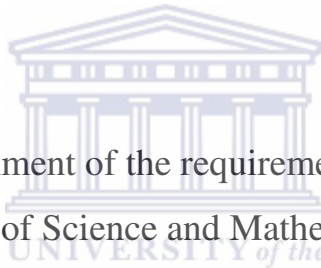


**STUDENTS' EXPERIENCES, LEARNING STYLES AND  
UNDERSTANDING OF CERTAIN CALCULUS CONCEPTS: A  
CASE OF DISTANCE LEARNING AT THE ZIMBABWE OPEN  
UNIVERSITY**

**CHIPO TSVIGU**



A thesis submitted in fulfilment of the requirements for the degree of Doctor  
Philosophiae in the School of Science and Mathematics Education, Faculty of  
Education, University of the Western Cape.

**SUPERVISORS:**

Professor Trygve Breiteig, University of Agder, Norway

Professor Jan Persens, University of the Western Cape, South Africa

**November 2007**

**Students' Experiences, Learning Styles and Understanding of Certain Calculus  
Concepts: A Case of Distance Learning at the Zimbabwe Open University**

Chipo Tsvigu

**KEYWORDS:**

Distance education

Learning styles

Learning materials

Learning experiences

Understanding in mathematics

Limit of function

Derivative of function

Felder-Silverman learning styles model

Representation

Information processing



# ABSTRACT

## **Students' Experiences, Learning Styles and Understanding of Certain Calculus Concepts: A Case of Distance Learning at the Zimbabwe Open University**

CHIPO TSVIGU

PhD thesis, School of Science and Mathematics Education, University of the Western Cape.

### **ABSTRACT**

This study attempts to understand how distance education practices influence the learning of calculus. Understanding student learning in a distance education environment is an important factor to consider in improving the learning experiences of those students who for one reason or the other opt not to study in conventional institutions of higher education. On one hand, understanding student learning may illuminate the influences that the learning environment has on student learning and on the other hand, it may inform on how learning experiences can be improved.

The aim of this study is to acquire a deeper understanding of the diverse manner in which distance students learn calculus. Specific focus is also placed on how the distance education context of the Zimbabwe Open University (ZOU) influences student learning. The study describes a group of students' experiences of learning calculus in the ZOU distance education environment. The study also describes the students' learning styles and relates these to their mathematical understanding of certain calculus concepts. The specific content topics of "limit of function" and "derivative of function" are used to view achievement and performance, thereby indicating the distance students' mathematical understanding.

The information processing learning theory is used as the theoretical framework for this study. The constructs of learning styles and mathematical understanding are used to illuminate the student's learning processes. The study used the Felder-Silverman learning styles model and Hiebert and Carpenter's notion of mathematical understanding to expound these constructs.

The distance education environment of the B.Sc. Mathematics and Statistics (BSMS) programme at the ZOU provided the context of the study and an interpretive case study

approach was adopted. A group of students registered in a first year first semester calculus course were studied. Data were collected from students based in four ZOU regional centres; namely Harare, Mashonaland Central, Mashonaland West, and Masvingo. These regional centres were conveniently selected for the study on the basis of proximity and accessibility. A total sample of twenty six students was involved and data for the in-depth part of the study emanated from five students who were purposively selected to participate in interviews. The interviewees were selected on the basis of their performance in a written calculus test. Data for this study were collected through use of learning journals, learning styles preference questionnaires, calculus tests and interviews. The data on students' learning experiences were predominantly qualitative in nature though supported by some quantitative data. The data on learning styles and mathematical understanding were also qualitatively analysed and presented case by case for the five interviewees.

The study established that in a distance education system, the type of learning environment has the potential to influence students' learning, both positively and negatively, of which the main contributing factor is the learning support system. The study found that the learning support system provided by the institution and distance educators can have an impact on student learning. With reference to the calculus course in the BSMS programme, the study identified specific aspects where the environment facilitated or deterred learning. The study also revealed that students have varied learning style preferences, and that the learning environment has the potential to impact on students' learning styles. Since learning styles occupy a central place when it comes to improving distance learning materials, the study further explored the relationship between the constructs of learning styles and mathematical understanding. The study revealed that students' learning styles can influence the students' mathematical understanding.

Improving students' learning in a distance education environment rests mainly on improving the learning materials and the support systems. A carefully designed and well supported instructional distance learning package can facilitate learning. Implications of the findings point towards the improvement of the distance teaching processes through the improvement of learning materials and the learning support systems for the BSMS distance education programme.

# DECLARATION

I declare that *Students' Experiences, Learning Styles and Understanding of Certain Calculus Concepts: A Case of Distance Learning at the Zimbabwe Open University* is my own work, that it has not been submitted for any degree or examination in any other university, and that all the sources I have used or quoted have been indicated and acknowledged by complete references.



Chipo Tsvigu

Signed:.....

Date: November 2007

## DEDICATION

- In memory of my late parents: My mother who passed on when I was still a toddler and my father who believed in empowering the girl child through education

- To my sister Mrs. Rainah Florence Chinemo for the motherly love she showed and

still shows me



- To my husband Obert for his love, care and affirmation

UNIVERSITY of the  
WESTERN CAPE

## ACKNOWLEDGEMENTS

*Thank you LORD God Almighty for giving me strength, good health and guiding me through the PhD journey: “I can do everything through Christ who gives me strength” Philippians 4:13.*

Reaching the destination of my long journey for my PhD studies has been made possible by the support and assistance of my academic mentors, family and friends.

- ❑ I am deeply indebted to my supervisors, Professor Trygve Breiteig and Professor Jan Persens. I would not have made it without the constant support, guidance and advice that you offered me throughout this study. Your professional guidance, timely advice, constructive criticism, untiring encouragement and patience really got me going and helped me come up with this thesis. I feel privileged to have worked with you as my supervisors and mentors as I gained a lot from this experience.
- ❑ I would like to acknowledge NUFU for the funding to support my studies through the Graduate Studies in Science, Mathematics and Technology Education (GRASSMATE) Project. I greatly appreciate the financial support.
- ❑ I wish to thank the GRASSMATE supervisory team, led by Professor Cyril Julie and Professor Oyvind Mikalsen. Not forgetting Dr. Mtetwa for the encouragement and advice that he always gave me. The interactions in the GRASSMATE ‘family’ moulded me both academically and professionally. It was a privilege for me to work alongside such great scholars.
- ❑ I would like to express gratitude to the Zimbabwe Open University (ZOU) management for all the support: affording me time, resources and space that enabled me to work on my studies. I greatly appreciate the permission they gave me to conduct the study at the ZOU. I also thank the Regional Directors whose regional centres were involved in this study; the support and assistance in my accessing the participants is greatly appreciated. To the colleagues in the Faculty of Science and the Department of Science, Mathematics and Technology, I thank you for the encouragement and understanding.
- ❑ I am greatly thankful to the ZOU BSc. Mathematics and Statistics Intakes 6 and 7 students who contributed to the research study, both in the pilot study and the main

study. I thank you for sparing your time and for sharing your learning experiences with me. Without your involvement, the study would not have been possible.

- ❑ I would not have managed to collect the data in a timely manner were it not for the involvement and assistance I got from some ZOU tutors. To the four tutors, Mr. K. Mutangi, Mr. A. Jaison, Mr. P. Chifamba and Mr. M. Magodora who served as my ‘Research Assistants’ in their respective regional centres, your time, patience, tolerance and cooperation are greatly appreciated.
- ❑ To all the fellow students and colleagues in the GRASSMATE project, I thank you for the discussions and sharing of experiences. You made my PhD studies ‘doable’.
- ❑ Special thanks go to the UWC Faculty of Education Librarian, Ms. Benita De Wet, who really made the distance between Harare and Cape Town short when it came to accessing literature. Thank you for all the advice and assistance with the literature searches.
- ❑ I thank the Curtin University of Technology (Perth, Bentley Campus) for allowing me the opportunity to access and use the Curtin University Library as a ‘Community Borrower’. The opportunity was timely and came at a crucial stage of the thesis writing.
- ❑ Special thanks go to my friends who spared their time to read drafts of this work. Dr. Yovita Gwekwerere read through various drafts of the thesis and offered invaluable suggestions. Dr. Charity Ndereya also read through excerpts of the work and gave useful suggestions. I greatly appreciate the input from Mrs. Mahes Pushpalingam who undertook the final proofreading.
- ❑ To the ‘big family’ and the ‘many friends’, I say thank you for your understanding, patience, assistance and encouragement. I marvelled at your readiness to assist whenever I needed help. You were prepared to listen to me when I went on and on talking about my study. Above all, I thank you for responding to my prayer requests. The constant prayers kept me going.
- ❑ Last but not least, my deepest thanks go to my husband Obert, and my sons Tendai Maxwell and Farai Ashel. I salute you ‘my TRIO’ for your love, support, patience and understanding. You braved my constant absences from home and my being ‘away from home’ even when I was present at home. Obert, you were always there for me, you encouraged me when I felt like giving up, and you gave me advice when I seemed lost, I wouldn’t have made it without you. I thank you ‘*Mwendamberi*’.



# TABLE OF CONTENTS

ABSTRACT .....	III
DECLARATION .....	V
DEDICATION .....	VI
ACKNOWLEDGEMENTS .....	VII
TABLE OF CONTENTS .....	IX
LIST OF ACRONYMS .....	XIV
LIST OF TABLES.....	XV
LIST OF FIGURES.....	XVI
<b>CHAPTER 1: BACKGROUND AND MOTIVATION.....</b>	<b>1</b>
1.1 INTRODUCTION.....	1
1.1.1 <i>The challenge for distance educators</i> .....	2
1.1.2 <i>A personal observation</i> .....	4
1.1.3 <i>Research into student learning</i> .....	6
1.1.4 <i>My point of view</i> .....	7
1.2 RESEARCH CONTEXT: DISTANCE EDUCATION.....	8
1.2.1 <i>The concept of distance education</i> .....	9
1.2.2 <i>The contextualisation of distance education</i> .....	12
1.3 DISTANCE EDUCATION AT THE ZOU .....	16
1.3.1 <i>Quality – a central concept at the ZOU</i> .....	16
1.3.2 <i>The ZOU model of distance education</i> .....	17
1.4 THE B.Sc. MATHEMATICS AND STATISTICS PROGRAMME .....	19
1.4.1 <i>About the BSMS programme</i> .....	19
1.4.2 <i>The calculus course at the ZOU</i> .....	19
1.5 RATIONALE OF THE STUDY .....	20
1.5.1 <i>Role of learning styles in distance education</i> .....	21
1.5.2 <i>Why the calculus concepts?</i> .....	22
1.6 OPERATIONAL DEFINITIONS OF TERMS .....	24
1.7 OVERVIEW OF THIS THESIS.....	26

<b>CHAPTER 2: FOCUS OF THE STUDY</b> .....	<b>28</b>
2.1 INTRODUCTION.....	28
2.2 STATEMENT OF THE PROBLEM .....	28
2.3 PURPOSE OF THE STUDY .....	29
2.3.1 <i>Aims of the study</i> .....	29
2.3.2 <i>Objectives of the study</i> .....	30
2.4 SIGNIFICANCE OF THE STUDY .....	30
2.5 DELIMITATIONS AND ASSUMPTIONS OF THE STUDY.....	31
2.5.1 <i>Delimitations of the study</i> .....	31
2.5.2 <i>Assumptions of the study</i> .....	31
2.6 CHAPTER OVERVIEW .....	32
<b>CHAPTER 3: REVIEW OF SELECTED LITERATURE</b> .....	<b>33</b>
3.1 INTRODUCTION.....	33
3.2 STUDIES ON LEARNING MATHEMATICS AT A DISTANCE.....	33
3.3 LEARNING STYLES .....	35
3.3.1 <i>Defining learning styles</i> .....	35
3.3.2 <i>Identifying learning styles</i> .....	36
3.3.3 <i>Learning styles and instruction</i> .....	38
3.4 STUDIES ON TEACHING AND LEARNING OF CERTAIN CALCULUS CONCEPTS	41
3.4.1 <i>Limit of function</i> .....	42
3.4.2 <i>Derivative of function</i> .....	52
3.4.3 <i>The function concept as prerequisite for understanding limits and derivatives</i> .....	56
3.5 CHAPTER OVERVIEW.....	59
<b>CHAPTER 4: THEORETICAL FRAMEWORK</b> .....	<b>60</b>
4.1 INTRODUCTION.....	60
4.2 INFORMATION PROCESSING MODEL OF LEARNING .....	61
4.2.1 <i>Information processing as a framework for learning theories</i> .....	62
4.2.2 <i>The computer as an analogy of the human memory system</i> .....	63
4.2.3 <i>Some criticisms of information processing theory</i> .....	70
4.2.4 <i>Information processing and learning styles- the link</i> .....	72
4.3 LEARNING STYLES THEORY .....	72
4.3.1 <i>Learning style models</i> .....	72
4.3.2 <i>A learning styles model for this study</i> .....	75
4.3.3 <i>Description of the Felder-Silverman learning styles model</i> .....	77

4.3.4	<i>Criticisms of the F-SLSM</i> .....	83
4.3.5	<i>Implications of the F-SLSM in mathematics distance learning materials</i> .....	84
4.4	MATHEMATICAL UNDERSTANDING.....	85
4.4.1	<i>About understanding</i> .....	86
4.4.2	<i>Conceptual and procedural knowledge</i> .....	87
4.4.3	<i>Mathematical understanding as an outcome of learning</i> .....	88
4.5	METACOGNITION AND ANDRAGOGY .....	91
4.5.1	<i>Metacognition</i> .....	91
4.5.2	<i>Andragogy</i> .....	93
4.6	CHAPTER OVERVIEW .....	94
<b>CHAPTER 5: METHODOLOGY.....</b>		<b>95</b>
5.1	INTRODUCTION.....	95
5.2	AN INTERPRETIVE RESEARCH ORIENTATION.....	95
5.3	RESEARCH DESIGN: CASE STUDY .....	96
5.3.1	<i>The case study research design</i> .....	97
5.3.2	<i>Motivation for a case study design in this study</i> .....	98
5.3.3	<i>Use of mixed methods in case studies</i> .....	99
5.3.4	<i>Overview of certain sources of data used in case studies</i> .....	100
5.3.5	<i>Generalisation in case studies</i> .....	102
5.4	RESEARCH INSTRUMENTS USED IN THIS STUDY .....	103
5.4.1	<i>The interview</i> .....	103
5.4.2	<i>Learning journals</i> .....	105
5.4.3	<i>Calculus Tasks Test (CTT)</i> .....	109
5.4.4	<i>Learning Style Preference Questionnaire (LSPQ)</i> .....	110
5.4.5	<i>The research question-method relationship</i> .....	112
5.5	THE PILOT STUDY .....	115
5.5.1	<i>Purpose of pilot study</i> .....	115
5.5.2	<i>The pilot study participants</i> .....	115
5.5.3	<i>The instruments used in the pilot study</i> .....	116
5.5.4	<i>Procedures and analysis of pilot data</i> .....	117
5.5.5	<i>Lessons learnt to inform the main study</i> .....	117
5.6	THE MAIN STUDY .....	120
5.6.1	<i>Data collection timeline</i> .....	121
5.6.2	<i>Selection of the participants of the study</i> .....	121

5.6.3	<i>The procedures</i> .....	124
5.7	ISSUES OF QUALITY FOR THIS STUDY .....	129
5.7.1	<i>Trustworthiness</i> .....	130
5.7.2	<i>Issues of ethics</i> .....	133
5.8	DATA ANALYSIS METHODS .....	138
5.8.1	<i>Data analysis of students' experiences with distance learning</i> .....	140
5.8.2	<i>Data analysis on learning styles and mathematical understanding</i> .....	141
5.9	CHAPTER OVERVIEW .....	147
<b>CHAPTER 6: STUDENTS' EXPERIENCES OF DISTANCE LEARNING .....</b>		<b>148</b>
6.1	INTRODUCTION .....	148
6.2	DEMOGRAPHIC INFORMATION OF THE PARTICIPANTS.....	149
6.3	CONTENT FACTORS.....	150
6.3.1	<i>Subject matter difficulties</i> .....	151
6.3.2	<i>Preferences of representation</i> .....	156
6.4	LEARNING MATERIALS FACTORS .....	160
6.4.1	<i>The course text (module)</i> .....	161
6.4.2	<i>Tutorial worksheets</i> .....	167
6.4.3	<i>Assignments</i> .....	168
6.4.4	<i>Students' suggestions for improvement on learning materials</i> .....	169
6.5	CONTEXTUAL FACTORS .....	170
6.5.1	<i>The social context</i> .....	171
6.5.2	<i>The physical context</i> .....	184
6.6	LEARNER BASED FACTORS .....	187
6.6.1	<i>Depth of learning</i> .....	187
6.6.2	<i>Metacognitive abilities</i> .....	189
6.6.3	<i>Learner perception of distance learning</i> .....	190
6.7	DISCUSSION .....	192
6.8	CHAPTER OVERVIEW .....	196
<b>CHAPTER 7: STUDENTS' LEARNING STYLES AND UNDERSTANDING.....</b>		<b>197</b>
7.1	INTRODUCTION.....	197
7.2	SOME CASE STUDIES.....	198
7.2.1	<i>Profile for Siphon (Student A)</i> .....	198
7.2.2	<i>Profile for Ray (Student B)</i> .....	208

7.2.3	<i>Profile for Tanya (Student C)</i> .....	220
7.2.4	<i>Profile for Kundai (Student D)</i> .....	233
7.2.5	<i>Profile for Tino (Student E)</i> .....	243
7.3	GENERAL DISCUSSION.....	253
7.4	CHAPTER OVERVIEW .....	254
<b>CHAPTER 8: REFLECTIONS AND CONCLUSION .....</b>		<b>256</b>
8.1	INTRODUCTION.....	256
8.2	REFLECTIONS ON THE RESEARCH PROCESS .....	257
8.2.1	<i>Reflections on the data collection stage</i> .....	257
8.2.2	<i>Reflections on the writing process</i> .....	260
8.3	REFLECTIONS ON FINDINGS: SUMMARY AND IMPLICATIONS .....	261
8.3.1	<i>Summary of research findings</i> .....	261
8.3.2	<i>Implications of the findings</i> .....	264
8.4	LIMITATIONS OF THE STUDY.....	271
8.5	DIRECTIONS FOR FUTURE RESEARCH.....	272
8.6	CONCLUSION .....	275
<b>REFERENCES .....</b>		<b>277</b>
<b>LIST OF APPENDICES .....</b>		<b>295</b>
APPENDIX I: REQUEST FOR PERMISSION-PRO VICE-CHANCELLOR .....		296
APPENDIX II: LETTER OF PERMISSION FROM THE REGISTRAR .....		297
APPENDIX III: REQUEST FOR PERMISSION - REGIONAL DIRECTORS.....		298
APPENDIX IV: PERMISSION FROM REGIONAL DIRECTOR (MASH CENTRAL).....		299
APPENDIX V: LETTER OF INTRODUCTION TO STUDENTS .....		300
APPENDIX VI: LEARNING JOURNAL .....		302
APPENDIX VII: CALCULUS TASKS TEST .....		308
APPENDIX VIII: LSPQ -LIMIT OF FUNCTION .....		316
APPENDIX IX: LSPQ - DERIVATIVE OF FUNCTION .....		322
APPENDIX X: INTERVIEW GUIDE .....		328
APPENDIX XI: DATA SETS ON CD IN PDF FORMAT .....		335

## LIST OF ACRONYMS

BSMS	Bachelor of Science in Mathematics and Statistics
CTT	Calculus Tasks Test
CDE	Centre for Distance Education
DE	Distance Education
DL	Distance Learner(s)
DSMT	Department of Science, Mathematics and Technology
F-S	Felder-Silverman
F-SLSM	Felder-Silverman Learning Styles Model
GRASSMATE	Graduate Studies in Science, Mathematics and Technology Education
IP	Information Processing
LJ	Learning Journal
LS	Learning Style(s)
LSPQ	Learning Style Preference Questionnaire
LTM	Long-Term Memory
RC	Regional Centre
RD	Regional Director
RPC	Regional Programme Coordinator
SIS	Sensory Information Storage
STM	Short-Term Memory
UCDE	University College of Distance Education
WM	Working Memory
ZOU	Zimbabwe Open University

## LIST OF TABLES

Table 1.1: Trend of Student Enrolment - BSMS Programme .....	4
Table 1.2: Scenarios for Distance Learning .....	11
Table 1.3: Nature of the Time-Place Scenarios for the BSMS Programme.....	20
Table 4.1: Summary of Felder-Silverman Learning Style Preference Characteristics .....	81
Table 5.1: A Research Question-Method Matrix for this Study .....	113
Table 5.2: Constructing a Case Record .....	139
Table 6.1: Demographic Data of the Participants (n = 26) .....	150
Table 6.2: Difficult Subject Matter Areas - Limit of Function .....	152
Table 6.3: Difficult Subject Matter Areas - Derivative of Function .....	153
Table 6.4: Reasons for Difficulties.....	154
Table 6.5: Ways of Overcoming Difficulties .....	155
Table 6.6: Responses to LJ Item 7 .....	156
Table 6.7: Summary of Limit of Function LSPQ Responses.....	157
Table 6.8: Summary of Derivative of Function LSPQ Responses.....	158
Table 6.9: Learning Difficulties Related to the Module as Identified from LJs .....	165
Table 7.1: Characterisation of Learning Styles for Sipho.....	199
Table 7.2: Characterisation of Learning Styles for Ray .....	208
Table 7.3: Characterisation of Learning Styles for Tanya .....	220
Table 7.4: Characterisation of Learning Styles for Kundai .....	233
Table 7.5: Characterisation of Learning Styles for Tino.....	243
Table 7.6: Summary Matrix of Linkages .....	254
Table 8.1: Summary Layout of Findings.....	262

# LIST OF FIGURES

Figure 4.1: Components of information processing ..... 63

Figure 4.2: Analogy of information processing in computers and in humans ..... 64

Figure 4.3: Outline of the human memory system..... 65

Figure 4.4: Families of learning style models (Coffield et al., 2004, p.9) ..... 73

Figure 4.5: The Felder-Silverman Learning Styles dimensions..... 78

Figure 5.1: Sources of data for the constructs of this study ..... 114

Figure 5.2: Research process timeline ..... 121

Figure 5.3: Learning styles profiling analytical model ..... 143

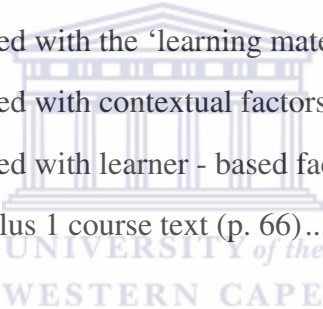
Figure 6.1: Subcategories associated with the ‘content factors’ category ..... 151

Figure 6.2: Subcategories associated with the ‘learning materials factors’ category ..... 160

Figure 6.3: Subcategories associated with contextual factors category ..... 171

Figure 6.4: Subcategories associated with learner - based factors category ..... 187

Figure 8.1: Extract from the Calculus 1 course text (p. 66)..... 269





# CHAPTER 1: BACKGROUND AND MOTIVATION

## 1.1 INTRODUCTION

Traditionally, university education has been conceived to take place in conventional institutions of higher education, where students physically attend the institutions for purposes of studying and learning. However, due to various reasons, some people may not be able to attend on-campus lessons despite the desire for further education. Distance education may therefore be better placed to meet the educational needs of such people. Trends in education indicate that university distance education (DE) is fast growing (Daniel, 1996a, 1996b; Jung, 2005, UNESCO, 2002) and may be viewed as an option or a complement to the conventional on-campus university system of education. Around the world there are more than 11 mega-universities, that is, distance education institutions that have student enrolments of more than 100 000 active students in degree level courses (Daniel, 1996a). Such a growth in distance education shows that distance education is playing an integral role in the provision of university education by meeting the needs of those students who are unable to, or opt not to attend the conventional institutions of education.

Contrary to conventional university education, distance education has the potential to take higher education to the “learner’s doorstep” and can still reach the student against competing priorities such as employment or family commitments. Apart from responding to the growing needs of the student, DE also responds to the education and training needs of a nation which are not possibly met by the traditional conventional programmes. For instance, the recognition and need for distance education in Zimbabwe is elaborated in a paper presented at the 2004 All-Africa Ministers’ Conference on Open Learning and Distance Education by Dr. Chombo, the Acting Minister of Higher and Tertiary Education for Zimbabwe at the time. An excerpt from the presentation is as follows:

*Distance education contributed immensely to the reduction of illiteracy, increased functional knowledge and increased number of people with higher academic and professional qualifications. The growth is predicted to continue for some years to come since members of the public are aware that it is an effective and economical*

*teaching/learning strategy. The demand for education in Zimbabwe is very high. Both employers and parents support the pursuit of education by the learners at various levels irrespective of whether the mode is distance education or conventional one. Employers send some of their employees to take up staff development courses through distance education because the student does not leave employment. (Chombo, 2004)*

In a developing nation like Zimbabwe, the role that distance education plays is therefore crucial, making DE not just an option but rather a necessity.

A definitive characteristic for distance education is the separation between the teacher and the students and amongst the students themselves (Keegan, 1990). Whereas, in a conventional programme, for the teaching and learning processes to take place, students are required to attend lectures, meet with their teachers on a face-to-face basis as well as meet with their peers. In a distance education programme, the teaching and learning processes take place when the students are physically separated from their teachers and from their peers. A teacher is always present to facilitate the teaching and learning process in a conventional programme whereas in distance education, the process is facilitated by use of some institutionally prepared learning materials and occasional tutorials.

In the light of the growing demand and popularity of distance education, some researchers in distance teaching and learning (e.g. Dzakiria, 2005; Jung, 2005; Marland, 1997; Tait, 2000) have been concerned about how institutions that run distance education programmes can facilitate quality learning. Challenges therefore emerge on how distance education programmes can assist students to optimise benefits from instruction.

### **1.1.1 The challenge for distance educators**

Apart from providing the student with physical access to education, distance education is also expected to facilitate student access to the respective subject matter by providing an educational environment that promotes learning. In as much as the responsibility to learn rests with the student, the responsibility to enable the student to access the required subject matter squarely rests with the distance education institution. Many a times distance students find themselves 'lost at sea' when faced with the demands of distance education, where they have to find their own way through the subject matter of a course that is provided in the learning materials. Marland (1997, p. 107-108) gave an example of exploring a new planet as an analogy to show the experience of new distance students, in the following manner:

*Imagine yourself on a space odyssey, about to descend on to a new planet, with no knowledge of the topography, vegetation, flora and fauna of that land, and charged with the responsibility of exploring and mapping it and finding your way across and within it. That would be a daunting task. Yet in many respects, that is similar to that task that confronts the distance learner who enrolls in a new subject and is expected to explore and gain mastery of it. They have to find out about the substance and structure of the subject, the main issues it addresses.*

Using the example further, Marland (1997, p. 108) put a challenge on the distance teacher

*the challenge for the distance teacher is how to assist the new 'explorers' find their way about the new 'continent of knowledge' so that they don't become lost or mired in conceptual swamps and abort the mission. What can be done to enable distance learners to find their way about the new subject and fulfil their own expectations (...).*

Analogically, in view of the teaching of university mathematics by distance education, the major challenge for the mathematics distance educators is to provide the distance students with an educational experience that promotes optimal learning.

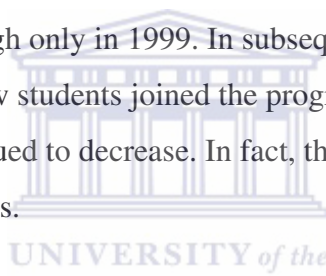
As a distance education university, the Zimbabwe Open University (ZOU) endeavours to assist its mathematics students to access the required mathematical knowledge through the use of some learning resources. However, like in any other academic discipline, the distance educators for mathematics face the challenging task of making the mathematical knowledge accessible to the students. This may raise issues related to learning resources such as content selection and organisation, communication of the content, facilitating cooperation amongst students, feedback to students, and the social aspects of learning. The challenge in the distance teaching of mathematics becomes more conspicuous considering that on one hand, in distance education, the students are 'quasi-separated' from the teacher and from their peers (Keegan, 1990). On the other hand, research has been indicating that students experience difficulties in understanding some basic mathematical concepts (see for instance, Bezuidenhout, 1998, 2001; Cornu, 1991; Dalby, 2001; Dorier & Sierpiska, 2001; Eisenberg, 1991; Fernandez, 2004; Ferrini-Mundy & Graham, 1994; Hahkionemi, 2004, 2006a, 2006b; Juter, 2005a, 2005b, 2006; Kannemeyer, 2003; Moru, 2006; Orton, 1983; Robert & Speer, 2001; Swinyard & Lockwood, 2007; Tall, 1993a).

Orton (1992) pointed out that teaching can take place without learning taking place for some students, whilst others manage to grasp the concepts, even though all would be occurring in the same learning environment. From a distance education research perspective, such

differences as observed above by Orton (1992), would call for research initiatives that focus on understanding how students learn in the distance education environment, how they respond to the distance education environment and why such differences exist. This may eventually illuminate the role of the distance education environment on student learning. Thus in turn, further inform on what aspects of the environment can possibly be improved so as to optimise learning.

### 1.1.2 *A personal observation*

Having been involved with the teaching of undergraduate mathematics courses and also with the writing of some course texts for the distance students at the ZOU, I became interested in understanding the kind of learning related problems that mathematics distance students experience. It has been my observation with the B.Sc. Mathematics and Statistics (BSMS) programme at the ZOU (refer to Table 1.1) that since the conception of the programme in 1999, student enrolments were high only in 1999. In subsequent years, the student enrolments were low. Even as new students joined the programme, the total number of students in the programme continued to decrease. In fact, the student numbers decreased to almost half in the course of 4 years.



**Table 1.1:** Trend of Student Enrolment - BSMS Programme

<b>Academic Year</b>	<b>Student enrolment</b>	<b>Level of study for the registered students</b>
1999/2000	778	Part 1 students only
2000/2001	872	Part 1 and 2 students only
2001/2002	437	Part 1, 2 and 3 students only
2002/2003	400	Part 1, 2, 3 and 4 students
2004/2005	405	Part 1, 2, 3 and 4 students

*Sources: 2004 ZOU Statistics at a glance brochure; 1992-2002 ZOU Enrolment Statistics*

The Zimbabwe Open University like any other institution of education, places priority on student retention. Such a trend as noted in the BSMS programme brings out challenges on how to retain more students in the programme than to lose them along the way. From an academic point of view, the first port of call would be to investigate and seek insights on the teaching and learning processes, asking questions such as: to what extent do the components of the instructional environment promote or deter learning, how do the students interact with

the distance learning environment, how do the students go about learning their courses and what really are the learning outcomes?

Although at one end, one finds students who struggle with their BSMS studies, at the other end, there are those who really excel in their studies. A thought provoking example on the issue of distance learning is the case of two former students of the BSMS programme who we will name Mr. X and Mrs. Y. The two students joined the BSMS programme at the same time. Both students frequented my office for varied reasons. Mr. X excelled through the programme and completed the programme within the stipulated time. He also managed to graduate with the best student award from his cohort. However, Mrs. Y's case was not such a success story. For the first three years of her studies, she was struggling with year one courses, and the worst being calculus related courses. Mrs. Y never managed to get past year two of her studies. She eventually temporarily dropped out of the programme with the intention of coming back after "a bit of resting". Among the courses that she never managed to pass was Calculus 1.

From the informal discussions I held with these students, it became apparent that both of them showed high interest in mathematics and each seemed to put a lot of effort in their studies. Mr. X claimed that his best reading time was during the night when there were no disturbances and would read and aim to get the best grades, which he definitely managed to get. Mrs. Y said she would sit for hours and hours reading the text and think that she has mastered the concepts only to find that on assessments she still did not perform well.

Here was an interesting situation of two students who joined the programme at the same time, had received the same type of learning resources, but after some few years, the students were many semesters apart in terms of academic advancement. Whilst one seemed to have excelled and progressed through the years, the other one seemed to struggle with first year courses.

Mrs. Y is not alone in that she never managed to pass Calculus 1. It has been my observation that of the four courses that are offered during the first semester of the first year, the Calculus 1 course presents a lot of challenges and is very unpopular with students. This is also reflected in the low pass rate for the course which is a cause for concern as calculus is an important course in mathematics and related subjects.

Such differences as observed in the case of Mr. X and Mrs. Y, though common with almost every group of students who join the BSMS programme, do trigger some reflections in me on what really could contribute to these differences. Could it be due to the different individual learner characteristics or could it be due to the sufficiency or insufficiency of the learning resources availed by the institution? How did the distance education environment influence the students' learning? To what extent was the students' learning potential fully realised? To what extent were the students' needs met in this distance education environment? What really were the students' needs? Were the needs the same or did they differ among individuals? These kinds of observations and reflections actually provided the conception of this study.

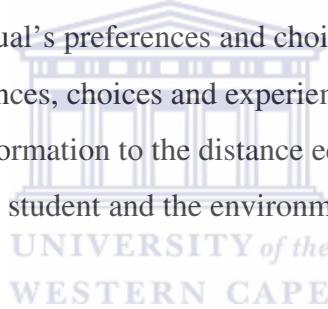
### **1.1.3      *Research into student learning***

Researching student learning of mathematics provides a sound footing towards improving teaching and learning in distance education systems. One way a distance educator can get to know how students learn in the distance education environment is by understanding the teaching and learning process from the perspective of the learner. Researchers on student learning posit that researching student learning informs the improvement of learning (Richardson, 2000; Marland, 1997; Ramsden, 1988). As more and more students get involved in the learning of mathematics at university level by distance education, it becomes imperative that mathematics distance educators consider research initiatives on how teaching can be improved and how learning can be optimised. This entails understanding how the students respond to the distance education environment in terms of learning, as well as gaining an understanding of how the distance education environment influences student learning. As pointed out by Dzakiria (2005, p. 99), researching student learning in a distance context enables the distance educators, practitioners and institutions "to get a balanced picture of what is 'right' and 'wrong' in their DE programmes, courses and administration".

Studies on student learning suggest that individuals receive, perceive and process information differently (Novin, Arjomand & Jourdan, 2003; Logan & Thomas, 2002; Felder & Henriques, 1995; Felder, 1993; Riding & Sadler-Smith, 1992; Felder & Silverman, 1988; Claxton & Murrell, 1987). For instance, in a given context, some students have strong preferences for verbal information and best grasp spoken or written information, while others prefer visual information and best grasp information presented as diagrams, pictures

or demonstrations. Some students prefer processing information actively whilst others prefer to think through the information and reflect upon it. Such preferences would form part of a student's learning styles. A fundamental concept that can be used to capture and describe how students learn is that of learning styles.

More elaborate discussions on learning styles will be provided in Chapters 3 and 4 as part of the literature review and theoretical framework for this study. However, the following definition on learning styles given by Keefe will suffice for the current purpose of an overview. Keefe (1985, p. 140) defined learning styles as “the composite of characteristic cognitive, affective and physiological behaviours that serve as relatively stable indicators of how learners perceive, interact with, and respond to the learning environment”. From the above definition, it is evident that learning styles are regarded as an individual's characteristic consistent approaches and preferences to receiving, perceiving and processing information in a learning situation. The main points to draw from this definition are that learning styles reflect an individual's preferences and choices in a learning situation. Such information on students' preferences, choices and experiences within their environment of learning yields very valuable information to the distance educator as the information serves as a point of contact between the student and the environment.



#### **1.1.4 My point of view**

Information on students' learning experiences and processes is indispensable as it potentially can explain how the environment influences learning. An understanding of the various distance students' learning style preferences and their learning experiences in the ZOU distance environment may provide information that can be useful if the knowledge is applied in the development and improvement of the learning resources. As a way of optimising the learning potential of the students, the learning resources and materials can be designed and coordinated in a way that does not cause learning conflicts but rather enhances learning by taking into consideration learner diversities in terms of learning styles. At this point, it is important to point out that exclusive matching of teaching to learning styles does not solve all learning conflicts, as there are other factors such as learner's previous background and motivation that can influence the amount and quality of learning (Montgomery & Groat, 1999). However, Montgomery and Groat also posit that being self-reflective and explicit about the role of learning styles can enhance student learning.

This study, which is based on student learning of calculus in distance education, seeks to gain an understanding of how students experience the BSMS distance education environment at the ZOU using the context of a calculus course. The ‘experience’ is examined in the light of students’ learning experiences in the distance education programme, students’ learning styles and their mathematical understanding of certain calculus concepts. The specific narrow content domain of limit of function and derivative of function concepts is used to understand more on the outcomes of the learning process.

Since this study is situated in the distance education context of the ZOU, the following section will provide a comprehensive discussion on distance education as the context of the study. This will put the ZOU distance learning environment into perspective.

## **1.2 RESEARCH CONTEXT: DISTANCE EDUCATION**

Kember, Ng, Tse, Wong and Pomfret (1996) alluded to that the distance education environment is usually perceived as a complex system. Since the learning process that is being investigated in this study is taking place within a complex environment, a comprehensive exposition of the context of the study will be helpful when it comes to understanding the students’ experiences of learning in the DE environment. Other than that, the complexity of the context of the study has some bearing on the way the research study is conducted and on how the research problem is perceived, hence the need to provide a more detailed account of the DE context.

In this section, the concept of distance education is discussed. A brief description of some underlying principles of distance education is also included so that the underlying benefits and complexities that are associated with distance education can be easily conceptualised. The distance education model at the Zimbabwe Open University is outlined as well as the distinctive features of the BSc Mathematics and Statistics Programme, which provides the context in which this study is conducted.



### **1.2.1 The concept of distance education**

#### **Defining distance education**

In the view of Keegan (1990), distance education includes both distance teaching and distance learning, where distance teaching is viewed from the teaching perspective and distance learning is viewed from the learning perspective. Keegan (1990) highlighted the following characteristics, which are essential for any comprehensive definition of distance education:

- the quasi-permanent separation of the teacher and the student throughout the duration of the teaching process,
- the quasi-permanent separation of a student from a learning group throughout the learning process,
- the influence of an educational organisation especially in the planning and preparation of learning materials,
- the use of mechanical or technical media such as print, audio-video cassettes or computers to carry the content matter of the course,
- the provision of two-way communication, so that the student can benefit from or initiate a dialogue.

One main characteristic of distance education is the separation between the teacher and the student, and the separation amongst the students themselves. Thus, learning in a distance education environment takes place when the teacher and their students are wholly or partially separated in terms of time, location or both (Rowntree, 1992) and is facilitated by some form of media.

#### **The nature of distance education learning materials**

The characteristics of distance education as per the definition by Keegan (1990) bring out the geographically dispersed nature of distance students. The physical distance between the 'human' teacher and the students, and amongst the students, can be very large. This implies the individualisation of student learning, the importance of careful pre-planning and preparation of the learning materials, as well as the careful planning of a learning support system. Lockwood (1998) emphasizes on using self-instructional materials, which are

supposed to provide among other features, expert and standardized content, individual learning, and active learning. A well designed system implies that the distance education learning materials must facilitate the students' accessing the subject matter by being self-explaining, self-guiding and by meeting their learning needs.

A common phenomenon in distance education as noted by Keegan (1990) is that educators prepare learning materials from which they may never personally have an opportunity to use directly with students. Someone else may use the materials to teach and evaluate students (Keegan, 1990). Such a situation can continue to happen for years after the learning materials have been developed. This gives a discerning feature between a distance education course and a course that is run in a conventional study programme. As the learning materials for a distance education course are pre-prepared, there is much less opportunity to immediately modify or adapt the learning materials to suit the needs of individual students once the materials are presented to the students (Mercer & Pettit, 2001). Whereas for a conventional programme, the opportunity to adapt teaching so as to suit learner's learning needs is readily available. Since distance education students mostly study on their own, the institution can only optimally respond to the learning needs of the students if the learning resources are well designed in a way that optimises the learning potential of most students. This once more brings into light the importance of understanding learner characteristics and experiences, as teaching may be improved and learning optimised as a result of applying what is known about student learning in the distance learning materials.

### **Scenarios of distance learning in terms of time - place separation**

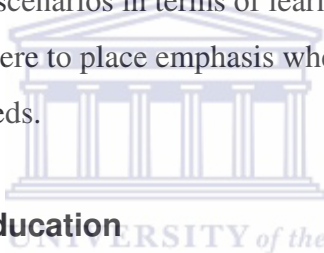
As already mentioned, in a distance education situation, teaching and learning occurs when there is separation in terms of place, time or both. The Commonwealth of Learning document on "Introduction to Open and Distance Learning" provide details of intersections that possibly can arise when time and place are viewed as continua resulting in four scenarios for distance learning.

**Table 1.2:** Scenarios for Distance Learning

	<b>Same time</b>	<b>Different time</b>
<b>Same place</b>	<i>Example:</i> Classroom teaching, face to face tutorials, seminars, workshops and residential schools.	<i>Example:</i> Learning resource centres which learners visit at their own time
<b>Different place</b>	<i>Example:</i> Audio conference and video conference, television with one-way video, two way audio; radio with listener-response capability, telephone tutorials conferences.	<i>Example:</i> Home study, computer conferencing, tutorial support by email and fax communication.

Source: COL (2000, p. 5-6)

In order to facilitate learning, distance education institutions and programmes use a combination of these scenarios. A salient feature of the ‘time and place separation’ scenarios as presented above, is that one can obtain a clear picture of how distance education institutions attend to each of the scenarios in terms of learning resources. Hence, enabling distance educators to identify where to place emphasis when it comes to improving student learning so as to meet learner needs.



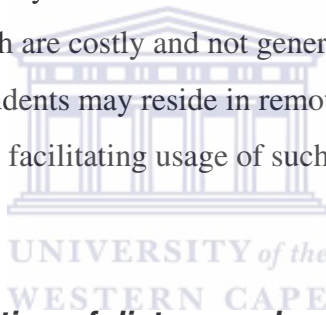
### **The evolution of distance education**

Internationally, distance education has since evolved from the pen and paper correspondence mode, situated in the ‘different time- different place’ scenario to the technologically computer based mode of distance education which can afford to combine all four scenarios. The worldwide technological advancement also has its influence on the history of distance education, which has seen distance education going through the technical evolution. Though different media and technologies drive the distance education evolution process, the technologies do not necessarily replace each other, but they are rather complementary. As such, the newer media and technologies have been incorporated and blended with the earlier versions in course delivery.

Various models of the evolutionary stages of distance education exist, and an example is the one described by Taylor (2001). Taylor (2001, p. 3) describes five generations of distance education mainly defined with regard to the media of instruction and the degree of flexibility and interactivity of the learning materials. A summarised description of the five generations follows: (i) First generation (*correspondence*) – text based correspondence with print text similar to that used in classroom situations and reliant on postal systems for communication;

(ii) second generation (*multimedia*) – mainly print based material characterised by self-instructional design and includes some form of media such as audiotape, videotape and computer based learning including interactive multimedia (iii) third generation (*tele-learning*) – includes audio teleconferencing, video teleconferencing, broadcast TV/radio; (iv) fourth generation (*flexible learning*) – includes online interactive multimedia, the internet and web based access to resources, computer mediated communication; (v) fifth generation (*intelligent flexible learning*) – as in (iv) above and also includes use of automated response systems and campus portals that provide access to institutional processes and facilities.

Whilst some distance education institutions have lived through to the fifth generation, most institutions especially in the developing countries of Africa still have yet to reach that stage (ADEA, 2002). The main reason being that at fifth generation level, distance education is strongly dependent on the availability of information and communication technology (ICT) resources and infrastructure, which are costly and not generally available in some countries. In addition to that, some of the students may reside in remote rural areas where opportunities and telecommunications networks facilitating usage of such media and technology are non-existent.



### **1.2.2      *The contextualisation of distance education***

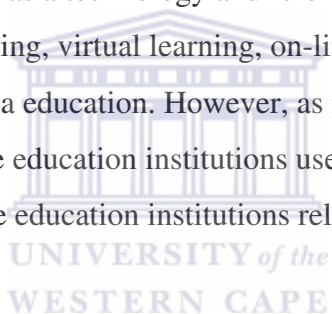
#### **The international perspective**

Currently, there is a trend where formerly single mode conventional institutions have changed to dual mode institutions offering both conventional and distance education programmes. Thus, while some countries have single mode DE universities, other countries offer distance programmes at their conventional universities. For instance, France was identified by Simonson, Smaldino, Albright and Zvacek (2006, p. 15) as an example of a country with no national distance teaching university but offering DE courses through 22 of its conventional universities.

Distance education programmes enjoy large student enrolments. As already mentioned, the world's mega-universities do offer their programmes by distance education methods. Amongst the world's mega universities are renowned institutions such as the UK Open University (UKOU) in the United Kingdom, the Anadolu University in Turkey, Indira

Ghandi National Open University in India, the China Central Radio and TV University in China, Korea National Open University in Korea and the University of South Africa (UNISA) in South Africa. The UNISA is the only Sub Sahara African distance education university that has student enrolments beyond the 100 000 mark. Such high figures on student enrolment in DE institutions serve as indicators of the crucial role that distance education is playing the world over.

Trends also suggest that the presence of the technological media has contributed to the growth of distance education worldwide (Daniel, 1996a, 1996b; UNESCO, 2002; Simonson *et al.*, 2006). Where resources for ICT permit, the technological media makes it possible for distance programmes to reach many dispersed students and to bridge the time-space gap. Thus, as ICT technologies get more advanced, distance course delivery and support systems also get more advanced. So much is the prevalence of technology in DE that some practitioners tend to describe DE as a technology and refer to it using ICT related terms such as distributed learning, tele-learning, virtual learning, on-line education, e-learning, interactive learning or multimedia education. However, as noted by Daniel (2004) and Guri-Rosenblit (2005), not all distance education institutions use high level ICT for teaching and learning purposes. Some distance education institutions rely on the print based material for teaching and learning.



### **The Sub-Saharan Africa perspective**

According to Leary and Berge (2007, p. 137), there are about 150 distance education programmes in the Sub Saharan Africa, of which twenty percent of these are offered by universities (Leary & Berge, 2007, p. 140). Most of the programmes are run by formerly single mode conventional institutions that are now running both conventional and distance education programmes. Among the single mode distance education universities in the region, the African Virtual University (AVU) and the University of South Africa (UNISA) can be identified as the two major distance education tertiary universities of the region. The AVU offers distance learning programmes in 27 countries inclusive of countries in the Sub Saharan Africa. Distance learning at AVU heavily relies on technology and is based on satellite transmissions and the use of the interactive telecommunications. The UNISA is recognised as one of the oldest distance education institution, and as already mentioned, it is one of the world's mega-universities, which enjoys large student enrolments. Although UNISA has a component of technological support of its course delivery, much of the

distance learning at UNISA does not depend on technology but rather still depends on the traditional print and postal systems (ADEA, 2002).

Despite the media and technological advances in the world, most of the single mode distance education universities in Sub-Saharan Africa including the UNISA, the Open University of Tanzania and the Zimbabwe Open University still use print based materials as the main mode of instructions. The printed materials may be supported in some instances by audio cassettes, video cassettes and face-to-face tutorials and other support services such as student counselling and library books. This is essential to note for this study. An apparent major background for this slowness in technological advancement for DE in the region is the lack of resources, infrastructure and expertise (ADEA, 2002; Leary & Berge, 2007; Simonson *et al.* 2006). Although DE in Sub Sahara Africa may appear to be marginalised in terms of incorporation of high level ICT for teaching and learning purposes, it is still seen as having the potential to capture many people for educational purposes. Daniel (2004) noted that DE is still relevant in Africa, and plays a significant and vital role in facilitating education in African countries. The 2004 conference held in South Africa where ministers of Education from different African countries participated illustrates the level of relevance in which governments place distance education in their respective countries (All-Africa Ministers' Conference in Open Learning and Distance Education, 2004).

In Zimbabwe, distance education still manages to reach a lot of students. In comparison to other institutions in Zimbabwe, the Zimbabwe Open University has enjoyed high student enrolments, registering above 23 000 students in 2001, approximately 19 000 in 2004 and almost 20 000 in the year 2005 (*2004 ZOU Statistics at a glance brochure, ZOU student enrolment statistics 1993-2002*). Such high enrolments serve as indicators of the crucial role that DE is playing in the education system of the country.

## **Distance education in Zimbabwe**

### *The evolution of university distance education in Zimbabwe*

The attainment of independence in Zimbabwe in 1980 brought about extensive and rapid expansion of the education systems. The Government of Zimbabwe then opened doors to education for many people who had failed to gain access to educational systems during the colonial era. The free 'primary education for all' policy adopted by the government in 1980 resulted in huge enrolments at primary level. This later translated to huge enrolments at

secondary school level, which also subsequently translated to an increase in the need for tertiary education. More than that, many people who had ‘missed the boat’ as a result of the colonial domination now had the opportunity to further their education. The country, then, witnessed a massive growth within all sectors of the education system including the establishment of more universities. However, the new universities that were set up were mainly catering for students using the conventional on-campus approach. These universities could not therefore cater for those people who were unable to attend the conventional institutions due to such reasons as work commitments, family commitments or too much competition for entrance into conventional institutions of higher education. A need to cater for those who wanted to further their education and attain tertiary qualifications whilst off-campus arose, and eventually led to the conception and growth of distance education initiatives at tertiary level in Zimbabwe.

While the initial distance education initiatives in the country focused on teacher education, the demand for distance education programmes in other disciplines such as the arts, social sciences, natural sciences as well as mathematics and statistics was still not being addressed and the gap was still very noticeable. This resulted in the establishment of the Zimbabwe Open University, the first distance education university in the country.

It is interesting to note that historically the Zimbabwe Open University was conceptualised from a teacher-training distance education programme. The experience gained from the distance education teacher-training initiatives provided background for the development of programmes in the non-teacher education disciplines. One such initiative was the Centre for Distance Education (CDE) that initially was a teacher training distance education programme. The CDE was established at the University of Zimbabwe in 1993. The need to move from programmes that directly relate to teacher education and focus on non-teacher education disciplines, led to the CDE becoming the University College of Distance Education (UCDE) in 1996. The UCDE was established as an affiliate college of the University of Zimbabwe and its mandate was to develop distance education programmes in a variety of disciplines so as to meet the changing knowledge base and the educational demands in the country.

### *The Zimbabwe Open University Mandate*

In March 1999, an Act of Parliament was passed which saw the UCDE receive its university charter, leading to the establishment of the Zimbabwe Open University. This is a single mode distance education university. As cited in the ZOU Act of 1998, the mandate of the university was to preserve, advance and transmit knowledge. Part of Chapter 25:20 of the ZOU Act, which is of interest to this study, accordingly reads as follows:

*(1)...The objects of the University are the preservation, advancement and transmission of knowledge through a distance education system, and so far as is consistent with these objects, the nurturing of the intellectual, aesthetic, social and moral growth of the students of the University*

*(2) For the achievement of its objects, the University shall, subject to this Act, have the following powers-*

*(a) to provide for research and courses of instruction, suitable to the needs of learners through a distance education system, and to take such other steps as may appear necessary and desirable for the advancement and dissemination of knowledge (...) (ZOU Act [Chapter 25:20], 1998, p. 245).*

Thus, amongst others, the university was mandated the power to provide instruction in a way that meets the needs of the students through the distance education mode.

In the following two sections, I will describe the distance education mode at the ZOU followed by a description of the BSc. Mathematics and Statistics programme as part of an exposition of the context of this study.

## **1.3 DISTANCE EDUCATION AT THE ZOU**

### **1.3.1 Quality – a central concept at the ZOU**

The institutional vision for the ZOU is “*to become a world-class open and distance learning university by 2009*”, (ZOU 2005-2009 Strategic Plan, p. 9), and the mission is to “*empower people through lifelong learning thereby enabling them to realise their full potential in an affordable and flexible manner while executing their various endeavours*” (ZOU 2005-2009 Strategic Plan, p. 10). From the ZOU Act, the ZOU vision and mission statements, it is evident that central to these statements is an emphasis on the role of the institution in facilitating student learning. Such phrases as “*...preservation, advancement and*



*transmission of knowledge...*” (ZOU Act), “...empower people...” (ZOU mission statement), “...enabling them to realise their full potential...” (ZOU mission statement) and “...become a world-class...” (ZOU vision statement) are evidence to the assertion of facilitating quality learning. Furthermore, the Zimbabwe Open University considers quality to be a central concept in all its plans for teaching and learning strategies as proposed in its 5-year strategic plan (ZOU 2005-2009 Strategic Plan). One of the specific key strategic areas of the strategic plan focuses on the ‘qualitative growth’ of the academic programmes that are offered by the institution. This brings into bearing the need to re-appraise the quality of student learning and how the distance education environment influences student learning.

### **1.3.2 The ZOU model of distance education**

The distance education model at the ZOU is in a way similar to that of other well known distance education universities in the region such as the University of South Africa (UNISA) and the Open University of Tanzania, in terms of course delivery where the main mode of course delivery is through the use of printed materials.

The Zimbabwe Open University follows the regional centres model. The majority of the students are drawn from all over the country, with a few coming from outside the borders of Zimbabwe. Those students within the country are serviced at the regional centres that are closest to where they reside. At the moment there are ten ZOU regional centres, which are located in the main towns of the country’s geographical provinces. In addition, there is the national centre in Harare, which serves as the administrative centre of the university.

Administrative units such as the Vice-Chancellery, the Registry, the Finance, the Faculty offices and the Academic Department offices are situated at the national centre. A noticeable feature across all the ZOU programmes, are the face-to-face tutorials, which are conducted at the regional centres.

Most administrative decisions related to academic activities are centralised. This includes such activities as the development of new programmes, evaluation and improvement of existing programmes, the designing and production of all learning materials such as writing of modules and workbooks or worksheets, setting of all assessments (assignments, in-class tests and examinations), examinations marking, processing of examination results, appointment of part-time and fulltime staff. All major timetable activities such as assignment

dates, in-class test dates, and examination dates are centrally generated from the national centre.

The regional centres which are headed by the Regional Directors (RD), in a way serve as 'the decentralised administrative units' of the university. Among other activities, the regional centres provide co-ordination of programmes, supervise the tutors, organise and manage the face-to-face tutorials, and manage the flow (collection, marking and feedback) of assignments and in-class tests. The regional centres also serve as examination and information centres. They also provide library facilities and student counselling services. Student registration processes take place at the regional centres. Upon registration, the student is issued with the tutorial package for a course, and that includes all the modules (course books), workbooks or worksheets, all the assignments, and the relevant timetables.

The academic year at the ZOU is divided into two semesters. Each semester comprises of 15 weeks, with twelve weeks of learning time and three weeks allocated to examination sessions. Each course is allocated six hours of face-to-face tutorials per semester. The first tutorial session of two hours is held at the beginning of the semester and serves for introductory purposes. The session is meant to cover such aspects as introducing the course, providing the students with a course overview and guiding the students on how to study for the course. The second session of two hours, which is normally held midway through the semester, provides another opportunity for tutor-student interaction. It also provides the tutor with an opportunity to monitor student progress, as well as to assist students in dealing with learning challenges that they would have experienced so far. The third and last session, which is also two hours long, is held at the end of the semester and is meant for revision purposes. Since these six hours of tutorials provide the formal opportunity that the students have of face-to-face contact with their tutors and with fellow students, the tutorials are deemed as a valuable component of this distance education model.

The Zimbabwe Open University offers a variety of programmes, which are housed in its three faculties, the Faculty of Commerce and Law, the Faculty of Science, and the Faculty of Arts, Education and Humanities. Amongst the programmes in the Faculty of Science is the B.Sc. Mathematics and Statistics (BSMS) Programme which is providing the context for this study. The following section provides detailed description of the BSMS programme.

## 1.4 THE B.Sc. MATHEMATICS AND STATISTICS PROGRAMME

### 1.4.1 *About the BSMS programme*

The B.Sc. Mathematics and Statistics (BSMS) programme is a four year degree programme, which offers a balanced number of courses in statistics and mathematics. The number of courses is balanced in the sense that in order for a student to pass and graduate, the student has to pass a minimum of 28 courses of which 14 are statistics and 14 are mathematics courses. A majority of the students are adult learners and are of varied mathematical experiences. Professional backgrounds of the students include primary and secondary school teachers, technicians, data analysts, statistical officers, meteorologists, immigration officers, customs officers as well as executive managers.

The courses in the BSMS programme are comparable to those in other conventional universities in the country. Normally in the first academic year, a BSMS student is expected to register for eight courses, four taken in each semester, of which in each semester, two of the courses are mathematics courses and the other two are statistics courses. The course Calculus 1 (MTD101) which serves as context for this study is one of the courses which the students encounter, in the first semester of the programme.

### 1.4.2 *The calculus course at the ZOU*

The Calculus 1 (MTD101) course is offered to undergraduate students who are registered in the first semester of the first year of the B.Sc. Mathematics and Statistics degree programme. The mode of teaching and learning is mainly through written materials and face-to-face tutorials. The ZOU learning resources for the BSMS Calculus 1 course include a ZOU produced course text book (module), ZOU produced problem worksheets, contact face-to-face tutorial sessions, and assessment feedback which is given on the tutor marked assignments and tests. Students are further encouraged to make use of some recommended textbooks.

The calculus course is divided into seven units and the course is run over one semester. As alluded to earlier, a maximum of six hours of contact face-to-face tutorial time is officially availed by the institution for the course though students are free to make their own arrangements for extra tutorials. The students are also encouraged to form study and discussion groups.

Students' progress is monitored and assessed by means of one take home assignment and one in-class test during the semester, and a three-hour written examination at the end of the semester. Both the examination mark and the semester coursework mark contribute to the final grade for the course. Although the academic department at the national centre generates all assessment tasks and marking guides, the tutors in their respective regions handle the tutorials and mark the two assessments (assignment and test). They are also encouraged to 'comprehensively communicate' feedback to the students on the marked assignments.

In respect of the time-place scenarios that were described in section 1.2 of this chapter, the time-place scenario for the BSMS programme can be presented as shown in the following table.

**Table 1.3:** Nature of the Time-Place Scenarios for the BSMS Programme

	<b>Same time</b>	<b>Different time</b>
<b>Same place</b>	Supported through face-to-face tutorial sessions and group meetings	Supported through Regional Centre visits and library facilities;
<b>Different place</b>	Not supported	Individual study /Home study supported through the learning materials.

## 1.5 RATIONALE OF THE STUDY

It is my point of view that in aiming to “empower”<sup>1</sup> the distance learner in mathematics, the educator is prompted to think of the outcomes of learning, which can be evidenced by the student’s performances in mathematics tasks. Similarly, in aiming to enable the students to “realise their full potential”<sup>2</sup> when learning mathematics, the educator is prompted to aim at understanding whether the students’ “full potential” is realised. Furthermore, one may seek to find out in what ways the institution facilitates or deters the students’ realisation of their full potential. In addition, helping students realise their “full potential” also prompts the educator to think of learner differences in terms of learning styles and the role the institution plays in catering for these differences. Having been a mathematics lecturer and a learning materials writer in the BSMS programme at the ZOU, and also having taken ‘cues’ from the

---

<sup>1</sup> “empower” – as stated in the ZOU mission statement.

<sup>2</sup> “realise their full potential” – as stated in the ZOU mission statement.

ZOU Act, mission and vision statements, I became curious to know and understand more about BSMS students' learning processes and how they experienced the programme.

To put this study into a context of a content subject, I consider the specific and narrow content domain of the calculus concepts: limit of function and derivative of function. Most of the available studies conducted on the teaching and learning of these calculus concepts were conducted in the context of traditional conventional programmes. Thus, there is less documented about students' learning of the limit of function and derivative of function concepts in a distance education environment. Not to mention within the context of the Zimbabwe Open University where such kind of research is virtually nonexistent. Such a gap in research on learning mathematics topics in DE environments was also noted by Krussel (2002, p. 199) who stated that "Only a handful of studies in any subject area explore the efficacy of various processes in promoting conceptual understandings in a distance environment. On many topics of importance in mathematics learning, there is no research at all". A notable gap thus exists with regard to research on students' learning processes of mathematical concepts in DE in general, and at the ZOU in specific, a situation which warrants for more in-depth research.



### **1.5.1 Role of learning styles in distance education**

The process of learning and teaching requires communication between the parties involved. The way mathematics is communicated to a university student either through a textbook, teacher or self-instructional materials contributes to the way the student learns and understands the concepts. Many times pedagogical materials are presented to a distance learner with much less opportunity to optimise the materials to the individual learner's needs. Felder (1993), for instance, reinforces this by saying that a student who has strong preferences for visual information may find text based materials harder to deal with than a student with strong verbal skills. If individuals have their own habitual ways of representing and structuring information for learning, then, as noted by Felder and Silverman (1988), facilitating instruction consistent with the learning styles contributes to more effective learning. Thus, learning style research may play a crucial role in distance education in as far as informing on attending to learner diversities of the distance learners.

The recognition that individuals may have specific learning style preferences, has become an important consideration for designing and preparing of learning materials in distance

education environments. Several researchers (Diaz & Cartnal, 1999; Dzakiria, Razak & Mohammed, 2004; Liu & Ginther, 1999; Logan & Thomas, 2002; McLoughlin, 1999; Rowntree, 1992) have argued for the incorporation of learning styles in production of DE learning materials.

Findings from research on students' learning styles may be useful for two main purposes. On one hand, the information may assist the distance educator by supporting and motivating instructional decisions (Felder & Brent, 2005; Felder & Spurlin, 2005). This provides the distance educator with additional information that can be useful in making the environment more efficient and successful in terms of facilitating learning. On the other hand, learning styles information can be useful information for the students themselves. The students may become aware of their learning style preferences and be able to understand what works best for them on different stages and issues. Alternatively, they may develop strategies of managing and acquiring information presented in their less preferred ways.

As already mentioned in subsection 1.1.4, the purpose for conducting a study on students' learning styles is not to exclusively match instruction to students' learning styles, but to use the information in ways that can support, inform and enhance learning to people of diverse learning characteristics. Exclusive matching assumes that students' learning styles are fixed and thus may not allow students to shift to accommodate and try other learning styles. Such fixedness may not be possible in distance education environments, especially the print based distance learning materials where teaching and learning materials are pre-prepared and pre-printed without having any information of who the students are and what their characteristics are. Section 3.3 of this thesis will present a further discussion on the aspect of matching instruction to students' learning styles.

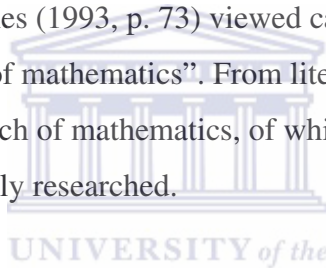
### **1.5.2      *Why the calculus concepts?***

The choice of the concepts of limit and derivative as the mathematical context for this study was based on three main reasons. Firstly, these concepts are selected on the basis of their fundamental role in calculus and in the subject of mathematics in general. Secondly, the study seeks to analyse deeply students' mathematical performance underpinning their understanding. Such concepts may serve as indicators of understanding and as evidence of the outcome of the learning process. Thirdly, current research has indicated that students

experience difficulties when developing the limit of function and derivative of function concepts. Some of these difficulties will be elaborated in Chapter 3.

### **Relevance of calculus**

The relevance of mathematics cuts across many academic disciplines including the natural sciences, social sciences, computing field, business field, engineering as well as the medical field. Calculus, as a branch of mathematics, is also relevant to other disciplines other than the mathematical sciences. Ferrini-Mundy and Lauten (1994, p. 120) describe calculus as “a critical landmark in mathematical preparation of students intending to pursue nearly all areas of sciences...”. Golden (n.d, p. 1) describes calculus as “the inescapable gateway to all higher level courses”, while Tall (1997, p. 289) says “it is both a climax of school mathematics and a gateway to further theoretical developments”. Golden and Tall, thus, illuminate the position of calculus in terms of the transition between elementary mathematics and advanced mathematics. Barnes (1993, p. 73) viewed calculus as a “tool that helps the student to appreciate the power of mathematics”. From literature cited above, it appears that calculus is an indispensable branch of mathematics, of which the teaching and learning of its components need to be continually researched.



### **The limit of function and derivative of function concepts**

The concepts of limit of function and derivative of function are fundamental concepts of calculus. Other calculus concepts dealt with in mathematical analysis, differential calculus, integral calculus and approximation theory are defined mathematically using the limit of a function. Research has shown that students have difficulties with understanding this fundamental concept (see for instance, Bezuidenhout, 2001; Cornu, 1991; Ferrini-Mundy & Graham, 1994; Juter, 2005a, 2005b, 2006; Monaghan, 1991; Swinyard & Lockwood, 2007; Tall, 1993a; Williams, 1991). The derivative of function concept is also a central concept in mathematics. Besides being fundamental to the calculus concepts identified above, the derivative is also fundamental in such mathematical areas as differential equations, optimisation methods, mathematical modelling and Laplace transforms. Ironically though, research indicates that students have difficulties conceptualising the derivative of function concept (Bezuidenhout, 1998; Habre & Abboud, 2006; Hahkioniemi, 2006a, 2006b; Hauger, 2000; Heid, 1988; Ferrini-Mundy & Graham, 1994; Orton, 1983). The fact that the

mathematical definition of the derivative of function is derived using the limit of a function concept makes it more difficult for students to understand conceptually.

The current chapter provides the background, the context and the rationale for this study. The specific statement of the research problem and the aims of the study are the subject of discussion of Chapter 2.

## 1.6 OPERATIONAL DEFINITIONS OF TERMS

For the purposes of this study, the following definitions will apply. However, definitions for learning styles and distance education are further elaborated in respective parts of this thesis.

**Distance education** – An educational process in which teaching and learning takes place when the student is separated by space or time or both from the teacher or from fellow students. The instruction may be done through the use of some instructional materials/resources that are prepared and designed by the institution responsible for the education.

**Distance learning** – a term that emphasises the learning aspect in a distance education system. In this thesis, the terms *distance education* and *distance learning* may be used interchangeably.

**Distance teaching** – a term that emphasises the teaching aspect in a distance education system.

**Distance student** – A student who is taking a course in a distance education environment. Although in certain contexts the terms distance student, distance learner, student, and learner may have differing emphasis, in this thesis these terms may be used interchangeably.

**Learning style** – Keefe (1985, p. 140) defined learning style as “the composite of characteristic cognitive, affective and physiological behaviours that serve as relatively stable indicators of how learners perceive, interact with, and respond to the learning environment”.

**Students’ experience of learning** – refers to the way the students interact and respond to the learning environment, how they perceive the environment, what they learn and how they learn it.

**Programme** – A collection of courses that lead to a certain qualification.



**Traditional conventional programme** – A programme in which teaching and learning occurs when the students meet and are in face-to-face contact with the teacher in a classroom environment. In the thesis, this term and the term *conventional programme* are used interchangeably.

**Traditional conventional institution** – A single mode learning institution where teaching and learning occurs when the students meet and are in face-to-face contact with the teacher in a classroom environment. In the thesis, this term and the terms *conventional institution*, or *conventional learning environment* are used interchangeably.

**Learning resources** – refers to the educational materials, resources, facilities that are used for the teaching and learning process.

**Learning materials package** – refers to the educational materials that are used for the teaching and learning processes in a course. For the BSMS programme, the materials are printed materials that are produced and packaged in-house.

**Face-to-face tutorial** – A face-to-face tutorial session is an institutionally organised contact session where the students and the tutor meet at a specified venue and at scheduled times for learning purposes, the main purpose being to assist students tackle problematic issues of their studies. In this thesis the terms tutorial and face-to-face tutorial will be used interchangeably.

**Module** – A module is a course of study which is examined. However, colloquially at the ZOU (including staff and students of the BSMS programme), the term *module* refers to the course book which contains the subject matter for the course. This is produced within the institution and comes as part of the learning materials package. In order to be aligned and consistent with participants' responses in the data, the later 'colloquial' meaning of *module* will be adopted through out the thesis unless otherwise stated.

**Tutor** – A person employed by the university either on a part-time or full-time basis whose primary purpose is to ensure that students get the most out of the educational experience, by providing additional academic interpretation and backup of the course material. This usually involves providing face-to-face tutorials. The tutor should be well knowledgeable with the course content.

**Student support** – Student support refers to other services that are meant to improve the students’ learning environment and serve to assist students by complementing the learning materials as well as supporting the students’ learning activities.

**Regional centre** – A centre established by the university for the purpose of managing, administering, and coordinating student learning activities such as student registration processes, coordination of tutorial activities, administration of assignments, handling of examinations, provision of counselling services, and provision of library facilities.

## 1.7 OVERVIEW OF THIS THESIS

This thesis comprises of eight chapters. In Chapter 1, I presented arguments on why it is important to research into the learning of mathematics in a distance education environment. In light of the importance of distance education, it is crucial for distance educators to understand how students respond to the distance learning environment. This motivates for understanding how the distance education environment facilitates or deters student learning. In the chapter, I discussed the importance of understanding student learning processes as a way towards improving the effectiveness of distance teaching. Part of Chapter 1 is also devoted to describing the context of study, which generally is distance education, and specifically, the distance education environment of the B. Sc. Mathematics and Statistics programme which houses the Calculus 1 course at the Zimbabwe Open University.

In Chapter 2, I present the aims of this research study as well as the research questions and significance of the study.

In Chapter 3, I present a review of selected related literature in terms of the main notions considered in this study; learning styles, the teaching and learning of calculus concepts and distance learning of mathematics.

In Chapter 4, I present the theoretical framework for the study, in which I consider viewing student learning through the information-processing model. Learning styles theory and mathematical understanding are blended as ‘components’ of the information-processing model and these constructs are used to ‘zoom’ into the students’ learning processes and outcomes. In this respect, the Felder-Silverman learning styles model and the Hiebert and

Carpenter's framework on mathematical understanding are also described and discussed in this chapter.

In Chapter 5, I describe the research design, methodology, procedures and the research methods employed in this study. I describe case study as the research design. I also provide descriptions of the instruments that were used for data collection. Most of the data were qualitative although some quantitative data were also generated. The data analysis methods used in the study are also described in this chapter.

Chapter 6 presents the findings and discussion of the study pertaining to students' experiences of learning in the BSMS distance education environment. Data for this chapter were analysed through the constant comparative approach.

Chapter 7 presents findings and discussion in the form of illustrative cases pertaining to students' learning styles and their mathematical understanding of the limit of function and derivative of function. Data for this chapter were analysed case by case. In this chapter, I qualitatively profile learning styles for five students in accordance to the Felder-Silverman learning styles model. I also outline mathematical understanding of limit of function and derivative of function concepts for the five students.

Chapter 8 presents my reflections of the research process, implications of the study, as well as the conclusion. In the chapter, I also present suggestions for further research.

## **CHAPTER 2: FOCUS OF THE STUDY**

### **2.1 INTRODUCTION**

The purpose of the previous chapter was to orientate the reader to this study's research problem by presenting the background and comprehensive context of the study. The current chapter serves to outline the research questions, purpose, significance and assumptions of the study.

### **2.2 STATEMENT OF THE PROBLEM**

The research problem for this study is focused on understanding the influence of the distance education environment on student learning processes using a context of a first year undergraduate calculus course at the Zimbabwe Open University. As previously discussed in Chapter 1, one way of understanding the influence of a learning environment on student learning is by obtaining insight of the teaching and learning process from the perspectives of the students. There is therefore a need to understand how the distance students respond to the learning environment and thus obtain perspectives from the students' lived experiences. This enables one to obtain the information as related by the students who are the intended beneficiaries of the educational programme. An understanding of how students respond to the learning environment will provide relevant information on the extent to which the environment supports or deters learning. In the scope of this study, the 'responses' can be enlightened through insight and understanding of the students' experiences of the learning process, the students' learning styles and the learning outcomes. An understanding of how learning style preferences relate to the learning outcomes is also important as it is an integral part of a student's response to a learning environment. In this study, learning outcomes are represented by students' mathematical understanding of certain calculus concepts. Issues about students' understanding of mathematics relate to how students learn, and give further insight into how the students internalise and externalise the mathematical knowledge.

In a broad sense, the question to be addressed in this study is "How does the B.Sc. Mathematics and Statistics distance education environment influence student learning, with

specific reference to the learning of an undergraduate first year calculus course?” The specific research questions that guide this study are as given below:

- (i) How do first year mathematics distance students experience distance learning at the ZOU, with specific reference to the learning of calculus?
- (ii) How can distance students’ experiences of learning calculus be used to inform on the students’ preferred learning styles?
- (iii) What relationships exist between students’ preferred learning styles and learning outcomes as represented by how students understand the limit of function and derivative of function concepts?

## **2.3 PURPOSE OF THE STUDY**

The purpose of this research study is to gain insight into the distance learning of mathematics at the Zimbabwe Open University. The context of this study is the ZOU BSMS programme. The research questions stated in the previous section imply some aims and objectives for this study and these are discussed further.

### **2.3.1 Aims of the study**

The study seeks to understand how a distance learning environment may influence student learning. The study aims to describe and analyse first year calculus distance students’ experiences of the learning process as well as to bring out aspects of the DE environment that might have supported or deterred students in their learning processes. The study also aims to identify students’ preferred learning styles as profiled from the students’ descriptions of their learning experiences and to further relate these learning styles to the learning outcomes. The study will outline learning outcomes as represented by the students’ mathematical understandings of the limit of function and derivative of function concepts. The mathematical understanding will be documented from the students’ behaviour and performance on carefully selected mathematical tasks. Furthermore, the study will explore and bring to light possible relationships that can exist between students’ learning styles and mathematical understanding.

### **2.3.2 Objectives of the study**

The specific objectives of the study are:

1. To generate insights and understandings of how first year mathematics students at the ZOU engage in learning activities in the distance education environment. This includes gaining insight on how they learn, what they learn, when they learn, what they prefer in terms of learning and why.
2. To characterise ZOU mathematics students' learning styles in some detail.
3. To characterise students' mathematical understanding of some calculus concepts; specific to this study are limit of function and derivative of function concepts.
4. To establish any possible patterns and relationships existing between the students' learning styles and their mathematical understanding.

## **2.4 SIGNIFICANCE OF THE STUDY**

The research questions are significant as they are related to the following:

1. The study is pertinent to ZOU administrators and educators as findings may impact on the development of teaching material, effective implementation of current and future DE programmes, improvement of the existing programme as well as informing faculty on support services for distance learners.
2. The study may give input to mathematics distance educators in their endeavour to diversify instructional resources and strategies. The study may directly have a significant impact on the B.Sc. Mathematics and Statistics programme at the Zimbabwe Open University as it may inform and illuminate aspects of the programme design that support or that deter learning. The study may also inform material developers, course designers and course tutors on what kind of instructional techniques would be effective and suitable for the ZOU BSMS students as they study calculus. Furthermore, the study may inform faculty on the role of learning styles in a distance education calculus course.

3. The study may also contribute to a body of knowledge on mathematics learning in distance education environments, thus in a way make the findings and output of this study useful for programmes in similar educational contexts.

## **2.5 DELIMITATIONS AND ASSUMPTIONS OF THE STUDY**

### **2.5.1 *Delimitations of the study***

The following delimitations were taken into consideration in order to make the research project manageable:

- The study will not analyse the learning resources (e.g. the module, the tutorials, the textbooks) in their individual components. The learning resources are looked at as a package and not as individual components since distance students have access to other various sources of content. Of interest to the study are the students' experiences, learning styles and mathematical understanding of certain calculus concepts, which emanate from interacting with these resources.
- The study will focus on the mathematical understanding of only two concepts from the entire calculus course. That is, focus is placed on understanding the limit of function and derivative of function concepts.
- Although tutors handle the face-to-face tutorials and may employ different teaching strategies, the tutors' teaching strategies are not analysed on an individual basis.

### **2.5.2 *Assumptions of the study***

The following assumptions were made during the study:

- Participants respond in as truthful a manner as possible.
- Other potent influences on students' choices in a learning situation are reasonably stable.

- The institutional position that I held within the ZOU would not influence participants' responses to the research instruments in any way.

## 2.6 CHAPTER OVERVIEW

This chapter served to provide the focus of the study. The study aims to gain insight of the influence of the distance learning environment on students' learning of a first year calculus course. The chapter outlined the research questions, the purpose, significance, delimitations and assumptions of the study. Chapter 3 will provide a review of selected related literature.

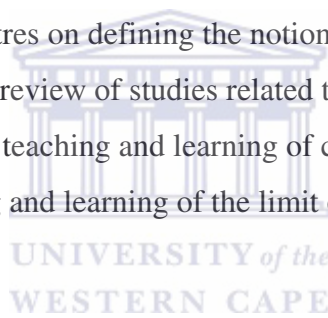




## **CHAPTER 3: REVIEW OF SELECTED LITERATURE**

### **3.1 INTRODUCTION**

In Chapters 1 and 2, the motivation, context, purpose, aims and research questions of this study have been detailed. The aim of this current chapter is to present a review of selected literature that is relevant to this study. The literature is reviewed in relation to the three main aspects of this study, the learning styles, the teaching and learning of certain calculus concepts and the distance education which provides the context for this study. Selected key issues of each of these aspects are discussed as they relate to this present study. Literature on distance education is discussed in relation to studies on learning mathematics at a distance. Literature on learning styles centres on defining the notion of learning styles, instruments for identifying learning styles and a review of studies related to the role of learning styles in instruction. The literature on the teaching and learning of calculus concepts mainly focuses on studies related to the teaching and learning of the limit of function and the derivative of function concepts.



### **3.2 STUDIES ON LEARNING MATHEMATICS AT A DISTANCE**

This section presents a review of selected literature that relates to distance education of which specific focus of the literature pertains to the learning of mathematics in distance education environments. The intention of the section is to illuminate on some of the research studies that have been conducted in the teaching and learning of mathematics in distance education.

Hubbard (1990) describes an experience of a mathematics course. Though Hubbard's (1990) study was conducted in a conventional institution, the mathematics course described in the study had a special feature similar to that offered in a distance education environment. The similarity lay in that there were no lectures for the students except for one hour a week of tutorials and a weekly thirty minute test. The students thus depended on instructional materials. Hubbard reports that the approach was successful and that positive comments and

reactions from the students were noted, as this approach enabled students to develop good study habits.

A study by Lawless (2000) focused on workload, study time and learning approaches for students in a mathematics distance education course. The study established that heavy workloads negatively affected the students' learning. Other factors such as student attitude, motivation, and nature of course materials and of learning activities also influenced the time spent studying. Taylor and Mohr (2001) described the development, and design of a distance education package for mathematics DE instruction intended to address mathematics anxiety in educationally disadvantaged students in a mathematics course. The developed instruction was student-centred and included components such as topics presented in-context, use of informal mathematical language, grouping of content and activities, appropriate tutor responses and reflective diaries. Taylor and Mohr report that the instruction helped students overcome feelings of mathematics anxiety and this in turn helped build mathematics confidence in the students.

Considering the current trends of technological advancement, the usage of the technological media in the teaching and delivery of distance education programmes has also found its way in mathematics courses. Arnold, Shiu and Ellerton (1996) alluded to that distance education materials such as printed learning materials, computer software packages and student support form essential components of the mathematics instructional package. Although printed materials still remain the medium of instruction for some distance education programmes, for some programmes, instruction has since moved from printed material being the main medium of instruction to technologically-enriched instruction. Arnold *et al.* (1996) also noted that text materials have challenges in catering for interactive and active learning while technology on the other hand, can have a potential to fill such gaps. Harman and Dorman (1998) observed that any technologies used in the distance teaching of mathematics must attend to motivation, enthusiasm for the subject, interactivity as well as satisfying the needs for visual, algebraic and geometric representations. Similar views of the versatility of the computer regarding representations and motivation for mathematics in DE environments are shared by Krussel (2002), and Abdul Karim and Ufuktepe (2002).

