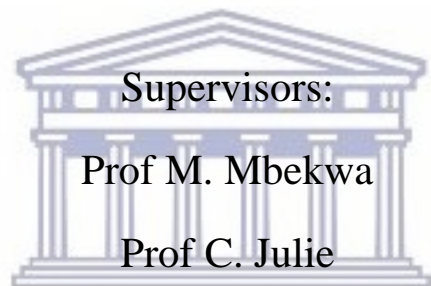


An analysis of Grade 12 national examinations marking memoranda on the topic of measurement in Mathematical Literacy using a mathematical modelling framework.

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A thesis submitted in partial fulfilment of the requirements for a M.Ed. degree in the School of Science and Mathematics Education, University of the Western Cape.



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March 2017

DECLARATION

I declare that AN ANALYSIS OF GRADE 12 NATIONAL EXAMINATIONS MARKING MEMORANDUM ON THE TOPIC OF MEASUREMENT IN MATHEMATICAL LITERACY USING A MATHEMATICAL MODELLING FRAMEWORK is my own work that has not been submitted before for any degree or examination in any university, and that all the sources I have used or quoted have been indicated and acknowledged by complete references.

MBULELO BALI

March 2017



Signed:



ACKNOWLEDGEMENTS

To my late parents Mzwandile and Nobantu Bali. Thank you very much for everything.

Thanks to my family, friends, colleagues and everybody who supported me in whatever way.

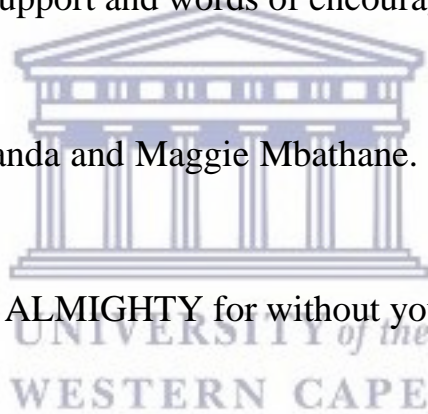
To my supervisors, Prof M. Mbekwa and Prof C. Julie. Thank you very much for affording me the opportunity to grow under your guidance. Your support in various forms throughout this project is beyond measure – May the LORD bless you.

Thanks to the National Research Foundation (NRF) for the financial support and ensuring that the project is a success.

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My proof-readers, Margaret Janda and Maggie Mbathane. Thanks for the job well-done in such a short period of time.

Most of ALL thanks GOD the ALMIGHTY for without your blessings nothing is possible.



Abstract

The research study conducted an in-depth analysis of the national marking memoranda on the topic of measurement in Mathematical Literacy. The object of analysis was the grade 12 national examinations. The purpose of Mathematical Literacy, according to the Department of Basic Education (DBE), is to equip learners with competencies that will enable them to use elementary mathematical concepts and skills to make sense of, participate in and contribute to the twenty-first century world characterised by numbers, numerically based arguments and data represented and misrepresented in a number of ways. With such competencies learners will in future become responsible individuals, contributing workers and participative critical citizens on social and political issues.

Both qualitative and quantitative research methods were used. The mathematical modelling framework was used for content analysis for the identification of competencies that are awarded marks in the examination. The main reason for the use of the mathematical modelling framework is the existence of both empirical and theoretical evidence that the development of mathematical modelling competencies results in the development of Mathematical literacy. Competencies identified through content analysis were then quantified as percentages of the total marks awarded for all competencies in the examination papers analysed.

The analysis reveals that in the main mathematical processes are highly prioritised in learner assessment and critical competence is hardly assessed. This is attributed to a prescribed taxonomy for questions to include in the national examinations which is mathematically based. The researcher strongly suggests the incorporation of mathematical modelling as content in the curriculum in order to address some of the shortcomings in the new subject and assist with the achievement of the intended aims of the subject in the curriculum.

Key words: Mathematical Modelling, Competence, Mathematical Modelling framework, Mathematical Literacy, Mathematical literacy, Measurement, Assessment, Word problems, Descriptive research, Qualitative data, Qualitative content analysis, Quantitative data analysis.

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Chapter 1

Introduction

1.1. Background

Mathematical Literacy is a new subject introduced in the South African post compulsory schooling phase in 2006. Post compulsory schooling phase is the last three years of schooling called Further Education and Training (FET) phase which runs from Grade 10 to 12. The subject is intended to develop learner's competencies that will allow them to make sense of, participate in and contribute to the 21st century world characterised by numbers, numerically based arguments and data represented and misrepresented in a number of ways. Furthermore, it is hoped that by doing Mathematical Literacy learners will develop a critical competence that will enable them to become critical citizens who participate meaningfully in social, economic and political issues (Department of Basic Education, 2011). Such competencies will be developed through teaching and learning characterised by the provision of opportunities to analyse real life authentic problems and devise ways to work mathematically to solve such problems.

Arguments for the development of competencies in learners similar to those mentioned for Mathematical Literacy in the South African school curriculum through the use of mathematical knowledge to solve real life problems have been made world-wide with different names to the subject to be used as noted by Jablonka (2003): "There are a number of perspectives on numeracy or mathematical literacy that vary with respect to the culture and the context of the stakeholders who promote it." (Jablonka, 2003, p.76). For example, the terms Quantitative Literacy and Qualitative Reasoning are used in the United States of America (USA) to denote the subject. The term numeracy is also common among most school mathematics curricula for many countries. Functional Mathematics in England has similar intentions. Perhaps the most common and unifying term used for the subject is that found in the Organisation for Economic Co-operation and Development/Programme for International Student Assessment, commonly known as OECD/PISA, framework. The OECD/PISA calls the subject Mathematical literacy and its aims are to develop capacity in learners to understand the role played by mathematics in the world on a number of personal and social issues.

Albeit there are disagreements among the protagonists for this subject regarding a common name for the subject, they all agree that mathematical modelling is central to the development of competencies the subject aims to develop in learners. Mathematical modelling is an umbrella term used to denote the process of translating between the real world and the world of mathematics in both directions.

In other words, the modelling process provides a meaningful way of connecting the real world and mathematics so that the necessary competencies can develop in learners. As noted by Cai, Mok, Reddy & Stacey (2016) the PISA process for the development of Mathematical literacy is synonymous to the development of mathematical modelling sub-competencies – a set of skills necessary to successfully carry out the mathematical modelling process. Many mathematics education scholars and researchers, for example Blum (2002), Julie (2006), De Corte, Verschaffel & Greer (2000) and Christiansen (2006) have suggested mathematical modelling as a vehicle for achieving Mathematical literacy aims. For example, Christiansen notes: “It seems evident that in order to truly understand how mathematics can be used and what effects thereof are, learners must engage in modelling of complex phenomena themselves, and critical reflection thereon.” (Christiansen, 2006, p.9). Julie (2006) further argues that the development of reflective knowledge requires learners to engage in activities with a specific goal of critiquing models. English (1999) also emphasises the benefits of allowing learners to critique models instead of just using readily available ones to obtain a solution. Such benefits include the ability to interpret information and critique assumptions made during the creation of a model – critical competence. A similar view is shared by Blum and Borromeo Ferri (2009) when they state that preparation for responsible citizenship and participation on social issues requires competency in mathematical modelling.

Curriculum revision also took place in South Africa after 1994 resulting in a subject clearly separated from mathematics – Mathematical Literacy. Unlike in the OECD/PISA and suggestions from mathematics educationists mathematical modelling skills are not explicitly mentioned in the subject curriculum statement as requirements to achieve the intended aims of the subject. Yet the general aims of the subject in the country are similar to those of the ‘subjects’ with the same intentions world-wide. The question of interest here is the nature of competences assessed in the new subject Mathematical Literacy and how these competences compare to those developed through mathematical modelling?

1.2 Rationale

According to the OECD/PISA the word *literacy* in the name Mathematical literacy emphasizes that school mathematical knowledge and skills do not constitute the primary focus in the assessment of learners in the subject. The emphasis is on the functional use of mathematical knowledge to solve real life problems in a variety of contexts. This is similar to the notion of *mathemacy* proposed by Skovsmose (2001) in which the focus of learning mathematics is extended beyond arithmetic and related knowledge and skills to include developing in learners the ability to apply their mathematics to deal with real life situations in which mathematics may be useful.

The importance of this ability to function in the twenty-first century world full of information represented in numbers is also expressed by Jablonka (2003). Evans (2000) takes this utility argument further and explains the exact nature of processes involved in the functional use of mathematical knowledge by an individual in his notion of numeracy. All these arguments point to the legitimacy of the act of introducing a subject in the FET phase of schooling in South Africa that will develop these competencies in learners.

The subject Mathematical Literacy has been introduced to empower learners with competences required for an individual to be functionally literate or numerate or more appropriately, mathematically literate. Since its introduction the subject has been attracting a large number of learners compared to mathematics. Learners in the FET band have to choose between Mathematics and Mathematical Literacy. Over the past 5 years there has been an increase of 30 624 learners taking Mathematical Literacy in matric and a decrease of 39 418 learners taking Mathematics (Department of Basic Education, 2014 Technical Report). The attractiveness of the subject to FET learners is also due to the fact that Mathematical Literacy, like any of the other school subject including mathematics, has a credit value of 20 in the National Senior Certificate (Department of Education Government Notice, 2008). Credit values of subjects are used to measure learner suitability for entry into certain post - school learning programmes.

With such value in both its aims and status an in-depth analysis of competencies the subject aims to develop in learners as shown in learner assessment is necessary. This is particularly because the curriculum statement for the subject, known as the curriculum and assessment policy statement (CAPS), mentions the use of elementary mathematics to interrogate real world authentic contexts to solve familiar and unfamiliar problems, make decisions and communicate such decisions (Department of Basic Education, 2011. *Curriculum and Assessment Policy Statement (CAPS) Grades 10 – 12: Mathematical Literacy*) as a vehicle to achieve the subject aims.

Since there is a strong connection between mathematical modelling competencies and those competencies envisaged for Mathematical Literacy, one would expect assessment of mathematical modelling competences to be the focus in learner assessment in the subject of Mathematical Literacy. The rationale for this study is to find the nature of competencies assessed in Mathematical Literacy examinations at grade 12 level. As the title of the thesis indicates, the analysis will focus on the topic of Measurement in the Grade 12 examination papers. The reason for choosing Grade 12 examinations is that, this examination is externally set, marked and moderated and is therefore a credible indicator of competencies learners should develop in learning Mathematical Literacy. The national examination for Mathematical Literacy, Grade 12, has two papers.

Paper 1, called the “Basic skills” paper assesses proficiency of content and/or skills, whilst paper 2, the “Applications” paper assesses ability to use both mathematical and non-mathematical techniques/considerations to explore familiar and unfamiliar contexts (Department of Basic Education 2011. *Curriculum and Assessment Policy Statement (CAPS) Grades 10 – 12: Mathematical Literacy*). Clearly questions assessing mathematical modelling competences are unlikely to feature in paper 1 since the paper assesses proficiency in basic mathematical skills. For this reason, the analysis will only be done on paper 2, the Applications paper, focussing on the topic of Measurement.

The topic of Measurement is the focus for two reasons. Firstly, it is for manageability of the project and the level of analysis the project aims to conduct. There are five topics in the subject and so analysing questions and answers for all of them would produce something beyond the scope of the project. The second reason is that Mathematical Literacy utilises elementary mathematics. This means all the mathematics that has been dealt with in Grades before Grade 10 is such that at Grade 10 level the knowledge of that mathematics is elementary. Measurement includes all basic concepts such as perimeter, area, surface area and volume and their formulae. All these concepts are dealt with in lower Grades and are thus elementary mathematical knowledge in the FET phase. Knowledge of these fundamental concepts presents a suitable ground for the assessment of Mathematical Modelling competencies. The research problem is stated in the next section.

1.3 Research problem and research questions

The research was motivated by the following question:

What competencies are assessed in the national applications paper for Grade 12 on the topic of measurement in Mathematical Literacy and how do these compare to those Mathematical Modelling aims to develop in learners?

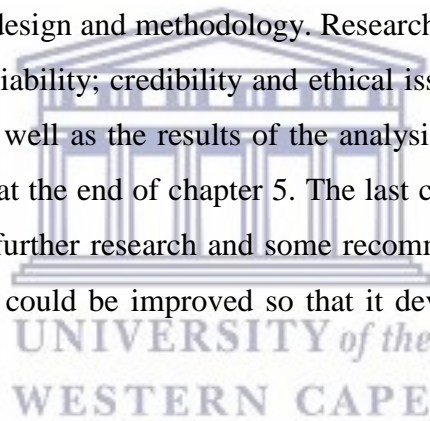
To answer this question an in-depth analysis of a set of five question papers on the topic of measurement was done. The analysis was conducted using a Mathematical Modelling framework. The framework is an adaptation of a framework developed by Stillman, Galbraith, Brown & Edwards (2007) to identify learner blockages as they progress from one step to another in the modelling process. Both the original and the adapted frameworks are presented and discussed in chapter 2. It is hoped that such an analysis will provide some answers to the following research questions:

1. What competencies are assessed in the national examinations for grade 12 in Mathematical Literacy?
2. How do these competencies compare to those of mathematical modelling?

1.4 The structure of the thesis

This thesis is organized as follows. The next chapter gives a discussion of both theoretical and research literature relevant to this research. The theoretical literature discussion provides an in-depth review of concepts, theories and contexts relevant to this study. This literature is used to produce a conceptual framework for capturing and interrogating the object of focus – the Mathematical Literacy Grade 12 national examination questions and their marking memoranda on the topic of measurement. In the same chapter research literature is also discussed. The purpose of this literature discussion is to locate the present research amongst researches of a similar nature conducted before. In particular the research on the role of mathematical modelling in making meaningful connections between the real world and mathematics conducted by Brown and Schafer (2006); the research on the nature and role of contexts used in Mathematical Literacy textbooks in South Africa by Mbekwa and Julie (2007); and the research by Venkat, Graven, Lampen and Nalube (2009) on the lack of problem solving in Mathematical Literacy national examination questions are of great relevance to this study. All three research studies mentioned above are discussed in detail in the next chapter.

Chapter 3 is dedicated to research design and methodology. Research type and design; sampling methods; data analysis methods; validity; reliability; credibility and ethical issues are all discussed in this chapter. Chapter 4 presents data analysis as well as the results of the analysis. A full analysis of all data items is provided from appendices A to E, at the end of chapter 5. The last chapter, is dedicated to the discussion of findings, recommendations for further research and some recommendations to policy makers on how the subject Mathematical Literacy could be improved so that it develops the competencies it claims to develop in learners.



Chapter 2

Literature review

2.1 Introduction

In this chapter a discussion of both theoretical and research literature which is used in this research is presented. According to Punch (2009), the difference between theoretical and research literature is as follows: “Whereas research literature concentrates on findings from empirical research, theoretical literature includes relevant concepts, relevant theories and theoretical contexts, and discursive and analytical literature that contains ideas and information relevant to the topic.” (Punch, p. 94). There are two main objects of interest in this research, namely, Mathematical Literacy and mathematical modelling. Both the research and theoretical literature whose object of analysis is either Mathematical Literacy or mathematical modelling will be discussed. The main purpose of the literature reviews in a thesis, as Mouton (2001) suggests, is to learn from other scholars how they have theorized and conceptualized issues, what they found from research and the methods they used. However, this does not mean that all published literature about Mathematical Literacy or mathematical modelling is presented here. Only literature relevant to the research question is discussed.

2.2 Theoretical literature

In this section theoretical literature on mathematical modelling and Mathematical literacy is presented. The discussion focusses mainly on the definitions of these two theoretical constructs, the various perspectives on mathematical modelling and Mathematical literacy and the relationship between the two.

2.2.1 Mathematical Modelling

Theoretical literature on mathematical modelling will be presented under the following sub-headings: Mathematical Modelling and Applications of Mathematics; Mathematical Modelling Perspectives and School Mathematics; and Mathematical Modelling Competencies and their Assessment.

2.2.1 (a) *Mathematical Modelling and Applications of Mathematics*

What is mathematical modelling? As an attempt to answer this question, let us begin with the English meaning of the word *model*. The Oxford on line dictionary (2015) gives the following meanings to the word:

- i) A copy of something usually smaller than the original object.
- ii) A particular design or type of a product.
- iii) A simple description of a system used for explaining how something works or calculating what might happen, etc.
- iv) Something that can be copied by other people.
- v) A person whose job is to wear and show new styles of clothes.
- vi) A person who is employed to be photographed, drawn or painted by a photographer or an artist.

From the six meanings above one deduces two main types of models, namely, concrete models e.g. (i), (ii), (v) and (vi) and abstract models e.g. (iii) and (iv). Clearly (v) and (vi) are not appropriate for the purpose of this discussion as they both refer to human beings. Definitions (i), (ii) also fall out of this discussion since they refer only to concrete models and not all models are concrete. Also (iv) is too generic and can even involve copying someone's behaviour or lifestyle and is therefore not suitable for this discussion. Meaning (iii), a simple description of a system used for explaining how something works or calculating what might happen, etc. is the only meaning of the word model relevant to this discussion. The next question is when does a model become mathematical? It becomes mathematical when its descriptions are expressed using mathematical nomenclature and syntax (Arleback, 2009).

From this discussion mathematical modelling then simply becomes a process of making a mathematical model. Kang & Noh (2012) agree with this simple definition and elaborate on it further. "Modelling is a cyclical process of creating and modifying models of empirical situations to understand them better and improve decisions." (Kang & Noh, 2012, p1). For Kang & Noh (2012), models are purposeful interpretations, descriptions, explanations, predictors or symbols that are used to construct, manipulate or predict the systems being modelled. Examples of mathematical models include graphs, tables, formulae, equations, etc. Perhaps a more theoretical, general and classical definition of a mathematical model is that of Niss (2012). He starts by drawing a line between the realm of mathematics (containing mathematical objects, relationships, questions and possible answers, etc.) represented by M and the domain outside of mathematics, call it D , containing its own objects, relationships, questions and answers, etc. Then a mapping (translation) f , called the mathematisation of D by means of M , relates the elements of D with those of M . A mathematical model is, therefore, a triple (D, f, M) and contains the characteristics of the three components.

The diagram below illustrates the structure.

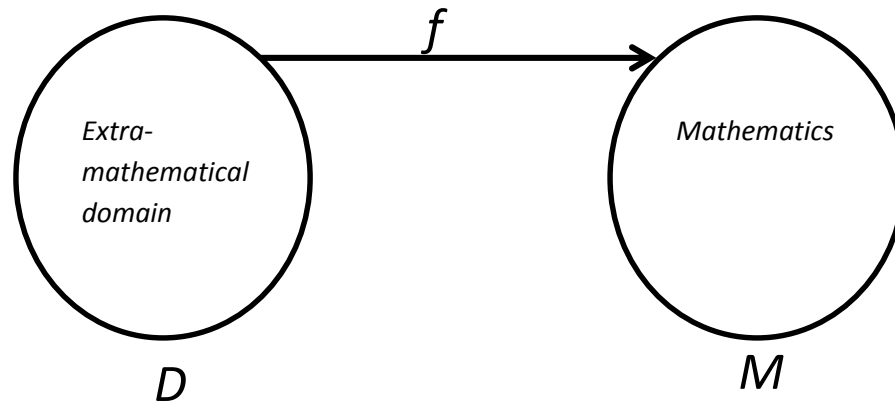


Figure 2.1: A mathematical model

Niss's (2012) classical definition of a mathematical model can be regarded as a mathematisation of a more general definition of a model once given by Davis (1979). He describes a model as a product of singling out elements from reality, symbolising these elements, defining and portraying them as a system of variables (1979). Once established this system can be used as an aid for describing, explaining and forecasting whatever is being modelled. Davis's definition of a model is useful for categorising models depending on the relationship among variables in the system (model). For example, Blum and Niss (1991) distinguish between two types of mathematical models depending on how mathematics in the model is used. They call these normative models in which mathematics is used to establish certain norms about value judgements, and descriptive models in which mathematics is used to describe the situation being represented by the model. Similar to Davis's (1979) identified three uses of models, Julie (2004) identifies three different kinds of models. These he calls descriptive, predictive and prescriptive models. This discussion here is not only about the different kinds of models that one can identify but serves to show the importance and usefulness of models, in particular, mathematical models in life generally. Regarding the process of introducing a mathematical model to an extra-mathematical situation, Niss (2012) calls it mathematical modelling. Stillman (2012) agrees with Niss when she states: "With mathematical modelling the focus becomes: where can I find some mathematics to help with this problem?" (Stillman, 2012, p.3). If a mathematical model already exists to solve a problem in an extra-mathematical context, then the process is simply called an application of mathematics, says Niss (2012). Blum (2002) summarises the difference between modelling and applications as follows:

The term ‘modelling’, on the one hand, focuses on the direction from reality to mathematics and, on the other hand and more generally, emphasises the processes involved. The term ‘application’, on the one hand, focusses on the opposite direction from mathematics to reality and, on the other hand and more generally emphasises the objects involved...(Blum, 2002, p.153).

