EFFICIENT MONTE CARLO METHODS FOR PRICING OF ELECTRICITY DERIVATIVES

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KEYWORDS

Electricity derivatives

Forwards and futures contracts

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Monte Carlo Methods

Options

Risk management

Spreads and Swaps

Variance reduction techniques

ABSTRACT

We discuss efficient Monte Carlo methods for pricing of electricity derivatives. Electricity derivatives are risk management tools used in deregulated electricity markets. In the past, research in electricity derivatives has been dedicated in the modelling of the behaviour of electricity spot prices. Some researchers have used the geometric Brownian motion and the Black Scholes formula to offer a closed-form solution. Electricity spot prices however have unique characteristics such as mean-reverting, non-storability and spikes that render the use of geometric Brownian motion inadequate. Geometric Brownian motion assumes that changes of the underlying asset are continuous and electricity spikes are far from being continuous. Recently there is a greater consensus on the use of Mean-Reverting Jump-Diffusion (MRJD) process to describe the evolution of electricity spot prices. In this thesis, we use Mean-Reverting Jump-Diffusion process to model the evolution of electricity spot prices. Since there is no closed-form technique to price these derivatives when the underlying electricity spot price is assumed to follow MRJD, we use Monte Carlo methods to value electricity forward contracts. We present variance reduction techniques that improve the accuracy of the Monte Carlo Method for pricing electricity derivatives.

DECLARATION

I declare that **Efficient Monte Carlo Methods for Pricing of Electricity Derivatives** is my own work, that it has not been submitted for any degree or examination in any other university, and that all the sources I have quoted have been indicated and acknowledged by complete reference.

Linda Nobaza October 2012

Signed………………………

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LIST OF ACRONYMS

Btu	British thermal units
FERC	Federal Energy Regulatory Commission
FGR	Flowgate Rights
FTR	Financial Transmission Rights
GBM	Geometric Brownian Motion
IPP	Independent Power Producers
ISO	Independent Systems Operator
KWH	Kilo-Watts per Hour
LMP	Location Market Price
LSE	Load Serving Entity
MMBtu	Million British thermal units
MRJD	Mean-Reverting Jump-Diffusion
MW	Mega Watt
MWh	Mega Watt hour
OTC	Over-the-Counter
PDE	Partial Differential Equation
SDE	Stochastic Differential Equation
SMD	Standard Market Design

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LIST OF PUBLICATIONS

Following technical report is generated out of this thesis whose revised form will be submitted as a research paper for publication to an international journal:

- Linda Nobaza and Kailash C. Patidar, Improved Monte Carlo techniques to price electricity derivatives, Report Nr. UWC-MRR 2012/15, University of the Western Cape, South Africa, 2012.

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Chapter 1

GENERAL INTRODUCTION

Deregulation of the electricity industry is sweeping through Europe and North America. Deng and Oren [28] in countries such as the US, UK, Germany, Spanish and Scandinavian countries, electricity industry is deregulated. There is typically more than one player throughout the electricity value chain, namely, generation, transmission and distribution. With competition, electricity prices fluctuate according to supply and demand interactions. This fluctuation of electricity prices poses risk to participants in the industry. For example, power generating companies are not guaranteed to receive electricity unit prices that are above the cost of generating one unit of electricity as it is the case in regulated electricity markets. Therefore, participants in a deregulated electricity industry, particularly large participants, have incentive to exercise risk management.

Electricity derivatives have been introduced in deregulated electricity markets precisely for risk management. These derivatives are largely adopted from known financial derivatives. There are exchanges where these derivatives are traded including exchanges for spot market.

The pricing of electricity derivatives has been the focus of research over the past three decades. Most of the research has been dedicated to modelling of electricity spot prices in order to accurately price these derivatives. Recent attempts are found for example in [8], [54] and [69]. This research is further divided to statistical models and fundamental models. In [69] for example, this fundamental approach was used while others use a statistical approach.

The statistical approach attempts to model the electricity spot price process directly. This approach depends on a set of parameters that describe the properties of the process of the underlying electricity spot price while fundamental electricity price models are based on competitive equilibrium models for the electricity market. In fundamental models, electricity prices are obtained from a model for the expected production costs of electricity and expected consumption of electricity [75].

In this thesis we use the statistical approach to model the electricity spot price behaviour. These models are adopted from financial models. In [27] Deng, Johnson and Sogomonian use standard geometric Brownian motion and mean-reverting process to model the evolution of the electricity spot price. Some researchers have gone beyond the standard geometric Brownian motion and proposed Jump-Diffusion model, see, e.g. [76].

Electricity spot prices possess some unique characteristics such as spikes, mean-reverting, non-storability and seasonality. Non-storability of electricity leads to the breakdown of the no-arbitrage argument as we know it in commodity derivatives. In particular, electricity is a flow variable and therefore it is impossible to construct a convenience yield. Electricity spikes makes it unreasonable to assume that changes in electricity spot price are continuous as this is the basic assumption of the geometric Brownian motion particularly in short term.

We use Mean-Reverting Jump-Diffusion (MRJD) process to describe the evolution of electricity spot prices. MRJD is capable of accounting for both mean-reverting and spikes found in electricity spot prices. The use of MRJD is found in [12] and [76]. We then use Monte Carlo Methods to evaluate the value of electricity forward contracts. Furthermore, variance reduction is very important in Monte Carlo simulation. Monte Carlo simulation entails simulating various states of a variable and returns its arithmetic mean. It is necessary for the mean to be as close as possible to the actual observable value and simulation can be very laborious in nature. Therefore, a balancing act is required between the effort of reducing variance in computation and the time it takes to complete the computation. This thesis therefore presents efficient Monte Carlo methods for pricing of electricity derivatives.

1.1 Research Motivation

For the past two decades, the focus of research in electricity derivatives has been on modelling of electricity spot prices. Little research has been dedicated to numerical methods. Recent attempts can be found in [1], [27], [28] and [72]. However, none of these researchers have used efficient Monte Carlo methods. The main purpose of this thesis is, therefore, to reduce this apparent gap and present efficient Monte Carlo methods for pricing electricity derivatives. Secondly, it aims to contribute to South Africa's knowledge production on deregulated electricity markets in order to accumulate requisite skills should deregulation of electricity sector become a reality in South Africa.

South Africa supplies two-thirds of Africa's electricity. According to the Department of Energy, South Africa is one of the four cheapest electricity producers in the world. Eskom (South Africa's state-owned electricity utility) supplies about 95 per cent of South Africa's electricity. Eskom also owns and operates the national electricity grid and supplies about half of electricity directly to customers. The remainder of electricity distribution is undertaken by about 188 local authorities [31].

In 2003, the South African government approved private-sector participation in the electricity industry and decided that future power generation capacity will be divided into two groups, 70 per cent generated by Eskom and the remaining 30 per cent generated by independent power producers (IPP's). This decision by Cabinet was precipitated by rapid increases in electricity demand during the beginning of the 21st century. In the same year, 2003, Eskom implemented a revised business model to prepare for capacity requirements and split its business into regulated and non-regulated divisions.

In 2008, Eskom was hit by capacity constraints that resulted in country wide power cuts. These power cuts threatened South Africa's economy and the timing of these power cuts coincided with world financial crises. This phenomenon was like a double sword in the South African economy. As a result of this crisis, many voices were calling for reforms in the South Africa's electricity sector including deregulation of the sector.

In its comment on the Amendments to Electricity Regulation Act 4 of 2006, Free Market Foundation (2012) calls for reforms in South Africa's electricity sector to allow for competition in the generation and distribution of electricity. It calls for deregulation of the sector. This call for deregulation is also supported by Solidarity, one of the largest trade unions in South Africa [70].

Given these proposals for deregulation of the South Africa's electricity industry and the fact that South African government has privatised some of the States Owned Enterprises (SOE) in the past such as Telkom SA, it is reasonable to conclude that deregulation of South Africa's electricity sector can be a reality in the future. In fact, the 2003 decision by South African government to allow for private-sector participation of up to 30% in power generation may be reasonably seen as a step towards deregulation of the electricity sector in South Africa.

It is therefore crucial to accumulate knowledge and technical skills surrounding deregulated electricity market.

1.2 Literature review on pricing electricity derivatives

Deregulated electricity markets are characterised by highly volatile electricity spot prices due to market forces. This high volatility of electricity spot prices creates incentives for large industry players to exercise risk management in order to hedge the risk thereof.

Deng and Oren [28] presented electricity derivatives and risk management. These researchers review different types of electricity financial instruments and the general methodology for utilizing and pricing such instruments. In particular, they highlighted the roles of these electricity derivatives in mitigating market risks and structuring hedging strategies for generators, load serving entities, and power marketers in various risk management applications. Finally, they point out the existing challenges in current electricity markets for increasing the breadth, liquidity and use of electricity derivatives for achieving economic efficiency.

In [63] Pinedo and Conejo focus on managing the financial risks of electricity producers using options. They propose a multi-stage stochastic model to determine the optimal selling strategy of a risk-averse electricity producer including options, forward contracts, and pool trading. Through a detailed case study they highlighted the advantages of an option versus a forward contract to hedge against the financial risks related to pool prices and unexpected unit failures.

Pricing of electricity derivatives however is the main challenge. In fact pricing any derivative is largely dependent on the modelling of the underlying asset and electricity derivatives are no different. The pricing of electricity derivatives depends on how well you can model the behaviour of the electricity spot price the underlying asset. In the literature, electricity spot prices are characterised by the following features:

- **Price Sparks-** electricity spot prices exhibit occasional price spikes due to supply shocks such as transmission constraints and unexpected outages.

- **Seasonality-** it is well known that electricity demand exhibits seasonal fluctuations ([35], [49] and [62]). These fluctuations often arise due to changing climate conditions, such as temperature and the number of daylight hours.

- **Mean-Reversion-** Electricity prices tend to fluctuate around values determined by the cost of production and the level of demand.

- **Non-Storability-** electricity cannot be stored and once generated it needs to be consumed almost immediately.

Giovanni and Gigli [43] empirically explored the possible causes behind electricity price jumps in the Nordic electricity market, Nord Pool. They used a time-series model (a mixed GARCH-EARJI jump model) capturing the common statistical features of electricity prices to identify price jumps. They defined a categorical variable distinguishing positive and negative jump. Then they explored causes for the jumps through the use of ordered probity models in a second stage. The empirical results indicate that the structure of the market plays an important role in whether shocks in the demand and supply for electricity translate into price jumps.

In [12] Blanco and Soronow put electricity spot price jumps into a proper context. They suggest that electricity spot prices do not jump but spike. That is, electricity prices do not jump to a new level and stay there, but rather they quickly revert to their previous levels. Within a very short space of time, prices can increase substantially and then drop back to the previous level. Price spikes are noticeable frequent during on-peak hourly or daily prices.

Given the above characteristics of electricity spot prices, a price process chosen to evaluate electricity derivatives should at least capture all these characteristics in order to ensure credible results. The first natural candidate to model electricity spot prices could be the geometric Brownian motion that is so widely and successfully used to model financial derivatives.

The use of geometric Brownian motion provides a closed-form solution through the application of Black-Scholes formula. However, the main weakness of the geometric Brownian motion is that it cannot capture some of the electricity spot price characteristics such as spikes and mean-reversion. Other researchers have applied the Black-Scholes formula on the mean-reverting price process. This practice allows for the capturing of the mean-reverting character of electricity spot prices.

Schwartz [71] discussed mean-reversion for commodities on general economic grounds. He compared three models of the stochastic behaviour of commodity prices that take into account mean reversion in terms of their ability to price existing future contracts, and their implication with respect to the valuation of other financial and real assets. The first model was a simple one-factor model in which the logarithm of the spot price of the commodity is assumed to follow a mean reverting process. The second model took into account a second stochastic factor, the convenience yield of the commodity, which was assumed to follow a mean reverting process. The third model included stochastic interest rates. The results of the analysis revealed a strong mean reversion in the commercial commodity prices.

In [27], Deng et al. used geometric Brownian motion and mean reverting price processes to model valuate spark and location spread options. They present and apply a methodology for valuing electricity derivatives by constructing replicating portfolios from electricity futures and the risk-free asset. Futures-based replication was made necessary by the non-storable nature of electricity, which ruled out the traditional spot market, storage-based method of valuing commodity derivatives. Using the futures-based approach, they derived valuation formulae for both spark and locational spread options for both geometric Brownian motion and mean reverting price processes. These valuation results are in turn used to construct real options-based valuation formulae for generation and transmission assets. Finally, they used valuation formula derived for generation assets to value a sample of assets that have been recently sold, and the theoretical values calculated are compared to the observed sales prices of the assets

It should be noted that the geometric Brownian motion and mean reverting price processes are unable to capture electricity price spikes. For example, Either and Doris [32] suggested that accounting for spikes in electricity derivatives is crucial. They posit that the inability to factor-in spikes lead to over-valuation for in-the-money options and under-valuation for out-of-the-money options.

Although to incorporate a jump component looks like a natural solution to model electricity spot prices, it comes with its own challenges. First, the luxury of having a closed-form solution in the form of a Black-Scholes formula as applied by Deng et al [27] is virtually impossible. Secondly, a pure jump model as presented in Merton [58] is inadequate because each jump is supposed to be permanent which is not the case with electricity price spikes.

Weron et al [77] addressed the issue of modeling spot electricity prices. After summarizing the facts about spot electricity prices such as seasonality, mean reversion and jumps, they reviewed a number of models proposed in the literature. Afterwards they fit jump diffusion and a regime switching model to spot prices from the Nordic power exchange and discussed the pros and cons of each one.

In [9], Benth et al conducted an empirical analysis of three widely used models for the electricity spot price process. Firstly, the jump-diffusion model, they adjusted to incorporate the most important characteristics of electricity prices. Secondly, a threshold model was proposed by Roncoroni [66] and further developed by Geman and Roncoroni [42] and is an exponential Ornstein–Uhlenbeck process driven by a Brownian motion and a state-dependent compound Poisson process. It was designed to capture both statistical and path wise properties of electricity spot prices. Thirdly, the factor model was proposed by Benth et al [7].

Albanese et al [1] presented a numerical algorithm for pricing electricity derivatives for jump-diffusion processes based on continuous time lattices. The algorithm was based on approximating the generator of the underlying price process on a lattice of prices, resulting in an approximation of the stochastic process by a continuous time Markov chain. They numerically studied the rate of convergence of the algorithm for the case of the Merton jump-diffusion model and applied the algorithm to calculate prices and sensitivities of both European and Bermudan electricity derivatives when the underlying price follows a stochastic process which exhibits both fast mean-reversion and jumps of large magnitude.

A model based Monte Carlo pricing of energy and temperature Quanto options was presented by Caporin et al [21]. Since the specific features of energy and weather time series do not enable the use of a closed-form solution based on the Black-Scholes pricing approach, nor other more advanced continuous time series methods that extend the Black-Scholes approach, unless other strong and unrealistic assumptions, these authors proposed a Monte Carlo approach based on a bivariate time series model. Their approach takes into account the average and variance interdependence between temperature and energy price series. Furthermore, their approach included other relevant empirical features, such as periodic patterns in average, variance, and correlations.

In this thesis, we use the Mean-Reverting Jump-Diffusion (MRJD) process to describe the evolution of electricity spot prices. This MRJD process is capable of accounting for both mean-reverting and spikes found in electricity spot prices. We then use Monte Carlo methods to evaluate the value of electricity forward.

Furthermore, variance reduction is very important in Monte Carlo simulation. Monte Carlo simulation entails simulating various states of a variable and returns its arithmetic mean. It is necessary for the mean to be as close as possible to the actual observable value and hence the variance reduction plays a vital role in such simulations. These simulations can be very laborious in nature. Therefore, a balancing act is required between the effort of reducing variance in computation and the time it takes to complete the computation. Efficiency should be accounted for because variance reduction is an add-on computation to the Monte Carlo simulation. This thesis therefore presents efficient Monte Carlo methods for pricing the electricity derivatives.

1.3 Outline of the thesis

Chapter 2 presents an overview of the electricity market. Regulated and deregulated electricity markets are discussed with particular emphasis on deregulated markets. The structure of the latter is discussed in detail including the behaviour of electricity spot prices in the market.

Chapter 3 deals with the application of Black-Scholes and Binomial models in valuation of electricity derivatives. Three different types of price processes are also discussed. The Black-Scholes formula is used to valuate cross commodity options. In addition, the use of one-step and two-step binomial models to invest in transmission lines is presented.

Chapter 4 presents efficient Monte Carlo methods for pricing the electricity derivatives. The mean reverting jump diffusion process is used to model electricity spot prices. Thereafter, Monte Carlo methods are used to valuate electricity futures. Furthermore, we show how variance reduction techniques can be used in the valuation of electricity derivatives.

Chapter 5 contains comparative numerical results. We look at different underlying price processes and compare their results in pricing electricity derivatives. We highlight their pros and cons and we recommend the use of efficient Monte Carlo methods in pricing electricity derivatives.

Finally, Chapter 6 provides concluding remarks and scope for future research.

Chapter 2

THE ELECTRICITY MARKET

2.1 Introduction

The purpose of this chapter is to provide an overview of the electricity markets, in particular, deregulated electricity market.

The electricity industry is characterised by two main markets, regulated and deregulated markets. Both markets can be divided into three main groups corresponding to the three stages of vertical inter-dependence process that is required to produce, transmit and deliver electricity to the end-user. These three stages are the generation stage, transmission stage and the distribution stage. However, it is the market set up of these three stages that differentiate them between the two markets.

In regulated markets, one firm is responsible for generation, transmission and distribution of electricity and usually this firm is owned by the state. While in deregulated markets, the three stages are unbundled and the market is competitive among many players. Deregulated markets are further characterised by two main models, wholesale and retail. The wholesale model is competitive only in the generation stage. The retail model is competitive in all stages from generation to the retail stage. Furthermore, deregulated electricity markets can be classified into three markets: the spot market, the forward/future markets and the options market. Below we provide a brief overview of the electricity industry.