Use of technology in the delivery of distance education mathematics courses has been reported to enhance the learning of mathematics (Harman & Dorman, 1998; Abdul Karim & Ufuktepe, 2002). Harman and Dorman discussed learning mathematics in a

videoconferencing and audiographics-enhanced environment. They reported that the media combination employed had enhanced learning. Their study showed that students benefited from the interactivity as well as from the immediate availability of the voice, vision and face-to-face contacts in the teaching and learning sessions. Abdul Karim and Ufuktepe (2002) noted that a web based software *WebMathematica* encouraged interactivity in the learning of mathematics. However, despite the noted success in the above-mentioned studies, some limitations were noted and included those regarding travelling expenses and time taken to travel to the centres (Harman and Dorman, 1998), connectivity to the internet server (Abdul Karim & Ufuktepe, 2002) and wastage of time if there is no proper planning (Abdul Karim & Ufuktepe, 2002).

### **3.3 LEARNING STYLES**

Learning styles theory emphasizes that different individuals receive, perceive and process information in different ways. The theory implies that how much and what individuals learn has to do with whether the educational experience is geared toward their particular style of learning. In Chapter 1, the definition of learning styles was discussed briefly. In this section, more detail is provided on the notion of learning styles with an emphasis on the definition and review of some literature. Chapter 4 will provide a discussion of learning styles with a focus on the particular learning styles model that will be used in this study.

#### **3.3.1 *Defining learning styles***

The term learning styles does not have a single prescriptive definition. In fact there are various definitions of learning styles that can be found in literature. For instance, Dunn and Dunn (1993, p. 2) refer to learning style as “the way in which each learner begins to concentrate on, process and retain new and difficult information”. Felder (1996, p. 18) defines students’ learning styles as “characteristic strengths and preferences in the ways they take in information”. Vermunt (1996, p. 2) defines learning style as “a coherent whole of learning activities that students usually employ, their learning orientation and their mental model of learning”. However, an earlier definition formulated by Keefe and a taskforce of the National Association of Secondary School Principals (NASSP) (Keefe, 1985; Keefe & Ferrell, 1990) appeared to be more comprehensive and more appropriate for a discussion that is situated in a distance education context. The definition recognises three domains of style -

the cognitive, the affective and the physiological domain. Keefe (1985, p. 138) justified and explained these domains as follows:

- cognitive elements are internal to information processing habits;
- affective elements are motivational processes and are preferential in nature. They are viewed as the typical modes arousing behaviour and respond to matching strategies;
- physiological elements are rooted in learner reactions to the environment and are responsive to instructional matching.

Keefe (1985) therefore defines learning style as

*The composite of characteristic cognitive, affective and physiological behaviours that serve as relatively stable indicators of how learners perceive, interact with, and respond to the learning environment (p. 140).*

From this definition, it is evident that learning styles are an individual's characteristics, consistent approaches and preferences to receiving, perceiving and processing information in a learning situation. The main points to draw from the definition are that learning styles reflect an individual's preferences and choices in a learning situation.

Although some researchers (Liu & Ginther, 1999; Pillay, 1998) use the terms learning style and cognitive style interchangeably, there is, however, an existing distinction between these two notions. The definition by Keefe provides a clearer distinction between the two with learning styles denoting a broader term that includes cognitive styles as well as affective and physiological styles (Keefe & Ferrell, 1990). For the purposes of this study, the notion of learning styles will be adopted as it is broader than the cognitive styles.

### **3.3.2 Identifying learning styles**

The aspect of the instruments that are used to identify students' learning styles is essential and cannot be ignored when we discuss learning styles in distance education. While on one hand, from an educator's point of view, instructional decisions can be made on the basis of these learning styles profiles, on the other hand, from a student's point of view, learning decisions can be taken on the basis of these learning style profiles. This implies that it is important to profile the learning styles using instruments that yield accurate information. As such, the appropriate questions to address right now are how can someone profile students' learning styles properly and what kind of LS identifying instruments can be used?

## Learning Styles Inventories/Indexes

A common type of instrument available for identifying learning styles is the inventory or index of learning styles. Examples of such instruments include the Honey and Mumford Learning Styles Questionnaire, Myers-Briggs Type Indicator (MBTI), the Kolb's Learning Styles Inventory, Grasha-Reichmann Student Learning Styles Scales (GRSLSS), the Felder-Solomon Index of Learning Styles, and Vermunt's Inventory of Learning Styles. The learning styles inventories are presented in the form of self-report surveys. Construction and development of the items of the learning styles inventories is based on some theoretical model of learning styles. For instance, the Felder-Solomon Index of Learning Styles is based on the Felder-Silverman learning styles model (Felder & Silverman, 1988), the Kolb's Learning Styles Inventory is based on the Kolb's learning style cycle (Kolb, 1984), and Vermunt's Inventory of learning styles is based on Vermunt's learning styles model (Vermunt, 1996). As already mentioned, a more elaborate discussion of learning styles models will be provided in Chapter 4, where specifically, focus is made on the model used in this study.

Many learning styles inventories have been widely used in educational research to assess individual students' learning styles. In the case of most self-report scales the students rate themselves according to the provided scales. As much as learning style inventories serve as invaluable sources of information on student learning, reservations have been raised in relation to the use of these instruments in educational contexts. For instance, in the case of the Felder-Solomon ILS, a major criticism resulted from the study by van Zwanenberg, Wilkinson and Anderson (2000), who concluded that the Felder-Soloman ILS lacked validity. van Zwanenberg *et al.* (2000) pointed out that there was little evidence available to support the use of the ILS as a measure of learning styles especially with the intentions of predicting performance. Bacon (2004) also pointed out that the items of the Felder-Solomon ILS had poor reliability.

In a literature review on learning styles that was conducted by Coffield, Moseley, Hall and Ecclestone (2004), the authors observed that many of the instruments used to measure learning styles fail to meet the basic criteria as measures of learning styles, and that the instruments only capture learner impressions rather than learner behaviour. Coffield *et al.* (2004, p.40) further recommend against making pedagogical interventions solely on the basis of a learner classification that is grounded on learning style instruments. Gregorc

(1979) alludes to that learning style inventories might be exclusive as the items focus on certain variables at the expense or exclusion of other possibilities and that might provide limited or guided responses. Furthermore, as noted by Gregorc, (1979) and Coffield *et al.* (2004), there are some students who have an innate tendency to lie on any self-reporting instrument and this may therefore distort the true reflection of the student's profile. Care therefore needs to be taken on making decisive instructional materials designs on the basis of the learning style inventories/indexes as the instrument might not provide sufficient information.

### **Other approaches of profiling of learning styles of calculus students**

In order to gain more meaningful and accurate information on learning style profiles, further phenomenological interrogations are required. For instance, information can be solicited from students' writings, learning journal entries or verbal accounts of their learning experiences. Gregorc (1979) recommends the use of interviews and conversations in diagnosing students' learning styles. Similar recommendations were made by Lewis (n.d) who also recommended use of observational methods and interviews to establish learning styles.

In an environment like distance education where it is difficult to observe individual students in learning situations, reflective learning journals can be used. Langer (2002) advocates the use of structured learning journals, as they allow the investigator an opportunity to receive information in a specific format. At the same time, the journal approach allows one to compare students' responses and reflections, and to obtain feedback on specific discussions and learning activities. Analysing and interrogating students' written descriptions of the activities they engage in, such as overcoming learning difficulties, can illuminate some learning style preferences. Although such analysis can be a long and tedious approach of profiling student learning styles, the approach may give a better manifestation of the actual learning styles as opposed to the inventories/indexes.

### **3.3.3 Learning styles and instruction**

One of the implications of identifying students learning styles is that the information may be useful for instructional purposes. Many researchers agree that learning styles play a crucial role in education, be it in a conventional education setup or distance education environment.

However, what may differ is how the information may be used, whether or not to match, mismatch or to balance instruction with learning styles.

### **Matching or mismatching**

A major point of debate arises on how learning styles information may be applied to instruction. It is apparent that the topic of matching or mismatching instruction to learning styles is controversial and has attracted debate in the education arena, with some supporting matching, others supporting mismatching and yet some settling for a balance of both.

Coffield *et al.* (2004, p. 121) cited a study conducted by Smith, Sekar and Townsend (2002) who concluded that for every study supporting matching, there is one study rejecting it. They came up with this conclusion after finding nine studies supporting matching and nine others supporting mismatching in a review of empirical studies on learning styles.

According to Pillay (1998), matching instruction to preferred ways of perceiving and processing information frees up ‘cognitive resources’ that can be directed to learning.

Claxton and Murrell (1987) suggest that matching of students’ learning styles is appropriate in working with new college students as well as with poorly prepared students. Such an approach allows students to access information in a format that matches their learning style, consequently eliminating the need by the student to engage in searches to ‘reorganise’ given information. Kramer-Koehler, Tooney and Beke (1995) acknowledge that adjusting instruction so as to offer students a variety of learning environments that match their learning styles may be beneficial to students’ learning by providing them with an opportunity to better understand what best fits their own needs.

Studies have also linked the matching of instruction with learning styles to enhanced learning. Bacon (2004) found that when instruction matched students’ learning styles, students’ learning was enhanced. Dunn and Dunn (1993) provided examples of empirical studies that showed that students’ learning was enhanced when instruction matched their learning styles. Larkin and Budny (2003) contended that the adoption of a learning style approach increased student interest and motivation to learn. Ford and Chen (2001) conducted a study comparing performance of students with matched instruction and that of students with mismatched instruction and concluded that the group where the instruction matched learning styles performed better.

As we discuss matching instruction to learning styles, it is essential that we also look at the other side of the coin, and consider what Coffield *et al.* (2004, p. 122) refers to as “deliberate mismatching” of instruction and learning styles. Authors such as Claxton and Murrell (1987), Friedman and Alley (1984), and Kolb (1984) argue that there is some benefit in learning as a result of deliberate mismatching. Kolb (1984) says mismatching enables the student to gain experience in resolving tensions and conflicts in learning situations. However, Felder (1993) warns against extreme mismatching and stresses that this may result in learning problems and may discourage students from continuing with their study programmes.

It has also been argued that strictly adhering to any of the extremes of either matching or mismatching instruction to learning styles may not be beneficial to students. Felder and others (Felder, 1993; Felder & Brent, 2005; Felder & Silverman, 1988) advocate for a balance so that learners experience both matches and mismatches of instruction to their learning styles. In an instructional sequence that is balanced in terms of learning styles, instruction is designed in such a way that learners sometimes encounter tasks in their preferred learning styles and sometimes in their less preferred styles to which they have to adapt and adjust their styles.

In view of balanced instruction, although it is important for a teacher to know the distribution of the learners’ learning styles, the point as noted by Felder (1993) and Felder and Brent (2005) is not to place each and every learner into one or another of the learning style categories and then exclusively teach them to that categorisation, but rather to address all sides of the categories. In any case, it may be impossible to instantly adjust and match or mismatch instruction to each individual student’s learning styles needs. More so, if it is in a distance education environment where the materials are pre-prepared and where learning takes place when the students are separated from their teachers and from each other.

### **Studies on learning styles in mathematics**

In a statement that can be taken to link quality of instruction and learning styles, Felder (1993, p. 287) stated “all points raised by Tobias about the poor quality of introductory college science instruction can be expressed directly as failures to address certain common learning styles”. A similar view may be inferred to the discipline of mathematics where students are known to encounter learning problems. As already discussed, it has been widely



recommended that incorporating learning styles in teaching processes can help enhance learning. Thomson and Mascazine (1997) argued for incorporation of learning styles in mathematics. Learning styles have been incorporated in the teaching of mathematics by Knisley (n.d) and Lewis (n.d). Knisley incorporated learning styles in the teaching of university mathematics courses whereas Lewis incorporated learning styles in a computer based primary level mathematics course. Knisley (n.d) proposed a four stage model of learning mathematics that incorporated learning styles and was grounded in the four stages of the Kolb learning styles model. Knisley (n.d) concluded that the model improved instruction methods and that it also enhanced student learning. The situation for Lewis (n.d) was somewhat different as the study showed that matching instructional methods to learning styles did not produce any differences in performance amongst the students in the control and experimental groups.

Studies have also revealed linkages that can possibly exist between learning styles and other factors encountered in learning mathematics. Steyn and Maree (2002) linked learning styles and study orientation. Knisley (n.d) also made linkages between learning styles and content. He noted that learning styles may be a function of the content as well as being a function of understanding. Lawless (2000) linked workloads and study time to learning styles and approaches in distance mathematics students. Lawless found that almost half of the group of respondents who indicated that they wanted to 'learn the course', put more time on study and tended to adopt learning approaches that enhanced conceptual understanding. The other group, who indicated that they just wanted to 'pass the course', put less time and tended to adopt approaches that promote surface learning. Sloan, Daane and Giesen (2002) linked learning styles and mathematics anxiety in elementary pre-service teachers. Sloan *et al.*(2002) established a relationship between mathematics anxiety and the global learner who approach learning in intuitive ways. They attributed anxiety to teaching sequences which normally do not address the needs of the intuitive and global learners.

### **3.4 STUDIES ON TEACHING AND LEARNING OF CERTAIN CALCULUS CONCEPTS**

Extensive research has been carried out on the teaching and learning of calculus, (see for instance, Asiala, Cottrill & Dubinsky, 1997; Aspinwall & Miller, 2001; Bezuidenhout, 2001, 1998; Cornu, 1991; Eisenberg, 1991; Ferrini-Mundy & Graham, 1994; Hahkioniemi, 2004, 2006a, 2006b; Juter, 2005a, 2005b, 2006; Leinhardt, Zaslavsky & Stein, 1990; Moru, 2006;

Orton, 1983; Robert & Speer, 2001; Swinyard & Lockwood, 2007; Tall, 1993a; Williams, 1991, 2001; White & Mitchelmore, 1996). A majority of the studies mentioned above show efforts made by mathematics educators to understand and aid students' learning of the specific calculus concepts of limit of function and derivative of function. Also a majority of the studies were discussed in the context of conventional educational settings. The limited availability of literature related to the teaching and learning of the limit of function and derivative of function concepts in distance education environments was notable during the literature search.

This section presents the literature review of the limit of function and derivative of function concepts, from both the learning and teaching perspectives only. On a lesser extent, selected literature related to the function concept is presented since functions are central to both the limit and derivative concepts. However, for purposes of this current study, only the literature on functions which relates to how students' understanding of the function concept influences the teaching and learning of limits and derivatives is presented.

The literature on learning the limit of function mainly focuses on students' difficulties of learning the concept, their conceptions and/or misconceptions, beliefs and attitudes towards the limit of function concept. Literature on the learning of the derivative of function mainly focuses on students' difficulties of learning the concept. For both limit of function and derivative of function concepts, a literature review on instructional strategies is presented and focuses on enhancing students' conceptual understanding.

### **3.4.1      *Limit of function***

Much of the literature on limits presents a state of agreement amongst researchers who share the opinion that students have difficulties learning the limit of function. Several researchers, such as Bezuidenhout (2001), Cornu (1991), Cottrill *et al.*, (1996), Fernandez (2004), Juter (2005a, 2005b, 2006), Monaghan (1991), Morash (1990), Moru (2006), Swinyard and Lockwood (2007), Szydlik (2000), Tall (1993a), Tall and Vinner (1981), and Williams (1991, 2001) have investigated various aspects of the teaching and learning of the limit of function concept. Researchers (e.g. Cornu, 1991; Ferrini-Mundy & Graham, 1994; Juter 2005a; Morash, 1990) acknowledge that students have no problems with handling routine limit problems but have problems with non-routine problems. Most of the studies on limit of function that are available provide descriptions of students' difficulties in understanding the

concept, with some studies such as Cottrill *et al.* (1996), Fernandez (2004), Morash (1990) and Swinyard and Lockwood (2007) also presenting some perspectives of teaching the limit of function concept with an aim to enhance students' understanding.

## Aspects of student learning

### *The formal definition and symbolism*

Definitions are generally meant to readily provide meanings and clarity of issues. However, this is not the case with the limit of function definition which researchers have considered unrevealing. Researchers have linked students' learning difficulties of the limit of function concept with its formal definition. Vinner (1991) acknowledged that in the learning of mathematics, the mathematical definitions are a major cause of conflicts between the structure of mathematics and the process of concept acquisition. Cornu (1991) specifically attributes difficulties in the teaching and learning of the limit concept to the formal mathematical definition of the concept.

The concept of the limit of a function passed through many developments including constructions by mathematicians such as Wallis (1616-1703), Bolzano (1781-1848), Cauchy (1789-1857) and Weierstrass (1815-1897)<sup>3</sup>. Weierstrass also provided a symbolic representation of the definition which is normally presented as

$$\lim_{x \rightarrow a} f(x) = L \Leftrightarrow \forall_{\varepsilon > 0} \exists_{\delta > 0} \text{ such that for all } x \in D_f, \text{ with } 0 < |x - a| < \delta \Rightarrow |f(x) - L| < \varepsilon \text{ where}$$

$D_f$  is the domain of  $f$ . This definition, commonly referred to as the  $\varepsilon$ - $\delta$  definition of the limit of function, may be read verbally as 'A function  $f(x)$  has limit  $L$  as  $x$  approaches  $a$  if and only if for every  $\varepsilon > 0$ , there exists a  $\delta > 0$ , such that for any  $x$  in the domain  $D_f$ ,  $0 < |x - a| < \delta$  implies that  $|f(x) - L| < \varepsilon$ '. Generally the  $\varepsilon$ - $\delta$  definition is 'rich' in notation, symbols and the use of other everyday language words and phrases for mathematical meanings.

However, research suggest that the 'richness' of the definition may not be advantageous to students but rather may be a major source of cognitive conflicts for students (Cornu, 1991; Juter, 2006; Morash, 1990; Swinyard & Lockwood, 2007; Tall 1993a). Researchers have noted that it is possible for students to do mathematics that involves limits and to perform tasks that involve finding limits, although in most instances, students never actually grasp the meaning and relevance of the formal definition. Cornu (1991) noted that the cognitive

---

<sup>3</sup> The formal definition of limit of function is also referred to as Weierstrass' definition of limit of function in some texts.

aspects of the mathematical content of the concept cannot be generated purely from the  $\epsilon$ - $\delta$  definition. Hence, for most students, remembering the definition is one thing and acquiring the concept is another. Swinyard and Lockwood (2007) reiterate that the formal definition does not provide a mechanism for finding the limit  $L$  but rather it provides a mechanism for verifying that a given number  $L$  is indeed the required limit. However, Swinyard and Lockwood (2007) emphasise on the distinction between the process of finding limits which depends on procedures and techniques, and the process of verifying limits which is dependent on the formal definition. Given such a distinction, Swinyard and Lockwood (2007) attribute students' difficulties in understanding the purpose of the formal definition to the conceptual gap that is created when the students fail to recognise the existing differences between the two processes of finding and verifying limits.

Morash (1990) pointed out that the  $\epsilon$ - $\delta$  definition gives little clue as to what the limit concept is and he attributed the difficulties associated with learning this concept to the “complex logical structure” of the  $\epsilon$ - $\delta$  definition (p. 183). Observations on the limitations of the formal definition that are due to the  $\epsilon$  and  $\delta$  symbols were also indicated in other studies (Fernandez, 2004; Juter, 2005a) where students queried the relevance of the  $\epsilon$  and the  $\delta$  in the limits definition. In other instances, students also failed to comprehend the relationships between the individual components and symbols in the definition such as the symbols  $\epsilon$ ,  $\delta$ ,  $f(x)$ ,  $L$  and  $a$  (Fernandez, 2004; Juter, 2005a; Swinyard & Lockwood, 2007). The study by Fernandez further specified students' problems with interpreting the inequalities involved in the definition and explicating why the inequalities regarding  $x$  and  $f(x)$  were asymmetrical.

The formal definition involves mathematical quantifiers such as the universal and existential quantifiers, as well as the implication predicate. Studies have attributed students' difficulties with comprehending the formal definition for limit of function to these mathematically quantified components of the definition (Cornu, 1991; Cottrill *et al.*, 1996; Juter, 2005a; Swinyard & Lockwood, 2007; Tall, 1993a). Cottrill *et al.* (1996) allude to the fact that quantifications in the formal definition make the limit concept inaccessible to students. Studies by Juter (2005) and Swinyard and Lockwood (2007) reveal that students fail to appreciate the role of the quantifiers in the definition. The students experienced problems with determining the appropriate quantifiers for the symbols  $\epsilon$  and  $\delta$  and failed to recognise the difference between the quantification structures such as ‘ $\forall \exists$  i.e. for every/there exists’ and ‘ $\exists \forall$  i.e. there exists/for every’ (Juter, 2005a; Swinyard & Lockwood, 2007).

Whilst it is evident from research that the  $\epsilon$ - $\delta$  definition is not very practical when it comes to techniques for obtaining the limit values  $L$ , some studies do acknowledge the role of the formal definition in the limit of function notion. For instance, Fernandez (2004, p. 43) views the formal definition as addressing the questions “why limits exist and why they are what they are”, which may allow for depth in understanding the concept. Swinyard and Lockwood (2007) posit that the formal definition provides useful information on how one might validate candidates for the limit value  $L$ , a process which is essential in the study of existence and uniqueness of limits of functions.

### *The language used in limits*

The limit of function is one example of a mathematical concept that uses everyday language phrases to refer to mathematical processes. However, for some students, the words and phrases may hold connotations that are very different to the intended mathematical purposes. For instance, students may have had experiences in everyday life where the word limit is involved as in such cases as speed limit, boundary, border, least value or highest point. Such everyday language connotations may therefore interfere with students’ understanding of the mathematical notion of limits.

Several studies (Cornu, 1991; Ferrini-Mundy & Graham, 1994; Juter, 2005a; Kim, Sfard & Ferrini-Mundy, 2005; Monaghan, 1991; Moru, 2006; Tall, 1993a; Williams, 1991) acknowledge that the language used in limits is a source of difficulty in students learning of the limit concept. Kim *et al.* (2005) report on the influence of everyday language on limits and infinity with a specific focus on the words ‘infinite’ and ‘infinity’. They conclude that colloquial language has a potential to impact on students’ usage of the terms in mathematical contexts.

The  $\epsilon$ - $\delta$  definition features everyday language words such as ‘for every’, ‘there exists’, ‘if and only if’, and ‘implies’ which in the language of mathematics have special meanings. However, these mathematical meanings are quite different from the everyday language meanings. Such situations, as noted by Cornu (1991) give rise to conceptual obstacles which may cause serious learning difficulties. When dealing with the limit of function concept, there are also instances when different phrases are considered to be synonymous irrespective of the nuances in the everyday language. Phrases such as ‘tends to’, ‘nears to’, ‘approaches’, ‘reaches’, ‘converges’, ‘gets closer to’ and ‘limit’ are commonly used when dealing with the

limit of function concept. Though these phrases have different meanings in everyday language, in calculus they hold the same mathematical meaning. However, some students hardly recognise the mathematical equivalence in these phrases and they stick to the colloquial meanings of these phrases. Such interferences of the everyday language meanings with the mathematical meanings of the phrases lead to students experiencing confusion and learning conflicts.

The study by Ferrini-Mundy and Graham (1994) noted that students' understanding of the concept "seemed deeply intertwined" with the everyday language, which created problems for the students. Monaghan (1991) focused on the four phrases 'tends to', 'approaches', 'converges' and 'limit', and he concluded that the language used in limits was a source of difficulties for students' learning of the concept. Though to a mathematician, these four phrases can be used interchangeably, the phrases held different connotations for the students in Monaghan's (1991) study. The students considered the term 'limit' to be more precise and specific than the other phrases and the term was more associated with bounds such as in speed limit. The other three phrases were considered to be vague with 'tends to' and 'approaches' often being seen as similar, and since both phrases are based on action words the phrases were given a dynamic interpretation of "getting to a limit" (Monaghan, 1991, p. 23). The students were unsure of what 'converges' meant in terms of limits and they associated the phrase with lines merging. Williams (2001) shows that the interferences in language meanings resulted in some students perceiving the limit of a function as a process of approaching a limit and therefore a never ending process. In such cases the focus is shifted from the limit as a number to limit as a process. This led to the incorrect viewpoint that was also noted by other researchers (Cornu, 1991; Juter, 2006; Moru, 2006; Szydlik, 2000; Tall, 1993a) that the limit of a function is unreachable. Such misconceptions arise from the students focusing on such phrases as 'getting closer to', 'reaching', and 'approaching'.

### *Students' conceptions of the limit of function*

Research studies have focused on different conceptions that students may hold either during or after the formal teaching of the limit of function concept and how this relates to students' understanding of the concept (Bezuidenhout, 2001; Ferrini-Mundy & Graham, 1994; Juter, 2005a, 2006; Williams, 1991, 2001). These studies have shown that some conceptions that students hold can at times interfere with the students' conceptual understanding of the

concept, if the conceptions are inadequate or inappropriate. Cornu (1991, p. 154) acknowledged that before any formal teaching of a concept occurs, students already have a number of ideas about the topic, which he referred to as “spontaneous conceptions”. In the case of limit of function, spontaneous conceptions may for instance relate to the everyday language phrases that are used in the teaching of the concept. As already mentioned in the previous subsection, the everyday language meanings may interfere with the ‘new’ mathematical meanings.

Bezuidenhout (2001) found that some students associate finding limits with the plug-in-and-substitute approach. However, such an approach leads to the misconception that in order to determine the limit of a function, the function must always be given algebraically. Similar findings were made by Ferrini-Mundy and Graham (1994, p. 38) where one student actually stated “maybe if I knew what the function  $f(x)$  was like ... like a polynomial ... and you plugged in the values”. Explicit and algebraically expressed functions that are frequently involved in limit of function problems provide space for students to believe that the limit of a function at a point can always be obtained by substitution and that  $\lim_{x \rightarrow a} f(x)$  is the same as  $f(a)$ .

Despite the fact that students approach limit problems from an algebraic perspective, studies have revealed other challenges that are inherent with this algebraic conception of limit. Tall (1993a) identifies algebraic manipulations to be one source of difficulties in finding limits. Tall (1993a) noted that although most students prefer algebraic manipulations, the misconceptions and errors can still be transferred to other mathematical tasks that require the knowledge, if their knowledge on algebraic manipulations is inadequate. For instance, little knowledge on algebraic manipulations of polynomials, trigonometric formulae, logarithmic or exponential function becomes a limitation when that knowledge is required to simplify an expression for purposes of finding limits. Juter (2005a) found that algebraic manipulation errors affected students’ processes of finding limits to given functions.

The ‘dynamic view of limits’ is another conception that researchers (Bezuidenhout, 2001; Cornu, 1991; Williams, 1991) found to be prevalent with students and could hinder students’ progression to a formal understanding of the concept. The ‘dynamic view of limits’ refers to the informal viewpoint that the function values approach the number  $L$  more and more closely as  $x$  moves towards a point  $a$ . In other words, it is the viewpoint that the limit of function can be determined by evaluating the function at a series of points that are

successively closer to the point of interest (Williams, 1991, p. 235). Again, such kind of a conception requires the existence of the function in its algebraic form which is not always the case. Furthermore, the substitutions may not always give the correct idea about limits (Bezuidenhout, 2001; Williams, 1991). Students having such a viewpoint of limit of function are prone to experiencing difficulties understanding the concept. Bezuidenhout (2001) pointed out that those students who hold such a misconception have difficulties appreciating the relevance of the  $\epsilon$ - $\delta$  definition. Spontaneous conceptions associated with the dynamic viewpoint of limit can also be difficult for students to get rid of even after formal instruction. For instance, as found by Williams (1991), the dynamic conception was resistant to change even after the students were taken through a series of intervention sessions given as treatments aimed to improve their understanding of the concept.

#### *Affective variables in relation to the learning of limits*

Research has also focused on affective variables as encountered in contexts of mathematics learning (Breiteig, Grevholm & Kislenko, 2005; Duffin & Simpson, 2000b; Malmivuori, 2001; McLeod, 1992). Affective variables take into consideration all aspects of emotions and feelings that may influence the students' learning of mathematical concepts.

Classifications of affective variables in mathematics include aspects such as attitudes, beliefs, emotions, motivations and moods. McLeod (1992) further provides a classification that distinguishes between beliefs, attitudes and emotions, and classifies in terms of stability of affective responses within a context of mathematics education. Beliefs and attitudes are considered to be relatively stable than the other variables that are easily changeable. Because of this stability, beliefs and attitudes often become attractive notions for research in mathematics learning.

Studies have given specific attention to students' beliefs and attitudes in the learning of mathematics in general and the limit of function concept in particular (Szydlik, 2000; Juter, 2005b; Mohammad Yusof & Tall, 1994, 1999). Szydlik (2000) focused on the students' beliefs about mathematics and the role that these beliefs play in the students' conceptual understanding of the limit of function. The study by Szydlik (2000) revealed that students' beliefs about how mathematical truth and validity are established can affect their conceptual understanding of the limit of function. In that study, those students whose sources of conviction were internal and relied more on intuition, logic, consistency, or empirical evidence for mathematical truths showed coherent and appropriate conceptions of the limit



of function. Comparatively, the other students whose sources of conviction were external and relied more on the authority of instructors or textbooks for mathematical truths, showed incoherent and inappropriate conceptions of the limit of function. In a study by Juter (2005b), the belief that learning mathematics was about solving problems was prevalent among the students. Such a belief would lead students to put more effort on problems and solution methods rather than on theory and understanding the concepts (Juter, 2005b).

Williams (1991) found that students' attitudes toward mathematical truth interfered with their understanding of limits. Students valued simplicity and practicality in terms of problems likely to be encountered in the learning process much more than the mathematical formality. As such, students would rely on the view of limits that was simple and most expedient at the expense of the conceptual entities. Juter (2005b), Mohammad Yusof and Tall (1994, 1999) specifically investigated the interactions between students' attitudes to mathematics and the learning of mathematics. The study by Mohammad Yusof and Tall (1994, 1999) was conducted in a general university mathematics course that encouraged problem solving and reflection whilst Juter's (2005b) study specifically focused on the limit of function concept. Juter's (2005b) study revealed that students with positive (confident) attitudes towards mathematics performed better in solving limit of function problems. However, it was not possible to specify which one was dependent on the other from the study. On one end it could be possible that students with positive attitudes can successfully solve limit of function problems. On the other end, it could be possible that students who are capable of solving limit problems have positive attitudes towards mathematics (Juter, 2005b). In Mohammad Yusof and Tall (1994, 1999), the problem-solving enriched mathematics course resulted in attitudinal changes from negative to positive attitudes. However, the changes were found to be temporal and short-lived as the attitudes tended to change back after some time. Considering the difficulties experienced with learning limits of function, Juter (2005b) recommends that students ought to have strong positive experiences with the concept in order for them to change their attitudes and maintain the change.

### **Some perspectives for teaching the limit concept**

Although the majority of studies available on limit of function focus on students' difficulties with the concept, there are also a few available studies that provide teaching perspectives aimed at addressing the difficulties experienced with learning limits, (Bezuidenhout, 2001; Cottril *et al.*, 1996; Fernandez, 2004; Morash, 1990; Swinyard & Lockwood, 2007).

Morash (1990) specifically focused on the formal definition and suggested a different way of looking at the  $\epsilon$ - $\delta$  definition that could be employed as a way of assisting students understand the meaning and relevance of the  $\epsilon$ - $\delta$  definition. According to Morash (1990, p.183), the  $\epsilon$  and  $\delta$  have something more intelligible to say about the number  $L$  that fails to satisfy the definition than about a number that satisfies it. Under such a proposition, Morash (1990, p. 184) thus recommends working with the logical negation of the  $\epsilon$ - $\delta$  definition, and hence focus on examining the “graphic reality behind the failure of a number  $L$  to satisfy the  $\epsilon$ - $\delta$  definition”. The negation of the  $\epsilon$ - $\delta$  definition would therefore be

$$L \neq \lim_{x \rightarrow a} f(x) \Leftrightarrow \exists_{\epsilon > 0} \forall_{\delta > 0} \exists_{x \in D_f} \text{ such that } [(0 < |x - a| < \delta \text{ and } (|f(x) - L| \geq \epsilon)]$$

Morash (1990) thus posits that such a negative formulation of the definition is much more concrete than the positive formulation in that it is easier for one to fix a positive value  $\epsilon$ , thus fixing the epsilon-gap. It then becomes easier to choose an arbitrary  $\delta$  after fixing the  $\epsilon$ , after which one can easily choose and fix an  $x$  whose corresponding  $f(x)$  is not within the distance  $\epsilon$  from the  $L$ . According to Morash (1990), such an approach would enable the students to understand the notion of the epsilon-gap and hence realise the relationship between the  $\epsilon$ , the  $L$ , the  $f(x)$  and the point  $a$ .

Other studies on limits argued for teaching practices that convey to the students that understanding of the conceptual aspects of the content is as important as understanding the procedural aspects of the content. Whilst Morash (1990) provides ideas aimed at assisting students in understanding the theoretical technicalities of the  $\epsilon$ - $\delta$  definition, researchers such as Bezuidenhout (2001), Fernandez (2004), Swinyard and Lockwood (2007) and Williams, (1991) argue that in order to develop instruction that benefits students, teachers should tape students’ ideas, misconceptions and views of the concept and then use these ideas to develop teaching strategies. Bezuidenhout (2001) recommends that students need to be made to realise that the mathematical symbols carry meaning and that having skills in manipulating the algebraic expressions does not imply an understanding of the meanings of the symbols involved. Instruction should convey that understanding the conceptual aspects is as important as understanding the procedural aspects of the concept. Bezuidenhout (2001) further recommends that as a way to enhance conceptual understanding of the calculus concepts, teachers need to be aware of the nature of their students’ conceptual problems such that the information would enable the teachers to develop specific teaching strategies that address such problems. A similar view was also aired in an earlier study by Williams (1991)

who posited that improving students' understanding of limits requires instruction that accounts for the diverse conceptions of the limit concept that students hold and value. Such an approach was employed and found to be beneficial by Fernandez (2004) who first elicited students' misconceptions and perceptions of the  $\epsilon$ - $\delta$  definition and then used these ideas to design a lesson aimed at addressing such misconceptions and perceptions. Swinyard and Lockwood (2007) recommended engaging students in activities that are designed to foster their "reinvention"<sup>4</sup> of the formal definition of the limit. According to Swinyard and Lockwood (2007), engaging the students in reinventing the definition may help the students to clarify their own misconceptions about the definition, and at the same time, enabling the teacher to learn more about how students reason about the formal definition. This would, eventually allow the development of an instructional sequence intended to support students' reinvention of the definition.

Other researchers, such as Cottrill *et al.* (1996) discuss about addressing students' difficulties with the limit of function concept through the lens of the APOS<sup>5</sup> theory as applied in a computer enriched environment. Cottrill *et al.* (1996) propose a computer tool called "*LimitProcess*" which they developed based on a 'genetic decomposition' of the limit of function concept and grounded in the APOS learning theory. The computer tool intended to help students learn and understand the limit concept. Although Cottrill *et al.* (1996) do not provide extensive investigation on the effectiveness of the tool, preliminary observations point towards benefits of usage of the tool. In other studies, Buyukkoroglu *et al.* (2006) and Guzel and Alkan (2006) showed that computer enhancement had positive impact on students' understanding of the limit concept. Such observations could be attributed to the strengths and contributions of the computer to visualisations. Researchers acknowledge the strength of technology in teaching mathematics which is embedded in its capability to perform many tasks. Such tasks include facilitating easy manipulations of multiple representations and supporting visualisations, facilitating interactive learning as well as cooperative learning (Kaput, 1992; Kaput & Thompson, 1994; Tall, 1993b). However, as much as positive effects were noted from using computer enriched environments in the

---

<sup>4</sup> Gravemeijer (2000) cited in Swinyard and Lockwood (2007) defines guided reinvention as a process by which students formalise their informal understanding and intuitions.

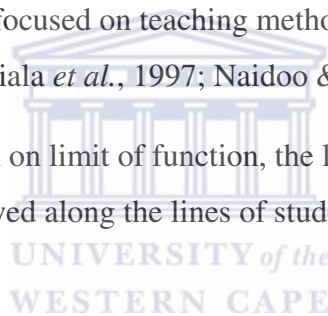
<sup>5</sup> APOS is acronym for Action, Process, Object and Schema. APOS theory postulates that individuals deal with mathematical situations and problems by constructing mental actions, processes and objects and then organising them in schemas, thus enabling them to make sense of the situation and to solve the problem (Asiala *et al.*, 1996).

teaching of the limit concept with some studies, this is not always the case as negative effects can also be observed. Monaghan, Sun and Tall (1994) noted both positive and negative effects of the computer with regards to learning limits. Care, therefore, needs to be taken when the teaching of limits involves computer enriched environments.

### **3.4.2 Derivative of function**

A number of studies have been presented on the teaching and learning of the derivative of function concept (Asiala, Cottrill, Dubinsky, & Schwingendorf, 1997; Bezuidenhout, 1998; Ferrini-Mundy & Graham, 1994; Habre & Abboud, 2006; Hahkioniemi, 2004, 2006a, 2006b; Heid, 1988; Naidoo & Naidoo, 2007; Orton, 1983; Serhan, 2006; White & Mitchelmore, 1996). Whilst some of the studies specifically focus on student learning and identify some difficulties that students experience with learning the derivative of function concept (Bezuidenhout, 1998; Ferrini-Mundy & Graham, 1994; Orton, 1983; White & Mitchelmore, 1996), others have focused on teaching methods that aim to enhance students' understanding of the concept, (Asiala *et al.*, 1997; Naidoo & Naidoo, 2007; Serhan, 2006).

Similar to the previous subsection on limit of function, the literature on derivative of function is demarcated and reviewed along the lines of student learning and some aspects of teaching.



#### **Aspects of student learning**

A majority of studies conducted on students' learning of the concept suggest that students have no challenges with performing routine differentiation processes (Ferrini-Mundy & Graham, 1994; Hahkioniemi, 2004, 2006b; Orton, 1983; Serhan, 2006). At the same time, many of the studies reveal that patterns of inadequacies do emerge regarding the conceptual understanding of the derivative. Some of the inadequacies arise from the different perspectives of the derivative, that is, as related to quotient differences, rates of change and slopes of tangent lines.

In this subsection, I review studies in relation to students' difficulties with understanding the derivative of function concept.

### *Relating the derivative to the limit of the difference quotient*

The derivative of a function can be considered as the limit of a difference quotient associated with the function  $f$ . The notion of limit is therefore encountered in such forms

as  $f'(x_0) = \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h}$  or  $f'(x_0) = \lim_{x_i \rightarrow x_0} \frac{f(x_i) - f(x_0)}{x_i - x_0}$  for the formal definition of derivative of

function  $f$  at point  $x$  or  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  for the derivative of function  $f$  at every point  $x$ .

However, students fail to understand the significance of both the difference quotient and the limit in the derivative of function definition (Hauger, 2000). Researchers have acknowledged that students' limitations with understanding the limit of function concept have the potential to hinder the students' conceptual understanding of the derivative of function concept (Hauger, 2000; Heid, 1988; Hahkioniemi, 2006a, 2006b; Orton, 1983; Tall, 1993a).

As indicated in the previous subsection on limits, several studies have reported that students have difficulties understanding the limit concept. In that light, the difficulties that students experience with the limit concept may be easily transmitted to other mathematical concepts that involve the limit concept such as the derivative. Orton (1983) found that the subjects of his study on students' understanding of differentiation made a lot of errors and scored lowly in those questions that involved limits. The students also scored lowly in questions that involved  $\delta$  symbolisms and had difficulties in making sense of the symbols used in derivatives and differentiation.

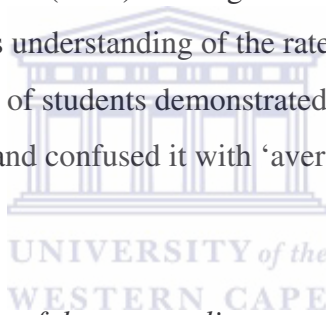
Hahkioniemi (2006a, 2006b) focused on students' understanding of the limit of the difference quotient. Procedural understanding was predominant amongst the students. The study revealed that some of the students considered the limit of the difference quotient to be a recollection and application of the formula in the textbook. The study by Viholainen (2006) shows that even if the students had a good geometrical interpretation of the derivative, they still had problems with the limiting process of the difference quotient.

### *Relating the derivative to instantaneous rate of change*

While the derivative can be viewed in terms of limit of a difference quotient, the difference quotient can also be viewed as an average rate of change of the dependent variable ( $y = f(x)$ ) over the width of the interval of the independent variable. In that way, the derivative of a

function at a point, also referred to as the instantaneous rate of change of a function, can be defined as the limit of the average rates of change, as the width of the interval for the independent variable gets smaller and smaller. Studies show that students have difficulties in establishing the linkages between derivatives, average rates of change, and instantaneous rates of change. Zandieh and Knapp (2006) identified some ‘misstatements’ regarding the derivative of function that were common with their interview subjects. One example of a misstatement that they identified is “the derivative is the change”. Such a statement is mathematically inaccurate and reflects the students’ misconception of the statement “the derivative is a rate” (Zandieh & Knapp, 2006, p.12).

The study by Orton (1983) revealed that his subjects showed problems with the notion of rate of change. They were not able to connect the concepts of rate of change and instantaneous rate of change. They also were not able to relate the notion of rate of change to the derivative concept. Bezuidenhout (1998) investigated students’ errors and misconceptions related to students understanding of the rate of change. Bezuidenhout (1998) reported that a substantial number of students demonstrated some misunderstandings regarding average rate of change and confused it with ‘average value of a continuous function’ and ‘arithmetic mean’.



#### *Relating the derivative to the slope of the tangent line*

One other common ‘misstatement’ that Zandieh and Knapp (2006) identified in their study is that “the derivative is the tangent line” (p. 11). Such kind of a statement reflects the misconception that the students hold regarding the geometric interpretation of the derivative, where they view the derivative as a line. The geometric interpretation of the derivative concept is anchored in the relationship between the derivative of a function at a point  $x_0$  and the slope of the tangent line to the graph of  $f$  at that point. Ferrini-Mundy and Graham (1994) noted that students were not able to form a relationship between the tangent line and the derivative of function concept. Orton (1983) noted that some of the subjects involved in the study were not able to interpret the graphical representation of a function.

The students in the study by Hahkioniemi (2006a, 2006b) had problems interpreting the difference quotient used in the definition of the derivative in terms of the other related notions such as the slope of a secant or average rate of change. However, on the contrary, the two students in the study by Viholainen (2006) understood very well the visual meaning of

the derivative and difference quotient although they had problems with the limit component. Viholainen (2006) attributes this awareness to geometric meanings of the derivative to the kind of instruction that may have an emphasis on the visualisations. Observations such as those made by Orton (1983), Ferrini-Mundy and Graham (1994), and Zandieh and Knapp (2006), show that students may at times have problems with the graphical representation of the derivative concept, as well as relating the graphical representation to the symbolic representation.

### **Some perspectives for teaching the derivative concept**

To improve the learning of the derivative of function concept, more and more research has focused on alternative methods of teaching the concept that may aid students' conceptual understanding. Although as observed in the previous subsection on limits, the role of technology in teaching is still a debatable issue. A majority of the studies that focus on enhancing students' understanding of the derivative concept emphasise the use of technology for the teaching and learning process. This may involve using computers (Aksoy, Bulut & Mirasyedioglu, 2006; Asiala *et al.*, 1997; Heid, 1988; Naidoo & Naidoo, 2007) or the graphing calculators (Serhan, 2006; Berry & Nyman, 2003) or both (Habre & Abboud, 2006).

Researchers have reported positive effects of technology on students' learning of the derivative which can be attributed to the interactive nature of technology-enriched learning environments, as well as its capability to foster visualisation, and multiple representations manipulations. For instance, Habre and Abboud (2006) who emphasised visualisation in their study concluded that encouraging visualisation through technology assisted some of their subjects to gain a complete understanding of the derivative. Hauger (2000) advocates adopting numerical approaches in the instruction of rates of change. Hauger (2000) argues that numerical approaches enhanced by computer spreadsheets, whereby the average rates of change are computed over successively smaller and smaller intervals can assist students to appreciate the relationship between the average rate and the instantaneous rates of change.

In an earlier study by Heid (1988), the computer aided concept development of derivatives in the experimental group of the study. These students were able to focus on a deeper understanding of derivatives rather than on the mere computations. They were able to analyse relationships between functions and their derivative functions as well as

relationships between slopes of secant lines to graphs of functions and derivatives. The students also benefited from the opportunity to explore the derivative by examining problems in a variety of representations such as graphs, tables, formulas and applied contexts.

Serhan (2006) reported an experimental study that used a graphing calculator to emphasise connections between symbolic, visual and numeric representations in students learning derivatives. For the experimental group, concepts were introduced numerically by tables of values, visually by graphs and algebraically by explicit formulas of which all were enhanced by a graphing calculator. The control group, on the other hand, only relied on a textbook and the instructor. Serhan (2006) concluded that the use of the graphical calculator and the emphasis on representations and the connections between them positively influenced students' learning of the derivative. Aksoy *et al.* (2006) made similar observations in an experimental study that used a computer algebra system in teaching the derivative.

A study by Naidoo and Naidoo (2007) revealed that computers enhanced students learning of differential calculus. The computer tool aided their subjects in understanding the concepts by providing the students with opportunities to engage in interactive and cooperative learning strategies.

Asiala *et al.* (1997) constructed a genetic decomposition of the derivative of function concept in the framework of the APOS theory in which a computer programme was developed on the basis of the genetic decomposition. The findings of the study revealed that an instructional treatment designed on the basis of the genetic decomposition had the potential to enhance students' understanding of the derivative. The experimental group whose instruction was based on the genetic decomposition were more successful in developing a graphical understanding of a function and its derivative as compared to the control group whose instruction was based on the usual traditional methods.

### **3.4.3 The function concept as prerequisite for understanding limits and derivatives**

#### **Review of selected studies on the function concept**

Research in calculus has also focused on how students' understanding of particular concepts interacts with the students' understanding of related concepts (Robert and Speer, 2001). The function concept is one example of a mathematical concept that has the potential to influence



students' understanding of the concepts of limit of function and derivative of function. The centrality of the function concept in calculus has led to extensive research being conducted on students' learning processes of the concept in general (Akkoc & Tall, 2005; Eisenberg, 1991; Janvier, 1987; Keller & Hirsch, 1998; Leinhardt, Zaslavsky, & Stein, 1990; Sajka, 2003) and in relation to other calculus concepts such as the limit of function and derivative of function.

The function concept has various ways in which it can be presented, graphically, algebraically or symbolically, numerically, and verbally. The range of different ways in which the function can be represented in instruction influences the ways learners acquire and apply the knowledge of what they are learning.

Research has shown that a major cause of learning difficulties for students regarding the function concept as encountered in calculus is due to the students' lack of flexibility in shifting between the various forms of representations of a function (Even, 1998; Lesh, Post & Behr, 1987). This also impacts on how learners develop their individual preferences of the function. Keller and Hirsch (1998) found that students do have learning preferences for representations of the function concept and that these preferences vary between contextualised and non-contextualised settings. Research has also shown that using a variety of representations enhances relational understanding of concepts. Greeno and Hall (1997), for instance, maintained that forms of representation are useful as they aid in the construction of understanding and in communicating information. In a study on difficulties associated with functions, Eisenberg (1991) highlighted that students seem to think of the function concept in only a symbolic representational mode where functions and their associated notions are not conceived visually. The study concluded that the unwillingness to stress visual aspects in mathematics is a serious impediment to students' learning. Stewart (1994) emphasised 'the rule of four' which implies presenting information in the four modes of representations, viz, the verbal, graphical, symbolic and numerical representations to be a way of facilitating learning.

### **Studies relating the learning of limits and derivatives to the function concept**

Research has provided insight on how students' understanding of the function concept could have an influence on students' understanding of concepts such as the limit of function and

the derivative of function (Asiala *et al.*, 1997; Bezuidenhout, 1998, 2001; Ferrini-Mundy & Graham, 1994; Williams, 1991; White & Mitchelmore, 1996; Szydlik, 2000).

Studies have attributed students' difficulties in understanding certain aspects of calculus to the students' difficulties with understanding the function concept. For instance, White and Mitchelmore (1996) attributed the difficulties that their subjects experienced with calculus concepts such as limit and derivative of function to the notion of variables. They found that the difficulties were due to the students' underdeveloped conceptions of variables. The students treated variables as symbols to be manipulated rather than quantities to be related. The study by Szydlik (2000) revealed that students' conceptions of functions had an influence on their understanding of the limit of function.

Ferrini-Mundy and Graham (1994) noted that students could easily determine the limit of a function as  $x \rightarrow a$  where the function  $f(x)$  was algebraically expressed, but showed little understanding with functions given graphically. Such limitations can be attributed to students' inflexibility with forms of representations of functions where students perceive a function to be a rule of correspondence that can only be given by a formula. Bezuidenhout (2001) pointed out that most calculus textbooks encourage algebraic exercises that are based on the representation of the function as a formula at the expense of the other representations. Such an approach creates conflicts in students as it promotes a habit of following the algebraic representation in mathematical situations and neglects the other approaches such as graphical, numerical. This results in students' failure to look at the mathematical situation from the other perspectives. For instance, some students in the study by Bezuidenhout (2001) experienced problems shifting between forms of representations of the function that was under study. When asked to find the limit of a function that was given numerically, some of the students first constructed what they felt was an algebraic formula for the given data, and then they evaluated the function value to obtain the limit. Some of the students used the given numerical data to construct what they felt was the graph for the function and then obtained the limit from the graph. In both instances, this indicates that students have their own conceptions of what a function should be (a formula or a graph) when finding limits. This also shows that the students hold the forms of representations of the same function as independent of each other.

### 3.5 CHAPTER OVERVIEW

This chapter presented a discussion of selected literature that is related to the main notions of this study. The chapter began with a review of literature related to learning mathematics in a distance education environment. This was followed by a presentation of literature related to learning styles which included defining the notion of learning style, followed by a review of studies on application of learning styles in instruction. The final section consisted of a review of selected literature related to the teaching and learning of certain calculus concepts, in which learning difficulties and suggestions for teaching the limit of function and derivative function concepts were discussed. Chapter 4 will present the theoretical framework underpinning this present study.



# CHAPTER 4: THEORETICAL FRAMEWORK

## 4.1 INTRODUCTION

This chapter presents the theoretical framework for this study. McShane (1991, p. 27) describes a framework as a “general set of assumptions and constructs that are shared by particular theories”. The purpose of a theoretical framework in an empirical inquiry is to provide a theoretical structure that guides the study including aspects of how to view and explain the phenomenon under inquiry and how to interpret the data. The theoretical framework therefore provides the ‘lenses’ through which one should view and think about the phenomenon or construct under study.

This study focuses on the learning of calculus in a distance education university and addresses the research questions, as stated in Chapter 2. In an attempt to address the research questions, it is important to describe the various components of a theoretical framework that can assist one to understand the essential facets of distance learning including learning styles and understanding of mathematical concepts. This chapter thus discusses the theories and models that are pertinent to this study.

In this study, students’ learning styles and their mathematical understanding are looked at in order to understand the learning of mathematics in the ZOU distance education environment. Thus, learning styles theory and mathematics understanding serve as theoretical constructs that can inform about mathematics distance learning processes. These are however viewed within the broader framework of the information processing model for learning theories. Out of a range of learning theories the choice of information processing theory was deliberate as this would inform on cognitive processes such as acquiring information, remembering, retrieval of information, and making decisions. Students receive external information as input, process it, internalise it and then produce output in the form of learning outcome, changed thinking and behaviour power. Understanding how students engage in such processes is crucial to distance learning where the goal is to provide a learning environment that assists students to optimally benefit from the instruction when they engage in individual learning.

## 4.2 INFORMATION PROCESSING MODEL OF LEARNING

Learning is an issue which depends on the individual, with students studying within the same educational context experiencing learning differently. Some students manage to ‘master the concepts easily’ and some ‘hardly get it at all’. Tall (1978, p. 50) explained the functioning of the brain in a learning process in the following way:

*The brain is a hive of activity; it takes in information, processes it, distinguishes between things, sees similarities, makes deductions, forgets things, remembers them later and has mental blockages and leaps of insights.*

Although learning may appear as a multifaceted and complex process, it is the point of departure of this study that the differences could be attributed to the different ways in which the students receive, manage, handle, process, reflect upon, and internalise the subject matter available to them.

Researchers such as Stewart and Atkin (1982), Felder and Silverman (1988), and Mayer (1989), Orton (1992) and Boulton-Lewis (1997) describe learning in terms of how students process information. For instance, Orton (1992) highlighted that learning being a mental activity might be modelled and understood if viewed in terms of the ‘functioning of the brain as a processor of information’. Orton (1992, p. 3) says,

*We might therefore understand more about learning if we knew more about the functioning of the brain as a processor of information. The brain receives information, interprets it, stores it, transforms it, associates it with other information to create new information and allows the information to be recalled.*

Stewart and Atkin (1982, p. 323) share a similar view when they say,

*the problem of understanding how humans learn can be considered as essentially the problem of understanding how information is stored in memory, how transformation of this stored information may occur and how stored information is retrieved for use in further learning.*

Felder and Silverman (1988, p. 674) also highlight that

*Learning in a structured educational setting may be thought of as a two-step process involving the reception and processing of information. In the reception step, external information (observable through the senses) and internal information (arising introspectively) become available to students, who select the material they will*

*process and ignore the rest. The processing step may involve simple memorisation or inductive or deductive reasoning, reflection or action and introspection or interaction with others. The outcome is that the material is either “learned” in one sense or another or not learned.*

The above quotes by Orton, Stewart and Atkin, and Felder and Silverman capture well the essence of viewing learning from an information processing perspective. The emphasis of information processing (IP) is on how learners select, perceive, process, encode and retrieve information from memory. Such kind of information is of relevance in a distance education environment for the design of learning resources so that the resources enhance learning.

This study uses a learning styles model, the Felder-Silverman learning styles model that is defined from an information processing perspective. I will however first of all discuss the information processing approach to learning, before discussing the learning styles model that I used in this study.

#### **4.2.1 Information processing as a framework for learning theories**

Miller (2003) and Proctor and Vu (2006) provide historical accounts of the cognitive revolution which led to the conception of the information processing approach. Whilst Proctor and Vu (2006) provide a historical overview of research that was conducted and is related to the development of the information processing, Miller (2003) provides a personal account from his own experiences of being involved with the developments of the cognitive revolution. The cognitive revolution came to being as a reaction to the behaviourist approaches. Whilst the behaviourist perspective focuses on the behaviour of the learner, the information processing focuses on the thought process behind the behaviour.

Information processing can be considered as a general framework of human cognition.

McShane (1991, p.10) espouses this when he says:

*information processing is an approach taken by different theorists who derive their theoretical constructs from a common loosely related framework. It is possible to construct theories within the information processing framework but it would be a mistake to assume that there is a single grand theory of information processing.*

Cobb (1987, p. 3) also noted that “the information processing approach is not a unitary one”. Hence, it is difficult to talk of and describe an information processing theory but rather talk of information processing as a framework of theories or for theories.

A characteristic feature of the information processing theory is that they divide the cognitive system into three main components: input, processing and storage, and output. IP theory further explores the way in which the components receive (input), transform, manipulate, store and then output the information. Information processing theories use the computer as an analogy for human memory system and acknowledge the central role that memory plays in learning. The stage model of IP by Atkinson and Shiffrin (1968) is one such information processing model and it proposes that information is processed in three stages: sensory memory, short-term memory and long-term memory.

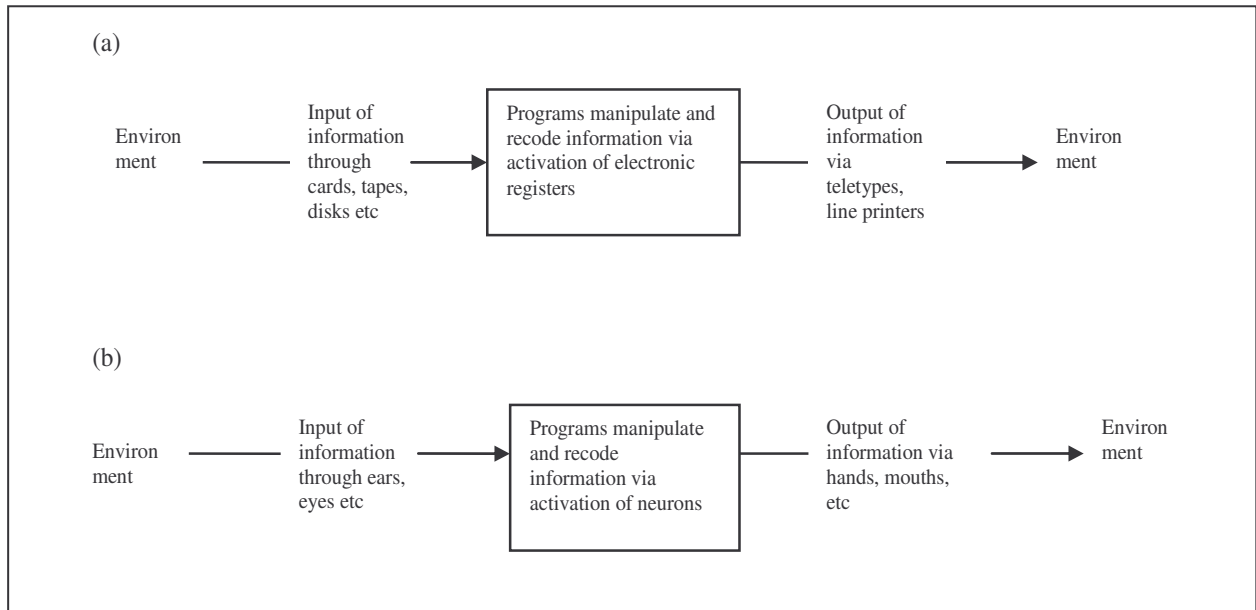
#### **4.2.2      *The computer as an analogy of the human memory system***

For the computer, there are three main components of information processing: the input component, the processing component and the output component. Figure 4.1 below shows a simple flow chart illustrating the components.



**Figure 4.1:** Components of information processing

The input component involves receiving some ‘unprocessed’ information from the environment. The output component involves outputting the processed information from the information processing system to the environment. A lot of activities and processes do take place during the processing stage, that is, between the input and the output stages. These include the storage of the input and output information, the transformation and manipulation of the input and all the organisational processes. Orton (1992, p. 168) presented a simple unpacking of the processes of how a computer works and illustrated the input, the control, processing unit, the storages and the output. Loftus and Loftus (1976, p. 5-6) presented a schematic analogy of the human mind and the computer as information processing systems.

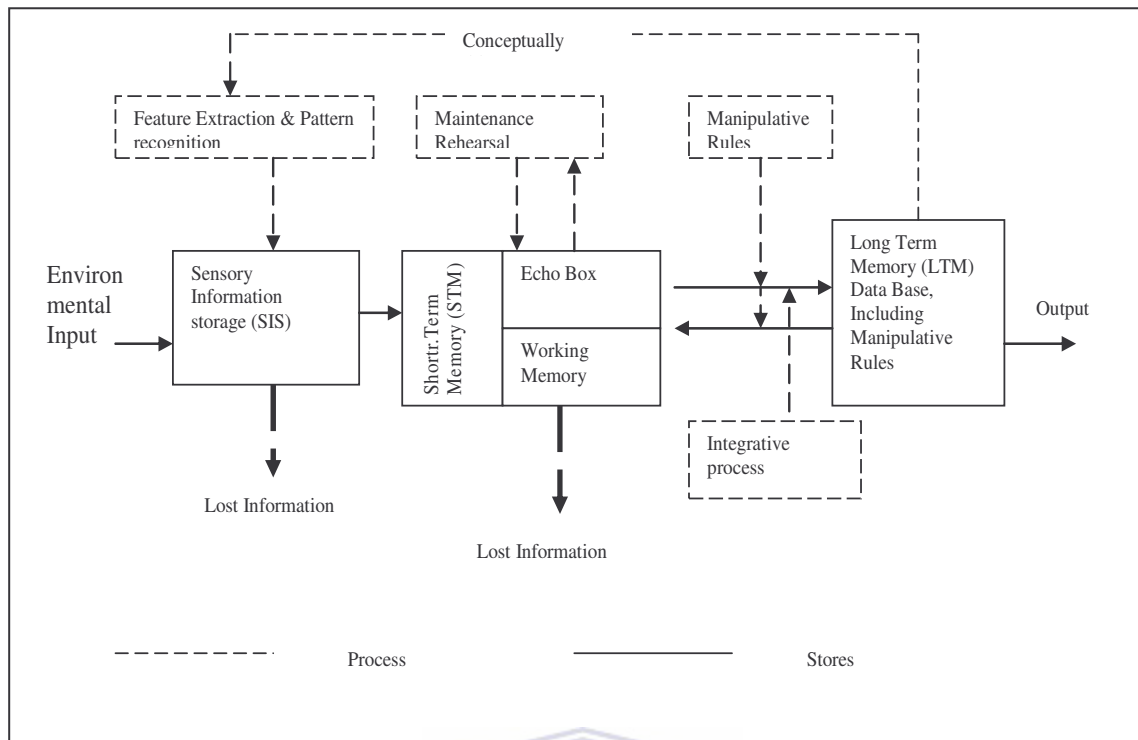


**Figure 4.2:** Analogy of information processing in computers and in humans  
 (a) computers, and (b) Human (adapted from Loftus & Loftus, 1976, p.6)

For information processing theories, the idea is that cognitive processes could be described in a way similar to the way a “computer functions” whereby the processing of information follows a certain path along which information is taken through the memory system, processed and stored and then recalled and reactivated when necessary.

Similarly, the three IP components of input, processing and output are also encountered in the human memory system. The mind takes in information, performs some kind of operations on it to change its form and content, stores it within, then retrieves it and generates responses to it when needed as the output.





**Figure 4.3:** Outline of the human memory system (adapted from Stewart & Atkin (1982, p.325))

Stewart and Atkin (1982, p. 325) presented the components of a memory system in a way analogical to that of the computer system. Analogically a memory system consists of

- the input component, in which information must become ‘encoded’ as mental symbols; (*input*)
- the *processing* component which comprises of a data base where all information and rules required for manipulation of this information are stored; (*memory stores*) and a set of control processes that allows inputs to be stored and outputs to be produced (*processes*);
- and the *output* component.

Figure 4.3 above, which presents a diagram adopted and adapted from Stewart and Atkin (1982, p. 325), provides an outline of the human memory system. The diagram outline includes the memory types and also indicates the control processes that are responsible for the manipulation and transformation of the information. For the purposes of this study, the

diagram was adopted also to include possible points where information is lost or forgotten, as shown.

We observe two features of the model, *the memory stores* and *the control processes* that are essential in an information processing theory. We will briefly zero into these two features in order to model how the memory system functions. Orton (1992, p. 167-168) argues that “memory is seen to be the key to learning, for the objective is storage within and ready recall from, long term memory”. Byers and Erlwanger (1985, p. 259) also acknowledge the role that memory plays in the learning of mathematics by saying “one of the outcomes of learning is remembered knowledge”. Thus, remembering knowledge is a function of the memory system. Tall (1978), in his paper titled, “The dynamics of understanding mathematics” also acknowledges the role of the brain in mathematical understanding as follows:

*Any useful classification of mathematical understanding will only prove itself if can unambiguously describe categories of mathematical understanding and non-understanding in a way that exhibits the realities of the situation: remembering, forgetting, mental blocks, leaps of insight (...)* (p. 52).

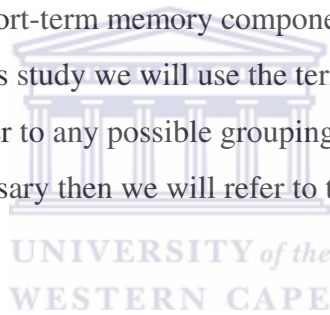
### **Some primary stages to processing information**

- Attention- The process of selecting information.
- Encoding-The process of translating information. Encoding involves gathering and representing information to be attended to.
- Retention- The process of storing or holding the information; Retention can occur in the short term memory or long term memory. However, information stored in the short term is retained for short periods of time.
- Retrieval- The process of recalling the information when appropriate.

### **Memory Stores**

The purpose of the memory stores is to store and retain information. However, the duration and the permanence of the retention vary in accordance to the type of storage. In a simple model, the human memory system is presented as having two types of memory, the *short-*

*term memory (STM)* and the *long-term memory (LTM)*. An example of one such model is presented by Orton (1992, p. 168). However, different researchers unpack and label the components of the short term memory with slight differences. Stewart and Atkin (1982, p. 323-325) for instance, presented a human memory system to be comprising of three components, the '*sensory information storage (SIS)*', the '*short-term memory*' which comprises of two functional features the *working memory (WM)* and what they call the '*Echo Box*' and the *long term memory*. Byers and Erlwanger (1985) also referred to the human memory system to comprise of three components, the short-term memory that consists of the SIS and the 'echo box', the working memory and the long term memory. The underlying fact however, is that the SIS, the Echo Box and the working memory can be viewed as components of the short term memory. Because of the importance of these memory components in perceiving, receiving and processing, transforming and retaining information, we will explain the functions of the long term memory and also separately explain the functions of these short-term memory components: SIS, WM and the Echo Box. However, for the purposes of this study we will use the term *short term memory* where short term memory will implicitly refer to any possible grouping of the three short term memory components, unless where necessary then we will refer to the specific short term memory component.



### *The Short Term Memory*

As alluded to earlier, we will view the short term memory to comprise the three components: the sensory information storage, the echo box and the working memory. We are therefore going to look at these components separately as summarised from Stewart and Atkin (1982) and Byers and Erlwanger (1985).

- The Sensory Information Storage – This is the first step in information processing and holds and maintains detailed 'sensory' information but for a very brief time period. SIS is a time limited and transient information store and has no rehearsal (instant replay) features. SIS functions in perceptual processing, pattern recognition and feature extraction from the LTM.
- The Echo Box – This is involved with remembering things for a very short while. New information is quickly lost if not rehearsed. The rehearsal is done so as to maintain the

information just for that while that it is in use e.g. when one repeats a telephone number long enough to dial it.

- The Working Memory – The working memory stores information during processing, e.g. for the time it takes to participate in a conversation or to solve a mathematical problem. It passes information between the long-term memory and the other components of short term memory. This means that the working memory does allow for new material to enter the long term memory (integrative process). It also functions in thinking, when information is recalled from LTM, brought out and worked on. It is in this component of STM that information is held while integrative processes are working on it. In mathematics learning, the working memory is a crucial component of STM. This is where a solution to a mathematics problem is organised by “constructing a suitable representation of the problem from data supplied by short term memory and relevant information retrieved from long term memory” (Byers & Erlwanger, 1985, p.268). The organisation of the working memory is therefore largely determined by the task at hand.

The main function of the short-term memory is therefore to select external information. It also serves as the site of storage when one is attempting to organise and transmit the information to the long-term memory. Information that is sensed, perceived and attended to is encoded in the STM. However, short term memory is characterised by very small capacity, ease of retrieval and by rapid turn over, though of course the degree of each variation depends on whether the process of encoding is taking place in the SIS, ‘Echo Box’ or working memory.

### *The Long-Term Memory*

Long-term memory has unlimited storage capacity and has a permanent organisation of memory structures though subject to modification due to learning. The LTM holds information until it is needed. When needed, the information is then retrieved. During retrieval, the information is located at the appropriate time and reactivated for use in a current task.

### **Control Processes**

Control processes act upon information and serve to expedite the flow of information among the memory stores.

### *Integrative Processes*

Integrative processes allow for new material to enter the long term memory from the short term memory. The new knowledge is committed to the long term memory, resulting in new long-term memory structures. Thus there is integration and modification of the already stored information to form new structures.

### *Maintenance Rehearsal Processes*

The rehearsal process is a control process within the STM where information is maintained through rehearsing it. Rehearsal processes only maintain information for a brief period of time.

### *Feature extraction and pattern recognition process*

This is a control process typical of the SIS. It functions with the perceptual, pattern recognition and feature extraction from the LTM.

### *Manipulative Rules*

In addition to the above-mentioned control processes are the manipulative rules. According to Stewart and Atkin (1982, p. 324) manipulative rules are the rules of logic such as “generalisation/induction, deduction, class inclusion and the substantive rules of inference of particular disciplines”. Stewart and Atkin (1982) also acknowledge the importance of these rules in learning by highlighting that the correct execution of these rules is important for storage of information (learning) and for problem solving.

### *Forgetting*

One important aspect that Stewart and Atkin (1982) did not explicitly include but is crucial to learning, is the one acknowledged by Carlile and Jordan, (2005), which is forgetting (losing information). The thickly dotted lines in Figure 4.3 indicate processes when information can most likely be lost. Most commonly, information is lost when one is not able to access the information when it is needed, in other words, when the information is simply forgotten. Forgetting information is likely to occur when there is decay of information. Decay of information occurs when the information is not attended to and

eventually fades away. Information can also be lost as a result of the interference between new and old information, that is, when either of them blocks access to the required information.

### **4.2.3 Some criticisms of information processing theory**

While information processing is a widely used theory for viewing learning processes and for predicting consequences of information processing activities taking place in the human mind, the theory does not go without criticisms. In this subsection, I discuss some of the criticisms raised against information processing theory.

One major criticism of IP is attributed to the fact that in IP, computer processing is taken as an appropriate analogy of the human mind, an analogy that Mayer (1996, p. 158) refers to as “incomplete”. Much of the criticism arising from this analogy is due to the fact that humans do not process information in the same way as a computer. The analogy results in a simple, rigid and mechanistic view of cognition which treats the mind as a fixed device awaiting to receive input for processing (Mayer 1996, Baddeley, 1998). Other noted criticisms (Mayer 1996; Baddeley, 1998; Schunk, 2000) were directed to the analogical division of the human mind into discrete components - sensory memory, short-term memory and long-term memory. Mayer (1996, p. 158) argues that the lines dividing these components are not firm, whilst Schunk (2000, p. 122) raises concern that the model does not fully specify how information moves from one memory store to the other. Although there are executive control processes that are defined for an IP model, the role of these control processes is considered vague or appears to be downplayed by the role of the memory stores in the IP model, (Schunk, 2000; Mayer, 1996). One other criticism of the IP is that it concentrates on processes that occur within the head of the individual to the exclusion of the learning context. As noted by Mayer (1996), focusing on the rational side of human learning implies that the IP ignores the affective, social, biological and environmental aspects of the learning process.

Another major criticism raised against IP is that on performing an activity, the theory does not address how processing of information develops. IP fails to address the role of the activity in cognition. Since in IP, cognition is interpreted in terms of performing computations on symbols, the model tends to ignore situations in which learners actively construct new knowledge (Mayer, 1996). As pointed out by Schunk (2000) the model is

vague when it comes to what really is learned and how it is learned. Criticisms of the IP theory that are related to knowledge construction gave rise to the third metaphor of learning theories, the constructivist view of learning. The constructivist view of learning is based on the premise that learners construct their own knowledge on the basis of their own experiences and internal knowledge structures. Constructivism was therefore conceptualized as a reaction to the IP view of learning.

A constructivist related criticism that is particular relevance to this current study is the fact that IP is defined and places emphasis on remembering and memory, and fails to address the aspect of how knowledge is constructed. This may have direct impact on distance learning. For a distance learner, who most of the time is involved in individual learning and who relies on pre-prepared learning materials, learning should not only be viewed from the input, output and processing of information perspectives. Rather it should also be viewed from the perspective of the learner constructing their own knowledge and making connections between new knowledge and existing knowledge. Beyond the inputting and the processing of information, IP does not directly shed light on how to provide learners with experiences that can help them discover on their own the underlying concepts and patterns. In a classroom environment a teacher can be readily available with tasks, activities and examples that can help the learner link the 'input' with the other knowledge structures, but in a distance environment the learner is alone save for the learning materials.

Because IP does not explain many aspects of a learning process as expected for some educational contexts, many may see IP as an incomplete explanation of the human cognition. However, irrespective of the criticisms, the IP theory still remains central to the development and applications of learning theories. As mentioned earlier on, the IP theory was developed as a reaction to the behaviourists, and the constructivist perspective originated as a reaction to the IP theories. Mayer (1996, p. 151) acknowledged that irrespective of the limitations, IP remains a legacy for the constructivist view of learning by "serving as a historic bridge" from the behaviourist to the constructivist conception of human learning. A similar view was shared by Cobb (1987, p. 35) who noted that the constructivist theory can be seen to serve as a prism through which to refract information processing theories. Proctor and Vu (2006) agree with Miller (2003) on that the IP today still has the appeal and is as vibrant today as it was 50 years ago, with Miller (2003, p. 144) stating:

*But the original dream of a unified science that would discover the representational and computational capacities of the human mind and their structural and functional realisation in the human and their structural and functional realization in the human brain still has an appeal that I can not resist.*

#### **4.2.4 Information processing and learning styles- the link**

Studying students' learning styles and approaches to learning facilitate a way of understanding the IP memory and control processes. As observed by Carlile and Jordan (2005), "work carried on levels or types of learning also draws on our knowledge of short term and long term memory stores". Carlile and Jordan (2005) elaborated on this using the 'surface' and 'deep' learning concepts. They referred to 'surface learners' as those "who try to retain the information held in short term memory because of information overload" and 'deep learners' as those who "attempt to understand and encode material so that it be transferred to long term memory and more effectively learnt" (Carlile & Jordan, 2005, p.18).

Since learning processes in an IP model involve selecting, organising and integrating information, we can find some learning styles models where learning is considered from an IP perspective. One example of such a learning styles model is the Felder-Silverman learning styles model. This learning styles model is constructed in such a way that it addresses the learning phenomenon in a way consistent with the information processing framework. In this study, I use the Felder-Silverman learning styles model to 'zoom' into the student's learning processes.

### **4.3 LEARNING STYLES THEORY**

Section 1.1 and section 3.3 of this thesis presented a definition of learning styles and a review of some selected literature respectively. The current section presents a discussion on learning styles models and a description of the learning styles model that is used in this study.

#### **4.3.1 Learning style models**

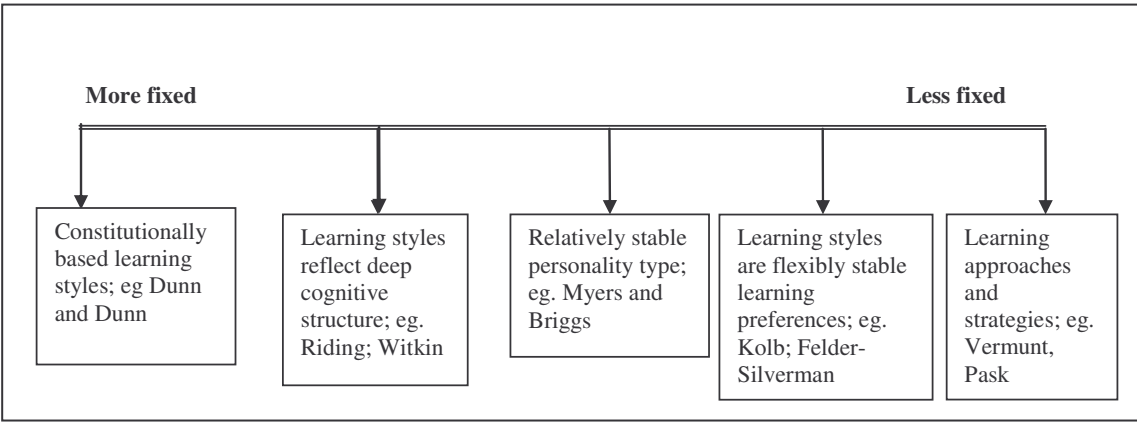
Learning style models are built on the assumption that individuals learn differently. Thus, basic objectives of a learning styles model may be to classify and characterise students' preferences, strategies or types of learning. Felder and Silverman (1988, p. 674) posit that a



learning styles model “classifies students according to where they fit on a number of scales pertaining to the ways they prefer to receive and process information”.

A glance at the literature of learning styles yields vast quantities of information on learning styles models. A review on learning styles models conducted by Coffield, Moseley, Hall and Ecclestone (2004) identified 71 learning styles models from learning styles literature published during the period 1902 to 2002. The variety of learning styles models present different descriptions and classifications of learning styles. However, as noted by Coffield *et al.* (2004), whilst some of the models are adaptations of the already existing learning styles models, some of the models offer new constructs which as well are just new labels for those already existing constructs. Nonetheless, such a variety of learning styles models shows that there are varied ways in which to view learning styles though all share the common premise that people learn differently.

Coffield *et al.* (2004) in their literature review brought another typology of classifying learning style models whereby they classified LS models into five ‘families’. The ‘families’ are organised on a continuum ranging from ‘more fixed’ to ‘less fixed’ learning styles models, of which the main organising factor is the degree to which the various authors of the learning styles models believe that learning styles are fixed. Such kind of placement helps in illuminating the degree of fixedness and flexibility of particular learning style model in relation to the other models. This also informs on the implications of the learning styles models on teaching and learning processes. Figure 4.4 below shows the diagrammatic placement of the ‘families’ of learning style models as adapted from Coffield *et al.* (2004).



**Figure 4.4:** Families of learning style models (Coffield *et al.*, 2004, p.9)

As shown on the above diagram, on the left hand end of the continuum is the ‘family’ labelled ‘constitutionally based learning styles’. This ‘family’ comprises of models that are hinged on the idea that learning styles are more fixed, and “should be worked with rather than changed” (Coffield *et al.* 2004, p. 10). Such fixedness of the learning styles model informs of a view that emphasises how important it is for instruction to be tailored to match learners’ learning styles. On the extreme right hand end, there is the ‘learning approaches and strategies’ family of models which is based on the premise that learning styles are less fixed and are modifiable. This points to the possibility of learning styles being dependent on external factors, such as curriculum, context, culture of a course or institution and lived experience (Hall, 2005, p. 54). Such flexibility informs of a view that emphasises how important it is for teachers to make learners aware of the ways in which they learn best and in making them aware of the available alternatives which could extend their repertoire of learning styles.

While there is a variety of learning style models available, as per the justification presented in subsection 4.3.2, I chose the Felder-Silverman learning styles model (F-SLSM) to provide a theoretical base in which to identify and explain the learning style preferences for the mathematics distance education students of this study. The model illuminates students’ learning styles preferences, thus acts as a tool for me to ‘zoom’ into the distance students’ learning processes in mathematics. In line with the classification of Coffield *et al.* (2004), the F-SLSM is placed in the fourth family of learning styles models that’s labelled as the “flexibly stable learning preferences”. Such kind of placement helps in illuminating the degree of fixedness and flexibility of the F-SLSM in relation to other learning style models. I will however discuss more on the fixedness and flexibility of the F-SLSM in subsection 4.3.3 of this chapter when I present a more detailed description of the model.

Because of the importance of the F-SLSM for this study, I therefore devote subsections 4.3.2 and 4.3.3 of this chapter to discussing certain aspects of the F-SLSM including the origins of the F-SLSM, justification why I settled for this model as well as detailing characteristics of the model’s learning styles dimensions.

### 4.3.2 *A learning styles model for this study*

This subsection presents a discussion on the origin of the Felder-Silverman learning styles model and also presents the reasons on why I found the model to be the suitable learning styles model for this study.

#### **The origin of the F-SLSM**

The Felder-Silverman Learning styles model was developed by Richard Felder and Linda Silverman and was first published in 1988 (Felder & Silverman, 1988). The model was developed to address learning differences within the context of engineering education at university level. The F-SLSM was developed from an information processing learning theory perspective whereby the authors viewed learning in a structured educational system as a two-step process that involves receiving and processing of information. According to Felder and Silverman (1988, p. 674), the reception step is whereby external and internal information become available to students and students select the information to process and ignore the rest, and the processing step is whereby students process information and move towards understanding.