Blum and Borromeo Ferri (2009) do not distinguish between *applications* and *modelling* as they define mathematical modelling as the process of translating between the real world and mathematics in both directions. For Blum and Borromeo Ferri the process involves two translations, namely, from the extra-mathematical domain, D , to mathematics, M , (*mathematisation*) and from mathematics back to the extra-mathematical domain (*interpretation*). With this definition, mathematical modelling can be represented diagrammatically as follows:

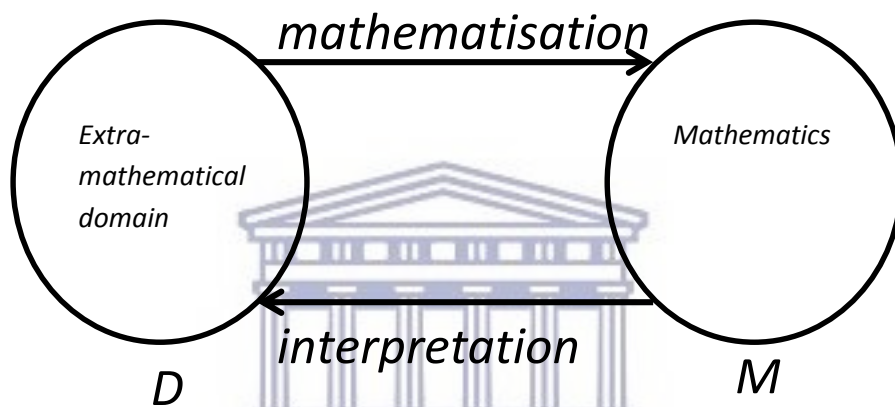


Figure 2.2: Mathematical modelling

A similar definition is also provided by Mischo and Maaß (2012) when they define mathematical modelling as solving complex, realistic and open problems with the help of mathematics. In this thesis, the term mathematical modelling will be used in the sense of Blum & Borromeo Ferri (2009), i.e. a process of translating between real world and mathematics in both directions. Albeit there are many meanings of mathematical modelling as noted by mathematics educationists such as Galbraith & Stillman (2006), Mischo & Maaß (2012) and others, all protagonists of mathematical modelling agree on the cyclic nature of the process.

The cycle consists of seven steps used to show activities involved in successfully carrying out the modelling process. The most commonly used representation of the modelling cycle is shown below.

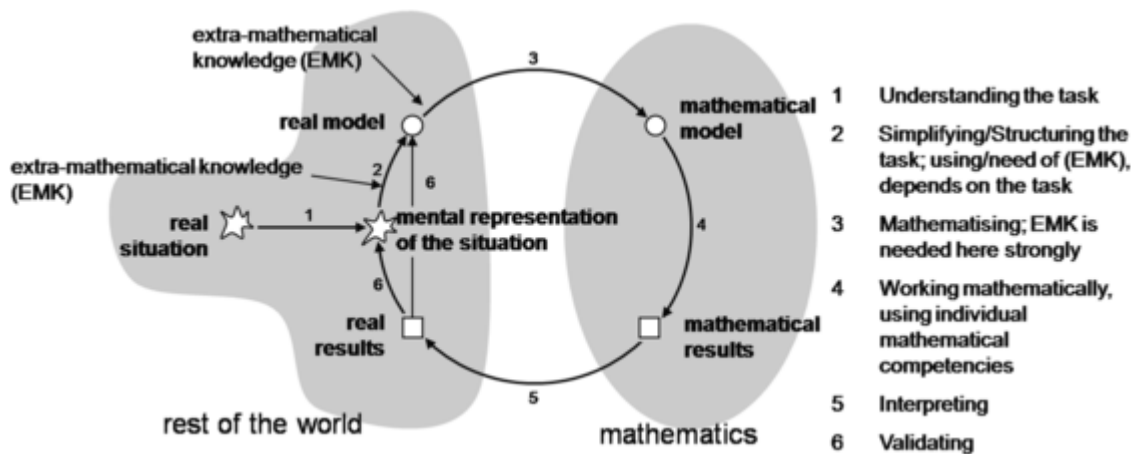


Figure 2.3: Mathematical modelling cycle from Blum & Borromeo Ferri (2009), p. 46.

In the cycle the first step is *understanding the situation* in which the task is embedded. Then the situation is *simplified, structured and made more precise*. This process involves making assumptions, identifying significant variables and their relationships, etc. The end result of the process is the *real model of the situation* which must be translated to the mathematical world through the process of mathematisation. Mathematisation constitutes step three of the process and results in a mathematical model which must be used to perform mathematical operations such as calculations, simplifications, solving equations, etc. Then the process *working mathematically* constitutes the fourth step and its end products are mathematical results or answers. The mathematical answers must be *interpreted* in terms of the parameters of the real situation from which the problem emerged. It may be necessary to validate the results in terms of their appropriateness to the situation and perhaps re-visit the whole modelling process and produce a modified or a new model.

The modelling cycle above is useful for research purposes, as noted by Blum and Borromeo Ferri (2009), in the sense that it specifies the various abilities and skills required to build a *mathematical modelling competency*. The verbs that describe the steps spell out clearly what actions are necessary to execute each step while the nouns (represented by circles and squares) give the end-product for each step.

2.2.1 (b) *Mathematical modelling perspectives and school mathematics*

For centuries mathematics has always been part of the school curriculum for many countries across the world. Mathematical knowledge has been stable and structured over time and in all societies there have been various reasons put forward by interest groups to justify the teaching of mathematics in schools. Some of these include the humanitarians who believed in the teaching of mathematics for its own sake and knowledge of mathematics, as a discipline, as a sign of intelligence. Others believe that mathematics should be taught so that its knowledge is useful in dealing with other school subjects, life and the world of work (Muller & Burkhardt, 2007). This utilitarian perspective to the teaching and learning of mathematics is also noted by Blum (2002) when he says: “For instance, one essential answer (of course not the only one) to the question as to why all human beings ought to learn mathematics is that it provides a means for understanding the world around us, for coping with everyday problems, or for preparing for future professions.” (p.151). Mathematical modelling would, therefore, be useful if its inclusion in the school mathematics curriculum results in the attainment of goals (humanitarian or utilitarian) for the teaching and learning of mathematics in schools.

Niss (2012) identifies two main arguments for including mathematical modelling in school mathematics. The first argument is that mathematical knowledge should be acquired for the purposes of applications, models and modelling. In other words, school mathematics teaching should pay attention to the utilisation of mathematical knowledge in extra-mathematical contexts for extra-mathematical purposes. This argument is in agreement with the utilitarian goals of mathematics teaching and learning. The second idea which Niss (2012) gave the slogan: “applications, models and modelling for the learning of mathematics”, is that the teaching and learning of mathematics in schools should use mathematical modelling to assist learners acquire mathematical knowledge and consolidate knowledge that they already have. In both cases the process of mathematical modelling is a vehicle either for utilising already learnt mathematical knowledge in extra-mathematical contexts or for facilitating learning and consolidation of mathematical knowledge. In her analysis of mathematical modelling approaches in school mathematics across the European countries, Borromeo Ferri (2013) identifies three main perspectives of mathematical modelling in school mathematics. These are realistic, educational and epistemological perspectives. On the one end there is a realistic perspective in which modelling is primarily an activity for solving authentic problems and not for developing mathematical theory. On the other end lies epistemological modelling in which modelling is an activity for learning and developing mathematical knowledge. Clearly, there is a strong similarity between these perspectives and those of Niss (2012) discussed above. But what about the third perspective – the educational? Borromeo Ferri (2013) answers: “The educational modelling perspective can be seen as an approach between epistemological and realistic modelling.

On the one hand, learning processes are structures, and on the other hand the understanding of mathematical concepts is promoted.” (p.22). Simply put, in the educational modelling perspective, emphasis is on both the modelling process as well as learning and consolidation of mathematical theory. Borromeo Ferri’s (2013) perspectives on the inclusion of mathematical modelling in school mathematics could be traced back to Blum & Niss’s (1991) arguments for the inclusion of modelling, applications and problem solving in mathematics instruction. Blum (2011) later referred to these arguments as justifications for including modelling and applications in everyday teaching of mathematics. According to Blum & Niss (1991) there are five arguments for the inclusion of applications, modelling and problem solving in everyday teaching and learning of mathematics. They are: formative, critical competence, utility, picture of mathematics arguments and promoting mathematics learning. Let us look at a brief description of each one of these.

Formative argument:

According to this argument, the inclusion of applications, modelling and problem solving in mathematics teaching will promote the development of useful competencies and attitudes in learners that will make them good future citizens. Such competencies and attitudes include creativity, problem solving strategies, open-mindedness, self-reliance and confidence.

Critical competence argument:

For Blum & Niss (1991) critical competence enables learners to critique, judge, recognise, understand, analyse and assess actual uses of mathematics in society. This will later enable learners become critical, private and social citizens who participate meaningfully on matters of life and society. Including applications, modelling and problem solving in mathematics teaching will help learners develop this important critical competence.

The utility argument:

The relationship between this argument and the realistic perspective to modelling proposed by Borromeo Ferri (2013) is succinctly captured by this extract from Blum & Niss (1991): “The utility argument emphasises that mathematics instruction should prepare students to utilise mathematics for solving problems in or describing aspects of specific extra-mathematical areas and situations, whether referring to other subjects or occupational contexts.” (p. 43).

The picture of mathematics argument:

According to this argument, mathematics is a human activity where creative mathematical processes lead to the creation of new mathematics or uses of already existing mathematical knowledge.

Applications, modelling and problem solving in mathematics teaching and learning will present this picture of the subject.

Promoting mathematics learning:

This argument is self-explanatory. The inclusion of applications, modelling and problem solving in the teaching of mathematics will help learners acquire and learn mathematical concepts, notions, methods and results. These will then help learners to think mathematically, select and perform mathematical techniques within and outside of mathematics. Clearly, this argument is in agreement with the epistemological perspective of mathematical modelling identified by Borromeo Ferri (2013).

The existence of these perspectives and arguments has resulted in different approaches in the teaching and learning of mathematical modelling across different countries of the world. It is also one of the reasons for mathematical modelling to be so ubiquitous in mathematics curricular documents of so many countries across the world. Julie (2002), brings a different perspective to the inclusion of mathematical modelling in school mathematics. He proposes that mathematical modelling be treated as content in its own right with emphasis on the competencies the process develops in learners. Julie explains: “Mathematical modelling as content entails the construction of mathematical models for natural and social phenomena without the prescription that certain mathematical concepts or procedures should be the outcome of the model-building process.” (Julie, 2002, p.3). In this way, relevance of school mathematical knowledge to learner’s lives could be achieved, argues Julie. Julie’s notion of modelling as content is crucial for successful teaching, learning and assessment of mathematical modelling because it focusses teaching and learning on the various modelling competencies. In this way there would be a reduction in the number of curricular documents claiming to have modelling as one of the competencies the curriculum aims to develop and yet teaching and learning focusses on other aspects such as mathematical techniques or interrogation of context.

2.2.1 (c) *Mathematical modelling competencies and their assessment*

In building up towards the definition of mathematical modelling competency, Jensen (2007) starts by defining the term *competence*. He defines competence as someone’s insightful readiness to act in response to a given challenge. If the challenge is mathematical, then the competence is said to be mathematical. Following this approach, Jensen defines modelling competency as someone’s readiness to carry out all the steps of a mathematical modelling process in a given situation. In a similar manner, Maaß (2006) defines modelling competencies as follows: “Modeling competencies include skills and abilities to perform modeling processes appropriately and goal-oriented as well as the willingness to put these into action.” (Maaß, 2006, p. 117).

In agreement with Maaß (2006), Niss, Blum & Galbraith (2007) define mathematical modelling competency as the ability to identify relevant questions and variables in a given situation; make assumptions and formulate relations between identified variables; translate these into mathematics and solve the mathematical problems that emerge; interpret and validate the resulting solution in terms of the situation as well as the ability to critique models. Clearly Jensen's definition of mathematical modelling competency is a summary of that given by Niss, Blum & Galbraith. The only addition is that of the ability to critique existing models. Fredj (2013) distinguishes between terms competency and competence as follows: "competency (plural competencies) is used as a term for standards to be achieved, while competence (plural competences) is used for an individual's skills." (Fredj, 2013, p.416). With this distinction it becomes clear that Niss, Blum & Galbraith's definition of mathematical modelling competency sets standards for an individual to be declared competent in mathematical modelling while the individual skills to carry out all the mathematical modelling steps constitute mathematical modelling competence.

At the level of a learner, Fredj (2013) identifies three categories of modelling competencies. He calls these implicit, explicit and critical modelling. With implicit modelling learners engage in a mathematical modelling activity without referring to it as mathematical modelling, while in explicit modelling learners are fully aware of the activity they engage in as mathematical modelling and know its aims. Fredj explains critical modelling: "Critical modelling refers to student's ability to reflect critically on the use and role of mathematical modelling in different subjects and in society." (Fredj, 2013, p. 416).

Henning & Kuene (2007) propose that mathematical modelling competence develops through three levels. For Henning & Kuene competence cannot be observed directly but can be inferred from observing student behavior as they work on modelling tasks. The three levels of modelling competence are differentiated as follows: at level 1, called recognize and understand modelling, learners are able to recognize, describe, characterize and distinguish among the different phases of the modelling process. At level 2, the independent modelling level, learners are able to solve a mathematical modelling problem independently and adjust their techniques and models according to changes in the context at hand. The third level is the most advanced level at which learners understand the concept of modelling. It is characterized by the learner's ability to critically analyse modelling, characterise the criteria for model evaluation, and reflect on the course of modelling and applications of mathematics. This level is called meta-reflection on modelling.

For the purpose of this research mathematical modelling competency will be used in the sense of Niss, Blum & Galbraith (2007), that is, to mean the ability to independently perform all the steps of the mathematical modelling cycle in a given context as well as the ability to critique existing models. Mathematical modelling competences will be understood as referring to all skills and knowledge necessary to execute all steps of the mathematical modelling cycle. In what follows, a discussion of mathematical modelling assessment is provided.

As Lingefjard (2002) notices, assessment of mathematical modelling is not easy to accomplish. One main reason for this is the existence of various perspectives on mathematical modelling. These perspectives in turn result in different opinions as to what to prioritize as key competencies when assessing learners. Eric, Dawn, Wanty & Seto (2012) also note this challenge: “Depending on the perspective that mathematical modelling takes and the goals to be fulfilled, the development and assessment of modelling competencies may appear different but at times overlapping.” (Eric et al, 2012, p. 150). For example, if realistic modelling is the main perspective, the focus of learner assessment will be on the ability to use mathematics to interrogate the problem. On the other hand, if the perspective is epistemological, emphasis in assessment will be on mathematical knowledge. In his analysis of mathematical modelling assessment in the Swedish national course tests (NCT), Fredj (2013) noted that certain aspects of mathematical modelling were given more priority than others. In the Swedish school mathematics curriculum, mathematical modelling is one of the topics that must be covered. Fredj (2013) summarises his findings as follows: “Frequently occurring aspects, such as to use an already existing model to calculate a result, were put in favour over other aspects that occurred sparsely or were left out, such as to critical assess conditions and validate results.” (Fredj, 2013, p.414). At the level of problems used in learner assessment of mathematical modelling competencies, the more complicated and open a problem is, the more complicated it is to assess the solution to it, says Lingefjard (2002).

Kang & Noh (2012) identify three levels of modelling problems. This classification is based on ambiguity and completeness of information provided in the problem statement. Level 1 problems are clearly stated with all the information required to formulate a model provided. Kang & Noh explain the expected solution process for such problems: “Students are expected to search for the needed information that is hidden in the problem, recall the (implicitly or explicitly) called for procedure, and carry it out correctly.” (Kang & Noh, 2012, p.7). Level 2 problems have some ambiguity about what needs to be done to solve them and often do not contain all the information needed to complete the task. Students need to devise meaningful ways to collect relevant data and produce reasonable answers. The last level is level 3 questions which contain open-ended information which is often incomplete and/or redundant. Students must first analyse the task to determine what needs to be done and suggest possible solution strategies and carry out these strategies.

It must be noted that all three levels of questions/problems discussed are modelling problems and require execution of all mathematical modelling steps to solve.

With the definition of mathematical modeling competency adopted in this thesis, assessment of mathematical modelling means assessment of all mathematical modelling sub-competencies and the ability to critique or compare existing models. This means assessment of each of the skills involved in moving from one step to next in the modelling cycle. Below are some examples of assessment tasks that could be used for mathematical modelling assessment.

Example 1. (Adapted from OECD/PISA, 2003)

A rectangular field of size 100m by 50m was reserved for the audience of a rock concert. The field was full with all the fans standing. Which one of the following is likely to be the best estimate of the total number of people attending the concert? Explain your choice.

- (a) 2000
- (b) 5000
- (c) 20000
- (d) 50000
- (e) 100000

Example 2 (Adapted from OECD/PISA, 2012)

7 girls share 2 pizzas equally and 3 boys share one pizza equally.

Does each boy get the same amount as each girl? Clearly show how you found your answer.

Example 3 (Adapted from: Blum & Borromeo Ferri, 2009, p.46)

Ms. Jones lives in Bloemfontein, 30 km away from Aliwal North in the Eastern Cape a coastal province. In Aliwal North petrol costs R12, 25 per liter while the cost of the same type of petrol in Bloemfontein is R12, 75 per liter. Is it worthwhile for Ms. Jones to drive her VW golf from Bloemfontein to Aliwal North to fill up petrol? Give reasons for your answer showing all necessary calculations.

There are numerous examples of problems suitable for the assessment of mathematical modelling. The next challenge becomes the choice of an assessment instrument to use. An assessment instrument is simply a list of criteria that the response must satisfy in order to be awarded a point or a mark. It could take the form of a marking memorandum as it is always the case with traditional tests and examinations in mathematics, or it could be a rubric. In whatever form, the assessment instrument must be valid, reliable, fair, less-time consuming and allow the assessor to grade learners.

Rubrics have been favoured over marking memoranda by some researchers in the field of mathematical modelling. Examples include Leong (2012), Eric et al (2012), Anhalt & Cortez (2015) and many others. Such rubrics include all the steps of mathematical modelling as criteria for scoring. An example of a rubric suggested by Leong is given below.

Process	Score	Weight	Total
Identifying Variables		1	
1. State the variables in the model			
2. State problem clearly			
3. State important features		3	
Formulating a Model			
1. Creates a model			
2. Clearly states all assumptions			
3. Describe relationships between variables			
Mathematical Operations		2	
1. Correct use of mathematics			
2. Analyses relationships between variables			
3. Performs operations on the variables and relationships.			
Interpreting the Results			
1. Reaches solution			
2. Interprets solution			
3. Evaluates model and solution		3	
Validating the conclusion			
1. Revises the model based on the problem			
2. Interprets solution based on the revised model			
3. Improves the model		2	
Reporting on Conclusions			
1. Summarises the results			
2. Reasons about assumptions		1	

Figure 2.4: A modelling cycle scoring rubric by Leong (2012)

The rubric is only a suggestion and can be modified according to the needs of the assessor. It is clear that this rubric for learner assessment of mathematical modeling competencies has been developed directly from the mathematical modelling cycle shown in figure 3 above. This modelling cycle is not only useful for the development of instruments for classroom purposes but is also useful in research. For example Eric, Dawn, Wanty & Seto (2012) used the modelling cycle to develop a framework for assessing mathematical modelling competencies of Primary 5 students in Singapore.