2.2 An overview of the electricity industry

The electricity industry can be divided into three main groups corresponding to the three stages of vertical inter-dependence process that is required to produce, transmit and deliver electricity to the end-user. These three stages are:

Generation- Electricity generation stage is where the actual production of electricity takes place. Electricity generation is the process of generating electric energy from other forms of energy. This process involves the conversion of chemical, solar, hydro, atomic or mechanical energy to electricity. Since electricity is non-storable, generation capacity is usually a constraint because of varying electricity demand. In deregulated markets, hedging strategies are used by retailers and power generators to avoid electricity black-outs[1]. Typically, a retailer may enter into a contract with a power generator to guarantee a certain amount of electricity to be supplied by the power generator at a predetermined price and period.

Transmission- Electricity transmission is the bulk transfer of high voltage electricity from a generating power plant to substations located near population locations for distribution. High voltages are used in transmission in order to counter-act voltage drops[2] caused by resistance in the transmission lines. Electricity transformers are used to step up voltage from the power generating plant for transmission and to step down voltage from sub-stations for distribution to customers. The capacity of transmission lines is usually limited. Furthermore, transmission lines are very capital intensive. Transmission constraints somewhat compels industry players such as distributors to use transmission hedging strategies in order to ensure delivery of energy to their customers.

Distribution- electricity distribution is the final stage in the delivery of electricity to the end-user. Traditionally, distribution and retailing are bundled together. Recently, more especially in deregulated markets, retail has been unbundled from distribution. In such a market setting, retail becomes the final stage in the delivery of electricity to the end-user. Typically, retail companies buy electricity from distribution companies

[1] Black-out means loss of electricity in the whole or part of a transmission network.
[2] Voltage drop is the reduction of voltage due to the resistance in the transmission line.

and their main business is advertising, branding, contract bundling, metering and billing for end users. Like transmission, distribution is very capital intensive and the capacity is again limited. Industry players are concerned with having enough electricity supply to serve their customers and use hedging strategies in order to ensure that they have enough electricity supply.

2.2.1 Regulated market

There is usually one utility company owned by the state such as Eskom in South Africa. The utility company is responsible for generation, transmission and distribution of electricity. Regulated markets have some variations whereby it is possible to have some Independent Power Producers (IPP) but usually these IPP's produce a small quantity of electricity compared to the utility company owned by the state. For example, Eskom is generating about 95 per cent of South Africa's electricity requirements [31].

In the regulated market, electricity prices are regulated by independent regulators and these regulators account to the state. Under this regulatory regime, utility companies are allowed to earn regulated rate of return that is above their marginal cost of capital [27]. Typically, regulators would approve the construction costs of a power generating plant, the costs would be passed onto the consumers through regulated electricity prices over the life of the investment irrespective of the fluctuation in the market value of the investment over time due to changing energy prices, improving technology, and evolving supply and demand conditions [27]. Thus the power generating company bears minimal investment risk.

2.2.2 Deregulated Markets

Deregulated electricity markets exist in the US, UK, Germany, Spanish and Scandinavian countries. These markets are competitive in character. In such markets, there are usually more than one producer, distributor and retailer. Deregulated electricity markets have two main models, namely:

1. **Wholesale Competition-** The distributor buys electricity direct from the IPPs and the IPPs compete to supply electricity to the distributor. The distribution company have

monopoly over final customers. There is open access to transmission lines. A power pool exchange is established to facilitate electricity price changes.

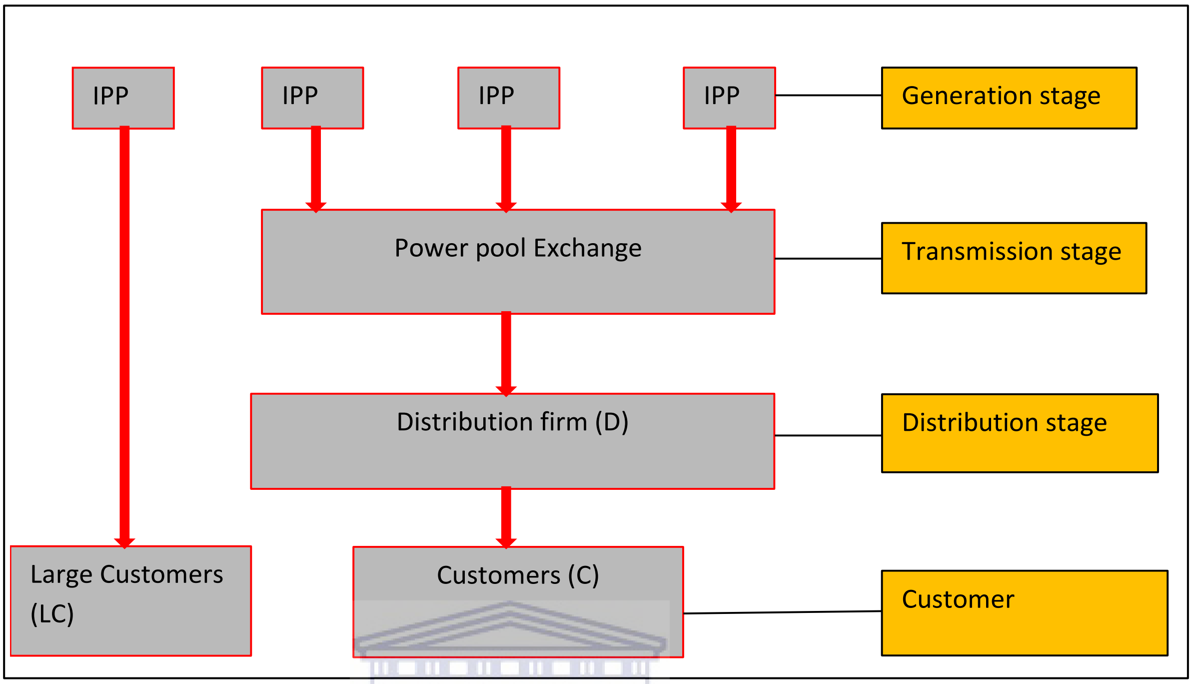

Figure 1.1 Illustration of wholesale electricity market model

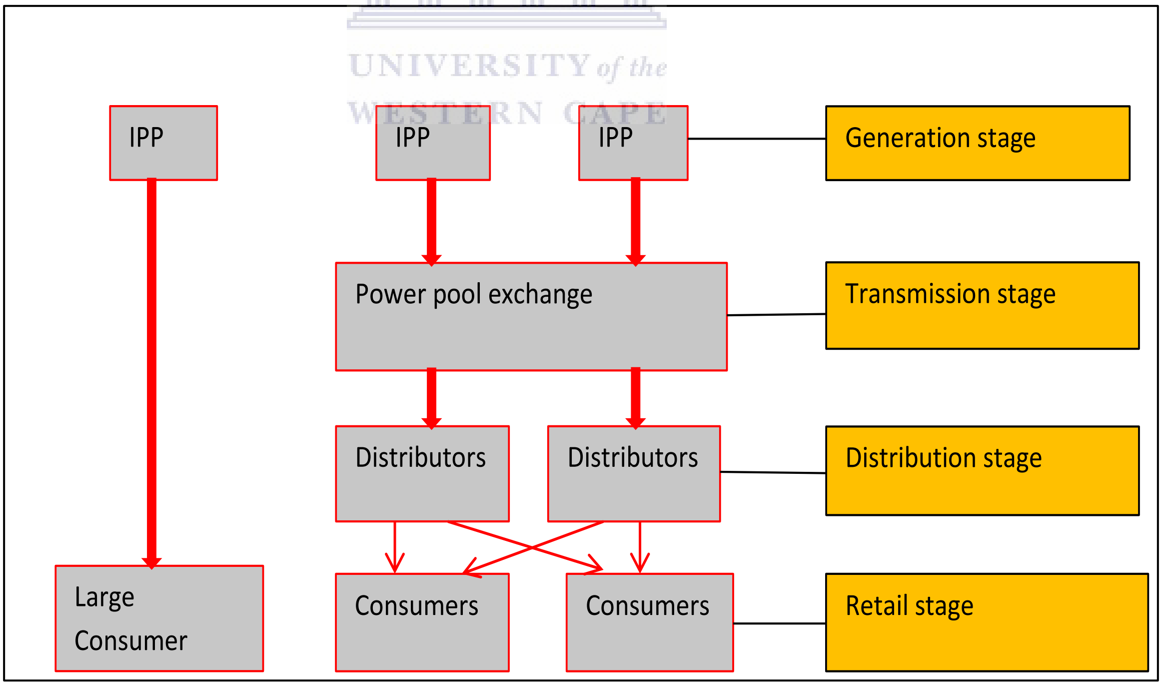

Figure 1.2 Illustration of a retail electricity market model

2. **Retail Competition-** All customers have choice of supplier. There is open access to the transmission and distribution network. Distribution is separate from retail activity and the retail industry is competitive.

In deregulated electricity markets, generation, transmission and distribution is unbundled. All Independent Power Producers supply electricity to the transmission grid. As we have indicated above, some electric energy is lost during transmission and this loss depends on various factors such as the transmission distance and consumptions levels. Thus electricity that is supplied to the grid must be corrected for transmission losses. Accounts of the quantity of electricity each producer delivers to the grid and each consumer uses are kept at any given time.

Once electricity is delivered to the consumer, it is no longer possible to separate supplies from different producers. When a consumer uses electricity, it is impossible to distinguish its source. If a consumer changes an electricity supplier, the physical flow of electricity does not change in the network. Thus the transmission tariff paid by the consumer does not change irrespective of who is the supplier.

Electricity is traded in the spot markets at several exchanges. Deregulated electricity markets are characterised by fluctuations in the electricity spot prices due to market forces of supply and demand. Production, transmission and distribution companies are exposed to the risk of fluctuations in the electricity spot prices. For example, the electricity spot price may fall below marginal cost of production due to disequilibrium in supply and demand. This phenomenon of electricity prices falling below marginal cost of production would result in producers supplying energy at a loss.

Consequently, the volatility of electricity spot prices creates incentives for large players in deregulated electricity prices to exercise risk management in order to hedge price risk. For example, a producer would want to enter in a bilateral agreement with its customers to supply electricity on a predetermined price per kilowatt hour (kwh) that is at least equal to or larger than its marginal cost of production.

To facilitate the need for risk management, exchanges that mimic those found in financial markets are established in deregulated markets. These exchanges are further categorised as Spot Markets, Future/forwards markets and Options Markets.

2.2.2.1 The Spot Market

As we have mentioned above, deregulated electricity market exist in the US, UK, Germany, Spanish and Scandinavians countries. Of these, the Scandinavian spot market is more matured hence it has been widely researched. This section makes reference to the Nordic market in describing the electricity spot market.

In the spot market, physical delivery of electricity takes place and it is actually a day-ahead market because electricity cannot be stored. Each day, spot prices and volume for each hour of the following day are determined. This is necessary because the Independent System Operator (ISO) or the Independent Power Producer (IPP) would require adequate time to match the demand side. While the ISO or IPP may have flexibility in its ability to adjust its supply capacity to match the demand, issues such as transmission constraints need to be considered.

Physical electricity spot prices in the Nordic market are set according to the market equilibrium where the supply and demand curves of all the market participants are matched. The prices for the one-hour periods are determined from the bid and ask prices given by the market participants [75]. Thus in this market, spot prices are largely determined by the demand side while to a lesser extent they can also be determined by the constraints in the supply and transmission side. In turn, electricity demand is very much dependent on other variables such as weather and peak times. It therefore follows that electricity spot prices would invariably exhibit the character of these variables.

2.2.2.2 Characteristics of electricity spot prices

Electricity spot prices are somewhat different from other spot prices set in the financial markets. In particular, electricity prices are characterised by seasonality as well as unanticipated large changes known as jumps or spikes. Technically, however, electricity prices do not jump but spike. That is, electricity prices do not jump to a new level and stay there, but rather they quickly revert to their previous levels.

Within a very short space of time, prices can increase substantially and then drop back to the previous level. Price spikes are noticeably frequent during peak hours. For example, peak hours can be observed in any market in the early hours of the morning when people are preparing for work and evenings when most people are preparing supper.

The main characteristics of electricity spot prices can be summarised as follows:

- **Price Sparks:** Electricity spot prices exhibit occasional price spikes due to supply shocks such as transmission constraints and unexpected outages. Transmission constraints usually occur during peak hours when the demand for electricity rises substantially or high consumption periods such as winter. On the other hand, unexpected outages such as the ones occurred in South Africa in 2006, put constraints on the remaining generators thereby resulting in a supply shock. However, as the time horizon increases and the data are aggregated, the spikes are less and less pronounced and their effects are usually neutralised in the data.

  Non-storability of electricity is the main effect of electricity spot price spikes. Electricity to be delivered at a specific hour cannot be stored at a reservoir for delivery at that particular hour. Electricity has to be consumed at the same time as it is produced hence the occurrence of extreme load fluctuations caused by sudden high demand during peak hours or unexpected outages can lead to price spikes.

- **Seasonality:** It is well known that electricity demand exhibits seasonal fluctuations (see [35], [49] and [62]). These fluctuations often arise due to changing climate conditions, such as temperature and the number of daylight hours. Winter, for instance, is characterised by high electricity demand due to low temperatures that require households to heat their houses. In some countries, the supply side shows seasonal variations in output [78]. Hydro units[3], for example, are heavily dependent on the rainfall which varies from season to season. These seasonal fluctuations in demand and supply eventually translate into seasonal behaviour of electricity prices, in particular, spot prices.

- **Mean-Reversion:** Electricity prices tend to fluctuate around values determined by the cost of production and the level of demand. Thus they are generally regarded to be mean-reverting or anti-persistent. This mean-reversion of electricity prices can partly

[3]Electricity generators that use water as a prime-mover (turning turbines) in the process of electricity generation.

be explained by price spikes already mentioned and the economic theory of perfect competition. In competitive markets, economic profits incentives new entrance and thereby pushing prices down around the cost of production.

- **Non-Storability:** Electricity cannot be stored and once generated it needs to be consumed almost immediately. This effectively implies that electricity prices do not follow smooth process as prices of other commodities.

2.2.2.3 Electricity Forward/Future market

As mentioned above, electricity cannot be stored and therefore there is a delay between the actual transaction and the delivery of electricity, hence, there is a need for future/forward contracts. Forward and futures prices are the result of supply and demand for hedging and speculation. IPPs hedge by selling (going short) and power marketers and power-intensive industry hedge by buying (going long). Speculators, which only include producers and power marketers, enter both sides of the market depending on their expectations and risk-taking ability [37]. Thus hedgers would ordinarily use forward contracts with physical delivery while speculators would be expected to use futures that are settled financially with no physical delivery.

Electricity forwards and futures can be traded over the counter (OTC) or through the exchange. Like the usual financial forwards/futures, time to maturity and forwards/futures price is specified in the contract. However since electricity is non-storable and therefore cannot be delivered in real time, the delivery period is also specified. The delivery of a certain amount of electricity takes place over a period of time rather than at a specify point in time. Thus, according to Fleten and Lemming [37], forward/future prices can be viewed as a portfolio of basic forward/futures contracts each with different time to maturities, one for each point in time during the delivery period.

Furthermore, the non-storability of electricity result in the break-down of the no-arbitrage assumption at least when it relates to the cost of carry relationships. The convenient yield does not exist because electricity cannot be stored. In [37], Fleten and Lemming also show that electricity futures and spot prices do not converge by using data collected from the Nord pool. But they could still employ the no-arbitrage assumption in electricity derivatives based on the convergence of the expected spot price and future prices.

In [14], Botterud studied Nord Pool's futures market. He used hypothesis testing to analyse the returns on future contracts over various holding periods. This hypothesis testing also included portfolio of futures contracts. The hypothesis was that the future price equals the expected future spot price,

$$F_t = E[S_t].$$

The result of the test found no sufficient evidence to reject the hypothesis.

Botterud et al [15] conducted a similar analysis but this time merely observing future prices and spot price in the Nord Pool for the period 1996 to 2001. Their results showed that future prices follow the same trend as the spot price at least in future contracts with a short time to delivery. They deduce that it is reasonable to believe that the market expects the future prices to resemble the expected spot price. Later on, many researchers use this no-arbitrage assumption when evaluating electricity derivatives.

Fleten and Lemming [37] argued that since electricity forwards/futures concern electricity delivery over a specified period of time $[T_1, T_2]$, the relevant statistic for the contract is not the spot price at T_1, but the average spot price during the interval $[T_1, T_2]$. Thus the value of the forward/futures contract would be the difference

$$F(T_1, T_2) - \sum_{t=T_1}^{T_2} S_t / (T_2 - T_1).$$

That is, the difference between the closing future price at T_1, $F(T_1, T_2)$ and the average of the spot price S_t in interval $[T_1, T_2]$ is paid to the buyer and is charged to the seller if this difference is positive. If this difference is negative, the seller gains and the buyer lose. It can therefore be argued that forwards/futures contracts offer a perfect hedge against the price risk in the interval $[T_1, T_2]$ for a constant amount of electricity held throughout the period.

2.2.2.4 Electricity Options market

The electricity industry had been utilising the idea of options through embedded terms and conditions in various supply and purchase contracts for decades, without explicitly recognising and valuing the options until the beginning of the electricity industry restructuring in the UK, the US and the Nordic countries in the 1990s [28]. As stated earlier, electricity derivatives are generally adopted from financial derivatives. Thus an electricity option can be adopted from each financial option by replacing the underlying asset of a financial option with an electricity option (see [45] for introduction to various kinds of financial options). Below, we describe a sample of electricity options that are commonly used in the electricity market.

