THE ECONOMETRICS OF STRUCTURAL CHANGE: STATISTICAL ANALYSIS AND FORECASTING IN THE CONTEXT OF THE SOUTH AFRICAN ECONOMY

by

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BELLVILLE

DECLARATION

I, Gilbert Robert Wesso, declare that "The Econometrics of Structural Change: Statistical Analysis and Forecasting in the Context of the South African Economy" is my own work and that all the sources I have used or quoted have been indicated and acknowledged by means of complete references.

Date: 11/03/94 **Signature:** ....

To my wife, Merl and daughter, Anita-Melissa

PREFACE

Structural change is of fundamental importance in econometric model building. Statistics and econometrics provide the tools for the identification of change, for estimating the onset of a change, and for assessing its extent and relevance. Statistics and econometrics also have developed models that are suitable for picturing the data-generating process in the presence of structural change by assimilating the changes in the models.

The need for such methods became obvious when, as a consequence of the oil price shock amongst others, the results of empirical analysis suddenly seemed to be much less reliable than before. Nowadays, economists agree that models based on fixed structures that attempt to picture reality over longer periods are gross oversimplifications.

After an euphoric period in the 1960's, when shifts were unknown, model builders in economics became aware of a need of model diagnostics and, in particular, for methods to detect - and cope with - structural changes. Statisticians' interest in the subject grew slowly after a few early contributions, especially Quandt's paper (1958) on switching regression. In various areas related problems were discussed and methods were developed in and adapted from areas such as probability theory (the change point problem), continuous sampling inspection (the

CUSUM technique), and engineering (recursive estimation and filtering).

Various research groups were created to analyse the econometrics of structural change. In 1983, the International Institute for Applied Systems Analysis (IIASA) in Laxemburg/ Austria initiated an ambitious project on "Economic Growth and Structural Change". These two issues and their interrelation pose a great challenge for economic theory. A research project, guided and supervised by Wilhelm Krelle, was started in the form of a joint IIASA - University of Bonn venture. The empirical base of the project covered all important countries and regions. Prominent economists and econometricians participated in the project, partly as members of a central group in Bonn and partly as members of country or regional groups. At the heart of the project was a highly aggregated world model established by the central group, which related results of country and regional groups to guarantee consistency.

Another IIASA Working Group was established in 1983 to research "Statistical Analysis and Forecasting of Economic Structural Change". In 1985 and 1986, three workshops took place where current and new statistical methods were presented and discussed. Issues such as the identification of structural change, which models to use in the presence of structural change and how to take into account structural change when forecasting future developments, were points of discussion.

The main results of the activities of the IIASA - University of Bonn Project and the IIASA Working Group were published in two books edited by Krelle (1989) and Hackl (1989).

Several other conferences and seminars dealing with the "structural change" problem were also organised. On January 26, 1973, the National Bureau of Economic Research (NBER), Computer Research Center for Economics and Management Science, hosted a symposium on the problem of time-varying parameter structures. Due to the wide interest in the topic, it was decided to publish revised versions of the papers in a special issue of the "Annals of Economics and Social Measurement" (1973). Statisticians and econometricians were also invited to the international conference on "Economic Structural Change: Analysis and Forecasting" that took place in May 1989 in Stockholm under the auspices of IIASA; Another seminar on the topic of "Econometrics and Structural Change" was held in Montreal, Canada during October 1992 (organisers of this seminar were Eric Ghysels and Allistair Hall). Many other research papers on the topic were also presented at other meetings and conferences such as the symposiums organised by the Institute of International Forecasters (IIF) and the American Statistical Association (ASA).

Nevertheless, in practical model building exercises, the methods recommended by statisticians seem not to be extensively used. Amongst the reasons for this may be the scarcity of user friendly computer programs and also the relative lack of systematic surveys in the field. Some systematic surveys, however, have been published recently. Broemeling and Tsurumi's (1986) book is the most comprehensive volume on the subject, despite the fact that it is based on a Bayesian paradigm. The same basis underlies a 1982 supplement of the "Journal of Econometrics", edited by Broemeling, which also covers a broad area of the subject matter. Specialised books have been published on multiphase regression (Schulze, 1986), spline function-based models (Poirier, 1976) and the analysis of residuals (Hackl, 1980). Several volumes contain special sections on model building, such as those by Broemeling (1985) and by Krämer and Sonnberger (1986). Accessible bibliographies have been compiled by Shaban (1980) and by Johnson (1977, 1980). Moreover, since 1989, a new journal entitled "Structural Change and Economic Dynamics" attempts to provide a forum for methodological discussions.

The purpose of this particular study is sixfold, namely: (a) to discuss forecasting and structural changes in the South African economy; (b) to provide a complete overview of aspects of testing for structural change; (c) to provide a simple, unified and systematic treatment of the alternative forms of time-varying and random coefficient models; (d) to investigate economic forecasting under conditions of structural change; (e) to apply the above methods empirically in the context of the South African economy; and (f) to provide recommendations for future econometric research. Thus, the thesis has two important characteristics: (i) the topics covered in textbooks, journals and articles are discussed in the context of the random coefficients model as opposed to the constant coefficient models (an assumption which is restrictive and often unnecessary); and (ii) an application of a suitable procedure in estimating the parameters of a random coefficient regression model for South Africa.

The thesis is divided into six parts:

The introductory part discusses not only the role of statistics in the detection and assimilation of structural change, but also the relevance of structural stability tests in the evaluation of econometric models. The historical development of these methods are reviewed as well as some of the possible causes of coefficient variation in South Africa.

The next part provides some reflections on forecasting in situations of structural change. The crisis in forecasting and econometric modelling is discussed and some fundamental issues of economic forecasting in South Africa are focused on.

The third part deals with the identification of structural change. The chapters combined under this heading are concerned with detection of parameter nonconstancy. The procedures discussed range from classical methods, such as the Chow and CUSUM tests, to new concepts, particularly those based on maximum likelihood statistics. Several sections assess the conditions under which these methods can be applied and their robustness under such conditions.[1]

Econometric model building in the presence of structural change is discussed in the fourth part. This part addresses models that are in some sense generalisations of constant parameter models, so that they can assimilate structural change. This is one of the most important parts in this study and it surveys a wide variety of random coefficient models for easy reference (see fn. 1). An overview of forecasting methods which can be used under conditions of structural change is provided. Chapter 6 reviews a model-based approach to adaptive estimation of regression parameters and discusses in detail the use of the Kalman filtering technique in this case. Chapter 7 deals with changing and random coefficient models. It comprehensively reviews the related literature and provides some guidelines for model choice. The final chapter under this part is concerned with autoregressive conditional heteroscedastic (ARCH) models which are used to model a series of which the variance is

[1]Reference to extensions and/or modifications of the original tests and models will be given in summary form only.

changing with time. Although relatively new, this model is quickly gaining wide acceptance amongst applied econometricians.

The fifth part discusses theoretical models for fixed investment, production and exchange rates as well as the empirical estimation of such models in the South African economy. The method for empirical investigation also forms part of this section.

South Africa proved to be particularly well suited as a test-bed to explore the effects of and remedies for structural instability in econometric relationships (see Smit and Wesso, 1989). A final part, therefore, deals with real-life structural change situations in South Africa and is limited to the estimation of a suitable model for private fixed investment, production and exchange rates, all of which are treated in any standard macro-econometric model.

It is hoped that this study will contribute to stimulate the interest of South African statisticians and econometricians in this topic and will help to improve models for analysing real-world phenomena and consequently the reliability of results.

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α, β, τ, K; .... Greek and Latin characters (upper or lower case) denote known or unknown constants (or population parameters).

a, b, y, z; .... Lower case Latin characters denote quantities subject to a probability distribution.

α, a, π, p; .... Unknown parameters (θ) and their estimates ($\hat{\theta}$) are denoted as far as possible by corresponding (Greek and Latin) characters.

t = 1, ... T; ...... Latin characters (lower case) are also used as subscripts for numbering of variables, equations or observation periods. In such cases the range of the subscript is often from 1 to a maximum denoted by the corresponding Latin capital letter.

f, F, Φ, ϕ; .... Greek and Latin characters are used without distinction to denote distribution functions.

$\mathbf{y}$, $\boldsymbol{\beta}$, X, Ω; .... Vectors are denoted by bold lower case type and matrices by italic Latin or normal Greek characters.

α_g, Y_{II}, M_{yz}; ...... In some cases, explained in the text, subscripts denote sets of elements, rows or columns, to be included in a submatrix or subvector of the matrix or vector denoted by the symbol to which these subscripts are attached.

a_{12}, β_{ij}, x_i; ...... Scalar elements of matrices or vectors are denoted in the normal manner by subscripts following the lower case character corresponding to the character denoting the matrix or vector.

PART I

INTRODUCTION AND PROBLEM STATEMENT

CHAPTER 1

THE ANALYSIS OF ECONOMIC STRUCTURAL CHANGE

1.1 INTRODUCTION

One of the assumptions of conventional regression analysis is that the parameters are constant over all observations. It has often been suggested that this may not be a valid assumption to make, particularly if the econometric model is to be used for economic forecasting. Apart from this it is also found that econometric models, in particular, are used to investigate the underlying interrelationships of the system under consideration in order to understand and to explain relevant phenomena in structural analysis. The pre-requisite of such use of econometrics is that the regression parameters of the model is assumed to be constant over time or across different cross-sectional units.

To put this more precisely, consider the standard regression framework

$$\mathbf{y} = X\boldsymbol{\beta} + \mathbf{e} \tag{1.1}$$

where $\mathbf{y}$ is a (T x 1) vector of observations on the dependent variable, X is a known (T x K) matrix of observations on the K predictors (including the intercept term); $\boldsymbol{\beta}$ is a (K x 1) regression vector to be estimated and $\mathbf{e}$ (T x 1) comprises the vector of error terms, which are assumed to be identically and independently distributed (henceforth iid).

The usual ordinary least square (OLS) estimate of the regression vector $\boldsymbol{\beta}$ is given by

$$\hat{\boldsymbol{\beta}} = (X'X)^{-1}X'\mathbf{y} \tag{1.2}$$

When such a regression model is fitted to a set of data, certain implicit assumptions are made. These include the assumption that the same regression model applies to the whole data set, that is, that there is no change of regime over the data. It is also assumed that all important variables are included in the predictor set. If any testing or estimation of the model parameters is to be done (as is usually the case), the assumption of normality of the errors is also invoked.

The standard assumptions of (1.1) can be mathematically expressed as

$$E(\mathbf{ee}') = \sigma^2 I \quad \text{and} \tag{1.3}$$

$$E(\mathbf{e}) = \mathbf{0} \tag{1.4}$$

or in large samples

$$\operatorname*{plim}_{T \to \infty}(1/T)\, X'\mathbf{e} = \mathbf{0} \tag{1.5}$$

Failure of (1.3), which is the stochastic specification, in general leads to inefficient, but still consistent parameter estimates, whereas a wrong specification of the structural part of the model (failure of 1.4), typically renders parameter estimates inconsistent or even, for a lack of well-defined parent parameters, almost meaningless. Such deviations include omissions of relevant independent variables from the equation, incorrect functional forms, structural breaks or correlation between regressors and error terms introduced through simultaneous equations or errors in the variables.

The standard linear regression model has been a very attractive model to use in econometrics and if econometricians can uncover stable economic relations which satisfy at least approximately the assumptions of this model, they deserve the convenience of using it. Sometimes, however, econometricians are not lucky or ingenious enough to specify a stable regression relationship, for example when the relationship being studied is gradually changing. Under such circumstances, an option would be to specify a linear regression model with stochastically evolving coefficients.

The economics profession increasingly recognises that the classical regression assumption of constant parameters is dubious. John Maynard Keynes (1938)[2] once remarked that:

[2]Keynes' comment on a proof copy of Jan Tinbergen's Business Cycles in the United States of America, as quoted in Moggridge (1973:286).

> "The coefficients arrived at are apparently assumed to be constant for 10 years or for a longer period. Yet, surely we know that they are not constant. There is no reason at all why they should not be different every year."

Indeed, if there were not this recognition, why adjust the constant terms or add factors to improve forecasting and policy simulations? By tampering with their models, econometricians implicitly acknowledge more variability in their models that they can capture by autoregressive errors. Furthermore, according to Swamy (1988:2), the necessity to apply such first aid to classical estimated models with or without deterministic shifts in coefficients appears to have increased over the past decade.

In attempting to achieve a deeper understanding of the nature of the economic system, economists employ models to represent the relationships between relevant variables. While such models, of necessity, can only be approximate characterisations of the process that actually generated the data, the hope is that they capture the important features of the relationship, and that they do this with a constant structure. Clearly, models that have continuously, or frequently, changing structure, which is unpredictable, are of limited value. Hence, economists who seek to characterise aspects of the economic system in a simple model, require a large degree of constancy or stationarity, and yet it is known that there are important changes in economic behaviour from time to time.

Evans (1983: 4), in an analysis of the performance of US econometric models, once remarked that:

> "...the record would appear to be quite clear on one point; errors have sprung, not from bad judgment or bad guesses about the principal exogenous variables, but apparently from the underlying structures of the model equations themselves. The models could not properly respond to changes in monetary policy, government spending, high interest rates, or inflationary expectations."

The complexity of the economic system , our limited knowledge of it, and the scarcity of data can result in the attendant inadequacies of econometric models manifesting themselves in apparent structural change. Indeed, one feature of macroeconometric models is the common occurrence of periods of predictive failure which could be the result of model misspecification or regime change.

Before discussing in Sections 1.3 and 1.4 some of the reasons of structural change in South Africa, the concept of structural change in econometric modelling is defined in Section 1.2. In Section 1.5 a historical background will be given, reviewing some of the important contributions in the field of econometrics and structural change. Section 1.6 offers some introductory concepts with regard to alternative ways of testing for structural change, and discusses the role of such tests statistics in the evaluation of econometric models. Section 1.7 provides a topology of structural change and of the models and approaches used to represent it. Section 1.8

explains the role of statistics in modelling structural change and offers some conclusions.

1.2 THE CONCEPT OF STRUCTURAL CHANGE

The concept of "structural change" has been used in the economic literature in two distinct contexts. The first context, and also the more commonly discussed in this thesis, uses it in a historical or temporal sense. In such a context the "change" that takes place involves altering the environment in which the relationship holds. In such instances time is often an independent variable. The effects of World War II on share market prices falls into this category.

The second context is non-temporal in nature and arises from an independent variable reaching a certain prescribed level. For example, considering an individual's income as a function of the number of formal years of education completed, it is reasonable to expect a threshold effect after ten or twelve years as a result of receiving a diploma.

Structural change has been of major concern in economics. Different theories on economic development and growth assume that economic relationships and processes are changing over time, and these changes are often explained and discussed in a descriptive way, without being statistically estimated and tested. An alternative perspective of structural changes comprises changes in the composition of the output vector of an economic system or changes in the composition of instrumental as well as exogenous input vectors. Among important reasons for structural changes in economic systems which manifest in parameter shifts are personal changes in behaviour, changes in the technical progress with corresponding changes in production function parameters, political changes, disinvestment and sanctions, domestic unrest, decentralisation of public functions, drought and moves from fixed to floating exchange rate regimes. Within macroeconomics the so-called Lucas (1974) critique claims for parameter changes. Here the idea is that the parameters of macroeconomic models are determined by the expectations of economic agents concerning future economic policy. If a policy changes, so do the expectations and related model parameters.

The statistical perspective of structural change is related to the model structure. There is no unambiguous definition of the concept "structure", although within the theory of systems, it indicates the relations among the variables in a system. A system is, of course, any arbitrarily selected set of variables interacting with each other and with an environment. Relevant variables are defined by the purpose of the study and by relevant theories. Thus, the actual "system at work" consists of a finite number of variables. Variables not included will represent the environment. In modelling economic systems the identification requires some knowledge of the relation between the system and the variables of the environment that have a special influence on the system. In the modelling process these variables are characterised as exogenous, while the rest of the environment is represented in the model by random disturbances. In a regression model framework the change in one or more of the parameters indicate structural changes. Poirier (1976) distinguishes between some kind of general structural variability and structural change, and emphasises that just considerable and low frequency variability should be associated with structural change.

It is extremely difficult to distinguish general misspecification problems from the problem of structural change. In order to do that, some identifying theoretical knowledge is necessary. If the modelling process, with respect to structural change, is not done correctly , it will introduce misspecifications, with possible consequences such as residual autocorrelation, heteroscedasticity, etc. Thus, any strategy of diagnostic checking, at least one based on residual analysis, must be influenced by the existing theoretical knowledge and hypotheses about structural change.

Apparently, structural change is a relative concept, and statements about it are restricted to the actual system at work and to the way it is manifested through a specified model. The concept is, however, an integral part of each model-building process, and the role of statistical analysis is to detect its presence, to find ways to assimilate it in models and to find methods of statistical inference that are robust to its presence.

1.3 THE SOUTH AFRICAN EXPERIENCE

South Africa has been a relative late entrant in the macro-economic model building era. The first models generated great interest and the results were awaited with high expectations. Recently, however, the attitude towards econometric modelling has changed to one of restrained cynism, based mainly on the perception of inaccurate forecasts.[3] In this, the South African experience echoes the prevailing world climate towards macro-econometric modelling.

South Africa during the past two decades have experienced a number of social and economic changes, some abrupt and some of a more gradual nature, which may be the cause of parameter instability in econometric equations. One may even argue on an *a priori* basis that the South African economy may be particularly well suited as a test-bed to explore the effects of and remedies for structural instability in econometric relationships.

[3]Smit and Wesso (1986) reported on the forecasting accuracy of a number of South African forecasters. It is extremely difficult to judge the accuracy of econometric model forecasts as such, because of the interaction between forecaster and model and the use of subjective judgement in arriving at the final forecast.

During the previous decade model builders had to deal with, among others, the effects of two major energy crises, the Soweto riots and the introduction of a general sales tax. These changes were of a rather abrupt nature and were usually incorporated in econometric models by means of dummy variables.

The user of any macro-econometric model of the South African economy was also challenged to deal with the possibility of time-varying coefficients induced by, among others, high inflation rates in recessionary times, drastic changes in the exchange rate and a new market oriented approach towards monetary policy resulting in record interest rates. On the labour front new labour laws were introduced and the country experienced the growth of a strong trade union movement. The future might also hold in the possibility of nasionalising some of the privately-owned enterprises when a new government comes into power.

Politically speaking, the country is experiencing a period of ongoing constitutional changes. The escalating violence restricted foreign investment and the declaration of a state of emergency during the 1980's impacted on business confidence and led to more stringently applied international sanctions.

Finally, the past decade also experienced both a serious drought and floods, high levels of consumption expenditure by the government and lately a drop in public investments expenditure.

While it is not the purpose of this study to relate observed structural changes in specific equations to the underlying causes - for which purpose a far more detailed analysis is required - it is important to note that all of these events could cause econometric models to be structurally unstable. The so-called 'transfer mechanisms' through which these events carry over in parameter instability are mainly five-fold (Raj and Ullah, 1981) and are discussed in the next section.

1.4 CAUSES OF COEFFICIENT VARIATION

Even when the underlying "true" parameters are stable, situations arise in which the parameter variation approach may prove valuable. By their very nature, econometric models are abstractions involving simplifications imposed by available data. Such simplifications and abstractions often result in misspecifications which in turn influence forecast accuracy. Effects of such misspecifications can be countered by introducing an appropriate parameter variation structure. Important types of misspecifications which arise in the construction of forecasting models include omitted variables, proxy variables, aggregate data and non-linearities.

The omission of important explanatory variables can arise from inadequate theoretical frameworks, unavailable data, or the desire for simplicity. Such excluded variables often relate to structural changes resulting from taste evolution, technological developments, changes in institutional arrangements, and the like. The effects of such excluded variables are presumed to be random with a distribution which has a time-invariant mean and variance. Such variables will not alter the parameter effects of included variables, provided the omitted variables are orthogonal to those that are included. However, time series for such omitted variables may exhibit nonstationary behaviour, and they are often not orthogonal to the included variables. Under these circumstances, the estimated effects of included variables can be expected to change with time. At a minimum it seems reasonable to expect that excluded variables with non-zero effects will result in time variations in the intercept or constant term.

Due to data limitations, proxy variables are often employed in the construction of econometric models. Such proxy variables are invariably introduced into dynamic representations which involve expectations formation patterns and measures of capital. Unfortunately, these proxy variables will only imperfectly capture changes in the economic behaviour of the true variable, and the relationship between the "true" variable and its proxy may change in time (see Raj and Ullah, 1981).

Under these circumstances, changes in the true variables which measure the actual economic stimuli induce instability in the estimated parameters of the proxy variables.

For aggregate data, the possibility of parameter instability has been demonstrated widely. Since aggregate data are measured by weighting the relative importance of the heterogeneous sets of micro-units, the parameters in the estimated aggregate equation will remain constant only so long as these weights do not vary. With time series data, the assumption of constant weights (i.e., relative importance of the individual components of the aggregate remains unchanged) is indeed unlikely to be satisfied. Hence, since shifts in the aggregation weights are the rule rather than the exception, parameter effects associated with the aggregate variables, in the estimated model, will vary across time (Zellner, 1962).

Another potential cause of parameter variation is the inappropriate specification of functional forms. Rausser, Mundlak and Johnson (1982) argued for example that, if under the pretext of a Taylor series expansion a linear relationship is estimated as an approximation to a nonlinear specification, the assumption of constant parameters for the simplified equation is reasonable only if the observed explanatory variables remain within some narrow range. For variations beyond this range, it is a simple matter to demonstrate the nature of parameter variation for the simplified equation -

moreover, the secular evolution of many economic time series strongly suggest the rejection of any model that is based upon the assumption of narrow sample ranges. The approximation of highly nonlinear "true" relationships by simpler functional forms, along with observations outside a narrow sample range, provides perhaps the strongest motivation for a varying parameter structure.

Economic theory substantiates varying parameter models. In many situations, economic theory allows for the expectation that relationships will vary over time. For example, changes in economic policy result in changes in the economic environment of the economic units. On the assumption that those units act according to the rules of rational behaviour, changes in economic policy result in changes in the parameters of equations describing their behaviour. Econometric models, therefore, are inappropriate tools for long-term policy evaluation precisely because they assume a stable structure. In his well-known critique of econometric policy evaluation, Lucas (1976: 41) made the following point about the structure of econometric models:

> "Given the structure of an econometric model consists of optimal decision rules of economic agents, and that optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any changes in policy will systematically alter the structure of econometric models."

Indeed, dynamic economic theory and the theory of rational behaviour provide no arguments for model formulations with constant parameters.

Finally, it is frequently found that the relationship is properly specified, but is different for some subsets of the sample. It is clear that in this case common parameters represent none of the existing subsets in the available sample. Division of the sample and the introduction of more than one regression regime can improve the accuracy of the model and resulting forecasts.

The cumulative implication of these observations is that with the progress of time the specified parameters of estimated econometric models may be expected to change. An investigation of the simplifying assumptions implicit in the maintained hypothesis should provide a starting basis for isolating parameter changes, if any.

1.5 ECONOMIC STRUCTURAL CHANGE: A HISTORICAL PERSPECTIVE

Wald (1947: 586) first observed that the coefficients in a regression equation can be random. He stated:

> "In some problems it seems reasonable, to assume that the regression coefficients $\beta_1,\ldots,\beta_p$ are not constants but chance variables."

This was apparently one of the first suggestions that in some contexts it may be meaningful to make the assumption that parameters in a model are themselves random variables. Unfortunately, Wald did not suggest a way to estimate such a model. A few years later, Rubin (1950) and Hurwicz (1950) examined models with random parameters and suggested maximum likelihood (ML) methods as a means of estimation. Rubin's model consisted of a single linear equation with K parameters, each of which is normally distributed with unknown mean and variance. He derived a set of maximum likelihood equations for this model which seemed to be very complex and difficult to solve in practice. Hurwicz (1950) looked at a simple system of two equations with one random parameter and its associated likelihood function, offering no practical solution to the estimation problem presented. Unlike Rubin (1950), who allowed for some random variation in the coefficients, Kendall (1953) restricted them to a deterministically evolving pattern.

Even though the papers of Wald, Rubin and Hurwicz do not yield useful procedures for estimation, they did bring the arbitrary nature of the constant coefficient assumption to the attention of analysts. In the well-known econometrics text by Klein (1953: 216), it was also noted in a cross-sectional economic framework that:

> "Individuals differ greatly in behaviour, and it may not be possible to obtain observations on a sufficiently large number of variables so that each unit may be considered to behave according to the same structural equation."

Yet, the problem of parameters being treated as random variables yielded no published research until the middle of the sixties, except for a paper by Theil and Mennes (1959), dealing with multiplicative randomness in time series regression analysis. Since the middle sixties economists and other users of regression analysis have shown increased interest in varying parameter regression models. This interest stemmed from the realisation that conventional constant parameter models often do not accurately describe the physical phenomena under investigation. Mundlak (1963) presented a good rationalisation for random intercept models while the work of Nerlove (1965), Rao (1965), Zellner (1966), Fisk (1967) and Theil (1968) provided further motivation and understanding of the nature of the problems of inference from the random coefficient model. Many of these articles were either concerned with the interpretation of random coefficients in aggregate economic relationships (Nerlove, Theil and Zellner) or with models of little applicability to business or economic problems, such as those used by Fisk (1967) and Rao (1965) for the analysis of growth curves.

A more rigorous and fruitful study regarding the estimation problem was taken up by Hildreth and Houck (H-H) (1968) who analysed a model very similar to that of Rubin (1950) and suggested several consistent estimators of the means and variances of the random β's. In the same year, a pair of doctoral dissertations (Swamy and Rosenberg, 1968) were completed which, to some extent, served as launching pads for much of the current work on random parameter estimation. In addition, the interest of social scientists in problems of control theory has generated efforts to relate varying parameter regression to existing models in the field of engineering automatic control. Swamy (1968) examined a model with cross-sectional time series data under the assumption that the coefficients vary across the cross-sectional units. A closely related model, based upon the work of Kalman (1960) in the field of engineering (called Kalman filtering), was also considered by Rosenberg (1968, 1973). Burnett and Guthrie (1970), Swamy (1971) and others suggested potential improvements for fitting historical observations as well as predicting new values when the model allows for variation in the regression coefficients.

The refinement of this earlier work, undertaken in the seventies, has been explosive, resulting in over thirty published works and at least six doctoral dissertations. These range from applied studies using random coefficient methods to the examination of more complex random parameter specifications. The work of Cooley (1971) and Cooley and Prescott (1973, 1976) with respect to the adaptive coefficients model popularised the random coefficients model with applied researchers.

In recent years, much effort has been devoted to the creation of a more general approach to the problem of parameter variation, including proper estimation and testing techniques for new types of models. Quandt's papers (1958, 1960) initiated research to find new methods of uncovering and handling the instability of regression models. A number of techniques have been developed (see Tsurumi, 1980) to test the parameter constancy of a regression relationship. Some of the more popular tests used by applied econometricians are Quandt's log-likelihood ratio test (Quandt, 1958), the Chow test (Chow, 1960) and the CUSUM and CUSUM of squares test (Brown, Durbin and Evans, 1975). Many alternatives, modifications and extensions of these tests have been suggested in later publications by, amongst others, Gujarati (1970), Goldfeld and Quandt (1973), Poirier (1973 and 1976), Hackl (1980), McCabe and Harrison (1980), Dufour (1982), Ploberger et al. (1983, 1991 and 1992), Krämer et al. (1986) and Andrews (1989, 1990, 1991 and 1992).

A number of surveys have also been written during the last ten years in the area of structural change and econometric modelling. Amongst these are books by Raj and Ullah (1981) and a contribution by Chow (1983) to the "Handbook of Econometrics". Other prominent articles, books and references in the field of varying parameter regression during the eighties include Judge et al. (1980, 1982, 1985 and 1989), Harvey and Phillips (1982), Nicholls and Pagan (1985), Broemeling and Tsurumi (1987), Schulze (1987), Hackl and Westlund (1985 and 1991) and Hackl (1989). Several volumes contain special sections on varying parameter model building, such as those by Broemeling (1985) and Krämer and Sonnberger (1986). Accessible bibliographies have been compiled by Shaban (1980) and by Johnson (1977, 1980). An updated annotated bibliography forms part of this study.

1.6 DETECTING THE PRESENCE OF STRUCTURAL CHANGE

The problem of testing for parameter constancy was tackled for the first time in the regression model context in the late 1950's. A large body of literature on various aspects of this problem has been published since. Modern econometric practice advocates that a given specification should be subject to a rigorous testing procedure and it is now becoming routine to test for misspecifications such as omitted variables, serially correlated disturbances, heteroscedasticity, incorrect functional form and structural change.