From this framework these researchers developed a rubric with specific levels for expected learner performance. Of importance to note is that Eric et al (2012) acknowledge the fact that mathematical modelling is not a linear process and as such some of the activities of the process are carried out throughout the modelling cycle. Eric et al explain: "... we see aspects of *validating and verifying* as situated within the *formulating, solving and interpreting* elements of the modelling process as without validating and verifying, revisions cannot be made towards improving the models." (Eric et al, 2012, p.156). As a result, the framework used by these researchers does not include *verifying and validating* as a distinct competence. An example of a rubric for assessing learner competence used by Eric et al is shown in figure 5 below.

Competences	Level 1	Level 2	Level 3
Assumptions	<ul style="list-style-type: none"> . No assumptions made . Incorrect assumptions 	<ul style="list-style-type: none"> . At least 2 assumptions made and explained. . Assumptions made relevant. 	<ul style="list-style-type: none"> . Comprehensive list of relevant assumptions.
Interpretation of task and solution using real world knowledge	<ul style="list-style-type: none"> . No evidence or only one real world constraint. 	<ul style="list-style-type: none"> . Evidence of 2 real world considerations. 	<ul style="list-style-type: none"> . Evidence of 3 or more real world considerations.
Mathematical reasoning and computation	<ul style="list-style-type: none"> . 1 variable considered. . Appropriate use of mathematics with some minor errors. .Mathematical reasoning somewhat logical. .Attempted recommendations but not well substantiated. 	<ul style="list-style-type: none"> . 2 variables considered. . Appropriate use of mathematics. .Mathematical reasoning is logical. .Recommendations substantiated with strong mathematical reasoning. 	<ul style="list-style-type: none"> . 3 or more variables considered. . Appropriate use of mathematics. .Mathematical reasoning is logical and computations clear and accurate. .Recommendations substantiated with strong mathematical reasoning.

Figure 2.5: Rubric for assessing modelling competencies as used by Eric et al (2012)

In a similar manner Stillman, Galbraith, Brown & Edwards (2007) use the modelling cycle in figure 3 above to develop a framework for the identification of student blockages during completion of a mathematical modelling task. This framework provides details of the processes involved in moving from one step to next in a modelling cycle. The ability to carry out each of these processes successfully is equivalent to being competent in mathematical modelling. In short the specified processes in the framework provide us with a list of skills necessary to successfully solve a mathematical modelling task. Figure 6 below shows the framework.

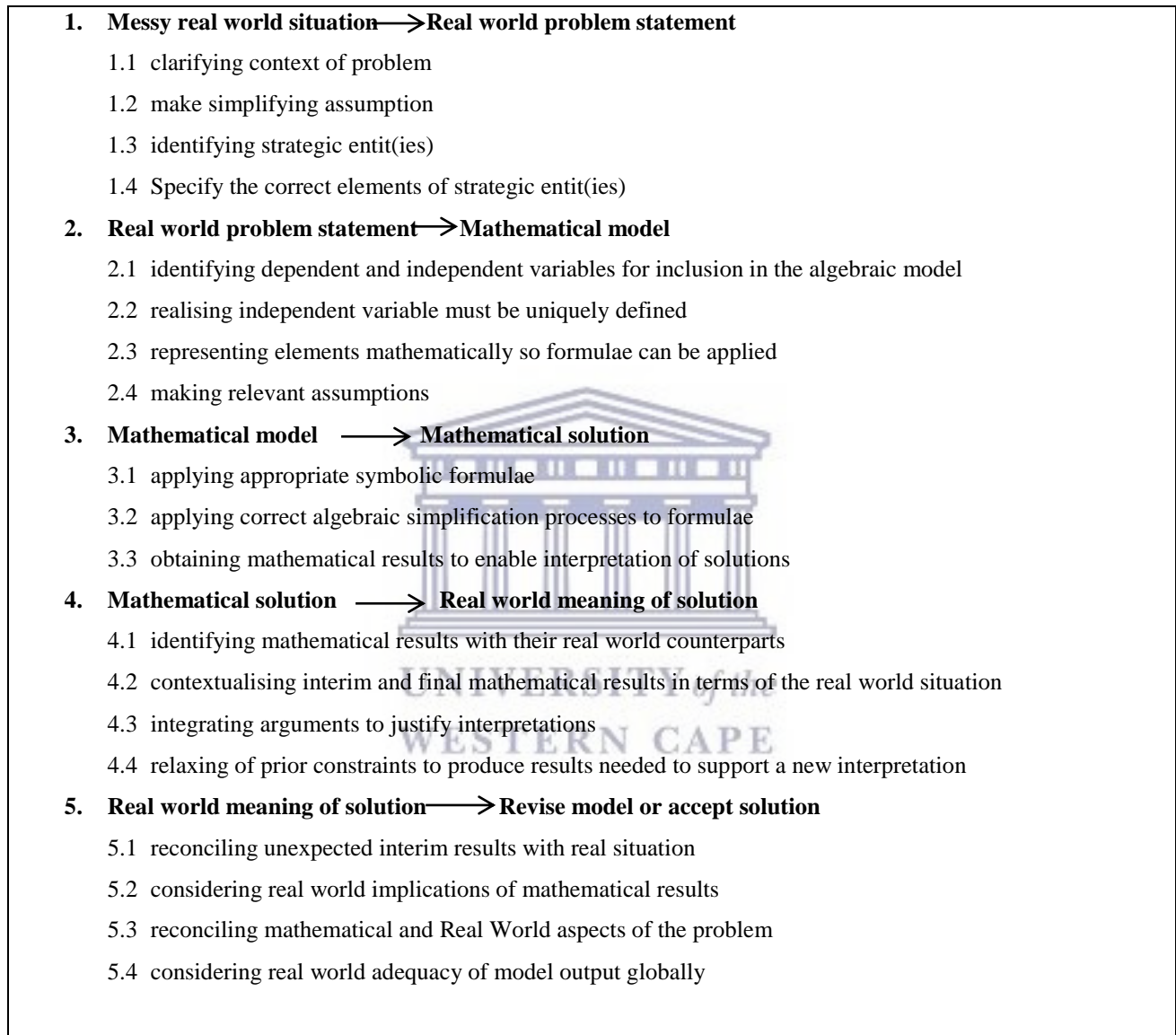


Figure 2.6: A mathematical modelling framework for identifying student blockages in transition, adapted from Stillman et al (2007)

As can be observed from the framework above the numbers 1 -5 represent the various stages of the modelling cycle, their starting objects and finished products. The statements under each stage indicate competencies required in order to successfully execute each stage.

The three examples just discussed demonstrate the usefulness of the mathematical modelling cycle shown in figure 3 above for classroom and research purposes. They also show the flexibility of the modelling cycle in accommodating the needs of the assessor or researcher as the frameworks by Eric et al (2012) and Stillman et al (2007) show. The framework by Stillman et al (2007) is very useful in this thesis as it is used for content analysis – a data analysis method used in this research. It must be noted that mathematical modelling sub-competencies can also be listed as criteria for mark allocation in a marking memorandum. This means that it is not only rubrics that are useful in assessing mathematical modelling.

2.2.2 Mathematical literacy

To avoid confusion in this discussion the term Mathematical literacy (MI) will be used to refer to the international theory of the subject, e.g. in OECD/PISA and Mathematical Literacy (ML) refers to a school subject in the South African FET phase curriculum. The theory on Mathematical literacy will be discussed under the following sub-headings: Mathematical literacy perspectives; Mathematical literacy and Mathematical Modelling; Word problems and school mathematics; Mathematical Literacy in the South African school curriculum; and learner assessment in Mathematical Literacy.

2.2.2(a) Mathematical literacy perspectives

There are numerous definitions of the term Mathematical literacy and as a result the term has many synonyms depending on where one finds the term in this world. For example, Houston, Tenza, Hough, Singh and Booyse (2015) identify four other synonyms to the term. These are Quantitative Literacy (mainly in the USA and Hong Kong), Qualitative Reasoning (mainly in the USA), Numeracy (worldwide) or Functional Mathematics (England). Houston et al (2015) go further in their analysis of this subject and find that all these ‘subjects’ have common quantitative skills they aim to develop in learners. These skills include: computational skills; application of mathematical content; reasoning skills; statistical analysis and application skills; and communication skills.

Perhaps a brief description of each of these forms of mathematical knowledge or ‘subjects’ may be necessary at this stage. Burkhardt (2007) defines Quantitative Literacy as “thinking with mathematics about problems in everyday life”. In other words, Quantitative Literacy entails applications of mathematical knowledge in dealing with everyday life issues. According to Houston et al (2015) Quantitative Reasoning involves three abilities, namely: calculating in a fixed and familiar context, solving problems in a particular applied context and reasoning about relationships.

Evans defines Numeracy as “the ability to process, interpret and communicate numerical, quantitative, spatial, statistical, even mathematical information in ways that are appropriate for a variety of contexts, and that will enable a typical member of the culture or subculture to participate effectively in activities that they value.” (Evans, 2000b, p.236). Functional Mathematics entails sufficient understanding of a range of mathematical concepts and skills and the ability to know how and when to use them. With these definitions saying the same thing but in different ways, the existence of common knowledge and skills among these disciplines, as identified by Houston et al is not surprising.

Julie (2006) sees these definitions as a continuum in which basic mathematical abilities occupy the one end and critical mathematics education sits on the other end. At the lower end of this continuum, Julie identifies what Kilpatrick (2001) calls Mathematical Proficiency. For Kilpatrick Mathematical Proficiency has five strands which are interwoven and interdependent. They are: conceptual understanding; procedural fluency; strategic competence; adaptive reasoning; and productive disposition. Successful learning of mathematics means attainment of Mathematical Proficiency, says Kilpatrick. At the top end of the continuum lies mathemacy. Skovsmose defines the term as follows: “Mathemacy refers not only to mathematical skills, but also to a competence in interpreting and acting in a social and political situation structured by mathematics.” (Skovsmose, 2001, p.123). Christiansen (2006) agrees with Julie’s analysis when she says, “Some writers see mathematical literacy as a narrowly defined competence, which can be demonstrated on word problems or even ‘pure’ calculations. At the other end of the spectrum we see strong links to a critical or democratic competence.” (Christiansen, 2006, p.6). It is this critical or democratic competence which Skovsmose calls mathemacy. Christiansen (2006) goes on and identifies three aspects of mathematical literacy. The first aspect is using mathematics to gain insights into oppression, inequalities and exploitation. The second is the development of awareness of the effects of applying mathematical models in society and lastly the awareness of how mathematics is used as ‘gate keeper’ to limit access to certain careers reserved for the privileged.

Steen, Turner & Burkhardt (2007) define mathematical literacy as ‘the capacity to make effective use of mathematical knowledge and understanding in meeting challenges in everyday life’. The ability to use mathematical knowledge in contexts outside mathematics is one of the skills needed by a 21st century learner in order to function as an individual as well as to participate meaningfully as a member of society. Mathematical literacy empowers learners in exactly the same way as literacy in language does (Steen et al, 2007). Jablonka (2003) agrees: “It is indisputable that in today’s society the ability to deal with numbers and to interpret quantitative information is an important component of literacy in addition to speaking, writing and reading.” (p.76).

Perhaps a unifying definition for all mathematical literacy perspectives is that provided in the Organisation for Economic Co-operation and Development (OECD) Programme for International Student Assessment (PISA) commonly known as OECD/PISA. In this international programme mathematical literacy is defined as:

Individual's capacity to identify and understand the role that mathematics plays in the world, to make well-informed mathematical judgements and to engage in mathematics in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen (OECD/PISA, 1999, p.41).

From all these definitions of mathematical literacy Steen, Turner & Burkhardt (2007) give the following important characteristics of the subject:

mathematical literacy is more than arithmetic or basic skill; it is quite different from traditional school mathematics in the sense that it is inseparable from its contexts and has no special content of its own but finds appropriate content for the context at hand; just like in writing and speaking the level of complexity depends on the level of sophistication of the issue being analysed. (Steen et al 2007, p.2).

The appropriateness of content or/and contexts chosen for inclusion in the curriculum for Mathematical literacy depends on the purpose of the subject for that country or interest group. Similarly, competencies prioritised in learner assessment depend on the purpose for the inclusion of the subject in the school curriculum.

2.2.2(b) *Mathematical modelling and mathematical literacy*

Following the PISA definition of mathematical literacy, Cai, Mok, Reddy & Stacey (2016) suggest the following processes as key to using mathematics to meet a real world challenge.

- Formulating situations mathematically (Formulate)
- Employing mathematical concepts, facts, procedures, and reasoning (Employ)
- Interpreting, applying, and evaluating mathematical outcomes (Interpret)

All these three are processes of Mathematical Literacy, say Cai et al (2016).

The diagram below shows the PISA processes of mathematical literacy.

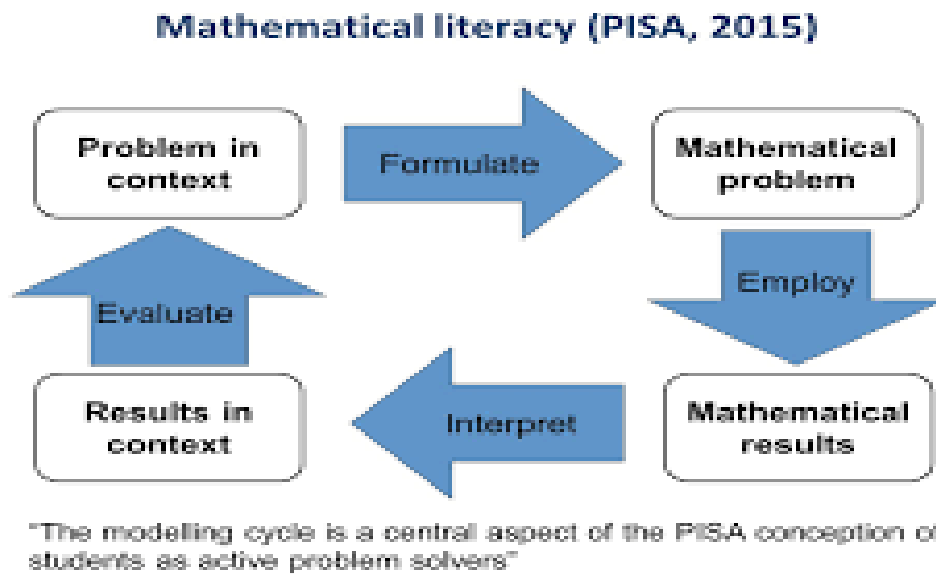


Figure 2.7: The PISA 2012/2015 processes of mathematical literacy from Cai et al(2016),

p. 12

This diagram shows that the development of mathematical literacy is synonymous to the development of mathematical modelling competency. It is, therefore, impossible to teach learners Mathematical literacy, in the sense of PISA, without teaching them mathematical modelling. To be precise, mathematical literacy competencies are similar to those of mathematical modelling. Considering Niss's (2012) definition of a mathematical model as a triple (D,f,M) , where D represents real life knowledge, M the realm of mathematics and f the relationship between the objects of real life and those of mathematics, it becomes clear that to be mathematically literate one must possess not only the mathematical knowledge and that of the real world but also the knowledge to enable formation of meaningful relations between these two domains. In other words, it is possible for an individual to know mathematics and be familiar with a context presented but lack the skill of connecting the mathematical knowledge to the context. Niss's definition then suggests that mathematical modelling can assist an individual to make meaningful connections between mathematics and real world. Steen et al (2007) agree with this view when they comment that the reason most adults use little of the mathematics they learnt from secondary school is the lack of additional modelling skills that would enable them to do so. Brown & Schafer (2006) summarise this argument when they state: "To learn mathematical literacy, it is important to master the mathematics used, as well as to develop familiarity with the different contexts. But it is also necessary to develop the skills needed to be able to effectively relate mathematics and context." (p.50). But does mathematical modelling not only provide opportunity for meaningful connections between mathematics and real world while neglecting the development of critical competence on the part of the learners?

The answer is no. Mathematical modelling does provide an opportunity for learners to critique models they themselves developed or are presented to them thus developing critical competence. Christiansen (2006) agrees: “It seems evident that in order to truly understand how mathematics can be used and what the effects thereof are, learners must engage in modelling of complex phenomena themselves, and engage in critical reflection thereon.” (Christiansen, 2006, p.9). The critical reflection Christiansen is referring to is one of the major components of mathematical literacy mentioned in the PISA definition of the subject. It is the same reflection that Skovsmose (2001) is referring to in his notion of mathemacy. But as Julie (2006) points out the reflective knowledge required for critical reflection requires learners to engage in activities whose purpose is the development of the ability to critique models. English (1999) also suggests this approach of model critiquing for the development of critical competence when she argues:

...if pupils are encouraged to be critical of the mathematical model they have constructed to solve a problem, rather than just find ‘the’ solution, they may begin to develop the skills to interpret information they are presented with more critically, recognise the hidden model and question the assumptions made and the possibility of bias in the interpretation. (English, 1999, p.120).

All these arguments point towards the conclusion that mathematical modelling is central to the development of mathematical literacy.

2.2.2 (c) *Word problems and school mathematics*

The development of competencies required for a learner to be mathematically literate requires use of non-mathematical contexts in the teaching and learning of mathematics. But for decades, as Bonotto (2002) notes, word problems are the only way of connecting mathematical knowledge to reality. The following questions now arise, are all word problems suitable for the development of mathematical literacy in learners? If not which kinds of word problems are suitable for the development of competences required for a 21st century learner to be mathematically literate?

There is a lot of published research on the nature of word problems in school mathematics and their associated challenges. In trying to find out if learners made use of real life considerations when solving mathematics word problems in the classroom, De Corte, Verschaffel and Greer (2000) found that learners excluded real-world knowledge when dealing with mathematics word problems. Learners simply extract numbers from the given ‘story’ in the problem and perform arithmetic operations with these numbers to get a solution. The reason for this, argue the researchers, is that some of the word problems school learners must solve require simple applications of the most obvious arithmetic operations with the given numbers. Such problems result in what the authors call suspension of sense-making by which they mean that learners simply ignore the role contexts play in shaping the solution to a problem.

Suspension of sense-making is not a cognitive deficit on the part of the learners but a product of learner's experiences of school word problems. De Corte et al (2000) suggest that standard word problems found in many classrooms be replaced with mathematical modelling problems in which the method of finding a solution is not so obvious and that will 'force' learners to make use of context information in trying to find solutions. In this way meaningful connections between mathematics and reality can be made.

Even in cases where real life problems are presented to learners, the expectation is for learners to use some part of real life considerations when solving the problems. This is because the assessor already has a fixed answer for the solution which prioritises certain solution paths. This was the finding of the research on mathematics word problems used in the UK to assess 11-12 year old's ability to answer realistic word problems. The research was conducted by Cooper and Harries in 2002.

In his analysis of the types of word problems found in schools mathematics textbooks, Pollak (1969) identified six types of word problems. The first kind he calls immediate use of mathematics in everyday life. Pollak (1969) explains the nature of these problems"

When we check the computation of the sales tax, when we try to figure out how much paint it will take for a living room, when we figure a recipe for a different number of people, when we try to build or move a bookcase, or buy a rug of the right size, or win a little money at poker, or plant tomatoes, we are forever using mathematics in everyday life. (Pollak, 1969, p.393).

The second type are word problems that use words from everyday life and pretend, in varying degrees to be applications. These problems require a certain amount of translation from English to Mathematics to formulate mathematical relationship between the given objects. Then the problem is solved using mathematical techniques. To succeed in these kinds of problems, argues Pollak, a learner needs to practice the translation along with the related mathematical technique.