2.2.2.4.1 Plain call and put electricity options

Electricity options offer the buyer the right, but not the obligation, to buy or sell a fixed amount of underlying electricity at a pre-determined price (strike price) and pre-determined expiration time (exercise time). Electricity call and put options have similar payoff functions as those of regular call and put options on financial assets and other commodities. The payoff of an electricity call option is:

$$Payoff\ of\ an\ electricity\ call\ option = \max(S_T - K, 0),$$

where S_T is the electricity spot price at time T and K is the strike price.

The underlying of electricity call and put options can be exchange-traded electricity futures or physical electricity delivered at major power transmission inter-ties such as the ones found in California-Oregon Border and Palo Verde in the Western U.S. power grid [27]. According to Deng et al [27], electricity call and put options are the most effective tools available to merchant electricity plants and electricity marketers for hedging price risk because electricity generation capacities can be viewed as call options on electricity, particularly when generation costs are fixed.

2.2.2.4.2 Spark spread or heat rate linked electricity options

Spark spreads are cross-commodity options paying out the difference between the price of electricity sold by generators and the price of the fuel used to generate it. Here, fuel is the input commodity used to generate electricity and therefore the definition could be extended to other commodities that are used to generate electricity. The amount of fuel that a particular generation asset requires to produce one unit of electricity depends on the asset's efficiency or asset's heat rate (Btu/MWh). Asset's heat rate is the number of British thermal units (Btu) of the fuel measured in millions required to generate one megawatt hour (MWh) of electricity. Thus the lower the heat rate required to producing one megawatt hour of electricity the more efficient is the facility. The spark spread associated with a particular heat rate is defined as the current price of electricity less the product of the heat rate and the current fuel price. Thus the lower the heat rate, the lower the fuel price and the higher the electricity price, the larger the spark spread.

It is reasonable to assume particularly under a deregulated market that only assets that have a positive spark spreads under prevailing market conditions will be operated. A close look at the payoff of a spark spread call function below indicate that $\mathrm{K_H.G_T}$ is cost of generating electricity and therefore it is financially viable only if the cost of generating one megawatt hour unit is less than the price of one megawatt hour. This leads naturally to the definition of the prevailing market implied heat rate H as *number of million British thermal units (MMBtu) needed for a marginal generating plant to generate one MWh of electricity.* This is given by

$$H = \frac{S_E}{S_G} \times \frac{10^{-3} MMBtu}{MWh},$$

where S_E is the spot price of electricity per Megawatt hour (MWh) and S_G is the spot price of the generating fuel per $MMBtu$.

The holder of a European-spark spread call option written on fuel G at a fixed heat rate K_H (strike price) has the right, but not the obligation, to pay at the option's maturity K_H times the unit price fuel price G at maturity time T and receive the price of one unit of electricity. Let S_E^T and S_G^T be the unit spot prices of electricity and fuel at time T, respectively.

Denote the value the option at time t by $C_1(S_E^t, S_G^t, t)$. Then the payoff of the option at maturity time T is

$$C_1(S_E^t, S_G^t, t) = max(S_E^T - K_H . S_G^T, 0).$$

On the other hand, the holder of a European-spark spread put option written on fuel G at a fixed heat rate K_H gives the option holder the right but not the obligation to pay the price of one unit of electricity and receive K_H times the unit price of fuel G at maturity time T. Denote the value of the put option at time t by $P_1(S_E^t, S_G^t, t)$. Then the payoff at time t is

$$P_1(S_E^t, S_G^t, t) = (K_H . S_G^T - S_E^T, 0).$$

2.2.2.4.3 Locational spread electricity option

Substantial differences frequently exist between the price of electricity at different locations due to transmission costs and constraints. Transmission costs are largely related to voltage drop that occurs when electricity is transmitted. Voltage drop is a function of resistance of the conductor and current that is passing through the conductor. The voltage at the end of the transmission is equal to the voltage generated at the plant less the voltage drop. Due to the positive correlation between resistance and distance, the longer the transmission lines, the larger the voltage drops and consequently the higher transmission cost. Furthermore, transmission constraints may relate to the non-storability of electricity which forces real time delivery. We refer to these differences in electricity prices at different locations as locational spreads.

A European call option on the locational spread between location one and location two with maturity T gives its holder the right but not the obligation to pay the price of one unit of electricity at location one at time T and receive the price of one unit of electricity at location two. Let S_i^T be the unit price of electricity at location i ($i = 1,2$) at time T. Denote the value of the option at time t by $C_2(S_1^t, S_2^t, t)$. Then the payoff of the option at time T is

$$C_2(S_1^t, S_2^t, t) = max(S_1^T - S_2^T, 0).$$

A European locational spread put option can be defined in a similar way.

2.2.2.4.4 Electricity callable and putable forwards

Callable and putable forwards are introduced in [40] and [41] to mimic the interruptible supply contracts and the dispatchable independent power producer contracts.

In callable forward contract, the buyer of the contract long one forward contract and short one call option with a buyer-selected strike price. The seller of this forward contract holds the opposite position and can exercise the call option if the electricity price exceeds the strike price, effectively cancelling the forward contract at the time of delivery. The buyer gets an "interruptibility" discount on the forward price, which is equal to the option premium at the time of contracting continuous compounding to the delivery time.

In a putable forward, the buyer longs one forward contract and one put option with a seller-selected strike price. The seller holds the opposite short positions. The buyer exercises the put option if the electricity price drops below the strike price at the maturity time, effectively cancelling the forward contract. At the beginning of the contract, the buyer is required to pay a capacity availability premium over the forward energy price, which equals the put option price at that time, continuously compounded to the maturity time.

2.2.2.4.5 Electricity swing options

Electricity swing options are adopted from their well-known counterparts in the natural gas industry [48]. Swing options have the following defining features:

- These options may be exercised daily or up to a limited number of days during the period in which exercise is allowed.
- When exercising a swing option, the daily quantity may vary (or swing) between a minimum volume and a maximum daily volume. However, the total quantity taken during a time period such as week or a month is required to be within certain minimum and maximum volume levels.
- The strike price of a swing option may be either fixed through its life or set at the beginning of each time period based on some pre-specified formula.
- If the minimum of the taken quantity of any contract period is missed by the buyer, then a lump sum penalty or a payment making up the seller's revenue shortfall is required to be paid.

2.2.2.4.6 Electricity swaps

Electricity swaps are financial contracts that enable their holders to pay a fixed price for underlying electricity, regardless of the floating electricity prices, or vice versa, over the contracted time period. They are used to provide short-to medium term price certainty up to a couple of years. Electricity swaps can be viewed as a strip of electricity forwards with multiple settlements dates and identical forwards price for each settlement [27]. According to the study in [27], this is a reasonable observation because in both forwards and swaps, delivery period and price is specified. While the former is settled at expiry date and electricity is delivered over the specified time, the latter is settled hourly or daily or yearly with the delivery of the electricity over the same period. That is, the payoff of electricity forward, $F(t, T_1, T_2)$, is settled at time T_1 for the delivery of electricity over the period $[T_1, T_2]$. On the other hand, the payoff of a swap is settled at specified periods over the delivery period $[T_1, T_2]$.

Electricity locational basis swaps are commonly used to lock in a fixed price at a geographic location that is different from the delivery point of a future contract. A holder of an electricity locational basis swap agrees to either pay or receive the difference between a specified futures contract price and another locational spot price of interest for a fixed constant cash flow at the time of the transaction. These swaps are effective financial instruments for hedging the basis risk on the price difference between power prices at two different physical locations [28].

2.2.2.5 Structured transactions

Structured bilateral transactions are powerful tools for power market participants to share and control a variety of risks including pricing and quantity risks over a potentially long time horizon [28].

2.2.2.5.1 Tolling electricity contracts

A tolling agreement has similar characteristics to a common electricity supply contract signed between a buyer and an owner of a power plant. However, there are notable differences. For an upfront premium paid to the plant owner, it gives the buyer the right to either operate or control the schedule of the power plant or simply take the generated electricity during pre-specified time periods subject to certain constraints. Often, there are often other contractual limitations in the contract on how the buyer may operate the power plant or take the output electricity. For example, a tolling electricity contract always has a clause on the maximum allowable number of power plant restarts. As a result, these constraints make the pricing of these contracts a very challenging task.

2.2.2.5.2 Load-serving full-requirement electricity contract

Large electricity consumers often prefer a power supply contract with flexible consumption terms. Specifically, they desire to pay affixed rate per unit of energy for the actual consumption quantity, regardless of the quantity being high or low [28]. Such a contract is termed as a load –serving full requirement contract. Electricity Load Serving Entity (LSE) signs a full-requirement contract with a customer and then utilizes futures contracts to lock in a fixed quantity of electricity supply at a fixed cost for hedging the expected energy consumption of the customer (see [2] and [81]). The contract fixes the price per unit paid by the customer for the actual consumption quantity. Thus the power demand by the customer is allowed to fluctuate. It then follows naturally that the LSE is at the risk of under or over hedging because almost surely the consumption quantity of the customer will be different from the amount hedged by the futures contracts. When the electricity spot price is high (low), the total demand for the electricity is likely to be high (low) as well [28]. This concept is derived from the law of demand and price of the economic theory.

It follows then that if the market price of electricity is higher than the fixed contract rate for serving electricity, probabilities are that the customer's energy consumption level is considerably higher than the hedged quantity. Consequently, the LSE is under-hedged relative to its load obligation and therefore must buy electricity in the open market to serve its customer at a loss because the wholesale spot price most likely exceeds the contracted price paid by consumers [28]. Conversely, when the electricity spot price is low, the LSE faces the risk of being over-hedged and having to sell the surplus in the spot market or settle it financially at a price below its long-term contract price.

The uncertainty in customers load and the positive price-load correlation that causes the under-and over- hedging exposures faced by an LSE as illustrated above is called volumetric risk. To hedge the volumetric risk, the LSE can purchase an electricity option on the consumption quantity of its customers. Unfortunately, such an option is usually unavailable in the marketplace [28]. However, notwithstanding the fact that perfect hedging may not be possible, weather derivatives that exploit the correlation between load and temperature can be used (see [20] and [64]).

2.2.2.6 Financial derivatives on electricity transmission capacity

Open access to transmission networks is very important for the electricity wholesale markets and retail competitions to be possible and efficient. This open access to transmission networks would, logically, calls for efficient utilisation of the transmission networks and adequate investment thereof. Furthermore, intuitively, rights would be required for using the transmission networks and rules would be needed for rationing transmission usage when networks become congested [28]. According to [28] , there are two major proposals for using financial instruments as transmission rights in United States:

a) the point to point financial transmission rights (FTRs),
b) the flowgate rights (FGRs).

These financial instruments are outlined in the Standard Market Design (SMD) established by the Federal Energy Regulatory Commission (FERC). The FTRs and FGRs, are therefore the electricity derivatives with their values derived from the network transmission capacity.

2.2.2.6.1 Financial transmission rights (FTR) and FTR options

In electricity market that employs locational market price (LPM), a point-to-point FTR is specified over any two locations in the power transmission grid. An FTR gives its holder the right to receive compensation for transmission congestion charges that arise when the grid is congested. The congestion payoff associated with one unit of FTR is equal to the difference between the two locational prices of one unit of electricity resulted from the re-dispatch of generators out of merit in order to relieve transmission congestion. The primary markets for the FTR trading are auctions conducted by the independent system operators (ISO) of power markets.

On the other hand, an FTR option gives its holder the right of the FTR settlement without the obligation to pay when that settlement is negative. The payoff (settlement) of an FTR option equals to the positive part of the corresponding two-sided point-to-point FTR.

2.2.2.6.2 Flowgate rights (FGRs)

Flowgate rights are defined over all transmission elements such as lines, transformers, or linear combinations of the two aforementioned. Each transmission element has two elemental flowgates, one in each direction. An elemental flowgate has a rated capacity in megawatts in its pre-specified direction corresponding to the capacity of an underlying transmission element. For an example, a holder of a flowgate right might want to transmit power from point A to point B. The rights would specify the size of the power that need to be transmitted and the capacity of the transmission line to be used. Thus, flowgate rights are link-based transmission rights for hedging transmission risks. The value (payoff) of flowgate rights are established through auctions conducted by the ISOs. The spot price upon which the settlement of flowgate rights is based is given by the real time shadow price on the corresponding constrained element such as transmission lines and transformers. These constrained elements are in-turn determined by the security constrained economic dispatch algorithm employed by an ISO. Since these shadow prices are non-negative, FGRs are inherently defined as options.

2.3 Summary

In this chapter, we have presented the structure of the electricity market. Electricity markets are mainly divided into two, regulated and deregulated markets. The vertical structure of either market is made up of three stages: the generation stage, transmission stage and the distribution stage. We further discussed the deregulated market. This is a competitive market whereby there is more than one producer, distributor and supplier. Deregulated electricity markets have two main models: wholesale competition model and retail competition model. In the wholesale competition model, the completion is on the generation stage; the distributor has a monopoly over the end user. In the retail competition model, there is a competition throughout the value chain.

We also looked at how deregulated electricity markets work. All independent power producers supply electricity to the transmission grid. Accounts of the quantity of electricity each producer delivers to the grid and each consumer uses are kept at any given time. The transmission tariff paid by the consumer does not change irrespective of who is the supplier due to the fact that when a consumer uses electricity, it is impossible to distinguish its source.

Deregulated markets can be divided into three markets: spot market, forwards/futures market and options market. In the spot market, physical delivery of electricity takes place and it is actually a day-ahead market because electricity cannot be stored. The electricity spot price is characterised by spikes, seasonality, mean-reverting and non-storability. Non-storability of electricity leads to the breakdown of the no-arbitrage argument based on the convenience yield that characterise the valuation of commodity derivatives.

Electricity forwards/futures market contracts are instruments that are used for hedging and speculation purposes. Furthermore, since electricity is flow variable (non-storable), spot prices as we know them in other commodities do not exist. There is a delay (usually an hour) between the transaction between the buyer and seller and the actual delivery of electricity. Thus the spot market can be viewed as a forward market with hourly maturity.

The electricity options market has a financial setting whereby participants use this market for risk management. Typically, power generators and distributors would hedge against electricity price risk. Power generators for example would hedge to ensure that they receive an electricity unit price that is above or at least equal to the electricity unit cost.

In the next chapter, we present the application of Black-Scholes and Binomial models in valuation of electricity derivatives. Three different types of price processes are also discussed. The Black Scholes formula is used to valuate cross commodity options. In addition, the use of one-step and two-step binomial models to invest in transmission lines is presented.

Chapter 3

NUMERICAL METHODS FOR PRICING OF ELECTRICITY DERIVATIVES

3.1 Introduction

Pricing any derivative is largely dependent on the modelling of the underlying asset and electricity derivatives are no different. The pricing of electricity derivatives depends on how well you can model the behaviour of the electricity spot price, the underlying asset. There have been several attempts on the modelling of the electricity spot prices. Most recent ones are for example [8], [54] and [69]. These approaches proposed in these works are divided into statistical and fundamental approaches. Accordingly, the governing models are named as statistical models or fundamental models

Statistical models depend on a set of parameters that describe the properties of the process of the underlying electricity spot price while fundamental electricity price models are based on competitive equilibrium models for the electricity market.

The statistical approaches attempt to model the electricity spot price process directly. Parameters of the price processes are estimated from the available historical market data. Recently, Weron [79] used an implied risk parameter from the traded Asian-style electricity options and futures. These models are adopted from other financial models. Deng et al [27] use a standard geometric Brownian motion to model the evolution of the electricity spot price. Some researchers such as Kou [54] have gone beyond the standard geometric Brownian motion and proposed Jump-Diffusion models.

In fundamental models electricity prices are obtained from a model for the expected production costs of electricity and expected consumption of electricity, see, e.g. Vehviläinen and Pyykkönen [75]. One approach is to calculate the theoretical equilibrium price of the whole market. Others model the supply function directly. The fundamental models would naturally require large data sets and this may be difficult to collect and maintain. In [75], the authors combine statistical and fundamental models simultaneously in order to exploit the favourable sides from each.

In this chapter, we use the standard geometric model and the mean-reverting model (Ornstein-Uhlenbeck processes) to model the evolution of electricity spot prices. We then apply the Black Scholes formula for the pricing of spark spreads and location spreads. We also show how the Binomial model can be used in pricing electricity derivatives.

3.2 Application of Black Scholes model on electricity derivatives

In Chapter 1, we described the vertical structure of the electricity market. The structure is made up of three stages, namely the generation stage, transmission stage and the distribution stage. In a deregulated electricity market, risk management is necessary in all three stages and all three stages have different risks. In the generation stage, IPPs are concerned with both capacity and cost recovery. Firstly, IPP's are required to deliver a certain amount of electricity to honour contractual agreements with their customers. Secondly, due to competition, no IPP is able to determine or even influence the spot price of the electricity. Thus IPPs or electricity generators are price takers. This phenomenon poses a threat on their cost recovery because of the fluctuating electricity spot price.

Spark spreads or heat rate-linked derivatives are designed to cover electricity generators from electricity spot price fluctuations. Spark spreads are cross-commodity options paying out the difference between the price of electricity sold by generators and the price of the fuel used to generate it. Here, fuel is the input commodity used to generate electricity and therefore the definition could be extended to other commodities that are used to generate electricity.

The amount of fuel that a particular generation asset requires to produce one unit of electricity depends on the asset's efficiency or asset's heat rate. Asset's heat rate is the number of British thermal units (Btu) of the fuel measured in millions required to generate one megawatt hour (MWh) of electricity. Thus the lower the heat rate required to produce one megawatt hour of electricity the more efficient is the facility. The spark spread associated with a particular heat rate is defined as the current price of electricity less the product of the heat rate and the current fuel price. Thus the lower the heat rate, the lower the fuel price and the higher the electricity price, the larger the spark spread.