Parameter constancy is a key assumption in standard econometric models. If it is violated, inference about the parameters as well as any policy implications drawn from the model may be misleading, while the accuracy of post-sample forecasting is also affected. Not surprisingly, testing parameter constancy in linear models has attracted considerable attention in the literature. For a recent bibliography, see Hackl and Westlund (1989); for surveys, see Krämer and Sonnberger (1986, Chapter 4), and Tsurumi (1988).

As argued before, one of the most important requirements for statistical and econometric modelling is the existence of relatively constant relationships between variables. In econometric modelling the aim is usually to represent these statistical regularities parametrically in the form of marginal propensities and elasticities. An important check on model adequacy is therefore a test of hypothesis of parameter constancy.

Indeed, some of the most commonly used methods for detecting structural change are those associated with testing hypotheses of parameter constancy and the absence of predictive failure. The second group of commonly used methods for detecting structural change are those employing recursive estimation and associated graphical display techniques. Methods that are of special interest for economists are those that allow one to test hypotheses of parameter constancy in linear regression relationships, simultaneous equation models and time series models, corresponding to the most commonly applied types of models. Comprehensive discussions about these methods can be found in Hackl (1980), Chow (1984), Judge et al. (1985), Anderson and Mizon (1989), Dziechciarz (1989) and the bibliography by Hackl and Westlund (1989).

A standard assumption underlying some parameter constancy tests is that if the parameters change, they change only once during the observation period, so that the linear model contains a structural break. Probably the best-known test statistic for the hypothesis of parameter constancy is the analysis of variance F-test statistic. The test statistic is an optimal one, in the context of the normal linear regression model, for testing the constancy of the regression coefficients across two or more regimes, each of which has sufficient data points to allow reliable estimation of regression coefficients, conditional on the error variances being constant across regimes (see Chow, 1960). It is therefore sensible to test the hypothesis that the error variances are constant before testing the hypothesis of regression coefficient constancy. Indeed, the fact that the variance ratio and the analysis of variance test statistics are statistically independent (see Phillips and McCabe, 1983) means that it is possible to control the probability of a Type I error for this testing procedure. In situations where there are insufficient observations to allow separate estimation of the model parameters for each regime, Chow (1960) proposed an alternative test statistic that is optimal for the hypothesis that there is no change in the conditional mean of the dependent variable across the regimes. This test statistic, therefore, is effectively a test for the absence of predictive failure, and has different power properties than the analysis of variance test statistic; on this point, see Anderson and Mizon (1983), and Breusch (1986).

The statistics typically used for testing parameter constancy and the absence of predictive failure, particularly when used as a part of the diagnostic checking of model adequacy, require the potential break points to be nominated by the researcher. However, when there are no obvious discontinuities, the selection of arbitrary breakpoints could produce misleading information about a model's properties. Hence, to locate a more precise point of switching, Quandt's log-likelihood ratio test is usually employed (see Brown, Durbin and Evans, 1975). Once the switching point has been located by the Quandt test, it is possible to apply the Chow test.

The fact that the classical Chow test assumes the possible breakpoint to be known may often be too strong an assumption in practice and considerable efforts have been made in the literature to develop tests releasing the assumption. Brown, Durbin and Evans (1975) derived the CUSUM and CUSUM of squares (CUSUM-SQ) tests based on recursive residuals assuming that the breakpoint or breakpoints were unknown and that all regressors were independent of the disturbances. Ever since the seminal paper by Brown et al. (1975), the value of computing recursive residuals and analysing them using CUSUMS and related test statistics has been increasingly appreciated, so much so that recursive estimation is now a standard estimation technique in a number of econometrics computer programs (for example, the IAS System, Forecast Master Plus, PC Give, SAS and TSP).

Recently, the theory of weak convergence and functional central limit theorems have also become important in the statistical theory of testing parameter constancy. Sen (1980) and Ploberger, Krämer and Kontrus (1989) considered a test called the fluctuation test based on comparisons between parameter estimates from the partial samples and those of the complete sample of which the regressors of the model were assumed stationary.

Another strand of literature is concerned about the case in which the alternative to constancy is that the parameters are stochastic and fluctuate according to some time series model (e.g., the Watson-Davies test).

A well-known, and obvious, method for spotting structural breaks could be the inspection of graphs of variables plotted against time. The recent development of econometric software for use on personal computers has, by incorporating powerful and easy-to-use graphing options, added significantly to the tools available to econometricians. The routine inspection of the graphs of the major variables involved in a modelling exercise should provide valuable information about trend, seasonal, and cyclical behaviour, as well as identifying potential structural breaks.

Finally, the question of robustness of statistical tests with respect to the underlying assumptions has found interest in certain research done on structural change problems (see references in Chapter 4). The question might be raised whether certain procedures are robust with respect to parameter nonconstancy. For example, Perron (1989), has shown how standard tests of the unit root hypothesis[4] against trend stationary alternatives cannot reject the unit root hypothesis if the true data generating mechanism is that of stationary fluctuations around a trend function which contains a one-time break. Hendry and Neale (1991) investigated the robustness of the DF (Dickey-Fuller) and the ADF (Augmented Dickey-Fuller) tests when there is a shift in the intercept of an AR (autoregressive) process. Their Monte Carlo study revealed interesting results. Such regime shifts can mimic unit roots in stationary time series; consequently, a unit-root test should be accompanied by a diagnostic test for parameter constancy.

Once potential breakpoints have been isolated, it has to be decided whether these are genuine structural turning points or the result of model misspecification. In the former case, the

[4]Testing the unit-root hypothesis is beyond the scope of this study and, therefore, only some introductory concepts will be given.

model has to be modified to assimilate the structural change; and as the wide range of models in the following section makes clear, there are indeed many alternative ways of modelling structural change.

1.7 PARAMETRIC MODELS OF STRUCTURAL CHANGE

In previous sections it has been suggested that the assumption of constant parameters over all the observations may not be valid. In cross-sectional studies there can be heterogeneity in the parameters across different cross-sectional units. In time series studies there can be variation in the parameters over time.

Several models to tackle such problems are suggested in the literature. Although some brief comments will be given on nonparametric models with time-varying parameters, the main focus of this study will be on parametric models and the issue of changing parameters (a summary of alternative varying and random coefficient models for the linear model is depicted in Diagram 7.1, Chapter 7). Econometricians assign parametric models with changing regression parameters to essentially one of the following three categories:

(i) varying, but nonstochastic parameters;

(ii) random parameters from a stationary process; or

(iii) random parameters from a nonstationary process (Kalman filter type models).

The problem of random parameters has received increasing attention because of an ever-growing body of evidence that the usual regression assumption of stable parameters often appears invalid. For instance, the apparent benefits for improved forecasting resulting from "adjusting the constant term" in regression equations is an indication that more variability is present than can be captured by ordinary autoregressive error terms. It seems, therefore, that more attention should be devoted to modelling regression processes where the regression parameters themselves are subject to various sorts of perturbations.

Some models postulate that there are one or more discrete jumps in the true parameter values. Here the econometrician seeks to estimate the points when the changes occur and the associated sets of coefficients. The parametric structure in this instance could vary, not as a random variate, but systematically with the influence of the omitted effects. These models represent an improved alternative over dummy variable models because the number of parameters to be estimated is reduced. Another type of parameter variation, known as "switching regression", has been proposed by Quandt (1958 and 1972). In these models observations at time t are assumed to be generated by one of several possible regression equations (regimes), producing discontinuous shifts in the regression over time. Models of this type have been employed in economic studies by Fair and Jaffee (1972), Hamermesh (1970) Ferreira (1975) and Swamy and Mehta (1975). These models stand in contrast to the more nearly continuous parameter variations assumed in the random and varying coefficient models adopted in papers by Swamy (1970, 1971, 1973, 1974 and 1980), Rosenberg (1973), Cooley and Prescott (1973 and 1976) and others.

The rationales for using varying parameters are several. For one, the "true" coefficients themselves can often be viewed directly as the outcome of a stochastic process; e.g., some sort of autocorrelated random event with a given mean. A related model assumes a cross-section of individuals who possess the same regression regime across time, but whose individual behaviour at a given point in time can be viewed as a random sample from a population of coefficients, again with a given mean. This view gives rise to a cross-sectionally random coefficient model. Second, even when the underlying parameters are stable, situations arise in which a time-varying coefficient approach will prove to be effective. Such is the case when there are specification errors, such as excluded variables or linear approximations of nonlinear forms. These can often be more appropriately modelled as random coefficients rather than using the simpler additive error term. Another difficulty that lends support to certain time-varying parameter models arises when the underlying data are aggregates. Shifts in aggregation weights among the micro components will often cause trends and/or other forms of autocorrelated behaviour to appear in the macro-regression equation parameters (see Section 1.3).

The Kalman (1960) filter is a technique that has only recently begun to spill over to economics from the engineering literature on optimal control systems where the object is to estimate the parametric state of a control model at various points in time so that corrective action in its time path may be taken. Since the early 1970's (see e.g., Sarris, 1973), Kalman filtering has been used and evaluated as a procedure for estimating econometric models with time-varying parameters.[5] A major limitation of the Kalman filter, however, is its frequent reliance on knowledge of the parameters of the stochastic process associated with the random coefficients. While engineers are often able to specify these parameters from direct physical information, econometricians are seldom so fortunate, and the identification and estimation problems are much more severe in an economic context. Other authors specific in the field of automatic control are Aoki (1967, 1987), Athans (1972), Zellner (1971) and Zellner et al. (1975).

[5]Quite loosely, Kalman techniques arise when one attempts to find an optimal (least mean square error) estimator for the "state" $\mathbf{B}_t$ in a linear model $y_t = \mathbf{x}_t' \mathbf{B}_t + e_t$ in which the parametric state is assumed to obey a first order transition equation $\mathbf{B}_t = T\mathbf{B}_{t-1} + \mathbf{v}_t$, where $\mathbf{v}_t$ is some appropriately specified stochastic term and T is a matrix of constants.

Several research contributions which extend the use of the Kalman filter have been published recently (see Hackl and Westlund, 1989). Watson and Engle (1983), for example, presented two methods for estimating the unknown parameters of dynamic unobserved component models. Both are algorithms for maximising the likelihood function - the first is based on the method of scoring and the second is the so-called EM (estimation and maximisation) algorithm. Each iteration of EM requires a Kalman filter and smoother, followed by straightforward regression calculations. Schneider (1991) continued this research and used scoring and the EM method for estimating hyperparameters in a state-space model. A descriptive interpretation of Kalman filtering (the so-called flexible least squares (FLS) approach) is also described by the author.

As with conventional regression models, Bayesian inference has also been applied to the study of varying parameter regression. These applications occur in various contexts and are sometimes tailored to the needs of the model under investigation. The decision analyst expresses a subjective opinion about the regression model through the choice of prior probability density functions (pdf). Swamy and Mehta (1975) applied the Bayesian approach to estimating the parameters in the seemingly unrelated regression model of Zellner (1962). Early work on Bayesian inference are those of Sarris (1973), Rosenberg (1973), Froehlich (1973), Singh and Ullah (1974), Akkina

(1974), Raj (1975), Hsiao (1975), Welsch and Kuh (1976), Singh (1976), Cooley, Rosenberg and Wall (1977), Barry (1977), Dent and Hildreth (1977) and Garbade (1977). In Swamy (1973) a time series of cross sections is studied and is perhaps the most general application of the Bayesian approach in this area. When time variation of the regression coefficients is given a special structure the application of Bayesian analysis can yield more tractable results. The most widely used structure is that employed in Kalman filter models. In a problem in univariate time series forecasting, Harrison and Stevens (1971) applied the Kalman filter to a simple Bayesian model but relaxed the assumption that the specification parameters are known. Instead, they assigned a prior pdf to the population parameter θ and used the subsequent computations to revise this pdf. A book by Krishnaiah and Miao (1988), a monograph by Broemeling and Tsurumi (1987) and the references therein and an article by Broemeling, Cook and Choy (1991) summarise the achievements of the Bayesian methodology.

Recently, econometricians have made important contributions to the structural change literature, including a wide range of models which are more generally applicable. Andrews (1990) considered general nonlinear models with weakly dependent data, Chu (1989) examined models with trending regressors and Hansen (1990) studied cointegrated regressions.

Some progress has also been made in the field of time series analysis and structural change. Many time series exhibit a changing trend or a changing autocorrelation function structure; that is to say, they have certain nonstationary characteristics that cannot be modelled by the usual ARMA representation. Broemeling (1989) introduced changing parameter ARMA processes as a way to model a time series. Robinson (1991) discussed the estimation of time series models that are possibly nonlinear in parameters, which change smoothly but nonparametrically over time. As is the case for simultaneous equations models, research on time series models with time-varying parameters is very rare.

Hamilton (1989) has suggested modelling the trends in nonstationary time-series as a stochastic switching Markov process. The nonlinearities with which his work is concerned arise if the process is subject to discrete shifts in regime. His basic approach is to use Goldfeld and Quandt's (1973a) Markov switching regression to characterise changes in the parameters of an autoregressive process. Extensions of Hamilton's work can be found in Ghysels (1992) who presented a general class of Markov switching regime time series models of growth cycles and seasonals.

Increasing attention has been paid to the cointegration concept in econometric model building over the last few years. Cointegration theory and tests have been developed by Granger

(1986) and Engle and Granger (1987) for models with constant parameters and whose relevance for business cycle research was examined in a paper by King, Plosser, Stock and Watson (1987). Granger and Lee (1991) extended the idea of cointegration to time-varying parameter (TVP) regression.

A more recently developed class of models involves changes in the conditional error variances, and these also offer scope for econometric analysis of the effects of economic structural change. The ARCH model of Engle (1982), and the many related extensions of it, provide a rich class of models capable of capturing the effects of volatility changes.

The above range of parametric models are capable of being used to characterise economic structural changes and although wide, it is by no means exhaustive. In addition, the class of models of potential relevance in modelling economic structural change can be further widened by considering nonparametric and semi-parametric models (not forming part of this study). The use of prior information concerning known institutional or market changes, plus any information about the nature of the change gained in using statistical techniques to detect structural change, will greatly facilitate the choice of an appropriate type of model to assimilate the structural change. However, even with the limitations on the range of models to be considered induced by using knowledge of the general type and location in time of the structural change, there will be considerable scope for choosing alternative models. No matter what class of models is considered, statistical inference can assist the econometrician in detecting the presence of structural change; locating the breakpoints between regimes; and assimilating economic structural change into econometric models and discriminating between alternative representations of it.

One of the aims of this study is to highlight the variety of models available, their usefulness, their assumptions, their limitations and possible contradictions. It will be demonstrated that stochastic coefficient modelling rests on firmer philosophical and logical foundations than an econometric methodology which involves conventional constant coefficient modelling.

1.8 THE ROLE OF STATISTICS IN MODELLING STRUCTURAL CHANGE

Statistics plays an important role in the analysis of economic structural change. Ignoring structural change and other forms of model misspecification induces predictive failure and parameter nonconstancy in models. Thus, statistical tests for the presence of these phenomena are important constituents in the evaluation of model performance. Statistical tests for the null hypothesis of no structural change, which have power against a wide range of alternative hypotheses involving structural change, are essential in applied modelling.

Statistical methods for detecting the presence of different regimes, the number of regimes, and their location in time, are also major aids in the analysis of economic structural change. While prior knowledge of institutional changes, and of economic theory associated with change, will always have a crucial role in the analysis of structural change, very often it is statistical analysis using recursive estimation or the Kalman filter that identifies structural change. Univariate and bivariate graphical methods have also proved valuable, though care must be exercised in interpreting information produced by such procedures, especially when it is used to infer multivariate relationships.

Statistics has also provided an extremely rich collection of models capable of representing structural change. These models range, on the one hand, from those incorporating deterministic shift dummy variables, to the more advanced changing and random coefficient models. This diversity of models capable of characterising structural change means that there is unlikely to be a single, uniquely best approach for modelling a particular economic phenomenon within a structural change framework, and so statistics again must provide methods for comparing the properties of alternative models.

Finally, it might be thought that "shocks" to the economic system, changes in institutional arrangements, and changes in government economic policies are all irritants to the economic modeller, because such changes cause econometric models to fail. On the contrary, these are precisely the changes that are required to "prove" models, and by helping to identify model inadequacies; they are also instrumental in model improvement. For, as argued in Hendry and Mizon (1985), econometric evaluation is essentially a destructive activity - hazarding models to failure - but it is destruction with a constructive purpose. New and improved models are typically sought and discovered when existing models are plagued with changing structures.

PART II

ECONOMIC FORECASTING IN SITUATIONS OF STRUCTURAL CHANGE

CHAPTER 2

STRUCTURAL CHANGES IN THE SOUTH AFRICAN ECONOMIC AND POLITICAL ENVIRONMENT

2.1 HISTORIC DEVELOPMENT

Structural changes in South Africa are reflected both in growing pressures emanating from the international economy, as well as in greater domestic pressures resulting from changes in the relative size and strength of social factors.

The 1950's and 1960's were pleasant decades for South African macroeconomic policymakers. Growth proceeded steadily at 4 to 5 per cent per annum (Moll, 1992), political resistance was muted, forms of fiscal and monetary management were reasonably successful, and the world economy was stable and flourishing. This was an era of 'conservative' Keynesianism in South Africa (De Kock 1981: 322-23), characterised by active demand management and considerable state economic intervention, but with a careful watch kept on inflation and the growth of the money supply, and with government budget deficits kept to modest levels. The money supply was restricted through moral suasion and various direct controls on the amount of credit banks could provide. These controls were tightened during the 1960's. Inflation rates were low, averaging under 3 per cent per annum between the mid-1950's and 1969 (Moll, 1992).

The South African economy has been subject to a series of unrelated exogenous shocks during the past two decades: the oil price rises of 1973 and 1979, the gold price drop in 1981, and the capital flight cum sanctions experiences of 1976 and 1985. These shocks are held to account for intense cyclical fluctuations over the period (see De Kock, 1986).

A confluence of political quiescence, macro-economic restraint and steady growth came under threat in the 1970's. A combination of events - slower and more erratic growth in the world economy, the breakdown of the Bretton Woods system, sharp changes in oil and gold prices, and political resistance - put South African economic managers under considerable pressure. While the economy continued growing until mid-1974, aided by a rise in the gold price, certain policies had to be modified or restructured.

The economy went through a painful period of adjustment to changing world conditions in the mid-1970's, made worse by capital outflows and investor scepticism after the political crisis of 1976-77. The Soweto riots of 1976, and the response to it, clearly signalled that the problems facing the economy were not simply temporary, and that greater efforts were needed to reshape the growth model. The state appointed a series of Commissions of Inquiry during 1977, with most reporting by early 1979.

The late 1970's marked a sea-change in the South African political economy. Some of the systems of social control developed in the days of high apartheid (for instance, those regarding labour relations and urbanisation) were relaxed or restructured, accompanied by a shift towards the increased use of 'market-orientated methods' in economic policy. This shift implied a more limited economic role for the state; the use of monetary policies affecting markets indirectly rather than directly (largely via changes in interest rates); some relaxation of financial regulations and exchange controls; a managed float of the exchange rate; and considerable deregulation of the economy at a microeconomic level (De Kock, 1981; De Kock Commission, 1985; Le Roux, 1986: 6-9). Most of the important macroeconomic policy modifications were introduced between 1979 and 1983, following the recommendations of the Commission of Inquiry into the Monetary System and Monetary Policy in South Africa (chaired by Dr Gerhard de Kock).

In the second half of the 1980's stagnation returned, and it was acknowledged that the initial attempts to restructure racial Fordism had been unsuccessful. The crisis was not yet resolved, although the second phase of the crisis saw the disappearance of most elements of racial Fordism. The third phase of the crisis was ushered in by the dramatic political developments of late 1989 and early 1990, which opened the way to the ending of apartheid- the last important element of the old growth model. This would finally make it possible for a new growth model to emerge in the future.

P.W. Botha's aggressive Rubicon speech of June 1985 sparked off world-wide political protest, investor confidence plummeted locally and internationally, capital fled the country, and the balance of payments came under severe pressure. The last straw was the refusal of foreign banks to continue lending to South Africa, leading to the debt standstill and restructuring arrangements of 1 September 1985, and the re-imposition of the dual exchange rate system. This 'debt crisis' was ultimately a liquidity constraint (De Kock 1989b: 270), since South Africa's capacity to repay its foreign debt eventually was not at issue, but it none the less drastically destabilised the macroeconomic environment.

South Africa enters the 1990's with a legacy of economic problems that makes a long and depressing list. The 1980's were marked by stagnation in output growth; inflation entrenched at over 13 per cent per annum (Gelb, 1991); a weak rand; a permanent decline in foreign exchange reserves; and historically low personal savings ratios. There has been massive and growing unemployment, with no net creation of new jobs in the manufacturing sector through the 1980's. Other serious difficulties have emerged in the labour market: wage increases, measured by employers (and sometimes the state) relative to productivity growth, were perceived as being 'too high'. Workers, on the other hand, see their wage gains as 'too low', when measured by their purchasing power. Growing poverty has been expressed as well in the severe shortages of essential consumer items, most prominently housing.

A range of views has been put forward to explain these problems. Implicit in most of these views is the assumption that the market economy itself is essentially stable. In other words, it is essentially self-correcting in response to disturbances, if left to itself. The basic cause of the difficulties, in this view, is inappropriate intervention in the economy, in the form of government policy or, more broadly, politics. One group sees apartheid as the major culprit, either directly because of the limits it has placed on the operation of the 'free market', or else indirectly because of the impact of sanctions imposed by other countries. A second view rests on similar assumptions about the stability of the market economy, but argues that recent fiscal and monetary policy has been too lax in responding either to shocks emanating from the world economy or to pressures to raise fiscal spending.

In contrast, another approach which can be taken rests on the idea that capitalist economies are inherently subject to extended phases of decline and disruption, alternating with periods of stability and more sustained growth. In other words, the economic system endogenously generates instability, and intervention is consequently required to restore stability.

In conclusion, one might say that much of the 'reform' process in South African politics since the late 1970's can be read as an effort to adapt and shore up the old growth model. But 'reform' involved structural changes which were only partial in their extent: the efforts to preserve the racial definition of much of the institutional structure placed narrow limits on policy options. Furthermore, in many instances the 'reform' policies themselves exacerbated the difficulties, by making the economy more vulnerable to 'shocks' from the international economy, or by deepening the economic material hardships faced by the poor and so stimulating political conflict.

2.2 FIXED INVESTMENT

Many of the large investment projects undertaken by the public sector in South Africa, especially during the 1970's, were for infrastructural and strategic purposes, that is, largely motivated by broad social and political - rather than just economic - considerations.

If investment spending increases for no apparent economic reason, then we attribute the increase to an unexplained change in the optimism of investors about the returns from investment. That change in investment behaviour can be regarded as a disturbance to the system. Changes in the optimism of investors are sometimes described as changes in their 'animal spirits', (Keynes, 1936: Chapter 22) - a term that suggests that there may be little rational basis for those spirits.

In South Africa some shifts in the investment function have been caused by the discovery of mineral deposits that require a large amount of investment for their successful mining, and by the undertaking of large projects (such as Sasol II and III) aimed at greater self-sufficiency.

Shifts in the demand for money, on the other hand, may affect the interest rate, and thus indirectly affect the rate of investment which then constitute a possible source of private sector economic disturbance. South Africa has also failed, even long before the debt 'standstill' of 1985, to attract sufficient foreign capital, on a regular basis, to compensate for a frequent shortfall of domestic saving (or domestic investment).

The underlying politico-economic situation in the 1980's and perceptions about the future growth and stability of the South African economy contributed to the decline in real fixed investment spending by the private sector between 1981 and 1985. Political unrest in South Africa and the proclamation of a State of Emergency during the 1980's provoked an intensified disinvestment campaign and partial economic sanctions against the country. South Africa has been faced with a more serious disinvestment campaign since 1985, which caused a lot of structural changes in the economy as a whole.

A series of dramatic developments in the mid - 1970's spelled the end of South Africa's relative ignorance of serious threats of sanctions. There was the collapse of the Republic's detente policy, the invasion of Angola, the Soweto riots, Steve Biko's death and the bannings of 19 October 1977. There was also a major revival in bilateral punitive actions against South Africa, with the new Carter Administration.

On 1 August 1985 a bill was passed by the House of Representatives of the US Congress which proposed a ban on new investments by US firms in South Africa. It was signed by President Reagan on 9 September 1985. Amongst the measures announced were the following (Bloom, 1986: 8):

(i) A ban on the export of nuclear goods or technology to South Africa;

(ii) a prohibition on US loans to the South African Government except those that improve economic opportunities or educational-, housing- and health facilities open to all races;

(iii) a ban on the sale of computers and computer technology to South African Government agencies that enforce apartheid;

(iv) a ban on the importation of Krugerrands;

(v) a ban on the importation into the US of any military goods manufactured by South Africa;

(vi) a ban on export assistance to any US company employing more than twenty five people in South Africa but failing to adhere to Sullivan's code[6] ;

(vii) an increase of $8,0 million in scholarship funds for black South Africans and an extra $1,5 million to support human rights programs; and

(viii) a directive that US agencies in South Africa buy more goods and services from black-owned businesses.

High profile publicity was also given to Nobel price winner Bishop Desmond Tutu as well as to events in South Africa during 1985 such as the Uitenhage police killings on 21 March (the Sharpeville anniversary), the declaration of a State of

[6]See Bloom (1986: 36-38) for more detail on Sullivan's code.

Emergency on 21 July and intensive television coverage of the violent unrest situation. Last, but not least, was the widely televised Rubicon speech of President P.W. Botha in mid-August 1985.

The objectives of the main disinvestment advocates in the US have ranged all the way from the desire to send a strong signal of disapproval of apartheid to the declared aim of creating conditions of economic breakdown conducive to a violent revolution.

European governments were initially little concerned with the American disinvestment campaign, especially after the Nkomati Accord was signed by South Africa and Mosambique in early 1984. Most were watching changes in South Africa with restrained optimism. The Uitenhage killings, however, marked something of a turning point as far as the condemnation it brought from European capitals. Pressure for sanctions built up and a meeting of European Economic Community (EEC) foreign ministers followed on 10 September 1985 where a package of sanctions was endorsed which include, among others, the following (Bloom, 1986: 10):

(i) A rigorously controlled embargo on imports and exports of arms and para-military equipment;

(ii) the withdrawal of military attache's;

(iii) a refusal of all co-operation in the military sphere;

(iv) the prohibition of all new nuclear collaboration;

(v) an end to oil exports; and

(vi) discouragement of cultural and scientific agreements except where they contributed to the ending of apartheid, and the freezing of official contracts and international agreements on sport and security.

Positive measures were also endorced like, for example, an adaptation, reinforcement and publicity of the EEC Code of Conduct for firms doing business in South Africa.

A widely quoted study of Spandau (1979: 132-151), who used an input-output model, found that even if an investment boycott was 50 per cent successful in 1976, South Africa's gross domestic product would have declined by only 1,5 per cent and unemployment would have risen by 90 000. He concluded that a trade boycott would have been far more damaging because, even if it were 20 per cent successful, it would have cost the balance of payments more than R2 billion, raise White unemployment by more than 150 000 and Black unemployment by some 500 000.

Cooper (1983: 16), however, pointed out that Spandau's work is not immune from technical shortcomings such as the fact that input-output models rely on fixed coefficients which make them poor instruments of anything but static economic analysis. Cooper's conclusion is that we lack an adequate macro-economic model with sufficient predictive power, partly because of uncertainty and lack of information in general and also partly due to the fact that the parameters of such a model are not known a priori.

The question of whether and how sanctions work has long been approached by economists. In this respect, Cooper's (1985: 294) view is that effective sanctions undoubtedly affect the basic structure of the economy and thus reduce its performance.

2.3 MANUFACTURING AND PRODUCTION

Industrialisation can be regarded as a process of structural change characterised by the growth of the share of manufacturing industry in the Gross Domestic Product (GDP). Resources, including labour, are shifted out of the primary sectors of the economy into the more productive manufacturing sector. The pattern of industrial growth itself changes as certain manufacturing subsectors grow more rapidly than others.

The less sophisticated consumer goods industries, which amount to little more than labour-intensive processing, become less important, while the more advanced sectors in the field of consumer durables, and of capital and intermediate goods, grow in prominence. The latter require skilled labour and higher levels of capital intensity.