A third type consists of problems that use words from other disciplines. These problems pretend to come from other scholarly or engineering disciplines but tend to be translation and subsequent mathematical techniques. The reality of the application is often neglected. The difference between this type and the one above is the source of statements that need translation. Statements from this type are derived from the contents of the scholarly disciplines while statements from the second type come from the everyday. The fourth kind are problems of whimsy. These use words from either daily life or other disciplines but with no real application intended. The focus is on finding mathematical relationships between objects and then perform mathematical calculations.

For example, consider the following problem.

Problem

Two ships A and B depart from the same point in the harbour at 90° to each other. Ship A sails at a constant speed of $4m.s^{-1}$ and ship B at $3m.s^{-1}$. How far from each other are the two ships after 20minutes?

Clearly, in this example the focus is on mathematical concepts such as distance, time speed relationship and the theorem of Pythagoras. The words ‘ship’ and ‘harbour’ are taken from daily life but are not required in the solution process.

All the above word problems simply require direct translation of the story into mathematical terms and the application of a mathematical technique. The fifth type is what Pollak calls genuine applications in real life. In this case a situation arises out of a real life context. The situation is usually messy and requires analysis to understand and formulate a clear problem to solve. The problem translated into mathematical terms and a model is created that could be used to find a specific solution to the given problem or solve related problems. These problems help learners to focus on the process of problem solving rather than the answer. The process for solving these kinds of problems is very similar to mathematical modelling. An example of such problems is example 3 given under the examples of mathematical modelling problems in section 2.1.3 above. In some cases, argues Pollak, learners may discover that some situations do not even need mathematics to understand.

The sixth and the last type are genuine applications in other disciplines. This is similar to type five above. The only difference is that the situation that needs interrogation and analysis to understand originates from a scholarly discipline.

The classification of word problems in school mathematics above reveals the following:

- School mathematics word problems are not the same.
- All mathematical modelling problems are word problems, but not all word problems are mathematical modelling problems.
- Not all word problems are suitable for making meaningful connections between mathematics and real life and hence develop mathematical literacy in learners.
- Mathematical modelling problems help develop mathematical literacy.

This classification of school word problems and their related shortcomings is used in the discussion of the types of questions used for learner assessment on the topic of measurement in the national Applications paper for Mathematical literacy in grade 12.

2.2.2 (d) *Mathematical Literacy in the South African School Curriculum – a focus on the topic of measurement*

The subject Mathematical Literacy in the South African school curriculum is intended to equip learners with knowledge and skills that will enable them to use elementary mathematical concepts and skills to manage their everyday lives, make useful contributions at the workplace, and critically participate in social and political issues. (Department of Basic Education, 2011). The striking relationship between competences Mathematical Literacy in South Africa aims to develop in learners and the first three arguments for the inclusion of mathematical modelling in school mathematics instruction presented by Blum and Niss (1991) is particularly useful in this study. This relationship may be summarized as follows:

- A self-managing person requires general competencies and attitudes such as ability to explore, creativity, problem solving abilities as well as open- mindedness, self-reliance and confidence. These are the competences of the formative argument for the inclusion of mathematical modelling in mathematics instruction.
- The ability to make meaningful contributions at the workplace can also be directly linked with the utility argument; and
- Critically participating in social and political matters is directly related to the development of critical competence.

The definition of Mathematical Literacy in the South African school curriculum tends to focus more on the benefits of the subject than on the contents of the subject itself. In other words, the definition provides justification or argument for the inclusion of the subject in the school curriculum instead of stating what the subject entails. This is very similar to the PISA definition of mathematical literacy which also focuses on the benefits of the subject rather than its content. For example, capacity to identify and understand the role that mathematics plays in the world; make well-informed mathematical judgments; engage on issues as a constructive, concerned and reflective citizen are benefits embedded in Mathematical literacy. There is a clear relationship between the PISA definition of Mathematical literacy and that given for Mathematical Literacy in the South African school curriculum in terms of purpose.

In the South African context, content for teaching, learning and assessment in the subject is organized in topics. The same topics are used for all three grades in the FET phase. There are two main topics in the curriculum, namely, *Basic Skills* and *Applications* topics.

Much of the content in the Basic Skills Topics comprises elementary mathematical content and skills that learners have already been exposed to in their first ten years of schooling (e.g. different number formats and conventions, calculating percentages, drawing graphs from tables of values, and so on). The Department of Basic Education (DBE) explains the contents of the applications topics.

...The Applications Topics contain contexts related to scenarios involving daily life, workplace and business environments, and wider social, national and global issues that learners are expected to make sense of, and the content and skills needed to make sense of those contexts. (Department of Basic Education, 2011. *Curriculum and Assessment Policy Statement (CAPS), Mathematical Literacy*, 2011, p.13).

The table below gives a summary of content arrangement in the subject.

Table 2.1: Content organisation as per CAPS, Mathematical Literacy

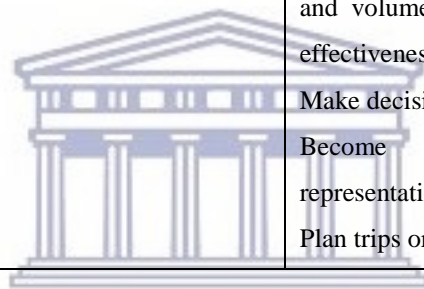
Basic Skills Topics	Applications Topics
Patterns, relationships and representations	Finance
Number and calculations with numbers	Measurement
Interpreting and communicating answers and calculations	Maps, plans and other representations of the physical world
	Data handling
	Probability

As stated above, learners have already been exposed to content in the Basic Skills Topics. They are expected to use this content to interrogate and make sense of the contexts in the Applications Topics. Furthermore, topics under Applications contain some content that learners need to know in order to be able to solve problems in those topics. Albeit specific these topics are stated, UMALUSI, the Department of Basic Education's examination and quality assurance body, emphasizes generic exit outcomes for the FET phase for each of the applications topics.

For the applications topic of measurement, the focus topic for this study, table 2.1 above can be extended using UMALUSI's generic exit outcomes for FET to produce a third column as table 2.2 below shows.

Table 2.2: The topic of measurement and its exit outcomes in the FET phase: UMALUSI (2014)

Basic Skills Topics (elementary mathematics to use)	Measurement (Applications topic)	Exit-level outcomes for FET (content/skills/competencies)
Patterns, relationships and representations	Conversions(of measuring units)	Use, recognize and convert appropriate units
	Measuring distance	Gain useful spatial and visual orientation
Number and calculations with numbers	Measuring mass	Solve spatial problems
	Measuring volume	Make informed decisions relating to space and shape
Interpreting and communicating answers and calculations	Measuring temperature	Gain practical experience in using measuring instruments
	Calculating perimeter, area and volume	Recognise relationships between Fahrenheit and Celsius temperature scales
	Time	Recognise impact of temperature in everyday life
		Estimate lengths, areas, time and quantities of materials
		Solve practical problems of perimeter, area and volume involving quantities and cost-effectiveness
		Make decisions relating to cost-effectiveness
		Become familiar with diverse representations of time
		Plan trips or projects using time constraints



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The last column in the table above is very crucial in this study. It is this column that is used to judge the quality of the grade 12 end-of the year national examinations in Mathematical Literacy on the topic of measurement in particular. In other words, this column gives a summary of competencies the subject Mathematical Literacy aims to develop in learners through the topic of measurement.

(e) Learner assessment in Mathematical Literacy

Assessment is the process of gathering, interpreting and synthesizing information about learner’s state of learning in order to make informed judgements. The Department of Basic Education (DBE) (2011) defines assessment as a “continuous process of identifying, gathering and interpreting information about the performance of learners, using various forms of assessment.” (Department of Basic Education, 2011. *Curriculum and Assessment Policy Statement (CAPS), Mathematical Literacy*, 2011, p. 96).

From the two definitions two characteristics of assessment emerge, namely, assessment is a process and it is for judging learner performance. Fredj (2013) identifies three objectives of assessment. They are formative, summative and evaluative. Fredj explains the difference amongst the three: "...assessment as an aid for learning (formative assessment), assessment as a guide for certifying student's performance (summative assessment) and assessment to monitor the quality of institutions or educational programs (evaluative assessments)." (Fredj, 2013, p.416). The impact of assessment on other role players in education is far reaching. For example, Boesen, Lithner & Palm (2010) argue that assessment tasks are cornerstone in student's work in mathematics in the sense that they influence students by directing them to a particular content and specific ways of information processing. Boesen et al go on and explain the impact assessment tasks have on teachers and textbook writers: "The types of tasks, and thus the competencies that are valued, in the tests may influence the work of teachers and textbook writers." (Boesen et al, p.90). The strength of the influence of assessment on student learning is even high when such assessments are set nationally. This is the reason why the object of analysis in this project is the national examination for grade 12.

For Mathematical Literacy, learner assessment mark has two components in grade 12. There is continuous assessment, commonly known as school-based assessment (SBA) and the national examinations at the end of the year. The SBA mark consists of marks obtained by the learner throughout the course of the year on various forms of assessment. These are assignments, project/investigation, tests and internal examinations. The SBA mark constitutes 25% of the learner's final mark in the subject. The remaining 75% is made up of learner performance in the national examinations. So national examinations have a much high contribution towards learner's final mark and therefore play a much bigger role in deciding whether a learner passes the subject or not. This is another reason this thesis is interested in the national examination. The national examination for Mathematical Literacy consists of two papers, the *Basic Skills paper* and the *Applications Skills paper* respectively called paper 1 and paper 2. Paper 1 assesses competence in basic mathematical skills whilst paper 2 assesses ability to explore both familiar and unfamiliar contexts and solve problems. Paper 2 with special focus on the topic of measurement is an object of analysis in this study.

2.2 Research literature

Since Mathematical Literacy is a new subject in the South African school curriculum, very little research has been done on the subject. However, studies of analytical nature have been done by Mbekwa and Julie (2007), where they analyzed the nature of contexts found in Mathematical Literacy textbooks. These researchers found that contexts used in Mathematical Literacy come from different categories.

They identified contexts that are mathematical; financial; socio-political; geographical; life science & environmental; and technological. The researchers also found that context engagement was not made explicit in the textbooks and in the main contexts were used for the development, application and practice of mathematical ideas. The study does not analyse learner assessments to see if context engagement level is similar to that observed in the textbooks. It is hoped that the analysis proposed in this study will assist in taking this idea of context engagement further into learner assessment.

Venkat, Graven, Lampen and Nalube (2009) conducted a research on how the Mathematical Literacy taxonomy for questions affects the nature of questions used in the national examinations. According to the Department of Basic Education, CAPS, Mathematical Literacy (2011), examination questions should be set at four different taxonomy levels. The table below clarifies the difference between taxonomy levels as well as percentage allocation of marks per taxonomy for each examination paper.

Table 3. *Taxonomy levels of questions for examination papers in Mathematical Literacy*

Taxonomy level	Paper 1	Paper 2
Level 1: Knowing	60% ($\pm 5\%$)	-----
Level 2: Applying routine procedures in familiar contexts	35% ($\pm 5\%$)	25% ($\pm 5\%$)
Level 3: Applying multi-step procedures in a variety of contexts	5% (minimum)	35% ($\pm 5\%$)
Level 4: Reasoning and reflection	-----	40% ($\pm 5\%$)

These researchers found that shortcomings in this taxonomy structure have resulted in the absence of problem solving questions in the national examinations. These kinds of questions, argue Venkat et al (2009) are central to the development of competencies that are aligned with curriculum aims of the subject. The researchers chose two questions from the 2008 national examinations and analysed these using taxonomy levels to arrive at the conclusion. This thesis aims to focus attention on a particular topic, in this case measurement, and conduct an in-depth analysis of what competencies are assessed over a period of time. Instead of looking at problem-solving alone, which is also part of mathematical modelling, the study aims to look at competencies assessed using a broader mathematical modelling framework.

Another research of interest which is of relevance to this thesis is the study conducted by Brown & Schafer (2006). The researchers used a mathematical modelling approach for the training of mathematical literacy teachers. Teachers were given two modelling tasks. In one task they were asked to design a pattern for tiling a floor with given dimensions and determine the number of tiles required. In another task, teachers were asked to design parking spaces in accordance with certain procedures and rules.

Although some teachers struggled to complete the tasks in the given space of time the project was a success and most teachers enjoyed it. The researchers also found that knowledge of mathematical content as well as context familiarity is not enough to succeed in mathematical modelling. The ability to effectively relate mathematics and context is as important as well, argue Brown & Schafer (2006). The research shows that the mathematical modelling approach is useful in developing competencies Mathematical Literacy aims to develop in learners.

2.3 Conclusion

This chapter presented a review of literature relevant to the research problem. Both the theoretical and research literature were discussed. The theoretical literature focused on concepts and terminology that were used throughout this study whilst the research literature presented some research conducted related to the research problem. Theoretical literature was used in the construction of a framework for the analysis of data as well as during the discussion of the findings from data analysis. Some of this theory such as types of word problems in school mathematics and the kinds of mathematical modelling problems was used as a framework for the discussion of types of questions found in the Mathematical Literacy examinations for paper 2 in South Africa. The theory related to research methodology and data analysis is presented in the next chapter.



Chapter 3

Research design and methodology

3.1 Introduction

In this chapter a discussion of the design of this research as well as methods for data collection and analysis are provided. Research methodology related issues and how these are addressed in this study also form part of this chapter. The discussion is organized into two main headings, namely, research type and design, data collection and analysis methods; and validity, reliability, credibility and ethical issues.

3.2 Research type and design, data collection & analysis methods

In this section the discussion focusses on research type and design, sampling and data collection, data analysis methods and illustrative example for data analysis.

3.2.1 Research type and design

The type of research used in the study is descriptive research. This categorization is consistent with Boudah's (2011) explanation of the aim of descriptive research. In Boudah's terms, "...the researcher's purpose [of descriptive research] is to understand and report the characteristics of a current or past situation." (Boudah, 2011, p.12). The situation in this case is the nature of competencies assessed in the national examinations in the applications paper in Mathematical Literacy. The research design is qualitative and takes the form of a case study. Like any qualitative research this case study follows processes, namely, sampling, coding and interpretation (Boudah, 2011). Sampling processes will be dealt with in the next section while coding will be discussed in detail under data analysis section.

3.2.2 Sampling and data collection

Boudah (2011) defines a population as a group with certain identifying characteristics. Sometimes this group may be too large for research purposes and a small subgroup representing the large group has to be chosen. This process of choosing a small representative subgroup from a population is called sampling. Since the research follows a case study approach and is qualitative the sampling used is critical case sampling. Boudah (2011) explains what it is: "In critical case sampling, the researcher chooses the situations or participants because of their uniqueness or how important they are to the issue." (Boudah, 2011, p.141). Critical case sampling is useful in making qualitative research strategic and purposeful. Applying this theory of sampling to this particular research, the population is the set of all answers in a grade 12 national examination memorandum.

The topic of measurement is also examined in paper 1 in the national examinations but the purpose of that paper is not problem solving and hence one cannot expect assessment of any modelling competencies in the paper. It is for this reason paper 1 marking memoranda are not chosen and only that of paper 2 are. The sample for analysis, answers on the topic of measurement, has been chosen using critical case sampling. The uniqueness of this topic arises from the fact that questions on the topic are necessarily context-based. This makes it possible for mathematical modelling competencies to be assessed in some of the questions. Moreover, mathematical content in the topic of measurement is elementary school mathematics and includes applications of basic formulae for calculating perimeter, area, surface area and volume. All these formulae are examples of mathematical models that can be used in the process of modelling. Therefore, an analysis of competencies assessed in a question involving a concept or concepts from the measurement topic using a mathematical modelling framework is feasible. Hence the choice of answers from the topic of measurement is strategic and purposeful. The set of answers in the sample constitutes data for analysis. Answers for questions on the topic of measurement for national examinations November 2014, February/March 2015, November 2015, February/March 2016 and June 2016 were analysed. Again the choice of years is strategic since the latest curriculum revision concluded in 2011 and the first grade 12 examinations based on the revised curriculum, called CAPS, were written in 2014. In the next section data analysis methods used are discussed.

3.2.3 Data Analysis Methods

The data analysis strategy used here is qualitative content analysis. The strategy is a natural choice since data to analyse is qualitative. Hsieh & Shannon (2005) define qualitative content analysis as a method of analysis for the subjective interpretation of the content of text data through the systematic classification process of coding and identifying themes or patterns. Content analysis, as Mayring (2000) points out, ‘embeds the text into a model of communication within which it defines the aims of analysis.’ Category development follows a deductive category application approach, since categories are derived directly from the theory of mathematical modelling through the mathematical modelling framework. Mayring (2000) explains the process: “The main idea here is to give explicit definitions, examples and coding rules for each deductive category, determining exactly under what circumstances a text passage can be coded with a category.” (p.5). Deductive category application approach is the same approach as what Stemler (2001) calls a priori coding. In dealing with a priori coding categories are established prior to the analysis based upon some theory, explains Stemler (2001). In this case the theory upon which category establishment is based is mathematical modelling. Perhaps one needs to explain the meaning of the word *category* at this stage. By category it is meant a group of words with similar meaning or connotations. (Weber, 1990, p37, cited in Stemler, 2001).

Coding units then are these words or sentences or even paragraphs that belong to a category. In content analysis it is crucial for categories to be both exhaustive and mutually exclusive. This simply means that all words/sentences/paragraphs should belong to a category and that no word/sentence/paragraph is assigned to two different categories at the same time. The product of category formation, definition and coding is a coding agenda which basically is the framework for text data analysis. The framework to be used in this study has been developed from the mathematical modelling framework of Stillman et al (2007) discussed in the previous chapter following the content analysis process. Figure 3.1 below shows the resulting data analysis framework.

Framework for content analysis

No.	Category	Definition (competences)	Coding Rules	Task	
				Question X	
				Key word/s	Marks
1	Production of a problem statement from a messy real world situation.	<ol style="list-style-type: none"> 1. Write down the 'givens' and the 'required to find'. 2. Make assumptions 3. Simplify the problem by writing appropriate relationships 	Satisfy both 1 and 2 OR Satisfies 3 only		
2	Mathematical model from real world problem statement.	<ol style="list-style-type: none"> 1. Identify dependent and independent variables for inclusion in the model 2. Uniquely defining variables 4. Represent relationships between quantities mathematically. 	Satisfy 1, 2 and 3 3 only acceptable		
3	Mathematical solution from the mathematical model.	<ol style="list-style-type: none"> 1. Applying formulae (model) correctly 2. Correct algebraic simplifications 3. Obtain mathematical results/solution 	Satisfy 1,2 and 3		
4	Interpret mathematical solution in terms of the real world.	<ol style="list-style-type: none"> 1. Match mathematical results with their world counterparts 2. Interpret mathematical results in terms of real situation 	Satisfy both 1 and 2		
5	Accept solution or validate or critique and revise model.	<ol style="list-style-type: none"> 1. Accept/ write down final solution 2. Reconcile unexpected mis-match between mathematical results and real world expectations 3. Revise the model 	Satisfies 1 or 2 or 3		

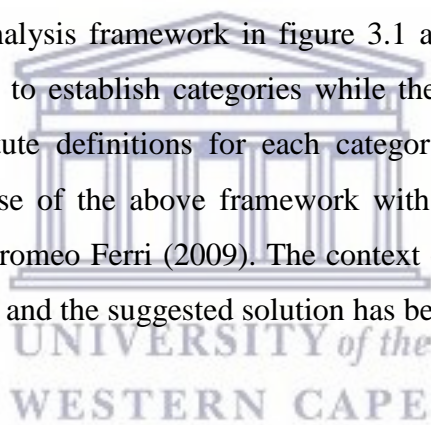
Figure 3.1 Content analysis framework based on Mathematical Modelling

Table 3.1 below gives some detailed explanation of coding rules used in the content analysis framework as well as clarification of codes used in the national marking memoranda for examinations.

