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Investigating an integrated teaching methodology as a means to prepare students for university studies in Mathematics

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A minithesis submitted in partial fulfilment of the requirements for the degree of Magister Scientiae (Mathematics Education) in the Department of Mathematics, University of the Western Cape.

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Investigating an integrated teaching methodology as a means to prepare students for university studies in mathematics.

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Integrated lessons

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ABSTRACT

Investigating an integrated teaching methodology as a means to prepare students for university studies in mathematics.

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A key issue for the success of students entering a first year mathematics course at tertiary level is whether or not they have an integrated understanding and view of the mathematical concepts acquired at school. Various integrated applications from first year mathematics suggest that a compartmentalised view of mathematics would be detrimental to any student's chances of passing mathematics at this level. In Differential Calculus for example, other concepts within mathematics are needed, such as geometry, graphs, etc. This study tries to assess whether learners do have an integrated understanding of mathematics at grade 12 level. With the use of assessment instruments it was found that learners found it hard to integrate their knowledge and concepts in mathematics even after integrated lessons were given to an experimental group. The research suggests that learners would be able to benefit from an integrated methodology of teaching mathematics if it is the objective of the course in general.

November 2005

<u>DECLARATION</u>

I declare that *Investigating an integrated teaching methodology as a means to prepare students for university studies in Mathematics* is my own work, that it has not been submitted before for any degree or examination in any other university, and that all sources I have used or quoted have been indicated and acknowledged as complete references.

REGINALD RAYMON CEASAR **November 2005**

Signed:……………………………..

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To my Lord, for giving me the opportunity and strength to study.

Chapter 1

Background

1.1 Introduction

Since 1994, education in South Africa has become a major concern for everybody. This chapter will look at some of the problems facing education currently and also look at the type of results being produced in general and specifically in mathematics. This will hopefully set the scene for a better understanding of the challenges facing the country.

In his speech at the opening of parliament in 2004, President Thabo Mbeki stressed government's determination to get the skills development strategy working. In a wide-ranging interview with the Financial Mail (*FM* 2004), the Minister of Education, Ms. Naledi Pandor, said that the department has given itself until 2007 to start injecting highly skilled and trained workers into the economy. She said that the challenge starts at school level which, despite 10 years of government intervention, still produces poor matric passes, (Naidoo: 2004).

So why would the South African government be so serious about skills development? The answer lies in the fact that the world around us is changing. Ordinary citizens are faced with the challenge of making sense of the world that they find themselves in. More and more information required by ordinary citizens is represented in a more scientific way. Communication takes place on an advanced level compared to about ten years back. This is of course a global trend and if South Africa is to integrate successfully into this global society, the country needs to adapt to meet new challenges. Since the global information technology revolution, the skills needed by the workforce shifted from manual, physical labour, to that of analysing, optimizing and critical thinking. In order to produce workers of such calibre, governments had to change their schooling curriculum and especially that of mathematics. This subject lends itself to enhancing the skills required in this new era we find ourselves in.

1.2 So what is the current status in terms of the need for more skilled people in South Africa?

A report by Andre Kraak (2004) from the Human Science Research Council, highlighted the need for more skilled labour in South Africa. Diagram 1.1 below, gives an estimation of the type of jobs companies had difficulty to recruit workers for in 2000.

Continued dependence on intermediate level skills

Occupational areas in which difficulties are experienced in recruiting qualified personnel, 2000

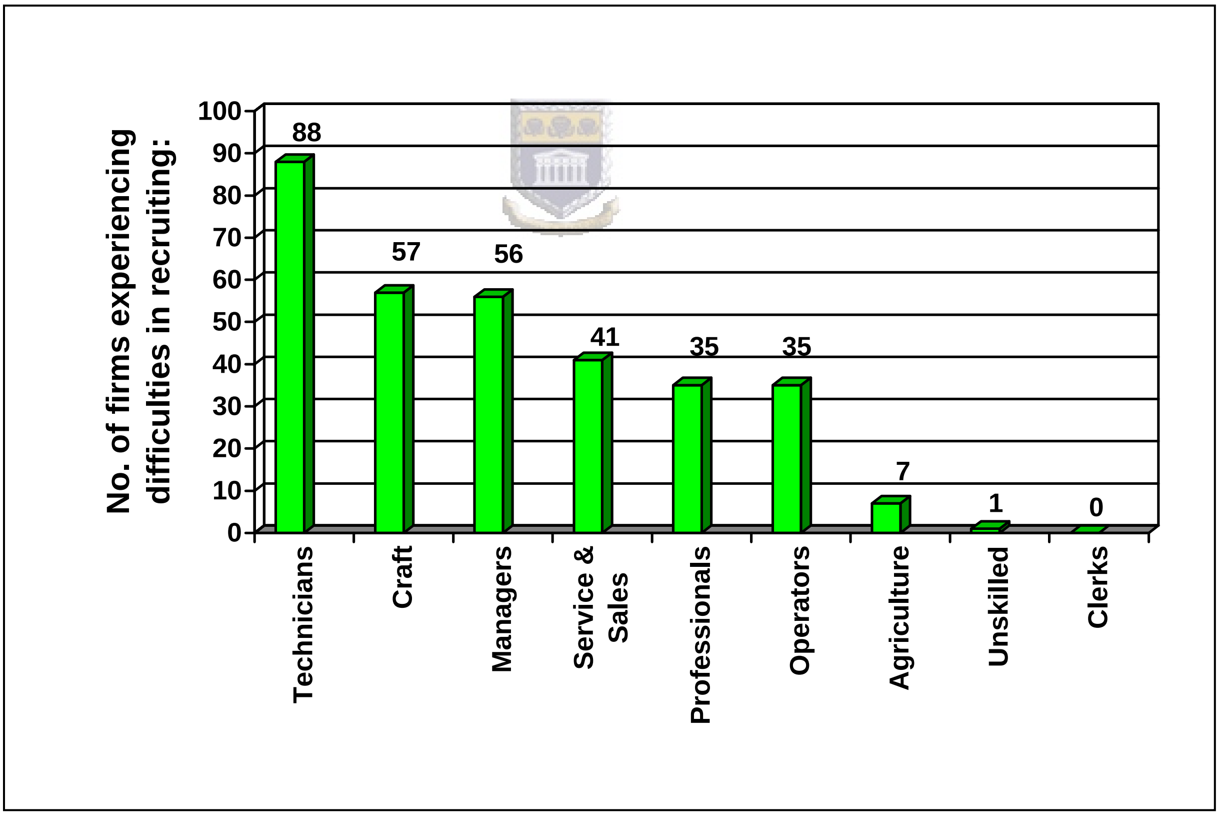


Diagram 1.1**:** Statistics for recruiting qualified personnel in 2000 – Kraak (2004)

From diagram 1.1, it is evident that South Africa experienced a shortage of skilled workers in 2000. Firms had little to no difficulty in recruiting workers for unskilled labour in 2000. In table 1.1 below, shortages are estimated for essential professions for the years 2001 to 2006.

Number of workers needed to meet new and replacement demand in selected occupations, 2001-2006

Occupations	Annual shortages rate as a percentage of the total number of professionals employed
Academics	3.6
Doctors	3.0
Nurses	4.6
Computer-related professionals	4.1
Teachers	4.1
Engineers	3.4
Engineering technologists	3.7
Managers	3.2

Table 1.1: Estimated annual shortages of professionals employed between 2001 to 2006 – Kraak (2004)

According to Kraak (2004), this does not constitute a skills crisis. It can however become a crisis if the country does not have the workers to meet these new and replacement demands. Workers retire, go look for greener pasture abroad etc, thus creating a need in these respective occupations. From diagram 1.1 it is evident that fewer workers in the lower skills category are needed and more for the intermediate and high skilled labour.

1.3 So how many people enter in the labour market at low skills, intermediate and high skills levels?

Diagram 1.2 (below) by Kraak (2004) attempted to answer the above question.

MANIFESTATION OF DYSFUNCTION

Average annual through-flow of school-leavers entering the youth labour market for the first time, 2000-2002

Supply-side dynamics: inflows

Summary: An estimated 1 011 000 youth leave school each year and enter the labour market for the first time.

An estimated 551 000 youth drop out of school between Grade 1 and Grade 11 per year.

An estimated 170 000 Grade 12 students fail the Senior Certificate examination per year.

An estimated 290 000 Grade 12 students pass the Senior Certificate examination per annum, with and without exemption.

Supply-side dynamics: outflows

An estimated 90 000 to 120 000 school-leavers enter private HE, public and private FET, and pre-employment training.

An estimated 65 000 Grade 12 school-leavers enter Higher Education each year.

Summary: An estimated maximum of 185 000 school-leavers enter higher education and other forms of post-school study each year.

Demand-side dynamics: The total number of school-leavers seeking jobs

Summary: an estimated 826 000 youth arrive on the labour market each year, having completed Grade 12 or having dropped out of education and who now seek a job.

Employment rates of new entrants into the labour market, by race:
29 per cent of African new entrants will get jobs.
50 per cent of Coloured new entrants will get jobs.
70 per cent of Indian new entrants will get jobs.
75 per cent of White new entrants will get jobs.

Diagram 1.2: Manifestation of dysfunction in South African schools – Kraak (2004)

From diagram 1.2 it is evident that only 6,4 % (65 000 out of the 1 011 000) enter Higher Education each year while 18,3 % (185 000 out of the 1 011 000) enter Higher Education or other forms of post-school study each year. That means that 18,3 % of school leavers might enter the job market at intermediate or high skills level. A whopping 81,7 % (826 000 out of the 1 011 000) learners enter the labour market at low skills level having passed grade 12 or dropped out of the education system.
The last part of diagram 1.2 on the employment rate by race reveals that those entering the labour market at the low skills level do not have equal chances of finding employment.

The Sunday Times reported (Ntsingila: 2004) that research reveals that white matriculants were 4 times more likely to find a job after leaving school than their black counterparts. Experts attribute the trend to white students having access to better schools, a general decline in mathematics and science standards and the continued uneven distribution of education resources. In this article, Paul Lindall, a senior researcher at the development policy research unit at the University of Cape Town was quoted saying:
"It is most likely that a higher proportion of the white and Indian graduates have matriculation passes in mathematics and science. In the case of African matriculants, the data will probably show a large proportion who have poorer skills in mathematics and science."

1.4 What type of results were produced at school level?

Diagram 1.3 (below) by C. Lotter (2004) from the Matriculation board, describes the quality of the results recently produced by the education system.

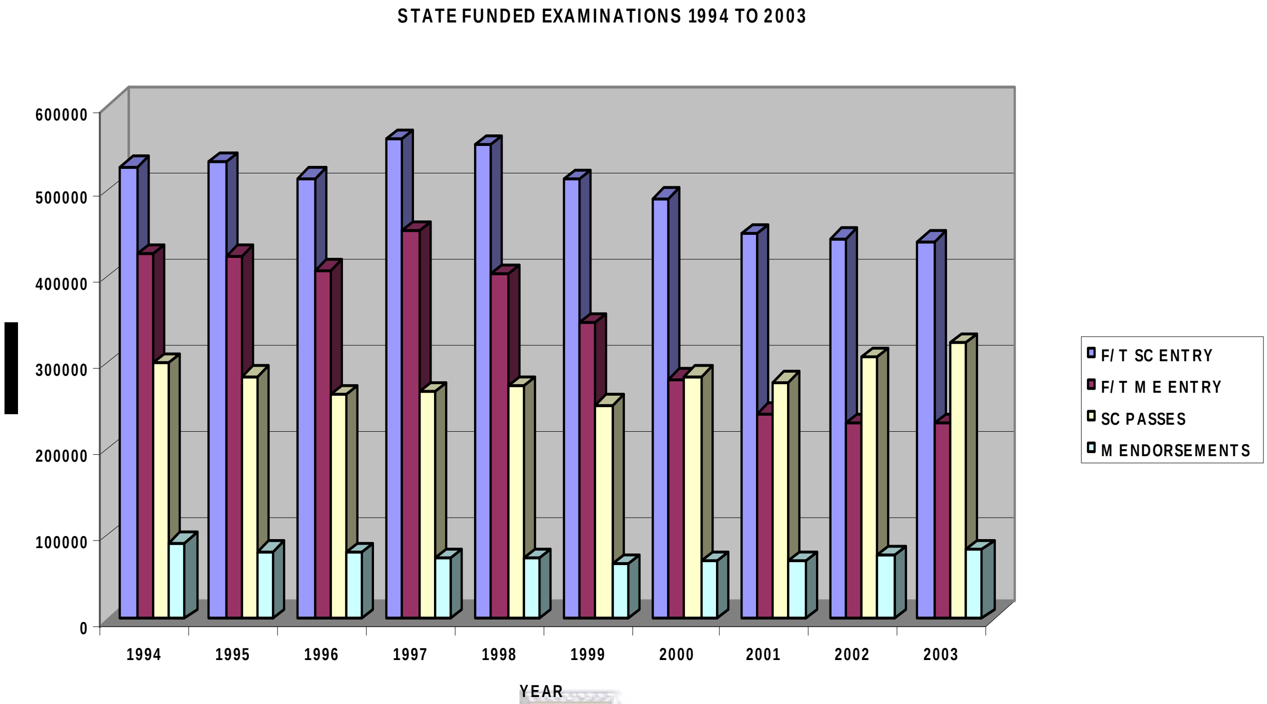


Diagram 1.3: State funded examination results – Lotter (2004)

Although the over-all pass rate has improved, the number of matric endorsements still remained fairly low. Less than 30 % of those candidates who wrote the matric examination passed with matric endorsements. It was highlighted that the average drop-out rate between grades 1 to 11 is about 3 %. This is alarming since 3 % of the total school going population will give you a considerable number. These learners enter the labour market on an annual basis at the low skilled level which is already saturated.

As this study is about mathematics, the discussion will continue by looking at the type of results produced in mathematics by our current education system. Diagram 1.4 (below) by C Lotter (2004), illustrates what mathematics results produced in South Africa were.

Mathematics 1997-2003
(C Lotter, Matriculation Board)

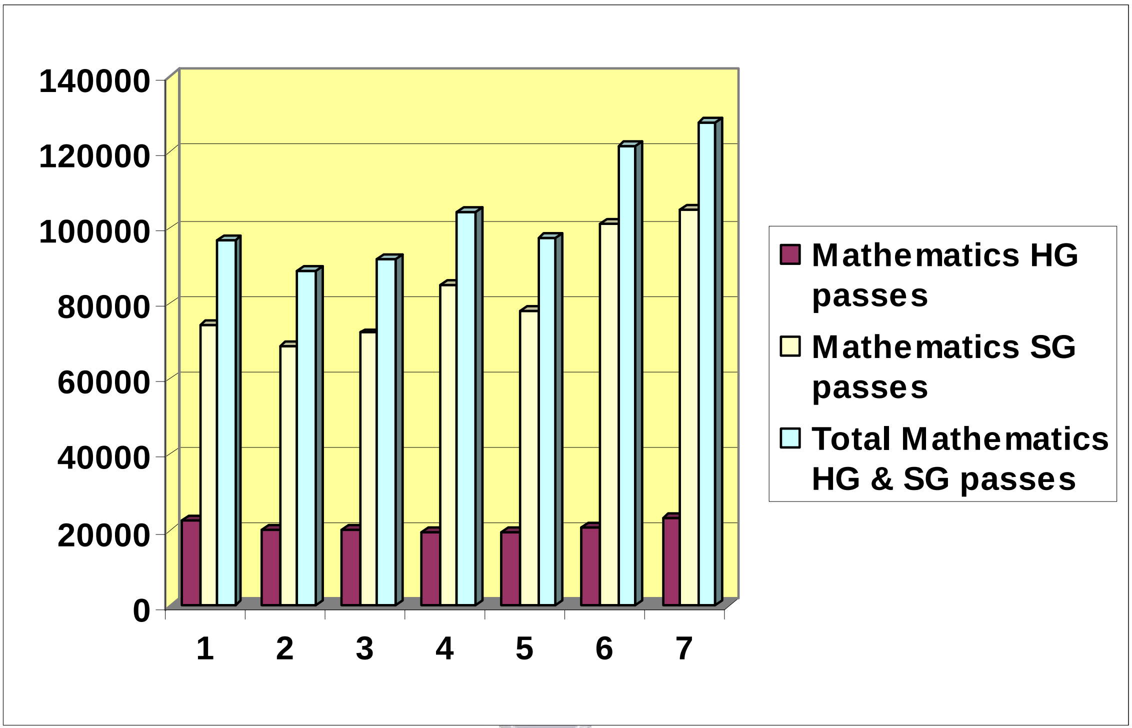


Diagram 1.4: Mathematics results from 1997 up to 2003

According to "Die Burger" (Rademeyer: 2005, p. 1), 24 143 candidates passed mathematics on the higher grade during the 2004 examination. From diagram 1.4 we can see that the number of students passing mathematics as a subject has increased. This was because more candidates passed the subject on standard grade. This number increased between 1997 to 2003, indicated by the numbers 1 to 7 in diagram 1.4. The number of candidates passing the subject on higher grade however did not change much over the period from 1997 to 2003. Over this period the percentage of learners, passing mathematics on higher grade, dropped from about 20 % in 1997 to about 16,7 % in 2003.

In diagram 1.5 (below) the Institute for Justice and Reconciliation (Naidoo: 2004), looked at the matric results and specifically the mathematics results produced by the different population groups.

Diagram 1.5: Matric mathematics results produced in 2004

NUMBER CRUNCHING

Population-group share of matriculants by performance category 2004

	African %	Coloured %	Indian %	White %	Unknown %	Total %
Matric-aged cohort	83,2	7,8	2,2	6,8		100
Matric candidates	77,4	7,2	3,8	10,5		100
Drop-outs	88,0	8,2	0,9	3,8		100
Matric passes	71,2	8,7	4,9	14,2	1,1	100
Endorsements	51,4	6,7	10,9	29,2	1,8	100
A-aggregates	8,4	4,1	18,8	65,5	3,2	100
Maths passes	65,2	7,0	6,7	19,8	1,2	100
HG maths passes	34,9	5,4	14,3	43,2	2,2	100
HG maths passes: A, B, C or D	22,4	5,3	16,3	53,3	2,7	100

Source: INSTITUTE FOR JUSTICE AND RECONCILIATION TRANSFORMATION AUDIT 2004

Diagram 1.5 shows that the majority (77,4 %) of the matriculants come from the African population group. Out of those who passed, 71,2 % were from this population group. This did not look too bad at all. Now looking at which population group produced the better quality results, the breakdown for matric endorsements will be studied. Although 71,2 % of those who passed were from the African population, only 51,4 % of those passes were matric endorsements. Most of the mathematics higher grade passes, 43,2 %, came from the white population group which only made out 10,4 % of all the matric candidates. Diagram 1.5 also shows that the better quality results for mathematics higher grade were also produced by the white population group. 53,3 % of those who obtained symbols A, B, C and D on higher grade were from this population group.

1.5 What are possible reasons for the above inequalities?

The Business Day (Blaine: 2004) reported that experts said mother tongue learning will raise standards and that language barriers were to blame for tertiary failures. One of the experts interviewed, Prof. Zubeida Desai, from the University of the Western Cape, favours mother tongue instruction until at least grade seven. She cited research which showed that it takes seven years to become sufficiently fluent in a second language before it can be used as medium of learning. Anstrom (1997: 24) prescribes standards of mathematics education which:

"…emphasise communication and discourse within the context of mathematical problem solving. The standards explicitly recommend that teachers pose questions and design tasks that engage students' thinking and ask students to clarify and justify ideas orally and in writing."

Anstrom (1997: 24) further states that:

"Command of mathematical language plays an important role in the development of mathematical ability. The importance of language in mathematics is often overlooked in the mistaken belief that mathematics is somehow independent of language proficiency. However, particularly with the increased emphasis placed on problem-solving, command of mathematical language plays an important role in the development of mathematical ability. Mathematics vocabulary, special syntactic structures, inferring mathematical meaning, and discourse patterns typical of written text, all contribute to the difficulties many second language students have when learning mathematics in English."

In South Africa, matric examination papers are set in two languages only (except those of languages), that is Afrikaans and English and these are the languages used by most secondary schools as their medium of instruction. That means learners whose mother tongue was either English or Afrikaans did have an advantage. The apartheid government of the past, spent huge amounts of money to set up infrastructures for these two languages.

Unfortunately, the same has not happened for any of our country's African languages. Looking at diagram 1.5 again, most African populations are not using Afrikaans or English as their mother tongue. They are however expected to write examinations in their second or third language. Tuition takes place in English, Afrikaans or the language that is preferred by the teacher or specific community. Whether taught in English or Afrikaans, these learners are still faced with a language barrier which they have to overcome. Subjects like mathematics and Physical Science are inherently difficult while the problem is compounded by a language barrier.

The other reality is that South African schools are still segregated between the 'haves and the have nots'. According to the encyclopaedia, Microsoft Encarta (2000), 61 % of South Africa's income in 2000 was still in the hands of the white population group which makes up about 13 % of the total population in this country. This has had a huge impact on the differences in the quality of the matric results across the different population groups. Students from this population group attend well resourced schools. Money can be used to appoint more teachers to keep the number of learners in a class low. Extra teachers can be appointed to teach subjects like mathematics or physical science on higher grade. They can use their money to attract well qualified teachers and to buy expensive resources such as software, internet access, etc. If a learner has a problem in a subject, parents pay extra money for private tutoring. These are just some of the things money can buy.

On the other side of the spectrum we have learners attending schools with overcrowded classes. Resources are scarce and a lot of the teachers are under-qualified. These learners also come from the sector of society where unemployment figures are very high. Going home, these learners have to face other social problems, such as malnutrition, AIDS, abuse, violence, etc.

For those who manage to get into Higher Education, the struggle is still not over. In the Sunday newspaper, "RAPPORT" (Mkhabela and Malan: 2004, p. 1), the National Minister of Education, Ms. Naledi Pandor, made public the percentage of first year level students who were failing. In her report, she indicated that almost 40% of students entering first year at tertiary level fail. She also revealed that this costs the taxpayer about one billion rand per annum. In her statement, she made particular mention of previously disadvantaged students at previously advantaged institutions, who are unable to pass. This suggested that learners were not well prepared for higher education by the school system. When considering students that study Mathematics, the question that comes to mind is**:**

1.6 Do students have basic skills and knowledge when they enter the first year mathematics class?

To demonstrate how the dysfunction of the education system in South Africa manifests itself in mathematics, the results of a diagnostic test administered by the Department of mathematics at the University of the Western Cape in 2003 will be looked at. A Mathematics Diagnostic Test was suggested by this department to draft a report, by Fish (2004), in order to put the current crisis in student performance (particularly at the first-year level) in context. A mathematics Diagnostic Test was administered and it was hoped that the following information would be gained from the results:

- the level of competency of students in the knowledge and skills in mathematics skills which were identified as essential for success in the mathematics mainstream course (MAM 111 and 121));
- the extent to which the test can predict performance in MAM 111 & 121 (this could influence the design of a future Placement Test, and can open up discussion around teaching, assessment and other relevant practices);
- the extent to which the test taps abilities which were tapped by the matric exams, which augured well for them to be used as an additional selection instrument, or in the event that formal matric exams were discarded, as an alternative selection instrument.

1.6.1 Test content and level

On the recommendation of two experienced first-year lecturers, the following topics were identified for inclusion in the test: arithmetic, algebraic expressions/equations, relations and graphs, exponents, surds and logs, trigonometry, and "concept" and algebraic manipulation. The test comprised of 30 multiple-choice items. Each question had five possible answers labelled a), b), c), d), and e), but only one of them was correct. The use of calculators or any other calculating aids was prohibited. Students were given one hour to answer all the questions. The test items, with the exception of the item on logs, require competency at the Grade 11 or lower level as shown in Table 1.2 below:

Table 1.2: Number of questions and grades in which mastery is required – Fish (2004)

No. of questions	Grade in which mastery is required
8	8
5	9
8	10
8	11
1	12

The following is an example of a test item (mastery of this item is required in Grade 9):

$$\frac{a^4 - b^4}{a-b}, \ \textit{where } a \neq b, \textit{ can be simplified to}$$

(a) $a^3 + b^3$ *(b)* $(a+b)(a^2 + b^2)$ *(c)* $a^3 - b^3$

(d) $(a+b)^2(a-b)$ *(e)* $(a-b)(a^2 + b^2)$

As in good testing practice, an attempt was made to sequence the questions according to their level of difficulty. The proficiency test appears in Appendix I, while the mathematics profile and performance of the students in 2003 appear in Appendix II.

1.6.2 Results of the proficiency test

113 students registered for MAM 111 and/or 121 wrote the test at the end of October 2003. It should also be borne in mind that these students exclude those that have either failed or dropped out, and can therefore be classified as the "better" students.

The following table (Table 1.3) summarises the results obtained by the students:

Table 1.3: Break–down of the overall results – Fish (2004)

% range	No. of students	% of students
75.0 – 100.0	8	7.1
70.0 – 74.9	6	5.3
60.0 – 69.9	12	10.6
50.0 – 59.9	16	14.2
40.0 – 49.9	26	23.0
30.0 – 39.9	27	23.8
20.0 – 29.9	16	14.2
10.0 –19.9	2	1.8
0.0 – 9.9	0	0
Total	113	100.0

It is cause for concern that of the 113 students that wrote the test, only 8 (7.1%) obtained 75% or above, and 71 (62.8%) obtained less than 50%. This implied that only 7.1% of the students were very competent in the basic knowledge and skills in mathematics identified as essential for success in MAM 111 and 121.

From the histogram below (Diagram 1.6) it is clear that the results were skewed to the right.

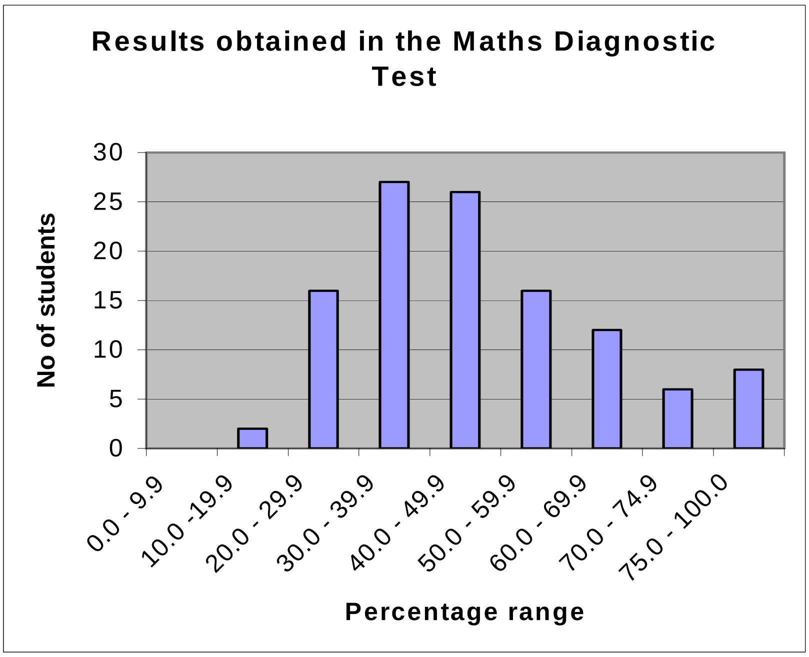


Diagram 1.6: Histogram for Maths Diagnostic Test – Fish (2004)

Table1.4 below describes the basic competencies tested in the various items and the percentage of the students that displayed these. The values in brackets show the percentage of abstentions. Those competencies displayed by fewer than 50% of the students are highlighted.