A disturbing feature of the growth experience during the 1980's is the fall in net fixed investment. In the manufacturing sector net fixed investment became negative during the late 1980's (that is industrialists in general, and not only foreigners, were disinvesting), with fixed capital stock falling by 8,7 per cent from 1984 to 1988 (McCarthy, 1992). Various factors can be associated with the collapse of industrial investment, but in broad terms it is the outcome of a Keynesian shift of investment functions in the face of declining confidence levels due to political uncertainty and social unrest (see Black, 1991: 169-170).

The recession from 1981 was a major factor in the re-emergence of political conflict during the early 1980's. This in turn was important in undermining the restructuring of the labour market, at least in so far as the intentions of policy-makers were not met. The trade-union movement could not be easily bought by wage-productivity trade-offs, this at a time of recession and growing unemployment. However, the trade unions became more militant, and strikes for wage hikes were frequent.

The emergence of a powerful trade-union movement, based upon skilled and semi-skilled African workers, was part of a process of increasing class differentiation within the black population in the course of the crisis. Slower growth and retrenchments, together with township resistance, placed unemployment and urbanisation on the agenda as essential issues to be addressed in a new growth model.

Import substitution succeeded in changing the composition of manufacturing output, but the economy remained dependent on imports. The average propensity to import during the first half of the 1980's was virtually the same as at the time of the introduction of an explicit strategy of import substitution in the 1920's (McCarthy 1988: 13).

The policy of import-substituting industrialisation, however, did have an important influence on the sectoral composition of manufacturing production. The salient feature of the structural change was the increase in the share of heavy industry (dominated by chemicals, metals and machinery) in manufacturing output (value added) from 36,4 per cent in 1925 to 44,1 per cent in 1946 and 64,3 per cent in 1985 (McCarthy, 1989).

These structural changes have been accompanied by an increasing tendency towards capital intensity, both in fixed capital per worker and in capital per unit of output. Important contributing factors are probably those that caused the price of labour, as perceived by businessmen, to be relatively high compared to the price of capital. Amongst these may also be: the low and frequently negative levels of real interest rates during the latter half of the 1970's and the early 1980's; the tax advantages attached to investment, initially introduced to stimulate investment in the wake of the Sharpeville tragedy of 1960 that caused a collapse in investor confidence; the over valued rand which, prior to the sharp depreciation of the currency, caused the cost of imported capital goods to be relatively low; and the increase in industrial action by workers which saw the man-days lost due to work stoppages increase from an average of 4308 in 1970-1971 to 993 615 in 1985-1986 (McCarthy, 1989).

It is reasonable to assume that supply-side measures alone cannot be expected to give inward industrialisation the required momentum. The unemployed masses have to be enabled to exercise an effective demand for housing and other labour-intensive goods. This will require redistribution. Whether and to what extent this redistribution should take place through the restructuring of current government spending, the tax system, or deficit spending have not been clearly indicated.

The 'redistribution-through-growth' approach has been described as neo-liberal export-oriented growth. This strategy is favoured by government, as reflected for example in the central theme of the 1991 Budget Speech ('Equity through Growth and Stability') and is applauded by the larger part of the business community. The strategy focuses on the restructuring of the economy, and of the manufacturing sector in particular, mainly through the market system in 'a spirit of' getting the prices of goods, money and factors of production right (Gelb, 1991: 29).

Should industrial development again be used as a major method of achieving particular distributionist objectives, this will require radical restructuring of the South African manufacturing sector. The outcome of efforts to restructure the sector will depend on many factors and constraints, of which not the least will be the need to create an efficient and profitable manufacturing sector.

2.4 INTERNATIONAL DEVELOPMENTS

During the 1960's, the supposedly 'clear-cut and stabilising rules' of the international monetary game were progressively disrupted, and US industrial hegemony, which formed the basis of the stability of the international order after the Second World War, also disintegrated. In addition, the financing of the war in Vietnam created a massive increase in the world supply of dollars, and this made the fixed exchange-rate system increasingly fragile. This monetary order was replaced in the

1970's by a more ambiguous and less stable system. Not only had US industrial supremacy declined, but the international monetary system had changed in two fundamental ways. Firstly, flexible exchange rates were introduced and, secondly, what Kahn (1991) referred to as the 'international debt economy' emerged. Together these factors altered the nature of the 'international monetary constraint', that is, the constraint imposed by the international monetary system on national economies.

The beginning of the 1970's saw the end of the reproductive boom in South Africa. This coincided with the period of 'mutational crisis' in the international economy after the breakdown of the Bretton Woods system. Under this system each country pegged its currency to the dollar, which in turn was convertible into gold at a fixed price of $35 per ounce (Kahn, 1991). This meant that the international economy operated on a fixed exchange-rate system, and exchange-rate policy was directed at maintaining a fixed link with the dollar. The role of central banks was to buy up dollars when there were excess supplies and sell dollars during periods of excess demands. The dollar was the major reserve currency by means of which international debts were settled.

The rise in the gold price had, until late 1981, cushioned the impact on South Africa of the second oil shock. There was no such relief, however, from the recessionary effects on the international economy of the monetarist deflationary policies adopted by the United States Federal Reserve Bank in 1981, in an attempt to halt accelerating inflation. These policies produced an international debt crisis. For South Africa, the immediate consequence was the collapse of the gold price, and severe balance of payments instability which persisted until 1986.

The nature of the balance of payments adjustment process and the adjustment of shocks depend in part on the nature of the exchange-rate regime. Under the Bretton Woods system, South African policy was directed at maintaining the exchange rate at a predetermined level, and the Reserve Bank bought and sold foreign currency to accommodate any excess supplies and demands. Since the breakdown of the system in 1971, countries have had to decide on the degree of flexibility of their exchange rate. In a world of floating exchange rates, even if a country decides to peg its own currency to a key currency it will effectively be floating against all third-party currencies. Part of South Africa's exchange-rate policy is exchange-control policy. Foreign-exchange controls were first instituted in 1961 after the Sharpeville shootings, which precipitated a large outflow of capital.

The sensitivity of capital flows to political crises has made the South African capital account extremely vulnerable to shocks, and these tend to require even more severe adjustment than in the case of the current account. Such shocks occurred in the aftermath of Sharpeville (1960), Soweto (1976) and again in 1985 following events in the Vaal Triangle. Events like these had the effect of restricting South Africa's access to international money markets and at the same time resulted in 'capital flight' (large outflows of capital in response to political or economic uncertainty), which could not be offset by recourse to increased borrowing.

Internal unrest, culminating in the declaration of a state of emergency in central magisterial districts on 20 July 1985, and foreign perceptions on the political future of the country, led to the withdrawal by foreign banks of credits to South African banks and other business enterprises and to a capital outflow. The 1985 shock was more severe in that not only were no loans forthcoming but existing loans were suddenly recalled on a large scale. In this case the adjustment was brought about partly through deflation - although it was induced more so by the collapse of local investor confidence than by deliberate government policy - and partly through depreciation, which saw the rand collapsing to 35 US cents (Kahn, 1991).

On 1 September 1985 the South African authorities announced a "standstill" in respect of a large portion of the country's foreign debt and the reintroduction of exchange control over non-residents in the form of the financial rand. An agreement followed between South Africa and its creditors on the scheduled repayment of the foreign debt in respect of which the standstill had been declared. Subsequently, South Africa found itself in the position of being a capital-exporting less developed economy. Since 1984 South Africa has experienced a net capital outflow of R25 billion, made up of R13 billion debt repayment, of which just more than R3 billion represented payments "inside the net" of scheduled repayments, and a further net outflow of R12 billion (McCarthy, 1989).

2.5 THE ROAD AHEAD

The economic challenges facing the new South Africa are formidable. The economy has been weakened by a decade of sluggish growth, low investment and industrial conflict, while gains from the ending of apartheid will only gradually be reaped. A new regime, facing the difficult task of democratic consolidation and economic reconstruction, will need to find some 'critical path' that maintains momentum on both fronts (Whitehead, 1990: 1141). The potential conflicts between them are evident. Extensive attention paid to democratisation (for example through the large-scale redistribution of assets) could lead to wealthy skilled people fleeing the country, low investment, and slow growth. Attempts at economic reconstruction which do not provide widespread benefits within a few years, could provoke ongoing industrial and social conflict, again undermining growth and stability.

The future prosperity of the country will also be threatened by the almost certain reluctance of private foreign investors to reinvest capital in a country from which it has previously been withdrawn, particularly if that country is suffering structural economic damage and political instability. Any government that then comes to power will inherit this situation, and will find it as difficult as the present government to correct the problems.

The social support for the post-apartheid state will include many people who suffered under the apartheid system and who will be eager to reap immediate economic benefits from the new era. Whites, investors and foreign observers will be wary of the new regime for several years, and may emigrate, or avoid investing in South Africa, if they are uneasy about macroeconomic management prospects for the near future. The goals of macroeconomic policy are likely to include difficult redistributive tasks and may be based on flawed or over-optimistic visions of how economies work. By the same token, policies may be erratic or short-lived, due to government weakness or the need to ensure immediate political support and minimise white opposition to fall in real income.

It is clear that South Africa's position as a net exporter of capital and the restrictions on access to foreign markets are severe obstacles in the way of the structural adjustments that need to be effected. Adjustments that lead to a gradual drift in the direction of a siege economy, with visions of autarky at the end of the road, will aggravate the situation by leading to negative growth, sharp declines in the domestic and foreign value of the currency, greater inequality and absolute poverty and therefore more social and political instability.

Heavy pressures will also be brought to bear on the post-apartheid state to make reparation for the suffering of apartheid by redistributing income - particularly by raising public sector employment and increasing social spending and real wages.

The question is: What are the redistribution options on the table in South Africa? As in the 1940's, South Africa appears to be facing two alternative paths. Both are articulated in terms of a 'growth plus redistribution' framework, but they reflect the interests of different combinations of classes and groups. Their separate growth paths have strongly contrasting implications for the nature, extent and time-scale of redistribution. The achievement of a constitutional settlement, and its terms, will naturally be a crucial factor in determining which of the two alternatives is ultimately pursued. It would reorganise (some of) the interests of different groups in society, in the process of transforming the balance of forces amongst them. It is also possible that a constitutional settlement could shift the balance between social forces in such a way as to prevent the emergence of any viable coalition linked to one of other accumulation strategy, and thereby prevent a new growth model from emerging.

Whether this redistribution should take place via a redistribution of government expenditure, or through deficit spending or the tax system, is open to debate. In the end, however, the success of growth with or through redistribution will depend on the nature of redistribution strategies and their influence on the dynamism of the economy.

The major black political movements in the 1990's are suspicious of market processes because of the multitude of ways in which markets have been rigged against blacks in the past, because they have been influenced by state planning and anti-capitalist ideologies, and because supporters of markets in South Africa have been extraordinarily narrow minded in advocating them. Implicit in ANC policy, for example, is a powerful impulse to expand the role of the state in the development process, in most cases involving direct state economic intervention or constraints on market functioning.

The South African economic crisis is therefore not yet resolved: we are still in the midst of the transition. The ending of apartheid - the last important element of the old growth model - through the introduction of a non-racial democratic state may finally open the way for a future growth model to emerge, though there is no guarantee this will occur.

CHAPTER 3

FORECASTING AND THE ASSUMPTION OF CONSTANCY

3.1 INTRODUCTION

Statistical forecasting is based on a misleading premise: the assumption of constancy. This assumption of constancy of patterns and/or relationships or structural stability in the data, is either ignored outright, brushed aside as unimportant, overlooked as useless, or at least is not made explicit. The practical implications are that false expectations arise which cannot be fulfilled. The accuracy of statistical forecasting cannot, after all, exceed the information content of the data.

For most economic systems, the assumption of parameter constancy is imposed in the face of noncontrollable effects and many important unobservable influences. The constancy assumption may adequately describe data from physical, natural and most engineering related applications, but it fails to capture the essence of business and economic data and structures which change continually and is inherently unstable. Thus, approaches which are successful in the hard sciences cannot be automatically transplanted to those that are of a social nature.

Economists, of course, have recognised that different data sets often result in noticeably different coefficient estimates. Perhaps the best example of this is the typical treatment of pre - and postwar data. To account for the difference in effects between pre - and postwar data, the general practice has been to introduce dummy variables to represent possible shifts in intercept and slope parameters. Although the dummy variable approach is indeed convenient, in many instances it can lead to grossly inaccurate forecasts. Neglecting issues of complexity, such specifications may be suboptimal. In time series regressions, it is appealing to view the data from, say 1980 as more relevant to forecast for 1990 than data from the early 1950s. Relevant taste, expectation formation patterns and sociological and environmental phenomena in 1980 were likely far different than 1950; and it would appear that in this sense the year 1980 contains more valuable information for forecasting. The following is a quotation by Hall (1971: 29-34):

> "The unforeseen event will occur again, just as the oil embargo of 1973 and the worldwide recession of 1975... and no econometric model or economist has the power to accurately forecast such uncertainties... despite or use of increasingly sophisticated tools, it's still a lot like a rolling dice."

In the opinion of the author, certain fundamental changes need to be made in statistical forecasting, in order that it becomes relevant and applicable to business and economic issues. Central to these changes is the fact that data need not be structurally stable and that forecasting must be mainly concerned with future, post-sample, predictions, rather than with what has already happened - that is, the accuracy of fitting a model to existing data. Forecasting should not be judged on the simple accuracy criterion but its role should be enlarged and be concerned with its ability to improve decision making within organisations (Makridakis, 1981: 308).

Recently the blind acceptance of the structural stability of econometric relationships has become the focus of expert criticism. Evans (1983: 4), in an analysis of the performance of US econometric models, argued that :

> "... the record would appear to be quite clear on one point; errors have sprung, not from bad judgement or bad guesses about the principal exogenous variables, but apparently from the underlying structures of the model equations themselves. The models could not properly respond to changes in monetary policy, government spending, high interest rates, or inflationary expectations."

The disillusionment with structural stability of econometric models gave rise to various theoretical responses (already discussed in previous chapters), all dealing with the problem of time-varying parameters. Amongst these are also sophisticated techniques for combining forecasts (Diebold and Pauly, 1987 and 1989). Empirical applications of all these techniques, however, are still scarce and comparisons are almost non-existent (Baudin, Nadeau and Westlund, 1984: 64).

3.2 THE NEED AND LIMITATIONS OF STATISTICAL FORECASTING

Today there is little doubt that there are many disappointed and frustrated users of forecasts, and others who have discarded the use of formal forecasting methods as irrelevant (Hogarth and Makridakis, 1981). At the same time the forecasting industry is flourishing and high interest is maintained, as shown by the number of recent books (see Makridakis, 1990) and articles published, the number of people attending conferences, and the opportunities for consulting in business and government.

In stable economic and environmental conditions, forecasting involves the continuation of established patterns/relationships in which case forecasting is usually fairly accurate. In turbulent environments, as during the 1970s, forecasting errors can be serious, McNees (1979). How could high inflation rates have been predicted during the 1974/1975 recession if such a phenomenon had never happened before? Moreover, how could the major 1974/1975 world recession have been foreseen on economic grounds when a major cause for it was political, the result of the Arab-Israeli War and the oil embargo? Models are not expected to possess prophetic powers.

Paradoxically, there is little interest in forecasting when stable conditions prevail and high interest when changes are frequent and uncertainty great. But this is precisely when forecasting accuracy is at its lowest level, inevitably inducing dissatisfaction. Unfortunately, however, forecasters have not only failed to communicate this point to users but have also failed to make explicit the alternative to statistical forecasting. All empirical evidence showed that human forecasters do not necessarily produce more accurate results than models (see Makridakis, 1981).

Today, with interest in this field growing rapidly, forecasting is plagued by many problems. The challenge, however, is not to attempt to blame who or what has been wrong but rather to enlarge the role of forecasting in order to be capable of dealing with real life data which often involves nonrandom changes from established patterns or relationships.

3.3 THE ROLE OF FORECASTING

The traditional role of statistical forecasting has been 'extrapolative'. That is, some model based on past data has been developed, which subsequently has been used to 'project' the past patterns beyond the sample data, thus, providing forecasts for the future. This usually works well as long as the established patterns/relationships do not change. However, if changes do occur, traditional statistical forecasting cannot deal with this situation because the assumption of constancy will not hold. The resulting errors do not have to follow previous patterns: they can be non-random, their variance can be wider, and/or they can be non-symmetric.

If an enlarged role for forecasting is accepted, then the problem becomes how to continue forecasting when systematic changes from established patterns are involved. Unfortunately, from a traditional model point of view, very little can be expected. Systematic changes from established patterns (e.g. the oil embargo) are non-repetitive or if they are (e.g. recessions) the length between two successive occurrences is not necessarily constant. Furthermore, even when repetitive events are involved, the effect of change cannot always be quantified because of the multitude of additional factors involved, making the isolation and measurement of single factor's influence impossible.

Forecasters and end users must accept the inevitable, this being the inability to forecast statistically when the assumption of constancy does not hold. If this is accepted, there are several choices: the first being to devise ways of knowing as soon as possible when systematic changes from past relationships are taking place. This is referred to as monitoring (or structural stability testing) and should be integrated into any forecasting system. If monitoring indicates some systematic change and the errors become non-random, this is when action must be taken in the form of varying-parameter estimation or adjustments to the quantitative forecasts.

The need for adjustment, however, requires a knowledge, and understanding of how things relate to and influence each other. Without such an understanding no meaningful adjustment is possible when systematic changes from established patterns/relationships occur. The dichotomy of forecasting continuations versus systematic changes in established patterns/relationships is critical on several grounds. Firstly, there is ample empirical evidence suggesting that where predictive accuracy is the issue, statistical models are superior to human judgement. Interestingly, it is not statisticians but psychologists who have been researching this area and advocating such a view. The conclusions are indisputable in the light of empirical evidence, available both in the psychological and forecasting literatures (see Makridakis, 1981).

Libby (1976) stated that human predictive ability has been found to be inferior to that of models in all cases, with one exception. Goldberg (1976), who reversed these exceptional findings by simply transforming the data, disputed this claim. Complaining, for instance, about the pure predictive ability of statistical forecasting makes little sense when the alternative, i.e. human judgement, can be even worse. On the other hand, traditional statistical models are not adequate when systematic changes from established patterns occur. In this case, the only alternative is either adjustments by humans who must, however, understand the nature of the system to which the forecasts refer or the application of models which are designed to assimilate structural change.

Therefore, a procedure is needed in order to avoid inconsistencies and to initiate prompt and effective action. More important is to understand the various factors affecting forecasting and the role of uncertainty while predicting the future.

3.4 THE CRISIS IN ECONOMIC FORECASTING

Econometricians and other forecasters have been concerned with intricate refinements of data collection techniques, statistical analysis and estimation procedures in order to search for the causes of and solutions to the problems in economic forecasting. The problems with inaccurate economic forecasting may lie much deeper and attempts at further technical refinements could miss the point. This is supported by Fourie (1986: 309) who conceded that "...economic forecasting harbors an inherent weakness which stems precisely from its presumed strength - its so-called scientific character".

Economic forecasters, because of a scrupulous emulation of the natural science ideal, have lost site of an important aspect of reality. This has led to an unfortunate distortion and disregard for the nature of economic relations. The implications of this, and for economic forecasting in particular, seem to be most unfortunate. Fourie (1986: 309) concluded that economic relationships and phenomena seem not to be subjected to the kind of fixed, exact, constant and immutable laws that are found in, for example, physical and chemical processes. The material with which scientists work differs fundamentally in that "... the material of physics possesses constancies and an absence of significant historical change and development, which the material of economics does not possess" (Hutchison, 1977: 37).

Georgescu - Roegen (1967: 18), argued forcefully and convincingly that economic processes are not mechanical. He goes on to conclude that economic relations are also not amenable to mathematical formalisation. Even Popper (1965), who appeared to be considerably impressed by the apparent scientificity of Economics, conceded that there is a fundamental problem in that the coefficients of economic equations are not constant and that the presumption of exactness and constancy that the mathematical expression of economic relationships convey is, in this respect, very misleading. This non-exactness and variability of economic phenomena and relationships affect economic forecasting at the root.

In the estimation phase of any econometric research, much historical data, covering several years, are required - large and sophisticated models require longer series and the data series employed must clearly be the result of repetitive measurements on the same phenomenon. The estimation procedure must, therefore, assume structural constancy of the phenomena during the estimation period. This is not only a convenient assumption, but also imperative for the formal validity of the statistical process. Streissler (1970: 25) went further to conclude that: "Without repetition in society no stochastic equation describing it can be conceived; but does society repeat itself? The sampling theory used in econometrics implies that phenomena we measure may, indeed, vary, but that their underlying pattern of variation remains basically constant. It assumed situations analogous to those in the physical world, where the characteristics of say, helium atoms ... are basically given once and for all."

The idea that Streissler is conveying above is that it is highly likely that an economic phenomenon under consideration will change over time and that the observed phenomena of the past will not necessarily continue to manifest itself in exactly the same manner.

In econometric forecasting the coefficients established by observing patterns in the past are utilised directly to predict the future course of variables. The statistical validity and significance of such a procedure, moreover, depends crucially on the existence of a constant probability distribution function with respect to the relevant phenomena. Similarly in so-called ARIMA time-series models, historical trends are directly extrapolated into the future on the assumption of the continued and unchanged existence of the trend pattern. However accurate these models describe the past, no claim can be made that they will continue to describe the future. It is, therefore, obvious to review the constancy assumption of econometric models due to the contradiction between assumption and reality which is a fatal weakness of economic forecasting and one of the principal causes of its problems.

Econometric forecasters, however, do at times acknowledge the problem of the constancy assumption in their models for which reason they use judgmental add-factors or constant term adjustments. Furthermore, when the possibility of structural changes is explicitly acknowledged, it is often seen as sudden discontinuous changes in the real world to be dealt with by the use of dummy variables. This implies that economic forecasting is in effect departing from the assumption of constancy and exactness. Such subjective fine-tuning also seriously undermines the original claim to scientific objectivity, which could raise questions such as "... is the array of hundreds of equations not needlessly costly if it has to be corrected by judgement any way? Why all the effort at generating highly computerised, intricate and sophisticated so-called scientific forecasts in the first place?" (Fourie, 1986: 312).

Even though the questions raised above are hardly new, satisfactory answers are still wanting. Moreover, these issues can never conveniently be passed by. On the contrary, facing them and dealing with them honestly can only contribute to a better understanding of forecasting.

3.5 THE CRISIS IN MACRO-ECONOMETRIC MODELLING

Econometricians are fully aware of the advancement in science, with no doubt of the fact that they are indeed practicing it for a very long time already. However, they do realise that they are dealing with some highly complex non-linear, interdependent, multivariate processes of disequilibria which are subjected to individual expectations and adjustments, external shocks, unmeasurable phenomena and parameters which have to be estimated from inaccurate data.

It is against this background that a formidable scientist, as an independent observer from the outside, evaluate the field of econometrics as being one with many shortcomings. Sir Peter Medawar (1984: 304) concluded:

> "...we cannot wonder that economic predictions are often grievously mistaken. It is not their wrongness so much as their pretensions to rightness that have brought economic predictions and the theory that underlies them into well- deserved contempt. The dogmatic self-assurance and the asseverative confidence of economists are additional causes of grievance - self-defeating traits among people eager to pass for scientists."

The argument of Medawar, a Nobel prize winner in the field of tissue transplantation and well known expert in scientific methodology, contains two elements that need to be addressed. The first statement deals with the accuracy of economic forecasts and the second statement with the partly-covered insinuation that economic forecasters cannot pass as scientists.

The one empirical fact in the forecasting profession is that forecasts are always wrong. The question now is whether, given the track record of a specific forecasting technique, an alternative exists that shows a better forecasting record. Such a comparison is relevant given the fact that econometric modelling is approximately one million times more expensive than smoothing techniques and about five hundred times more expensive than stochastic time-series modelling (Sullivan et al., 1977: 36-39).

The unofficial and impartial referee on the accuracy of economic forecasts is McNees (1976: 37-48) who stated that: "Forecasts not based on econometric models appeared to be generally as accurate or more accurate than econometrically based forecasts." In 1982 he tested three, according to him, generally accepted ideas with regard to econometric forecasts. Firstly, that model forecasts are generally poor and specifically worse than forecasts based on judgment; secondly, that time-series extrapolation perform better than model forecasting and finally, that available models are of no use as a basis for policy simulation. Although no strong confirmation on any of the above statements could be found, an element of truth is captured in each statement when reformulated.

One should not get the impression that criticism of econometric forecasts is only to be found outside the field of the econometric profession. Malinvaud (1981: 1364) summarised the dissatisfaction as follows: "One would have to be deaf not to hear the many protests which are being directed against it: those from our fellow citizens and from their governments, those from our colleagues the economists, and even those from certain of us econometricians."

The second aspect of Medawar's comment concentrates on the scientific nature of econometrics. Smit (1986: 9) described science as a system of methods and conceptual schemes which could lead to the explanation of natural processes. It is a public process and uses a system of concepts, called theories, to interpret and consolidate observations, called data. Theories can originate from inductive processes while testing and proofs are deductive in nature. In the non-exact sciences it is found that testing is done with the help of statistical techniques. Objectivity and the possibility to disprove a hypothesis are key elements in the practicing of science. Observations, on the other hand, are theory-dependent and, therefore, the rejection of hypotheses can be qualitatively rationalised so that discredited theories keep on existing despite reasonable alternatives. It is this system of data-based elucidation that separates science from dogmaticism.

The falsification procedure of Popper (1965) requires the formulation of hypotheses and ever stricter testing of it, so that every new theory accepted should pass the tests where previous theories failed. The text book approach to econometrics, on the other hand, presumes the existence of a prior economic model which gets exposed to economical, statistical and econometrical criteria for judging its validity. The similarity between the approach of Popper (1965) and that of the text book is quite clear. The text book approach can indeed be regarded as a step towards the falsification procedure. There are, however, gaps between practice and theory of which the following are a few examples: data are not always available in such a form as prescribed by theory, which then lead to the use of proxy variables; theory does not specify the length and structure of lagged distributions, with the result that a pure empirical approach is enforced upon a model builder; statistical tests lose validity under certain conditions and the inferential procedures collapse; a single equation conforming to all criteria of fit, might perform poorly in model context and must, therefore, be replaced by an alternative which again might not satisfy all a priori criteria. Many more examples exist that could give rise to a situation where one is forced to deviate from the text book approach. It is no wonder that Keynes was of the opinion that: "If the method cannot prove or disprove a qualitative theory and if it cannot give a quantitative guide to the future, is it worth while? For assuredly, it is not a very lucid way of describing the past" (Hendry, 1980: 396). This problem of exposing the underlying structure necessitates a further obstacle in the falsification procedure, namely the satisfaction of the forecasting accuracy criterion of the model. This has led to the rejection of many false models during the past decade - which in a sense satisfied the pre-requisite for the practicing of science. It is a historical fact that more time is spent on estimation than testing models. Hendry (1980: 403) also remarked that: "The three golden rules of econometrics are test, test and test, that all three are broken regularly in empirical application is fortunately easily remedied. Rigorously tested models which adequately describe the available data, encompassed previous findings and were derived from well based theories, would greatly enhance any claim to be scientific."

Although no final answers have been reached yet, it is indeed true that still more advanced scientific methods are currently practiced in econometrics. A price has already been paid insofar the complexity of the subject is concerned; it is becoming ever less accessible to the person who has not specialised in the field.

3.6 PROSPECTS AND REQUIREMENTS OF FORECASTING

Views ranging from unqualified faith to cautious optimism to deep pessimism is found in the professional and popular literature and the degree of dissatisfaction with economic forecasting may even be expected to increase in the foreseeable future.

The non-constancy of economic phenomena is clearly not a problem to be overcome by further refinement of constancy-assumption techniques and it is highly doubtful whether any significant improvement in the forecasting record can be achieved. Even Klein (1981: 52-56) concluded that econometric forecasting techniques may have reached their zenith and that one cannot expect the forecasting record to be improved significantly.

Changes in the tempo and direction of technological change occur almost daily, with highly uncertain but far-reaching social and economic consequences. The increasing interdependence and complexity of modern industrialised economies create conditions increasingly resistant to theoretic explanations, not to mention mechanistic explanations.

