her own thinking processes. Her ability to apply her thinking to problem solving will then be the key to success in life.

This concludes my experimental investigation. In the next chapter the results of the experimental investigation will be discussed.

Chapter 5

Results and discussions

Introduction

In this chapter the researcher gives and discusses the results of the pre-test, post-test and the two interviews. The pre-test was done to determine the pupils' general standard of performance while solving problems. The main aim of the post-test was to determine how exposure to mediation (including heuristics) can change pupils' problem solving abilities.

The experimental investigation was conducted with 20 pupils in Phase 2 (Substandard B). Four word problems were done in the pre-test and four slightly different word problems were done in the post-test. Each of the twenty pupils completed these word problems. The interviews were conducted with two of the pupils in the experimental group.

These eight word problems will be used as the basis for the format of my analysis. The results of each word problem will firstly be given and then discussed individually to find out if there are any differences in the performance in the pre-test and post-test. Thereafter the results of the whole group will be analysed and lastly, the two interviews will be analysed.

Questions 1 to 4 will form the pre-test and Questions 5 to 8 the post-test.

Marks were allocated so that every pupil had a fair chance to score marks. The marks were allocated out of 20. First the pupil could score 8 or 4 or 0 marks.

- **8 marks** if the pupil only wrote down a correct answer. If the pupil could explain how the problem was solved, she could score more marks.
- **4 marks** for a pupil who diagramatically solved the problem, but took a wrong route that resulted in a wrong answer.

Then additional marks were allocated as indicated below.

- **2 marks** for a pupil who broke the problem into parts that she could manage, to make it easier to solve the problem.
- **2 marks** for a pupil who drew a picture to represent the word sum. Pupils who drew a picture to represent the problem tended to arrive at a possible solution.
- **2 marks** for a pupil who used written symbols. Pupils used written symbols as cues to help them remember the process of the route they took to solve the problem.
- **3 marks** for the pupil who checked for correctness. Some pupils did it mentally. Those who wrote it down, were more successful in rectifying a wrong route.

- **3 marks** for a pupil who put down an indication on how she solved the problem. This is an important aspect of problem solving. It lets pupils reflect on their thought processes.

Analysis of the 4 word problems in the pre-test

Results

Table 1 reflects the analysis of question 1.

<table>
<tr><th>RAW SCORES OUT OF 20</th><th>NUMBER OF PUPILS</th><th>PERCENTAGE OF PUPILS</th><th>NUMBER OF PUPILS WHO SCORED BETWEEN</th><th>PERCENTAGE</th></tr>
<tr><td>0</td><td>3</td><td>15.0</td><td rowspan="6">0-5
15</td><td rowspan="6">± 75.0</td></tr>
<tr><td>1</td><td>1</td><td>5.0</td></tr>
<tr><td>2</td><td>6</td><td>30.0</td></tr>
<tr><td>3</td><td>0</td><td>0.0</td></tr>
<tr><td>4</td><td>3</td><td>15.0</td></tr>
<tr><td>5</td><td>2</td><td>10.0</td></tr>
<tr><td>6</td><td>2</td><td>10.0</td><td rowspan="5">6 - 10
5</td><td rowspan="5">± 25.0</td></tr>
<tr><td>7</td><td>3</td><td>15.0</td></tr>
<tr><td>8</td><td>0</td><td>0.0</td></tr>
<tr><td>9</td><td>0</td><td>0.0</td></tr>
<tr><td>10</td><td>0</td><td>0.0</td></tr>
<tr><td>11</td><td>0</td><td>0.0</td><td rowspan="5">11 - 15
0</td><td rowspan="5">± 0.0</td></tr>
<tr><td>12</td><td>0</td><td>0.0</td></tr>
<tr><td>13</td><td>0</td><td>0.0</td></tr>
<tr><td>14</td><td>0</td><td>0.0</td></tr>
<tr><td>15</td><td>0</td><td>0.0</td></tr>
<tr><td>16</td><td>0</td><td>0.0</td><td rowspan="5">16 - 20
0</td><td rowspan="5">± 0.0</td></tr>
<tr><td>17</td><td>0</td><td>0.0</td></tr>
<tr><td>18</td><td>0</td><td>0.0</td></tr>
<tr><td>19</td><td>0</td><td>0.0</td></tr>
<tr><td>20</td><td>0</td><td>0.0</td></tr>
</table>

Table 1 : Summary of the results of problem 1 in the pre-test

The result of the problem in the pre-test indicates that 15, or ± 75%, of the 20 pupils scored between 0 and 5 out of the possible 20 marks, while 5, or ± 25%, scored between 6 and 7 out of 20 marks. None of the pupils scored between 8 and 20 marks.

Discussion

When we look at the analyis of the results of problem 1 we find that even though the 20 pupils consisted of mixed abilities, none of the pupils could score more than 50%. The obvious reason for this low score is that these pupils did not know how to solve a word problem. They did not know what strategies to use to solve a problem. It has been my experience that pupils in the Junior Primary phase lack the necessary skills needed to solve a word problem on their own. From this we can conclude that these pupils must be given the necessary strategies that they can use as tools which would make them more successful in the problem solving approach to maths.

Results

Table 2 reflects the analysis of question 2.

RAW SCORES OUT OF 20	NUMBER OF PUPILS	PERCENTAGE OF PUPILS	NUMBER OF PUPILS WHO SCORED BETWEEN	PERCENTAGE
0	3	15.0	0 - 5	
1	2	10.0		
2	13	65.0	19	± 95.0
3	1	5.0		
4	0	0.0		
5	0	0.0		
6	0	0.0	6 - 10	
7	0	0.0		
8	1	5.0		
9	0	0.0	1	± 5.0
10	0	0.0		
11	0	0.0	11 - 15	
12	0	0.0		
13	0	0.0		
14	0	0.0	0	± 0.0
15	0	0.0		
16	0	0.0	16 - 20	
17	0	0.0		
18	0	0.0		
19	0	0.0	0	± 0.0
20	0	0.0		

Table 2 : Summary of problem 2 in the pre-test

Analysis of problem 2 in the pre-test yielded the following results: 19, or ± 95%, of the 20 pupils scored less than 5 out of the possible 20 marks and only 1, or 5%, of the pupils scored 8 out of the possible 20 marks. Again, none of the pupils could score between 9 and 20 marks.

Discussion

What is very noticeable from the results of problem 2 in the pre-test is that the scores were very low, i.e. ± 95% scored less than 4 out of 20 marks. When one looks at the problem posed and considers that we are dealing with Grade 2 (Sub. B) pupils in the third term, it would not be an unrealistic expectation that at least ± 50% of the pupils should be able to solve it. As the analysis showed, this was not to be because pupils tended to grab at every number they came across and would either plus or minus it. They do not try to establish what is being asked and what is wanted. They must be encouraged to inhibit their impulsive responses and to collect the necessary data for decision making before reacting.

Results

Table 3 reflects the analysis of question 3.

RAW SCORES OUT OF 20	NUMBER OF PUPILS	PERCENTAGE OF PUPILS	NUMBER OF PUPILS WHO SCORED BETWEEN	PERCENTAGE
0	8	40.0	0 - 5 15	± 75.0%
1	0	0.0		
2	2	10.0		
3	3	0.0		
4	2	10.0		
5	0	15.0		
6	2	10.0	6 - 10 5	± 25.0%
7	0	0.0		
8	0	0.0		
9	0	0.0		
10	3	15.0		
11	0	0.0	11 - 15 0	± 0.0
12	0	0.0		
13	0	0.0		
14	0	0.0		
15	0	0.0		
16	0	0.0	16 - 20 0	± 0.0
17	0	0.0		
18	0	0.0		
19	0	0.0		
20	0	0.0		

Table 3 : Summary of problem 3 in the pre-test.

If we look at Table 3, it can be seen that all of the 20 pupils obtained less than 11 out of 20 marks. Of the 20 pupils 5, or 25%, scored between 6 and 10 marks and 15, or 75%, scored between 0 and 5 marks and 8 of the 20 pupils scored 0.

Discussion

The pupils found word problem 3 very confusing; here no numbers were given and they could not impulsively + or -. In order to clarify the problem, the pupils had to invent a visual manner of representing the heights of the three children. Only one pupil made a drawing of the different heights of the three children. It thus becomes evident that children in the Junior Primary phase lack these necessary skills or lack the knowledge to represent the data diagramatically to become successful problem solvers in Mathematics.

Results

Table 4 reflects the analysis of question 4.

RAW SCORES OUT OF 20	NUMBER OF PUPILS	PERCENTAGE OF PUPILS	NUMBER OF PUPILS WHO SCORED BETWEEN	PERCENTAGE
0 1 2 3 4 5	4 0 9 0 1 1	20.0 0.0 45.0 0.0 5.0 5.0	0 - 5 15	± 75.0
6 7 8 9 10	0 1 2 0 0	0.0 5.0 10.0 0.0 0.0	6 - 10 3	± 15.0
11 12 13 14 15	1 1 0 0 0	5.0 5.0 0.0 0.0 0.0	11 - 15 2	± 10.0
16 17 18 19 20	0 0 0 0 0	0.0 0.0 0.0 0.0 0.0	16 - 20 0	± 0.0

Table 4 : Summary of problem 4 in the pre-test.

The results of problem 4 in the pre-test are slightly better. Two of the 20 pupils scored more than ± 50%. They scored 11 and 12 out of 20 marks. 15, or ± 75%, of the pupils scored between 0 and 5 out of 20 and 3, or ± 15%, of the pupils scored between 7 and 8 out of 20 marks.

Discussion

The mistakes that were made re-emphasised that mediation is necessary because the answers these pupils obtained showed that they lack the necessary skills to solve problems. Some only made small, careless errors, while others lacked the skills and strategies, they did not even know where to begin. Some just didn't know what to do, they seemed to be confused by the number of people mentioned in the word problem.

Conclusion of the pre-test analysis

What is very noticeble from Tables 1, 2, 3 and 4 is that these pupils cannot be left on their own to solve a word problem. No problem can be solved without first understanding thoroughly what the problem is about. An interesting point can be raised as to the role of reading. The statement of a problem often includes words and descriptions of events or situations that, in their verbal form, are difficult to interrelate. In these cases mediation is necessary to help pupils.

Analysis of the 4 word problems in the post-test

Results

Table 5 reflects the analysis of question 5.

RAW SCORES OUT OF 20	NUMBER OF PUPILS	PERCENTAGE OF PUPILS	NUMBER OF PUPILS WHO SCORED BETWEEN	PERCENTAGE
0	0	0.0	0 - 5	
1	0	0.0		
2	7	35.0	7	± 35.0
3	0	0.0		
4	0	0.0		
5	0	0.0		
6	1	5.0	6 - 10	
7	1	5.0		
8	1	5.0	4	± 20.0
9	1	5.0		
10	0	0.0		
11	0	0.0	11 - 15	
12	1	5.0		
13	4	20.0	7	± 35.0
14	2	10.0		
15	0	0.0		
16	2	10.0	16 - 20	
17	0	0.0		
18	0	0.0	2	± 10.0
19	0	0.0		
20	0	0.0		

Table 5 : Summary of problem 5 in the post-test

Looking at table 5, it is evident that there is a remarkabe improvement. 9, or 45%, of the 20 pupils scored between 12 and 16 out of 20 marks. 7,or 35%, scored 2 out of 20 marks.

Discussion

When these results are compared with table 1 that refers to Question 1 in the pre-test, we note that there is a remarkable improvement. 9

pupils scored between 12 and 16 marks, which means that 45% of the pupils scored more than 50%. By comparison, it seems that pupils in the Junior Primary phase fare much better when exposed to mediation. Some of the pupils worked out a plan of action, by breaking the goal into manageable parts they solved this problem, which at first seemed impossible. These pupils' feelings about their own competence are significantly enhanced.

Results

Table 6 reflects the analysis of question 6

RAW SCORES OUT OF 20	NUMBER OF PUPILS	PERCENTAGE OF PUPILS	NUMBER OF PUPILS WHO SCORED BETWEEN	PERCENTAGE
0	1	5.0	0 - 5 7	± 35.0
1	4	20.0		
2	1	5.0		
3	1	5.0		
4	0	0.0		
5	0	0.0		
6	0	0.0	6 - 10 3	± 15.0
7	1	5.0		
8	2	10.0		
9	0	0.0		
10	0	0.0		
11	0	0.0	11 - 15 5	± 25.0
12	3	15.0		
13	0	0.0		
14	2	10.0		
15	0	0.0		
16	0	0.0	16 - 20 5	± 25.0
17	1	5.0		
18	3	15.0		
19	0	0.0		
20	1	5.0		

Table 6 : Summary of problem 6 on the post-test

It is interesting to note that exactly 10, or 50%, of the pupils scored more than ± 50% and 1, or 5%, of them scored full marks. 7, or 35%, of the pupils scored between 0 and 3 out of 20 marks and 3, or 15%, scored between 7 and 8 out of 20 marks.

Discussion

We note that 50% of the pupils scored more than 50% in the post-test whereas all of these pupils scored less than 50% in the pre-test. It was evident here that pupils who scored high marks expressed self-control. They didn't impulsively plus or minus the three numbers in the problem sum. They tackled the maths problem with care and precision. Some of the pupils asked the meaning of the word "tiles" before they tried to work out a plan. Some pupils also gave a detailed outline of how they solved the problem. The obvious reason for this improvement in results is that the pupils who were exposed to mediation had now acquired the necessary skills and strategies (e.g. from Project Odyssey representation, breaking problem into steps) to solve a word problem.

Results

Table 7 reflects the analysis of question 7.

RAW SCORES OUT OF 20	NUMBER OF PUPILS	PERCENTAGE OF PUPILS	NUMBER OF PUPILS WHO SCORED BETWEEN	PERCENTAGE
0	0	0.0	0 - 5: 6	± 30.0
1	0	0.0		
2	3	15.0		
3	0	0.0		
4	3	15.0		
5	0	0.0		
6	1	5.0	6 - 10: 5	± 25.0
7	1	5.0		
8	2	10.0		
9	0	0.0		
10	1	5.0		
11	0	0.0	11 - 15: 9	± 45.0
12	9	45.0		
13	0	0.0		
14	0	0.0		
15	0	0.0		
16	0	0.0	16 - 20: 0	± 0.0
17	0	0.0		
18	0	0.0		
19	0	0.0		
20	0	0.0		

Table 7 : Summary of problem 7 in the post-test

Analysis of question 7 yields the following results: 9, or 45%, of the 20 pupils scored 12 out of 20 marks. 11, or 55%, of the pupils scored less than 50% and 6 of the 11 pupils scored between 2 and 4 marks out of 20 and 5, or 25%, scored between 6 and 10 marks out of 20.