The original Felder-Silverman learning styles model incorporated five dichotomous dimensions intended to capture students' learning preferences with regards to *perceiving information* (sensing or intuitive), *input of information*<sup>6</sup> (visual or auditory), *organising information* (deductive or inductive), *processing information* (active or reflective) and *understanding information* (sequential or global) (Felder and Silverman, 1988). The authors of the model have since made two modifications to the original model in relation to the organisation and input dimensions. The organisation dimension was dropped from the model because of pedagogical reasons associated with teaching at university level. The input (visual/auditory) dimension was changed to input (visual/verbal) as the term 'auditory' was considered to be much more inclined to sound and spoken words to the exclusion of written texts (Felder, 2002; Felder & Henriques, 1995; Felder 1993). The present study is based on the updated F-SLSM. Subsection 4.3.3 of this chapter, presents detailed descriptions of the updated F-SLSM dimensions.

---

<sup>6</sup> In the F-SLSM the 'input' dimension is sometimes referred to as the 'receiving' dimension

By nature of its design, the learning styles dimensions of the F-SLSM are not original to the model (Felder & Silverman, 1988, p.675) but are an amalgam of dimensions from other learning models. However, this combination of the dimensions is unique to the F-SLSM (Felder and Spurlin, 2005, p. 103). In several studies (Felder, 1993; Felder & Henriques, 1995; Felder & Silverman, 1988; Felder & Spurlin, 2005) Felder and others are explicit in acknowledging the original learning theories on which the F-SLSM dimensions are based. The sensing/intuitive dimension is analogous to the sensing/intuitive dimension of the Myers-Briggs Type Indicator (MBTI) whose dimensions are based on Carl Jung's theory of psychological types. This dimension also parallels the concrete/abstract dimension of Kolb's learning styles model. The processing (active/reflective) dimension is a component of Kolb's active experimentation/reflective observation dimensions. This dimension also borrows from Myers-Briggs model's extrovert/introvert dimensions. The input (visual/verbal) and the understanding (sequential/global) dimensions are rooted in cognitive studies of information processing and learning styles, such as Pask's holist/serialist, Leteri's analytic/global dimension and several others that are detailed in Felder and Silverman (1988) and Felder and Spurlin (2005).

### **Why opt for the F-SLSM for this study**

At its conception, the F-SLSM was designed for engineering education in a conventional education. However, studies show that the usefulness of the F-SLSM has since extended to various other subject disciplines, such as the languages (Felder & Henriques, 1995), medical discipline (Laight, 2004), business and management (Bacon, 2004), science and engineering related disciplines (Brown & Pluske, 2007; Tanner & Allen, 2004) and mathematical sciences (Roebber, 2005). Studies have also shown that, the model has been applied to educational contexts other than the conventional face-to-face systems. For instance, Graf, Viola, Kinshuk and Leo (2006), Kinshuk and Lin (2004), and Mourtos and McMullin (2001) considered the model in developing technologically enriched learning systems.

I found the F-SLSM to be the appropriate learning styles model in which to study and interpret students' learning for this study which is situated in a distance education environment. Several reasons contributed to this decision. Firstly, since the Felder-Silverman model was designed with dimensions that are "particularly applicable to engineering education" (Felder & Silverman, 1988, p.674), I am of the opinion that the model applies well to mathematical related subjects because the academic nature of

engineering is in a way similar and related to that of mathematics. This position is illuminated by the fact that mathematics is indispensable in engineering, and by the symbiosis existing between mathematics and engineering as indicated in some studies concerning engineering education, such as Alpers (2006), Willcox & Bounova (2004) and the SEFI Mathematics working group (Mustoe & Lawson, 2002).

Secondly, the categories of the model appeal to me as I am interested in the learning of university level mathematics in a distance education environment. In searching for ways to facilitate distance learners with an optimal learning environment, I find that the four dimensions of the model, viz, perception, input (receiving), processing and understanding are relevant for this study as they may inform on the sufficiency of both the learning materials and the learner support systems. The dimensions directly have some implications on how mathematics subject matter could be presented in distance learning materials. Whilst the perception and input dimensions may differentiate on the format and mode of presentation of information in the learning materials, the processing and understanding dimensions are pertinent as they bring in the aspect of interactivity of the materials and sequencing of information in the learning materials

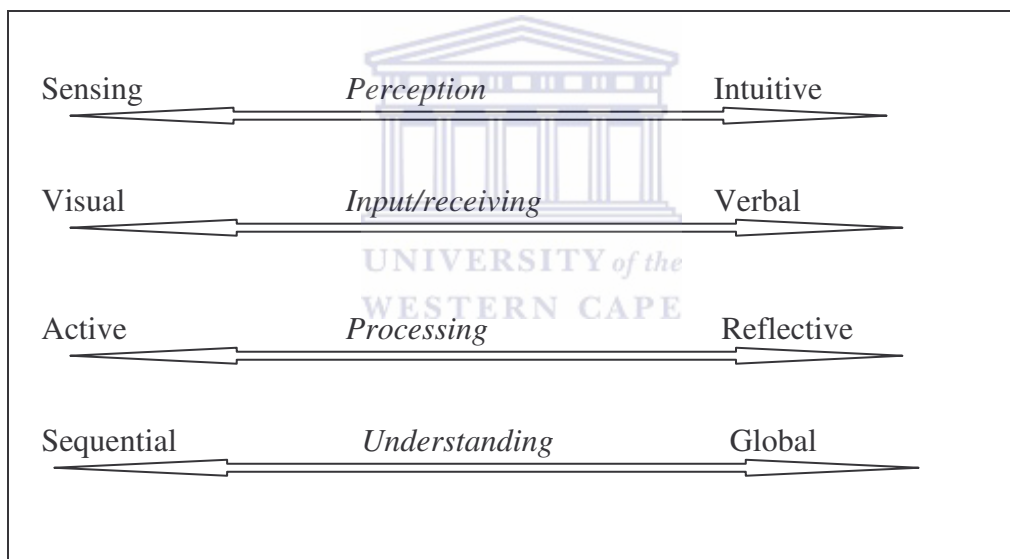
Thirdly, is the fact that the F-SLSM has the potential to classify students into more groups. This would provide me the opportunity to capture a wider spectrum of learning style characteristics amongst the students. Using the F-SLSM, would also enable me to elicit the distance students' experiences of learning in the DE environment. Fourthly, is the fact that the F-SLSM was developed for university level students, a similar educational level to the kind of students that I intend to involve in this study.

### **4.3.3 Description of the Felder-Silverman learning styles model**

In this subsection I provide a detailed description of the F-SLSM so as to create a framework within which to identify and explain the learning style preferences for the group of calculus students involved in the study. The F-SLSM is concerned with how students preferentially perceive information (sensing/intuitive), take in external information (visually/verbally), process information (actively/reflectively) and progress towards understanding (sequentially or globally). According to Felder (1993, p. 286), the dimensions of the F-SLSM may be defined in terms of answers to the following questions:

1. **Perception:** What type of information does the student preferentially perceive? Sensory or intuitive?
2. **Input or Receiving:** Through which mode does the student most effectively receive external information? Visual or verbal?
3. **Processing:** How does the student prefer to process information? Actively or reflectively?
4. **Understanding:** How does the student progress toward understanding? Sequentially or globally?

Figure 4.5 below shows a diagrammatical elaboration of the dichotomous situation of the F-SLSM dimensions.



**Figure 4.5:** The Felder-Silverman Learning Styles dimensions.

Felder and Silverman posit that a student’s learning style may be characterised by specific preferences in each of these dimensions. For example, a student’s learning style may be characterised by the following preferences sensing/visual/reflective/sequential on the four F-SLSM dimensions. Potentially there are 16 ( $2^4$ ) such possible combinations. However, these are extremities based on the extreme poles of the dimensions. Felder and Silverman (1988) further explain that although the dimensions are dichotomous, the preferences within a dimension are continua and may not be exclusively either/or. Furthermore the placement on the continua is descriptive and not normative. According to Felder (1993, p. 286) “a student’s preference of a dimension on a given scale may be strong, moderate or almost

nonexistent, may change with time, and may vary from one subject or learning environment to another”. This informs that Felder and Silverman considered learning styles to be less fixed, changeable and to have the possibility of depending on other external factors such as the context, course, time, and experiences.

A careful study of the F-SLSM shows that the dimensions of the model have discerning characteristics. As in any learning styles model, some characteristics of the F-SLSM are more representative of their dimensions than others. Graf *et al.* (2006), for instance, empirically identified the most representative characteristics of the F-SLSM dimensions. As noted by Graf *et al.* (2006), knowledge of such detail about a learning styles model becomes handy when it comes to applying the model, for instance, in materials design or in identifying learning styles from learner behaviours on the basis of the model.

In the following two subsections I present some characteristics of the Felder-Silverman learning styles model dimension based on a summary of some of Felder and Silverman’s studies and publications, (Felder & Silverman, 1988, 2002; Felder, 1996; Felder & Henriques, 1995; Felder, 1993). I present the characteristics both descriptively and in tabulated format. A tabulated summary is necessary for clarity and easy reference purposes since the F-SLSM serves as the main analytical framework for the learning styles profiling of Chapter 7. The detailed information would be of importance in deriving learning styles from the students’ descriptions, experiences and preferences of learning calculus in the distance education environment.

## **Characteristics of F-SLSM dimensions**

### *Perception Dimension: Sensing and Intuitive Learners:*

- Sensing learners (sensors) tend to like learning facts, while intuitive learners (intuitors) often prefer discovering possibilities and relationships.
- Sensors like solving problems by well-established methods and dislike complications and surprises; intuitors like innovation and dislike repetition.
- Sensors are more likely than intuitors to resent being tested on material that has not been explicitly covered in class.

- Sensors tend to be patient with details and good at memorising facts and doing hands-on work; intuitors may be better at grasping new concepts and are more comfortable than sensors with abstractions and mathematical formulations.
- Sensors don't like courses that have no apparent connection to real world; intuitors don't like "plug-and-chug" courses that involve a lot of memorization and routine calculations.

*The Input (receiving) Dimension: Visual and Verbal Learners*

Visual learners remember best what they see—pictures, diagrams, flow charts, time lines, films and demonstrations. Verbal learners get more out of words—written and spoken explanations. Everyone learns more when information is presented both visually and verbally.

*The Processing Dimension: Active and Reflective Learners*

- Active learners tend to retain and understand information best by doing something active with it--- discussing or applying it or explaining it to others. Reflective learners prefer to think about it quietly first.
- “Let’s try it out and see how it works” is an active learner’s phrase; “Let’s think it through first” is the reflective learner’s response.
- Active learners tend to like group work more than reflective learners, who prefer working alone.
- Sitting through lectures without getting to do anything physical but take notes is hard for both learning types, but particularly hard for active learners.

*The Understanding Dimension: Sequential and Global Learners*

- Sequential learners tend to gain understanding in linear steps, with each step following logically from the previous one. Global learners tend to leap in large jumps, absorbing material almost randomly without seeing connections, and then suddenly “getting it”. Since they do not learn in a steady or predictable manner, global learners tend to feel ‘out-of-step’ with fellow students and incapable of meeting expectations of their teachers.



- Sequential learners tend to follow logical stepwise paths in finding solutions; global learners may be able to solve complex problems quickly or put things together in novel ways once they have grasped the big picture, but they may have difficulty explaining how they did it.

### A tabulated summary of the F-SLSM dimensions characteristics

We are now able to give a tabulated summary (Table 4.1) of some of the characteristic preference attributes of the Felder-Silverman learning style classification scheme. For the sake of overview, the summary may be set up in a table, where the students may be regarded as stereotypes.

**Table 4.1:** Summary of Felder-Silverman Learning Style Preference Characteristics

DIMENSION	Characteristic attributes	Characteristic attributes
<b>PERCEPTION</b>  <i>Sensing and Intuitive</i>	Sensing Learners tend to: <ul style="list-style-type: none"> <li>• Be concrete thinkers. They like facts, data, observations and experimentation;</li> <li>• Be methodical. Like solving problems by “standard” methods and dislike surprises;</li> <li>• Be practical; Prefer connections to real world;</li> <li>• Rely on memorisation, drill and practice as learning strategies;</li> <li>• Be comfortable learning and following rules and standard procedures;</li> <li>• Be patient with details but do not like complications;</li> <li>• Are not comfortable with symbols but are comfortable with numerical examples;</li> <li>• Be patient with repetitions;</li> <li>• Be careful and maybe slow;</li> </ul>	Intuitive Learners tend to <ul style="list-style-type: none"> <li>• Like abstractions and prefer concepts and interpretations. Oriented towards theories and meanings. They deal better with mathematical concepts, principles and theories;</li> <li>• Be innovative. They don’t mind complexity;</li> <li>• Be imaginative;</li> <li>• Be good at grasping new concepts;</li> <li>• Like variety, can accommodate exceptions to rules and procedures;</li> <li>• Are impatient with details but welcome complications;</li> <li>• Be comfortable with symbols;</li> <li>• Be impatient with repetitions;</li> <li>• Be quick and maybe careless;</li> </ul>
<b>INPUT</b>  <i>Visual</i>	Visual Learners tend to get more information from what they see and they prefer visual	Verbal Learners tend to prefer verbal (spoken or written) explanations and discussions.

<i>and Verbal</i>	representations such as pictures, diagrams, graphs, flowcharts, films, demonstrations.	They are comfortable with context and word problems.
<b>PROCESSING</b> <i>Active and Reflective</i>	<p>Active Learners</p> <ul style="list-style-type: none"> <li>• Prefer doing something in the external world with the information first, e.g. discussing it, explaining it to others, applying it or testing it in some way;</li> <li>• Prefer trying things out first; “Let’s try it out and see how it works” is an active learner’s phrase;</li> <li>• Prefer group work and discussions;</li> <li>• Do not derive much benefit sitting in simple lectures;</li> </ul>	<p>Reflective Learners</p> <ul style="list-style-type: none"> <li>• Prefer examining and manipulating the information introspectively;</li> <li>• Prefer thinking things through first; “Lets think it through first” is the reflective learner’s response;</li> <li>• Prefer working alone or in pairs;</li> <li>• Do not derive much benefit sitting in simple lectures;</li> </ul>
<b>UNDERSTANDING</b> <i>Sequential and Global</i>	<p>Sequential Learners</p> <ul style="list-style-type: none"> <li>• Are orderly and logical. Prefer material presented in small and logically connected chunks.</li> <li>• Are linear. Tend to learn in linear steps, with each step following logically from the previous one. They learn in small incremental steps Tend to learn better when material is presented in a steady progression of complexity and difficulty.</li> <li>• Can work with course materials when they understand it partially or superficially;</li> <li>• May lack on seeing the interrelationships between the material and other topics, subjects and disciplines.</li> <li>• May be strong in convergent thinking and analysis;</li> <li>• Present neatly laid out work with each step clearly following the preceding one. Tend to follow stepwise paths in finding solutions;</li> </ul>	<p>Global Learners</p> <ul style="list-style-type: none"> <li>• Are systems thinkers. Learn in large leap; They learn better by jumping directly to more complex and difficult material;</li> <li>• Are holistic. They need the big picture before the details; May be able to solve complex problems quickly once they have grasped the big picture;</li> <li>• Have difficulty working with course materials when they understand it partially or superficially;</li> <li>• Once they have the big picture they often see connections no one else sees e.g. connections between topics, subjects and disciplines.</li> <li>• May be better at divergent thinking and synthesis;</li> <li>• Present correct problem solutions, which however might be incomprehensible to anyone else. They may be unable to explain their problem solving processes.</li> </ul>

#### **4.3.4 Criticisms of the F-SLSM**

As much as I considered the F-SLSM to be a useful learning styles model for this current study, the general domain of learning styles as well as the F-SLSM has received some criticisms. The learning styles theory is not without contestation. While on the one hand there are concerns related to the adequacies or inadequacies of learning styles theory in providing theoretical bases for learner diversities in terms of learning, on the other hand, concerns are also raised about conceptual clarity and quality of learning styles models.

An internet search that I performed did not yield any studies with direct criticisms of the F-SLSM as a model of learning styles. Rather, much of the criticisms that are available are directed to the Index of Learning Styles (ILS) which is an instrument that was designed by Felder and Soloman in 1998 to characterise learning preferences according to the F-SLSM. Although the F-SLSM ILS is not used in this current study as an instrument, some criticisms of the ILS were extended to the model by some researchers. For instance, van Zwanenberg, Wilkinson and Anderson (2000) used the instrument and concluded that the instrument was not of much use in predicting performance in learners due to the low internal reliability of the scales on the ILS. van Zwanenberg *et al.* (2000) further directed criticism to the F-SLSM by saying “doubts also remain over the conceptualisation of learning styles model which underlies the ILS, both in the bipolarity of the scales and their definition” (p. 379).

Criticisms can also be levelled against the F-SLSM as a learning styles’ model emanating from the nature of its design. One of the major observed criticisms relates to the fact that the dimensions of the model are an adaptation of other learning theories such as the Kolb’s learning styles model dimension and Myers-Briggs personality type, which on their own have also been criticised as learning styles models. Because of its adapted nature, the F-SLSM also adopted the criticisms of its original models. For instance, the criticism placed against Kolb and MBTI models that these models utilise dichotomous dimensions to categorise learning preferences gets transferred to the F-SLSM (van Zwanenberg *et al.*, 2000). Learning styles models that employ such kind of dichotomous categorisations of learning preferences have been considered as having a weakness of over-looking some learner tendencies as well as over-simplifying human variations.

Another example of an indirect criticism of the F-SLSM is noted in Coffield *et al.* (2004). Coffield *et al.* did not categorise the F-SLSM as one of the ‘major’ learning styles model. Of the 71 learning styles models that Coffield *et al.* (2004) identified, 13 were categorised as

‘major’ learning styles in terms of their importance and contributions in the field of learning styles. The F-SLSM was not classified as one of the major learning styles models. Rather, Coffield *et al.* (2004, p. 1) classified the F-SLSM among the other 58 models which are regarded not major as they “lack influence on the field as a whole”. They argued that this was so because many of the 58 models “consist of rather minor adaptations of one of the leading models” (Coffield *et al.*, 2004, p. 1).

Although in general learning styles theory has received some criticisms, as noted by Coffield *et al.*, (2004), learning styles still remain the most common construct which have been sufficiently researched upon to advance an understanding of how individuals engage in learning activities. In addition, the models also enlighten how internal and external factors affect individuals’ learning processes.

#### **4.3.5 Implications of the F-SLSM in mathematics distance learning materials**

Felder and Silverman (1988) advocate for a balance between the extremes in each of the learning style dimensions. The availability of advanced ICT may facilitate the ‘balance’ by enriching some components of the materials such as audio, visual and the interactive components.

In mathematics, a balance in learning styles can be achieved by using a variety of representations when explaining concepts. For instance, when preparing and presenting learning materials for a concept in calculus, there is a need to balance and blend the abstract concepts such as theories and principles with concrete information such as facts, tabulated data from experiments or real life situations, and thus take care of the perception dimension. Intuitive learners can be catered for by the formal mathematics in the form of mathematical theories, proofs, symbols and equations, whilst the sensing learners can be catered for by numerical and tabulated information. In order to take care of the input/receiving dimension, there is need to blend and balance the visual and the verbal information. Learning materials can, for instance, be presented with some diagrams, graphs or sketches accompanying the symbolic and textual information so as to reach both the visual and the verbal learners. To take care of the varied needs in the processing dimension, materials can be presented in ways that will engage both the active and reflective learners. This implies providing work with exercises and activities for balanced practicing and including questions that probe individual student reflection. Similarly the understanding dimension of the F-SLSM can be taken care

of, for instance, by complementing the usually encountered sequential presentation of concepts with the inclusion of overviews, by relating the information being presented to other concepts or courses as well as by providing activities that call for students' creativity.

As mentioned earlier on in section 3.3, to accommodate different learning style preferences, many research based mathematics curricula present information in a variety of representations. A function, for instance, can be represented in its graphical, verbal (contextual), symbolic (algebraic) and numerical forms. In the same way some calculus concepts, can be presented using a variety of modes of representations. For example, movement and motion can be described with words, pictures or graphs, equation form or in tabular form. The limit of a function concept can also be explained using the graph of the function, the algebraic equation, the tabulated function values (numerical) or the verbal explanations. Furthermore, some mathematical principles and laws such as laws of limits can be stated both verbally and symbolically. For example, the 'Sum Law' of limits can be presented in the symbolic form as,

$\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$  or in the verbal form as *the limit of a sum of functions is the sum of the limits* (Stewart, 1994).

A balanced instruction where a variety of representation is used enhances the student's concept development and mathematical reasoning. This may occur, for instance, as the students attempt to establish the links between the representations or as they attempt to evaluate the soundness of the solutions obtained from the same problem, though presented in different forms.

#### 4.4 MATHEMATICAL UNDERSTANDING

Since learning processes refer to the way students encode information, the way the information is presented to the students affects the way the students learn, as it tends to affect the ways the students select, attend to, organise and integrate the information. In a learning process, it is essential to consider the learning outcome. The nature of the mathematical knowledge that the student acquires as a result of the learning processes can be taken to represent the learning outcome. In this study, I consider the students' mathematical understanding of the concepts that are under study to represent the outcome of the learning process.

#### 4.4.1 **About understanding**

The importance of mathematical understanding is elaborated by the prevalence of studies on students' understanding of certain mathematical concepts (e.g. Bezuidenhout, 1998, 2001; Ferrini-Mundy & Graham, 1994; Habre & Abboud, 2006; Hahkioniemi, 2006b; Naidoo & Naidoo, 2007; Szydlik, 2000), as well as those studies where the notion of mathematical understanding is the object of study (e.g. Byers, 1980; Duffin & Simpson, 2000a, 200b; Hiebert & Carpenter, 1992; Hiebert & Lefevre, 1986; Pirie & Kieren, 1994; Sierpinska, 1990, 1992, Skemp, 1971, 1976). As noted by Sierpinska (1990, p. 25), studies on understanding may attempt to answer questions such as: what is understanding? what does it mean to understand a mathematical notion? how do people come to understand mathematical notions and why don't they understand?

Although it is difficult to come up with a single definition on understanding, it can possible to discuss understanding on the basis of the “numerous claims made on its behalf” (Hiebert & Carpenter, 1992, p. 74) such as the types, kinds or levels of understanding. For instance, Sierpinska (1990, 1992) focuses on ‘acts of understanding’ and ‘processes of understanding’. Pirie and Kieren (1994) discuss understanding in terms of growth and levels of understanding. Duffin and Simpson (2000a, p. 420) identified and described understanding in terms of three components, (a) building understanding - which refers to the formation of connections between internal structures, (b) having understanding- which refers to the state of connections at any particular time and (c) enacting understanding - which refers to the use of the connections available to solve problems. Skemp (1976) considered two kinds of understanding as relational understanding which he described as ‘knowing both what to do and why’, and instrumental understanding which he described as ‘knowing rules without reasons’. Hiebert and Carpenter (1992) describe understanding in terms of connections between pieces of information.

Hiebert and Carpenter (1992, p. 67) define understanding in terms of the way the information is represented and structured in the following way,

*A mathematical idea or procedure or fact is understood if it is part of an internal network. More specifically, the mathematics is understood if its mental representations is part of a network of representations.*

Thus for Hiebert and Carpenter (1992), understanding in mathematics can be viewed in terms of connections between ideas, facts or procedures and the degree of understanding is

determined by the number of and strength of connections. From a teaching perspective, Hiebert and Carpenter (1992) propose that teaching environments should be designed to help students build internal representations between pieces of information.

A definition on understanding that centres on connections between pieces of information links well with possessing knowledge that is rich in relationships or that is isolated. Hiebert and Lefevre (1986), as discussed in subsection 4.4.2 below, describe such kind of knowledge in terms of conceptual knowledge and procedural knowledge. For the purposes of this study, ‘conceptual understanding’ is understanding taken in the sense of Hiebert and Carpenter (1992), where pieces of information can be considered as conceptual knowledge as per the Hiebert and Lefevre (1986) definition of conceptual knowledge, and ‘procedural understanding’ is understanding considered in relation to procedural knowledge.

The following subsection will discuss conceptual and procedural knowledge as defined by Hiebert and Lefevre (1986). Subsequent subsections are presented on consequences of understanding representations in mathematical understanding and externalising knowledge.

#### **4.4.2 Conceptual and procedural knowledge**

Hiebert and Lefevre (1986) describe two types of mathematical knowledge in terms of conceptual knowledge and procedural knowledge. These have distinguishing characteristics of the richness in connections and linkages between ideas or pieces of information.

According to Hiebert and Lefevre (1986, p.4), conceptual knowledge is characterised as knowledge that is rich in relationships that cannot exist as isolated pieces of information. Development of conceptual knowledge is achieved by the construction of relationships between pieces of information. The linking process can occur between two pieces of information that already have been stored in memory or between an existing piece of knowledge and one that is newly learned. Procedural knowledge on the other hand is defined in two parts, as knowledge consisting of the form and symbolic language of mathematics, and as knowledge consisting of rules, algorithms or procedures used to complete a mathematical task (Hiebert & Lefevre, 1986, p. 6). Procedural knowledge can exist as isolated pieces of information, and development of procedural knowledge requires some form of input, whereby the initial procedure operates on the input to produce an outcome. A re-conceptualisation of procedural knowledge has been proposed by Star (2005) who argues for the broadening of the procedural definition to ‘deep procedural knowledge’. According

to Star (2005, p. 408) deep procedural knowledge would include knowledge of procedures associated with comprehension, flexibility and critical judgement, hence, bringing in a component of quality to that type of knowledge. As much as conceptual and procedural knowledge seem separate and distinct, Hiebert and Carpenter (1992) argue that these types of knowledge interact and support each other in learning situations.

#### **4.4.3 Mathematical understanding as an outcome of learning**

The emphasis of an instruction is to support students' construction of relationships between pieces of information and the reason why rather than encouraging just executing commands without reasoning. Meaningful learning enables the student to attend to the relevant information and to build connections internally between pieces of relevant information. Hiebert and Carpenter (1992) refer to it as building understanding and they refer to as creating rich networks of knowledge. This is similar to Duffin and Simpson's (2000a, 2000b) component of building understanding. Hiebert and Lefevre (1986) posit that with meaningful learning, meaning is generated as relationships between units of knowledge are recognised or created. Thus, Hiebert and Lefevre (1986) propose that meaningful learning produces conceptual knowledge. Rote learning on the other hand, produces knowledge that is absent in relationships where the material that is learnt by rote is stored in memory as isolated bits of information. However, the isolated information may be linked to conceptual knowledge resulting in meaningful learning.

#### **Consequences of understanding**

Hiebert and Carpenter (1992) noted that those students who learn mathematics with understanding are capable of retaining what they learn and transferring what they learn to novel situations. Some consequences of understanding summarised from Hiebert and Carpenter (1992, p. 74-77) include:

- *Understanding is generative* - to have the potential to integrate new knowledge into the existing knowledge and construct useful connections.
- *Understanding promotes remembering*- well-connected information is easily remembered since retrieval of information is enhanced if it is connected to a larger well-connected network.



- *Understanding reduces the amount of information that must be remembered* - if something is understood, it is represented in a way that connects it to a network, hence reduces the number of items that must be remembered.
- *Understanding enhances transfer*-the potential to apply the knowledge in a productive and meaningful way.

### **Representations in understanding**

'Representation' as described in the works by Kaput (1987) and Janvier (1987) shows that it involves something standing for something else. Kaput (1987) refers to it also in terms of the "representing world" and the "represented world", thus emphasising on that there is something that is representing something else.

Representations may be classified into two types, internal (mental) and external representations. According to Hiebert and Carpenter (1992, p. 66), the external representations take the form of physical materials, pictures, symbols etc. In the case of mathematics, a concept can have one or more modes in which it can be represented. For instance, with regards to mathematical notion of function, possible external modes of representations of the function would include the symbolic, the graphical, the verbal or the numeric representations. External representations can either refer to the representation that an individual interacts with when they take in information or the representation in which information is externalised from the person's mind (Hiebert & Carpenter, 1992).

Internal (mental) representations refer to the mental structures of the information in the individual's mind. Internal representations are not easily observable. However, Hiebert and Carpenter (1992) claim two assumptions regarding internal representations. Firstly, there is a claim that a relationship exists between internal and external representations. Hiebert and Carpenter (1992, p. 66) suggest that the nature of internal representations is influenced by the nature of external representations, as shown in the following excerpt, "the external representations with which a student interacts makes a difference in which the student represents the quantity or relationship internally". Another claim from Hiebert and Carpenter (1992) is that internal representations can be usefully connected to one another. The internal connections can be stimulated by building connections between the external representations. When relationships between the internal representations are formed, the representations may

link and connect well to form new networks of knowledge. To elaborate on the structure of the networks, Hiebert and Carpenter (1992) use the metaphors of “vertical hierarchies” and “webs”. With ‘vertical hierarchies’ networks, other representations subsume others, and with ‘webs’ networks, viewed analogically with spider webs, the nodes stand for pieces of represented information and the threads between the nodes (as in spider webs) stand for the connections between the pieces of information (Hiebert & Carpenter, 1992, p. 67). More connections between the representations in the knowledge networks of a particular student may imply that the student is able to manifest conceptual understand regarding the concept under study.

In this study, the notion of representation is used in the context of external representations. Within the context of distance learning, external representations are important because on one hand, they relate to the representations which the students interact with in the learning materials and on the other hand, external representations may be used to reveal how the students hold the information internally, that is, to reveal the internal representations.

### **External manifestations of knowledge**

The descriptions of understanding presented above in the earlier parts of this section show that understanding is something that happens internally in a person’s mind. As noted by Hiebert and Carpenter (1992, p. 66) a “useful way of describing understanding is in terms of the way an individual’s internal representations are structured. But then the internal connections and representations are not observable by an ‘outsider’ without any form of externalisation. Even though Hiebert and Carpenter (1992) acknowledge that assessing understanding is a highly inferential process, they also posit that “the way in which a student deals with or generates an external representation reveals something of how the student has represented that information internally” (p. 66). A similar view point is carried by Duffin and Simpson (2000a, 200b) in their description of enacting understanding. Duffin and Simpson’s enacting understanding involves usage of available connections in trying to solve a problem or a task. As such what a student understands can possibly be observed through what Duffin and Simpson (2000a, p. 421) refer to as the “external manifestation” of the knowledge. According to Duffin and Simpson (2000a, p.423), the internal characteristics of knowledge manifest themselves externally in a number of physical ways, such as speaking, writing and drawing. Thus, this alludes to that it can be possible to observe an individual’s understanding in mathematics through an analysis of the individual’s ‘external

manifestations of the knowledge' such as the individual's written work, performance in a task, or in the person's behaviours such as what they say, how they react when faced with certain tasks. In this study, students' understanding of the limit of function and derivative of function concepts were observed through the students' writings and performances in some selected tasks related to the concepts.

## 4.5 METACOGNITION AND ANDRAGOGY

### 4.5.1 *Metacognition*

This section discusses metacognition as a factor that contributes and relates to students learning and may be linked to information processing model. When looking at student learning in distance education, one has to consider discussing about metacognition since metacognitive abilities enable the student to control, manage or monitor the cognitive processes that the student engages in when learning. Thus, metacognitive abilities give potential for students to benefit from instruction. Irrespective of the mode of instruction, the student has got own ways of regulating and managing his or her own learning. The control of own learning may include to repeat, go back or change approach and so on. Below we will look closer into this issue.

Metacognition, a construct that is usually referred to as “thinking about thinking”, originated in the context of the cognitive information processing studies. Flavell (1976, p. 232) described metacognition as follows:

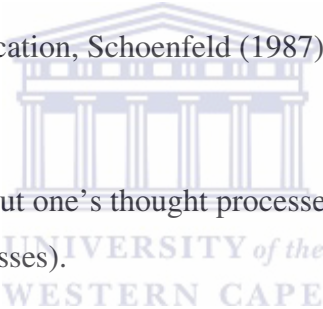
*Metacognition refers to one's knowledge concerning one's cognitive processes and products or anything related to them (...). Metacognition refers among other things, to the active monitoring and consequent regulation and orchestration of those processes in relation to the cognitive objects or data on which they bear, usually in the service of some concrete (problem solving) goal or objective.*

Flavell (1976; 1987) describe metacognition as consisting of the two components- *metacognitive knowledge* and *metacognitive regulation* processes. Dart (1997), in an overview on metacognition, provided succinct explanations of these two components. According to Dart (1997, p. 32-33), metacognitive knowledge includes “one's knowledge about learning”, incorporates “understanding about the task characteristics that influence how the learner will approach the learning activity” and as well includes “the information

about effective cognitive strategies”. Metacognitive regulation, on the other hand involves two elements; metacognitive awareness and metacognitive control. Metacognitive awareness “results from the learner’s conscious self-interrogation in relation to the learning task and activity”, whilst metacognitive control involves “the key operations of planning, monitoring/directing and evaluating” the learning activities. Dart (1997, p. 33) also clearly explained the components of metacognitive control by saying,

*Planning involves task analysis, identifying relevant prior knowledge, goal setting, selecting appropriate cognitive strategies (conditional knowledge), anticipating possible obstacles to successful completion of the task, and predicting ways of overcoming these. Directing/monitoring includes checking to see that learning is proceeding according to plan and assessing understanding or lack of understanding so that corrective strategies can be used if needed. Evaluating comprises assessment of both processes used to reach the goal and the product from these processes.*

In the context of mathematics education, Schoenfeld (1987), described metacognition by using three components

- 
- Knowledge about one’s thought processes (that can be used to control cognitive processes).
  - Self-regulation: How well does one keep track of what they are doing when solving problems and how well the input coming from these observations is used to guide problem solving actions.
  - Beliefs and intuitions: What ideas does one bring and how does this shape the way they do mathematics.

From the above descriptions of metacognition, it is apparent that it is not easy to completely split the two constructs – cognition and metacognition – as these depend upon each other. The link between cognition and metacognition cannot therefore be ignored in a framework on student learning. Whilst on the one hand, cognition refers to the process by which a person acquires, processes, retains and recalls information, metacognition, on the other hand, refers to a person’s awareness and ability to monitor and control such cognitive processes.

When learners ask themselves questions relating to cognitive processes and decide on answers to these questions, they engage in both cognitive and metacognitive strategies.

Livingston (1997), in elaborating the link between cognition and metacognition, used an example and says if a learner engages in a self questioning strategy while reading, in one way the learner gains knowledge of the concept, and on the other, the learner is controlling his or her approach to, progress and the outcomes of learning. The “rehearsals” processes on the IP model for instance can serve as a good example of both cognitive and metacognitive processes.

Metacognition therefore guides the flow of information through the cognitive system. It has an influence on the decisions made in learning. It influences on the choice, use and maintenance of cognitive strategies. Thus, metacognition helps the learner make informed decisions about how to categorise, attend to, organise and interpret information. Cognitive and metacognitive strategies are therefore dependent upon each other. This interdependency was pointed out by Livingston (1997) who emphasized that any attempt to examine one without acknowledging the other would not provide an adequate picture of the learning process.

#### **4.5.2 Andragogy**

This section discusses andragogy as a concept that can inform more the teaching and learning processes in DE environments. When discussing student learning in a DE environment such as the Zimbabwe Open University, where most of the students are adult learners, one has to consider andragogy and self-directed learning as these inform on adult learning. Knowles (1980) posit that adults have distinct and unique learning characteristics and that they learn differently as compared to non adults. Knowles (1980, p. 43) describes andragogy as “the art and science of helping adults learn”. This can be contrasted with pedagogy which is normally related to the art and science of teaching children. Andragogy and pedagogy should however “not be seen as dichotomous, but rather as two ends of a spectrum with a realistic assumption (about the learners) in a given situation falling in between the ends” (Knowles, 1980, p. 43).

Andragogy provides a set of assumptions that are premised on the characteristics of adult learners. The five assumptions underlying andragogy as presented by Knowles (1980, 1984) are:

- *Self-concept*:-the adult learner has an independent self-concept and is self-directed;

- *Experience*:- the adult learner has accumulated a reservoir of experiences that serves as a rich resource for learning;
- *Readiness to learn*:- the adult learner's readiness to learn is closely related to their changing social roles;
- *Orientation to learning*:- the adult learner is problem-centred and their perspective of time and curricular changes from one of postponed application to immediate application of knowledge;
- *Motivation to learn*:- the adult learner is motivated to learn by internal factors and not by external factors.

An emphasis from Knowles' assumptions is that as people mature they become more independent, more self-directed, and take responsibility for their learning decisions. Instruction for adults therefore needs to focus more on the process of learning than on the content. With specific reference to distance education, Burge (1988) suggested that andragogy can effectively contribute to the design and development of distance education programmes through informing on the quality of learning materials, tutoring and counselling services.



## 4.6 CHAPTER OVERVIEW

This chapter presented the theoretical framework that would guide this study. The study is situated in an information processing perspective of learning. A learning styles model, the Felder Silverman learning styles models is proposed as the learning styles model to guide the discussion on learning styles. Hierbert and Carpenter's (1992) notion on procedural and conceptual understanding is proposed as the theory in which mathematics understanding can be viewed. The next chapter, Chapter 5 will provide a discussion of the methodology for this study.

## **CHAPTER 5: METHODOLOGY**

### **5.1 INTRODUCTION**

The goal of this research study is to gain insight into student learning of mathematics at the Zimbabwe Open University. Focus of the study is placed on the influence of the distance learning environment on student learning processes, that is, how the learning environment supports or deters learning. An interpretive instrumental case study is used to illuminate the learning processes drawn from some first year mathematics distance students who were learning calculus at the ZOU. In Chapter 4, I proposed the constructs of learning styles and mathematical understanding as apposite constructs that could be used to ‘zoom’ into the learning processes of the mathematics distance learners. The object of this study is on understanding the distance students’ experiences with the distance learning environment, identifying the students’ learning styles from these experiences, as well as exploring the possible relationships and influences of the students’ learning styles on their mathematical understandings of the limit of function and derivative of function. The study also presents an opportunity to gain insights into the students’ perspectives of the BSMS distance learning environment.

This chapter outlines the methodology that was used to address the research questions which are stated in Chapter 2. In the chapter, I make a case for an interpretive case study approach. Accordingly, in the chapter, I discuss the research instruments used in the pilot study and in the main study. I also discuss the procedures used to collect the data, the procedures used to analyse the data and issues related to the quality of the study.

### **5.2 AN INTERPRETIVE RESEARCH ORIENTATION**

In view of the three basic orientations to educational research, the positivist, the interpretive and the critical research (Cohen, Manion & Morrison, 2000; Merriam, 1998), an interpretive orientation was adopted for this study. Three main reasons contributed to my decision on using the interpretive approach.

Firstly, the nature of the research problem, where I intended to understand the influence of the learning environment on student learning meant that I needed to investigate student learning processes. The need to understand students' learning processes from the students' perspectives motivated my choice of decision. An interpretive study would therefore enable me to understand the phenomenon from the standpoint of the students, what Cohen *et al.* (2000, p. 22) refer to as “to get inside the person and to understand from within”, thus enabling me to examine the distance learning from the perspective of the students. An interpretive approach would also enable me to listen to student voices, thus capture how the students were learning from the students' actual and lived experiences.

Secondly, an interpretive approach is appropriate for this study which is situated in a distance education environment as there would be a possibility of producing a lot of context bound data. As mentioned by Wellington (2000) and Cohen *et al.* (2000), interpretive studies encourage researchers to look at a context as a dynamic whole. Thus, this enables me to study the phenomenon in its natural settings, contrary to the positivist studies that may view the context as manipulable and as separate from the participant.

Thirdly, an interpretive approach for this study would provide indications on how students learn, as well as provide indications on the role of the distance education environment on student learning. An opportunity for understanding this role from the students' perspectives would be significant as it would inform faculty of reasons for improving the BSMS programme in which the calculus course is housed.

To bring out the richness and the constraints of the research setting, which is the BSMS programme, the interpretive study is carried out following a case study design. Hence, case study design is the subject of discussion for the next section.

### **5.3 RESEARCH DESIGN: CASE STUDY**

Mouton (2001, p. 49) states that research design addresses the question “What type of study will be undertaken in order to provide acceptable answers to the research problem or research questions?”. Yin (2003, p. 19) points out that research design addresses “the logic that links the data collected (and conclusions to be drawn) to the initial questions of study”. These and other descriptions of research design are indicators that the research design plays



a central role in an empirical study as it provides the 'route taken' to address a research problem.

There are various research designs that are available to a researcher when one is planning on carrying out a research study. These include among others, the survey, the experimental, the historical, the developmental, archival analysis and the case study research design, with each one having its own advantages and disadvantages. Although at times the research designs are not mutually exclusive, Yin (2003, p. 5), identified three conditions that could significantly contribute to the choice of a research design, and these are:

- a) the type of research question posed, depending on whether the question is a “who”, “what”, “where”, “how” and “why” type of question,
- b) the extent to which an investigator has control over actual behavioural events, and
- c) the degree of focus on contemporary as opposed to historical events.

For this study, these conditions were applicable in coming up with the decision to conduct the study as a case study.

Case study design is the subject of discussion of this section. In the following subsections, I concentrate on certain aspects of the case study research design.

### **5.3.1 The case study research design**

According to Yin (2003, p. 6-8), case study can be a favourable design when a “how” or a “why” research question is being asked about some contemporary events, whilst at the same time, the researcher has little or no control over the behavioural events.

Bassey (1999, p. 47) gave the definition of case study inquiry as “the study of a singularity conducted in its natural settings”. Yin (2003) provided a more comprehensive definition of case study. He defined a case study as,

*An empirical inquiry that*

- *Investigates a contemporary phenomenon within its real-life context, especially when*
- *The boundaries between phenomenon and context are not clearly evident (Yin, 2003, p. 13).*

Two essential features of the case study approach emerge from Bassey (1999) and Yin's (2003) definitions. These are the investigation of a contemporary phenomenon occurring within a certain context and the importance of the context or environment in which the event is taking place. Bassey (1999) refers to the context as the "natural settings", whilst Yin (2003) refers to it as the "real-life context". Thus, in a case study research both the phenomenon under study and the context of the study are of significant importance.

Stake (2000, p. 437) elaborated on types of case studies. He emphasized that the type of case study depends upon the purpose or outcome of the inquiry. Stake distinguished between three types of case studies: intrinsic, collective or instrumental case study. An intrinsic case study is undertaken to gain a deeper understanding of a particular case in its particularity and ordinariness, a collective case study is the study of a number of cases (multi-site) undertaken in order to gain fuller understanding of a particular phenomenon, and the instrumental case study is when the case is examined in order to provide insight into an issue.

This study, which focuses on students' learning of calculus in a distance education university, is conducted as a single-site instrumental case study.

### **5.3.2 Motivation for a case study design in this study**

Stake (2000, p. 435) pointed out that "case study is not a methodological choice but a choice of what is to be studied by whatever methods, we choose to study the case". The decision to use a case study design as opposed to other research designs is therefore not an issue of case study being a method of inquiry, but an issue of what the researcher wants to do and how it is to be done.

Several issues motivated my decision to follow a case study research design. Firstly, the nature of the research problem for this study motivated the case study research design. The study attends to "how" and "what" research questions which can be better addressed through a case study (Yin, 2003). Coupled with this, is the need to understand a complex "contemporary phenomenon" such as the distance learning of calculus in a "bounded system" (Yin, 2003, p.13) of the BSMS programme at the Zimbabwe Open University. As discussed earlier on in Chapter 1, distance education is a special mode of teaching and learning where learning takes place when the student is separated from the teacher and other students. A case study would therefore assist in capturing the situatedness of the research problem, as well as the complexity of the process of learning in a distance education

environment. As pointed out by Ary, Jacobs and Razavieh (1990), in a case study, emphasis can be placed on understanding why the individual does what he or she does and how the individual responds to the environment. A case study design for this study would result in an in-depth study, thus provide opportunities to involve the students and understand the problem from the perspective of the people who are actually living the experience of learning mathematics at the ZOU.

Secondly, since the Zimbabwe Open University is the only distance education university in Zimbabwe, it presents a unique and typical case of distance offered university level education in the country. The uniqueness and typicality of the ZOU warrants a case study approach so as to gain a deep understanding of the challenges and benefits of DE in the country. Moreover, the university is a relatively young institution, of which most of the programmes including the BSMS programme are new on the market, thus having a lot of potential for improvement in terms of providing quality programmes to the students.

Thirdly, I needed to obtain an in-depth understanding of the learning processes for students in only one ZOU programme, the BSMS programme. A case study would enable me to focus on the BSMS programme, which was a programme of interest and convenience to me since I was involved in it on a day to day basis. A case study would therefore holistically and meaningfully bring out the strengths and weaknesses of the BSMS programme. Furthermore, I could focus on a narrow content domain of limit of function and derivative of function. Hence, provide opportunities to understand how students learn and what kind of mathematical understanding they hold for such fundamental calculus concepts.

Fourthly, the possibility of using several sources of data in the study (Yin, 2003) also motivated the case study design. Using a combination of several data collection methods provides rich data in which to ground the findings. The use of a variety of sources of data usually provides depth for the study, with the intensive probing characteristic of case studies leading to the discovery of previously unsuspected relationships. Other than that, the use of several sources of data is convenient for the study's quality, since the methods can be complementary and serve well for triangulation purposes.

### **5.3.3 Use of mixed methods in case studies**

Whereas case study is a common research design that is used in qualitative research, this does not necessarily mean that all case studies have to rely on qualitative methods. Yin

(2003, p. 15) pointed out that “instead, case studies can be based on any mix of quantitative and qualitative evidence”. Merriam (1998, p. 19) singled out and presented case study as a qualitative research, though she also acknowledged that case studies can be of mixed methods by saying “while case studies can be very quantitative and can test theory, in education they are more likely to be qualitative”. Therefore, the choice of whether to use qualitative or quantitative or a combination of both types of research methods in a study depends on the purpose and the research questions of the study. Several researchers, (Creswell, 2005; Yin, 2003; Miles & Huberman, 1994; Patton, 1980) acknowledge the use of mixed methods when conducting research in an educational setting. Creswell (2005) additionally emphasises the importance of indicating the extent to which qualitative or quantitative research methods are used in a study that uses mixed methods.

The nature of this study, which seeks students’ responses to a distance learning environment in the form of students’ experiences, learning styles and their understanding of some calculus concepts, predominantly would call for a qualitative investigation. Such an investigation would make it possible to interrogate the actual activities that students engage in as they learn calculus. It also allows the study to capture these activities as related by the students themselves. The present study, however, employed a mixed method approach. In line with the recommendation from Creswell (2005), qualitative research methods were used to a major extent whilst quantitative methods were used to a minor extent. This combination of methods served the purpose of bringing out richer explanations of the students’ learning processes.

#### **5.3.4 Overview of certain sources of data used in case studies**

There are various sources of data that can be considered when designing a case study research. Yin (2003, p. 85) identified sources of data that are most commonly used in case studies. These include archival records, documentation, interviews, direct observation, participant observation, and physical artifacts. However, Yin (2003) acknowledged that not all of these sources may be relevant in the same case study, and that no single method has a complete advantage over all the others, but that the methods are complementary.

The current study mainly benefited from information that was obtained from interviews and documents that were generated specifically for the study. For that reason, in the following subsections, I purposely single out and provide an overview of interviews and

documentation as data collection methods in case studies. More detailed and elaborate descriptions of the specific research instruments that are used in the study are presented in section 5.4 when I describe the data collection instruments for this study.

### **Interviews as data sources**

Several researchers (Best & Kahn, 2006; Creswell, 2005; Yin, 2003; Fontana & Frey, 2000; Merriam, 1998; Patton, 1980) have written about the strength of interviews as a source of data in studies with a qualitative orientation. In describing interviews, Patton (1980, p. 196) pointed out that “the purpose of interviewing is to find out what is in or on someone else’s mind”. Fontana and Frey (2000, p. 645) asserted that “interviewing is one of the most common and powerful ways in which we try to understand our fellow human beings”. Thus, interviews provide depth and allow researchers “to enter into the other person’s perspective” (Patton, 1980, p. 196). Case study research, which aims for depth, can therefore benefit from data collected through the use of interviews.

Interviews vary by format, as they can be structured, semi-structured or unstructured (Merriam, 1998; Fontana & Frey, 2000). The structured interview aims to elicit information from members of a group of participants in a consistent and standard manner. The unstructured interview is much more qualitative and uses open-ended questions thus eliciting information in an in-depth and flexible manner. The semi-structured interview is the amalgam of the structured and unstructured interview. As such, semi-structured interviews combine the consistency of structured interviews with the freedom and flexibility in unstructured interviews.

A common form of interviewing is the individual one-on-one interview, in which the interviewer asks questions and records responses from one individual at a time. Other forms of interviewing include the focus group interviews, telephone interviews, mailed or self-administered questionnaires and electronic e-mail interviews, (Creswell, 2005; Fontana & Frey, 2000). Whereas one-on-one and focus group interviews are face-to-face, the other forms are not.

The present study benefited from semi-structured one-on-one interviews.

## **Documentation as data sources**

When documentation is included as a data source in case studies, a common type of documents that are referred to is the kind of documents that are not produced specifically for purposes of the research but are already present in the study setting before the commencement of the study. This includes such documents as public records, administrative documents, meeting minutes, personal records, print press clippings, evaluation reports and other written reports. Yin (2003), acknowledged the use of such types of documents, though he expressed concern since these documents would have been developed for purposes other than the research being studied. Merriam (1998) additionally acknowledged another type of documentation, the “researcher-generated documents”. According to Merriam (1998, p. 119), researcher-generated documentation relates to documents that are prepared by the researcher or generated for the researcher by the participants after the study has begun, and these are generated specifically for the research study. Such documents are beneficial to the study as they are generated specifically for the study and hence enable the researcher to learn more about the phenomenon being investigated.

The current study benefited from researcher-generated documents in the form of students’ written learning journals, questionnaires and calculus tests.

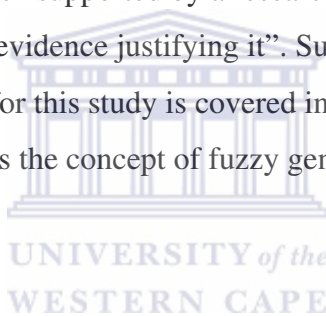
### **5.3.5 Generalisation in case studies**

Case study, by nature of its design, characteristically has depth and as a result reveals strengths since it exudes reality, depicts uniqueness of the case, and can be both illuminating and insightful. However, despite having the depth, case studies lack the breadth and as a result have limitations when it comes to extending the findings to produce generalisations. As pointed out by Merriam (1998, p. 208) case study research is carried out because “the researcher wishes to understand the particular in depth and not to find out what is generally true of the many.”

The issue of generalisation normally centres on the possibility to generalise from the research findings and make inferences for a wider population on the basis of the empirical data collected from samples. Yin (2003, p. 32) refers to this as ‘statistical’ generalisation, and he emphasises that this kind of generalisation is inappropriate for case study research since a case is not a sampling unit. He further discusses what he terms the ‘analytic’ generalisation. In ‘analytic’ generalisation, Yin (2003) emphasises the role of theory in case

study, and posits that though case studies are not generalisable to populations, they are generalisable to theoretical propositions.

Bassey (1999) introduced the concept of ‘fuzzy’ generalisation of which its descriptive definition allows for an element of uncertainty. In the definition, Bassey (1999, p. 12) states that fuzzy generalisation arises from studies of singularities and claims that “it is possible, or likely or unlikely that what was found in the singularity will be found in similar situations elsewhere”. Such a definition indicates that fuzzy generalisation contains in-built qualifiers for uncertainty. For instance, instead of a generalising statement such as ‘*x in y circumstances will result in z*’, one can use a ‘fuzzy’ generalisation statement such as ‘*x in y circumstances may result in z*’ (Bassey, 2001, p. 5), of which the ‘*may*’ serves as the qualifier for uncertainty. The qualifier is thus an indicator that there are other uncontrollable variables that may influence the process *z*. In that regard, Bassey (2001, p. 5) emphasises that fuzzy generalisations must be “supported by a research account which makes clear the context of the statement and the evidence justifying it”. Such an assertion hinges well on the notion of *transferability*, which for this study is covered in section 5.7 where aspects of this study’s quality are discussed. It is the concept of fuzzy generalisation that may be considered appropriate for this study.



## **5.4 RESEARCH INSTRUMENTS USED IN THIS STUDY**

This section describes the research instruments that were used during the data collection for the main study. Data were collected through the use of interviews (Int), and researcher generated documents. The documents were in the form of learning journals (LJ), learning style preference questionnaires (LSPQ), and calculus tasks tests (CTT). The construction and structure of each of these are detailed in the following subsections.

### **5.4.1 The interview**

#### **One-on-one interview**

The study benefited from semi-structured one-on-one interviews, which is a face-to-face strategy of interviewing and requires the presence of both the participant and the interviewer. Considering that the distance education students are geographically dispersed, other forms of non-face-to-face interviews such as the telephone, e-mail or mailed

interviews could have been used. However, e-mail and mailed interview questionnaires were options that were not considered for reasons related to the technological and postal communication challenges in some of areas where students lived. Though the telephone interview might seem to be a possible option, it had its own limitations as well.

Several reasons contributed to why I found one-on-one interviews to be more appropriate for this study than the telephone interviews. Firstly, it was the need to grab an opportunity for establishing rapport with the research participants while at the same time capturing in-depth information from the participants. A one-on-one interview would create an opportunity whereby I could hold a face-to-face discussion with the interviewee, a situation any researcher could take advantage of to gain more information. Secondly, since in a telephone interview there would be lack of direct contact with the participant (Creswell, 2005), I wouldn't be sure on whether the telephone respondent is the intended respondent. Thirdly, it was the intention to avoid over-reliance on telephones since some of the distance students are based in rural and remote areas where there is limited access to telephone communication systems. In addition, it was not possible to know beforehand who would be selected for interviews and whether or not they had access to a telephone.

### **The interview guide**

This study employed semi-structured interviews based on an interview guide that I constructed. The interview guide is appended in this thesis (Appendix X). The interview guide was designed to serve a dual purpose; (a) to capture the students' experiences of learning in the distance education environment, and (b) to capture the data in such a way that aspects of students' learning styles could be read from the students' descriptions of their experiences. The interview guide also helped to guide time usage during the interview, as well as guide on conforming the questions in so that they could capture relevant information for the study.

The interview guide comprised of five sections, A, B, C, D and E. The first page of the guide contained the basic details of the interview such as the date, venue, and time of the interview as well as general reminders. The sections of the interview guide were organised in such a way that each section had a specific focus for the study. However, because of the intricacy of the learning styles construct, most sections were framed around the learning styles dimensions of the F-SLSM.

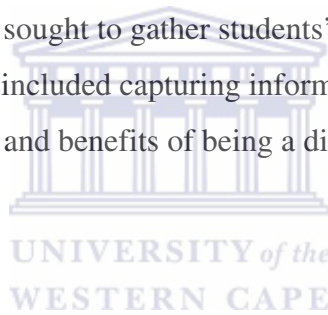


### *Section A*

Section A of the guide sought to gather information on the students' experiences of learning in the distance education environment. In order to cater for the learning styles aspect of the study, the question items in subsections A1, A2, A3, and A4 were framed in such a way that each subsection would attempt to capture characteristic attributes of a particular dimension of the F-SLSM. Thus, A1 intended to capture experiences related to the Perception dimension, A2 intended to capture experiences related to the Input/Receiving dimension, A3 intended to capture experiences related to the Processing dimension and A4 intended to capture experiences related to the Understanding dimension. These were frames to guide the data collection only, though on analysis it would be possible to capture aspects of one dimension from the other subsections, depending on the given responses.

### *Section B*

Section B of the interview guide sought to gather students' experiences of learning in the distance education context. This included capturing information on learning strategies adopted, challenges experienced and benefits of being a distance learner in the BSMS programme.



### *Sections C, D and E*

Sections C, D and E of the guide sought to obtain further clarifications of the LJ, LSPQ and CTT entries, where necessary. Each of Sections C and E comprised of one question. Section C sought further clarifications with regard to students' responses to the learning journal and Section E sought further clarifications with regard to entries in the CTT. Section D sought further clarifications to responses related to the LSPQ. This section was split into three subsections according to the mathematical issue studied. D1 was linked to the LSPQ for limit of function, D2 was linked to that for derivative of function, and D3 mainly focused on further probing students' preferences for forms of representation.

## **5.4.2 Learning journals**

### **Defining learning journal**

A learning journal is a form of a diary made up of a learner's self-report, recordings and reflections of learning processes. For example, for a day's learning session, students can

diarise what they learnt, how they learnt it, what they found difficult, and how they overcame difficulties. In educational settings, learning journals have been developed to serve as learning tools, data collection tools in research or both purposes (Kember, 2004; Langer, 2002; Morrison, 1996; Kember *et al.*, 1996; McCrindle & Christensen, 1995). For purposes of this study, the learning journal is mainly used as a data gathering tool, although it may serve learning purposes for the students.

Learning journals can be created in various formats as they can be presented in the form of written texts, audio or even video recording. Langer (2002) pointed out that, the significant organising concept of a learning journal is in its design and structure. Langer (2002, p.340-341) identifies three types of learning journals that have often been used in education:

- *The unstructured journal* - allows students to produce their own format. Using their own design, students tend to use a free writing format, open to a range of content and structure design. The unstructured nature of a journal makes it difficult to compare with other formats.
- *The structured journal* - carries an imposed form of constraint regarding the manner in which it is written. Students can follow a template, which serves to provide guidance to students on approaching and developing journals.
- *The dialogue journal* - where there is written dialogue between two or more parties in the journal.

In an environment like distance education where it is difficult to observe individual students in learning situations, learning journals may be useful. Robson (2002) supported use of learning journals as research tools. He pointed out that in situations where it would be difficult or impossible for direct observation to take place, the journal could serve as a proxy for observation. Langer (2002) also pointed out that if structured, learning journals can provide a researcher with an opportunity to receive information in a specific format. Thus, the structured journal can allow the researcher to compare students' responses, reflections and feedback on specific learning activities. As much as learning journals can be used as sources of data in a research study, like any other data collection method, the approach may present data collection problems if the respondents are not willing to cooperate. However,

this can be minimised if respondents are aware of the benefits of filling in or writing learning journals.

### **The construction of the learning journal (LJ) for this study**

For purposes of this study, I developed a structured learning journal. The guidelines for the learning journal were constructed in such a way that responses would reveal some learning experiences as well as reveal some learning style attributes of the Felder-Silverman learning styles model (F-SLSM). Thus, the guidelines were developed with a specific purpose, that is, to capture some dimensions of the students' learning styles from the experiences of the students in a learning situation. Ideas on the type, structure and design of the learning journal were adapted from other researchers (Langer, 2002; McCrindle & Christensen, 1995; Morrison, 1996). A sample of the learning journal that was used in this study can be found in Appendix VI.

The journal instrument comprised of four structured learning journals labelled L1, L2, D1 and D2. L1 and L2 were used to make journal entries for learning experiences associated with the limit of function concept whilst D1 and D2 were used to make journal entries for the derivative of function learning experiences. The four journals L1, L2, D1 and D2 were similar in structure, though the content being referred to was different for each of them. The split in content was done so as to prevent students from making congested entries in the LJs.

Each learning journal comprised of seven items. Items 1 to 6, which are open-ended questions, mainly sought insight into the students' reflections on the content as well as experiences associated with learning the limit and derivative of function concepts. The items were also meant to reveal the content that students had difficulties with, why they experienced these difficulties and how they overcame the difficulties.

Responses to items 1 to 6 aimed to reveal the learning processes and activities that students engaged in, in order to overcome the difficulties. This consequently would reveal some learning style preferences. In the following paragraph, I highlight what each of the LJ items intended to capture. However, it is crucial to point out that although items 1-6 intended to capture and reveal attributes of certain F-SLSM dimensions, it is possible that attributes pointing to other dimensions would emerge from any of the items depending on the particular student's responses.

Items 1, 2 and 3 read as follows:

- Item 1: *What do you think are the most important new things you have learnt in these section(s)?*
- Item 2: *Describe the new things that you have learnt.*
- Item 3: *Please give an example of a mathematical task that you think you are now able to solve after working with this section(s) that you could not do before.*

Responses to items 1, 2 and 3 intended to highlight aspects of the content that the student had learnt. In the framework of the Felder-Silverman learning style model, the items intended to capture preferences for the perception dimension, and to a lesser extent also capture preference aspects of the input/receiving dimension.

As for the following items,

- Item 4: *What did you find difficult in these section(s)?* Responses are aimed at capturing the perception and input/receiving dimension.
- Item 5: *Why did you find this difficult?* Responses are aimed at capturing the perception, the input/receiving as well the understanding dimensions.
- Item 6: *How did you try to overcome the difficulties?* Responses to this item would help in identifying aspects of the processing dimension.
- Item 7 gives a closed question “*What do you think helped you most to learn the main concepts?*” In order to obtain responses to this question, guided responses were provided for the student to choose from on a multiple choice list. The student was asked to choose the best three responses out of a list of sixteen. Item 7 intended to capture aspects from all four learning style categories. In the end, the most dominant preferences emerging from the responses to item 7 (across all the four journals L1, L2, D1 and D2) would contribute towards the learning style profiling for that particular student.

During the development of the instrument, the LJ was discussed and constructively criticised with my supervisory team. The LJ was also pilot tested.

### 5.4.3 **Calculus Tasks Test (CTT)**

A calculus tasks test (CTT) that I developed was used to capture students' understanding of certain calculus concepts. The CTT comprised of mathematical tasks based on the calculus concepts of limit of function and derivative of function that were selected and designed for my particular research purpose. The CTT is appended in this thesis (Appendix VII). The primary aim of the CTT was to gain meaningful information regarding students' understanding of the limit of function and derivative of function concepts. Sources of the question items used in this test originated from both the research of others (Williams, 2001; Asiala *et al.*, 1997; Ferrini-Mundy & Graham, 1994) and calculus textbooks (Ostebee & Zorn, 2002; Hughes-Hallet *et al.*, 1994; Stewart, 1994; Thomas & Finney, 1992).

The CTT comprised of two sections A and B whereby Section A contained question items on limit of function and Section B contained question items on derivative of function. Section A comprised of 10 question items related to limit of function and were numbered L1 to L10. Section B comprised of 7 question items related to derivative of function and were numbered D1 to D7. On both sections, each question item except for items L1 and D1 comprised of three parts, viz, the question, the solution and the explanation part. On the 'solution part', respondents wrote their solution to the mathematical problem and on the 'explanation part' respondents explained these solutions.

Questions L1 and D1 sought the definition of the respective concept. The CTT comprised of both routine and diagnostic questions. However, on constructing the instrument, special attention was made to include more the diagnostic questions than routine questions. Responses to question would therefore illuminate a student's understanding of the concepts under study. On how to determine students' mathematical understanding, Hubbard (1997, p. 795) argues that "well thought out questions can reveal a great deal about students' understanding and misunderstanding particularly if asked to give reasons for their responses". Thus, for the CTT, alongside the solution for each of the question items, was the section for an explanation in which the respondent was asked to respond to such questions as "explain why this is so?" or "why do you think this is so?" or "explain how you obtained the answer?" Such questions intended to avail the students with an opportunity to reflect on their answers.

During the development of the CTT, constructive criticism on the instrument was received from my supervisory team. Two fellow graduate student colleagues in the same PhD

programme as I am also commented on the instrument. The set of items was also given to a colleague who is a mathematics lecturer at another university in Zimbabwe for checking. Respective items were revised and improved upon in accordance with the advice and suggestions given. The instrument was also pilot tested and improved accordingly.

#### **5.4.4 Learning Style Preference Questionnaire (LSPQ)**

Data for this study was also collected through the use of a two-part learning styles preference questionnaire (LSPQ) that I developed. Samples of the two parts of the LSPQ are also attached in the thesis (Appendix VIII and Appendix IX). The LSPQ was a mathematical tasks based questionnaire that I designed to elicit students' learning style preferences on the basis of their preferences of the mathematical representations. The tasks used in the instrument only addressed the limit of function and derivative of function concepts.

#### **The LSPQ and the F-SLSM**

The LSPQ was developed so as to provide information on student preferences on how they would like information presented. As illustrated in section 4.3 of Chapter 4, the authors of the F-SLSM associated preferences of certain formats of information presentation to certain learning styles. The LSPQ would therefore inform or give indications of the students' preferences of forms of mathematical representation. This may provide useful information for the learning styles profiling process in terms of the F-SLSM, as dominant preferences would indicate the learning styles. Information on students' preferences of formats of presentation would also provide opportunities of understanding students' experiences with the content that is under study, vis-à-vis, the way the content is presented in the module.

#### **The construction of the LSPQ**

The LSPQ was in two parts. Part 1 contained question items on limit of function and Part 2 contained question items on derivative of function. Part 1 comprised of two sections A and B. Section A gathered data on participants' demographics. Section B gathered data on representation preferences and comprised of three question items based on the limit of function concept. The question items for Part 1 were numbered L1, L2 and L3. Part 2 comprised of only one section, Section A, which gathered data on preferences for representations for the derivative of function concept. The section comprised of three

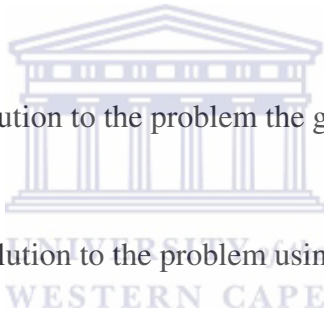
question items based on the derivative of function concept, and these were numbered D1, D2, and D3.

The questions in the LSPQ were structured in such a way that respondents should solve the given problem using each of the following four forms of the solution: (i) algebraic/analytical form, (ii) graphical form, (iii) numerical form and (iv) use of the definition. Throughout the LSPQ, each question item comprised of five parts, the ‘main part’ of the question, followed by four other parts to the question, labelled A, B, C, and D. Below I describe the structures of each of these five parts:

#### *Main part of question item*

The ‘main part’ of each question item had four response items labelled (i), (ii), (iii) and (iv)

- item (i) required a solution to the problem in the analytical/algebraic form of representation,
- item (ii) required a solution to the problem the graphical (visual) form of representation,
- item (iii) required a solution to the problem using the numerical/tabular form of representation and
- item (iv) required a solution to the problem using the definition.



Respondents were asked to solve each problem using each of the forms of representation options (i)-(iv). The solutions to the mathematical tasks were fairly simple. The main reasons for requesting the respondents to solve the tasks were to provide the respondents with an opportunity to have a feel ‘in loco’ of the solution using each of the forms of representation.

#### *Part A*

Part A of an LSPQ item was an immediate follow up of the ‘main part’ of the question. After going through items (i) to (iv) in the main part of the question, the respondents were asked to choose from (i), (ii), (iii) and (iv) an option they would use to best describe or solve the problem. Ideas for this part of the LSPQ on choices of preferences were adapted and modified from Keller and Hirsch (1998).

### *Part B*

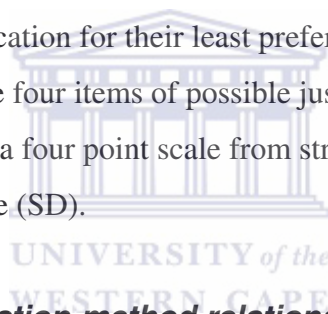
In Part B, respondents were asked to rank their preference choices on a four point scale starting from the most preferred to the least preferred representation. Ideas on ranking were adapted and modified from Maull and Berry (2000).

### *Part C*

Part C sought for respondents' justification for their most preferred choice. It comprised of closed questions where there were five items of possible justifications. Respondents were asked to indicate their choices on a four point scale ranging from strongly agree (SA), agree (A), disagree (DA) to strongly disagree (SD).

### *Part D*

Part D sought for students' justification for their least preferred choice. It comprised of closed questions where there were four items of possible justifications. Respondents were asked to indicate their choices on a four point scale from strongly agree (SA), agree (A), disagree (DA) to strongly disagree (SD).



#### **5.4.5 The research question-method relationship**

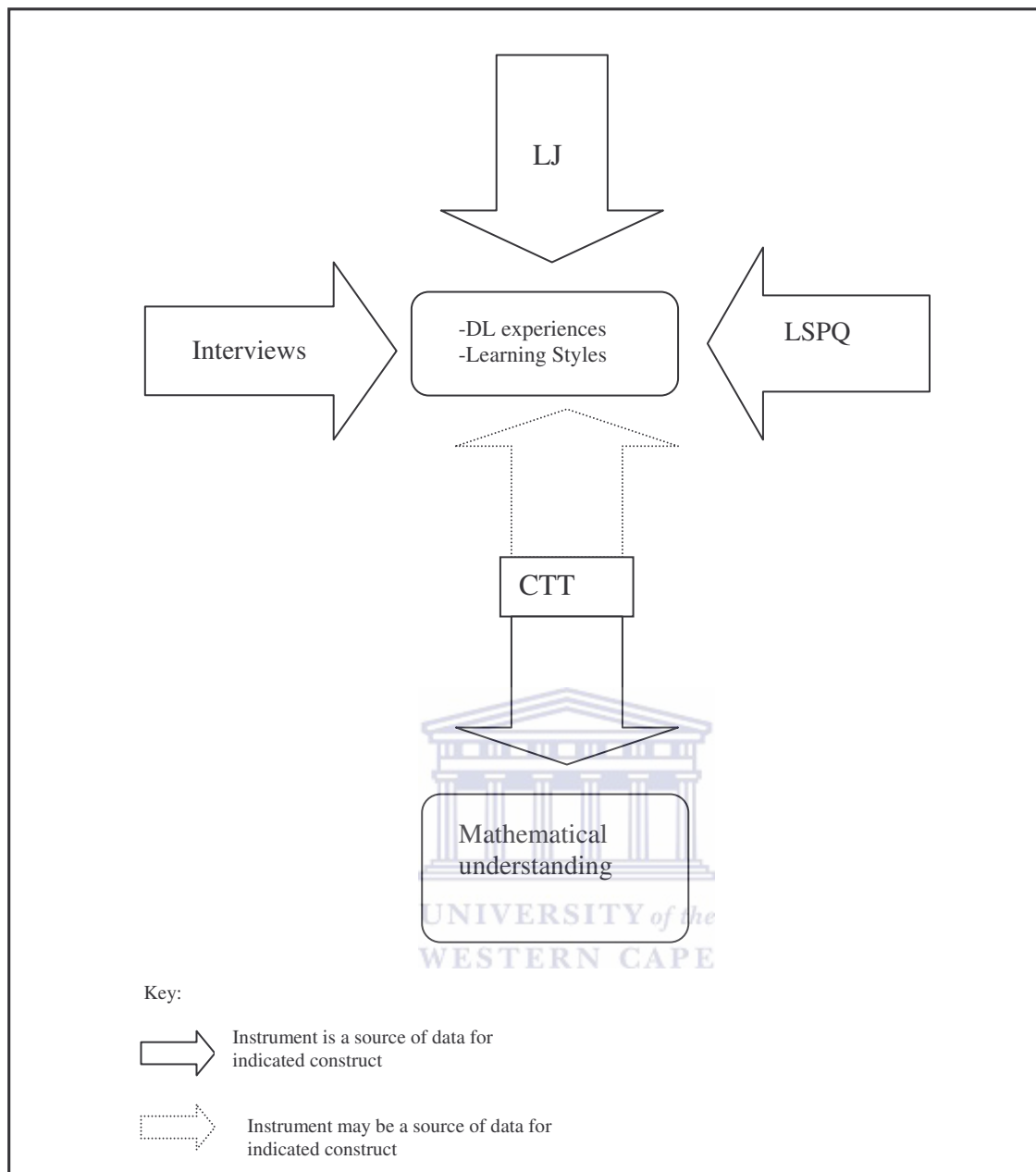
To elaborate on which research instruments were used to collect data for which specific research questions, I constructed a “research questions-method matrix” (Wellington, 2000, p. 50). The matrix is given in Table 5.1.



**Table 5.1:** A Research Question-Method Matrix for this Study

<b>METHOD</b>	<b>LJ</b>	<b>LSPQ</b>	<b>Interview</b>	<b>CTT</b>
<b>RESEARCH QUESTION</b>				
How do first year mathematics distance students experience distance learning at the ZOU with specific reference to the learning of calculus?	•	•	•	
How can distance students' experiences of learning calculus be used to inform on the students' preferred learning styles?	•	•	•	
What relationships exist between students' preferred learning styles and learning outcomes as represented by how students understand the limit of function and derivative of function concepts?				•

While Table 5.1 provides a matrix to clarify the research question-method relationships, Figure 5.1 below presents a pictorial diagram of the 'construct-source of data' relationship.



**Figure 5.1:** Sources of data for the constructs of this study

As shown in both Table 5.1 and Figure 5.1, only one method (the CTT) was used to address the research question related to the mathematical understanding construct, and more than one method was used to address the other two research questions which are related to the distance learning experiences and learning styles. A combination of methods was necessary for both the distance learning and the learning styles as these methods may provide sufficient data to allow information on students' experiences and learning styles to emerge from the data.

## **5.5 THE PILOT STUDY**

Several researchers (Yin, 2003; Cohen, Manion & Morrison, 2000; Wellington, 2000; Bell, 1999) highlight the importance of conducting a pilot study in any research study. A pilot study for this study was conducted on a small scale in 2004, during the September to December 2004 semester. This section presents a discussion on the pilot study as well as how the pilot study informed the main study. Part of the results for the pilot study were written in an article which is due to appear in a book chapter (Tsvigu, Persens, Breiteig & Ndalichako, in print)

### **5.5.1 Purpose of pilot study**

The pilot study was conducted for several reasons. Firstly, the pilot study was conducted with an aim to refine the research instruments. This mainly centred on determining the clarity of the instructions on the research instruments, clarity of the questions and items on the instruments as well as checking the relevance of the questions with respect to the constructs of learning styles and mathematical understanding. Secondly, the pilot study aimed to try out how the instruments would be administered in the field as well as determining on average, the time period that would be required by a participant to complete the instruments. Thirdly was the need to obtain some empirical data and hence gain indications on how the data from the main study would be managed and analysed.

### **5.5.2 The pilot study participants**

Eleven B.Sc. Mathematics and Statistics students from the 2004 group (Intake 6) from the ZOU Harare regional centre who were registered for the (MTD101) Calculus 1 course were involved in the pilot study. The decision to involve BSMS Intake 6 students for the pilot study was motivated by the need to use a group of students that is experiencing the learning of calculus in the same distance education environment and under more or less the same conditions as the prospective research subjects. The choice of a sample similar to the one that is to be used in the main study conforms to Bell's (1999) recommendation that a pilot study ought to be conducted on a group that is similar to the one that will form the population of a main study. No specific sampling criteria were used for selecting the pilot study participants. The students participated in the pilot study on a voluntary basis after a

debriefing of the purpose of the study. For convenience purposes, only students from the Harare regional centre were involved in the pilot study.

### **5.5.3      *The instruments used in the pilot study***

The data for the pilot study was collected using the learning journals, the learning styles preference questionnaire and calculus tasks test. This subsection presents the discussion on the structures of the instruments that were used in the pilot study and subsection 5.5.5 gives a discussion of the lessons learnt from the pilot study inclusive of the related adjustments of the instruments.

#### **The LJ**

The construction of the LJ was similar to the one that has been described in subsection 5.4.2 which was used in the main study, except that for item 7, students were choosing five items in the pilot study and in the main study these were reduced to three.

#### **The LSPQ**

The construction of the Learning Styles Preference Questionnaire that was used in the pilot study slightly differed in terms of structure with the one described in subsection 5.4.4 that was used in the main study. As in the main study, each question item on the LSPQ comprised of the main section of the question which had four parts labelled (i), (ii), (iii) and (iv), followed by four other parts to the question, labelled A, B, C, and D. Differences were however in the structure of the main section of each question, and in parts C and D, while parts A and B remained the same for both the pilot and the main study.

For the pilot study, items (i), (ii), (iii) and (iv) comprised of a pre-worked solutions of given mathematical tasks, in which each solution was presented in its analytical/algebraic, numerical, graphical, and verbal forms of representation. Respondents were expected to choose from these pre-worked solutions their most preferred and least preferred forms of representations for each task. Part C and D for the pilot study were open questions where students were asked to write out their justification for their choices for their most preferred and least preferred representations.

## **The CTT**

The items in the Calculus Tasks Test were similar to the ones in the main study, however some of the items included in the pilot study were very simple and less diagnostic in terms of diagnosing students' understandings. These were accordingly adjusted. The structure of the paper for the pilot study CTT was such that every question was followed by a blank space for writing the solution. This format resulted in a bulky question paper and hence had to be adjusted.

### **5.5.4 Procedures and analysis of pilot data**

The pilot study was conducted during the September to December 2004 semester. Initially sixteen students had volunteered to participate, though only eleven continued with the study. Some learning journals (each set comprising of learning journal L1, L2, D1, D2) were issued to sixteen Intake 6 BSMS students from the ZOU Harare regional centre, during the students' first tutorial session. The students were given specific instructions to fill in the LJs as they were studying limit of function and derivative of function concepts during the course of the semester. The students were also given a date on which to return the LJs and that same date was also set as the date on which the students would write the CTT and the LSPQ. Only eleven students returned the LJs and as a result only 11 students wrote the CTTs and the LSPQ. Furthermore, four students did not have complete information on one or more of the instruments; hence these sets could not be used for the complete trial LS profiling process. The LJ data for three students were qualitatively analysed. LS profiles emerging from the LJs for two students are included in the book chapter mentioned above. Data from the CTT and the LSPQ were closely looked at and shortcomings of the instruments were identified and recommended for further improvement.

### **5.5.5 Lessons learnt to inform the main study**

The pilot study served to inform and reshape the way the main study was to be conducted. The pilot revealed that it was necessary to adjust the instruments so that they captured the essential aspects of the students' learning processes. It also informed on certain aspects of the field procedures and on how important it was to have complete sets of data from the respondents. In the next subsection I present a discussion of the lessons learnt from the pilot study in terms of the research instruments and of the field procedures.

## Research instruments

The following recommendations and adjustments were made on the research instruments:

### 1. Interviews

One of the major lessons learnt from the pilot study was that there was a need to include interviews as a data collection strategy. The interviews would aim to get students to talk about their experiences of the distance education environment. A shortcoming of the instruments used in the pilot study was that the data was not sufficient enough to support the interpretation of an individual student's learning processes. The most intricate of the processes was the learning styles as it required data to cover the learning styles dimensions of the Felder Silverman learning styles model. Although it was possible to identify emerging LS profiles emerging from the LJ data, the data was not sufficient to build a strong case of the emerging LS. Hence it was found necessary to include interview data so that all the data would be complementary. Furthermore, since most of the instruments used during the pilot study produced data in the form of written text, an interview would provide an opportunity for gaining in-depth information by acquiring the spoken verbal perspective of the student learning processes.

An interview guide, as described in section 5.4 of this chapter, was therefore developed. Though the interview guide was developed after the pilot study had been carried out, the guide was trial tested before taking it out to the field. The guide was pre-tested on two colleagues whom I worked with, one worked as a full time lecturer and the other as a part time lecturer. The main aim of pre-testing was to check on the clarity of the questions and to determine if the questions were adequate in providing LS related information. There were no major adjustments recommended following the pre-test except for comments on improving the clarity of some of the interview questions.

### 2. LJ

There were no major adjustments for the LJ. The only change was on item 7 as pointed out earlier on.

### 3. CTT

It was observed that some of the mathematical tasks on the CTT were too simple and too straightforward, and hence were not diagnostic enough to enable one to outline a student's mathematical understanding for the concepts under study. These items were removed and were replaced with more diagnostic mathematical tasks, which would reveal students' conceptions and misconceptions of the concepts under study. For instance, items with *piece-wise defined functions* or *division by zero* or use of *infinity* on limit of function tasks were included in the CTT. The layout of the question paper was also changed so as to reduce on the bulkiness of the question paper. A three column row was used for each question in which the left column was smaller and contained the item number, the middle column contained the question and the right column provided the blank space for the solution. Furthermore, an additional feature seeking for the student's explanation of the solution was included on the solution blank space.