It is reasonable to assume, particularly under a deregulated markets that only assets that have positive spark spreads under prevailing market conditions will be in production. A close look at the payoff of a spark spread call function below indicates that $\mathrm{K_H.G_T}$ is the cost of generating electricity and therefore it is financially viable only if the cost of generating one megawatt hour unit is less than the price of one megawatt hour. This leads naturally to the definition of the prevailing market implied heat rate H as

$$\frac{S_E}{S_G} \times 10^{-3} MMBtu/MWh,$$

where H is the number of *MMBtu* needed for a marginal generating plant to generate one MWh of electricity, S_E is the spot price of electricity per Megawatt hour (MWh) and S_G is the spot price of the generating fuel per $MMBtu$.

Definition 3.1 The holder of a European-spark spread call option written on fuel G at a fixed heat rate K_H (strike price) has the right, but not the obligation, to pay at the option's maturity K_H times the unit price fuel price G at maturity time T and receive the price of one unit of electricity. Let S_E^T and S_G^T be the unit spot prices of electricity and fuel at time T, respectively. Denote the value the option at time t by $C_1(S_E^t, S_G^t, t)$. Then the payoff of the option at maturity time T is

$$C_1(S_E^t, S_G^t, t) = max(S_E^T - K_H . S_G^T, 0).$$

Naturally, power generators would buy European spark spread put options in order to at least receive $K_H . S_G^T$ if $K_H . S_G^T - S_E^T$ is negative. However, power marketers would be expected to buy European spark spread call options in order to receive $max(S_E^T - K_H . S_G^T, 0)$ when S_E^T is greater than $K_H . S_G^T$.

On the other hand, transmission stage has its own cost and capacity constraints. In Chapter 1, we mentioned that, in a deregulated market, open access to transmission lines is necessary. We also mentioned that transmission lines are intensive capital assets and the capacity and have capacity constraints. Location spreads options are designed to hedge price differences that exist in different locations due to difference in transmission costs and capacity constraints.

Substantial differences frequently exist between the price of electricity at different locations due to transmission costs and constraints. Transmission costs are largely related to voltage drop that occurs when electricity is transmitted. Voltage drop is a function of resistance of the conductor and current that is passing through the conductor. The voltage at the end of the transmission is equal to the voltage generated at the plant less the voltage drop. Due to the positive correlation between resistance and distance, the longer the transmission lines, the larger the voltage drops and consequently the transmission cost is higher. Furthermore, transmission constraints may relate to the non-storability of electricity which forces real time delivery.

Black [10] derived a formula to valuate European call and put options under the assumption that stock prices follow a geometric Brownian process. Deng and Oren [28] valuated spark spread and location European call and put options for both geometric Brownian motion process and mean-reverting price processes (Ornstein-Uhlenbeck process).

3.2.1 Geometric Brownian motion on electricity spot prices

Under a Geometric Brown Motion (GBM) the electricity spot price $X_t \equiv \log S_t$ follows the stochastic differential equation (SDE)

$$\frac{dS_t}{S_t} = udt + \sigma dB_t,$$

where u,σ are assumed to be constant and $\{B_t, t \geq 0\}$ is a standard Brownian motion. Under an equivalent risk neutral measure (equivalent martingale measure) Q, we get

$$\frac{dS_t}{S_t} = rdt + \sigma dB_t,$$

where r is the risk neutral rate that will be assumed constant. Appling Ito's lemma [57] to the above SDE in terms of $X_t \equiv logS_t$ we get the following SDE

$$d \log S_t = \left(r - \frac{1}{2}\sigma\right) dt + \sigma dB_t.$$

From the above SDE, the mean and variance are computed as

$$log\ S_T = \log S_0 + \left(u - \frac{1}{2}\sigma^2\right)T$$

$$Var(\log S) = \sigma^2 T,$$

and then from the above SDE, the Black-Scholes-Merton differential equation is derived as

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf.$$

Solving this differential equation leads to the Black-Scholes formula for the pricing of European call on a non-dividend paying asset:

$$c = S_0 N(d_1) - ke^{-rT} N(d_2),$$

European put on a non-dividend paying asset:

$$p = ke^{-rT} N(-d_2) - S_0 N(-d_1),$$

where

$$d_1 = \frac{\ln\left(\frac{S_0}{k}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},$$

$$d_2 = \frac{\ln\left(\frac{S_0}{k}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},$$

$$or\ d_2 = d_1 - \sigma\sqrt{T}.$$

In the above, $N(d_2)$ is the risk-adjusted probability that the option would be exercised. $N(d_1)$ is the factor by which the present value of the contingent receipt of the electricity spot price exceeds the current electricity spot price.

Since electricity is non-storable, it is reasonable to assume that electricity is a non-dividend paying asset and therefore Black-Scholes Formula can be readily applied to spark spreads and location spreads. However, electricity has unique characteristics compared to other commodities such as mean-reversion. Electricity spot prices tend to revert to a level that reflects production cost and the major drawback of GBM is that it cannot capture this mean-reversion of electricity spot prices. The simplest model to capture the mean-reverting characteristic of electricity is given by an Ornstein-Uhlenbeck process.

3.2.2 Ornstein-Uhlenbeck process on electricity spot prices

Ornstein-Uhlenbeck process is able to capture the mean-reverting behaviour of electricity spot prices. In this process, the price process satisfies the SDE:

$$d\log S_t = k(u - \log S_t)dt + \sigma dB_t,$$

where $k > 0$ is the magnitude of the speed of reversion to the long-run mean log-price, u is considered to be constant under an equivalent risk neutral measure (equivalent martingale measure) Q.

Solving the above Ornstein-Uhlenbeck SDE (see Appendix) and assuming $k = 1$, we get the expression for the process $\{X_t, t \geq 0\}$ where $X_t \equiv \log S_t$:

$$X_t = X_0 e^{-t} + \mu \int_0^t e^{-(t-s)}\, ds + \sigma \int_0^t e^{-(t-s)}\, dBs,$$

$with$ $\{X_t, t \geq 0\}$ and $0 \leq s \geq t$

Thus the conditional expectation and conditional variance under the probability measure, Q and filtration, $\mathcal{F}$ will be

$$E_Q[X_t|\mathcal{F}_s] = \mu + (X_0 - \mu)e^{-t}$$

and

$$Var_Q[X_t|\mathcal{F}_s] = (1 - e^{-2t})\frac{\sigma^2}{2}.$$

The above discussion implies that we have captured the mean-reverting behaviour of electricity spot prices while at the same time we can be able to apply the Black-Scholes formula to valuate spark spread options and location options. A closer look at the Ornstein-Uhlenbeck process reveals that this process is an extension of the GBM with mean-reverting parameter. However, so far we have not accounted for price sparks which are the very important characteristics of electricity spot prices. The Ornstein-Uhlenbeck process cannot capture price sparks but many researchers such as Weron [78] propose a Mean-Reverting Jump Diffusion (MRJD) process in order to account for both the mean-reverting and spikes behaviour of electricity spot prices. Black-Scholes formula however cannot be applied in this process.

3.3 Application of Black-Scholes formula on Sparks/Locations Spreads Option

As noted above, electricity is non-storable. Thus traditional storage-based methods of constructing replicating portfolio for commodities cannot be used to value electricity derivatives. In Chapter 1, it was noted that the so called electricity spot market is in fact a futures market with small maturity periods such as Nord Pool hourly futures. In [27] a method is presented that replicates electricity derivatives by dynamically trading futures contracts of the appropriate maturity. This method is based on the assumption that the futures contract must converge to the then current spot price. Thus, the underlying asset of the electricity derivative is no longer the electricity spot price but the electricity futures price. To illustrate the method, the method is used to derive the replicating strategy for spark and location spreads under a Geometric Brownian motion process and a Ornstein-Uhlenbeck process.

3.3.1 Valuation of spark and location spread options: geometric Brownian process

As mentioned above, the underlying asset here is the futures price. In the case of a spark spread, there are two futures prices namely the futures price of electricity and the futures price of the generating fuel. Thus the two Geometric Brownian processes would be presented as

$$\frac{dF_E}{F_E} = u_E dt + \sigma_E dB^1$$

and

$$\frac{dF_E}{F_G} = u_G dt + \sigma_G dB^2,$$

where B^1 and B^2 are two Brownian motion processes with instantaneous correlation ρ. And u_E, u_G, σ_E and σ_G are assumed to be constant.

The location spreads can be modelled the same way through

$$\frac{dF_E}{F_{E,1}} = u_{E,1}dt + \sigma_{E,1}dB^1$$

and

$$\frac{dF_E}{F_{E,2}} = u_{E,2}dt + \sigma_{E,2}dB^2.$$

Now denote the value of a spark call option which matures at time T by $V(x, y, t) \equiv C_1\left(F_E^{t,T}, F_G^{t,T}, K_H, T - t\right)$ and let $F_*^{t,T}$ represent the price at time t of the electricity futures contract with maturity date T. By constructing an instantaneously risk-free portfolio using the electricity and generating fuel futures contracts and the riskless asset, it follows that C_1 must satisfy the partial differential equation (PDE):

$$-V_t + \frac{1}{2}\left[x^2V_{xx}\sigma_x^2 + y^2V_{yy}\sigma_y^2 + 2xyV_{xy}\sigma_x\sigma_y\rho\right] = 0,$$

with boundary conditions

$$V(x, y, t) = \max(x - y, 0)\,, V(x, 0, t) = x \; and \; V(0, y, t) = 0.$$

Proposition 3.2.1.1 the Black-Scholes closed form solution for the value of a spark spread call option is

$$C_1\left(F_E^{t,T}, F_G^{t,T}, K_H, T - t\right) = e^{-r(T-t)}\left[F_E^{t,T}N(d_1) - K_HF_G^{t,T}N(d_2)\right],$$

where

$$d_1 = \frac{ln\left(\frac{F_E^{t,T}}{K_HF_G^{t,T}}\right) + v^2\,\frac{(T-t)}{2v}}{\sqrt{T-t}},$$

$$d_2 = d_1 - v\sqrt{T-t}$$

and

$$V^2 = \sigma_E^2 + \sigma_G^2 - 2\rho\sigma_E^2\sigma_G^2.$$

Proposition 3.2.1.2 the Black-Scholes closed form solution for the value of a location spread call option, C_2 is

$$C_1\left(F_{E,1}^{t,T}, F_{E,2}^{t,T}, K_L, T-t\right) = e^{-r(T-t)}\left[F_{E,1}^{t,T} N(d_1) - K_L F_{E,2}^{t,T} N(d_2)\right],$$

where

$$d_1 = \frac{ln\left(\frac{F_{E,1}^{t,T}}{K_L F_{E,2}^{t,T}}\right) + v^2 \frac{(T-t)}{2v}}{\sqrt{T-t}},$$

$$d_2 = d_1 - v\sqrt{T-t}$$

and

$$V^2 = \sigma_{E,1}^2 + \sigma_{E,2}^2 - 2\rho\sigma_{E,1}^2\sigma_{E,2}^2.$$

3.3.2 Valuation of spark and location spread options: Ornstein-Uhlenbeck process

Here the future price processes of electricity F_E and of the relevant generating fuel F_G follow the Ornstein-Uhlenbeck process (mean-reverting process):

$$\frac{dF_E}{F_E} = \propto_E (u_E(t) - \ln F_E)dt + \sigma_E(t)F_E B^1$$

and

$$\frac{dF_G}{F_G} = \propto_G (u_G(t) - \ln F_G)dt + \sigma_G(t)F_G B^1,$$

where $\sigma_E(t)$ and $\sigma_G(t)$ are functions of time t, $u_E(t)$ and $u_G(t)$ are the long-term means, $\propto_E$ and $\propto_G$ are the mean-revering coefficients (magnitude of speed of mean-reversion) and, B^1 and B^2 are two Brownian processes with instantaneous correlation ρ.

Now denote the value of a spark call option which matures at time T by $V(x,y,t) \equiv C_1\left(F_E^{t,T}, F_G^{t,T}, K_H, T-t\right)$ and let $F_*^{t,T}$ represent the price at time t of the electricity futures contract with maturity date T.

Proposition 3.2.2.1 The Black-Scholes closed form solution for the value of a spark spread call option is

$$C_1\left(F_E^{t,T}, F_G^{t,T}, K_H, T-t\right) = e^{-r(T-t)}\left[F_E^{t,T}N(d_1) - K_H F_G^{t,T}N(d_2)\right],$$

where

$$d_1 = \frac{ln\left(\frac{F_E^{t,T}}{K_H F_G^{t,T}}\right) + v^2\frac{(T-t)}{2v}}{\sqrt{T-t}},$$

$$d_2 = d_1 - v\sqrt{T-t}$$

and

$$V^2 = \frac{\int_t^T [\sigma_E^2(s) + \sigma_G^2(s) - 2\rho\sigma_E^2(s)\sigma_G^2(s)]}{T-t}.$$

Proposition 3.2.2.2 The Black-Scholes closed form solution for the value of a location spread call option, C_2 is

$$C_2\left(F_{E,1}^{t,T}, F_{E,2}^{t,T}, K_L, T-t\right) = e^{-r(T-t)}\left[F_{E,1}^{t,T}N(d_1) - K_L F_{E,2}^{t,T}N(d_2)\right],$$

where

$$d_1 = \frac{ln\left(\frac{F_{E,1}^{t,T}}{K_L F_{E,2}^{t,T}}\right) + v^2\frac{(T-t)}{2v}}{\sqrt{T-t}},$$

$$d_2 = d_1 - v\sqrt{T-t}$$

and

$$V^2 = \frac{\int_t^T \left[\sigma_{E,1}^2(s) + \sigma_{E,2}^2(s) - 2\rho\sigma_{E,1}^2(s)\sigma_{E,2}^2(s)\right]}{T-t}.$$

3.4 Application of binomial tree method on the pricing of financial transmission rights

Cox and Rubinstein [24] and Hull later [45] derived a binomial model for the pricing of financial derivatives. The model is derived under the assumption that the evolution of the underlying asset follows a geometric Brownian motion. Contreras et al [22] derived a binomial model for the pricing of Financial Transmission Rights (FTR). The nodal prices of electricity are determined by the interaction of supply and demand conditions subject to the physical constraints of the electricity grid. This is an apparent deviation from the tradition binomial model where the nodes are determined under an assumption of the Brownian motion. The supply and demand approach as presented by these researchers decomposes the nodal electricity prices into a large number of small binomial steps. This decomposition is the same as the one used in [24] and [45].

To discuss these approaches, firstly the one-step model is presented followed by the two-step model

3.4.1 Application of one step binomial model on the pricing of financial transmission rights

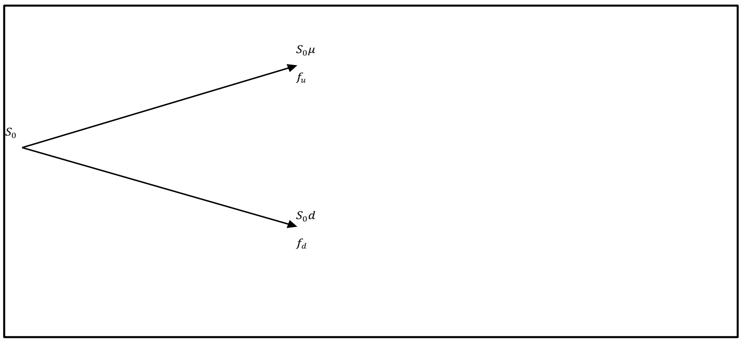

Figure 3.1 One-step binomial model

Consider the pricing of an option to invest in transmission assets. Suppose that the investment horizon is denoted as time t. An option to invest in transmission assets can be treated as a call option on a non-dividend paying asset where at the end of the investment horizon, the value of the option is worth

$$f = \max(S_T - K, 0),$$

where S_t the electricity spot price at time T, K is the strike price and f is the value of the option. Now denote the movement of electricity spot prices with u, the up movement and d, the down movement. Thus u and d represent the evolution of the electricity price at time interval Δt. Electricity spot prices move up and down with probabilities p and $(1-p)$, respectively. Note further that f_u is the payoff from the option if the electricity spot price moves up and f_d is the payoff from the option if the electricity spot price moves down. Contreras et al [22] obtained parameters $p, u\ and\ d$ from the supply and demand interaction. Jaillet et al [47] derived these parameters from the Brownian motion process.

Under the assumption of a risk neutral measure, the expected return from the investment must equal the risk-free rate. Thus the value of the option will be

$$f = e^{-rT}[pf_u + (1-p)f_d], \tag{3.1.1}$$

where

$$p = \frac{e^{rT} - d}{u - d}.$$

See [45] for the detailed proof:

It is therefore apparent that the value of the option is evaluated backwards where the value of the option today is the future payoffs discounted at the risk-free rate.

3.4.2 Application of two-step binomial tree for the pricing of financial transmission rights

We can extend the analysis of a generalised one-step binomial tree into a two-step binomial tree as indicated in Figure 3.2 below, where

f_{uu} : the payoff from the option after two up movement of the stock price.

f_{dd} : the payoff from the option after two down movement of the stock price.

f_{ud} : the payoff from the option after one up movement followed by one down movement of the stock price. This payoff is the same as the payoff from the option after one down movement followed by one up movement of the stock price and which can be represented by

f_{du}.