Discussion

These results show that there is a remarkable improvement in the results of question 7 of the post-test. The reason is that some of the pupils used the strategies they acquired in the mediation sessions and could thus acquire higher marks when attempting to solve word problems. Pupils in the E-group used the strategy of Representation. This strategy helped the pupils to actually "see" who is the eldest because no numbers were given in this word sum. They also applied the Postpone strategy by giving the first and the second pupil in the word sum an age and then later working out the age of the third child.

Results

Table 8 reflects the analysis of question 8.

RAW SCORES OUT OF 20	NUMBER OF PUPILS	PERCENTAGE OF PUPILS	NUMBER OF PUPILS WHO SCORED BETWEEN	PERCENTAGE
0	1	5.0	0 - 5 5	± 25.0
1	0	0.0		
2	3	15.0		
3	1	5.0		
4	0	0.0		
5	0	0.0		
6	2	10.0	5 - 10 5	± 25.0
7	2	10.0		
8	0	0.0		
9	1	5.0		
10	0	0.0		
11	0	0.0	11 - 15 4	± 20.0
12	2	10.0		
13	0	0.0		
14	1	5.0		
15	1	5.0		
16	0	0.0	16 - 20 6	± 30.0
17	0	0.0		
18	6	30.0		
19	0	0.0		
20	0	0.0		

Table 8 : Summary of problem 8 in the post-test.

Analysis of the results of question 8 shows that exactly 10, or 50%, of the 20 pupils scored higher than 50%. 5, or 25%, of the pupils scored between 0 and 3 out of 20 marks and 5, or 25%, scored between 6 and 9 out of 20 marks.

Discussion

As in questions 5, 6, and 7, there is also a remarkable improvement in this question of the post-test. Most of the pupils in the experimental group used the strategy of breaking the problem sum into manageable parts. They also used the Postpone strategy, leaving part of the sum aside and dealing with it at a later stage. Looking back at how the problem was solved, made them aware that they had actually successfully solved the problem. This enhanced a feeling of competence.

Comparison of the results of the pre-test of the experimental and control groups

The main purpose is to compare the results of the pre-test of the experimental and control groups. This can assist towards the identification of problem areas in the new approach to teaching mathematics, i.e. problem centred approach to mathematics. It would also be possible to determine whether all pupils are able to solve a problem without any mediation.

In comparing the results of the pre-test, the distinction is made between scores achieved by the experimental group and the control group. (During the pre-test, pupils were not yet labelled as experimental and control groups. This labelling only occurred when pupils were divided into groups to receive mediation.)

Table 9 reflects the analysis of question 1.

RAW SCORES OUT OF 20	EXPERIMENTAL GROUP				CONTROL GROUP			
	NUMBER OF PUPILS		PERCENTAGE OF PUPILS		NUMBER OF PUPILS		PERCENTAGE OF PUPILS	
0	2		20.0		1		10.0	
1	0		0.0		1		10.0	
2	0	6	0.0	±60%	6	9	60.0	±90%
3	0		0.0		0		0.0	
4	3		30.0		0		0.0	
5	1		10.0		1		10.0	
6	2		20.0		0		0.0	
7	2		20.0		1		10.0	
8	0	4	0.0	±40%	0	1	0.0	+10%
9	0		0.0		0		0.0	
10	0		0.0		0		0.0	
11	0		0.0		0		0.0	
12	0		0.0		0		0.0	
13	0	0	0.0	0%	0	0	0.0	0%
14	0		0.0		0		0.0	
15	0		0.0		0		0.0	
16	0		0.0		0		0.0	
17	0		0.0		0		0.0	
18	0	0	0.0	0%	0	0	0.0	0%.
19	0		0.0		0		0.0	
20	0		0.0		0		0.0	

X = 4.3 X = 2.5

Table 9 : Summary of problem 1 in the pre-test.

20, or 100%, of a sample of 20 pupils scored less than 8 out of 20 marks and 3, or 15%, scored 0 out of 20 marks. Although both groups achieved below expectation, the experimental group performed slightly better than the control group in this question. 4, or 40%, of the experimental group scored between 6 and 7 marks whereas only 1, or 10%, of the control group could score 7 marks.

Discussion

The main purpose of this question was to determine whether pupils made use of any strategies to solve word problems without teacher direction or interaction. The results indicate that pupils experienced serious difficulties in solving a word problem. Most of the pupils just took the two numbers mentioned in the word problem and added them together to get an answer. It is also evident from the results that the pupils had difficulty with the word problem because they did not understand it. According to Sternberg (1986:44), the pupil must first define the nature of the problem. If the pupil is unable to do this she will find it impossible to solve the problem (which includes the case where a solution is found for a problem other than the one that was posed).

Results

Table 10 reflects the analysis of question 2.

RAW SCORES OUT OF 20	EXPERIMENTAL GROUP				CONTROL GROUP			
	NUMBER OF PUPILS		PERCENTAGE OF PUPILS		NUMBER OF PUPILS		PERCENTAGE OF PUPILS	
0	2		20.0		1		10.0	
1	1		10.0		1		10.0	
2	5	9	50.0	±90.0	8	10	80.0	± 100.0
3	1		10.0		0		0.0	
4	0		0.0		0		0.0	
5	0		0.0		0		0.0	
6	0		0.0		0		0.0	
7	0		0.0		0		0.0	
8	1	1	10.0	±10.0	0	0	0.0	0.0
9	0		0.0		0		0.0	
10	0		0.0		0		0.0	
11	0		0.0		0		0.0	
12	0		0.0		0		0.0	
13	0	0	0.0	0.0	0	0	0.0	0.0
14	0		0.0		0		0.0	
15	0		0.0		0		0.0	
16	0		0.0		0		0.0	
17	0		0.0		0		0.0	
18	0	0	0.0	0.0	0	0	0.0	0.0
19	0		0.0		0		0.0	
20	0		0.0		0		0.0	

X = 2.2 X = 1.7

Table 10 : Summary of problem 2 in the pre-test.

20, or 100%, of the 20 pupils scored between 0 and 8 out of 20 marks. 19, or 95%, of the pupils scored less than 4 marks and only 1, or 5%, scored 8 out of 20 marks and 3, or 15%, of the pupils scored 0 out of 20 marks.

Discussion

From the average percentages of the two groups it is clear that the majority of pupils had problems in solving this problem. It can thus be concluded that the pupils from both groups performed poorly in solving this problem. What became evident is that some of the pupils understood the problem, because it was a situation which they had already experienced. It was an everyday problem, but most of them had difficulty in getting started.

The main purpose of question 2 was to determine the pupils' skills in dealing with a real-life situation problem. The results of the two groups, as reflected in table 10, indicate that most pupils experienced serious difficulties in solving a word problem without mediation.

Results

Table 11 reflects the analysis of question 3.

RAW SCORES OUT OF 20	EXPERIMENTAL GROUP				CONTROL GROUP			
	NUMBER OF PUPILS		PERCENTAGE OF PUPILS		NUMBER OF PUPILS		NUMBER OF PUPILS	
0	4		40.0		6		60.0	
1	0		0.0		2		20.0	
2	0	9	0.0	± 90%	0	9	0.0	± 90%
3	3		0.0		0		0.0	
4	2		20.0		1		10.0	
5	0		30.0		0		0.0	
6	1		10.0		1		10.0	
7	0		0.0		0		0.0	
8	0	1	0.0	± 10%	0	1	0.0	± 10%
9	0		0.0		0		0.0	
10	0		0.0		0		0.0	
11	0		0.0		0		0.0	
12	0		0.0		0		0.0	
13	0	0	0.0	0%	0	0	0.0	0%
14	0		0.0		0		0.0	
15	0		0.0		0		0.0	
16	0		0.0		0		0.0	
17	0		0.0		0		0.0	
18	0	0	0.0	0%	0	0	0.0	0%
19	0		0.0		0		0.0	
20	0		0.0		0		0.0	

X = 1.3 X = 1.2

Table 11 : Summary of problem 3 in the pre-test.

20, or 100%, of the pupils scored less than 50% and 20, or 100%, scored less than 7 out of 20 marks. 10 of the 20 pupils scored 0 out of 20 marks. Although both groups performed poorly in this question, the experimental group scored only slightly better than the pupils in the control group.

Discussion

The purpose of question 3 was to determine the pupils' ability to solve a word problem where there are no numbers involved. It is clear from table 11 that all the pupils from both groups encountered difficulties in solving a word problem where no numbers are to be used. These pupils need strategies in solving word problems.

Results

Table 12 reflects the analysis of question 4.

RAW SCORES OUT OF 20	EXPERIMENTAL GROUP				CONTROL GROUP			
	NUMBER OF PUPILS		PERCENTAGE OF PUPILS		NUMBER OF PUPILS		PERCENTAGE OF PUPILS	
0	2		20.0		2		20.0	
1	0		0.0		0		0.0	
2	2	8	20.0	±80%	7	9	70.0	±90%
3	0		0.0		0		0.0	
4	2		20.0		0		10.0	
5	2		20.0		0		0.0	
6	0		0.0		0		0.0	
7	0		0.0		0		0.0	
8	2	2	10.0	±20%	1	1	10.0	±10%
9	0		0.0		0		0.0	
10	0		0.0		0		0.0	
11	0		0.0		0		0.0	
12	0		0.0		0		0.0	
13	0	0	0.0	0%	0	0	0.0	0%
14	0		0.0		0		0.0	
15	0		0.0		0		0.0	
16	0		0.0		0		0.0	
17	0		0.0		0		0.0	
18	0	0	0.0	0%	0	0	0.0	0%
19	0		0.0		0		0.0	
20	0		0.0		0		0.0	

X = 3.8 **X = 2.2**

Table 12 : Summary of problem 4 in the pre-test.

20, or 100%, of the pupils scored less than 50%; they scored between 0 and 8 out of 20 marks. 3, or 30%, of the pupils scored the highest marks which were only 8 out of 20 marks. If we compare question 4 in the pre-test with questions 1, 2 and 3, the pupils performed slightly better in this question. But the overall performance in this question was weak.

Discussion

The main purpose here was to determine whether pupils worked out a plan before they solved the problem. From these results it is clear that most of the pupils rushed into a solution without proper consideration of the situation as a whole. Polya (1962) views the first step of solving a problem, i.e. understanding the problem, as most important. Without understanding, it is impossible to attain a correct solution.

What becomes evident from the results of the pre-test questions is that pupils, especially weak ones, need direction and interaction with the teacher to succeed in the problem solving approach to maths. The more capable pupils were able to solve the problems with little teacher direction and interaction.

Results show that the experimental group scored slightly higher marks in all of the pre-test questions. During the pre-test, the groups were not yet labelled as separate groups. After the pre-test and before the marking of the pre-test, the experimental group was chosen. The group was chosen so that both groups were more or less equal in terms of age, gender and intellectual ability.

Comparison of the results of the post-test of the experimental and control groups

Results

Table 13 reflects the analysis of question 5 in the post-test.

RAW SCORES OUT OF 20	EXPERIMENTAL GROUP				CONTROL GROUP			
	NUMBER OF PUPILS		PERCENTAGE OF PUPILS		NUMBER OF PUPILS		PERCENTAGE OF PUPILS	
0	0		0.0		0		0.0	
1	0		0.0		0		0.0	
2	0	0	0.0	0%	7	7	70.0	±70%
3	0		0.0		0		0.0	
4	0		0.0		0		0.0	
5	0		0.0		0		0.0	
6	0		0.0		1		10.0	
7	0		0.0		1		10.0	
8	1	1	10.0	±10%	0	3	0.0	±30%
9	0		0.0		1		10.0	
10	0		0.0		0		0.0	
11	0		0.0		0		0.0	
12	1		10.0		0		0.0	
13	4	7	40.0	±70%	0	0	0.0	±0%
14	2		20.0		0		0.0	
15	0		0.0		0		0.0	
16	2		20.0		0		0.0	
17	0		0.0		0		0.0	
18	0	2	0.0	±20%	0	0	0.0	0%
19	0		0.0		0		0.0	
20	0		0.0		0		0.0	

X = 12.4 X = 3.6

Table 13 : Summary of results of problem 5 in the post-test.

If we look at the statistics for the control group, we find that 70% scored 2 out of 20 marks and 3 scored between 6 and 9 out of 20 marks. Thus 10, or 100%, of the control group scored less than 50%. On the other hand, 9, or 90%, of the experimental group scored between 12 and 16 out of 20 marks and only 1, or 10%, of this group scored below 50%.

Table 14:

Means of raw scores in pre-test and post-test

QUESTION 5				
		$\bar{X}$	$\bar{X}$	DIFFERENCE
GROUP	N	PRE-TEST	POST-TEST	PRE-POST
Experimental	10	4.3	12.4	8.1
Control	10	2.5	3.6	1.1

Discussion

We can thus see that there is a remarkable improvement in the scores for the experimental group who received mediation. Language played an important role in this question. Pupils had to read with understanding to be able to solve this problem. This word sum will be very confusing if the pupils did not understand what was asked. One pupil of the experimental group who had difficulty in reading with understanding only scored a mark of 8 out of 20 marks whereas those who had a good reading ability scored higher marks.

Results

Table 15 reflects the analysis of question 6 in the post-test.

RAW SCORES OUT OF 20	EXPERIMENTAL GROUP				CONTROL GROUP			
	NUMBER OF PUPILS		PERCENTAGE OF PUPILS		NUMBER OF PUPILS		PERCENTAGE OF PUPILS	
0	0		0.0		1		10.0	
1	0		0.0		4		40.0	
2	0	0	0.0	0%	1	7	10.0	±70%
3	0		0.0		1		10.0	
4	0		0.0		0		0.0	
5	0		0.0		0		0.0	
6	0		0.0		0		0.0	
7	0		0.0		1		10.0	
8	1	1	10.0	±10%	1	2	10.0	±20%
9	0		0.0		0		0.0	
10	0		0.0		0		0.0	
11	0		0.0		0		0.0	
12	3		30.0		0		0.0	
13	0	4	0.0	±40%	0	1	0.0	±10%
14	1		10.0		1		10.0	
15	0		0.0		0		0.0	
16	0		0.0		0		0.0	
17	1		10.0		0		0.0	
18	3	5	30.0	±50%	0	0	0.0	0%
19	0		0.0		0		0.0	
20	1		10.0		0		0.0	

X = 14.9 X = 3.8

Table 15: Summary of the results of problem 6 in the post-test.