Urgent demands for more comprehensive and accurate forecasts are, however, created by the increasing complexity of modern society. Businessmen and politicians do have an understandable need for forecasts and will always find somebody willing to forecast if paid enough or if promised enough publicity. Economists, however, have certain responsibilities in this regard insofar as they have to reflect critically on the practice of forecasting. Economists should, therefore, avoid creating false impressions of their abilities and of the accuracy of their forecasts. Worswick (1972: 84) observed, in general terms, that "the more the impression is allowed to persist that economics is an exact science, or if not already one, then with the aid of mathematical models and the computer is about to become one, the more damage will be done to the subject when it fails to live up to the exaggerated expectations." One way of achieving this is that forecasters should at least refrain from making point forecasts and start forecasting confidence intervals. The accompanying significance level and its interpretation will at least convey a more appropriate message to the public.

A final requirement is that econometricians should be more forthright about their forecasting record. If, however, this is done, the public is normally confronted with vague press references to past successes in forecasting or a forecaster may point to his success in having forecast the current level of one variable without any reference to the accuracy of his forecast of other important variables. Wesso and Smit (1986) have shown in a study of forecasting accuracy that not one of the South African economic forecasters whose forecasts were investigated, had shown a forecasting track record of constantly good or bad forecasts. Such studies could undoubtedly lessen the negative criticism which is currently expressed by the public about the forecasting profession.

Klein (1986 :30) noted that "forecast testing is one of the most rigorous and relevant tests of a model. Predictive testing is important because objective standards for model validation, if implemented, determine credibility. If a model has not passed forecasting tests under conditions of frequent replication, the outside-user world has little to go on for judging the model's overall credibility." Unfortunately in South Africa there is no tradition of regularly subjecting forecasting models publicly to outside sample period performance tests.

Equally true is the fact that the forecasting models should be subjected to tests for structural stability over time, because the underlying structure of the model could influence the correctness of economic forecasts. Nowadays there are no excuses for not performing such tests for it can be done quite

In view of possible errors, like for example wrong predictions of the values of the exogenous variables, parameter changes, specification errors, and the fact that the impact of important non-economic factors, such as droughts and political disturbances, cannot be captured in a constant parameter model, econometricians have to either blend qualitative information with their judgement and forecasting experience to adjust their forecasts or apply varying parameter regression techniques as described in Part IV of this text.

3.7 FORECAST EVALUATION AND COMPARISON

The forecast system consists of two phases: model building and forecasting. These two phases are not quite as separable as it might appear from the earlier discussions. In fact, forecasts may also be used to validate and compare models.

The residuals, residual autocorrelations, mean square errors, and so on, are used to select models at the diagnostic checking stage. Diagnostic checking, which is part of the modelling phase, is very useful. However, the residuals or the historic one-step-ahead forecast errors depend on estimates of unknown parameters. Thus, the selected model might fit the data from which the estimates are calculated very well. However, when the forecasts are compared with future data that are not used for estimation, the agreement need not be as good. Hence, forecasts are compared with future data that are not used for estimation, the agreement need not be as good. Hence, comparisons of forecasts with actual observations can be useful additional tools for model evaluation and selection (see Box and Tiao, 1976). In some practical situations it may be unreasonable to expect many ex-post sample observations. However, in long series one can use the initial part for model construction and the remaining part as a holdout period for forecast evaluation and comparison. Such an approach is later pursued in the empirical investigation section.

Good forecasting models should lead to small uncorrelated one-step-ahead forecast errors. Various informal checks based on these errors can be performed (see Abraham and Ledolter, 1983: 372-373). These checks can be used to compare different forecasting methods. The comparison can be carried out in basically three summary statistics that are based on standard symmetric loss functions: the mean error (ME), the mean absolute error (MAE) and the root mean square error (RMSE). These are defined as follows:

$$ME = \frac{1}{n} \sum_{i=1}^{n} (\hat{y}_{T+i} - y_{T+i}) , \tag{3.1}$$

$$MAE = \frac{1}{n} \sum_{i=1}^{n} | \hat{y}_{T+i} - y_{T+i} | , \tag{3.2}$$

$$\text{RMSE} = \left[\frac{1}{n} \sum_{i=1}^{n} (\hat{y}_{T+i} - y_{T+i})^2 \right]^{\frac{1}{2}} \tag{3.3}$$

where $\hat{y}_{T+i}$ is an i-step-ahead forecast of y_{T+i} in some period i after the terminal period T of the fitting period. The RMSE is generally a good substitute for an averaged within-sample residual sum of squares (see Swamy et al., 1988:31). The RMSE is, therefore, regarded as the principle criterion for comparing forecasting models in this study. Root mean square error is, however, an inappropriate criterion if the dependent variable is governed by a non-normal stable Paretian process with infinite variance. In such cases it is useful to include MAE which is a useful criterion when the distribution has fat tails, even if the variance is finite (see Meese and Rogoff, 1983:12). Mean error provides another measure of robustness. By comparing MAE and ME one can ascertain whether a model systematically over- or underpredicts. These two statistics measure forecast bias and should be close to zero. Root mean square error measures forecasting accuracy; methods that yield small values for these statistics should be chosen.

Although out-of-sample comparisons have considerable intuitive appeal, formal tests of whether these differences are statistically significant generally require restrictive assumptions (see Granger and Newbold, 1977:281). But this limitation to the experimental design does not turn out to be crucial for the interpretation of the results.

Besides the summary statistics mentioned above, use can also be made of the so-called Akaike Information Criteria (AIC) (Akaike, 1976) and Schwarz or Bayes Information Criteria (BIC) (Schwarz, 1978). The AIC is computed as the log likelihood function for the model and fitted data, using the total number of observations less twice the number of independent parameters. Experimentally, minimisation of the AIC to determine model order has been proved sound and efficient. According to Goodrich and Stellwagen (1990), the AIC statistic is very close to the BIC, but does not penalise model complexity as severely. Thus it will sometimes opt for more complex models than the BIC. By rewarding goodness-of-fit to the historical data and penalising model complexity, they both provide a relative measure of expected out-of-sample forecasting performance. Selecting the model that minimises the AIC and BIC will generally lead to the most accurate forecasts. There is not a great deal of evidence available as to which statistic is superior, but research has shown that for business data the BIC leads to better out-of-sample forecasts than the AIC (see Koehler and Murphree, 1986).

PART III

IDENTIFICATION OF STRUCTURAL CHANGE

CHAPTER 4

PARAMETRIC TESTING FOR STRUCTURAL CHANGE

4.1 INTRODUCTION

> "The three golden rules of econometrics are test, test and test."
>
> D. Hendry (1980)

Linear regression is by far the most widely used means of data description, analysis and prediction in economics and elsewhere. Partly because of this popularity, it is very often used in situations where its assumptions do not apply. Much too often, empirical "laws" are produced, whether unintentionally or by some sort of data mining, which have little to do with reality. As a result, statistical analysis nowadays tends to be either greatly discounted or completely ignored. The golden days of empirical econometrics are definitely over.

As a consequence, recent years have witnessed a remarkable growth of interest in testing econometric models. While it took more than a quarter of a century for the first serious article on testing to appear in Econometrica (Chow, 1960), the predominance of testing within theoretical econometrics can hardly be overlooked in more recent volumes. In the 1980-1989 period alone, over a hundred articles and notes appeared with a focus on testing econometric models (see Hackl and Westlund, 1989).

The problem of structural shifts in regression arises in various fields. In economics, there is often reason to suspect that some model parameters have changed due to political events (change of government, new tax laws, etc.). One cannot for instance be confident that key parameters like the marginal propensity to consume were the same in the Great Depression as in the 1960s as is often done in empirical studies. The common features of such situations is the natural ordering of the data points, either by size, age or time. The respective model is assumed to fit the first segment of the data, and the problem is to test whether this model covers the whole data set.

Part of this study is to survey and apply some recent contributions in this area. This study confines itself to situations where the null hypothesis is that the assumptions of the standard linear regression model apply.

A simple varying parameter regression model will form the basis of further discussion. Consider

$$y_t = \mathbf{x}_t' \boldsymbol{\beta}_t + e_t \tag{4.1}$$

with t = 1,...,T.

For the T equations, y_t is the observation on the dependent variable at time t and $\mathbf{x}_t$ is the column vector of observations on K regressors. The first regressor, x_{1t}, is taken to be equal to unity for all values of t if the model contains a constant. The column vector of the $\boldsymbol{\beta}_t$ parameters is written with subscript t to indicate that it may vary over time. The error term $e_1,\ldots,e_t$ is assumed to be iid $N(0, \sigma_t^2)$ for $t = 1,\ldots,T$. This study will mainly be concerned with detecting time variations in the β's although a procedure is given in Chapter 8 which permits the investigation of variance changes. It is also assumed that the independent variables are non-stochastic.

As pointed out in Chapter 1, failure of assumption 1.3, which is the stochastic specification, in general, leads to inefficient, but still consistent parameter estimates, whereas a wrong specification of the structural part of the model (i.e., failure of assumption 1.4), typically renders parameter estimates inconsistent or even, for a lack of a well-defined parent parameter, almost meaningless. Such failures of the standard assumptions could cause parameters to change.

It may be useful at this stage to place this study in some sort of overall perspective within the literature dealing with the problem of testing for parameter shifts. Essentially the approaches to this problem fall into two groups:

(i) those which assume the knowledge that a parameter shift has taken place; and

(ii) those which assume no such knowledge but attempt to investigate whether or not, and the points at which, such shifts have taken place.

Among the more important contributions in the former category are those due to Quandt (1958, 1972) and Farley and Hinich (1970) on the one hand, and Chow (1960) and Fisher (1970) on the other hand. In the second category one may mention the work of Brown, Durbin and Evans (1975). The work by Farley and Hinich is primarily a generalisation of that by Quandt, just as Fisher's approach is a generalisation of Chow's. Some of these tests are based upon either the Wald (W), Lagrange multiplier (LM)- or Likelihood ratio (LR) tests and it is, therefore, more appropriate to first discuss the general W, LM and LR tests before turning to tests like the Quandt, Chow, BDE, etc.

Graphical plots for some of these tests, which are discussed later in the chapter, also provide useful tools to check the assumption of structural stability.

4.2 LIKELIHOOD RATIO (LR), LAGRANGE MULTIPLIER (LM) AND WALD (W) TESTS

Tests of misspecification provide a means of assessing "goodness of fit". No specific alternative hypothesis is entertained at the outset, although a knowledge of the power of the tests against particular alternatives is obviously useful insofar it suggests ways of modifying the model. Tests of specification, on the other hand, employ a specific alternative hypothesis, H_1.

The objective here is to consider the possibility that the parameters of a regression model may be stochastic rather than fixed. It is shown in Chapters 6 to 8 how the coefficients of a stochastic parameter model can be estimated. In addition the aim is to test hypotheses about such models. For example, it is useful to have available a test of the null hypothesis that the constant parameter model is appropriate against the alternative of stochastic parameters.

Perhaps the most widely applied hypothesis testing procedure for stochastic parameters is the likelihood ratio test. Suppose that a statistical model involves a set of unknown parameters. A null hypothesis may specify a set of K equality constraints on these parameters, although they are free to take any values under the alternative hypothesis. Parameter estimates can be obtained through the method of maximum likelihood. Coefficient estimates are those values for which the likelihood function - that is, the joint probability density function of the observations, viewed as a function of the parameters - is a maximum. If L_1, denotes the maximum value of the likelihood function under the alternative hypothesis, and L_0 the maximum value of the likelihood function when the parameters are forced to satisfy the K constraints imposed by the null hypothesis then the likelihood ratio test statistic is defined as

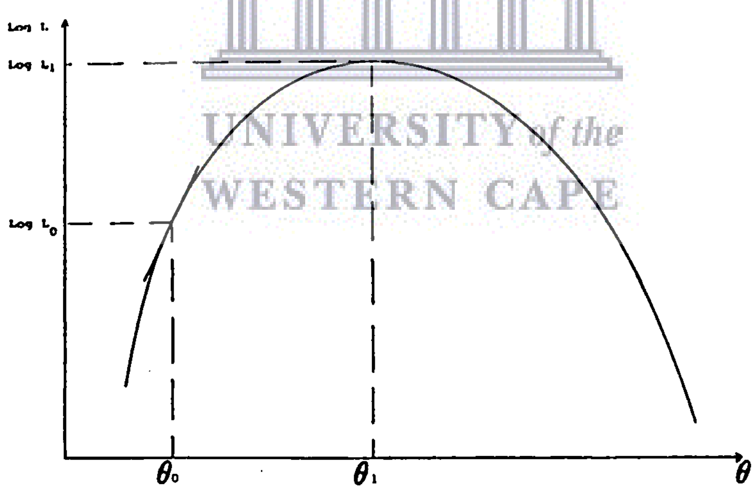
$$LR = 2\ (\log L_1 - \mathrm{Log}\ L_0) \qquad (4.2)$$

It is known that, in large samples, this statistic has, under the null hypothesis, a chi-square distribution with K degrees of freedom. The null hypothesis is rejected for large values of the test statistic. Harvey (1981: 163) has shown that, for large samples, the likelihood ratio test has strong optimality properties.

An alternative test procedure, called the Lagrange multiplier test (LM), due to Rao (1948) and Silvey (1959), is also available. The LM procedure is applicable to testing nested hypotheses. It also leads to a test statistic with the same asymptotic distribution as the LR statistic under the null hypothesis, and it is consistent. The distinguishing feature of the LM test is that it only entails the estimation of the restricted model (under H_0). On the other hand, in order to carry out a LR test, the model must be estimated under the alternative hypothesis, which in some practical applications can greatly increase its computational burden.

To see the relationship between the LR and LM tests , suppose we have a statistical model with just a single unknown parameter θ, and that the null hypothesis specifies that this parameter takes the specific value θ_0. Having collected data, it is found that the maximum likelihood estimate of θ is θ_1. Figure 4.1 shows a graph of the log likelihood function with a simple null hypothesis. This function has a maximum log L_1 at $\theta = \theta_1$; its value at $\theta = \theta_0$ is log L_0. The greater the difference between log L_1 and log L_0, the more suspicious one would be about the null hypothesis - that θ is equal to θ_0 - and it is on this difference that the likelihood ratio is based.

FIGURE 4.1: LOG-LIKELIHOOD FUNCTION WITH A SIMPLE NULL HYPOTHESIS



Source: Newbold and Bos (1985: 22)

At its maximum point, the slope of the log likelihood function is, of course, zero. Moreover, the further the hypothesised value θ_0 is from the maximum likelihood estimate θ_1, the higher in absolute value will be the slope of the likelihood function at this hypothesised value. This suggests that a test of the null hypothesis can be based on the slope of the log likelihood function at θ_0. The Lagrange multiplier test is based on the derivative of the log likelihood function evaluated at the hypothesised value θ_0. It can be shown that, for large sample sizes, this derivative has, under the null hypothesis, a normal distribution with mean zero.

Details of the LM test, and of its application to some econometric problems, are given by Breusch and Pagan (1980).

The form of the LR implies, as noted earlier, that the model must, in general, be estimated both under H_0 and H_1. Wald (W) tests on the other hand, may be carried out on the basis of the unrestricted model only, and so they are particularly appealing when the restricted model is difficult to estimate. The Wald test also shares optimality properties with both the LR and ML tests. The Wald principle also provides a convenient approach to the testing of restrictions in both linear and nonlinear regression models. As with the LM test, the Wald test is also applicable to testing nested hypotheses.

In comparing these three classical test procedures, there are essentially two issues at stake. The first is computational convenience, and the second is power. Harvey (1981: 174) noted that this test issue cannot be resolved unequivocally since in general all three tests are asymptotic. This must be borne in mind when assessing the results of Monte Carlo experiments carried out for special cases. Although no general results exist regarding the properties of the three tests, it can be shown that under certain circumstances the test statistics obey the inequality $W \geq LR \geq LM$ (see Berndt and Savin, 1977 and Breusch, 1979).

Thus, for example, the Wald test will tend to have a higher power than the LM test, but at the expense of a greater probability of making a Type I error (α).

When the sample size is large, the ambiguity over α is resolved since the test statistics all have the same distribution under the null hypothesis. Furthermore, if the parameters are 'near' to the values taken under H_0, it can still be shown that the test statistics have the same asymptotic distribution (see Harvey, 1981). However, this result by no means support the notion that the three tests will have similar power in small samples. As regards computational convenience, the LR statistic is very simple to construct but suffers from the disadvantage that it requires maximum likelihood estimates for both restricted and unrestricted models. This suggests that as far as computer time is concerned, the LM and W tests are both preferable, the choice being dependent on whether the restricted or unrestricted model is easier to estimate.

With the linear regression model, it is common in econometrics to test for a one-time structural change occurring at a given time point using an F-test (often referred to as a Chow test). This test, for example, can be extended to W, LM-like and LR-like tests in general parametric models, whether estimation is by maximum likelihood or not (see Andrews and Fair, 1988). Another example is the Quandt likelihood-ratio test for structural change which uses a standard likelihood ratio approach for deciding between two hypotheses (H_0 versus H_1). In the next few sections some of these tests for structural stability which were designed around the above principles will be examined.

4.3 THE QUANDT-RATIO TEST

The problem of establishing the timing of a change in regression parameters was first considered by Quandt (1958, 1960) who proposed a test for no change versus one change based upon a likelihood ratio. Quandt (1958) has suggested that the position in time of a switch from one regime to another can be determined by a direct examination of the likelihood function.

In attempting to estimate the parameters of a linear regression system obeying two separate regimes, it is necessary first to estimate the position of the point in time at which the switch from one regime to the other occur. The Quandt-ratio is a maximum likelihood procedure which estimates the most likely point at which a switch from one relationship to another has occurred.[7] It is assumed that the error terms are independent of each other and independent of the explanatory variable. The implication is that autocorrelation has to be dealt with before applying this test.

The null hypothesis is that the assumptions of the standard linear regression model (see Equation 1.1) hold. Under the alternative, Model (4.1) is split into

$$y_t = \begin{cases} \mathbf{x}_t' \boldsymbol{\beta}_1 + e_{1t} & \text{if } t \in \{T_1\} \\ \mathbf{x}_t' \boldsymbol{\beta}_2 + e_{2t} & \text{if } t \in \{T_2\} \end{cases} \qquad (4.3)$$

[7]It is important to note that the maximum likelihood procedure may have the disadvantage that maxima are flat, which means that much of the same maximum value could be reached over a fairly wide range of time points. In certain cases it is important to narrow down the point of change with great accuracy and, therefore, some further research on this subject is desirable.

or in convenient matrix notation

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \end{bmatrix} \qquad (4.4)$$

where contrary to the Chow test, the disturbance variance σ^2 is allowed to vary across the regimes under the alternative. T is the length of the estimation interval, T_1 is the first set of observations and $T_2 = T - T_1$ the remaining set of observations corresponding with $\mathbf{y}_1$ and $\mathbf{y}_2$ respectively.

The logarithm of the likelihood function for a given value of $T = T_1 + T_2$ is given as:

$$L(t) = - T\log(2\pi)^{\frac{1}{2}} - T_1\log\hat{\sigma}_1 - T_2\log\hat{\sigma}_2 - T/2 \qquad (4.5)$$

which is a function of t alone; $\hat{\sigma}_1$ and $\hat{\sigma}_2$ are the standard errors of estimate for a regression taking T_1 and T_2 observations, respectively. The value of the likelihood function (4.5) is calculated for all possible values of t = K+1,...,T-(K+1), after which the t value that corresponds to the maximum maximorum (maximum of the maxima) is selected to be the maximum likelihood estimate. The likelihood ratio test is used in testing the hypothesis (H_0) that no switch in the regression regime occurred against the simple alternative (H_a) that one switch took place at time t.

The Quandt likelihood ratio θ(t) is defined as:

$$\theta(t) = \ln \left[\frac{\text{max. likelihood given } H_o}{\text{max. likelihood given } H_a} \right] \qquad (4.6)$$

$$= \tfrac{1}{2}T_1 \ln \hat{\sigma}_1^2 + \tfrac{1}{2}T_2 \ln \hat{\sigma}_2^2 - \tfrac{1}{2}T \ln \hat{\sigma}^2 \qquad (4.7)$$

This is a standard likelihood ratio statistic for deciding between the two hypotheses H_o and H_a. In Equation (4.7) $\hat{\sigma}_1^2$, $\hat{\sigma}_2^2$ and $\hat{\sigma}^2$ are the residual sums of squares when the regression is fitted to the first T_1 observations, the remaining T_2 observations, and the whole set of T observations respectively. The data point r with the lowest Quandt-ratio is used to split the sample, except where this point occurs very close to the beginning or end of the sample period. The null hypothesis of parameter constancy is therefore rejected whenever the θ(t) statistic gets too small. Apart from the point with the lowest Quandt-ratio, all local Quandt minima can be identified for later testing by means of the Chow (1960) test. The appropriate rejection region follows the well known fact that -2 θ(t) has, under H_o, an asymptotic chi-squared distribution with (K+2) (see Quandt, 1958: 878). The Quandt test is therefore only an asymptotic test. The behaviour of θ(t) against t has also been found to shed some light on the stability of the regression, and is particularly useful as an indicator of whether changes have occurred as the result of an abrupt or gradual transition.

The power of the test depends on the magnitude of the switch (Quandt, 1958: 880), which means that the closer the switch is to either endpoint, the lesser the power of the test. The implications of deviations from the standard normal assumptions on the test are not clear yet.

4.4 THE CHOW TEST AND RELATED METHODS

4.4.1 THE CHOW TEST FOR PARAMETER STABILITY

A classical problem in econometrics is testing whether the coefficients of a regression model are the same in two or more separate subsamples. To be useful as a device either for forecasting or for the evaluation of alternative policy measures, a model should be built from structural relationships that prove statistically 'robust' over various periods of both the past and the future. Consequently, econometricians have spent considerable effort developing techniques to test the sturdiness of regression coefficients across different time periods. In the case of time-series data, where the subsamples generally correspond to different economic environments, such as different exchange-rate or policy regimes, such tests are generally referred to as tests for structural change. They are equally applicable to cross-sectional data, where the subsamples might correspond to different groups of observations such as large firms and small firms, rich countries and poor countries or men and women. Evidently there could well be more than two such groups of observations.

The classical F-test for the equality of two sets of coefficients in linear regression models is commonly referred to by economists as the Chow test, after the early and influential paper by Chow (1960). The Chow test is perhaps the most familiar and certainly the most frequently used. The classic approach is to partition the data into two parts, possibly after re-ordering. Another exposition of this procedure is given by Fisher (1970).

The problem of testing for structural shifts is closely related to what in econometrics is known as "switching regression" (see e.g. Goldfeld and Quandt, 1973; Poirier, 1973). The discussion on the Chow test is concerned with the case where the switch from one regime to another occurs on the basis of the time index. Once the switching point has been located by the Quandt test (discussed in the previous section), it is possible to apply the Chow test. The data point with the lowest Quandt-ratio is used to split the sample, which of course inflates the size of the test well above the nominal α-level of the Chow test.

The Chow test rejects the null hypothesis (H_o: $\beta_1 = \beta_2 = \beta$) of parameter stability whenever the fit of the equation can be significantly increased by splitting the regression into two parts as defined in Equation (4.3).

The sum of squares of the residuals under H_o equals the sum of squares of residuals under the alternative hypothesis (H_a: $\beta_1 \neq \beta_2$) plus the sum of squares of the deviation between the two sets of estimates of y under these two hypotheses. The ratio between the latter two sums, adjusted for their degree of freedom, follows an F distribution if the null hypothesis is true. If both parts in Equation (4.3) are fitted separately H_o is rejected whenever

$$F^* = \frac{[\Sigma e_p^2 - (\Sigma e_1^2 + \Sigma e_2^2)]/K}{(\Sigma e_1^2 + \Sigma e_2^2)/(T - 2K)} \tag{4.8}$$

is too large, where Σe_p^2, Σe_1^2 and Σe_2^2 are the residual sums of squares from the regression of the entire T observations, the first T_1 observations and the remaining T_2 observations respectively. F^* has an F - distribution with (K, T-2K) degrees of freedom under the null hypothesis of no structural change. This test applies when both X_1 and X_2 have full column rank (which implies that $T_1 \geq K$ and $T_2 \geq K$ in Equation (4.3)) and is based on fitting both regressions separately.

The test proceeds on the usual assumptions that the residuals associated with the specification are normally and independently distributed with zero mean and constant variance. In much time series work, however, disturbances are either heteroscedastic or serially dependent (or both). In such cases it is interesting to ask whether inferences drawn from the Chow test are seriously affected. Under the same model Nyblom and Mäkeläinen (1983) derived a family of locally most powerful tests. In a simple case they also made detailed comparisons between one of their tests and the Lamotte - McWhorter (1980) tests which are based on the Cooley-Prescott random walk model of parameter variation (see discussion in Section 4.9).

Another drawback of the statistic (4.8) is the requirement that $T_1 \geq K$ and $T_2 \geq K$. The formula (4.8) can therefore not be applied when a possible shift occurs very early or very late in the sample. For such cases, Chow (1960) has suggested the following alternative test statistic

$$C^* = \frac{[\Sigma\, e_p^2 - \Sigma\, e_1^2] \,/\, T_2}{(\Sigma\, e_1^2) \,/\, (T_1 - K)} \qquad (4.9)$$

which has under H_0 an $F(T-T_1, T_1-K)$ distribution. The statistic (4.9) can of course also be computed when (4.8) applies, but will then lead to a less powerful test. When $T_1 = T - K$, we have $\Sigma e_2 = 0$ and $C^* = F^*$.

4.4.2 ALTERNATIVES TO THE CHOW TEST

The multi-step Chow test procedure can be substantially abridged by the use of dummy variables. If structural changes occur at known points in time, then the changes in the coefficients of the relevant variables can be represented by dummy variables. Gujarati (1970a and 1970b) and Dufour (1980) have developed dummy variable interpretations of the predictive Chow test.

Although the overall conclusion derived from the Chow and dummy variable tests in any given application is the same, there are many advantages to the dummy variable method. Rewriting Equation (4.1) and pooling the T_1 and T_2 observations, the regression function

$$y_t = \mathbf{x}_t' \; [(1-D_t)\,\boldsymbol{\beta}_1 + D_t\,\boldsymbol{\beta}_2] + (1-D_t)\,e_{1t} + D_t\,e_{2t} \qquad (4.10)$$

with $t = 1,\ldots,T,$

which is equivalent to estimating Equation (4.3). $D_t = 0$ for observations in the first period (T_1) and 1 for observations in the second period (T_2). The introduction of the dummy variable *D* in the multiplicative form enables us to differentiate between slope coefficients of the two periods, just as the introduction of the dummy variable in the additive form enables us to distinguish between the intercepts of the two periods.

The advantages of the dummy variable technique over the Chow test can be readily seen. Firstly, regression (4.10) provides a computationally very convenient method for obtaining direct evidence on one of the main consequences of structural stability (large prediction errors) jointly with a whole array of predictive tests. Secondly, the Chow test does not explicitly tell us which coefficient, intercept or slope, is different or whether both are different in the two periods - one can get a significant Chow test because only the slopes are different or only the intercepts are different or both are different. In this respect, the dummy variable approach has a distinct advantage, for it not only tells if two regressions are different, but also pinpoints the source(s) of the difference - whether it is due to the intercept or the slope, or both. Thirdly, without the dummy variable method, one would need to perform two extra regressions or to compute the t statistics explicitly, which may be quite burdensome. Furthermore, since pooling increases the degrees of freedom, it may improve the relative precision of the estimated parameters.

Ashley (1984) introduced the stabilogram (STAB) test, which is also based on dummy variables, to detect parameter variation over several subsets. This test is equivalent to the Chow test whenever there are just two subperiods. The STAB test is fairly simple to use (a straightforward application of covariance analysis) and assumes little about the form of the instability, thus eliminating the need for separate tests against outliers, discrete parameter jumps, deterministic parameter drift, etc. This test yields a plot of the estimated parameter variation over time; the author calls this plot a "stabilogram". The stabilogram plays a role analogous to that of the correlogram in Box-Jenkins modelling - it can be used to formally test the need for further specification modifications and it can also be used informally to suggest the form such modifications should take place. In its simplest form the test is applied to one coefficient at a time. In cases where the stability of all coefficients is tested simultaneously, the STAB statistic is equivalent to the "homogeneity" statistic described in Brown, Durbin and Evans (1975: 156) and implemented in their TIMVAR program.

4.4.3 EXTENSIONS OF AND MODIFICATIONS TO THE CHOW TEST

The original Chow test can also easily be extended to more than two groups. The same steps are followed than before except that there will be as many individual regressions as the number of periods or groups and σ^2 and the degrees of freedom will have to be estimated appropriately (see Gujarati, 1988: 445).