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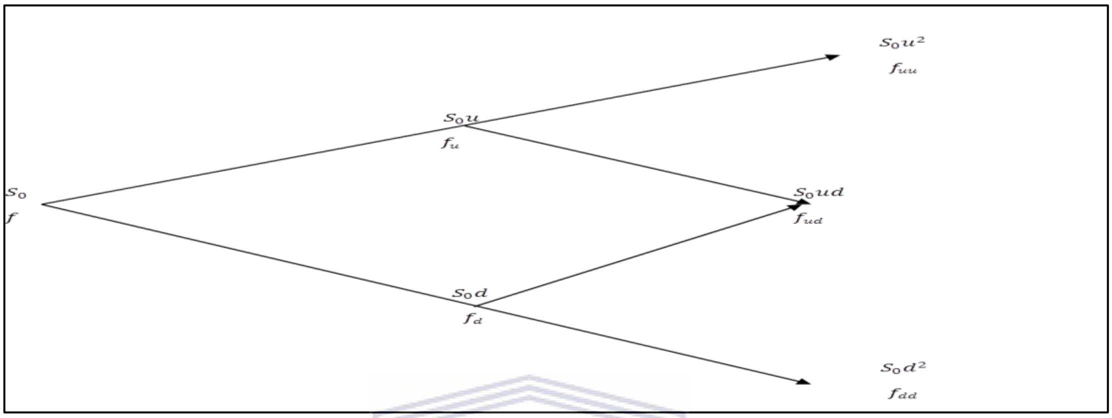

Figure 3.2 Two-step binomial tree

Application of Equation (3.1.1) moving backwards gives

$$f = e^{-r\Delta t}[pf_u + (1-p)f_d], \qquad (3.1.2)$$

where

$$f_u = e^{-r\Delta t}[p + (1-p)f_{ud}] \qquad (3.1.3)$$

and

$$f_d = e^{-r\Delta t}[pf_{ud} + (1-p)f_{dd}]. \qquad (3.1.4)$$

Upon simplification, we obtain

$$f = e^{-2r\Delta t}[p^2 f_{uu} + 2p(1-p)f_{ud} + (1-p)^2 f_{dd}], \qquad (3.1.5)$$

where

p^2, $2p(1-p)$ and $(1-p)^2$ in Equation (3.1.5) are the probability that the upper, middle and lower final nodes will be reached. The time interval between the nodes is denoted by Δt.

Thus the model can be replicated to evaluate a setting with steps greater than two by constructing large number of small binomial steps.

3.5 Summary

In this chapter, the Black-Scholes formula is used to evaluate spark and location spreads for both the GBM and the mean-reverting process. We have also presented a binomial model for the pricing of electricity derivatives. We have derived it on a transmission call option. In the case of electricity derivatives, the parameters of the binomial model can be obtained from supply and demand interaction as well as from the Brownian process.

We noticed both the Geometric Brownian motion and the mean-reverting process cannot adequately capture all the main characteristics of electricity derivatives. The Geometric Brownian motion cannot account for both the mean-reverting and the spike characteristics of electricity spot prices. While the mean-reverting process can account for the mean-reverting characteristic, it cannot account for the spikes in electricity spot prices.

In the following chapter a Mean-Reverting Jump Diffusion Model (MRJD) is proposed to model the behaviour of electricity spot prices.

Chapter 4

EFFICIENT MONTE CARLO METHODS FOR PRICING ELECTRICITY DERIVATIVES

4.1 Introduction

A lot of research is dedicated for developing models that fairly capture the behaviour of electricity spot prices. By inference and interpretation, many of these researchers agree that "Jump diffusion with mean-reversion or Mean-Reverting Jump Diffusion process (MRJD)" is a reasonable model to capture the behaviour of electricity spot prices, see for example [12] and [78]. In the previous chapter we were able to use the Black-Scholes formula to analytically evaluate options for both GBM and the mean-reverting process. In the case of MRJD, however, Black-Scholes formula cannot be used. Black-Scholes formula is derived based on the assumption that the evolution of the underlying asset is continuous and follows a standard normal distribution. However, the evolution of electricity spot prices with spikes cannot be considered continuous. Numerical methods that are normally used in exotic derivatives such as Binomial, Finite difference and Monte Carlo methods lend themselves naturally in evaluating electricity derivatives under the assumption of MRJD. We however prefer Monte Carlo methods to price electricity derivative where the evolution of the spot price is described by MRJD due to the fact that this method can handle even multidimensional problems. For Binomial and finite difference methods, computation can be tedious for such multi-dimensional problems.

Monte Carlo method is a simulation method that computes an arithmetic mean from many simulation runs. Inherently, this exercise produces variance and it is obvious that variance reduction would be necessary in order to return a mean that is close to actual variable observed. Normally, large simulation runs are required to return a mean with small variances. Thus any variance reduction technique should also account for efficiency in terms of labour required to do the simulation.

In this chapter, firstly we present the arbitrage argument in the pricing of electricity derivatives by looking at the forwards/futures contracts. Secondly, we use Mean-Reverting Jump-Diffusion process to describe the evolution of electricity spot prices. Thirdly we present Monte Carlo methods to evaluate a forward contract. Then, we present variance reduction techniques that account for efficiency.

4.2 Arbitrage argument on electricity forwards/futures contracts

Electricity forward contracts represent the obligation to buy or sell a fixed amount of electricity at a pre-determined contract price, known as the forward price, at certain time in the future called maturity or expiration. Electricity forwards are non- standardised supply contracts between a buyer and seller whereby the buyer is obliged to take a fixed amount of electricity and the seller is obliged to supply. Electricity futures have the same definition as electricity forwards except that electricity futures are highly standardised contracts with specifications such as the contract size, transaction requirements and settlement procedure. Furthermore, electricity futures are only traded on the organised exchanges while electricity forwards are usually traded over-the-counter in the form of bilateral transaction.

Non-storability of electricity make electricity forwards different from other financial and commodity forwards contracts. Electricity is a pure flow variable (energy per time) measured in Kilowatts per hours (kWh) or Mega-Watts per hour (MWh). The non-storability of electricity leads to :

- Delivery of a specified constant electricity level takes place over a period of time rather than specific point in time. The value of electricity forwards would therefore be

$$F(t, T_1, T_2) = F(T_1, T_2) - \sum_{T_1}^{T_2} S_t / (T_2 - T_1).$$

  That is, the value of the forward contract is the difference between the closing forward price at time T_1, $F(T_1, T_2)$ and the average of the spot price S_t in $[T_1, T_2]$. Thus this difference is paid to the buyer and is charged to the seller and if the difference is negative, the seller gains and the buyer loose.

- The breakdown of the relationship which prevails at equilibrium between spot and future price on the financial instruments. Convenience yield also cannot be determined. Brennan and Schwartz [19] incorporated convenience yield in the valuation of commodity derivatives and established in particular that the relationship between the spot price S_t and the future price $F(t,T)$ of a contract of maturity T is

  $$F(t,T) = S_t e^{(r-y)(T-t)}.$$

  The spot price and the forward price in the electricity market however do not converge because the "no arbitrage" argument used to establish this relation is not valid since it requires that the underlying instrument be bought at time t and held until expiration of the future contract (buy and hold strategy), see Figure 4.1.

Botterud [14] studied Nord Pool's futures market and the results of the showed that the futures price equals the expected future spot price. Botterud et al [15] conducted a similar analysis but this time merely observing future prices and spot price in the Nord Pool for the period 1996 to 2001. They deduced that it is reasonable to believe that the market expects the future prices to resemble the expected spot price.

Many researchers use such no-arbitrage assumption when evaluating electricity derivatives. Deng et al [27] uses the no-arbitrage assumption based on the convergence of the expected electricity spot price and the futures price.

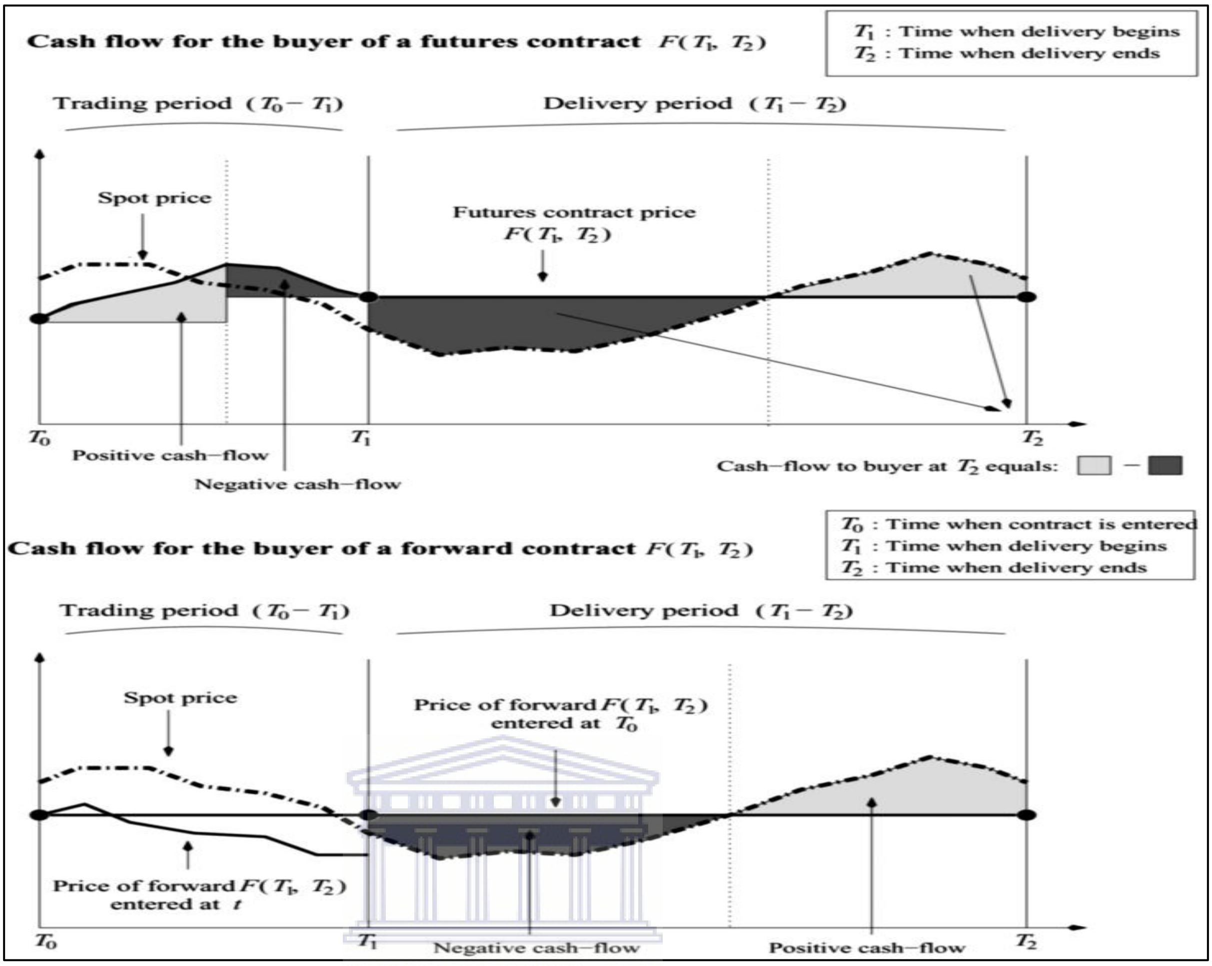

Figure 4.1 Source [37].The structure of future contracts (top figure) and forward contracts (bottom figure) supplied by Nord Pool.

4.3 Application of mean-reverting jump diffusion process on electricity spot prices

Electricity prices are somewhat different from other prices set in the financial markets. In particular, electricity prices are characterised by abrupt and unanticipated large changes known as jumps or spikes. Technically, however, electricity prices do not jump but spike. That is, electricity prices do not jump to a new level and stay there, but rather they quickly revert to their previous levels. Within a very short space of time, prices can increase substantially and then drop back to the previous level.

A lot of literature has been devoted on the stochastic process of electricity price movements. Many of these pointed out the inadequacy of the diffusion process in accurately reflecting the evolution of electricity spot prices. The diffusion process fails to capture the mean-reversion and spikes found in the evolution of electricity spot prices. Subsequently researchers such as

Weron [78] propose a Mean-Reverting Jump-Diffusion (MRJD) process in order to closely capture the evolution of electricity spot prices.

The MRJD is in fact a mean-reverting process with jump factor. To fix the ideas, we first present the mean-reverting process. In this, the price process satisfies the SDE:

$$d\log S_t = k(u - \log S_t)dt + \sigma dB_t,$$

where $k > 0$ is the magnitude of the speed of reversion to the long-run mean log-price, u is considered constant under an equivalent risk neutral measure (equivalent martingale measure) Q.

Solving the above Ornstein-Uhlenbeck SDE, we get the expression for the process $\{X_t, t \geq 0\}$ where $X_t \equiv \log S_t$:

$$X_t = X_0 e^{-t} + \mu \int_0^t e^{-(t-s)}\, ds + \sigma \int_0^t e^{-(t-s)}\, dBs$$

$with$ $\{X_t, t \geq 0\}$ and $0 \leq s \geq t$.

Thus the conditional expectation and conditional variance under the probability measure, Q and filtration $\mathcal{F}$ will be

$$E_Q[X_t|\mathcal{F}_s] = \mu + (X_0 - \mu)e^{-t}$$

and

$$Var_Q[X_t|\mathcal{F}_s] = (1 - e^{-2t})\frac{\sigma^2}{2}.$$

Now we add the jump factor so that a complete MRJD model can be expressed as follows.

$$dlogS_t = k(u - logS_t)dt + \sigma dB_t + \log(1 + J)\, dP_t,$$

where

μ: the mean reversion level or long run equilibrium price,

$logS_t$: the spot price at time t,

k: the mean reversion rate,

$\log(1+J)$: the log-return jump size with log-return mean μ_j and variance σ_j^2.

P_t: a simple Poison jump process with jump rate λ.

Parameters u, σ, μ_j *and* σ_j are assumed to be constant. The jump process Jdp has a mean $E[J]\lambda dt$ and variance $E[J^2]\lambda dt$. Also dP_t has a discrete distribution :

$$p_n(\lambda dt) = Prob[dP(t) = n] = \frac{e^{\lambda \Delta t (\lambda \Delta t)^n}}{n!}\ ,\ \ n = 0{:}\infty.$$

The above model has two parts

1) $k(u - logS_t)dt + \sigma S_t dB_t$ is the Mean-Reverting Diffusion process /Ornstein-Uhlenbeck process,
2) Jdp is the Poison Process.

The Ornstein-Uhlenbeck process would thus capture the fact that electricity prices tend to fluctuate around values determined by the cost of production and the level of demand (mean-reversion and seasonality of electricity spot prices). On the other hand, the Poison process would capture the electricity price sparks.

In this model, the continuous lognormal diffusion process $B(t)$ and the Poison process $P(t)$ are assumed to be independent. Blanco and Soronow [12] argued that this is not the case for the electricity derivatives. For example, prices are highly unlikely to spike overnight when the demand is very low. Thus a MRJD that takes cognisance of the existence of some kind of correlation between $B(t)$ and $P(t)$ would be more appropriate in modelling electricity spot prices.

4.4 Application of Monte Carlo methods in pricing electricity derivatives

The electricity spot market in its strictest definition would mean a simultaneous deliver of asset with the payment by the buyer of that particular asset. However such a transaction is not possible with electricity. The supply of the electricity would require some time to match its capacity with demand. As a result, the so called electricity spot market such as Nord Spot Market in Europe settles the delivery on an hourly basis. In essence this is a forward market with one hour delivery.

Since forward prices are tradable instruments; options, spread, swings, and weekly, monthly and yearly forward contracts are written on these forward prices. (see [27] for the Black-Scholes solution for the value of a range of cross-commodity derivatives, including sparks and locations spread options in which the underlying price process follow a Geometric Brownian Motion (GBM) and mean reverting).

An asset that assumed to exhibit jumps such as electricity means that derivatives written on this asset may not be valued by the Black-Scholes formula. The Black-Scholes formula assumes that asset prices changes are continuous and follow a Geometric Brownian Motion. We use Monte Carlo methods in valuating electricity derivatives. Indeed, Either and Doris [32] conclude that neglecting jumps can lead to over-valuation for in-the-money options and under-valuation for out-of-the-money options. Du [30] employed Monte Carlo methods on mean-reverting jump diffusion process to evaluate electricity derivatives and confirmed earlier findings by Either and Doris [32].

By the general Monte Carlo simulation, suppose we want to estimate$Y$, the expected value of some random variable X, i.e.,

$$Y = E[X].$$

Further suppose that we are able to generate the value of independent random variables having the same probability distribution as X. Each time we generate a value, we say that a simulation run is completed. Suppose we perform k simulation runs and generated the values of $X_1, X_2 \ldots, X_k$. If we let

$$\bar{X} = \frac{1}{k}\sum_{i=1}^{k} X_i,$$

be their arithmetic average, then $\bar{X}$ can be used as an estimator of Y. The expected value of $\bar{X}$ would be

$$E[\bar{X}] = \frac{1}{k}\sum_{i=1}^{k} E[X_i] = Y,$$

and its variance would be

$$v^2 = var(X).$$

Now

$$Var(\bar{X}) = Var\left(\frac{1}{k}\sum_{i=1}^{k} X_i\right)$$

$$= \frac{1}{k^2} Var\left(\sum_{i=1}^{k} X_i\right)$$

$$= \frac{k}{k^2}\sum var(\bar{X}_l) \text{ (By independence)}$$

$$= {v^2}/{k}.$$

It also follows from the central limit theorem that, for large k, $\bar{X}$ will have an approximately normal distribution. Hence then $\bar{X}$ will tend to be near Y. This highlight the importance of variance reduction in Monte Carlo methods which we are going to deal with in the next chapter.

4.4.1 Monte Carlo methods in Pricing Electricity Forwards

Earlier in this chapter, we discussed electricity forwards. We presented the value of electricity forwards as follows

$$F(t, T_1, T_2) = F(T_1, T_2) - \sum_{T_1}^{T_2} S_t/(T_2 - T_1).$$

Here S_t= $E[S]$

Suppose $S_i = S_0 \exp(X_i)$ where $i = 1, 2, \dots \dots . n$, and

$$(X_t - X_0) = k(u - X_t)\Delta t + \sigma\Delta B(t) + \text{J}\Delta P(t).$$

Assuming a week forward contract, we can compute $\sum_{T_1}^{T_2} S_t/(T_2 - T_1)$ as follows, assuming a week forward contract.

$$S_1 = \frac{1}{k} S_0 e^{\sum_{i=1}^{k} X_i},$$

$$S_2 = \frac{1}{k} S_1 e^{\sum_{i=1}^{k} X_i},$$

• •

• •

$$S_7 = \frac{1}{k} S_6 e^{\sum_{i=1}^{k} X_i},$$

We then have $S_{(T_2 - T_1)} = \frac{1}{7} \sum_{i=1}^{7} S_i$.