The statistic for the control group shows that 9, or 90%, of the pupils scored less than 50%. Only 1, or 10%, of this group scored more than 50%. Here there is a slight improvement in their score; in the discussion I will mention why. The experimental group again fared quite well: 9, or 90%, scored above 50% and one of these pupils scored 100%. Only 1, or 10%, of this group scored less than 50%. I must mention here that this is not the same pupil who scored less than 50% in question 5.

Table 16

Means of raw scores in pre-test and post-test

QUESTION 6				
		X	X	DIFFERENCE
GROUP	N	PRE-TEST	POST-TEST	PRE-POST
Experimental	10	2.2	14.9	12.7
Control	10	1.7	3.8	2.1

Discussion

In this question language also plays an important role. Some of the pupils did not understand the meaning of the word "tiles". This word was not part of their vocabulary. But pupils of the experimental group who sought help were referred to the tiles on the floor and on the roofs of their houses. This interaction made the situation so clear that most of them drew a picture of a roof with the tiles neatly packed in rows. Without mediation, those pupils would never have been able to attain such high marks for this question. We must remember that most of our pupils come to school with informal mathematical knowledge and vocabulary (Charles and Lester, 1984). However, many children are not as fortumate and should therefore be provided with opportunities to experience and develop this knowledge and vocabulary in a variety of contexts, and here mediation plays a vital role.

In the results, I mentioned that 1, or 10%, of the control group scored more than 50%, i.e. 14 out of 20 marks. This pupil was definitely influenced by the experimental group. The pupils in my class tend to play "school-school" in the afternoon. Here they tend to repeat the work we have done on the mat during the day and pupils from the experimental group must have carried over some of the strategies they acquired. This is actually a good thing as it shows how effective peer teaching is.

Results

Table 17 reflects the analysis of question 7 in the post-test.

RAW SCORES OUT OF 20	EXPERIMENTAL GROUP				CONTROL GROUP			
	NUMBER OF PUPILS		PERCENTAGE OF PUPILS		NUMBER OF PUPILS		PERCENTAGE OF PUPILS	
0	0		0.0		0		0.0	
1	0		0.0		0		0.0	
2	0	0	0.0	0%	3	6	30.0	±60%
3	0		0.0		0		0.0	
4	0		0.0		3		30.0	
5	0		0.0		0		0.0	
6	0		0.0		1		10.0	
7	0		0.0		1		10.0	
8	1	2	10.0	±20%	1	3	10.0	±30%
9	0		0.0		0		0.0	
10	1		10.0		0		0.0	
11	0		0.0		0		0.0	
12	8		80.0		1		10.0	
13	0	8	0.0	±80%	0	1	0.0	±10%
14	0		0.0		0		0.0	
15	0		0.0		0		0.0	
16	0		0.0		0		0.0	
17	0		0.0		0		0.0	
18	0	0	0.0	0%	0	0	0.0	0%
19	0		0.0		0		0.0	
20	0		0.0		0		0.0	

X = 11.4 **X = 5.1**

Table 17 : Summary of the results of problem 7 in the post-test.

When we look at the analysis of the results of question 7 in the post-test we find that the experimental group did not fare as well as expected. 10, or 100%, scored only between 8 and 12 marks out of 20. The control group's score was also very low: 9, or 90%, of the pupils scored between 2 and 8 out of 20 marks and 1, or 10%, scored 12 out of 20 marks.

Table 18

Means of raw scores in pre-test and post-test

QUESTION 7				
		X	X	DIFFERENCE
GROUP	N	PRE-TEST	POST-TEST	PRE-POST
Experimental	10	1.3	11.4	10.1
Control	10	1.2	5.1	3.9

Discussion

The reason the experimental group did not fare so well was that the problem posed did not involve numbers. They found this problem very confusing. Some pupils in the (E) group who made a drawing to represent the situation, successfully solved this problem. What I found most amazing was that one pupil actually gave the first child in the problem sum an age and from there on she worked out who is the eldest and who is the youngest.

Here again, one of the (C) group attained a high score. I questioned her about how she had solved the problem and it became clear that this pupil had also been influenced by some of the pupils in the (E) group.

Results

Table 19 reflects the analysis of question 8 in the post-test.

RAW SCORES OUT OF 20	EXPERIMENTAL GROUP				CONTROL GROUP			
	NUMBER OF PUPILS		PERCENTAGE OF PUPILS		NUMBER OF PUPILS		PERCENTAGE OF PUPILS	
0	0		0.0		1		10.0	
1	0		0.0		0		0.0	
2	0	0	0.0	0%	3	5	30.0	±50%
3	0		0.0		1		10.0	
4	0		0.0		0		0.0	
5	0		0.0		0		0.0	
6	0		0.0		2		20.0	
7	0		0.0		2		20.0	
8	0	0	0.0	0%	0	5	0.0	±50%
9	0		0.0		1		10.0	
10	0		0.0		0		0.0	
11	0		0.0		0		0.0	
12	2		20.0		0		10.0	
13	0	4	0.0	±40%	0	0	0.0	0%
14	1		10.0		0		0.0	
15	1		10.0		0		0.0	
16	0		0.0		0		0.0	
17	0		0.0		0		0.0	
18	6	6	60.0	±60%	0	0	0.0	0%
19	0		0.0		0		0.0	
20	0		0.0		0		0.0	

X = 16.1 X = 4.4

Table 19 : Summary of the results of problem 8 in the post-test.

Analysis of question 8 yields the following results: 10, or 100%, of the control group scored below 50% whereas 10, or 100%, of the experimental group scored above 50%. This means that none of the (C) group scored more than 9 out of 20 marks and none of the (E) group scored less than 12 out of 20 marks. The highest score for the (C) group was 9 out of 20 marks and the lowest 0 out of 20 marks whereas the highest score for the (E) group was 18 out of 20 marks.

Table 20

Means of raw scores in pre-test and post-test

QUESTION 7				
		X	X	DIFFERENCE
GROUP	N	PRE-TEST	POST-TEST	PRE-POST
Experimental	10	1.3	11.4	10.1
Control	10	1.2	5.1	3.9

Discussion

What is very noticeable from the results of question 8 is that when we compare these results of the (E) group with the results of the (E) Group in the pre-test (table 12), we find a remarkble improvement in the pupils' problem solving abilities. All of the (E) group could successfully solve this problem.

We see that these pupils made use of strategies to solve the problem. Most of them made a drawing and broke the sum into manageable parts. It was, of course, first necessary to work out a plan on how to break the problem sum into parts. This, most of the (E) group did successfully. Most of them also applied the looking back strategy. Those who did not were unable to realise that they had not completed every component of the problem sum. What is also evident is that pupils in the (E) group's planning behaviour had improved tremendously. One pupil actually dramatised the situation posed by the problem. This is an exceptionally useful technique for the Junior Primary pupils.

Table 21

The problem areas where mediation was most successful for the experimental group.

Question		Types of Problem sums	PRE-TEST		POST-TEST		DIFFERENCE		ORDER OF SUCCESS	
Pre	Post		E-group	C-group	E-group	C-group	E-group	C-group	E-group	C-group
1	5	Two or more numbers	X=4.3	X=2.5	X=12.4	X=3.6	X= 8.1	X=1.1	4	4
2	6	Real life situation	X=2.2	X=1.7	X=14.9	X=3.8	X=12.7	X=2.1	1	3
3	7	No numbers	X=1.3	X=1.2	X=11.4	X=5.1	X=10.1	X=3.9	3	1
4	8	Work out a plan	X=3.8	X=2.2	X=16.1	X=4.4	X=12.3	X=2.2	2	2

Analysis of the post-test results for the experimental group yields the following results. When a hierarchy is formed to establish in what areas mediation was most successful, it will be in the following order:

- ☐ Mediation was most successful in problems situated in **real life situations**;
- ☐ Followed by problem sums which compelled pupils to **work out a plan**;
- ☐ And then by problem sums which contained **no numbers**;
- ☐ Finally, mediation was least successful in problem sums which contained **two or more numbers** where pupils tended to act impulsively and just added or subtracted the numbers.

The results of the order of success in the post-test are very interesting. The control group was most successful in question 7.

This could be attributed to peer group teaching occurring after school. It is in this question where one of the control group was influenced by members of the experimental group and attained a score of 12 out of 20 marks. (Refer to Appendix 4.)

The control group fared very poorly in the post-test. When we look at the results of tables 13, 14, 15 and 16, it becomes clear that through mediation pupils fared much better in solving problems in mathematics.

Analysis and discussion of the results of interviews

The data for the interviews as well as the two pupils' raw data are enclosed as appendices. The interviews are reported as utterances between the interviewer (I) and the respondent (R). The reponse is to the interviewer and the lines (L) are numbered according to the interviewer's utterances.

The analysis of the interviews with two pupils illustrates beyond doubt that mediation plays a crucial role in the problem solving approach to mathematics. Before the two pupils were exposed to intensive mediation, they were unable to solve the word problems (R1:L1; R2:L1) in the pre-test. But after the mediation sessions both pupils successfully solved their word problems. This contrasts with their "I cannot do it" in the pre-test.

The researcher found it fascinating that when pupils were asked whether they could now solve the problem which they had been unable to solve in the pre-test, both pupils answered with confidence that they could now do so (R1:L12; R2:L10). They were in no doubt as to the nature of the question. They knew exactly what strategies to use to solve the problem (R1:L7,L8; R2:L7,8,9).

In the pre-test the pupils showed no planning behaviour. In contrast to this, an exposure to mediation brought about noticeable changes in planning behaviour. After mediation sessions, these pupils confronted the word problem systematically. They successfully broke the sum into manageable parts, which was not evident before. The only mistake that was made was, when they put it all back together, they tended to get mixed up, which resulted in a wrong answer. Through mediation they overcame this stumbling block. (R1:L3,4; R2:L10) shows how these pupils worked out a plan. They defined which steps to take and they chose steps which they could manage.

Internal representations and images are put down on paper (R1:L3; R2:L4,7). Here, in addition to concrete models, visual thinking is developed. By representing the problem, the pupils are able to translate each proposition from the problem into an internal representation (Sternberg, 1986) which is very helpful for the pupils in the Junior Primary phase. The pupils write down what is in their minds by making a drawing that represents the term of the problem.

During the mediation phase, the experimental group was encouraged to select information needed to solve the problem and not to be impulsive. In (R1:L3,5; R2:L7,10) the pupils displayed self control and attempted to solve the problem with care and precision. They defined the problem carefully, selected the relevant cues and applied it to the problem. Here they inhibited impulsive behaviour. Cognitive processes like comparing and selecting were displayed.

Conclusion

This study set out to investigate how mediation could be used as a tool in the Problem Centred Approach to mathematics. The post-test results have shown that with mediation pupils move towards becoming autonomous problem solvers.

This evidence confirms the need for a different approach which will encourage cognitive development. The use of mediation was an attempt to empower pupils to become autonomous problem solvers. This attempt did bear fruit as shown in the post-test and the interviews. The respondents see themselves as autonomous learners, as illustrated by the fact that they are confident and motivated, that they can now solve problems that before seemed impossible. The experimental group showed a definite change in their planning and solving of problem sums.

Chapter 6

Conclusion

Introduction

The results of the experimental investigation were very encouraging. Students were enthusiastic and evidence was found of changes in the manner in which they tackled a problem. Results from the comparison of experimental and control groups, and interviews with pupils indicate that mediation promoted the improvement of pupils' problem solving performance.

Why were the problems in the pre-test so hard to solve? Why is it so difficult for some pupils to solve "real" problem sums? In spite of the notion that pupils construct their own knowledge, why do some students greet "real" problem sums with moans, fearful faces, and incorrect answers? The obvious conclusion is that they lack suitable general problem solving strategies.

Problem solving is a complex mental process involving visualisation, imagination, manipulation, abstraction, and the association of ideas. There is no easy way to learn these techniques, nor can pupils be left on their own to discover them. The best way would be through mediation. It should be a way of providing the child with tools which she may use should the need arise.

When mediation was applied to problem solving pupils were very enthusiastic. Their planning behaviour improved tremendously, they were eager to work on problems that led to remarkable levels of achievement. Formal instruction in appropriate solution behaviour was never consciously employed, yet problem solving ability improved in virtually all the children in the experimental group. Evidence was also found of changes in their manner of tackling a problem. Pupils became more confident about their problem solving abilities. Interviews with both pupils supported this observation.

Among the most important benefits of mediation is that the mediational sessions improved pupils' willingness to engage in problem solving. Pupils gained confidence in their ability to succeed in problem solving and were beginning to learn "how to think". This study also provided some evidence of the effectiveness of mediation in enhancing the conceptual abilities and cognitive skills of pupils. Even the pupils in the control group attained a higher score in the post-test. This could be the result of "peer group teaching" after school when the pupils play "school-school". Developmental growth, especially in Junior Primary, can also play a role.

The key feature of such mediation is metacognition, or the conscious awareness of processes of cognition and the monitoring and controlling of these processes. This applies to both teacher and pupil and their interaction. Thus the teacher, as mediator, should be aware of the underlying dimensions of her interaction with the pupil that are most likely to promote or hinder learning and performance. Thus mediation should suit each individual child's needs (Confrey, 1984), which implies Piagetian constructivism.

In general I have achieved my goal, i.e. that with mediation pupils' problem solving abilities can be improved significantly. The major goal of mediation was thus to improve pupils' ability to perform tasks they were unable to perform well prior to mediational sessions.

I still sometimes feel insecure teaching mathematics my way, that is with "mediation". I will, however, continue to apply mediation to the problem centred approach in mathematics because of the rewarding results. It really fills me with pleasure to see a pupil's face light up with pleasure when after struggling for some time and using different strategies, she rushes up to me to show me that she has "successfully" solved the problem. My insecurity is partly due to the fact that I'm doing it "my way" and not the prescribed way. Sometimes I'm filled with fear when I think of what can happen if my pupils pass on to the next phase and they cannot cope with their "new" teacher's method. I believe that this way of teaching problem solving gets the best from all of my pupils. And knowing that success breeds success, it is my belief that through mediation a child can go from being a poor performer in problem solving to an autonomous problem solver who can function well above the level that might be expected of her. Thus, when mediation is applied to problem solving, there is enormous enthusiasm from pupils as well as remarkable levels of achievement in solving problems. Isn't this what we are striving for – that maths should be fun?!