### 4. LSPQ

Adjustments on the LSPQ were made on the main section of each question and on parts C and D of the paper. Instead of providing pre-worked solutions for items (i), (ii), (iii) and (iv) for students to choose their most and least preferred options from, the instrument was adjusted such that the solutions were provided by the students. Thus alongside each item was a blank space available for the student to write out the solution. This enabled the student to get a feel of the solution in each of the four forms of representation. In part C and D, instead of having open questions, these were changed to closed items which were similar for each of the questions items. The closed items provided possible justifications from which the respondents to rank the response on a four point scale ranging from strongly agree to strongly disagree. The decision to use closed questions was motivated by the need to focus students' responses on the justification of their preferences as most responses given to the open questions during the pilot study did not yield meaningful information. However, the few meaningful responses to the open questions generated ideas for the closed questions items.

## Field procedures

Three issues of importance regarding the field procedures emerged from the pilot study experience and from the discussions held with some of the pilot study participants:

1. The sequencing of the administration of the research instruments was important.
  - The LJ should be dispatched to the regions early enough in the semester, before the students' first tutorial meetings so as to maximise on student participation and to give students sufficient time with the journals.
  - Allow a minimum gap in time of at least six weeks before the CTT and the LSPQ are written. By then the students are assumed to be well acquainted and comfortable with the content.
  - The CTT should be written before the LSPQ so as to minimise chances of students' responses being biased on forms of representation which were the main focus of the LSPQ. In addition, since the CTT comprised of more challenging and engaging mathematical tasks than the LSPQ, it would be better to allow students to write the CTT first when the students would still be "feeling mentally fresh" (to use the phrases of one pilot participant).
2. The pilot study influenced facilitated the making of an informed decision with regard to selection of participants for interview. That is, the decision to select interview participants on the basis of their mathematical understandings as reflected on the CTT was arrived at after the pilot study.
3. The pilot study also provided an estimate of the time that a participant would require completing the CTT or LSPQ.

## 5.6 THE MAIN STUDY

The study progressed in three stages. The first stage was at the beginning of the semester with the selection of the participants and the distribution of the learning journals.

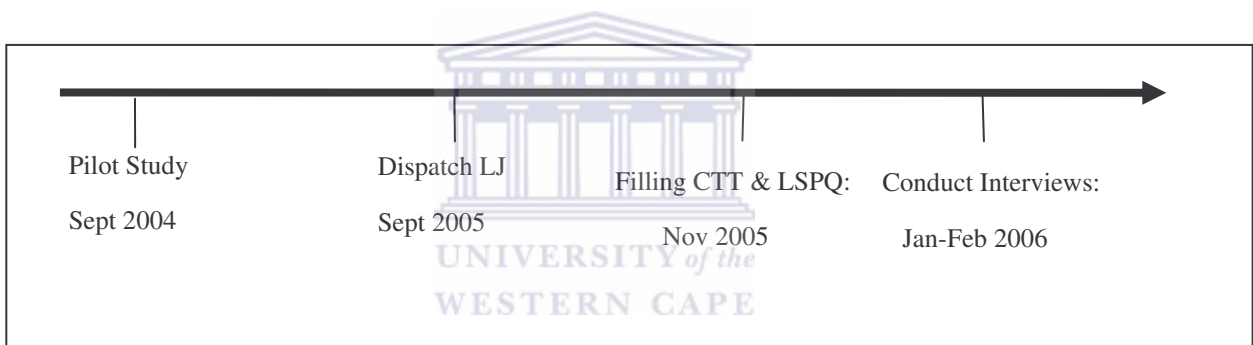


Participants were expected to complete the learning journals during the semester as they were learning the limit of function and derivative of function concepts. The second stage of the study was the administration and the writing of the CTT and the LSPQ and the third stage involved the selection of the participants for interviewing followed by the interviewing process. This section discusses the research process through the three stages, from the selection of the participants to conducting of the interviews.

### 5.6.1 Data collection timeline

The initial plans were to collect data for the study during the first semester of 2005, that is, the period September to December 2005. However, the interview process stretched to February 2006.

Figure 5.2 below gives the timeline for the research process for this study.



**Figure 5.2:** Research process timeline

### 5.6.2 Selection of the participants of the study

The plan of the study was to collect data during the first semester for the students attending first year of the B.Sc. Mathematics and Statistics programme at the ZOU. The study involved Intake 7 distance education students who were registered for the course Calculus 1 (MTD101) in the B.Sc. Mathematics and Statistics degree programme at the ZOU. Calculus 1 is a course that is offered during the first semester of the students' first year of study.

Since distance education involves students who are geographically dispersed and are situated in various parts of the country, it would be practically impossible to involve all the students in this study of a qualitative nature. Hence, there was need to select and involve only a sample of students in the study. There were three levels of sampling involved in this study.

There was the selection of the regional centres to involve in the study, then selecting from these regional centres a group of students that would participate in the study, then finally select from that group, the participants for interviewing.

### **Selection of ZOU regional centres**

It was necessary to first select the regional centres to involve in the study since operationally since students at the ZOU are clustered by regional centres. It is in these regional centres where student learning activities are managed and where students occasionally travel to. It was therefore meaningful to involve the students in the study in their normal coordinated operating frameworks as this would make the coordination of the students for the research purposes easier and more manageable.

Four ZOU regional centres namely Harare, Mashonaland Central, Mashonaland West and Masvingo were involved in the study. Convenience sampling was used to select the ZOU regional centres. Two factors mainly contributed to the decision to conveniently sample the regional centres. First was the aspect of accessibility to the regional centre in terms of distance, since the researcher intended to travel to the regional centres for interviews. Coupled with accessibility was the availability of someone who would be readily available and prepared to assist in coordinating the students for research purposes in that regional centre. As pointed out by Cohen *et al.* (2000, p. 102) on convenience sampling, “The researcher simply chooses the sample from those to whom she has easy access”. However, a notable disadvantage of convenience sampling is that generalisations of findings are limited as the sample does not represent any group other than itself.

### **Selection of students**

#### *Selection of research participants*

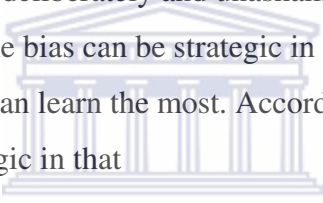
Twenty-six students from the four ZOU regional centres volunteered to participate in the study by virtue of their returning the research instruments. Since by design, the study required continuity by tracking a student through the three data collection instruments and possibly an interview, it was important that the students volunteer their participation in the study at the initial stage of the research. Allowing the subjects to volunteer also enabled me to gain the students’ cooperation. For those students who opted out of the study, outright at

the beginning or during the course of the research process, their decision not to participate was respected without prejudice.

### *Selection of interview participants*

Purposive or purposeful sampling was used to select the participants for the interviews. From the twenty-six volunteer participants, a smaller group of six participants was purposefully selected to participate in interviews, though as it will be explained later in the thesis, interview data for only five participants is reported in the study.

In purposive sampling, the sample is selected on the basis of certain criteria and for specific purposes. Cohen *et al.* (2000, p. 103) elucidate this by saying “In purposive sampling researchers handpick the cases to be included on the basis of their judgement of their typicality. In this way they build up a sample that is satisfactory to their specific needs.” Purposive sampling is therefore “deliberately and unashamedly selective and biased” Cohen, *et al.* (2000, p. 104). However, the bias can be strategic in that it can allow the researcher to select a sample from which one can learn the most. According to Patton (1980, p. 101), purposive sampling can be strategic in that



*Instead of studying some representative sample of clients in a program, decision makers may decide that they can learn the most by studying and understanding the unusual cases in the program: a few more of the people who are really struggling and a few of the people who are really doing well. In many cases more can be learned from intensively studying extreme cases than can be learned from trying to determine what the average case is like.*

Hence, as pointed out by Patton above, in order to do purposive sampling certain information must be known about variations among cases.

In this study, selection for the interview participants was restricted to those participants who had a complete set of data comprising of the LJ, the LSPQ and the CTT so that there would be sufficient data to build on for the profiling. Selection was then further narrowed down on the basis of the information originating from the participants’ responses in the calculus tasks test which reflected their understandings of the limit and derivative of function concepts. Two of the selected interview participants had very good responses to the calculus tasks test and seemed to show a thorough understanding of the concepts limit of function and

derivative of function. The other two seemed to be struggling with understanding the concepts and had poor responses to the CTT and the last two were of average performance.

Purposive sampling was employed at this stage of the study as it provided the researcher with an opportunity to strategically select on a group of participants from which there was a potential to learn the most about the phenomenon of learning calculus in the BSMS distance environment. However, despite the fact that purposive sampling does satisfy the needs of the research, care has to be taken when intending to generalise since data from a purposefully selected sample cannot be generalised to a wider population.

### **5.6.3      *The procedures***

This subsection discusses the procedures that were involved during the research process. Since the research was conducted in an environment where students are geographically dispersed and where students happen to meet as a group on rare occasions, the dispatch and administration of the research instruments for this study had to be carefully planned so as to ensure that the data collection process was not derailed in any way. One major concern during the data collection process was to ensure that by the end of the second phase of the study, there were complete data sets for as many individuals as possible. Each data set would comprise of LJs, LSPQ, and CTT responses for that particular participant. This would in turn provide a larger pool of participants to consider for selection for further in-depth interrogation. Another concern was to ensure that all research related expenses incurred by the participants and the RPCs were reimbursed. Apart from the reimbursements of the expenses, participants were also offered a small monetary token of appreciation.

At the beginning of the data collection process, all participating students were issued with unique research identification codes, which they were requested to remember and to consistently use for the purposes of the research. The codes served a dual purpose, to enable the researcher to keep track of the students' responses in all the instruments on analysis as well as to preserve the anonymity of the students' records. Because of the nature of the study, where follow-up interviews would be required for some selected but yet still unknown participants, the Regional Programme Coordinators (RPCs) were requested to maintain a record of the participants, their research identification codes as well as their contact details. This was done to ensure that the student would be reachable in the event that further information or a follow up interview were required for the study. However, in line with

educational research ethics, and as will be detailed in section 5.7, all personal details were treated with confidentiality.

### **Instrument Administration**

Four Regional Programme Coordinators from the targeted regional centres assisted the researcher in coordinating the research participants as well as administering the research instruments in their respective regions. Throughout the various stages of the study, the instruments were administered, handed out and collected from the participants by the RPCs, and all this was done in person. Administering the instruments in person was beneficial to the study in several ways, among others it was to: (a) use that opportunity to brief students about the study, (b) to optimise on instrument return, and (c) to establish rapport with the participants.

It was necessary to ensure that the instruments were timeously dispatched to the regional centres. Any delays in dispatching the instruments would have downstream negative effects on the study. For instance, delays in the dispatch of the LJs would decrease the chances of having the instruments administered in person, consequently losing out on some participants, as it would be more difficult to reach the students once they went back to their respective homes. Secondly, there was need to ensure that the instruments which required supervision and students' independent responses were well coordinated and run at more or less the same time so as to eliminate any chances of students sharing questions and responses. Failure to ensure coordination would result in compromising the findings of the study.

#### *Dispatching the research instruments to the RPCs*

Courier services were used to dispatch the packs of research instruments to and from RPCs in Masvingo and Mashonaland West regional centres, whilst for the other two regional centres the packs were hand delivered to the RPCs. Each pack contained instructions for the RPC, on what was to be done or how to administer the instruments. In addition to that, a verbal follow up discussion was held with the RPCs, either in person or on the phone.

The research instruments were dispatched to the regional centres in two batches, with the learning journals being dispatched early September 2005 and the LSPQ and the CTT being dispatched early November 2005. Dispatching the instruments in two batches was deliberate so that the students would receive the LJs at the beginning of the semester, by the first

tutorial session and then write the CTT and the LSPQ after they had gone through the calculus concepts. Timing for the distribution of the LJs at the first tutorial session was vital since tutorial sessions provide that rare occasion where the RPCs and the students physically meet at a central venue. Furthermore, students rarely miss attending the first tutorial. This provided the study with opportunities to maximise on student participation as well as informing the students about the research study in person.

The data collection for the study progressed in three phases. These are discussed in the following subsections:

#### *Administering the learning journals*

The first phase of data collection involved administering the learning journals. A total of forty learning journals were dispatched to the four regions, of which thirty two students collected and agreed to fill them in. However, only 26 students returned the learning journals with 25 of them returning the complete sets of learning journals, L1, L2, D1 and D2, and one student returned only learning journal L1.

At the beginning of the semester, each of the 'prospective' research participants was issued with four blank learning journals, labelled journal L1, L2, D1 and D2. The participants were asked to fill in the LJ after going through the limit of function and the derivative of function sections in the module. It was emphasised to the participants that journal completion was an individual's responsibility and not a shared activity. The participants were also encouraged to complete all items of the learning journals and as accurately as possible.

Although on designing the LJ, a date in October was set *a priori* for when the students were to return the LJs, this date was shifted to November 2005. The new date was timeously chosen to be in sync with the weekend when all the students were most likely to be in the regions writing their in-class tests for the BSMS programme. Nevertheless, the shift on submission dates for LJs benefited the study in three main ways. Firstly, this facilitated an opportune time to maximise on the return rate of the LJs, as the LJs could be collected from the participants in person at a time when almost all students would be in their regional centres. Secondly, the extension allowed the students to have the LJs for a longer period, thus availing the participants more time to interact with the content and the learning environment. Thirdly, as the students were preparing for their in-class tests, they could identify and relate more with the LJ entries, such as the new things learnt, the difficulties

experienced, why they experienced the difficulties, and how they overcame the difficulties, and consequently provide information that was more meaningful for the study.

### *Administering the CTT and the LSPQ*

The second phase of the data collection for the research study involved the administration of the CTT and the LSPQ instruments. Plans were to have these run under supervision, in one day and at the same time across the four regional centres. This meant that the participants were expected to travel to their respective regional centres on a specified weekend for that purpose. For the sake of flexibility and accommodating student's schedules, the date for writing these instruments could not be pre-set before the commencement of the semester as is the usual case in the DE environment, but had to be negotiated and agreed upon by those involved during the course of the semester.

The CTT and the LSPQ were written in November 2005 on a Saturday morning. The decision to have these instruments written on a weekend and on the same day and under supervision had a dual purpose. Firstly, it was to minimise the chances of participants making repeated trips for purposes of the research, thus minimising both interference with the participants' schedules and travel expenses. Secondly, it was to minimise chances of students sharing responses since the instruments required independent responses. Since learning styles and mathematical understanding are individual constructs, it was essential that these instruments be written under supervision so that participants could produce independent responses that would serve well for purposes of this research.

However, both the CTT and the LSPQ were not constrained in terms of time. On average the participants required two hours to complete the CTT and one and half hours to complete the LSPQ. Participants wrote the CTT before responding to the LSPQ. This sequencing was deliberate as a recommendation from the pilot study.

### *Conducting the interviews*

The third phase of data collection involved conducting interviews with the selected students. I conducted all the interviews. Semi-structured one-on-one interviews were conducted with six participants and all interviews were audio taped. However, the recording of one of the interviews was not successful due to the malfunction of the tape recorder.

An interview guide (refer to Appendix X) was used to guide the interview discussion. On average, an interview session lasted one hour. The interviews were held during the months of January 2006 stretching to February 2006. Although I had planned for the interviews to take place up to the week of 20<sup>th</sup> December 2005, this was not possible as some participants indicated that they were busy with their end of semester examinations (that were written in December 2005). Some indicated that they were busy with preparations and or travelling for the Christmas festive season. Therefore, all the interviews were rescheduled for the beginning of the year 2006. Two participants postponed the interviews to later dates, one citing work commitments and the other one citing family commitments. This resulted in repeated trips to the interview venues.

As the researcher and residing in Harare, I had to travel long distance journeys so as to meet those participants residing out of Harare for interview purposes. Four of the interviews were held during weekends, as this was the best time when the participants were available. The other two interviews were held during week days. One of the interviews was held at the student's home and the other five were held at ZOU offices, either at the regional centre or at the national centre. Interview venues were agreed upon by both the participant and me, mainly on the basis of accessibility, availability of space and availability of a quiet place suitable for recording purposes.

Interviews for all participants were held behind closed doors and were held one-on-one between the participant and me. Although the language of communication during the interviews was set to be English, participants were allowed to communicate in their local language, which in this case was Shona. Three of the participants conversed in English throughout the interview, two extensively conversed in Shona as they indicated that they could express themselves clearly in their local language and one participant used both languages.

### **Coming up with interview transcripts**

At the beginning of each interview permission was sought from the participant to record the interviews. Five of the interviews were successfully recorded and were clearly audible. The recording of one interview was not successful as there was a technical problem with the tape. The tape was inaudible on many parts of the discussion and hence it presented problems on



transcription. This interview was therefore not considered further in the study. As such, interview data from five participants is reported in the study.

For each recorded interview, I first listened to the tape so as to familiarise myself with the data. I then fully transcribed the tape verbatim and typed out each interview transcript in MS Word. It was necessary to have the transcripts verbatim so as to avoid losing any information that could be useful for the study of learning processes. To improve on the transcriptions I had to listen several times to the tapes and compare with the typed text. Where it was deemed necessary and where the explanations from the participants were not clear, I included comments and explanations and this appears as the italic text in parenthesis in the transcripts.

With regards to the interviews which were conversed in Shona, I first of all transcribed them in the language of discussion, which is Shona, and then translated them to English. Since my local language is Shona, I personally did the translations from Shona to English. As a way of validating the translations, two different persons, both knowledgeable in the Shona language, checked and verified the translations. A colleague from the same work place as me and another one who was on the same PhD programme with me did the checking. Relevant comments and suggestions were made and these were incorporated on the transcripts.

The transcripts for the five students are appended in this thesis (refer to Appendix XI ). For those transcripts that required translation, the original transcript in Shona is also appended alongside the English version of the transcript. Some excerpts of the transcripts are used in Chapters 6 and 7 to elucidate some findings where necessary. In both the transcripts and the excerpts used in the chapters on findings, the notation “(…)” means that some text has been left out.

## **5.7 ISSUES OF QUALITY FOR THIS STUDY**

Traditionally, discussions on the quality of an empirical research study are centred on the concepts of validity, reliability, objectivity and ethics. Although the principles of validity and reliability are acceptable, they are seldom used in qualitative studies. In a qualitative study the concepts of validity, reliability and objectivity are replaced by the concept of trustworthiness. Lincoln and Guba (1985) have noted that the notions of validity and reliability have their foundations rooted in the positivist paradigm. They suggested that for a

qualitative study, these notions must be replaced by criteria that are more rigorous and trustworthy.

This section discusses trustworthiness and ethics as aspects of quality for this study.

### **5.7.1 Trustworthiness**

Trustworthiness involves the four notions of credibility, transferability, dependability and confirmability, (Guba & Lincoln, 1989; Lincoln & Guba, 1985) as parallels to the notions of internal validity, external validity, reliability and objectivity that are normally encountered as criteria for quality in a positivist study.

The following subsections discuss credibility, transferability, dependability and confirmability as aspects of trustworthiness of this study.

#### **Credibility**

Credibility can be considered to be the parallel to internal validity in the positivist paradigm. Credibility of a study is about how another person could recognise that the findings and interpretations of the study are credible. Lincoln and Guba (1985) suggest that credibility can be addressed by attending to persistent observation, prolonged engagement in the field, triangulation, member checking and peer debriefing. The following discussion will dwell on four of these that were found most applicable for this study.

#### *Persistent observation and prolonged engagement in the field*

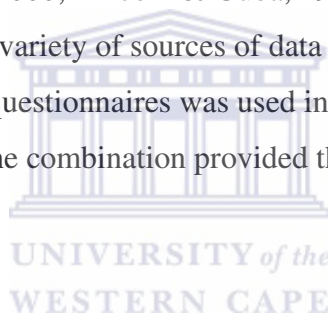
The research process for this study incorporated some persistent observations starting from the preparations for data collection through the data collection process. Throughout the process of development of the research instruments, there was constant engagement and interaction with the research instruments by my supervisory team, my colleagues and myself. The constructive criticisms from my supervisory team and colleagues facilitated an opportunity for me to persistently reflect on the nature of the 'would be' data. The piloting of the instruments accordingly provided an opportunity for me to have a feel of the preliminary data.

The data collection process for this study was conducted within one semester. However, the interviews were held during the students' vacation period after the semester had ended. The

time could be considered to be relatively short for an interpretive case study. This was however due to the nature of the study which focused on student learning of calculus, a course which is offered in one semester at the ZOU. Furthermore, there was the need to fit into the students' busy schedules; a longer and prolonged interruption with the students' schedules might have spoiled the cooperative relationship that existed between the participants, RPCs and me. Despite these time constraints, the data that was collected through the various sources of data employed in this study was intense and plentiful.

### *Triangulation*

Researchers identify four types of triangulation that may be used in research namely (a) data triangulation (in terms of time, space and person triangulation), (b) investigator triangulation, (c) theory triangulation and (c) methodological triangulation, (Yin, 2003; Wellington, 2000; Cohen *et al.*, 2000; Lincoln & Guba, 1985). This study benefited from methodological triangulation. A variety of sources of data in the form of interviews, learning journals, calculus tasks and the questionnaires was used in the study. These sources of data complemented each other, and the combination provided the study with enough opportunity to triangulate the findings.



### *Member checks of raw data*

The raw data of this study comprised of written texts from the learning journals, calculus tasks test, learning styles preference questionnaire and transcripts of the interviews. Member checks of the interview transcripts occurred to a certain degree since I could only access two participants to check their transcripts. I did not manage to have the other three students to check their interview transcripts as they were inaccessible either in terms of distance or time. The two students who were accessible had an opportunity to read through the transcripts. No major concerns were raised.

### *Peer debriefing*

Peer debriefing was held throughout this study. My thesis supervisory team and colleagues constantly interrogated the research process. The constant questioning of the research methods, findings, and assertions provided valuable insights into the study which challenged me to clarify claims I made about the study.

## **Transferability**

Transferability is the parallel to external validity or generalisability. It refers to the extent to which research findings could be generalised or transferred to other contexts or settings. By design, the current case study does not seek to generalise findings to a wider population. Any generalisations made from this study depend on the relationship between the reader and this text, and should be viewed in the realm of fuzzy generalisations as discussed in section 5.3.5 of this chapter. However, transferability in this study has been enhanced through the use of thick descriptions (Merriam, 1998) of those facets that are crucial to understanding the findings. Thick and comprehensive descriptions have been provided for the research context, the research assumptions as well as the data collection process. These thick descriptions serve to enable readers "... to determine how closely their situations match the research situations and hence, whether findings can be transferred" (Merriam, 1998, p. 211). Furthermore, thick descriptions of the data have been used to emphasise the voices of the participants.



## **Dependability**

Dependability is the parallel to the concept of reliability in a study that is based on a positivist paradigm. Traditionally reliability is based on the assumptions of repeatability and is concerned with whether another researcher could reach the same findings and conclusions if the same study is conducted all over again.

However, in a qualitative study, dependability refers to the stability of findings over time. It emphasises the need for the researcher to account for an ever changing context within which the research occurs. In this study, dependability is evidenced through an audit trail of the data sets for five participants that are appended as Appendix XI. The data sets include the participants' learning journal entries, LSPQ responses and interview transcripts. Including the data sets also serves "to enable other investigators to review the evidence directly and not be limited to the written case study reports" (Yin, 2003, p. 102). Furthermore, dependability for this study was also enhanced through the triangulation of the methods.

## **Confirmability**

Confirmability is the parallel to the concept of objectivity in a positivist oriented study. It refers to the degree to which findings could be confirmed or corroborated by others. In addition to questioning the integrity of the findings, confirmability also questions whether

the findings are free from bias or not, since in a qualitative study the integrity of the findings is rooted in the data. In order to help satisfy confirmability, it is necessary to make the research process as ‘trackable’ as possible by leaving an audit trail (Guba & Lincoln, 1989; Merriam, 1998). The audit trail may also include explaining how the data were collected and how the findings were arrived at. For this study, a ‘trail’ that is mentioned in the earlier sections of this chapter also contributes to the purpose of confirmability. In addition, the appended raw data enhances the confirmability of the study.

### **5.7.2 Issues of ethics**

In any research study it is important to take into consideration issues of ethics. Consideration of ethical issues enhances the quality of a research study. Wellington (2000, p. 54) described ethics as “the moral principles, guiding conduct, which are held by a group or even a profession”. Thus, research ethics are principles concerned with the way researchers act or behave in a research process. Ethics serve to protect the research participants, the researcher, the institution or organisation in which the study is being carried out, as well as the profession. Since in the education profession research usually provides situations where “people are studying people” (Wellington, 2000, p. 54), it becomes important that an educational researcher observes and adheres to the ethical guidelines of the discipline. Merriam (1998) and Wellington (2000) therefore emphasise that ethics are the researcher’s responsibility.

In the literature, several authors (Ary, Jacobs & Razavieh, 1990; Best & Kahn, 2006; Christians, 2000; Cohen *et al.*, 2000; Creswell, 2005; Merriam, 1988, 1998; Wellington, 2000) have discussed the importance of considering ethics in an educational research study. Common issues on ethics that are dealt with in the literature include gaining access to the participants, protection of subjects against harm, informed consent, respecting participant privacy and guarding against deception. Merriam (1998) further discussed on ethical issues at dissemination stages, while Best and Kahn (2006) further underscored guarding against participant coercion. Coercion can happen when participants are coerced or when undue influence is applied on the participants so that they can participate in a study (Best & Kahn, 2006, p. 54).

Cohen, *et al.* (2000) pointed out that each stage in the research process may be a potential source of ethical problems. Hence, as observed by Merriam (1988, 1998) and Wellington

(2000), ethics should be observed through out the study, starting from the planning stage, data collection stage, write up stage and the dissemination stage. In this study, ethical considerations were considered at various stages of the research process and are as described in the following subsections.

## **Ethical considerations prior to data collection**

### *Sanctioning of research activities at the ZOU*

The Zimbabwe Open University has a policy that encourages its staff members to conduct research. Any research that positively affects the teaching and learning processes at the institution is encouraged. The 2005-2009 institutional strategic plan has as one of its goals, the qualitative growth of programmes. This goal mainly hinges on researching and improving the quality of the running programmes, including the BSMS which has run for a cycle of more than 5 years. Being an internal member of the university and the Programme Leader for the BSc. Mathematics and Statistics (BSMS) programme, as well as the substantive Chairperson of the Department of Science, Mathematics and Technology that houses the BSMS programme, I was sanctioned to be actively involved in the review and evaluation process of the BSMS programme. Furthermore, I also sought permission from the university management to carry out the research study at the institution, and the permission was granted as described below.

### *Accessing the student*

As the researcher, I submitted a request to carry out the research study at the ZOU to the Pro-Vice Chancellor responsible for academic affairs at the university. A clearance letter authorising me to carry out the research study at the ZOU was received from the Registrar's office. The correspondences to the Pro-Vice Chancellor and the letter of permission from the Registrar are appended in this thesis as Appendix I and II respectively.

In order to access the students at the regional centre level, I also sent letters of requests (Appendix III) for permission to involve the Intake 7 BSMS students in the research study to the four ZOU Regional Directors (RDs) who are in charge of the regional centres involved in the study, namely, Harare region, Mashonaland Central region, Mashonaland West region and Masvingo regional centres. The RDs gave their permission. A written response was

received from the RD of Mashonaland Central region (Appendix IV), whilst for the other three regions verbal responses were given either in person or over the phone. Verbal consent was also obtained from the four Regional Programme Coordinators (RPCs) who assisted in coordinating the students in their respective regional centres for research purposes.

## **Ethical considerations during data collection**

### *Informed consent*

The participants in this study were informed on what the research was about through the letter of introduction (refer to Appendix V). The ‘prospective’ participants were served with the letters of introduction at the beginning of the semester. I dispatched the letters to the regional centres and these were distributed to the participants by the RPCs. Overall, the letter served to provide relevant information to the prospective participant, such as (a) introducing myself to the participants in my capacity as the researcher, (b) explaining the purpose of the research to the participants, (c) outlining the data collection procedures, (d) guaranteeing participant anonymity and confidentiality and (e) providing my personal contact details. Although there were no consent forms that were signed by the participants, the participants’ consent was assumed *a priori* once they volunteered to participate in the study.

### *Informed refusal*

As highlighted by Cohen *et al.* (2000, p. 51) “informed consent implies informed refusal”, where ‘refusal’ can happen when the subject refuses to take part in the study right from the beginning or when a participant withdraws after the research has commenced. Respect was given to those students who did not want to participate or contribute to the study, as participation in the study was voluntary. Furthermore, the ‘letter of introduction’ also contained information advising the participants’ freedom to withdraw from the study at any time, and with no prejudice at all.

### *Guarding against participant deception*

To guard against possible deception of participants, the ‘letter of introduction’ provided an honest and fair explanation of the purpose of the study and the procedures of the study.

Furthermore, those students who were interviewed were once more briefed on the purpose of the study. In order to guarantee that the students did not incur any expenses related to the study, a promise to reimburse the students on the expenditures was put in writing and this was honoured during the research.

#### *Guarding against coercion*

Since I was conducting the study in a programme in which I was serving as both the Programme Leader and Departmental Chairperson and intended to involve students from this programme as research participants, there was need to take precaution against coercing the students into participating in the study. Therefore, I minimised possibilities where individual students would feel coerced to participate in the study, by personally not being involved in the process in which the students indicated their voluntary participation in the study. Rather, the RPCs played a significant role at this stage as they managed the participant 'selection' process. This kind of space allowed the students the opportunity to freely volunteer or to decline their participation in the study without my interference.

#### *Guaranteeing participant anonymity and confidentiality*

In this study precaution was taken to protect the confidentiality of both the data and the participants who are students in the ZOU BSMS programme. The participants were assured in the 'letter of introduction' that the study data were only going to be used for research purposes and would be treated with confidentiality and anonymity. In addition the RPCs, who were instrumental in the administration and collection of the LJ, the CTT, and the LSPQ from the participating students in their respective regions, were advised to also verbally emphasise this to the students. The RPCs, as well, were in turn reminded to respect anonymity and maintain confidentiality of both the data and the students. They were advised to handle the information on the data collection instruments as strictly confidential information. The RPCs were also advised to put the collected instruments into packs, seal them immediately after collection and then send them to me either by courier or hand deliver them with no delay. Furthermore, I kept the data for the research confidential and no unauthorised persons had access to the data.

As already mentioned in section 5.6 of this chapter, all the participants were issued with research identification codes. Two levels of coding were used in the study, at data collection



and on data analysis. Hence, the names of the participating students remained anonymous. Although some of the RPCs organised the interview meetings, they were not privileged to the interview data. Interviews were held between the participants and me only and deliberations from the interviews were not disclosed to anyone, but were solely used for purposes of the study. Though the names of the students who participated in the interviews were known to me, these were treated with confidentiality and remained anonymous throughout the study.

### **Ethical considerations at write-up stage**

As described in the preceding subsection, due care was taken in this study to protect the participants' anonymity and confidentiality at both the data collection and reporting stages. In reporting the data, both the student name and research identification code were concealed and pseudonyms were used so as to camouflage the students' identity. Furthermore, specific information such as names of regional centres, names of persons, names of places, time of interview, venue of interview or any such information that could lead to the participant's identification, have been purposely taken out from the raw data.

### **Ethical issues at dissemination stage**

The research report for this study will be published in the form of a PhD thesis submitted in fulfilment of the requirements for PhD degrees at the University of the Western Cape. This information and intention were revealed to the involved persons, that is, the participants and the ZOU authorities at various points of the study. The participants were informed of these intentions in the letter of introduction, the LJ, the CTT and during the interview. The ZOU, being the institution in which the research was taking place, were informed that the study would culminate in a PhD thesis and there were no objections. In the letter of permission that I received from the ZOU registrar, the registrar on behalf of the institution requested for a copy of the final thesis (Appendix II).

Therefore as outlined above, attempts were made to honour ethical considerations of conducting a research study and I therefore foresee no opposition to the publication of the research report.

## 5.8 DATA ANALYSIS METHODS

This section presents the methods of data analysis that were employed in this study. The study sought to understand student learning in the distance education environment through interrogating the students' learning experiences, learning styles and mathematical understanding. Specifically, the study aimed to find out how the distance education environment was influencing student learning. As outlined earlier on, the data for this study comprised of data from learning journals, learning styles preference questionnaires, calculus test, and interviews.

As a first step to the data analysis, a 'case record' (Patton, 1980) or what Yin (2003) refers to as a 'case study data base', was constructed. Merriam (1998, p. 194) describes a case record (case data base) to be "the data of the study organised so the researcher can locate specific data during intensive analysis". The process of constructing the case record for this study followed the one proposed by Patton (1980, p. 304) and is as presented in Table 5.2 below;



**Table 5.2:** Constructing a Case Record

<b>Step One:</b>	<p><i>Assemble the raw data.</i></p> <p>These data consists of all the information collected about a person or program for which a case study is to be written.</p>
<b>Step Two:</b>	<p><i>Construct a case record.</i></p> <p>This is a condensation of the raw data organising, classifying, and editing the raw case data into a manageable and accessible package.</p>
<b>Step Three:</b>	<p><i>Write a case study narrative.</i></p> <p>The case study is a readable, descriptive picture of a person or program making accessible to the reader all the information necessary to understand that person or program. The case study is presented either chronologically or thematically (sometimes both). The case study presents a holistic portrayal of a person or program.</p>

(adopted from Patton, 1980, p. 304)

The data analysis for this study was carried out in three parts:

1. *Part 1* of the analysis involved the preliminary scanning of the written texts, with special focus on the responses to the calculus tasks test. This was done at the stage when the interviewees were selected. Six out of the twenty-six participants were purposively selected to participate in the interviews on the basis of their responses to the calculus tasks test.
2. *Part 2* of the analysis involved the analysis of the data obtained from all the participants of the study so as to get insights of the students' experiences of learning in the BSMS distance education environment. The data used at this level comprised of the data from all the LJs, LSPQs, and the interviews.

3. *Part 3* of the analysis was the qualitative analysis of the sets of data for each of the five interviewed participants. The data comprising of the interview transcripts and the written texts was grouped for each participant to form the data sets. Profiling of the learning styles only commenced after the compilation of the data sets. The data were then holistically analysed for each individual participant so as to produce profiles of the participant's learning styles and mathematical understanding of the concepts under study.

To provide opportunity for familiarisation with the data during the analysis stage, the data were read and re-read. For purposes of clarity I will present the data analysis methods in two parts, using the two perspectives of:

- *Students' experiences of learning in the distance education environment* in which data was thematically analysed and presented and is reported in Chapter 6. In Chapter 6, I thus present findings and discussions based on the generated themes on students' experiences with the distance education environment.
- *Students learning styles and their mathematical understanding* in which data was analysed and presented case by case for the five students who were interviewed. This is reported in Chapter 7. In Chapter 7, I thus present cases of five students, of which each case includes a profile of the student's learning styles and their mathematical understanding of limit and derivative of function concepts.

### **5.8.1 Data analysis of students' experiences with distance learning**

Students' experiences were mainly taped from the qualitative data. However, quantitative data were used to illuminate some aspects of the experiences such as students' preferences of forms of representation of the content as well as identifying the difficult content areas. Some of the data from the LJs and from the LSPQs were presented using quantitative forms, mainly as frequencies and percentages. The LSPQ data were counted and then converted to percentages whilst the LJ data was first organised by question, categorised into themes and then quantified. Data from the interviews were analysed using a constant comparative approach (Merriam, 1998).

Open coding of the interview data was done in accordance with the procedures outlined by Strauss and Corbin (1990). The data were read line by line, stanza by stanza and then categorised into themes. Approaching the data with questions such as '*What is this?*', '*What*

*does it represent?*' (Strauss & Corbin, 1990) helped to generate the categories of the data. Incidences of learning related to students' experience of learning within the BSMS distance learning environment at the ZOU were identified from the data to form the emerging categories or themes. Where necessary exemplar quotes were used to illustrate the themes. To protect the students' confidentiality, pseudonyms were used where quotes are used. The pseudonyms are consistently used to refer to the same student throughout the thesis.

### **5.8.2 Data analysis on learning styles and mathematical understanding**

Findings for this part of the study were presented as cases, where in each case a particular student's learning style, as well as the student's mathematical understanding of limit and derivative of function were profiled. Data from five students were used for this part of the study. The data comprised of sets of data that were collected from each participant and the data emanated from the students' learning journal (LJ) entries, learning styles preference questionnaire (LSPQ) entries, CTT, and the interviews (Int). As such, five data sets were constructed and these are appended in the thesis (refer to Appendix XI).

#### **Data analysis on learning styles profiling**

In this subsection I present the data analysis methods for the learning styles profiling process. The data used for LS profiling mainly emanated from the LJs, the LSPQ and the interviews. Multiple sources of data were used for the learning styles profiling process so that "a comprehensive picture" (Patton; 1980, p. 305) of the student's learning styles could clearly emerge from the data.

In order to profile students' learning styles using the students' descriptions of their experiences of learning in the DE environment, a constant comparative approach was used. However, in this case, the intention was not to generate categories from the data but to fit data items into the pre-set categories of the Felder-Silverman Learning styles model, and thus coming up with a profile of the particular student's learning styles. The F-SLSM as outlined in Chapter 4 is therefore used as the analytical model for profiling the learning styles. Pre-set categories borrowed from the F-SLSM were used so that the data would be presented in a manner congruent with an already existing learning styles model. In using preset categories, data are "scanned and sorted to fit borrowed categories" Merriam (1998, p. 183). Merriam (1998) further noted the dangers of using preset categories in qualitative

research as it hinders the generation of categories. However, for purposes of this study, where there was a need to profile students' learning styles according to a learning styles model, it was apposite to use the pre-defined categories of the learning styles model as pre-set categories for the data analysis. Below, I describe the actual learning styles profiling process.

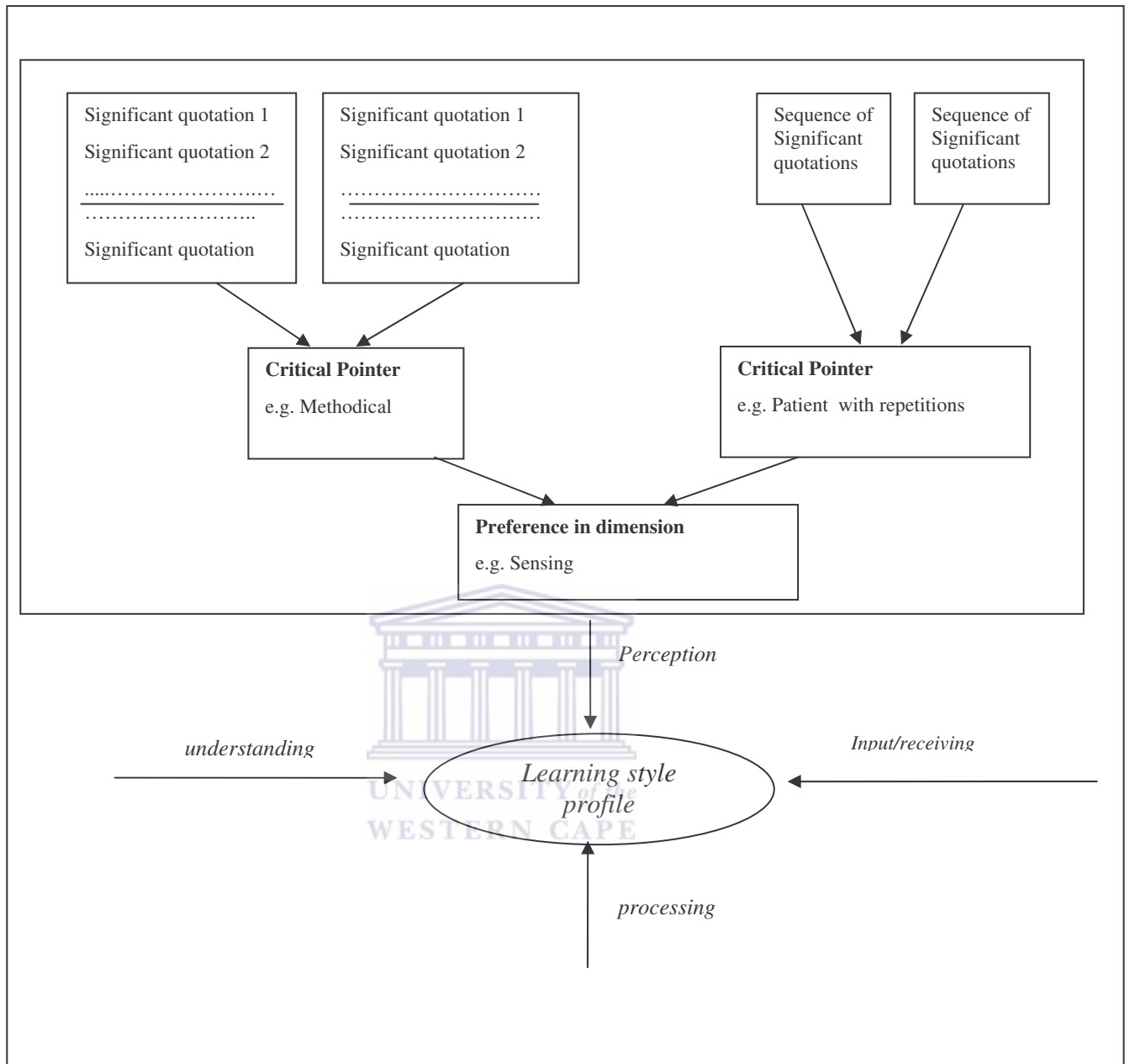
### *The LS profiling process*

To begin with, case records which in this study I will refer to as *data sets* were constructed for each of the five interviewees. The data sets composed of data emanating from the three data sources, viz, LJ, LSPQ and interviews. A coding system resulting in the identification of the characteristic pointers to the dimensions of the Felder-Silverman's learning styles model was applied on the data sets. The characteristic pointers (referred to as '*critical pointers*' in this thesis) are therefore the pre-set categories of the coding process and are linked to the characteristic attributes of the Felder- Silverman learning styles model dimensions. A collection of these critical pointers aided in pointing to the learning style dimension.

The critical pointers were arrived at after identifying and grouping statements made by a particular student which were thematically similar in terms of the F-SLSM dimensions. Thus in order to identify a critical pointer for a learning style dimension, a quotation or a group of quotations made by the student (referred to as '*significant quotations*' in this thesis) that are thematically similar were identified from the data and these served as indicators of the critical pointers. The statements are labelled as 'significant' as they are significant to the study in the sense that they contribute to the building of the student's learning styles profile in accordance to the Felder Silverman learning styles model. Figure 5.3 illustrates the coding process by means of a diagram.

### *Diagrammatic presentation of the LS profiling process*

The diagrammatic illustration in Figure 5.3 results in the sensing preference of the perception dimension in an LS profiling process.



**Figure 5.3:** Learning styles profiling analytical model

To explain the diagram above, a group of thematically similar quotations selected from the data leads to a critical pointer. The identified critical pointer is then fit to the LS model as a characteristic attribute to a particular dimension of the F-SLSM. The availability of many critical pointers pointing to the same dimension was interpreted as showing the strength of the student’s preference tendencies for that dimension.

## **The mathematical understanding outlining process**

For each of the participants whose learning styles have been profiled, I present a description of the participant's understanding of the calculus concepts of limit of function and derivative of function. The mathematical understanding is interpreted in terms of Hiebert & Carpenter's (1992) framework of mathematical understanding. Hiebert and Carpenter (1992) presented a view that equates understanding in mathematics with the quality of the mental representations.

To investigate the mathematical understanding, we therefore analyse students' written responses to the Calculus Tasks Test. Asking the students to write out responses to the mathematical tasks was a way to allow the students to externalise their internalised mathematical knowledge, hence, reveal their conceptions and misconceptions about the concepts. For the purposes of this study, I considered individual students' conceptions, errors and misconceptions in the CTT responses to reflect the kind of mathematical understanding the student held.

For the sake of making the CTT data manageable for data analysis purposes, the students' CTT responses to questions L6, L7, L9 and L10 for limit of function and D2, D3, D4 and D7 for the derivative of a function concept were mostly referred to for the analysis. Whilst on the one hand the main reason for being selective on the CTT items that were used for analysis was to make the data manageable, on the other hand, the choice of the items was mainly determined by how informative the items were for the purposes of the study. The rest of the questions were either providing the same kind of information or were routine and less informative. However, responses to some of these 'left out' items are referred to in situations where they enrich the analysis.

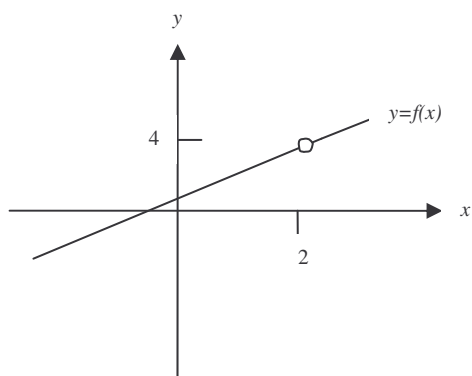
Although a full copy of the Calculus Tasks Test question paper is appended in Appendix VI, for easy and convenient reference's sake, I am going to write out the CTT items that were selected for the data analysis (that is, L6, L7, L9, L10, D2, D3, D4 and D7) in the following subsections.

### *Limit of function CTT items that were selected for analysis*

The limit of function CTT items that were selected for analysis purposes read as follows:



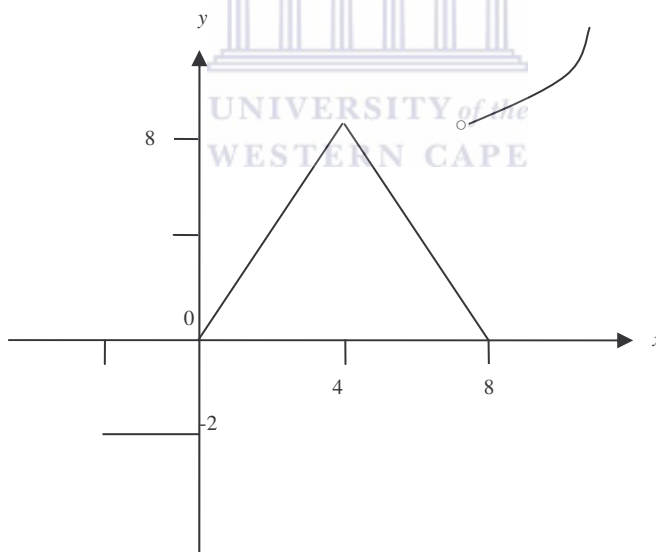
- **Item L6:**



Find  $\lim_{x \rightarrow 2} f(x)$ ; Why do you think this is so?

- **Item L7:** Find  $\lim_{x \rightarrow \infty} \frac{(x+3)(2x-1)}{x^2}$ ; Why do you think this is so?

- **Item L9:**



a) Find  $\lim_{x \rightarrow 4} f(x)$

b) Find  $\lim_{x \rightarrow \infty} f(x)$

- **Item L10:** Consider the function  $f(x)$  given as follows  $f(x) = \begin{cases} 1-x & \text{for } x \leq 1 \\ x^2 & \text{for } x > 1 \end{cases}$

a) Determine  $\lim_{x \rightarrow 1} f(x)$  if it exists;

b) Determine  $\lim_{x \rightarrow 0} f(x)$  if it exists

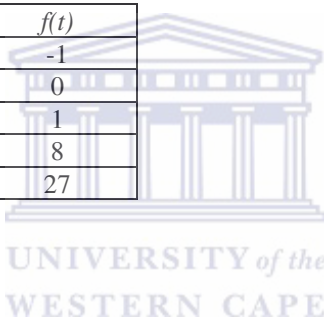
Provide explanations in each case.

*Derivative of function CTT items that were selected for analysis*

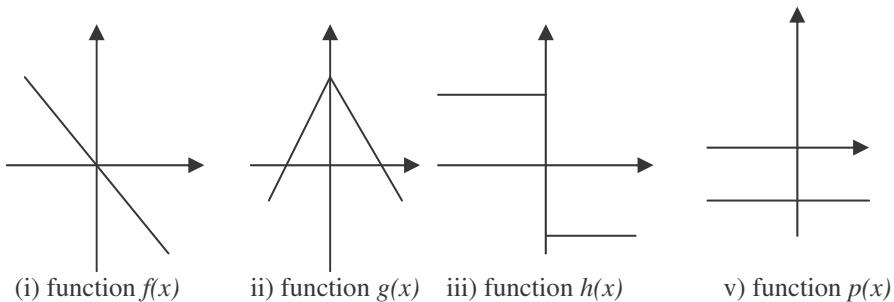
The derivative of function CTT items that were selected for analysis purposes read as follows:

- **Item D2:** a) Find  $f'(x)$  for  $f(x) = 3x + 8$  at  $x_0 = 2$ ;  
 b) Find  $f'(x)$  for  $f(x) = 9x^2$  at  $x_0 = 3$ ;  
 c) Find  $f'(x)$  for  $f(x) = x^3 + 8$  at  $x_0 = 2$
- **Item D3:** Let some values of  $f(t)$  be as given in the table. Estimate  $f'(1.5)$ . Explain how you obtained the solution?

$t$	$f(t)$
-1	-1
0	0
1	1
2	8
3	27

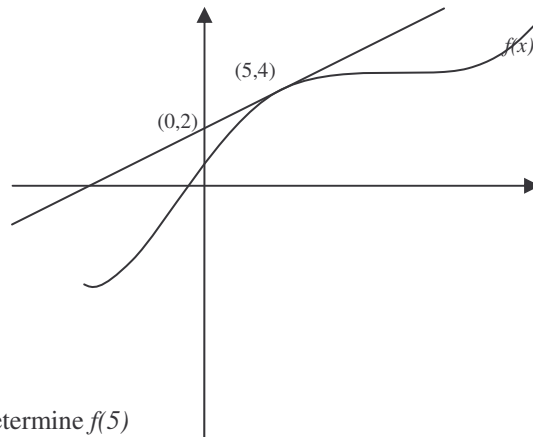


- **Item D4:**



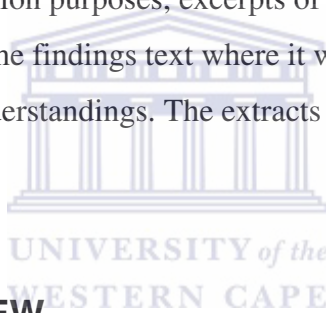
- Indicate the pairs, i.e., show which is the function and which is the derivative function.
- Explain your solutions in each case.

• **Item D7 :**



- a) Determine  $f(5)$
- b) Determine  $f'(5)$

Findings for the mathematical understanding were presented and interpreted in the form of descriptive accounts. For discussion purposes, excerpts of the original students' writings from the CTT were included in the findings text where it was deemed necessary and illuminating on the students' understandings. The extracts were included either as scanned texts or as typed out texts.



## 5.9 CHAPTER OVERVIEW

This chapter discussed the methodology and methods that were employed in this study. The chapter identified case study as the research design that was adopted for this study. The single case study is informed by an interpretive methodology. The sampling procedures involved in the study were convenient, purposive as well as voluntary sampling. The chapter also discussed the construction of the specific instruments that were used to collect the data for the main study. This also included a discussion of the pilot study and how it informed the main study. The chapter also provided a detail of how the data collection was conducted. Issues of quality in relation to ethics and trustworthiness of the study were also discussed.

Multiple sources of data were used to collect the data and these resulted in mainly qualitative but with some quantitative data. The data analysis methods that were employed in this study were also discussed in this chapter. The findings of the study are discussed in Chapters 6 and 7 of this thesis. The next chapter, Chapter 6 will thus present findings related to students' experiences with the distance education environment.

# CHAPTER 6: STUDENTS' EXPERIENCES OF DISTANCE LEARNING

## 6.1 INTRODUCTION

This chapter presents insights into first year mathematics distance students' experiences of learning calculus in the BSMS distance education environment at the ZOU. Throughout the chapter, where reference to subject matter is required, reference is made to the narrow content domain of limit of function and derivative of function.

This chapter responds to the research question:

How do first year mathematics distance students experience distance learning at the ZOU, with specific reference to the learning of calculus?

Following the methods of data analysis that are explained in Chapter 5, the findings on students' experiences were divided into categories and subsequent subcategories. Four main categories of students' experiences were identified and these revolved around the themes of:

- *content* related experiences, which refer to how the students experienced and interacted with the subject matter;
- *learning materials* related experiences, which refer to how the students interacted with the printed learning materials environment;
- *context* related experiences, which refer to how the students experienced and interacted with the environment in which the learning was occurring, that is, interactions with the BSMS distance education context; and
- *learner* based experiences, which refer to individual student characteristics and orientation of learning.

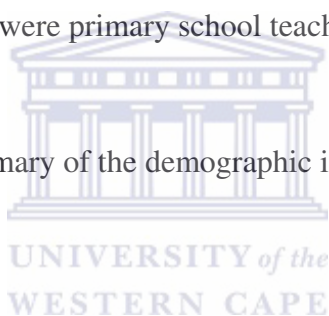
The data used in the study were mainly qualitative but were complemented with some quantitative data. The qualitative data emanated mainly from interview responses of five participants while the quantitative data emanated from contributions of all 26 participants.

Findings from the qualitative data are supported with quotations from the interviews. The pseudonyms Kundai, Ray, Siphon, Tanya and Tino<sup>7</sup> are used in reporting the quoted data so as to ensure student anonymity. These pseudonyms are consistently used in the thesis and refer to the same persons across all chapters where they are referred to.

## 6.2 DEMOGRAPHIC INFORMATION OF THE PARTICIPANTS

The information on student demographics was collected through the LSPQ. However, for those participants who were interviewed, where necessary, more details were obtained during the interviews. The participants were adult learners, of varied professional backgrounds though a majority of them were school teachers. Of the six interviewees, there were three males and three females. Three of the interviewees were staying in an urban area and the other three were based in a rural area. Coincidentally, all six interviewees were teachers, of which three of them were primary school teachers and the other three were secondary school teachers.

The following table gives a summary of the demographic information of the twenty six participants of this study:



---

<sup>7</sup> Female participants- Kundai and Tanya; Male participants - Ray, Siphon and Tino

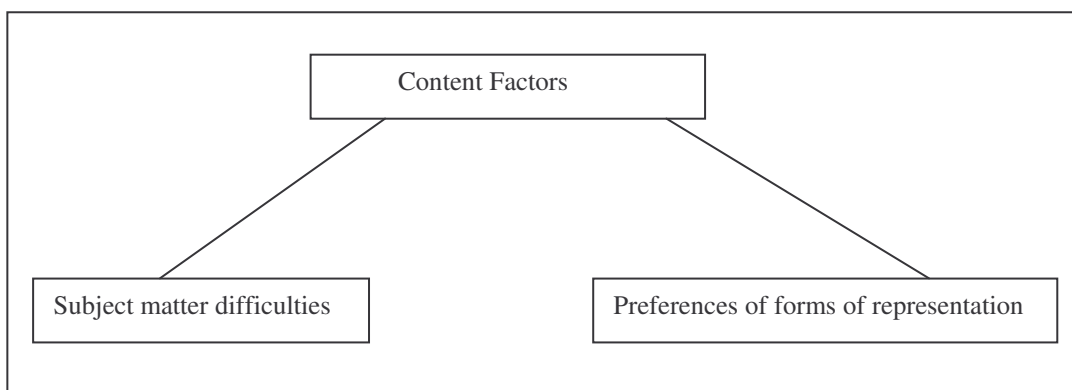
**Table 6.1:** Demographic Data of the Participants ( $n = 26$ )

ITEM	Characteristic	Count	%
<b>GENDER</b>	Male	17	65
	Female	9	35
<b>AGE</b> (years, to nearest whole year)	21-25	6	23
	26-30	4	15
	31-35	6	23
	36-40	10	38
<b>Where do you stay?</b>	Rural	14	54
	Urban	10	38
	Peri-Urban	2	8
<b>Profession</b>	Teacher	21	81
	Technical	4	15
	Unemployed	1	4

### 6.3 CONTENT FACTORS

Moore (1989) identified three types of interaction in a teaching and learning experience. One of the identified interactions is learner-content interaction which focuses on the interaction between the learner and the content. Moore (1989, p.1) acknowledges the importance of content related interaction by saying “without it there can not be education...” Thus, the subject matter of a course is an important aspect of any learning experience.

The data of this study showed that the subject matter and the way it is structured have a strong influence on how students engage in learning. Participants of the study identified some calculus content areas which they felt were difficult to learn. The purpose of this section is to present findings that pertain to what I have categorised as ‘content factors’. The ‘content factors’ serve to illuminate the students’ experiences with the subject matter. Figure 6.1 diagrammatically shows the two subcategories associated with the ‘content factors’ categories that were identified from the data. These are ‘subject matter difficulties’ and ‘preferences of forms of representation’.



**Figure 6.1:** Subcategories associated with the ‘content factors’ category

### **6.3.1 Subject matter difficulties**

In this subsection, I present aspects of the content area which the participants identified as difficult, as well as discuss why these aspects were identified as difficult areas. Much of the data used in this subsection emanated from the student entries of the learning journals<sup>8</sup>, though complemented with data from the interviews. Responses for each of the LJ items 4 and 5 were grouped together, which made it possible to obtain a clear and holistic picture of the difficult content areas and why they were considered as such. Though responses from the learning journals were qualitative in nature, in the form of written texts, the data were quantified and then commented upon. The data from the LJs were not given as percentages as there were possibilities of participants giving multiple responses. As such, data that emanated from LJ entries and presented in Tables 6.2, 6.3, 6.4 and 6.5 are given as frequencies only.

#### **What concepts did students find difficult? (LJ item 4)**

This subsection summarises responses to the LJ question “What did you find difficult in these sections?” i.e. LJ item 4.

#### *Identified difficult areas for limit of function concept*

The participants frequently cited content areas that involve the formal definition of the limit of function concept as difficult. This was followed by areas that involve determining limits

---

<sup>8</sup> Learning journal entries to item 4 and item 5

of specific non-standard functions. Table 6.2 presents the identified areas and the frequency each was cited across responses from all the participants.

**Table 6.2:** Difficult Subject Matter Areas - Limit of Function

Identified difficult subject matter area (limit of function)	Frequency <sup>9</sup>
Definition of limit of function:	
- Understanding and applying the definition in proofs	21
- Linking the $f(x)$ , the $\epsilon$ and the $\delta$	3
Determining limits of specific 'non standard' functions:	
- Trigonometric functions	11
- Piecewise defined functions	3
Applying theorems to solve problems e.g. use of squeeze theorem	5

The students who were interviewed admitted to having difficulties in understanding and applying the definition. Siphon, for instance, was explicit on how the  $\epsilon$ - $\delta$  definition was confusing to him. He stated:

Siphon: Yaah. Definition of limit, it became clear the second time but the first time it was not so clear to me, because of these other notation. I don't know what you call them, u-u-hm, the delta, the epsilon-delta definitions. A-a-ah, it appeared I didn't understand what was going on.

Tino echoed similar sentiments about the  $\epsilon$ - $\delta$  definition,

Tino: In fact, I really didn't persist on following up on the links between the  $x$ , *epsilon* and the *delta*, like really getting to know exactly on where the link is, what it is and where it is coming from.

Such findings on students experiencing difficulties with the formal definition of limit of function corroborate the findings of other researchers such as Cornu (1991, Fernandez, (2004), Juter (2005a), Morash (1990) and Swinyard and Lockwood (2007).

### *Identified difficult areas for derivative of function concept*

The most dominant areas that were identified as difficult for the derivative concept mainly relate to differentiation of specific functions, applying rules and procedures and the use of the definition to show differentiability of given functions. Table 6.3 presents a summary of the identified difficult areas as well as their respective frequencies. As noted previously with

<sup>9</sup> It was possible for students to make multiple responses



the limit of function, there were possibilities for participants to make multiple responses; hence, the frequencies are not put into percentages.

**Table 6.3:** Difficult Subject Matter Areas - Derivative of Function

Identified difficult subject matter area (derivative of function)	Frequency
Differentiation of specific 'non standard' functions	
- Implicit functions	17
- Trigonometric functions	6
- Hyperbolic functions	6
Use of definition (first principles) to show differentiability of functions (including piecewise defined functions)	15
Applying rules: chain rule and product rule	6
Linking continuity and differentiability of functions	6
Understanding theorems and proofs on differentiation	3
Applications of derivatives	4
No difficulty	2

Data from the interviews did not reveal any major challenges with the derivative of function, except on applying the definition. However, Tino had the following comment about applying the definition:

Tino: Myself, I normally liked to use the calculation method; I found that to be easy for me. I used to try using first principles, but then that's when I'd end up getting confused, like when applying the  $x_0 + h$ .

The chain rule was mentioned as one of the differentiation rules that presented problems to students. Similar findings were made in an earlier study by Morgan (1990). Morgan (1990) found that his subjects experienced problems applying the chain rule.

### Reasons for difficulties

The following table, Table 6.4 presents a summary of responses to LJ item 5. For item 5, participants were required to respond to the question "Why did you find this difficult?" The responses from both limit and derivative LJs were combined on analysis and were thematically grouped together. The most common causes of difficulties were related to the social isolation which is due to the distance education environment. Difficulties were also

related to mathematical abstractions and to the insufficiency of worked examples in the learning materials. The data is again presented only as frequencies as there were possibilities for multiple entries from participants.

**Table 6.4:** Reasons for Difficulties

<b>Reasons for difficulty –limit and derivative of function</b>	<b>Frequency</b>
Difficulties related to isolation: -Having no one to discuss/share ideas with/explain concepts	21
Difficulties related to definitions, theorems, and proofs	19
Lack of worked examples	18
Difficulties working with specific functions; e.g. trigonometric functions; implicit functions; piecewise-defined functions	9
Application related problems: applying the theorems/definition	8
New concept, had not done it before	7
Method related problems: standard method to follow. Method used difficult to follow.	7
Unavailability of solutions to problems and exercises	5
Failure to link concepts	4
Could not graph the function	3
No access to other information, including textbooks	3
Module not clear,	3

### **Ways of overcoming the difficulties (item 6)**

Table 6.5 gives the participants' responses to LJ item 6 and the associated frequencies. Item 6 required the participants to explain how they attempted to overcome the difficulties that they experienced.

**Table 6.5:** Ways of Overcoming Difficulties

<b>Ways of overcoming difficulties</b>	<b>Frequency</b>
Group discussion and sharing ideas with colleagues	38
Researching and consulting other textbooks/sources	23
Work out/ solve a number of problems	16
Follow similar worked examples	12
Repeat the concepts several times	11
Consulting with the tutor	9
Reading on my own	7
Difficulty not overcome	3

The responses to item 6 of the LJ indicated in the table above show that discussions with colleagues, further researching and solving more mathematical tasks were popular responses with students as ways of overcoming difficulties. This was a consistent observation with responses to item 7 of the LJ which is shown in Table 6.6. Item 7 sought responses to the question “ What do you think helped you learn the main ideas in the section(s)?”

**Table 6.6:** Responses to LJ Item 7

No.		Limit of function		Derivative of function	
		possible responses (50)	(%)	possible responses (50)	(%)
1	Relating the concepts to previous sections/units of the course	13	26	10	20
2	Working out each problem solution one step at a time	18	36	19	38
3	Working out problems on my own	16	32	15	30
4	Discussing with my colleagues	29	58	34	68
5	Thinking quietly on my own	5	10	2	4
6	Proving the theorems	7	14	6	12
7	Relating the concepts to real life situations	1	2	3	6
8	Use of graphs when finding limits/differentiating	2	4	0	0
9	Applying the definition	11	22	7	14
10	Memorising the method of solution	1	10	5	10
11	Using tabulated data to obtain the limit of functions	2	4		
	Practising on several problems (derivatives)			14	28
12	Listening during the tutorials	10	20	11	22
13	Linking the exercises to the theory	4	8	1	2
14	Following the given procedure of solution method	24	48	17	34
15	Explaining the concepts or tasks to someone else	1	2	3	6
16	Making an overview of what I've learnt	2	4	3	6

Table 6.6 comprises of responses to item 7 which was a closed question. Respondents were required to indicate choices on what helped them learn the main ideas of the topics under consideration. Responses related to group discussion and solving problems were the most popular.

### 6.3.2 Preferences of representation

This subsection discusses students' learning preferences regarding forms of representation for the limit of function and derivative of function concepts. The subsection presents a quantitative summary of the responses from the learning styles preference questionnaires

(LSPQ). The LSPQ served to illuminate students' preferences regarding four approaches to solutions, namely the analytical, graphical, tabular and use of the definition. These preferences would give pointers of the participants' learning styles preferences according to the Felder-Silverman learning styles model.

The responses to the two LSPQ are presented in Tables 6.7 and 6.8, following which some comments are made regarding the most preferred and the least preferred forms representations. Table 6.7 presents summary results from the LSPQ for limit of function and Table 6.8 presents summary results from the LSPQ for derivative of function. In each of the tables, the data is presented as both frequencies and percentages. The percentages were based on 78<sup>10</sup>, with 78 being the maximum number of possible responses.

### Limit of Function LSPQ Responses

**Table 6.7:** Summary of Limit of Function LSPQ Responses

<b>Rank Option</b>	<b>Rank 1: Most Preferred</b> (78 possible entries)	<b>Rank 2:</b> (78 possible entries)	<b>Rank 3:</b> (78 possible entries)	<b>Rank 4: Least Preferred</b> (78 possible entries)
Option (i) Analytic/Algebraic	60% (47)	23% (18)	9% (7)	8% (6)
Option (ii) Graphical	26% (20)	47% (37)	23% (18)	4% (3)
Option (iii) Tabular	11% (9)	24% (19)	39% (30)	26% (20)
Option (iv) Use of the definition	3% (2)	5% (4)	30% (23)	49 (63%)

From Table 6.7 we observe that the algebraic representation, which mainly involves calculations and use of rules and procedures to solve problems, was the most common response for the most preferred representation, with 60% of the responses. Only 8% of the responses showed the algebraic approach as the least preferred representation and 23% and 9% of the responses placing it on rank 2 and rank 3 respectively. This observation indicates that the algebraic representation was a popular preferred representation.

<sup>10</sup> 78=26x3. There were 26 participants and an LSPQ had 3 items, in which each respondent ranked his or her preference for the representation option. Therefore in one LSPQ, for each representation there is a possible maximum of 78 entries for a particular rank. For instance, there are 78 possible entries in which option (i) could be ranked as most preferred (rank 1).

We also observe that of the 78 possible responses, only 3% of these responses indicate the use of the definition as the most preferred representation and 63% responses indicated the use of the definition as the least preferred representation, with 5% and 30% placing the option on rank 2 and 3 respectively. Thus, pointing to the fact that the use of the definition was an unpopular approach with the students. The graphical and tabular representations were averagely ranked. However, the graphical approach was more preferred than the tabular approach.

### Derivative of Function LSPQ Responses

The LSPQ responses for the derivative of the function concept, also showed a trend on choice of options that is similar to the one discussed above from the limit of function LSPQ. Responses from the derivative of function LSPQ are as shown in Table 6.8.

**Table 6.8:** Summary of Derivative of Function LSPQ Responses

<b>Rank Option</b>	<b>Rank 1: Most Preferred</b> (78 possible entries)	<b>Rank 2:</b> (78 possible entries)	<b>Rank 3:</b> (78 possible entries)	<b>Rank 4: Least Preferred</b> (78 possible entries)
Option (i) Analytic/Algebraic	78% (61)	13% (10)	5% (4)	4% (3)
Option (ii) Graphical	13% (10)	49% (38)	19% (15)	19% (15)
Option (iii) Tabular	1% (1)	28% (23)	39% (30)	31% (24)
Option (iv) Use of the definition	8% (6)	9% (7)	37% (29)	46% (36)

The analytical approach was ranked as the most preferred option with 78% of the responses in favour of the option. Only 4% responses indicated the analytical approach as the least preferred option with the other 18% coming from rank 2 and rank 3. Only 8 % of the responses ranked the use of the definition as the most preferred option, against 37% for rank 3 and 46% for rank 4, which is the least preferred option. Thus we again observe that analytical approach was the favoured preference against the use of the definition for the derivative of function, which was an unpopular choice with 8% of the responses showing it as most preferred option and 46% as least preferred. Similar to the limit of function, LSPQ responses we observe that the graphical and tabular representations were averagely ranked, though with the graphical approach being the more preferred of the two.

The finding from the LSPQs about the unpopularity of the formal definitions is consistent with that observed in LJ responses as presented in the previous section.

### Forms of representation preferences from interviews

Some interview responses also indicated that students have specific preferences when it comes to forms of representation for the limit of function concept. The following extracts illuminate on this. Siphso, for instance was consistent with his preference for graphs, and some of the responses that he gave during the interviews were as follows:

- Siphso: When finding a limit of a function. It's easy to arrive at an answer using graphs.
- Siphso: You can interpret information easily using graphs. It's easier that way.
- Siphso: Yaah, graphs present information. You do the graph thing, it's easy to interpret the information from the graph. It's easy.

Ray was explicit with the preference for the tabular approach. He stated,

- Ray: Maybe that one depends with the nature of the question or the concept itself. For an example, I was just showing you that table on limits of function. For most of the questions on limits, I would put them in the form of a table, that makes it a lot easier for me.
- Ray: Because you could work it out and get the answers and compare the answers, right by yourself and check it and see if your answers are right or wrong. And you can see by yourself that this is right or this is wrong.
- Ray: Yaah, the moment you start seeing the 2.999999, 2.999999998, 2.999999999999999 (repeats the nines and laughs).

Kundai was also consistent with the preference for the algebraic approach, which she referred to as "the direct" method.

- Kundai: As for me I didn't like the definition. I liked the direct method of looking for the limit. This approach of using the definition is something else. I have never enjoyed it.
- Kundai: For derivatives I didn't like using first principles. Still, I liked using the direct methods to find the derivatives.
- Kundai: People of course have their own preferences, but as for myself, I find this (*referring to what the student calls the direct method*) to be much better, because it is very straightforward

However, Kundai also showed discomfort with too much symbolic notation,

- Kundai: Frankly speaking, like when I opened the module intending to read on the series (sic) (*by series student is referring to sequences*), I just don't know what happened really, but the presentation, it really put me off.

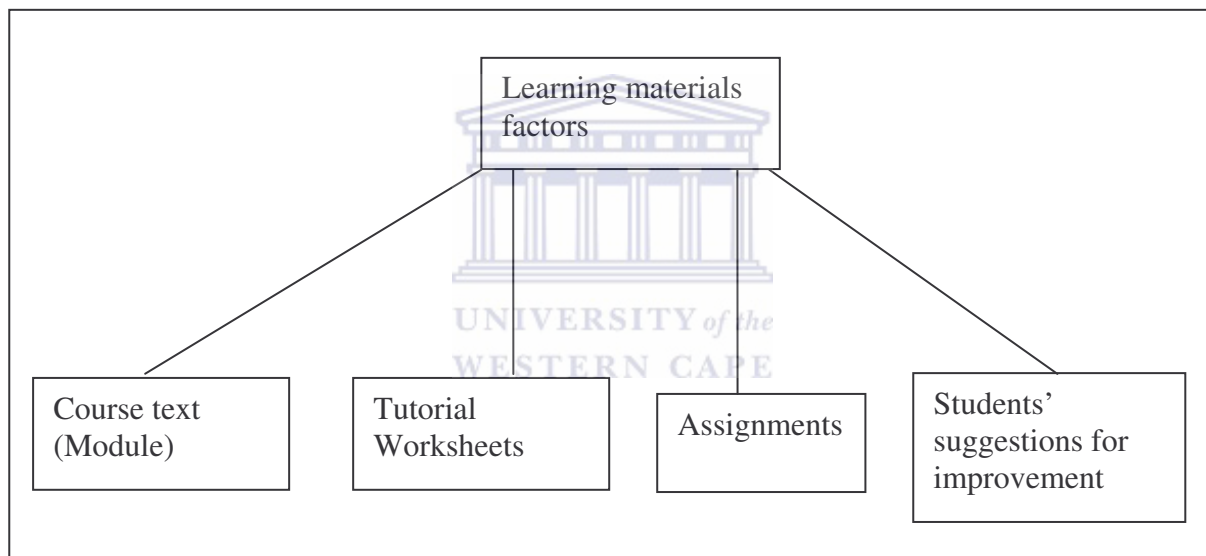
### Comments of preference responses in relation to LS

The responses from the two LSPQs and the interviews indicate that students do show different preferences for different forms of representation. For both limit of function and

derivative of function concepts, ‘the algebraic approach’ was frequently selected as the most preferred option, and ‘using the definition’ was frequently selected as the least preferred option. Generally, on the F-SLSM, the algebraic approach is an approach popular among sensing learners who do not prefer abstractions, and using definitions is an approach popular among intuitive learners who are abstract and theory oriented (Felder & Silverman, 1988).

## 6.4 LEARNING MATERIALS FACTORS

A category related to the printed learning materials environment was identified from the data. This category refers to the medium of instruction, which for the BSMS programme is through printed learning materials. Four subcategories were identified and these are as presented diagrammatically in Figure 6.2.



**Figure 6.2:** Subcategories associated with the ‘learning materials factors’ category

The category was considered as a very important aspect of distance education environment since it is possible that the medium of instruction may place constraints on how students interact with the content. In a distance education program, it is hoped that the medium of instruction facilitates that didactic conversation (Holmberg, 1986) that is expected to take place between the learner and the content. The clarity of the learning materials and the relevance of the learning tasks play a crucial role in motivating the students to learn. The ‘learning materials factors’ category as described in this section, therefore, provides findings related to the students’ experiences and engagement with the medium of instruction. Chapter

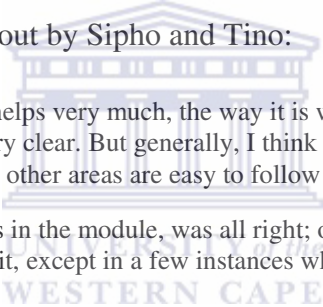


1 of this thesis detailed the learning materials that are availed for the BSMS programme Calculus 1 course.

The data of this study revealed that students rely heavily on the institutionally availed learning package that is the institutionally prepared course text (commonly referred to as the *module*), the problem worksheets and the assignments.

#### **6.4.1 The course text (module)**

The data showed that the participants of this study mainly depended on the course text (*module*) as the main source of the subject matter. Such a finding is not common with those students who are learning in a conventional programme, as they have a teacher readily available. Some of the participants indicated that where the text in the module was not clear they reverted to other textbooks. Generally the group of students appreciated the course text as they felt that the book was helpful. However, there were sections of the course text that were not clear. This was pointed out by Siphon and Tino:

- 
- Siphon: That book helped. It helps very much, the way it is written, except in some areas where some other areas are not very clear. But generally, I think its helpful and it contains a lot of information and some other areas are easy to follow (...)
- Tino: The material, which is in the module, was all right; one could work on it even when you are alone. You could use it, except in a few instances where one could experience some difficulties.

The following subsections discuss specific aspects of students' experiences with the module with respect to text supporting devices and general presentation of the text in the module.

#### **Guiding and supporting devices in the module**

Guiding and supporting devices in the text refer to aspects of the course module that are supposed to guide and support the student in the learning process. Such text devices include the overview section, the objectives section, the worked examples, and the self-assessment exercises.

##### *Overview and Objectives sections*

The participants of this study provided mixed though invaluable responses regarding the usage of *the Overview* and *the Objectives* sections of the module. All but one of the interviewed students indicated that these sections were important and helpful for their learning. The justifications why students considered these to be important sections of the

module revolved around themes such as (a) highlighting important areas of the subject matter to the student, (b) guiding the student through the unit, (c) checking for progress, (d) checking for understanding of concepts and (e) informing the student about the concepts covered and how they are sequenced.

Kundai was articulate about the role of overviews and objectives sections in highlighting areas of importance. She stated

Kundai: I think these sections are important, they are useful in that they can be used to guide you to know more about the topic. For instance, say you are reading on a chapter, on the objectives section, they list all the important things such that when you are studying or you are solving problems, you'd know that this is more important so you can give it more attention when reading. At the end of the chapter, when you are doing the exercises, then you can also check on whether you mastered the concepts or achieved the objectives of what they are testing.

Similar responses were obtained from Tino and Tanya,

Tino: Yaah, it (*objectives section*) was useful in a way because it highlighted the important areas of the unit and how they are sequenced in concept (...)

Tanya: At least those pages they help us to actually work. They guide us in a chapter. (...) you have to check on your progress. Did I understand this, did I understand that concept. Am I able to solve this kind of problem? So I think that this page helps to guide us.

Sipho was more depth oriented and elaborate on how the objectives and overview sections were useful for assessing progress and understanding,

Sipho: (...) Ya-ah, they are very helpful, they tell you about what's coming. These also test you on whether you have understood on what you have been learning. Have you fully understood or whether there is some concepts that maybe you need to research more on, some concepts that are not clear? Those (*objectives*) are the guidelines for that. Because you need to go back now and look at those objectives and say have I achieved (a), (b), (c) and (d). You are now looking at your performance and now that makes it easy for you to even make some research regarding a certain concept only but not the whole topic.

Ray seemed indifferent to the overview and objectives sections in the module, though he admitted that there are moments when he did find meaningful purpose for these sections, especially when he experienced problems.

Ray: If there are no problems, they (*the overview and objectives sections*) are not necessary really, because I would just go through the unit. If I can read and understand the unit, and if I can work out the problems from the module then I know it's done.

The same student also stated,

Ray: Yaah, sometimes when the going gets tough, maybe I'd come back to the objectives and go through the objectives and start all over again. Then in this case I'm forced to go through the objectives of the unit (*pause*), such that when I finish reading, then I would say did I achieve the objective?

As much as the overview and objectives sections are important sections in a text, it is interesting to note that students place importance on these sections for varied reasons as highlighted above. Carefully written overviews and objectives do play a significant role on students' learning, otherwise without the objectives students wouldn't know what they are learning. This is explicated by Siphon in the following way:

Siphon: Yes. It's not learning. You see, learning without the objectives set, a-a-ah you will be learning nothing, because you don't know what you are learning, you cannot even evaluate yourself on whether you are doing the right things. But when you go to the objectives if they are there, they guide you now, they tell you where you are going. So I think they are very helpful, these objectives.

Such a sentiment was also echoed by Tino who said,

Tino: (...) But then maybe if they are left out, then on reading you might miss some important concepts, because you are not guided. You might miss the stage by stage guide.

#### *Worked examples and self assessment exercises*

The data of this study revealed that students value course text supporting devices such as worked examples and self assessment exercises. The findings presented in section 6.3 showed that solving mathematical problems was amongst the top ranked activities that students engaged in to overcome difficulties. Students valued the benefits they gained from the practice on problem solving in coming to understand concepts. Siphon elaborated the importance in the following way:

Siphon: Yaah, this topic on limits, I would suggest that it should have more examples so that people understand what limits are, because I have seen that, the topic limits is appearing in almost every course that we are doing. In every course, we have to cover some problems regarding limits. So, it's very very (*repetition as a way of emphasis*) important that we understand the basics. If you solve many problems, you understand what limits are really. It can make it a lot easier to learn calculus that way, because it's mainly more of the limits.

However, as can be read from the above quote, Siphon seemed disappointed that the course text does not contain sufficient worked examples. This concern was consistent across all the interviewees. These students felt that the quantity of worked examples in the module were not sufficient and needed to be increased.