Then compute $F(t, T_1, T_2) = F(T_1, T_2) - \sum_{T_1}^{T_2} S_t / (T_2 - T_1)$.

As we have mentioned above, the expected value of $\bar{X}$ would be

$$E[\bar{X}] = \frac{1}{k} \sum_{i=1}^{k} E[X_i] = Y.$$

Its variance is

$$Var(\bar{X}) = Var\left(\frac{1}{k} \sum_{i=1}^{k} X_i\right).$$

A numerical example (simulated in excel) incorporating this Monte Carlo simulation is presented below.

Table 4.4.1 parameters of MRJD process

Initial Price	50.00	
Time (T)	7	days
Drift (μ)	20.00%	
Volatility (σ)	40.00%	
Intensity (λ) Lambda	0.50	number of jumps per year
Jump size (κ) Kappa	0.2%	as % of previous spreads level
% spreads level after jump (J)	100.2%	as % of previous spreads level
γ Gamma = ln(1+κ)	0.001998	Drift of ln (J)
Standard deviation log of J (σ')	50.00%	
J (+σ')	1.65201871	
J (-σ')	0.60774372	
Δ T	0.0220	
Mean Reverting Speed	0.15	

Table 4.4.2 Excel numerical results of electricity spot prices using MRJD

Period	Time	Norminv(Rand(),0,1)	Rand()	Price(1000 runs)	Jumps
0	0	0.71891064		50.00	
1	1	-0.36462745	0.00083719	53.93	4.90
2	2	-0.15773325	0.10254109	53.52	4.90
3	3	0.30506471	0.53986724	54.59	4.90
4	4	-0.49451575	0.82353606	53.10	4.90
5	5	-0.11627421	0.94602879	52.82	4.90
6	6	-0.5687199	0.36478717	51.16	4.90
7	7	0.71503219	0.67132181	53.48	4.90

Table 4.4.2 shows the expected closing electricity spot prices for the next 7 days. For each day we run 1000 simulation and compute an arithmetic average. Table 4.4.2 further shows expected jumps for the next 7 days.

In an estimation exercise, variance represents an error of estimation. Reducing the variance therefore becomes very important in order for an accurate result. Hence in what follows, we will discuss variance reduction techniques.

4.5 Application of Variance Reduction Techniques on electricity derivatives

In this section, we first discuss the rational of variance reduction in achieving the main objective of improving the computational efficiency of Monte Carlo simulation. Subsequently, we discuss some specific variance reduction techniques such as, method of antithetic variables, control variate technique, moment matching and stratified sampling.

4.5.1 Variance reduction and efficiency improvement on electricity derivatives

Suppose that we want to compute a parameter Y, the price of an electricity derivative. Further suppose that using Monte Carlo simulation we can generate simulations of sequence $(\bar{X}_1, \mathrm{i} = 1,2, \dots)$, independent and identically distributed (i.i.d) where each $\bar{X}_1$ has expectation Y and variance σ^2. As in previous section, an estimator of Y based on n replications is then the sample mean

$$\frac{1}{n}\sum_{i=1}^{n}\bar{X}_1.$$

By the central limit theorem[4], for large n this sample mean is approximately normally distributed with mean Y and variance $\frac{\sigma^2}{n}$. Thus the error in the indicator is proportional to the standard deviation, $\frac{\sigma}{\sqrt{n}}$. It follows therefore that reducing the variance σ^2 by a factor of 10, for example, with everything remains constant, would yield an error reduction equivalent to increasing the number of samples by a factor of 100, see [15] for further clarification.

Now suppose that we have a choice between two types of Monte Carlo estimates which we denote by $(\bar{X}_i^{(1)}, i = 1, 2, \dots)$ and $(\bar{X}_i^{(2)}, i = 1, 2, \dots)$. Further suppose that both $\bar{X}_i^{(1)}$ and $\bar{X}_i^{(2)}$are unbiased, so that $E\left[\bar{X}_i^{(1)}\right] = E\left[\bar{X}_i^{(2)}\right] = Y$ and $\sigma_1 < \sigma_2$, where $\sigma_j^2 = Var\left(\bar{X}^{(j)}\right)$, $j = 1,2$. It follows then that a sample mean of n replications of $\bar{X}^{(1)}$ gives a more precise estimator of Y than does a sample mean of n replication of $\bar{X}^{(2)}$. This conclusion is based on the fact that the standard deviation of the first sample is less than the standard deviation of the second sample.

However, Botterud et al [15] argue that "this analysis oversimplifies the comparison because it fails to capture possible differences in the computational effort required by the two estimators. Generating n replications of $\bar{X}^{(1)}$ may be more time-consuming than generating n replications of $\bar{X}^{(2)}$". In essence they argued for a more holistic approach that combines variance reduction and efficiency. Thus having a smaller variance is not sufficient to prefer one estimator over another.

To compare estimators with different computational requirements and different variances, the authors in [15] argued as follows: suppose the work required to generate one replication of $\bar{X}^{(j)}$ is a constant $b_j, j = 1, 2$. With computation time t, the number of replications of $\bar{X}^{(j)}$ that can be generated is $\frac{t}{b_j}$. For simplicity, we assume that $\frac{t}{b_j}$ is an integer. Thus the two estimators available with computing time t are

$$\frac{b_1}{t}\sum_{i=1}^{t/b_1}\bar{X}^{(1)} \text{ and } \frac{b_2}{t}\sum_{i=1}^{t/b_2}\bar{X}^{(2)}.$$

---

[4] This theorem states that the sum of a large number of independent random variables, all having the same probability distribution, will itself be approximately a normal random variable.

By the central limit theorem, for large t, these are approximately normally distributed with mean Y and standard deviations

$$\sigma_1\sqrt{\frac{b_1}{t}} \text{ and } \sigma_2\sqrt{\frac{b_2}{t}}.$$

That is, for large t, the first estimator should be preferred over the second estimator if

$$\sigma_1^2 b_1 < \sigma_2^2 b_2.$$

The above equation therefore provides a reasonable basis for trading-off estimator variance and computational requirements.

4.5.2 Application of antithetic variance reduction technique on electricity derivatives

Antithetic variable technique involves calculating two values of a derivative and taking their average as the price of the derivative. Suppose we want to compute the price of an electricity derivative and denote it by C. We then simulate the first value C_i using the Z_i, independent samples from the standard normal distribution. The second value $\bar{C_i}$ is calculated by changing the sign of all the Z_i in the first simulation run. The sample value of the derivative calculated from the simulation run is,

$$C_{AV} = \frac{1}{n}\sum_{i=1}^{n}\frac{C_i + \bar{C_i}}{2},$$

where C_{AV} is the unbiased estimator of the price of the derivative. The argument for preferring C_{AV} is that the random inputs obtained from the collection of antithetic pairs $(Z_i \, and - Z_i)$ are more regularly distributed than a collection of $2n$ independent samples [17]. Precisely, the sample mean over the antithetic pairs always equals the population mean of zero, whereas the mean over finitely many independent samples is almost surely different from zero.

As we indicated in Section 4.4.1 that a more holistic approach that incorporate efficiency is desirable. We shall then look at the efficiency of the antithetic variable technique. Since C_i and $\bar{C_i}$ have the same variance,

$$Var\left(\frac{C_i + \bar{C}_i}{2}\right) = Var(C_{AV}) = \frac{1}{2}\left(Var(C_i) + Var(\bar{C}_i) + Cov(C_i, \bar{C}_i)\right).$$

Thus $Var(C_{AV})$ should be $\leq$ $Var(\bar{C}_i)$ if $Cov(C_i, \bar{C}_i) \leq Var(C_i)$. But it is important to note that C_{AV} uses twice as many replications as $\bar{C}_i$, therefore, we must account for differences in computational requirements. Thus we require

$$2Var(C_{AV}) \leq Var(\bar{C}_i),$$

which, requires that $Cov(C_i, \bar{C}_i) \leq 0$. That is, C_{AV} simulation run should be $\leq \frac{n}{2}$ for n simulation run of $\bar{C}_i$. Thus one can reasonable choose C_{AV} over $\bar{C}_i$ if the following conditions are met simultaneously:

$$Var(C_{AV}) \leq Var(\bar{C}_i)$$

and the C_{AV} simulation run is at most half the $\bar{C}_i$ simulation run. This process will reduce variance while at the same time it is time efficient.

4.5.3 Application of control variate technique on electricity derivatives

Control variate technique is among the most widely applicable, easiest to use, and an effective variance reduction techniques [15]. This technique relies on one or more auxiliary random variables called controls and uses information about these variables to reduce the variance of the estimator of concerned variable. In our case, control variate technique calls for the use of the information of the underlying asset, X (electricity spot price), to reduce the variance of the estimator $E[Y]$ for the price electricity derivative.

According to Borogovac and Vakali [13], an effective control needs to satisfy two requirements:

- X needs to be correlated with Y and
- $E[X]$ needs to be available to the user, i.e., known.

Numerical results of Kemna and Vorst [50] indicate that X and Y are indeed strongly correlated. The second requirement can also be easily achieved if the mean of the underlying asset can be analytically evaluated.

Suppose further that we analytically evaluate the mean of the underlying electricity spot price,

$$\mu_x = E[X].$$

Then instead of using the value of $\bar{Y}$ as the estimator, we can use one of the form

$$\bar{Y}^{cv} = Y + c(X - \mu_x),$$

where $\bar{Y}^{cv}$ is the control variate estimator of Y, c is the constant to be specified and $(X - \mu_x)$ is the known error. Thus $\bar{Y}^{cv}$ adjusts the straight forward estimator $\bar{Y}$ according to the difference between the known value X and the evaluated or observed value μ_x. Note that $\bar{Y}^{cv}$ also estimates Y, i.e.,

$$E[\bar{Y} + c(X - \mu_x)] = E[\bar{Y}] + cE[X - \mu_x] = Y + c(\mu_x - \mu_x) = Y.$$

The best estimator of $\bar{Y}^{cv}$ is obtained by choosing c to be the value that minimises

$$Var\big(\bar{Y} + c(X - \mu_x)\big).$$

Now

$$Var\big(\bar{Y} + c(X - \mu_x)\big) = Var(\bar{Y} + cX)$$

$$= Var(\bar{Y}) + Var(cX) + 2Cov(\bar{Y}, cX)$$

$$= Var(\bar{Y}) + c^2Var(X) + 2cCov(\bar{Y}, X).$$

We then differentiate the last equation with respect to c, set the derivative equal to 0 and solve for c. By just observing this equation, it is clear that its second derivative with respect to c is positive. Thus the value of c that minimises $Var\big(\bar{Y} + c(X - \mu_x)\big)$ is

$$c^* = -\frac{Cov(\bar{Y}, X)}{Var(X)}. \qquad (4.4.1)$$

Substituting this value back gives

$$Var\big(\bar{Y} + c^*(X - \mu_x)\big) = Var(\bar{Y}) - \frac{Cov^2(\bar{Y}, X)}{Var(X)}.$$

Dividing both sides of this equation by $Var(\bar{Y})$ leads to

$$\frac{Var\left(\bar{Y} + c^*(X - \mu_x)\right)}{Var(\bar{Y})} = 1 - corr^2(\bar{Y}, X). \tag{4.4.2}$$

Since

$$corr(\bar{Y}, X) = \frac{cov(\bar{Y}, X)}{\sqrt{Var(\bar{Y})Var(\bar{X})}},$$

is the correlation between $\bar{Y}$ and X. It can be easily observed that left hand side of (4.4.2) is the proportion of $Var(\bar{Y}^{cv})$ to $Var(\bar{Y})$. It follows that if this proportion is less than one, we have then managed to reduce the variance of the estimator of Y. From the right hand side of (4.4.2), it is clear that if there is some positive correlation between $\bar{Y}$ and X, then almost surely this proportion is less than one.

The quantities $Cov(\bar{Y}, X)$ and $Cov(X)$, which are needed to determine c^* are not usually known. Ross [67] suggests that these quantities can be estimated from the simulated data. If k simulation runs produce the output $\bar{Y}_i$ and X_i where $(i = 1, \ldots \ldots, k)$ then, letting

$$\hat{Y} = \sum_{i=1}^{k} \frac{\bar{Y}_i}{k} \; and \; \hat{X} = \sum_{i=1}^{k} \frac{X_i}{k},$$

be the sample means, $Cov(\bar{Y}, X)$ is estimated by

$$\frac{\sum_{i=1}^{k}\left(\bar{Y}_i - \hat{Y}\right)\left(X_i - \hat{X}\right)}{k-1}, \tag{4.4.3}$$

and $Cov(X)$ is estimated by the sample variance

$$\frac{\sum_{i=1}^{k}\left(X_i - \hat{X}\right)^2}{k-1}. \tag{4.4.4}$$

Combining (4.4.1), (4.4.3) and (4.4.4) we obtain the estimator of c^* as

$$\hat{c}^* = \frac{\sum_{i=1}^{k}\left(\bar{Y}_i - \hat{Y}\right)\left(X_i - \hat{X}\right)}{\sum_{i=1}^{k}\left(X_i - \hat{X}\right)^2},$$

and this produces the following controlled simulation estimator of Y

$$\frac{1}{k}\sum_{i=1}^{k}(\bar{Y}_i + \hat{c}^*(X_i - \mu_x)).$$

4.5.4 Application of moment matching methods on electricity derivatives

Moment matching variance reduction technique also known as quadratic re-sampling was first proposed by Barraquand [5]. It involves adjusting the sample taken from the standardised normal distribution so that the first, second and possibly higher moments are matched. Suppose we sample from a normal distribution with mean zero and standard deviation 1 to calculate the change in the value of an electricity derivative. Suppose that the samples are X_i $(1 \leq i \leq n)$. To match the first two moments, we calculate the mean of the sample, m and the standard deviation of the sample, s. We then define the adjusted samples X_i^* $(1 \leq i \leq n)$ as follows

$$X_i^* = \frac{X_i - m}{s}.$$

These adjusted samples have the correct mean of zero and the correct standard deviation of 1. We then use these adjusted samples for all our calculations.

A major disadvantage of moment matching is that the X_i^*'s are not independent as mentioned Botterud et al [15]. In the standard Monte Carlo, confidence intervals for the true value of the derivative Y could be estimated from the sample mean and variance of estimator. This is impossible because the n values of the estimator, $\hat{Y}$, are not independent. To over-come this challenge, authors in [15] suggest that moment matching be applied to independent batches of runs and estimate the standard error from the batch means.

4.5.5 Application of stratified sampling on electricity derivatives

Stratified sampling involves sampling representative values rather than random value from a probability distribution. Thus stratified sampling seeks to make the inputs to simulation more regular than random inputs. In particular, stratified sampling forces certain empirical probability to match theoretical probability. This is similar to the moment matching technique that forces empirical moments to match theoretical moments.

In [15], Botterud et al used the following example to illustrate stratified sampling.

Consider the generation of 100 normal random variates as inputs to a simulation. The empirical distribution of an independent sample $X_1 \dots \dots .. X_{100}$ will look only roughly like the normal density. The tails of the distribution, often the most important part, will inevitably be under represented. Stratified sampling can be used to force exactly one observation to lie between the $(i-1)^{th}$ and i^{th} percentile; $i = 1 \dots \dots \dots 100$. Thus produce a better match to the normal distribution. One way to implement this generates 100 independent random variates $U_1 \dots \dots U_{100}$, uniform on $[0, 1]$ and sets

$$\hat{X}_i = N^{-1} \frac{i+U_i-1}{100} \text{ where, } i = 1 \dots \dots \dots 100$$

and N^{-1} is the inverse of the cumulative normal distribution. This works because $\big((i + U_i - 1)/100\big)$ falls between the $(i-1)^{th}$ and i^{th} percentiles of the uniform distribution and are preserved by the inverse transform.

This would force the empirical probability to match theoretical probability. However, as with the moment matching technique, the $\hat{X}_i$'s are no longer independent and therefore it is impossible to compute confidence intervals. To overcome this challenge, Botterud et al [15] suggested the same procedure of batching the runs.

4.6 Summary

In this chapter, we presented the arbitrage argument in the valuation of electricity derivatives. We presented the finding of Botterud [14] about the converging of expected electricity spot prices and futures prices. We then concluded that an arbitrage argument can be assumed in the valuation of electricity futures/forward contracts based on the convergence of expected electricity spot prices and future/forward prices.

We then used the MRJD process to describe the evolution of electricity spot prices. We showed that MRJD is infact the combination of mean-reverting process and poison process. We argued that the celebrated Black-Scholes formula cannot be used to valuate derivatives with an underlying asset that exhibit jumps. Black-Scholes formula is derived under the assumption that the evolution of the underlying asset is continuous and normally distributed. However, we argued that changes in electricity spot prices are far from being continuous. We therefore used Monte Carlo methods for the pricing of electricity derivatives. We used Monte Carlo methods in pricing electricity forward contract that has a 7 day maturity. We then discussed the importance of employing efficient variance reduction techniques to improve the efficiency of the Monte Carlo methods. We successfully demonstrated various variance reduction methods that can be used in valuation of electricity derivatives and accounted for efficiency.

In the next chapter, we present comparative numerical results.