Table 1.4: Success rate of the basic competencies – Fish (2004)

Basic competency	**% of successful students**
Calculating the values for which a rational function is undefined	85.8 (0.9)
Simplifying an algebraic expression	84.1 (2.7)
Identifying the relationship between variables given a table of values	77.9 (0.0)
Adding and subtracting integers	68.1 (0.0)
Solving a linear inequality	66.4 (15.0)
Simplifying an exponential expression	65.5 (3.5)
Determining whether a point lies on a given line	64.6 (6.2)

Identifying the graph of a given line	62.8 (0.9)
Determining whether two given lines are parallel, perpendicular or equal	57.5 (4.4)
Ordering and subtracting decimal fractions	50.4 (2.7)
Solving a quadratic equation involving fractions	50.4 (14.2)
Factorising an expression containing a common factor and a trinomial	**46.0 (8.8)**
Finding the trigonometric expression for a side in a right-angled triangle	**42.5 (2.7)**
Dividing two decimal fractions	**42.5 (10.6)**
Solving a simple trigonometric equation	**42.5 (7.1)**
Simplifying a radical expression	**42.5 (18.6)**
Changing the subject of a formula	**38.9 (5.3)**
Evaluating a trigonometric expression using special angles	**38.9 (12.4)**
Determining a trigonometric ratio in a right-angled triangle	**38.9 (20.4)**
Finding the sign of the coefficients in a quadratic equation given its graph	**37.2 (3.5)**
Determining the square root of an expression	**35.4 (0.0)**
Using function notation	**34.5 (8.0)**
Determining the relationship between two variables given the relationship between their logs	**31.9 (15.0)**
Factorising an expression involving the difference of two squares	**30.1 (8.0)**
Determining the domain of a function involving a square root	**30.1 (1.8)**
Simplifying a trigonometric expression using trigonometric identities	**30.1 (8.0)**
Simplifying a trigonometric expression using special angles	**27.4 (18.6)**
Determining a set of linear equations from a word problem	**22.1 (19.5)**
Simplifying an expression involving surds	**18.6 (12.4)**
Multiplying and adding fractions according to the order of operations	**8.8 (1.8)**

Grouping the test items according to the topics identified, yields the following results (Table 1.5).

Table 1.5: Mean percentage passing per topic – Fish (2004)

Grouping	Mean %
Arithmetic	43.2
Algebraic expressions/equations	57.7
Exponents, surds and logs	36.2
Relations and graphs	60.0
Trigonometry	37.6
"Concept" and algebraic manipulation	28.8

From Tables 1.4 and 1.5 it was clear that all the topics identified, with the possible exception of algebraic expressions/equations and relations and graphs, need urgent attention. In 19 of the 30 test items fewer than 50% of the students answered correctly, and in 10 items more than 10% of the students abstained. It needs to be emphasised that the test items, with the exception of the item on logs, required mastery well before Grade 12, and hence well before the commencement of tertiary studies. This has serious implications not only for the curriculum and teaching of MAM 111 and MAM 121, but for that of other subjects such as Physics, Chemistry and Computer Science as well, since these may assume an even higher level of competency in the basics of mathematics than required in the test.

The following table (Table 1.6) gives some descriptive statistics for the groupings of test items as well as for the test as a whole. These statistics confirmed the huge backlog in the students' knowledge of exponents, surds, logs and trigonometry, and their inability to solve problems.

Table 1.6: Statistical information for Maths Diagnostic Test – Fish (2004)

Variable	**Mean**	**Std dev**	**Minimum**	**Maximum**
Arithmetic	43.2	26.2	0	100
Algebraic express./equations	57.7	23.1	0	100
Exponents, surds, logs	36.2	26.2	0	100
Relations and graphs	60.0	26.3	0	100
Trigonometry	37.6	28.2	0	100
"Concept" & algebraic man.	28.8	29.1	0	100
Diagnostic test (overall)	45.6	18.3	13.3	100

1.7 Future changes in Education

In 2006, the new National Curriculum Statement will be introduced in grade 10. So, by 2008 the first cohort of learners, who complete further education and training, will receive their NSC (National Senior Certificates). This certificate will be issued in three pathways:

1. NSC General
2. General Vocational NSC, aimed at the college sectors.
3. The NSC aimed at the trade, occupational and professional sectors.

Minimum levels of attainment are prescribed. Assessment will be school-based and external for the grade 12. Candidates scoring 60 (30 – 39 %) or 70 (40 -49 %) credits, will receive an NSC. No differentiation between higher or standard grade will be offered. (Amoore, Griesel, Pityana: 2004). The question that Griesel (2004) asked was whether such learners will be considered for admission and placement in degree studies at a public institution and, if so, what level of support will need to be provided to ensure success in degree studies.

According to Griesel (2004), the value of the new curriculum and the NSC in preparing learners for higher education, will only be known once the first group of students finishes their tertiary education studies. Griesel (2004) suggested:

1. minimum levels of attainment in fundamental and core subjects;
2. additional rules of combinations; and
3. admission and placement tests that are not curriculum-specific but assess the levels of literacy and numeracy, attained, given that these competencies must serve as threshold requirements for entry into higher education studies.

Griesel (2004) says that there are three major advantages to setting benchmarks for higher education study:

1. Applicants, parents, schools, colleges and the public at large will be clear on the expected entry levels to higher education institutions and how they were assessed;
2. Higher education institutions will be clear on the entry level competencies of their students and what needs to be included in "responsive" curriculum programmes; and
3. The formal schooling system will receive useful feedback on the critical competencies students need to develop in order successfully to engage with higher education study.

These, and many other revelations, put tremendous challenges and pressure on tertiary institutions which they cannot ignore. Mogens Niss (1998), outlined very nicely why tertiary institutions cannot afford to be passive in their quest to get more learners to pass mathematics. The following appropriate quote comes from pages 2 and 3:

"In former times, university students were expected to assume responsibility for their success or failure. Students who passed the exams, had 'it', and those who failed, lacked 'it'. This implied that lecturers of mathematics could concentrate on the delivery of their teaching, whereas the individual student's learning of what was taught was not the business of the lecturer but entirely of the student him/herself. The outcome of learning was finally gauged in tests and examinations, and students were filtered accordingly. Today, universities can no longer afford to concentrate their main efforts on students who can. We have to cater for students who are actually able to learn mathematics, if properly assisted where didactical and pedagogical attention is paid to their background, situations, prerequisites and needs. Should we forget … will remind us and to blame us for our autistic arrogance and for our (co)-responsibility of waste of human potential. Whether we perceive these as facts in a hostile or ill-informed world that have to be counteracted, or as a genuine challenge that has to be met, this – second element – points to the need of trying to understand what it is and what it takes to learn mathematics, including the processes involved therein, in particular for students who experience difficulty in this endeavour, and to invent and investigate ways of teaching that are more beneficial and effective to average students than the ones traditionally employed."

1.8 Conclusion

This study endeavours to make a contribution towards the search for possible reasons why first year mathematics students in particular, find it hard to deal with the transition from school level mathematics to that being taught at first year university. The next chapter illustrates the integrated way in which mathematics is approached at first year level and also highlights how the topics covered at school level are used at first year level. Chapter 3 looks at the important issues raised in literature on integrated teaching methodology in Mathematics. Chapter 4 looks at a pre-test, three integrated lessons and a post-test which was administered to a group of learners. The last chapter reviews all the findings of the previous chapters and makes recommendations and suggestions for future studies of this nature.

Chapter 2

The Problem statement

2.1 Introduction

From discussions with experienced lecturers at the University of the Western Cape's Mathematics Department, Prof. Fray, Ms. N. Myburgh and Dr. L. Kannemeyer, the following problems pertaining to first year mathematics students were identified:

- Students appear to have a compartmentalised view of mathematics. They treat mathematical concepts as separate units which have little or no link with each other.
- Because of the above-mentioned, they struggle to cope with mathematics at first year level where different concepts are integrated. At this level, mathematics is no longer treated as separate units but within a particular problem, knowledge from different fields in mathematics can be drawn from. They are expected to have their mathematical knowledge, which they gained from their schooling experience, and be able to draw from any part of that knowledge at any point in time.
- Students lack the ability to integrate graphical representation and algebraic manipulation.
- They were also unable to communicate their ideas both verbally and in writing when they were required to solve problems.

To demonstrate what is meant by the above statements, problems from different 1st year mathematics topics will be looked at.
Here are some examples of the integrated application of mathematical knowledge gained as school level. (Anton: 1988; Stewart: 1997)

2.2 Examples

2.2.1 Improper integrals

In dealing with improper integrals, first year students are confronted with the problem of evaluating certain limits such as $\lim_{t \to \frac{\pi}{2}^-} \ln\left|\sec t + \tan t\right|$.

Consider the example $\lim_{t \to \frac{\pi}{2}^-} \ln\left|\sec t + \tan t\right|$.

In order to evaluate this limit, the student needs to know the behaviour of both the secant and tangent functions at $\frac{\pi}{2}$. This requires knowledge of, amongst others, the graphs of the two functions. But students often don't know how to sketch the graph of $y = \sec x$. This can easily be overcome by making use of the fact that $\sec x = \frac{1}{\cos x}$. But then it is required of the student to know how to deduce the graph of $\sec t$ from the graph of $\cos t$. More generally, it would mean that students were able to deduce the graph of $y = \frac{1}{f(t)}$ from the graph of $y = f(t)$. Now the graphs of $\cos t$ and $\sec t$ are

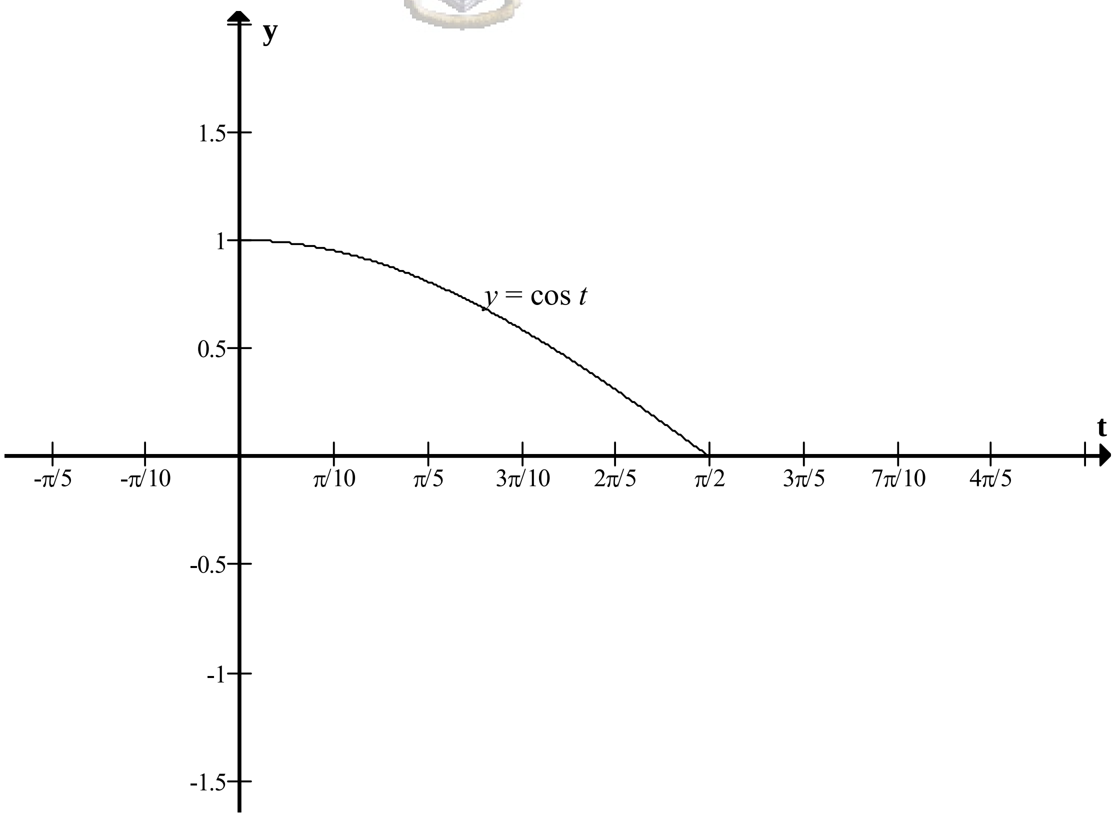


Figure 2.1

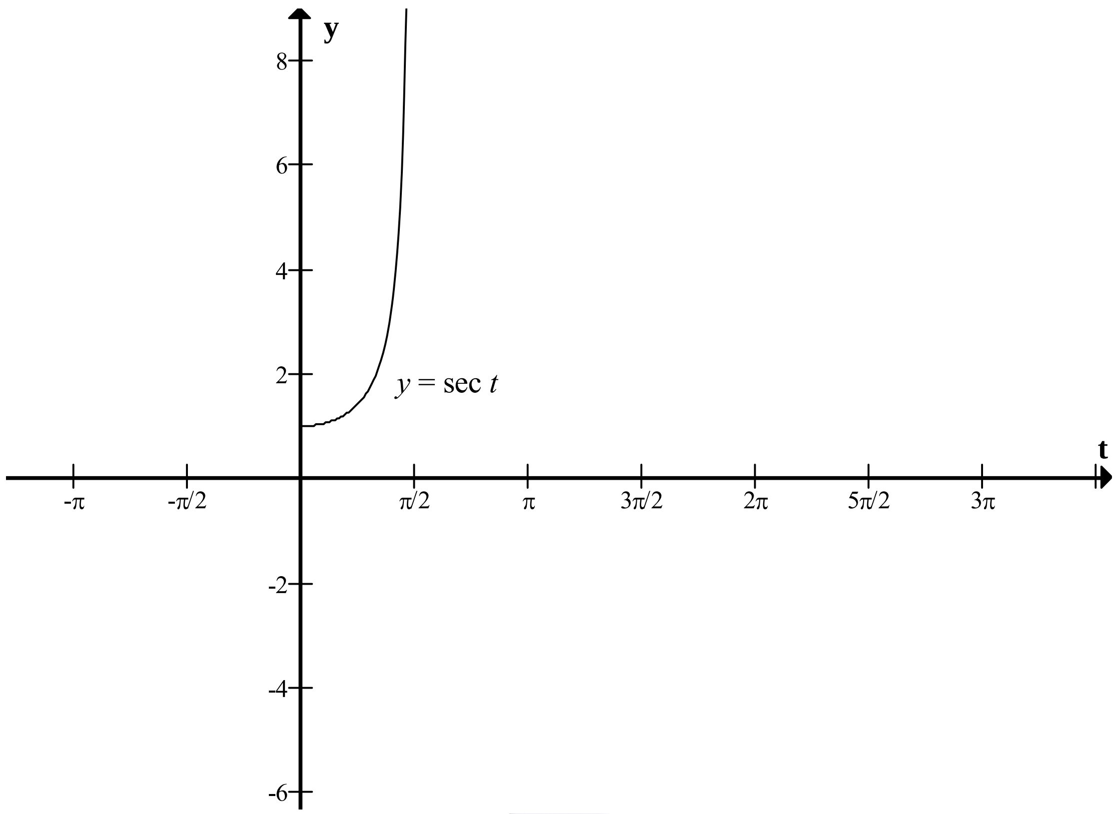


Figure 2.2

The graph of $\tan x$ should be familiar to students.

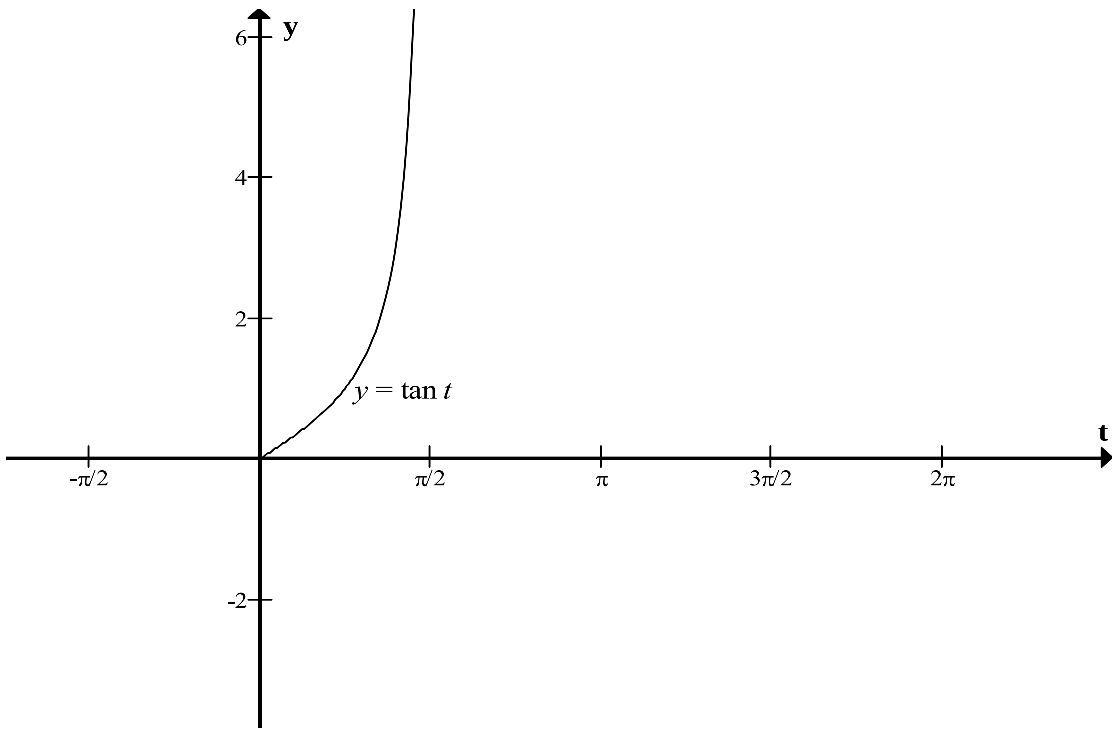


Figure 2.3

Looking at the graphs it can be seen that both $\sec t$ and $\tan t \to \infty$ if $t \to \frac{\pi}{2}^{-}$.

Therefore

$$\sec t + \tan t \to \infty .$$

Finally, the logarithmic function comes into the picture. The graph of $\ln t$ is

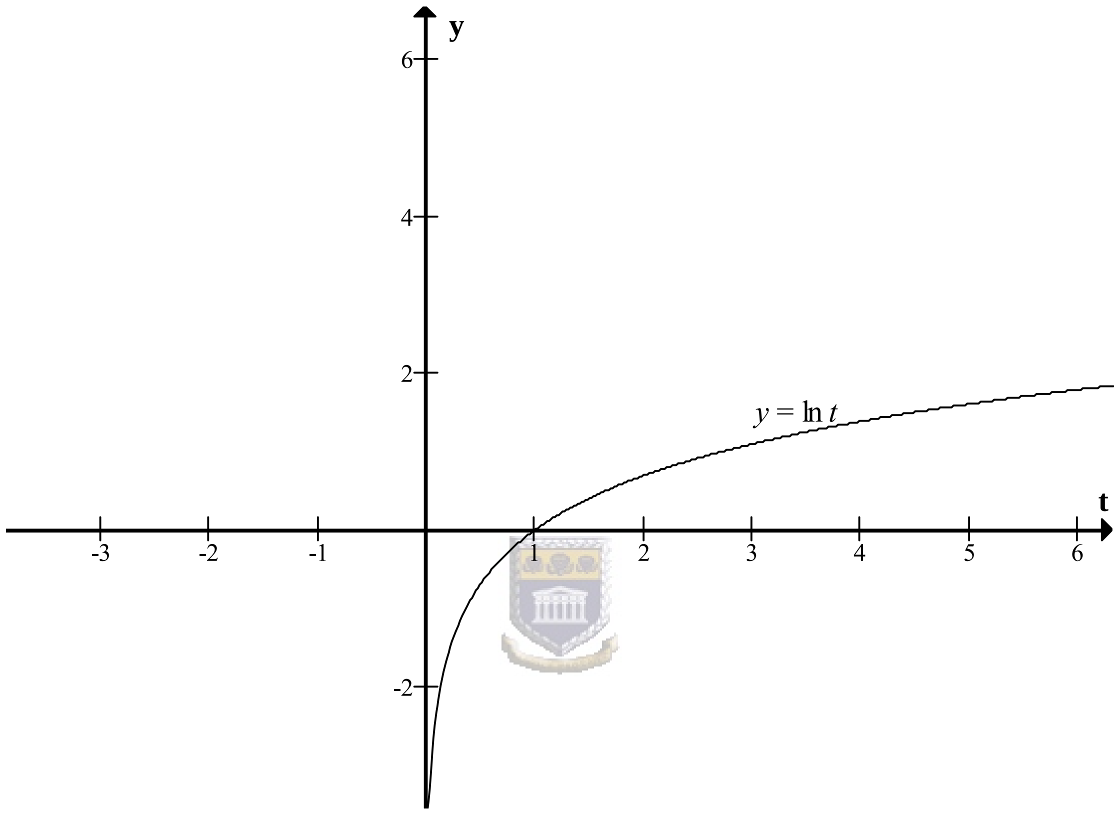


Figure 2.4

$\ln t \to \infty$ as $t \to \infty$. Therefore

$$\lim_{t \to \frac{\pi}{2}^{-}} \ln\left|\sec t + \tan t\right| = \infty$$

Another example, $\lim_{t \to 1^+} \left(-\ln|t-1| \right)$, requires the learners to be able to draw the graph of $|t-1|$.

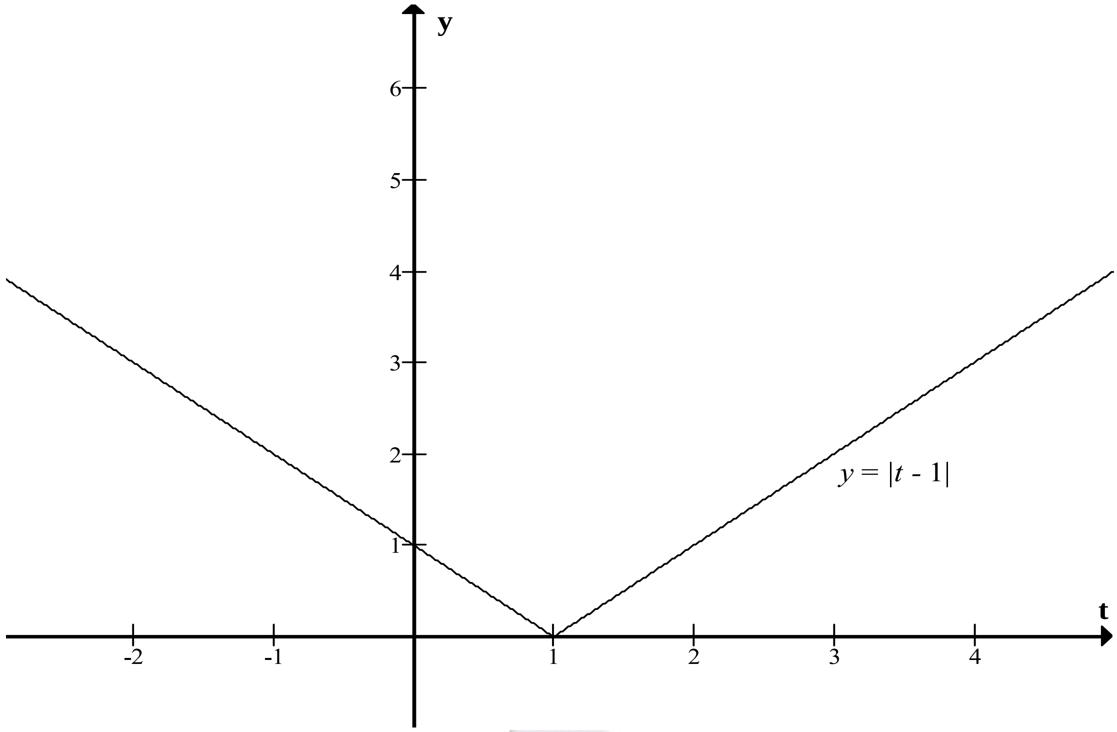


Figure 2.5

From the graph learners can see that $|t-1| \to 0$ if $t \to 1^+$. From the graph of $\ln t$ it can be seen that $\ln|t-1| \to -\infty$ if $t \to 1^+$. Hence

$-\ln|t-1| \to \infty$ if $t \to 1^+$.

In these examples, students must be able to use graphs to evaluate the behaviour of different functions at given points.

2.2.2 Methods of integration

a) In evaluating $\int \frac{1}{\sqrt{x^2 + 8x + 25}}\, dx$, students must be able to perform the following steps:

Step 1: Complete the square, to get

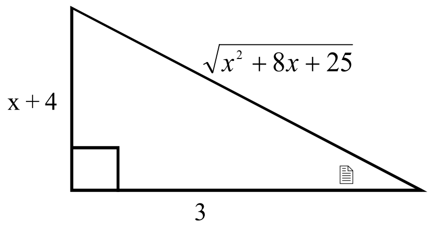
$$x^2 + 8x + 25 = x^2 + 8x + 16 + 25 - 16$$
$$= (x+4)^2 + 9$$

Hence $\displaystyle\int \frac{1}{\sqrt{x^2+8x+25}}\,dx = \int \frac{dx}{\sqrt{(x+4)^2+9}}$

Step 2: Identify a suitable trigonometric identity

In this case: $\tan^2\theta + 1 = \sec^2\theta$ (by putting $x+4 = 3\tan\theta$)

Step 3: Write other trigonometric ratios of θ in terms of x. In order to do this, the following right-angled triangle is completed using Pythagoras and the fact that $\tan\theta = \dfrac{x+4}{3}$.



For example, $\sin\theta = \dfrac{x+4}{\sqrt{x^2+8x+25}}$. It is clear from the above that in evaluating integrals a lot of knowledge gained at school is applied in an integrated way.

2.2.3 <u>Applications of definite integrals</u>

Finding the area enclosed by a number of curves is an application of definite integrals in the first year Calculus course. For example, a typical question is the following:

Find the area of the region R bounded by the graphs of

$y - x = 6$, $y - x^3 = 0$ and $2y - x = 0$

To start off, students are faced with a graphing problem in order to visualise what the region looks like. This is indicated in the sketch below.

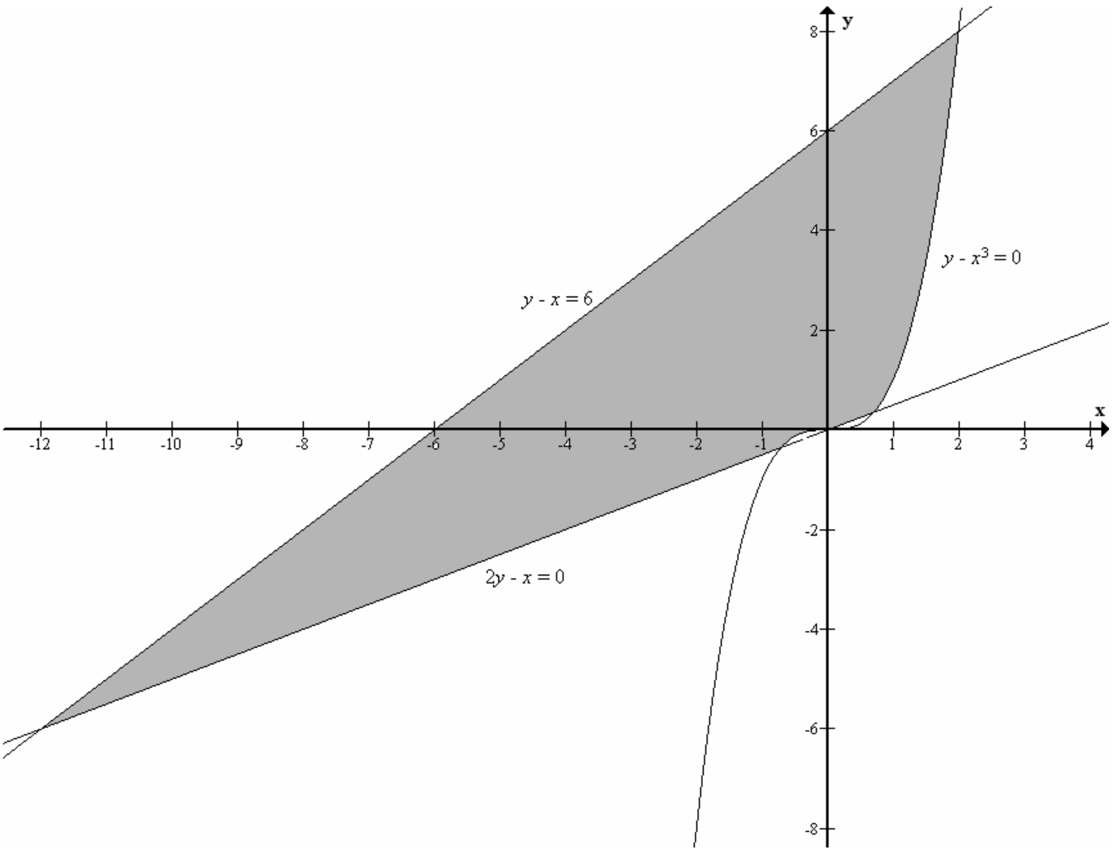


Figure 2.6

The formula for calculating areas under curves requires that the limit of integration be calculated.

Thus, the next hurdle is to find the points of intersection of the different graphs. This requires the ability to use the skills for solving simultaneous equations to find these points of intersection. Students will then come to the points A(-12; -6), the origin, and B(2; 8). In order to find the coordinates of B, the equation $x^3 - x - 6 = 0$ must be solved for x. From their ability to interpret graphs, students must be able to see that the lower boundary consists of $2y - x = 0$ for $-12 \le x \le 0$ and $y - x^3 = 0$ for $0 \le x \le 2$, while the upper boundary only consists of $y - x = 6$ for $-12 \le x \le 2$. All of the above steps are mathematical tools that students acquired at school.