Despite the concerns on the quantity of the worked examples in the module, Ray and Kundai brought in a dimension of the quality of the worked examples. They felt that the worked examples were not challenging enough and were mostly not relevant to the self assessment activities. Both Ray and Kundai alluded to that some of the worked examples that are in the module are not challenging enough. Challenging activities have the potential to encourage

students to engage deeply with the subject matter, whereas, activities that are not challenging may not do so. Kundai articulated this in the following way:

Kundai: The way the module is written seems okay, but if I had an opportunity I would add more challenging examples in the modules. I can say the kinds of problems that are solved in the module right now are very easy to understand, but the difficult ones are not solved in these modules. So, at times after having worked through a topic, the easy problems I can solve them. But then there are those ones that are more challenging which one cannot solve. If there was at least one or more solved examples of the more challenging problems, then that would assist us when we need to deal with the more challenging problems.

In other words, Kundai is saying that the module illustrates how to solve the easy tasks, but does not do the same with the difficult tasks. Ray echoed similar sentiments:

Ray: (...) But now, when it comes to the other problems, they are hardly any challenging worked problems in this book, because in this book (*he points to the module*) they don't give as much examples as possible.

Ray further referred to the relevance of self assessment exercises to what is covered in the text.

Ray: Maybe you get just one example and then the next example it's a different problem altogether from what you were doing in the first example and obviously tougher than what you were doing.

Ray: Eehm. Maybe with the modules, for them to have some worked examples within the modules and then they give exercises with the relevant questions, which are more or less the same with the ones that would have been given in the example. I think we will learn better that way, than a situation where there is a question here, and there's another question here, then there is another one there, then when it comes to the exercises it's completely different

One can easily sense Ray's frustration from his experience with the in-text exercises since he could not make out the connection between the worked examples and the self assessment exercises in the module. Relevant activities have the potential to motivate students to learn.

Another major concern for this group of students was that the module did not have worked solutions for the self assessment activities. The major recommendation that these students gave to combat this gap on worked problems was that the course should have an accompanying workbook. The workbook can contain more worked examples, more self assessment exercises and worked solutions to the exercises. Students felt that a workbook would be handy as it could guide them in their individual learning. Tino had the following to say:

Tino: Yes, I mean the workbook. At least it would be easy and won't be heavy on us. It will also help us not to lose direction when we are reading.

Tanya echoed similar sentiments in the following way:

Tanya: Through working out and looking at the solutions. If I work out a solution, then I look for the solution in the workbook. You find that if there are some concepts which you would have not grasped properly and when you make mistakes, it will be easy to identify where your mistake is, rather than if you continue working on your solution, thinking that you are doing the right thing, only to be disappointed at the end of the day when you realise that what you have been doing all along is not right. So I think if there is a provision of a workbook or something similar with worked out solutions then that's better. At least you know how things are.

### Some drawbacks

The concept 'drawbacks' refers to the hindrances that students experienced that are linked to the learning materials. Such drawbacks could be noted in both the interviews and the LJs. Some students in their LJ entries attributed certain difficulties that they experienced to the module. Responses such as the following were frequent throughout the LJs

**Table 6.9:** Learning Difficulties Related to the Module as Identified from LJs

the proofs in the module are difficult to understand
proofs are not clearly stated in the module
methods explained in the module are too shallow to solve the given problems
explanations given in the module are not clear
It was very difficult to know whether I got it right or wrong since the module has no answers
There are no clear questions which start from the known to the unknown
There is no laid out pattern on how to find the limits, you are just given the facts
There are not enough worked examples to give me practice on the topic

Data from the interviews provided illumination to some specific drawbacks related to the module and these are presented below.

#### *The presentation of the text*

The format of presentation of the text was a drawback to learning for some of the students. For instance, Kundai referred to how the presentation for limit of function de-motivated her as the text was too symbolic. Kundai stated:

Kundai: When I started reading the calculus module, the units at the beginning of the module were okay. But then when I started reading on that other unit, is it the series or something like that, (*student in this case opens and refers to unit on sequences in module*). Yes, these ones, I just don't know but the presentation is different, they have there own presentation, which is just de-motivating. Such that after that, for me to actually proceed and actually work on the other later topics in the course, a-ah it was difficult. So as for me, the issue here is that after having tried to read that unit, I couldn't really get down to do calculus, it just put me off. Because of

that, I wasn't paying much attention to it since I just considered it to be a difficult course. But maybe it's my attitude, I don't know.

Ray seemed frustrated in that the concepts were presented from the unknown to the known or from the abstract to the simple. Ray was not comfortable with that approach since calculus was a new subject for him. The following extracts from the interview with Ray illuminate his discomfort:

Ray: The examples given are not clear and there are no clear questions which start from the known to the unknown; (LJ entry)

Ray: (...) It's unlike in (*student names other courses*) where some of the concepts were not completely new, but we were starting from the known to the unknown. But here (*in calculus*) we are starting from the unknown to the known.

Ray: You know when you learn these mathematical concepts, you learn them in stages and not from nowhere to the abstract. But, you can go from the simple to the abstract and it's best that way.

Sipho showed a preference for graphs, which he observed, was not sufficiently catered for in the module. Sipho stated:

Sipho: Yaah, even in the module it is clear if you follow the graph. The definition of the derivatives using the graph and the examples given graphically it is easy to follow that way though unfortunately there are not many such problems in the module.

It is therefore observable that the way content is presented in the text does interfere with the learning process. In such instances students have to rearrange their learning orientation. Whereas some can manage to do that without any learning conflict, others like Kundai and Ray end up de-motivated.

### *Errors in text*

For some students errors in the text were a cause of frustration. Tanya sounded frustrated with the errors that she encountered in another course, and constantly referred to the experience in this interview

Tanya: (...) at times you find that there are some mistakes in the modules. Not necessarily pertaining to calculus but generally. I am just generalising. But I know that we are human beings and at one time or the other we are bound to make mistakes. There are some questions that may have mistakes, so maybe (*hesitates and laughs*) as human beings mistakes are always bad for learning (...)

Tanya seemed perturbed with the experience. On responding to another question later on in the interview, she again referred to the issue of errors in modules in the following way:

I: What do you feel about the module itself? Any challenges?

Tanya: Like I was saying, in some of these workings in the modules there are some mistakes, and I remember in one other subject, we had to make some corrections (*to the text*), and that was confusing. (...)

Although Tanya was quick to point out that her experience with in-text errors was not in calculus, it is clear that encountering errors in the modules was a bad experience for her, and as she pointed out, it can be a confusing experience. Although the experience mentioned above was not in calculus, it was worthy highlighting in this thesis, as in-text errors can have negative effect on student learning. It is clear that in-text errors in the main text of a course can be distracting especially when students depend on that book. This may result in students' lack of confidence in the text.

#### **6.4.2 Tutorial worksheets**

Students in the BSMS programme receive tutorial worksheets as part of the tutorial package for calculus. These tutorial worksheets comprise a set of questions for the respective units in the module. The set of problems is supposed to beef up the exercises in the module. The purpose of worksheets is to provide the student with an additional base of problems to practice on for problem-solving purposes. The interviewed students of this study acknowledged that the worksheets were helpful as they served as a guide to “cover ground” when studying as mentioned by Siph0. In addition, the worksheets also help to provide “a varied base of problems to be worked on” as noted by Tino.

However, of major concern to some of these students was the fact that the problems in the worksheets did not have worked solutions.

Siph0: Yes, those can help especially when we have the worksheets, then we have the tutorials, then we have the assignments, all these they do help us. Yaah, they help us to cover some ground. But then in Calculus we have got a worksheet, but the worksheet now does not supply solutions. That's now where the tutorials have got to be important, because when we cover some ground in the worksheets, we need some clarifications here and there. But when we are given these worksheets for the first time and we only meet once and forget about it, it becomes very difficult.

When asked why the solutions were essential, the reasons given were mainly for clarification and verification purposes.

Tino: So that you can refer and check whether what you are doing is right or wrong.

Siph0: I can be able to move on but there are some areas where you need them (*the solution*), if you are learning alone. You need to find out whether what you are doing is exactly in line with this kind of learning.

Hence, as already explained in the previous subsection, the call for a workbook in calculus was once more reinforced. Students were of the opinion that the workbook would attend to an existing gap since the workbook would comprise of problems and their worked solutions.

### **6.4.3 Assignments**

BSMS calculus students are expected to submit one take home assignment and one in-class test, which is written under supervision as part of the coursework assessment. For discussion purposes, I will use the term ‘*assignment(s)*’ to refer to either or both of these forms of assessment, unless where specific reference is required. The students of this study spoke very highly of the assignments. Varied reasons emerged from the data on why students found these assignments to be useful. The students felt that the assignments served as a guide for students on what and how to study. In addition, the assignments helped the students acquire techniques of solving mathematical tasks. Tino indicated this in the following way

Tino: (...) Then the assignments would also give us direction, so they are useful also.

Tino: So that I could be guided on what to read at the end of the course. From the practice I would also get to feel the steps involved on solving problems, that is from this step you move to that one, then to that one and so on

Kundai echoed similar sentiments about assignments though with an emphasis on mastering the questioning technique.

Kundai: They were useful because they could guide me on studying, plus I could also get a feel of the questioning technique (...)

Furthermore, the assignments served as a motivator for studying for the course as pointed out by Kundai.

Kundai: (...) Also as I was doing the assignment, I would read for that course. So for me they were important.

The same aspect of motivation can be read from the following statement from Sipho.

However, for Sipho it was the feedback that he obtained from the tutor on his assignment which motivated him to reflect more on what he had written down.

Sipho: (...) I remember one problem that I worked out, I think it was on the assignment, I stated a certain theorem and then the tutor replied, “You did not research much on this theorem”. But that helped me very much, because I looked at my problem and at the comment and I moved back to look at that theorem and yes, I had misquoted it (...)



Sipho's comment shows that students value the comments that tutors give as feedback on assignments. Similar findings were made in other studies by Bhalalusesa (2006) and Dzakiria (2005).

Assignments were also helpful as a means of self assessment. Sipho elucidated this by saying

Sipho (...) do those assignments to your best because those assignments make people see their areas of weakness.

Contrary to the positive benefits mentioned above, some of the students considered assignments as good sources of information to be memorised. The possibility where some question items repeatedly appeared in more than one assessment paper created a good opportunity for memorising and reproducing information, as candidly stated by Tanya to "cheat" as well.

Tanya: Yes, sometimes. You find with these exams (assignments) if you look through, you get up to exactly the same problem that you once had. If you are very good at that, then reproduce it and (*then whispers*) cheat.

Tanya: I am saying with these studies, what I have found out, you might get a question on an assignment or on a test, and you find that it will be the same. So if you are really studying and you are very good at numbers, or are good at memorising or remembering, you find sometimes that the problem which you have in an assignment is also in the test, so at times you just reproduce it.

Ray also echoed the same sentiment on memorising

Ray: You know there are certain things that we used to ignore, especially these proofs and so forth. Then the moment we realised that the same things, which are coming in the assignment are the same things which are coming in the tests, then we realized that we must go through it. Then, that's when we started to be serious about proofs and we were trying to memorise the theorems because we realised that these are things that are coming from time to time.

It is apparent that students place great value on the assignments. Thus these assignments play a crucial role in students' learning, not only for assessment purposes, but also for the learning process. However, from a distance educator's point of view, the papers need to be well thought and carefully compiled. In addition, due care has to be taken on preparing these assignments so as to minimise instances where students aim to regurgitate the information.

#### **6.4.4 Students' suggestions for improvement on learning materials**

During the interviews, students gave what they considered to be invaluable suggestions for improving the learning materials for the calculus course for the BSMS programme. As already mentioned in subsection 6.4.1 one of the suggestions that emerged is that of

improving the base of worked examples and self assessment exercises by introducing a workbook for calculus.

Other suggestions were related to the ways in which the information is presented in the text.

Sipho and Tino suggested presenting information using various forms of representation.

Sipho: (...) if they would use and define tables and then state the formulas and then give as much examples so that we can follow those examples and then later on you can apply those learnt formulas to certain other concepts which are not in the modules (...)

Tino: There is nothing on graphs in the module. Maybe if all these approaches were included in the module, then we get to encounter them, maybe we could find some of them easy to use. Then the one on tabulation, yes it's included (*in the module*) but then it's time consuming.

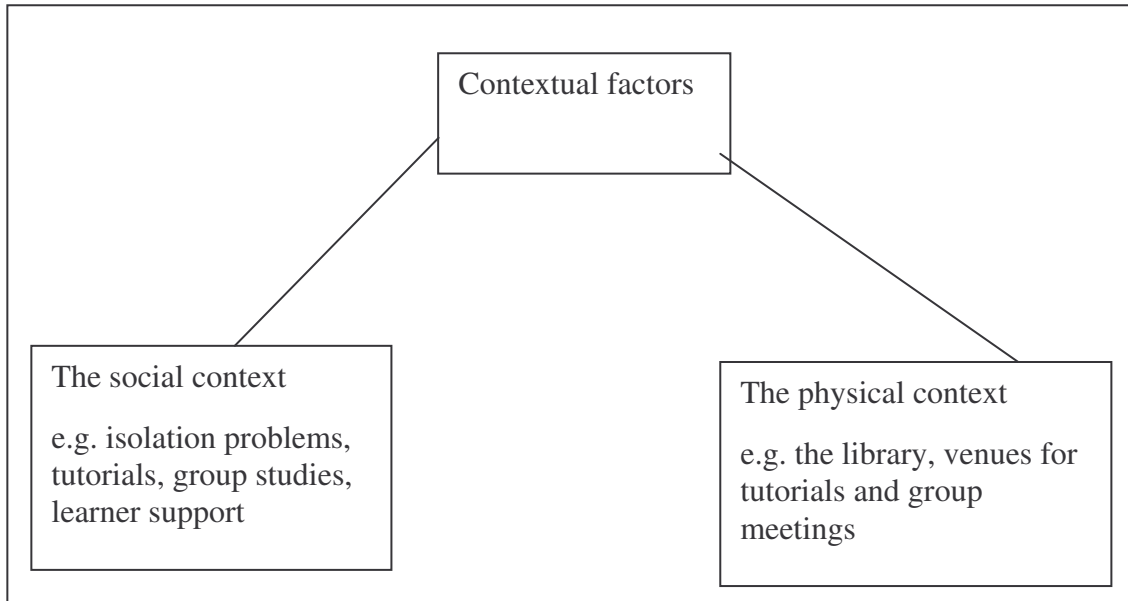
Tanya was explicit about presenting the concepts in a step-by-step approach and about explaining all the information in a clear way.

Tanya: Yaah, the concepts, like I have been saying that I met this animal calculus here for the first time, so with the module for some of the concepts, you know there are some stages, say as you are working on a problem, there are some stages they assume that you know. So from somebody from a layman's point of view, you wouldn't know exactly what is happening. So I think with their examples, if they had actually worked them step-by-step, so that any other student might know what exactly is happening and can follow those steps up. So as it is right now you find that a concept or a step is written, and then they get to the answer. How did they get to the answer? How did they cancel? How did they divide? How did they do this and that? So I think if they could improve on the steps on the examples so that when I am alone and I am doing it that way, I think I will be able to solve the problems.

UNIVERSITY of the  
WESTERN CAPE

## 6.5 CONTEXTUAL FACTORS

The open coding revealed 'contextual factors' as another category of students' experiences in the distance education environment. Contextual factors referred to issues directly related to the distance education background of the BSMS programme. Two subcategories, social context and physical context were identified from the data, and these are indicated diagrammatically in Figure 6.3 below



**Figure 6.3:** Subcategories associated with contextual factors category

### 6.5.1 *The social context*

A characteristic of distance education is the separation between the students and their teachers and amongst the students themselves, in terms of location, space and time. In the context of the BSMS programme at the ZOU, students had opportunities to meet with their tutors and with fellow students during tutorials. The students were also encouraged to organise themselves into study groups. However, despite these opportunities the data of this study show that being separated by space and time was a cause of frustrations for the students. Nevertheless the data also reveals that despite the challenges of distance, the students benefited from such measures as institutionally organised tutorials and self organised study groups. This subsection will present these findings.

#### **Problems with isolation**

##### *Isolation related to separation by distance*

The findings show that the interviewees of this study missed on the social interaction aspect of learning, a situation which is not normally experienced in the conventional setting. Feelings of social isolation and the resultant frustration could be noted with the participants of this study. The issue of feeling isolated by distance learners is consistent with the findings

of others (Dzakiria, 2005; Lyall & McNamara, 2000), though their studies were conducted in disciplines other than mathematics.

In LJs entries, a common response that students gave regarding why they encountered learning difficulties was related to social isolation. Responses such as “there was no one to consult”, “no one to share or discuss with”, or “no one to explain the concepts” were common in the LJs. During the interview, Tanya was very explicit in this regard. She said:

Tanya: Sometimes I wouldn't understand either, and as for consulting, I wouldn't have anyone near me to consult with and then it would be very difficult for me to go ahead.

Sipho, similarly expressed his frustration in the following way,

Sipho: (...) when you go out there without knowing anything, following the module alone, there will be some things of course that you can do, but, there are some other areas where you need clarification and it becomes difficult when you are alone.

Ray also echoed similar sentiments; though for him the problem was compounded by the fact that the content was new to him.

Ray: But now, to start from scratch I think that's where the problem is, where you are starting from nowhere. You have got the book, yes, but no one to ask but yourself. You read it by yourself until you get the concept. *(pause)* But now I am not into that kind of learning. But maybe as time goes on, I'll get used to it.

The data show that the students do acknowledge and attribute the isolation to the separation in terms of distance, as pointed out by Tanya, Ray and Sipho below.

Tanya: We live very far and in different places such that when I need help I wouldn't see anyone.

Ray: In our class we all come from different places and now to meet maybe over the week or during a working day, it is a problem.

Sipho: (...) it is very difficult to meet some other group members because of the distance.

I: It's the problem of the distance now?

Sipho: Yes it's the distance; we are so far apart

From the above statements it is apparent that the students are aware of the fact that as distance learners they are supposed to be separated with their teachers and peers.

### *Time and related logistical issues*

Students in a distance education environment have the flexibility of learning at their own pace and time (Keegan, 1990). This implies that they have that opportunity to learn around their normal work and family commitments. The role of the distance education institution like any other educational institution is to try to lay down time frames and systems in which

the learning of a course should take place. However, in a distance education institution, since students learn at their own pace and time this implies that they are separated by time; their instruction doesn't occur at the same time other than when they meet for the face-to-face tutorials. Students are however expected to meet the same objectives within specific timeframes.

This subsection presents findings focusing on how the systems and timeframes influenced the distance students' learning processes for this group of BSMS students. Aspects that emerged from the data with regard to systems and timeframes concerned the number of tutorials, duration of tutorial session and duration of semester. Of major concern to the interviewed students was the total number of hours availed for the face-to-face tutorial sessions, per course, per semester. Students felt the time allocated for the face-to-face tutorial sessions was insufficient both in terms of the duration of the tutorial session and the number of tutorial sessions in a semester.

The two hour long tutorial sessions was a cause for concern amongst some of the research students as they felt that the two hours per session was insufficient. Tanya put across her dilemmas, compromises and consequences of having "limited" time in a tutorial session in the following way:

Tanya: Yaah, actually with our programme, I should like to say it's the time. You know at times you find that you have got so limited a time in tutorials so much that at times you find that or you feel that if I ask this type of question maybe its not very useful, and say I shouldn't do that at the expense of others. So you find that at times you just go over a tutorial without actually understanding what is happening in the group. Taking into mind also that maybe you are hindering the progress of the tutorial or something like that.

Students also expressed dissatisfaction with the current situation where the institution avails them three two-hour sessions of face-to-face tutorials in a semester per course. The students wanted the number of hours increased. Justifications as to why the students wanted more time availed for the face-to-face tutorial sessions included; (a) getting opportunities to cover more content, (b) getting assistance from tutors and (c) discussing with colleagues. Proposals of what the students felt was the sufficient number of tutorial sessions included "every fortnight", "every week", and "12 hours". Tanya called for more time for a tutorial session so that students could have space for consultation with tutors after a tutorial session. Ray was even contemplating having meetings during the week days after work.

Ray: If I had a choice, I would prefer a situation whereby after work, maybe I meet 3 or 4 people, we sit down together, maybe with the assistance of a tutor here and there, not just one or two. So when I get home, these things will still be there (*he points to his head*).

Indicators of the students' dependence on human contact when learning persisted throughout the interviews. In related issues the interviewees expressed concern over the duration of the semesters, which they felt was very short. They felt that the time was insufficient for them to accomplish the learning targets. In this regard, students gave proposals as to what they felt was an ideal duration of a semester instead of the current 12 week learning semester. Kundai for instance proposed "Maybe sixteen weeks, then examinations can come after that", and Ray suggesting that the semester starts earlier in August instead of in September.

Ray: (...) as far as I am concerned, I think the time its not enough. I think maybe if the course would start in August and then you have enough time for tutorials. Actual lessons really are better. But now by the time we were trying to settle down, the assignments were due the following week, and before we could do anything the due date was right by the door there

Students felt the tutorial and semester times were rather congested and insufficient for them to learn. The views by Ray and Tanya presented below show how the congested schedules could not allow them to settle into a study pattern

Ray: Yaah, in the tutorials because you go there once, the second time you go you are told that the next time you will be meeting for the last time, the next time you are submitting your exams (*assignments*). So tutorials as far as I am concerned, I am not satisfied. (*student repeats the sequence again*) It's like you meet now, and then meet the second time, the next time you are submitting your exams (*assignments*), the next time you are going for revision, the next time you are writing exams. Can you see that?

Tanya echoed similar sentiments. She stated,

Tanya: (...) Because time is one factor, which for sure presented problems for me. It really was insufficient, such that really making a decision on what to do, it was very difficult. So it was really difficult for me to zero in to a concept.

Kundai further highlighted how this congested time schedule impacted on the way she prepared her take home assignments.

Kundai: I think the time we were given to do the assignments was very short. Most of us had not covered much in the module by the assignment submission due dates. So we ended up just finding answers without actually understanding how the problems could be solved since we had not finished reading the module by that time. And also we were not even prepared to hand in the assignments by the due dates.

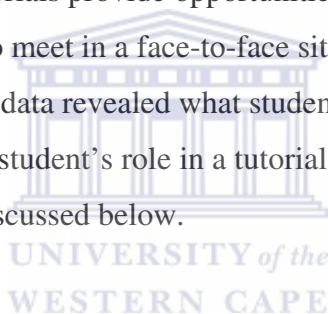
Justifications for a longer semester given by this group of students included giving students more time to interact with the learning materials and to work on their own. It is interesting to note that although the students are aware of the fact that they are distance learners, they continue to call for more contact time with tutors and colleagues. The separation in terms of time was a cause of frustration amongst the students.

As evidenced in the above excerpts by Tanya and Kundai logistical problems that are time related also affected the students' learning processes. Tanya elaborated how the 'limited time' affected her throughout the discussion. Kundai elaborated how the limited time hindered her preparations for assignments.

The fact that the students constantly called for more contact face-to-face hours and that they were prepared to be in learning situations with the tutors and colleagues readily available could be revealing that these students were not quite ready for the separation in terms of time as expected. However, there is also another viewpoint. This may also indicate areas where the institution could improve with regard to orienting and preparing the students for distance learning, as most of them would be experiencing DE for the first time.

### **Institutionally organised tutorials**

The institutionally organised tutorials provide opportunities for students to meet as a group with fellow students as well as to meet in a face-to-face situation with a tutor who is also an expert in the subject matter. The data revealed what students prefer for a tutorial session, the role of the tutor in a tutorial, the student's role in a tutorial and the nature of interactions in tutorial sessions and these are discussed below.



#### *Preferred type of tutorial session*

The interviewees of this study indicated a preference for a tutorial session in which students are involved in active learning. The preference for active participation in tutorials illuminates a connection with the active learning style preference in the F-SLSM. When asked on what they considered as an ideal tutorial session or what kind of tutorial session they preferred to participate in, students' comments seemed to indicate that they prefer to be part of learning sessions in which they are actively involved, solve problems and engage in discussions. They also showed preferences for tutorial sessions where the tutor guides them through the session rather than lecture to them. Some of the responses given by the students were as follows:

Sipho: (...) one which is more of a discussion, and the tutor probes students with questions and then allows people to discuss certain concepts rather than him just explaining things. A-a-ah, it makes things difficult.

Ray: If I am just passive, then it means I haven't learnt anything.

I: You mean you want to be active?

Ray: Yeah! Yeah. When I am active, because when I read about it, I think about it, I talk about it, that way I get the concept. Because when I am quiet it means maybe I don't understand it.

Tanya: Uhm, myself from what I know in this subject of ours (*calculus*), it's sort of a practical course and you learn by doing it. So if you just listen, you wouldn't know exactly what is happening, so I think note-making and problem solving are supposed to be done in a tutorial.

Kundai: For example, if I bring my own problem to the tutorial, then we try to solve that problem as individuals. Then, if we all fail to solve it, then the tutor can then explain and work out the problem on the board for us as a class. But first each individual must individually solve the problems brought by each of the other students, and then we report back afterwards.

Whereas some of the students were part of tutorial sessions where they actively participated, others in actual fact did not experience such tutorial sessions as the tutor adopted a lecture like approach in the tutorial. Kundai and Ray showed frustration with their experience in which the tutorial sessions did not engage them actively but were rather conducted in lecture like manner.

Kundai: Yes, for example, let's say it's a question on definition of limit of a function, then the tutor would write for us the limit or write the entire definition of the limit on the chalkboard board and yet we have this in our modules already. Also, the tutor would for instance take a worked example from the module then start doing it again in the tutorial, I felt that this was time wasting. If only we could give our own questions on the things that were difficult for us, then start solving them in the tutorial.

Similar sentiments were echoed by Ray where the tutor solved the problems for the students;

Ray: By piling, I mean the tutor giving more and more problems for solving. On just one question you find the whole chalk board from here up to there, from here up to there, (*student elaborates width and height of board using own hands*). Of which if the concepts were explained right from the beginning then that would mean the tutor will have less work, because you are starting from the known to the unknown, unlike where you are starting from the unknown to the known.

It is evident that the students were not comfortable with tutorial sessions which were conducted in a lecture like manner as they felt that this did not promote their learning. Tino associated tutorials conducted in a lecture like manner with memorisation, which is not beneficial for learning.

I: How do you feel about lecture-like tutorials?

Tino: There isn't much benefit on the learner, because he (*the learner*) will just be listening without participating in anything, so it will be something similar to what we discussed earlier on, on memorisation.

An opinion that is shared by Tanya, who says after being "lectured to" its easy to forget the concepts.

Tanya: I wouldn't like that myself, because I know at times you come to the lecture and you are lectured to, you go home, you find a different situation then you tend to forget. So If I write something down and I work something down, I will easily remember, o-oh you can do this and that. So lecturing cannot work for me, it's not very good.



However, as much as the students found benefit from tutorials, some aspects of the subject matter were said to be too difficult such that tutorial sessions would not be of much help.

Sipho, for example identified proofs to theorems:

Sipho: (...) some of the tutorials I think they are okay, you can understand some concepts but they are also some areas where some tutorials don't give much help. As far as proofs are concerned, they don't give much help.

### *Role of tutor and role of student in tutorials*

Tutorial sessions provided opportunities for both teacher-student and student-to-student interaction. Tutors play a significant part in a distance education environment since they facilitate the student learning in a face-to-face situation. When asked what the role of the tutor should be in a tutorial, the participants of this study indicated that the role of the tutor should be to assist students in difficult content areas, to highlight areas of importance in the course, to give information to the students, to clarify issues as well as to provide for an active learning environment by facilitating discussion.

On further inquiry about their experiences in tutorials, comments from some of the interviewees seemed to show that the encounters they had with their tutors were beneficial to their learning. The students appreciated the role of the tutor during tutorials and they found the tutors to be helpful. Tutors were helpful in clarifying difficult concepts. The following excerpts serve as evidence to this.

Sipho: Yes. The tutor that we had, helped us to understand and use it (*the definition*), he took us through that definition but at first it was really difficult.

Tino: Because the tutor would assist us where we had difficulties.

The role of the tutor was also seen as directing and guiding the learning processes,

Tanya: In a tutorial session, I would like to think that the role of the tutor is to explain to us and help us with the problems that we face when we were reading. You know, at times being corrected on what you have been doing. If I work out on a problem and I don't get it, then I ask him "where did I exactly go wrong?" then he will be in a position to correct me and explain the things to me.

Ray: My role is to participate as much as possible, because the tutor is just there to assist me in the learning process and nothing more. That's why I was talking of lessons. Because you will be there in the tutorial saying "I didn't understand this one", and then the tutor says "Ok this is how you do it. Basically you do it this way". So in other words it's just guiding. He (*pauses*), he is not teaching me the concepts as such but he is telling me.

The tutor was also seen to play an important role through giving feedback to assessments. The quality of the feedback comments is significant to student learning. For instance, as pointed out by Siphon in the following excerpt:

Siphon: (...) I remember one problem that I worked out, I think it was on the assignment, I stated a certain theorem and then the tutor replied, "You did not research much on this theorem". But that helped me very much, because I looked at my problem and at the comment and I moved back to look at that theorem and yes, I had misquoted it (...)

In this case the comment from the tutor was instrumental to learning. To Siphon the feedback was meaningful as it enabled him to reflect on what he had written on a piece of assessment.

However, for Kundai the experience was different. Although she acknowledged that the tutor was at times helpful, feelings of frustration with her tutorials could be sensed because she failed to get full benefit of the tutorial sessions as per her expectations.

Kundai: (...) the tutor would for instance take a worked example from the module then start doing it again in the tutorial, I felt that this was time wasting. If only we could give our own questions on the things that were difficult for us, then start solving them in the tutorial.

For Kundai a tutor should facilitate and assist students in areas of difficulty. She stated:

Kundai: I think in a tutorial we should bring our problems, those questions that were difficult for us, then we discuss them with the tutor. Its not just a matter of him taking us through the same worked examples that are in the module.

In the case of Kundai the approach the tutor used to conduct the tutorial did not result in any meaningful learning on the part of the student. It also appears as if the tutor was not well oriented with what was expected of him as a distance tutor. As such his approach to tutoring was at times found to be incongruent with expectations of his students.

When asked about the role of the student in a tutorial session students gave responses such as "to participate as much as possible", "discuss with others", "share ideas", "bring difficult problems", "asking questions", "solving questions", "listen to the tutor" and "assist others".

The data showed that tutorial sessions also provide good opportunities for student-to-student interaction. Students could meet, discuss and share ideas with other students. Comments from Tino and Ray were as follows

Tino: (...) Maybe it's assisting each other. If there is anyone who has a problem about a certain concept and if I know the area then we just assist each other.

I: You do that in a tutorial?

Tino: Yes. If someone does not understand something and is asking for help, if I know the area then I can assist.

For Ray, student-to-student interaction was a source of encouragement. The comments obtained from other students enabled him to confront difficult concepts.

Ray: (...) You meet some friends and talk. Tutorials were useful sometimes. Maybe that gave me the courage to sit down and study the limit concept, because, I was talking to some of my friends and they were telling me that there are very easy, they are not as hard as such. If you can sit down and study you will get it. Of course I tried but at least I managed to get something.

Tutorials therefore provided opportunities both for tutor-student and student-student interactions. It is apparent that a tutorial session in which the tutor-to-student interaction was good and where the students were actively engaged in the learning process benefited the students. Similarly, the student-to-student interaction during tutorials was found to be beneficial. From the above discussions it is evident that the tutors therefore played significant roles during tutorial sessions by creating conducive learning environments. However it is of importance that the tutors be well acquainted with DE as well as be aware of the crucial role that they play in facilitating learning.

### **Learner support**

The open coding of the data also informed on the type of support the students received from both within and without the institution. Two subcategories of learner support were identified and labelled as internal support and external support.

#### *Internal support*

Internal support referred to the support that the students received internally from within the institution structures. The internal support issues that arose from the data mostly concerned the tutors. As indicated earlier, the importance and role of the tutor in supporting students' learning processes during the tutorials did not go unnoticed by some of the students. However, a major concern raised by the students related to the unavailability of the tutors beyond tutorial session times. Unlike in conventional institutions where fellow students easily and instantly access their teachers, the situation is quite different in distance education where students and teachers are separated in terms of distance.

However, some participants of this study expressed disappointment over the fact that their tutors were not instantly available when they needed them. Tanya felt strongly about the fact that tutors were not instantly available, she said:

Tanya: Like I have been saying that these modules and their examples, suppose I am studying at home, and I find that challenge that I have no one to consult at that moment. Again from what I have discovered, these tutors you hardly find them. Because you have a problem and you want some help, you wouldn't be able to find them (*the tutors*). Again as we have got these group studies, you might not be able to have them the times you want to. So you find that at times you are forced to leave your studying because you cannot go anywhere when you don't know anything. (...)

Interestingly, the students are aware of the fact that their tutors are part-time tutors who also have other responsibilities besides tutoring. The part-time tutor appointment was seen as interfering with how the tutors availed themselves when students needed their assistance.

Kundai, for instance had the following to say

Kundai: Some of the problems were queer. Yes, the tutor would at times help us but he was also busy himself, in one instance he told us "I am busy since I am also a student"

Tanya echoed similar sentiments. She further added how an unclear institutional policy on how tutors could be reached in times of need compounded with who would pay the tutor for that 'extra' service perturbed her. She stated:

Tanya: As I have been saying, time and the availability of the tutors. I am not very sure of how we should consult with them. From what I know some are said to be part timers (*part-time tutors*). So it would be very difficult. At one time I remember I wanted help but I couldn't find it in time. (...) Actually we had a problem as a group and then we tried to consult with the lecturer (*tutor*) but we couldn't get hold of him. So it would be okay if our tutors would be available. If they could have a schedule, which we could follow, I think it would help us.

Tanya: (...) And maybe if they could make some consultation provision so that we could see these tutors. You find that at times they give us their numbers (*phone*) and everything, but we are not very sure, are we going to pay them or are they there for us. So these are some of the things that we are not very sure about. Because at one time or another, I need some help but because I don't have that money then I won't be able to get the help. If we are to pay anything then, it would be better to be informed on how much?

The constant call for consultation hours by some of the students in this study is an indication that the students were expecting to have more time with the tutors than what was availed.

Tanya felt very strongly about having a time slot for consultation with the tutor. She even suggested the following:

Tanya: (...) So maybe if you could make some provision for consulting time maybe after tutorials, or sometime lets say during the week or any other time and say if you have got any problems, come at this time, between this time and that time. I think it would help us.

Notable in the data of this study is the absence of comments relating to support emanating internally from within the institution other than that which was offered by the tutors.

However, the role of the tutor in supporting students' learning processes during tutorials was appreciated by the students.

### *External support*

External support referred to the support that the students received external of the institution. The data reveals that students also benefited from support of others external to the institutional structures. The nature of support identified in this study was varied and included support in terms of resources as well as with the actual learning process. Students were assisted with books, learning materials and private tutoring.

As observed earlier on, peer support emanating from the study groups was appreciated and students considered this kind of support as being an important part of their learning as it provided opportunities for sharing and generating ideas. Apart from peer support, some of the students benefited from the support of family members and from other members of the community such as tutors or former students of the programme. For instance, Tino managed to get textbooks from other sources. Tanya and Ray mentioned how family members helped them in their studies. Tanya acknowledged the help of her sisters with photocopying materials.

Tanya: Of late, we have been organising past exam papers and looking for photocopier machines. Like I have got some sisters who have got access to them (*photocopiers*), so I could provide the others with those photocopied materials and we discuss them.

In search of explanation to difficult concepts, Ray involved his brothers as they would explain difficult concepts to him.

Ray: (...) I said I will find somebody who is doing the course to explain or I will ask some of my young brothers to explain the concepts to me.

Ray further organised private tutoring, which in turn was costly for him.

Ray: (...) Well, sometimes I've even gone to the extent of looking for tutors and give them money so that they explain the concepts, and I'd say (*hesitates*), I've given out a lot of money.

This was also similarly echoed by Kundai who would engage tutoring services from former students of the BSMS programme and pay them. She stated:

Kundai: What helped us is that we would find someone else to assist us, maybe a former student in the programme, and then we pay for the service.

### **Self organised study groups**

Study groups were commonly mentioned by the students as a good source for peer support and were a useful strategy for combating isolation, which they experienced as distance learners. Data from the interviews and the LJs showed that the students were of the opinion

that group meetings were important and were beneficial to their learning. However, some of them were quick to point out challenges they experienced associated with the group meetings.

### *Benefits of study groups*

Self-organised groups proved to be another good source for student-to-student interaction, where students had the opportunity to discuss problems, to share ideas and to prepare for tutorial sessions. Some of the responses from the interviewed students in support of groups are given below:

- Sipho: You exchange ideas where you are not clear, where you have gone wrong, somebody can correct you, and other information can come in during group discussion. It is very important.
- Tino: I find them to be useful. They are useful for instance, for mastering the concepts. If one member of the group has mastered some of the difficult concepts, and another one has also mastered some other concepts, then when you meet as a group, things can become easy.
- Ray: Ahmm, being by myself, it's a problem to me, but a group it's okay. Because obviously I'm sure you will have someone who will explain one or two things unless where you are all by yourself?
- Tanya: Like I have been saying, the discussions, you know, we could meet before the tutorials. Maybe I would be having some problems, at least if we are together, maybe we can share some problems, as what is difficult for me can be easier for somebody else. So these group discussions they actually help me before I go to the tutorials. And when I actually go to the tutorial it's such a step ahead.

The role of the student in a study group was to participate actively, to ask questions, to share ideas and resources such as providing resources and extra materials.

The data also revealed that most frequently, the group meetings were either 'assessment' driven or 'tutor tasks' driven or 'opportunity of grouping' driven. Students intensified on group meetings when they were preparing for assignments or examinations, or had tasks given to them by their tutors or when the opportunity to meet cropped up. Some responses given by the participants in this regard are as follows:

- I: What prompted the meetings?  
Ray: You know when we were about to write exams (*laughter*)
- I: What motivated the meetings? What prompted the meetings?  
Tanya: (...) we scheduled it for ourselves, like around our exam time, we actually spend about a week meeting every day.
- I: What prompted the meetings when you used to meet then?  
Sipho: The work that we had, that we were given by the tutors prompted us to meet and discuss some issues. (...)

'Opportunity of grouping' driven meetings were mainly motivated by opportunities, such as the availability of group members before or after a tutorial session, or the availability of an empty classroom before or after a tutorial session. Kundai pointed this out in the following manner:

Kundai: We would meet consistently, because at times we'd meet after a tutorial session, at times we'd just agree where to meet.

Kundai: (...) Sometimes we would meet after tutorials, that's when we would look for an open and empty classroom and then meet there with our group at the tutorial venue.

Thus, in general, the students felt that group studies contributed positively to their learning and was a significant part of their learning processes. However, contrary to these beneficial experiences from study groups, students also experienced some challenges. These are discussed below.

#### *Challenges experienced with study groups*

As much as the students found benefit with study groups, the data revealed that students also experienced some challenges, which hindered their learning activities. Four different types of challenges were identified; challenges related to (a) distance, (b) unavailability of venues, (c) insufficient time and (d) failure to get instant assistance when facing problems in groups.

The following comments from three students are evidence of challenges that were related to the nature of distance education where the students are geographically dispersed.

Sipho: O-o-oh this time, it is very difficult to meet some other group members because of the distance.

Ray: In our class we all come from different places and now to meet maybe over the week or during a working day, it is a problem. So you find that in most cases we meet during the tutorials and maybe when we are under pressure like before the exams. Yaah, I think we met about one or two times beyond the tutorials.

Tanya: Again as we have got these group studies, you might not be able to hold them the times you want to. So you find that at times you are forced to leave your studying because you cannot go anywhere when you don't know anything. (*student is saying she would fail to progress*)

Thus, for Sipho, Ray and Tanya the distance between them and their group mates was prohibitive as they would fail to meet. Furthermore, in the case of Ray, the challenge was compounded by the unavailability of venues for the group meetings,

Ray: Yes. Well we wanted to submit exams (*assignments*), so we met a week or a few days before we submitted the assignments. But where to meet was the problem you see.

Tino experienced challenges that were due to insufficient time. As such his group could not meet frequently, though they would take advantage of the tutorial schedules to meet.

Tino: Not frequently though. Maybe because of too much pressure, we wouldn't get the time for that. The time was just insufficient.

Tino: The only time we managed to meet was during and after the tutorials.

Another challenge was due to the fact that the students would fail to get instant assistance when they encountered difficult concepts. Kundai and Tanya elucidate some frustrations related to this which they experienced in their groups:

Kundai : We used to meet as a group but nothing would materialise in terms of learning. As a group, we would even meet with the intentions to solve the problems then we end with nothing after failing to solve the problems. But for the other courses, we would try to solve the problems in the group. But as for calculus, we would come up with nothing, a-a-ah it was difficult for us.

Tanya: (...) Actually we had a problem as a group and then we tried to consult with the lecturer (*tutor*) but we couldn't get hold of him. So it would be okay if our tutors would be available. If they could have a schedule, which we could follow, I think it would help us.

From the above comments, it is apparent that in order to overcome these challenges, the students would also take advantage of the tutorial schedules and meet in their study groups “beyond tutorial sessions” and hence use the tutorial rooms. Making such arrangements was strategic on the part of the students since on a tutorial day more students would be available for a study group discussion as they would have travelled for the tutorial session in any case. Furthermore, there was the opportunity of using the tutorial rooms for their group discussions, “before or after the tutorials” They could also use the time to meet with the tutors since the tutor would have come to conduct tutorial.

### **6.5.2      *The physical context***

In this study, physical context refers to the environment in which learning was carried out with specific focus on the physical resources and facilities that the institution availed to the students for learning purposes. Normally, in a distance education institution students have the flexibility of choosing where to study from, and are thus not confined to a common room for learning purposes unless under special arrangements as in the case of tutorials. However, the data revealed that physical environment is an important aspect of distance education as it has a potential for influencing student learning activities. Two main aspects of the physical environment were identified from the data - library facilities and resources, as well as venues for tutorials and self-organised group meetings.



## The library

The group interviewed showed that they made use of the university libraries available to them and they found the libraries to be very useful for their learning. The library was mainly used as a source of textbooks to supplement the module as is indicated by the following quotations:

Kundai: (...) you only get to solve the problems after referring to other sources maybe from the library or from other textbooks.

Ray: What I have done in the past, I've tried to go to the library to get some books with the same concepts to assist me.

Ray: I will tell him/her to work through the module, but not to rely on the module only but also to get some textbooks from the library. Because in most cases you find it was much easier to follow examples in the textbook than to follow examples in the module.

This finding from interviews was consistent with some of the entries to LJs in which students mentioned the library as a reference point when they experienced difficulties. Despite the mentioned positive experiences related to the library, some students were concerned and had reservations as they were not satisfied with the limited provisions of text books in the library.

For instance, although Sipho appreciated the services of the library which he found to be very useful, he sounded concerned that it was not always possible to get what one wanted from the library,

Sipho: Yes. It is very useful. Although sometimes you don't get much of what you want, but it's useful in some other cases.

Ray had the following to say:

Ray: (...) You go to the library and you try to find some books on calculus. At times you can't find them or you don't find what you want also.

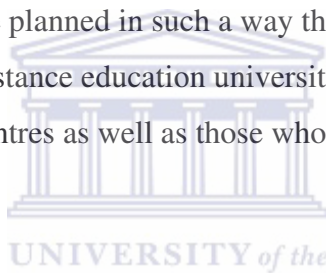
Similar findings on students' dissatisfaction with the provisions of the library facilities at the ZOU were also noted by Benza, Chitsika, Mvere, Nyakupinda and Mugadzaweta (1999) in a study conducted using students from three subject disciplines other than mathematics and statistics.

Some students found the distance to the library to be prohibitive such that they failed to benefit from the facilities in the library. Such an observation was prevalent in the interviews as well as the LJs. As a response to the question on "why did you find it difficult", one student wrote "being far away from the library ...". Another example is the following excerpt from an interview with Tino:

Tino (...) It was a problem of time because I am staying far, so I would find it difficult to get to the library on time.

In the case of Tino, whilst on the one hand the distance was a constraint as he would fail to reach the library on time, on the other hand, this could also be taken to reflect on the library opening times which apparently were not favourable for some of the distance students. Thus, early closing hours for the library were inconvenient to the distance learner.

Although students used the module as the main text in their learning processes, it is evident that students needed to be well supported in terms of learning resources such as textbooks. In instances where students need to do further research as well as in instances when the module fails to satisfy the needs of the learner, then the library should come in handy. A well equipped library provides an environment that is conducive for the learning process. Because of the importance of the library as a source of learning resources and also taking into consideration the nature of the student who is a distance learner, the opening and closing hours of the library can be planned in such a way that the needs of all the learners are catered for. Thus, a library in a distance education university has to cater for the needs of those from nearby the regional centres as well as those who travel from afar.



## Venue

Students showed appreciation of the tutorial venues. However, of major concern to some of them was the unavailability of space for group discussion meetings. Such a situation is rarely experienced by conventional students who have easy access to classrooms at their institutions. Findings described earlier in this chapter revealed that some of the students were calling for institutionally organised central venues for group meetings.

The data showed the students met either indoors or outdoors for the group discussions. Such places as the park, under a tree, privately organised venue, and use of the tutorial venue before or after the tutorial session were mentioned as places where students met for group discussions. For some students this lack of facilities was a constraint on their learning activities as they would have to change their learning strategies. For some this would mean holding on to a misconception until the next tutorial session.

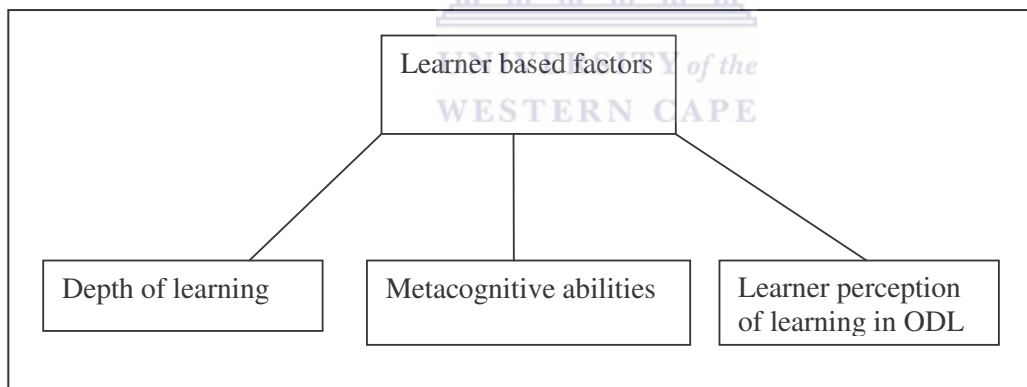
Ray felt very strongly that it was the responsibility of the institution to provide facilities where students could meet for group discussion purposes. For instance, Ray stated:

Ray: But if you had a complex (*by 'complex' student is referring to building structures*), I mean if ZOU had a complex where we could meet and discuss then there is no problem. We could organize our meeting there, but now under a tree you are not comfortable. Sometimes it starts raining, and then you move away from the tree and things like that.

## 6.6 LEARNER BASED FACTORS

At the centre of any teaching and learning experience is the student, that is, the person to whom instruction is directed and the person who engages in learning activities. The category 'learner based factors' was arrived at after considering that students have particular characteristics and engage in varied strategies when response to the environment. The category on 'learner based factors' enlightens further how the subjects of this study actually interacted with the content.

This category will dwell on subcategories – student's (a) depth of learning, (b) metacognitive abilities and (c) perception of distance education. The subcategories in turn aim to reveal more of the learning processes and activities that the distance learners engaged in when learning calculus in the ZOU distance education environment.



**Figure 6.4:** Subcategories associated with learner - based factors category

### 6.6.1 *Depth of learning*

Data of this study showed that some the students would search for depth of content whilst some would just go through the content with an aim to reproduce the information. For instance, Tino and Sipho would aim for depth when learning. They would focus on proofs and deriving formulas, while Tanya and Ray would aim to get information for memorisation and reproduction.

Sipho: The most important thing to do is in deriving the formula. Then you will be able to use it.

Sipho: I make sure that the theories and the proofs given, I understand them and then I always apply them to solving some problem and some other examples now and again, that makes me to be able to understand these concepts better.

Sipho: What I mean here is, if I am able to derive or prove a certain theorem and I am now able to solve some other examples using the information that I have learnt, that now makes me understand some concepts.

Sipho was conscious of the long term benefits of deriving formulas and understanding theorems and proofs. Both Sipho and Tino would aim to gain information for long term use hence the reference on 'future use' of the information. Tino who shared a similar orientation to learning as Sipho pointed out the following:

Tino: Because it (*deriving*) is useful. Once you can derive the formula, you won't have problems when you want to use it later on. By deriving you manage to master the concepts for future use, later on

Both Tino and Sipho were not supportive of memorisation because of the short term gains.

Tino was explicit in this regard:

Tino: Because memorising won't help them in their studying. They just benefit for that time only, but later on in life they won't have benefited anything. They will just be at the same level with those who would have not done the concepts.

Tino: I believe that if a person has not memorised but has mastered the concepts properly, then a-ah, the concepts can stay here (*pointing to his head*). But then memorising it's just temporary; it's just short lived.

Tino: At the end, those people don't have the proper understanding of the concepts. Although of course, they memorise just for survival so that they manage to write the examinations (*laughs*), but they do so without having a deep understanding of that subject.

Sipho echoed similar sentiments:

Sipho: (...) Not much. Because memorising like in calculus there are a lot of formulas, a lot of them. Now if you memorise all of them, a-a-ah it becomes difficult. But when you know how to derive it, I think it's easy that way.

While Sipho and Tino were critical about memorisation and couldn't see any benefit in memorising information, others like Tanya, Kundai and Ray were rather surface oriented and would rely on memorisation, search for algorithm. Kundai elucidated on memorisation in the following way:

I: So, in your opinion do you think by memorising one learns something?

Kundai: Yes, I think you will learn something. Because (isn't it) you are supposed to understand the definition, so when you understand it you'll have learnt it. Then you can memorise and you keep on using it

Kundai: For you to be able to use that formula, it has to be in your memory. Like in an examination situation, isn't it, you are supposed to recall the formula from your memory and then use it to respond to the questions. So it must not just end there in your head, like, "now that it is in my head, so I must leave it like that," you have to keep on using it, when you are reading

Tanya would aim to “follow” the solution steps,

I: By grasping the concept what do you mean?

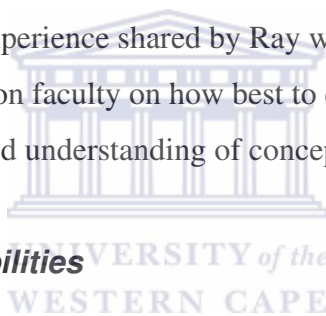
Tanya: I mean actually being able to follow the steps and make the necessary steps to solve the problem

With that approach, it’s no surprise therefore that she alluded to memorising whole solutions to problems. This is evidenced in the following excerpt:

I: What kind of things would you memorise, say with limit of function or derivative of function?

Tanya: Maybe the actual format of solving the problems, the step by step and even the (*hesitates and whispers*) the problem solution itself sometimes. (*laughs*).

The findings on the students’ orientation for depth of learning presented under this category gives an insight of the distance students’ methods of learning and thus provide information that would be invaluable to faculty when it comes to the design of learning materials, that is, design materials that would aim to foster depth in all learners and dissuade reproduction and surface oriented approaches. The information by Tanya that she could memorise the whole solution to a question, and the experience shared by Ray when he reproduced a solution from the course text place challenges on faculty on how best to ensure that students encountered a learning experience that promoted understanding of concepts.



## 6.6.2 Metacognitive abilities

Metacognitive activities relate to the regulation of cognitive abilities. This subcategory would provide insight on how the distance students regulated their learning.

### Self regulation of learning

The interview data revealed how the students regulated their learning. Students like Siphon were quite explicit on how they controlled their learning processes. Siphon stated:

Siphon: I don’t think so, because with learning, you say you are learning when you ask a question to something that is not clear to you. That way you can also assess yourself whether you are discussing something meaningful in the group. But when you just become a participant, listening only, a-a-ah, I don’t think that’s helpful.

Siphon: I don’t follow page by page, but if I have a problem, say I am studying derivatives, I just go to that section and read about the derivatives and I also make some research concerning that topic only without even going to other topics. That’s how I do it.

### Repetition as a metacognitive strategy

For some students repeating things is a metacognitive strategy that they use to reinforce information into their memory. Some students easily remember and understand the

information better if they repeatedly echo it in their memory. However, the strategy is conducive for reproduction and as noted by Felder and Silverman (1988), for some students it's a strategy that is used so as "just to be on the safe side". This kind of strategy was observable with Ray, Tanya and Kundai,

- I: What's your best strategy to calculus learning?  
Ray: (...) Maybe it is to go over and over and over the work again. Because with Calculus, that's the area where I found that most of the concepts were completely new. It's unlike in Linear Maths, unlike in Probability Theory or Applied Statistics where some of the concepts there were not completely new, but we were starting from the known to the unknown. But here (*in calculus*) we are starting from the unknown to the known.  
I: So going over and over again is your best bet?  
Ray: Yes. It's going over and over again until I understand the concept.

Tanya also adopted a similar strategy. This is indicated in the following excerpt:

- I: Ok. What do you do in order to remember something in calculus?  
Tanya: Like I was saying, I usually read over again what I have done, and I try to work on some of the examples that were done before  
I: Are you comfortable going back and forth trying to correct a mistake for the same problem?  
Tanya: I wouldn't mind. Yaah, I don't mind. I really, really (*repetition of 'really' as a way of emphasis*) don't mind. You know sometimes I could spend two hours on a problem. So I wouldn't mind doing it for as long as I grasp the concept

Kundai also adopted a similar approach;

- Kundai: If I am enjoying the topic then I can repeat it many times. But if I am not, then maybe I leave it, and then come back to it some other time.  
I: You don't mind the repeating?  
Kundai: No. I don't for as long as I am getting the things right.

However, Kundai would further aim to apply what she would have read.

- Kundai: If I continually keep on reading and then practically apply what I've read to solving problems then I find that the things just stick in my memory. But as for definitions I never really read them.

It is interesting to note that for Ray, Tanya and Kundai understanding a concept would be reached after repetitive processes, "understanding the concepts" (Ray), "grasping the concept" (Tanya) and "getting the things right" (Kundai).

### **6.6.3 Learner perception of distance learning**

The perception of the student about distance education was identified as an important category for 'learner based factors' since the learner perception would have an influence on how the learner approached learning. All the interviewed students felt that studying by distance required consistency and hard work. Siphon's response would in a way summarise the other responses from all the other interviewees in this regard,

Sipho: (...) once you start BSMS you will be doing it everyday. You make it a programme that you do it everyday so that you don't forget much of the things that you have learnt. And you also attend the tutorials. They are helpful. They clarify some other areas that are not very clear in the module. And of course you have to do some assignments. Do those assignments to your best because those assignments make people see their areas of weakness.

Kundai acknowledged the fact that the learning materials are not shared. She appreciated that she can learn on her own, at her own time, with her own materials. She said,

Kundai: Okay. The learning materials, I think they are all right, in the sense that they are not shared and I could get time to use them on my own unlike in cases where you have to share with someone and you need to give each other chances to use the modules when studying. But in this case, each one had own modules, so I had the opportunity to use them anytime I needed to. So I think ZOU is good in that they offer learning materials to each individual.

However, some students like Ray perceived and expected the distance education institution to conduct courses in the same way as the conventional institutions, where students are taught in classroom environment. In his LJ responses, Ray gave such responses as “the university does not provide proper tuition, that makes the learning of new concepts even more difficult”, “the fact there are no proper lectures makes it more difficult”. Ray's perception of distance education explicitly comes out when he says,

Ray: (...) Eehm, maybe you as Zimbabwe Open University, maybe you only require those people who haven't been to formal schools. But for people like us who have been to formal schools, then it becomes a problem, we expect to be taught how to do one, two, three, four concepts, and then you go home by yourself and then you do some of the things you were taught and that kind of thing.

Ray: (...) I think there is need for actual lessons like any of the other universities, where you go and learn the principle concepts. You know, a student from ZOU and a student from the (*name of university supplied*), there is a difference in the true sense of it. Because the ZOU student is half cooked as compared to the student from the (*name of university supplied*) where for that one there are lessons. I mean actual lessons where the students are taught the concepts. But with us here it's a different story altogether (...)

It is clear that Ray came into the institution with an expectation of being taught. It is apparent that Ray's orientation or attitude to distance education is affecting Ray in his learning processes as he gets frustrated by the social isolation. The first year of study is always a stressful year for the new students as they have to adjust to the routine of distance learning and at the same time try to master the new content. It is therefore imperative for the institution to have available support systems and orientation packages that would take care of students such as Ray, and to ensure that such students are not lost from the programme on the way.

## 6.7 DISCUSSION

This chapter presented findings on students' experiences of learning in the BSMS distance education environment. The findings show that the distance education environment can have an influence on student learning. Both effective and deterrent influences were identified, and these were centred on the content, the context and the learners themselves.

Part of the learning process includes how students interact with the content that is presented to them as part of their distance education curriculum. The category on content factors identified in this chapter pointed to aspects of the content on limit of function and derivative of function, which students found to be challenging during their learning. Notable content areas that were identified as difficult were concepts that are abstract in nature such as the mathematical definitions, theorems, proofs and concepts that use 'non standard functions'. Research conducted by others (Bezuidenhout, 1998, 2001; Cornu, 1991; Ferrini-Mundy & Graham, 1994; Hahkioniemi, 2006a; Juter 2005a, 2006; Moru, 2006; Orton, 1983; Tall & Vinner, 1981; Williams, 1991) has shown that the calculus concepts under consideration in this present study have been extensively researched upon, and that the findings of this study are consistent with the findings of others: Students do experience challenges in understanding the limit of function concepts and the derivative of function especially within non-routine problems. However, these other studies have been conducted using students in conventional systems of education, where the learning support system is different from that for distance education systems. The challenges for distance learners are magnified by the fact that learning takes place in isolation where there is no teacher who is readily available to assist with difficult content areas. This isolation coupled with the fact that the students are normally expected to read about the concepts on their own so as to obtain all the explanations and clarity from the course texts greatly magnifies the difficulty.

A key to enhancing students' understanding of the subject matter points to the clarity of the learning materials. While in a conventional education system students are guided by their lecturers in the subject matter, in a distance education environment, students rely on the learning materials. This study confirms the research findings of others (Bhalalusesa, 1999, 2001; Rahman, 2002) that distance students depend heavily on the institutionally availed learning resources. Since the learning materials serve as the main vehicle that carries the subject matter in a distance education environment, the learning materials therefore become a predominant feature of the teaching and learning process in distance education setup. The



way materials are written can facilitate or deter students' learning. Kundai's reaction to some aspects of limit of function in the course book, serves as an example of how the way content is presented to students can affect their learning process. It is therefore crucial that the materials are written in ways that encourage learning and promote depth in understanding of the concepts rather than to promote rote learning.

Although the separation between the students from their teachers and from the other students is a distinguishing feature of distance education (Keegan 1990), the resulting isolation presents challenges for most distance students. The findings of this study showed that students were frustrated with the isolation. This finding is consistent with the findings from other researchers (Dzakiria, 2005; Lyall & McNamara, 2000). These studies were conducted in a technology supported environment and still students experienced challenges with the isolation. This shows that irrespective of the medium of instruction, "feelings of isolation" (Dzakiria, 2005, p. 99) remain an issue in distance education for as long as the learning systems are not well supported to combat the isolation. Some of the participants of this present study felt that the institution did not sufficiently cater for them to combat the isolation, hence the constant call for more opportunities to meet with their tutors or to meet with fellow students or the request for libraries to be open for longer periods.

As ways to combat the isolation, students benefited from such activities as participating in tutorial sessions, self organised groups and self initiated tutoring. The BSMS programme at the ZOU organises tutorial sessions where students get the opportunity to meet with tutors. The participants of this study gave high ratings for tutorial sessions. The tutorials provided good opportunities for tutor-to-student and student-to-student interaction, something which the students appreciated. However, the tutors may have an effect on the learning experience and on what is learnt. This study, revealed that the tutorials only became meaningful to the students when the tutors facilitated an active oriented tutorial session and also encouraged student-to-student interactions, unlike where the tutor adopted a lecture like approach students as evidenced by Kundai and Ray's experiences in tutorials. Concerns related to the quality of face-to-face tutorials in distance education are not unique observations. Other researchers such as Bhalalusesa (2001), Dzakiria (2005) and Rahman (2002) noted similar concerns raised by their research subjects. The role of the tutor in the distance learning process is not limited to tutorials only, it also extends to the marking of assignments and the nature of feedback comments that they give to the students. Feedback comments on

assignments can be considered as a form of dialogue between the individual student and the tutor (Lentell, 2003, Tait, 2004). This study revealed that meaningful feedback comments on students' assignments helped students in their learning process, such a finding is consistent with one made by Bhalalusesa (2006). Considering the quality of tutors' contributions to learning points to the importance of professional development and training tutors in distance education methods that the tutors can assist students to optimally benefit from the teaching and learning processes (Dzakiria, 2005; Mercer & Petit, 2001; Tait 2004). The need to train tutors is magnified by the multiple roles of the tutors which include marking and grading of assignments, conducting tutorial sessions and attending to student queries (Tait, 2004).

This study also revealed that it is not only the physical separation that presents problems for the distance learners, but also the geographical separation from the institution. Separation from the other institutional facilities also presents challenges for students. The isolation from the institution for instance, prohibited students from easily accessing the university library facilities. The data of this study showed that students found the university libraries useful though they felt that the provisions of textbooks were limited. Although the participants of this study indicated that they benefited from the using the libraries, some of them felt that the opening times of the libraries was restrictive. For instance, as mentioned by Tino, because he has to travel long distances, he would find the library closed by the time he gets to the regional centre. The fact that the libraries would be open during the normal working hours of the day when most of the students would be at work made it more difficult for the students to optimally benefit from the library. Concerns about students' dissatisfaction with insufficiently equipped libraries or limited access to libraries in a distance education environment are not unique to this study. Other studies (Benza *et al.*, 1999; Rahman, 2002) noted similar findings. The need for sufficient library support services in a distance education environment is obvious. A library is a crucial component of the student support system in a distance education environment as it facilitates the distance students in accessing information (Tait, 2000; Mizoue, 2003). In addition, the library facilitates the distance students' development of the requisite knowledge by themselves (Mizoue, 2003).

The students also experienced problems with organising venues for group meetings and with getting in touch with tutors when the need arose.

One barrier to students' learning that emerged from the data was the insufficiency of time to enable the students to fully engage and interact with the content. Participants of this study

reported that the time afforded was insufficient for them to be able to cope with the requirements of the course, which included the coursework as well as preparations for the examinations. The time factors included the duration of the semester which students felt was too short, the tutorial sessions which students felt needed to be run frequently as well as the length of the tutorial session which students felt could be increased so that they could have more time to consult with the tutors. The students were not ready for ‘separation in terms of time’ where it was evident that some students like Tanya and Ray were not able to meaningfully learn whilst away from a group, hence the constant call for more consultation time and more time with tutors and with fellow students.

One other deterrent factor tied with the insufficiency of time is the aspect of workload. Taking a heavy workload in a congested time schedule resulted in students engaging in a surface approach to learning. This placed constraints on how students attended to other activities of the course such as assignments. For instance, data from Ray, Tanya and Kundai was evident to this, with the comments by Kundai “... so we ended up just finding answers without actually understanding how the problems could be solved since we had not finished reading the module by that time” and “I just wrote something, but then assignments to some courses submitted with incomplete solutions” showing how the workload influenced her. Kember (2004) and Kember *et al.* (1996) refer to studies they conducted on workload. They found workload to be related to surface approach of learning, when students fail to cope with the workload. Though these studies were conducted with university students in conventional settings, workload remained an issue in conventional systems. Workload can still be considered an issue in distance education as evidenced by the findings from the current study.

As much as the experiences identified in this chapter inform about how the students responded to the BSMS distance learning environment at the ZOU, the experiences also bring about insights on the role of the learning environment on student learning. It is observable that some of these experiences relate to and inform on students’ learning styles. For instance, students’ preferences on forms of representation of mathematical concepts would indicate preferences for particular dimensions on the F-SLSM. Preferences for concrete and numerical information as revealed in the case of Ray relate to the sensing category and preferences for abstract information relates to the intuitive category of the perception dimension. Preferences for graphical information as in the case of Siphso would

relate to the visual category and the preference for actively participating in learning activities points to the active category of the processing dimension. The fact that the students recommend the use of multiple representations in the course texts illuminates on the need to cater for learner diversities in terms of learning styles.

## 6.8 CHAPTER OVERVIEW

This chapter aimed to provide insights into the distance learning experiences of mathematics students in the BSMS environment at the ZOU, using the context of a first year calculus course. The focus of studying the students' experiences was to illuminate on the influences of the distance learning environment on student learning processes. The findings of this study as presented in this chapter revealed that the learning environment can influence learning processes in both effective and deterrent ways. An instructional system that is carefully designed and well supported has the potential to effectively influence students' learning processes, whilst one that is not well supported can inhibit meaningful learning. In this chapter, a number of environment related factors that were influential to student learning were identified and described, and these revolved around content related factors, context related factors and learner related factors. The students' experiences also informed about the students' learning styles. Learning styles is the focus of the next chapter where, for some students, I profile their learning styles preferences from the data and relate these to the learning outcome, as represented by the students' mathematical understanding of the limit and derivative of function concepts.

# CHAPTER 7: STUDENTS' LEARNING STYLES AND UNDERSTANDING

## 7.1 INTRODUCTION

Chapter 6 discussed students' experiences with the distance learning environment, with the intention of illuminating students' responses to the learning environment, which would consequently inform on the influence of the learning environment on students learning. However, individual differences in reactions to the environment also exist and these can be interpreted in terms of the learning styles construct. While Chapter 6 served to illuminate how students experienced distance learning, dominant learning style preferences could be identified for each of the individual students who were interviewed. The focus of this current chapter is to present and discuss findings related to students' learning styles and mathematical understandings. The findings are presented by individual cases. Considering individual cases made it possible to capture how individual students interacted and responded to the distance education environment. In this way it was possible to explore deeply how students learn and hence identify the students' learning style preferences from the students' experiences with the DL environment.

This chapter, serves a threefold purpose. Firstly, it aims to identify and describe learning styles for selected first year mathematics distance students. Secondly, it will describe the students' mathematical understanding of the limit and derivative of function concepts. Thirdly, it aims to relate the two variables of learning style and mathematical understanding.

The chapter will thus attempt to address the two research questions:

- How can distance students' experiences of learning calculus be used to inform on the students' preferred learning styles?
- What relationships exist between students' preferred learning styles and learning outcomes as represented by how students understand the limit of function and derivative of function concepts?

The following section presents the case by case analysis and discussion of students' learning styles and the associated mathematical understandings. The learning styles are interpreted in terms of the Felder-Silverman learning styles model.

## 7.2 SOME CASE STUDIES

This section presents some illustrative case studies of five students' learning style profiles and the mathematical understandings of the limit and derivative of function concepts for the five students.

The construction of case study record summary on learning styles followed the schematic process that is proposed in Figure 5.3 of Chapter 5 whereby some 'significant quotations' and their corresponding 'critical pointers' are identified from the raw data. For each of the students, a summary of the case record on the learning styles preference is presented in tabular form followed by a narrative description of the case. Quotations from the students' writings and or interview transcripts are used to support the narrative descriptions of the emerging learning styles. The sources of these quotations are also indicated next to the statement. For example the notation (Int, xxx) means that the source of the statement is the interview transcript, stanza number xxx; and also the notation (LJ, L1, item 3) means that the source of the statement is learning journal L1, item 3.

### 7.2.1 *Profile for Sipho (Student A)*

This section provides the learning style and mathematical understandings profile for Sipho (Sipho is not his real name). Sipho, who is referred to as Student A (St A) in the transcript, is a primary schoolteacher by profession and is working in a rural school. Sipho was selected to be interviewed on the basis of 'good' responses to the Calculus Tasks Test.

#### **Learning Styles Profile**

Table 7.1 below gives a case record summary of the characterisation of Sipho's learning styles.

**Table 7.1:** Characterisation of Learning Styles for Siphon

Dimension	Significant Quotations	Critical Pointers	Preference
Perception	<ul style="list-style-type: none"> <li>- I liked most using the definition and theorems and applying the theorems to examples. (Int, 001)</li> <li>- I make sure that the theories and the proofs given, I understand them and then I always apply them to solving some problem and some other examples now and again, that makes me to be able to understand these concepts better. (Int, 058)</li> </ul>	<i>Theory oriented</i>	Intuitive
	<ul style="list-style-type: none"> <li>- The most important to do is deriving the formula, and then you will be able to use it. (Int, 012)</li> <li>- If you just use it (<i>the formula</i>) without knowing where it is coming from you can have some complications. (Int, 015)</li> <li>- But when you know how to derive it, I think it's easy that way. (Int, 018)</li> <li>- So, it's very very (<i>repetition of 'very' as a way of emphasis</i>) important that we understand the basics. (Int, 070)</li> <li>- And he can now derive it and then can now apply it, but knowing from first principle that this is where this is coming from. (Int, 090)</li> </ul>	<i>Meaning oriented</i>	
	<ul style="list-style-type: none"> <li>- (...) like in calculus there are a lot of formulas, a lot of them. Now if you are to memorise all of them, a-a-ah it becomes difficult. But when you know how to derive it, (...) its easy that way (Int, 018)</li> </ul>	<i>Discomfort with memorisation</i>	
	<ul style="list-style-type: none"> <li>- Consulted other texts (LJ, L1 &amp; L2, Item 6)</li> <li>- You know, I have to research further on my own so that I understand some of the proofs. (Int, 032)</li> <li>- It's learning what is in the module and doing some further research on the topic given. That's how I learn this course. (Int, 057)</li> </ul>	<i>Does further researches in order to understand (innovative)</i>	
Input/ Receiving	<ul style="list-style-type: none"> <li>- When finding a limit of a function. It's easy to arrive at an answer using graphs. (Int, 021).</li> <li>- You can interpret information easily using graphs. It's easier that way (Int, 022).</li> <li>- My best is interpreting using the graph (Int, 025).</li> <li>- Yaah, graphs present information. You do the graph thing it's easy to interpret the information</li> </ul>	<i>Preference for Graphs</i>	Visual

	from the graph (Int, 075).		
<b>Processing</b>	<ul style="list-style-type: none"> <li>- How Siphso overcame difficulties: Discussed certain problems with colleagues (LJ, D1 &amp; D2, item 6).</li> <li>- It's (<i>an ideal tutorial its</i>) one, which is more of a discussion, and the tutor probes people or students with questions and then allowing people to discuss certain concepts rather than him just explaining things (Int, 033).</li> <li>- That kind of conversation where you exchange from questions, answers and so on. That tutorial is very helpful (Int, 034).</li> <li>- Group discussion is very important (Int, 037).</li> <li>- Yaah, in a group I learn much better than when I am alone (Int,039).</li> <li>- But when you meet as a group you share ideas, and something may come out from that group (Int, 047).</li> <li>- Those tutorials are very important. When you go out there without knowing anything, following the module alone, there will be some things of course that you can do, but, there are some other areas where you need clarification and it becomes difficult when you are alone. (Int, 062)</li> </ul>	<p><i>Active oriented</i></p> <p><i>Finds benefit from groups/tutorials</i></p>	Active
<b>Understanding</b>	<ul style="list-style-type: none"> <li>- (...) they (<i>the objectives</i>) are very helpful, they tell you about what's coming. (Int, 051)</li> <li>- learning without the objectives set, a-a-ah you will be learning nothing, because you don't know what you are learning, (Int, 052)</li> <li>- But when you go to the objectives if they are there, they guide you now, they tell you where you are going. (Int, 052)</li> </ul>	<p><i>Holistic/Big picture first followed by parts</i></p>	Global
	<ul style="list-style-type: none"> <li>- I don't follow page by page, but if I have a problem, say I am studying derivatives, I just go to that section and read about the derivatives and I also make some research concerning that topic only without even going to other topics. That's how I do it. (Int, 050)</li> </ul>	<p><i>Can learn in large</i></p>	
	<ul style="list-style-type: none"> <li>- I have seen that that topic limits is appearing in almost every course that we are doing. In every course, we have to cover some problems regarding limits. (Int, 070)</li> </ul>	<p><i>Links concepts/topics</i></p>	



The summary of the case record that is given in the table above is indicating that the emerging learning style profile for Siphon across all the four dimensions of the F-SLSM is: **Intuitive, Visual, Active and Global.**

i) *Perception dimension: Intuitive*

Siphon is an intuitive learner. From the summaries of the case record it is evident that Siphon shows tendencies of being abstract and meaning oriented. He shows comfort working with theories, understanding issues using first principles, researching further on his own on issues he doesn't understand and he shows discomfort with memorisation. All these serve as characteristics of the intuitive learner in accordance to the Felder Silverman learning styles model.

The learning journal entries that Siphon provided in response to items 1 and 2 are biased towards theorems, definitions and deriving things using first principles. For instance, in journal L1, Siphon indicated the most important things that he learnt for the limit of function topic to be “*proving limits using the definition of limits*” and “*proving limits using theorems*”<sup>11</sup>. This was notably consistent with his response in learning journal L2, when he says “*using definition of limits to prove (limits) for given function*”. In journals D1 and D2 which have entries pertaining to the derivative of a function concept, Siphon wrote “*finding derivatives of functions from first principles*” and “*applying rules for differentiation*” to be the most important to him. Such responses highlight that when engaging in some learning activity, Siphon puts his focus on the abstract materials and tends to search for meanings in whatever he would be reading. For instance, when working with formulae, Siphon says he attempts to understand where the formula is coming from (deriving formula) and believes that deriving the formula from first principles is much more important than just knowing the facts. This is consistently noted in his responses to the LSPQ for derivative of function where he indicated the most preferred representation to be the use of the definition. The following interview excerpt shows the level of importance that Siphon places on working from first principles:

I: What do you think is more important to know when you are learning calculus? Is it knowing on how to use the formula or rules, or is it deriving them?

---

<sup>11</sup> By ‘proving limits’ the student is referring to use of the definition or theorems in solving the ‘show that’ or ‘prove that’ kind of problems for limit of function.

- Sipho The most important to do is deriving the formula, and then you will be able to use it.
- I: You say you derive it then you will be able to use it?
- Sipho: Yes.
- I: Can you not separate the two?
- Sipho: No, you cannot. Because once you derive a formula, you will know what it is, and then you can apply it to some examples, and that will make things a lot better.
- I: What about using or applying without deriving?
- Sipho: (*Hesitates*) E-e-eh, yes, it can work and you can apply it. But, there are certain areas which need that knowledge of knowing where certain formulas are coming from. If you just use it without knowing where it is coming from you can have some complications.

Sipho shows tendencies of being conceptual and tends to have a preference for meanings, such that he tries not only to learn the facts of mathematics as given but he actually attempts to understand why things are as they are. Sipho showed some discomfort using analytical methods or applying rules and theorems just like that. He had the following to say about his ranking of the analytical method in his LSPQ response:

- I: And you ranked it (*the analytical method*) lower.
- Sipho: It can even be least preferred. The danger is you can misrepresent information. I remember one problem that I worked out, I think it was on the assignment, I stated a certain theorem and then the tutor replied, "You did not research much on this theorem". But that helped me very much, because I looked at my problem and at the comment and I moved back to look at that theorem and yes, I had misquoted it. You see that's the danger, you misquote the theorem you can misrepresent the information.

The search for meaning was also noticeable in the discomfort that Sipho had about memorisation. Although Sipho said he memorises "certain formulas", he believes that there is not much benefit in memorising:

- Sipho: (...) There are some certain formulas where you feel you cannot derive that formula, you can memorise it and then apply it.
- I: Do you feel there is benefit in memorising?
- Sipho: No. Not much. Not much (*shaking head*). Because memorising like in calculus there are a lot of formulas, a lot of them. Now if you are to memorise all of them, a-a-ah it becomes difficult. But when you know how to derive it, I think it's easy that way.

ii) *Input/Receiving dimension: Visual*

Sipho shows characteristics of a visual learner. The summary of the case record indicates that Sipho shows tendencies of being non-verbal and he showed strong preferences for the graphical material. Sipho finds much benefit and understands information better if it is presented graphically. He said the following about graphical information:

- I: Do you find a place somewhere for using graphs or pictures in the modules
- Sipho: Yes.

I: Where?

Sipho: When finding a limit of a function. It's easy to arrive at an answer using graphs.

I: Okay.

Sipho: You can interpret information easily using graphs. It's easier that way.

I: Which would be easier for you, say you are given a function  $f(x) = \frac{1}{x}$  and you were asked to find the limit of  $f(x)$  as  $x \rightarrow 0$ . What would be easier for you to graph it or to use the analytical approach? Which approach quickly comes into your mind?

Sipho: The graph. It's easy for me to use a graph to such a function. Yaah, I can follow what the interpretation I have for the conclusion is, using the graph. It's easier that way.

The preference for graphical information is dominant for Sipho and he acknowledges that it is easier for him to interpret information if it is presented using graphs. Even on his LSPQ entries for limit of function, Sipho consistently indicated his most preferred representation to be the graphical one. Sipho also noted with frustration that the module does not have sufficient problems that elaborate the derivative of function concept using graphs.

I: You were really consistent with the graph?

Sipho: Yaah, even in the module it is clear if you follow the graph. The definition of the derivatives using the graph and the examples given graphically it is easy to follow that way though unfortunately there are not many such problems in the module.

Sipho also showed tendencies of being non-verbal. Hearing about a concept first does not make it easy for him to understand the information. He still has to work on it at his own pace.

I: Please enlighten me, would you understand a concept easier if you first hear about it in a lecture like situation or if you read about it on your own.

Sipho: U-u-uhm, myself it's not easy for me to understand a concept when I hear about it. But, maybe if I am to follow up, later on at my own pace, then I can understand it.

I: So if it's said to you in a lecture like situation, you say you cannot understand it?

Sipho: Not really that I can't, but I can pick up things because in a lecture, maybe because of time, you won't have the time to master the main concepts, you see. So I can pick up some other certain concept but not all of them. But when I follow up now, later on, I can understand the concept.

Sipho said he does not find much benefit from the lecture approach. About lectures Sipho indicated,

I: How do you feel about the lecture approach personally?

Sipho: A lecture is important. Of course, it gives you some insights of what exactly is called a limit, if you are learning about limits for instance. In a lecture you try to understand things, but then it does not clarify things. But when you follow up now, later on you are now adding to what you have understood, that becomes a bit clear.

Although Siphon said lectures are important, he said this is so because the lecture gives insight of the concept, though not clarifying things. This could be due to his earlier observation that there is insufficient time in a lecture to master the main concepts. In order to understand the information he still has to work at his own pace.

(iii) *Processing dimension: Active*

Siphon shows characteristics of an active learner. Most of his responses related to how he solves problems and difficulties indicated that he is a learner who prefers to engage actively with the information rather than introspectively. From the summary of his case record, it is evident that Siphon benefits from the discussions that take place in groups, and this could be in the tutorial group, the study group or just the casual discussion with colleagues. For instance, he pointed out in his entries for learning journals D1 and D2 that the discussions with colleagues helped him clarify issues and hence overcome some difficulties.

Siphon placed great value on group sessions as he sees the group as an opportunity for brainstorming issues, clarifying concepts, sharing ideas, as well as arguing about the concepts with colleagues.

I: What is your opinion about group work or group discussions?

Siphon: It is very important. Group discussion is very important.

I: Why?

Siphon: You exchange ideas where you are not clear, where you have gone wrong, somebody can correct you, and other information can come in during group discussion. It is very important.

He acknowledged that he felt better learning in a group than when he is alone. In a group he has the chance to explain what he has learnt, to give information and to discuss with others.