Implications and suggestions

My interpretation of data suggests some additions in the problem centred approach to mathematics.

1. **Recommendation for further research**

   This research should be followed by quantitive research where the experimental and control groups are kept completely apart to avoid cross influencing between pupils. This can be done by utilising separate classes in preferably separate schools, but the pupils must come from the same statistical population. The pre-test and the post-test should be graded by using this research as an example. It could be argued that during the post-test the experimental group should not receive mediation. By doing the research this way we might get a clearer picture on how mediation can improve pupils' problem solving abilities.

2. There seems to be a great **need to educate teachers** on how to apply the problem centred approach to mathematics. There need to be more workshops where teachers have "hands on" experiences. There should be guidelines on how to teach problem solving effectively. For example,

- Problem solving **strategies can specifically be taught**, and when they are, not only are they used more, but also pupils perform successfully more frequently.

- **No one strategy** is suitable for solving all problems. Some strategies are used more frequently than others, with various strategies being used at different stages of the problem solving process. Teaching a variety of strategies (in addition to an overall plan for how to go about problem solving) provides children with a repertoire from which they can draw as they meet a wide variety of problems. They should be encouraged to solve different problems with the same strategy and to discuss why some strategies are appropriate for certain problems.

- Pupils need to be faced with problems in which the way to solve them is not apparent, and they need to be encouraged to test **many alternate approaches**.

- Children's problem solving achievement is related to their developmental level. Thus, they need problems at **appropriate levels of difficulty**.

It is important that children develop skills and strategies needed to solve problems and to apply mathematical procedures to everyday situations.

3. **Mathematics should be fun**

Confrey (1984) suggests successful implementation of instructional approaches that focus on the problem solving process, encouragement, independence, persistence, and flexibility, "requires change in students' conceptions of mathematics". Thus students may not be able to become better problem solvers unless they change their beliefs about mathematics. Sometimes pupils have a fear of problem sums which has been built up by their misunderstanding, non-understanding and failure during previous problem solving sessions, and this fear has to be resolved before any learning of mathematics can be attempted.

Our aim should be that students achieve at least minimally competent levels of performance, and competent performance is not necessarily synonymous with expert performance. The teacher must thus make it explicit that there is no "right way" of solving a problem, everyone's method is acceptable.

4. There should be a growing interest in the role of metacognition in problem solving

Teachers should be aware of the role of metacognition in problem solving. We now recognise that most of the difficulties pupils have in solving mathematical problems do not only stem from failure to know vocabulary and understand the language of problem statement - important as these may be, successful problem solving in a given domain depends upon the possession of a large store of organised knowledge about that domain, techniques for representing and transforming the problem, and metacognitive processes to monitor and guide performance (Silver, 1985). It can be argued that learning to solve a problem is the most significant learning that occurs in any mathematics class - and the most difficult. Thus, in teaching problem solving, teachers must analyse closely all the steps required in the process so that they can reach all pupils, and the pupils themselves must recognise the steps if they are to solve problems successfully. Children should learn by doing and thinking about what they do.

5. Algorithms should have a place in the curriculum

Algorithms are tools, not educational ends in themselves, and they have a proper role in the curriculum as long as they contribute to effective solving and to the understanding of mathematical applications. But the teacher must take note that premature exposure to symbolic algorithms can easily cripple conceptual understanding. It is imperative that the concepts be nurtured through meaningful instruction. Teaching algorithms by rote makes no sense. If teachers try to give all of mathematics a routine algorithmic flavour, then learning suffers. Teachers must aim for the timely teaching of algorithms, paced to fit with each child's cognitive development and tied closely to each child's understanding of the mathematical meaning behind the algorithmic steps (Dirkes, 1990).

6. **Language and mathematics**

Particularly problems with no numbers illustrate the need for further investigation of the role of language in mathematics.

7. **Mediation should be an essential component of the problem centred approach to Mathematics**

Mediation is necessary for almost all pupils, but the amount, quality, intensity, frequency, and duration of what is needed for adequate cognitive development will vary as a function of individual differences and the feedback from pupils to regulate their problem solving behaviour and to select mediational strategies rather than memorising particular mediational sequences (Haywood, 1993).

When the teacher mediates the learning experiences of her pupils it includes, but is not limited to, such functions as stimulus selection, focusing on relevant aspects of a stimulus complex, repeating exposure to important stimuli, perceiving and understanding similarities and differences, sequential relationships, dimensionality, antecedents and sequences, commonalities in experience and such operations as comparing, categorising, relating past, present and future, and grasping the idea of generalisability of experience to new situations. Briefly Arbitman-Smith, Haywood and Bransford (1984) describe mediational teaching as:

- Supply the information that may be needed to learn relationships or lead to the discovery of a solution.
- Ask questions, i.e. elicit rather than give answers.
- Guide children's learning by arranging and directing sequences to experience in a developmental fashion.
- Bring about induction of explanatory rules by arranging and by calling attention to similarities among isolated events.
- Guide deduction of application of rules.
- Build the confidence of pupils by communicating belief in their competence as problem solvers.
- Maintain a metacognitive emphasis, i.e. focus attention on the pupils' own thinking processes and encourage them to do so.

This study has been able to investigate and confirm that MEDIATION in the problem centred approach to mathematics is necessary so that the learner acquires the fundamental cognitive functions in order to become an autonomous problem solver. In conclusion, the researcher thus argues that there is a strong need for Mediation as an essential component in the Problem Centred Approach to Mathematics in the Junior Primary phase.

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WESTERN CAPE EDUCATION DEPARTMENT

SYLLABUS FOR MATHEMATICS

JUNIOR PRIMARY PHASE

IMPLEMENTATION DATE:
January 1996

CONTENT

1. INTRODUCTION

1.1 AIMS

1.1.1 Societal aims

This syllabus is aimed at fostering and developing the following societal aims:

* to work towards the reconstruction and development of the South African society and the empowerment of its people;
* to develop equal opportunities;
* to contribute towards the development of the society's cultures;
* to encourage democratic, non-racial and non-sexist teaching practices;
* to create an awareness of and a responsibility for the protection of the total environment.

1.1.2 General Teaching and Learning aims

This syllabus is aimed at enhancing the following teaching and learning aims:

* to develop independent, confident and self-critical citizens;
* to develop critical and reflective reasoning abilities;
* to develop creativity and problem solving abilities;
* to develop fluency in communicative and linguistic skills, e.g. reading, writing, listening and speaking;
* to encourage a supportive learning environment where discussion and the sharing of ideas take place;
* to develop the necessary understanding, values and skills for sustainable individual and social development;
* to understand knowledge as a contested terrain of ideas;
* to contextualise the teaching and learning in a manner which matches the experience of their pupils

1.1.3 Specific aims of mathematics education

The syllabus is aimed at fostering and developing the following specific aims of mathematical education:

* to enable learners to gain mathematical knowledge and skills;
* to enable learners to apply mathematics in daily life and to other subjects;
* to develop insight into spatial relationships and measurement;
* to enable learners to construct mathematical concepts and patterns by experimentation, discovery and conjecture;
* to develop number sense and computational abilities and to judge the feasibility of results by estimation;
* to develop the ability to reason logically, to generalise, specialise, organise, draw analogies and prove;
* to enable learners to recognise a real-world situation as amenable to mathematical representation, choose and design an appropriate mathematical model, solve the problem mathematically and interpret the result in the real-world situation;
* to develop the ability to understand, interpret, read, speak and write mathematical language;
* to develop an inquisitive attitude towards mathematics;
* to develop an appreciation of the place of mathematics and its widespread applications in society;
* to provide basic mathematical preparation for future study and careers;
* to create an awareness of and an appreciation for the contribution of all peoples of the world to the development of mathematics.

1.2. NATURE OF JUNIOR PRIMARY MATHEMATICS

School mathematics is aimed at the exploration of the subject discipline and the mastery of knowledge and skills at an appropriate developmental level with a view to cope with mathematical situations in everyday life, preparing for further study and entering the world of work. Mathematics should also serve as an instrument for developing thinking skills, e.g. understanding, reflecting (*Cf.* Glossary: p.31), analysing, synthesising and evaluating, as well as problem solving and communication skills. It is important to create sufficient opportunities for pupils to reflect on what they are thinking and doing in mathematics and to discuss this in an interactive situation. **In the teaching of mathematics, emphasis should be placed on establishing a positive attitude towards the subject and on the development of a positive self-image for all learners.**

Problem solving (*Cf.* Glossary: p.31) is used as a medium for teaching and learning mathematics. Problem situations that learners experience as meaningful should be presented to them. Knowledge and skills are developed through solving these problems. Existing knowledge should be utilised and new knowledge (for the learner) should be developed rather than knowledge being merely transferred from the teacher to the learner.

The mathematical ideas which pupils already have when they start school should be taken as the point of departure for developing their number concept and their computational (*Cf.* Glossary: p.31) and problem-solving skills. This should take place within a stimulating, co-operative and secure environment. **Since a conceptual knowledge of number facts and number properties is part of the development of effective computational skills, a knowledge of number facts (bonds and tables) and properties is developed by working with numbers rather than expecting learners to memorise particular facts.** They should therefore be given sufficient opportunities to develop and practise the construction of number facts. The development of number concept and computational skills should, for the sake of continuity in the curriculum, be integrated with measuring (lengths and other physical quantities).

Many aspects of the world can be understood, described and represented through the application of spatial sense and geometry. Knowledge of shapes and solid bodies, their properties, the relationships amongst them and the effect of change on the shape of figures and bodies, is essential for the development of spatial sense and mathematical ability. Activities aimed at developing spatial skills offer exciting and stimulating opportunities for facilitating the formation of concepts.

1.3. PRINCIPLES FOR THE TEACHING, LEARNING AND APPLYING OF MATHEMATICS

* Progression, differentiation and continuity are essential.
* Teaching/learning situations should always link up with the life experience and the developmental stage of learners.
* Active participation by the pupils in classroom activities is a prerequisite for realising the aims of the curriculum.
* Learners are expected to explain and justify their way of thinking.
* Learners' autonomy (*Cf.* Glossary: p.31) is essential.
* Problem-solving serves as the central focus for the development of knowledge, skills and attitudes.
* The level of number concept and computational skills of learners should be the point of departure for the development of mathematical knowledge and skills.

* A balance should be maintained between the development of measuring skills,number concept, computational skills, problem-solving skills and spatial sense.

* Learners should be allowed to progress at their own rate. No restriction should be placed on learners' development and progress.

1.4. OBJECTIVES FOR TEACHING MATHEMATICS

The mathematics curriculum is directed at the development of:

- A positive, enquiring attitude to, and an appreciation for mathematics and a view of knowledge as a contested terrain of ideas
- Number concept and number properties
- Functional and effective computational methods and the recording thereof
- Spatial concepts and relationships as well as measuring skills
- Logical thinking and reasoning skills
- Problem-solving skills (*Cf.* Glossary: p.31)
- Communication skills in mathematics, e.g. recording, explaining, justifying, etc.
- Independent, self-critical, confident learners
- The ability to solve the specified range of different problem types (*Cf.* 2.4.2: p.21)

To realise these objectives the teacher has the following responsibilities:

- to create a positive classroom culture
- to accept the fact that the learner and not the syllabus determines progress
- to facilitate a risk-taking attitude when solving mathematical problems
- to allow for learner autonomy
- to allow learners to work in the number ranges in which they feel comfortable
- to provide suitable challenging activities
- to assist learners in recording their own computational strategies in an acceptable way

1.5 OUTLINE OF THE MATHEMATICS SYLLABUS

1.5.1 The mathematics syllabus is divided into the following sections:
(These sections should not be dealt with in isolation but should run concurrently)

- Introduction
- Measurement and physical quantities
- Number concept
- Problem solving
- Geometry and spatial sense
- Graphs

1.5.2 The syllabus has not been structured into separate year units. Since progress is largely determined by the needs of individual pupils and class groups, planning should be done at classroom level.

2. PROVINCIALISED SYLLABUS (Grade 1, Grade 2, Standard 1)

2.1 INTRODUCTION

2.1.1 Remarks:

- Learners should have a thorough understanding of the concepts listed in this section for their mathematical development.
- Vocabulary should be introduced in a natural way through practical activities and should not be memorised by the learners.
- Learners should not be expected to read the vocabulary.
- Complete mastering of the concepts specified in this section should not be required from all learners before starting with the other sections of the syllabus.
- It is essential that learners should be able to sort and classify objects according to common properties, e.g. shape, size, length, line, colour, etc. and at a later stage according to a combination of properties, e.g., give me all the small red ones.

2.1.2 Position

CONTENT	NOTES/EXAMPLES/OBJECTIVES
* below, above; bottom, top; on; under one beneath the other, on top of each other * first, last, next, middle, between * in front of, behind, front, back, after, one behind the other * up, down; over, one over each other; put with, together; next, next to, next to each other, beside * in, inside; out, outside; left, right	* Spatial concepts should be consolidated using the learners' awareness of their own bodies and their position in relation to objects in their environment * To understand the relative position of an object in respect of another object * Learners should have a perception of direction

2.1.3 Colour

CONTENT	NOTES/EXAMPLES/OBJECTIVES
* red, yellow, green, blue, white, black, brown, orange, purple, etc. * darker than, lighter than	* Activities: classifying, sorting, discussing, comparing, recording

2.1.4 Size

CONTENT	NOTES/EXAMPLES/OBJECTIVES
* small, little, big; smaller than, bigger than; smallest, biggest; the same as	* compare objects of different sizes * vocabulary should be extended to deal with the learners' needs, experience and local conditions * vocabulary derived from classroom discussions, excursions, etc. * activities: classifying, sorting, discussing, comparing, recording, sequencing

2.2 MEASUREMENT

2.2.1 Remarks:

- Measurement activities should relate to real life situations; it should therefore form an integral part of number concept development, solving word problems and other classroom activities throughout the year.

- Where applicable, estimation should be integrated with measurement activities. This can lead to meaningful questions like: Whose estimation was the closest? How far were you out?