Chapter 5

COMPERATIVE NUMERICAL RESULTS

5.1 Introduction

In earlier chapters, we mentioned the main characteristics of electricity prices :

- **Price Sparks:** electricity spot prices exhibit occasional price spikes due to supply shocks such as transmission constraints and unexpected outages.
- **Seasonality:** it is well known that electricity demand exhibits seasonal fluctuations (see [35], [49] and [62]). These fluctuations often arise due to changing climate conditions, such as temperature and the number of daylight hours.
- **Mean-Reversion:** Electricity prices tend to fluctuate around values determined by the cost of production and the level of demand.
- **Non-Storability:** electricity cannot be stored and once generated it needs to be consumed almost immediately.

Price process models are the soul of derivatives pricing. If the prices process chosen to price any derivative does not fully capture the main characteristics of the underlying asset price, the results from the model are likely to be unreliable. It follows the same logic when pricing electricity derivatives. In order to accurately price electricity derivatives, the price process chosen must capture all the main characteristics of electricity prices. In this chapter, we present comparative obtained by Black-Scholes formula, binomial trees and Monte Carlo methods for pricing of electricity derivatives. In order to do this comparison, we look at the following price process: (a) Geometric Brownian Motion (b) Mean-reverting process (c) Jump-diffusion and (d) Mean-reverting jump-diffusion.

Before we proceed, note that the distribution of electricity prices exhibit heavy tails due to generally unanticipated extreme changes in the spot prices known as spikes or jumps. Within a very short period of time, spot prices can increase substantially and then drop back to previous levels when the prime mover such as the weather phenomenon or outage is over

[77]. Figure 5.1 depicts these spikes clearly. These spikes account for a large part of the total variation of changes in spot prices [78].

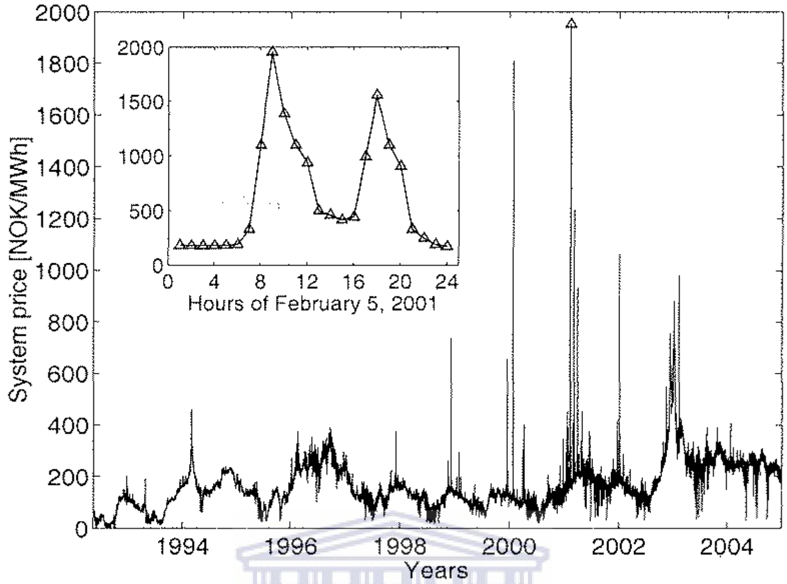

Figure 5.1: source [78]. Hourly system price for the spot market (Elspot) at the Nordic power exchange Nord Pool from May 4, 1992 until December 31, 2004.

Furthermore, Figure 5.1 also exhibits strong seasonality and mean-reverting characteristics. It is therefore apparent that any price process chosen to price an electricity spot price derivative that does not capture the spikes would most likely be unreliable.

5.2 Numerical results of geometric Brownian motion on electricity spot prices

Under a Geometric Brown Motion the electricity spot price $X_t \equiv \log S_t$ follows the stochastic differential equation

$$\frac{dS_t}{S_t} = \mu dt + \sigma dB_t,$$

where μ, σ are assumed to be constant and $\{B_t, t \geq 0\}$ is a standard Brownian motion. By design, B_t is supposed to capture the randomness of electricity spot prices. Note that a random process $B_t, t \in [0, T]$ is a Brownian motion if

- B_t has both stationary and independent increments: $\forall\ 0 \leq s \leq T$, the increments $B_s - B_0$ and $B_T - B_t$ are independent random variables,

- $\forall\ \omega \in \Omega$, the price path $t \mapsto B_t(\omega)$ is a continuous function,

- For $0 \leq s \leq t, B_t - B_0$ is normally distributed with mean $\mu(t-s)$ and variance $\sigma^2(t-s)$. That is $B_t - B_0 N(\mu(t-s)), \sigma^2$ (t − s), μ and σ are real numbers and $\sigma \neq 0$.

The break-down of the Geometric Brownian motion in modelling electricity spot prices is easy to identify. Firstly, the price spikes make the electricity price path to be discontinuous as it is evident in Figure 5.1. Secondly, the price spikes result in the distribution of electricity prices exhibiting heavy tails. Thus to assume a normal distribution would understate the existence of the price spike which we have stated above that the variation in electricity spot prices is largely due to spikes. Thirdly, the deterministic part of the SDE clearly does not capture the mean-reverting characteristic of electricity spot prices. Thus it can be concluded that using the Black-Scholes formula to price electricity spot prices would most likely yield to unreliable results.

Deng et al [27] applied the Black-Scholes formula in the valuation of electricity generation and transmission. They assumed a Geometric Brownian motion and presented a methodology for valuing electricity derivatives by constructing portfolios with futures contracts and risk free asset. Their methodology assumed that the future price processes of electricity and the appropriate generating fuel, denoted respectively by F_e and F_b, follow a Geometric Brownian motion processes

$$dF_e = \mu_e F_e dt + \sigma_e F_e dB^1,$$

$$dF_g = \mu_g F_g dt + \sigma_g dB^{2,}$$

where B^1 and B^2 are two Brownian motion processes with instantaneous correlation ρ and μ_e, μ_g, σ_e and σ_g are assumed to be constants. The methodology set the scene perfect for the application of the Black-Scholes formula. They also presented a Black-Scholes closed form solution for Spark spread options. However, the short-coming of this methodology was the assumption that electricity future price processes and the appropriate generation fuels follow a Geometric Brownian motion process. Note that the Geometric Brownian motion processes cannot capture mean-reversion and spikes of electricity spot prices. The evolution of electricity future prices resembles that of electricity spot prices. Subsequently, this leads to the mispricing of these derivatives. Inability to factor-in spikes leads to over-valuation for in-the-money options and under-valuation for out-of-the-money options as pointed out by Either and Doris [32].

Table 5.2.1 shows a forecast of electricity spot prices for the next 100 days. Using a Geometric Brownian motion with the following parameters:

- $P_0 = R10$, closing electricity spot price at day 0
- $\sigma = 20\%$, volatility
- $\alpha = 10\%$, drift parameter
- $r = 8\%$, risk-free rate

We use the following excel functions:

- RAND(): Uniform Random Variable Between 0 and 1
- NORM. S. INV(Rand()): Standard Normal Random Variable N(1,0)

Then we compute

$$P_t = \exp(-r * t) * P_0 * (\exp(\alpha - 0.5\sigma^2)\Delta t + NORM.S.INV\big(Rand(\quad)\big) * SQRT(t) * \sigma$$

Table 5.2.1 illustration of the electricity spot prices using GBM

Days	t	SQRT(t)	Norm.S.INV (Rand())	Pt
0	0	0	-1.44	10
1	0.01	0.10	0.58	9.96
2	0.02	0.14	0.18	9.95
3	0.03	0.17	3.72	9.89
4	0.04	0.20	0.44	9.89
5	0.05	0.22	-1.04	10.27
6	0.06	0.24	0.63	10.25
95	0.95	0.97	0.28	10.40
96	0.96	0.98	-1.41	10.33
97	0.97	0.98	0.24	10.25
98	0.98	0.99	-1.53	10.20
99	0.99	0.99	-1.67	10.17
100	1.00	1.00	0.03	10.15

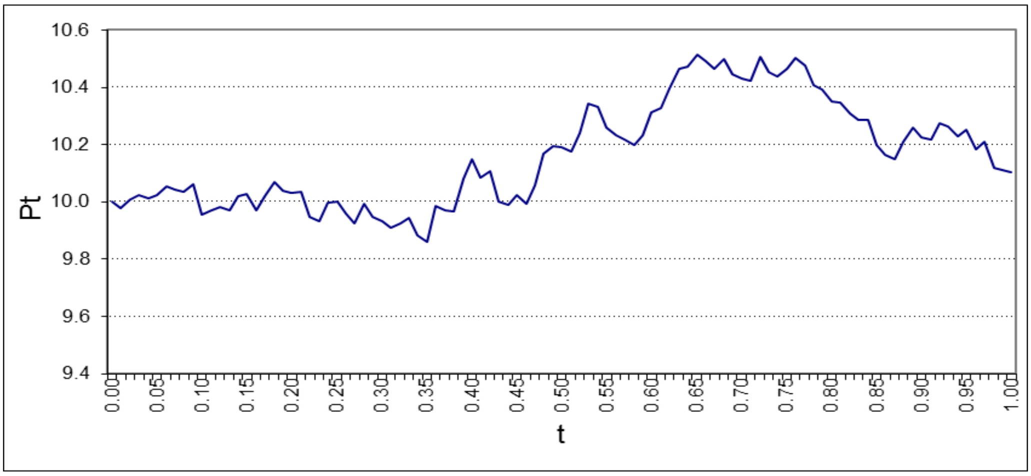

Figure 5.2.1 Electricity spot prices (from Table 5.2.1) using GBM

Figure 5.2.1 depicts electricity spot prices for the next 100 days. Comparing Figure 5.2.1 and Figure 5.1, it is apparent that the former does not capture mean-reverting and spikes that characterise electricity spot prices.

5.3 Numerical results of Ornstein-Uhlenbeck process on electricity spot prices

Ornstein-Uhlenbeck process is able to capture the mean-reverting behaviour of electricity spot prices. In this process, the price process satisfies the SDE:

$$d \log S_t = k(\mu - \log S_t)dt + \sigma dB_t,$$

where k>0 is the magnitude of the speed of reversion to the long-run mean log-price, μ is considered constant under an equivalent risk neutral measure Q. The intuition behind this SDE is that deviations of the price from the equilibrium level $(\mu - \log S_t)$ are corrected at the rate k and subject to random perturbations σdB_t. It is therefore easy to see that this SDE captures the mean-reverting behaviour of electricity spot prices. In fact, a closer look at the Ornstein-Uhlenbeck process reveals that this process is an extension of the GBM with mean-reverting parameter.

Deng et al [27] applied the Black-Scholes formula to the Ornstein Uhlenbeck process by assuming electricity future price processes, F_e, and the appropriate generating fuel, F_g follow mean-reverting processes

$$dF_e = \pi_e(\mu_e - lnF_e)F_e dt + \sigma_e F_e dB^1,$$

$$dF_e = \pi_g(\mu_g - lnF_g)F_g dt + \sigma_g F_g dB^2,$$

where μ_e and μ_g are long-term means, π_e and π_g are the mean-reverting co-efficients, and B^1 and B^2 are two Brownian processes with instantaneous correlation ρ.

Table 5.3.1 shows a forecast of electricity spot prices for the next 100 days. We use the Ornstein Uhlenbeck process with the following parameters:

- $S_0 = R10$, closing electricity spot price at day 0
- $\mu = 10.1$
- $k = 0.369$
- $\sigma = 20\%$
- $\alpha = 10\%$
- $r = 8\%$

and we compute

$$S_t = \exp(-r * t) * S_0 * (\exp k(\mu - lnS_t)\,\Delta t + NORM.S.INV(Rand(\quad)) * SQRT(t) * \sigma.$$

Table 5.3.1 Illustration of electricity spot prices using Ornstein-Uhlenbeck process

Days	t	SQRT(t	Norm.S.INV (Rand())	Pt
0	0	0	-0.72	10
1	0.01	0.10	0.90	10.01
2	0.02	0.14	0.36	10.01
3	0.03	0.17	0.93	10.01
4	0.04	0.20	1.46	9.98
5	0.05	0.22	-0.77	9.96
6	0.06	0.24	0.77	9.94
95	0.95	0.97	-1.12	10.11
96	0.96	0.98	-0.66	10.09
97	0.97	0.98	0.45	10.08
98	0.98	0.99	-0.14	10.05
99	0.99	0.99	0.43	10.05
100	1	1	-1.44	10.06

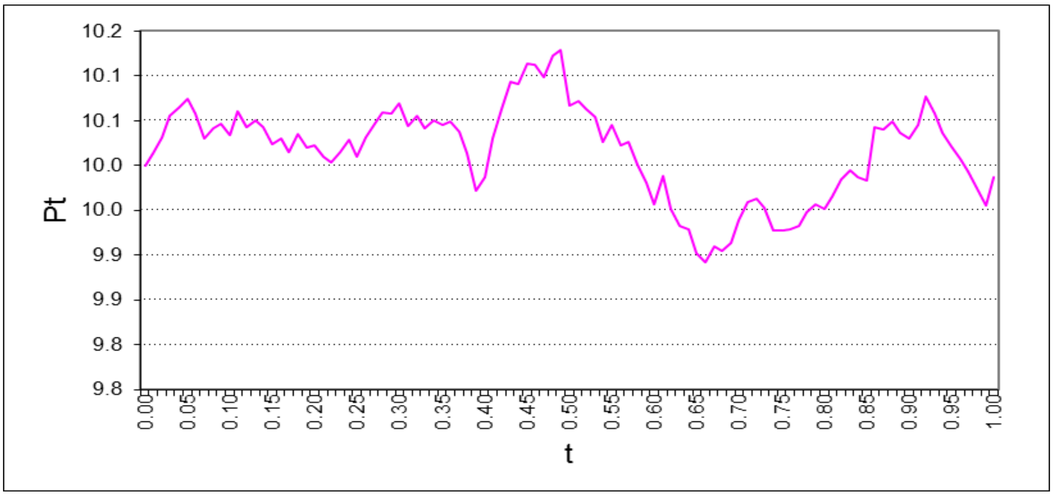

Figure 5.3.1 Electricity spot prices (from Table 5.3.1) using Ornstein-Uhlenbeck

Figure 5.3.1 depicts electricity spot prices for the next 100 days. Comparing Figure 5.3.1 and Figure 5.1, it is apparent the latter does not capture spikes that characterise electricity spot prices. It however captures the mean-reverting characteristic of electricity spot prices.

5.4 Numerical results of mean-reverting jump diffusion process on electricity spot prices

Mean-Reverting Jump Diffusion Process (MRJD) is in fact a mean-reverting process with a jump factor and can be represented as follows:

$$dlogS_t = k(\mu - logS_t)dt + \sigma dB_t + \log(1+J)\, dP_t,$$

where

μ: the mean reversion level or long run equilibrium price,

$logS_t$: the spot price at time t,

k: the mean reversion rate,

$\log(1+J)$: the log-return jump size with log-return mean μ_j and variance σ_j^2,

P_t: a simple Poisson process jump process with jump rate λ.

Parameters μ and σ are assumed to be constant. The jump process Jdp has a mean $E[J]\lambda dt$ and variance $E[J^2]\lambda dt$. Furthermore dP_t has a discrete distribution:

$$p_n(\lambda dt) = Prob[dP(t) = n] = \frac{e^{\lambda\Delta t(\lambda\Delta t)^n}}{n!}, \quad \forall\, n = 0\!:\!\infty.$$

The model has two parts:

1. $k(\mu - logS_t)dt + \sigma S_t dB_t$ is the Mean-Reverting Diffusion process /Ornstein-Uhlenbeck process and
2. Jdp is the Poisson Process.

The Ornstein-Uhlenbeck process would therefore capture the fact that electricity prices tend to fluctuate around values determined by the cost of production and the level of demand (mean-reversion and seasonality of electricity spot prices) and the Poisson process would capture the electricity price sparks.

Weron [79] proposed a jump-diffusion model which recovers the main characteristics of electricity spot price dynamics in the Nordic market, including seasonality, mean-reversion and spiky behaviour. He showed how the calibration of the market price of risk to actively traded futures contracts allows for efficient valuation of Nord Pool's Asian-style options written on the spot electricity price.

Table 5.4.1 shows a forecast of electricity spot prices for the next 100 days. Using process with t MRJD with the following parameters:

- $S_0 = R10$, closing electricity spot price at day 0
- $\mu = 10.1$
- $k = 0.369$
- $\sigma = 20\%$
- $\alpha = 10\%$
- $r = 8\%$
- $\lambda = 3\%$,

we compute

$S_t = \exp(-r * t) * S_0 * k(u - lnS_t)\,\Delta t + NORM.S.INV\big(Rand(\quad)\big) * SQRT(t) * \sigma + if(rand(\quad) < 3\%, 1{,}0) * Poisson.S.Dist(2{,}4, True).$

Table 5.4.1 Illustration of electricity spot prices using MRJD process

Days	t	SQRT(t)	Norm.S.INV (Rand())	Jump: If((rand()<0.03,1,0)	Pt
0	0	0	1.29	0.00	10
1	0.01	0.10	0.47	0.00	10.01
2	0.02	0.14	-1.09	0.00	10.00
3	0.03	0.17	1.00	0.00	9.98
4	0.04	0.20	0.24	0.00	9.96
5	0.05	0.22	0.33	0.00	9.94
6	0.06	0.24	0.50	1.00	10.40
95	0.95	0.97	-0.68	0.00	9.05
96	0.96	0.98	-0.18	0.00	9.03
97	0.97	0.98	-0.08	0.00	8.98
98	0.98	0.99	1.24	0.00	8.89
99	0.99	0.99	0.50	0.00	8.87
100	1	1	-0.38	0.00	8.85

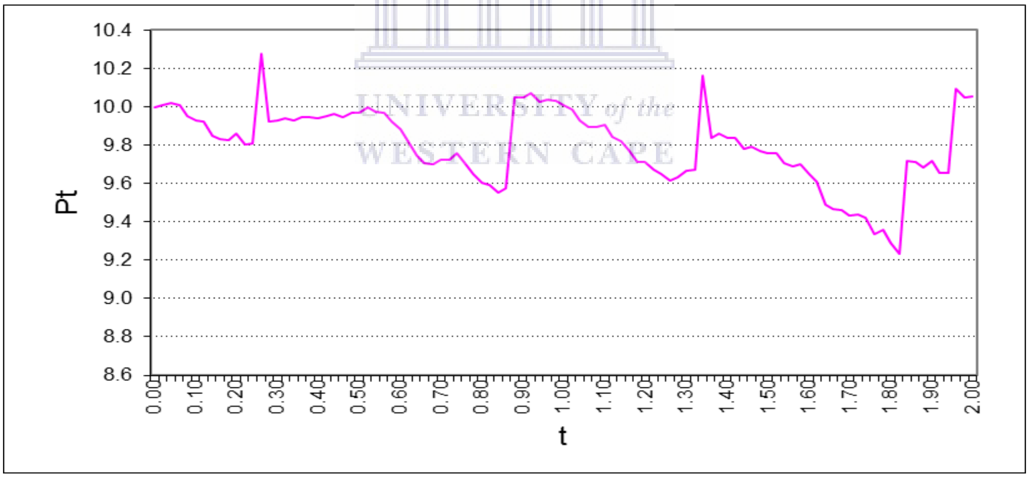

Figure 5.4.1 Electricity spot prices (from Table 5.4.1) using MRJD process

Figure 5.4.1 depicts electricity spot prices for the next 100 days. It is apparent this captures all characteristics of electricity spot prices as shown in Figure 5.1.