Table 3.1: Explanation of codes and coding rules

Coding rules as used in content analysis framework	Codes used in the national marking memoranda for examinations
<p>Explanation of coding rules:</p> <ul style="list-style-type: none"> • Verbs that are used for mark allocation are used for category placement. • A solution may not show the formula being applied. In such a case the formula is implied and the answer belongs to category 2. • If the formula is provided in the question, the answer belongs to category 3. • Rounding an answer according to context belongs to category 4. Any rounding to a specified number of decimal places belongs to category 3. • Marks given for conversion of units belong to category 3. 	<p>Explanation of codes used in the national marking memoranda:</p> <ul style="list-style-type: none"> • M means Method • MA means Method with Accuracy • CA means Consistent Accuracy • A means Accuracy • D means Define • S means Simplification • SF means Substitution into a formula • R means Rounding Off • RT/RD/RG/RP means Reading from a Table or Graph or Diagram or map or Plan. • NP means No Penalty for rounding OR omitting units.

As it can be observed from the analysis framework in figure 3.1 above, the steps of the mathematical modelling process have been used to establish categories while the individual mathematical modelling sub-competencies or skills constitute definitions for each category. There are five categories in the framework. Let us illustrate the use of the above framework with a mathematical modelling problem example taken from Blum and Borromeo Ferri (2009). The context of the problem has been modified to make it appropriate to South Africa and the suggested solution has been produced by the researcher.



Example

Problem (Adapted from Blum & Borromeo Ferri, 2009, p.46)	Suggested solution
<p><i>Mrs. Stone lives in Bethlehem, 20km from the border of the Eastern Cape province where immediately behind the border there is a petrol station. In the Eastern Cape (a coastal province) petrol cost R11.10/litre whereas in Bethlehem the cost is R11.35/litre. To fill up her VW golf she drives to the Eastern Cape.</i></p> <p><i>Is it worthwhile for Mrs. Stone to drive to the Eastern Cape to fill up her car? Give reasons for your answer.</i></p>	<p>Step 1</p> <p><i>Givens:</i> 20km distance between Bethlehem and E.Cape, petrol price E.Cape = R11.10/litre, Beth = R11.35/litre</p> <p><i>Assumptions:</i> this is an economics problem, worthwhile means does she save money by going to the E.Cape to fill-up?</p> <p>She buys 30litres of petrol to fill up a 40litre tank.</p> <p>Fuel consumption of the golf is 5.2litres per 100km</p> <p>Step 2</p> <p>Cost of filling up = litres x price per litre</p> <p>Cost of travelling = fuel consumption x distance x price per litre + litres x price/litre(E.Cape)</p> <p>Step 3</p> <p>Cost of filling-up in Beth = 30litres x R11.35/litre = R340.50</p> <p>Cost of filling-up in E.Cape = 5.2l/100km x 40km x R11.35(assuming that fuel in the car was bought in the Bethlehem) + 30litres x R11.10</p> <p style="text-align: right;">= R23.61+R333 = R356.61</p> <p>Step 4</p> <p>It costs less to fill-up in the E.Cape than in Bethlehem. But the cost of travelling makes it more expensive to travel to the E.Cape to fill up. The results would hold for any amount of petrol purchased.</p> <p>Step 5</p> <p>It is not worthwhile for Mrs. Stones to travel to the E.Cape to fill-up for the reasons in step 4 above.</p>

A closer look at the question and the suggested solution reveals the following:

1. The question can be categorised as level 2 mathematical modelling type of question according to Kang & Noh (2012) classification.
2. Mathematical modelling sub-competencies from each modelling step can be identified.

This task could be assessed by a marking memorandum or a rubric both with specified mark allocations. Mark allocation specificity is achieved by allocating a mark/s to each of the sub-competencies.

The framework for content analysis in figure 3.1 above allows for key words (which represent sub-competencies) to be identified from the marking memoranda and the mark allocated to each key word recorded. These key words belong to the different categories in the framework and the results of item analysis can be recorded in the following table.

Table 3.2: Summary table for data analysis

<i>Item</i>	<i>Production of a problem statement from a messy real world situation.</i>	<i>Mathematical model from real world problem statement.</i>	<i>Mathematical solution from the mathematical model.</i>	<i>Interpret mathematical solution in terms of the real world.</i>	<i>Accept solution or validate or critique and revise model.</i>
e.g. Nov.2014 exam					

The above table is derived directly from the content analysis framework which also derived from the mathematical modelling framework. Therefore, the summary table for data analysis is itself mathematical modelling framework for recording the results of data analysis. It gives quantitative data in the sense that the amount of marks allocated to different categories in the content analysis framework are recorded as percentages. Chapter 4 gives a detailed illustration of the data analysis process. A complete analysis of all data items is given in appendices A to E, at the end of chapter 5.

3.3 Validity, reliability, credibility and ethical issues

In this section issues of validity and reliability are discussed together because of the relationship between these two concepts. Credibility and ethical issues are each discussed separately.

3.3.1 Validity and reliability issues

Boudah (2011) defines validity as the degree to which the conclusions drawn from a research study come from the study itself and not from chance or error. In a similar manner Brink (1993) distinguishes between internal and external validity. For Brink internal validity refers to the extent to which research findings are a true reflection of reality while external validity is a measure of the generalisability of findings. Validity, whether internal or external, is associated with quantitative and quasi-experimental studies. Since the study presented here is none of the two, validity is addressed differently. Before addressing validity issues for his research let us look at another related concept - reliability.

Brink (1993) defines reliability as the extent to which a research method can yield consistently the same results over different periods of testing. Boudah (2011) follows this definition when he defines reliability as the degree to which a study can be repeated with similar results. Just like validity, reliability is a concept mostly associated with quantitative and quasi-experimental research. For a qualitative study the meanings of these two terms are slightly adjusted to suit the situation as Noble & Smith (2015) explain:

In the broadest context these terms are applicable [to qualitative research], with validity referring to integrity and application of the methods undertaken and the precision in which the findings accurately reflect the data, while reliability describes consistency within the employed analytical procedures. (Noble & Smith, 2015, p.34).

With these adjusted meanings, Noble & Smith (2015) suggest the incorporation of methodological strategies that increase the 'trustworthiness' of the findings. Boudah (2011) notes this when he states that many researchers who use qualitative methods use the idea of trustworthiness. The issue of trustworthiness is closely linked to the concept of credibility which is concerned with establishing the truth value of the study. One method of establishing the truth value is through inter-rater reliability or agreement. Inter-rater agreement is the degree of agreement among raters. It is useful for refining and increasing consistency in tools for judgements. In this study data analysis instrument's reliability was addressed through inter-rater reliability. The original framework for content analysis was given to knowledgeable peers (Mathematical Literacy subject advisors) to analyse the same question and its solution. The results of analysis of at least three peers are then compared to check for consistency. The analysis framework was then refined, i.e. making coding rules more specific thus increasing mutual exclusiveness of categories. The refined framework was given back to the experts (subject advisors) and the average percentage agreement between 2 randomly chosen pairs of experts was approximately 98% (97.76%). The final data analysis framework is a product of modifications after the analysis of the results from peers. Although credibility is closely related to trustworthiness and hence can be addressed through truth values increases, this only addresses credibility of the chosen data analysis methods. Credibility issues regarding the researcher and sources of data are discussed in the next section.

3.3.2 Issues of credibility

According to Patton (1999), credibility of qualitative enquiry depends on three distinct elements. These are rigorous methods for doing field work that produce high quality data; credibility of the researcher; and philosophical belief in the value of qualitative inquiry. The first element is addressed in the study through the choice of the assessment task to use as a source of data. National externally set examinations are a credible source of data for the following reasons:

- Examination question papers and marking memoranda are set by an independent external panel of examiners and moderators chosen across the country through legal and credible means of advertising, criteria specification and careful screening.
- Draft papers are set and presented for external moderation by the independent standards and quality assurance body, UMALUSI. Continuous discussions between the examination setting panel and UMALUSI moderators take place throughout the year to ensure the quality of the product.
- After examination papers have been written by all provinces, provincial moderators meet the examination panel and UMALUSI moderators to iron out any issues with the marking memoranda and produce a final memorandum to be used by all provinces to mark learner responses.

The final marking memoranda that are used as a source of data in this study are, therefore, credible.

Patton (1999) goes on and contends that the credibility of the researcher depends on training, experience, track record, status, and self-presentation. Boudah explains the importance of this credibility element in qualitative research: “Though quantitative methodology includes procedures for decisions within the study, the researcher processes the data in a unique way, based upon training, experience, bias, and other factors.”(Boudah, 2011, p.76). To address this credibility element, the researcher’s credibility may be summarised as follows:

The researcher is a provincial head of the subject Mathematical Literacy for the Western Cape Province. He holds an B.Sc.(HON) Degree in Mathematics specialising in Group Theory, Ring Theory, Measure Theory, General Topology, and Functional Analysis; a B.Sc. (Ed) Degree specialising in Mathematics, Chemistry and Education both from the University of Transkei; a PG Diploma in Curriculum Studies specialising in Curriculum, Pedagogy and Society; Changing frameworks in curriculum and education; Educational studies in mathematics; Science education; and Advanced research methods from the University of Cape Town. He has been involved in the education field on a number of capacities, namely, a High school mathematics and physical sciences teacher, a University junior lecturer, a mathematics subject advisor, a teacher trainer, a school text book author in mathematics (Platinum Mathematics, grade 7, published by Maskew Miller Longman, SA), mathematics (Headstart Mathematics, grade 9, published by Oxford University Press, SA), Mathematical Literacy(Via Afrika Mathematical Literacy, grade 10 – 12 series, published by via Afrika Publishers), and Mathematical Literacy Study Guide also published by via Afrika Publishers . All these books are nationally approved and included in the national catalogue for South African schools. Currently the researcher is managing, monitoring and supporting the implementation of the subject Mathematical Literacy in the Western Cape. He is also the current chairperson of the mathematics provincial subject committee.

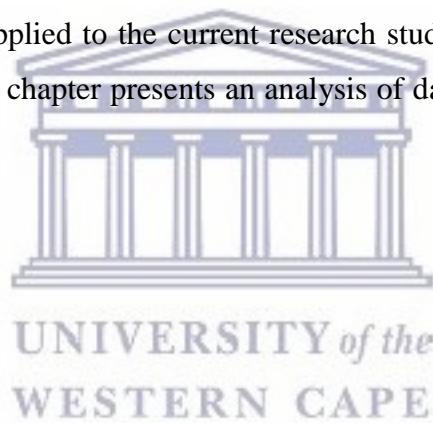
All this information is presented here to address the credibility of the researcher element of credibility.

3.3.3 Ethical issues

Data sources for this study are the grade 12 national examination question papers and their marking memoranda. Although these papers are published in the departmental website after the examination results have been released, and therefore become public domain knowledge, permission was formally requested from the Western Cape Education Department (WCED) to use these papers for research purposes. After following correct procedures, permission to use the papers was granted in writing by WCED which is a provincial branch of the national Department of Basic Education (DBE) that owns the papers.

3.4 Conclusion

In this chapter a number of issues related to research and methodology were discussed. Sampling and data collection; data analysis methods and their use; validity, reliability, credibility and ethical issues were explained. Effort was made to start the discussion of each issue with some underlying theory about the issue. The theory would then be applied to the current research study to show that legitimate means are used to address the issue. The next chapter presents an analysis of data items using methods discussed in this chapter.



Chapter 4

Analysis of data

4.1 Introduction

In this chapter analysis of data items is presented. Data items are the actual questions and their answers selected for the purposes of this research from the five nationally set examination papers. As mentioned in chapter 3, the five national examination papers chosen are November 2014, March 2015, November 2015, March 2016 and June 2016. A full analysis of questions on the topic of measurement is done for each of these examinations according to the following process. Questions and their answers as presented in the national marking memorandum from one examination are presented. This is immediately followed by an analysis of each answer using a mathematical modelling framework. The process continues until all the questions on the topic of measurement from all five examinations are analysed. At the end of the analysis a summary table showing the distribution of competencies assessed in all these papers is presented. Section 4.2 below shows this process by analysing one data item - questions and answers from the November 2014 examination. A complete analysis of all data items is given in appendices A – E at the end of chapter 5. The last section presents a summary of the results of data analysis.

4.2 Analysis and presentation of the results of the measurement question for one of the examinations.

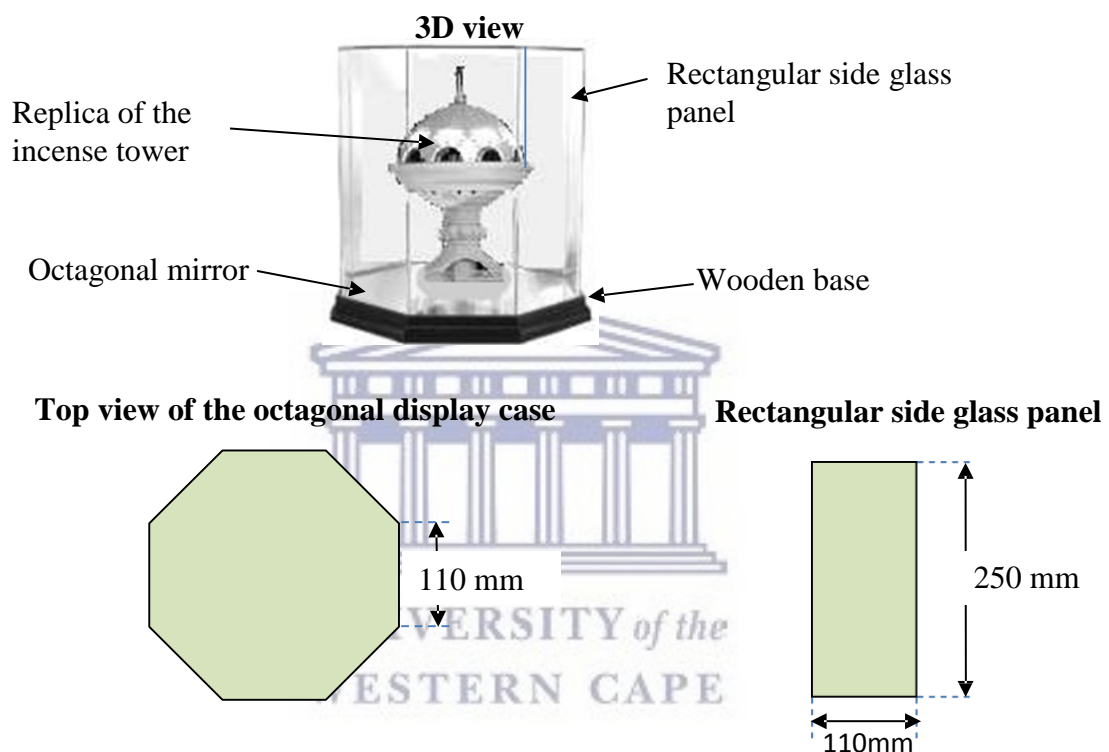
As mentioned earlier in this section a full analysis of a question and its memorandum is presented. The question is based on the topic of measurement and is taken from the November 2014 national examination, Mathematical Literacy, Paper 2. The actual question and its memorandum are given in figure 4.1 below. Table 4.1 that follows gives the analysis of the memorandum.

QUESTION

4.3 Jackie bought a replica of the giant incense tower she saw in Muscat as a souvenir. She displays the replica in an octagonal glass display case with a wooden base as shown in the picture below. On top of the base is an octagonal mirror to enhance the display of the incense tower.

The inside dimensions of the identical rectangular side glass panels of the display case is 110 mm by 250 mm.

The inside surface area of the octagonal top is 0,058 423 m².



NOTE: All eight sides of the octagon are equal in length.

The following formula may be used: $TSA = P \times H + K$, where:

TSA = The total inside surface area of the octagonal display case, excluding the mirror

P = The perimeter of the octagonal base

H = The height of the rectangular side glass panels

K = The inside surface area of the octagonal top

4.3.1 Jackie would like to tint the inside of the glass using a special type of spray paint. This paint is sold in 250 ml spray cans.

The following information is printed on the side of the spray can:

- 100 ml of spray paint can cover $0,07 \text{ m}^2$ of glass per coating.
- Apply two coats.

Calculate the number of spray cans of paint needed to tint the glass of the display case. (8)

4.3.2 The scale of the replica is 1: 164.

Calculate the actual height, in metres, of the tower if the height of the replica inside the display case is only 1 cm less than the height of the side glass panels. (3)



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Solution:

4.3.1

$$\begin{aligned}
 \text{TSA} &= P \times H + K \\
 &= 8 \times 110 \text{ mm} \times 250 \text{ mm} + 58\,423 \text{ mm}^2 \quad \checkmark \text{SF} \\
 &= 220\,000 \text{ mm}^2 + 58\,423 \text{ mm}^2 \\
 &= 278\,423 \text{ mm}^2 \quad \checkmark \text{S} \\
 &= 0,278\,423 \text{ m}^2 \quad \checkmark \text{C} \\
 \text{For } 0,07 \text{ m}^2 \text{ one needs } 100 \text{ ml of paint} \\
 \therefore 1 \text{ m}^2 \text{ one need } \frac{100}{0,07} \text{ ml} \quad \checkmark \text{M} \\
 &= 1\,428,57 \text{ ml} \\
 \\
 \therefore 0,278423 \text{ m}^2 \text{ need} &= 1428,571429 \times 0,278423 \\
 &= 397,7471429 \text{ ml} \\
 &\approx 397,75 \text{ ml} \quad \checkmark \text{CA} \\
 \text{Two coats} &= 2 \times 397,75 \text{ ml} \\
 &= 795,49 \text{ ml} \quad \checkmark \text{CA} \\
 \\
 \text{Number of spray cans} &= \frac{795,49 \text{ ml}}{250 \text{ ml}} \\
 &= 3,18184 \\
 &\approx 4 \quad \checkmark \text{CA}
 \end{aligned}$$

1A total area of panels
 1SF substitution in formula
 1S simplification
 1C conversion to m²
 1M Method

1CA paint needed for 1 coat

1CA paint needed for 2 coats

1CA rounding up

L4

4.3.2

$$\begin{aligned}
 \text{Height} &= 240 \text{ mm} \times 164 \quad \checkmark \text{MA} \\
 &= 39\,360 \text{ mm} \quad \checkmark \text{CA} \\
 &= 39,36 \text{ meters} \quad \checkmark \text{C} \\
 \therefore \text{The height of the actual tower is approximately } &39,4 \text{ m} \\
 &\text{OR} \\
 \text{Height} &= 25 \text{ cm} - 1 \text{ cm} = 24 \text{ cm} = 0,24 \text{ m} \quad \checkmark \text{MA} \quad \checkmark \text{C} \\
 \text{Actual height} &= 0,24 \times 164 = 39,36 \text{ m} \quad \checkmark \text{CA}
 \end{aligned}$$

1MA correct height
 1CA correct answer in mm
 1C conversion

OR

1MA correct height
 1C conversion
 1CA correct answer in m
 NPR

L2

(3)

Figure 4.1: November 2014, Mathematical Literacy, Paper 2 – Measurement

Table 4.1: Assignment of marks to the different competencies as per the memorandum of marking – November 2014 examinations

<i>Framework for content analysis memorandum: Measurement</i>				<i>Mathematical Literacy Exam marking</i>			
No.	Category	Definition (competences)	Coding Rules	November 2014			
				Question 4.3.1		Question 4.3.2	
				Key word/s	Marks (8 marks)	Key word/s	Mark (3)
1	Production of a problem statement from a messy real world situation.	1. Write down the ‘givens’ and the ‘required to find’. 2. Make assumptions 3. Simplify the problem by writing appropriate relationships	Satisfy both 1 and 2 OR Satisfies 3 only				
2	Mathematical model from real world problem statement.	1. Identify dependent and independent variables for inclusion in the model 2. Uniquely defining variables 3. Represent relationships between quantities mathematically.	Satisfy 1, 2 and 3 3 only acceptable				
3	Mathematical solution from the mathematical model.	1. Applying formulae (model) correctly 2. Correct algebraic simplifications 3. Obtain mathematical results/solution	Satisfy 1,2 and 3	<i>Perimeter</i> <i>Substitution</i> <i>Simplification</i> <i>Conversion to m²</i> <i>Proportion</i>	1 1 1 1 1	<i>Correct height</i> <i>Conversion</i> <i>Answer</i>	1 1 1 [3]

				<i>Proportion proportion</i>	1 [7]		
4	Interpret mathematical solution in terms of the real world.	<ol style="list-style-type: none"> 1. Match mathematical results with their world counterparts 2. Interpret mathematical results in terms of real situation 	Satisfy both 1 and 2	<i>Rounding up</i>	1 [1]		
5	Accept solution or validate or critique and revise model. (This category occurs in all answers in the memo since they all have 'an answer'.)	<ol style="list-style-type: none"> 1. Accept/ write down final solution 2. Reconcile unexpected mis-match between mathematical results and real world expectations 3. Revise the model 	Satisfies 1 or 2 or 3				
Explanation of coding rules:				Explanation of codes used in the national marking memoranda:			
<ul style="list-style-type: none"> • Verbs that are used for mark allocation are used for category placement. • A solution may not show the formula being applied. In such a case the formula is implied and the answer belongs to category 2. • If the formula is provided in the question, the answer belongs to category 3. • Rounding an answer according to context belongs to category 4. Any rounding to a specified number of decimal places belongs to category 3. • Marks given for conversion of units belong to category 3. 				<ul style="list-style-type: none"> • M means Method • MA means Method with Accuracy • CA means Consistent Accuracy • A means Accuracy • D means Define • S means Simplification • SF means Substitution into a formula • R means Rounding Off • RT/RD/RG/RP means Reading from a Table or Graph or Diagram or map or Plan. • NP means No Penalty for rounding OR omitting units. 			