- Although the standard units of measurement should be introduced at the stage when a real need arises, there are great advantages in using a variety of measuring tapes, rulers, mass meters (scales), clocks and measuring containers.

- Millilitres may be introduced as early as possible because most learners are familiar with commercial containers.

- Activities concerning the handling of money are of great value. Teachers should ensure that learners understand the nature of transactions before they are expected to solve word problems involving buying and selling.

- Graphic representations could serve as an introduction to statistics and create opportunities for working with information and numbers in a meaningful way; it should therefore be used throughout the year in all standards.

- It is essential that learners should be able to sort, classify and compare objects according to common properties, e.g. shape, size, length, line, colour, etc.

2.2.2 LENGTH

CONTENT	NOTES/EXAMPLES/OBJECTIVES
Vocabulary relevant to length:	
* Long/longer than/longest; short/shorter than/shortest; the same length as; as long as wide, narrow high,low; far, near	* informal introduction through activities and discussion - comparing two unequal lengths - matching lengths - ordering objects of different lengths in the correct order of length * measuring with arbitrary units, e.g. parts of the body (handspans, palms, footprints, paces) and objects such as dusters, pieces of wool/string, etc. - recognise that measurements with these units differ * measuring the length and/or the height of an object with a suitable selected unit * by setting word problems in context
* Centimetre, metre millimetre	* the concept of <u>measuring length</u> using different units * practical acquaintance with the metre (m) and the centimetre (cm), and the relationship 1m = 100cm * the measurement of given line segments with length given in cm and mm * estimation of lengths * compare different lengths * record measurements * compare and order straight lines of different lengths * the information can be recorded as a graph
* Assignments using centimetres only, or metres only, or millimetres only awareness of kilometres	* assignments and relevant computations using cm, m and mm (without stressing that 1000 mm = 1m) * be familiar with the relationship between the units (convert metres to centimetres and vice versa) * draw lines the correct length * calculate units of length * understand and solve contextual word problems which involve units of length
* Word problems	

2.2.3 MASS

CONTENT	NOTES/EXAMPLES/OBJECTIVES
* measure; heavy, light; heavier than, lighter than; heaviest, lightest; balance	* informal introduction through activities and discussion - comparing objects - experimenting to discover that the heavier object may not be the larger object - balance two objects of the same mass - balance one object against a set of objects - ordering objects according to their mass from the lightest to the heaviest or from the heaviest to the lightest * acquaintance with terminology
* the mass of, kilogram, massmeter, gram	* use of a massmeter * recording of measurements * compare, estimate, discuss and measure the mass of different objects * calculations involving units of mass * understand and solve contextual word problems which involve units of mass * the information can be recorded as a graph
* The measurement of the mass of objects Practical acquaintance with a kilogram (kg) and a gram (g) Assignments and relevant computations	* measurement of mass in kilograms and grams * recording of measurements * estimation followed by measurement of mass * compare and order the mass of different objects * assignments using kilograms only, or grams only by setting word problems in context

2.2.4 CAPACITY

CONTENT	NOTES/EXAMPLES/OBJECTIVES
* measure; full, empty; the same amount as, as much as; more than, less than	* informal introduction through activities and discussion * recognise when a container is empty, when it is full and when it is approximately full or half full * compare the capacities of the same and different containers
* capacity, litre, millilitre	* acquaintance with terminology * measure with reasonable accuracy the capacity of everyday containers with reasonable accuracy * estimate and then measure the capacity of everyday containers * record measurements
* - the measurement of volumes of fluid	* without stressing that $1\ell = 1000\ m\ell$
- practical acquaintance with the litre (ℓ) and the millilitre ($m\ell$)	* measurement of volume in litres and millilitres
- estimation of volumes	* estimate, measure and compare capacity
- assignments and relevant computations - using one unit only	* calculations involving units of capacity * assignments using millilitres only, or litres only by setting word problems in context
* word problems	

2.2.5 TIME

CONTENT	NOTES/EXAMPLES/OBJECTIVES
* a long time, a short time; day, night; names of the days of the week; yesterday, today, tomorrow - hours, minutes, seconds - early, late	* compare the intervals of time: i.e. the relationship of 'long' to more time and 'short' to less time * associate happenings with day-time and night-time * relate activities to the time of the day * use the terms yesterday, today and tomorrow correctly * informal introduction through activities and discussion * use the terms correctly when referring to time * know the position of numbers on the traditional clock face; the direction in which the hour hand of the clock moves
* day, week, month, year, clock, watch	* the reading of time in hours on traditional and digital clocks * name the days of the week and the months of the year in the correct order * relate everyday occurences to months of the year
* - daylight hours, night time hours - reading of time	* reading the traditional and digital clock in hours and minutes
- interpreting the clock face - hour, half-hour, half an hour, quarter-hour (quarter of an hour)	* half an hour and quarter of an hour (only when the pupil understands the concept of a quarter of 60) state the time correctly in half hours and quarter hours from watches and clocks * practical experience of the passing of time with reference to duration * value of time units
- estimation - names of the months of the year and seasons - number of days of the year/leap year - cultural calendars/festivals/public holidays	* judge the time it takes to do common tasks

2.2.6 MONEY

CONTENT	NOTES/EXAMPLES/OBJECTIVES
* Recognition of coins and notes currently used	* recognition of coins and notes on sight * comparing: appearance, and value * ordering from most to least value and vice versa * getting a "feel" for realistic prices, e.g. "How much does a pencil cost?" "What is it worth?" * Games and activities, e.g. shopping and banking games (understanding buying and selling)
* Know the value of all coins and notes available 100c = R1 practical assignments and word problems involving money	* value of coins and notes * identify a set of coins equal in value to a given coin * identify a set of coins equivalent in value to the cost of a specific article
* The correct notation for coins and notes in circulation.	* identify the value and relevant properties * using money in practical applications, e.g. problem solving
Equivalents for coins and notes	* e.g. R1 = five 20c coins = ten 10c coins, etc. * grouping of coins in rands * determining change
* Word problems	

2.3 NUMBER CONCEPT

2.3.1 Remarks:

- Learning about numbers through counting activities, with concrete materials, solving problems (the different types of word problems) and discussion.

- For the purpose of developing number concept (counting, number names and number symbols), pupils should work in higher number ranges than those used in calculations.

- Written work in the number interval 1 to 10 is limited to the writing of closed number sentences, e.g. 5 + 3 = 8.

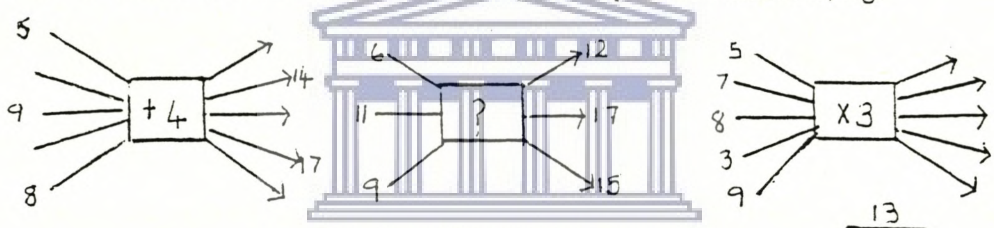
- The number sentence should suit the structure of the problem as each individual pupil interprets the problem, e.g.

  I have 10 apples and give 6 to my friends. How many apples do I have left? 6 + ___ = 10 or 10 - 6 = ___ or 10 = 6 + ___

- Learners should be allowed to use counters.

- Learners should be encouraged to use paper and pencil in informal ways.

- Flowcharts and tables should be used extensively in computational exercises, e.g.



15 =

10	9		11	13		
5		7			8	14

13	
7	–
12	–
–	9
–	8
6	–
–	2
10	–

- ***The number ranges as used in this document only indicate those activities which are deemed suitable for numbers of a particular size (one-, two- or three digit numbers) and are not to be regarded as working units.***

- **Non-contextual problems, e.g. number sentences and flow diagrams, should be used only after the relevant concepts and meanings of the operations have been developed through word problems.**

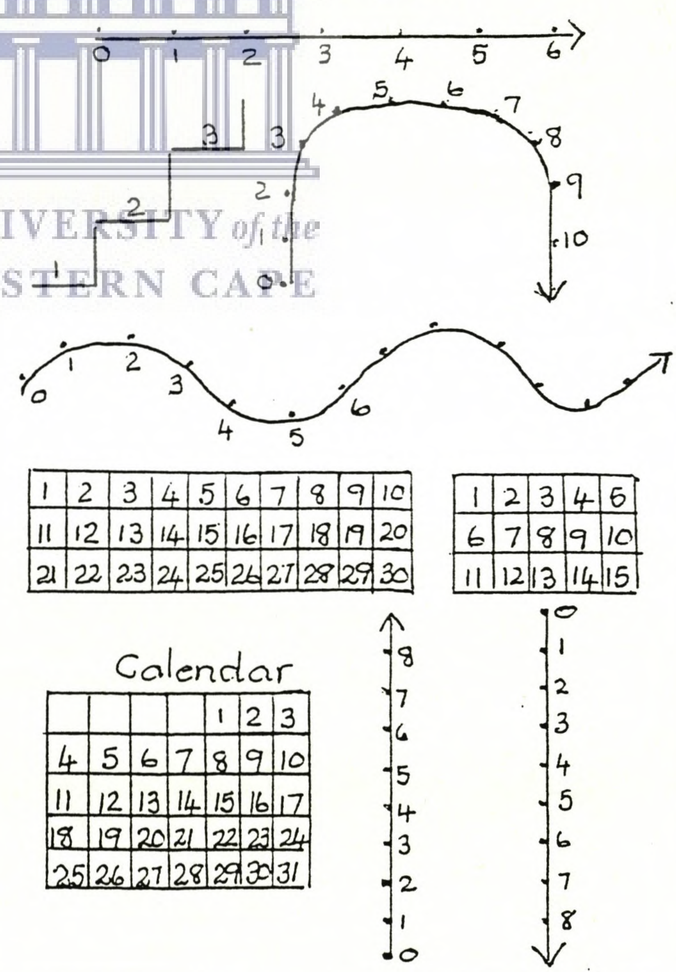
  It is suggested that non-contextual division problems should not be posed in the junior primary phase.

- The fraction concept is introduced and developed through word problems relating to the learners' environment.

2.3.2 NUMBER RANGE 0 - 10

CONTENT	NOTES/EXAMPLES/OBJECTIVES
1. Concepts	
more, less; more than, less than; as many as, the same as; how many, put with, fewer than, most, least, just as many as; different; something, nothing; a little, a lot, none; over-estimate, under-estimate; greater than; number line	* Arrangement and comparison of groups of objects: - recognise more or fewer objects in a group and arrange the groups of objects accordingly - compare two groups of objects telling whether they have fewer, more or the same number of objects
2. Counting	
(a) Subitizing (looking at a collection of objects and associating the number name with it immediately without counting the objects)	* up to 5
(b) Rote counting	
(c) Counting of objects - counting up to at least 20	All counting exercises should be preceded by the question "How many?". Learners should be required to estimate the number of objects to be counted and then to confirm whethe they are correct or not. * the principle of "one more/one less" * counting on from; counting back from * estimating of the number of objects and verifying b counting * associate the spoken number name with the object being counted * it is imperative that the groups of objects be arranged in different ways (in ordered patterns as well as at random).
(d) Skip counting	
- correctly count in 2's and 5's to at least 20	* Know the order of number names * with body parts and other objects
- know the order of number names	* e.g. 2, 4, 6, 8, ...
3. Number symbols	
Recognise that a numeral is a symbol to denote a number name.	* recognition of numerals and spoken number names

4. Cardinal number	
- Knowing the "how many" of a number (numerosity) *This knowledge cannot be forced*	* by handling objects and determining their number
- Understand that the number of objects in a group does not vary even though the group is rearranged	* arranging groups of physical objects with corresponding numerals * matching the correct number symbol to a group of objects * small addends and minuends e.g. 4+1, 4+2, 4+3 4-1, 4-2, 4-3
- *It is not necessary for learners who know the "how many" of number to reconstruct the number with counters or counting actions before they can use the number*	
5. Comparison and sequence	* comparison of groups of objects for **equality and inequality**, e.g. by matching in one-to-one correspondence
6. Ordinal number	
- The ability to arrange numbers in the correct order	* ordination of groups of objects (ascending and descending) * ordinal numbers: first, second, last, etc. * the building of a variety of number lines, e.g.



7.	**Regrouping** Arranging the same number of objects in different ways	* e.g. in groups of 4 9: **** **** * in groups of 5 9: ***** **** in groups of 3 9: *** *** ***
8.	**Doubling and halving**	* double 4 is 8 (4+4 = 8), double 3 is 6 (3+3 = 6) half of 8 is 4, half of 6 is 3 half of 5 is ___ * applying the doubling and halving strategies for computational purposes should be discussed, e.g. double 4 = 4 + 4 = 8 therefore 4 + 5 = 9 half of 10 = 10 - 5 = 5 therefore 10 - 6 = 4
9.	**Number facts (bonds)**	* bonds should not be drilled but developed - make use of the learners' knowledge of (i) doubling and halving (ii) small addends and minuends and (iii) regrouping, to build up additions and subtraction tables: - e.g. "bonds of 8" (i) 4 + 4 = 8 therefore 5 + 3 = 8 therefore 6 + 2 = 8 therefore ... (ii) 7 + 1 = 8 therefore 6 + 2 = 8 therefore 5 + 3 = 8 therefore ... (iii) xxxxxxx x 7 + 1 = 8 xxxxxx xx 6 + 2 = 8 xxxxx xxx 5 + 3 = 8 etc. (The learners will start recognising certain number patterns when these strategies are written down and are discussed).
10.	**Nought (zero)** - The ability to arrange numbers in the correct descending order until 0 is reached - Understanding that adding 0 with any number does not change the value of the number - Understanding that taking away (subtracting) 0 from any number does not change the value of the number	* as a number - .... _ 5 4 3 2 1 0 - 6 + 0 = 6 4 + 0 = 4 - 6 - 0 = 6 4 - 0 = 4

11. **Context free problems** (not word problems) - e.g. 4 + 2 = □ 6 - 3 = □ (This should be done only after the learners have solved word problems and have learned to record what they have done and the teacher knows that they understand the operations).	* use flow diagrams and tables (*Cf.* Remarks: p.12)
- multi-step problems, e.g.	* 3 + 2 + 4 4 + 3 - 2 etc.