It is only in the last part of the solution that learners are faced with the new concept of integration whereby they have to evaluate the following:

$$Area = \int_{-12}^{0} (x + 6 - \frac{1}{2}x)dx + \int_{0}^{2} \left(x + 6 - x^3\right)dx$$

2.2.4 <u>Applications of Differentiation (Graphing problems)</u>

The following question from 1st year mathematics deals with applications of differentiation, specifically the determination of the relative extrema of the function $f(x) = x^{\frac{2}{3}}(x^2 - 8)$.

To determine the derivative of *f*, the students can either use the product rule, which is a new tool from 1st year mathematics, or they can remove the brackets and then determine the derivative, which is a grade 12 procedure. They will then end up with $f'(x) = \frac{8}{3}x^{\frac{5}{3}} - \frac{16}{3}x^{-\frac{1}{3}}$. In order to find the critical points of the function, they now have to solve for $\frac{8}{3}x^{\frac{5}{3}} - \frac{16}{3}x^{-\frac{1}{3}} = 0$. It requires a good knowledge of surds and exponents to firstly simplify this to $\frac{8}{3}x^{-\frac{1}{3}}\left(x^2 - 2\right) = 0$ and then to solve for *x*. Next, they have to determine where the function is increasing and decreasing by evaluating the following inequalities: $x^{-\frac{1}{3}}\left(x^2 - 2\right) < 0$ and $x^{-\frac{1}{3}}\left(x^2 - 2\right) > 0$. An inequality question of this nature is unfamiliar in grade 12 but is easily accessible when $\frac{x^2 - 2}{\sqrt[3]{x}}$ is considered using a sign table. This only requires knowledge of the behaviour of $\sqrt[3]{x}$.

2.2.5 <u>Piece wise functions</u>

The following example is taken from first year mathematics and is useful as an example of a function which is or is not differentiable at a certain point. A major problem is coping with a function defined by different formulas at different parts of its domain.

Consider the example $f(x) = \begin{cases} y = x + 1, x < 0 \\ y = x^3, x \geq 0 \end{cases}$.

To sketch the graph of the function, students have to tap into their knowledge of graphs (lines, parabolas, third degree polynomials, etc.) to draw each of the individual pieces of the graph. This produces the following result:

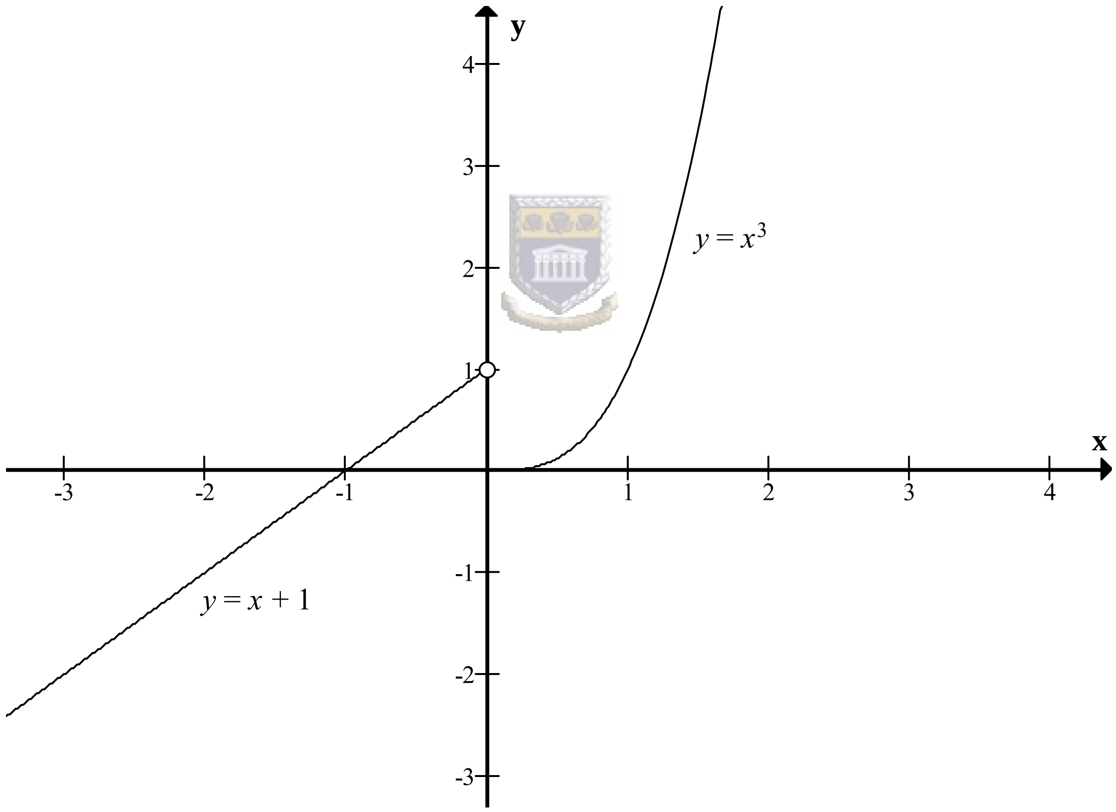


Figure 2.7

To see that this function is not differentiable at $x = 0$, students have to calculate the limits $\lim_{x \to 0^-} f'(x) = 1$ and $\lim_{x \to 0^+} f'(x) = \lim_{x \to 0^+} 3x^2 = 0$. It is clear that $\lim_{x \to 0} f'(x)$ does not exist.

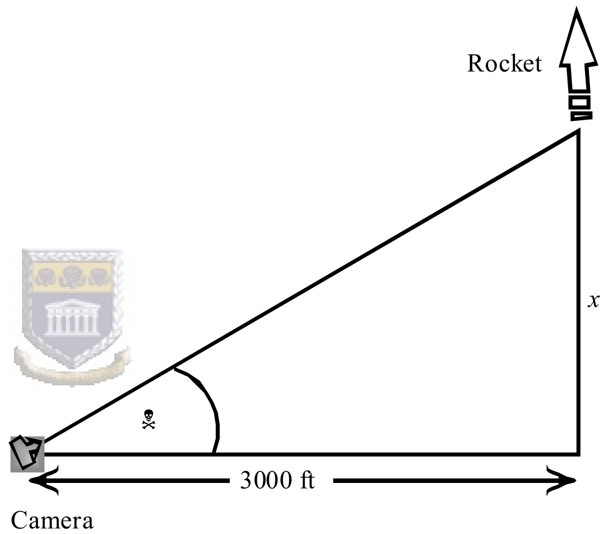
2.2.6 Applications of Differentiation

The following examples look at the mathematical knowledge and skills required to solve problems dealing with applications of differentiation.

Examples:

2.2.6.1 Find the radius and height of the right-circular cylinder of largest volume that can be inscribed in a right-circular cone with radius 6 units and height 10 units.

2.2.6.2 If the rocket shown in the accompanied figure is rising vertically at 880 ft/sec when it is 4000 ft up, how fast must the camera elevation angle change at that instant to keep the rocket in sight?



In both these examples, students' real understanding of the concept of rate of change is being challenged. They must also be able to use their geometry knowledge. In example a) similar triangles are used to generate a relation (an equation) between the height and the radius of the cylinder while in example b) students must be able to apply the theorem of Pythagoras to the given triangular shaped diagram.

2.3 Conclusion

All the above examples come from a range of 1st year level mathematics topics and it is clear that a good school mathematics background is required in order to cope with mathematics at this level. As can be observed from these examples, the mathematical skills and understanding of mathematical concepts developed from school level are being used extensively in each of these topics. These concepts from school level are no longer the main topic that is being studied and developed, but are needed and used to develop other new concepts. Students are not explicitly asked to solve simultaneous equations or similar triangles, to find the maximum or the area bounded by a couple of graphs. They are expected to be able to know that these tools can be used in these unfamiliar ways. The main emphasis is no longer on solving simultaneous equations, drawing lines or parabolas or solving an inequality, but these tools are used to find, say, the area bounded by graphs or to find the extrema of a particular function or to evaluate limits or improper integrals. Textbooks often simply write down the answers of these steps without any further explanations, making it difficult for students, reading the textbook, to understand how answers were obtained. The above problems suggest a much more integrated approach to the teaching of mathematical concepts at the school level.

In order for learners to be able to deal with mathematics of this nature, they must be able to visualise whatever answers they got from algebraic manipulation and also be able to argue mathematically about their solution or methods employed to solve a particular problem. Basic concepts from school such as solving of equations, graphs, Pythagoras, etc. are assumed by the lecturers or textbooks as secondary issues.

2.4 Hypothesis

This study will investigate a teaching strategy that will lend itself to develop an integrated understanding of certain sections of the school curriculum.

This will be done by employing an assessment strategy which allows us to assess, in a valid and reliable way, the knowledge, insights, abilities, and skills related to the understanding and mastering of mathematics in its essential aspects. (Niss: 1993). According to Niss (et al, 1993), assessment addresses the outcome of mathematics teaching at the student level. When tests are considered to be a way of judging student performance they are special forms of assessment and are thus subsumed under the assessment category.

This will be followed by an integrated teaching methodology which will be followed by another assessment strategy to test whether the teaching strategy was effective. This process is outlined in chapter 4 while the results will appear in the last chapter.

Chapter 3

Literature Review

3.1 Introduction

In this chapter various aspects of an integrated teaching methodology will be looked at from the literature point of view. There are a number of ways in which mathematics can be integrated, namely:

- mathematics integrated with real life situations;
- mathematics integrated with other subjects, referred to as the integrated curriculum;
- mathematics as a subject seen as an integrated whole.

This study focuses on the last association with regards to the integration of mathematics as an integrated subject and also on effective ways in which mathematics can be taught in an integrated manner.

3.2 Definitions

A basic definition is offered by Humphrey, Post and Ellis (1981) when they state, "An integrated study is one in which children broadly explore knowledge in various subjects related to certain aspects of their environment". Shoemaker (1991) defines an integrated curriculum as "…education that is organised in such a way that it cuts across subject-matter lines, bringing together various aspects of the curriculum into meaningful association to focus upon broad areas of study." He also views teaching and learning in a holistic way and reflects on the real world, which is interactive. Kathy Lake (1994, p 3) advocates that any definition of integrated mathematics should support the idea of lifelong learning through the development of abilities and the move away from compartmentalised/ departmentalised subject matter to a view which include:

- a combination of subjects;
- an emphasis on projects;
- sources that go beyond textbooks;
- relationship among concepts;

- thematic units as organising principles;
- flexible schedules;
- flexible student grouping.

3.3 Background

According to Humphrey (1981) it is taken for granted that learners will eventually work out for themselves how things fit together. We forget that they will learn what we teach. If we teach connectedness and integration, it is most likely that this is what they will learn. On the other hand, if we teach separation, discontinuity or in compartments, then this is what learners will learn. Benjamin (1989) relates this with the trend towards global interdependence of a complex system; the increase in pace and complexity and the expanding body of knowledge needed by workers to solve inter-related problems. The above changes and challenges faced by a worker who enters these complex systems, forces us to look at our view of the nature of mathematics. If it is viewed as merely a memorisation of formulas, rules and procedures, then a teaching strategy such as drilling, recitation and flashcards would be appropriate methods of teaching mathematics.

Perkins (1991) argues that the 21st century requires a flexible use of knowledge beyond a superficial understanding of multiple isolated events to that of insight, developed by learning, that things are connected or integrated. He is further concerned about the teaching for the transfer of thoughtful learning. Connecting things up with integrated ideas will develop a broader and deeper sense of understanding. Perkins (1991) also says that there is natural alliance between those teaching for better understanding and towards integration and the development of meaningful learning which will ultimately lead to increased student success.

3.4 Preparing mathematics for students

Jerome Bruner (1960) says that: "the fundamental ideas of each subject can be taught to any individual at any age in some honest way". The major question thus is: What can be taught at different levels of schooling and what can be done to make it happen? Mathematics is a human activity, i.e. humans create it in legitimate mathematical institutions (i.e. in mathematical academic conditions). So how do we adapt this formal mathematics so that it can be accessible to a broader audience? The French didactition (Michele Artigue, 1994) referred to it as didactical transposition or didactical engineering. Others called it elementarization of the mathematics from its source to the group targeted to acquire a whole range of significant conceptual and procedural knowledge and the ability to transfer that knowledge from the specific context in which it was presented to new and apparently different settings.

Morgan Niss (1994) suggests three steps to decide what should be taught at a particular level of schooling. He called for *justification* of the topic that is earmarked for school education. Secondly, he suggested that there should be looked at whether a certain section can be taught and *how possible* it is within the constraints of the social environment that the intended learners find themselves in. Thirdly, he looked at the *implementation* of this section, i.e. whether resources can be prepared to make this possible. Curriculum development is the tool used to bridge the gap between professional mathematicians and the broader audience who is targeted to acquire new concepts, principles, techniques and reasoning. According to Uwe-Peter Tietze (1994, pg. 43) a curriculum must include aims, content, methods, and assessment procedures. In developing a curriculum, one must justify aims, content, and methods with rational and inter-subjective argument. There are also different forces that determine the purpose that education should have. Societal and political pressures are also aggravated by the ever-changing technological world in which curriculum development takes place. In the new era, education can determine economic growth or not. The teacher alone determines the effectiveness of curriculum by his or her decisions, behaviour, attitudes, and cognitive processes, no matter how carefully the curriculum has been developed. Recent research placed more emphasis on everyday curriculum in the classroom and on teaching ideas.

In the South African context, we still have a long road to travel, in terms of curriculum development. There are a lot of forces acting on this process which makes it questionable at this moment, namely:

- The *National Guidelines* states exactly what kinds of problems are examinable. This causes the focus to be on the preparations for the final question paper, rather than to enter the next level of the academic arena with the proper understanding of the relevant topics. Questions asked in the final examinations are predictable and all of this lends itself to rhetorical learning.
- The *government* also rewarded schools that obtained a good over-all pass percentage. This has a huge impact on what is currently happening in classrooms. Students are encouraged to take mathematics on standard grade, if they are deemed a possible threat to tarnish the prestige of the school by taking higher grade.
- The public schools are designed and *resourced* to offer mathematics on standard grade. Teachers who want to offer higher grade mathematics, have to do so in their own time as most of these schools do not have the financial resources to appoint additional teachers.
- Our curriculum is still very much *problem-centred* and little to no room is made for contextual problems.
- No clear objectives of the mathematics curriculum are provided in terms of the integrated nature of mathematics as a subject.

3.5 Psychology of integrated learning

Cromwell (1989) argues that the brain processes and organises information. It will add and absorb new information based on its previous experiences and meaning. It processes many things at the same time and a holistic approach to learning makes the recall of information easier. Shoemaker (et al: 1991) states that the brain seeks patterns and finds meaning through these patterns.

Caine and Caine (1991) connect neuro-psychology and educational methodologies and states that the search for patterns and meaning is a basic process of the human brain. Their research also suggests that the brain might reject facts learned in a fragmented way and presented in isolation. According to Caine and Caine (et al: 1991), learning takes place faster and more thoroughly when presented in a meaningful context. The search for pattern and context is universal but each and every learner has his/her own style of learning. To provide for these diverse ways of learning, students have to be presented with a number of choices.

Shoemaker (et al 1991) suggests that the brain researches towards interdisciplinary learning, experiential learning and teaching which is responsive to the student's learning style. Kathy Lake (et al: 1994, p6) argues that global economy and international connectedness are pushing the ability to make connections and to solve problems by looking at multiple perspectives. Another argument for integration by Lipson, (1993) is that it represented a way to avoid fragmented and irrelevant acquisition of isolated facts. Lipson (et al: 1993) further says that integrated teaching:

- helps students to apply skills;
- brings about faster retrieval of knowledge;
- leads to integrated knowledge if taught with multiple perspectives;
- encourages depth and breadth;
- promotes positive attitudes;
- provides more quality time for exploration of issues beyond the scope of the curriculum.

3.6 Technology and mathematics teaching

Textbooks, paper and pencil, blackboards, transparencies, videotapes and computer software are all different forms of technological tools that are currently being used in mathematics teaching. Each of those tools has its advantages and drawbacks.

In recent years, most research has been focused on the use of the computer as an instrument for mathematics teaching. The computer programs used for this purpose range from programming (Logo), spreadsheets (Excel) to software applications (Cami and Master Maths).

Each of those mentioned have their advantages and drawbacks which users must be aware of. For example, the problems experienced with the use of Logo as a programming tool range from user friendliness to knowledge of programming itself. The problem can easily become a programming one rather than the main task of solving a mathematical problem. According to Niss (1994), less attention must be given to tasks that can be readily done by machines thus creating more room for attention to conceptual thinking and planning in any tool environment. When a programming tool is used to solve a mathematical problem, teachers must refrain from giving learners model solutions thereby depriving learners of constructing their own problem solution.

Creating a computer environment for the learning of mathematics must have cognitive growth of learners as its main consideration. This environment must be built around it. The micro-world is a computer-based interactive learning environment where the pre-requisites are built into the system and where learners can become active, constructing architects of their own learning (Papert, 1980). Children are often highly creative in a computer environment but long-term curriculum objectives require external guidance and support. Weaker students can re-enter the curriculum without necessarily first closing all gaps (Hillel, Lee, Laborde, & Linchevski: 1992).

The following question has been subjected to a lot a discussion. How do we prevent learners from using the computer for doing algebra while they are supposed to be learning algebraic manipulations?
More fundamentally: Should we prevent them? Later in life they will hopefully have a computer at their disposal whenever they need one, so why not at school level.
Trying to answer this, one is led to the old issue about the relationship between skills and understanding: whether and to what extent are manipulations necessary for conceptual understanding (Nesher: 1986). Two approaches are generally employed:
1. Develop curricular material for use of general computer algorithms.
2. Design specific computer tools for use in educational settings.

Another question that is also a very important one in learning mathematics in a computer environment is: Should mathematics depend on the tools or should the tools depend on the mathematics to be learned? The latter is the widely accepted view where the mathematical concepts should be the primary objective to be considered when designing a tool.

Ready made software can mainly be categorized as using the drill-and-practice method or the intelligent tutorial system. The table below gives comparisons between these methods.

Table 3.1: Comparison between drill-and practice and intelligent tutorial systems

Drill-and-practice	Intelligent tutorial system
1. Learners know whether answer was right or wrong.	1.Compared to being taught by a private teacher.
2. They have no access or insight to the mathematics content behind the problems	2. Must 2.1 be an expert on subject matter. 2.2 Know how to present a particular topic. 2.3 have an idea of the student knowledge and skills.
3. The learning environment is fixed and closed with little flexibility to allow creativity on the part of the learner.	

According to Reusser (1991), "intelligence" should not be concentrated in the computer, but rather be spread out across the entire pedagogical setting, with the learner at its centre. Question: "To whom does the learner turn to when having difficulty to solve the task?"

Generally there are three ways to address this problem:

1. Teacher;
2. Pairing learners at computers;
3. Using a program with an intelligent tutorial component.

Any computer aided learning of mathematics should be designed to allow learners to shape their own process of learning while being supported. It should also reduce the number of unproductive errors thus helping learners to acquire a better understanding of mathematical concepts.

In South Africa, software packages such a Cami, Master Maths, Maths Trek, etc. are being marketed extensively. We have yet to establish a body with integrity which will serve as “watchdogs” to protect the broader public from making investments which will be of little or no benefit to their children. Every year, hundreds of thousands of rands are spent on technology (specifically computers) in schools to assist learners to excel in mathematics. The question that comes to my mind is: Wouldn’t it be better to use the money to employ more qualified teachers who can be used to teach higher grade mathematics? Have we looked at cheaper options such as programmable calculators which price at ±R300 each? Would such a tool (programmable calculator) not enhance a better understanding of mathematics? There are various free mathematics software packages available on the internet, such as Graphmatica, which we have not yet started to explore.

3.7 Aspects of effective mathematics teaching

Sorensen (2003, p2-3) emphasises that mathematics is an integrated whole, a study of structures and relationships between things and a way to understand the world around us. Another view is that mathematics is a set of rules, formulae, models and the relationships between the models, etc. (Strausss: 1996, p5). Strauss argues that teachers with this view will teach mathematics as an abstract subject with no relation to everyday situations or problems. Zemelman, Daniels and Hyde (1998) say that the role of the mathematics teachers is that they have to help develop students’ mathematical power. This power will make mathematics personally meaningful and useful to such an extent that they will develop the confidence to understand and apply mathematics. Stein (2001) adds by saying that the teacher must develop mathematical and problem-solving “habits of mind” with their learners.

Stein states that too many teachers were concerned with students’ rote use of procedures rather then understanding concepts and the development of what he calls higher order thinking skills.

Zemelman (1998) says that:
"Without true understanding of the underlying concepts, guarantees serious problems with learning other concepts." When the focus is on the understanding of mathematical ideas, students are "far more likely to study mathematics voluntarily and acquire further skills as they are needed."

Sorensen (et al: 2003, p3) says that the following principles appear frequently in literature about effective teaching:

- deriving of concepts;
- the use of cooperative learning;
- encouragement of frequent mathematics communication;
- multiple representations and multiple strategies;

3.7.1 Deriving concepts

Sorensen (et al: 2003, pp. 7-8) argues that concepts must as far as possible be derived by the students. If the concept is too difficult, the teacher must derive and explain it and not say that the proof of the concept is beyond the scope of the syllabus. If the latter happens, students will revert to memorising rather than understanding. She further says that:
"Mathematical ideas are formed through a process of analyzing problems, trying a number of strategies to solve them, evaluating the strategies' effectiveness, looking for novel strategies, and verifying that a particular strategy is valid."
According to Sorensen, textbooks hamper such approaches as they mostly show only the last steps or might present one strategy to prove why it works.

Battista (2000) says that the understanding of new ideas depends heavily on the cognitive structures students have developed previously. He further says that time must be allocated to discuss the meaning of making sense of mathematical ideas; how to make sense and how to know when sense has been made.

3.7.2 Cooperative Group work

Reynold and Muijs (1999) indicate that whole-class instruction and teacher led discussions are most effective for the transfer of basic skills. Group work is more effective when a teacher wants to establish higher order thinking skills. During group discussions each member can talk about his/her own understanding of ideas and compare it with the thinking of the other members of the group. They also argue that group work can reduce mathematics anxiety and help learners to overcome their insecurities about problem-solving, especially when they realise and see that others are also struggling. Reynold and Muijs say:

"It is insufficient to put students in groups and let them get on with it"

Battista (et al: 2000) suggests that all students must be engaged as partners to make the group reach its full potential. Sorensen (2003, p9) feel that teachers must give clear instructions and have measures in place by which each group member be held accountable.

Grouws and Cebulla (2000) feel that not just any task is appropriate for group work. The work must centre around an idea or concept and must be interesting to the learners. The teacher must be aware of the progress that each of the groups are making and should be in the position to tell whether the groups could or could not manage the task. Closure to a group activity is of utmost importance by making sure that the key idea is arrived at. If the groups could not manage the task, the teacher should have a whole class discussion to bring about closure. Classroom activities should alternate between direct teaching and individual work time. Group work can also become more effective if learners are allowed to work as individuals on the task before they are engaged in collaborative group work. This will bring different ideas together when the group is established.

3.7.3 Communication in mathematics

Another advantage of cooperative group work is that it encourages communication amongst students. Effective mathematics teachers “create a classroom environment of mutual trust and respect in which students can critique mathematical thinking without personally criticizing their peers”. (Pagulee, 2001)

Stein (et al: 2001) adds that the “culture of niceness” will hinder the potential of good mathematical discussions and teachers should show the way for learners in terms of criticizing logically and reasoning without attacking each other. Teachers must listen to their learners and ‘revoice’ a learner’s unclear or imprecise statement to bring out the original thought of the learners, thus allowing the ownership of the idea to remain that of the learner.
According to Piaget (1936) and Vygotsky(1930) there are two major perspectives on the role of social interaction in the process of learning:

Table 3.2: Comparison between constructivism and the activity theory

Piaget (constructivism)	Vygotsky (activity theory)
Considers learning as a result of interaction between individuals and their environment.	Based on social relations.
New objects and situations are integrated into the individual’s scheme	Interaction (learning) between individuals takes place.
Individuals have to make an effort to adjust to the new situation	Learning also takes place inside the individual.

The two distinct approaches question the relationship (interaction) between the teacher and the learner and also have implications on the role of the teacher. The perception of mathematics also plays an important role. Is it seen as an objective truth (societal treasure) or as a practice of shared mathematizing? Group work is developed in curricula and used by teachers to introduce an unseen problem to a classroom.
Studies show that answers produced in a group are generally better than individual work. But this does not come without problems. The mathematical

problem also has a social (second) side to it. Each member brings to the group different points of view that were gained from different past experiences. Proposals are rejected, improved and transformed by the interaction in a group.

A good functioning group has to systematically organise these different solutions in order to solve the problem. In this environment some learners can regress while others might be eager to win the social side of the problem while loosing sight of the mathematical problem. Technology (for example computers), if collaborative learning is applied, promotes discussion between learners. They are faced with short actions in order to get the desired effect on the screen.
There are different variables to consider when the teacher wants to use group work.

- The choice of partners must not create a serious social problem within the group.
- The type of task that the learners will engage in must pose them with a new situation. It must tap into their existing knowledge and promote communication. Room must be made for learners to describe and justify their solutions.
- The length of the task will have to be taken into consideration. Learners need time to internalize the problem and to relate it to their existing experiences. Proposals become solutions amongst other solutions.

According to Margolinas (1993), the previous knowledge of the student takes the role of *validity criteria*. In some experiments (Rivinia, 1991; Polivanova, 1991; Roubtsov, 1991), group work was organised by giving different subtasks to each partner – but these tasks were not independent, and students had to coordinate their solutions in order to achieve the whole task.

3.7.4 Multiple representation and strategies

Sorensen et al (2003) argues that mathematics teachers present concepts and show how to solve problems in various ways to help all learners to get the most out of mathematics instructions. Stein (et al: 2001) suggests that multiple representations help students make "personal meaning" out of mathematics concepts and give them the "opportunity to think in diverse ways". Teachers should also choose problems that can be solved in more than one way thus improving the "student's flexibility of thinking". Group work can also be utilised to have different members solve a single problem using different methods. Learners must also be allowed to evaluate these different strategies and to determine which one is appropriate or effective in certain situations.

The power of multiple representations and strategies was being clearly emphasised by Joe Crosswhite (1970, p322) when he said:
"Mathematics is undoubtedly the most tightly knit and cumulative of all subject-matter areas. The teacher who fails to capitalise upon this not only robs himself of one of the real advantages of being a teacher of mathematics, but also ignores one of his/her major responsibilities."

Crosswhite (1970, p. 320) further adds that the teacher usually works in clear specifications when it comes to subject matter. He/she needs to decide how to adapt the subject matter to the type of learners he has. The teacher also takes primary responsibility for the teaching strategies he/she will employ. All these suggested that the teacher should be equipped with various strategies to teach different components of the curriculum. Other factors which will influence the strategies or teaching methodologies that the teacher of mathematics will employ include o*bjectives* of the mathematics course and *goals* of separate units in the course.

The teaching methodology depends on the long term objectives and the short term goals of the course. According to Crosswhite (1970, p. 321) objectives and goals should take precedence over methods.

Strauss (1996, pp. 6 – 7) suggests that teaching objectives might be something which is explicitly stated by the teacher or only exists in his/her thoughts. His/her understanding of the objectives of mathematics will influence every aspect of the teaching process. Strauss further adds that the teacher's objectives can be observed during his/her teaching or by looking at the tests and examinations. This shows what the teacher considers as important elements of mathematics. Some reasons why teachers must formulate objectives include:

a) Objectives evaluate the effectiveness of teaching. It guides teachers and students toward the expected result of the course.
b) Objectives provide guidelines to develop alternative teaching activities.
c) Objectives help to broaden the concept of what teaching of mathematics entails.

If the long-term objective of the teacher is to establish mathematics as an integrated source of knowledge, then he/she will endeavour to use every opportunity to mould his/her learners to this objective.

Strauss highlights three types of objectives, namely:

a) content objectives, which involve knowledge of mathematics;
b) process objectives, which deal with the acquisition of mathematical work methods;
c) affective objectives, which endeavour to instil appreciation for mathematics as a subject.