Lee (1991) showed how the classical Chow test can be modified with a Kalman (1960) filter algorithm and then used sequentially for the detection of the timing of possibly more than a single structural change in regression parameters. His test, called the "Chow-Fisher test", involves the sequential construction of conditional F-test statistics using the recursive residuals generated by a Kalman filter. At each recursion of a Kalman filter a Chow-Fisher F-test statistic is obtained conditioned on test results acquired on earlier observations. The sequential Chow-Fisher F test was found to be reasonably robust when the assumption of iid errors was violated. It is particularly encouraging that the test retains almost all of the power it has for nonstochastic regressor models when applied to dynamic regression models with iid errors

Interesting papers by Gordon and Smith (1988) and Hamilton (1989) have also focused on modelling multiple structural changes in dynamic settings. Their frameworks allow inferences to be drawn about the timing of changes in regime. Gordon and Smith's (1988) multistate Bayesian method treated structural change as the realisation of a stochastic forcing variable. Their method can accommodate change in intercept and slope parameters and model outlying observations. The method requires a prior knowledge of the variance of the stochastic forcing variable and utilises transition probabilities at each recursion that depend upon the prior probabilities of each state. In Hamilton's (1989) model the parameters of an autoregression were viewed as the outcome of a discrete-state Markov process where probabilistic inferences are drawn about the timing of changes in regime. Hamilton's model pertains to the statistical properties of a univariate time series and is not a regression model.

For the classical linear regression model the Chow (1960) test commonly is used and for the linear simultaneous equation model the Lo and Newey (1985) or Hodoshima (1986) extension of the Chow test can be used. Somewhat surprisingly, however, more general cases have not been considered extensively in the literature. An exception is the work of Anderson and Mizon (1983) on the nonlinear simultaneous equations model. Andrews et al. (1985) proposed a simple large sample Chow test for stability of coefficients in a system of linear simultaneous equations. Harvey and Phillips (1989) further demonstrated that analogous tests can also be constructed in static simultaneous equation models when equations are estimated by common K-class estimators, e.g., OLS, 2SLS and LIML. The tests are based on the residuals obtained when the estimated endogenous part of a simultaneous equation is regressed on all the exogenous variables in the system. Salkever (1976) and Dufour (1982a) gave a more detailed account of the Chow test's role as a test for structural change. Dufour (1982a), in particular, provided explicit and easily applicable solutions to problems of undersized samples in subperiods, thereby generalising the predictive Chow test. Their results are further generalised to dynamic and simultaneous models by Pagan and Nicholls (1984).

Andrews and Fair (1988) extended the classical Chow test for structural change in linear regression models to a wide variety of dynamic, simultaneous and nonlinear models, estimated by a variety of different procedures. Wald, Lagrange multiplier-like and likelihood ratio-like tests statistics were introduced by the authors. The results allowed also for heterogeneity and temporal dependence of the observations.

4.4.4 EVALUATION OF THE CHOW TEST

4.4.4.1 THE USE OF THE CHOW TEST UNDER HETEROSCEDASTIC CONDITIONS.

Part of the maintained hypothesis of the Chow test is that the error variances are the same of the two regressions (i.e., $\sigma_1^2 = \sigma_2^2 = \sigma^2$). If this is not the case, the Chow test may be inaccurate, in the sense that the true size of the test (under the null hypothesis) may not equal the prescribed α-level. A partial analytical solution to the case of heteroscedastic disturbances has evolved in an exchange between Toyoda (1974), Jayatissa (1977) and Schmidt and Sickles (1977).

Toyoda (1974: 601-608) has investigated the accuracy of the Chow test under conditions of heteroscedasticity, using an approximation to the distribution of the test statistic (F^*) for the Chow test. The approximation is made by taking any well-chosen weighted average of σ_1^2 and σ_2^2 and is approximately distributed as F(K, f) where

$$f = \frac{[(T_1 - K)\sigma_1^2 + (T_2 - K)\sigma_2^2]^2}{(T_1 - K)\sigma_1^4 + (T_2 - K)\sigma_2^4} \qquad (4.11)$$

and K the number of regressors in Model (4.1). This test proved to be well behaved for variations of variances if at least one of the two sample sizes is very large. Jayatissa (1977) suggested similar amendments to the Chow test in the case of heteroscedasticity. Schmidt and Sickles (1977: 1293-1298) examined Toyoda's approximation and concluded that the approximation is of reasonable accuracy only when the two variances and sample sizes are of the same order of magnitude.

For comparing several regressions under heteroscedasticity, Dufour (1991) introduced simple exact bounds for the null distribution of a general Wald-type statistic for testing any set of linear restrictions linking the coefficients of regression, using an extension of Kimball's inequality. The bounds proposed are based on central Fisher distributions and are much easier to compute than the earlier bounds proposed by Ohtani and Kobayashi (1986) and Farebrother (1989), especially when more than two regressions are considered.

MacKinnon (1989) has shown that it is remarkably easy to test for structural change, of the type that the classic F or Chow test is designed to detect, in a manner that is robust to heteroscedasticity of possibly unknown form. He designed a modified version of the Chow test for both linear and nonlinear regression models by using a variant of the Gauss-Newton regression. The results of a number of Monte Carlo experiments, designed to see how well these tests perform in finite samples, are presented. The Monte Carlo evidence suggests that, for the linear case, the ordinary F-test can be so misleading that it is clearly dangerous to ignore the possibility of heteroscedasticity when testing for structural change.

4.4.4.2 THE ROBUSTNESS OF THE CHOW TEST TO AUTOCORRELATED DISTURBANCES

In a context of time-varying parameter models one also needs to take care of the influence of model misspecification on the hypothesis of no structural change. In such a situation, in fact, it is not always easy to distinguish between departure from the null hypothesis concerning structural parameters and the changes in residual variances.

This raises the problem of what to do and how to proceed once one has found structural variation in model parameters. Up to now the natural answer to this question has been that of splitting the sample into subsamples for which the stability assumption regarding structural parameters continues to hold. In many cases this is an artificial way of facing the problem, and in any case one has to assure that the observed parameter variation is not a consequence of the model misspecification.

If the null distribution was robust to autocorrelation, there would be no need to circumvent this problem, either by eliminating the disturbance autocorrelation via some Cochrane-Orcutt-type transformation or by searching for more robust alternatives, as White (1980) has done successfully for heteroscedasticity.

However, care must be exercised in considering the meaning of structural parameter changes in the presence of autocorrelated residuals. The basic theoretical question arises in two different ways: in the first the structural change is simply a result, that is an artificial consequence, of the serial correlation that affects the model when it is specified over the whole period. The second way takes as fundamental the structural change itself which, if not taken into account in a model , possibly causes serially correlated errors. It seems quite obvious that a correct procedure for facing the problem of testing structural parameter variation in the presence of serial correlation cannot leave these considerations apart.

Corsi, Pollack and Prakken (1982), however, considered the mean of the test statistic (4.7) and found that it was affected by the first order autocorrelation coefficient rho (ρ). They derived an expression for the asymptotic displacement of the expectation of the test statistic when the Chow test is applied to models characterised by first-order serially dependent disturbances. In assuming a stationary AR(1) disturbance process

$$e_t = \rho e_{t-1} + \epsilon_t \qquad (4.12)$$

the null hypothesis is enlarged to allow for a non-zero (ρ) in the range (-1, 1). The problem can then be rephrased as to whether or not the size of the test remains intact when the nuisance parameter (ρ) takes values different from zero. The authors showed via Monte Carlo studies that the true size of the test exceeds the nominal one by considerable margins.

Corsi et al. (1982) further stated that the error autocorrelation causes an artificial departure from the hypothesis of structural stability and suggested that, if the errors are serially correlated in the entire model, the transformation of the error structure term by the same autoregressive filter before applying the Chow test to the whole period. Unfortunately, they assumed the same autoregressive residual coefficient for the two subperiods, which, in turn, is identical for the whole period in the entire model. This way of filtering data cuts off all possibilities of having a different coefficient for the two subperiods. In general this does not seem to be a justifiable assumption because when a structural parameter change happens, it is not expected to leave the autocorrelation coefficient unchanged.

Krämer (1989) has also shown that the Chow test is extremely nonrobust to autocorrelation and that the true size can even be as large as one for the special case of an AR(1) disturbance process. This serves as a warning that the Chow test might wrongly indicate a structural change when the real culprit is "only" autocorrelation. Some empirical evidence is available in Corsi et al. (1978, 1982) and Krämer and Sonnberger (1986, Chapter 6), where the significance of the Chow test declined drastically after applying a Cochrane-Orcutt transformation to the data. Krämer (1989) concluded that the Chow test can be trusted only when there is reason to assume that the disturbances are indeed independent. The Chow test, therefore, calls for amendments similar to those suggested by Jayatissa (1977) in the case of heteroscedasticity. The recent results by Newey and West (1987) may also be used to construct autocorrelation - consistent versions of any F-test, and thus for the Chow test in particular. Such issues are, however, beyond the scope of this study.

Generally speaking, the problem of testing the structural change of parameters, when there is no certainty that the usual independence assumption on the residuals holds, has to be thought of as a simultaneous test on parameter shifts and autocorrelation coefficients, permitting the rolling together of both types of hypotheses. Indeed, the error terms in structural equation models represent a kind of poorly understood effect, due to various sorts of misspecification. In such a situation a simultaneous treatment of parameters and errors seems quite natural, as has been suggested by Quandt (1975) and Smith (1975).

4.4.4.3 INDETERMINACY OF THE CHOW TEST WHEN THE OBSERVATIONS ARE INSUFFICIENT

Rea (1978: 229) argued that when the number of observations in one of the models is less than the number of regression coefficients, the Chow test is incapable of testing the hypothesis of equality against that of inequality. It can never be concluded from the Chow test itself that the two sets are equal, although at times it may be possible to conclude that they are unequal.

If the T_2 observations of Model (4.6) are less than the K regression coefficients, the null hypothesis of the Chow test is $X_2(\boldsymbol{\beta}_2 - \boldsymbol{\beta}_1) = \mathbf{0}$ (Chow, 1960: 293). This hypothesis is not, however, equivalent to the hypothesis $\boldsymbol{\beta}_1 = \boldsymbol{\beta}_2$. In particular, since the rank of the (T_2 x K) matrix X_2 is less than K, there must exist at least one $\beta^*_1 \neq \beta^*_2$ such that $X_2(\beta^*_2 - \beta^*_1) = 0$. Hence, acceptance of the null hypothesis of the Chow test cannot necessary be interpreted as acceptance of the hypothesis $\beta_1 = \beta_2$. Nevertheless, rejection of the null hypothesis also implies the rejection of the hypothesis $\beta_1 = \beta_2$, thereby indicating that the only determinate result from the Chow test is the inequality of β_1 and β_2.

Rea suggested that it may be possible to resolve the indeterminacy in these cases where either prior information suggests a certain relationship between β_1 and β_2 or by increasing the number of observations in the undersized sample or to restrict the test of equality to subsets containing no more than T_2 of the regression coefficients. Dufour (1982) provided further explicit and easily applicable solutions to such problems of undersized samples, thereby generalising the predictive Chow test.

4.4.5 CONCLUSION

The likelihood ratio tests of Chow (1960), Quandt (1960) Fisher (1970), and others check for a parameter break at a single point in time.

The major problem with all these approaches is the difficulty in defining the segments since the timing of the structural change is rarely known. Coefficients in a response function might also vary between each time period rather than between a number of time periods. Parameter variation may be stochastic but, for instance, required to follow an autoregressive process of low order such as a first-order Markov process. Alternatively, parameter variation may be systematic with the parameters themselves being functions of observable variables. In such cases it is necessary to apply tests which will uncover gradual or continuous changes rather than a sudden change in the model parameters. These are discussed in the next section.

4.5 THE BDE CUSUM AND CUSUM OF SQUARES (CUSUM-SQ) TESTS

4.5.1 INTRODUCTION

In this section situations are considered where neither the number nor the timing of possible shifts in the regression parameters are known. A well known procedure to detect such types of deviations from the standard linear regression model, which has since gained wide acceptance in empirical econometrics and elsewhere, is the CUSUM tests by Brown, Durbin and Evans (henceforth BDE) (1975: 149-155).

The BDE tests are based on recursive residuals which are obtained by sequentially estimating OLS regression parameters as the time index is advanced. More formally, assume a model of the form

$$y_t = \mathbf{x}_t' \boldsymbol{\beta}_t + e_t \tag{4.13}$$

where y_t = the dependent variable;

$\mathbf{x}_t'$ = a column vector of K independent variables;

$\boldsymbol{\beta}_t$ = a column vector of regression parameters;

and e_t = error term.

The recursive residuals w_r are obtained by computing for each period over the interval T the standardised prediction error of R_{it}, when R_{it} is predicted from the preceding t-K observations. The recursive residuals are, therefore, defined as

$$w_r = \frac{\mathbf{y}_r - \mathbf{x}_r' \hat{\boldsymbol{\beta}}_{r-1}}{[1 + \mathbf{x}_r' (X_{r-1}' X_{r-1})\mathbf{x}_r]^{\frac{1}{2}}} \tag{4.14}$$

with $r = K+1,\ldots,T$

where $R_{it} = \mathbf{y}_r - \mathbf{x}_r' \hat{\boldsymbol{\beta}}_{r-1}$;

$\hat{\boldsymbol{\beta}}_{r-1} = \hat{\boldsymbol{\beta}}$ as estimated from periods K to r-1;

X_{r-1}' = a matrix composed of the column vectors $\mathbf{x}_i$, i.e., $X_{r-1}' = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{r-1})$;

and K = number of independent variables.

Under H_0, $\hat{\boldsymbol{\beta}}_r = (X_r' X_r)^{-1} X_r' \mathbf{y}_r$ with matrix $(X_r' X_r)$ assumed to be non-singular and with covariance matrix equal to $\sigma^2(X_r' X_r)^{-1}$; $\boldsymbol{\beta}$ is based on the first r (r > K) observations. Thus, each recursive residual w_r represents the (standardised) discrepancy between the actual value of the dependent variable at time t and the optimal forecast using only the information contained in the previous t-1 observations.

The BDE method of using recursive residuals in detecting structural weaknesses in the estimated regression parameters can be outlined as follows:

For a data set of t points, discard the last data point and fit the model to t-1 points, thus obtaining $\hat{\boldsymbol{\beta}}_{t-1}$. The recursive residual is then defined as the standardised residual of the last observation from the new line, the standardisation making the variance σ^2. Consequently, discard the second to last point as well, and fit the regression model to the first t-2 points. The standardised residual of this second to last point from the new line of data is then w_{t-1}. Continue omitting the points in this way, obtaining $w_t, w_{t-1}, \ldots, w_{t-K}$. It is obvious that only t-K recursive residuals can be calculated, as at least K points are needed for the fitting of a K-parameter regression. The same operation can be executed, starting with K+1 data points and adding one data point at a time. The recursive residuals may therefore, be interpreted as showing the effect on the model of successively deleting (or adding) observations from (or to) the data set.

If β_t is constant up to time $t = t_0$ and differs from this constant value from then on, then

$$E(w_r) = 0 \text{ for } r \leq t_0 \quad \text{and}$$
$$\neq 0 \text{ for } r > t_0 \qquad (4.15)$$

This suggests examination of plots intended to reveal departures of the mean of the w_r's from zero as one travels along the series through time.

The calculation of the recursive residuals may appear to be a time-consuming operation, involving the fitting of r-K regressions, but the use of the updating formulas derived by Plackett (1950: 149-157) and Bartlett (1951: 107-111) allows this to be done in a very economical manner. These updating formulas are:

$$(X_r' X_r)^{-1} = (X_{r-1}' X_{r-1})^{-1} - \mathbf{dd}'/(1 + \mathbf{x}_r'\mathbf{d}) \qquad (4.16)$$

where

$$\mathbf{d} = (X'_{r-1} X_{r-1})^{-1} \mathbf{x}_r$$

$$\hat{\boldsymbol{\beta}}_r = \hat{\boldsymbol{\beta}}_{r-1} + (X'_r X_r)^{-1} \mathbf{x}_r(\mathbf{y}_r - \mathbf{x}'_r \hat{\boldsymbol{\beta}}_{r-1})$$

and $$S_r = S_{r-1} + w_r^2, \qquad \text{with } r = k+1,...,T.$$

where S_r is the residual sum of squares based on r observations.

Although McCabe and Harrison (1980) and Ploberger and Krämer (1992) have generalised these tests for structural stability by using ordinary OLS residuals, the objections against using OLS residuals instead of recursive residuals in applied econometric work are still numerous. BDE showed that if the null hypothesis, H_O: $\beta_r = \beta$, $\sigma_r^2 = \sigma^2$ holds, the w_r are independent and $N(0, \sigma^2)$ distributed. The use of recursive residuals avoids the serial correlation and nonnormality problems associated with ordinary OLS residuals and thus forms a much more powerful test. The recursive residuals also seem preferable for detecting the change in a model over time since until a change takes place, the recursive residuals behave exactly as on the null hypothesis. It also does not have the problem of deficiencies in one part of the data being smeared over all the residuals. Cook and Weisberg (1982) reported that ordinary OLS residuals are highly susceptible to masking and swamping problems with multiple outliers. Although it is possible for outliers occasionally to remain unidentified, their presence can still be detected by the normal probability plot failing to pass through the origin. This is due to another advantage that recursive residuals have over ordinary residuals - they are not constrained to sum to zero. Recursive residuals also allow for testing for a change of regime, something for which ordinary residuals are not well suited. BDE further argued that the plot of the OLS residuals, or their squares against time is also not a very sensitive indicator of small or gradual changes in the regression parameters.

It was suggested by Page (1954: 100-114) that the cumulative sum (or CUSUM) should be used for detecting small changes in the parameter coefficients. This suggests that instead of plotting the individual least squares residuals, z_t, the CUSUMS $Z_r = \hat{\sigma}^{-1} \Sigma z_t$, $r = 1,...,T$, should be plotted, in which the division of the estimated standard deviation is used to eliminate the irrelevant scale factor. Brown et al. (1975: 151) pointed out that the difficulty with this suggestion is that there seems to be no way of assessing the significance of the departure of the observed graph of Z_r against r from the mean-value line $E(Z_r) = 0$. The intractability of the problem arises from the fact that in general the covariance function $E(Z_r Z_s)$ does not reduce to a form that is manageable by standard Gaussian process techniques (Mehr and McFadden, 1965: 505-522). BDE showed that for the simple case of regression on a linear time trend with zero intercept, the covariance function is in an unmanageable form. An alternative is to consider the standardised CUSUM of squares residuals $\hat{\sigma}^{-2} \sum_{t=1}^{r} z_t^2$. Although more tractable, BDE pointed out that this is still difficult to deal with and preferred to make the transformation to recursive residuals given in the next section.

Thus, evidence points towards the transformation of ordinary residuals as defined in (4.15) to CUSUMS of residuals. Some of the problems with OLS residuals, however, were solved by Ploberger et al. (1992). A discussion follows in Section 4.5.4.

4.5.2 THE BDE CUSUM TEST

The CUSUM test for structural change, in the form proposed by Brown, Durbin and Evans (1975), has become a standard diagnostic in linear regression models.

The CUSUM quantity is defined as

$$W_r = \frac{1}{\hat{\sigma}} \sum_{j=K+1}^{r} w_j \qquad (4.17)$$

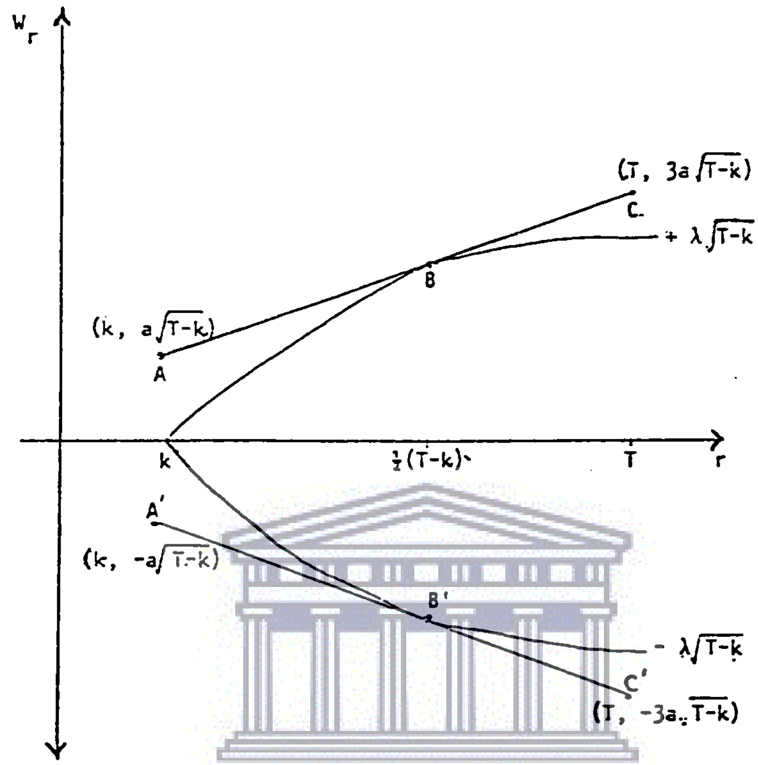
and can be plotted against r for r = K+1,...,T; $\hat{\sigma}$ denotes the estimated standard deviation defined by $\{S_T/(T-k)\}^{\frac{1}{2}}$ with S_T being the residual sum of squares of the full model assuming H_O to be true. In testing the significance of the departure of the sample path of W_r from its mean value line ($E(W_r) = 0$), a pair of lines, lying symmetrically above and below the line $W_r = 0$, is drawn (see Figure 4.2 below) such that the probability of crossing one or both is α, the required significance level.

From the properties of the w_r's under H_O the sequence $W_{k+1},\ldots,W_r$ is a sequence of approximately normal variables such that:

$$\begin{aligned} E(W_r) &= 0, \\ var(W_r) &= r - K \quad \text{and} \\ cov(W_r, W_s) &= \min(r, s) - K \end{aligned} \tag{4.18}$$

This proves to be a good approximation of W_r if σ is assumed to be known. In order to derive the test, W_r is approximated by the continuous Gaussian process (Z_t, $K \le t \le T$) with the above mean and covariance functions and with $var(Z_t) = t - K$. W_r weakly converges to the Brownian motion process (Garbade, 1977: 56), starting from zero at time $t = K$.

FIGURE 4.2: GRAPHICAL PRESENTATION OF THE BDE CUSUM TEST



Source: Wesso (1989: 127)

The crossing probability of W_r under H_0 is not constant for all t, therefore the procedure adopted is to choose the family of pairs of straight lines halfway between t = K and t = T which are tangent to the curves $\pm A(t-K)^{½}$ (see Figure 4.2 above). This leads to the family of pairs of straight lines through the points $A[k; \pm a(T-K)^{½}]$ and $B[T; \pm 3a(T-K)^{½}]$, where *a* is the parameter to be solved, using Brownian motion theory. The probability that the point (r, W_r) lies outside any given line in this family is a maximum for r halfway between r = K and r = T. The probability that a sample path Z_t crosses a member of this family is $\frac{1}{2}\alpha$. It is assumed that the probability of W_r crossing both lines is negligible.[8] The function of these lines is to provide tests as well as yardsticks against which to assess the observed behaviour of the sample path. The lines can be used to provide a formal test of significance by rejecting the null hypothesis if the sample path travels outside the region between the straight lines. Gambetta et al. (1982) suggested that it is logical to integrate the Chow test with the CUSUM plot. The plot can be used for locating the structural break and subsequently the Chow test may be applied to provide a formal significance test for the structural shift.

The null hypothesis is therefore rejected if $|W_r| > [a(t-K)^{\frac{1}{2}} + 2a(r-K)(T-K)^{\frac{1}{2}}]$ for any $r \in (K+1, T)$, where the scalar a is chosen to obtain the desired confidence level. Garbade (1977: 56) has found that these values of a are exact only for a continuous Brownian process with Gaussian noise.

[8]See comments of T.W. Anderson in the discussion of the BDE paper (Brown et al., 1975: 175).

4.5.3 THE BDE CUSUM OF SQUARES (CUSUM-SQ) TEST

The CUSUM of squares test uses the squared recursive residuals w_r^2, and it is based on the plot of the quantities

$$s_r = [\sum_{j=k+1}^{r} w_j^2] / [\sum_{j=k+1}^{T} w_j^2] = S_r/S_t \qquad (4.19)$$

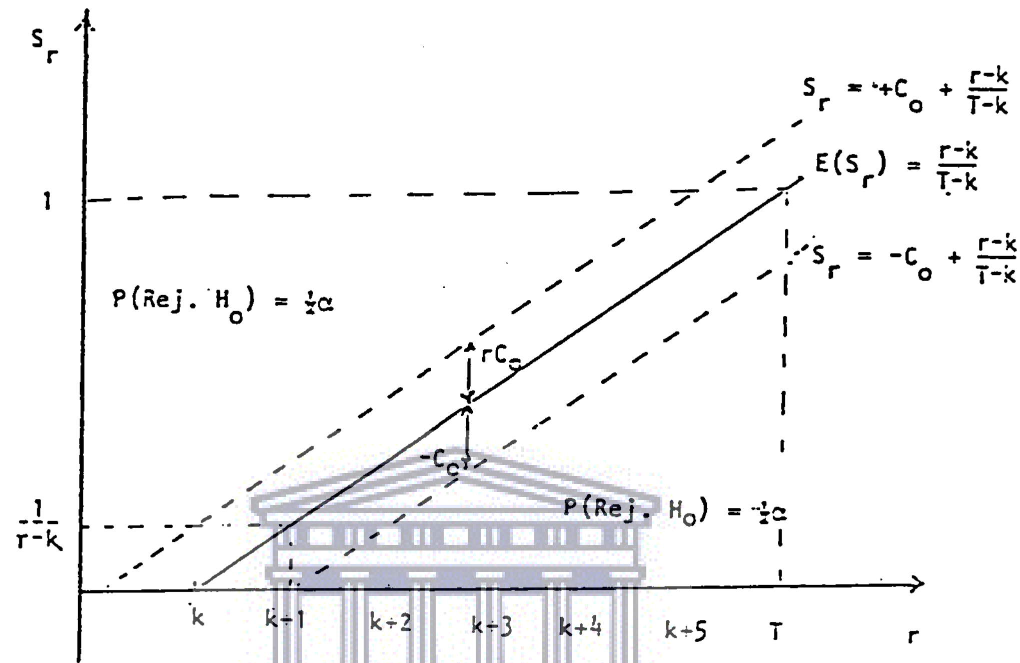
The test provides a useful complement to the CUSUM test, particularly when the departure from constancy of the $\boldsymbol{\beta}_t$'s is haphazard rather than systematic.

Under H_o, s_r has a beta distribution with parameters $\alpha = -1 + (T-K)/2$ and $\beta = -1 + (r-K)/2$ (see Garbade, 1977: 57), and therefore s_r has a mean value of:

$$E(s_r) = (r-K)/(T-K) \qquad (4.20)$$

A pair of lines $s_r = \pm C_o + E(s_r)$ are drawn parallel to $E(s_r)$ (see Figure 4.3), such that the probability that the sample path crossing one or both lines is α, which is the required significant level.

FIGURE 4.3: GRAPHICAL PRESENTATION OF THE BDE CUSUM OF SQUARES TEST



Source: Wesso (1989: 130)

The critical values of C_O are given by

$$^{+}C_O = \max_{i=1,\ldots,T-(K+i)} (s_{K+1} - i/(T-K)) \quad \text{and}$$

$$^{-}C_O = \max_{i=1,\ldots,T-(K+i)} (i/(T-K) - s_{K+1}) \qquad (4.21)$$

which are the significant maximum positive and negative deviations of all the s_r's from $E(s_r)$. The values

$$C_O^+ = \max_{j=1,\ldots,\frac{1}{2}(T-K)-1} (s_{K+2j} - 2j/(T-K)) \quad \text{and}$$

$$C_O^- = \max_{j=1,\ldots,\frac{1}{2}(T-K)-1} (2j/(T-K) - s_{K+2j}) \qquad (4.22)$$

are used as approximations for $+C_O$ and $-C_O$, for T-K even. A table of values is available for choosing C_O, (Durbin, 1969: 1-15). If $|s_r - [(r-k)/(T-k)]| > C_O$ for any $r \in (K+1, T)$ the null hypothesis is rejected. A similar procedure can be adopted when T-K is odd.