2.3.3 NUMBER RANGE 0 - 34

CONTENT	NOTES/EXAMPLES/OBJECTIVES
1. **Counting**	* estimate and count estimation should include decisions on whether the estimate is less than/more than the exact answer * counting with and without objects * learning the number names (words to 20) and matching the number symbols to at least 104 * from any given number * counting on and back * pictures * measuring: counting centimetres * skip counting: in twos and tens (starting at any number) * building up various number lines * bridging 10, 20, 30, e.g. 28, 29, <u>30</u>, 31, 32 * patterns and relationships should be recognised and discussed
2. **Number names and number symbols**	* place value: 37 = 30 + 7
3. **The term "even and odd"**	* investigations involving odd and even numbers, e.g. - odd numbers end in digits 1,3,5,7 or 9 while even numbers end in 2,4,6,8 or 0 - patterns predict the outcome: - adding 2 even numbers → the answer is an even number - adding 2 odd numbers → the answer is an even number - adding an odd number to an even number → the answer is an odd number - do the same with subtraction
4. **Comparison and sequence**	* compare numbers to determine smaller and larger numbers, differences and patterns
5. **Ordinal number**	* extension of number lines * random numbers rearranged
6. **Doubling and halving**	* see number range 0 - 10 * recognising doubles and near doubles, e.g. 14 + 17 = (14 + 14) + 3 = 28 + 3 * recognising halves and near halves, e.g. 28 - 17 = (28 - 14) - 3 = 14 - 3 * recognising patterns
7. **Addition and subtraction**	* estimate - evaluate - improve * small addends and minuends * adding and subtracting decade numbers, e.g. +10, +20, +30 and -10, -20, -30 * using flow diagrams and tables

8. **Decimal decomposition**	* e.g. 25 = 20 + 5 (place value) * using decimal decomposition for doubling and halving of multi-digit numbers, e.g. double 18 = 18 + 18 = (10 + 10) + (8 + 8) = 20 + 16 = 36 half 29 = (half 20) + (half 9) = 10 + 4½ = 14½
9. **Other decompositions**	* e.g. 21 = 14 + 7 21 = 7 + 7 + 7 21 = 18 + 3
10. **Number patterns**	* e.g. 2 + 1 = 3 12 + 1 = 13 22 + 1 = 23 32 + 1 = 33 * investigate when dealing with - small addends and minuends - adding and subtracting decimal numbers - doubling and halving - decimal decompositions - other decompositions
Building up and constructing of multiplication, addition and subtraction tables	* building up addition, subtraction and multiplication tables: - addition and subtraction tables basic addition combinations up to 10 + 10 and subtraction up to 20 - 10 (Follow the same procedures as for numbers in the range 0 - 10) - multiplication write down the patterns which are formed when doing doubling, e.g. double 1 = 1 + 1 = 2 double 2 = 2 + 2 = 4 double 4 = 4 + 4 = 8 double 8 = 8 + 8 = 16, etc. when counting in tens, e.g. +10 +10 +10 0 10 20 30 (3 times "plus ten" is equal to 30) * Routine written work - flow diagrams - tables - context free problems, e.g. 23 + 6 = □

2.3.4 NUMBER RANGE 0 - 104 - 1004

CONTENT	NOTES/EXAMPLES/OBJECTIVES
1. **Counting**	* estimate and count * counting,number names, number symbols to at least 1004 * note the relationship between number sequences, e.g.: 10, 20, 30 and 310, 320 and 330 * Skip counting: counting in 3s, 5s, 20s, 25s, 50s, etc.; starting at any number * bridging 40, 50, 60, 70 ...., 100, ...,1000 (e.g. 97, 98 99 100, 101, 102 103) * patterns and relationships should be recognised and discussed
2. **Decomposition of number**	* Decomposition of number: decimal: 212 = 200 + 10 + 2 (place value) other: 212 = 120 + 80 + 12
3. **Number patterns** (see notes for previous number ranges and extend activities to larger numbers)	
4. **Addition** } 5. **Subtraction** }	* using larger numbers applying known strategies * addends and minuends +10, +20, +100, etc., -10, -20, -100, etc.
6. **Multiplication**	* Number patterns: 10 x 2 10 x 3 10 x 4 20 x 2 20 x 3 20 x 4 40 x 2 40 x 3 40 x 4
7. **Number facts**	
building up/construction of multiplication tables	* e.g. counting in 2's and 3's asking "How many 2's?" How many 3's?" * doubling and halving activities: doubling a number means x2 halving a number means ÷2 (write down the pattern and complete the table) * the x2 table is "used" to build up/construct the x4 table, e.g. 2 x 2 = 4 2 x 4 = 8 because (2 x 2) x 2 = 8 * using known facts, e.g. 10 x 2 = 20 therefore 9 x 2 = (10 x 2) - 2 10 x 2 = 20 therefore 11 x 2 = (10 x 2) + 2 * context free problems, e.g. 39 + 12 = ☐ * flow diagrams and tables

2.4 UNDERSTANDING and SOLVING PROBLEMS

2.4.1 Remarks

* Learning opportunities aimed at the development of number concept and computational skills should be characterised by:

  - an atmosphere of trust, mutual support and the sharing of knowledge

  - solving relevant word problems right from the beginning of Grade 1

  - an awareness of the stages of development of number sense and their related computational strategies, e.g.

    Level 1: Counting all
    The need to reconstruct the number with counters or drawings

    Level 2: Counting on
    The ability to use the number in its abstract form

    Level 3: Changing the original number and task to make it easier
    Breaking up of numbers or changing, e.g.
    51 = 50 + 1 = 30 + 21
    19 + 19 = 20 + 20 - 2

  - encouragement of pupils to attempt to solve new problems before seeking help

  - the free availability of concrete aids (e.g. counters) and informal writing materials from the beginning.

* Division, addition, subtraction and multiplication problems should be alternated on a daily basis from Grade 1

* Computational strategies and recording skills are refined

  - as learners' number concept develops

  - through reflection, discussion and comparison of strategies among learners

* The learners develop the different meanings of the four basic operations through solving meaningful problems which are dased on the different problem types.

* Social interaction is an integral part of problem solving.

2.4.2 Problem types

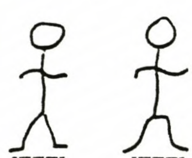
2.4.2 (a) Division and multiplication

These problem types are used in all different number ranges		
1.	Sharing with remainder:	*Share 5 pencils equally between 2 children*
2.	Sharing without a remainder:	*Share 8 sweets equally among 4 children*
3.	Sharing with remainders that lead to fractions:	*Share 10 sausages among 3 friends so that they all get the same amount and nothing is left.*
4.	Grouping with and without a remainder:	*How many packets of 6 apples can be made up from 14 apples?*
5.	Repeated addition (bicycle wheels, hands, fingers, etc.)	*How many wheels do 4 bicycles have?*
6.	Rate:	*Themba drinks 3 cups of milk every day. How many cups of milk does he drink in a week?*
7.	Grids:	*Mr Daku plants 3 rows of cabbage plants, 6 plants in a row. How many plants are there?*
8.	Combinations:	*Mary has 3 skirts and 4 T-shirts that match. In how many different ways can she dress if she wears one skirt and one T-shirt at a time?*

2.4.2 (b) Addition and subtraction

These problem types are used in all different number ranges			
	CHANGE **Join**		**CHANGE** **Separate**
1.	Connie had 5 marbles. Jim gave her 8 more marbles. How many marbles does Connie have altogether?	2.	Connie had 13 marbles. She gave 5 marbles to Jim. How many marbles does she have left?
3.	Connie has 5 marbles. How many more marbles does she need to have 13 marbles altogether?	4.	Connie had 13 marbles. She gave some to Jim. Now she has 8 marbles left. How many marbles did Connie give to Jim?
5.	Connie had some marbles. Jim gave her 5 more marbles. Now she has 13 marbles. How many marbles did Connie have to start with?	6.	Connie had some marbles. She gave 5 to Jim. Now she has 8 marbles. How many marbles did Connie have to start with?
	COMBINE		**COMBINE**
7.	Connie has 5 red marbles and 8 blue marbles. How many marbles does she have?	8.	Connie has 13 marbles. Five are red and the rest are blue. How many blue marbles does Connie have?
	COMPARE		**COMPARE**
9.	Connie has 13 marbles. Jim has 5 marbles. How many more marbles does Connie have than Jim?	10.	Connie has 13 marbles. Jim has 5 marbles. How many fewer marbles does Jim have than Connie?
11.	Jim has 5 marbles. Connie has 8 more than Jim. How many marbles does Connie have?	12.	Jim has five marbles. He has 8 fewer marbles than Connie. How many marbles does Connie have?
13.	Connie has 13 marbles. She has 5 more marbles than Jim. How many marbles does Jim have?	14.	Connie has 13 marbles. Jim has 5 fewer marbles than Connie. How many marbles does Jim have?
	EQUALIZE		**EQUALIZE**
15.	Connie has 13 marbles. Jim has 5 marbles. How many marbles does Jim have to win to have as many marbles as Connie?	16.	Connie has 13 marbles. Jim has 5 marbles. How many marbles does Connie have to lose to have as many marbles as Jim?
17.	Jim has 5 marbles. If he wins 8 marbles, he will have the same number of marbles as Connie. How many marbles does Connie have?	18.	Jim has five marbles. If Connie loses 8 marbles, she will have the same number of marbles as Jim. How many marbles does Connie have?
19.	Connie has 13 marbles. If Jim wins 5 marbles, he will have the same number of marbles as Connie. How many marbles does Jim have?	20.	Connie has 13 marbles. If she loses 5 marbles she will have the same number of marbles as Jim. How many marbles does Jim have?

2.4.3 CONCEPTS

CONTENT	NOTES/EXAMPLES/OBJECTIVES
	* **A variety of problems should be given.** * **Practical and oral work through discussion** * **Estimate/evaluate/improve** * **Recording of computations:** *Where the need arises learners should be allowed to use drawings when recording their methods. As/when refinement occurs symbols should be introduced.*
1. **Addition** *Typical terminology* how many, add together, altogether, how many short, add, add to, plus, etc. * of more than two numbers * two-digit numbers * two-digit and single-digit numbers * three-digit numbers * three-digit and two-digit numbers	**Typical solution strategies which learners may construct:** * counting all * counting on: 8 + 7 = 8,9,10 ..., * counting on from the largest number * using known number facts: 8 + 7 = 7 + 7 + 1 etc.
2. **Division** *Typical terminology* half, half of, pair, thirds, two thirds, fifths, four fifths, fourths (quarter), a whole, a part of, etc. (*Cf.* Problem types: 2.4.2 p.21) * one-digit by one-digit numbers * two-digit by one-digit and two-digit numbers * three-digit by three-digit, two-digit and one-digit numbers	* sharing: 8 sweets shared between 2 children  xxxx xxxx 4 each * grouping: 8 sweets divided into groups of four: xxxx xxxx 2 groups of 4 8 sweets divided into groups of two: xx xx xx xx 4 groups of 2

	Division problems with remainders	* remainders should be evaluated carefully in order to arrive at sensible solutions
3.	**Fractions**	* division problems with remainders leading to halves (divided into two equal parts), thirds (divided into three equal parts), fourths, fifths, etc. (see problem types)
4.	**Subtraction** *Typical terminology* take away, seperate, give away, lose, how many left over, subtract, how many do you need, how many more, how many to start with (*Cf.* Problem types: 2.4.2 p.22) * of more than two numbers * two-digit numbers * two-digit and single-digit numbers * three-digit numbers * three-digit and two-digit numbers	* by physically taking away and counting the rest * by counting on * by using known number facts e.g. 9 - 4 = 5 because 5 + 4 = 9
5.	**Multiplication** *Typical terminology* times, multiply by, groups of (*Cf.* Problem types: 2.4.2 p.21) * the multiplication of one- and two-digit numbers by one- and two-digit numbers	* repeated addition e.g. 2 + 2 + 2 = 3 x 2 * grouping e.g. 3 groups of 2 is 6 * using the doubling strategy as explained for number concept E.g. 3 x 4; 13 x 4; 23 x 13
6.	**Multi-step problems**	* a combination of more than one problem type, e.g. Linda buys 2 toffees at 15c each and 3 pieces of fudge at 30c each. She pays with R5. How much change does she get?

7. **Reading and understanding of word problems**	* Getting practice in the reading of written word problems and learning to interpret the information before start solving the problem. Estimate/predict what the outcome might be → test the estimation/prediction
8. **Designing and solving own problems**	* based on practical situations * formulated to match given number sentences * formulated to match given problem situations

2.5 GEOMETRY AND SPATIAL SENSE

2.5.1 Remarks

* Mathematical activities should contribute to the development of an awareness of natural and geometric shapes through **observation and discussion, the introduction** of the names of the shapes, and the **discovery** of the properties of the shapes.

* A display of various shapes in the classroom can create an interest in space and shape and help to increase the learners' awareness of the aesthetic qualities of geometrical shapes.

* Children should get ample opportunity to become acquainted with shape by handling concrete material. Shapes made from cardboard or plastic or cut from lino or vinyl tiles as well as drawings of these shapes can be used.

* The vocabulary of shape should be introduced in a natural way and need not be memorised by the pupils.

* Apparatus such as Lego, plasticine, clay shapes and pastry shapes is useful to develop an awareness of space and shape.

* The names of plane shapes (square, rectangle, triangle, circle) and the solid shapes (cube, cuboid, sphere, cylinder and pyramid) should be introduced gradually.

* Activities which relate to the spatial orientation of learners offer exciting stimulating opportunities for enjoying mathematics and for supporting the formation of concepts, communication and problem solving while simultaneously developing the potential of the learners.