Goals on the other hand are achieved by looking at the different components of the curriculum. By achieving each of these individual goals, the broader objective of the course will be reached. The following are the functions of establishing goals:

a) It serves as guidelines for lesson planning;
b) It serves as guidelines for selection and organisation of lesson content;
c) It determines learners' activities;
d) It must have the broader objective of the course in mind;
e) It must allow for adequate evaluation. (Strauss: 1996, pp. 8 – 9)

3.8 Teacher education – research on teaching

The objective in any teacher-training program should rest primarily on the development of teachers' mathematical knowledge and to develop practical competence to improve the current practice by recognizing the shortcomings in the current mathematical education system.

This, of course is not an easy task as there are numerous challenges facing teaching of mathematics, one of the main being that this teaching takes place within a social context that might be different from what the teacher is used to. Other factors like syllabi, textbooks, length of the periods, assessment, structure of the classroom and intellectual capabilities of different learners can both be limiting or supportive in nature. Thus, the three main areas for research and reflection include:

a) Teachers' cognition and behaviours.

b) Relating research and practice in mathematics education.

c) Development of models and programs for teacher education.

Many researchers see didactics of mathematics in its relation to teachers as an opportunity to bridge the gap between theory and practice. The question that frequently comes up is: "What kind of knowledge, experiences and understanding of mathematics should a mathematics teacher have in order to bring life to theory?", has also become a major question for research. Wittmann (1989) argues that any course on elementary mathematics must be rich in relationships to history, culture and the real world but must be organised in a problem and process-oriented way which leaves room for variety in representations and teaching and learning formats. Hans-Joachim Vollrath (1994) suggests that teachers' academic mathematical knowledge should be restructured, re-analysed and evaluated from a didactical point of view. He also puts a lot of emphasis on the understanding of concepts and what it means to understand concepts. Is there any optimal way of teaching mathematical concepts?

Wittmann (1981) suggests that when teaching concepts the following has to be taken into consideration:

a) the students: their cognitive structures, their intellectual abilities, their attitudes, and their needs.

b) the concepts: different types of concepts, logical structure of definitions, context, development of concepts.

c) the teacher: their personality, their intentions, their background.

Innovations in curriculum development and teaching methods have to take the teachers and their professional knowledge into account. They must have additional knowledge to appropriately discuss mathematics with their students. According to Shulman (1987), pedagogical content knowledge should definitely be considered when constructing a program for teachers' professional development. This is the knowledge that separates the teacher from the pure mathematician.

Both content knowledge and pedagogical content knowledge of mathematics are equally important. New approaches to teach different sections of the curriculum must be devised to obtain higher levels of achievements from all learners. The question that should be asked is: What will the teacher be doing when he or she enters the classroom? This question, in my opinion, will guide whoever when constructing a course for teacher training.

3.9 Conclusion

This chapter highlights some of the important aspects of integrated mathematics. There are various other challenges which must be taken cognisance of when talking about an integrated course in mathematics. The aspects raised will also be used during our search for the effectiveness of an integrated teaching methodology. In the next chapter, instruments will be developed which will lend themselves to:

a) the issue of multiple representation of concepts;

b) cooperative group-work and the psychological aspects involved in it;

c) technology and mathematics teaching.

Chapter 4

Learning from teaching

4.1 Introduction

In chapter 2 some examples of how different topics in high school mathematics are applied, in an integrated way, to solve problems in the first year mathematics course at the University of the Western Cape. This raises the question whether students entering university can apply the mathematics learnt at school in this integrated fashion. In this chapter some instruments which assess whether learners are able to integrate their mathematical knowledge) as well as lessons (that enhance integrated learning and teaching) are developed. The design of assessment instruments and teaching units will be at the core of this chapter.

4.2 Design Theory

Erich Wittmann (1995, pp. 355-374) suggests the following structure for the research in mathematics education:

i) a core and

(ii) other related fields located around the core.

The core defines and gives stability to the field of mathematics education. Developing the core would mean advancement in mathematics education and development in the related fields would mean development in the core. So what does the core consist of? According to Wittmann (1995), it consists of a variety of components which include:

- analysis of mathematical activities,
- development of local theories,
- exploration of possible contents to make it accessible to learners,
- critical examination and justification of contents in view of the general goals of mathematics teaching,
- research into the pre-requisites of learning and into the teaching/learning processes,

- development and evaluation of substantial teaching units and curricula,
- development of methods for planning, teaching, observing and analysing of lessons,
- inclusion of the history of mathematics education.

The core must start from mathematical activities as an original and natural human activity.
Wittmann (1995) further suggests a very broad use of the word mathematics which includes mathematics developed and used in science, engineering, economics, computer science, statistics, industry, commerce, craft, art, daily life and so forth. Specialised mathematics is certainly an essential part of mathematics but the converse is also equally true that a great deal is also contributed by other societal sources.

The core is aimed at an interdisciplinary, integrative view of different aspects and at constructive developments. The specific character of the core is the constructive development of and research into mathematics teaching. Here, the field of mathematics education is assigned to the larger class of "design science".

The field of mathematics education competes with other more "established fields" for funding where measure of quality is based on scientific principles and also where research has a direct or visible influence on people's lives. (Examples: medicine, engineering, etc.). The problem is: How do we decide whether a certain design is of high quality or not; i.e. How do we establish scientific standards in the field of mathematics education? This problem is solved by looking at the core as a constructive development of research into mathematics teaching. The knowledge that already exists in the core will serve as a yardstick to measure the quality of designs. This is of course a very important consideration as future development of mathematics education depends on funding from different sources. The design of teaching units will be a major feature in this chapter. Through the design of teaching units, knowledge is no longer an exposition from the teacher to the learner. Room is created for active interaction between learners and teachers.

Erich Wittmann (1995) characterises “substantial teaching units/designs” as follows:

1. They represent central objectives, contents and principles of mathematics teaching.
1. They are rich sources of mathematical activities.
2. They are flexible and can be adapted to different mathematical ecologies.
3. They also involve other aspects of mathematics teaching, such as: psychology, pedagogy, etc.

The design of teaching units has never been a focus of research in mathematics education in the past but rather incidental investigations and presentations. This was also due to the fact that the design of teaching units was not thought of as academically respectable work, but done by teachers and authors of textbooks. In order to overcome this fundamentally incorrect view, Wittmann (1995) suggests that role players from a wide spectrum be involved and that specialists in special centres for research and development be at the forefront of it all. Teachers however can not be left out of this, but can make important contributions as members of research teams.

The theory of the didactical situation, Michele Artigue (et al: 1994), on the other hand aims to model a teaching situation so that it can develop and be managed in a controlled way. The didactical system consists of the teachers, the students and the knowledge that needs to be taught.

According to Artigue, didactitians are particularly interested in the mutual interaction between these three components. Their aim is to develop a conceptual and methodological means to control the interacting phenomena and their relation to the formation and functioning of mathematical knowledge in the student. Their concern is thus mainly with a teaching object that already exist. So the following questions might be guide to analyse an existing teaching object.

- Why is there a need to change the current teaching object?
- What new aims should be included?
- What difficulties can be foreseen and what can be done to overcome it?
- How can the validity of the solutions be determined?

Artigue suggests the following steps to review a teaching object:

i.) analyse the teaching object as it already exist;

ii.) determine its inadequacies;

iii.) outline a structure of the reformed project;

Didactitians view the teaching system as a dynamic system which is affected by constraints. They view the teaching object to be changed as the equilibrium of the dynamic system. This equilibrium, and its constraints that affect it, has to be studied. Constrains might include:

- the role of the teacher;
- time management;
- the content treated, etc;

These constraints can be varied and didactitians has to observe whether the new equilibrium occurs at a point which can be judged as more satisfactory. Effectiveness, power and adaptability to different context are just a few considerations which can be used as a yardstick.

Artigue further cautions that changes can not be made unless the theory of transposition has been considered. This theory refers to the origin of the mathematical knowledge and the gradual process whereby it made its way into primary and secondary institutions. Artigue argues that it is not a mere elementarisation of knowledge fabrication in institutions of higher learning so that it fits with the previous knowledge of the students. The theory of transpositions also includes the analysis of a teaching system in an open manner. This analysis might include:

- the institution where the knowledge was created;
- the target group or institutions;

- the awareness of the economical and ecological aspects of the knowledge to be taught;
- the viability in terms of the expertise; physical resources; laws that govern functioning of teaching, etc;

Didactitians must be able to foresee the changes that the knowledge can possibly undergo. Artigue says that the knowledge must be able to live and develop without too drastic changes which might corrupt it.
In the sections that follow, the teaching units for this study are discussed in detail.

4.3 Method

Two groups of grade 12 learners participated in this study. Both these two groups of learners had experienced mathematics as a subject consisting of rules and formulae which had to be rote-learnt. In this situation all knowledge emanates from the teacher and invariably involves a lot of drill and practice on the part of the learners. Teaching is focussed on examinations rather than the learning and understanding of mathematical concepts which can be applied to solve practical problems. Both groups wrote a pre-test based on their grade 11 knowledge of Quadratic Theory and grade 12 knowledge of Differential Calculus. After the pre-test, one group was subjected to a series of lessons in which topics were presented in an integrated way, whilst the other group (the control group) continued to be taught in the traditional way. The lessons were followed by a post-test to determine whether the integrated teaching methodology was an effective teaching strategy.
The pre- and post-tests, answers and the marking memoranda of these tests are given in Appendix III.

4.4 Integrated Instruction

In this section a brief description is given of the three lessons that were designed. Lesson 1 deals with the reading off from graphs, the domain and range of functions and their restrictions, estimates for the roots of $f(x) = k$, k a constant and $f(x)$ a quadratic function and estimates for the solution of quadratic inequalities. Lesson 2 deals with translation of parabolas, methods of solving quadratic equations and the relation between the derivative and the gradient of tangent lines. Lesson 3 elaborates on the graphs of functions and their derivatives. Applications of quadratic functions are also discussed in this lesson. The lessons are on disks in the envelope attached to the inside of the back cover of this mini-thesis and can be viewed by inserting the disks into a DVD-rom drive or CD-rom drive with the software program power DVD or a normal DVD player connected to a television set.

4.4.1 Lessons outline:

4.4.1.1 Grade Level(s): 11, 12

4.4.1.2 Subject(s):

- Mathematics/Algebra

4.4.1.3 Topics:

- Quadratic Theory and Differential Calculus

4.4.1.4 Duration: Three lessons of about 45 minutes each.

4.4.2 Description: The lessons were a follow-up of the written pre-test. They served as a strategy to assess whether an integrated teaching methodology will be effective or not.

4.4.3 Goals: The learners will be able to use and interpret graphs together with algebraic manipulation as a means to solve problems.

4.4.4 Objectives: To integrate learner's fragmented/compartmentalised knowledge of quadratic theory and Differential Calculus.

4.4.5 Materials: Khanya computer room, Netops (Computer software which allows the teacher to control different stations), White board with different colour white board markers, Graphing software (Graphmatica), video camera and worksheets (activities).

4.4.6 Procedure: The learners were grouped around between 15 to 20 stations to allow for cooperative learning. Learners had note books, pencils, pens and calculators to record notes.

4.4.7 Assessment: The lessons provided a means for alternative assessment. Activities were incorporated within the lesson to reinforce concepts. Questions were asked and the lessons continued after satisfactory responses. A post-test was administered to assess whether learners could apply their fragmented knowledge in an integrated way.

4.5 Lesson 1

4.5.1 Aims:

To employ an integrated teaching methodology to enhance students' ability to read off from graphs

4.5.1.1 the intervals where $f(x)$ changes from negative, zero to positive.

4.5.1.2 the domain and range of functions and their restrictions.

4.5.1.3 the number of roots of equations of the form $f(x) = k$.

4.5.2 Objectives:

To expose learners to an integrated teaching methodology dealing with inequalities, roots, domain and range of functions.

4.5.3 Procedures:

The aspects covered and the strategies that were employed are outlined below:

4.5.3.1 Intervals where $f(x)$ changes from negative, zero to positive.

- As an introduction, the function $f(x) = -2x - 4$ was written on the white board. Learners gave different values for *x* and the corresponding *f*(*x*)-value was calculated. These values were written in coordinate form and indicated as points in the Cartesian plane.
  The graph of $f(x) = -2x - 4$ was then drawn with Graphmatica and a pointer was moved over the line and learners could see how the coordinates changed when the pointer was above, on and below the *x*-axis.

The general strategy was that $f(x) > 0$ for $x < -2$, i.e. where the graph is above the x-axis; $f(x) < 0$ for $x > -2$, i.e. where the graph is below the x-axis and $f(x) = 0$ for $x = -2$, i.e. where the graph cuts the x-axis. This is illustrated in figure 4.1 below.

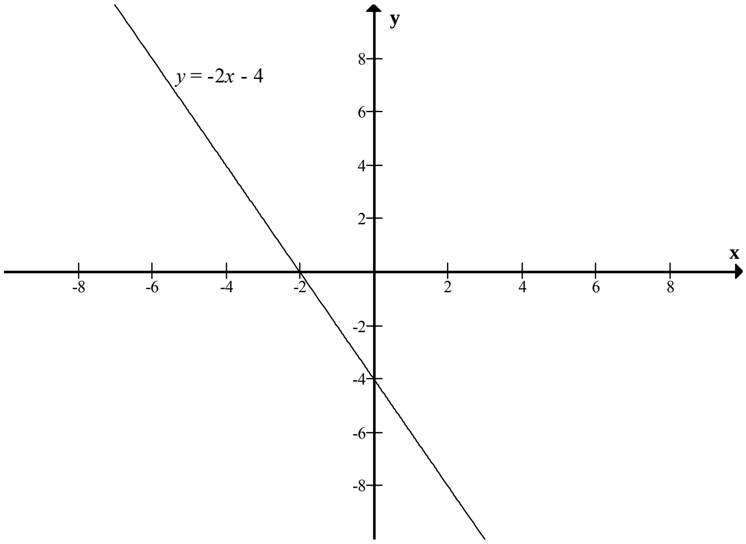


Figure 4.1

- The next example was the graph of the function $g(x) = -2x^2 - 5x + 3$. Again the pointer was moved over the curve and learners could read off from the graph the answers to the following inequalities: $-2x^2 - 5x + 3 > 0$ and $-2x^2 - 5x + 3 < 0$. Learners observed that $g(x) > 0$ for $-3 < x < \frac{1}{2}$ (graph above the x-axis); $g(x) < 0$ for $x < -3$ or $x > \frac{1}{2}$ (graph below the x-axis).

- The discussion was expanded by looking at the solution of inequalities of the form $-2x^2 - 5x + 3 > 3$; $-2x^2 - 5x + 3 < 3$ and $-2x^2 - 5x + 3 = 3$ graphically. Learners could see from the graphs of $g(x) = -2x^2 - 5x + 3$ and $y = 3$ that the inequality $-2x^2 - 5x + 3 > 3$ was true when the pointer was moved along the curve $g(x) = -2x^2 - 5x + 3$ but was above the line $y = 3$. Similarly, $-2x^2 - 5x + 3 < 3$ was true when the pointer was below the line $y = 3$.

This is illustrated in figure 4.2 below

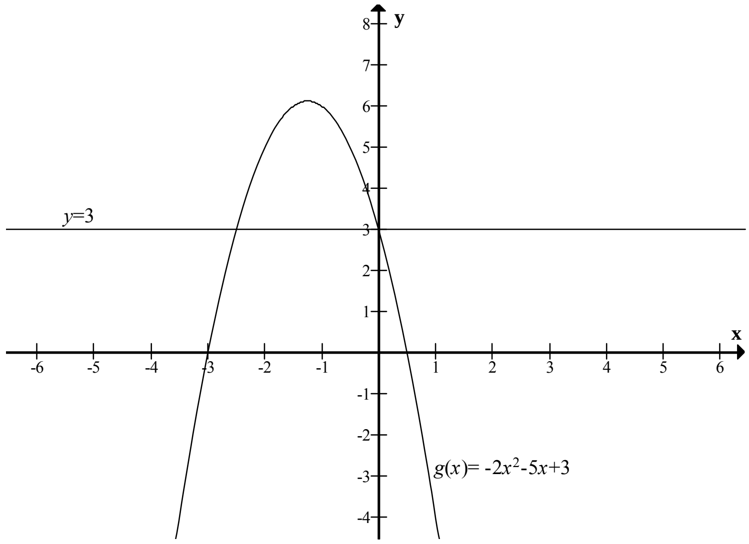


Figure 4.2

- Any of the above inequalities would normally be solved by using algebraic methods. For example:

  Solving for x in $-2x^2 - 5x + 3 < 0$, would involve the following steps:

  $$-2x^2 - 5x + 3 < 0$$
  $$2x^2 + 5x - 3 > 0$$
  $$(2x - 1)(x + 3) > 0$$

  Hence $x < -3$ or $x > \dfrac{1}{2}$ by using the fact that for all real numbers a and b, if $ab > 0$, then either $a > 0$ and $b > 0$ or $a < 0$ and $b < 0$. Alternatively, learners would use the following sign table to solve the inequality.

		-3		½	
$(2x - 1)$	-	-	-	0	+
$(x + 3)$	-	0	+	+	+
$(2x - 1)(x + 3)$	+	0	-	0	+
$-(2x - 1)(x + 3)$	-	0	+	0	-

Table 4.1: Sign Table

The answer to $-2x^2 - 5x + 3 < 0$ would then be read off from the table as $x < -3$ or $x > \frac{1}{2}$.

- As consolidation of the concepts, learners were allowed to work on question 1.1 of Activity 1 (see Appendix IV) and feedback was provided.

4.5.3.2 The number of roots of functions of the form $f(x) = k$

- Here the function $f(x) = -2x^2 - 5x + 3$ and its graph were considered. Various horizontal lines of the form $y = k$ were drawn and the number of roots was related to the number of times the line intersected the parabola.
- For $-2x^2 - 5x + 3 = 3$, the normal algebraic method was used to solve the equation and learners observed what the solutions looked like graphically on the computer screen.

  The familiar procedure where $-2x^2 - 5x + 3 = 3$ is solved for *x*, involves the steps:

  $$-2x^2 - 5x + 3 = 3$$
  $$-2x^2 - 5x = 0$$
  $$2x^2 + 5x = 0$$
  $$x(2x + 5) = 0$$

  The final answer is: $x = 0$ or $x = -2\frac{1}{2}$. The graphical view of the answer is illustrated in figure 4.3 below.

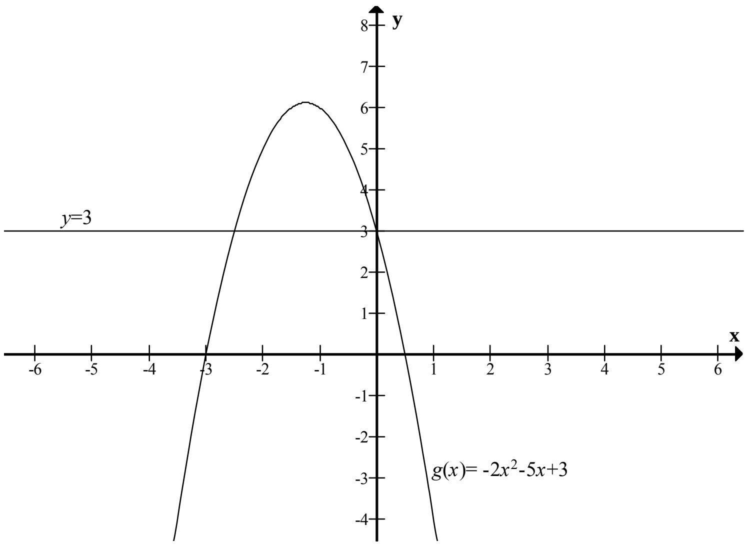


Figure 4.3

The idea of no roots was illustrated by drawing lines like $y = 8$ and $y = 7$. Learners could see from the graphs that $-2x^2 - 5x + 3$ does not intersect the lines $y = 8$ and $y = 7$. The familiar procedure to solve $-2x^2 - 5x + 3 = 8$ algebraically would involve the steps:

$$-2x^2 - 5x + 3 = 8$$
$$-2x^2 - 5x - 5 = 0$$
$$2x^2 + 5x + 5 = 0$$

As the last quadratic equation cannot be factorised, learners use the quadratic formula to solve for x to get:

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(-2)(-5)}}{2(-2)}$$

Since the discriminant, $\Delta = b^2 - 4ac$ in this case is equal to -15, they would conclude that the equation has no real solutions.

- The line $y = 5$ was drawn and learners could see that the line intersects the graph of $f(x) = -2x^2 - 5x + 3$ at two negative x-values. In this way the notion of two negative roots was conveyed.
- The lines $y = 2$ and $y = -1$ were drawn and in this case learners could see the x-values of the points of intersection were positive and negative. In this way the notion of positive and negative roots was conveyed.
- The line $y = 6\frac{1}{8}$ was drawn. This line is a tangent line to the graph of $f(x) = -2x^2 - 5x + 3$.
- Learners could see that the line and the parabola only intersected once and thus had only one root. Some of the scenarios are illustrated in figure 4.4 below.

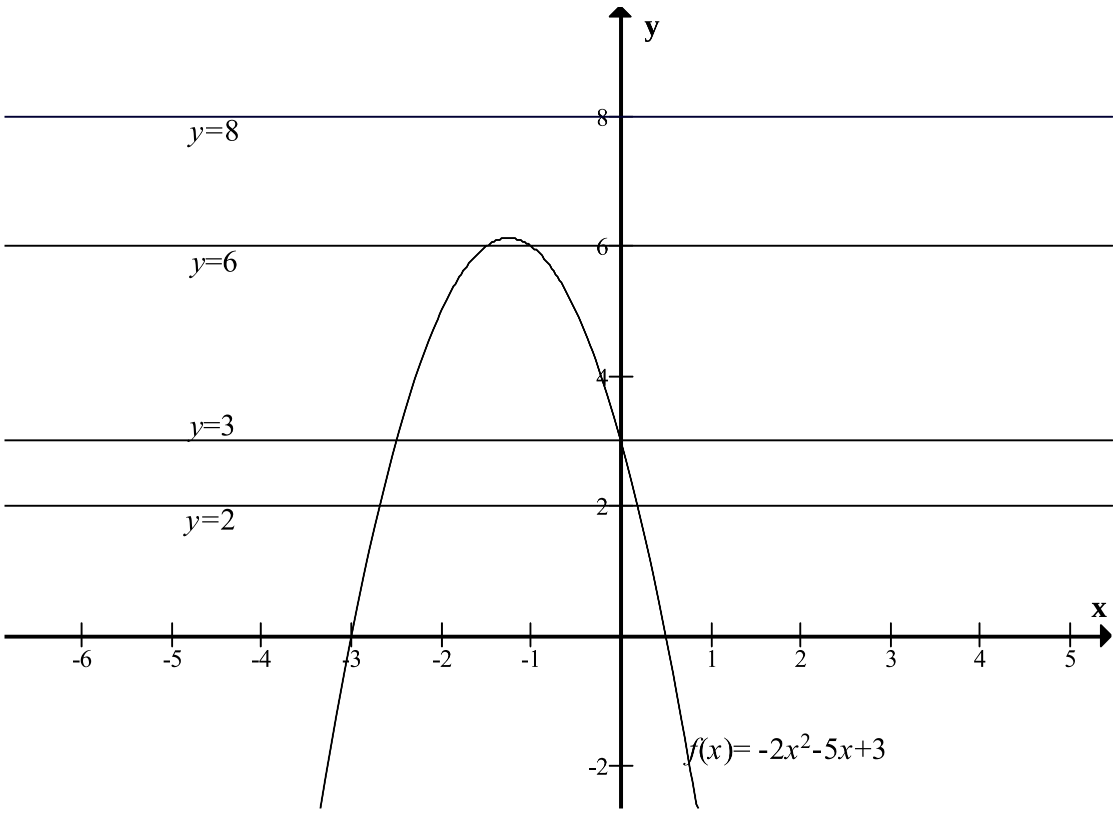


Figure 4.4

- As consolidation of the concepts, learners were allowed to work on question 1.3 of Activity 1 (see Appendix IV) and feedback was provided.

4.5.3.3 The domain and range of functions and their restrictions:

- Learners were asked to explain the meaning of the domain and range of a function and after none of the learners could answer correctly, this was explained to them. A simple explanation of domain as the *x*-values for which the function is defined, that is, the values of *x* for which $f(x)$ exists while range is the set of function values $f(x)$, was given.
- With the aid of Graphmatica, various graphs or portions of graphs were drawn and learners had to give the domain and range of the functions or their restrictions. For example, learners were asked to give the domain and range in each of the cases illustrated in figure 4.5 below, where $f(x) = 8$, $g(x) = -2x^2 - 5x + 3$ and $h(x) = -2x^2 - 5x + 3$.

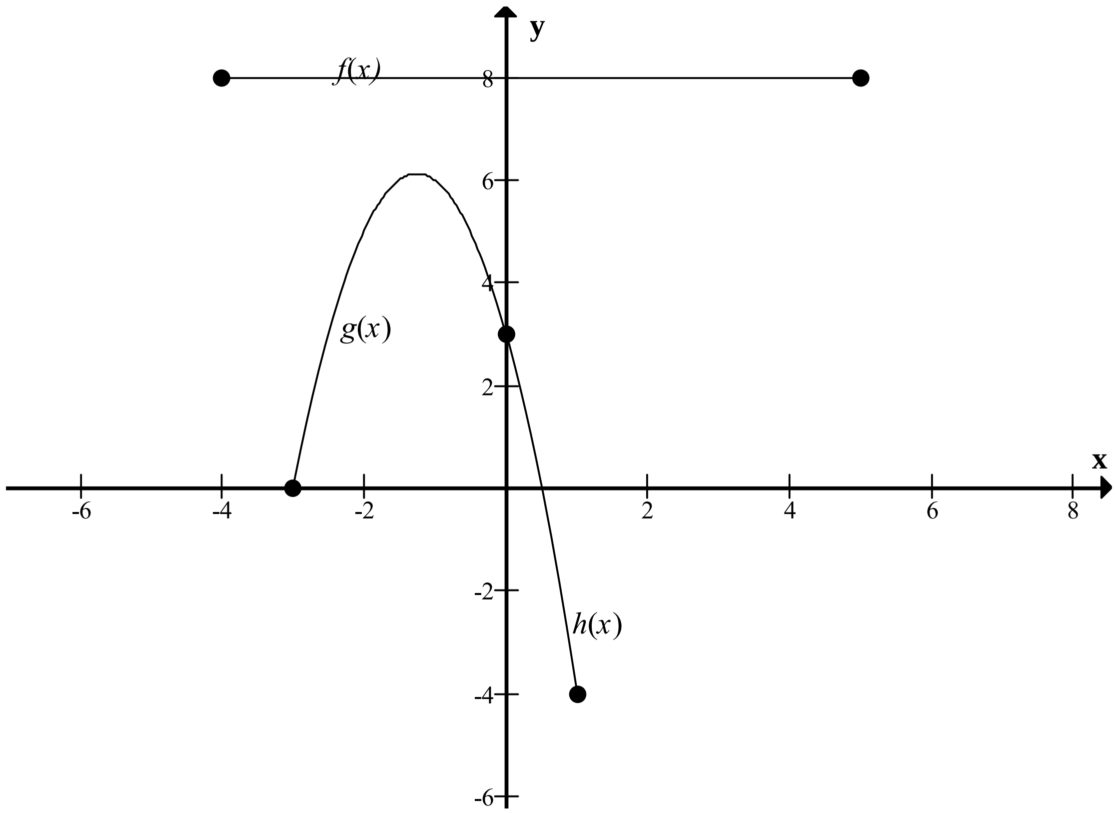


Figure 4.5

The correct answers are:

For $f(x) = 8$ the domain is [-4; 5] and the range is $\{8\}$

For $g(x) = -2x^2 - 5x + 3$ the domain is [-3; 0] and range is [0; 6.15]

For $h(x) = -2x^2 - 5x + 3$ the domain is [0; 1] and range is [-4; 3]

- As consolidation of the concepts, learners were allowed to work on question 1.2 of Activity 1 (see Appendix IV) and feedback was provided.

4.6 Lesson 2

4.6.1 Aims:

To employ an integrated teaching methodology to enhance learners' ability to visualise with the aid of graphs by considering:

4.6.1.1 the translation of graphs in both vertical and horizontal directions.

4.6.1.2 the estimation of irrational roots of equations of the form $f(x) = k$, k a constant.

4.6.1.3 the intervals where the function or graph is increasing, decreasing or turning.

4.6.2 Objectives:

4.6.2.1 To integrate learners' knowledge of functions and graphs to improve their understanding of the translation of graphs.