It may sometimes be appropriate to consider a one-sided test if it is, for example, assumed that $\beta_t = \beta^*$ for $r \le t_0$ and $\beta_t = \beta^{**} + \beta^*$ for $r > t_0$ while $\sigma_t^2 = \sigma^2$ for all t, which implies that $E(w_r^2) = \sigma^2$ for $r \le t_0$ and $E(w_r^2) > \sigma^2$ for $r > t_0$. One would expect the departure from the null hypothesis to be indicated by a tendency for the sample path s_r to lie below the mean value line, and would therefore use a one-sided test. For this purpose, one would take the significance value of C_O to be α, instead of $\frac{1}{2}\alpha$. However, whether the one - or two-sided situations are envisaged, it is advisable to regard the lines constructed for testing H_O as yardsticks against which to assess the observed sample path, rather than providing formal tests of significance.

4.5.4 ALTERNATIVES TO THE BDE TESTS

The power of both the CUSUM and the CUSUM-SQ tests depend very much on the timing of possible shifts. For example, when the parameters change only late in the sample, neither test has much time to pick this up and the power will be very low. One way to avoid this, already suggested by Brown et al. (1975), is to reverse the order of observations, i.e. to run the recursive estimation process backwards. A related test by Schweder (1976) still uses the recursive residuals as computed in the forward manner and only does the summing up backwards. There is some evidence that this is preferable whenever the regression relationship remains constant for more than K initial periods. If a single shift occurs at T^*, where $T^* >$ K+1, the first $T^* - (K+1)$ recursive residuals contribute only noise to the forward CUSUM (similarly to the forward CUSUM of squares), which does not happen when the summation is done backwards. It is, therefore, essential that forward residuals are used for the backward CUSUMS, since otherwise the initial recursive residuals would again only contribute noise to the test statistic.

A related procedure, due to Bauer and Hackl (1978), Hackl (1980) and Westlund et al. (1989), is based on moving (MOSUM) rather than cumulated sums of recursive residuals. The main reason for using recursive residuals in the MOSUM (and CUSUM) technique is the simplicity of their distributional properties as compared to the conventional OLS residuals that are used in other approaches. The MOSUM test relies on the quantities

$$M_t = \frac{1}{\sigma} \sum_{r=t-G+1}^{t} w_r \tag{4.23}$$

$t = K+G,\ldots,T,$

where G is a fixed number of terms included in each moving sum (MOSUM), and σ is the familiar estimate for the standard deviation σ of the disturbances. Since there is always a fixed number of terms in M_t, the relative importance of recursive residuals that due to a zero mean do not contribute to a significant test statistic is automatically limited. There is however no general rule for choosing G, and the distribution under H_0 of $\max_{K+G\leq t\leq T} M_t$ appears rather unmanageable.

The same holds for the MOSUM of squares tests (MSQ), which is based on the quantities.

$$MSQ_t = \sum_{r=t-G+1}^{t} w_r^2 \Big/ \sum_{r=K+1}^{T} w_r^2 \tag{4.24}$$

where $t = K+G,\ldots,T$.

Under H_o, the MSQ_t's have a beta $B(G/2,(T-(K+G))/2))$ distribution with constant mean value $E(MSQ_t) = G/(T-K)$. Again, however, it is hard to evaluate whether or not the maximum departure of the M_t's from this mean value line is significant. In particular, the arguments that Brown et al. (1975) have used to evaluate the null distribution of the CUSUM-SQ do not apply here.

It is also known that the distribution of the CUSUM-SQ as well as the MOSUM-SQ test statistics are skewed, a fact that further complicates the characterisation process (see Krämer et al., 1986: 57). The degree of skewness subject to the alternative hypothesis H_a is not theoretically known, but can numerically be estimated. Westlund et al. (1989) have shown, by focussing on expectations and variances of the test statistic, that the degree of skewness is considerable, above all for the MOSUM-SQ statistic, which is always positively skew. The CUSUM-SQ test statistic is shown to be positively skewed up to the parameter shift period, after which the skewness will be negative, and the size partly decreases.

Westlund et al. (1989) studied the distributions of the CUSUM-SQ and MOSUM-SQ test statistics by using two linear regression models - the intercept model, M1, and the simple regression model, M2, with an intercept and one regressor. Figures 4.4 and 4.5 illustrate the general picture with respect to observed skewness for a few cases. The regression coefficients in these models either change instantaneously, PM1, or gradually, PM2. In the case of M2, either one or both of the regression coefficients are varying.

FIGURE 4.4: SKEWNESS OF THE MOSUM-SQ IN CASE OF A SUDDEN CHANGE

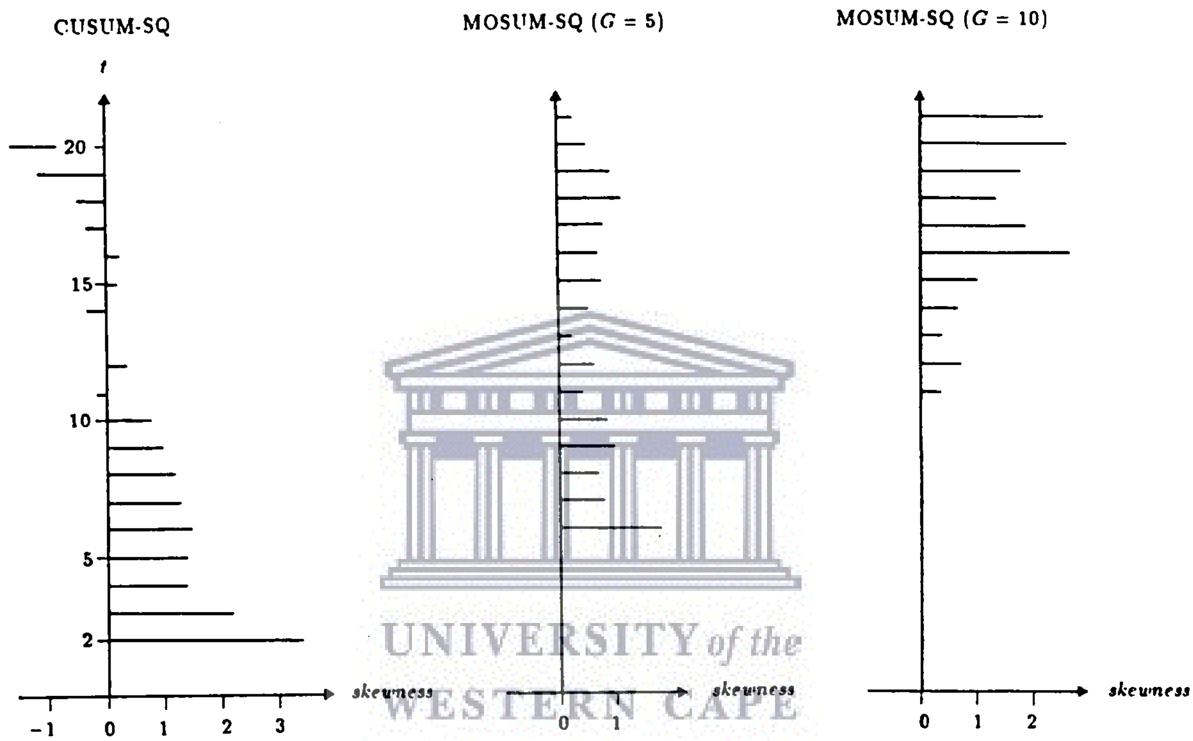
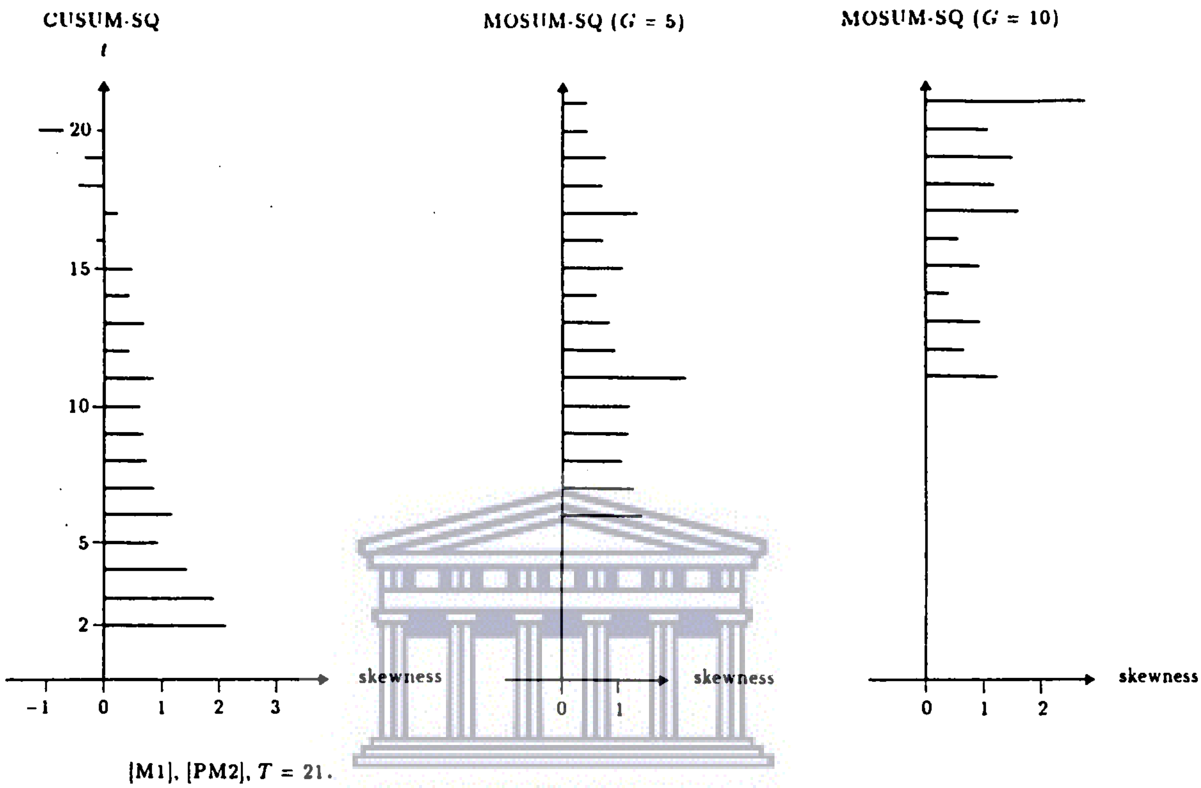


Figure 8.20: [M1], [PM1], $T = 21$.

Source: Westlund and Törnkvist (1989: 123)

FIGURE 4.5: SKEWNESS OF THE MOSUM-SQ IN CASE OF A GRADUAL CHANGE

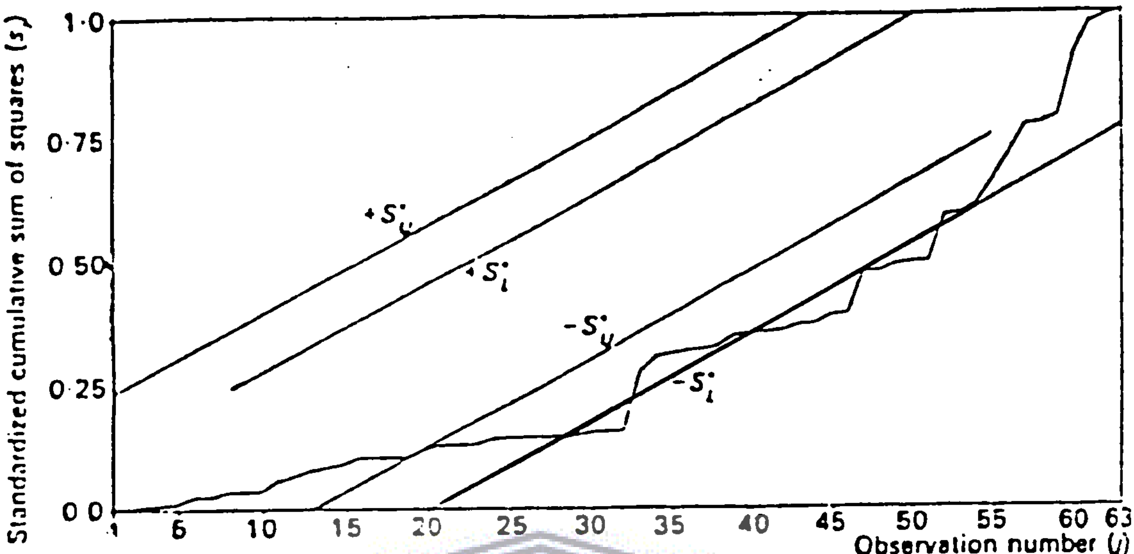


Source: Westlund and Törnkvist (1989: 124)

The basic conclusions to be drawn from this analysis are as follows. When the structural variability is *a priori* known to have caused an instantaneous parameter change, the results given by Westlund et al. (1989) seems to show that the time dating problem is reasonably solvable by CUSUM-SQ as well as MOSUM-SQ procedures. It was found that the time dating ability increases with the length of the time series and also with the size of the parameter change. In the case where it cannot be *a* *priori* decided whether structural stability variability implies instantaneous or gradual parameter changes, it is rather difficult to identify the type of parameter model, particular when applying the CUSUM-SQ test statistic.

On the other hand, McCabe and Harrison (1980) have suggested to stay with the CUSUM-SQ statistic as defined in (4.19), but to use OLS rather than recursive residuals. The main advantage of this procedure is its computational simplicity. However, since the distribution of the OLS residual vector $\hat{u}$, and also the distribution of the OLS-based CUSUM of squares statistics S_r ($r = K+1,...,T$) depend on the regressor matrix X, the null hypothesis of the S_r's is now much harder to evaluate. In Figure 4.6 McCabe and Harrison suggested bounding random variables S_L and S_U of which the distributions do not depend on X, such that $S_L \leq S_r \leq S_U$ for all r ($K+1 \leq r \leq T$).

FIGURE 4.6: CUSUM OF SQUARES OF ORDINARY LEAST SQUARES RESIDUALS.



Source: McCabe and Harrison (1980: 146)

With the McCabe-Harrison method there are four rather than two critical lines of the form suggested by BDE for any significance level α, with rejection resulting from crossing either (or both) of the outer lines. The null hypothesis of parameter constancy is accepted when the sample CUSUM-SQ plot stays within the inner lines, and the test is inconclusive when the plot crosses the inner lines only. This inconvenience, which the procedure shares with the Durbin-Watson test for autocorrelation, has prohibited widespread acceptance in empirical work.

Krämer, Ploberger and Schlüter (1990) also extended the BDE CUSUM test for the constancy of the coefficients of a linear regression model, which is usually based on recursive residuals, to ordinary least squares (OLS) residuals. They have shown how to modify the test statistic, derive its limiting distribution under H_O, and have compared the finite sample power of the two versions of the test via Monte Carlo experiments.

A generalisation of the CUSUM test to OLS residuals, which are dependent and heteroscedastic even under H_O, has been made for the first time (except for a very special case by MacNeill, 1978) by Ploberger and Krämer (1992). In fact, both BDE (1975: 151) and McCabe and Harrison (1980: 142) argued that a least squares variant of the CUSUM test poses "intractable problems", due to an alleged difficulty in assessing the significance of the departure of the cumulated OLS residuals from their mean value zero. Ploberger et al. showed that it is no more difficult to derive the limiting distribution for a CUSUM test based on OLS residuals than for a CUSUM test based on recursive residuals. While the CUSUMS of the recursive residuals, properly standardised, tend in distribution to a standard Wiener process (Sen, 1982); Krämer, Ploberger and Alt (1988) showed that the OLS-based CUSUMS tend in distribution to a Brownian bridge (or "tied-down-Brownian-motion"; see Billingsley, 1968: 64)]. The authors also demonstrated that the CUSUM test based on OLS residuals has higher (local) power for certain types of structural change than the one based on recursive residuals; and it also reacts to structural shifts which occur late in the sample, which are likely to go unnoticed by the standard version of the test - though neither variant is uniformly superior to the other.

Most procedures discussed thus far have been developed without regard to any particular parameterised alternative hypothesis. Once such an alternative is specified, any one of the Wald-, Lagrange multiplier or Likelihood Ratio principles can be employed to derive yet another test. Investigators faced with the prospect of potential regime shifts in regression relationships often adopt random walk time varying parameter (TVP) models. Proceeding along these lines, Garbade (1977) has investigated the alternative that the $\boldsymbol{\beta}_t$'s in (4.1) are stochastic and follow a random walk with zero drift through time:

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \mathbf{p}_t \qquad (4.25)$$

$t = 2,\ldots,T,$

where the $\mathbf{p}_t$'s are NID($\mathbf{0},\sigma^2 P$) and independent of the regression disturbances e_t. Such variable parameter models are discussed at length in the Kalman filter literature (see e.g., Athans, 1974 or Cooley and Prescott, 1976). The null hypothesis of parameter constancy is here obviously equivalent to H_0: $P = 0$.

Garbade suggested a likelihood ratio procedure to test whether or not this is true. Again, the null distribution of the test statistic is hard to evaluate and is in particular not well approximated by a chi-squared distribution in small samples. LeSage (1992) pointed out, however, that the problem with TVP estimation procedures is that they produce an estimate for the variance of the parameters based on the entire data sample that "average" variability over time. This is undesirable in the face of rapid discontinuous shifts in the parameters, or outliers, since averaging is unrepresentative of the true parameter variability during periods of abrupt shift or transient observations. This conclusion is quite general in that econometric researchers often adopt a random walk TVP model to accommodate potential regime shifts in a relationship to be estimated.

There are also other procedures that explicitly rely on parameter estimates as indicators of possible structural shifts. Brown et al. (1975), for example, have proposed to fit a regression to G successive observations ($G \geq K$) and to move this segment along the series, as a supplement to their CUSUM and CUSUM-SQ tests. The graphs of the resulting estimates of the elements of $\boldsymbol{\beta}$ are then supposed to provide further evidence of departure from constancy (although this is not a formal test). The disturbance variance σ^2 may also be estimated separately for each data segment and plotted against time.

Dufour (1982) has suggested a direct parameter based analogue of the CUSUM test. His point of departure is the relationship:

$$\hat{\beta}_t = \hat{\beta}_{t-1} + (X_t' X_t)^{-1} \mathbf{x}_t (y_t - \mathbf{x}_t' \hat{\beta}_{t-1}) \qquad (4.26)$$

which produces a link between recursive parameter estimates and recursive residuals. The test is based on standardised first differences of recursive parameter estimates. Under H_O, the changes in the parameter estimates as one proceeds with the recursive process are independent and normal with mean

$$E(\beta_t - \beta_{t-1}) = 0 \qquad (4.27)$$

and covariance matrices

$$\mathrm{cov}(\beta_t - \beta_{t-1}) = \sigma^2 f_t^2 (X_t' X_t)^{-1} \mathbf{x}_t \mathbf{x}_t' (X_t' X_t)^{-1} \qquad (4.28)$$

$t = K+1, \ldots, T,$

with normalisation factor $f_t^2 = 1 + \mathbf{x}_t' (X_{t-1}' X_{t-1}) \mathbf{x}_t$ which is used to ensure that all the recursive residuals have the same variance σ^2.

Dufour provided some evidence that the first differences in parameter estimates can be much more revealing concerning structural change than the recursive residuals.

4.5.5 EXTENSIONS OF AND MODIFICATIONS TO THE BDE TESTS

A major drawback of the BDE tests, as noted by several discussants of the paper and elsewhere, has been the requirement that all regressors be independent of the disturbances. In particular, this excludes lagged dependent variables on the right-hand side of the equation - a severe restriction, given that tests for structural change are typically designed for time series data. Krämer, Ploberger and Alt (1988: 1355-1369), therefore, investigated the CUSUM test for structural change when lagged dependent variables appears among the regressors in a linear model. They have shown that both the modified test suggested by Dufour (1982), and the straightforward CUSUM test retain their asymptotic significance levels in dynamic models, and found that the power depends crucially on the intensity of the shift. Krämer et al. (1989) concluded that there is not much reason in practice to use the Dufour procedure, and that one may stay with the straightforward CUSUM test in dynamic models.

The CUSUM test also requires that the cumulated sums of recursive residuals must be standardised by some estimate of the disturbance standard deviation σ, where

$$s = [\{1/(T-K)\} \sum_{t=1}^{T} \hat{e}_t]^{\frac{1}{2}} \qquad (4.29)$$

where $\hat{\mathbf{e}} = \mathbf{y} - X\hat{\boldsymbol{\beta}}$ is the vector of OLS regression residuals from the full sample. Harvey (1975) pointed out that, depending on the type of structural change, the OLS residuals will often follow a very erratic pattern which will lead to a gross overestimation of σ^2 when using (4.29). Ceteris paribus, any overestimation of σ^2 will reduce the power of the test, since it is obvious from (4.17) that the cumulated sums of the recursive residuals are then less likely to cross the critical lines. Harvey suggested a modification to the estimate of σ which mitigate this overestimation and thus increase the power of the test.

A crucial aspect in the application of tests and the interpretation of the test outcomes is, of course, the robustness of the test statistic with respect to the various assumptions. Krämer et al. (1985) have shown that normality of the disturbances is not necessary in order to establish the null distribution of the CUSUM test. Ploberger and Krämer (1986: 341-344) have shown further that the asymptotic null distribution of CUSUM-SQ is not robust against deviations from normality, and proposed therefore a modification to the CUSUM-SQ which overcomes this deficiency. Cox (1975) in the discussion of the BDE tests suggested a modification of the significance lines in the graph of CUSUM of squares of recursive residuals that takes into account the effects of serial correlation. Husková (1991) introduced robust modifications of the CUSUM and MOSUM procedures for testing the constancy of regression relationships over time that are based on robustified versions of recursive residuals and M-estimators. These M-tests are introduced under general assumptions.

Giles (1981: 323-326) has developed CUSUM and CUSUM-SQ tests for parameter stability in an equation which forms part of a structural simultaneous equation model. The case of possibly multiple switching of parameters at unknown sample points, where the application of Two Stage Least Squares (2SLS) estimation is feasible, is considered. Giles tests are in the tradition of the single-equation literature and is easy to apply. However, they also inherit the weaknesses of the traditional CUSUM and CUSUM-SQ tests - the associated procedures is shown to lack some appeal, and their power may vary substantially depending on H_a.

4.5.6 EVALUATION OF THE BDE TESTS

It is clear that the BDE tests simultaneously test the null hypothesis of time-independent regression parameters and time-independent error variances. This leaves the alternative hypothesis in a rather vague form, because neither does it distinguish between abrupt or gradual changes in the regression parameters, nor between homoscedasticity and parameter constancy.

Power considerations are also difficult due to the complex structure of the alternative hypothesis. It is, however, difficult to decide the likely usefulness of the proposed tests in the absence of any investigation of their power under a range of alternative hypotheses. The CUSUM test is often criticised for its alleged low power as compared to, e.g., the CUSUM of squares test. This belief probably originated from the devastating Monte Carlo results in Garbade (1977) and is echoed in, e.g., McCabe and Harrison (1980), Ashley (1984) and Johnston (1984: 392). On the contrary, Krämer and Ploberger (1986) showed that under mild regularity conditions, the CUSUM test has in general nontrivial local power, whereas under the same conditions the local power of the CUSUM of squares test is no larger than α. Krämer et al. (1988) demonstrated that the poor performance of the CUSUM test in Garbade's Monte Carlo study is mostly due to his particular experimental design and does not generalise to other models.

A related issue concerns the optimality of the CUSUM test. Since it was designed with no particular type of structural shift in mind, one cannot expect it to be optimal against any particular narrow alternative. The CUSUM test is for instance easily outperformed by the Chow test when there is a single shift at a known point in time. On the other hand, this ranking is easily reversed when an incorrect location for the shift is assumed in the Chow test. Harvey and Collier (1975), suggested that if $\hat{\sigma}^2 = \Sigma(w_t-\bar{w})^2/(T-k-1)$, where the arithmetic mean of the recursive residuals $\bar{w} = \Sigma\, w_t/(T-k)$, the CUSUM test is likely to be more powerful under certain alternative hypotheses. This alternative definition of $\hat{\sigma}^2$ does not alter the distribution of w_t under the null hypothesis.

Also, if structural change is orthogonal to the regressors or occurs rather late in the sample period, no version of the CUSUM test will detect it. Krämer et al. (1988) suggested that, if such prior information is available, it is better to switch to another procedure such as Ploberger's (1983) fluctuation test.

Furthermore, these tests assume the classical error structure in the linear regression model, and the effect of departures from this assumption, e.g. autocorrelation, and the assumption of nonstochastic regressors, is not well understood. Ploberger (1989), however, considered the limiting behaviour of the CUSUM-SQ test for suitably defined sequences of local alternatives describing heteros-cedasticity. He showed that under certain circumstances the asymptotic behaviour of the CUSUM-SQ test can be computed even for alternatives describing changes of the conditional variance of the error term (e.g., ARCH-processes) and that the CUSUM-SQ test has only trivial local power for these alternatives (in contrast with the nontrivial local power of the CUSUM test). The assumption of nonstochastic regressors is obviously not true in most practical cases. The failure of this assumption leads not merely to minor technical difficulties or small biases, but radically affects the null hypothesis of parameter constancy and indeed the applicabi-lity of regression analysis on the whole (Ehrenberg, 1975).

A further point is that the recursive residuals are independent only when disturbances are normal. Phillips and Harvey (1974: 935-939) pointed out that when there is a departure from normality, recursive residuals may perhaps be no more effective than the ordinary least-squares residuals. There is also some evidence in a paper by Johnson and Bagshaw (1974: 103-122), which suggests that the CUSUM tests are not robust to departures from independence. Thus, the CUSUM test certainly merits further investigation.

Other drawbacks emerged when the BDE tests were applied. Firstly, LaMotte and McWhorter (1980) claimed that the test is neither exact, nor even conservative.[9] The authors have found that a 5 percent BDE CUSUM-SQ test wrongly rejects the null

[9]Conservative tests are tests for which only an upper bound on the size is known.

hypothesis of coefficient stability 7,3 percent of the time in their simulations. Secondly, the BDE tests and variants thereof constitute what might be called a global stability test. They do not allow one to identify the particular source of instability once global instability is detected. The test, therefore, inherently considers the stability of all K coefficients simultaneously; it is not possible to focus on a subset of coefficients whose stability may be either in greater doubt or of more intense interest.[10] Garbade (1977) pointed out this problem in a simulation study comparing CUSUM, recursive residuals, Cooley and Prescott tests and ML estimation of random walk variances. Thirdly, results reported by Ashley (1984), LaMotte et al. (1980) and Garbade (1975) showed that the BDE tests are substantially lower in power than the alternative tests suggested by them. These results are plausible in view of Farley, Hinich and McGuire's (1975) proof that the BDE CUSUM-SQ test is inconsistent.

Recently, Swamy, Conway and LeBlanc (1989) remarked that the BDE tests, which are based on recursive residuals, are not unique. A different set of T-K recursive residuals are obtained depending on which K of the T residuals are set equal to zero. Computing these residuals usually means that the population variance is arbitrarily set equal to $\sigma^2 I$. Therefore, the values of a Type I (α) and Type II (β) error for

[10]The BDE test shares this disadvantage with the tests on the recursive residuals suggested by Harvey (1976).

the test of the null hypothesis of parameter constancy against the alternative hypothesis that the coefficient vector changes at some unknown periods based on the CUSUM (or CUSUM-SQ) depend on the value of the co-variance and also which K of the T residuals are set equal to zero. For this reason, two different econometricians working with two different recursive residuals for the same model and data can come up with two different pairs of values of (α, β) for the CUSUM (or CUSUM-SQ) test. These pairs of values may give contradictory conclusions. It is also clear that the CUSUM (or CUSUM-SQ) test cannot detect shifts in coefficients in any period if the recursive residuals of that period is equal to zero. Swamy et al. (1989) pointed out that even in large samples, the CUSUM (or CUSUM-SQ) test does not give correct conclusions because, under the alternative hypothesis of parameter changes at some unknown periods, in some unknown manner, the power, ($1 - \beta$), of the test does not tend to 1 as the sample size tends to ∞. This discussion and the discussion in the previous paragraphs show that the BDE stability tests are not always informative and can be misleading. The seductive danger of the BDE and other stability tests is that sometimes they pretend to a kind of relevance which their logical machinery cannot justify.

It is therefore evident that the problem of discriminating between rejections of the null hypothesis of stability due to non-well behaved errors, and rejection due to incorrect assumptions about the structural part, has not been solved.