<http://etd.uwc.ac.za/>

4.3 Summary of the results

In this last section of the chapter a summary of the results from content analysis of all data items is presented. The data in the summary table is quantitative and is used in chapter 5 in which results, recommendations and limitations of this research are discussed. A complete analysis of all data items is given in appendices A – E at the end of the last chapter.



Table 4.2: Summary of the results

<i>Exam</i>	<i>Production of a problem statement from a messy real world situation.</i>	<i>Mathematical model from real world problem statement.</i>	<i>Mathematical solution from the mathematical model.</i>	<i>Interpret mathematical solution in terms of the real world.</i>	<i>Accept solution or validate or critique and revise model.</i>
November '14 [11 marks]	0	0	10	1	0
March 2015 [13 marks]	0	1	12	0	0
November 2015 [26 marks]	0	2	22	2	0
March 2016 [15 marks]	0	2	12	1	0
June 2016 [15 marks]	0	2	13	0	0
Total	0	7	69	4	0
Percentage	0%	8.75%	86.25%	5%	0

Grand total for all the marks: 80

Number of examinations: 5

4.4 Conclusion

The focus of this chapter was the presentation of data analysis. The chapter started by providing clarity on how data items were analysed by content analysis using a mathematical modelling framework. The same framework is used to record the findings of content analysis in quantitative form. In the next chapter, both qualitative data in the content analysis framework and quantitative data in the summary table are used as a basis of the discussions.



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Chapter 5

Discussion of the results and conclusion

5.1 Introduction

In this chapter the results of data analysis presented in chapter 4 are discussed. In the discussion, reference is made to the complete analysis of all data items given in appendices A – E as well as to the summary table given in the previous chapter. The literature, both theoretical and research, discussed in chapter 3 will be used in this discussion. At the end of the discussion an attempt is made to answer the research questions posed in chapter 1. The chapter is organised into four sections as follows. The first section deals with the discussion of the results in the context of the aims of the subject Mathematical Literacy and the purpose of examination paper 2. Towards the end of this discussion an attempt is made to answer the research questions that resulted in this research. The second section discusses some recommendations for further research and policy makers. These recommendations are based on the findings from data analysis and relevant research and theoretical literature. The third section discusses some limitations of the present study. Finally, the chapter ends with a brief summary of the discussions and conclusions.

5.2 Results of data analysis and their discussion

The following observations are based on the analysis of data presented in the previous chapter. The analysis of this data was done using content analysis methods as explained in chapter 3 of this thesis. A summary table based on the mathematical modelling framework is also given towards the end of chapter 4. Discussions in this chapter are therefore based on both the qualitative results from the content analysis framework and the quantitative results on the summary table. Before the presentation and discussion of the findings the following statements regarding content and purpose of Mathematical Literacy in the FET phase of schooling, competencies the subject aims to develop in learners, progression from Grade 10 – 12 and the purpose of examination paper 2 may be necessary to put the discussion in context.

These statements have been randomly selected from the curriculum and assessment policy statement (CAPS), 2011, for the subject.

The mathematical content of Mathematical Literacy is limited to those elementary mathematical concepts and skills that are relevant to making sense of numerically and statistically based scenarios faced in everyday life of individuals (self-managing individuals) and the workplace (contributing workers), and to participating as critical citizens in social and political discussions. In general, the focus is not on abstract mathematical concepts. (Department of Basic Education, CAPS, Grade 10 – 12, Mathematical Literacy, 2011, p. 8).

On the issue of competencies to develop in learners to make them mathematically literate CAPS (2011) says:

Learners who are mathematically literate should have the capacity and confidence to interpret any real-life context that they encounter, and be able to identify and perform the techniques, calculations and/or other considerations needed to make sense of the context. (Department of Basic Education, CAPS, Grade 10 – 12, Mathematical Literacy, 2011, p. 9).

But how does progression in terms of knowledge taught and learned occurs from Grade 10 to 12 in the FET phase? CAPS answers:

One of the ways in which Mathematical Literacy develops across the grades is in terms of mathematical concepts and skills. For example, in Grade 10 learners are expected to be able to work with one graph on a set of axes; in Grade 11 two graphs; and in Grade 12 two or more graphs on the same set of axes. ... Progression also occurs in relation to the nature, familiarity and complexity of the context in which problems are encountered. (Department of Basic Education, CAPS, Grade 10 – 12, Mathematical Literacy, 2011, pp. 11 – 12).

The examination paper 2 seems to be the paper in which these problem solving skills will be assessed as CAPS (2011) pronounces:

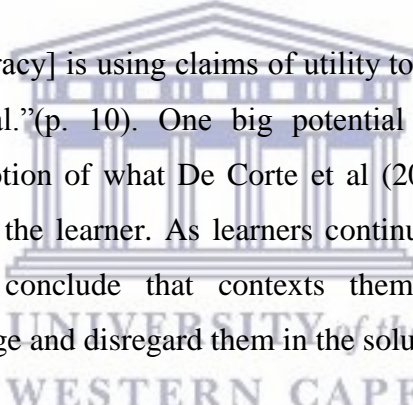
The intention of this examination paper is to assess the ability to identify and use a variety of mathematical and non-mathematical techniques and/or considerations to understand and explore both familiar and unfamiliar authentic contexts. (Department of Basic Education, CAPS, Grade 10 – 12, Mathematical Literacy, 2011, p. 107).

It is against the background of these statements that this research was conducted. These statements will be referred to during the course of discussion in this chapter. The analysis of the national marking memoranda for Mathematical Literacy, Paper 2 for questions on the topic of measurement reveals the following:

- All questions are context-based: This is consistent with the purpose of the paper as stated in the CAPS document (see fourth extract above).
- Of the five examination papers analysed 86.25% of the total marks on the topic of measurement are awarded for mathematical calculations, techniques and skills. Only 8.75% of the total was awarded for introducing a mathematical model (e.g. formula, graph, etc.) that can be used to relate the given quantities in a problem. In almost all cases models are provided in the form of formulae and conversion factors/relations and learners are simply expected to apply the given formulae to arrive at the required solution. This finding is consistent with the observation by Mbekwa & Julie (2009) on the role of contexts used in textbooks for Mathematical Literacy. These researchers concluded that context usage in these textbooks served the purpose of developing, applying and practising mathematical ideas and skills. As noted by Venkat et al (2009) the provision of formulae provides extensive scaffolding such that reasoning which is an important component of problem solving is completely removed. As such some ‘smart’ learners would know that a formula is provided to be used. So they make sure that they substitute correct values with correct measurement units into the formula and simplify. They are likely to get more than 50% of marks allocated to that question without making any reference to the given context.

- 5% of the marks were allocated to the skill of interpreting the solution in terms of context. Such interpretation is usually limited to some commentary based on the calculated answer (e.g. November 2015, question 3.4 and March 2016, question 5.1.3) or inferred from rounding-off choices (rounding up or down to the nearest whole number according to context).

- In some questions the role of context in the solution process is so invisible such that the question is more of a mathematics nature than mathematical literacy. Examples of such questions include appendix A, question 3.4; appendix B, question 1.5; appendix D, question 5.1.1 and 5.1.2 and appendix E, question 1.2.1. This is inconsistent with the purpose of the subject and in particular that of paper 2 given in the CAPS document. The mathematical focus of these questions is in agreement with Christiansen (2006)'s argument about the nature of the subject Mathematical Literacy when she claims:


 "It [Mathematical Literacy] is using claims of utility to justify itself, yet its content is distinctly mathematical." (p. 10). One big potential danger with these kinds of questions is the promotion of what De Corte et al (2000) call suspension of sense-making on the part of the learner. As learners continue to encounter these kinds of questions they may conclude that contexts themselves are only carriers of mathematical knowledge and disregard them in the solution process.

- Some of the questions (e.g. November 2014, question 4.3.2, November 2015, question 3.3.1 and March 2016, question 5.1.2), the so-called level 2 questions do not meet the criteria set for paper 2 – assess ability to use both mathematical and non-mathematical techniques to explore familiar and unfamiliar contexts. Their inclusion in this paper can be traced to the prescribed taxonomy levels of questions for paper 2. The prescribed taxonomy levels are mathematically based and pose a challenge to both the aims of the subject and its assessment. In trying to meet the requirements of taxonomy levels distribution of questions in a question paper (see chapter 2, section 3 for details), examiners modify questions by asking for quantities on the right hand side of the given formula so that learners have to change the subject of the formula to get the answer (e.g. March 2015, question 1.5 and March 2016, question 5.1.1).

They also give measurement of the same quantity, for example length, in different units e.g. cm and mm, so that learners have to do unit conversion calculations before substituting into the given formula or convert units at the end of the calculation. In some cases (e.g. November 2015, question 3.3.2) multiple calculations of the same type (e.g. areas of different objects, windows, doors, walls, etc.) are used to adjust questions to level 3 of the taxonomy. All these efforts are made to adjust taxonomy levels of questions. Such mathematical adjustments make the context less- authentic and real thus defeating the purpose of the paper and that of the subject as Venkat et al (2009) note:

However, to allow mathematical progression to dominate the assessment of a curriculum that is oriented towards quantitative and mathematical reasoning for real-life situations appears akin to the “tail wagging the dog” and runs the risk of diminishing the emphasis on understanding everyday contexts that is central to the curriculum rhetoric. (Venkat et al, 2009, p.49).

- Types of questions used in the topic of measurement for paper 2 vary from whimsical problems (e.g. November 2014, question 4.3.2; March 2015, question 1.5; March 2016, question 5.1.1 & 5.1.2) to immediate use of mathematics in daily life (e.g. November 2014, question 4.3.1; March 2015, question 5.2.3; November 2015, question 3.3.2 and 3.4 and March 2016, question 5.1.3). This classification is based on Pollak’s (1969) analysis of the types of word problems found in school mathematics textbooks. (See chapter 2 for a detailed discussion).
- Questions categorised as immediate applications of mathematics (see examples in the bullet above) can easily be modified to mathematical modelling problems of level 1 according to Kang & Noh (2012) classification of modelling problems. This can be done by removing the given formulae (models) in the question and allow learners to search for the appropriate formulae themselves.

- The skill *production of a problem statement from a messy real world situation* is not assessed since all the problems in the paper are straight and clear questions with mathematical formulae provided where necessary. The same is true for the skill of *critiquing existing models*. No questions in the papers assesses this important skill for a critical citizen who participates meaningfully on social and political issues.

With this presentation and discussion of the results, I'm now in a position to answer the research questions this project is trying to answer. Let us recap these questions as they are stated in chapter 1.

1. What competencies are assessed in the national examinations for Grade 12 in Mathematical Literacy?
2. How do these competencies relate to those of mathematical modelling?

To answer the first question, let us recall that the national examination for Grade 12 in Mathematical Literacy consist of two papers. The purpose of paper 1, the basic skills paper, is clearly stated in the CAPS (2011) document:

The intention of this paper is to assess understanding of the core content and/or skills outlined in the CAPS document in the context of authentic real-life problems. Although questions will be contextualised, the focus is primarily on assessing proficiency in a range of content topics, techniques and/or skills. (Department of Basic Education, CAPS, Grade 10 – 12, Mathematical Literacy, 2011, p. 105).

With this clearly stated intention for the paper one cannot expect any problem solving questions in this paper, other than demonstration of mastery of basic mathematical calculations and techniques. Paper 2 is intended to assess problem solving through context engagement. But the prescription of taxonomy levels of questions for this paper results in the assessment focused more on mathematical skills – just like Paper 1. The reason for this, as Venkat et al (2009) point out, is that the taxonomy for the levels of questions for Mathematical Literacy is more mathematical than mathematical literacy. So to answer research question 1, in the main mathematical knowledge and skills are the main competencies assessed in the Grade 12 national examinations for Mathematical Literacy.

Questions categorized as level 4, whose purpose is to assess reasoning and reflection, are reduced by excessive scaffolding in the form of providing formulae and conversion factors to the level of just substituting correct values and perform correct mathematical procedures – a mathematical skill. As the discussion of the results show, some questions assess immediate applications of mathematics in real life but again prioritize mathematical techniques and skills during mark allocation. Little attention is given to learners developing or selecting appropriate models to use to solve the problem at hand. The same can be said regarding interpretation of answers in terms of the context from which the problem emerged.

Mathematical knowledge and skills also form part of mathematical modelling competencies or sub-competencies. As the results show there are some mathematical modelling competencies assessed in the national examinations but not all mathematical modelling competencies are assessed. This answers research question 2. In the next section some recommendations for further research and improvement of the subject are discussed.

5.6 Some recommendations for further research and improvement of the subject

As Julie (2006) says, Mathematical Literacy is here to stay for a number of social, economic and political reasons. These reasons emanate from the utilitarian approach adopted by the subject. But if the status quo remains we may be living in a ‘false world’ of assumed benefits of introducing the subject Mathematical Literacy in the FET phase and yet none of the competencies the subject aims for are developed in learners. South Africa took the lead by introducing this important subject as a separate subject from Mathematics. In most countries (e.g. Singapore, Sweden, etc.) of the world mathematical literacy or quantitative literacy is part of the school mathematics curriculum. It is usually described as mathematical modelling whose aims are exactly the same as those of mathematical literacy. For example Eric et al (2012) say about the Singaporean school mathematics curriculum:

A distinct feature that sets mathematical modelling apart from traditional problem solving in the Singapore curriculum is that modelling provides a platform for students to ‘deal with ambiguity, make connections, select and apply appropriate mathematics concepts and skills, identify assumptions and reflect on the solutions to real-world problems, and make informed decisions based on given or collected data.’(p.147).

There is a strong relationship between the benefits of mathematical modelling and those of mathematical literacy. In fact, even the critical competency can be realized through the model critiquing sub-competence of mathematical modelling and Julie (2002) agrees: “In essence the envisioned *critical competence* can only be realized in the mathematical applications and modelling component of the Mathematical Literacy curriculum.” (p.192). Skovsmose (1990) summarises competencies/knowledge/skills learners develop when engaging in modelling tasks in the classroom as mathematical knowledge itself; technological knowledge about how to build and use mathematical models; and reflective knowledge for discussing the nature of models and criteria used in their construction, applications and evaluation – critical competence.

The PISA 2012/2015 processes of Mathematical literacy discussed by Cai et al (2016) are precisely mathematical modelling processes. This means developing Mathematical literacy in learners is synonymous to developing mathematical modelling competencies. Similarities between arguments advocated by Blum (2011) for the incorporation of mathematical modelling in everyday classroom teaching of mathematics and the aims of Mathematical Literacy in the South African school curriculum also point towards the strong relationship between mathematical modelling and Mathematical Literacy. It is against this background that the following recommendations for the improvement of the subject are made:

- South Africa should lead the way towards making mathematical modelling as content a major component of the Mathematical Literacy curriculum. Since the subject is completely separated from Mathematics a great opportunity exists for what Borromeo Ferri (2013) calls realistic modelling in which modelling is treated as an activity for solving authentic real-life problems and not for developing mathematical theory. As it has been argued above the development of mathematical modelling competencies through appropriate modelling activities for learners will result in the development of the envisaged competencies for Mathematical Literacy learners. This will move the subject further towards the upper end of the Mathematical literacy continuum in terms of Julie (2006) classification. Furthermore, the challenge of teach ability of Mathematical Literacy in the classroom highlighted by Julie (2006) would be addressed by teaching Mathematical Literacy through mathematical modelling.

Julie (2006) further contends that the difficulty in teaching Mathematical Literacy is due to lack of epistemic dependence on experts and lack of experimentation with hypothetical teaching trajectories. Fortunately, mathematical modelling, as Blum & Borromeo Ferri (2009) contend, is teachable. The two researchers suggest four implications, based on empirical findings, for successful teaching of mathematical modelling in the classroom. These are:

1. The substance for quality teaching is constituted by appropriate modelling tasks. When treating modelling tasks, a permanent balance between maximal independence of students and minimal guidance by the teacher ought to be realized.
2. It is important to support student's individual modelling routes and to encourage multiple solutions. To this end, teachers have to be familiar with task spaces and to be aware of their own preferences for special solutions.
3. Teachers have to know a broad spectrum of intervention modes, also and particularly strategic interventions.
4. Teachers have to know ways how to support adequate student strategies for solving modelling tasks. (Blum & Borromeo Ferri, 2009, p. 54).

Blum & Borromeo Ferri (2009) further suggest that teachers should stick to the seven step schema presented in chapter 2 when teaching mathematical modelling as this schema is indispensable for research and teaching.

- The CAPS document should be revised to remove contradictions in the document and the creation of artificial boundaries for content progression across grades in the FET phase. For example, the first extract from the CAPS document above claims that the content for the subject Mathematical Literacy is limited to elementary mathematical concepts and skills. The fact that these mathematical concepts and skills are elementary means that learners have already been exposed to them in the previous Grades and now they form part of their common sense knowledge. Therefore, content progression in Mathematical Literacy does not exist. In fact, this claim is consistent with the observation made by Steen et al (2007) when they conclude: "It [mathematical literacy] is quite different from traditional school mathematics in the sense that it is inseparable from its contexts and has no special content of its own but finds appropriate content for the context at hand..." (p.2).

The boundaries set for content progression such as one graphs on a set of axes for Grade 10, two graphs for Grade 11 and three or more graphs for Grade 12 are just artificial. To elaborate more on this point, for the topic of Data Handling content progression from Grade 10 to 12 is achieved by stating that Grade 10 learners should deal with one set of data, Grade 11 learners with two sets of data and Grade 12 learners with multiple sets of data (see CAPS page 24). But in drawing one graph on a set of axes in Grade 10, learners already work with two sets of data (data for the vertical axes and data for the horizontal axis) and working with two data sets in the topic of Data Handling is prescribed for Grade 11 in the same curriculum. Therefore, the contradiction can be attributed to the artificial nature of content boundaries between Grades and these should be removed.

- The classification of learner assessment questions using the current taxonomy should be removed from the curriculum. As discussed in the previous section, the taxonomy is based on mathematical knowledge, skills and techniques and hence poses a danger to defeat the whole purpose of the subject. As such, Steen et al (2007) have the following to say about complexity levels in mathematical literacy: "...just like in writing and speaking the level of complexity depends on the level of sophistication of the issue being analysed." (Steen, Turner & Burkhardt, 2007, p.2). This statement concurs with Kang & Noh (2012) classification of modelling problems which is based on ambiguity and completeness of information provided in the problem. It is strongly recommended that paper 2 examinations should contain the three different levels of modelling questions identified by Kang & Noh so as to assess the curriculum stated objectives of this paper.
- Make Mathematical Literacy compulsory for all learners. There is no logical argument for learners taking pure mathematics not to take Mathematical Literacy. The two subjects are completely different both in terms of content and purpose. The Purpose of school mathematics is to introduce and guide learners into the practice of mathematics. Mathematics is about structures and relationships between structures and these do not necessarily relate to real life situations. In fact, as Houston et al (2015) argue, doing mathematics does not make one mathematically literate. These researchers suggest that Mathematical Literacy be compulsory for all learners.