4.6.2.2 To integrate learners' knowledge of quadratic theory, graphs and functions to improve their understanding of the irrational roots of equations of the form $f(x) = k$

4.6.2.3 To relate the rates of change of functions with the intervals where the graphs increase, decrease or turn.

4.6.3 Procedures:

The aspects covered and the strategies that were employed are outlined below:

4.6.3.1 The translation of graphs in both vertical and horizontal directions:

- The graph of $f(x) = x^2 + x$ was drawn with the aid of Graphmatica. The vertical movement of the graph was demonstrated by drawing $g(x) = f(x) - 2$, $h(x) = f(x) - 4$ and $d(x) = f(x) + 4$. Learners were asked to observe what was happening to the graph as a positive constant was added or subtracted. In this way they observed the graph was shifting upwards when a positive constant was added and downwards when a positive constant was subtracted. The algebraic representation of the answer was also given on the white board. In this way $y = f(x) - 2$ and $f(x) = x^2 + x$ - 2 described the same thing. This is illustrated in figure 4.6 below.

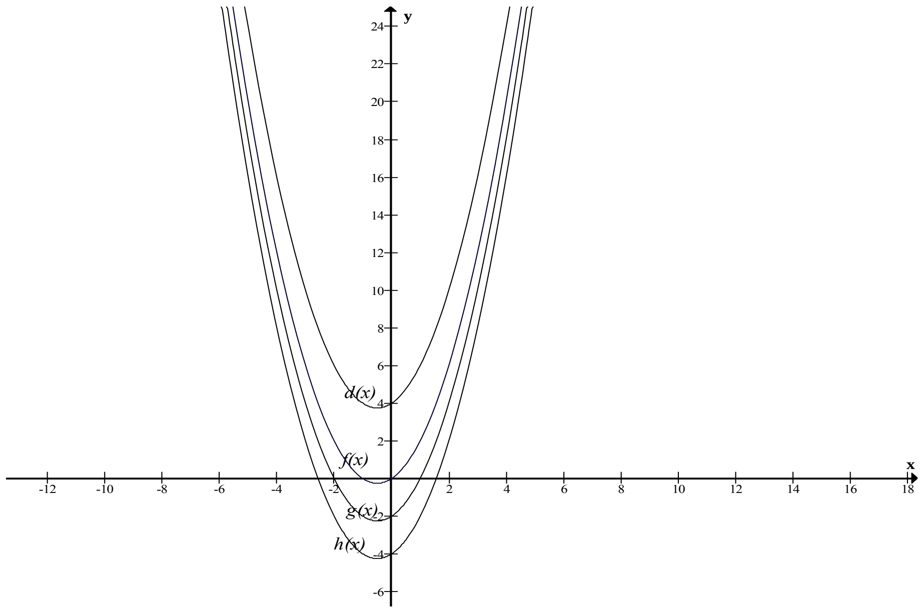


Figure 4.6

- In the same way learners observed the horizontal shifting of the graph when the graphs of $g(x) = f(x+3)$, $h(x) = f(x-2)$ and $j(x) = f(x-5)$ were drawn. Learners saw that $y = f(x+3)$ produced movement to the left whereas $y = f(x-2)$ and $y = f(x-5)$ produced movement to the right. The horizontal movement of the graph is illustrated in figure 4.7 below.

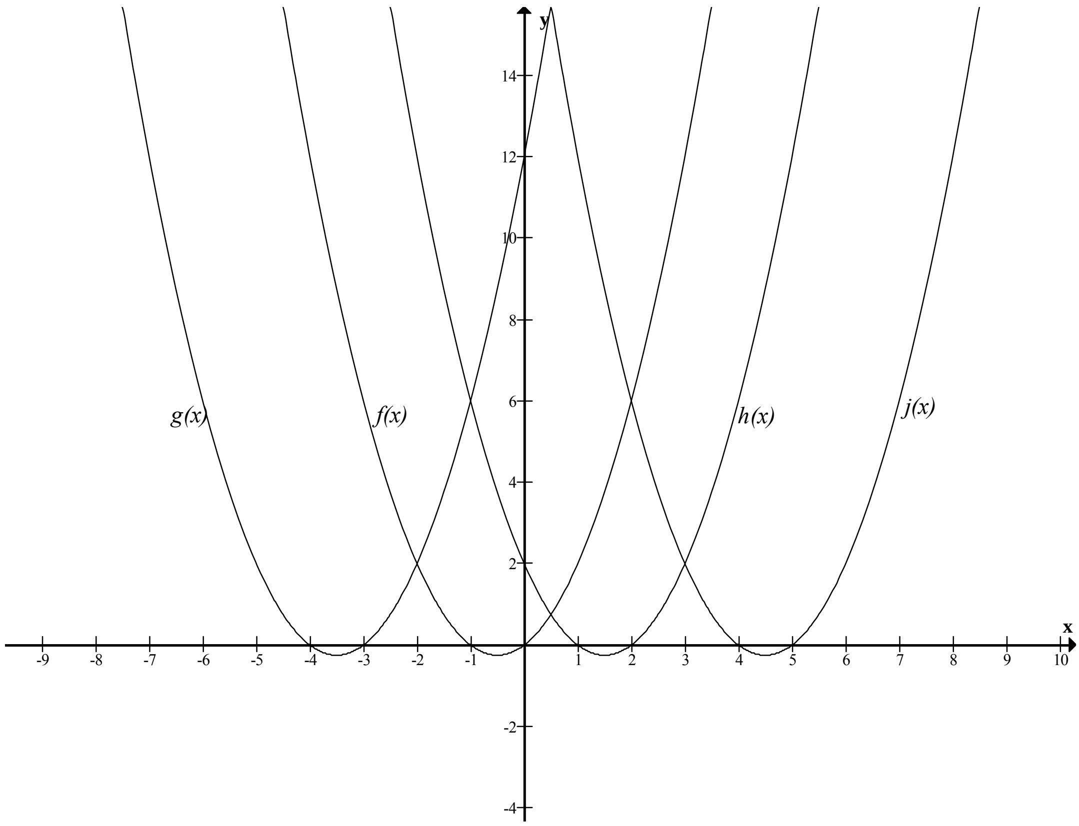


Figure 4.7

- To complement this, learners were asked to evaluate $g(x-1)$ where $g(x) = x^2 + x$ and no learner gave a satisfactory answer. The notion of function values was then revised by looking at $g(-1); g(5); g(k)$ and it was only after this discussion that they had an idea what to do with $g(x-1)$. They were made aware that, knowing the graph of $g(x) = x^2 + x$, they could now predict what the graph of $g(x-1)$ was without going through long calculations.
- As final consolidation, learners had the opportunity to predict the movement of the graphs $y = f(x+2)+3$ and $y = f(x-1)+2$. At this point of the lesson interesting interactions between learners in the groups and what was displayed on the computer screen, could be observed.

4.6.3.2 Finding solutions of irrational roots:

- The graph of $f(x) = x^2 + x$ was drawn and learners had to identify two consecutive integers where, $x^2 + x = 2$ and $x^2 + x = 3$.
- In the case, $x^2 + x = 6$, learners could easily read the answers x = -3 or x = 2 from the graph. For the example, $x^2 + x = 3$, they realised that an answer lies between 1 and 2.
- The equation was firstly solved by using the quadratic formula as follows:

  $$x^2 + x = 3$$
  $$x^2 + x - 3 = 0$$
  $$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-3)}}{2(1)}$$

  i.e. the solutions are $x = \frac{-1+\sqrt{13}}{2}$ or $x = \frac{-1-\sqrt{13}}{2}$, giving the approximate solutions $x =$ 1.3 or $x = -2.3$.

- Another method was by making some intelligent guesses. Learners could see that $x^2 + x = 3$ for x between 1 and 1.5 on the positive side of the x-axis. Their first guess was $x = 1.3$. The corresponding $f(x)$ value was calculated as follows

  $$f(x) = x^2 + x$$
  $$f(1.3) = (1.3)^2 + 1.3$$
  $$= 2.99$$

  This produced a point or coordinate which is below the y = 3 line.
- The next guess was $x = 1.4$. The corresponding $f(x)$ value was calculated as follows

  $$f(x) = x^2 + x$$
  $$f(1.4) = (1.4)^2 + 1.4$$
  $$= 3.36$$

  This produced a point or coordinate which is above the y = 3 line.
- The third and last guess was x = 1.32, and learners could see how their guesses generated function values closer and closer to 3.

- These two methods were complemented with the graphs of $f(x) = x^2 + x$ and the horizontal line $y = 3$. Learners could see whether their guesses generated answers above, below or on the line. This is illustrated in figure 4.8.

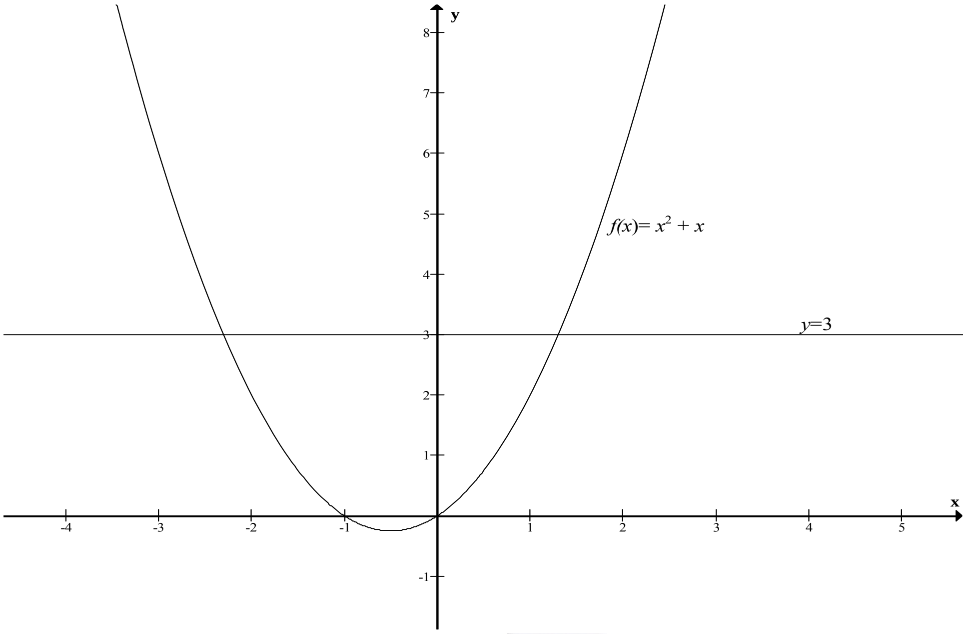


Figure 4.8

- As consolidation of the concepts, learners were allowed to work on Activity 2 (see Appendix IV) and feedback was provided.

4.6.3.3 Intervals where the function or graph is increasing, decreasing or turning:

- As an introduction, learners were asked to give the derivative of the function $y = -2x - 4$. After a while the answer $\frac{dy}{dx} = -2$ was given. They were made aware that $y = -2x - 4$ is a straight line of the form $y = mx + c$ with gradient, $m = -2$. They could see that $\frac{dy}{dx} = gradient$ of the line. As a follow-up learners were asked to give the derivative of the constant function $y = 4$. They responded that $\frac{dy}{dx} = 0$. They were again made aware that the answer of zero again corresponds to the gradient of the horizontal line $y = 4$.

- Various tangent lines were drawn at different points on a given quadratic curve. Learners could see how the gradient of the tangent lines changed from negative to zero to positive values. They were made aware that the gradients of the tangent lines became smaller as points closer to the turning point were used. This is illustrated in figure 4.9 below.

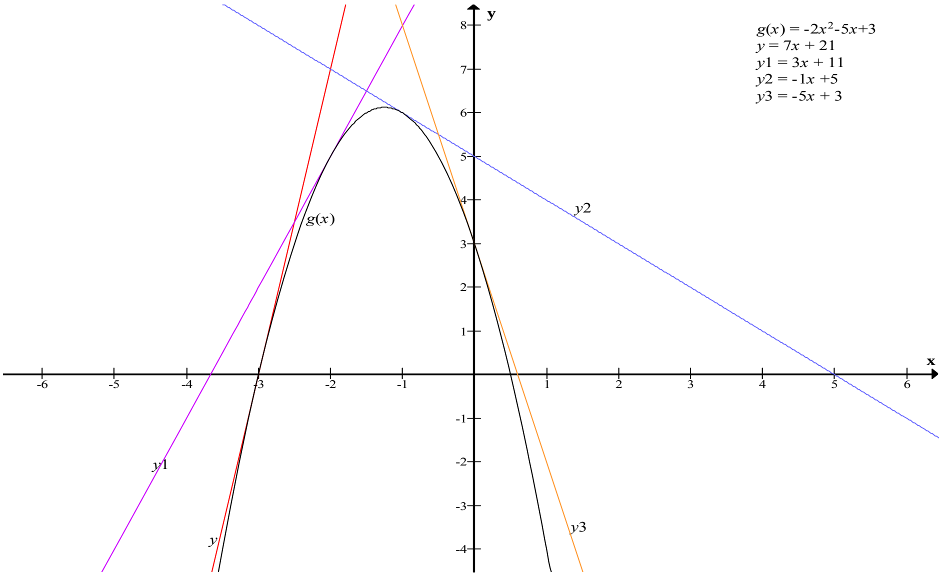


Figure 4.9

- Learners observed that the slope of the tangent line was positive where the graph was increasing, negative where the graph was decreasing and zero at the turning points.
- As consolidation of the concepts, learners were allowed to work on question 1.4 of Activity 1 (see Appendix IV) and feedback was provided.

4.7 Lesson 3

4.7.1 Aims:

To employ an integrated teaching methodology to enhance learners' ability to visualise with the aid of graphs:

4.6.1.1 the intervals where the derivatives of functions change from positive, to zero to negative.

4.6.1.2 the graph of the derivative function.

4.6.1.3 the solution of maxima or minima problems.

4.7.2 Objectives:

To apply the knowledge of lessons 1 and 2 to solve minima/ maxima word problems and to integrate the learners' fragmented knowledge of Quadratic Theory and Differential Calculus.

4.7.3 Procedures:

The aspects covered and the strategies that were employed are outlined below:

4.7.3.1 Intervals where the derivatives of functions change from positive to zero to negative:

- The function $f(x) = x^2 - 2x - 3$ was given and learners were asked to give the derivative of the function. $f'(x) = 2x - 2$ was given as an answer. Various x-values were substituted into the derivative function. For example: $f'(1) = 0; f'(2) = 2; f'(-4) = -10$.
- With the aid of Graphmatica tangent lines were drawn at $x = 1, 2, -4$ to the curve of $f(x) = x^2 - 2x - 3$ and learners could observe that the previous answers correspond to the slope of the tangent lines to the curve. This is illustrated in figure 4.10.

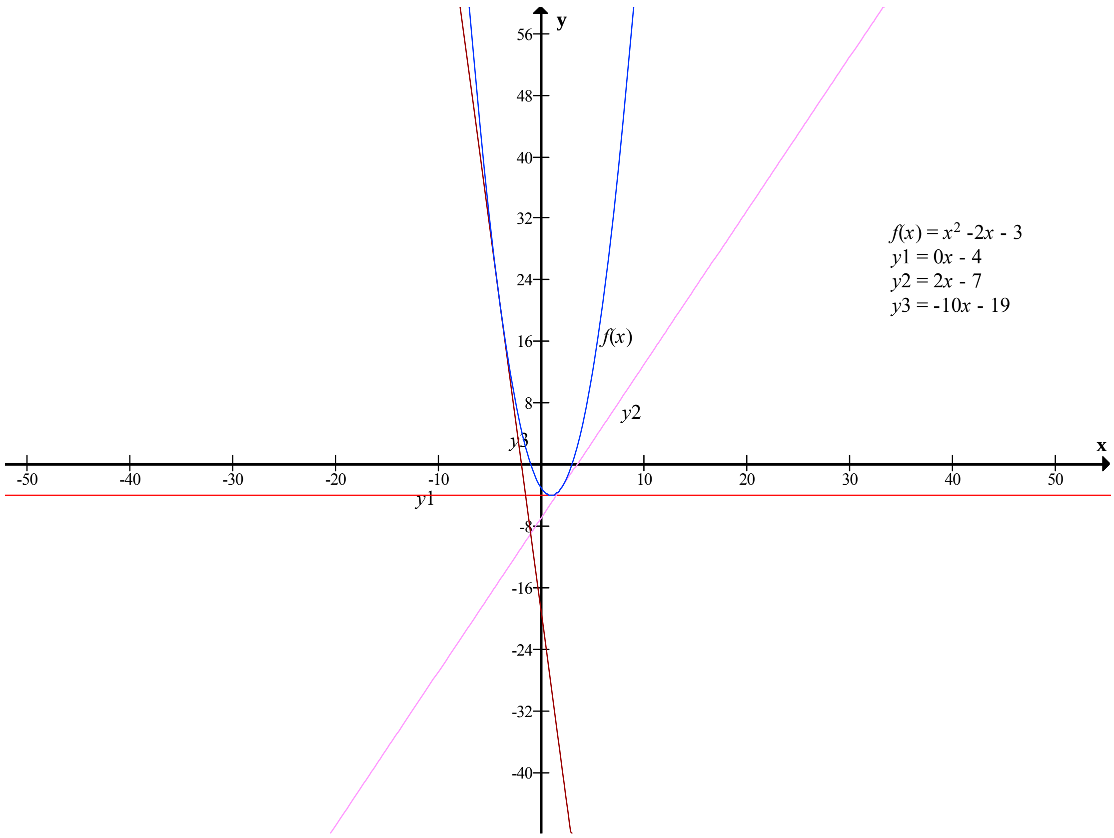


Figure 4.10

- Using the knowledge of the previous two lessons, the graph of the derivative function was discussed. This was not understood very well by the learners. Various graphs and the graphs of their derivatives were drawn. This was related to finding the intervals where the graphs were increasing, i.e. where the derivative was positive, decreasing, i.e. where the derivative was negative and turning where the derivative was zero. This aspect requires further attention.
- One of the examples is illustrated in figure 4.11.

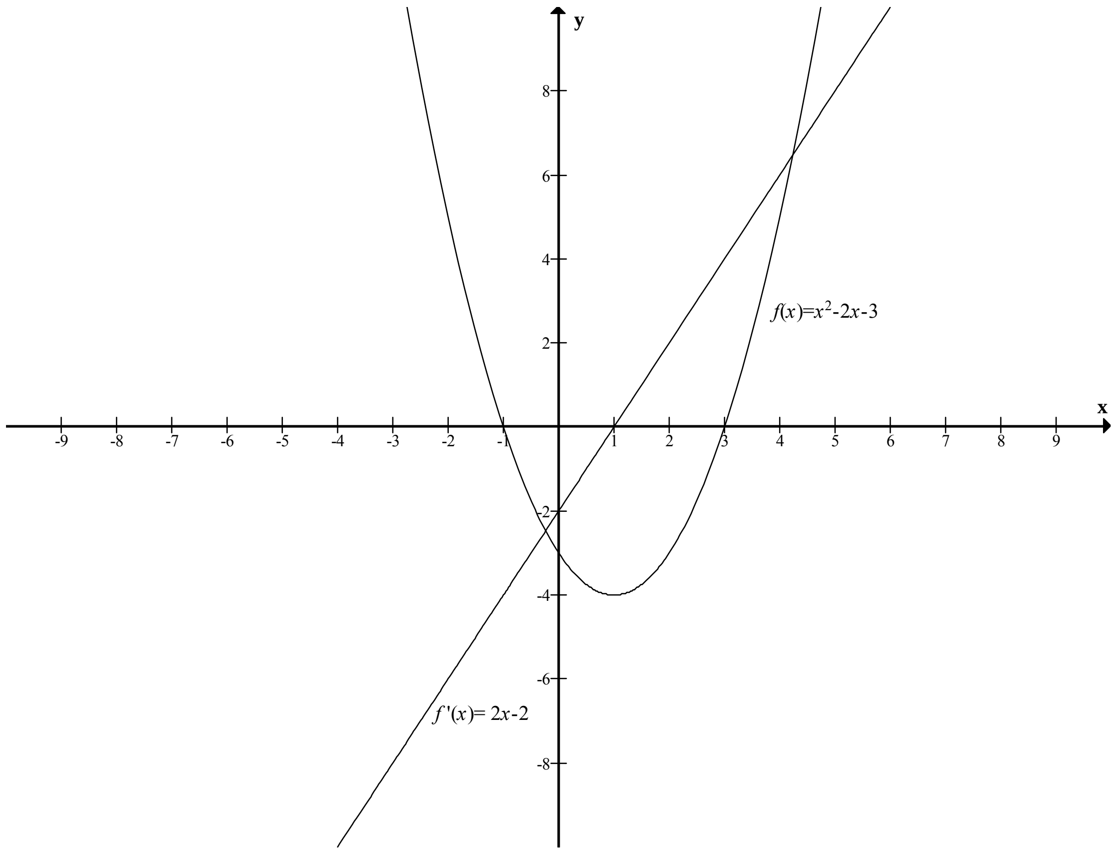


Figure 4.11

4.7.3.2 Various methods to solve maxima or minima problems:

- Learners were referred to ball throwing, which was observed the previous day as a real life example of motion in a parabolic path. They were also reminded, with the aid of the illustration below, that the tangent line at the minimum or maximum is a horizontal line which had a zero gradient. With the use of the graph they could see that $\frac{dy}{dx} = 0$ can be used to calculate the minimum or maximum of a function. This is illustrated in figure 4.12 for the graph of $f(x) = x^2 - 2x - 3$. The tangent at the turning point has an equation of $y = -4$ with gradient equal to zero.

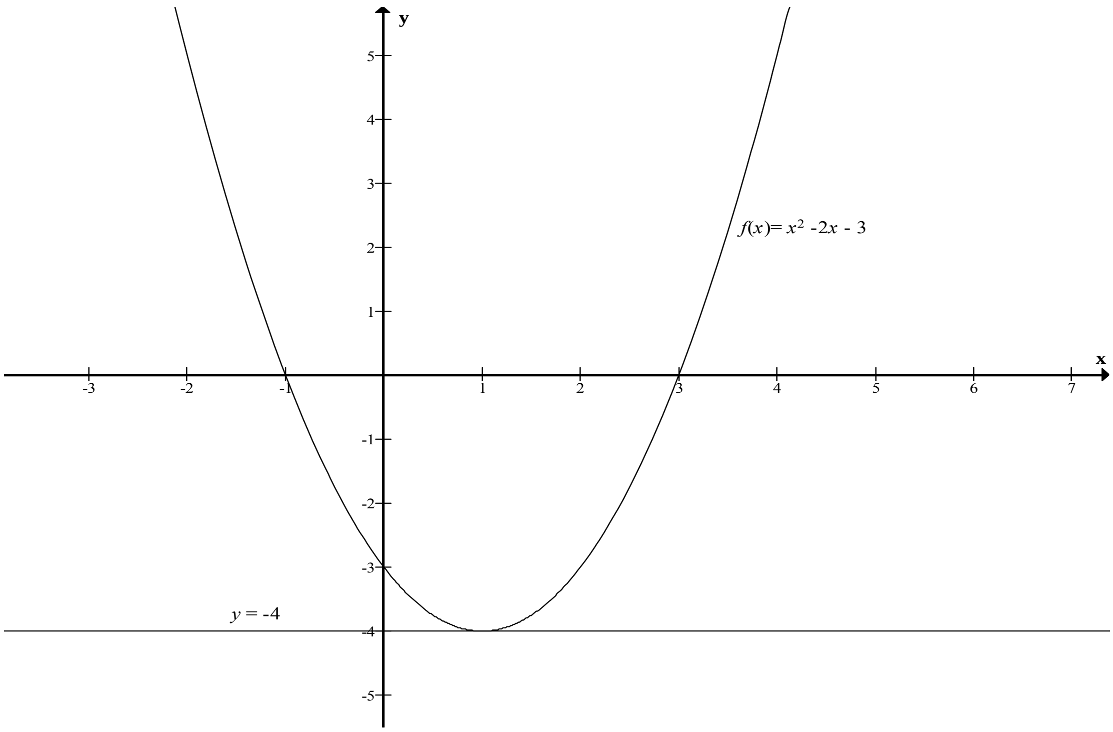


Figure 4.12

- It was explained to the learners that the maxima or minima of functions could be found by three methods, two algebraic and one with Calculus. They could use $x = \frac{-b}{2a}$ and substitute this to get the corresponding function value, or they could complete the square or they could find the derivative $f'(x)$, of the function and by solving for x in $f'(x) = 0$. This was demonstrated using the function $f(x) = -2x^2 - 5x + 3$ as follows:

  Method 1: The axis of symmetry for $f(x) = -2x^2 - 5x + 3$ is

  $$x = -\frac{b}{2a}$$
  $$= -\frac{(-5)}{2(-2)}$$
  $$= -\frac{5}{4}$$

  The corresponding y-value was calculated using:

  $f\left(-\frac{5}{4}\right) = 6\,{}^{1}\!/\!{}_{8}$ or by using the grade 11 formula of:

$y = \frac{4ac - b^2}{4a}$ to get the same answer of $6\,{}^{1}\!/_{8}$.

Method 2: By completing the square in the following way,

$$f(x) = -2x^2 - 5x + 3$$
$$= -2\left(x^2 + \frac{5}{2}x - \frac{3}{2}\right)$$
$$= -2\left[x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2 - \frac{3}{2}\right]$$
$$= -2\left(x + \frac{5}{4}\right)^2 + 6\,{}^{1}\!/_{8}$$

learners would be able to deduce that the maximum is at $x = -\frac{5}{4}$ and $y = 6\,{}^{1}\!/_{8}$.

Method 3: By using the fact that the gradient of the tangent line at the minimum or maximum is a horizontal line, we did the following:

$$f(x) = -2x^2 - 5x + 3$$
$$f'(x) = -4x - 5$$

At the minimum or maximum, $f'(x) = 0$. Therefore

$$-4x - 5 = 0$$
$$x = -\frac{5}{4}$$

The corresponding *y*-value was calculated using:

$$f\left(-\frac{5}{4}\right) = 6\,{}^{1}\!/_{8}.$$

- As consolidation of the concepts, learners were allowed to work on, question 1.5 and 1.6 of Activity 3 (see Appendix IV) and feedback was provided.

4.8 Motivation for the approach

Some of the characteristics of "substantial" design of teaching units were outlined earlier in the chapter. In all the lessons, the concepts were explained by employing strategies which can also be transferred to other mathematical "ecologies". Thus, in lesson one, a function was described as being positive where the graph is above the *x*-axis. This fact can be used for any other graph. In the same lesson the number of roots was related to the number of intersections between two graphs. This again is very much transferable to other mathematical "ecologies".

This was the general strategy in all three lessons. Learners were grouped around computers to allow for cooperative learning. Within such an arrangement, various psychological aspects come into play. Some of these were mentioned in chapter 3. Substantial teaching units are also rich in activities. The learners were involved in the lessons by employing strategies which ranged from worksheets, answering questions, making predictions and taking guesses.

Learners were exposed to different ways of understanding various concepts. For example, the chalk and talk method of teaching was complimented by using graphical representation of functions.

The lessons were centred on the use of technology which, according to SIMMS (2005, p. 3), is an important aspect of an integrated mathematics curriculum. The central objective of these three lessons was that it endeavoured to integrate the fragmented knowledge of the learners. This is also one of the characteristics of a substantial teaching unit according to Wittmann (1995).

4.9 Conclusion

The instruments discussed in this chapter were used to determine the effectiveness of an integrated teaching methodology. The next chapter analyses the results in order to establish whether learners can benefit from this kind of teaching methodology. Recommendations and suggestions will also be made to assist future studies of this nature.

Chapter 5

Results

5.1 Introduction

This chapter displays the results of the pre- and post tests. Comments and recommendations about the whole experience of doing research of this nature in the current climate of schooling at grade 12 level will also be made.

5.2 Background and procedure

Two groups of grade 12 learners from two different schools were used in this investigation. One school was the experimental group whilst the other was the control group. The schools are very different as the one is a high school situated in a black township and the school's matric pass percentage has been ranging from 50 to 65 percent the past three years. This school was the experimental group referred to as school A for practical purposes. The other school is also serving a worker's class community but their pass rate had been hovering between 80 and 95 percent. This school was the control group referred to as school B.

Both schools A and B were given the pre-test, so at that stage both groups of learners had not been exposed to this form of integrated teaching. They entered the pre-test with only their past experiences of classroom mathematics. It might also be worth while to mention that these tests were written towards the end of the third quarter and at that stage both schools were busy with revision in their respective mathematics classes. A total of 75 learners wrote the pre-test, 40 from school A and 35 from school B. At school A, three integrated lessons of about 45 minutes each were administered. Thereafter the learners from both schools A and B were given a post test. What will follow are the results of the pre- and post-tests. The results of the pre-test for schools A and B were combined as there was no difference between the participants at that stage of the procedure. The post-test results for school A and school B were kept separated as integrated lessons were administered only to school A.