I: Do you feel you learn better in a group than when you are alone?

Siphon: Yeah, in a group I learn much better than when I am alone.

I: Suppose you are involved in a group activity and I am part of that group, what would I see you doing in that group.

Siphon: Myself, I would be taking part in explaining and giving information of what I know, and then also arguing some other concepts from fellow friends and then in discussing as we go.

Siphon was of the view that in a group, if one is just listening and not participating actively, then for that individual no learning is taking place. He thus fails to see any meaningful learning taking place without active participation. He had the following to say:

I: What do you think about those group members who want to listen, those who are in the group but they want to listen?

Sipho: Just listening without contributing? Yaah, there are some people who are like that. I don't think they will be learning much.

I: Okay?

Sipho: I don't think so, because with learning, you say you are learning when you ask a question to something that is not clear to you. That way you can also assess yourself whether you are discussing something meaningful in the group. But when you just become a participant, listening only, a-a-ah, I don't think that's helpful.

Although, Sipho is an advocate for group meetings, he was failing to meet with his group mates because of the distance between the group members. We sense the frustration of that 'limitation' in the following:

Sipho: The problem that I have is, I used to have a certain group, and we could meet during the school term every fortnight. There is a certain place where we could meet. But during the time when we are on holidays or breaks we would meet nearly every day and discuss some problems.

I: And now?

Sipho: O-o-oh this time, it is very difficult to meet some other group members because of the distance.

I: It's the problem of the distance now?

Sipho: Yes it's the distance; we are so far apart.

(iv) *Understanding dimension: Global*

The case record indicates that Sipho shows tendencies that characterise global learners, that of being holistic, being able to learn in jumps and thinking of links between concepts.

Sipho tends to be holistic and looks for the big picture first followed by the parts. He finds the overview and objectives of a topic to be "very helpful as they tell you what's coming". To him once he gets the big picture from the overview and objectives, he can now use them as guidelines and then fit in the other activities such as checking on understanding and identifying the concepts that need to be researched on further, and attending to them.

I: Please say something about your opinion regarding the sections in the module "What this unit is all about" and the objectives section "by the end of this unit you should be able to". Do you find these sections to be useful to you?

Sipho: (...) yaah, they are very helpful, they tell you about what's coming. Those also test you on whether you have understood on what you have been learning. Have you fully understood or whether there is some concepts that maybe you need to research more on, some concepts that are not clear? Those (*objectives*) are the guidelines for that. Because you need to go back now and look at those objectives and say have I achieved (a), (b), (c) and (d), you are now looking at your performance and now that makes it easy for you to even make some research regarding a certain concept only but not the whole topic.

For Sipho there is no learning that can take place if there are no objectives set because you don't know what you are learning. He looks at objectives in a global sense whereby you need to know where you are going in order to come up with/determine the route

I: Would it be difficult for you to use the module without these sections?

Sipho: Yes. It's not learning. You see, learning without the objectives set, a-a-ah you will be learning nothing, because you don't know what you are learning, you cannot even evaluate yourself on whether you are doing the right things. But when you go to the objectives if they are there, they guide you now, they tell you where you are going. So I think they are very helpful, these objectives.

As noted in the case record summary Sipho also showed some tendencies of being able to learn in leaps rather than in sequence. Furthermore, as quoted above he says he can pick on certain concepts that require further research and attend only to them.

## Mathematical Understanding

Sipho serves as an example of a student who showed conceptual understanding of the topics under study. An analysis of Sipho's CTT responses reflected a good understanding of the concepts. He showed mature conceptions of the limit and derivative of function concepts. When asked to "explain why", Sipho explained and argued for his written responses.

- *Limit of Function*

Sipho's strength with graphs helped him to solve some problems, such as L6, L9 and L10. He had no challenges responding to item L6, where he wrote the  $\lim_{x \rightarrow 2} f(x) = 4$  and made his justification using the graph. Items L9 (a) and (b) which were based on a graphical problem did not present problems for Sipho with the following responses and justification;

- a)  $\lim_{x \rightarrow 4} f(x) = 8$ ; Justification: *The graph shows that  $x$  approaches 4 from the positive and negative directions. The limit is therefore 4.*
- b)  $\lim_{x \rightarrow \infty} f(x) = \infty$ ; Justification: *The graph shows that the value of  $x$  increases, therefore the value  $f(x)$  approaches  $\infty$ .*

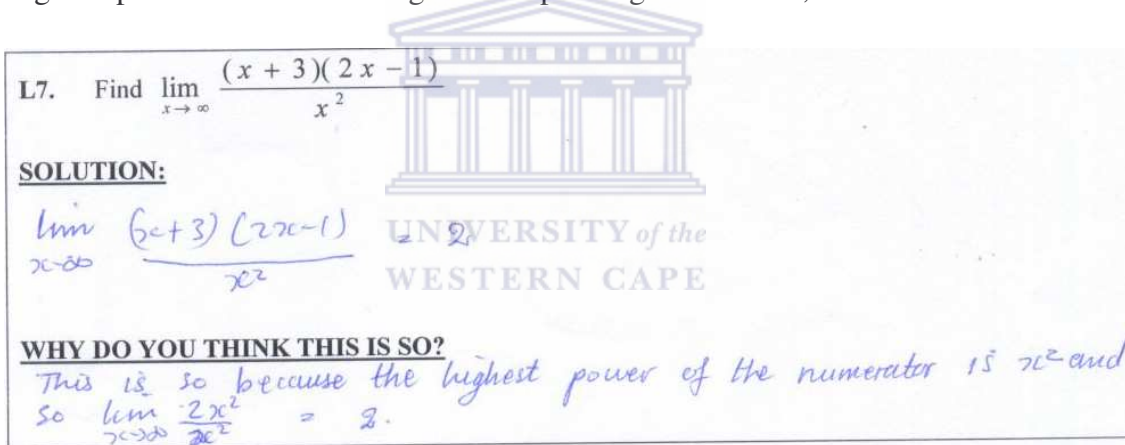
For item L10, which had a piecewise defined function Sipho made a sketch of the graph and used the graph of  $f(x)$  to come up with the solution. For the justification, he wrote,

**SOLUTIONS (Provide explanations in each case)**

- a)  $\lim_{x \rightarrow 1} f(x)$  = As  $x$  turns 1 from positive and negative direction  $f(x)$  is undefined. Therefore limit does not exist
- b)  $\lim_{x \rightarrow 0} f(x)$  = limit does ~~not~~ exist again and it is 1, graphically  $f(x)$  is 1 as  $x$  approaches 1 from positive and negative direction.

We observe from the above extract that he justified his responses in a meaningful way, where he considered both the positive and negative direction for the piecewise defined function. This showed a thorough understanding of the limit of function concept.

For most students item L7 was done sequentially following the procedure for finding limits to rational functions. However, Siphon approached the problem from a conceptual angle. Siphon wrote the following when explaining his solution,



L7. Find  $\lim_{x \rightarrow \infty} \frac{(x+3)(2x-1)}{x^2}$

**SOLUTION:**

$$\lim_{x \rightarrow \infty} \frac{(x+3)(2x-1)}{x^2}$$

**WHY DO YOU THINK THIS IS SO?**

This is so because the highest power of the numerator is  $x^2$  and so  $\lim_{x \rightarrow \infty} \frac{2x^2}{2x^2} = 2$ .

- *Derivative of Function*

Item D2, which was a routine question, did not present problems for Siphon. For his justification for D2 (a), Siphon wrote “differentiated from first principles” and for (b) and (c) it was “through the usual differentiation process”. Siphon managed to work out item D3 which had tabulated data. This item presented problems to most students but Siphon approached it from a theoretical perspective by applying the Mean Value Theorem to solve it, a theorem in calculus which is mostly avoided by many students.

Siphon correctly paired the functions in item D4 accompanied with abstract and meaningful justifications. For instance, for the pair  $g(x)$  and  $h(x)$  he explained as follows

$g(x) = -|ax+b|$ . Differentiating the function gives the function  $h(x)$ .

### Discussion on Sipho's profiles

The above excerpts show that Sipho's strengths with graphs and abstract orientation helped him to solve and interpret the mathematical problems. Sipho's learning style profile revealed that he did not rely on rote learning strategies such as memorisation because of the short-term gains. Instead his intuitive approach to learning in which the search for meaning dominated actually played an important role in his learning and resulted in long-term gains. Because of his intuitive, visual and global tendencies, Sipho could comfortably interpret information from graphs, comfortably apply theorems and he could afford to use non-routine methods to solve would be routine problems. The way Sipho tackled some of the problems, and the easiness with which he justified and argued for his responses across the items, indicates that Sipho had a good grasp of the concepts and hence showed conceptual understanding of the concepts.

#### 7.2.2 Profile for Ray (Student B)

This section provides the learning style and mathematical understanding profile for Ray (Ray is not his real name). Ray, who is referred to as Student B (St B) in the transcript, is a secondary school mathematics teacher and is based at an urban school. Ray was selected to participate in the interview on the basis of 'poor' responses to the Calculus Task Test.

### Learning Styles Profile

Table 7.2 given below gives a case record summary of the characterisation of Ray's learning styles.

**Table 7.2:** Characterisation of Learning Styles for Ray

Dimension	Significant Quotations	Critical Pointers	Preference for dimension
Perception	<ul style="list-style-type: none"> <li>- <i>Reasons for difficulty:</i> Concepts are completely new to me. (LJ; L1; item 5); It is new to me (LJ, D1, item 5); The questions are completely different from the examples (LJ, L2, item 5);</li> <li>- (...) you make a mistake here and you try to find out where the problem is and you can't find it. You try to do this and that, then that way it becomes a problem and it will put you off and</li> </ul>	<i>Dislikes surprises and complications</i>	Sensing



	<p>you stop struggling (Int, 037)</p> <ul style="list-style-type: none"> <li>- Frankly speaking sometimes you throw away the module. You try other things (Int, 054)</li> </ul>		
	<ul style="list-style-type: none"> <li>- Through repeated attempts (LJ, L1, item 6)</li> <li>- I had to go through the given examples several times (LJ,L2, item 6)</li> <li>- Going through the problems over and over again (LJ, D2, item 6)</li> <li>- Maybe the definition you'd go through it several times (Int, 005);</li> <li>- As I mentioned earlier on, that you could read the definition several times so that you can understand it properly (Int, 009);</li> <li>- Then I sat down again and I said, let me do it and take my time (Int, 028);</li> <li>- when I make mistakes, sometimes it's a challenge, but you keep on trying, (Int, 037);</li> <li>- I don't mind, even trying the same problem several times, as long as I know that I am doing this for that long (Int, 039);</li> <li>- It's to going over and over again until I understand the concept, (Int, 127);</li> </ul>	<i>Does not mind repeating</i>	
	<ul style="list-style-type: none"> <li>- I'd say those are questions (<i>problems</i>) which were both simple and were easy to follow and those questions which were refined in the module with the examples (Int, 007).</li> <li>- That's why I'm saying we want questions, which are more or less the same, not exactly the same (Int, 050)</li> <li>- I would prefer a situation where someone would work out the problem for me first, whilst I am watching and then I will do similar problems by myself (Int, 101).</li> </ul>	<i>Preference for standard methods</i>	
	<ul style="list-style-type: none"> <li>- Calculating. I'd be very comfortable with that, than to say prove this, prove that this is equal to that (Int, 016).</li> <li>- then go through it, maybe getting my tables done, you know and study the pattern and things like that, than memorising (Int, 044)</li> </ul>	<i>Preference for non abstract materials</i>	
<b>Input/ Receiving</b>	<ul style="list-style-type: none"> <li>- There is no one to explain to me, the abstract concepts (LJ, L1, item 5);</li> <li>- The fact that there are no proper lectures makes it more difficult (LJ, D2, Item 5);</li> <li>- Overcome difficulties by trying to get some other people to explain the concepts (LJ, D1, Item 6)</li> </ul>	<i>Requires someone to explain concepts</i>	<b>Verbal</b>

	<ul style="list-style-type: none"> <li>- I said I will find somebody who is doing the course to explain or I will ask some of my young brothers to explain the concepts. So later on as I was talking to some of my friends, they said, they (<i>limits</i>) are very easy. (Int, 028)</li> <li>- I think there is need for actual lessons like any of the other universities, where you go and learn the principle concepts (...), I mean actual lessons where the students are taught the concepts (Int, 058).</li> <li>- I understand a concept better if the concept is explained to me first then I do it by myself later on. That way I will understand the concept better.(Int, 075)</li> </ul>		
<b>Processing</b>	<ul style="list-style-type: none"> <li>- You meet some friends and talk (<i>during tutorial sessions</i>). Tutorials were useful sometimes. Maybe that gave me the courage to sit down and study the limit concept, (...) (Int, 063)</li> <li>- My role (<i>in a tutorial</i>) is to participate as much as possible (Int, 067)</li> <li>- I have no problems with that, and I mean I am comfortable with being in such groups, (Int, 082).</li> <li>- Just to participate and discuss with the other students, (<i>student's role in a group</i>). Maybe this one knows this, and the other one knows that, then each could try and explain what they know and the other one explains also what they know. Then you come to a general consensus. (Int, 092).</li> </ul>	<i>Finds benefit from group discussions</i>	<b>Active</b>
<b>Understanding</b>	<ul style="list-style-type: none"> <li>- As long as I know that I am doing this for that long, and I am going there, then get there right up to the end of the problem (Int, 039).</li> <li>- But initially when we got these modules, then, we'd try to go through from Chapter 1 through to the last chapter. (Int, 103)</li> <li>- But later on we realised that we haven't learnt anything, then, I had to go through chapter by chapter again (Int, 103).</li> </ul>	<i>Preference for linear sequencing</i>	<b>Sequential</b>
	<ul style="list-style-type: none"> <li>- there is no follow up to the given examples (LJ, L2, item 5);</li> <li>- But when you try to do the second one and the third one, there is no relationship between the second and the third example (Int, 009).</li> <li>- You find the first example its ok, but the second one is completely different from the first one, and to link the two, becomes even more difficult (Int, 010)</li> </ul>	<i>Searches for logical sequencing</i>	
	<ul style="list-style-type: none"> <li>- There are no clear questions, which start from the known to the unknown ((LJ, D2, item 5)</li> </ul>	<i>Prefers steady progression of complexity and</i>	

	<ul style="list-style-type: none"> <li>- You know when you learn these mathematical concepts, you learn them in stages and not from nowhere to the abstract. But, you can go from the simple to the abstract and it's best that way. (Int, 051)</li> <li>- If the concepts were explained right from the beginning then that would mean the tutor will have less work, because one is starting from the known to the unknown, unlike where you are starting from the unknown to the known. (Int, 070)</li> </ul>	<i>difficulty</i>	
--	--	-------------------	--

The above summary of the case record indicates that the emerging learning style for Ray is **Sensing, Verbal, Active and Sequential**.

(i) *Perception dimension: Sensing*

The summary of Ray's case record that is given in the table above shows that Ray displays tendencies of being a sensor. He dislikes complications, is patient with repetitions, prefers working with standard methods, shows discomfort working with abstract materials and shows tendencies of memorising though he doesn't say so explicitly. These all serve as characteristic indicators to the sensing dimension on the Felder Silverman learning styles model.

Ray shows discomfort with challenges and with those situations that result in complications. He also dislikes surprises. He easily gets frustrated or gives up when he encounters the unexpected, such as when he makes mistakes or when he encounters new or difficult material.

Ray: Sometimes when I make mistakes, sometimes it's a challenge, but you keep on trying. But at times it will put you off. But in most cases it's a challenge. You make mistakes here and there, you discover the mistake then it's a challenge again. But the moment you are stuck completely, you make a mistake here and you try to find out where the problem is and you can't find it. You try to do this and that, then that way it becomes a problem and it will put you off and you stop struggling.

I: Then you stop completely?

Ray: Yeah.

The fact that Ray is a distance learner and most of his studying has to take place whilst he is alone makes studying difficult for him. More frustrating for him is when he has to tackle the challenging concepts when he is studying alone. For instance, Ray's frustration of studying alone and having to experience new materials (alone) is illustrated below,

Ray: (...) Aaah, Initially it was tough. At first I couldn't understand the topic because (*pause*) for the greater part of the course, I mean the first semester, we did most of these things by ourselves. When we went for the tutorials, I think we did Chapter 1 and part of Chapter 2. So we didn't do the limits part. So you had to do it by yourself and sometimes you needed to ask friends who weren't there. So it was tough. It was really challenging in actual fact. And also taking into consideration that some of the things we were doing, for me, they were completely new. So you know when you are learning new things by yourself it can be very difficult.

Ray tends to be patient with details and does not mind repeating things over and over again. Even in his responses during the interviews, Ray tended to give very detailed and wordy responses and at times he would repeat phrases as a way of emphasising his position. An example would be:

Ray: Yaah, in the tutorials because you go there once, the second time you go you are told that the next time you will be meeting for the last time, the next time you are submitting your exams (*assignments*). So tutorials as far as I am concerned, I am not satisfied. (*student repeats the sequence again*) It's like you meet now, and then meet the second time, the next time you are submitting your exams (*assignments*), the next time you are going for revision, the next time you are writing exams. Can you see that?

When problem solving, Ray indicated that he does not mind to make repeated attempts at solving a problem for as long as he would be making a headway with the solution. For instance, Ray's patience with repetitions is elaborated in the following:

I: If you encounter a mistake, do you mind when you keep on trying to solve that problem?  
Ray: No I don't. A-ah as long as I am able to work it out. You find that you make this mistake, you correct the mistake then you proceed or you start again. A- a- h, I don't mind, even trying the same problem several times, as long as I know that I am doing this for that long, and I am going there, then get there right up to the end of the problem. I don't mind.

As per the FS learning styles model, sensors tend to like well-defined problems that can be solved using standard and routine methods of solution. Ray is indicating this preference, as he clearly points out that he prefers to work with problems that are identical to previously worked-out examples. It would appear that Ray has set 'his standard method' to be the already worked out method of solution to a problem. The following statement illustrates how dominant and important this is for Ray,

I: (...) how would you want the information to be presented to you? I mean what would best enable you to learn?  
Ray: Eehm. Maybe with the modules, for them to have some worked examples within the modules and then they give exercises with the relevant questions, which are more or less the same with the ones that would have been given in the example. I think we will learn better that way, than a situation where there is a question here, and there's another question here, then there is another one there, then when it comes to the exercises it's completely different.  
I: So you don't like that? The different ...(*interjected*).  
Ray: I don't feel comfortable. I just don't feel comfortable with that at all.

We note the frustration in Ray, if it so happens that the problem deviates from ‘his standard problem’ and is not similar to what he knows, maybe because this results in his standard method of solution not being readily transferable to a different type of problem, hence the frustration.

Ray: (...) Because, I say, if I do the next one, maybe I will go to the exercises and see if I can do similar (*with emphasis*) questions. If I don’t find a similar question, then it means I won’t be interested in doing other questions that are there because I will get stuck.

I: But maybe the purpose is to let you think differently and to look at different problems.

Ray: Yes. That’s why I’m saying we want questions, which are more or less the same, not exactly, the same.

Ray insisted that he learns better and feels comfortable when he is presented with problems that are similar to previously worked out examples. However, we observe that this presents him with a good opportunity to reproduce solutions,

Ray: (...) Just like when we do mathematical induction, you know that one.

I: Yes, I know it. Is that what you like?

Ray: No. I don’t like it. Fortunately for me the one that came in the exams was coming straight from the module (*laughs and pauses*). So at least I could do that one, there were no problems. We once solved it.

I: So you remembered it?

Ray: Yeah. I remembered how to do it; the first part right up to the last part.

Although Ray says he doesn’t memorise, we note that in fact he does memorise. The availability of similar problems eventually presents a good grounding for memorising information from the module.

Ray: You know there are certain things that we used to ignore, especially these proofs and so forth. Then the moment we realised that the same things, which are coming in the assignment are the same things which are coming in the tests, then we realized that we must go through it. Then, that’s when we started to be serious about proofs and we were trying to memorise the theorems because we realised that these are things that are coming from time to time.

Another Felder Silverman learning style pointer for sensing learners that is dominant with Ray is the discomfort with abstract materials. Instead he showed strong preference for the concrete information including solving problem involving calculations and numerical examples.

Ray: For an example, I was just showing you that table on limits of function. For most of the questions on limits, I would put them in the form of a table, that makes it a lot easier for me.

I: You mean the table?

Ray: Yaah. If I get a question on limits, if I can put it in the form of a table, then I know, that it’s done. But if I can’t, then it means that there are problems.

I: What if it’s not a simple function?

Ray: Oh, those complicated ones now, that's why I was saying those ones now become a problem.

Ray acknowledged that the tabular approach helped him to understand the limit of a function, as the table would help him 'see' the right answers.

I: Why did you find the tabular approach easier to work with?

Ray: Because you could work it out and get the answers and compare the answers, right by yourself and check it and see if your answers are right or wrong. And you can see by yourself that this is right or this is wrong.

I: Then you start seeing the limit? How do you see that? Do the tables really show you the limit?

Ray: Yaah, the moment you start seeing the 2.999999, 2.999999998, 2.9999999999999999 (*repeats the nines*)

Preference for tabular information is associated with sensing learners on the F-SLSM.

(ii) *Input/Receiving dimension: Verbal*

The above summaries of the case record for Ray indicate that Ray is a **verbal** student.

Verbal learners prefer verbal information either in the form of verbal or written text. It is evident from the case record that Ray prefers to have new information verbally explained (spoken) to him first. Ray acknowledges that he understands a concept better if it's explained to him first.

I: Do you understand a concept easier if you hear about it first or do you want to work on it on your own first?

Ray: I understand a concept better if the concept is explained to me first, then I do it by myself later on. That way I will understand the concept better.

When responding to the question (item 5) in the learning journals on why he had difficulties, Ray attributed this to lack of lectures and or the unavailability of someone to explain the new concepts.

Learning Journal L1: The concepts are completely new to me and there is no one to explain to me the abstract concepts

Learning Journal D1: The University does not provide proper tuition that makes the learning of new concepts even more difficult.

Learning journal D2: The fact that there are no proper lectures makes it more difficult.

The idea of having someone explain things to him is quite strong in Ray, so much that he makes every effort and takes advantage of every opportunity to get someone to explain concepts to him, and this includes family and friends.

Ray: So like as I was doing the limits, as I mentioned earlier on, that, initially when I tried to do them I couldn't understand anything. Then I shelved them and said "I'd do them later on". I said I will find somebody who is doing the course to explain or I will ask some of my young

brothers to explain the concepts to me. So later on as I was talking to some of my friends, they said, they (*i.e the limits*) are very easy. (...)

Furthermore, oblivious to the fact that he is in a distance education environment, Ray makes a call for “actual lessons” (lectures) where the principle concepts are taught. He says:

Ray: (...) I think there is need for actual lessons like any of the other universities, where you go and learn the principle concepts. You know, a student from ZOU and a student from the (*name of university supplied*), there is a difference in the true sense of it. Because the ZOU student is half cooked as compared to the student from the (*name of university supplied*) where for that one there are lessons. I mean actual lessons where the students are taught the concepts. But with us here it's a different story altogether. (...).

The following shows the frustration Ray is experiencing with individual learning as he is expected to do in the distance education environment.

I: What really gave you the problems?

Ray: Everything.

I: Everything including?

Ray: Everything right from the word go. Eehm, maybe you as Zimbabwe Open University, maybe you only require those people who haven't been to formal schools. But for people like us who have been to formal schools then it becomes a problem, we expect to be taught how to do one, two, three, four concepts, and then you go home by yourself and then you do some of the things you were taught and that kind of thing.

I: Uhm?

Ray: But now, to start from scratch I think that's where the problem is, where you are starting from nowhere. You have got the book, yes, but no one to ask but yourself. You read it by yourself until you get the concept. (*pause*). But now I am not into that kind of learning. But maybe as time goes on, I'll get used to it.

I: Okay. So you mean you want someone around to tell you?

Ray: No, to assist me, not to tell me.

I: O-oh to assist!

Ray: Yeah, just to explain the concepts. Just to explain the concepts, (*emphasised*) then I know I can do it by myself.

### (iii) *Processing dimension: Active*

Ray shows characteristics of an active learner. He tends to take an **active** approach to learning. He prefers working in groups, be it in a study group or tutorial group.

I: Please tell me how do you learn best? Do you like to study by yourself or do you want to be part of a group?

Ray: Ahmm being by myself, it's a problem to me, but a group it's okay. Because obviously I'm sure you will have someone who will explain one or two things unless where you are all by yourself?

Most of Ray's responses pertaining to how he solves problems and difficulties indicate that he is the kind of learner who prefers to engage actively with the information rather than

introspectively. He says that being active enables him to “get the concept” and then “it will last longer”.

Ray: Those who want to sit and listen?

I: Yes.

Ray: Well, it would seem like they are participating but they are not taking an active role as such.

I: Is that you? Is that what you do?

Ray: No, (*emphatically*) I don't feel comfortable with that. In actual fact I won't let it go that way.

I: What do you mean?

Ray: Because if you have read about something, if you think about it, and talk about it then at least you are certain about it.

I: So you mean you don't ... (*interjected*).

Ray: If I am just passive, then it means I haven't learnt anything.

I: You mean you want to be active?

Ray: Yeah! Yeah. When I am active, because when I read about it, I think about it, I talk about it, that way I get the concept. Because when I am quiet it means maybe I don't understand it.

I: Its only when you talk about it that ...(*interjected*).

Ray: Yeah, then it will even last long.

The summary of the case data indicates that Ray prefers to work in groups, and he tends to benefit more from the interaction in the group activities.

Ray: Just participate and discuss with the other students. Maybe this one knows this, and the other one knows that, then each could try and explain what they know and the other one explains also what they know. Then you come up with a general consensus on whether you are right or wrong. There are no answers (*in the module*) so you have to see what he is talking about or she is talking about, and see who is right and who is wrong. Then the general consensus comes first.

Because of his active orientation to learning, Ray shows some frustration caused by the mismatch that exists between his expectations as a student and what is expected of him as a distance student. Since in a distance education environment students are separated from each other, they are expected to engage in individualised learning. However, Ray shows some discomfort when he has to study alone. This is elaborated in the following excerpt:

I: So you are not a loner?

Ray: I've tried and am trying that because most of the times I am by myself. I go to work, finish work at about 4-5. Go home, get to the book. So in most cases I am by myself; but that's not what I want.

I: If you had a choice?

Ray: If I had a choice, I would prefer a situation whereby after work, maybe I meet 3 or 4 people, we sit down together, maybe with the assistance of a tutor here and there, not just one or two. So when I get home, these things will still be there (*he points to his head*).



(iv) *Understanding dimension: Sequential*

Ray shows tendencies of being a sequential learner. The summary of the case data indicates that Ray prefers information that is linear and logically sequenced that follows a steady progression of complexity and difficulty. He prefers materials that are presented from the simple to the abstract and or from the known to the unknown. For instance, Ray acknowledges this as shown in the following excerpt:

Ray: Yeah. Then from there I can now do the complex ones. Because if I can do the simple ones, it means I can do those complex problems. But now, if I cannot do the simple problems, then with me, it means that those challenging problems I can't even do them. (...)

It is noted that on several instances, Ray complained about the information in the learning materials being presented from the unknown to the known, a situation that caused some learning conflicts for him.

Whereas some students find benefit from overviews and objectives of a written text, this was not the case with Ray who is not holistic but prefers the information to be presented in small logically connected chunks.

I: (...). Please just share with me your opinion about the following sections in the module, "What this Unit is all about", and the objectives section "By the end of this unit you should be able to"

Ray: At first I had to read them right from the start, but sometimes I was just ignoring them for the other units (*chapters*).

I: Meaning that you didn't find these useful?

Ray: Yaah, sometimes when the going gets tough, maybe I'd come back to the objectives and go through the objectives and start all over again. Then in this case I'm forced to go through the objectives of the unit (*pause*), such that when I finish reading, then I would say did I achieve the objective?

For Ray the overview and the objectives sections are of little significance, as he indicated that he would still go through the chapter without them anyway.

I: Would you find it difficult really, to go through a module without these sections?

Ray: Which sections?

I: "What this unit is about" and also the "Objectives" section?

Ray: If there are no problems, they (*the overview and objective sections*) are not necessary really, because I would just go through the unit. If I can read and understand the unit, and if I can work out the problems from the module then I know its done.

## Mathematical Understanding

Ray gave very brief answers that lacked supporting evidence as to how he arrived at that answer. For some questions he did not respond to the justification prompt and in cases where

he responded, the justifications were not detailed and were not meaningful to the solution of the problem but rather revealed some misconceptions. Ray serves as an example of a student having or aiming at procedural understanding and who is found lacking conceptual understanding.

### Limit of Function

Ray's response to L7 was brief with just a one-digit answer, "2" a brief explanation as indicated in the following extract.

L7. Find  $\lim_{x \rightarrow \infty} \frac{(x+3)(2x-1)}{x^2}$

**SOLUTION:**

2.

**WHY DO YOU THINK THIS IS SO?**  
As  $x$  approaches in finite our answer is almost 2.

Handwritten calculations:  
 $\frac{13}{19} = \frac{130}{177} = \frac{247}{100}$   
 $\frac{(13)(19)}{100} = \frac{247}{100} = 2.47$   
 $\frac{(103)(199)}{10000} = \frac{20497}{10000} = 2.0497$

However, his side working reveals that Ray resorted to the numerical approach of finding limits whereby he evaluated using  $x=10$  and  $x=100$ , though limited to two evaluations, as a result he viewed the limit as an approximation, (not as a finite number) as evidenced by his justification where he said the "answer is almost 2". It can also be observed that the use of the enumeration (tabular) approach here is consistent with what he said during interviews, where he said that tabulating the function made it easy for him to obtain the limit of a function.

In responding to L6, Ray provided the one digit answer "4" which is the correct answer. However, the justification that he provided indicates that though he got the correct answer, it was for the wrong reason, as he wrote that

For the straight line graph, when  $x=2$ ,  $y=4$ .

As found in the studies of others (Bezuidenhout, 2001; Juter, 2006; Moru, 2006), students can produce correct answers but these answers do not necessarily come from the right mathematical ideas or procedures.

The above response by Ray to L6 indicates that he seemed to merely read off the function value from the graph, hence holding the misconception that  $\lim_{x \rightarrow a} f(x) = f(a)$ , a notion that is

correct only for continuous functions by definition. The identification of the misconception that  $\lim_{x \rightarrow a} f(x)$  is always the same as  $f(a)$  is consistent with the findings of other researchers such as Bezuidenhout (2001) and Ferrini-Mundy and Graham (1994).

In response to L9 (a) Ray similarly obtained the correct answer but for the wrong reasons. However, as for L9 (b) we note a further misconception that of failing to interpret the limit as  $x \rightarrow \infty$  using the graph. Ray wrote the brief answers and justifications for L9 as follows,

- a) 8. Justification: *the highest value when  $x=4$  is 8.*
- b)  $-\infty$ . Justification: *As  $x$  tends to infinite, our answer approaches  $-\infty$ .*

For item L10 Ray just wrote the one-digit answers “0” for both part (a) and (b), with no justification for both parts, even though the answer for part (b) was “0”. One would assume that Ray blindly obtained the answers for both part (a) and (b) by merely substituting for the  $x$  value into the function irrespective of the nature of the function.

Ray’s responses to L6, L9 and L10 indicated a limited understanding regarding non-routine problems, more so if they were using a graphical approach.

### *Derivative of Function*

Similar observations were made with regards to the items on the derivative of function with brief answers, justified answers for routine items like D2, and no justifications for non-routine questions. Ferrini-Mundy and Graham (1994) and Orton (1983) also noted that students were comfortable with routine differentiation problems and encountered problems with non-routine problems.

Ray also managed to correctly pair the functions in item D4 but with no meaningful justification for the answers.

### **Discussion on Ray’s profiles**

A failure to respond to the “why questions” as is observed in Ray’s responses usually reflects a deficiency in the conceptual understanding of the information. A profile of Ray’s learning style indicated that his style supported rote learning hence reinforcing procedural understanding. We noted that he depended on people explaining things to him, on memorising information, didn’t mind repeating things (processes) over and over again, and would take information sequentially as presented. We also noted that he relied on the “show

me how to do it so that I understand it” approach to studying. It is evident that Ray’s views on mathematics and mathematics learning are a collection of methods and algorithms where he expects to receive instructions on what to do and how to carry out the task. As such he preferred the self evaluation exercise problems to be similar to previously worked out examples. This would enable him to easily adopt and adapt the solution method, and where necessary just to plug into the solutions. The presence of any new material would frustrate him.

Repeating things is a metacognitive skill that some students use to reinforce information into memory. Some students easily remember and understand the information better if it is repeatedly echoed in their memory. However, for some it is a strategy that is used so as “just to be on the safe side” (Felder, 1993), without any thorough grasp of the concepts as is the case with Ray, who revealed procedural understanding of the concepts. Ray also serves as an example of a student who expresses frustration with studying alone. It is evident that the social isolation that is due to the DE frustrated him in his learning endeavours.

### 7.2.3 Profile for Tanya (Student C)

This section provides the learning style and mathematical understanding profile for Tanya (Tanya not her real name). Tanya, who is referred to as Student C (St C) in the transcript, is a primary school teacher and is based at an urban school. Tanya was selected to participate in the interview on the basis of ‘poor’ responses to the Calculus Task Test.

### Learning Styles Profile

Table 7.3 below gives a case record summary of the characterisation of Tanya’s learning styles.

**Table 7.3:** Characterisation of Learning Styles for Tanya

Dimension	Significant Quotations	Critical Pointers	Preference
Perception	<ul style="list-style-type: none"> <li>- (...) it was wholly a new thing, such that it was really difficult for me. At one time, I had decided to drop out (Int, 007)</li> <li>- So you find that at times you are forced to leave your studying because you cannot go anywhere when you don’t know anything. So sometimes you are forced to leave them for a later day.</li> </ul>	<i>Dislikes surprises and complications</i>	<b>Sensing</b>

	<p>(Int, 042)</p> <ul style="list-style-type: none"> <li>- Yaah, those ones were difficult for me, that's why I left them out. (Int, 104)</li> </ul>		
	<ul style="list-style-type: none"> <li>- read and re-read the area (LJ, L2, item 6)</li> <li>- Yaah, I don't mind. I really (...) don't mind. You know sometimes I could spend two hours on a problem. (Int, 020)</li> <li>- (...) I usually read over again what I have done, (Int, 065)</li> <li>- usually I go back to what I know, and then go back again. So I sort of reverse, go back and forth, read about what I know, work on what I know, then go back, (Int, 077)</li> </ul>	<i>Does not mind repeating things</i>	
	<ul style="list-style-type: none"> <li>- I couldn't see a similar example. Since there are no answers, I couldn't get it right. (LJ, D1, item 5)</li> <li>- I try to work on some of the examples that were done before. (Int, 065)</li> </ul>	<i>Preference for use of standard methods and similar problems</i>	
	<ul style="list-style-type: none"> <li>- I don't memorise often but if need be, well, I do that and I don't mind that. (Int, 022)</li> <li>- I do memorise. I do memorise. I can favour memorising, and I can do that (Int, 024)</li> </ul>	<i>Preference for memorisation</i>	
<b>Input/ Receiving</b>	<ul style="list-style-type: none"> <li>- I find that challenge that I have no one to consult at that moment. (Int, 042)</li> <li>- With me if I read it first, then sometimes I don't understand it, but if I get into the tutorial and that same aspect is done, maybe I will get the light out of it. (Int, 043)</li> <li>- (...) usually I want to have the lecture first, (Int, 044)</li> <li>- (...) the role of the tutor is to explain to us and help us with the problems that we faced when we were reading. (Int, 048)</li> <li>- (...) then I ask him "where did I exactly go wrong?" then he will be in a position to correct me and explain the things to me. (Int, 048)</li> <li>- (...) so that's when I am saying it will be very difficult for me to study ahead without someone to consult with (Int, 054)</li> </ul>	<i>Requires someone to explain concepts</i>	<b>Verbal</b>
<b>Processing</b>	<ul style="list-style-type: none"> <li>- what I know in this subject of ours (<i>calculus</i>), its sort of a practical thing and you learn by doing it. (Int, 050)</li> <li>- (...) so I think note making and problem solving is supposed to be done in a tutorial. (Int, 050)</li> </ul>	<i>Active oriented/doing something with the information first</i>	<b>Active</b>

	<ul style="list-style-type: none"> <li>- So If I write something down and I work something down, I will easily remember (Int, 051)</li> </ul>		
	<ul style="list-style-type: none"> <li>- (...) I could manage to solve some of the problems which I was working on, and this I also managed because of the study group that I was involved in. (Int, 013)</li> <li>- So these group discussions they actually help me before I go to the tutorials (Int, 057)</li> <li>- In a study group in fact everybody's role is sharing. Like what we have been doing. (...) (Int, 058)</li> <li>- (...) when I face problems then I go to my group and we work it out and help each other. (Int, 068)</li> </ul>	<i>Preference and benefits from working in a group</i>	
<b>Understanding</b>	<ul style="list-style-type: none"> <li>- being able to follow the steps and make the necessary steps to solve the problem. (Int, 021)</li> <li>- Maybe the actual format of solving the problems, the step by step and even the problem solution itself. (Int, 027)</li> <li>- I am that kind of person who reads page by page. I do read page by page. I can't jump between because I think I would miss something. (Int, 070)</li> <li>- I think it would be of benefit to all the students if we are given examples which are worked step by step (Int, 081)</li> </ul>	<i>Linear sequencing/Step by step approach</i>	<b>Sequential</b>
	<ul style="list-style-type: none"> <li>- (...) because I believe that some of these topics are interlinked. So you cannot go ahead when you cannot deal with the first steps in the first chapters. (Int, 042)</li> <li>- Like I said, I start from what I know and develop into the new concept (Int, 079)</li> </ul>	<i>preference for logical sequencing of complexity</i>	

The above case record summary for Tanya indicates the emerging learning style for Tanya on the F-SLSM to be **sensing, verbal, active and sequential**.

(i) *Perception dimension: **Sensing***

Tanya shows tendencies that characterise her as a sensing learner. From the case record above we note that she is patient with repetitions, dislikes complications, shows discomfort with abstract materials, looks for standard methods and does not mind memorising information.

When studying, Tanya does not mind repeating things and this includes repeated attempts at solving problems as well “reading and re-reading the area” as mentioned in both her learning journal and interview responses. The following elaborates just how patient Tanya is with repetitions,

I: So what do you do when you make a mistake?

Tanya: Sometimes you wouldn't find the mistake until you get to the final solution. So from there maybe you can go back, and you find out yourself where exactly you have gone wrong, then you can correct your mistakes from there.

I: Are you comfortable going back and forth trying to correct a mistake for the same problem?

Tanya: I wouldn't mind. Yeah, I don't mind. I really really (*repetition of 'really' as a way of emphasis*) don't mind. You know sometimes I could spend two hours on a problem. So I wouldn't mind doing it for as long as I grasp the concept.

As shown in the above text, Tanya would make repeated attempts at solving a problem and would spend a lot of time (she mentioned “two hours”) trying to work out the solution to the problem. However, we also observe that she would only comfortably do so for as long as she understood what was going on.

The condition that Tanya gave at the end of the above quotation “I wouldn't mind for as long as I grasp the concept” indicates that she has some discomfort working with complicated things and always takes the easy way out. For instance, when faced with new and difficult situations as a distance education student, Tanya contemplated dropping out.

I: What was really difficult for you?

Tanya: You know, I left school a long time ago, so when I started here, it was wholly a new thing, such that it was really difficult for me. At one time, I had decided to drop out, and then I said no, let me just go on. I think it will make sense some time somehow.

In addition, when she experienced difficulties when studying, Tanya would easily give up and move on to the other subjects or topics, despite being aware of the consequences of such an action.

Tanya: (...) So you find that at times you are forced to leave your studying because you cannot go anywhere when you don't know anything. So sometimes you are forced to leave them for a later day. And by so doing the progress is hindered. Maybe you might go to the other subject or the other topic. But then, you might get problems because I believe that some of these topics are interlinked. So you cannot go ahead when you cannot deal with the first steps in the first chapters. So I think its very difficult for us (...).

Since Tanya didn't like complications she would take the easy way out, when faced with difficult concepts. For example she repetitively mentioned that one of the textbooks that she referred to when facing difficulties was an 'A' level textbook. Instead of referring to other university level calculus textbooks, she used one which was at a lower level. The content in

the 'A' level textbooks is less abstract and is much lower than the university level calculus in terms of depth.

According to the FS learning styles model, sensing learners display a discomfort with abstract materials. Tanya constantly showed discomfort with abstract materials including definitions. Her LSPQ responses also indicated the "use of the definition" as the least preferred option. Interview responses such as the following also highlight Tanya's discomfort with definitions and theorems:

Tanya: there were so many definitions and these theorems. I couldn't follow them.

Tanya: Like I have been saying, there are so many definitions and at times you are prone not to understand it. So I wouldn't be likely to go back to definitions because of the wording or maybe the understanding of the definition. So I'd rather use the other methods than I will later go back to the definition?

Tanya: (...) these definitions at times they become too much and unmanageable

When asked about her opinion regarding working from first principles Tanya acknowledged the benefits but admitted that she would do what can enable her to give feedback.

I: How do you feel about using procedures or a formula without knowing where it's coming from?

Tanya: Yaah, most times you are forced to do that, but of course you need to know where its coming from. But at times given the situation you are forced just to do what is there, in order to give feedback.

Other characteristics for the sensors that we observe in Tanya are the tendency to follow standard methods and the tendency to memorise information. Tanya prefers to work on problems that are similar to previously worked-out examples. Her persistent call for someone to first work out and explain the things for her or the call for workbooks during the interview is evidence to this as this would allow her an opportunity to follow the solution when faced with similar problems. It is apparent that she has set her own standard methods of solution to be based on the methods that were previously used on exemplary problems. For instance, in her learning journal (LJ, D1, item 5) response she indicated that she experienced some difficulties because she couldn't find similar examples. The preference for worked out problems also presented good basis for Tanya to memorise and reproduce information as elaborated in the following statements that she made

Tanya Yes, sometimes. You find with these assignments if you look through, you get up to exactly the same problem that you once had. If you are very good at that, then reproduce it and (*then whispers*) cheat.

Tanya: I am saying with these studies, what I have found out, you might get a question on an assignment or on a test, you find that it will be the same. So if you are really studying and you are very good at numbers, or are good at memorising or remembering, you find



sometimes that the problem which you have in an assignment is also in the test, so at times you just reproduce it.

When asked about her opinion regarding memorisation, the initial impression one would get is that Tanya doesn't memorise information but on further probing it was clear that Tanya deeply relied on memorisation.

- I: Okay. How about memorising things when you are learning calculus? What's your opinion?
- Tanya: U-u-uhm, to be honest I don't memorise often but if need be, well, I do that and I don't mind that.
- I: What do you mean here? You don't memorise or you do memorise?
- Tanya: I do memorise. But it depends, like as it is, sometimes with pressure of reading, you find that you might be prone not to be able to memorise. You find that you want to memorise and you have got a lot on your hands, and when memorising if you just miss one concept, everything would just be mixed up. And by so doing you get everything wrong. So at one time or another, in a relaxed atmosphere, sometimes memorising is OK. It depends on what you want.

The only reason that prevented Tanya from memorising information is the "pressure of reading" where because of these pressures she 'feared' to get things mixed up, otherwise in a relaxed learning atmosphere, she wouldn't mind memorising. Furthermore, she alluded to even memorising the solution to an exercise,

- I: Personally are you one for memorising or not?
- Tanya: I do memorise. I do memorise. I can favour memorising, and I can do that
- I: You favour memorising?
- Tanya: Yes, yes. I do favour memorising. I can do that but because of the pressures, like I have just been saying, if you just miss one thing, everything you'll get mixed up. That's the difficulty with memorising but otherwise I would prefer it.
- I: And do you think by memorising you learn something?
- Tanya: Yaah, in a way I do. Yes, in a way I do learn something.
- I: What kind of things would you memorise, say with limit of function or derivative of function?
- Tanya: Maybe the actual format of solving the problems, the step by step and even the (*hesitates and whispers*) the problem solution itself, sometimes.

It is interesting to note that whilst too much pressure prevented Tanya from memorising, for other people, too much pressure would be good basis to memorise.

(ii) *Input/Receiving dimension: Verbal*

According to the Felder-Silverman learning style model, verbal learners prefer information that comes in the form of words; it can be oral or written words or both. Tanya showed characteristics of verbal learners and this included attributes such as the preference for someone to first explain the concepts to her (as evidenced on the case record summary in

Table 7.4), the preference for group discussions and preference for individual consultation with the tutors. What's more, the interview responses show Tanya to be wordy, comfortably giving long and detailed responses during the interview.

To further elucidate Tanya's preference for verbal information we observe that she persistently made a call for the introduction of consultation hours so as to facilitate students' individual "face-to face" meetings with the tutors. The following interview excerpts about consultation illustrate how highly Tanya valued tutor-student consultations,

Tanya: (...) But if I get it from the tutor, face-to-face, I think it will help me. So maybe if you could make some provision for consulting time maybe after tutorials, or sometime lets say during the week (...)

Tanya: (...) and I find that challenge that I have no one to consult at that moment. (...)

Tanya: (...) It would also be better if we are given time to consult before the tutor goes away.

Tanya: so that's when I am saying it will be very difficult for me to study ahead without someone to consult with so that the problems can be attended to.

Tanya: (...) Of course consulting we cannot do that, and it becomes a problem.

Tanya: I wouldn't have anyone near me to consult with and then it would be very difficult for me to go ahead.

Consultations hours could be a source of verbal information. The idea of having such student-tutor consultations was so strong for Tanya such that when experiencing learning challenges, it would be difficult for her to move on without the consultations. A preference for oral explanations for concepts presented learning conflicts for Tanya who being in a distance education environment, is expected to learn whilst in separation with the teacher and or with the other students in terms of time and space.

Although Tanya showed tendencies of being verbal, she indicated that she preferred to work with graphs especially for finding limit of a function.

### (iii) *Processing dimension: Active*

Tanya shows tendencies that characterise her as an active learner. She preferred to actively engage (to do something) with the information. Tanya is of the view that calculus is a "practical" subject and that "you learn by doing it",

I: What's your opinion about those who just prefer to sit and listen in a tutorial session? There are some people like that.

Tanya: Uhm, myself from what I know in this subject of ours (*calculus*), its sort of a practical thing and you learn by doing it. So if you just listen you wouldn't know exactly what is happening, so I think note making and problem solving is supposed to be done in a tutorial.

Tanya is of the opinion that no learning takes place with those group members who reflect quietly and are inactive in a group activity, and even considers such students as having an attitude problem.

I: Again there are those who prefer to sit, listen and reflect quietly?

Tanya: Yaah, there are some who are like that but well you wouldn't mind much about them because you will be learning. Just leave them as they are. If they come, if they don't, if they contribute, if they don't, well, just leave them like that. You cannot change a person, the attitude of a person needs time.

Although earlier on Tanya called for lectures, she has a different opinion on tutorials and wouldn't like tutorials to be conducted in a lecture-like approach. The following extract from the interview underscores Tanya's active orientation,

I: How do you feel to be part of a lecture-like tutorial?

Tanya: Uhm, I wouldn't like that myself, because I know at times you come to the lecture and you are lectured to, you go home, you find a different situation then you tend to forget. So If I write something down and I work something down, I will easily remember, o-oh you can do this and that. So lecturing cannot work for me, it's not very good.

One main characteristic of active learners as per FS learning style model is the preference for group work and group discussions, a characteristic that is dominant with Tanya. The following interview extract shows that Tanya advocated for group studies. She played a steering role in setting up the study group that she was involved in,

I: Was that okay with you?

Tanya: No it wasn't, that's why I had to advocate for these groups. Actually I was one of the organisers for the groups. I could actually supply some of these things so that we could get going. Because I knew I didn't know much.

Tanya indicated that the group sessions and the tutorials were useful for her. For Tanya group sessions are for sharing and brainstorming ideas.

I: You have indicated about being part of a study group, how do you feel about study groups?

Tanya: Okay, aa-ah they are so helpful. To me they have been very very (*repetition of 'very' as a way of emphasis*) helpful.

I: In what way?

Tanya: Like I have been saying, the discussions, you know, we could meet before the tutorials. Maybe I would be having some problems, at least if we are together, maybe we can share some problems, as what is difficult for me can be easier for somebody else. So these group discussions they actually help me before I go to the tutorials. And when I actually go to the tutorial it's such a step ahead.

From the above excerpt we observe that Tanya appreciated the group discussions and used them to prepare for the tutorial sessions. However, she is frustrated with the fact that at times she failed to fully benefit from the tutorials because of insufficient time.

Tanya: (...) You know at times you find that you have got so limited time in tutorials so much that at times you find that or you feel that if I ask this type of question maybe its not very useful, and say I shouldn't do that at the expense of others. So you find that at times you just go over a tutorial without actually understanding what is happening in the group.

In the following excerpt we observe that once more time and distance between the students were a constraint when it came to group meetings,

Tanya: Yeah, time is a factor. Time is very limited. Of course consulting we cannot do that, and it becomes a problem. You see at times you want to work very fast regarding that we have groups that we study with. We live very far and in different places such that when I need help I wouldn't see anyone. So with these tutors you sometimes are not able to get hold of them.

In learning journal D2, Tanya indicated that she was unable to overcome some difficulties because she was not able to meet with her colleagues to discuss. Tanya's active orientation and preference to engage in group discussions in a way caused some learning conflicts for her as a distance learner. From the above interview extract and the response in learning journal D2 we observe Tanya's frustration with the physical separation that exists between her and the other students as well as between her and the tutors. This however is in contrast with the expectations of distance education where students are supposed to be separated from each other and from the teacher.

(iv) *Understanding dimension: Sequential*

Tanya is a sequential learner. From the summary of the case record we observe that Tanya showed tendencies that characterise sequential learners on the FS learning style model. Tendencies such as preference for linear and step-by-step explanations of concepts as well as preference for logical sequencing of concepts are dominant with Tanya.

When asked about her opinion regarding the learning materials that she uses when studying calculus, Tanya called for a sequential and step-by-step presentation of the information. For instance, she suggested that the examples in the calculus module could be improved if the examples were worked out step by step and all steps of the solution method were included in the text.

I: What do you feel about the module itself? Any challenges?

Tanya: (...). Yaah, they are okay, but only that they should be beefed up. Say some of the information, which like I have been saying, some of the steps are not given there. You know some of these students, some have done A level maths and some have not done it, so I think it would be of benefit to all the students if we are given examples which are worked step by step so that they help each and every student who is reading the module.

To elaborate on how the step-by-step approaches could be included in the modules Tanya suggested the following

Tanya: Yaah, the concepts, like I have been saying that I met this animal calculus here for the first time, so with the module for some of the concepts, you know there are some stages, say as you are working on a problem, there are some stages they assume that you know. So from somebody from a layman's point of view, you wouldn't know exactly what is happening. So I think with their examples, if they had actually worked them step-by-step, so that any other student might know what exactly is happening and can follow those steps up. So as it is right now you find that a concept or a step is written, and then they get to the answer. How did they get to the answer? How did they cancel? How did they divide? How did they do this and that? So I think if they could improve on the steps on the examples so that when I am alone and I am doing it that way, I think I will be able to solve the problems.

Tanya was convinced that the inclusion of the details would assist each and every student since it assisted her, a fallacy that what works for her should work for everyone. She was very conscious of the 'missing' steps and stages in a solution. Furthermore, we observe that the absence of these details from the text caused some frustration for her as a distance learner hence making it difficult for her to study alone.

When asked about how she uses the learning materials, Tanya commented on the module and said,

Tanya: (...) I am that kind of person who reads page by page. I do read page by page. I can't jump between because I think I would miss something.

Tanya goes through the module in a sequential manner, following the order in which the chapters are presented, and reading page by page. We also observe that she follows the sequence of presentation in the module so as not to miss any information.

Tanya seems to be indifferent to the overview and objectives sections and finds them to be of little importance, other than serving as guidelines for self-evaluation. She only gets to read them anyway since she reads the module page by page.

I: For you, is there a difference really?

St C: The difference for me is not much. Like I have been saying, I read page by page, so that page on objectives I also read it.

Whilst for the global learners these sections would provide the big picture about a topic, this is not the case for Tanya who just uses them for self-evaluation purposes.

Another characteristic for sequential learners that is dominant with Tanya is the preference for logical sequencing of complexity of the information. Tanya alluded to wanting to learn things starting from what she knows moving on to the unknown, as evidenced by the following interview extracts,

Tanya: What I do is when I am learning new things, usually I go back to what I know, and then go back again. So I sort of reverse, go back and forth, read about what I know, work on what I know, then go back, I implement the concept of which I know.

Tanya: Like I said, I start from what I know and develop into the new concept. So I start with limit of function.

## Mathematical Understanding

Tanya serves as an example of a student having a less well-articulated understanding of the calculus concepts with a procedural understanding. Tanya showed a limited grasp of the concepts under study and that of other fundamental mathematical skills and processes.

- *Limit of Function*

An inspection of the errors that Tanya made indicates that she holds several misconceptions regarding the concept of limit of function. Her response to L6 revealed that she had problems interpreting the limit of the function from the graph, as she erroneously expressed the limit of the function as a Cartesian coordinate (2,4). On responding to the question “why do you think this is so” for item L6 Tanya wrote “On the  $x$  axis the limit is at 2 and on the  $y$  axis the limit is at 4”.

Tanya also considered the method of substitution as the only way of finding the limit of a function and hence substituted even when it was inappropriate to do so. Similar observations of substitution were made by Bezuidenhout (2001). However as observed by Bezuidenhout (2001), the substitution approach may not always work. For instance, when solving problem L7 Tanya treated  $\infty$  as a number and hence substituted for  $x$  using  $\infty$  in the expression. In addition she further cancelled throughout by  $\infty$ . The following extract shows Tanya’s algebraic manipulations.

L7. Find  $\lim_{x \rightarrow \infty} \frac{(x+3)(2x-1)}{x^2}$

**SOLUTION:**  $\lim_{x \rightarrow \infty} \frac{(\infty+3)(2\infty-1)}{\infty^2} = (3)(1) = \underline{\underline{3}}$

**WHY DO YOU THINK THIS IS SO?**  
I have substituted  $x$  for  $\infty$  then cancelled and got  $\underline{\underline{3}}$

From the above excerpt, it is evident that Tanya also holds some misconceptions regarding algebraic manipulations such as the use of infinity and division. This is evidenced by the way she erroneously cancelled throughout by infinity. In the justification of her solution to L7, Tanya used substitution by infinity as part of the explanation, indicating that for her this was a normal mathematical process. This confirms that she performed the manipulations blindly disregarding the conceptual aspects of infinity. The symbols were her focus rather than the concept. Limitations in algebraic manipulations were also observed in other studies by Juter (2005a) and Tall (1993a)

The need to substitute when finding limits was so strong with Tanya and this repeatedly emerged from most of her solutions. For instance for L9 and L10 Tanya wrote

<p>L9.</p> <p>Answer the following questions using the above</p> <p>a) Find <math>\lim_{x \rightarrow 4} f(x)</math></p>	<p><b>a) SOLUTION:</b></p> $\lim_{x \rightarrow 4} f(4) = 1$ $\lim_{x \rightarrow 4} f(8) = 2$ <p><b>WHY DO YOU THINK THIS IS SO?</b></p> <p>I have substituted <math>x</math> for 4 and 8 hence I got 1 and 2 as a result of dividing by <math>x \rightarrow 4</math></p>
--	--

Her response to L10 is as indicated in the following excerpt,

**SOLUTIONS (Provide explanations in each case)**

a)  $\lim_{x \rightarrow 1} f(1-x) \leq 1 = 1-1 \leq 1$   
 $= 0 \leq 1$   
 As  $x \rightarrow 1$  then substituting for  $x = 0 \leq 1$

b)  $\lim_{x \rightarrow 0} f(x^2) \geq 1 = f(0)^2 \geq 1$   
 As  $x \rightarrow 1$  then substituting for  $x = \infty \geq 1$

A quick scrutiny of Tanya's CTT written responses on limits would give listings of mathematical symbols, which were neither mathematically coherent nor meaningful as

exemplified by her response to L10. Such errors and misconceptions serve as indications of Tanya's existing deficiencies in understanding the relationships between the mathematical processes, ideas and concepts about limits.

- *Derivative of Function*

Although most students did not experience any challenges with the section on derivative of function, it would appear Tanya did experience some problems. Even item D2 that was just a routine question where students were asked to find the derivatives of the given functions at the given points with the method of solution not specified, Tanya erroneously solved it. It would appear that she couldn't even perform the basic differentiation processes. Because of her orientation towards substitution and "as an easy way out" Tanya jumped to substitute for  $x_0$  through out and evaluated the function values oblivious of the requirements of the question (i.e. to find the derivative). Tanya didn't respond to item D7, and when asked during the interview she admitted that she left it out because she didn't know what was going on.

### **Discussion on Tanya's profiles**

It emerged from the data that Tanya depended on rote learning and operated at the symbolic level. Though she is unaware of it, Tanya's goal for learning was surface, not to understand the information but to go through the material and 'catch up'. Tanya was that kind of person who would use repetition as a way to overcome challenges, who would want someone (colleagues and tutors) to explain and show things to her, and would take things sequentially as presented. In a way, repeating things served as a way of committing information to memory, though with short-term gains.

The fact that Tanya constantly complained of the pressure of work is evidence that she could not cope up with the workload as was expected of her. Hence, she resorted to rote learning, and was not concerned with the deeper understanding of concepts. It appears that for Tanya the mathematical concepts and processes were just pieces of information which were not connected in any way. Unfortunately, Tanya's learning style resulted in her being more surface and procedure oriented, such that she tended to use some procedures even where they were not appropriate.



## 7.2.4 Profile for Kundai (Student D)

This section provides the learning style and mathematical understanding profiles for Kundai (Kundai is not her real name), who is referred to as Student D (St D) in the transcript.

Kundai is a primary school teacher by profession and is based at an urban school. Kundai was selected to participate in the interview on the basis of ‘average’ responses to the Calculus Tasks Test.

### Learning Styles Profile

Table 7.4 given below gives a case record summary of the characterisation of Kundai’s learning styles.

**Table 7.4:** Characterisation of Learning Styles for Kundai

Dimension	Significant Quotations	Critical Pointers	Preference
Perception	<ul style="list-style-type: none"> <li>- (...) such that initially after I had failed to understand it, I even put away the module. (Int, 003)</li> <li>- (...) I couldn’t really get down to do calculus, it just but put me off. Because of that, I wasn’t paying much attention to it since I just considered it to be a difficult course. (Int, 067)</li> <li>- I don’t like those problems where I have to explain things, I only want to solve problems, because it’s faster for me. (Int, 103)</li> </ul>	<i>Dislikes complications</i>	Sensing
	<ul style="list-style-type: none"> <li>- There is no laid out pattern on how to find the limits you are just given the facts. (LJ, L2, item 5)</li> <li>- As for me I didn’t like the definition. I liked the direct method of looking for the limit. This approach of using the definition is something else. I have never enjoyed it. (Int, 010)</li> <li>- (...). In fact what I actually understood is the direct method. (...) It’s that method which when you are given a function you just substitute some values where its defined and then you just come up with the limit. (Int, 014)</li> <li>- For derivatives I didn’t like using first principles. Still, I liked using the direct methods to find the derivatives. (Int, 015)</li> <li>- A-aah, I don’t know. It (<i>first principles method</i>) just gives me problems. I guess it’s too long. (Int, 018)</li> </ul>	<i>Methodical and not abstract oriented</i>	

	<ul style="list-style-type: none"> <li>- I never bothered about that (<i>deriving formulae</i>) myself. I just considered them to be some of the facts in mathematics which are just given to us to use. I never worried myself on how they were derived (Int, 079)</li> </ul>		
	<ul style="list-style-type: none"> <li>- I just go back to where it started and follow all the steps checking where exactly the mistake is until I identify the mistake then I correct it. (Int, 025)</li> <li>- If I am enjoying the topic then I can repeat it many times (Int, 026)</li> <li>- If I continually keep on reading (Int, 097)</li> </ul>	<i>Does not mind repeating</i>	
	<ul style="list-style-type: none"> <li>- There are not enough examples to give me practice on the topic. (LJ, D1, item 5)</li> <li>- Overcame difficulty by reading it (<i>definition of limit</i>) regularly and trying to solve many problems using it. (LJ, L1, item 6)</li> <li>- Overcame difficulty by solving as many problems as possible (LJ, L2, item 6)</li> <li>- Overcame difficulty by more practice and following up examples (LJ, D1, item 6)</li> <li>- I mean to continue practising and solving problems regularly (Int, 085)</li> </ul>	<i>Practicing on examples and problems</i>	
	<ul style="list-style-type: none"> <li>- U-uhm, memorising? There are times when you have to, especially for formulas (<i>lowers voice</i>), in the end you have to memorise them (laughs). (Int, 029)</li> <li>- (...) after practising using the formulas then you can memorise. But, maybe proofs also one can memorise, (...) (Int, 029)</li> <li>- Then you can memorise and you keep on using it. (Int, 032)</li> </ul>	<i>Finds benefit from memorisation</i>	
<b>Input</b>	<ul style="list-style-type: none"> <li>- I would read about it first. If I watch someone solving the problem, it would appear as if the concepts are clear to me and yet they are not clear. The things don't even stick in my mind. (Int, 070)</li> <li>- Overcame difficulty by reading it (<i>definition of limit</i>) regularly (...) (LJ, L1, item 6)</li> </ul>	<i>Preference for textual material</i>	<b>Verbal</b>
	<ul style="list-style-type: none"> <li>- A-ah, I just don't enjoy drawing things. Its not that it is difficult, no it's not difficult. It's just that myself I just don't enjoy drawing the graphs. (Int, 100)</li> </ul>	<i>Discomfort with graphs</i>	
	<ul style="list-style-type: none"> <li>- (...) those questions that were difficult for us, then we discuss them with the tutor (Int, 048)</li> <li>- if we all fail to solve it, then the tutor can then explain and work out the problem (...) (Int,</li> </ul>	<i>benefits form explanations</i>	

	053)		
<b>Processing</b>	<ul style="list-style-type: none"> <li>- you can use the module for a week or some weeks before the tutorial, you get time to think things through (Int, 040)</li> <li>- But first each individual must individually solve the problems brought by each of the other students, and then we report back afterwards. (Int, 053)</li> <li>- Because in the end, you'd realise that if you work individually, it would be much better than when we met as a group. (Int, 060)</li> <li>- As for me, I didn't find them (<i>group activities</i>) useful for calculus. It was much better for me if I read on my own at home. (Int, 061)</li> <li>- So I think its better if you do it individually following the stages, then if you get stuck somewhere and cannot proceed on your own, you can then look for help from someone. (Int, 070)</li> <li>- Myself, I prefer to study alone by myself. (Int, 080)</li> <li>- I never benefited from the group. Also myself I also want to go there when already I have gone through and I have the problems and questions to ask on the things that I don't understand. (Int, 081)</li> <li>- I keep on trying to solve the problems on my own. (Int, 082)</li> </ul>	<i>Prefers to work as an individual first</i>	<b>Reflective</b>
<b>Understanding</b>	<ul style="list-style-type: none"> <li>- (...). But then, it's not easy to master other concepts well when you have skipped out a certain topic because you couldn't understand it (Int, 068)</li> <li>- Me, I read from page to page. But then if there is a section that is difficult for me, I just skip it so that I am not stuck (...) and then I proceed and return to them when I get help (...). Otherwise for the other things I just go from page to page. (Int, 071)</li> </ul>	<i>Linear</i>	<b>Sequential</b>

The above summary of Kundai's case record indicates that the emerging learning style for Kundai on the basis of the F-SLSM is **sensing, verbal, reflective and sequential**.

(i) *Perception dimension: Sensing*

Kundai shows characteristics of a sensing learner. The summary of her case record gives pointers to some characteristic tendencies for sensors. She dislikes complications and

surprises, and she easily gives up when faced with difficult mathematical content. She illustrates tendencies of being uncomfortable with abstract materials and this includes her discomfort to work with definitions, theorems or working from first principles. With regards to use of definitions, Kundai responded as follows

I: How do you feel about using the definition of limits? (silence)

I: You remember the  $\epsilon$ - $\delta$  definition of limit of function?

Kundai: You mean like this one (*student opens the module and refers to the  $\epsilon$ - $\delta$  definition written in the module on page 65*)

I: Yes, this one.

Kundai: As for me I didn't like the definition. I liked the direct method of looking for the limit. This approach of using the definition is something else. I have never enjoyed it.

I: Oh, why is it like that? (*student laughs, but does not respond*).

I: What's wrong with using the definition of limit of a function? Why didn't you like it?

Kundai: A-a-ah, I just feel that it takes a lot of time. The other method is direct such that you just find the limit of the function and then solve the problem quickly.

Kundai showed discomfort working with definitions. Furthermore, in her response to the LSPQ for limits, she indicated the use of the definition as her least preferred option for solving the problems on limits. When asked why, Kundai said she couldn't keep the definition in her head.

I: The least preferred choice for finding limit of function was 'use the definition', maybe you can also share on why this was least preferred?

Kundai: Uhm, I was not good at keeping definitions in my head (sic), so I never liked that approach. The definitions were just not there in my head.

However, Kundai is rather methodical and prefers working with procedures and tasks that follow laid out patterns, (rules and procedures), which she refers to as the 'direct method'.

On interrogating what she calls the 'direct method', Kundai responded as follows

I: By the direct method what do you mean? Which one is the direct method?

Kundai: (*Opening the module*) I don't know what you call the method but it deals with this kind of problems, (*referring to the examples on page 74; Examples on page 74 involve the evaluation of limit of function*).

I: So this is the direct method?

Kundai: Yes, and this is better.

She further explained the 'direct method' as

Kundai: (...). In fact what I actually understood is the direct method. I don't know, if you have understood the method I am referring to when I am saying the direct method? It's that method which when you are given a function you just substitute some values where its defined and then you just come up with the limit.

For Kundai, the ‘direct method’ is easy, straightforward, faster and makes it easy for her to understand concepts. However, she acknowledges that this ‘direct’ approach has limitations when it comes to attacking challenging problems, when she says

Kundai: Not even a single (*challenging*) example is solved (*in module*), because most of the problems solved in here (*pointing to the module*), they are ‘direct’. To me the way they are solved in here, its very easy for me to comprehend, but then when I want to try to solve the other problems in the exercises, I fail to solve them.

Another important characteristic tendency for sensors that Kundai displayed is her patience with repetitions (especially when problem solving) and her tendency to memorise information.

Kundai: Okay, you mean mistakes that come by when I am studying? If that’s the case then I just go back to where it started and follow all the steps checking where exactly the mistake is until I identify the mistake then I correct it.

I: And how many times are you prepared to go back and forth?

Kundai: If I am enjoying the topic then I can repeat it many times. But if I am not, then maybe I leave it, and then come back to it some other time.

I: You don’t mind the repeating?

Kundai: No. I don’t for as long as I am getting the things right.

In a way, when reading and solving problems she uses repetitions as a strategy of retaining information in her memory. For instance, this is elaborated in the following quotation,

I: What do you do to retain the definitions in your head?

Kundai: If I continually keep on reading and then practically apply what I’ve read to solving problems then I find that the things just stick in my memory. But as for definitions I never really read them.

Kundai benefited from “*solving regularly as many problems as possible*” and as recorded in her learning journals, this helped her overcome some learning difficulties. In one way or another, tendencies to repeat things and “continuously practising” to solve problems and applying formulas provide favourable ground for memorising information. Below is Kundai’s opinion about memorisation

I: What do you think about memorising?

Kundai: U-uhm, memorising? There are times when you have to, especially for formulas (*lowers voice*), in the end you have to memorise them (*laughs*). But for the other topics, there is no need to memorise really. I think formulas only are the ones that one can memorise, after practising using the formulas then you can memorise. But, maybe proofs also one can memorise, but as long as you are able to use it, and you also continue practising using it.

Thus, for Kundai memorisation has a place in her learning activities. Equally important to her is the idea of constantly using or applying what has been memorised, a position that she

keeps on alluding to in the interviews. Kundai says there is need to keep on using the memorised information so as not to forget it,

Kundai: Yes. It's clear, if it's a formula you are to keep on using it, so that you will not forget it.

I: Is that so?

Kundai: For you to be able to use that formula, it has to be in your memory. Like in an examination situation, isn't it, you are supposed to recall the formula from your memory and then use it to respond to the questions. So it must not just end there in your head, like, "now that it is in my head, so I must leave it like that," you have to keep on using it, when you are reading.

I: So, in your opinion do you think by memorising one learns something?

Kundai: Yes, I think you will learn something. Because *handiti* (isn't it) you are supposed to understand the definition, so when you understand it you'll have learnt it. Then you can memorise and you keep on using it.

Kundai seems to be uncomfortable with symbols and tends to be easily put off by a lot of mathematical symbols. This gives us another characteristic pointer to the sensing dimension. During the interview, she related her frustration regarding the symbolic presentation of an earlier chapter in the module (Unit 2 on sequences), which she referred to as de-motivating.

I: Please, just tell me what really was difficult for you in calculus?

Kundai: When I started reading the calculus module, the units at the beginning of the module were okay. But then when I started reading on that other unit, is it the series or something like that, (*student in this case opens and refers to unit on sequences in module*). Yes, these ones, I just don't know but the presentation is different, they have their own presentation, which is just de-motivating. Such that after that, for me to actually proceed and actually work on the other later topics in the course, a-ah it was difficult. So as for me, the issue here is that after having tried to read that unit, I couldn't really get down to do calculus, it just but put me off. Because of that, I wasn't paying much attention to it since I just considered it to be a difficult course. But maybe it's my attitude, I don't know.

She went on further to elaborate how this symbolic presentation put her off and contributed to her developing a negative attitude towards the topic. Notably this eventually affected her learning of the calculus course as a whole.

Kundai: Frankly speaking, like when I opened the module intending to read on the series (sic) (*by series student is referring to sequences*), I just don't know what happened really, but the presentation, it really put me off. So what I did when we were approaching examination time, I was just skipping some of the things, and I became selective. I just said to myself, okay, at least let me do the other things properly and understand them, and this let me just leave it out. But then, it's not easy to master other concepts well when you have skipped out a certain topic because you couldn't understand it. Worse still, at times you skip the topic thinking it's difficult, only to realise later that it's not difficult after all. So after writing the tests, that's when I realised that okay, the questioning technique on these series (sic) was even different from the presentation in the module.

(ii) *Input/Receiving dimension: Verbal*

The summary of the case record indicates that Kundai shows tendencies of a verbal learner. According to the FS learning styles model, verbal learners prefer to receive verbal

information, where verbal can refer to oral or written text. Kundai is one student who prefers both oral and written text explanations. In her responses during interviews she tended to be very verbose, providing with ease detailed explanations and elaborations to her responses. However, she would rather have verbal explanations to concepts only after interacting with the content on her own through reading.

She is also comfortable with the written text and shows preferences to getting information through reading. The written text for the distance education environment enabled her to learn the things, as she says:

Kundai: I would read about it first. If I watch someone solving the problem, it would appear as if the concepts are clear to me and yet they are not clear. The things don't even stick in my mind. (...)

Kundai does not find much benefit from visual materials, a characteristic which is dominant with the verbal learners. On her LSPQ questionnaires, she averagely ranked the option on using the graph. On asking why, she plainly admitted that she is not comfortable with graphs,

St D: A-ah, I just don't enjoy drawing things. Its not that it is difficult, no it's not difficult. It's just that myself I just don't enjoy drawing the graphs.

(iii) *Processing dimension: **Reflective***

The summary of the case record shows that Kundai displays tendencies of being a reflective learner. As per the Felder-Silverman learning styles model, a characteristic tendency of the reflective learners is that they prefer to work alone. Kundai displayed this characteristic when she was learning calculus, she indicated that she preferred to first brainstorm or read the content alone before engaging in a group, be it in a study group or in a tutorial.

I: (...) Do you understand something better if you first hear about it in a lecture like scenario or if you read about on your own?

Kundai: I prefer to read about it first on my own. Its like if you have the module, you can read before hand, you can use the module for a week or some weeks before the tutorial, you get time to think things through, solve the problems, then for those things that you would've not understood, you can then understand them when the tutor is explaining. If he doesn't explain on what you want, you can still ask him, and then it becomes much more clearer.

Kundai believes that pre-reading a text well before the tutorial meetings allows one an opportunity to think things through and then maybe refer the difficult ones to the group. The fact that she had the module facilitated a conducive environment for the initial reading and

individual brainstorming. To her there's no learning that takes place without this pre-reading and brainstorming.

I: What if someone tells you about the concepts first, without you first reading about it on your own?

Kundai: I feel like I am not learning anything. It's good if you know what the topic is about, after reading for yourself beforehand. Otherwise you can easily forget the things.

The idea of studying alone is dominant in Kundai. She says she found no benefit at all from the group studies in calculus and would rather go to the group when she has questions to ask. Working alone would also enable her to work at her own pace and not be driven by the pace of the others, as some group members could be faster and some slower than her.

I: Please share with me on how best you like to learn. Do you like to study by yourself or do you want to be part of a group.

Kundai: Myself, I prefer to study alone by myself. I prefer going to the group or to the tutorial when I already have my own problems or questions to ask.

I: You mean you would be comfortable to study by yourself?

Kundai: I never had problems with that, because in calculus I never benefited from the group. Also myself I also want to go there when already I have gone through and I have the problems and questions to ask on the things that I don't understand. Also when working with others, some can be faster or slower than me, so I prefer that I work on my own and at my own pace.

Being a reflective learner, Kundai believes that others would benefit from individual brainstorming just like her. An ideal tutorial session for her should involve individual brainstorming of the difficult problems first and then the tutor's explanations or the group/class discussion following afterwards.

I: To you, what is the role of the tutor in a tutorial?

Kundai: For example, if I bring my own problem to the tutorial, then we try to solve that problem as individuals. Then, if we all fail to solve it, then the tutor can then explain and work out the problem on the board for us as a class. But first each individual must individually solve the problems brought by each of the other students, and then we report back afterwards.

Although Kundai shows strong tendencies of being reflective, she shows some traits of being an active learner. The fact that preferentially her learning strategy especially regarding mathematical problems involves working out solutions to problems shows that she actively interacts with the content, though after an initial individual brainstorming session.

(iv) *Understanding dimension: Sequential*

Kundai showed characteristics of a sequential learner. She preferred materials presented in a linear and sequential manner and in a logical order of complexity as evidenced by her



preference for the “direct method”. Kundai also preferred going through the module page by page in the sequential manner in the order in which it is presented.

Whereas global learners would benefit from the overview and objectives section as these sections provide the big picture, Kundai views these sections as important but for a different purpose though. For her the overview and objectives sections are important in that they serve as a guideline of what is important and for self-evaluation purposes. She says:

Kundai: I think these sections are important, they are useful in that they can be used to guide you to know more about the topic. For instance, say you are reading on a chapter, on the objectives section, they list all the important things such that when you are studying or you are solving problems, you’d know that this is more important so you can give it more attention when reading. At the end of the chapter, when you are doing the exercises, then you can also check on whether you mastered the concepts or achieved the objectives of what they are testing.

For her, skipping between pages would disturb the sequencing of the concepts and hence making it difficult to “master other concepts well when you have skipped out a certain topic”.

## Mathematical Understanding

Kundai serves as an example of a student having procedural understanding of the concepts. An analysis of the CTT responses showed that Kundai wrote brief unsupported answers to non-routine items, but was very elaborative and articulate with items that depended on procedures.

- *Limit of Function*

Kundai experienced problems interpreting the limit of the function from the graph. Her response to L6 reinforced her discomfort with graphical information, which she later on alluded to during the interviews. Instead of obtaining the solutions directly from the graph, Kundai constructed her own algebraic function of  $f(x) = 2x$  from the graph and then used that to obtain the limit of the function. She proceeded as follows

$$\text{If } f(x) = 2x, \quad \lim_{x \rightarrow 2} f(x) = 2(2) = 4$$

Kundai’s response to L7 points to usage of procedural knowledge regarding finding limits of rational functions. Her solution to L7, which is highly procedural, is shown in the following excerpt

L7. Find  $\lim_{x \rightarrow \infty} \frac{(x+3)(2x-1)}{x^2} = \lim_{x \rightarrow \infty} \frac{2x^2 - x + 6x - 3}{x^2} = \lim_{x \rightarrow \infty} \frac{2x^2 + 5x - 3}{x^2}$

**SOLUTION:**

$$\lim_{x \rightarrow \infty} \frac{\frac{2x^2}{x^2} + \frac{5x}{x^2} - \frac{3}{x^2}}{\frac{x^2}{x^2}} = \lim_{x \rightarrow \infty} \frac{2 + \frac{5}{x} - \frac{3}{x^2}}{1} = \underline{\underline{2}}$$

**WHY DO YOU THINK THIS IS SO?**  
 As  $x$  approaches  $\infty$   $\frac{5}{x}$  and  $-\frac{3}{x^2}$  approach 0  $\therefore$  the constants remain.

We observe from the above that she focused on the use of the procedure, for which step by step she eloquently laid out the solution.

- *Derivative of Function*

A similar pattern was observed in responses to the derivative of function items, where the response to item D2, which was a routine question, was well articulated and for item D3 which had tabulated data, she constructed an algebraic function  $f(t)=t^3$  and then performed the differentiation.

For item D4 she successfully matched the pairs of functions  $f(x)$  with  $p(x)$  and  $g(x)$  and  $h(x)$ . However, she avoided the explanations.

### Discussion on Kundai's profiles

Failing to explain “why” is usually associated with a lack of conceptual understanding. In general Kundai was ‘very shy’ of giving any written explanations, reasons or justification to her solutions and this was notable in responses to L9 and L10 and D7 where she would write brief one line answers with no explanations. During the interviews Kundai admitted that she was not comfortable with those problems that required explanations but she’d rather settle for what she called the “direct method” which mainly depended on following rules and procedures. The direct method also referred to involving explicitly algebraically expressed functions. The need for an algebraic expression for the function is evident in Kundai. This is evidenced by her constructing of algebraic expressions for those functions, which were given graphically (question L6) and in tabular form (question D3). Such findings corroborate findings of others (Bezuidenhout, 2001; Ferrini-Mundy & Graham, 1994) where students

showed discomfort working with function representations other than algebraically expressed functions.

To a large extent Kundai’s learning style contributed to her procedural understanding of the concepts. Her preference for the direct method, reliance on memorisation, discomfort with graphical information, and discomfort with abstract information and symbols provided a favourable grounding for the procedural understanding. Kundai was only interested in the “mathematical facts” and was not worried about the relationships between them. She also was not worried about where ‘the mathematical facts’ were coming from, hence deriving things from first principles and explaining things was not in her space of operation. This kind of learning style did not support the in-depth understanding of the concepts but resulted in procedural understanding.

### 7.2.5 Profile for Tino (Student E)

This section provides the learning style and mathematical understanding profile for Tino (Tino is not his real name) who is referred to as Student E (St E) in the transcript. Tino is a secondary school science teacher and is based at a rural school. He was selected to participate in the interview on the basis of ‘good’ responses to the Calculus Task Test.

#### Learning Styles Profile

Table 7.5 given below gives a case record summary of the characterisation of Tino’s learning styles.