2.5.2 Concepts

		CONTENT		NOTES/EXAMPLES/OBJECTIVES
1	-	Recognition of shape in nature, e.g. the shape of leaves, trees, bodies, clouds, shadows, etc.	*	Exploration of plane and solid shapes, awareness of shape (texture, sides, edges, faces, outlines, etc.)
	-	3D shapes: cones, cylinders, bricks, boxes, etc.	*	observe, handle, discuss, classify and sort
	-	2D shapes: square, different rectangles, different triangles, circle	*	recognising shapes in everyday life
	-	area, symmetry and vocabulary	*	awareness of area; first ideas of symmetry; use of informal vocabulary
	-	Vocabulary: round, corner, sides, straight, slant		
2	-	Handling and recognition of the following geometric shapes: ball (sphere), block (cube), cylinder, cone and pyramid		Activities:
			*	construction and drawing
	-	rectangle, rectangular; square, triangle, triangular; circle, round, circular; curved, oval	*	performing geometric transformations using apparatus and drawings
	-	2D shapes:	*	predicting the results of planned geometrical transformations
		pentagon (5 sides):	*	testing predictions
			*	describing simple geometrical transformations: slide, flip, rotate
		octagon (8 sides)		
		equilateral triangle (equal sides)		

3 - Vocabulary: diameter: diagonal: construct, vertical, horizontal (the use of relevant adjectives, e.g. symmetrical, vertical, horizontal)	Activities: * drawing on squared paper * measuring of lengths of sides in centimetres * tessellations (*Cf.* Glossary: p.31) * build or fold according to given specifications * design and build

2.6 DATA HANDLING

2.6.1 Remarks

* Information collected from the learners may be recorded by means of pictographs.

* In the beginning the pictures or drawings should be arranged horizontally only. Later the data should be recorded both horizontally and vertically.

* The theme for each pictograph should be one in which the learners have a special interest.

* Graphs should be made and discussed throughout the year.

* Learners should get the opportunity to learn to interpret the information of different graphs.

* Possible themes are:

  - The pets of learners
  - The names of learners and months in which they have their birthdays
  - The flowers in our gardens during each season of the year
  - Fruit on sale in the shops
  - The games or sport we play during the different seasons
  - My favourite motor car
  - Mass and height charts
  - The mass of various substances, e.g. wire nails, oats, rice
  - Time (in seconds) taken to run 20m
  - Weekly school attendance

2.6.2 Examples

How learners come to school

by car	*	*	*	*	*	*	*	*	*	
by bus	*	*	*	*						
by bicycle	*	*								
by train	*	*	*	*	*	*				
by taxi	*	*	*							
walk	*	*	*	*	*	*	*	*	*	*

Our favourite fruit

apricots	oranges	bananas	grapes	peaches

3. GLOSSARY

attainment targets

competency levels

autonomy

it means that learners:

- never experience any obligation to use specific operations or strategies in solving a problem, or specific methods of computation
- are allowed to choose methods according to their own individual judgements
- accept individual and collective responsibility to assess the feasibility of their answers

Cf. - refer to

computational

- that which is determined by mathematics, especially by numerical methods
- performance of mathematical or logical calculations

non-contextual problems

e.g. 4 + 3 = ☐ 6 + 7 + 4 = ☐ (not word problems)

problem solving

A variety of carefully selected and well-formulated problems, based on real situations are posed on a regular basis for learners to solve. Learners should be able to identify themselves with the problem-situation to enhance visualisation, interpretation and full understanding, which will lead to feasible solutions.

reflecting

to think about:

- what you have done
- why you have done it in a certain way
- how others have done it
- why one way of doing it may be better than another way

subitizing

looking at a collection of objects and associating the number name with it immediately without counting the objects

tessellations

tessellate:

- covering a surface with flat shapes that fit together exactly, e.g. you cannot tessellate with a circular tile, but you can tessellate with a rectangular tile

- a tessellating shape is a shape you can tessellate with

4. CONTINUATION AND PROGRESSION INTO SENIOR PRIMARY PHASE

PROVINCIALISED SYLLABUS: STD 2 - STD 4

4.1 READING, UNDERSTANDING AND SOLVING PROBLEMS			
4.1.1 Remarks	**4.1.2 Standard 2**	**4.1.3 Standard 3**	**4.1.4 Standard 4**
	1. Reading for understanding 2. Strategies for solving problems 3. Recording the solution 4. Evaluating the solution	1. Reading for understanding 2. Strategies for solving problems 3. Recording the solution 4. Evaluating the solution	1. Reading for understanding 2. Strategies for solving problems 3. Recording the solution 4. Evaluating the solution

4.2 CALCULATOR SKILLS			
4.2.1 Remarks	**4.2.2 Standard 2**	**4.2.3 Standard 3**	**4.2.4 Standard 4**
	1. Acquaintance with the keyboard 2. The calculator as a computational tool 3. The calculator as an investigational aid	1. Acquaintance with the keyboard 2. Investigating key sequences 3. The calculator as a computational tool 4. The calculator as an investigational aid	1. Using the calculator 2. Interpreting calculator answers

4.3 MEASUREMENT: Length, Mass, Capacity, Time, Money			
4.3.1 Remarks	**4.3.2 Standard 2**	**4.3.3 Standard 3**	**4.3.4 Standard 4**
	1. Length 2. Mass 3. Capacity 4. Time 5. Money	1. Length, Mass, Capacity 2. Perimeter 3. Time	1. Length, Mass, Capacity 2. Perimeter 3. Area 4. Time

4.4 WHOLE NUMBERS			
4.4.1 Remarks	**4.4.2 Standard 2**	**4.4.3 Standard 3**	**4.4.4 Standard 4**
	1. Number concept 2. Ordering and comparing 3. Approximation 4. Classification of numbers 5. Addition 6. Subtraction 7. Multiplication 8. Division	1. Numeration 2. Addition and subtraction 3. Multiplication 4. Division	1. Numeration 2. Addition Subtraction Multiplication Division

Interview 1

Naam:	**Grizilda**	**Vak:**	**Wiskunde**
Ouderdom:	**8 jaar**	**Stadium:**	**Probleemsomme**
Standerd:	**Graad 2 (Sub. B)**	**Datum:**	**20 September**

Probleem: **Ceril is ouer as Cole. Cole is jonger as Zelda, maar Zelda is ouer as Ceril. Wie is die oudste en wie is die jongste?**

Line 1: I: Ek sien jy kan nie die probleemsom oplos nie. Hoekom nie?

R: Ek verstaan nie die som nie. Dit is nie 'n som nie, daar is niks syfers nie.

Line 2: I: Wat wil die storiesom hê moet jy uitvind?

R: Wie is die langste en wie is die kortste?

Line 3: I: Nou hoekom kon jy dit nie uitwerk nie?

R: Juffrou ek weet dan nie hoe lank is al die kinders nie?(Trek moedeloos haar skouers op.)

Line 4: I: Kyk na die som. Dit vra wie is die oudste en wie is die jongste en dit gee ook geen syfers nie, maar jy het baie goed die probleem opgelos. Hoekom?

R: Juffrou het mos gesê as ek nie weet wat die antwoord is nie moet ek 'n tekening van die som maak. (Baie huiwerig. Sy voel onseker van haarself.)

Line 5: I: Hoe weet jy wat om te teken?

R: Ek teken wat die storiesom sê.

Line 6: I: Wat sê die storiesom?

R: Ceril is ouer as Cole. Cole is jonger as Zelda, maar Zelda is ouer as Ceril

Line 7: I: Vertel vir my hoe het jy die storiesom gedoen?

R: Ek het eers Ceril en Cole en Zelda se name neergeskryf.

Line 8: I: Wat het jy toe gedoen?

R: Ek het geteken hoe oud Ceril en Cole is.

Line 9: I: Wat dan van Zelda?

R: Ek het later vir Zelda geteken. Ek het eers vir Ceril ouer as Cole geteken.

Line 10: I: Hoe het jy dit gedoen?

R: Ek het vir Ceril 10 lyne gegee en vir Cole 7 lyne omdat hy jonger as Ceril is.

Line 11: I: Dis goed, gaan aan.

R: Zelda is ouer as Ceril daarom het ek vir Zelda 11 lyne gegee, nou is sy die oudste.

Line 12: I: Dis oulik gedoen. As ek nou vir jou die eerste storiesom gee om te doen sal jy dit kan doen?

R: Ja, Juffrou.(vol versekering.)

Interview 2

Naam:	**Angelo**	**Vak:**	**Wiskunde**
Ouderdom:	**8 jaar**	**Stadium:**	**Probleemsomme**
Standerd:	**Graad 2 (Sub. B)**	**Datum:**	**20 September**

Probleem: **Daar is 42 blokkies. Antonio het 5 blokkies meer as Eileen en 7 blokkies minder as Marco. Hoeveel blokkies kry elkeen?**

Line 1: I: Angelo, hoekom kon jy nie die probleemsom gedoen het nie? (Verwys na "pre-test")

R: Dis 'n lang som juffrou en ek kan nie onthou nie (baie moedeloos).

Line 2: I: Maar kyk! jy het dan so goed die probleem opgelos en dit is net so 'n lang som.

R: Ja! (Met 'n uitdrukking van redunasie op sy gesig.) Maar ek het geweet wat om te doen.

Line3: I: O! en wat moes jy doen?

R: Ek kyk eers hoeveel kinders is in my som, nou skryf ek hulle name af.

Line 4: I: Waarom doen jy dit so?

R: Dan kyk ek wat sê die sin van elke kind en nou werk ek my antwoord uit. (Vol selfversekering.)

Line 5: I: Wat sê die storiesom hoeveel blokkies het elke kind?

R: Nee juffrou die som sê nie, maar ek weet altesaam is daar 42 blokkies. (Baie gretig om te verduidelik.)

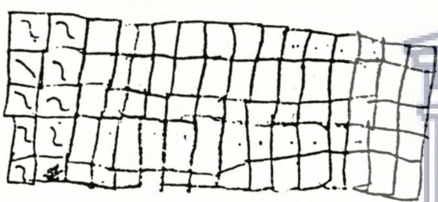
Line 6 I: Wat het jy met die 42 blokkies gemaak ?

R: Ek deel dit eers tussen die 3 kinders.

Line 7:	I:	Maar ek lees nêrens dat jy die blokkies moet deel tussen 3 kinders nie?
	R:	(Lang stilte) As elkeen dieselfde het dan kan ek vir Antonio 5 meer gee en vir Marco 7 meer.
Line 8:	I:	Dis reg, maar waar kry jy die 5 blokkies om vir Antonio 5 meer te gee?
	R:	Ek vat dit by Eileen want sy het nou 5 minder as Antonio.
Line 9:	I:	Hoekom gee jy vir Marco 7 meer?
	R:	Want Antonio het 7 blokkies minder as Marco. Ek het die 7 blokkies van Antonio gevat en vir Marco gegee. Nou het Marco 21 blokkies.
Line 10:	I:	Dis reg ja. Sal jy nou die probleemsom kan doen wat jy nie kon gedoen het nie.
	R:	Ja, juffrou. (Vol selfversekering.)

20 September 1995 Zaida

Die hondehok se dak het 16 teëls di die lang kant en 5 teëls aan die kort kant. As 9 van die teëls gebreek is, hoeveel teëls is heel?



(71)

1	8
2	1
3	2
4	—
5	2
6 -	1

(14)

18 September 1998 Berenice

Ceril is ouer as Cole. Cole is jonger as Zelda, maar Zelda is ouer as Ceril. Wie is die oudste en wie is die jongste.

Ceril Cole Zelda
O O O
9 7 10

ouer jongste oudste

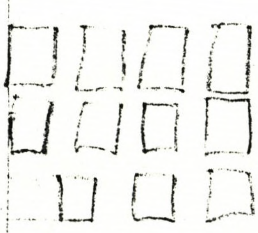
Zelda is die oudste en Cole is die jongste

1	3
2	2
3	2
4	—
5	—
6	—

12

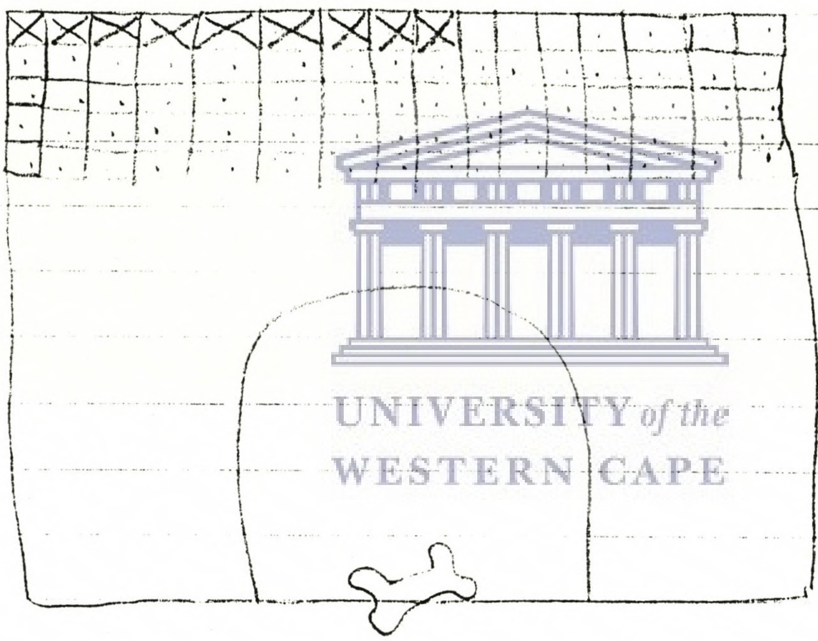
17 Augustus 1995 Kevin

'n Sjokolade het 4 blokkies aan die kort kant en 7 blokkies aan die lang kant. Cyril gee die helfte van die sjokolade vir sy maat. Hoeveel blokkies sjokolade bly oor?



18 September 1995 Kevin

Die hondehok se dak het 16 teëls aa die lang kant en 5 teels aan die kort kant. As 9 van die teëls gebreek is, hoeveel teëls is heel?



$80 - 9 = 71$

1	8
2	2
3	2
4	2
5	2
6	2

(18)

17 Augustus 1995 Moricia

Daar is 36 lekkers. Paul het dieselfde aantal lekkers as Mark. Cole het 5 lekkers meer as Paul en Mark. Hoeveel lekkers het elkeen?

12

(4)

1	0
2	4
3	0
4	0
5	0
6	0

18 September 1995 Moncia

Daar is 42 blokkies. Antonio het 5 blokkies meer as Eileen en 7 blokkies minder as Marco. Hoeveel blokkies kry elkeen?