In the next section some alternatives to the BDE tests which can be used to overcome some of the above problems when testing for structural change are discussed.

4.6 THE FLUCTUATION TEST

The Fluctuation test (henceforth FLUCT), developed by Ploberger (1983), Kontrus and Ploberger (1984) and Ploberger, Krämer and Kontrus (1989) is a test for structural stability of the regression coefficients with no prior knowledge about number and timing of possible structural shifts. Unlike CUSUM and CUSUM-SQ, it is based on successive parameter estimates themselves rather than on the recursive residuals. The name of the test is derived from the rule to reject the null hypothesis of parameter constancy whenever the recursive parameter estimates β_t ($t = k+1,...,T$) fluctuate too much.

Contrary to Dufour's (1982) procedure, the FLUCT test is based on levels, rather than the first differences, of successive recursive parameter estimates. A similar rule has also been suggested by Sen (1980), but only for single regressor models and under severe restrictions on the independent variable. The test compares parameter estimates from the partial samples to those of the complete sample. The basic idea is to reject the null hypothesis of parameter constancy whenever the estimates fluctuate too much. More precisely, the test rejects H_0 whenever there is excessive fluctuation in the quantities

$$\| \hat{\beta}_t - \hat{\beta}_T \|_\infty = \max_{i=1,\ldots,K} | \hat{\beta}_{it} - \hat{\beta}_{iT} | \qquad (4.30)$$

$t = K,\ldots,T,$

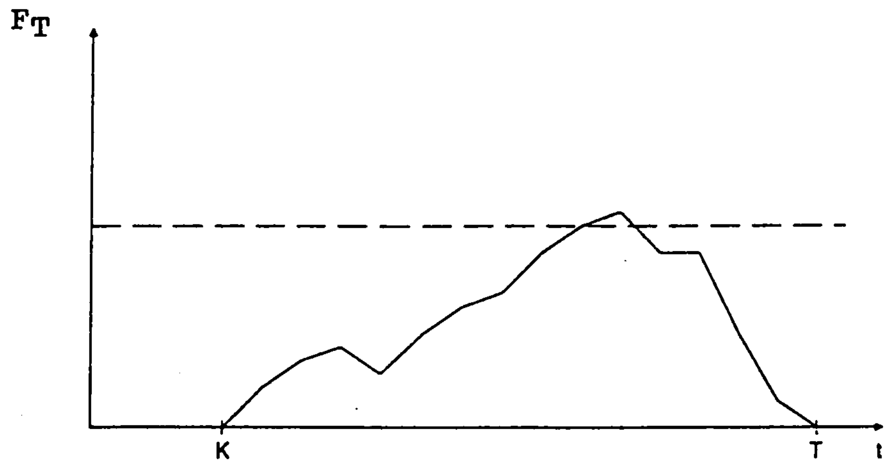
where $\hat{\beta}_T$ is the full sample estimate and $\hat{\beta}_t$ the OLS estimate from the first t observations; $\|.\|_\infty$ will in the sequel always denote the maximum norm. The test statistic is

$$S_T = \max_{t=K,\ldots,T} \| F_T \|_\infty \qquad (4.31)$$

where $F_T = [(t-k)/\hat{\sigma}(T-k)] (X_T' X_T)^{\frac{1}{2}} (\hat{\beta}_t - \hat{\beta}_T)$, and the null hypothesis is rejected whenever S_T is too large.

Figure 4.7 shows a typical sample trajectory of the F_T's corresponding to a significant level α (indicated by the broken line on the graph); (note that these trajectories always pass through the points (K,0) and (T,0) and are non-negative otherwise).

FIGURE 4.7: A TYPICAL SAMPLE PATH OF THE FLUCTUATION TEST



Source: IAS-System manual (1990)

This is determined from the fact (see Ploberger, 1983) that S_T has a well defined limiting distribution under H_O with distribution function

$$F(z) = \begin{cases} 0 & z \leq 0 \\ 1 - [\, 2 \sum_{i=1}^{\infty} (-1)^{i+1} \exp(2i^2 z^2) \,] & z > 0 \end{cases} \tag{4.32}$$

where i denotes the number of terms in the series when z > 0.

Ploberger et al. (1989) have shown that the limiting distribution of the test under the H_0 depends on $B_T(z) \xrightarrow{d} B(z)$, where $\xrightarrow{d}$ denotes convergence in distribution as $T \to \infty$. The test therefore cannot be applied to trended data; B(z) is a process known as the "Brownian Bridge". This process has well known boundary crossing probabilities.

Ploberger et al. (1989) also showed that the test has non-trivial power against many local alternatives, and that it compares favourably with both CUSUM and CUSUM-SQ tests. Monte Carlo experiments have shown that FLUCT is considerably more powerful than both CUSUM and CUSUM-SQ. Additional research, however, is called for to weaken the assumptions under which the limiting results apply and, in particular, to modify the test statistic to allow for trended data.

4.7 THE WATSON-DAVIES TEST

Watson and Engle (1985) considered the problem of testing for a constant regression coefficient against the alternative hypothesis that the coefficient follows a stationary first-order autoregressive process. The alternative is the so-called return to normality model proposed by Rosenberg (1973). For a single time varying coefficient the Rosenberg model (also discussed in Part 4) may be written as

$$y_t = \mathbf{x}_t' \tau + z_t \beta_t + e_t \quad (4.33)$$

$$(\beta_t - \bar{\beta}) = \phi(\beta_{t-1} - \bar{\beta}) + \mu_t \quad (4.34)$$

with $|\phi| < 1$ and $t = 1,\ldots,T$,

where $\mathbf{x}_t$ is a (K x 1) vector of explanatory variables, z_t is a scalar, τ is a (K x 1) vector of unknown constant coefficients, β_t is a time varying coefficient and e_t and μ_t are independent Gaussian white noise disturbances such that $\mathbf{e} \sim N(\mathbf{0}, rI_n)$ and $\mu \sim N(\mathbf{0}, qI_n)$.[11] This model, in which the coefficients follow a stable first-order Markov process, is a particularly attractive specification, incorporating some of the best features of the random walk and the random coefficients model. The coefficients vary around a constant mean, a feature present in the random coefficients model, but also possess some inertia, a feature found in the random walk model.

The parameters of the model can be estimated using the nonlinear maximum likelihood procedures described in Pagan (1980) and Watson and Engle (1983). When the coefficient β_t is constant, maximum likelihood estimation is greatly simplified - it reduces of course to ordinary least squares

[11]The results are only slightly modified if e_t follows an ARMA process (see Harvey and Phillips, 1979).

(OLS) estimation. Equation (4.34) implies that $\beta \sim N(\bar{\beta}, q(1 - \phi^2)^{-1})$ so that β_t is constant if and only if $q = 0$. The hypotheses of interest are therefore $H_0 : q = 0$ and $H_1 : q > 0$.

One is tempted to use a standard large sample test, a likelihood ratio, a Wald or a Lagrange multiplier test; however, these tests cannot be used in the usual fashion since the transition parameter, ϕ, is only identified under the alternative hypothesis but not under H_0. With $q = 0$ any $\phi \in (-1,1)$ yields the same value of the likelihood function. This implies that the information matrix will be singular under H_0, a violation of one of the standard regularity conditions required to derive the usual asymptotic distribution (and local equivalence) of the tests listed above. To overcome this problem Watson and Engle (1985) suggested the use of a test procedure proposed by Davies (1977) which can be applied to the varying coefficient problem.

Davies' approach involves applying Roy's Union-Intersection Principle (Roy, 1953: 220-238) to the class of test statistics one gets by assuming that the non-identified parameter, ϕ, takes a known value. In such cases the null hypothesis should be rejected if the LM test statistic is "large" when evaluated at any value of ϕ. Since the value of ϕ does not matter under the null, the LM test will have the correct size (asymptotically) regardless of the value of ϕ. With ϕ unknown, the test can be carried out using any arbitrary value of the parameter. The size of the test will not be affected by the value of ϕ chosen, but the power of the test will be affected.

Specifically, let $S(\bar{\phi})$ be the normalised element of the score vector corresponding to q evaluated under the null and assuming that $\phi = \bar{\phi}$ is known. Then, under the usual regularity conditions, $S(\bar{\phi})$ - the (sign corrected) square root of the LM statistic - is asymptotically a standard normal random variable, and the Davies test statistic is

$$D \equiv \{ \sup_{-1<\phi<1} S(\phi) \} \qquad (4.35)$$

This test statistic is asymptotically locally equivalent to the Wald test statistic and the (sign corrected) square root of the likelihood ratio test statistic. For a one-sided alternative the null is rejected when D is greater than some critical value CV.

Unfortunately, in the model under consideration a closed form solution for $\{\sup_{-1<\phi<1} S(\phi)\}$ is difficult to derive so that the test statistic D cannot be formed. However, it can be approximated by maximisation using a grid search over n different values of ϕ. Watson and Engle (1985) approximated the Davies test statistic by

$$AD = \max \{S(\phi_i);\ i = 1, 2, \ldots, n \qquad (4.36)$$

with $\{-1 < \phi_i < \phi_{i+1} < 1\}$.

The critical value CV for the test is such that

$$\text{prob}\ (AD > CV) = 1 - \text{prob}\ [S(\phi_i) < CV] \qquad (4.37)$$

$i = 1, \ldots, n$,

where $S(\phi_i)$ is a sequence of score statistics which depend only on the regressors and the OLS residuals. Under the usual regularity conditions the asymptotic distribution of $S(\phi_i)$ $(i = 1, \ldots, n)$ is multivariate normal so that this probability can, in principle, be calculated and a value of CV, equating the probability of a Type I error to the desired size of the test, can be found. Watson (1982) presented the derivation of $S(\phi)$ and regularity conditions which justified the assertion concerning asymptotic distributions made above and showed that

$$S(\phi) = \frac{S_1 + S_2(\phi)}{S_3(\phi)} \qquad (4.38)$$

where

$$S_1 = \tfrac{1}{2} \sum_{t=1}^{T} z_t^2 \{ (\hat{e}_t^2 / \hat{\tau}) - 1 \} ,$$

$$S_2(\phi) = 1/\hat{\tau} \sum_{t=2}^{T} \hat{e}_t \, z_t \sum_{i=1}^{t-1} \hat{e}_i z_i \phi^{t-i} \quad \text{and}$$

$$S_3(\phi) = \{ \tfrac{1}{2} \sum_{t=1}^{T} z_t^4 + \sum_{t=2}^{T} z_t^2 \sum_{i=1}^{t-1} z_i^2 \phi^{2(t-i)} - (1/2T) (\sum_{t=1}^{T} z_t^2)^2 \}^{\frac{1}{2}}$$

The interpretation of $S(\phi)$ is straightforward. If the alternative is true and the model is estimated by OLS, then we would expect the residuals to exhibit both heteroscedasticity and serial correlation. The first term in the numerator of (4.38) checks for heteroscedasticity, while the second term checks for serial correlation. The denominator is just a normalising constant.

Two extreme cases are helpful in interpreting the test statistic. If it is assumed that $\phi = 0$, so that only heteroscedasticity is present, then $S(\phi)$ is asymptotically equivalent to $\hat{\rho}\ (T)^{\frac{1}{2}}$, where $\hat{\rho}$ is the sample correlation between z_t^2 and $\hat{e}_t^2$. This is a version of the Langrange multiplier test for heteroscedasticity proposed in Breusch and Pagan (1979). Secondly, if $z_t = 1$ for all t, then the errors are homoscedastic, but generated by an ARMA (1,1) process if ϕ

$\downarrow 0$. In this case the test statistic is asymptotically equivalent to

$$T^{\frac{1}{2}}(1 - \phi^2)^{\frac{1}{2}} \sum_{i=1}^{T-1} \hat{\rho}_i \, \phi^i \tag{4.39}$$

where $\hat{\rho}_i$ is the i-th autocorrelation coefficient of the OLS residuals.

Unfortunately, Watson and Engle's test statistic has no closed form and is approximated by maximisation using a grid search. Furthermore, both its finite sample and asymptotic distributions are unknown under the null hypothesis although they provide a method of calculating a critical value whose asymptotic size can be bounded from above.

To overcome these problems, King (1987) suggested a different approach. Rather than testing for zero variance in the autoregressive process as Watson and Engle suggested, they proposed testing for lack of variation in the regression coefficient over time. This allows the construction of a locally best invariant (LBI) test whose critical values can be found exactly or approximated using standard computational techniques designed by King and Hillier (1985). This test is also LBI against the alternative hypothesis that the coefficient follows a random walk process.

4.8 OTHER TESTS FOR STRUCTURAL CHANGE

The parameter constancy assumption of the standard linear model is clearly unrealistic in many economic applications. Because the assumptions may be unrealistic, it is important to subject any estimated linear model to as many as possible relevant structural stability tests before the model is used for inference or forecasting purposes. Besides the tests discussed in the sections above, many other tests for the constancy of regression coefficients have also been proposed in the literature, some of which are not widely used in econometric applications due to a lack of commercialised computer software. The following tests can be regarded to fall in this category. Only introductory remarks will be given with regard to these tests because the goal of this study is to eventually apply only those tests for which commercialised computer software exist.

Hawkins (1977) and Worsley (1979) have suggested a likelihood ratio test statistic W for testing a sequence of observations for a shift in location which is sensitive to the assumption that the distribution of the population being sampled is normal. Talwar (1983) proposed a modification of this statistic which prove to be robust against "heavy tailed" distributions.

Andrews (1990) considered tests of parameter instability and structural change with unknown change point. The proposed tests are designed for a one-time change in the value of a parameter vector, but are shown to have power against general forms of parameter instability. He considered the LR-like test for one-time structural change with unknown change point, as well as the analogous Wald (W) and Lagrange multiplier (LM) tests. These tests apply to parametric models that do not exhibit deterministic or stochastic trends.

There also exists other methods to estimate structural breaks (see Krishnaiah and Miao, 1988) of which the Bayesian point of view and the nonparametric methods are the most influential. The monograph by Broemeling and Tsurumi (1987) and the references therein summarises the achievements of the Bayesian methodology. Until 1990 Bayesian studies of structural change in linear models have focused mainly on the shift point or on the transition function of the model. For the two-phase regression problem Broemeling, Cook and Choy (1991) in a Bayesian derived framework the marginal posterior density of the intersept in closed form starting from a proper joint distribution of the intersept, the regression coefficients and the disturbance variance that is assumed to be common to both regimes. It is shown that point and interval estimators can be obtained by means of numerical integration. A detailed survey of the nonparametric methodology was given by Csörgö and Horvàth (1988).

The statistical literature on change point problems is extensive (see "change point" in "Current Index to Statistics" and the review papers by Zacks, 1983 and Krishnaiah and Miao, 1988). Recent references include James, James and Siegmund (1987), Hawkins (1987) and Kim and Siegmund (1989) among others. Many of the results in the literature concern location models, other scalar parameter models or simple regression models. Most of the results only apply to tests of pure structural change. The most general results available appear to be those of Hawkins (1987), who considered Wald tests of pure structural change based on ML estimators in independent identically distributed (iid) scenarios. These results are still too restrictive, however, for many econometric applications.

In consequence, Andrews (1990) established results that allowed for dependent non-identically distributed (dnid) observations, estimation by methods other than ML, tests of both pure and partial structural change and tests based on W, LM and LR (or LR-like) test statistics. For example, stationary models estimated by the generalised method of moments (henceforth GMM) and dnid models estimated by ML are used to illustrate the general results.

Andrews (1990) also derived the asymptotic null distribution of the sequential likelihood ratio test (Quandt, 1960) of parameter constancy. He showed that the test has nontrivial local asymptotic power against all alternatives of nonconstant parameters. Ploberger et al. (1989) obtained a corresponding result for the fluctuation test.

Andrews and Ploberger (1992) expanded on Andrews' (1990) study by deriving asymptotically optimal results for tests of parameter constancy. Their results apply to tests of one-time structural change with unknown change point. King and Shively (1991) considered locally mean most powerful tests for problems of the same sort. They employed a transformation of parameters, which provides a useful alternative perspective. Their tests, however, have direct power only against very local alternatives. Chu and White (1991) extended the distribution results of Andrews (1990) to nonstationary regressors. They then derived tests for the constancy of the trend and the parameters in the cointegration relationship.

Andrews, Lee and Ploberger (1992) determined a class of finite sample optimal tests for the existence of a change point at an unknown time point in a normal linear multiple regression with known variance. Optimal tests for multiple change points were also derived. Simulations reported in Andrews et al. showed that an optimal exponential test dominates the likelihood ratio test over a fairly wide variety of alternatives.

Another strand of literature deals with the case in which the alternative to constancy is that the parameters are stochastic and fluctuate according to some time series model. LaMotte and McWhorter (1978) assumed that if the null hypothesis is not true the parameters follow a (Cooley-Prescott) random walk and constructed an exact F-test for testing against this alternative. Assuming the same alternative Nyblom and Mäkeläinen (1983) derived a family of locally most powerful tests. In a simple case they also made detailed comparisons between one of their tests and the Lamotte-McWhorter tests. All of these tests are based on the assumption that if the parameters are nonconstant they are deterministic. Nyblom (1989: 223-230) continued this work considering tests based on maximum likelihood estimation. He proposed large sample tests for detecting possible changes in parameters when the observations are obtained sequentially in time. In his test he assumes that changes in the observation-generating process occur through parameter variation in the form of a martingale which has the advantage of covering several types of departure from constancy (the so-called change point model and random walk process). These tests are therefore, designed for nonstationary alternatives. His results can be criticised in that (1) they direct power only against very local alternatives and (2) they only apply when there are no unknown parameters under the null hypothesis, which rarely occurs in practice. Hansen (1990) further extended Nyblom's (1989) work to cover other maximum likelihood estimators.

Shively (1988) developed an exact small-sample test for testing the hypothesis that a regression coefficient is constant against the alternative that is generated by a random walk process. His test proved to be mean- and scale invariant and approximates the most powerful invariant (MPI) test against any specific alternative compared to similar tests given in previous literature. Shively used a version of the Kalman filter to compute the test statistic and its distribution. Under the same model Nabeya and Tanaka (1988) suggested a locally best invariant (LBI) test under normality for the constancy of regression coefficients against the alternative hypothesis that one component of the coefficients follows a random walk. They discussed the limiting null behaviour of the test statistic without assuming normality under two situations, where the initial value of the random walk process is known or unknown. The locally best invariant (LBI) test for coefficient constancy, against the random walk alternative, in a regression which may contain non-varying coefficients was also investigated by Leybourne and McCabe (1989). The distribution of the LBI test depends critically on the exogenous variables of the model, even asymptotically. Leybourne and McCabe (1989) suggested a modification to the test statistic which has a known asymptotic distribution regardless of the regression effect. They explored the effects that different assumptions about the initial value of the random walk process have on the form and asymptotic distribution of the resulting test statistic. When this initial value is allowed to be random, it is shown that the test statistics are either exactly the same or possess the same asymptotic distributions as when the initial value is fixed.

As it is the case for simultaneous equations models, research on nonlinear regression models with time-varying parameter models is very rare. Recently tests for structural change for linear models were extended to a wide variety of models including dynamic, simultaneous and nonlinear models for a large class of inference methods. A generalised predictive testing procedure for structural stability was introduced by Dufour et al. (1991). The test is applicable to nonlinear dynamic simultaneous equations models and extends recent contributions by Andrews and Fair (1988), Ghysels and Hackl (1990a) and Andrews and Ploberger (1991). The methods considered are applicable when model coefficients can be taken as stable during a given (relatively large) subperiod (the estimation subsample) but the form and timing of possible structural changes during the second period (the prediction subsample) are left unspecified.

Recently, econometricians have made other important contributions to the structural change literature. Banerjee, Lumsdaine and Stock (1989), Perron (1989) and Zivot and Andrews (1989) developed various techniques for testing unit root and/or changing coefficients in time series regressions and provided empirical evidence on structural changes in economics. Other recent developments on testing for parameter constancy include the work of Bates (1990) and Perron (1990a, 1990b). An extensive bibliography of the literature on the testing of the change point problem in general has been compiled by, amongst others, Poirier (1976), Johnson (1977, 1980), Hinkley et al. (1980), Shaban (1980) and Hackl and Westlund (1989).

4.9 THE USE OF GRAPHICAL DISPLAYS IN THE ANALYSIS OF STRUCTURAL CHANGE

Validation of a regression model is mainly based on investigating to what extent the assumptions underlying the model specification are fulfilled. Relatively less attention is paid to the structural analysis of regression data and residuals. The drawback of many test procedures is that they are usually based on many assumptions that are often difficult to verify, and even when such procedures indicate model failure, they do not provide the real reason, owing to mutual relationships between certain types of failures. The typical example here is the indication of outliers, structural change and nonlinearity. Such failures can usually be detected by applying suitable tests, but very often only all these tests together indicate the failure. The use of graphical displays can be very useful in such a situation.

A number of tests for checking the specification of a regression model have been suggested earlier. As with the accompanying graphical procedures these tests are based primarily on OLS and recursive residuals. Recently developed techniques for graphical analysis of residuals have made these more legible. Wasilewski (1989) showed how graphical displays can be used to investigate nonconstancy of regression parameters. In order to make the graphical analysis more legible the author considered a number of transformations of OLS residuals and different smoothing procedures. The plots show a variety of features and make it possible to see how certain features relate to one another. In comparison with tests, they provide much more qualitative information (see, e.g., Anscombe, 1973; Daniel and Wood, 1971).

Owing to rather poor correspondence between theoretical regression disturbances and the OLS residuals, their use in judgment about model failures is rather limited. To make the graphical displays more legible, a number of transformations of OLS residuals and different smoothing procedures can be used.

In least squares regression, the internally studentised residuals r_t, $t = 1,\ldots,T$ are derived from the OLS residuals e_t as

$$r_t = e_t/s(1-h_{tt})^{\frac{1}{2}} \qquad (4.40)$$

where $s^2 = \sum_{i=1}^{T} e_t^2/(T-K)$, $h_{ts} = \mathbf{x}_t'(X'X)^{-1}\mathbf{x}_s$, and $\mathbf{x}_t'$ denotes the t-th row of the (T x K) matrix X. The quantity $r_t^2/(T-K)$ follows a Beta-distribution with parameters 1/2 and (T-K-1)/2, and $E(r_t) = 0$, $Var(r_t) = 1$, $Corr(r_t, r_s) = -h_{ts}/\{(1-h_{tt})(1-h_{ss})\}^{\frac{1}{2}}$ for $t \neq s$ (see Cook and Weisberg, 1982; Atkinson 1982).

According to Wasilewski (1989: 168), the residuals $r_t = e_t/s(1 - h_{tt})^{\frac{1}{2}}$ reflect a contamination of the t-th observation better than the OLS residuals, when the variance h_{tt} -> 1, but problems can still arise when h_{tt} -> 0.

The internally studentised residuals are often used as a replacement for the OLS residuals in graphical procedures, such as scatterplots or probability plots (see, e.g., Anscombe and Turkey, 1963; Cook and Weisberg, 1982; Atkinson 1982).

The so-called externally studentised residuals, denoted by t_t, are defined similarly to the internally studentised residuals, but the estimator of σ^2 is modified so that it does not make use of the t-th observation (see Wasilewski, 1989). Other residuals which can be used for diagnostic purposes are the predicted residuals and the recursive residuals. Both OLS residuals and studentised residuals are based on a fit of the regression model to all the data. The predicted residuals are, in contrast, interpreted as prediction errors, because the t-th predicted residual is based on a fit of the model to the data with the t-th case deleted. For a comprehensive study of the use of recursive residuals, see Galpin and Hawkins (1984) (and the discussion in Section 4.5).

To enhance the visual information on a scatterplot, three techniques for discovering and summarising smooth residual patterns can be used. The first one is suggested by Cleveland (1979), Cleveland and McGill (1984) and Cleveland and Kleiner (1975). They proposed a smooth plot in conjunction with a scatter plot of the data (x_t, y_t), $t = 1, \ldots, T$. These smoothed plots are designed to enhance the perception of the pattern of dependence of y on x.

For a given set of data points (x_t, y_t), $t = 1, \ldots, T$, corresponding smoothed values $\hat{y}_t$ can be obtained using locally weighted polynomial regression of degree d of y on x. This is done in four steps (see Cleveland, 1979). In order to aid the interpretation of a smoothed scatterplots for the studentised residuals, a confidence envelope around the smoothed curve can be plotted. This is achieved in a way proposed by Atkinson (1981) for probability plots, using small Monte Carlo experiments.

Two other smoothing techniques are based on combinations of running medians and running weighted averages. These were described by Tukey (1971) and Velleman and Hoaglin (1981) (see also Mosteller and Tukey, 1977).

To illustrate how helpful graphical displays can be used in investigating the nonconstancy of the regression parameters, two examples from Sonnberger (1986), one based on artificially generated data with a breakpoint and one based on real data from the Austrian economy are considered. Figures 4.8 - 4.11 are based on the data used in examples given in the test processor of the IAS-SYSTEM (see Sonnberger et al., 1986).

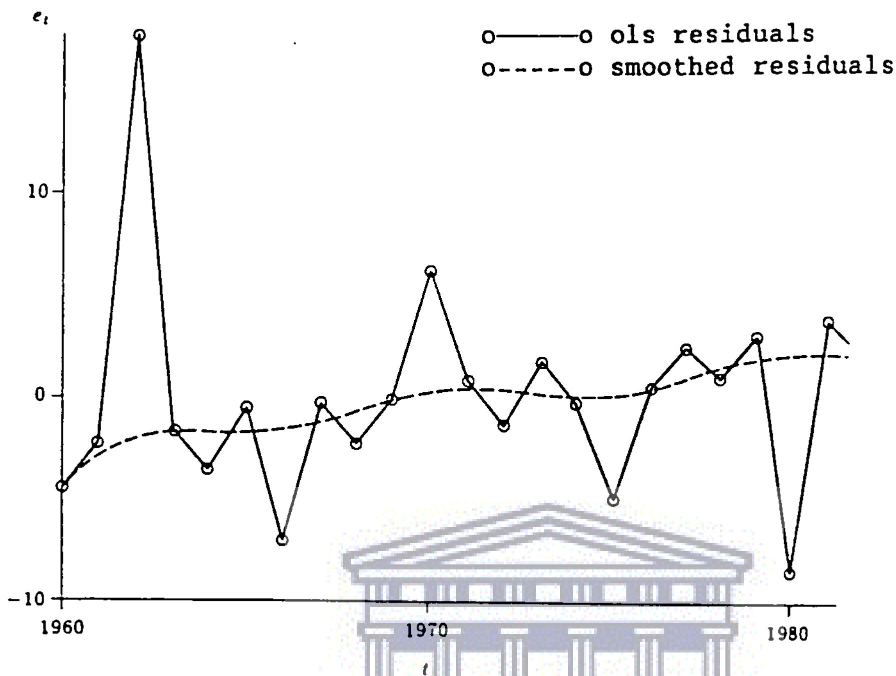
The 23 observations from 1960 to 1982 (t = 1,...,23) were used to estimate the parameters of

$$y_t = \beta_1 + \beta_2 x_{2t} + \beta_3 x_{3t} + e_t \qquad (4.41)$$

which is equivalent to that of Model (1.1); y_t is the dependent variable; x_{2t} denotes a trend variable; and x_{3t} an additional explanatory variable.

According to Sonnberger et al. (1986), the Quandt test indicated a change of the parameters in 1964. The CUSUM test did not reject the null hypothesis that the parameters remain constant over time. The CUSUM-SQ test indicated two regimes before and after 1964 (t = 5). The same phenomena was indicated by Ploberger's fluctuation test.