**Table 7.5:** Characterisation of Learning Styles for Tino

Dimension	Significant Quotations	Critical Pointers	Preference
Perception	<ul style="list-style-type: none"> <li>- (Enjoyed) finding the limit, by using the set methods and properties. (Int, 005)</li> <li>- (...) These ones (problems) on calculating were all right, they were better. (Int, 023)</li> <li>- Myself, I normally liked to use the calculation method, I found that to be easy for me. (Int, 024)</li> </ul>	<i>Methodical/Use of standard methods / procedures</i>	Sensing
	<ul style="list-style-type: none"> <li>- But then, I keep on working out and solving problems, and then it becomes much better. (Int, 029)</li> <li>- At times I’d keep on trying until I managed to</li> </ul>	<i>Does not mind repetitions</i>	

	<p>solve it (...) then you keep on trying to solve it (Int, 068)</p>		
	<ul style="list-style-type: none"> <li>- (Liked) That one of using the first principles (Int, 017)</li> <li>- (...) Even when I am proving the theorems, I like that, because that's the only way I can get the concepts to stick in my mind. (Int, 028)</li> </ul>	<i>Theory and abstract oriented</i>	<b>Intuitive</b>
	<ul style="list-style-type: none"> <li>- Memorisation doesn't work (Int, 038)</li> <li>- (...) They memorise just for survival so that they manage to write the examinations (Int, 039)</li> <li>- I use it (<i>memorisation</i>) but to a lesser extent. I don't rely on it.</li> <li>- But then memorising its just temporary, its just short lived (Int, 046)</li> </ul>	<i>Does not rely on memorisation</i>	
	<ul style="list-style-type: none"> <li>- I think its both but deriving the formula is more important (Int, 030)</li> <li>- (...) By deriving you manage to master the concepts for future use, later on. (Int, 033)</li> <li>- (...) but they do so without having a deep understanding of that subject (Int, 039)</li> <li>- I believe that if a person has not memorised but has mastered the concepts properly, then a-ah, the concepts can stay here (<i>the head</i>) (Int, 046)</li> <li>- I think it's much better if you know where it's (<i>formula</i>) actually coming from, so that when you are using it, you know where exactly it came from. (Int, 119)</li> </ul>	<i>Conceptual/ Meaning oriented</i>	
<b>Input/ Receiving</b>	<ul style="list-style-type: none"> <li>- I think if I first of all go to a lecture I will find it useful, rather than when I first read on my own. (Int, 062)</li> <li>- (...) its much better if someone explains the things first (Int, 062)</li> <li>- Aah, I would say the lecture. I think I would prefer the lecture and of course if accompanied with the text it's much better. (Int, 072)</li> </ul>	<i>Prefers someone to explain first</i>	<b>Verbal</b>
<b>Processing</b>	<ul style="list-style-type: none"> <li>- Discussed with my colleagues (LJ, D1 &amp; D2, item 6)</li> <li>- (...) But then if you discuss and you ask questions then it can be much better (Int, 081)</li> </ul>	<i>Benefits from discussions</i>	<b>Active</b>
	<ul style="list-style-type: none"> <li>- There isn't much benefit on the learner, because he will just be listening without participating in anything, so it will be something similar to what we discussed earlier on, on memorisation. (Int, 073)</li> </ul>	<i>Active oriented</i>	

	<ul style="list-style-type: none"> <li>- I think they are not benefiting anything just by listening only (...) (Int, 081)</li> <li>- If there is anyone who has a problem about a certain concept and if I know the area then we just assist each other. (Int, 077)</li> <li>- I find them (<i>group studies</i>) to be useful. They are useful for instance, for mastering the concepts (...) (Int, 082)</li> <li>- A-aah, individually it's (<i>understanding a concept</i>) a bit difficult; it's not simple at all. (Int, 088)</li> </ul>		
<b>Understanding</b>	<ul style="list-style-type: none"> <li>- Yaah, like in my case I was reading the module from the first page to the last. (Int, 091)</li> <li>- I made sure that I followed the sequence, like from Chapter 1 then I move on to Chapter 2, then I move on to the next one. (Int, 092)</li> <li>- Yaah, it was useful because it highlighted the important areas of the unit and how they are sequenced in concept (Int, 093)</li> <li>- You might miss the stage by stage guide. (Int, 096)</li> <li>- I would also get to feel the steps involved on solving problems, that is from this step you move to that one, then to that one and so on (Int, 104)</li> </ul>	<i>Linear and preference for stepwise approaches</i>	<b>Sequential</b>
	<ul style="list-style-type: none"> <li>- Then the assignments would also give us direction, so they are useful. (Int, 103)</li> <li>- So that I could be guided on what to read (...)</li> <li>- It will also help us not to lose direction when we are reading (Int, 108)</li> </ul>	<i>Logical sequencing</i>	
	<ul style="list-style-type: none"> <li>- <i>Important new things learnt:</i> The use of limits in calculating derivative. (LJ, D1, item 1)</li> <li>- I really didn't persist on following up on the links between the <math>x</math>, <math>\epsilon</math> and the <math>\delta</math>, like really getting to know exactly on where the link is, what it is and where it is coming from. (Int, 009)</li> <li>- Aah I am not able to. I can't make any link between these two. (Int, 098)</li> </ul>	<i>Not persistent in the search for links between concepts</i>	

The above summary of Tino's case record indicates the following learning style **intuitive, verbal, active and sequential** as the emerging dominant learning style for Tino.

(i) *Perception dimension: Intuitive*

Tino has characteristic tendencies for both the intuitive and sensing attributes for the perception dimension. However, the preferences are not balanced and his preference for the **intuitive** category is stronger than that for the sensing category.

We observe from the case record summary that Tino is theory and abstract oriented and does not rely on memorisation. He also shows tendencies of being conceptual and meaning oriented. Tino's learning journal entries show that he is comfortable with the abstract and theoretical components of the course. His learning journal entries 'for the most important new things that he learnt' are mostly abstract oriented and indicate his consistent search for meaning, where he gave such responses as "meaning of  $\epsilon$  and  $\delta$  and their relationship", "calculation of  $\epsilon$  in determining the limit", "proving limits using the sandwich theorem", "finding derivatives using first principles", and proving rules for differentiation, continuity and differentiability of functions were given. Furthermore, Tino's responses to the learning journals' item 3 (on the mathematical task) also pointed to this abstract orientation as three of the four responses that he gave were related to use of the first principles. Entries such as "evaluating of limits using the definition", "evaluating derivatives using first principles" and "deriving and using rules for differentiation" were given.

Tino's learning journal entries also give pointers for preferences to the sensing dimension such as being methodical, not minding repetitions applications of rules where he hinted on evaluations of calculating of limits and derivatives. However, Tino is mindful of the benefits of balancing the intuitive preference with the sensing preferences,

- I: Which one would you be comfortable with, to be working on problems where you are supposed to be proving things or where you are supposed to be calculating?
- Tino: I think I am comfortable with both. Even when I am proving the theorems, I like that, because that's the only way I can get the concepts to stick in my mind.

He insists that deriving a formula and using the formula are equally important aspects of calculus learning, which tend to complement each other. In the following quotation we observe how consistent Tino is with this position, though when asked to make a choice of preference between the two he opted for deriving a formula as more important.

- I: What do you think is more important to you in the learning of calculus, is it knowing how to use the formula or is it deriving the formula?
- Tino: I think it's both, but deriving the formula is more important.

- I: Okay?
- Tino: In fact, you must know the two aspects, because they are both important.
- I: If you were to decide on one of these, which one would you decide on?
- Tino: I would go for the deriving.
- I: Why would you go for the deriving?
- Tino: Because it is useful. Once you can derive the formula, you won't have problems when you want to use it later on. By deriving you manage to master the concepts for future use, later on.
- I: What do you mean by "mastering the concepts"?
- Tino: The major aspects on a certain concept will be easy to remember.

On the Felder-Silverman learning styles model, tendencies to memorise information are common and strong with sensing learners. Contrary to those learners who prefer to memorise information Tino strongly believes that memorisation is not beneficial to learning, as this does not enhance "proper understanding of concepts". The following illustrates his opinion on memorisation

- I: What do you think about people who memorise or what do you feel about memorising when learning calculus?
- Tino: Memorising doesn't work, because you won't get far with memorising. Because at times the questions differ, and if you memorise and you get slightly different questions then you can easily get lost.
- I: So what do you think about those who memorise?
- Tino: At the end, those people don't have the proper understanding of the concepts. Although of course, they memorise just for survival so that they manage to write the examinations (laughs), but they do so without having a deep understanding of that subject.

The above quotation also supports an earlier observation that Tino is someone who is in search of meaning and deeper understanding of concepts, and hence, for him memorising would not be useful.

- I: Now, just a direct question related to you, do you memorise or rely on memorising?
- Tino: I use it, but to a lesser extent. I don't rely on it.
- I: And you find it beneficial?
- Tino: It's not that useful. But then for those few instances when I get difficulties, that's when I at times memorise.
- I: So the few times that you memorise, what do you memorise?
- Tino: Sometimes its when I am proving something, or say it's the proof to a theorem which is giving me problems, that's when I memorise and then later on I make an attempt to apply it when solving problems.

Although, from the above Tino admits that he uses memorisation, he says he does so to a lesser extent but only as a way to 'pass over' some difficult problems. The fact that he immediately declares that memorisation is not useful and that he does not rely on it, indicates the extent of the frustration that he experiences when he has to memorise. Tino

believes that information acquired through memorisation is ‘short-lived’ and can be easily forgotten

- I: As a teacher, what do you think when your students memorise?
- Tino: A-ah, we actually try to discourage them. In fact we discourage them.
- I: Why is that?
- Tino: Because memorising won't help them in their studying. They just benefit for that time only, but later on in life they won't have benefited anything. They will just be at the same level with those who would have not done the concepts.
- I: What do you mean by benefiting “later in life”?
- Tino: I believe that if a person has not memorised but has mastered the concepts properly, then a-ah, the concepts can stay here (*pointing to his head*). But then memorising it's just temporary; it's just short lived.

(ii) *Input/Receiving dimension: Verbal*

Tino shows tendencies of a **verbal** learner. He prefers to have someone verbally explain information to him. When asked about his opinion regarding the lecture-like scenarios, Tino said,

- I: Now, tell me do you understand a concept easier if you first hear about it in a lecture-like scenario or if you read about it on your own?
- Tino: I think if I first of all go to a lecture I will find it useful, rather than when I first read on my own.
- I: Okay, so you'd rather have a lecture first?
- Tino: Yes, it would be easy.

He is very clear and explicit on the role that oral explanations play in his learning. In the following excerpt Tino explains that first hearing from someone who knows the concepts would help him in understanding the concepts.

- I: Why would you find it to be easier?
- Tino: When it comes to grasping concepts.
- I: What do you mean, “when it comes to grasping concepts”?
- Tino: I mean that if you first hear about the concepts from someone who knows them, then maybe it won't be difficult to understand them when you are working on your own.
- I: For you what's wrong with reading on your own first?
- Tino: There is nothing wrong really, but it's just that it's difficult, it's much better if someone explains the things first.



(iii) *Processing dimension: Active*

Tino is active oriented towards learning. He shows characteristics of active learners such as having a preference for group discussion (either in study groups or tutorial groups), and prefers to be active when involved in group activities. He believes group sessions to be useful “for mastering the concepts”. Tino commented as follows about group work,

I: Group work or group studies, what do you think about them?

Tino: I find them to be useful. They are useful for instance, for mastering the concepts. If one member of the group has mastered some of the difficult concepts, and another one has also mastered some other concepts, then when you meet as a group, things can become easy.

I: And what is your role in a study group? What do you do?

Tino: Group is for sharing ideas.

Thus, for Tino, group sessions provide a platform for brainstorming concepts, discussion and sharing of ideas. Since Tino is based at a rural school and as a result is separated from the other students most of the time, Tino is not comfortable with such a situation. He however, admits to having accepted the situation since “there is no other way out”.

I: And study groups?

Tino: We didn't have any groups often; It was merely studying individually.

I: Were you comfortable studying alone always?

Tino: Aaah, not very comfortable, but there was no other way out.

Tino also finds tutorial sessions to be equally important, as he sees tutorial sessions as a platform for assisting each other in areas where there are difficulties.

I: And your role in a tutorial? What do you do yourself in a tutorial?

Tino: What can I say? Maybe it's assisting each other. If there is anyone who has a problem about a certain concept and if I know the area then we just assist each other.

I: You do that in a tutorial?

Tino: Yes. If someone does not understand something and is asking for help, if I know the area then I can assist.

Furthermore, Tino says that tutorial sessions need not be conducted as lectures as lectures do not encourage student participation,

I: How do you feel about lecture-like tutorials?

Tino: There isn't much benefit on the learner, because he will just be listening without participating in anything, so it will be something similar to what we discussed earlier on, on memorisation.

Because of this active orientation to learning, Tino feels that those members of a group (study or tutorial) who are not active during a group session do not benefit anything by just “listening only” as “their misconceptions would still remain unattended to”.

- I: I want you to think about those who just sit in tutorials, what is your opinion about them?
- Tino: I think they are not benefiting anything just by listening only, because if you have misconceptions, then those misconceptions will still remain unattended to. But then if you discuss and you ask questions then it can be much better. So I think they are wasting their time (laughs).

The above quotation clearly shows the importance that Tino attaches to being active and the subsequent benefits (clarification of misconceptions). While it is evident that Tino is an advocate for active learning, unfortunately, he does not frequently get the opportunity of being part of a group himself, the main prohibitive reason being the distance and “insufficient time”.

- I: Are you part of a group? Or can I say during the past semester where you part of a group?
- Tino: Not frequently though. Maybe because of too much pressure, we wouldn't get the time for that. The time was just insufficient.

When asked what he does to remember information in calculus, Tino says “ I usually work the problems” which is a pointer for actively engaging with the content.

(iv) *Understanding dimension: Sequential*

Tino shows tendencies of a sequential learner. The summary to his case record shows some characteristic tendencies that are associated with sequential learners, that include being linear and sequential when learning, following logical progression of concepts, looking for “directions” and “stage by stage guides” when learning.

Strong pointers for the sequential category emerge from how Tino uses and interacts with the learning materials. Regarding use of the learning materials, Tino prefers to use the materials following the sequence in which they are presented in the module. For example, when reading the module, Tino says he would read from the first page to the last page, following the sequencing of the text from the first chapter to the last. Furthermore, he sees the tutorial assignments as giving him direction and guiding him on what to read and the worksheets as providing an opportunity to feel the step by step approaches involved on solving problems.

- Tino: Yaah, the worksheets are quite useful. I also think that they give us a varied base of problems to be worked on. Then the assignments would also give us direction, so they are useful also.
- I: Direction for what?
- Tino: So that I could be guided on what to read at the end of the course. From the practise I would also get to feel the steps involved on solving problems, that is from this step you move to that one, then to that one and so on.

When asked about his opinion regarding the overviews and objectives sections in the module, Tino seems to take a neutral position.

I: Would you find it difficult to use the module without these sections?

Tino: I am not sure, because I just used them. But then maybe if they are left out, then on reading you might miss some important concepts, because you are not guided. You might miss the stage by stage guide.

Because of his 'linear' orientation to learning, Tino views the overview and the objectives sections with neutrality and mainly as providing "direction" and giving the "stage by stage guide" on the important areas of the text. He thus considers the highlights from the overviews and objectives as 'tools' that help and make it easy for him to identify the important concepts in the text. This neutrality is contrary to the global learners who view these sections as important and would use them as a way of obtaining the big picture of the section.

We also observe another pointer to the sequential category, the fact that Tino does not persist on following up on the links between concepts. Although he is conscious that there are such possibilities, he would not persist on understanding the links. For instance, at one point in the interview he admitted not pursuing on the relationship between  $\epsilon$  and  $\delta$  in the limit definition. Also in the learning journal entries (LJ, D1, item 1), that he filled in during the semester he indicated that the most important new thing that he learnt on derivatives was about the "use of the limit of function in calculating derivative". However, when asked after the semester had ended (during the interview), Tino responded as follows

I: Would you be in a position for example to link the derivative of a function to limit of a function? (...)

Tino: Aah I am not able to. I can't make any link between these two.

I: Nothing in the line of definition of derivative of function being defined using the limit of function?

Tino: (*silence*). Its possible but it never appeared to me like that.

## Mathematical Understanding

Tino's responses showed a good grasp of the concepts. He provided coherent and meaningful justifications when responding to the "why" question. Tino served as an example of a student with a conceptual understanding of the concepts.

### Limit of Function

In responding to L6, Tino correctly interpreted the solution from the graph  $\lim_{x \rightarrow 2} f(x) = 4$  and provided the following justification:

The limit is obtained by extrapolating at  $x=2$  as we approach from both sides, the +ve and the -ve side.

The above explanation indicates that Tino was conscious of the need to consider the left and right side limits when dealing with functions with points of discontinuities. A similar explanation he gave with L9 (a) where he explained using left and right side limits,

$\lim_{x \rightarrow 4} f(x) = 8$ . Justification: As  $x$  approaches 4 from the negative side  $f(x)$  tends to 8 and as  $x$  tends to 4 from the positive side we obtain 8.  $\lim_{x \rightarrow 4} f(x) = 8 = \lim_{x \rightarrow 4^-} f(x)$ .

He correctly justified the limit in L9 (b),  $\lim_{x \rightarrow \infty} f(x) = \infty$  in terms of the increasing function using the graph.

When responding to L10 Tino again referred to the right side and left side limits. However he did not conclude in his justification whether the limit for  $\lim_{x \rightarrow 1} f(x)$  exists or not.

SOLUTIONS (Provide explanations in each case)

a)  $\lim_{x \rightarrow 1^-} (1-x) = \lim_{x \rightarrow 1^-} 1 - \lim_{x \rightarrow 1^-} x = 1 - 1 = 0$  OR  $\lim_{x \rightarrow 0} (1-x) = \lim_{x \rightarrow 0} 1 - \lim_{x \rightarrow 0} x = 1 - 0 = 1$

b)  $\lim_{x \rightarrow 0} x^2 = \lim_{x \rightarrow 0} x \cdot x = \lim_{x \rightarrow 0} x \cdot \lim_{x \rightarrow 0} x = 0 \cdot 0 = 0$  OR  $\lim_{x \rightarrow 1^+} (1-x) = \lim_{x \rightarrow 1^+} (1-x^2) = \lim_{x \rightarrow 1^+} (1-x) \cdot (1+x) = 0 \cdot 2 = 0$

### Derivative of Function

When responding to D2(a) Tino used the first principle approach and obtained the limit from the definition. However, he solved D2 (b) and (c) he using the usual rules of finding derivatives. In his response to D7(b) Tino gave the response  $f'(5) = 2/5$  and provided justification that linked the derivative of  $f$  at a given point with the gradient of the tangent line at that point, an abstract relationship which most students fail to master. Tino wrote “ $f'(x) = 2/5$  which is found by calculating the gradient of the slope at  $x = 5$ ”. In line with the classification made by Asiala *et al.* (1997, p. 414) for their research subjects’ responses to

that same question (D7), Tino, in my opinion, would belong to the first group of students that showed “a reasonable understanding” of the relationship between the derivative of a function at a point and the slope of the tangent line to the graph of the function at that point.

### **Discussion on Tino’s profiles**

The learning style profile for Tino revealed that for the perception dimension, although he showed strong intuitive tendencies, this was balanced with the characteristics for the sensing category. However, the intuitive tendencies enabled him to gain a thorough grasp of the concepts, hence he could easily answer to the “why” question and respond to the justifications where required.

## **7.3 GENERAL DISCUSSION**

The findings of this study confirm the fact that individual students exhibit different and particular learning preferences (Felder & Silverman, 1988). The learning experiences as described by the students provided information that revealed the variations in students’ learning preferences and showed that students vary in the ways they perceive, process, and understand information. The preferences exude particular individuals’ preferred learning styles, with some preferring to be in a highly interactive learning environment with a lot of social interaction and others like Kundai preferring to first brainstorm problems in isolation and then join a group after an individual brainstorming. Some students like Ray prefer concepts that are logically sequenced with an orientation that learning mathematics is about learning a collection of methods with instructions of what and how to solve problems. Whilst students like Tanya are surface oriented, are not meaning oriented but rather are too focused on symbols to the extent of inappropriately using symbols and procedures, other students like Sipho require the big picture first before working on the small parts and aim for depth and meaning in all learning activities. Students like Kundai seem to be ‘put off’ with too much mathematical symbols whilst others like Tino indicates that they are comfortable with both the symbols and the theories. All these and many other individual characteristics and preferences of learning affect the nature of interaction that can occur between the learner and the learning environment.

The aim of every teaching episode is to develop students’ deep understanding of the subject matter. However, we have observed that for some students their learning style preferences

promote learning and understanding of the mathematical concepts, whereas for others, the learning styles are rather detrimental to the process of learning. For instance the tendencies showed by Tanya of “favouring memorising” and tendencies showed by Ray of “wanting someone to explain information concepts first” results in limitation when it comes to knowledge transfer in a distance learning context. But the intuitive tendencies showed by Siphso and Tino who exhibit an emphasis in ‘searching for meaning’ and depth in what they learn enhance and facilitate knowledge transfer in this situation where there is no teacher to explain the concepts.

The data from this study tends to suggest that, for these students, the learning style preferences did have an influence on the individual’s mathematical understandings of the concepts under study. The following table summarises the findings of this chapter regarding the learning styles and the consequential learning outcomes whereby the learning outcome is represented by the individual students’ mathematical understandings of the limit function and derivative of the function concept.

**Table 7.6:** Summary Matrix of Linkages

<b>Student</b>	<b>Learning Styles Profile</b>	<b>Reason for selection</b>	<b>Mathematical Understanding</b>
Siphso	Intuitive, Visual, Active, Global	Good CTT responses	Conceptual
Ray	Sensing, Verbal, Active, Sequential	Poor CTT responses	Procedural
Tanya	Sensing, Verbal, Active, Sequential	Poor CTT responses	Procedural
Kundai	Sensing, Verbal; Reflective; Sequential	Average	Procedural
Tino	Intuitive; Verbal; Active; Sequential	Good CTT responses	Conceptual

## 7.4 CHAPTER OVERVIEW

This chapter presented the findings and discussion of the study by individual cases. The findings of this chapter were presented by individual cases so as to get a feel of how the students experienced their learning in the distance education environment at the ZOU. The data was qualitatively analysed so as to produce the individual students’ learning styles and mathematical understanding profiles. An attempt was made to establish a relationship between individual students’ preferred learning style and their mathematical understandings through the scrutiny of each of the student’s understandings of the limit of function and

derivative of function concepts. The chapter suggests that learning styles preferences can possibly influenced the students' mathematical understandings.

This chapter showed how an understanding of students' experiences in the DE environment did illuminate on students learning styles, which were subsequently linked with the students' learning outcomes. The next chapter, Chapter 8 presents my reflections of the process of this research study as well as outlining the implications of the findings of this study that are presented in Chapters 6 and 7 of this thesis.



## CHAPTER 8: REFLECTIONS AND CONCLUSION

### 8.1 INTRODUCTION

In this study, I investigated the influence of the distance education environment on student learning processes using a context of a first year undergraduate calculus course at the Zimbabwe Open University. My goal was to understand how a distance learning environment may influence student learning. In the thesis, I described and analysed first year calculus distance students' experiences of the learning process as well as brought out certain aspects of the DE environment that might have supported or deterred students in their learning processes. The thesis also identified students' preferred learning styles as given in the students' descriptions of their learning experiences, and further related these learning styles to the learning outcomes, which in the study were represented by the mathematical understanding of certain calculus concepts. The study, thus also showed students' mathematical understanding of a narrow content domain of the limit of function and the derivative of function. Further, it explored and brought to light possible relationships that existed between students' learning styles and mathematical understanding.

To gain insight into the influences of learning environments on students learning in a DE context, a predominantly qualitative study was conducted, and a case study design was used. The context of the study was the B.Sc. Mathematics and Statistics programme at the Zimbabwe Open University, and the research participants were first year, first semester students registered in the Calculus 1 (MTD101) course. The initial study sample involved 26 students and five of these students were purposely selected for interviews on the basis of their responses to a calculus tasks test. Data from the five students formed cases for the thesis.

In this chapter, I present my reflections of the research journey that culminated in the thesis. I also present my reflections on the research process and present a summary of the findings as well as implications of the study. In the chapter, I also depict some limitations of the study as well as make some recommendations for future research.



## 8.2 REFLECTIONS ON THE RESEARCH PROCESS

Chapter 5 explicated the methodology and explained how the research questions of this study were empirically attended to, using multiple sources of data. The sources of data comprised of learning journals (LJ), calculus tasks test (CTT), learning styles preference questionnaires (LSPQ) and interviews (Int). Conducting research with a qualitative orientation in a distance education institution where students are geographically dispersed was not as straightforward as I anticipated, but was rather a learning experience for me, full of challenges, discoveries and satisfaction.

In this section, I discuss my reflections on the research process which mainly covers the data collection stage, the data analysis stage and the overall thesis writing process. As I discuss my reflections, I attempt to point out some strengths, weaknesses, challenges and the lessons I learnt from the research process.

### 8.2.1 *Reflections on the data collection stage*

#### **Instrumentation**

This study used multiple sources of data with a concentration on qualitative data. Data on learning experiences were derived from the students' descriptions of their learning activities that were either in written form or orally through interviews. Students' learning styles were profiled from the qualitative data. Although there is plenty of learning styles inventory tests which are available that I could have used, I felt that if I used a learning styles inventory I would miss on the opportunity to understand the students' actual and lived experiences of learning calculus in the BSMS programme, and thus fail to capture how learning styles related to the learning environment.

The process of constructing the research instruments for this study was a long and winding journey for me. The main criteria that guided the construction of the instruments were that the instruments should capture the students' learning experiences as well as their learning styles in the sense of the Felder-Silverman learning styles model. There was the need to capture these aspects from the students' descriptions and to let the data speak out from the processes. In addition, there was also the intention to capture the students' learning outcomes in the form of mathematical understanding of the concepts of the limit of a function and the derivative of a function. Coming up with a combination of data collection

methods that would capture all these processes, as well as capture the intricate dimensions of the F-S learning styles model, more so, from a qualitative perspective was a challenge which I grappled with for some time. Although the process of constructing the research instruments was a challenge, it was a significant and beneficial process for me to go through since the process helped me to read more about and to sharpen my understanding of student learning in a distance context.

The multiple sources of data used in the study supplemented and complemented each other very well in bringing out the students' learning experiences and learning styles profiles. The combination of the LJ, the LSPQ and the interview was beneficial for this study in terms of data triangulation. Consistencies could be noted in the responses when analysing the data. The CTT illuminated students' understanding of the concepts of the limit of a function and the derivative of a function. Furthermore, this combination of instruments helped to illuminate on the views of distance learning held by the first year mathematics students who were involved in the study.

However, I feel the case study data could have been enriched if I had included data from documentations pertaining to the BSMS programme as a background to the students' learning environment. For instance, documents that have a bearing on student learning, such as the programme objectives, the course module, calculus course syllabi and objectives could have been scrutinised in detail. The specific chapter objectives on limit of function and derivative of function, as stated in the course text, could have been scrutinised closely as these also have a bearing on what and how students learn. Data from the students' written semester assessments could also have been analysed and corroborated with the data from the CTT on mathematical understanding. However, since the study was focused on student learning, I decided to focus on giving voice to the data and concentrated on those facets where the students were the primary source of data.

I feel that the combination of the LJ, LSPQ, CTT and the interview produced rich data for this study. The instruments generated a lot of information and I feel that beyond this thesis, there is still room for me to produce more research papers on student learning in a distance education environment.

## **Data collection process**

Collecting data on student learning processes was not an easy task due to the fact that distance students are dispersed in terms of location. The data collection process mainly relied on the students' availability and on how well-timed and well-coordinated the data collection activities were so as to avoid missing possible informants. The most intricate part of the data collection related to the data for profiling learning styles, where it was necessary for the participant to have filled all the research instruments. Although a pilot study had previously been carried out, the focus of the pilot study was more on the development of the instruments than on the actual data collection processes such as the distribution and collection of instruments to and from the participants. Furthermore, during the pilot study I involved a group of students who were all from one regional centre and was located in my hometown. These students were all easy to reach and to organise for research purposes. As a result, an oversight was made on the possible challenges that could be associated with conducting a research study that reaches out to students who are very much spaced apart and are based in various geographical parts of the country. The data collection pre-planning period (i.e. the period before I commenced the data collection process) was a learning period for me, full of discovering and problem solving, as I planned, reflected and strategised on how best to coordinate the data collection processes so as to eradicate any chances of losing any data. This also included taking into consideration the need to track a student's data across the full set of research instruments, that is, the LSPQ, LJ and CTT. Missing any part of the three instruments for a student would imply lost data in terms of complete LS profiling.

The nature of distance education where students are geographically dispersed, and where most of the students are mature, employed and have other responsibilities and commitments meant that it was not easy to reach and organise the participants for research purposes. The fact that the participants were first year students, in their first semester of study with most of them experiencing distance learning for the first time in life also meant that the students were mindful and conscious of devoting their time to an activity such as a research study. More so, the activity did not contribute to their final grading. As a result, during the pre-planning stage I had to be well-planned, strategic and timeous with regard to getting the research instruments to and from the participants. This was especially so in instances when the participants needed to be physically present for purposes of the study, and in instances when the data collection activity needed to be in sync (same day, same time) throughout the

four regional centres. For instance, as mentioned in Chapter 5, the dispatch of the learning journals had to be done very early in the semester so that the LJs were in the regional centres before the students' first tutorial meetings. Another instance was when students responded to the LSPQ and the CTT; the writing of these instruments had to be supervised by the RPCs and had to be in sync across all four participating regional centres so that chances of students sharing or discussing responses were eliminated.

The distance between the research participants and me also put some compromise on certain issues of rigour for the study. For instance, I did not manage to have three of the interview transcripts member checked as it was not easy for me to reach the participants for that purpose. More so, I had reservations on using the postal system; firstly, it would take more time for the papers to be posted to and from the participants, and secondly, it was the issue of privacy and confidentiality, where it was necessary to make sure that the right person received and read the papers.

The fact that I was a researcher internal to the institution was helpful. I was able to be in the picture of when students would next come together in their respective regional centres. This made it easier for me to make prior arrangements with the RPCs for the distribution, administration or collection of the instruments. As an example, the distribution of the LJs and their subsequent collection was done at a time when the students were having some scheduled activity. However, I had to negotiate with the students (through their RPCs) to avail a free Saturday so that they could write the CTT and LSPQ. I presume my being internally involved with the BSMS programme helped create a great deal of trust between me and those involved in the study, both students and the RPCs. Overall though, I feel the RPCs and the students were very cooperative, considering the fact that this study took some of their time away from their busy schedules.

### **8.2.2 Reflections on the writing process**

In this subsection, I present my reflections on the writing process. I refer to this as the *writing process* and not *writing stage* as I am in agreement with Henning, Gravett and van Rensburg (2002) that in a research study, writing is both a fundamental and continuous part of the research process. It is fundamental in that it helps in clarifying thoughts and in generating new ideas and it is continuous since it is a process of thinking.

In search of a way to bridge the gap that existed in my knowledge of the teaching and learning of mathematics at university level, as I transitioned from the domain of mathematics to mathematics education, I found the writing process of the thesis to be both a learning and rewarding experience. Having worked in a mathematics department and having been involved in the writing of some mathematics course books for the BSMS programme, I experienced tensions in me regarding how to approach the writing of a thesis in mathematics education. Ironically, I managed to resolve some of the tensions by introspection into my own learning styles.

Through the process of writing I managed to re-orient my perception of thesis writing. Initially I approached the writing of this thesis with absoluteness and definiteness, similar to the way I would do when writing a mathematical text. In mathematics, for instance, it is possible that a solution to a problem is ‘the solution’, a theorem is the theorem or ‘a proof’ is the proof. With such a background, I would therefore approach the reading and writing of parts of the thesis with an aim of producing absolute, finite and unique ‘answers’ similar to what I would do when solving mathematics problems. However, parallel to what was observed by Selden (2002) in her paper on mathematics and mathematics education, I personally have since realised that there are many perspectives of viewing and addressing a research problem in mathematics education. I have also realised that the writing of a thesis is a continuously ongoing process and not an end product in itself. Rather it is a process of thinking that uses written language (Henning *et al.*, 2002). More so, writing of a thesis does not necessarily have to be linear since there is a lot of reflecting and moving backwards and forwards between the sections of the document that needs to be done in the whole process.

### **8.3 REFLECTIONS ON FINDINGS: SUMMARY AND IMPLICATIONS**

This section presents my reflections of the research findings as presented in Chapters 6 and 7. I will also present reflections on the findings in the form of implications of the study on the teaching and learning of first year undergraduate mathematics in the BSMS programme at the ZOU.

#### **8.3.1 Summary of research findings**

Chapters 6 and 7 presented the findings of this study. Table 8.1 below shows the summary layout of the findings presented by theme and chapter.

**Table 8.1:** Summary Layout of Findings

Learner Experiences	Content factors	Subject matter difficulties	Chapter 6
		Preferences for forms of representation	
	Contextual factors	Social context	
		Physical context	
	Learner based factors	Depth of learning	
		Metacognitive abilities	
		Learner perception of DE	
	Learning styles and mathematical understanding	Learning styles	Chapter 7
		Mathematical understanding	

The findings of this study revealed that the distance education environment has a potential to influence student learning. The environment may positively or negatively influence a student's learning depending on the student's learning styles and the learning support system. The environment can have a positive influence if it promotes student learning, and negative influence if it deters student learning. Effective distance learning can be facilitated through a learning environment that is well supported to meet the learning needs of the students. The participants of this study indicated that generally Calculus 1 was a difficult course. Specifically, as an example, the limit of function concept was considered to be too abstract and too difficult for the students to understand without appropriate support.

The students who participated in this study showed feelings of isolation due to the separation from the others and from the institution. The isolation was a source of frustration to the students. The students considered tutorials to be an important component of their instructional package, as the tutorials were a source of teacher-student and student-student interaction. Students also appreciated the group studies that they engaged in. However, for both tutorials and group studies it was found that it is important that the students be well supported by their tutors especially when the students encounter problems. The tutorials were seen to be beneficial especially when the tutors actively engaged the students during the tutorial session. The findings of this study also revealed that students do value the feedback that comes from the tutors.

The findings also showed that the students benefited from using university libraries. However, the students felt the libraries should be well stocked and that the current library opening times were restrictive as the libraries are open during the normal working hours when the students would be at work. Those students who travelled long distances to the regional centres also pointed out that at most times, they would find the library closed by the time they arrived at the regional centres. Hence, recommendations to extend library opening times over working days and weekends.

The course text (module) was seen to be a major text for the students and they highly depended on the text. However, the students felt that the calculus course text was not self sufficient, mainly in terms of clarity and in terms of in-text supporting devices such as examples, activities, self assessment tasks, and worked solutions.

Students also appreciated other supporting learning materials such as the worksheets and the assignments. However, they insisted that more worked solutions be provided. The students also gave some suggestions for improvement on the learning materials that could be invaluable to their distance teachers. The suggestions centred on the introduction of a workbook for the calculus course, the incorporation of multiple representations, and the inclusion of clear step by step explanations in the texts. Gaining insights into the students' learning experiences also illuminated the learning preferences and characteristics that were dominant for the students.

Profiling and interpreting learning styles from qualitative data informed the study as to how students perceive, interact and respond to the learning environment. Thus, illuminate the students' learning styles in line with Keefe's (1985) definition of learning styles. The findings of this study showed that students have different learning styles preferences, a viewpoint that is consistent with Felder and Silverman (1988). The study identified learning styles profiles of five students and outlined mathematical understanding for each of these students. The findings of the study provided evidence of the existence of possible relationships between the two constructs, namely learning styles and mathematical understanding. The study therefore revealed that the mathematical understanding for a student was influenced by the particular student's learning styles. The study on one hand revealed that those students categorised as intuitive learners on the F-SLSM, that is, such students who were comfortable with abstract information, had a preference for depth, and were in search of meaning in their learning activities. They also tended to show a conceptual

understanding of the calculus concepts that were under study. On the other hand, those students categorised as sensing learners, that is, those students who were method focused, reproduction and surface learning oriented tended to show procedural understanding.

The findings from this study emphasise the importance of understanding how students learn. The findings also inform the designing of learning materials and support systems for the distance education environment. Carefully designed learning materials and support systems that cater for the distance students' learning needs may help promote learning. Some implications of this study's findings are discussed in the following subsections.

### **8.3.2      *Implications of the findings***

In endeavours to meet the needs of the distance learners as highlighted in the ZOU institutional mission statement (ZOU 2005-2009 strategic plan), faculty could benefit from gaining more insight into students' learning experiences and processes such as learning styles and the mathematical understanding and the associated relationships. The more insight distance educators and policy makers have about student learning, the more they are in a position to design instructional packages and systems that facilitate quality learning. A broad understanding by faculty of the influences of the learning context and of learner variables on student learning in the first year BSMS courses may inform faculty about those aspects of the learning environment that have a potential to facilitate learning and those which can deter learning. The findings of this study thus have direct implications for practice for the course Calculus 1, the BSMS programme and generally for the ZOU as a distance education university. The implications revolve around learner support systems and relevance of learning styles.

#### **Support systems**

Findings of the study show that the kind of support that distance educators and institutions provide for their students can either positively or negatively affect student learning processes. Issues related to feelings of isolation, the content, learning materials and time were major concerns among the ZOU distance education students involved in this study. The learning support systems therefore need to be as supportive as possible both in terms of combating the isolation and attending to learner diversities so as to make subject matter accessible.



*Tutors and tutorials:* As evident from the study tutorials should be retained. Tutorials were a popular feature with the students. Tutorials provide an opportunity for teacher-student and student-student interaction. However, it emerged from the findings that there is need to have the tutors well trained and developed in distance education methods so that they can conduct the tutorials in a beneficial way. Training would also enable the tutors to provide beneficial feedback to the students' assessments. It also emerged from the study that the institution needs to put concrete supporting systems in place, where part-time tutors are reachable for instance by phone, e-mail or physically in instances where students require assistance.

*Self organised group studies:* Students benefited from study groups. Group studies need to be encouraged for those students who can afford to meet. Group meetings, like tutorials, provided opportunities for student-student interaction. However, the findings revealed that students experienced challenges with regards to venues for holding the group meetings. It was difficult for students to find or organise rooms in which to meet as groups. The students also experienced challenges when they failed to access their tutors for assistance in times when they encountered learning problems within their groups. In such instances, the group meetings ended up unproductive.

*Learning materials:* The findings of this study revealed that the students depended heavily on the printed learning materials. The students appreciated the purposes of tests and assignments. However, it emerged from the data that there was need to 'beef up' the learning materials for the Calculus 1 course by introducing workbooks, presenting information using multiple representations, and improving on the in-text support devices in the course text (the module). The module could be improved so that it clearly explained and presented information step by step. The need for multiple representations and presenting information step by step points to the need to consider incorporation of learning styles in the course text.

*Library:* The data showed that students benefited from the library as a source for extra materials and for research purposes. Library materials were helpful especially in relation to information that was not included in the course text. However, there is need to adjust library opening times so that the libraries are open beyond normal working hours. This would give the students an opportunity to use the library resources after they come from work. This adjustment would also cater for students who live far away from regional centres, who because of the long travelling times often find the libraries closed.

*Tracking student learning:* Some of the challenges that participants of this study experienced could be averted if their learning activities were well tracked. It is important to track student learning processes by use of self-report techniques such as learning journals so as to consistently monitor the quality of student learning. Since observations are not easy to carry out in a distance education environment, students can be encouraged to write and fill in diaries in the form of learning journals and periodically send these to their tutors for progress checking. The information can assist the tutors to gain an understanding of the challenges that students would be experiencing before it's too late.

*Time and workload factor:* Time was also a big issue for participants of this study. The findings reveal students felt that the time available for the course was not sufficient. The students were of the opinion that the 12 week semester was too short, and as a result they ended up with a congested semester. Because of a congested schedule some of the students could not do justice to other essential learning activities such as assignment writing and studying. Another related issue concerned the 6 hours of tutorials which students felt was not sufficient. The students recommended that both the semester duration and the tutorial times be increased. As evidenced in the findings such an adjustment would help reduce many learning frustrations.

*Orient new students on distance learning:* It is evident from the findings of this study that students have varied learning preferences and varied perceptions of distance education. The students in this study being new to both university education and distance education, experienced challenges with the 'separation' as expected of them in distance education systems. In order for the distance institution to provide the new distance students with a learning environment that facilitates learning, careful consideration must be given to the special needs of these students who are encountering university education for the first time, with little or no experience of distance learning at all. It is imperative that the institution avails comprehensive orientation programmes to the new distance students so that they develop study skills, attitude and orientation that can help them optimise on the learning experience.

### **Awareness of various learning styles**

The more insight faculty and programme administrators have about learning styles and possible learning outcomes that are associated with particular learning style preferences, the

more likely that they will attend to learner diversities when developing learning materials. Apart from directly impacting on learning materials, information on learning styles can help distance educators to reflect on their roles in facilitating learning.

*Distance educators' awareness that students have varied learning styles*

Awareness on learning styles has the potential to change and improve distance teaching and learning processes by directly impacting on the learning materials. Learning materials developers can benefit from learning styles research, for instance, by designing the learning materials in such a way that the materials are flexible and balanced in terms of learning styles matching and/or mismatching of instruction to learning styles.

*Students' awareness of own learning styles*

Understanding learning styles is not only important in improving distance teaching, but the information on learning styles can also be important to the students by aiding them to reflect on their role in a learning process. For the students, an awareness of their own learning styles can enable them to take control of their learning processes by optimizing on their strengths. The knowledge of learning styles can also help students to understand the situations in which they learn best and how best they can adjust in instances of mismatches.

WESTERN CAPE

*Distance educators awareness of own learning styles*

Distance educators, teachers and material designers just like the distance students also have their own learning styles. As such the distance educators need to understand the influence of their own learning styles on their teaching styles. As they design and prepare learning resources, the distance educators must be aware that the learning materials should be able to reach more and diverse learners, including those students' whose learning styles may differ from theirs.

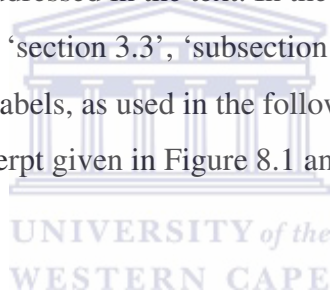
**A suggestion on incorporating learning styles in a calculus text: An example based on a text on limit of function**

As revealed in the findings of this study, students do have different learning style preferences in calculus. From a learning materials point of view, the learning styles can be catered for by attending to various aspects as identified by the learning styles model. This may include attending to multiple representations, sequence of the content as well as

attending to activities that aid on the interactivity of the materials. I will now attempt to illustrate how the suggestions on incorporating learning styles in mathematical textual learning materials can be implemented in practice. This suggestion however, should not be seen as a prescriptive formula as it is not exhaustive. It is also limited to the F-SLSM only.

This subsection therefore suggests how learning styles can be incorporated in a calculus text. To guide the discussion, I will refer to an extract of a text on the limit of function concept that I obtained from the BSMS Calculus 1 course book. I also base the discussion on the F-SLSM. The discussion is specifically oriented towards how to consider learning style principles in a calculus text. The excerpt used in the discussion deals with the introduction of the limit of function and is given in Figure 8.1.

In this subsection, I will discuss the contents of the excerpt as is presented in the course text in relation to the F-SLSM. I will also propose how to take care of aspects of the F-SLSM which seem not to be optimally addressed in the text. In the subsection, as I discuss the text, I make reference to labels such as ‘section 3.3’, ‘subsection 3.3.1’ or ‘subsection 3.3.2’. It is my intention to clarify that these labels, as used in the following paragraphs, only relate to the section/subsections of the excerpt given in Figure 8.1 and not the section/subsections of this thesis.



### 3.3 Limits

#### 3.3.1 Intuitive Ideas

Let us start by looking closely at the function

$$f(x) = \frac{x^2 - 9}{x - 3}.$$

- We notice that the domain of the function is the set of all real except the real numbers  $+3$ . Although  $f$  is not defined at  $x = 3$ , we can calculate the value of  $f$  near 3. The table below shows the values of  $f$  near 3.

$x$	2,9	2,998	2,9999997	3,0001	3,001	3,1
$f(x)$	5,9	5,998	5,9999997	6,0001	6,001	6,1

From the table, we see that as  $x$  approaches 3 from either the left hand side or the right hand side of 3, the values of  $f$  approach the number 6. That is, when  $x$  is near 3,  $f(x)$  is near 6. We then say that 6 is the **limit** of  $f$  as  $x$  approaches 3.

#### 3.3.2 Definition of a Limit

We see that  $f$  has limit  $l$  as  $x$  approaches  $x_0$ , if the closer  $x$  is to  $x_0$ , the closer  $f(x)$  is to  $l$ . Now **closeness** of  $x$  to  $x_0$  is given by  $|x - x_0|$  and that of  $f(x)$  to  $l$  is given by  $|f(x) - l|$ . The smaller the values of  $|x - x_0|$  and  $|f(x) - l|$  are the closer  $x$  is to  $x_0$  and  $f(x)$  is to  $l$  respectively. Note that we use **absolute differences**  $|x - x_0|$  and  $|f(x) - l|$  since we are considering approaches from **both** the left hand side and the right hand side of  $x_0$  and from above and below  $l$  respectively.

Thus  $f$  has limit  $l$  if  $|f(x) - l|$  tends to zero as  $|x - x_0|$  tends to zero i.e.  $|f(x) - l| \rightarrow 0$  as  $|x - x_0| \rightarrow 0$ . Let us now put this more precisely

- **Definition 3.1** A function  $f$  which is defined and single-valued for all values of  $x$  near  $x = x_0$  (with the possible exception of  $x = x_0$  itself) has limit  $l$  as  $x$  approaches  $x_0$  if given any  $\epsilon > 0$

(however small), we can find a positive number  $\delta > 0$  such that whenever  $|x - x_0| < \delta$ ,

$$|f(x) - l| < \epsilon.$$

**Figure 8.1:** Extract from the Calculus 1 course text (p. 66)

We note that after the heading “Limits” under section 3.3 in the extract, the text takes us to the subsection 3.3.1. We therefore observe the absence of an overview as an introduction to the section. In the framework of the Felder-Silverman model, availing an overview would attend to the needs of the global learners. At the same time, including in the overview, a sequential presentation of what is covered in the section would be beneficial to the sequential learners. The overview could also provide the connection between the concepts under consideration with the other concepts in the previous sections of the text or with other courses. Hence, cater for the global learners who prefer to relate what they are learning to previously encountered information.

Subsection 3.3.1 in the extract serves the purpose of introducing the concept of ‘limit of a function  $f(x)$ ’ to the learner. We observe that the function  $f(x)$  has been presented in both its algebraic form and tabular form. The tabular form is used to develop the concept of limit of the function  $f(x)$  as  $x$  approaches the point 3. We also note that written verbal explanations are used to reinforce and put in plain words the mathematical task under consideration, which is ‘limit of a function’. In the framework of the F-SLSM, the inclusion of tabulated data to enlighten the behaviour of the function  $f(x)$  near the point 3 would be appealing to sensing learners who have a preference for factual and tabulated data, whilst the intuitive learners can derive more benefits from the symbolic expression of  $f(x)$ .

However, what we miss from the text is the geometrical representation of the limit of function. A geometrical interpretation would have been useful in showing the behaviour of the given function nearer the point 3 and in showing how the concept can be developed from a graphical perspective. At the same time including the graph of the function and providing verbal explanations that elucidate the concept under consideration would provide an opportunity for the text to address the needs of both the verbal and the visual aspects of the input/receiving dimensions of the F-SLSM.

Analysing subsection 3.3.1 further, we miss the inclusion of other examples elaborating how this viewpoint of limit of function can be translated to various other functions. This could benefit all the dimensions of the model as the students think, reflect and practise as they attempt to understand the examples. Wrapping up the introductory section of 3.3.1 with a few exercises and activities could be beneficial for independent and self-assessment purposes and hence cater for almost all the dimensions of the model. The needs of both the active and reflective learners could be addressed depending on the nature of the exercises and activities.

The purpose of subsection 3.3.2 is to express and put across what has been discussed in 3.3.1 in mathematical and symbolic language. The beginning of section 3.3.2 is very much abstract as compared to section 3.3.1, a move that can be beneficial to the needs of the intuitive learners whilst at the same time edifying the sensing learners into appreciating the symbolic language. However, the introductory part of 3.3.2 could be elaborated further by use of graphical illustrations, showing the position of all the symbols involved  $f(x)$ ,  $l$ ,  $x$ ,  $x_0$ ,  $\varepsilon$  and  $\delta$  relative to each other, whilst at the same time capturing the needs of the visual

learners. The current format of presentation is very abstract and might for instance deter the sensing learners.

## 8.4 LIMITATIONS OF THE STUDY

This study was constrained by several limitations which mainly stemmed from two main causes. Firstly, it was the fact that the study was conducted in a distance education institution which from my experience proved to be a complex environment to carry out a qualitative research study. Secondly, it was the fact that the study is a case study and case study findings cannot be generalised. Chapter 5 presented a discussion on generalisation in this study.

While specific limitations regarding the data collection process are highlighted in context in Chapter 5, where data collection procedures are discussed, this section serves to point out other limitations pertaining to the study as a whole.

1. *Not all first year BSMS students were involved in the study:* It was not possible to involve all first year students in the study, due to the fact that distance education students are geographically dispersed throughout the country. Some students stay far away and in remote areas that are inaccessible, both in terms of road and telephone communication networks. As such, participants were selected from conveniently selected regional centres. The regional centres were conveniently selected on the basis of the proximity to me or accessibility of the area in terms of road and telephone communication networks. This kind of selection of participants consequently implied that some potential participants who could have richly informed this study were technically left out since their regional centres were not selected in the study.

2. *Financial constraints:* Conducting a qualitative study in a distance education environment where the key informants are DE students proved to be costly, especially where travelling was involved. Since this study relied on multiple sources of data, which were mainly collected at different points during the semester, there was need at times to organise the participants to meet on specified dates for research purposes only. Examples of such instances are when participants filled in the LSPQ and the CTT or when I conducted the interviews. Additional costs were incurred when I made repeated trips to interview venues. Overall, all research related expenses that were incurred by the participants, the RPCs and myself had to be well planned for.
3. *Time constraints:* Another related limitation was in terms of time. Since most of the DE students are mature students with other responsibilities at their work places and their families, the participants of this study were very mindful of their time. As such, at the beginning of the study, some students indicated that they could not be part of the study as they were very busy and could not afford to spare time for purposes of this study. Other students opted not to participate because they lived far from their regional centres, and travelling to the regional centres for purposes of this study would take a lot of their time.
4. *Findings of the study cannot be generalised:* This limitation was due to the fact that the study was conducted as a case study of a qualitative nature. Since the sampling strategies used in this study were not random, the sample of students used in the study was therefore not representative of the whole BSMS first year students. Rather the initial twenty-six students who volunteered to participate in this study came from conveniently selected four of the ten ZOU regional centres and interviewees for the in-depth interrogation were purposively selected. The findings of this study are therefore limited to the group of students who participated in the study and cannot be generalised.

## **8.5 DIRECTIONS FOR FUTURE RESEARCH**

There are several issues that arose from this present study that do warrant further research. Further research following this study would illuminate insights that may inform and aid



faculty in making informed decisions as they prepare learning materials and support services for the distance students. While some of the research issues emanate directly from the design of the study, some emanate as direct extensions of the findings of this study and some as new but important issues which subtly emerged in the data.

### **In-depth studies to explore on the role and nature of interactions in the BSMS DE environment**

A more in-depth research study could be carried out to explore the nature of interactions such as the student-tutor, student-content and student-student interactions and to explore on the role of these interactions on student learning in the BSMS distance education environment. Findings would provide information invaluable to the mathematics distance educators, administrators and instructional designers on improving the learning materials and learning support systems.

### **Further research on the links between learning styles and mathematical understanding**

One shortcoming of the current study is that the study does not deeply concentrate on the connectivity of the learning styles and the mathematical understanding. There is a need to further explore on the linkages that can possibly exist between the constructs of learning style and mathematical understanding. Research with such a focus would be important as it would give faculty insight on the role of learning styles in student learning within a distance education course. Several possible approaches to this strand of research do exist, of which a couple are mentioned below.

One possibility would be to conduct a quantitative study that could possibly bring out the links and relationships between LS and mathematical understanding using statistical methods. Alternatively, use a mixed methods approach that relies to a major extent on the correlational quantitative approach.

Another possibility would be to conduct a qualitative case study where the selection of the participants for interviews is based on students' dominant learning styles. With regards to the present study, the selection of the interviewees was based on the students' mathematical understanding of certain calculus concepts. It would be worthwhile to select the interviewees on the basis of their learning styles. That is, identify the learning styles first, and then

purposively select those students who exhibit some dominant learning styles and interrogate their mathematical understanding of certain calculus concepts.

### **Research focusing on relationships between learning styles, learning materials and mathematical understanding**

Further research could be carried out with a focus on understanding the possible relationships that can exist between the instructional design of the learning materials, learning styles and students' learning outcomes. For instance, research could be carried out to interrogate if the learning materials do address the learning styles needs and the learner differences. If so what are the kinds of mathematical learning outcomes? A research study with such a focus may be helpful as it would inform instructional designers on the role of learning styles in instruction. An experimental study could be designed or an action oriented developmental research could be embarked on whence the learning materials incorporating learning styles could be developed.



### **Similar study to this present study but explore using some BSMS courses other than Calculus 1**

There is a need to conduct a similar study to the present study but with a focus to explore other first year BSMS courses different to calculus. Courses such as Linear Mathematics (MTD102) and Mathematics Discourse and Structures (MTD104) that also normally present problems for students could be investigated. More research is necessary using different courses so that the distance course delivery methods may be designed to suit different courses.

### **Interrogate students' conceptions, preparedness and readiness for distance learning**

There is a need for research that interrogates the distance students' views of distance education as well as interrogating the students' preparedness and readiness to join the BSMS distance learning environment. This may add to the research based literature as well as inform the faculty on how best to support the BSMS students and how to bridge the existing gaps. The data emanating from interviews and learning journals of the present study showed that some of the students were either not ready or were not aware of the differences between distance and conventional education systems. Some of the students showed such propensities

where they expected to be taught just as in a conventional educational system. Special attention should therefore be placed on how learning materials address learner differences in terms of readiness for distance education. Further attention should be placed on the course content, so that bridging courses may be introduced so as to prepare students for higher level courses. Conducting research on students' readiness for course content is one way in which faculty may improve and strengthen the distance learning experience.

## 8.6 CONCLUSION

This study intended to address three research questions and the primary question centred on investigating how the ZOU BSMS distance education environment influenced student learning processes. Findings from this study on students' experiences of learning in the BSMS distance education environment revealed that the learning environment can influence student learning, and the influences can either be effective or deterrent to learning. The students' learning experiences also informed the students' learning styles. Using the content context of the limit of a function and the derivative of a function, it was possible to establish relationships between the constructs of learning styles and mathematical understanding. That is, the study showed that the kind of mathematical understanding that individual students held was dependent on the nature of the students' learning styles. Some learning styles seemed to promote conceptual understanding whereas others seemed to reinforce procedural understanding.

This study therefore argues that while the distance learning context has the potential to influence learning, the student's learning styles also potentially can influence the student's mathematical understanding. A notable bridging concept that may help improve the learner's experience of distance learning whilst at the same time attending to learning styles needs is through a carefully designed learner support system. Since this study has a predisposition towards calculus learning materials, the study also argues for catering for students' learning styles needs in calculus learning materials.

Although the present study was a case study that focused on a small group of students, some recommendations for the BSMS programme were raised as direct implications of this study. The combination of the implications and the identified suggestions for further research shows that there is still need to gain more knowledge, through research, about students'

learning in the ZOU BSMS programme through research. More research on student learning in the programme may contribute towards improving student learning, and may empirically contribute towards the university's mission to “*empower*” the distance learner and to enable the distance learner to “*realise their full potential*” (ZOU mission statement, ZOU 2005-2009 Strategic Plan).



## REFERENCES

- Abdul Karim, M.F., & Ufuktepe, U. (2002). Pedagogical issues in web*Mathematica* applications in distance learning mathematics. In A. Rogerson (Ed.), *Proceedings of the 4<sup>th</sup> International Conference of the Mathematics Education into the 21<sup>st</sup> century project*, Palermo, Italy. Retrieved June 7, 2007, from [http://math.unipa.it/~grim/Si\\_Karim.pdf](http://math.unipa.it/~grim/Si_Karim.pdf)
- Akkoc, H., & Tall, D. (2005). A mismatch between curriculum design and student learning: The case of the function concept. In D. Hewitt & A. Noyes (Eds.), *Proceedings of the 6<sup>th</sup> British Congress of Mathematics Education* (pp. 1-8), University of Warwick. British Society for Research into Learning Mathematics. Retrieved May 2, 2006, from <http://www.bsrlm.org.uk/IPs/ip25-1/BSRLM-IP-25-1-01.pdf>
- Aksoy, Y., Bulut, M., & Mirasyedioglu, S. (2006). The influence of Computer Algebra System (CAS) in teaching derivative concept. *Proceedings of the 3<sup>rd</sup> International Conference on the Teaching of Mathematics at the Undergraduate Level (ICTM3)*, Istanbul, Turkey. John Wiley and Sons.
- All-Africa Ministers' Conference on Open Learning and Distance Education, (2004). February 1-4, 2004, Cape Town, South Africa. Retrieved March 7, 2007, from <http://www.africaodl.org/conference/odl.htm>
- Alpers, B. (2006). The mathematical expertise of mechanical engineers. In M. Demlova & D. Larson (Eds.), *Proceedings of the 13<sup>th</sup> SEFI European Seminar on Mathematics in Engineering Education, Mathematics Working Group* (pp. 11-17). Kongsberg, Norway. Retrieved April 2, 2007, from [http://math.feld.cvut.cz/demlova/sefi/proceedBuskerud/SEFI\\_2006.pdf](http://math.feld.cvut.cz/demlova/sefi/proceedBuskerud/SEFI_2006.pdf)
- Arnold, S., Shiu, C., & Ellerton, N. (1996). Critical issues in the distance teaching of mathematics and mathematics education. In A.J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (pp. 701-753). Dordrecht: Kluwer Academic.
- Ary, D., Jacobs, L.C., & Razavieh, A. (1990). *Introduction to research in education*. (4<sup>th</sup> edition). Fort Worth: Holt, Rinehart and Winston.
- Asiala, M., Brown, A., DeVries, D., Dubinsky, E., Mathews, D., & Thomas, K. (1996). A framework for research and curriculum development in undergraduate mathematics education. *Research in Collegiate Mathematics Education II, CBMS Issues in Mathematics Education*, 6, 1-32.
- Asiala, M., Cottrill, J., Dubinsky, E., & Schwingendorf, K.E. (1997). The development of students' graphical understanding of the derivative. *Journal of Mathematical Behavior*, 16(4), 399-431.

- Aspinwall, A., & Miller, L.D. (2001). Diagnosing conflict factors in calculus through students' writings: One teacher's reflections. *Journal of Mathematical Behavior*, 20(1), 89-107.
- Association for the Development of Education in Africa (ADEA), (2002). *Distance Education and Open Learning in Sub Saharan Africa: A literature survey on policy and practice*. A report of the Working Group on Distance Education and Open Learning (WGDEOL). Retrieved June 21, 2007, from <http://www.adeanet.org/wgdeol/wgdeol/publications/open%20learning%201.pdf>
- Atkinson, R.C., & Shiffrin, R.M. (1968). Human memory: A proposed system and its control process. In K.W. Spence & J.T. Spence (Eds.), *Advances in the psychology of learning and motivation research and theory*, (Vol. 2, pp. 89-195). New York: Academic.
- Bacon, D.R. (2004). An examination of two learning style measures and their association with business learning. *Journal of Education for Business*, March/April, 205-208.
- Baddeley, A.D. (1998). *Human memory: Theory and practice* (revised edition). Boston: Allyn and Bacon.
- Barnes, M. (1993). Calculus for non-specialists. In M. Artigue & G. Eryvnyck (Eds.), *Proceedings of working group 3 on students' difficulties in calculus*, ICME-7 (pp. 73-74). Quebec, Canada.
- Bassey, M. (1999). *Case study in educational settings*. Buckingham: Open University Press.
- Bassey, M. (2001). A solution to the problem of generalisation in educational research: A fuzzy prediction. *Oxford Review of Education*, 27(1), 5-22.
- Bell, J. (1999). *Doing your research project: A guide for first time researchers in education and social science* (3<sup>rd</sup> edition). Buckingham: Open University Press.
- Benza, T.E.S., Chitsika, R., Mvere, F.S., Nyakupinda, D., & Mugadzaweta, J.G. (1999). A critical assessment of learner support services provided by the Zimbabwe Open University. *Pan Commonwealth Forum On Open Learning*. Bandar Seri Begawan, Brunei Darussalam. Retrieved December 21, 2006, from <http://www.col.org/forum/PCFpapers/benza.pdf>
- Berry, J.S., & Nyman, M.A. (2003). Promoting students' graphical understanding of the calculus. *Journal of Mathematical Behavior*, 22(4), 481-497.
- Best, J.W., & Kahn J.V. (2006). *Research in education* (10<sup>th</sup> edition). Boston: Pearson Education.
- Bezuidenhout, J. (1998). First-year university students' understanding of rate of change. *International Journal of Mathematical Education in Science and Technology*, 29(3), 389-399.

- Bezuidenhout, J. (2001). Limits and continuity: Some conceptions of first-year students. *International Journal of Mathematical Education in Science and Technology*, 32(4), 487-500.
- Bhalalusesa, E. (1999). The distance mode of learning in higher education: The Tanzanian experience. *Open Learning*, 14(2), 14-23.
- Bhalalusesa, E. (2001). Supporting women distance learners in Tanzania. *Open Learning*, 16(2), 155-168.
- Bhalalusesa, E. P. (2006). The dynamics of teaching at a distance in Tanzania: Reflections from the field. *Open Learning*, 21(1), 49-58.
- Boulton-Lewis, G. (1997). Information processing, memory, age and adult learning. In P. Sutherland, (Ed.), *Adult learning: A reader* (pp. 14-29). London: Kogan Page.
- Breiteig, T., Grevholm, B., Kislenko, K. (2005). Beliefs and attitudes in mathematics teaching and learning. In *Skriftserie. Konferanserapport No 3 – 2005* (pp. 129-137). Trondheim: Nasjonalt Senter for Matematikk i Opplæringen, NTNU. Retrieved June 6, 2007, from [http://fag.hia.no/lcm/papers/TB\\_BG\\_KK\\_Beliefs\\_rev.pdf](http://fag.hia.no/lcm/papers/TB_BG_KK_Beliefs_rev.pdf)
- Brown, E.J., & Pluske, J. (2007). An application of learning and teaching styles: A case study of science and engineering seminars. In Student Engagement, *Proceedings of the 16<sup>th</sup> Annual Teaching Learning Forum*, Perth: The University of Western Australia. Retrieved March 25, 2007, from <http://lsn.curtin.edu.au/tlf/tlf2007/refereed/brown.html>
- Burge, L. (1988). Beyond andragogy: Some explorations for distance learning design. *Journal of Distance Education*, 3(1), 5-23.
- Buyukkoroglu, T., Cetin, N., Deniz, A., Duzce, S.A., Mahir, N., & Ureyen, M. (2006, May 3). The effect of computers on teaching the limit concept. *International Journal for Mathematics Teaching and Learning*, [online journal]. Centre for Innovation in Mathematics Teaching (CIMT). Retrieved February 19, 2007, from <http://www.cimt.plymouth.ac.uk/journal/buyukkoroglu.pdf>
- Byers, V. (1980). What does it mean to understand mathematics? *International Journal of Mathematical Education in Science and Technology*, 11(1), 1-10.
- Byers, V., & Erlwanger, S. (1985). Memory in mathematical understanding. *Educational Studies in Mathematics*, 16(3), 259-281.
- Carlile, O., & Jordan, A. (2005). It works in practice but will it work in theory? The theoretical underpinnings of pedagogy. In G. O'neill, S. Moore, & B. McMullin, (Eds.), *Emerging issues in practice of university learning and teaching*. AISHE: Dublin. Retrieved November 11, 2005, from <http://www.aishe.org/readings/2005-1/>
- Carnwell, R. (2000). Approaches to study and their impact on the need for support and guidance in distance learning. *Open Learning*, 15(2), 123-140.

- Chombo, I.M. (2004). Country paper: The status of open learning and distance learning in Zimbabwe. A paper presented at the *All-Africa Ministers' Conference on Open Learning and Distance Education*, Cape Town, South Africa. Retrieved March 7, 2007, from <http://www.africaodl.org/conference/odl.htm>
- Christians, C.G. (2000). Ethics and politics in qualitative research. In N.K. Denzin & Y.S. Lincoln (Eds.), *Handbook of qualitative research* (2<sup>nd</sup> edition, pp. 133-155). Thousand Oaks, California: Sage.
- Claxton, C., & Murrell, P.H., (1987). Learning styles: Implications for improving educational practices. *Association for the Study of Higher Education*, Washington.
- Cobb, P. (1987). Information processing psychology and mathematics education - A constructivist perspective. *Journal of Mathematical Behavior*, 6, 3-40.
- Coffield, F., Moseley, D., Hall, E., & Ecclestone, K.(2004). *Learning styles and pedagogy in post-16 learning: A systemic and critical review*. London: Learning Skills Research Centre. Retrieved November 15, 2006, from <http://www.lsda.org.uk/files/PDF/1543.pdf>
- Cohen, L., Manion, L., & Morrison, K. (2000). *Research methods in education* (5<sup>th</sup> edition). London: RoutledgeFalmer.
- Commonwealth of Learning (COL), (2000). *An introduction to Open and Distance Learning*. Retrieved November 8, 2006, from <http://intranet.col.org:8090/jahia/webdav/site/myjahiasite/shared/docs/ODL%20Intro.pdf>
- Cornu, B. (1991). Limits. In D.O. Tall (Ed.), *Advanced Mathematical Thinking* (pp.153-166). Dordrecht: Kluwer Academic.
- Cottrill, J., Dubinsky, E., Nichols, D., Schwingendorf, K., Thomas, K., & Vidakovic, D. (1996). Understanding the limit concept: Beginning with a coordinated process scheme. *Journal of Mathematical Behavior*, 15(2), 167-192.
- Creswell, J.W. (2005). *Educational research: Planning, conducting and evaluating quantitative and qualitative research* (2<sup>nd</sup> edition). Upper Saddle River, New Jersey: Pearson Prentice Hall.
- Dalby, T. (2001). Theory and practice of teaching in a university mathematics learning center. *International Journal of Mathematical Education in Science and Technology*, 32(5), 691-696.
- Daniel, J.S. (1996a). *Mega-Universities and knowledge media: Technology strategies for higher education*. London: Kogan Page.
- Daniel, J. S. (1996b). New kids on the box: Distance education enters its third generation. In C. McBeath & R. Atkinson (Eds.), *Proceedings of the 3<sup>rd</sup> International Interactive Multimedia Symposium*, (pp. 1-9). Perth: Promaco Conventions



- Daniel, J. (2004). Distance education: Is it still relevant in 2004? A paper presented at the *All-Africa Ministers' Conference in Open Learning and Distance Education*, Cape Town, South Africa. Retrieved March 7, 2007, from <http://www.africaodl.org/conference/odl.htm>
- Dart, B. (1997). Adult learner's metacognitive behaviour in higher education. In P. Sutherland, (Ed.), *Adult learning: A reader*, (pp.30-43). London: Kogan Page.
- Denzin, N.K., & Lincoln, Y.S. (Eds.). (2000). *Handbook of qualitative research* (2<sup>nd</sup> edition). Thousand Oaks, California: Sage.
- Diaz, P.D., & Carnal, R.B. (1999). Students' learning styles in two classes: Online distance learning and equivalent on campus. *College Teaching*, 47(4), 130-135.
- Dorier, J., & Sierpinska, A. (2001). Research into the teaching and learning of linear algebra. In D. Holton (Ed.), *The teaching and learning of mathematics at university level: An ICMI Study* (Vol. 7, pp. 255-273). Dordrecht: Kluwer Academic.
- Duffin, J.M., & Simpson, A.P. (2000a). A search for understanding. *Journal of Mathematical Behavior*, 18(4), 415-427.
- Duffin, J., & Simpson, A. (2000b). Understanding their thinking: The tension between the cognitive and the affective. In D. Coben, J. O'Donoghue & G.E. Fitzsimons (Eds.), *Perspectives on adults learning mathematics* (Vol. 21, pp. 83-99). Dordrecht: Kluwer Academic.
- Dunn, R., & Dunn, K. (1993). *Teaching secondary students through their individual learning styles: Practical approaches for grade 7-12*. Boston: Allyn and Bacon.
- Dzakiria, H. (2005). The role of learning support in open and distance learning: Learner's experiences and perspectives. *Turkish Online Journal of Distance Education-TOJDE*, 6(2), 95-109, [online journal]. Retrieved March 15, 2006, from <http://tojde.anadolu.edu.tr/tojde18/>
- Dzakiria, H., Razak, A.A., & Mohammed, A.H. (2004). Improving distance courses: Understanding teacher trainees and their learning styles for the design of teacher training courses and materials at a distance. *Turkish Online Journal of Distance Education-TOJDE*, 5(1), 75-92, [online journal]. Retrieved July 16, 2004, from <http://tojde.anadolu.edu.tr/tojde13/>
- Eisenberg, T. (1991). Functions and associated learning difficulties. In D.O. Tall (Ed.), *Advanced mathematical thinking* (pp. 140-152 ). Dordrecht: Kluwer Academic.
- Even, R. (1998). Factors involved in linking representations of functions. *Journal of Mathematical Behavior*, 17(1), 105-121.
- Felder-Solomon Index of Learning Styles (ILS) (n.d.). Available online on <http://www.engr.ncsu.edu/learningstyles/ilsweb.html>
- Felder, R.M., & Silverman, L.K. (1988). Learning styles and teaching styles in engineering education. *Journal of Engineering Education*, 78(7), 674-681.

- Felder, R.M. (1993). Reaching the second tier – Learning and teaching styles in college science education. *Journal of College Science Teaching*, 23(5), 286-290.
- Felder, R. M., & Henriques, E. R. (1995). Learning and teaching styles in foreign and second language education. *Foreign Language Annals*, 28(1), 21-31.
- Felder, R.M. (1996). Matters of Style. *ASEE Prism*, 6(4), 18-23.
- Felder, R.M. (2002). Author's preface. Learning styles and teaching styles in engineering education, (1988). Retrieved April 15, 2004, from [http://www.ncsu.edu/felder-public/Learning\\_Styles.html](http://www.ncsu.edu/felder-public/Learning_Styles.html)
- Felder, R.M., & Spurlin, J. (2005). Applications, reliability and validity of the index of learning styles. *Journal of Engineering Education*, 21(1), 103-112. Retrieved May 25, 2005, from [http://www.ncsu.edu/felder-public/Learning\\_Styles.html](http://www.ncsu.edu/felder-public/Learning_Styles.html)
- Felder, R.M., & Brent, R. (2005). Understanding student differences. *Journal of Engineering Education*, 94(1), 57-72. Retrieved May 24, 2005, from [http://www.ncsu.edu/felder-public/Learning\\_Styles.html](http://www.ncsu.edu/felder-public/Learning_Styles.html)
- Fernandez, E. (2004). The students' take on the epsilon-delta definition of a limit. *PRIMUS: Problems Resources and Issues in Mathematics Undergraduate Studies*, 14(1), 43-54.
- Ferrini-Mundy, J., & Graham, K. G. (1994). Research in calculus learning: Understanding of limits, derivatives and integrals. In J.J. Kaput & E. Dubinsky (Eds.), *Research issues in undergraduate mathematics learning*, MAA Notes 33, (pp. 31-45). Washington DC: Mathematical Association of America.
- Ferrini-Mundy, J., & Lauten, D. (1994). Learning about calculus learning. *The Mathematics Teacher*, 87(2), 115-121.
- Flavell, J.H. (1976). Metacognitive aspects of problem solving. In L.B. Resnick (Ed.), *The nature of intelligence* (pp. 231-235). Hillsdale, New Jersey: Lawrence Erlbaum Associates.
- Flavell, J.H. (1987). Speculations about the nature and development of metacognition. In F.E. Weinert & R. H. Kluwe, (Eds.), *Metacognition, motivation and understanding* (pp. 21-29). Hillsdale, New Jersey: Lawrence Erlbaum Associates.
- Fontana, A., & Frey, J.H. (2000). The interview: From structured questions to negotiated text. In N.K. Denzin & Y.S. Lincoln (Eds.), *Handbook of qualitative research* (2<sup>nd</sup> edition, pp. 645-672). Thousand Oaks, California: Sage.
- Ford, N., & Chen, S.Y. (2001). Matching/mismatching revisited: An empirical study of learning and teaching styles. *British Journal of Educational Technology*, 32(1), 5-22.
- Friedman, P., & Alley, R. (1984). Learning/teaching styles: Applying the principles. *Theory into Practice*, 23(1), 77-81.

- Golden, K. M. (n.d.). *First lecture: What is calculus about and why should I study it?*  
Retrieved December 12, 2006, from  
[http://www.math.utah.edu/~dobson/teach/1250/First\\_Lecture.pdf](http://www.math.utah.edu/~dobson/teach/1250/First_Lecture.pdf)
- Graf, S., & Kinshuk, (2006). An approach for detecting learning styles for learning management systems. *Proceedings of the 6<sup>th</sup> IEEE International Conference on Advanced Learning Technologies (ICALT'06)*, (pp. 161-163), Kerkrade, Netherlands.
- Graf, S., Viola, S.R., Kinshuk, & Leo, T. (2006). Representative characteristics of Felder-Silverman learning styles: An empirical model. *Proceedings of the IADIS International Conference on Cognition and Exploratory Learning in Digital Age (CELDA'06)*, Barcelona, Spain. Retrieved December 22, 2006, from  
[http://wit.at/people/graf/publications/graf\\_viola\\_kinshuk\\_leo\\_CELDA06.pdf](http://wit.at/people/graf/publications/graf_viola_kinshuk_leo_CELDA06.pdf)
- Greeno, J.G., & Hall, R.P. (1997). Practicing representation: Learning with and about representational forms. *Phi Delta Kappan*, 78(5), 361-367.
- Gregorc, A.F. (1979). Learning/teaching styles: Potent forces behind them. *Educational Leadership*, 34(4), 234-236.
- Guba, E., & Lincoln, Y. (1989). *Fourth generation evaluation*. London: Sage.
- Guri-Rosenblit, S. (2005). 'Distance education' and 'e-learning': Not the same thing. *Higher Education*, 49(4), 467-493.
- Guzel, E. B., & Alkan, H. (2006). Constructing the limit concept by using the 7E learning cycle model. *Proceedings of the 3<sup>rd</sup> International Conference on the Teaching of Mathematics at the Undergraduate Level (ICTM3)*, Istanbul, Turkey. John Wiley & Sons.
- Habre, S., & Abboud, M. (2006). Students' conceptual understanding of a function and its derivative in an experimental calculus course. *Journal of Mathematical Behavior*, 25(1), 57-72.
- Hahkioniemi, M. (2004). Perceptual and symbolic representations as a starting point of the acquisition of the derivative. In M.J. Hones & A.B. Fuglestad (Eds.), *Proceedings of the 28<sup>th</sup> International Conference of the International Group for the Psychology of Mathematics Education (PME)*, (Vol. 3, pp. 73-80), Bergen, Norway.
- Hahkioniemi, M. (2006a). Associative and reflective connections between the limit of the difference quotient and limiting process. *Journal of Mathematical Behavior*, 25(2), 170-184.
- Hahkioniemi, M. (2006b). Is there a limit in the derivative?-Exploring students' understanding of the limit of the difference quotient. In M. Bosch, (Ed.), *Proceedings of the 4<sup>th</sup> Congress of the European Society for Research in Mathematics Education (CERME 4 (2005))*, Working Group 14 on Advanced Mathematical Thinking, (pp. 1758-1767), Sant Feliu de Guixols, Spain. Retrieved March 22, 2007, from [http://ermeweb.free.fr/CERME4/CERME4\\_WG14.pdf](http://ermeweb.free.fr/CERME4/CERME4_WG14.pdf)

- Hall, E. (2005). Learning styles - Is there an evidence for this popular idea? *Education Review*, 19(1), 49-56.
- Harman, C., & Dorman, M. (1998). Enriching distance teaching and learning of undergraduate mathematics using videoconferencing and audiographics. *Distance Education*, 19(2), 299-318.
- Hauger, G.S. (2000). Instantaneous rate of change: A numerical approach. *International Journal of Mathematical Education in Science and Technology*, 31(6), 891-897.
- Heid, M.K. (1988). Resequencing skills and concepts in applied calculus using the computer as a tool. *Journal for Research in Mathematics Education*, 19(1), 3-25.
- Henning, E., Gravett, S., & van Rensburg, W. (2002). *Finding your way in academic writing*. Pretoria: Van Schaik.
- Henson, K.T., & Borthwick, P. (1984). Matching styles: A historical look. *Theory into Practice*, 23(1), 3-9.
- Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1-27). Hillsdale, New Jersey: Lawrence Erlbaum Associates.
- Hiebert, J., & Carpenter, T.P. (1992). Learning and teaching with understanding. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65-100). New York: Macmillan.
- Holmberg, B. (1986). *Growth and structure of distance education*. London: Croom-Helm.
- Hubbard, R. (1990). Tertiary mathematics without lectures. *International Journal of Mathematical Education in Science and Technology*, 21(4), 567-571.
- Hubbard, R. (1997). Designing questions that develop understanding. *International Journal of Mathematical Education in Science and Technology*, 28(6), 793-801.
- Hughes-Hallet, D., Gleason A.M., Flath, D.E., Gordon, S.P., Lomen, D.O., Lovelock, D. *et al.* (1994). *Calculus* (International edition). New York: John Wiley and Sons.
- Janvier, C. (1987). Representation and understanding: The notion of function as an example. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 67-71). Hillsdale, New Jersey: Lawrence Erlbaum Associates.
- Jung, I. (2005). Quality assurance survey of mega-universities. In C. McIntosh (Ed.), *Perspectives on distance education: Lifelong learning and distance higher education* (pp. 79-95). COL/UNESCO.
- Juter, K. (2005a). Limits of function - How do students handle them? *Pythagoras*, 61, 11-20.
- Juter, K. (2005b). Students' attitudes to mathematics and performance in limits of functions. *Mathematics Education Research Journal*, 17(2), 91-110.