A total of 52 learners wrote the post-test, 28 from school A and 24 from school B.

What will follow are firstly the results of the pre-test followed by those of the post-test.

5.3 Results of the pre-test

Table 5.1 summarises the results obtained by the learners:

The number in brackets in the second column refers to the number of learners from school B.

Table 5.1: Summary of the pre-test results for school A and B

% range	No. of learners	% of learners
80.0 – 100.0	0	0
70.0 – 79.9	0	0
60.0 – 69.9	0	0
50.0 – 59.9	0	0
40.0 – 49.9	0	0
30.0 – 39.9	0	0
20.0 – 29.9	3(0)	4
10.0 –19.9	5(4)	12
0.0 – 9.9	32(31)	84
Total	75	100.0

From table 5.1 it is evident that most of the learners had no clue of what to do in the pre-test.

Table 5.2 shows the average percentage obtained in each question of the pre-test and shows the percentage of abstentions for each question.

Table 5.2: Average percentage and abstentions per question in pre-test for schools A & B

Question number	Average %	% Abstentions
1.1.1	33.3	16
1.1.2	24.7	13.3
1.1.3	22.67	24
1.2	0.7	28
1.3.1	4.7	17.3
1.3.2	49.3	9.3
1.4.1	42.7	10.7
1.4.2	1.3	32
1.5.1	4.7	28
1.5.2	0	53.3
1.5.3	0	56
1.6.1	0	42.7
1.6.2	0	57.3
1.6.3	0	57.3
1.6.4	5.3	54.7
1.7	4	62.7
1.7.1	6.7	62.7
1.7.2	1.3	68
1.8.1	7.3	64
1.8.2	2	68
1.8.3	8	68
1.9	0.7	77.3
2.1	7.3	48
2.2	3.3	54.7
2.3	3.3	60
2.4	6.7	61.3
2.5	9.3	68
2.6	0	70.7
2.7	2.7	78.7
2.8	1.3	49.3
2.9.1	0	73.3
2.9.2	0	73.3
2.10	0	90.7

Tables 5.1 and 5.2 confirm observations and comments from learners who sat down to write the pre-test. The age old fear of word problems was again confirmed as most of the learners did not attempt these problems. This is evident from the high percentages of abstention in Question 2. These percentages were much higher than the percentages from the other questions. Most of the learners could not explain how they came to choose a certain

answer especially when attempting question 1.1. This indicated that learners had no idea what graphs represented. They could not read intervals from graphs where the function was, for example, positive. They did not know that the function was positive where the graph was above the *x*-axis, negative where the graph was below the *x*-axis and zero where the graph cut the *x*-axis.

Question 1.3 was surprisingly poorly answered. This question specifically tried to assess whether learners could bring together what normal algebraic manipulation represented and what graphs displayed. An average percentage of 4.7 % for 1.3.1 and 49.3% for question 1.3.2 confirmed that learners could not tie the two views together. The percentage of 49.3 % for question 1.3.2 was obtained by learners when they solved the equation of $x^2 - 2x - 3 = -4$ by normal algebraic manipulation and not by finding a contradiction just by reading off from the graph, the *x*-values where the graph was equal to -4. Learners were thus looking for the familiar so that they could perform their drilled sequence of procedures.

The nature of the roots was assessed in Question 1.6. Again in this question, learners could not read off from the graph the number of roots for different values of *k* in $x^2 - 2x - 3 = k$. Most were tempted to calculate the discriminant. So they were lacking a visual understanding of the nature of roots.

The poor results of Question 1.8 also showed that learners did not know that the derivative was positive when the graph was increasing, zero at the turning point and negative where the graph was decreasing. Learners did not have a graphical understanding of Differential Calculus.

5.3.1 Comparison of the pre-test results of schools A and B

The Mann-Whitney U test was used to analyse the difference in the pre-test results of school A and B to determine whether there was a significant difference in the ability of the learners from the two different schools to deal with the pre-test. The result of the test is illustrated in Table 5.3.

Table 5.3: Mann-Whitney U test for the pre-test scores of schools A and B

				analysed with: Analyse-it + General 1.71
Test	**Mann-Whitney U test for pre-test**			
	Comparative analysis of the pre-test scores for School A and School B			
Alternative hypothesis	Scores by School A or B: A ≠ B			
n	75			
Scores by School A or B	**n**	**Rank sum**	**Mean rank**	**U**
A	40	1456.0	36.40	764.0
B	35	1394.0	39.83	636.0
Difference between medians	0.0			
95.1% CI	-2.0	to 1.0	(normal approximation)	
Mann-Whitney U statistic	764			
2-tailed p	0.4934	(normal approximation, corrected for ties)		

The initial hypothesis was that there was not a significant difference in the ability of learners to integrate their mathematical knowledge and concepts. From Table 5.3 the hypothesis is confirmed. The difference between the medians is 0.0 and the two tailed p value is greater than 0.05. Because $p > 0.05$, this indicated that there was no significant difference between the mathematical abilities of the learners from schools A and B in the pre-test scores. Therefore the initial hypothesis was accepted.

5.4 Results of the post-test for school A

Table 5.4 below summarises the results obtained by the learners from school A.

Table 5.4: Summary of the post-test results for school A

% range	No. of learners	% of learners
80.0 – 100.0	0	0
70.0 – 79.9	0	0
60.0 – 69.9	0	0
50.0 – 59.9	0	0
40.0 – 49.9	1	3.6
30.0 – 39.9	3	10.7
20.0 – 29.9	2	7.1
10.0 –19.9	2	7.1
0.0 – 9.9	20	71.4
Total	28	100.0

The above table shows an improvement in the performance of the learners compared to their performance in the pre-test. One learner scored a mark ranging between 40 and 49.9 % and three scored marks ranging between 30 and 39.9 %. The best scores in the pre-test ranged between 20 and 29.9%.

Table 5.5 gives a breakdown of the results of the learners in school A with regard to their performance in the post-test.

Table 5.5: Average percentage and percentage abstentions per question in the post-test for school A

Question number	Average %	% abstentions
1.1.1	23.2	0
1.1.2	14.3	0
1.1.3	19.6	3.6
1.2.1	14.3	25
1.2.2	17.9	17.9
1.3.1	7.1	10.7
1.3.2	10.7	17.9
1.3.3	7.1	28.6
1.4.1	17.9	14.3
1.4.2	0	42.9
1.4.3	0	46.4
1.5.1	12.5	3.6
1.5.2	12.5	7.1
1.5.3	10.7	17.9
1.6	19.6	35.7
1.7	7.1	14.3
1.7.1	17.9	14.3
1.7.2	7.1	10.7
2.1.1	14.3	14.3
2.1.2	16.1	10.7
2.1.3	12.5	14.3
2.2.1	14.3	14.3
2.2.2	5.4	28.6
2.2.3	0	25
2.3	4.5	35.7
3.1.1	17.9	10.7
3.1.2	21.4	7.1
3.2	3.6	14.3
3.3	12.5	25
3.4	0	32.1
3.5.1	3.6	21.4
3.5.2	3.6	21.4
3.6	19.6	25

Table 5.4 shows a slight improvement with respect to the number of learners who scored marks in the ranges other than 0 to 9.9 percent. None of the learners could however score more than 50 percent even after the integrated lessons were given to learners from school A. The percentages of abstentions were much lower than those of the pre-test as showed by Table 5.5. The average percentages for questions 1.4.2 and 1.4.3 for the post-test and questions 1.5.2 and 1.5.3 were zero even after the integrated lessons were

administered. This shows that the strategy employed in the lessons did not work or that more consolidation exercises were needed. Question 3 was also better answered. The percentages for abstention of the individual questions were lower and more learners scored marks in the post-test compared to the pre-test.

5.5 Results of the post-test for school B

What will now follow are the results of the control group for the post-test of school B. Table 5.6 below gives a summary of the results from the control group (no intervention).

Table 5.6: Summary of the post-test results for school B

% range	No. of learners	% of learners
75.0 – 100.0	0	0
70.0 – 74.9	0	0
60.0 – 69.9	0	0
50.0 – 59.9	0	0
40.0 – 49.9	0	0
30.0 – 39.9	0	0
20.0 – 29.9	0	0
10.0 –19.9	2	8.3
0.0 – 9.9	22	91.7
Total	24	100.0

Table 5.6 shows a similar result to that of the pre-test. Most of the learners' scores ranged between 0% and 9.9%. Only two managed to score results between 10% and 19.9%. Similar results were scored in the pre-test, i.e. 31 scored between 0 and 9.9% and 4 learners scored between 10 and 19.9%. This already suggests that no improvement was evident between the results of the pre- and the post-test for this group.

Table 5.7 gives a breakdown of the results for the learners in school B with regard to their performance in the post-test.

Table 5.7: Average percentage and percentage abstentions per question in the post-test for school B

Question number	Average %	% abstentions
1.1.1	0	0
1.1.2	8.3	0
1.1.3	22.9	0
1.2.1	20.8	12.5
1.2.2	2.1	20.8
1.3.1	0	0
1.3.2	4.2	0
1.3.3	0	12.5
1.4.1	18.8	0
1.4.2	1.7	29.2
1.4.3	0	29.2
1.5.1	4.2	0
1.5.2	8.3	4.2
1.5.3	6.3	4.2
1.6	25	8.3
1.7	0	33.3
1.7.1	4.2	29.2
1.7.2	0	25
2.1.1	0	20.8
2.1.2	0	16.7
2.1.3	0	20.8
2.2.1	0	33.3
2.2.2	0	33.3
2.2.3	0	11
2.3	4.2	41.7
3.1.1	4.2	16.7
3.1.2	0	11
3.2	0	20.8
3.3	6.3	29.2
3.4	6.3	54.2
3.5.1	0	37.5
3.5.2	0	37.5
3.6	0	50

The average percentages for almost all the questions were below 20%; with the majority of the percentage range between 0 and 10 percent. The percentages for abstention in individual questions were lower than those of the pre-test. This can possibly be attributed to the fact that learners had a fair idea what to expect after they wrote the pre-test. The average percentages for question 3, on the application of quadratic functions, were still very low. This suggests that this group still had no idea how to integrate their concepts in mathematics even after they were given the pre-test and had some idea of what the questioning in this study was like.

5.5.1 Comparison of the post-test results of schools A and B

The Mann-Whitney U test was used to analyse the difference in the post-test results of schools A and B to determine whether there was a significant difference in the ability of the learners from the two different schools to deal with an integrated assessment form. The results are illustrated in Table 5.8 below.

Table 5.8: Mann-Whitney U test for the post-test scores of school A and B

				analysed with: Analyse-it + General 1.71
Test	**Mann-Whitney U test for post-test** Comparative analysis of the post-test scores for School A and School B			
Alternative hypothesis	Scores by School A or B: A ≠ B			
n	52			
Scores by School A or B	**n**	**Rank sum**	**Mean rank**	**U**
A	28	812.5	29.02	265.5
B	24	565.5	23.56	406.5
Difference between medians	1.0			
95.0% CI	0.0	to 3.0	(normal approximation)	
Mann-Whitney U statistic	265.5			
2-tailed p	0.1900	(normal approximation, corrected for ties)		

The initial hypothesis was that there would be a significant difference in the ability of learners to integrate their mathematical knowledge and concepts especially after school A had been exposed to integrated lessons. From Table 5.8 a contradiction was established. The difference between the medians is 1.0 and the two tailed p value of 0.19 is greater than 0.05.

This improvement was however not large enough for the initial hypothesis to be contradicted. Therefore, the ability of the two groups to integrate their mathematical knowledge and concepts still remained more or less the same. On average, school A showed some improvement, but this was not statistically significant.

5.6 Overall comments of the results

The results do not show a significant change between those of the pre-test and the post test but mainly that more learners in school A felt more able and positive to at least answer the questions in the post-test. From interactions with some of the learners after the post test, some commented that:

- although the post-test included questions and graphs which were far different from the pre-test and those discussed in the integrated lesson, they felt that the questions did not feel as strange as when they wrote the pre-test.
- they had an idea what to look for, but had difficulty to write down the answers.
- it was their first time to link what was done in rote calculation with other methods and with graphs especially.

5.7 Comments about the pre and post-test

Looking at the concepts covered in the pre and post-tests, it can now be said that too many were dealt with in these tests. The tests were too long (4 pages) which put learners off initially from doing something that did not really mean anything to them. For us as teachers and researchers, the concepts seemed elementary but since learners had been programmed to expect questions asked in a certain way, it would then make sense to do less of these unfamiliar things. One of the possible reasons why questions 1.5 of the pre-test and 1.4

of the post-test were so poorly answered might have been that learners did not understand what was expected of them.

The fact that learners could write the test anonymously was well received and they felt more at ease to write answers knowing that it will not affect their year mark.

Basic inequality signs were also unfamiliar to some of the learners as some learners wanted to know what these signs meant. Question 1 was also strange to them because as the learners progressed with the question, they lost track of the fact that the questions still referred to the graph which they had to study. Few learners actually studied or used the graph but were rather looking for familiar clues to which they could apply their learnt procedures.

It was a rewarding experience to draft the pre- and post tests with experienced lecturers from tertiary level. One gets brain-washed by the predictable nature of questioning at high school level.

5.8 Comments about the lessons

The lessons had to be taught within the normal schooling program. Learners who could not attend these recorded lessons were given an opportunity to attend the same lessons after school. These lessons were a repeat of the recorded lessons and were thus not recorded. Thus, all the learners who sat down for the post test went through the three integrated lessons.

A steady pace had to be maintained and the focus had to be kept on integrating concepts as opposed to the consolidation of supposedly familiar concepts. Mastering certain procedures such as the completion of the square or using the quadratic formula was not the primary goal of the lessons, but rather to use these tools and knowledge and combine it with other concepts for the learners. Recapping was done at the start of lessons 2 and 3 to re-orientate learners with the previous day's work.

The applications of quadratic functions definitely needed more time since learners generally have a negative attitude towards word problems. Grouping learners in front of the computers worked very well. Learners found it confusing when they had to know what to look for when the function and derivative of the function changes from positive, to negative to zero. This I picked up when I explained how to draw and interpret the graph of the derivative function.

Ideally, if the learners were computer literate, the format of the activities could have changed to activities whereby they could have discovered concepts by themselves. From discussions with learners after the lessons they also commented that it was a first for them to sit through lessons of this nature and that they could grasp the concepts faster especially with the sketches of the different graphs on the screens. The video camera was distracting to some of the learners.

During any one of the lessons, the role of the teacher changed from exposition, to facilitator to computer operator.

5.9 Comments on contextual factors

Writing a test for research purposes was difficult to explain to learners. This was understandable as they were in a year where they wrote an average of 20 tests; not even mentioning the other forms of assessment which they were also confronted with. Some of the integrated lessons for this study had to take place after school which was very demanding on the learners. There were also unforeseen factors which had to be dealt with as they came up, like unscheduled tests in other subjects, curriculum advisors visiting classes, etc. On day two the weather was also very bad and quite a number of the learners did not or could not attend. Thus on day three, lessons from day two had to be repeated for those learners who could not attend.

5.10 Recommendations and suggestions for studies of this nature

- Electronic equipment in the form of computers (software and hardware) and the video camera formed a central part of this exercise. Double or triple check the main computer and sub-stations thoroughly before the start of each lesson. A simple thing like charging the battery of the video camera must be done before every lesson,
- Study leave must be taken when recording data of this nature, since it is difficult to go through a normal day of teaching and managing a school and also to teach lessons of this nature.
- This study targeted specifically the interface between grade 12 and tertiary level. Timing tests for research purposes for grade 12 was much more difficult than expected. Learners in grade 12 attend school for only three quarters. They do a minimum of six subjects; thus making them pushed for time and might be reluctant to take part in research of this nature.
- The third quarter was ideal for integrated lessons of this nature. Most of the curriculum had been covered and most of the time teachers were busy with revision. Learners were approaching a trial examination and hopefully most of them were busy with preparations.
- Learners have to know that they can also benefit from the process as they will be taught mathematics in an integrated way.
- Make sure that arrangements are made for the usage of learners and facilities such as the computer room.
- You need to be familiar with the software which will be used in the demonstrations. Things can go horribly wrong if you cannot work with the software packages such as Graphmatica and Netops.
- You need to be well prepared with ample examples and make sure that the activities, worksheets, etc. are all ready at the start of the lessons.
- It would have worked better if the activities were also copied on transparencies. The use of an overhead projector with transparencies could help to give more effective feed-back to the learners.
- A trial run for the pre- and post tests would have been a useful exercise to check whether the responses were that expected by the assessor.

5.11 Conclusion:

Although the results did not show a significant improvement in the scores of the post-test, it did have an impact on the learners' overall spirit when they wrote the post-test. The recordings also show some interesting interaction between learners and whatever was graphically displayed on the computer screen at that point in time. If learners were exposed to integrated lessons of this nature from early on in their mathematics journey, they would have dealt with the pre-test very differently. Problems encountered at first year level will then be reduced by greater degrees as learners would be better equipped when they enter learning at that level. The mathematics teacher community, i.e. experienced teachers, lecturers at tertiary institutions, non-government organisations and government structures must begin to work together to raise awareness of issues in mathematics education in order to close the gap between research and practice.

Appendix I

Mathematics Proficiency Test

1. For which values of x will $\frac{(3x-2)(x+1)}{(2x+1)(x+1)}$ be undefined?

   a) -1 and $\frac{2}{3}$
   b) – ½ and -1
   c) – ½ only
   d) – ½ and $\frac{2}{3}$
   e) -1, - ½ and $\frac{2}{3}$

2. The expression $2x-3(x-2)+2(y-2)$ can be simplified to
   a) $2y-x+2$
   b) $2y-5x-10$
   c) $2y+x+4$
   d) $2y-5x-2$
   e) $2y-x-4$

3. The table below gives the values of x and y.

x	-2	-1	0	1	2
y	8	1	0	-1	-8

   Which equation gives the relationship between *x* and *y*?
   a) $y=-4x$
   b) $y=-7x-6$
   c) $y=2x^2$
   d) $y=-x^3$
   e) $y=x^4$

4. $(-1+2-3)-4$ is equal to
   a) -8
   b) 8
   c) -6
   d) 6
   e) 10

5. $\dfrac{\frac{1}{2}}{\frac{7}{12}-\frac{1}{3}}$ is equal to

a) $\frac{1}{3}$
b) ½
c) $\frac{3}{4}$
d) 1
e) 2

6. The solution of the inequality $2x-7<5x+2$ is

a) $x<-3$
b) $x>3$
c) $x<3$
d) $x>-\frac{5}{3}$
e) $x>-3$

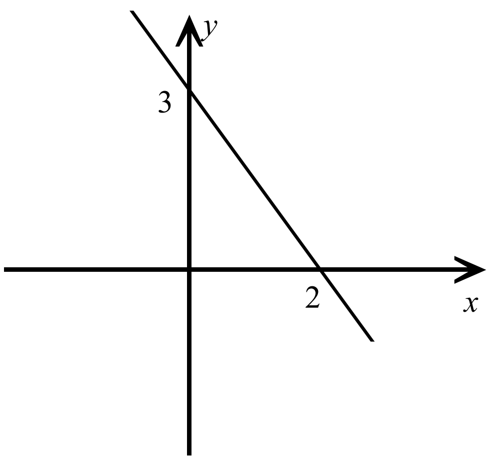
7. Which of the following points are on the line $x+2y-3=0$?

P(-3; 0) Q(3; 0) R(1; -1) S(- ½ ; 4) T(4; - ½)

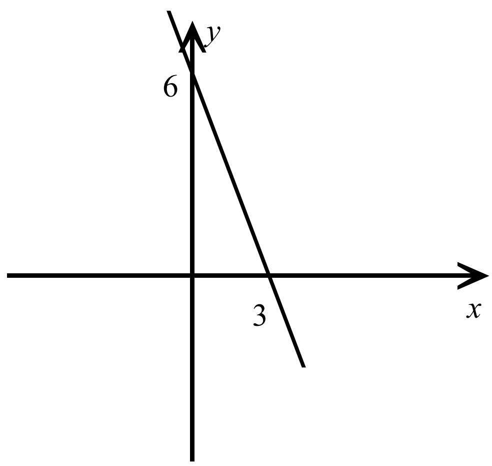
a) P and R
b) P, R and S
c) Q and S
d) Q and T
e) R and S

8. The graphs of $y=-2x+3$ is represented by

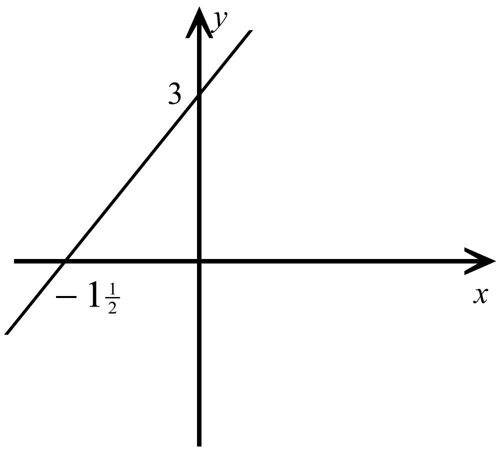
a)



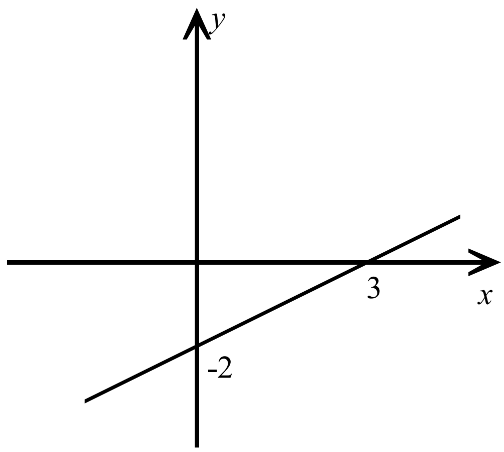
b)



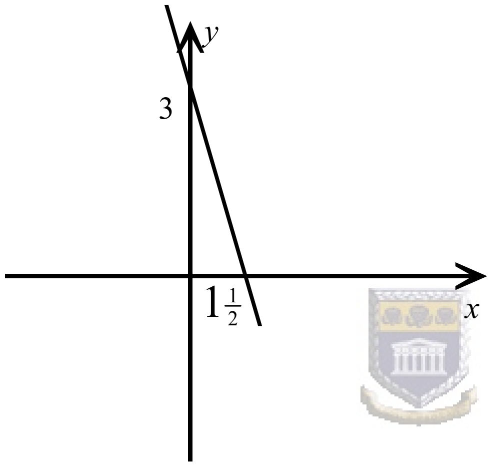
c)



d)



e)



9. The graphs of the equations $5x + 2y = 3$ and $y = -\frac{5}{2}x + 3$ are

   a) the same line
   b) two perpendicular lines
   c) two different parallel lines
   d) two lines intersecting at the origin
   e) two lines having the same *y*- intercept

10. Subtract the smallest from the largest of the numbers below.

0.301   0.31   0.5   0.867   0.87

   a) 0.557
   b) 0.56
   c) 0.566
   d) 0.569
   e) 0.571

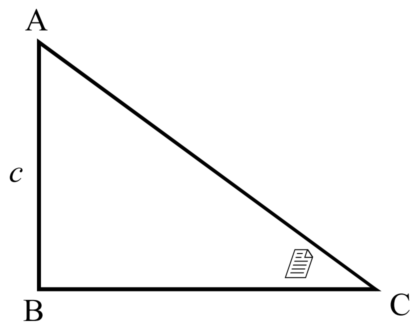
11. A solution of $\frac{x+5}{3}+\frac{3}{x+5}=\frac{5}{2}$ is

a) 0
b) -1
c) $-\frac{7}{2}$
d) -7
e) -5

12. Factorise $a^2(a-1)+(1-a)(2-a)$
a) $(a-1)(a^2+2)$
b) $(a-1)^2(a-2)$
c) $(a^2-1)(a+2)$
d) $(a-1)^2(a+2)$
e) $(a^2-1)(a-2)$

13. In the figure below πB = 90°

BC is equal to



a) $c\tan\theta$
b) $c\cot\theta$
c) $\frac{\tan\theta}{c}$
d) $c\sin\theta$
e) $\frac{\cot\theta}{c}$

14. $\frac{3.248}{0.08}$ is equal to

a) 0.0406
b) 0.406
c) 4.06
d) 40.6
e) 406

15. Solve for x if $2\sin x + 1 = 0$ and $0° \leq x \leq 360°$.

a) 30° only
b) 30° and 150°
c) 60° and 120°
d) 150° and 210°
e) 210° and 330°

16. $\sqrt{16a^8 + 4a^4b^4}$ is equal to

a) $2\sqrt{4a^4 + 2a^2b^2}$
b) $2a^2\sqrt{4a^4 + b^4}$
c) $4a^4\sqrt{4a^2 + b^4}$
d) $4a^2\sqrt{4a^4 + b^4}$
e) $2a^4\sqrt{4a^2 + b^2}$

17. If $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, then u is equal to

a) $\frac{fv}{f+v}$
b) $\frac{fv}{v-f}$
c) $f - v$
d) $\frac{f-v}{fv}$
e) $\frac{v-f}{fv}$

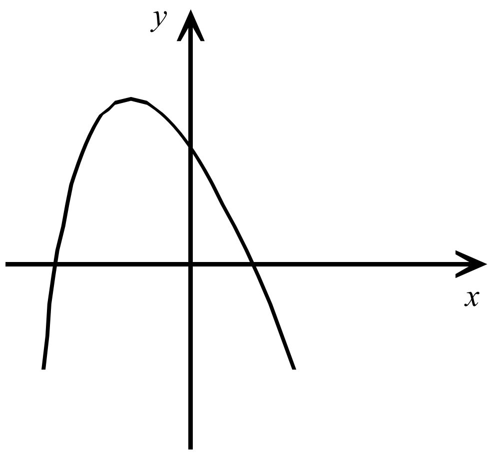
18. If $\theta = 30°$, then $\sin(2\theta + 90°)\cot(\theta + 30°)\cos(2\theta - 15°)$ is equal to

a) $-\frac{4}{\sqrt{3}}$
b) $-\frac{1}{\sqrt{2}}$
c) $\frac{1}{2\sqrt{6}}$
d) $\frac{\sqrt{3}}{4}$
e) $\frac{3}{2}$

19. If $\tan\theta = x$ and $0° < \theta < 90°$, then $\cos\theta$ is equal to

a) $\dfrac{1}{x+1}$
b) $\sqrt{x^2+1}$
c) $\dfrac{1}{\sqrt{x+1}}$
d) $\dfrac{1}{\sqrt{x^2+1}}$
e) $\dfrac{x}{\sqrt{x^2+1}}$

20. The graph of $y = ax^2 + bx + c$ is shown below.



Which one of the following is true?

a) $a > 0, b < 0, c > 0$
b) $a < 0, b < 0, c > 0$
c) $a < 0, b < 0, c < 0$
d) $a < 0, b > 0, c > 0$
e) $a > 0, b > 0, c > 0$

21. $\sqrt{100x^{100}}$ is equal to

a) $10x^{10}$
b) $10x^{50}$
c) $10x^{100}$
d) $100x^{10}$
e) $100x^{50}$

22. If $f(x) = x^2 + 1$, then $f(x+1) - f(x)$ is equal to

a) $2x+1$
b) 1
c) 2
d) 3
e) $2x$

23. $\dfrac{a^4 - b^4}{a - b}$, where $a \neq b$, can be simplified to

a) $a^3 + b^3$
b) $(a+b)(a^2 + b^2)$
c) $a^3 - b^3$
d) $(a+b)^2(a-b)$
e) $(a-b)(a^2 + b^2)$

24. $2^n 5^n$ is equal to

a) 10^n
b) 7^n
c) 10^{2n}
d) 7^{2n}
e) 10^{n^2}

25. For which values of x will $\sqrt{-x}$ be defined?

a) No real values.
b) No real values except 0.
c) All real values greater than or equal to 0.
d) All real values less than 0.
e) All real values less than or equal to 0.