Antonoi	Eileen	Marco
10	10	10
3	3	3
1	1	1
14 7 12	14 9	14 7 21

1	4
2	2
3	2
4	2
5	1
6	2
	13

5 14

Ek het die 42 blokkies Ek gee vir Antonoi 14 en Eileen 14 en Marco 14 en Antonoi vat van Eileen 5 weg en gee vir Marco 7

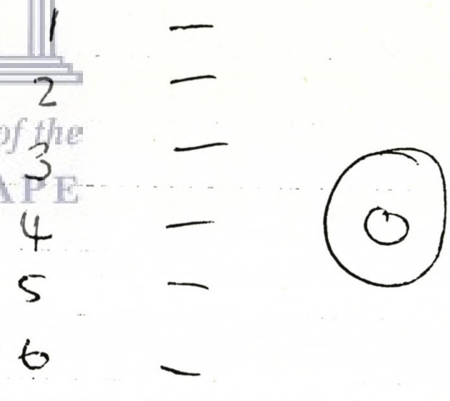
Antonoi kry 12 Eileen kry 9 Marco 21

18 Augustus 1995 Grizilda

Paul is korter as Mark, maar langer as Kevin, Kevin is korter as Paul maar langer as Cole. Wie is die langste en wie is die kortste?

Paul is die lanste.

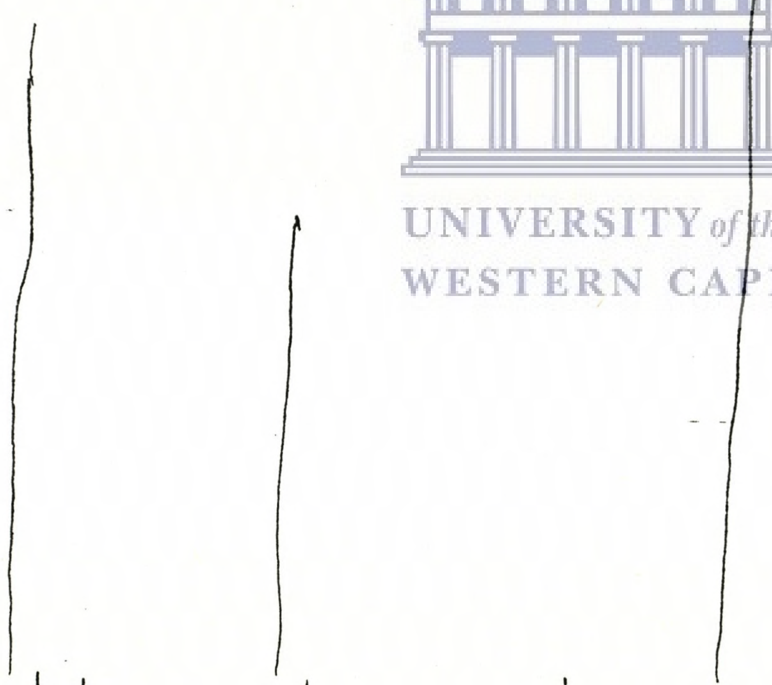
Kevin is die kortste.



19 September 1995 Grizilda

Ceril is ouer as Cole. Cole is jonger as Zelda, maar Zelda is ouer as Ceril. Wie is die oudste en wie is die jongste?

Ceril Cole Zelde



Zelde is die oudste.
Cole is die jonste.

1 8
2 2
3 –
4 –
5 2
6 –

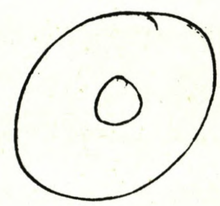
(12)

17 Augustus 1995 Angelo

Daar is 36 lekkers. Paul het dieselfde aantal lekkers as Mark. Cole het 5 lekker meer as Paul en Mark. Hoeveel lekkers het elkeen?

210

1 —
2 —
3 —
4 —
5 —
6 —



19 September 1995 Angelo

Daar is 42 blokkies. Antonio het 5 blokkies meer as Eileen en 7 blokkies minder as Marco. Hoeveel blokkies kry elkeen?

Antonio	Eileen	Marco
~~10~~	~~10~~	~~10~~
~~4~~	~~4~~	~~4~~
~~14~~ 7	~~14~~ 9	~~14~~
~~5~~	9	~~7~~
12		21

1	4
2	2
3	2
4	2
5	3
6	3

(16)

Daar is 42 blokkies ek gee elkeen 10 en daar is nog 12 toe gee ek elkeen 4 nou het elkeen 14 nou gee ek vir Antonio nog 5 meer

nou vat ek 5 weg van Eileen
nou het Antonio 12 en Eileen
het 9 Antonio het 7 blokkies
meer as Eileen Antonio het 12
en Eileen het 9 en Marco
het 21.

18 September 1995 Raymonde

Op 'n bord is 26 koekies Mary eet die helfte van die koekies en Joan eet 4 mind as Mary. Hoeveel koekies bly oor vir Anne?

18 0

21 September 1995 Raymonde

Mary het 28 lekkers Sy hou die helfte vir haar en gee 4 vir Sally. Hoeveel lekkers kan sy vir Mark ge

Mary Sally Mark

14 4 10 ½ van 28 - 14

20 = 10

8 - 4

15

1 8

2 2

3 2

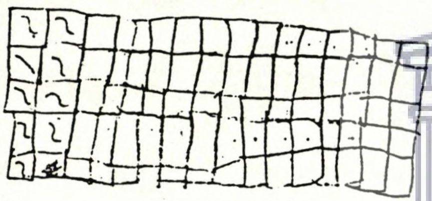
4 1

5 2

6 —

20 September 1995 Zaida

The roof of the dog's kennel has 16 tiles on the long side and 5 tiles on the short side. 9 of the tiles are cracked. How many whole tiles are there?



(71)

1	8
2	1
3	2
4	—
5	2
6 -	1

14

18 September 1995 Berenice

Ceril is older than Cole. Cole is younger than Zelda but Zelda is older than Ceril. Who is the eldest and who is the younges

Ceril	Cole	Zelda
O	O	O
9	7	10
oldest	youngest	eldest

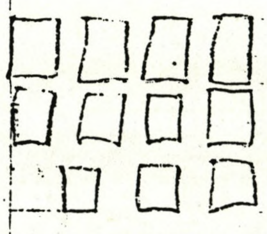
Zelda is the eldest and Cole the youngest

1 8
2 2
3 2
4 –
5 –
6 –

(12)

17 Augustus 1995 Kevin

A chocolate has 4 blocks on the short side and 7 blocks on the long side. Cyril gave half of the chocolate to his friend. How many blocks are left over?

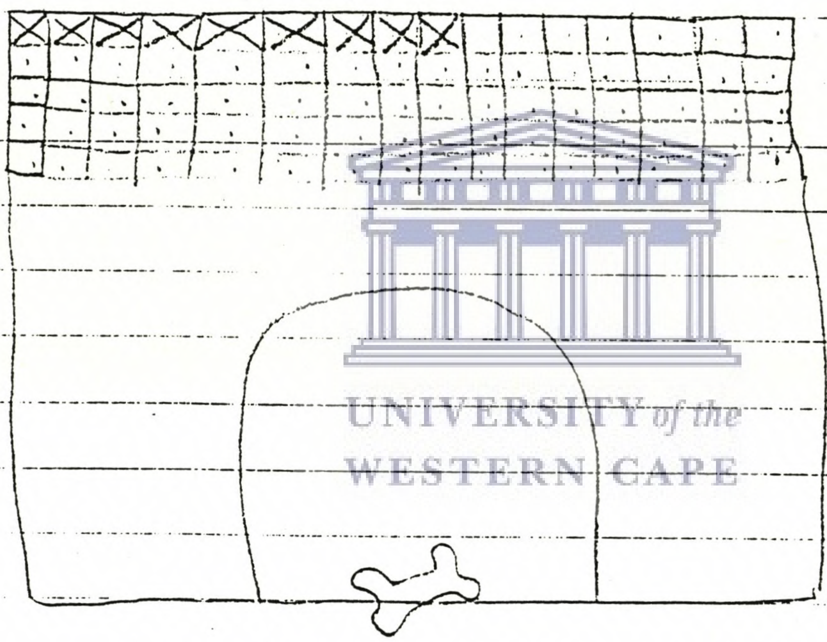


(3)

1 —
2 2
3 — (2)
4 —
5 —
6 —

18 September 1995 Kevin

The roof of the dog's kennel has 16 tiles on the long side and 5 tiles on the short side. 9 of the are cracked. How many whole tiles are there?



80 - 9 = 71

1	8
2	2
3	2
4	2
5	2
6	2

18

17 Augustus 1995 Moricia

There are 36 sweets. Paul has the same amount of sweets as Mark. Cole has 5 sweets more than Paul and Mark. How many sweet do each have?

12

X

1	0
2	4
3	0
4	0
5	0
6	0

18 September 1995 Moricia

There are 42 blocks. Antonio has 5 blocks more than Eileen and 7 blocks less than Marco. How many blocks do each one have?

Antonio	Eileen	Marco
10	10	10
3	3	3
1	1	1
~~14~~ 7 5 19 12	~~14~~ 9	14 7 21

1	4
2	2
3	2
4	2
5	1
6	2
	(13)

I have 42 blocks. I gave 14 to Antonio and 14 to Eileen and 14 to Marco. Antonio took 5 from Eileen and gave 7 to Marco. Antonio get 12, Eileen 9 and Marco 21.

18 Augustus 1995 Grizilda

Paul is shorter than Mark, but taller than Cole. Who is the tallest and who is the shortest?

Paul is the tallest.
Kevin is the shortest

1 —
2 —
3 —
4 —
5 —
6 —

0

19 September 1995 Grizilda

Ceril is older than Cole. Cole is younger than Zelda but Zelda is older than Ceril. Who is the eldest and who is the youngest?

Ceril Cole Zelde

(12)

1. – 8
2. – 2
3. —
4. —
5. – 2
6 —

Zelde is the eldest
Cole is the youngest.

17 Augustus 1995 Angelo

There are 36 sweets. Paul has the same amount of sweets as Mark. Cole has 5 sweets more than Paul and Mark. How many sweet do each have?

210

1 —
2 —
3 —
4 —
5 —
6 —

0

19 September 1995 Angelo

There are 42 blocks. Antonio has 5 blocks more than Eileen and 7 blocks less than Marco. How many blocks do each one have?

Antonio	Eileen	Marco
~~10~~	~~10~~	~~10~~
~~4~~	~~4~~	~~4~~
~~14~~	~~14~~ 9	~~14~~
~~5~~ 7	9	~~7~~
12		21

1	4
2	2
3	2
4	2
5	3
6	3

(16)

There are 42 blocks. I gave each child 10. There are 12 left over. I gave each one 4. Now every child has 14. I gave Antonio 5 more. I took 5 from Eileen. Now Antonio has 12 and Eileen has 9. Antonio has 12, and Marco 21.

18 September 1995 Raymonde

On a plate is 26 biscuits. Mary ate half of the biscuits and Joan ate 4 less than Mary. How many biscuits are left over for Anne?

18 0

21 September 1995 Raymonde

Mary has 28 sweets. She keeps half of the sweets for herself and she gives 4 to Sally. How many sweets are left over for Mark?

Mary	Sally	Mark
14	4	10

½ van 28 = 14

20 = 10

8 = 4

1 8
2 2
3 2
4 1
5 2
6 —

15

Interview 1

Name:	**Grizilda**	**Subject:**	**Mathematics**
Age:	**8 years**	**Stage:**	**Problem sums**
Standard:	**Grade 2 (Sub. B)**	**Date:**	**20 September**

Problem: Ceril is older than Cole. Cole is younger than Zelda. Who is the eldest and who is the youngest?

Line 1: I: You could not solve this problem. Why not? (refer to pre-test).

R: I don't understand this sum.It is not a sum, it has no numbers.

Line 2: I: What does the story sum want you to find out?

R: Who is the tallest and who is the shortest?

Line 3: I: Why could'nt you work that out?

R: I don't know the height of the children? (very despondent).

Line 4: I: Look at this story sum.You must find out who is the eldest and who is the youngest? This sum has no numbers, but you solve it. Why?

R: Teacher you said if I cannot solve a problem I must make a drawing of what the sum ask (very hesitant. She feel very unsure).

Line 5: I: How do you know what to draw?

R: I draw what the story sum says.

Line 6: I: What do the story sum say?

R: Ceril is older than Cole. Cole is younger than Zelda, but Zelda is older than Ceril.

Line 7: I: Tell me how you solved this story sum?

R: I first wrote down Ceril, Cole and Zelda's names.

Line 8: I: What did you do then?

R: I made a drawing of how old Ceril and Cole are.

Line 9: I: What about Zelda?

R: I drew Zelda last. I first drew Ceril older than Cole.

Line 10: I: How did you do that?

R: I gave Ceril 10 lines and for Cole I gave 7 lines because he is older than Ceril.

Line 11: I: That's good. Go on.

R: Zelda is older than Ceril that's why I gave Zelda 11 lines. She is the eldest.

Line 12: I: You did that very well. Will you be able to do the first story sum now?

R: Yes! Teacher.

Interview 2

Name:	**Angelo**	**Subject:**	**Mathematics**
Age:	**8 years**	**Stage:**	**Problem sums**
Standard:	**Grade 2 (Sub. B).**	**Date:**	**20 September**

Problem: There are 42 blocks. Antonio has 5 blocks more than Eileen and 7 blocks less than Marco. How many blocks do each one have?

Line 1: I: Angelo, why could'nt you solve this story sum?(refer to pre-test).
R: This sum is too long. I cannot remember.(very dispondent).

Line 2: I: Look! You solved this problem very well and this is also a long sum.

R: Yes! (He tried to reason).But I knew what to do.

Line 3: I: Oh! And what were you suppose to do?

R: I must first find out how many children is mentioned in the sum, then I write down their names.

Line 4: I: Why did you do it this way?

R: Now I find out what the sentence says about child and then I work out my answer.

Line 5: I: What does the story sum say, how much blocks does each child have?

R: No teacher, the sum does'nt say how much, but I know there are 42 blocks altogether.(very eager to explain).

Line 6: I: What did you do with the 42 blocks?

R: I shared it between the three children.

Line 7: I: But nowhere do I read that you must share the blocks between the 3 children.

R: (silence). If every child have the same amount then I can give Antonio 5 more and for Marco 7 more.

Line 8: I: That is correct , but where will you get the 5 blocks to give to Antonio?

R: I,ll take it from Eileen, then she will have 5 less than Antonio.

Line 9: I: Why do you give Marco 7 more?

R: Because Antonio has 7 blocks less than Marco. I took the 7 blocks from Antonio and I gave it to Marco. Now Marco has 21 blocks.

Line 10: I: That is correct. Will you be able to do the first story sum now?

R: Yes, teacher. (with self- confidence).