- Juter, K. (2006). Limits of function - Students solving tasks. *Australian Senior Mathematics Journal*, 20(1), 15-30.
- Kannemeyer, L. (2003). *The development of a reference framework for measuring students' understanding in a first year calculus course*. Unpublished doctoral dissertation, Department of Didactics, University of the Western Cape, South Africa.
- Kaput, J.J. (1987). Representation systems and mathematics. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 19-26). Hillsdale, New Jersey: Lawrence Erlbaum Associates.
- Kaput, J.J. (1992). Technology and mathematics education. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 515-556). New York: Macmillan.
- Kaput, J.J., & Thompson, P.W. (1994). Technology in mathematics education research: The first 25 years in the JRME. *Journal for Research in Mathematics Education*, 25(6), 676-684.
- Keefe, J.W. (1985). Assessment of learning style variables: The NASSP Task Force Model. *Theory into Practice*, 24(2), 138-144.
- Keefe, J.W., & Ferrell, B.G. (1990, October). Developing a defensible learning style paradigm. *Educational Leadership*, 57-61.
- Keegan, D. (1990). *Foundations of distance education* (2<sup>nd</sup> edition). London: RoutledgeFalmer.
- Keller, B.A., & Hirsch, C.A. (1998). Student preferences for representations of functions. *International Journal of Mathematical Education in Science and Technology*, 29(1), 1-17.
- Kember, D. (2004). Interpreting student workload and the factors which shape the students' perceptions of their workload. *Studies in Higher Education*, 29(2), 165-184.
- Kember, D., Ng, S., Tse, H., Wong, T.T.E., & Pomfret, M. (1996). An investigation of the interrelationships between workload, study time, learning approaches and academic outcomes. *Studies in Higher Education*, 21(3), 347-358.
- Kim, D.J., Sfard, A., & Ferrini-Mundy, J. (2005). Students' colloquial and mathematical discourses on infinity and limit. In H.L. Chick & J.L. Vincent (Eds.), *Proceedings of the 29<sup>th</sup> Conference of the International Group for the Psychology of Mathematics Education (PME)*, (Vol. 3, pp. 201-208), Melbourne, Australia.
- Kinshuk, & Lin, T. (2004). Application of learning styles adaptivity in mobile learning environments. *Third Pan Commonwealth Forum on Open Learning*, Dunedin, New Zealand. Retrieved April 2, 2007, from [http://www.col.org/pcf3/Papers/PDFs/Kinshuk\\_Lin\\_1.pdf](http://www.col.org/pcf3/Papers/PDFs/Kinshuk_Lin_1.pdf)
- Knisley, J. (n.d). *A four-stage model of mathematical learning*. Retrieved April, 12 2007, from <http://faculty.etsu.edu/knisleyj/MathematicsEducatorArticle.pdf>

- Knowles, M. S. (1980). *The modern practice of adult education: From pedagogy to andragogy* (2<sup>nd</sup> edition). New York: Cambridge Books.
- Knowles, M.S. (1984). *The adult learner: A neglected species* (3<sup>rd</sup> edition). Houston: Gulf.
- Kolb, D.A. (1984). *Experiential learning: Experience as the source of learning and development*. Englewood Cliffs, New Jersey: Prentice Hall.
- Kramer-Koehler, P., Tooney, N.M., & Beke D.P. (1995). The use of learning style innovations to improve retention. *Proceedings of the Conference of ASEE/IEEE Frontiers in Education*.
- Krussel, L. (2002). What's the difference? Teaching mathematics to standards in a distance learning environment. In A. Rogerson (Ed.), *Proceedings of the 4<sup>th</sup> International Conference of the Mathematics Education into the 21<sup>st</sup> century project*, Palermo, Italy. Retrieved June 7, 2007, from <http://math.unipa.it/~grim/SiKrussel.pdf>
- Laight, D.W. (2004). Attitudes to concept maps as a teaching/learning activity in undergraduate health professional education: Influence of preferred learning style. *Medical Teacher*, 26(3), 229-233.
- Langer, A.M. (2002). Reflecting on practice: Using learning journals in higher and continuing education. *Teaching in Higher Education*, 7, 338-351.
- Larkin, T.L., & Budny, D.D. (2003). Learning styles in the physics classroom: A research-informed approach. *Proceedings of the 2003 American Society for Engineering Education Annual Conference & Exposition*.
- Lawless, C. (2000). Using learning activities in mathematics: Workload and study time. *Studies in Higher Education*, 25(1), 97-111.
- Leary, J., & Berge, Z. (2007). Successful distance education programs in Sub-Saharan Africa. *Turkish Online Journal of Distance Education-TOJDE*, 8(2), 136-145 [online journal]. Retrieved June 21, 2007, from <http://tojde.anadolu.edu.tr/tojde26/>
- Leinhardt, G., Zaslavsky, O., & Stein, M.K. (1990). Functions, graphs, and graphing: Tasks, learning and teaching. *Review of Educational Research*, 60(1), 1-64.
- Lentell, H. (2003). The importance of the tutor in open and distance learning. In A. Tait & R. Mills (Ed.), *Rethinking learner support in distance education: Change and continuity in an international context* (pp. 64-76). London: RoutledgeFalmer.
- Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 33-40). Hillsdale, New Jersey: Lawrence Erlbaum Associates.
- Lewis, E. (n.d). *Modes of representation of ideas, computers and learning styles in K-6 mathematics*. Retrieved November 22, 2003, from <http://www.aare.edu.edu.au/94pap/lewie94.313>.
- Lincoln, Y., & Guba, E. (1985). *Naturalistic inquiry*. London: Sage.

- Liu, Y., & Ginther, D. (1999). Cognitive styles and distance education. *Online Journal of Distance Learning Administration*, 2(3). Retrieved March 10, 2003 from <http://www.westga.edu/~distance/liu23.html>
- Livingston, J.A. (1997). *Metacognition: An overview*. Retrieved June 8, 2004, from <http://www.gse.buffalo.edu/fas/shuell/cep564/Metacog.htm>
- Lockwood, F. (1998). *The design and production of self-instructional materials*. Open and Distance Learning Series. London: Kogan Page.
- Loftus, G.R., & Loftus, E.F. (1976). *Human memory: The processing of information*. Hillsdale, New Jersey: Lawrence Erlbaum Associates.
- Logan, K., & Thomas, P. (2002). Learning styles in distance education students learning to program. In J. Kuljis, L. Baldwin & R. Scobie (Eds.), *Proceedings of the 14<sup>th</sup> Workshop of the Psychology of Programming Interest Group*, (pp. 29-44), Brunnel University.
- Lyall, R., & McNamara, S. (2000). Influences on the orientation to learning of distance education students in Australia. *Open Learning*, 15(2), 107-121.
- Malmivuori, M.L. (2001). *The dynamics of affect, cognition and social environment in the regulation of personal learning processes: The case of mathematics*. Unpublished doctoral dissertation, Department of Education, University of Helsinki, Finland. Retrieved June 7, 2007, from <http://ethesis.helsinki.fi/julkaisut/kas/kasva/vk/malmivuori/thedynam.pdf>
- Marland, P. (1997). *Towards more effective open and distance teaching*. London: Kogan Page.
- Marton, F., & Säljö, R. (1976). On qualitative differences in learning. I- Outcome and Process. *British Journal of Educational Psychology*, 46, 4-11.
- Mauil, W., & Berry, J. (2000). A questionnaire to elicit the mathematical concept images of engineering students. *International Journal of Mathematical Education in Science and Technology*, 31(6), 899-917.
- Mayer, R.E. (1989). Models for understanding. *Review of Educational Research*, 59(1), 43-64.
- Mayer, R.E. (1996). Learners as information processors: Legacies and limitations of educational psychology's second metaphor. *Educational Psychologist*, 31(3/4), 151-161.
- McCrinkle, A.R., & Christensen, C.A. (1995). The impact of learning journals on metacognitive and cognitive processes and learning performance. *Learning and Instruction*, 5, 167-185.
- McLeod, D. (1992). Research on affect in mathematics education: A reconceptualization. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 575-593), New York: Macmillan.

- McLoughlin, C. (1999). The implications of the research literature on learning styles for the design of instructional material. *Australian Journal of Educational Technology*, 15(3), 222-241.
- McShane, J. (1991). *Cognitive development: An information processing approach*. Oxford: Blackwell.
- Mercer, N., & Pettit, J. (2001). The development of distance education in new environments. *The International Journal of Open and Distance Learning (The Zimbabwe Open University Journal)*, 1, 109-124.
- Merriam, S.B. (1988). *Case study research in education: A qualitative approach*. Oxford: Jossey-Bass.
- Merriam, S.B. (1998). *Qualitative research and case study applications in education* (2<sup>nd</sup> edition). San Francisco: Jossey-Bass.
- Miles, B.M., & Huberman, A.M. (1994). *Qualitative data analysis: An expanded sourcebook* (2<sup>nd</sup> edition). Thousand Oaks, California: Sage.
- Miller, G. A. (2003). The cognitive revolution: A historical perspective. *Trends in Cognitive Sciences*, 7(3), 141-144.
- Mizoue, C. (2003). Distance higher education and library services in Japan. In A. Tait & R. Mills (Eds.), *Rethinking learner support in distance education: Change and continuity in an international context* (pp. 115-122), London: RoutledgeFalmer.
- Mohammad Yusof, Y., & Tall, D. (1994). Changing attitudes to mathematics through problem solving. In J. Pedro da Ponte & J. Filipe Matos (Eds.), *Proceedings of the 18<sup>th</sup> Conference of the International Group for the Psychology of Mathematics Education (PME)* (Vol. 4, pp. 401-408), Lisbon, Portugal.
- Mohammad Yusof, Y., & Tall D. (1999). Changing attitudes to university mathematics problem solving. *Educational Studies in Mathematics*, 37, 67-82.
- Monaghan, J. (1991). Problems with the language of limits. *For the Learning of Mathematics*, 11(3), 20-24.
- Monaghan, J., Sun, S., & Tall, D. (1994). Construction of the limit concept with a computer algebra system. In J. Pedro da Ponte & J. Filipe Matos (Eds.), *Proceedings of the 18<sup>th</sup> Conference of the International Group for the Psychology of Mathematics Education (PME)* (Vol. 3, pp. 279-286), Lisbon: Portugal.
- Montgomery, S.M., & Groat, L.N. (1999). Student learning styles and their implications for teaching. *CRLT occasional paper*. Center for Research on Learning and Teaching, University of Michigan. Retrieved November 22, 2004, from <http://www.crlt.unimich.edu/occ10.html>
- Moore, M.G. (1989). Three types of interaction. *The American Journal of Distance Education*, 3(2), 1-6.



- Morash, R.P. (1990). Closing the epsilon gap. *International Journal of Mathematical Education in Science and Technology*, 21(2), 183-186.
- Morgan, A.T. (1990). A study of the difficulties experienced with mathematics by engineering students in higher education. *International Journal of Mathematical Education in Science and Technology*, 21(6), 975-988.
- Morrison, K. (1996). Developing reflective practice in higher degree students through a learning journal. *Studies in Higher Education*, 21(3), 317-332.
- Moru, E.K. (2006). *Epistemological obstacles in coming to understand the limit concept at undergraduate level: A case of the National University of Lesotho*. Unpublished doctoral dissertation, School of Science and Mathematics Education, University of the Western Cape, South Africa.
- Mourtos, N.J., & McMullin, K. (2001). A comparison of student learning in online and onground engineering courses. *Proceedings of the 4<sup>th</sup> UICEE Annual Conference on Engineering Education*, Bangkok, Thailand. Retrieved December 12, 2006, from <http://www.engr.sjsu.edu/nikos/pdf/bangkok.pdf>
- Mouton, J. (2001). *How to succeed in your Master's and Doctoral studies: A South African guide and resource book*. Pretoria: Van Schaik.
- Mustoe, L., & Lawson, D. (Eds.). (2002). Mathematics for the European engineer: A curriculum for the twenty-first century. *A report by the European Society for Engineering Education (SEFI) Mathematics Working Group*, Brussels. Retrieved April 2, 2007, from <https://learn.lboro.ac.uk/mwg/core/latest/sefimarch2002.pdf>
- Naidoo, K., & Naidoo, R. (2007). First year students' understanding of elementary concepts in differential calculus in a computer laboratory teaching environment. *Proceedings of the 2007 Applied Business Research (ABR) and The College of Teaching and Learning (TLC) Conference*, Hawaii, USA. Retrieved July 1, 2007, from <http://www.cluteinstitute.com/Programs/Hawaii-2007/Article%20276.pdf>
- Novin, A.M., Arjomand, L.H., & Jourdan, L. (2003). An investigation into preferred learning styles of accounting, management, marketing and general business majors. *Teaching and Learning*, 18(1), 24-31.
- Nulty, D.D., & Barret M.A., (1996). Transitions in students' learning styles. *Studies in Higher Education*, 21(3), 333-345.
- Orton, A. (1983). Students' understanding of differentiation. *Educational Studies in Mathematics*, 14(3), 235-250.
- Orton, A. (1992). *Learning mathematics: Issues, theory and classroom practice* (2<sup>nd</sup> edition). London: Continuum International.
- Ostebee, A., & Zorn, P. (2002). *Calculus: From graphical, numerical and symbolic points of view* (2<sup>nd</sup> edition). Boston: Houghton Mifflin.

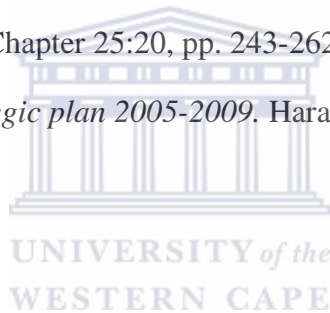
- Pask, G. (1976). Styles and strategies of learning. *British Journal of Educational Psychology*, 46, 128-148.
- Pask, G. (1988). Learning strategies, teaching strategies and conceptual or learning style. In R.R. Schmeck (Ed.), *Perspectives on individual differences: Learning strategies and learning styles* (pp. 83-100). New York & London: Plenum Press.
- Patton, M.Q. (1980). *Qualitative evaluation methods*. Beverly Hills, California: Sage.
- Pillay, H. (1998). An investigation of the effect of cognitive preferences on learning through computer based instruction. *Educational Psychology*, 18(2), 171-182.
- Pirie, S.E.B., & Kieren, T.E. (1994). Growth in mathematical understanding: How can we characterise it and how can we represent it? *Educational Studies in Mathematics*, 26(3), 165-190.
- Proctor, R.W., & Vu, K.L. (2006). The cognitive revolution at age 50: Has the promise of the human-information processing approach been fulfilled? *International Journal of Human-Computer Interaction*, 21(3), 253-284.
- Rahman, M.Z. (2002). *Student support: Bridging the gap between students and the university*. Unpublished doctoral dissertation, Faculty of Education, Deakin University, Victoria, Australia.
- Ramsden, P. (1988). Context and strategy: Situational influences on learning. In R.R. Schmeck (Ed.), *Perspectives on individual differences: Learning strategies and learning styles* (pp. 159-184). New York & London: Plenum Press.
- Richardson, J.T.E. (2000). *Researching student learning: Approaches to studying in campus based and distance education*. Buckingham: The Society for Research into Higher Education and Open University Press.
- Riding, R.J., & Sadler-Smith, E. (1992). Type of instructional material, cognitive style and learning performance. *Educational Studies*, 18(3), 323-340.
- Robert, A., & Speer, N. (2001). Research into the teaching and learning of calculus/elementary analysis. In D. Holton (Ed.), *The teaching and learning of mathematics at university level: An ICMI Study* (Vol. 7, pp. 255-273). Dordrecht: Kluwer Academic.
- Robson, C. (2002). *Real world research: A resource for social scientists and practitioner-researchers* (2<sup>nd</sup> edition). Oxford: Blackwell.
- Roebber, P.J. (2005, April). Bridging the gap between theory and applications: An inquiry into atmospheric science teaching. *Bulletin of American Meteorological Society*, 507-517.
- Rowntree, D. (1992). *Exploring open and distance learning*. The Open and Distance Learning Series. London: Kogan Page.
- Rowntree, D. (1997). *Making material-based learning work*. The Open and Distance Learning Series. London: Kogan Page.

- Sajka, M. (2003). A secondary school student's understanding of the concept of function: A case study. *Educational Studies in Mathematics*, 53(3), 229-254.
- Schoenfeld, A.H. (1987). What's all the fuss about metacognition? In A.H. Schoenfeld (Ed.), *Cognitive Science and Mathematical Education*. Hillsdale, New Jersey: Lawrence Erlbaum Associates.
- Schultz, J.E., & Waters, M.S. (2000). Why representations? *Mathematics Teacher*, 93(6), 448-453.
- Schunk, D.H. (2000). *Learning theories: An educational perspective* (3<sup>rd</sup> edition). New Jersey: Prentice Hall.
- Selden, A. (2002). Two research traditions separated by a common subject: Mathematics and mathematics education. *Technical Report No. 2002-2*, Tennessee Technological University. Retrieved November 18, 2006, from [http://www.math.tntech.edu/techreports/TR\\_2002\\_2.pdf](http://www.math.tntech.edu/techreports/TR_2002_2.pdf)
- Serhan, D. (2006, May 8). The effect of graphing calculators use on students' understanding of the derivative at a point. *International Journal for Mathematics Teaching and Learning*, [online journal]. Centre for Innovation in Mathematics Teaching (CIMT). Retrieved February 19, 2007, from <http://www.cimt.plymouth.ac.uk/journal/serhan.pdf>
- Sierpinska, A. (1990). Some remarks on understanding in mathematics. *For the Learning of Mathematics*, 10(3), 24-36.
- Sierpinska, A. (1994). *Understanding in mathematics*. London: The Falmer Press.
- Simonson, M., Smaldino, S., Albright, M., & Zvacek, S. (2006). *Teaching and learning at a distance: Foundations of distance education* (3<sup>rd</sup> edition). Upper Saddle River, New Jersey: Pearson, Merrill Prentice Hall.
- Skemp, R.R. (1971). *The psychology of learning mathematics*. London: Penguin Books.
- Skemp, R.R. (1976). Relational understanding and instrumental understanding, *Mathematics Teaching*, 77, 20-26.
- Sloan, T., Daane, C.J., & Giesen, J. (2002). Mathematics anxiety and learning styles: What is the relationship in elementary preservice teachers? *School Science and Mathematics*, 102(2), 84-87.
- Stake, E.R. (2000). Case studies. In N.K. Denzin & Y.S. Lincoln (Eds.), *Handbook of qualitative research* (2<sup>nd</sup> edition, pp. 435-454). Thousand Oaks, California: Sage.
- Star, J.R. (2005). Reconceptualising procedural knowledge. *Journal for Research in Mathematics Education*, 26(5), 404-411.
- Stewart, J. (1994). *Calculus: Concepts and contexts* (2<sup>nd</sup> edition). California: Brooks/Cole.

- Stewart, J.H., & Atkin, J.A. (1982). Information processing psychology: A promising paradigm for research in science teaching. *Journal of Research in Science Teaching*, 19(4), 321-332.
- Steyn, T. & Maree, J.G. (2002). A profile of first-year students' learning preferences and study orientation in mathematics. *Proceedings of the 2<sup>nd</sup> International Conference on the Teaching of Mathematics (at undergraduate level), ICTM2*. John Wiley. Retrieved December 22, 2006, from <http://www.math.uoc.gr/~ictm2/Proceedings/pap211.pdf>
- Strauss, A., & Corbin, J. (1990). *Basics of qualitative research: Grounded theory procedures and techniques*. Newbury Park, California: Sage.
- Swinyard, C.A., & Lockwood, E. (2007). Research on students' reasoning about the formal definition of limit: An evolving conceptual analysis. *Proceedings of the 10<sup>th</sup> Conference on Research in Undergraduate Mathematics Education (CRUME2007)*, San Diego, California. Retrieved June 7, 2007, from <http://cresmet.asu.edu/crume2007/papers/lockwood-swinyard.pdf>
- Szydlik J.E. (2000). Mathematical beliefs and conceptual understanding of the limit of a function. *Journal for Research in Mathematics Education*, 31(3), 258-276.
- Tait, A. (2000). Planning student support for open and distance learning. *Open Learning*, 15(3), 287-298.
- Tait, J. (2004). The tutor/facilitator role in student retention. *Open Learning*, 19(1), 97-109.
- Tall, D. (1978). The dynamics of understanding mathematics. *Mathematics Teaching*, 81, 50-52.
- Tall, D. (1993a). Students' difficulties in calculus. *Proceedings of working Group 3 on students' difficulties in calculus, ICME-7* (pp. 13-28). Quebec, Canada. Retrieved June 23, 2005 from <http://www.warwick.ac.uk/staff/David.Tall/pdfs/dot1993k-calculus-wg3-icme.pdf>
- Tall, D. (1993b). Technology and mathematics education: Computer environments for the learning of Mathematics. In R. Biehler, R.W. Scholz, R. Straber, & B. Winkelmann, (Eds.), *Didactics of mathematics as a scientific discipline-The state of the art* (pp. 189-199). Dordrecht: Kluwer Academic.
- Tall, D. (1997). Functions and Calculus. In A.J. Bishop *et al.* (Eds.), *International handbook of mathematics education* (pp. 289-325). Dordrecht: Kluwer Academic.
- Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, 12, 151-169.
- Tanner, K., & Allen, D. (2004). Approaches to biology teaching and learning: Learning styles and the problem of instructional selection - Engaging all students in science courses. *Cell Biology Education*, 3, 197-201.

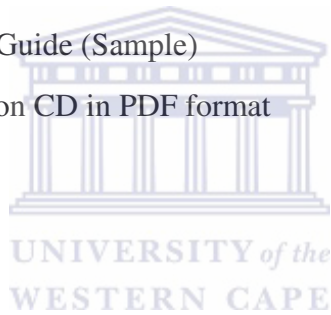
- Taylor, J.C. (2001). *Fifth generation distance education*. Report No. 40, Higher Education Series, Australia. Retrieved November 15, 2006, from <http://www.dest.gov.au/archive/highered/hes/hes40/hes40.pdf>
- Taylor, J.A., & Mohr, J. (2001). Mathematics for math anxious students studying at a distance. *Journal of Developmental Education*, 25(1), 30-36.
- Thomas, B. (Jr.), & Finney, R.L. (1992). *Calculus and analytic geometry* (2<sup>nd</sup> edition). New York: Addison-Wesley.
- Thomson, B.S., & Mascazine, J.R. (1997). Attending to learning styles in mathematics and science classrooms. ERIC Clearinghouse for Science, Mathematics and Environmental education, ED432440.
- Tsvigu, C., Persens, J., Breiteig, T. & Ndalichako, J. (in print). Learning styles in calculus in a distance education environment. In L. Holtman, C. Julie, Ø. Mikalsen, D. Mtetwa and M. Ogunniyi, (Eds.). *Some developments in research in science and mathematics education in Sub-Saharan Africa. Access, adaptation, localisation and adoption*. Cape Town: University of the Western Cape.
- UNESCO, (2002). *Open and distance learning: Trends, policy and strategy considerations*. Paris:UNESCO. Retrieved August 10, 2005, from <http://unesdoc.unesco.org/images/0012/001284/128463e.pdf>
- van Zwanenberg, N., Wilkinson, L.J., & Anderson, A. (2000). Felder and Silverman's Index of Learning Styles and Honey and Mumford's Learning Styles Questionnaire: How do they compare and do they predict academic performance? *Educational Psychology*, 20(3), 365-380.
- Vermunt, J.D. (1996). Metacognitive, cognitive and affective aspects of learning styles and strategies: A phenomenographic analysis. *Higher Education*, 31, 25-50.
- Viholainen, A. (2006). Relationships between informal and formal reasoning in the subject of derivative. In M. Bosch, (Ed.), *Proceedings of the 4<sup>th</sup> Congress of the European Society for Research in Mathematics Education (CERME 4 (2005)), Working Group 14 on Advanced Mathematical Thinking*, (pp. 1811-1820), Sant Feliu de Guixols, Spain. Retrieved March 22, 2007, from [http://ermeweb.free.fr/CERME4/CERME4\\_WG14.pdf](http://ermeweb.free.fr/CERME4/CERME4_WG14.pdf)
- Vinner, S. (1991). The role of definitions in the teaching and learning of mathematics. In D.O. Tall (Ed.), *Advanced mathematical thinking* (pp. 65-81). Dordrecht: Kluwer Academic.
- Vuma, D., & Mudehwe, L.M. (2000). *Calculus 1, MTD101* (2<sup>nd</sup> edition). Harare: Zimbabwe Open University Publication.
- Wellington, J. (2000). *Educational research: Contemporary issues and practical approaches*. London: Continuum International.

- Willcox, K., & Bounova, G. (2004). Mathematics in engineering: Identifying, enhancing and linking the implicit mathematics curriculum. *Proceedings of the 2004 American Society for Engineering Education Annual Conference and Exposition*. Retrieved April 2, 2007, from <http://raphael.mit.edu/WillcoxASEE04.pdf>
- Williams, S.R. (1991). Models of limit held by college calculus students. *Journal for Research in Mathematics Education*, 22(3), 219-236.
- Williams, S.R. (2001). Predications of the limit concept: An application of repertory grids. *Journal for Research in Mathematics Education*, 32(4), 341-367.
- White, P., & Mitchelmore, M. (1996). Conceptual knowledge in introductory calculus. *Journal for Research in Mathematics Education*, 27(1), 79-95.
- Yin, R.K. (2003). *Case study research: Design and methods* (3<sup>rd</sup> edition). Applied Social Research Methods Series, (Vol. 5). Thousand Oaks, California: Sage.
- Zandieh, M.J., & Knapp, J. (2006). Exploring the role of metonymy in mathematical understanding and reasoning: The concept of derivative as an example. *Journal of Mathematical Behavior*, 25(1), 1-17.
- Zimbabwe Open University Act*, Chapter 25:20, pp. 243-262.
- Zimbabwe Open University Strategic plan 2005-2009*. Harare: Zimbabwe Open University



## LIST OF APPENDICES

Appendix I:	Request for Permission - Pro Vice-Chancellor
Appendix II:	Letter of Permission from the Registrar
Appendix III:	Request for Permission - Regional Directors
Appendix IV:	Permission from Regional Director (Mash Central)
Appendix V:	Letter of Introduction to students
Appendix VI:	Learning Journal (Sample)
Appendix VII:	Calculus Tasks Test (Sample)
Appendix VIII:	LSPQ - Limit of function (Sample)
Appendix IX:	LSPQ - Derivative of function (Sample)
Appendix X:	Interview Guide (Sample)
Appendix XI:	Data Sets on CD in PDF format



## APPENDIX I: REQUEST FOR PERMISSION-PRO VICE-CHANCELLOR



**ZIMBABWE OPEN UNIVERSITY**

DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY

### MEMORANDUM

**To:** The Pro-Vice-Chancellor, Academic Affairs  
**From:** Mrs C. Tsvigu,  
Chairperson, Department of Science, Mathematics & Technology  
**Cc:** Executive Dean, Faculty of Science  
**Date:** 06 September 2005

**RE: Request for permission to carry out my research with B.Sc. Mathematics and Statistics Intake 7 students**

---

I write seeking your permission to carry out my research with the Zimbabwe Open University, BSc. Mathematics and Statistics Intake 7 students. I am registered for doctoral studies (student number 2259826) with the University of the Western Cape (UWC) in South Africa where I take part in the GRASSMATE (Graduate Studies in Science, Mathematics and Technology Education) project, a joint project housed at the UWC but of a sandwich nature between the University of the Western Cape (RSA) and University of Bergen (Norway).

I am due for data collection during the September 2005 to December 2005 semester. My thesis working title is:

*“Mathematics Distance Education students’ learning styles and their understandings of some calculus concepts: A case at the Zimbabwe Open University.”*

The main subjects for the study are Part 1, semester 1, BSc. Mathematics and Statistics, Intake 7 students who are registered for the course MTD101 Calculus 1. The study is of a qualitative nature and data will be collected through the use of students’ learning journals, learning styles preference questionnaires, interviews and calculus tasks test on limit of function and derivative of function concepts. For convenience purposes I intend to use Harare Region, Mashonaland West Region, Mashonaland Central and Masvingo Region. All data collected will be treated with confidentiality and will be used solely for the purposes of this study.

Thank you

C. Tsvigu



APPENDIX II: LETTER OF PERMISSION FROM THE REGISTRAR



**ZIMBABWE OPEN UNIVERSITY**

From the Office of the Registrar  
Mr T. E. S. Benza

22<sup>nd</sup> September 2005

Mrs C Tsvigu  
C/O Faculty of Science  
Zimbabwe Open University  
**HARARE**

Dear Mrs Tsvigu

**RE: APPLICATION FOR PERMISSION TO CARRY OUT RESEARCH WITH  
BSC MATHEMATICS AND STATISTICS INTAKE 7 STUDENTS**

Authority is granted for you to do your research covering aspects of the Zimbabwe Open University. The University will be interested to get a copy of your study.

Please be advised that all data collected should be treated with confidentiality and used only for the purpose of this study.

We wish you the best in your endeavours.

Thank you,

Yours sincerely,

  
**T.E.S. BENZA**  
**REGISTRAR**

/cg

4<sup>th</sup> Floor Stanley House Corner J. Moyo Avenue/1<sup>st</sup> Street Harare  
P. O. Box MP 1119 Mount Pleasant

Tel: 263-4-793002/3/7/9 / 251518 Fax: 263-4-703679

## APPENDIX III: REQUEST FOR PERMISSION - REGIONAL DIRECTORS



**ZIMBABWE OPEN UNIVERSITY**

DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY

### MEMORANDUM

**To:** The Regional Director  
**From:** Mrs C. Tsvigu,  
Chairperson, Department of Science, Mathematics & Technology  
**Cc:** Executive Dean, Faculty of Science  
**Date:** 06 September 2005

**RE: Request for permission to carry out my research with B.Sc. Mathematics and Statistics Intake 7 students**

---

I write seeking your permission to carry out my research with the Zimbabwe Open University, BSc. Mathematics and Statistics Intake 7 students. I am registered for doctoral studies (student number 2259826) with the University of the Western Cape (UWC) in South Africa where I take part in the GRASSMATE (Graduate Studies in Science, Mathematics and Technology Education) project, a joint project housed at the UWC but of a sandwich nature between the University of the Western Cape (RSA) and University of Bergen (Norway).

I am due for data collection during the September 2005 to December 2005 semester. My thesis working title is:

*“Mathematics Distance Education students’ learning styles and their understandings of some calculus concepts: A case at the Zimbabwe Open University.”*

The main subjects for the study are Part 1, semester 1, BSc. Mathematics and Statistics, Intake 7 students who are registered for the course MTD101 Calculus 1. The study is of a qualitative nature and data will be collected through the use of students’ learning journals, learning styles preference questionnaires, interviews and calculus tasks test on limit of function and derivative of function concepts. For convenience purposes I intend to use Harare Region, Mashonaland West Region, Mashonaland Central and Masvingo Region. All data collected will be treated with confidentiality and will be used solely for the purposes of this study.

Thank you

C. Tsvigu

## APPENDIX IV: PERMISSION FROM REGIONAL DIRECTOR (MASH CENTRAL)



ZIMBABWE OPEN UNIVERSITY

MASHONALAND CENTRAL REGION

P. Bag 984 • 209 Hay Road • Bindura • Zimbabwe

Tel +263-71-7161/7107/7484 • Mobile 011 410 505

Fax +263-71-6653

Email: [stevenvere@yahoo.com](mailto:stevenvere@yahoo.com) Website: <http://www.zou.co.zw>

### MEMORANDUM

TO : The Chairperson, Department of Science – C. Tsvigu

cc : Programme Coordinator – M. Magodora

FROM : Regional Director

REF : CM/rm

DATE : 19 September 2005



UNIVERSITY of the  
WESTERN CAPE

Re: REQUEST FOR PERMISSION TO CARRY OUT RESEARCH WITH BSc MATHEMATICS AND STATISTICS INTAKE 7

We acknowledge receipt of your communication of request.

The region grants you permission to carry out your research. You can also liaise with the Programme Coordinator to facilitate your collection of data.

Regards,

C. MAJONI (Mr.)  
Regional Director



*“Empowerment Through Open Learning”*  
The Zimbabwe Open University Is A Distance Teaching And Open Learning Institution

## APPENDIX V: LETTER OF INTRODUCTION TO STUDENTS



**ZIMBABWE OPEN UNIVERSITY**

### DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY

September 6<sup>th</sup> 2005

Dear ZOU BSc Maths and Statistics, Calculus 1 Student

I write seeking your permission to involve you in my research study. I am pursuing doctoral studies at the University of the Western Cape (RSA) and am about to collect data for my Ph.D. thesis. The study is meant to gather data on distance education students' learning styles and understandings of some calculus concepts, specifically the concepts of limit of function and derivative of function. The data will contribute towards the research findings for the thesis. It is my hope that the results of the study will give indications/pointers of one's learning style preferences and hence hint to mathematics distance educators on how best to attend to learner differences and difficulties in calculus learning.

I intend to collect the data through the use of:

- **Learning Journals** (which are diary of recordings of your learning processes and activities). The learning journal is a structured diary comprising of your self-report and reflections of your learning processes as you learn the limit and derivative of function concepts. I also believe that other than the benefits to the research study, making entries into the learning journal can also be beneficial to you as a participant as this can help you to reflect about your learning processes.
- **Learning style preference questionnaires**
- **Calculus tasks** on limit of function and derivative of function.
- **Interviews** focused on the emerging learning styles and understandings of the concepts.

You will be issued with blank structured learning journals at the beginning of the semester and are expected to make recordings as you learn the mentioned Calculus 1 concepts. The Calculus tasks and interviews will be held at the end of the semester.

All the data collected will remain confidential and will be solely used for the purposes of this study. Your name will not be used on any of the instruments,

however you will be allocated with a code which you will be expected to remember and use consistently through out for the purposes of the research study. For thesis writing and reporting purposes I will use pseudonyms. However, your Mathematics and Statistics Coordinator in your Region will assist by maintaining a record of how you can be easily reached or contacted for the purposes of the study e.g. interviewing.

Where there are costs incurred and are directly related to the study (e.g. transport), you will be reimbursed. Please also be advised that at any time, if you feel like you'd prefer to pull out of the study I will fully understand your decision.

If you have any questions about the study please call me on 011 802613 (mobile) or 795990/2 (business).

Your cooperation is greatly appreciated.

Thank you very much

Yours sincerely



C. Tsvigu

(Chairperson; Programme Leader BSMS and Researcher)

## APPENDIX VI: LEARNING JOURNAL



# ZIMBABWE OPEN UNIVERSITY

As indicated in my introductory note to you, I hereby seek your cooperation and assistance with the completion of the attached “Learning Journals”. The data will contribute towards my PhD studies. If the space provided in the journals is not sufficient, please feel free to write on a separate sheet and be careful to indicate and label the entry.

Please complete the journals only in relation to the mentioned concepts which are covered in your MTD101, Calculus 1 module, under the following sections:

- **Sections 3.3 of Unit 3** titled ‘Limits’; i.e. subsections **3.3.1 to 3.3.7** only.
- **Sections 5.2--5.4 of Unit 5** titled ‘Differentiation’ i.e. sections **5.2 to 5.4** only.

Please always remember to put your code in the box provided on each journal entry sheet.

The data collected will be treated with strict confidentiality.

Thank You

C. Tsvigu

# LEARNING JOURNAL

## 1) WHAT IS A LEARNING JOURNAL?

A *Learning Journal* is a diary comprising of a learner's self-report and reflections of his/her learning processes.

## 2) PURPOSE OF A LEARNING JOURNAL

The purpose of the Learning Journal is twofold:

- The learning can help you as a learner to analyse, assess and reflect upon your own learning process and thus enhance your learning.
- The journal can also help me as a researcher to follow and evaluate students' learning processes, the actions and thoughts they engage in when learning and thus get feedback on how students learn.

The learning journal is your personal reflection of your learning processes and hence should be completed individually. It will be treated anonymously in my thesis.

## 3) MAKING JOURNAL ENTRIES

You are encouraged to make notes separately as you study given sections in your module and then to make entries into the journal once satisfied with your learning of each section.

Please Use -:

- Journal **L1** to make entries for **sections 3.3.1 to 3.3.3**
- Journal **L2** to make entries for **sections 3.3.4 to 3.3.7**
- Journal **D1** to make entries for **sections 5.2**
- Journal **D2** to make entries for **sections 5.3 to 5.4**

## 4) COLLECTION OF JOURNALS

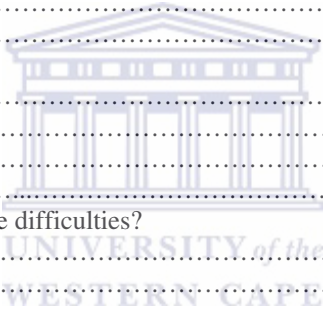
Your Regional Programme Coordinator will collect these journals on my behalf. The journals must be submitted by the weekend of 16<sup>th</sup> October 2005.

**OWN RECORDING IN A LEARNING JOURNAL**

enter student code here

**Journal Entry Number:** L1  
**Topic under Study:** LIMITS (sections 3.3.1 to 3.3.3).....  
**Concepts Under Study:** ..... **Date(s) of Entry:**.....

1. What do you think are the most important new things (concept, method, task type etc) that you have learnt in these section(s)?  
 .....  
 .....  
 .....
2. Describe the new things that you have learnt.  
 .....  
 .....  
 .....
3. Please give an example of a mathematical task that you think you are able to solve after working with this section(s) that you could not do before.  
 .....  
 .....  
 .....
4. What did you find difficult in these section(s)?  
 .....  
 .....  
 .....
5. Why did you find this difficult?  
 .....  
 .....  
 .....
6. How did you try to overcome the difficulties?  
 .....  
 .....  
 .....
7. What do you think helped you to learn the main ideas of the section(s) ?  
 (Please indicate your preference for choice of responses from those listed below.)



*Tick at most three from the following statements .Place your tick in the box on your right*

Item No.		Tick
1	Relating the concepts to previous sections/units of the course	
2	Working out each problem solution one step at a time	
3	Working out problems on my own	
4	Discussing with my colleagues	
5	Thinking quietly on my own	
6	Proving the theorems	
7	Relating the concepts to real life situations	
8	Use of graphs when finding limits	
9	Applying the definition of a limit of function	
10	Memorising the method of solution	
11	Using tabulated data to obtain the limit of functions	
12	Listening during the tutorials	
13	Linking the exercises to the theory	
14	Following the given procedure of solution method	
15	Explaining the concepts or tasks to someone else	
16	Making an overview of what I've learnt	



**OWN RECORDING IN A LEARNING JOURNAL**

enter student code here

**Journal Entry Number:** L2  
**Topic under Study:** LIMITS (sections 3.3.4 to 3.3.7).....  
**Concepts Under Study:** ..... **Date(s) of Entry:**.....

1. What do you think are the most important new things (concept, method, task type etc) that you have learnt in these section(s)?  
 .....  
 .....  
 .....
2. Describe the new things that you have learnt.  
 .....  
 .....
3. Please give an example of a mathematical task that you think you are able to solve after working with this section(s) that you could not do before.  
 .....  
 .....
4. What did you find difficult in these section(s)?  
 .....  
 .....
5. Why did you find this difficult?  
 .....  
 .....
6. How did you try to overcome the difficulties?  
 .....  
 .....
7. What do you think helped you to learn the main ideas of the section(s) ?  
 (Please indicate your preference for choice of responses from those listed below.)

*Tick at most three from the following statements .Place your tick in the box on your right*

Item No.		Tick
1	Relating the concepts to previous sections/units of the course	
2	Working out each problem solution one step at a time	
3	Working out problems on my own	
4	Discussing with my colleagues	
5	Thinking quietly on my own	
6	Proving the theorems	
7	Relating the concepts to real life situations	
8	Use of graphs when finding limits	
9	Applying the definition of a limit of function	
10	Memorising the method of solution	
11	Using tabulated data to obtain the limit of functions	
12	Listening during the tutorials	
13	Linking the exercises to the theory	
14	Following the given procedure of solution method	
15	Explaining the concepts or tasks to someone else	
16	Making an overview of what I've learnt	

**OWN RECORDING IN A LEARNING JOURNAL**

enter student code here

**Journal Entry Number:**

**D1**

**Topic under Study:**

**DERIVATIVES (section 5.2).....**

**Concepts Under Study:**

..... **Date(s) of Entry.....**

1. What do you think are the most important new things (concept, method, task type etc) that you have learnt in these section(s)?

.....

.....

.....

2. Describe the new things that you have learnt.

.....

.....

3. Please give an example of a mathematical task that you think you are able to solve after working with this section(s) that you could not do before.

.....

.....

4. What did you find difficult in these section(s)?

.....

.....

5. Why did you find this difficult?

.....

.....

6. How did you try to overcome the difficulties?

.....

.....

7. What do you think helped you to learn the main ideas of the section(s) ?

*(Please indicate your preference for choice of responses from those listed below.)*

*Tick at most three from the following statements. Place your tick in the box on your right*

Item No.		Tick
1	Relating the concepts to previous sections/units of the course	
2	Working out each problem solution one step at a time	
3	Using graphs to explain the concepts	
4	Working out problems on my own	
5	Discussing the problems with my colleagues	
6	Thinking quietly on my own	
7	Proving the theorems	
8	Relating the concepts to real life situations	
9	Applying the definition of a derivative	
10	Memorising the method of solution	
11	Practicing on several problems	
12	Listening during the tutorials	
13	Linking the exercises to the theory	
14	Following the procedures of solution method	
15	Explaining the concepts or tasks to someone else	
16	Making an overview of what I'd learnt	

**OWN RECORDING IN A LEARNING JOURNAL**

enter student code here

**Journal Entry Number:** D2  
**Topic under Study:** DERIVATIVE (sections 5.3 to 5.4).....  
**Concepts Under Study:** ..... **Date(s) of Entry:**.....

1. What do you think are the most important new things (concept, method, task type etc) that you have learnt in these section(s)?  
 .....  
 .....  
 .....
2. Describe the new things that you have learnt.  
 .....  
 .....  
 .....
3. Please give an example of a mathematical task that you think you are able to solve after working with this section(s) that you could not do before.  
 .....  
 .....  
 .....
4. What did you find difficult in these section(s)?  
 .....  
 .....  
 .....
5. Why did you find this difficult?  
 .....  
 .....  
 .....
6. How did you try to overcome the difficulties?  
 .....  
 .....  
 .....
7. What do you think helped you to learn the main ideas of the section(s) ?  
 (Please indicate your preference for choice of responses from those listed below.)

*Tick at most three from the following statements. Place your tick in the box on your right*

Item No.		Tick
1	Relating the concepts to previous sections/units of the course	
2	Working out each problem solution one step at a time	
3	Using graphs to explain the concepts	
4	Working out problems on my own	
5	Discussing the problems with my colleagues	
6	Thinking quietly on my own	
7	Proving the theorems	
8	Relating the concepts to real life situations	
9	Applying the definition of a derivative	
10	Memorising the method of solution	
11	Practicing on several problems	
12	Listening during the tutorials	
13	Linking the exercises to the theory	
14	Following the procedures of solution method	
15	Explaining the concepts or tasks to someone else	
16	Making an overview of what I'd learnt	

## APPENDIX VII: CALCULUS TASKS TEST

enter your allocated code here

### TEST ITEMS for STUDENTS UNDERSTANDING

Once again thank you for agreeing to be part of this study.

The information you provide will be treated confidentially and will be used only for the purposes of this research. It will **not be used** for any assessment other than for the research purposes.

This questionnaire comprises of Test Items that seek to elicit your understandings of the Limit of a Function and Derivative of a Function concepts. It comprises of section A and section B.

**Section A** comprises of questions on the Limit of a Function Concept

**Section B** comprises of questions on the Derivative of a Function Concept.

WESTERN CAPE

**SECTION A**

**LIMIT OF A FUNCTION**

**L1.**

a) Please describe in a few sentences, what you understand a limit of a function to be. In other words, describe what it means (for you) to say that the limit of a function  $f(x)$  as  $x$  approaches  $a$  is the number  $L$ ?

.....

.....

.....

.....

b) If possible, write down the formal definition of limit of a function  $f(x)$  as  $x$  approaches  $a$ .

.....

.....

.....

.....

**L2.** Determine  $\lim_{x \rightarrow 0} \frac{\sin x}{x}$  if it exists.

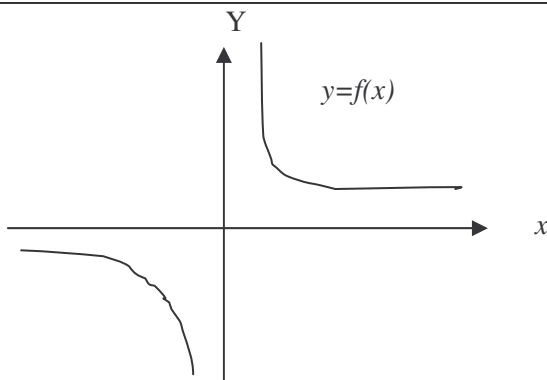
**SOLUTION:**



**EXPLAIN HOW YOU OBTAINED THE ANSWER:**

UNIVERSITY of the  
WESTERN CAPE

**L3**



Determine  $\lim_{x \rightarrow \infty} f(x)$  where  $f(x)$  is as shown in the graph above.

**SOLUTION:**

WHY DO YOU THINK THIS IS SO?

**L4**

A student was given a function  $F$  and asked to find the limit of  $F$  as  $x$  approaches 0. He plugged in numbers on each side of zero and made the following table.

What can you conclude about the limit of the function  $F$  as  $x$  approaches 0?

$x$	$F(x)$
-0.1	0.9
-0.01	0.99
-0.001	0.999
-0.0001	0.9999
-0.00001	0.99999
.....	.....
0.00001	1.00001
0.0001	1.0001
0.001	1.001
0.01	1.01

**SOLUTION:**

**EXPLAIN YOUR ANSWER:**

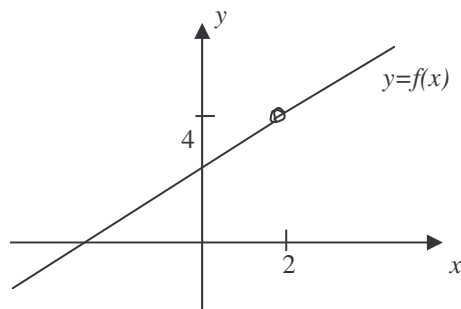
**L5.** Obtain the following limit  $\lim_{x \rightarrow \infty} \frac{1}{x}$

**SOLUTION:**



**WHY DO YOU THINK THIS IS SO?**

**L6.**



Find  $\lim_{x \rightarrow 2} f(x)$

**SOLUTION:**

**WHY DO YOU THINK THIS IS SO?**

L7. Find  $\lim_{x \rightarrow \infty} \frac{(x + 3)(2x - 1)}{x^2}$

**SOLUTION:**

**WHY DO YOU THINK THIS IS SO?**

L8. Determine  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$

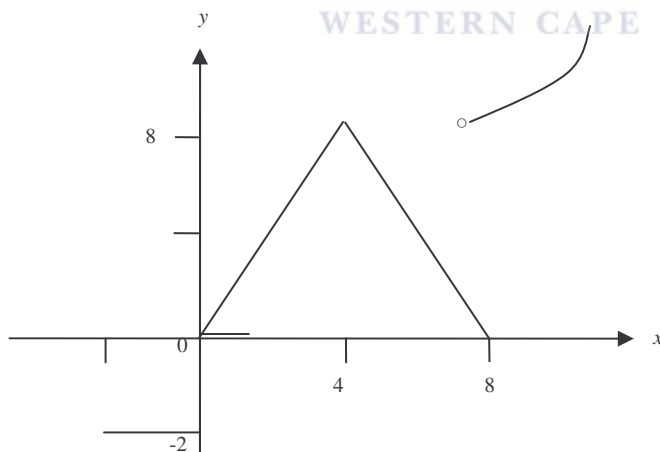
**SOLUTION:**

**WHY DO YOU THINK THIS IS SO?**



UNIVERSITY of the  
WESTERN CAPE

L9.



Answer the following questions using the above

a) Find  $\lim_{x \rightarrow 4} f(x)$

**a) SOLUTION**

**EXPLAIN WHY THIS IS SO?**

	<p>b) <math>\lim_{x \rightarrow \infty} f(x)</math></p>	<p><b><u>b) SOLUTION</u></b></p>   <p><b><u>EXPLAIN WHY THIS IS SO?</u></b></p>
	<p>c) <math>\lim_{x \rightarrow 0} f(x)</math></p>	<p><b><u>c) SOLUTION</u></b></p>   <p><b><u>EXPLAIN WHY THIS IS SO?</u></b></p>



**L10.** Consider the function  $f(x)$  given as follows  $f(x) = \begin{cases} 1-x & \text{for } x \leq 1 \\ x^2 & \text{for } x > 1 \end{cases}$

- a) Determine  $\lim_{x \rightarrow 1} f(x)$  if it exists.
- b) Determine  $\lim_{x \rightarrow 0} f(x)$  if it exists.

**SOLUTIONS (Provide explanations in each case)**

a)

b)



**SECTION B**

**DERIVATIVE OF A FUNCTION**

**D1.**

a) Please describe in a few sentences, what you understand a derivative of a function to be. In other words, describe what it means (for you) to say that the derivative of  $f(x)$  at point  $a$  is  $g(x)$

.....

.....

.....

.....

.....

b) If possible write down the formal definition of derivative of a function  $f(x)$  at point  $x = a$ .

.....

.....

.....

**D2.**

a) Find  $f'(x)$  for  $f(x)=3x+8$  at  $x_0 = 2$

b) Find  $f'(x)$  for  $f(x)=9x^2$  at  $x_0 = 3$

c) Find  $f'(x)$  for  $f(x)=x^3+8$  at  $x_0 = 2$

**EXPLAIN HOW YOU OBTAIN THE ANSWERS.**

a)

b)

c)

**D3.**

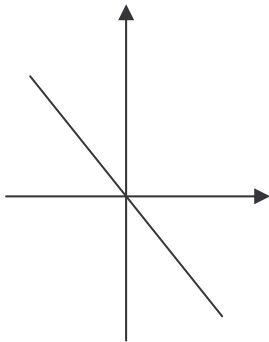
Let some values of  $f(t)$ , be given as in the table. Estimate  $f'(1.5)$

$t$	$f(t)$
-1	-1
0	0
1	1
2	8
3	27

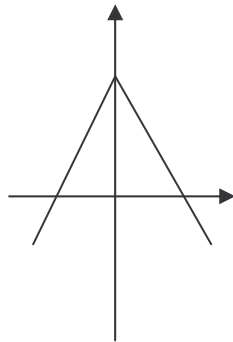
**SOLUTION**

**Explain how you obtained the solution.**

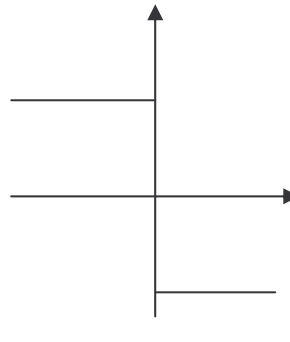
**D4.** Given below are two functions and their derivative functions. Match each function with its derivative function.



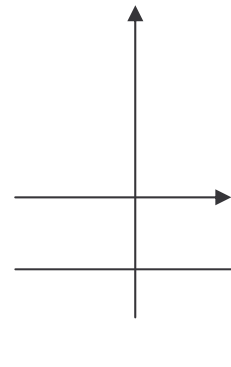
i) function  $f(x)$



ii) function  $g(x)$



iii) function  $h(x)$



iv) function  $p(x)$

- a) Indicate the pairs, i.e. show which is the function and which is the derivative function.  
 b) Explain your solutions in each case.

**SOLUTIONS:**

--Pair 1

Explain why this is so.

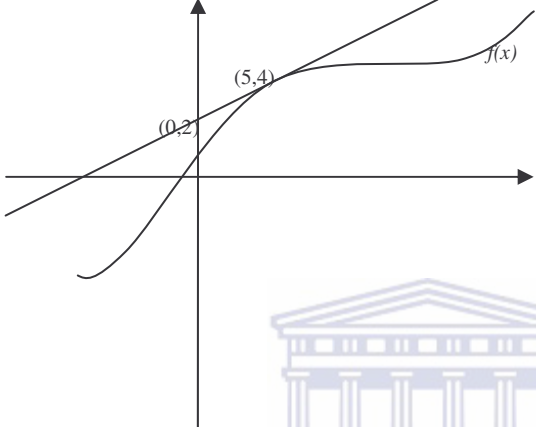


--Pair 2

UNIVERSITY of the  
WESTERN CAPE

Explain why this is so.

<p><b>D5.</b></p>	<p>The number of bacteria in a culture, <math>N</math>, was found to depend on time <math>t</math> in seconds, since it was first cultured. If <math>N=t^{1.8}</math>, find the instantaneous rate of increase in the number of bacteria after 3 seconds</p>	<p><b><u>EXPLAIN WHY IS THIS SO?</u></b></p>
-------------------	--	--

<p><b>D6.</b></p>	<p>Describe the relationship between the derivative of a function <math>g</math> at point <math>x=a</math> and the slope of the tangent line to the graph of the function where <math>x=a</math>.</p>	<p><b>SOLUTION:</b></p>
<p><b>D7.</b></p>	<div style="text-align: center;">  <p>The graph shows a curve <math>f(x)</math> and a straight line tangent to it at the point <math>(5, 4)</math>. The line also passes through the point <math>(0, 2)</math> on the y-axis. The x and y axes are shown with arrows.</p> </div> <p>a) Determine <math>f(5)</math></p> <p>b) Determine <math>f'(5)</math></p>	<p>a) <b><u>SOLUTION:</u></b></p> <p><b>WHY IS THIS SO?</b></p> <p>b) <b><u>SOLUTION:</u></b></p> <p><b>WHY IS THIS SO?</b></p>



## APPENDIX VIII: LSPQ -LIMIT OF FUNCTION

### LEARNING STYLES PREFERENCE QUESTIONNAIRE

#### (PART 1: Limit of a function representation preferences)

This questionnaire seeks to elicit your preferred representations with respect to the limit of a function.

**Section A.** seeks to obtain your demographic data.

**Section B** comprises of questions on the Limit of a Function Concept.

#### SECTION A

Gender: Male/Female *(Please tick appropriate)*

What is your age (to the nearest whole year)? *(Please tick one)*

< 20	
21-25	
26-30	
31-35	
36-40	
>40	

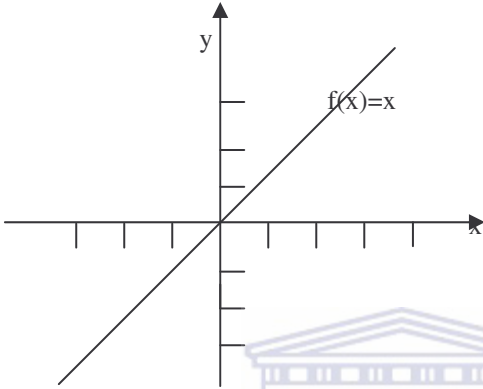
What is your profession? .....

Where do you stay? Urban/Rural/Peri-Urban *(Please tick one)*

## SECTION B:

## Limit of a Function

L1. Consider the following limit  $\lim_{x \rightarrow 3} x$

Item No.		Solution								
(i)	Find $\lim_{x \rightarrow 3} x$									
(ii)	Use the graph to find $\lim_{x \rightarrow 3} x$ 									
(iii)	Use of the tabular approach <table border="1" data-bbox="363 1070 831 1328"> <thead> <tr> <th>x</th> <th>f(x)=x</th> </tr> </thead> <tbody> <tr> <td> </td> <td> </td> </tr> <tr> <td> </td> <td> </td> </tr> <tr> <td> </td> <td> </td> </tr> </tbody> </table>	x	f(x)=x							
x	f(x)=x									
(iv)	Use the Definition									

A) To best explain the solution of this limit problem I would use:

(i)                      (ii)                      (iii)                      (iv)                      *(Please circle one)*

B) Please rank the above representations in the order of your preferences.

Start with the most preferred to the least preferred.

Most Preferred			Least Preferred

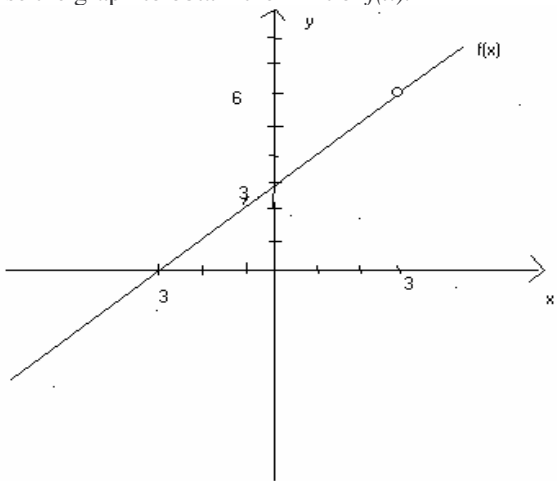
C) The following are possible justifications for your **most preferred** choice. Please indicate your choice of justification on a scale **Strongly Agree, SA; Agree, A; Disagree, DA; and Strongly Disagree; SD:**

	SA	A	DA	SD
The meaning of limit of the function is clear to me				
This is how I learnt the concept				
I can understand the concept fully				
I can easily explain this concept to someone else using this form				
I can easily master the idea of limit of a function from this form				

D) The following are possible justifications for your **least preferred** choice, please indicate your choice of justification on a scale **Strongly Agree, SA; Agree, A; Disagree, DA; and Strongly Disagree; SD:**

	SA	A	DA	SD
I can not link this to the limit of a function				
It means nothing to me				
I can not use this to obtain the limit of a function				
The idea of limit of a function can not be easily mastered				

L2. Consider the following limit  $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$

Item		Solution
(i)	Obtain $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$	
(ii)	Use the graph to obtain the limit of $f(x)$ . 	

(iii)	Use tabular approach	<table border="1"> <tr> <td>x</td> <td>f(x)</td> </tr> <tr> <td> </td> <td> </td> </tr> <tr> <td> </td> <td> </td> </tr> <tr> <td> </td> <td> </td> </tr> <tr> <td> </td> <td> </td> </tr> </table>	x	f(x)									
		x	f(x)										
(iv)	Use the definition												

A) To best explain the solution of this limit problem I would use:

- (i)                      (ii)                      (iii)                      (iv)                      *(Please circle one)*

B) Please rank the above representations in the order of your preferences.

Start with the most preferred to the least preferred.

Most Preferred				Least Preferred

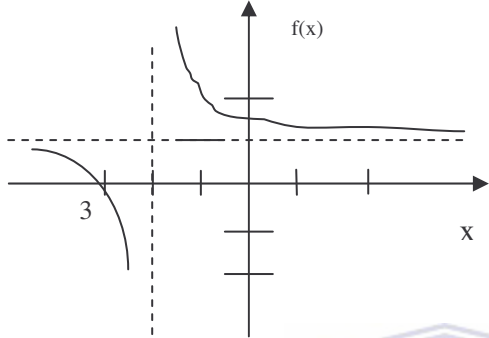
C) The following are possible justifications for your **most preferred** choice, please indicate your choice of justification on scale **Strongly Agree, SA; Agree, A; Disagree, DA; and Strongly Disagree; SD:**

	SA	A	DA	SD
The meaning of limit of this function is clear to me				
This is how I learnt the concept				
I can understand the concept fully				
I can easily explain this concept to someone else using this form				
I can easily master the idea of limit of a function from this form				

D) The following are possible justifications for your **least preferred** choice, please indicate your choice of justification on a scale **Strongly Agree, SA; Agree, A; Disagree, DA; and Strongly Disagree; SD:**

	SA	A	DA	SD
I can not link this to the limit of a function				
It means nothing to me				
I can not use this to obtain the limit of a function				
The idea of limit of a function can not be easily mastered				

L3. Consider the limit  $\lim_{x \rightarrow 0} \left( 1 + \frac{1}{x + 2} \right)$

Item. L3		Solution																
(i)	Obtain $\lim_{x \rightarrow 0} \left( 1 + \frac{1}{x + 2} \right)$																	
(ii)																		
(iii)	Use tabular approach <table border="1" data-bbox="403 981 890 1211"> <thead> <tr> <th>x</th> <th>f(x)</th> </tr> </thead> <tbody> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> </tbody> </table>	x	f(x)															
x	f(x)																	
(iv)	Use the definition																	

A) To best explain the solution of this limit problem I would use:

- (i)                      (ii)                      (iii)                      (iv)                      *(Please circle one)*

B) Please rank the above representations in the order of your preferences.

Start with the most preferred to the least preferred.

Most Preferred			Least Preferred

C) The following are possible justifications for your **most preferred** choice, please indicate your choice of justification on a scale **Strongly Agree, SA; Agree, A; Disagree, DA; and Strongly Disagree;**

**SD:**



	SA	A	DA	SD
The meaning of limit of the function is clear to me				
This is how I learnt the concept				
I can understand the concept fully				
I can easily explain this concept to someone else using this form				
I can easily master the idea of limit of a function from this form				

D) The following are possible justifications for your **least preferred** choice, please indicate your choice of justification on a scale **Strongly Agree, SA; Agree, A; Disagree, DA; and Strongly Disagree, SD:**

	SA	A	DA	SD
I can not link this to the limit of a function				
It means nothing to me				
I can not use this to obtain the limit of a function				
The idea of limit of a function can not be easily mastered				



## APPENDIX IX: LSPQ - DERIVATIVE OF FUNCTION

### LEARNING STYLES PREFERENCE QUESTIONNAIRE

#### (PART B: Derivative of a Function representation preference)

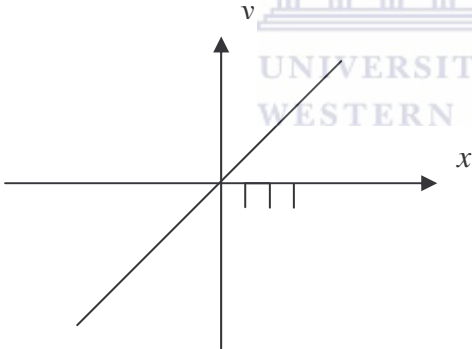
Thank you for agreeing to take part in my study.

The information that you provide will be treated confidentially.

This questionnaire seeks to elicit your preferred representations with respect to concepts regarding the derivative of a function.

#### SECTION A Derivative of a Function

D1. Consider the function  $f(x)=x$

Item		Solution												
(i)	Find the derivative of $f(x)=x$ , at $x=3$ .													
(ii)	Use the graph of $f(x)=x$ , to find the derivative of $f$ at $x=3$ . 													
(iii)	Use the tabular approach <table border="1" data-bbox="370 1581 821 1800"> <thead> <tr> <th><math>x_0</math></th> <th><math>h</math></th> <th><math>\frac{f(x_0 + h) - f(x_0)}{h}</math></th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </tbody> </table>	$x_0$	$h$	$\frac{f(x_0 + h) - f(x_0)}{h}$										
$x_0$	$h$	$\frac{f(x_0 + h) - f(x_0)}{h}$												
(iv)	Use the definition													

A) To best describe  $f'(x_0)$ , I would use:

- (i)                      (ii)                      (iii)                      (iv)                      *(Please circle one)*

B) Please rank the above representations in the order of your preferences. Start with the most preferred to the least preferred.

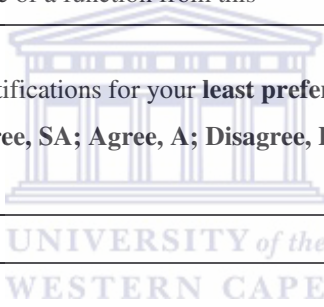
Most Preferred			Least Preferred

C) The following are possible justifications for your **most preferred** choice, please indicate your choice of justification on a scale **Strongly Agree, SA; Agree, A; Disagree, DA; and Strongly Disagree; SD:**

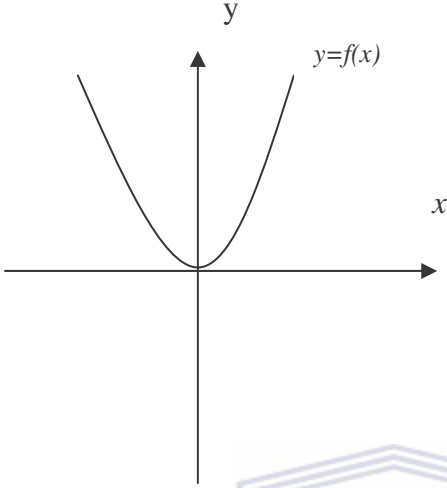
	SA	A	DA	SD
The meaning of derivative of the function is clear to me				
This is how I learnt the concept, though I don't understand it				
This is how I learnt it and I understand the concept fully				
I can easily explain this concept to someone else using this form				
I can easily master the idea of derivative of a function from this				

D) The following are possible justifications for your **least preferred** choice, please indicate your choice of justification on a scale **Strongly Agree, SA; Agree, A; Disagree, DA; and Strongly Disagree; SD:**

	SA	A	DA	SD
It though it means nothing to me				
I can not link this to the derivative of a function				
I can not use this to calculate the derivative of a function				
The idea of derivative of a function can not be easily mastered				



. D2 Consider the function  $f(x)=x^2$

Item		Solution																								
(i)	Determine $f'(x)$ at $x_0=2$																									
(ii)	Use the graphical approach. 																									
(iii)	Use the tabular approach <table border="1" data-bbox="371 1099 823 1379"> <thead> <tr> <th><math>x_0</math></th> <th><math>h</math></th> <th><math>\frac{f(x_0 + h) - f(x_0)}{h}</math></th> </tr> </thead> <tbody> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> </tbody> </table>	$x_0$	$h$	$\frac{f(x_0 + h) - f(x_0)}{h}$																						
$x_0$	$h$	$\frac{f(x_0 + h) - f(x_0)}{h}$																								
(iv)	Use the definition																									

A) To best describe  $f'(x_0)$ , for this function I would use:

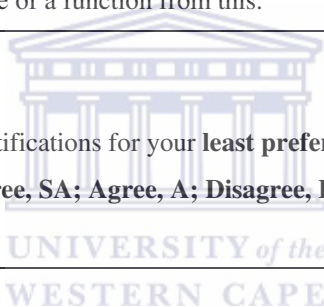
- (i)                      (ii)                      (iii)                      (iv)                      *(Please circle one)*

B) Please rank the above representations in the order of your preferences. Start with the most preferred to the least preferred.

Most Preferred			Least Preferred

C) The following are possible justifications for your **most preferred** choice, please indicate your choice of justification on a scale **Strongly Agree, SA; Agree, A; Disagree, DA; and Strongly Disagree; SD:**

	SA	A	DA	SD
The meaning of derivative of the function is clear to me.				
This is how I learnt the concept, though I don't understand it.				
This is how I learnt it and I understand the concept fully.				
I can easily explain this concept to someone else using this form.				
I can easily master the idea of derivative of a function from this.				



D) The following are possible justifications for your **least preferred** choice, please indicate your choice of justification on a scale **Strongly Agree, SA; Agree, A; Disagree, DA; and Strongly Disagree; SD:**

	SA	A	DA	SD
It means nothing to me.				
I can not link this to the derivative of a function.				
I can not use this to calculate the derivative of a function.				
The idea of derivative of a function cannot be easily mastered.				

**D3.**

Water is flowing into a tank at a constant rate, such that, for each unit increase in the time  $t$ , the depth ( $y$ ) of the water increases by 2 units. Determine the rate of increase in the depth of the water when  $t=2\frac{1}{2}$

Item		Solution
(i)	Use the analytical method	
(ii)	Use the graphical approach	
(iii)	Use the tabular approach	
(iv)	Use the definition	

**A)** To solve the above problem I would use:

- (i)                      (ii)                      (iii)                      (iv)                      *(Please circle one)*

**B)** Please rank the above representations in the order of your preferences. Start with the most preferred to the least preferred.

Most Preferred			Least Preferred

**C)** The following are possible justifications for your **most preferred** choice, please indicate your choice of justification on a scale **Strongly Agree, SA; Agree, A; Disagree, DA; and Strongly Disagree; SD:**

	SA	A	DA	SD
The meaning of derivative of a function is clear to me.				
This is how I learnt the concept, though I don't understand it.				
This is how I learnt it and I understand the concept fully.				
I can easily explain this concept to someone else using this form.				
I can easily master the idea of derivative of a function from this.				

**D)** The following are possible justifications for your **least preferred** choice; please indicate your choice of justification on scale **Strongly Agree, SA; Agree, A; Disagree, DA; and Strongly Disagree; SD:**

	SA	A	DA	SD
It means nothing to me.				
I can not link this to the derivative of a function.				
I can not use this to calculate the derivative of a function.				
The idea of derivative of a function cannot be easily mastered.				



## APPENDIX X: INTERVIEW GUIDE

### LEARNING STYLE PREFERENCES INTERVIEW SCHEDULE

#### Semi Structured Questions

Date of Interview: ..... Venue of Interview: .....

Interview Starting Time:.....Finishing Time:.....(estimated time 40-60 mins)

This (learning style) interview guide comprises of 4 Parts: Section A, B, C and D.

<b>SECTION A:</b>	General Learning Style Preferences
<b>SECTION B:</b>	Learning Styles in Context: Calculus Learning at ZOU
<b>SECTION C:</b>	Learning Journal (LJ) Responses
<b>SECTION D:</b>	Learning Style Preferences Questionnaire (LSPQ) in Calculus responses
<b>SECTION E</b>	Calculus Test Tasks (CTT) Items

---

#### **REMINDERS to Interviewer:**

1. Before the interview, interviewer must remember to:
  - identify the person to be interviewed;
  - arrange an appointment for interview and agree on venue and time;
  - remember to read through and remind oneself on the student's LJ responses LSPQ responses, & Calculus Tasks Test responses, then identify areas which need further probing;

2. **Interviewer:**
  - Introduce oneself
  - Request for interviewee's permission to use tape recorder during the interview

3. **Interviewer:** Remember to thank the interviewee, e.g. say  
*Thank you very much for agreeing to be part of my research study and for responding to the instruments that I have issued so far. I also want to thank you for allowing me this opportunity to interview you.*

4. **Interviewer must: OPENING STATEMENT**

- Explain why the respondent has been chosen for interviews eg. *I found that you have some interesting responses in the instruments that you have filled in so far and I felt that a follow up face-to-face interview with you would enrich my study.*
- Inform participant about purpose of interview;
- Explain to the interviewee, who the information is for;



- Guarantee confidentiality and anonymity to the respondent;
- Explain how the data will be used;
- Make it clear that if the student does not clearly understand the question he is free to ask for clarity.

**5. Interviewer:** Remember to probe and stimulate the interviewee to explain and clarify everything related to their styles of learning.

**NOTE:** Interviewer remember that some LS attributes could be elicited (could emerge) anywhere throughout the interview though the specific question are designed to seek out some other attributes of the student's best way of learning something.



## **SECTION A: GENERAL LEARNING STYLE PREFERENCES**

### **A1: Perception Dimension (Sensing/Intuitive)**

1. In the instruments (LJ, LSPQ, CTT) that you have filled in so far, we have been dealing with issues pertaining to the learning of some calculus concepts, i.e. Limit and Derivative of Function concepts. Could you please tell me briefly:
  - What aspects of the topics limit of function and derivative of function have you enjoyed/liked most? Please elaborate on why this is so? (*probe further on reason*).
  - What aspects of the topics that are under study (limit and derivative of function) that you didn't enjoy/disliked most? Please elaborate on why this is so? (*probe further on reason*).
2. Probe if it's not explicit: What do you think about the definition of limit of function/derivative of function? How do you feel about using the  $\epsilon$ - $\delta$  definition of limit of function? How about using analytical methods?
3. Please give me your opinion about such things like theorems and their proofs?
4. What do you think is more important to you in the learning of calculus: knowing how to use the symbols/formulae or deriving and proving formulae/theorems?
5. When studying calculus, how do you know that you have fully mastered or understood a topic? (*if need use reference to limit and derivative of function concepts*).
6. How do you feel when you make mistakes in problem solving? What do you do when you make mistakes when learning some new concept? (*Probe on redoing!!*)
7. What do you feel about memorising things when learning calculus? (*probe on whether student relies on memorising and what kind of information he prefers to memorise?*)
8. If you were given an opportunity to learn the limit of a function and derivative of a function concepts for the first time, at the ZOU how would you want the information to be presented to you so that you best learn these concepts? (*probe: what do you think would help you to learn better?*)

### **A2: Input Dimension (Visual/Verbal)**

1. Please share with me on what makes it really difficult for you to learn when you experience difficulties?
2. Do you understand a concept easier if you first hear about it in a lecture like scenario? Or if you Read about it or if you See something pictorial/graphical about it? Or only if you use the idea yourself e.g. working out a problem? Why is that so?
3. Do you skip graphs, charts pictures or diagrams in reading materials? Why?
4. Do you tend to avoid reading if you can get the same information in another way eg lecture, tape or video.
5. How do you feel about learning in a 'lecture like' tutorial session? Why is that?)

### **A3: Processing Dimension (Active/Reflective)**

1. What would you do to figure out how to remember some concepts in calculus?
  - First watch someone apply the theory and then later on try to follow on what he/she did?

- First read about the theory on your own and then try to solve the problem out on your own?
  - Get someone to discuss with and work on the problem together?
2. How do you learn best? Do you like to study by yourself (alone) or to be part of a group with other people? Why is this so? Can you elaborate further please?
  3. What do you do if you don't understand what you have just read in the module? (*probe further on what he/she does exactly*)
  4. What is your opinion about tutorial sessions? (*probe on the role of tutorials in the student's learning routine, does the student find tutorials useful to him/her, explain why?*) What role do tutorials play in your learning?
  5. Can you describe to me what you consider a good tutorial session should be like in your own opinion?
  6. What in your opinion should be the role of a tutor during a tutorial?
  7. If I were to be in the same tutorial group with you, what would I see you doing? Please share with me what is your role in a tutorial session? (*probe: explain what do you do during a tutorial session? Is this by choice*)
  8. What is your opinion about group work/being part of a study group where discussions and group interactivity are taking place?
  9. If I were to be part of your study group, what would I see you doing? Can you please share with me on what your role would be (*what kinds of things would you do*) if you were to be part of a group setting? (*probe on what exactly the student does*)
  10. Have you ever been involved in one such group setting for your calculus studying? If yes how frequent? What prompts the meetings? Where?
  11. If you encounter difficult parts, what do you do with difficult parts? (*This question can be linked to LJ entry e.g. I notice you have indicated in your Learning Journal entry that to overcome difficult parts pertaining to Limit of Function/Derivative of Function you normally.....* (interviewer describe what student wrote). *Can you briefly explain to me how really this contributed to your overcoming the difficulty? (probe into what it is exactly that helped the student overcome difficulties)?*

**A4: Understanding Dimension (Sequential/Global)**

1. What do you need to do in order to remember something in calculus?
2. Please enlighten me about how you go about using the module and other learning resources such as worksheets, assignments etc. (*Probe on: Do you start from the first page of the unit then move page by page or do you jump in between the sections? Why that preference?*)
3. Refer to attached paper: [Suppose you were asked by a colleague to show him/her why for the function  $f(x)$  given as  $f(x) = \dots$ , we have  $f'(x) = \dots$ . How would you go about explaining/proving this to him/her.]
4. Please talk to me about your opinion regarding the following sections of a ZOU module "What this Unit is about" section, and the Objectives i.e. "by the end of this Unit you should be able to" (*Probe: do you find the sections useful for your learning, explain why in either case*)
5. What do you think, would you find it difficult to use module without these sections? Please explain.
6. How often do you want to relate/link new information learnt to information that you learnt before? Please elaborate. (*Probe: For example what is the link between the limit of function and derivative of a function concepts.*)

**SECTION B:      LEARNING STYLES IN CONTEXT: Calculus  
Learning at ZOU**

- B1. Please share with me your best study strategy in calculus? (*Probe: the strategy that gives you maximum benefit? What do you do to best remember and use the information you are learning? What do you do in order for you to gain understanding of a concept? How did you learn the calculus course material so that it was easy for you to remember and use the information learnt?*)
- B2. What do you think are the most challenging factors (in terms of learning/academic factors) when it comes to distance learning in Mathematics at the Zimbabwe Open University. Please elaborate more on your response. (*Probe also for any frustrations, disappointments etc and what caused the frustrations and disappointments? Elicit more information about learning resources package i.e. module, tutorial worksheets, tutorials, assignments and their feedback if information is not explicitly coming out*)
- B3. What are the things in which you derived most benefit/ with distance learning at the Zimbabwe Open University? (*Elicit/Probe more information about learning resources package i.e. module, tutorials, tutorial worksheets, feedback assignments and tutorial if it's not explicitly coming out*)
- B4. Suppose you have a colleague who is coming in as a new student into the ZOU (BSMS) programme and has to learn these calculus concepts for the first time, what would be your advice to him/her on how he/she should go about learning these concepts so as to gain maximum benefit?
- B5. You have given a lot of info about BSMS. If you had the power to change things in the BSMS What things/areas would you propose for improvement (changes) in the BSMS, calculus course? How would you improve Calculus learning at the ZOU?

**SECTION C:      LEARNING JOURNAL INSTRUMENT RESPONSES**

**Interviewer:** Purpose of this section is to seek further clarification from journal entries. E.g. say: *In this section of the interview we are going to consider some of the responses which you gave in your learning journal entries, (i.e. the instrument that you received at the beginning of the semester), I would like to understand more about some of your responses:*

- C. In your learning journal you indicated that.....(*refer to what student said*) Can you explain to me what you mean when you say that.....

**SECTION D:      LEARNING STYLE PREFERENCE  
QUESTIONNAIRE (LSPQ) IN CALCULUS**

The purpose of this section is to seek further clarification on student's responses in the LSPQ.

The student should have filled in the Learning Styles Preference Questionnaire(s) (LSPQ), before the interview (preferably during the semester when learning process is still taking place since its content based). During the interview the researcher asks the student questions based on the student's responses to the preference items, especially their choices of

preferences on the four options available. Questions are mainly meant to probe the student further into explaining his/her preference choices.

**D1. LSPQ: LIMIT of FUNCTION**

- (a) I have noticed in your responses for the LSPQ for Limit of Function that you indicated.....(*highlight the choice from the student's responses*) as your most preferred choice of form for solving the given problems.
  - Can you please explain to me why you find this most preferable?
  - What do you mean?
- (b) You have also indicated that your least preferred choice for solving the given problems in Limit of Function is choice.....( *highlight the choice from the student's responses*).
  - Can you also please explain to me why you find this to be your least preferred choice?
  - What do you mean?
- (c) How do you feel about using the other available forms of representation for solving the Limit of Function problems? Would it be possible for you to use these in solving problems?
- (d) (Question asked in the event that the solution to the most preferred solution is incorrect) I would like you to have a re-look at the solution which you provided for this question, i.e. question ..... in the LSPQ that corresponds to your most preferred solution for this problem. Would you like to explain to me how you obtained this answer? (*probe further if necessary*)

**D2. LSPQ: DERIVATIVE of FUNCTION**

- (a) I have noticed in your responses for the LSPQ for DERIVATIVE of Function that you indicated.....( *highlight the choice from the student's responses*) as your most preferred choice of mode of representation for solving the given problems.
  - Can you please explain to me why you find this most preferable?
  - What do you mean?
- (b) You have also indicated that your least preferred choice for solving the given problems in Derivative of Function is choice.....( *highlight the choice from the student's responses*).
  - Can you also please explain to me why you find this to be your least preferred choice?
  - What do you mean?
- (c) How do you feel about using the other available forms of representation for solving the Derivative of Function problems? Would it be possible for you to use these in solving this problem?

- (d) Question asked in the event that the solution to the most preferred solution is incorrect) I would like you to have a re-look at the solution which you provided for question ..... in the LSPQ that corresponds to your most preferred solution for this problem. Would you like to explain to me how you obtained this answer? (*probe further if necessary*)

### **D3. FORMS of REPRESENTATION Preferences**

- (a) How do you feel about using tabulated data for limit of function/derivative of function? (*probe further*)
- (b) How do you feel about using the function graphs to obtain limit of function/derivative of a function? (*probe further*)
- (c) If for the first encounter you are given to use the fact that the first derivative  $f'(x)$  for  $f(x)=x^n$  is  $f'(x)=nx^{n-1}$ . Would you rather want to know how this comes about or would you rather just to apply it on a problem. What do you feel about understanding the definitions, the theorems and deriving the formulas from first principles? How about using the definitions? (*probe further*)
- (d) How do you feel about using the procedures and formulas without worrying about how they come to be? (*probe further*)

### **SECTION E: CALCULUS TEST TASKS**

(If there is any response of interest to learning style profiling that needs further clarification)

#### **ENDING NOTE**

You have been very helpful by responding to my questions. I'd be interested in any other ideas, thoughts and feelings about the Calculus course in the BSMS programme that you'd like to share with me, or any questions you'd like to ask me.

**THANK YOU VERY MUCH**

**APPENDIX XI: DATA SETS ON CD IN PDF FORMAT**