26. The expression $\dfrac{\sec\theta \operatorname{cosec}\theta}{(1 + \tan^2\theta)}$ can be simplified to

a) $\operatorname{cosec}\theta$
b) $\sec\theta$
c) $\cot\theta$
d) $\cos\theta$
e) $\tan\theta$

27. If $x = 90°$, then the value of $\dfrac{\sin 3x}{\sin x}$ is

a) -1
b) 0
c) 1
d) 3
e) undefined

28. A father is three times as old as his son. Five years ago he was four times as old as his son was. If the present age of the father is y years and that of his son is x years, which one of the following sets of equations is correct?

a) $y = 3x$
$y - 5 = 4x$

b) $x = 3y$
$x - 5 = 4y$

c) $x = 3y$
$y - 5 = 4x$

d) $y = 3x$
$y - 5 = 4x - 5$

e) $y = 3x$
$y - 5 = 4(x - 5)$

29. $\sqrt{\sqrt{11}+\sqrt{7}}.\sqrt{\sqrt{11}-\sqrt{7}}$ is equal to

a) 2
b) 4
c) $\sqrt{22}$
d) 6
e) $\sqrt{72}$

30. If $\log_{10} x + \log_{10} y = 0$, then

a) $x + y = 0$
b) $x + y = 10$
c) $xy = 1$
d) $xy = 0$
e) $xy = 10$

Appendix II

Mathematics Profile (2003 group)

Students' mathematics profile and performance in 2003

In 2003, 245 students registered for the mainstream mathematics course at the University of the Western Cape. The profile of all students registered is presented in Table 1.

Table 1: Profile of students registered for 1st year mathematics

Symbol	A	B	C	D	E	F	G	H	Total	%
HG	4	4	9	14	22	3	0	1	57	23.3
SG	16	21	46	53	37	14	1	0	188	76.7
									245	

After the first semester there were 234 students. Of the 234 students 173 were new students and 61 were repeaters. Table 2 below presents the profile of new students who had marks for the mainstream mathematics course in the first semester.

Table 2: Profile of new students

Symbol	A	B	C	D	E	F	G	H	Total	%
HG	4	4	7	11	19	3	0	1	49	28.3
SG	15	20	37	29	17	6	0	0	124	71.7
									173	

Of these 173 students, 152 (87.9%) wrote the final examinations and 21 (12.1%) dropped out. Of the 152 students who wrote the final examinations, 48 (31.6%) passed and 104 (68.4%) failed. Tables 3 and 4 present the profile in terms of matric mathematics results of students in these two groups.

Table 3: Profile of new, successful students

Symbol	A	B	C	D	E	F	G	H	Total	%
HG	4	4	7	8	8	2	0	0	33	68.75
SG	7	5	1	1	1	0	0	0	15	31.25
									48	

Table 4: Profile of new, unsuccessful students

Symbol	A	B	C	D	E	F	G	H	Total	%
HG	0	0	0	3	11	1	0	1	16	15.4
SG	7	13	31	17	14	6	0	0	88	84.6
									104	

The "dropouts" all had SG matric mathematics symbols as indicated in table 5 below.

Table 5: Profile of new "dropouts"

Symbol	A	B	C	D	E	F	G	H	Total	%
SG	1	2	5	11	2	0	0	0	21	12.1

From the above tables we can see that the HG pass rate was 67.4% (33/49), which includes all students with HG A, B, and C symbols. Less than half of the students with HG E passed, i.e. 8 out of the 19 students. The SG pass rate was 12.1% (15/124), 80% having SG A or B symbols.

Appendix III

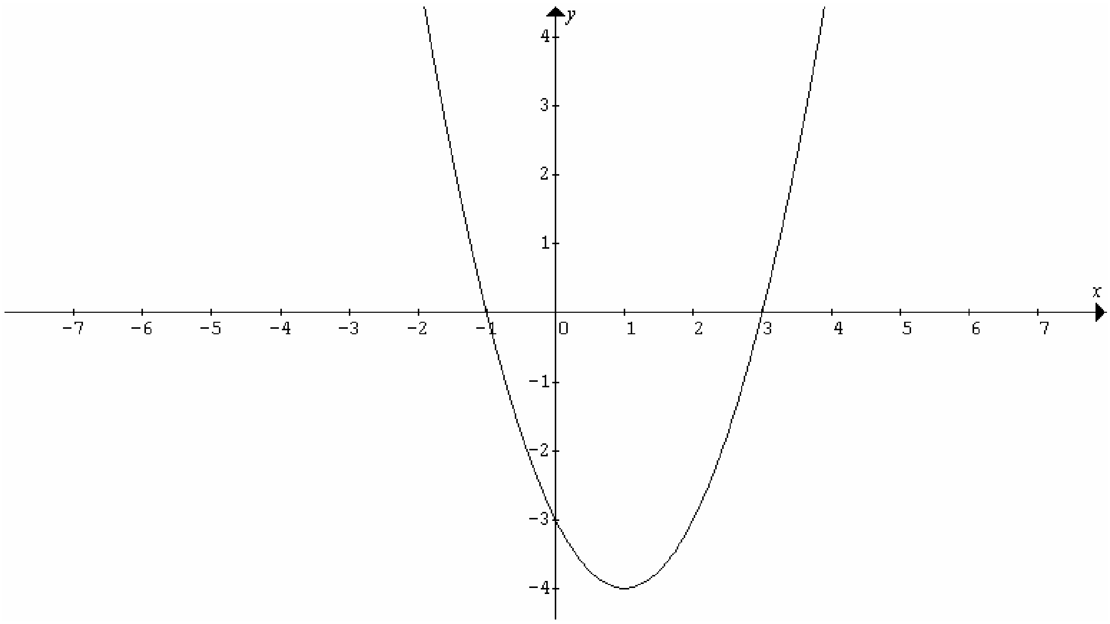
Pre- and post-tests with memoranda

Instruments

Pre-test

Question 1

Study and use the graph of $f(x) = x^2 - 2x - 3$ to answer the following questions.



1.1 Give any number a for which

1.1.1 $f(a) > 0$ and explain how you arrived at this answer. (2)

1.1.2 $f(a) = 0$ and explain how you arrived at this answer. (2)

1.1.3 $f(a) < 0$ and explain how you arrived at this answer. (2)

1.2 Based on your reasoning (answers) in 1.1, give all the values of x for which $f(x) < 0$. (2)

1.3 Check the answers given below by Siyabonga and Thandiwe for correctness. If an answer is incorrect, explain how you will use the graph to write down the correct answer.

1.3.1 Siyabonga does the following to find the *x*-value(s) where $f(x) \geq 0$:

$x^2 - 2x - 3 \geq 0$

$(x-3)(x+1) \geq 0$

$x - 3 \geq 0$ or $x + 1 \geq 0$

Siyabonga's answer : $x \geq 3$ or $x \geq -1$ (2)

1.3.2 Thandiwe finds the *x*-values for which $x^2 - 2x - 3 = -4$ as follows:

$x^2 - 2x - 3 = -4$

$(x-3)(x+1) = -4$

$(x-3) = -4$ or $(x+1) = -4$

Thandiwe's answer : $x = -1$ or $x = -5$ (2)

1.4 Let $t = x^2 - 2x - 3$ and find

1.4.1 the possible value(s) of *t* if $x \geq 2$. (2)

1.4.2 the domain and range of *f* if $f(x) = x^2 - 2x - 3$ for $x \geq 2$. (2)

1.4.3 Verify your answer in 1.4.1 by first setting $x^2 - 2x - 3 - t = 0$ and solving for *x* in terms of *t* using the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. (4)

1.5.1 Between which two positive *x*-values (integers) will $x^2 - 2x - 3 = 1$. (2)

1.5.2 Complete the following table by putting the smallest *x*-values of 1.5.1 into the spaces below. (Round the answer up to three decimal numbers)

x	….,15	….,2	…..,25	…...,3	…..,35
f(*x*)					

(5)

1.5.3 Study the above table and find between which two *x*-values will $x^2 - 2x - 3 = 1$. (2)

1.6 For which value(s) of *k* will $x^2 - 2x - 3 = k$ have:

1.6.1 two roots (2)

1.6.2 two positive roots (2)

1.6.3 no roots (2)

1.6.4 one root (2)

1.7 Write an explicit formula for the function $f(x) = x^2 - 2x - 3$ in the form $a(x-p)^2 + q$. (2)

Determine the following and explain how the graphs in 1.7.1 and 1.7.2 differ from that of *f*.

1.7.1 $f(x+2)$ (3)

1.7.2 $f(x - 3)$ (3)

1.8 Give any number p for which

1.8.1 $f'(p) > 0$ and explain how you arrived at this answer. (2)

1.8.2 $f'(p) = 0$ and explain how you arrived at this answer. (2)

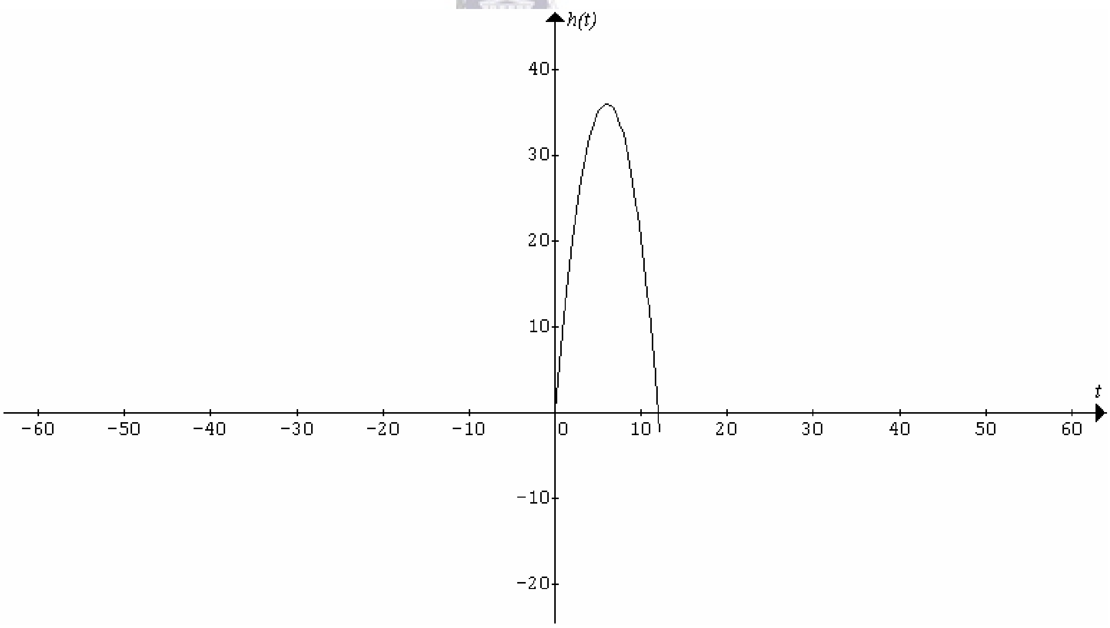
1.8.3 $f'(p) < 0$ and explain how you arrived at this answer. (2)

1.9 Based on your reasoning in 1.8, give all the values of x for which $f'(x) > 0$.

(2) /53/

Question 2

A rocket is fired vertically into the sky. After t seconds, the rocket reaches a height of $h(t)$ metres, where $h(t) = 12t - t^2$. The graph of $h(t)$ is illustrated below.



With the aid of the graph and by using calculations, where necessary, answer the following questions:

2.1 After how many seconds will the rocket reach a height of 20 metres?

(2)

2.2 For how long is the rocket higher than 20 metres? (2)

2.3 When will the rocket reach its maximum height? (2)

2.4 What is the maximum height reached by the rocket? (2)

2.5 After how many seconds will the rocket again be on the ground? (2)

2.6 What is the initial speed of the rocket? (2)

2.7 Determine the speed of the rocket after 3 seconds. (2)

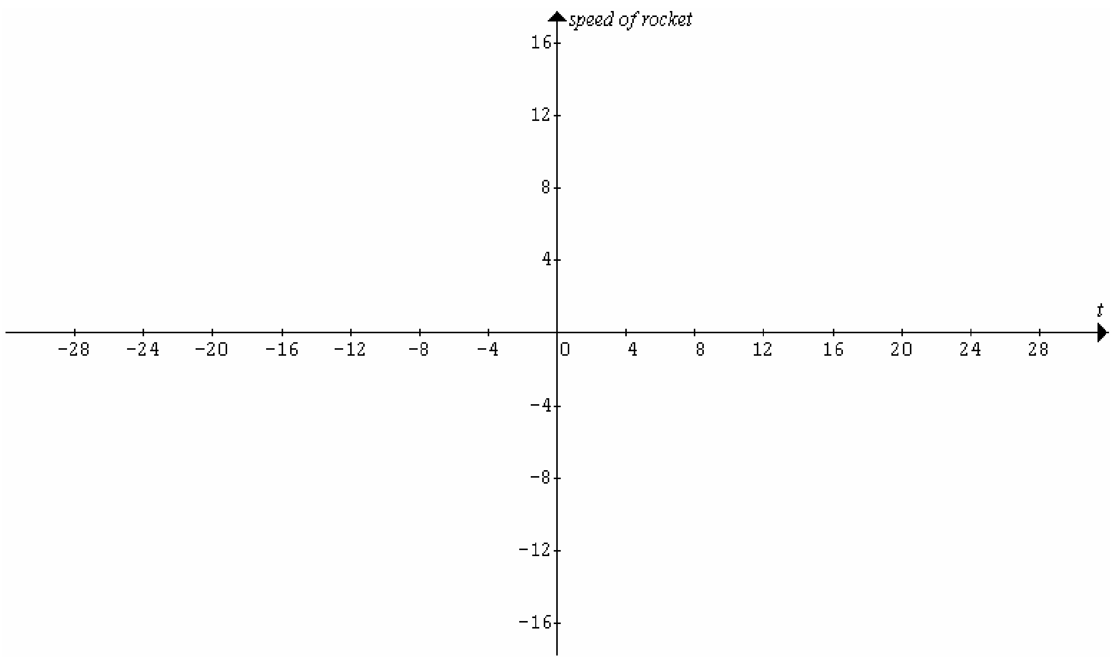
2.8 How does this relate to the gradient of the graph at $t = 3$ seconds? (2)

2.9 Over what time interval(s) did the speed of the rocket

2.9.1 increase? (2)

2.9.2 decrease? (2)

2.10 Sketch a graph (on the axes below) which will explain how the speed varies over the entire period from the time that rocket was fired into the sky, until it reaches the ground again.



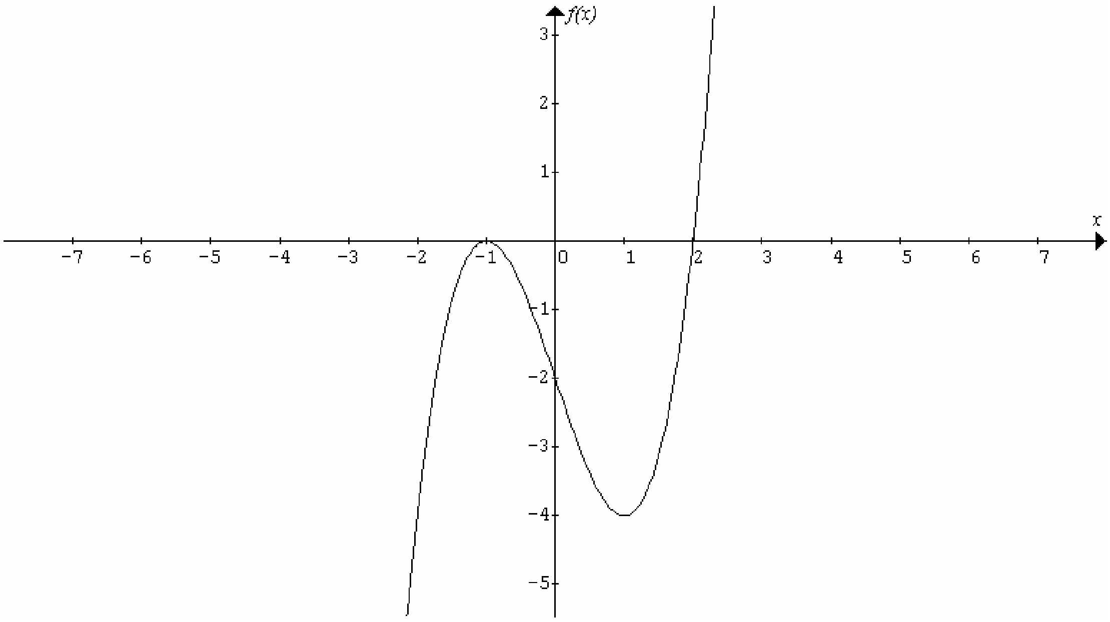
(2) /22/

/75/

Post-test

Question 1

Study and use the graph of $f(x) = x^3 - 3x - 2$ to answer the following questions.



1.1 For which value(s) of *x* will:

1.1.1 $x^3 - 3x - 2 > 0$ (2)

1.1.2 $x^3 - 3x - 2 < 0$ (2)

1.1.3 $x^3 - 3x - 2 = 0$ (2)

1.2 Let $t = x^3 - 3x - 2$ and find

1.2.1 the possible value(s) of *t* if $-1 \leq x \leq 1$. (2)

1.2.2 the domain and range of *f* if $f(x) = x^3 - 3x - 2$ for $-1 \leq x \leq 1$.(2)

1.3 Find the value(s) of *k* for which $x^3 - 3x - 2 = k$ have:

1.3.1 one positive root (2)

1.3.2 one negative root (2)

1.3.3 two positive roots and one negative root. (2)

1.4.1 Between which two positive *x*-values (integers) will $x^3 - 3x - 2 = -1$. (2)

1.4.2 Complete the following table by putting the smallest *x*-values of 1.4.1 into the spaces below. (Round the answer up to three decimal numbers)

x	….,7	…..,75	…..,8	.....,85	…..,9
f(*x*)					

(5)

1.4.3 Study the above table and find between which two *x*-values will $x^3 - 3x - 2 = -1$. (2)

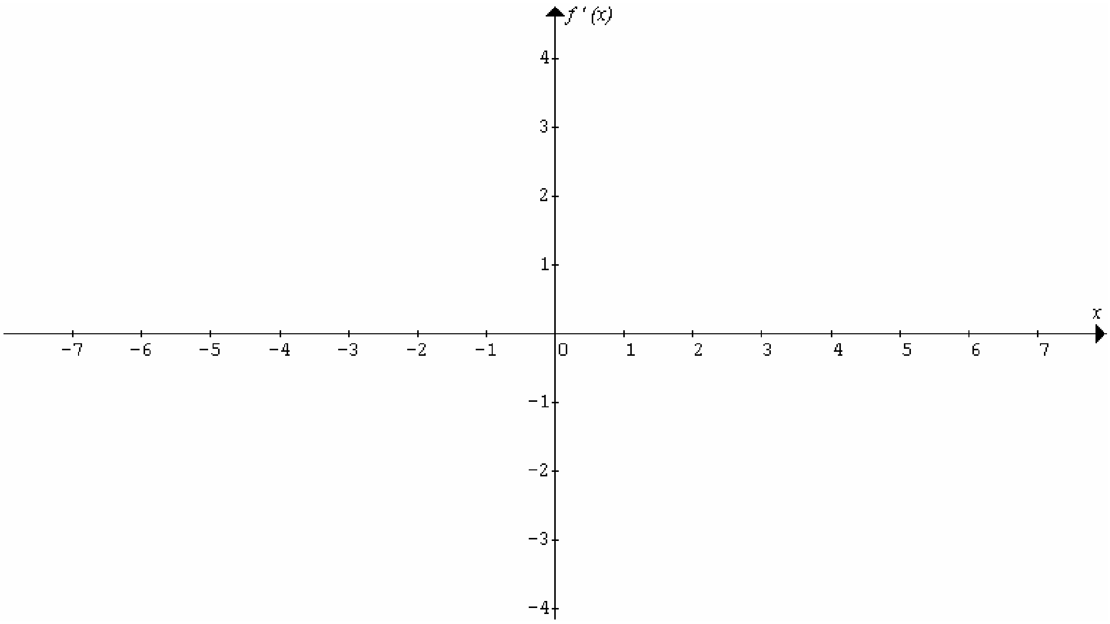
1.5 For which value(s) of *x* will

1.5.1 $f'(x) > 0$ (2)

1.5.2 $f'(x) < 0$ (2)

1.5.3 $f'(x) = 0$ (2)

1.6 Use your answers of 1.5 to draw the graph of $f'(x)$ on the axes below.



(2)

1.7 Write an explicit formula for the function $f(x) = x^3 - 3x - 2$ in the form $f(x) = a(x - x_1)(x - x_2)(x - x_3)$. (2)

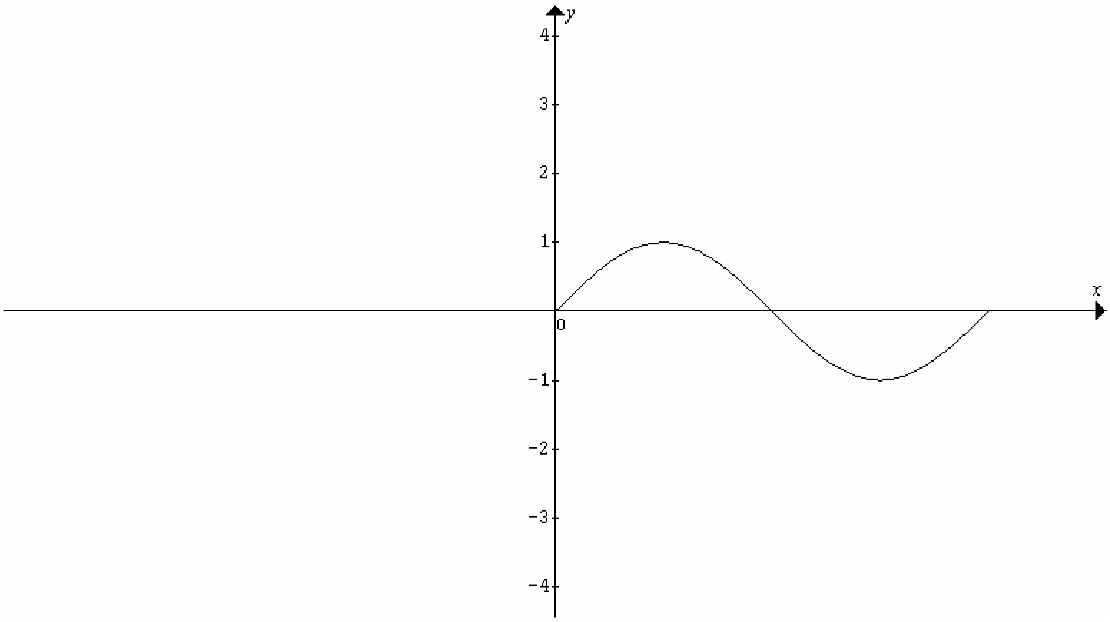
Determine the following and explain how the graphs in 1.7.1 and 1.7.2 differ from that of *f*.

1.7.1 $f(x+2)$ (3)

1.7.2 $f(x - 3)$ (3) /41/

Question 2

The sketch below illustrates the graph of $y = \sin x$ for $x \in [0°;360°]$.



2.1 For which value(s) of k will $\sin x = k$ have

2.1.1 no roots (2)

2.1.2 two positive roots (2)

2.1.3 two negative roots (2)

2.2 If $f(x) = \sin x$, for what value(s) of $x \in [0°;360°]$ is

2.2.1 $f'(x) = 0$ (2)

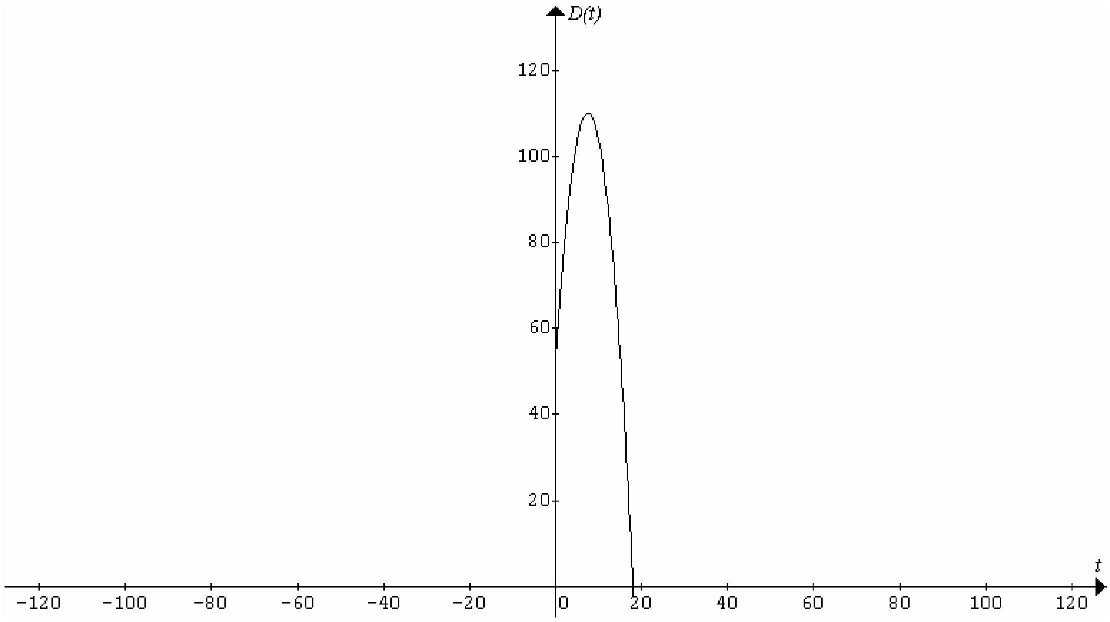
2.2.2 $f'(x) > 0$ (2)

2.2.3 $f'(x) < 0$ (2)

2.3 Write down the value(s) of $x \in [0°;360°]$ for which $\sin x > \frac{1}{2}$. (4) /16/

Question 3

A television reporting team that regularly attends political conferences has found that the combined effect of the applause and other noise in the hall produces a noise level of $D(t) = 54 + 15t - t^2$ decibels, t seconds after the end of the main speech.