Steen et al (2007) explain the nature of this subject: “Mathematical literacy is neither an expanded list of topics to be added to the mathematics curriculum nor is it just the basic skills part of a traditional mathematics program.” (Steen et al, 2007, p. 9). The subject requires open-minded thinking to understand the problem at hand and make meaningful connections between mathematical content and the context of the problem as noted by Brown & Schafer (2006). The skills Mathematical Literacy aims to develop in learners enable the learner to function with confidence in a world characterized by information presented in mathematical terms. Such information includes weather charts, information tables, data tables, exchange rate graphs, etc. Surely all learners need this kind of literacy including those taking pure mathematics. As such, Steen et al (2007) have the following warning if Mathematical literacy is not offered by those learners taking pure mathematics: “If it [pure mathematics] is offered as an alternative, it will surely remain the prestige track, with Mathematical literacy becoming a ‘sink’ subject, taken only by weak students, while the well-qualified adult population remains innumerate.” (p.10).

Let us conclude this section with some possible suggestions for further research.

- The suggestions made for curriculum and assessment modifications above are based on the analysis of competencies assessed in the topic of measurement using a mathematical modelling framework. It can be a good research to apply the same framework on marking memoranda for questions from other topics in paper 2 national examinations to identify competencies assessed in these topics as well.
- Analysis of data on learner performance on paper 2 national examinations reveals that learners perform poorly on level 4 questions, particularly those with minimum scaffolding. It may be a good ground for research to determine whether a mathematical modelling approach to teaching and learning of Mathematical Literacy could have a positive effect on learner performance in these kinds of questions.

5.7 Limitations of the study

The only limitation of this study is that the analysis of competencies assessed in the national examinations for the subject Mathematical Literacy was done only for questions on the topic of measurement. There are three more topics whose questions and solutions need to be analysed to reach a well-informed conclusion. However, the topic of measurement is naturally context-based and has a lot of readily available models (formulae for calculating different quantities, conversion factors and table, etc.) to use to assess mathematical modelling competencies. It is highly likely that other topics assess more mathematical knowledge/skills and techniques than problem solving competencies and critical competence.

5.8 Conclusion

In this chapter the presentation and discussion of the results of data analysis was done. The results indicate that learner assessment in the national examinations for the subject Mathematical Literacy prioritises mathematical knowledge and skills and focusses less on problem solving and critical competence – the skills the subject aims to develop in learners through the subject. This problem could be attributed to the existence of a prescribed taxonomy for classification of questions in the national examinations. This taxonomy classifies examination questions in a mathematical way using the mathematics knowledge structure. Some recommendations for subject improvement were also made. Finally, the limitations of the study were also discussed.

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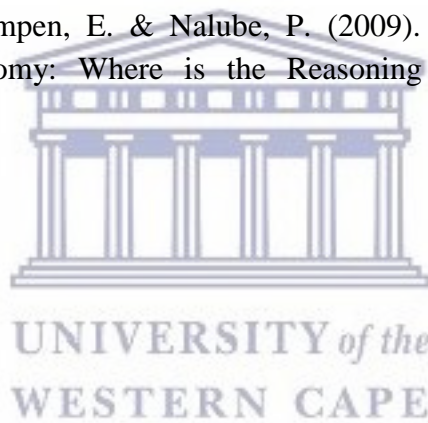
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Appendices

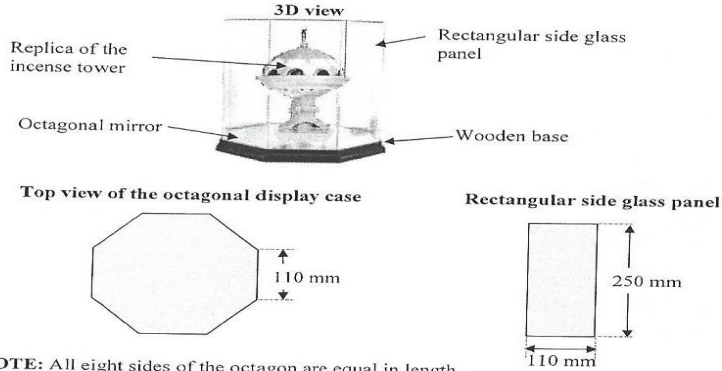
Appendix A: November 2014, Mathematical Literacy, Paper 2 - Measurement

NSC-examinations, November 2014: Mathematical Literacy, Paper 2 – Measurement

Question

4.3

Jackie bought a replica of the giant incense tower she saw in Muscat as a souvenir. She displays the replica in an octagonal glass display case with a wooden base as shown in the picture below. On top of the base is an octagonal mirror to enhance the display of the incense tower.
The inside dimensions of the identical rectangular side glass panels of the display case is 110 mm by 250 mm.
The inside surface area of the octagonal top is 0,058 423 m².



NOTE: All eight sides of the octagon are equal in length.

The following formula may be used: $TSA = P \times H + K$, where:

TSA = The total inside surface area of the octagonal display case, excluding the mirror
P = The perimeter of the octagonal base
H = The height of the rectangular side glass panels
K = The inside surface area of the octagonal top

4.3.1 Jackie would like to tint the inside of the glass using a special type of spray paint. This paint is sold in 250 ml spray cans.

The following information is printed on the side of the spray can:

- 100 ml of spray paint can cover 0,07 m² of glass per coating.
- Apply two coats.

Calculate the number of spray cans of paint needed to tint the glass of the display case. (8)

4.3.2 The scale of the replica is 1 : 164.

Calculate the actual height, in metres, of the tower if the height of the replica inside the display case is only 1 cm less than the height of the side glass panels. (3)

Solution

4.3.1

$$\begin{aligned} TSA &= P \times H + K \\ &= 8 \times 110 \text{ mm} \times 250 \text{ mm} + 58\,423 \text{ mm}^2 \quad \checkmark SF \\ &= 220\,000 \text{ mm}^2 + 58\,423 \text{ mm}^2 \\ &= 278\,423 \text{ mm}^2 \quad \checkmark S \\ &= 0,278\,423 \text{ m}^2 \quad \checkmark C \end{aligned}$$

For 0,07 m² one needs 100ml of paint

$$\therefore 1 \text{ m}^2 \text{ one need } \frac{100}{0,07} \text{ ml} \quad \checkmark M \\ = 1\,428,57 \text{ ml}$$

$$\therefore 0,278423 \text{ m}^2 \text{ need } = 1428,571429 \times 0,278423 \\ = 397,7471429 \text{ ml}$$

$$\approx 397,75 \text{ ml} \quad \checkmark CA \\ \text{Two coats} = 2 \times 397,75 \text{ ml} \\ = 795,49 \text{ ml} \quad \checkmark CA$$

$$\text{Number of spray cans} = \frac{795,49 \text{ ml}}{250 \text{ ml}} \\ = 3,18184 \\ \approx 4 \quad \checkmark CA$$

DEPARTMENT OF BASIC EDUCATION

1A total area of panels
1SF substitution in formula
1S simplification
1C conversion to m²
1M Method

1CA paint needed for 1 coat

1CA paint needed for 2 coats

1CA rounding up

L4

4.3.2

$$\begin{aligned} \text{Height} &= 240 \text{ mm} \times 164 \quad \checkmark MA \\ &= 39\,360 \text{ mm} \quad \checkmark CA \\ &= 39,36 \text{ meters} \quad \checkmark C \end{aligned}$$

\therefore The height of the actual tower is approximately 39,4m

OR

$$\begin{aligned} \text{Height} &= 25 \text{ cm} - 1 \text{ cm} = 24 \text{ cm} = 0,24 \text{ m} \quad \checkmark MA \quad \checkmark C \\ \text{Actual height} &= 0,24 \times 164 = 39,36 \text{ m} \quad \checkmark CA \end{aligned}$$

1MA correct height
1CA correct answer in mm
1C conversion

OR

1MA correct height
1C conversion
1CA correct answer in m
NPR

(3)

L2

Analysis table A: Assignment of marks to the different competencies as per the memorandum of marking – November 2014 examinations

Framework for content analysis			Mathematical Literacy Exam marking memorandum: Measurement				
No.	Category	Definition (competences)	Coding Rules	November 2014			
				Question 4.3.1		Question 4.3.2	
				Key word/s	Marks (8 marks)	Key word/s	Mark (3)
1	Production of a problem statement from a messy real world situation.	1. Write down the 'givens' and the 'required to find'. 4. Make assumptions 5. Simplify the problem by writing appropriate relationships	Satisfy both 1 and 2 OR Satisfies 3 only				
2	Mathematical model from real world problem statement.	4. Identify dependent and independent variables for inclusion in the model 5. Uniquely defining variables 6. Represent relationships between quantities mathematically.	Satisfy 1, 2 and 3 3 only acceptable				
3	Mathematical solution from the mathematical model.	4. Applying formulae (model) correctly 5. Correct algebraic simplifications 6. Obtain mathematical results/solution	Satisfy 1,2 and 3	<i>Perimeter</i> <i>Substitution</i> <i>Simplification</i> <i>Conversion to m²</i> <i>Proportion</i> <i>Proportion</i> <i>proportion</i>	1 1 1 1 1 1 1 [7]	<i>Correct height</i> <i>Conversion</i> <i>Answer</i>	1 1 1 [3]
4	Interpret mathematical solution in terms of the real world.	3. Match mathematical results with their world counterparts 4. Interpret mathematical results in terms of real situation	Satisfy both 1 and 2	<i>Rounding up</i>	1 [1]		
5	Accept solution or validate or	4. Accept/ write down final solution	Satisfies 1 or 2 or 3				

	critique and revise model. (This category occurs in all answers in the memo since they all have 'an answer'.)	5. Reconcile unexpected mis-match between mathematical results and real world expectations 6. Revise the model					
Explanation of coding rules: <ul style="list-style-type: none"> • Verbs that are used for mark allocation are used for category placement. • A solution may not show the formula being applied. In such a case the formula is implied and the answer belongs to category 2. • If the formula is provided in the question, the answer belongs to category 3. • Rounding an answer according to context belongs to category 4. Any rounding to a specified number of decimal places belongs to category 3. • Marks given for conversion of units belong to category 3. 			Explanation of codes used in the national marking memoranda: <ul style="list-style-type: none"> • M means Method • MA means Method with Accuracy • CA means Consistent Accuracy • A means Accuracy • D means Define • S means Simplification • SF means Substitution into a formula • R means Rounding Off • RT/RD/RG/RP means Reading from a Table or Graph or Diagram or map or Plan. • NP means No Penalty for rounding OR omitting units. 				



Appendix B: March 2015, Mathematical Literacy, Paper 2 – Measurement

NSC – examinations, March 2015: Mathematical Literacy, Paper 2 – Measurement

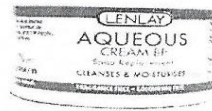
Question

1.5

One of the products that Lindiwe sells is an aqueous cream used for cleansing and moisturising, as shown in the picture alongside.

The aqueous cream is sold in 100 ml jars. The cylindrical jars are filled with cream to a height of 4 cm.

Picture of a cylindrical jar of aqueous cream



Calculate (in cm) the diameter of the jar.

You may use the formula:

Volume of a cylinder = $\pi \times (\text{radius})^2 \times \text{height}$, where $\pi = 3,142$

NOTE: 1 ml = 1 cm³

Solution

Volume of a cylinder = $\pi \times (\text{radius})^2 \times \text{height}$

$$100 \text{ ml} = 3,142 \times (\text{radius})^2 \times 4 \text{ cm} \quad \checkmark \text{SF}$$

$$100 \text{ cm}^3 = 12,568 (\text{radius})^2 \quad \checkmark \text{C}$$

$$\frac{100}{12,568} = \frac{12,568 (\text{radius})^2}{12,568} \quad \checkmark \text{MA}$$

$$7,956715468 = (\text{radius})^2$$

$$\sqrt{7,956715468} = \sqrt{(\text{radius})^2}$$

$$2,82076505 = \text{radius} \quad \checkmark \text{CA}$$

$$\begin{aligned} \text{Diameter} &= 2,82076505 \times 2 \text{ cm} \\ &= 5,6415301 \text{ cm} \quad \checkmark \text{CA} \end{aligned}$$

1SF substitution

1C converting to cm³

1MA simplifying

1CA radius

1CA diameter

(5)

Source: Department of Basic Education, RSA: National Senior Certificate Examinations (NSC)

NSC-examinations, March 2015: Mathematical Literacy, Paper 2 – Measurement

Question

5.2.3 On the plan the dimensions of the floor of Bedroom 2 are as follows:

Length = 33 mm and width = 28 mm

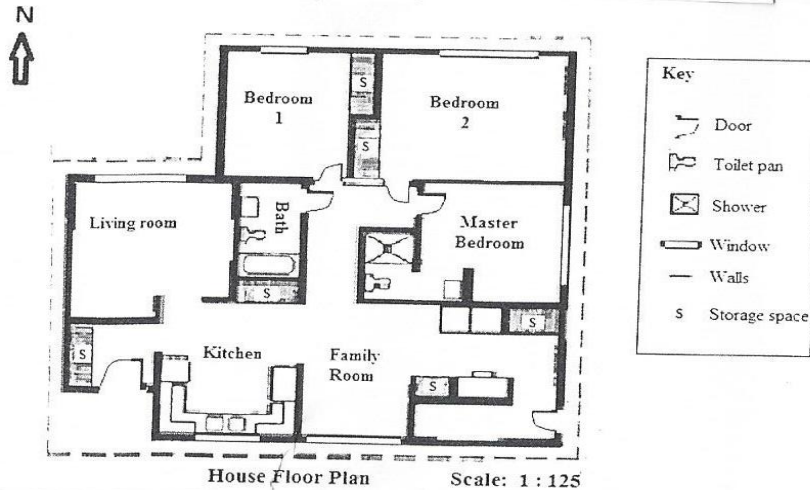
According to the building regulations of the local municipality the area of a window must be at least 11,5% of the floor area of the room.

The actual window is 220 cm wide. Calculate (to the nearest cm) the minimum height of the window.

You may use the formula:

Area of a rectangle = length × width

(8)



Solution

5.2.3

$$\begin{aligned} \text{Actual length} &= 33 \text{ mm} \times 125 \\ &= 4125 \text{ mm} = 412,5 \text{ cm} \checkmark \text{CA} \end{aligned}$$

$$\begin{aligned} \text{Actual breadth} &= 28 \text{ mm} \times 125 \\ &= 3500 \text{ mm} = 350 \text{ cm} \checkmark \text{CA} \end{aligned}$$

$$\begin{aligned} \text{Floor area of the room in cm}^2 &= \text{length} \times \text{breadth} \\ &= 412,5 \times 350 \\ &= 144\,375 \checkmark \text{CA} \end{aligned}$$

$$\begin{aligned} \therefore \text{minimum area of the window in cm}^2 &= 144\,375 \times 11,5\% \\ &= 16\,603,125 \checkmark \text{CA} \end{aligned}$$

$$\begin{aligned} \text{Area of the window in cm}^2 &= \text{width} \times \text{height} \\ 16\,603,125 &= 220 \times \text{height} \end{aligned}$$

$$\begin{aligned} \therefore \text{height in cm} &= \frac{16\,603,125}{220} \checkmark \text{M} \\ &= 75,46875 \\ &\approx 75 \checkmark \text{CA} \end{aligned}$$

IA using scale
ICA length
ICA breadth
IC converting
ICA area of room
ICA area of the window
IM finding the height
ICA rounding off

L4

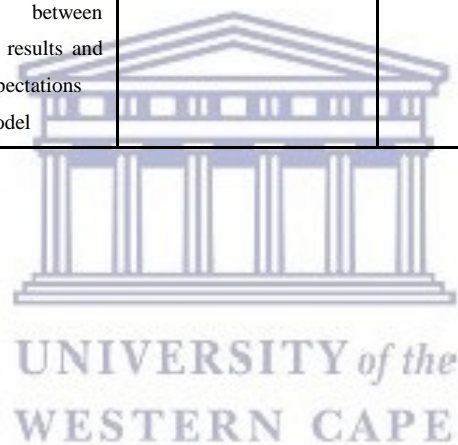
(8)

Source: Department of Basic Education, RSA: National Senior Certificate Examinations

Analysis table B: Assignment of marks to the different competencies as per the memorandum of marking – November 2014 examinations

Framework for content analysis			Mathematical Literacy Exam marking memorandum: Measurement				
No.	Category	Definition (competences)	Coding Rules	March 2015			
				Question 1.5		Question 5.2.3	
				Key word/s	Marks (5 marks)	Key word/s	Marks (8 marks)
1	Production of a problem statement from a messy real world situation.	<ol style="list-style-type: none"> Write down the 'givens' and the 'required to find'. Make assumptions Simplify the problem by writing appropriate relationships 	Satisfy both 1 and 2 OR Satisfies 3 only				
2	Mathematical model from real world problem statement.	<ol style="list-style-type: none"> Identify dependent and independent variables for inclusion in the model Uniquely defining variables Represent relationships between quantities mathematically. 	Satisfy 1, 2 and 3 3 only acceptable			<i>Finding the height</i>	1 [1]
3	Mathematical solution from the mathematical model.	<ol style="list-style-type: none"> Applying formulae (model) correctly Correct algebraic simplifications Obtain mathematical results/solution 	Satisfy 1,2 and 3	<i>Substitution</i> <i>Units conversion</i> <i>Simplifying</i> <i>Radius</i> <i>Diameter</i>	1 1 1 1 1 [5]	<i>Using scale</i> <i>Length(answer)</i> <i>Breadth</i> <i>Units conversion</i> <i>Area of the room</i> <i>Area of the window</i> <i>Rounding off</i>	1 1 1 1 1 1

							[7]	
4	Interpret mathematical solution in terms of the real world.	<ol style="list-style-type: none"> 1. Match mathematical results with their world counterparts 2. Interpret mathematical results in terms of real situation 	Satisfy both 1 and 2					
5	Accept solution or validate or critique and revise model. (This category occurs in all answers in the memo since they all have 'an answer'.)	<ol style="list-style-type: none"> 1. Accept/ write down final solution 2. Reconcile unexpected mis-match between mathematical results and real world expectations 3. Revise the model 	Satisfies 1 or 2 or 3					



Appendix C: November 2015, Mathematical Literacy, Paper 2 – Measurement

QUESTION

NSC-examinations, November 2015: Mathematical Literacy, Paper 2

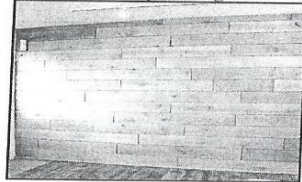
Mr Vermeulen intends to cover the FOUR interior walls of his living room with wood panelling. The northern side of his house is exposed to the sun during daytime.

Mr Vermeulen bought wood panels to cover the four interior walls of his living room, as shown in the photographs below.

Photograph of a living room wall with wood panelling



Close-up photograph of a wall with wood panelling



The interior dimensions of the living room floor are 7,04% less than the exterior dimensions, as shown on the floor plan.

3.3.1 Show that the interior dimensions of the living room floor are 3,3 m × 3,3 m. (3)

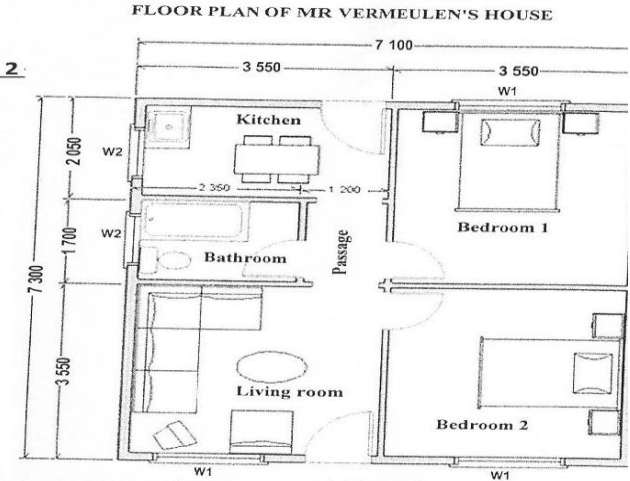
3.3.2 Determine the total surface area (to the nearest m²) of the interior walls of the living room that have to be covered with wood panels.