5.5 Numerical results of Binomial models on MRJD Electricity Spot Prices

Cox and Rubinstein [24] and Hull [45] derived a binomial model for the pricing of financial derivatives. The model is derived under the assumption that the evolution of the underlying asset follows a Geometric Brownian motion. Contreras et al [22] derives a binomial model for the pricing of Financial Transmission Rights (FTR) assuming no jumps. On the other hand, Giovanni and Gigli [43] presented a novel algorithm on the application of a binomial model in electricity derivatives that captures spikes of electricity spot spices. These researchers used a combination of the binomial model and Monte-Carlo methods. The trees of a binomial model are constructed as follows:

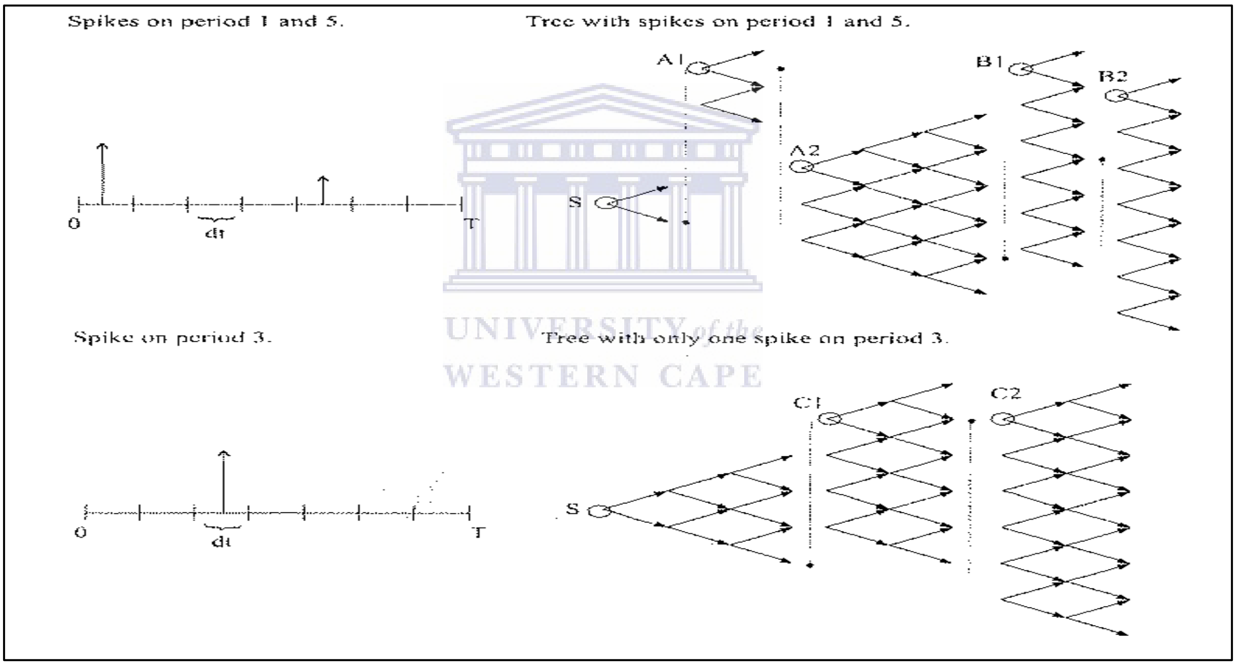

Figure 5.5 source [43]. The figure show two examples of spike process realizations over binomial trees. The number of levels affected by the spike as well as intensity of the spike magnitude is random.

The disadvantage of a binomial model in electricity derivatives can be readily identified Figure 5.5. As the time horizon becomes longer, high frequency of spikes would result in many nodes and jumps that need to be constructed. This exercise can be cumbersome and time consuming.

5.6 Numerical results using improved Monte Carlo methods on MRJD Electricity Spot Prices

In financial markets, Monte Carlo methods are recognised as very flexible tools for simulating future time series with which to evaluate cost and risk of various contracts. Possible future values can be generated by randomly sampling the relevant distributions, maintaining given volatility characteristics. While the Black-Scholes formula offers a closed form solution, it fails to account for spikes.

Table 5.6.1 shows a forecast of electricity spot prices for the next 15 days. Using improved Monte Carlo methods in which we have used the method of antithetic variables to improve the efficiency of the classical Monte Carlo method.

- $S_0 = R10$, closing electricity spot price at day zero
- $\mu = 10.1$
- $k = 0.369$
- $\sigma = 20\%$
- $\alpha = 10\%$
- $r = 8\%$
- $\lambda = 3\%$.

We compute

$$S_t = \exp(-r * t) * S_0 * k(u - lnS_t)\,\Delta t + NORM.S.INV\big(Rand(\quad)\big) * SQRT(t) * Sigma + if(rand(\quad) < 3\%, 1,0) * Poisson.S.Dist(2,4, True).$$

Table 5.6.1 Illustration of electricity spot prices using improved Monte Carlo method

						z	(z)	Antithetic
Days	t	SQRT(t)	NORM.S.INV(Rand())	Negative*NORM.S.INV(RAND())		Pt1	Pt2	(Pt1+Pt2)/2
0	0	0	-0.54	-0.31	0	10.0	10	10.00
1	0.01	0.10	-0.64	0.43	0	10.0	10.0	10.00
2	0.02	0.14	-0.39	-0.29	0	10.0	10.0	9.99
3	0.03	0.17	1.60	-1.30	0	10.0	10.1	10.04
4	0.04	0.20	-1.18	-0.47	0	10.0	10.1	10.05
5	0.05	0.22	-0.49	0.79	0	10.0	10.0	10.01
6	0.06	0.24	-1.32	0.41	1	10.0	10.2	10.12
7	0.07	0.26	-0.39	0.66	0	10.0	10.1	10.02
8	0.08	0.28	0.68	-1.16	0	10.0	10.1	10.02
9	0.09	0.30	-0.51	0.58	0	10.0	10.1	10.01
10	0.1	0.32	-0.48	-0.20	0	9.9	10.1	10.00
11	0.11	0.33	-0.52	0.70	0	9.9	10.1	10.01
12	0.12	0.35	0.66	-0.10	0	9.9	10.6	10.22
13	0.13	0.36	-0.99	1.06	0	9.9	10.2	10.03
14	0.14	0.37	0.86	-0.64	0	9.9	10.1	9.98
15	0.15	0.39	-1.37	-0.85	0	9.9	10.1	9.97

From Table 5.6.1, we compute the mean of P_{t1} and $(P_{t1} + P_{t2})/2$ using Excel routine MonteCarlito. We ran 10000 trials to compute the mean of P_{t1} and 5000 trials to compute the mean of $(P_{t1} + P_{t2})/2$.

Our results obtained from excel routine MonteCarlito are

Table 5.6.2 Monte Carlo simulation results

Pt1	
Mean	9.992
Standard deviation	0.062
Variance	0.004

Table 5.6.3 Monte Carlo simulation results with antithetic variance reduction technique

Method of Antithetic variables	
Mean	9.996
Standard deviation	0.055
Variance	0.003

We ran 10 000 trials for P_{t1} and 5000 trials for the use of Antithetic variables. The results show an improvement in variance when employing antithetic variance reduction with half the trials. Thus we have achieved efficient results using this variance reduction technique. These results are also confirmed by the histograms below. As expected Figure 5.6.2 has fatter tails than Figure 5.6.3.

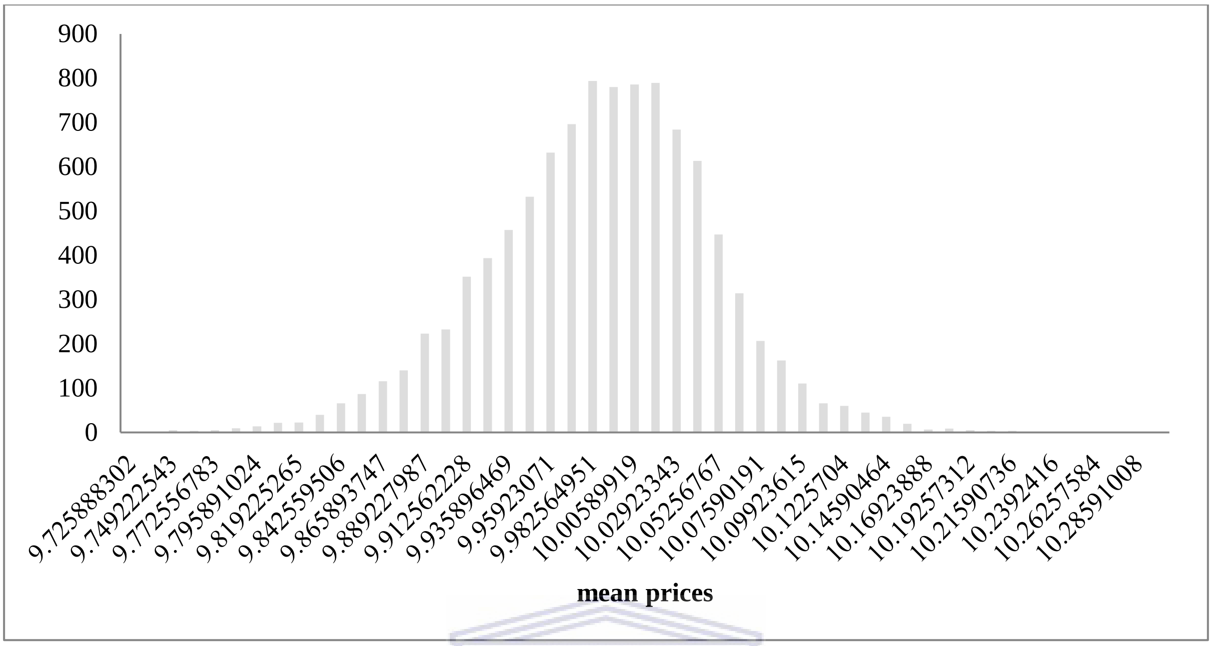

Figure 5.6.2 Results of 10000 trials for the computation of the mean of prices at time-1 (P_{t1}).

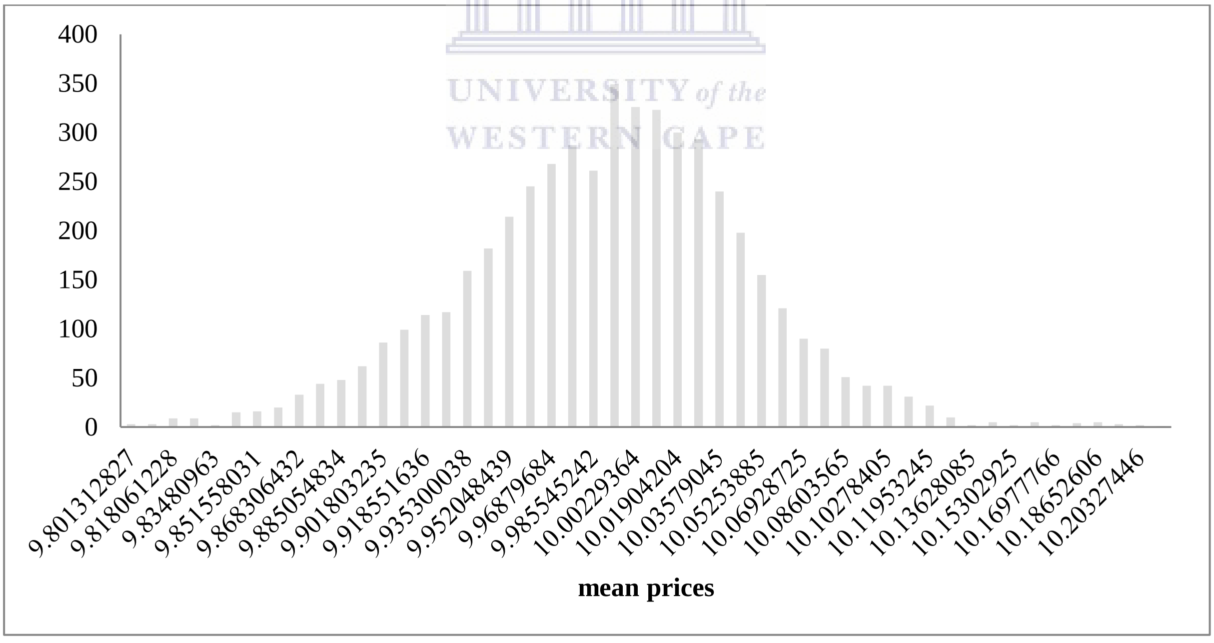

Figure 5.6.3 Results of 5000 trials for the computation of the mean of $(P_{t1} + P_{t2})/2$ using the variance reduction technique based on antithetic variables.

5.7 Summary

In this chapter, we presented comparative results obtained by the Black-Scholes formula, binomial trees and Monte Carlo methods for pricing electricity derivatives. We looked at the following price processes: (a) Geometric Brownian motion (b) mean-reverting process and MRJD. We also applied an improved Monte Carlo simulation together with MRJD for pricing electricity derivatives. MRJD process showed accurate results in capturing the evolution of electricity spot prices compared to other price processes. In addition, the method of antithetic variables resulted in improved efficiency of Monte Carlo method.

Chapter 6

CONCLUDING REMARKS AND SCOPE FOR FUTURE RESEARCH

In this thesis, we have provided a thorough discussion on electricity derivatives and numerical simulations. Firstly, we presented a classification of the regulated and deregulated electricity markets and then discussed in details the structure of the deregulated electricity markets. Pricing of such derivatives usually poses a great difficulty. To this end, we discussed how the classical Black-Scholes and Binomial methods were used. The drawbacks of these methods were also discussed up to some extent. We then presented results obtained by some improved methods such as the Ornstein-Uhlenbeck process and the mean reverting jump-diffusion processes. To further improve the quality of simulated results, we explored the usage of Monte Carlo simulations for pricing these derivatives. Needless to mention, this presentation would have been incomplete if we did not touch the variance reduction techniques. To this end, we would like to mention that we have used the method of antithetic variables as a variance reduction technique for pricing these derivatives and obtained improved results as can be seen from Chapter 5.

As far the **scope of future research** is concerned, we note that through this thesis work, the drift coefficient and the volatility parameter in the Brownian processes are assumed to be constant over the life of the derivative. Relaxing this assumption in the valuation of electricity derivatives would be an interesting focal point moving forward. Modelling of these co-efficient would be another challenging task. It would also be very crucial to look particularly at different volatility models for the pricing of electricity derivatives.

APPENDIX

Mean and variance of Ornstein-Uhlenbeck Process

$$dX_t = (\mu - X_t)dt + \sigma dB_t$$

$$dX_t + X_t dt = \mu dt + \sigma dB_t$$

Let $I(t) = e^t$. Then

$$d(e^t X_t) = \mu e^t dt + \sigma e^t dB_t.$$

Since

$$d(e^t X_t) = (e^t X_t - e^0 X_0)$$

$$\Rightarrow e^t X_t = X_0 + \mu \int_0^t e^s ds + \sigma \int_0^t e^s dBs.$$

$$\Rightarrow X_t = X_0 e^{-t} + \mu \int_0^t e^{-(t-s)} ds + \sigma \int_0^t e^{-(t-s)} dBs.$$

$$\Rightarrow E[X_t] = \mu + (X_0 - \mu)e^{-t}.$$

Since

Bt is a martingale with $E[Bs] = 0$ and $Var(X_t) = E[(X_t - E[X_t])^2]$,

we have

$$Var(X_t) = \left[\left(X_0 e^{-t} + \mu \int_0^t e^{-(t-s)} ds + \sigma \int_0^t e^{-(t-s)} dBs - \mu + (X_0 - \mu)e^{-t}\right)^2\right].$$

$$\Rightarrow Var(X_t) = E\left[\sigma \int_0^t e^{-(t-s)} dBs\right].$$

Symmetry implies

$$Y_t = \int_0^t f(Y_s) dBs, \qquad Y_t^2 = \int_0^t f^2(Y_s) ds.$$

Thus

$$Var(X_t) = \sigma^2 \int_0^t e^{-2(t-s)}\, ds.$$

$$Var(X_t) = \frac{\sigma^2}{2} e^0 - \frac{\sigma^2}{2} e^{-2t} = \frac{\sigma^2}{2}(1 - e^{-2t}).$$

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