3.2 After how many seconds will the noise level be

3.2.1 38 decibels? (2)

3.2.2 0 decibels (2)

3.3 How long was the noise level above 80 decibels? (2)

3.4 Determine the rate of change of the noise level with respect to time at $t = 9$ seconds. Is the noise level increasing or decreasing at $t = 9$ seconds? (2)

3.5 How does this answer relate to the gradient of the graph at $t = 9$ seconds? (2)

3.6 Over what time interval(s) did the rate of change of the noise level

- 3.6.1 increase (2)
- 3.6.2 decrease? (2)

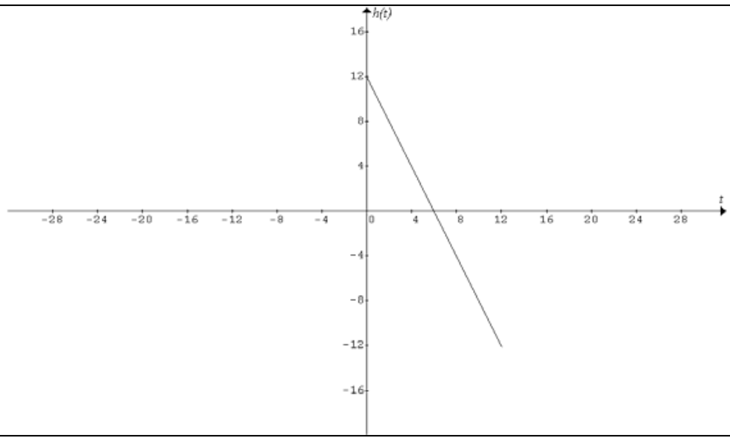
3.7 Determine after how many seconds the maximum noise level will be reached. Hence determine the maximum noise level. (4) /18/

/75/

Marking memoranda for pre and post-test

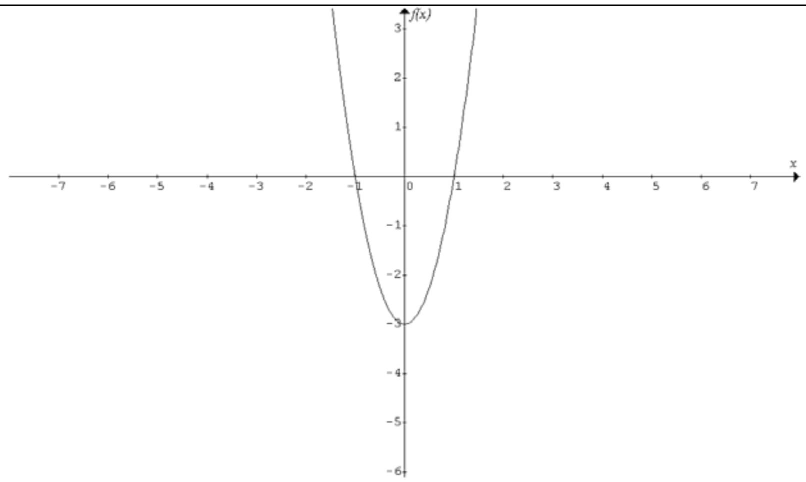
Pre-test

<table>
<tr><td>Question number</td><td>Answers</td><td>Mark allocation</td></tr>
<tr><td>1.1.1</td><td>▪ Any value for $a > 3$ or $a < -1$
▪ Reason: $f(a) > 0$ or point $(a; f(a))$ is above x-axis</td><td>% value
% reason</td></tr>
<tr><td>1.1.2</td><td>▪ $a = -1$ or $a = 3$
▪ Reason: $f(-1) = 0$ or $f(3) = 0$ or the graphs intersects the x-axis at -1 or 3.</td><td>% value
% reason</td></tr>
<tr><td>1.1.3</td><td>▪ Any value for $-1 < a < 3$
▪ Reason: $f(a) < 0$ or point $(a; f(a))$ is below x-axis</td><td>% value
% reason</td></tr>
<tr><td>1.2</td><td>$-1 < x < 3$</td><td>%% or 0</td></tr>
<tr><td>1.3.1</td><td>▪ Wrong
▪ Explanation: $x \geq 3$ is correct graph is above x-axis for $x \leq -1$</td><td>% wrong
% explanation</td></tr>
<tr><td>1.3.2</td><td>▪ Wrong
▪ Explanation: From graph $x = 1$ when $x^2 - 2x - 3 = -4$</td><td>% wrong
% explanation</td></tr>
<tr><td>1.4.1</td><td>▪ $t(2) = -3$
▪ $t \geq 3$</td><td>% $t(2) = -3$
% $t \geq 3$</td></tr>
<tr><td>1.4.2</td><td>▪ domain: $x \geq 2$
▪ range: $f(x) \geq -3$</td><td>% domain
% range</td></tr>
<tr><td>1.4.3</td><td>$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-3-t)}}{2}$
$\pm\sqrt{16+4t} \geq 4$
$16 + 4t \geq 4$
$t \geq -3$</td><td>% substitution
%simplifying
% squaring
% answer</td></tr>
<tr><td>1.5.1</td><td>For x between 3 and 4.</td><td>% % values</td></tr>
<tr><td>1.5.2</td><td><table><tr><td>x</td><td>3,15</td><td>3,2</td><td>3,25</td><td>3,3</td><td>3,35</td></tr><tr><td>$f(x)$</td><td>0.623</td><td>0.84</td><td>1.063</td><td>1.29</td><td>1.523</td></tr></table></td><td>% %% %% fo r each correct function value</td></tr>
<tr><td>1.5.3</td><td>Between $x = 3.2$ and $x = 3.25$</td><td>% % correct values</td></tr>
<tr><td>1.6.1</td><td>$k > -4$</td><td>%% or 0</td></tr>
<tr><td>1.6.2</td><td>$-4 < k < -3$</td><td>%% or 0</td></tr>
<tr><td>1.6.3</td><td>$k < -4$</td><td>%% or 0</td></tr>
<tr><td>1.6.4</td><td>$k = -4$</td><td>%% or 0</td></tr>
<tr><td>1.7</td><td>$f(x) = (x-1)^2 - 4$</td><td>%% or 0</td></tr>
<tr><td>1.7.1</td><td>▪ $f(x) = (x+1)^2 - 4$
▪ graph moves 2 units left</td><td>% % function
% reason</td></tr>
<tr><td>1.7.2</td><td>▪ $f(x) = (x-4)^2 - 4$
▪ graph moves 3 units right</td><td>% % function
% reason</td></tr>
</table>

1.8.1	▪ Any value for $p > 1$ ▪ Reason: $f'(p) > 0$ or the graph is increasing at p	% value % reason
1.8.2	▪ $p = 1$ ▪ Reason: $f'(p) = 0$ or the graph is turning at $p = 1$	% value % reason
1.8.3	▪ Any value for $p < 1$ ▪ Reason: : $f'(p) < 0$ or the graph is decreasing at p	% value % reason
1.9	▪ $x > 1$	%% or 0
2.1	▪ At $t = 2$ sec. and $t = 10$ sec.	%% or 0
2.2	▪ Between 2 and 10 seconds i.e. 8 seconds above 20 meters	%% or 0
2.3	▪ At $t = 6$ seconds	%% or 0
2.4	▪ $h(6) = 36$ meters	%% or 0
2.5	▪ $t = 12$ seconds	%% or 0
2.6	▪ $h'(0) = 12$ m.s^{-1}	%% or 0
2.7	▪ $h'(3) = 6$ m.s^{-1}	%% or 0
2.8	▪ The gradient of the tangent line at 3 seconds is also 6.	%% or 0
2.9.1	▪ $6 < t < 12$	%% or 0
2.9.2	▪ $0 < t < 6$	%% or 0
2.10	 ▪	%% or 0

Post-test

Question number	Answers	Mark allocation
1.1.1	$x > 2$	%%
1.1.2	$x < 2$ but $x \neq 2$	%%
1.1.3	$x = -1$ or $x = 2$	%%
1.2	$-4 \leq t \leq 0$	%% or 0
1.3.1	$k > 0$	%%
1.3.2	$k <$ -4	%%
1.3.3	$-4 < k < -2$	%%
1.4.1	Between $x = 1$ and $x = 2$	%%
1.4.2	x 1.7 1.75 1.8 1.85 1.9	% %% %% f or each correct

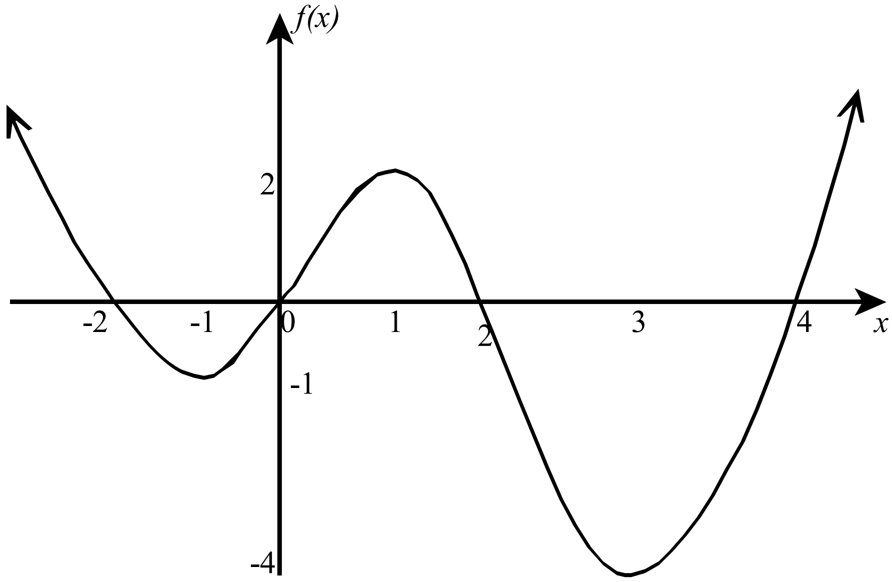
	$f(x)$ | -2.187 | -1.89 | -1.568 | -1.218 | -0.84	function value
1.4.3	Between $x = 1.85$ and $x = 1.9$	%%
1.5.1	$x < -1$ or $x > 1$.	%%
1.5.2	$-1 < x < 1$	%%
1.5.3	$x = -1$ or $x = 1$	%%
1.6		%%
1.7	$f(x) = (x+1)(x+1)(x-2$	%%
1.7.1	$f(x+2) = (x+3)(x+3)x$ Explanation: Graph moved 2 units left	%% function % explanation
1.7.2	$f(x-3) = (x-2)(x-2)(x-5)$ Explanation: Graph moved 3 units right	%% function % explanation
2.1.1	$k > 1$ or $k < -1$	%%
2.1.2	$0 < k < 1$	%%
2.1.3	$-1 < k < 0$	%%
2.2.1	$x = 90°$ or $x = 270°$	%%
2.2.2	$0 \leq x < 90°$ or $270° < x \leq 360°$	%%
2.2.3	$90° < x < 270°$	%%
2.3	$30° < x < 150°$	%% %%
3.1.1	$t = 16$ sec	%% or 0
3.1.2	$t = 18$ sec	%% or 0
3.2	The noise level was above 80 decibels between $t = 2$ and $t = 15$ seconds. That means it was above 80 decibels for 13 seconds	%% or 0
3.3	$D'(9) = -3$, the noise level was decreasing	% $D'(9) = -3$ % decreasing
3.4	The gradient of the tangent line at $t = 9$ seconds is also -3.	%% or 0
3.5.1	$0 \leq t < 9$	%% or 0
3.5.2	$9 < t \leq 18$	%% or 0
3.6	Maximum noise level was obtained after 7 ½ seconds when the maximum level was 110,25 decibels	%% for 7.5 sec. %% for 110,25 dec.

Appendix IV

Activities

Activity 1:

Consider the graph below, which represents some function *f* whose equation we do not know.



1.1 For which value(s) of x will:

1.1.1 $f(x) > 0$

1.1.2 $f(x) < 0$

1.1.3 $f(x) = 0$

1.2 Let $t = f(x)$ and find

1.2.1 the possible value(s) of t if $1 \leq x \leq 4$.

1.2.2 the domain and range of $f(x)$ for $1 \leq x \leq 4$.

1.3 Find the value(s) of k for which $f(x) = k$ have:

1.3.1 two roots

1.3.2 four roots

1.3.3 no roots

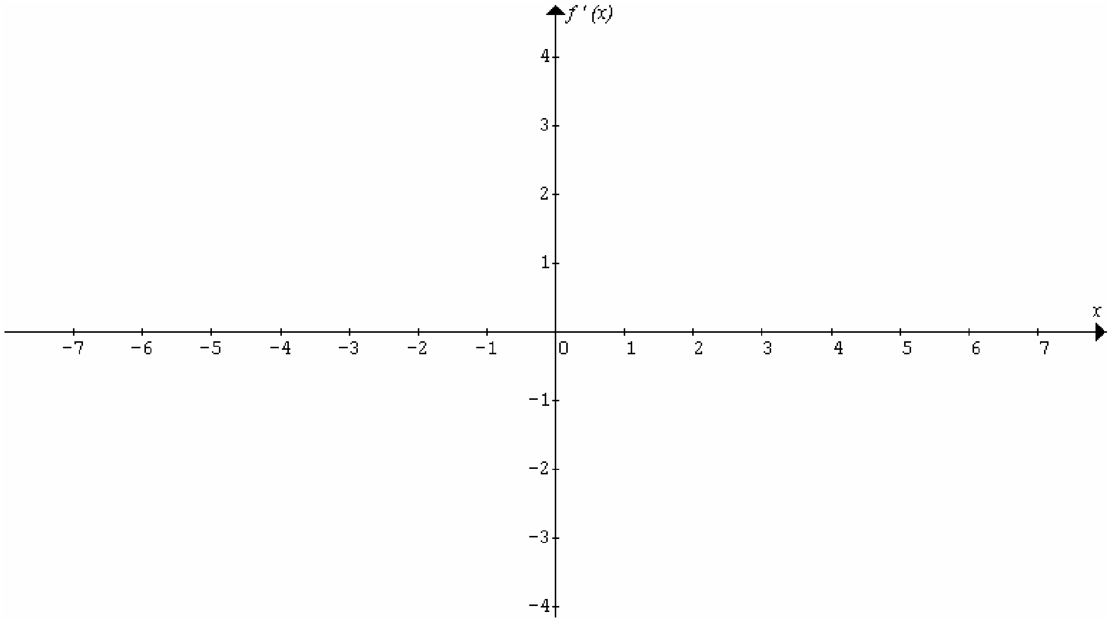
1.4 For which value(s) of x will

1.4.1 $f'(x) > 0$ (2)

1.4.2 $f'(x) < 0$ (2)

1.4.3 $f'(x) = 0$ (2)

1.5 Use your answers of 1.4 to draw a rough graph of $f'(x)$ on the axes below.

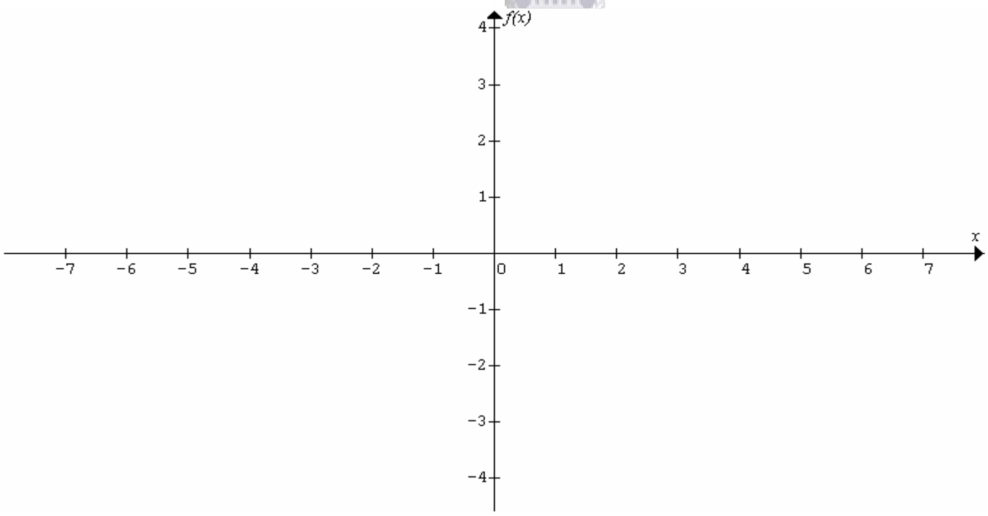


Explain how the graphs in 1.6.1 and 1.6.2 differ from that of *f(x)*.

1.6.1 *f*(*x*+2) (3)

1.6.2 *f*(*x* - 3) (3) /41/

1.7 Draw the graphs of 1.6.1 and 1.6.2 on the axis below. Label the graphs as *f*(*x*+2) and *f*(*x* - 3).

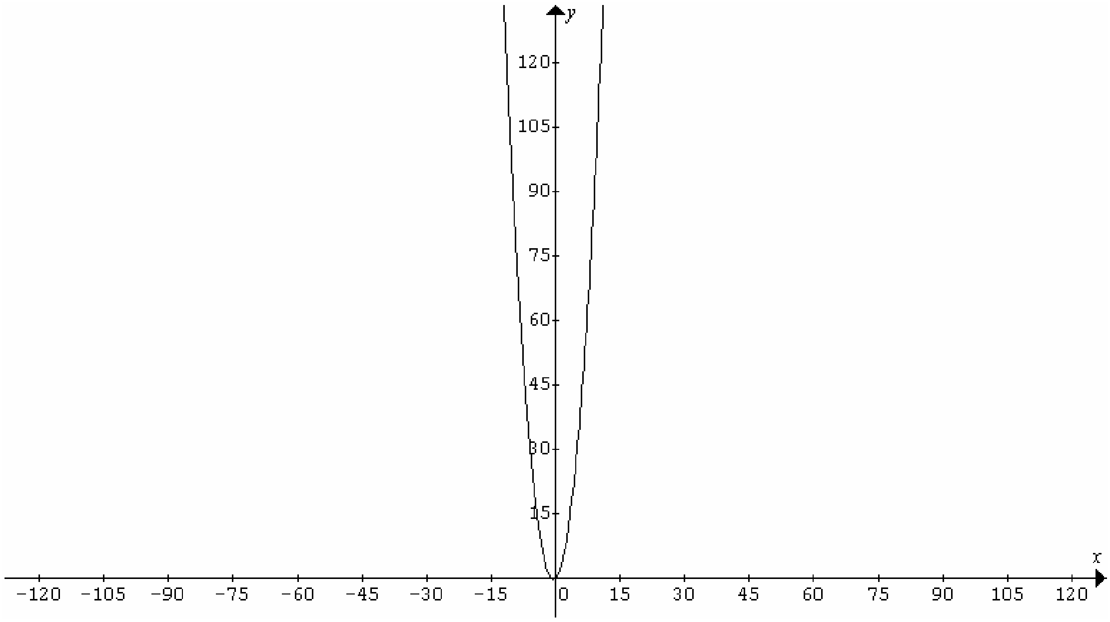


Activity 2:

Use the graph of $y = x^2 + x$ below to solve:

(a) $x^2 + x = 66$ (b) $x^2 + x = 45$

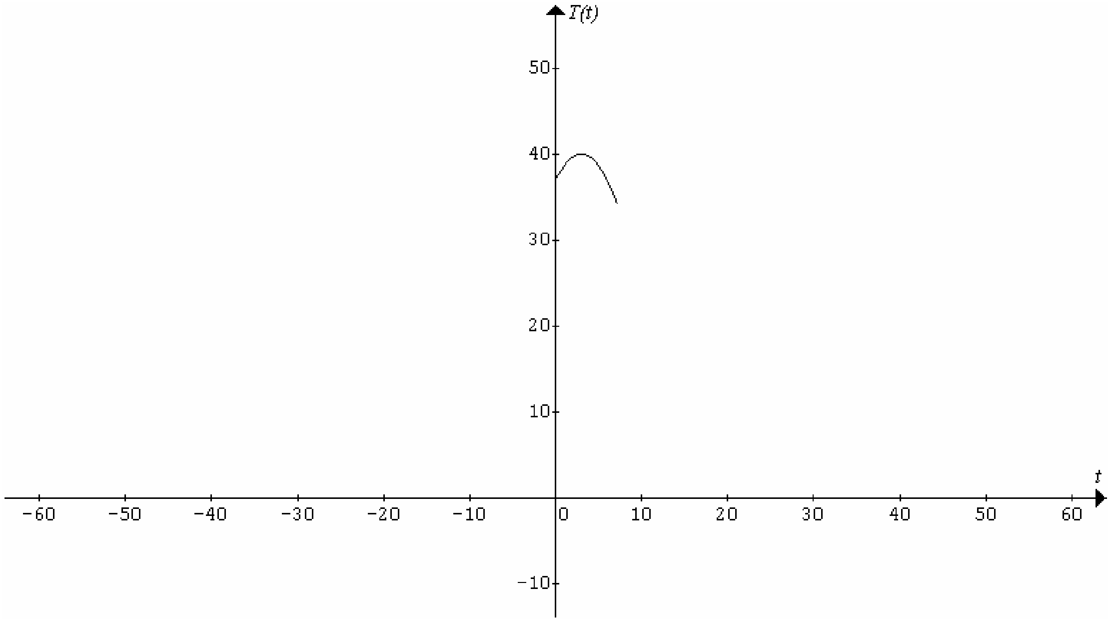
by choosing values for *x* and then calculating the corresponding *y*-values in the table below.



$x^2+x=66$	x	$x^2+x=45$	x

Activity 3: (Applications of quadratic functions)

1. The equation $T=-\frac{1}{3}t^2+2t+37$ models (gives) the temperature (T) of a patient in hospital for several days (t).



1.1 For how many days was the patient's temperature above 37°C?

1.2. For how many days did the temperature of the patient increase?

1.3. What was the temperature of the patient when he/she arrived at the hospital?

1.4. The patient was released when his/her temperature was 37°C again. How many days did the patient spend in the hospital?

1.5. Determine the day on which the patient had the highest temperature. (Use calculations)

1.6. What was the highest temperature? (Use calculations)

Bibliography

1. Amoore, H, Pityana NB, Griesel, H (2004), *The FETC, Higher Education and Future Enrolment,* SAUVCA.

2. Anstrom (1997: pp. 24 – 25), *U.S. National Council of Teachers of Mathematics guidelines.*

3. Anton, H (1988), Calculus, Third Edition.

4. Artigue, M(1994), Didactical engineering as a framework for the conception of teaching products, *Didactics of Mathematics as a Scientific Discipline*, Kluwer Academic Publishers.

5. Battista(2000, cited in Sorensen: 2003), Effective Maths. http://www.lakeforest.edu/images/userImages/blocker/Page_3947/rachelsorenson.pdf

6. Benjamin (1989, cited in Lake: 1994), *Integrated Curriculum,* School Improvement Research Series.

7. Blaine, S (2004), *Mother Tongue Learning Will Raise Standards, say Experts,* Business Times (South Africa), 3 September 2004.

8. Bruner, J (1960, cited in Winkelman: 1994), Preparing Mathematics for Students, *Didactics of Mathematics as a Scientific Discipline*, Kluwer Academic Publishers.

9. Caine and Caine (1991, cited in Lake: 1994), *Integrated Curriculum,* School Improvement Research Series. http://www.nwrel.org/scpd/sirs/8/c016.html

10. Cromwell (1989, cited in Lake: 1994), *Integrated Curriculum,* School Improvement Research Series. http://www.nwrel.org/scpd/sirs/8/c016.html

11. Crosswhite, J (1970), Implications for Teacher Planning. *The Teaching of Secondary School Mathematics*, The National Council of Teachers of Mathematics.

12. Desai, Z (2004), interviewed by Blaine, S (2004), *Mother Tongue Learning Will Raise Standards, say Experts,* Business Times (South Africa), 3 September 2004

13. Fish, W (2004), *Interim report on the performance of MAM 111 and 121 students in the Mathematics Diagnostic Test,* Department of Mathematics and Applied Mathematics, University of the Western Cape.

14. Griesel, H (2004), *National Benchmark Tests Project,* SAUVCA.

15. Grouws and Cebulla (2000, cited in Sorensen: 2003), Effective Maths.

http://www.lakeforest.edu/images/userImages/blocker/Page_3947/rachelsorenson.pdf

16. Hillel, Lee and Linchevski, (1992, cited in Dreyfus: 1994), The role of cognitive tools in Mathematics Education, *Didactics of Mathematics as a Scientific Discipline*, Kluwer Academic Publishers.
17. Holland, G (1994), Intelligent Tutorial Systems, *Didactics of Mathematics as a Scientific Discipline*, Kluwer Academic Publishers.
18. Humphrey (1981, cited in Lake: 1994), *Integrated Curriculum,* School Improvement Research Series. http://www.nwrel.org/scpd/sirs/8/c016.html
19. Humphrey, Post and Ellis (1981, cited in Lake: 1994), *Integrated Curriculum,* School Improvement Research Series. http://www.nwrel.org/scpd/sirs/8/c016.html
20. Kraak, A (2004), *Contextualising the school-leaving certificate*, FETC Workshop, 20 August 2004.
21. Laborde, C (1994), Working in small groups: A learning situation? *Didactics of Mathematics as a Scientific Discipline*, Kluwer Academic Publishers.
22. Lake, K (1994), *Integrated Curriculum,* School Improvement Research Series. http://www.nwrel.org/scpd/sirs/8/c016.html
23. Lindall, P (2004) interviewed by Ntsingila, F (2004), *White matrics get the first crack at jobs,* Sunday Times.
24. Lipson (1993, cited in Lake: 1994), *Integrated Curriculum,* School Improvement Research Series. http://www.nwrel.org/scpd/sirs/8/c016.html
25. Lotter, C (2004), *Policy Debates, Challenges for Higher Education and Collaboration between Higher Education institutions and the WCED,* FETC Workshop, 20 August 2004.
26. Margolinas (1993, cited in Laborde: 1994), Working in small groups: A learning situation? *Didactics of Mathematics as a Scientific Discipline*, Kluwer Academic Publishers.
27. Microsoft Encarta Encyclopedia (2000), *Way of Life,* Republic of South Africa.

28. Mkhabela, M and Malan, M (2004), *"By die 40% eerstejaars druip."*, "Rapport", pg. 2.

29. Naidoo, P (2004), *Pandor Labours on,* Financial Mail, (23-26).

30. Nesher (1986, cited in Dreyfus: 1994), The role of cognitive tools in Mathematics Education, *Didactics of Mathematics as a Scientific Discipline*, Kluwer Academic Publishers.

31. Niss, M (1998), Aspects of the Nature and State of Research in Mathematics Education, *IMFUFA* tekst nr. 351/98.

32. Niss, M (1993), *Cases of Assessment in Mathematics Education,* Kluwer Academic Publishers, pp. 1-7.

33. Niss, M (1994, cited in Winkelman: 1994), Technology and Mathematics Education, *Didactics of Mathematics as a Scientific Discipline*, Kluwer Academic Publishers.

34. Niss, M (1994), Mathematics in society, *Didactics of Mathematics as a Scientific Discipline*, Kluwer Academic Publishers.

35. Ntsingila, F (2004), *White matrics get the first crack at jobs,* Sunday Times.

36. Pagulee (2001, cited in Sorensen: 2003), Effective Maths. http://www.lakeforest.edu/images/userImages/blocker/Page_3947/rachelsorenson.pdf

37. Pandor, N (2004), interviewed by Naidoo, P (2004), *Pandor Labours on,* Financial Mail, (23-26).

38. Papert (1980, cited in Tall: 1994), Environments for learning, *Didactics of Mathematics as a Scientific Discipline*, Kluwer Academic Publishers.

39. Perkins (1991, cited in Lake: 1994), *Integrated Curriculum,* School Improvement Research Series. http://www.nwrel.org/scpd/sirs/8/c016.html

40. Piaget (1936, cited in Bussi: 1994), Theoretical and empirical approaches to classroom interaction, *Didactics of Mathematics as a Scientific Discipline*, Kluwer Academic Publishers.

41. Rademeyer, A (2004), "*Pandor sit voet dwars",* "Die Burger", pg. 1.

42. Reusser (1991, cited in Holland: 1994), Intelligent Tutorial Systems, *Didactics of Mathematics as a Scientific Discipline*, Kluwer Academic Publishers.

43. Reynold and Muijs (1999, cited in Sorensen: 2003), Effective Maths.

http://www.lakeforest.edu/images/userImages/blocker/Page_3947/rachelsorenson.pdf

44. Rivinia, Polivanova and Roubtsov (1991, cited in Laborde: 1994), Working in small groups: A learning situation? *Didactics of Mathematics as a Scientific Discipline*, Kluwer Academic Publishers.

45. Shoemaker (1991, cited in Lake: 1994), *Integrated Curriculum,* School Improvement Research Series. http://www.nwrel.org/scpd/sirs/8/c016.html

46. Shulman (1987, cited in Bromme: 1994), A psychological topology of teachers' professional knowledge, *Didactics of Mathematics as a Scientific Discipline*, Kluwer Academic Publishers.

47. Silver, E.A. (2003), Improving Education Research: Ideology or Science? *Journal for Research in Mathematics Education,* 106-109

48. SIMMS Integrated Mathematics (2005), About the Curricula. http://www.edc.org/mcc/csimms/asp.

49. Sorensen, R (2003), Effective Maths. http://www.lakeforest.edu/images/userImages/blocker/Page_3947/rachelsorenson.pdf

50. Stein (2001, cited in Sorensen: 2003), Effective Maths. http://www.lakeforest.edu/images/userImages/blocker/Page_3947/rachelsorenson.pdf http://www.nwrel.org/scpd/sirs/8/c016.html

51. Stewart, J (1997), *Concepts and Contexts*, Calculus.

52. Strauss, J (1996), Mathematics *and Science Education, Subject Didactics: Mathematics and Science,* RAUCEH.

53. Tall, D (1994), Environments for learning, *Didactics of Mathematics as a Scientific Discipline*, Kluwer Academic Publishers.

54. Tietze, U (1994), Mathematical curricula and the underlying goals, *Didactics of Mathematics as a Scientific Discipline*, Kluwer Academic Publishers.

55. Vollrath, H (1994), Reflections on mathematical concepts as starting point for didactical thinking, *Didactics of Mathematics as a Scientific Discipline*, Kluwer Academic Publishers

56. Vygotsky (1930, cited in Bussi: 1994), Theoretical and empirical approaches to classroom interaction, *Didactics of Mathematics as a Scientific Discipline*, Kluwer Academic Publishers.

57. Wittmann, E (1981, cited in Vollrath: 1994), Reflections on mathematical concepts as starting point for didactical thinking, *Didactics of Mathematics as a Scientific Discipline*, Kluwer Academic Publishers

58. Wittmann, E (1989, cited in Biehler: 1994), *Didactics of Mathematics as a Scientific Discipline*, Kluwer Academic Publishers.

59. Wittmann, Erich CH. (1995), Mathematics Education as a 'Design Science', *Educational Studies in Mathematics 29*, 355- 374.

60. Zemelman (1998, cited in Sorensen: 2003), Effective Maths. http://www.lakeforest.edu/images/userImages/blocker/Page_3947/rachelsorenson.pdf

61. Zemelman, Daniels and Hyde (1998, cited in Sorensen: 2003), Effective Maths. http://www.lakeforest.edu/images/userImages/blocker/Page_3947/rachelsorenson.pdf