The following formula may be used:

Area of a rectangle = length × width

(11)

Source: Department of Basic Education, RSA: National Senior



Solution

3.3.1	<p>The living room floor side</p> <p style="text-align: center;">✓MA ✓C ✓M</p> $= 3,550 \text{ m} - (3,550 \text{ m} \times 7,04\%)$ $= 3,3008 \text{ m}$ $\approx 3,3 \text{ m}$ $\therefore 3,3 \text{ m} \times 3,3 \text{ m}$	OR	<p style="text-align: center;">✓MA</p> $100\% - 7,04\% = 92,96\%$ <p style="text-align: center;">Side ✓C ✓M</p> $= 3,550 \text{ m} \times 92,96\%$ $= 3,3008 \text{ m}$ $\therefore 3,3 \text{ m} \times 3,3 \text{ m}$	<p>IC conversion</p> <p>1MA for subtracting</p> <p>1M multiplication</p> <p style="text-align: right;">(3)</p>	L2
3.3.2	<p>Area of 4 walls</p> <p style="text-align: center;">✓SF</p> $= 4 \times (3,3 \text{ m} \times 2,650 \text{ m})$ $= 34,98 \text{ m}^2 \quad \checkmark \text{CA}$ <p>Area of 2 door openings</p> $= 2 \times \text{length} \times \text{width}$ $= 2 \times 2,032 \text{ m} \times 0,750 \text{ m} \quad \checkmark \text{M}$ $= 3,048 \text{ m}^2 \quad \checkmark \text{CA}$ <p>Area of window</p> $= 1,511 \text{ m} \times 0,949 \text{ m} \quad \checkmark \text{M}$ $= 1,434 \text{ m}^2 \quad \checkmark \text{CA}$ <p>Area to cover with panelling</p> $= (34,98 - 3,048 - 1,5615 - 1,434) \text{ m}^2 \quad \checkmark \text{M}$ $= 28,9365 \text{ m}^2 \quad \checkmark \text{CA}$ $\approx 29 \text{ m}^2 \quad \checkmark \text{R}$	<p>Area of opening to passage</p> $= \text{length} \times \text{width}$ $= 2,082 \text{ m} \times 0,75 \text{ m} \quad \checkmark \text{M}$ $= 1,5615 \text{ m}^2 \quad \checkmark \text{CA}$	<p>1SF area wall dimensions</p> <p>1CA area of 4 walls</p> <p>2M door opening dimensions</p> <p>1CA area of opening to passage</p> <p>1CA 2 door openings</p> <p>1M window dimensions</p> <p>1CA area of window</p> <p>1M subtracting</p> <p>1CA area</p> <p>1R rounding</p>	L3	

NSC-examinations, November 2015: Mathematical Literacy, Paper 2 – Measurement

Question

3.4 Wood panels are sold per cubic metre (m^3) and not per length. The timber retailer gave Mr Vermeulen a quote of R5 000,00 per cubic metre, excluding 14% VAT.

Mr Vermeulen noted the following:

- Each wood panel is 2 m long, 150 mm wide and 12,5 mm thick.
- 4,5% more than the required number of panels is required due to cutting and wastage.
- Labour cost for panelling is R125,00 per square metre, including VAT.

He budgeted R6 000,00 for the cost of the panels and the labour.

Verify whether Mr Vermeulen's budgeted amount is enough to cover the cost of the panels and labour.

The following formulae may be used:

Area of a rectangle = length \times width

Volume of a rectangular prism = length \times width \times height

(12)
[31]

Solution

Ques	Solution	Explanation
3.4	Surface area of one panel = $2\text{ m} \times 0,15\text{ m}$ = $0,3\text{ m}^2$ ✓A	1A area
	Number of panels needed = $\frac{29\text{ m}^2}{0,3\text{ m}^2}$ = $96,666\dots \approx 97$ ✓CA	1CA from Q3.3.2 simplification
	Total panels needed to be purchased = $97 \times 104,5\%$ OR $97 \times 4,5\% = 4,365$ = $101,365$ ✓CA ≈ 5 ≈ 102 $97 + 5 = 102$ ✓CA	1CA number of panels 1R rounding
	Volume of 102 panels = $102 \times 0,0125\text{ m} \times 0,3\text{ m}^2$ ✓SF = $0,3825\text{ m}^3$ ✓CA	1C convert to metre 1SF finding volume 1CA volume in m^3
	Cost of panels excluding VAT = $0,3825 \times R5\,000,00$ = $R1\,912,50$ ✓CA	1CA cost excluding VAT
	OR Price of wood including VAT = $R5\,000\text{ per }m^3 \times 114\%$ = $R5\,700\text{ per }m^3$ ✓CA	
	Cost of the panels including VAT = $1,14 \times R1\,912,50$ = $R2\,180,25$ ✓CA	1CA cost incl. VAT
	OR Cost of the panels including VAT = $R5\,700 \times 0,3825$ = $R2\,180,25$ ✓CA	
	Labour cost = $29 \times R125,00$ = $R3\,625,00$ ✓CA	1CA labour cost (CA area from 3.3.2)
	Total cost = $R2\,180,25 + R3\,625,00$ = $R5\,805,25$ ✓CA	1CA total cost
	Budget is ENOUGH ✓O	1O conclusion

Source: Department of Basic Education, RSA: National Senior Certificate Examinations

No.	Category	Definition (competences)	Coding Rules	November 2015			
				Question 3.3.1		Question 3.3.2	
				Key word/s	Marks (3 marks)	Key word/s	Marks (11 marks)
1	Production of a problem statement from a messy real world situation.	<ol style="list-style-type: none"> Write down the 'givens' and the 'required to find'. Make assumptions Simplify the problem by writing appropriate relationships 	Satisfy both 1 and 2 OR Satisfies 3 only				
2	Mathematical model from real world problem statement.	<ol style="list-style-type: none"> Identify dependent and independent variables for inclusion in the model Uniquely defining variables Represent relationships between quantities mathematically. 	Satisfy 1, 2 and 3 3 only acceptable			<i>Subtracting unaffected areas (method)</i>	1 [1]
3	Mathematical solution from the mathematical model.	<ol style="list-style-type: none"> Applying formulae (model) correctly Correct algebraic simplifications Obtain mathematical results/solution 	Satisfy 1,2 and 3	<i>Units conversion</i> <i>Subtraction</i> <i>Multiplication</i>	1 1 1 [3]	<i>Substitution</i> <i>Area(4 walls)</i> <i>Area(2 door openings)</i> <i>Area passage</i> <i>Area(window)</i> <i>Area</i> <i>Rounding off</i>	1 1 2 2 2 1 1 [10]
4	Interpret mathematical solution in terms of the real world.	<ol style="list-style-type: none"> Match mathematical results with their world counterparts Interpret mathematical results in terms of real situation 	Satisfy both 1 and 2				
5	Accept solution or validate or critique and revise model. (This category occurs in all answers in the memo since they all have 'an answer'.)	<ol style="list-style-type: none"> Accept/ write down final solution Reconcile unexpected mis-match between mathematical results and real world expectations Revise the model 	Satisfies 1 or 2 or 3				

4	Interpret mathematical solution in terms of the real world.	<ol style="list-style-type: none"> 1. Match mathematical results with their world counterparts 2. Interpret mathematical results in terms of real situation 	Satisfy both 1 and 2	<i>Rounding –up</i> <i>Conclusion</i>	1 1 [2]
5	Accept solution or validate or critique and revise model. (This category occurs in all answers in the memo since they all have ‘an answer’.)	<ol style="list-style-type: none"> 1. Accept/ write down final solution 2. Reconcile unexpected mis-match between mathematical results and real world expectations 3. Revise the model 	Satisfies 1 or 2 or 3		



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Appendix D: March 2016, Mathematical Literacy, Paper 2 – Measurement

NSC-examinations, March 2016: Mathematical Literacy, Paper 2 – Measurement

Question

5.1 Mrs Dundee, an Australian farmer, has four silos on her farm in which she stores fertiliser, as shown in the photograph and diagram below. The silos are cylindrical with a roof section. Fertiliser is only stored in the cylindrical part of the silos.

PHOTOGRAPH OF FOUR SILOS

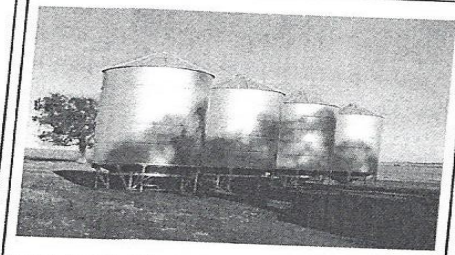
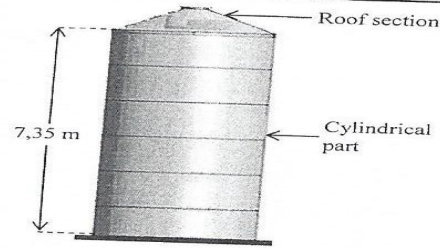


DIAGRAM OF ONE SILO



[Source: www.eicobulk.co.za]

The following formula and conversion rates may be used:

Volume of a cylinder = $\pi \times (\text{radius})^2 \times \text{height}$; using $\pi = 3,142$
 $1 \text{ m}^3 = 1\,000 \text{ } \ell$
 $1,3 \text{ kg} = 1 \text{ litre}$
 $1 \text{ gallon} = 3,7 \text{ litres}$

- 5.1.1 Calculate the diameter of a silo if the volume of the cylindrical part is 60 m^3 . (5)
- 5.1.2 Calculate the total maximum capacity (in gallons) of the four silos. (4)
- 5.1.3 After fertilising all her main fields, Mrs Dundee wants to use the remaining fertiliser for a wheat field, which is 1 055 acres in size.
- The capacity readings of the four silos are as follows:
- Silo 1: 15% full
 - Silo 2: $\frac{1}{4}$ full
 - Silos 3 and 4: empty
- Verify, showing ALL calculations, whether she will have enough fertiliser left in her silos for the wheat field if the spread rate is 22,65 kg of fertiliser per acre. (6)

Solution

Ques	Solution	Explanation
5.1.1	$\text{Volume of a cylinder} = \pi \times (\text{radius})^2 \times \text{height}$ $60 \text{ m}^3 = 3,142 \times (\text{radius})^2 \times 7,35 \text{ m} \quad \checkmark \text{ SF}$ $(\text{radius})^2 = \frac{60 \text{ m}^3}{3,142 \times 7,35 \text{ m}} \quad \checkmark \text{ M}$ $= 2,598111173 \text{ m}^2$ $\text{radius} = \sqrt{2,598111173} \quad \checkmark \text{ M}$ $= 1,611865743 \text{ m} \quad \checkmark \text{ CA}$ $\text{diameter} = 1,611865743 \text{ m} \times 2$ $= 3,223731486 \text{ m} \quad \checkmark \text{ CA}$	1S substituting 1M changing the subject 1M square root 1CA radius 1CA diameter (5)
5.1.2	$\text{Total capacity} = 4 \times 60 \text{ m}^3 \quad \checkmark \text{ M}$ $= 240 \text{ m}^3 \quad \checkmark \text{ C}$ $= 240\,000 \text{ } \ell \quad \checkmark \text{ M}$ $\text{Capacity in gallon} = \frac{240\,000}{3,7} \quad \checkmark \text{ M}$ $\approx 64\,864,86 \quad \checkmark \text{ CA}$	1M multiplying 1C convert to ℓ 1M dividing 1CA gallons (4)
5.1.3	$\text{Volume of fertiliser in silos} = (15\% \times 60 \text{ m}^3) + \left(\frac{1}{4} \times 60 \text{ m}^3\right) \quad \checkmark \text{ M}$ $= 9 \text{ m}^3 + 15 \text{ m}^3$ $= 24 \text{ m}^3 \quad \checkmark \text{ A}$ $\text{Fertiliser needed for wheat field} \quad \checkmark \text{ M}$ $= 1\,055 \text{ acres} \times 22,65 \text{ kg}$ $= 23\,895,75 \text{ kg}$ $= \frac{23\,895,75}{1,3} \text{ litre}$ $= 18\,381,35 \text{ litre} \quad \checkmark \text{ C}$ $\text{Volume of fertiliser needed} = 18\,381,35 + 1\,000$ $= 18,38... \text{ m}^3$ $\approx 18,4 \text{ m}^3 \quad \checkmark \text{ C}$	1M % and $\frac{1}{4}$ of 60 1A volume of silos 1M multiply by 22,65 1C convert to ℓ 1C conversion 1O deduction (6)
	She will have enough fertiliser for the wheat field. $\checkmark \text{ O}$	

No.	Category	Definition (competences)	Coding Rules	March 2016					
				Question 5.1.1		Question 5.1.2		Question 5.1.3	
				Key word/s	Marks (5 marks)	Key word/s	Marks (4 marks)	Key word/s	Marks (6 marks)
1	Production of a problem statement from a messy real world situation.	1. Write down the 'givens' and the 'required to find'. 2. Make assumptions 3. Simplify the problem by writing appropriate relationships	Satisfy both 1 and 2 OR Satisfies 3 only						
2	Mathematical model from real world problem statement.	1. Identify dependent and independent variables for inclusion in the model 2. Uniquely defining variables 3. Represent relationships between quantities mathematically.	Satisfy 1, 2 and 3 3 only acceptable					<i>Method (volume of fertilizer in all silos). Fertilizer needed</i>	1 1 [2]
3	Mathematical solution from the mathematical model.	1. Applying formulae (model) correctly 2. Correct algebraic simplifications 3. Obtain mathematical results/solution	Satisfy 1,2 and 3	<i>Substitution</i> <i>Changing the subject of the formula.</i> <i>Take square root.</i> <i>Radius</i> <i>Diameter</i>	1 1 1 1 1 [5]	<i>Multiplying</i> <i>Units conversion</i> <i>Dividing</i> <i>Gallons</i>	1 1 1 [4]	<i>Volume</i> <i>Units</i> <i>conversion</i> <i>Units</i> <i>conversion</i>	1 1 1 [3]
4	Interpret mathematical solution in terms of the real world.	1. Match mathematical results with their world counterparts 2. Interpret mathematical results in terms of real situation	Satisfy both 1 and 2					<i>Deduction</i>	1 [1]

5	Accept solution or validate or critique and revise model. (This category occurs in all answers in the memo since they all have 'an answer'.)	<ol style="list-style-type: none"> 1. Accept/ write down final solution 2. Reconcile unexpected mis-match between mathematical results and real world expectations 3. Revise the model 	Satisfies 1 or 2 or 3						
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Appendix E: June 2016, Mathematical Literacy, Paper 2 – Measurement

NCS –examinations, June 2016: Mathematical Literacy, Paper 2 – Measurement

Question

1.2

Some students painted one letter of the slogan #FeesMustFall on their T-shirts.

The letter E was painted on each T-shirt using fabric paint.

The spread rate of this particular fabric paint is 100 ml per 1 m² of fabric.

- 1.2.1 Calculate the height (in mm) of the fabric paint container if the volume of the container is 367,38 cm³.
You may use the following formula:
Volume of fabric paint container = $\pi \times \text{radius} \times \text{radius} \times \text{height}$,
where $\pi = 3,142$ (5)
- 1.2.2 Calculate the amount of fabric paint required to paint the letter E on four T-shirts. (7)
- 1.2.3 The students used braiding (edging) to place a border around the edges of the letter E.
Calculate the length of braiding (edging) needed to place a border around the edges of the letter E on one T-shirt. (3)

Solution

Ques	Solution	Explanation
1.2.1	<p>Volume of fabric paint container = $\pi \times \text{radius} \times \text{radius} \times \text{height}$</p> $367,38 \text{ cm}^3 = 3,142 \times 3 \text{ cm} \times 3 \text{ cm} \times \text{height}$ $367,38 \text{ cm}^3 = 28,278 \times \text{height}$ $367,38 \text{ cm}^3 \div 28,278 = \text{height}$ $\text{height} = 12,9917 \text{ cm}$ $= 129,92 \text{ mm}$	<p>1M calculating radius 1SF substituting into formula 1CA simplification 1M dividing by 28,278</p> <p>1C converting to mm NPR</p> <p style="text-align: right;">(5)</p>
1.2.2	<p>Area of one letter E = (length \times width) – (side \times side \times 2)</p> $= (29,5 \times 19,5) - (5,9 \times 5,9 \times 2)$ $= 505,63 \text{ cm}^2$ <p>Amount of paint needed for one letter E = $505,63 \div 10000 \times 100$ = 5,0563 ml</p> <p>Amount of paint needed for four letter E's = $5,0561 \times 4$ = 20,2252 ml</p> <p style="text-align: center;">OR</p> <p>Area of letter E = (length \times width) + (side \times side \times 3)</p> $= (29,5 \text{ cm} \times 13,6 \text{ cm}) + (5,9 \text{ cm} \times 5,9 \times 3)$ $= 505,63 \text{ cm}^2$ <p>Amount of paint needed for one letter E = $505,63 \div 10000 \times 100$ = 5,0563 ml</p> <p>Amount of paint needed for four letter E's = $5,0561 \times 4$ = 20,2252 ml</p>	<p>2M using formula for two areas 1CA calculating area</p> <p>1C converting to m² 1M converting to ml 1CA calculating paint</p> <p>1CA total volume NPR</p> <p style="text-align: center;">OR</p> <p>2M using formula for two areas 1CA calculating area</p> <p>1C converting to m² 1M converting to ml 1CA calculating paint</p> <p>1CA total volume</p> <p style="text-align: right;">(7)</p>
1.2.3	<p>Perimeter of letter E = $29,5 \text{ cm} + 19,5 \text{ cm} + 19,5 \text{ cm} + (9 \times 5,9 \text{ cm})$</p> $= 121,6 \text{ cm}$ <p style="text-align: center;">OR</p> <p>Perimeter of letter E = $2 \times 29,5 \text{ cm} + (2 \times 19,5 \text{ cm}) + (4 \times 5,9 \text{ cm})$</p> $= 121,6 \text{ cm}$	<p>1A reading all values 1M adding 1CA perimeter</p> <p style="text-align: center;">OR</p> <p>1A reading all values 1M adding 1CA perimeter</p> <p style="text-align: right;">(3)</p>

No.	Category	Definition (competences)	Coding Rules	June 2016					
				Question 1.2.1		Question 1.2.2		Question 1.2.3	
				Key word/s	Marks (5 marks)	Key word/s	Marks (7 marks)	Key word/s	Marks (3 marks)
1	Production of a problem statement from a messy real world situation.	1. Write down the ‘givens’ and the ‘required to find’. 2. Make assumptions 3. Simplify the problem by writing appropriate relationships	Satisfy both 1 and 2 OR Satisfies 3 only						
2	Mathematical model from real world problem statement.	1. Identify dependent and independent variables for inclusion in the model 2. Uniquely defining variables 3. Represent relationships between quantities mathematically.	Satisfy 1, 2 and 3 3 only acceptable			Using area formulae	2 [2]		
3	Mathematical solution from the mathematical model.	1. Applying formulae (model) correctly 2. Correct algebraic simplifications 3. Obtain mathematical results/solution	Satisfy 1,2 and 3	Calculating radius Substitution Simplification Dividing Converting to mm	1 1 1 1 1 [5]	Calculating area Converting to m ² Converting to ml Calculating paint Total volume	1 1 1 1 [5]	Reading values from a diagram Adding Perimeter	1 1 [3]
4	Interpret mathematical solution in terms of the real world.	1. Match mathematical results with their world counterparts 2. Interpret mathematical results in terms of real situation	Satisfy both 1 and 2						

5	Accept solution or validate or critique and revise model. (This category occurs in all answers in the memo since they all have 'an answer'.)	<ol style="list-style-type: none"> 1. Accept/ write down final solution 2. Reconcile unexpected mis-match between mathematical results and real world expectations 3. Revise the model 	Satisfies 1 or 2 or 3					
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