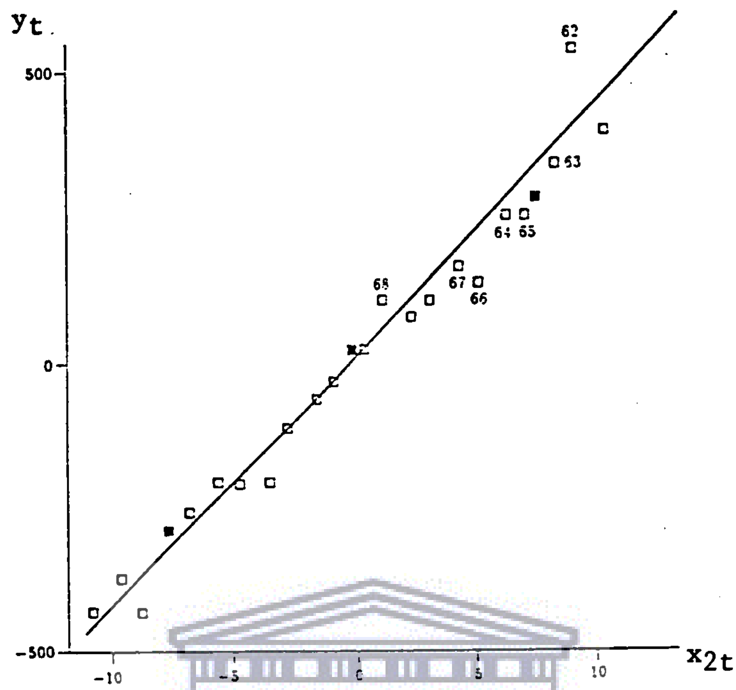
FIGURE 4.8: OLS RESIDUALS e_t AND SMOOTHED OLS RESIDUALS AGAINST THE YEAR t



Source: Wasilewski (1989: 176)

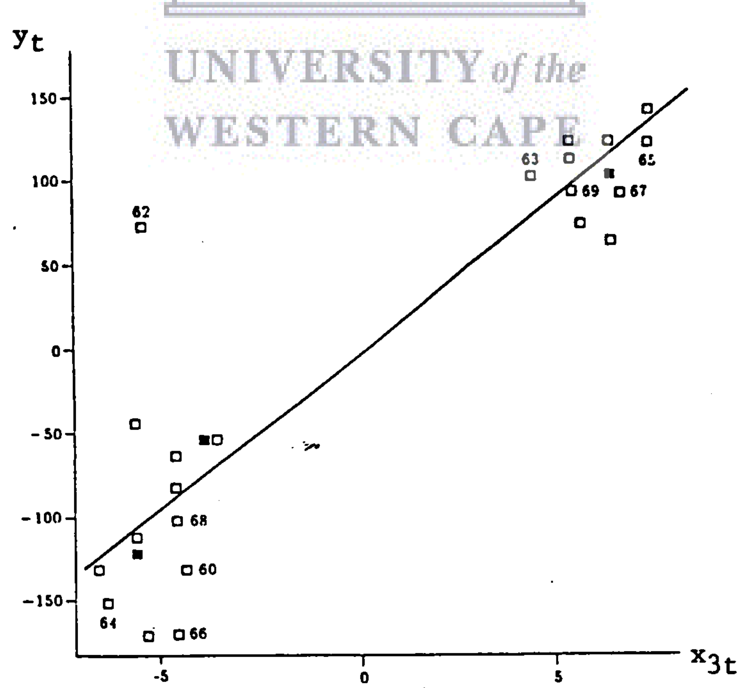
In Figure 4.8, examination of the scatterplot for OLS residuals (indicated by the solid line) and their smoothed values (indicated by the broken line) does not show any significant change in the configuration of the residuals. The smoothed curve seems to rise a bit, starting in 1965, but this may be due to the fact that there is an outlier in 1962 and probably also in 1980.

FIGURE 4.9: LEVERAGE PLOT FOR THE TREND VARIABLE x_{2t} AND DEPENDENT VARIABLE y_t



Source: Wasilewski (1989: 180)

FIGURE 4.10: LEVERAGE PLOT FOR THE EXPLANATORY VARIABLE x_{3t} AND DEPENDENT VARIABLE y_t



Source: Wasilewski (1989: 181)

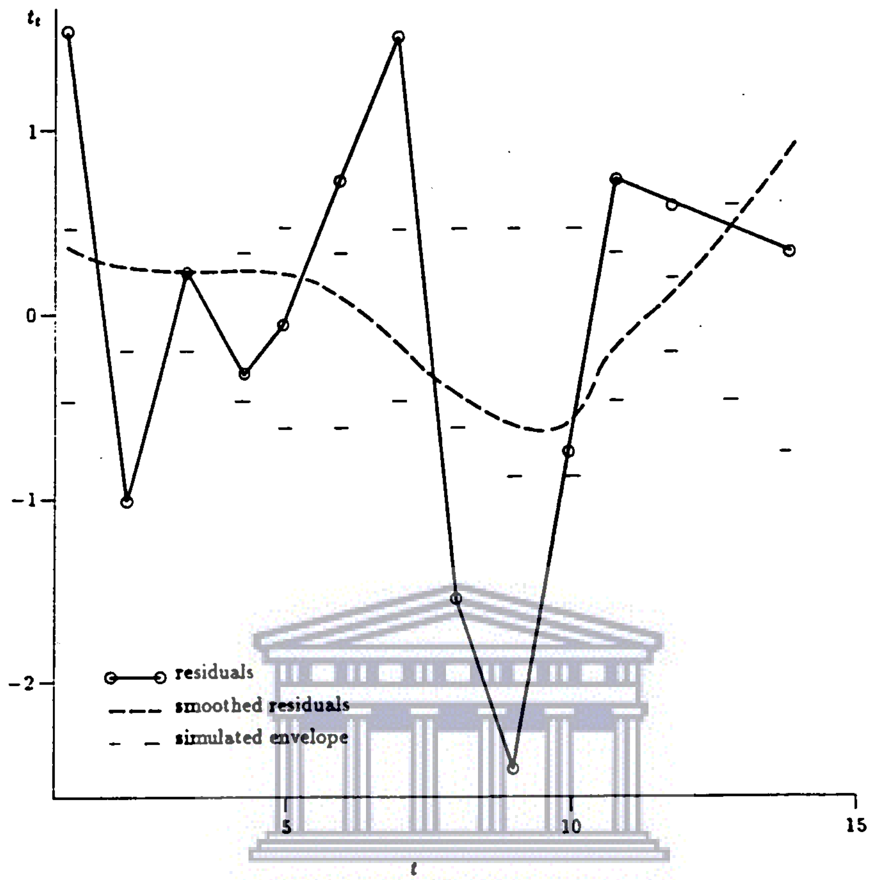
Analyses of the partial regression residuals (or "leverage") plots for the two exogenous variables (Figures 4.9 and 4.10) allows one to look for any subconfiguration of the given subset of successive data points. The solid points are median summary points for the left, middle and right thirds of the data according to the order of the x values. The leverage plot for the variable x_{3t} only indicates that the observation from 1964 to 1968 would lead to different parameter estimates than the global estimation process.

The graphical analysis for the second example is based on the following equation (using Model (1.1)) for the demand for labour in the Austrian building industry given in Sonnberger et al. (1983); 15 observations from 1965 to 1981 ($t = 1,\ldots,15$) were used to estimate the parameters of

$$\ln(L) = \alpha_1 + \alpha_2 TYD + \alpha_3 \ln(Q) + v_t \qquad (4.42)$$

where ln(L) is the logarithmic transform of employment in the construction sector; TYD is a trend variable; and ln(Q) is the logarithmic transform of the real output in the construction sector.

FIGURE 4.11: SCATTERPLOT OF EXTERNALLY STUDENTISED RESIDUALS t_t AND SMOOTHED VALUES AGAINST THE INDEX t



Source: Wasilewski (1989: 183)

A scatter plot of the externally studentised residuals (t_t) (Figure 4.11) shows that only an extremely negative residual for 1975 (t = 9), but also indicates two other relatively high positive residuals for 1967 and 1973. The same plot indicates a possible breakpoint in 1972 or 1973.

Phenomena to look out for in these plots are, for example, the largest residuals; progressive changes in variability of the residuals, a curved regression of residuals on fitted values or a number of cases; and the subsets of successive residuals with significantly different configurations. They also provide important qualitative information about the data structure and its effects in the estimation process (see, e.g., Anscombe, 1973; Daniel and Wood, 1971).

4.10 SUMMARY AND CONCLUSION

Many tests for the constancy of regression coefficients have been proposed. If the coefficients are suspected of discrete changes a Chow test or the test proposed by Quandt is appropriate. If the coefficients are suspected of changing smoothly through time, e.g., are generated by some economic process, another class of tests can be used. For these tests an ARIMA process can be used as a proxy for the true generating process. When a simple white noise process generates the random coefficient model, it can be tested using the Langrange multiplier test of Breusch and Pagan (1979). For coefficients suspected of following a random walk, tests have been proposed by Brown, Durbin and Evans (1975), Garbade (1977), Pagan and Tanaka (1979), LaMotte and McWhorter (1978) and a series of tests have been proposed and compared by Harvey and Phillips (1976), and Krämer (1989). Cooley and Prescott (1976) introduced a model where the coefficients follow an ARIMA (0,1,1) process, and proposed a likelihood ratio test. Convenient surveys of this literature are in, e.g., Pesaran, Smith and Yeo (1985), Hackl and Westlund (1985) or Krämer and

Sonnberger (1986: Chapter 4). In the case of a discrete jump in parameter variation, Ashley's simulation results indicated that the STAB test are clearly superior, in terms of effective power than, for example, the VPR test of Garbade (1977) and the LM test suggested by Lamotte and McWhorter (1978) which are both based on the Cooley-Prescott random walk model.

A problem with the CUSUM tests and variants thereof (see McCabe and Harrison, 1980; Hackl, 1980; Dufour, 1982 or Brown et al., 1975) and the Cooley-Prescott test (Cooley and Prescott, 1973) is the fact that they constitute what might be called a global stability test. To overcome this particular problem Watson and Engle (1985) proposed a very practical test in which the individual regression coefficients can be tested for either a stationary AR(1) - or a random walk process.

Many of the above procedures put forward by the authors, however, seem to require a great deal of informal use of personal judgment. Therefore, it is advised to interpret results of this whole battery of tests in the spirit of exploratory data analysis, i.e. as yardsticks rather than formal tests of hypotheses at nominal α - levels. This view of Brown et al. (1975: 150) is also supported by the often contradictory results obtained from the various tests in a study by Wesso and Smit (1989).

Having outlined most of the important tests for structural stability the next step is to consider the problem of estimating a linear model with random or varying coefficients using alternative specifications of these coefficients. This can be done in either a time-series analysis framework or in the context of econometric modelling.

PART IV

MODEL BUILDING IN THE PRESENCE OF STRUCTURAL CHANGE

CHAPTER 5

METHODS OF ESTIMATION AND PREDICTION UNDER CONDITIONS OF STRUCTURAL INSTABILITY: AN OVERVIEW

5.1 INTRODUCTION

After estimating a fixed coefficient model, the econometrician who adjusts the estimate of a constant term or the estimates of some other parameters like the autoregressive coefficient of the error process, while retaining the original estimates of the remaining parameters to obtain good forecasts, may violate one or more of the probability laws. If any probability law is violated in the process of drawing an econometric inference, the resulting inference will be incoherent. There is no logic associated with such an inference.

Over the last decade an impressive body of literature has been dedicated to the rationales for time-varying parameter models in time series studies and/or to the problems of estimating such models (see Hackl and Westlund, 1989). It is thus widely recognised that the traditional econometric assumption of fixed parameters over time may often lead to misspecified models and, by the same token, to the reduction in both estimation and prediction efficiency.

Also, as a result of these studies, numerous estimation techniques that can reveal and be adaptive to structural instabilities have now been developed and their respective drawbacks discussed.

5.2 SOME FUNDAMENTAL REFLECTIONS ON ECONOMIC MODEL BUILDING

Macro-econometric models have been used extensively in South Africa during the past two decades, not only as forecasting devices but also for simulating the effects of changes in economic policy. In recent years the attitude towards econometric modelling has been very negative based mainly on the perception of inaccurate forecasts (Smit and Wesso, 1986). Although theoretically-based and decidedly more elegant than the so-called barometric approaches to forecasting, the forecasts generated by macro-econometric models have, on the whole, not been any better than those generated by other techniques or approaches. In fact it has been found in the United States of America that even the most sophisticated macro-econometric models often perform no better than the most naïve, same-as-last-quarter forecasts, (Dornbusch et al., 1987: 259)

Although economists differ as to the nature and causes of economic fluctuations, they generally agree that the myriad factors which can affect the dynamics of the economy make economic forecasting a particular hazardous exercise. Many would therefore gladly refrain from venturing quantitive projections of the future course of economic activity. However, during the past two decades there has been a definite (and growing) demand for such forecasts, in the public as well as private sector, which has had to be met in some way.

If the economy were growing or declining steadily, forecasts could be made simply by extrapolating along the trend line. Similarly, if the economy were characterised by a regular cycle, forecasts could be made simply by extrapolating along the cycle path. Unfortunately, the path of the South African economy is decidedly irregular, with the result that the task of economic forecasting cannot be handled only by naïve extrapolation techniques.

An alternative approach, which is employed by many economists and forecasters, is the barometric approach, which include concepts such as 'leading indicators', 'anticipatory data' and 'consensus of observers' (Dornbusch et al., 1987: 257). Generally speaking, barometric procedures can be quite useful for short term forecasting, but they cannot be used to ascertain the long-run path of the economy, particularly if some significant change has just occurred. Moreover, they are of limited use to policy makers, because they do not explain why the economy is heading in the predicted direction nor do they indicate what policy should be prescribed. In view of these shortcomings many economists prefer to use macro-econometric models.

In a macro-econometric model specific numerical values are estimated for each of the various parameters on the basis of past behaviour of the economy. Once specific values have been assigned to the relevant exogenous variables (for e.g. government spending and the nominal money stock) the model can be solved to yield values for the endogenous (the unknowns). The question now is: How have these models performed during the past?

Economists and economic forecasting appear to be experiencing a confidence crisis. Derogatory references have been made about the forecasting profession and honesty forces one to admit that economists and forecasters are viewed with much scepticism. It is in this vein that Sir Peter Medawar reached the conclusion stated in Section 3.5.

To answer his criticism one is inclined first to focus on the inadequacy of current techniques, data problems or the criticism of the Rational Expectations theorists. Further considerations, however, show that one needs to reflect on methodology and to probe the fundamental nature of economic forecasting.

Frequently modern forecasting is based upon advanced scientific work involving empirical estimation of complicated mathematical models of the economy. Forecasting, and especially econometric forecasting, is the result of a direct extension of the main features of standard economics which implies, of course, that a critical analysis of forecasting involves probing questions about the nature of economics. Hutchison (1977: 8) argued that "The question of prediction in Economics involves, or brings together, most of the main questions as to what sort of subject Economics is: questions, that is of what economists can or should try or claim to do."

Economists tend to assume the existence of a permanent natural economic order. The presumption of exact, fixed cause-and-effect laws, that govern economic relations, is found in the absence of unnatural restrictions. Closely to this presumption is the prominent role of mathematics being most useful for the rigorous manipulation of exact economic laws and relationships. This also explains why, in forecasting, economists in their criteria, objectives and methods, have attempted to follow the natural sciences closely and explicitly. This is also supported by Hutchison (1977: 4) who pointed out that "Some economists and econometricians have attempted to define or describe more precisely the kind of predictions they were claiming to produce as 'scientific' in the same sense as the predictions of the most advanced natural sciences." This heritage is clear from the distinguishing features of standard economic forecasting techniques - as seen in the precise mechanistic specification of economic relationships with presumably exact, true underlying structures. These relationships are then also supplemented by stochastic disturbance terms with assumed "well behaved", usually normal probability distributions. Mathematical formalisation is central to econometric model specification, and sophisticated, rigorous statistical techniques are applied. The hypothesised model is then tested empirically and if not rejected, used to generate forecasts.

Most of the presumed requirements for science seem to be met with the exception of the success of and the respect shown to the natural sciences.

5.3 STRUCTURAL VERSUS NONSTRUCTURAL MODELS

The recent boom in the area of time series analysis has greatly advanced our ability to forecast economic time series. Economic time series analysis is particularly powerful because the "typical spectral shape" (Nerlove 1964; Granger, 1966) of economic variables is well described by certain parsimonious classes of models, such as the Box and Jenkins (1970) multiplicative seasonal ARIMA model or the popular "unobserved components" (trend, cycle, seasonal, irregular) model as developed by Persons (1919, 1925) and refined by Nerlove (1967); Engle (1978); Nerlove, Grether and Carvalho (1979); Harvey (1984), amongst others.

Short term forecasts from these models have generally been found to be superior to conventional econometric (structural) forecasts in term of mean squared prediction error and other reasonable criteria (see Diebold and Pauly, 1987). However, this fact does not mean that structural forecasts should be discarded in favour of nonstructural time series approaches, basically because of the following three reasons.

First, it is well known that structural econometric models enable behavioural simulation and the study of policy issues which are impossible to analyse with a nonstructural model. Second, it has been argued by McNees (1982) that the "economic fundamentals" and nonlinearities contained in most econometric models have the most impact in the medium to long run, making those models most useful for forecasting over longer horizons. Finally, recent results in the theory of combining forecasts suggest that, even for very short term forecasting, various candidate models such as nonstructural/time series, structural/econometric, and expert consensus may all prove valuable.

5.4 STRUCTURAL CHANGE AND THE COMBINATION OF FORECASTS

Forecasters are constantly challenged by continuous structural changes in the relationships of interest. As Makridakis et al. (1984) once wrote,

> "... in reality there are constant changes, structural shifts in the economy, changes in attitudes, political moves that alter established trends, new technological developments, and the like, which cause existing patterns to change and existing relationships to shift. Forecasting must, therefore, accept that structural changes in the data are and will be taking place. Otherwise, it will not be a relevant and practical field. The major question, then, becomes how the various methods perform under a continuously changing environment. There is little interest in knowing which methods perform the best in fitting a model to a set of data. The most important and relevant aspect of forecasting is to know the methods which can minimize the post-sample forecasting errors."

Naturally, model-builders seek to identify structural changes, in the process of model specification and they generally attempt to incorporate extraneous adjustments in the forecast to account for those shifts not yet modelled in a nonparametric way. Nevertheless, forecasters remain susceptible to changes in the environment. Furthermore, various candidate models, such as different structural econometric models, nonstructural time series models or expert consensus forecasts may turn out to be vulnerable to structural change in different degrees.

It has been said that the ultimate test of any econometric model is its performance as a forecasting device. Forecasting and simulation are the two most important uses of econometric models, and any new technique which significantly enhances their performance in one or both of these areas is a welcome addition to the economist's tool kit.

Diebold and Pauly (1989) suggested that the techniques of forecast combination can be used successfully to partially alleviate the effects of structural changes on forecasting performance. In their pioneering work, Bates and Granger (1969) showed that if a number of unbiased forecasts of the same variable are available, then it is rarely (if ever) optimal to seek out the best of the competing forecasts and use it alone. Rather, the forecasts can always be combined in such a way that the composite forecast has (asymptotic) variance less than or equal to any of the competing forecasts; in that sense, all sources of information may prove valuable. Similar reductions in mean squared error may be achieved for (possibly) biased forecasts via the regression-based technique of Granger and Ramanathan (1984).

The basic concept of combining has been extended in various directions. Of most important concern are those efforts that are directed toward allowing the combining weights to be flexible over time. Diebold and Pauly (1989) viewed the explicit modelling of nonconstancies in the combining weights as an attempt to compensate for the poor performance of the primary forecasts in situations of structural change of unknown form. In many situations, such an approach yields powerful increases in forecasting performance because the available primary forecasts do not adequately account for structural change. Furthermore, even if it is desired to model structural change explicitly in the primary forecasts, it is often difficult (or impossible) to locate and compensate for the changing structure, particularly in an ongoing forecasting organisation where timely forecasts must be produced.

Diebold and Pauly (1987: 21-40) presented alternative ways to model nonconstancy of weights within the framework of regression-based combining methods, which includes weighted least squares and various forms of varying - coefficient models. These models are more general than, and include as special cases, time-varying, variance covariance methods. They also outline testing procedures for various aspects of these models.

Recently, combining forecasts is also discussed by Makridakis (1991) as a strategy to be used to improve forecasting of structurally changing economic systems. He showed that model selection should be based on the actual out-of-sample forecasting performance. Different forecasting horisons are used to identify not only the preferred forecasting method but also "best" forecasting models. This concept will certainly improve the possibility to cope with structural change problems in economic forecasting.

5.5 COMBINING FORECASTS USING ARCH MODELS

Despite the extensive literature on ARCH and related models (see Chapter 8), relatively little attention has been given to the issue of forecasting in models where time-dependent conditional heteroscedasticity is present.

Economists frequently have available several different forecasts for a single quantity, each of which are based on a different information set. For example, forecasts of inflation may be available from a univariate ARIMA model, a single reduced form regression model or any of a number of large macro-econometric models. If one considers the following models;

$$y_t = f_1 (x_{t-1}, \theta_1) + e_{1t} \tag{5.1}$$

$E(e^2_{it}) = \sigma_1{}^2$

$$y_t = f_2 (z_{t-1}, \theta_2) + e_{2t} \tag{5.2}$$

$E(e^2_{2t}) = \sigma_2{}^2$ and $E(e_{1t}e_{2t}) = \sigma_{12}$

and supposes one observes unbiased forecasts f_{1t} and f_{2t} from these models - how does one choose a forecast for y_t?

The approach taken by Granger and Newbold (1977) was to consider the weighted average

$$f_{ct} = w\, f_{1t} + (1 - w)\, f_{2t} \tag{5.3}$$

They show that the variance of f_{ct} is minimised when

$$w = (\sigma_2^{\,2} - \sigma_{12}) \,/\, (\sigma_1^{\,2} + \sigma_2^{\,2} - 2\sigma_{12}) \tag{5.4}$$

Engle, Granger and Kraft (1984: 151-165) allowed these weights, used in combining the forecasts, to vary over the sample period by allowing the conditional covariance matrix of the forecast error to change. The behaviour of the covariance matrix is modelled as a bivariate version of the autoregressive conditional heteroscedasticity (ARCH) model introduced by Engle (1982). The procedure has merit as a pure statistical tool for more effectively combining forecasts. In addition, the way in which the weights vary over time may provide insight into situations where one model will dominate the other. At each point in time the conditional estimates of the variances and covariances are used to construct the optimal weights for combining the forecasts. Consequently, when one model is fitting well, its variance will be reduced and its weight will be increased.

This approach of Engle et al. (1984) make use of the full sample to produce a sequence of time-varying weights in a rigorous and systematic fashion, rather than simply (and artificially) basing the weight calculations on a recent subset of observations. While this approach represent a notable contribution, it has problems of its own. Diebold and Pauly (1989: 301) pointed out that first, it produces an extremely noisy weight sequence, as opposed to the smoothly changing weights argued for by Granger and Newbold (1974). Second, although ARCH - combined forecast does improve upon the individual forecasts, it does not compare favourably with a fixed-weight combination. Engle et al. (1984) noted that this may be due to misspecification of the diagonal bivariate ARCH-model that is used, and that further research in this area is needed.

5.6 SEQUENTIAL ESTIMATION

In making forecasts of future variables, econometricians often use sequential estimation techniques. This method involves fixing the starting date and the initial size of the sample and enlarging it by adding successive observations for re-estimation and prediction as new data become available (see, for example, Fromm and Klein, 1976: 9; and Mees and Rogoff, 1983 and 1985). It has been suggested that such a procedure improves forecast accuracy for two reasons: first, because a larger sample reduces the variance of fixed coefficient estimators; and second, because sequential estimation or "rolling" captures any variation in coefficients.

The logic underlying this conventional wisdom - that one should always use all the available observations in estimations and prediction - is more ambiguous than generally realised.[12] Without proper theoretical justification, like an explicit risk minimising motivation (favouring good predictions), a procedure that sequentially updates estimates of coefficients and predictions in a model, assumed to have constant parameters, is meaningless. Swamy and Schinasi (1989: 1-17) demonstrated that forecast accuracy is not necessarily improved when fixed coefficient models are sequentially re-estimated and used for prediction, after updating the database with the latest observation(s). It is argued that although sequential estimation may minimise the variances of predictors based on some classes of estimators, sequential estimation does not necessarily yield accurate predictions. A corollary of the demonstration is that a predictor of a value with a smaller variance need not be better than another with a larger variance of the same value.

[12]This conventional wisdom is also not fully supported by the asymptotic theory, some simple normal cases apart. For example, Lehmann (1983: 352-388) analysed various nonnormal situations showing that a parameter is more efficiently estimated even in large samples by discarding some sample observations than by using all the available observations.

Swamy and Schinasi (1989) argued that if the objective of estimation is forecast accuracy, then one should prefer predictions that are close to actual realisations to those that are close to some other quantities (such as the mean of the predictors). Consequently, one should select the predictor that has the highest probability of taking values close to actual realisations. Although it is difficult to derive predictors based on this general criterion, a necessary condition for a predictor to take values close to actual realisations with the highest probability is that its mean square error - MSE (i.e.. predictors expected squared deviation from the actual realisation) is a minimum. Sequential estimation may minimise the forecast variance among all predictors in a broad class of predictors, but minimising variance does not necessarily minimise the mean square error. This minimisation of MSE need not require, and may even exclude, the most recent data. Hence, for any given predictor, "rolling" may reduce its variance, but does not necessarily improve, and may even diminish, forecast accuracy (i.e., the distance from a prediction to the actual realisation).

Rather than "rolling" a fixed-coefficient regression, one may improve accuracy by using a different procedure that permits temporal changes in regression slopes. Swamy and Schinasi (1989) have shown that a prediction based on a nonsequential estimate of a stochastically varying coefficient model is superior to predictions based on several sequential estimates of the fixed-coefficient models, including a random walk model.

5.7 CONCLUSION AND FUTURE RESEARCH DIRECTIONS

There is merit in considering certain parameter effects as fixed in economic models. Their principal advantage is simplicity in providing insights about economic interrelationships, unclouded by the meticulous details of a changing real world. There is also, however, merit in obtaining more accurate representations, forecasts and economic policy evaluations. Hence, from a research strategy standpoint, the principal issue faced in deciding whether to employ a constant or varying parameter formulation depends critically upon the trade-off between inaccuracy and complexity.

The research is currently proceeding in a number of directions. First, systematically time-varying parameter models, such as the random parameter model, which can be conveniently estimated using the Kalman filter, are considered. The Kalman filter also facilitates real-time parameter "updating" and can readily handle both stationary (e.g. ARMA) and nonstationary (e.g., integrated ARMA) parameter shift.

Secondly, while the combined weights of Diebold and Pauly (1987a) enable quick adaptation to structural change, they may be unduly influenced by outliers, so that robust estimation methods, such as least absolute deviations or m-estimation, may prove useful for the combining equation. Recent experience with the combination of real macroeconomic time series (see Kang, 1986, or Clemen and Winkler, 1986) indicated that instability in the combining weights may be caused by severe multicollinearity of the primary forecasts due to overlapping information sets. In Diebold and Pauly (1987b) these issues are explored by using Bayesian shrinkage techniques to incorporate prior information into the estimation of combining weights.

CHAPTER 6

ADAPTIVE ESTIMATION AND STRUCTURAL CHANGE IN REGRESSION AND TIME SERIES MODELS

6.1 INTRODUCTION

The coefficients in statistical models, such as regression, time series, ARMAX and econometric models, are usually assumed to be constant. Under the assumption of parametric stationarity, statisticians have developed methods for the efficient estimation of the underlying parameters, for example least squares, maximum likelihood or Bayesian methods can be used to estimate the underlying coefficients.

In applied work it is often recognised and anticipated that relationships change over time. For example, the relationships before and after a certain event or intervention may differ; the behavioural characteristics may change and drift with time; and linear approximations to complex, nonlinear and poorly defined phenomena may exhibit time-varying structures.

If the parameters are expected to shift at a given point in time from one value, say $\boldsymbol{\beta}$, to some other value, say $\boldsymbol{\beta}^* = \boldsymbol{\beta} + \boldsymbol{\delta}$, then it is usually quite easy to formulate a more general model, estimate the change in the coefficients and test whether the change is significant (see e.g. the Chow test, discussed in Chapter 4).

Parameters may also change continuously over time - a topic which is addressed in this chapter. Instead of assuming that the coefficients are constant or shift from one value to another, is assumed that they drift and vary continuously over time. Recursive estimation plays an important role as recursive parameter estimates provide information on the existence of nonstationarity. Furthermore, it is quite easy to modify recursive estimation procedures such that more weight is given to the most recent observations and less to the ones in the distant past. Modifications may be heuristic, such as various weighted least squares approaches where the weights decrease with the age of the observation. Or, one can assume that the parameters follow certain stochastic models. For example, one can assume that the coefficients change smoothly according to a random walk ($\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \mathbf{v}_t$, where $\mathbf{v}_t$ are independent disturbances) and that one can derive at each time point t the optimal estimates of the coefficient $\boldsymbol{\beta}_t$ in this more general model.

The heuristic approach for modelling time-varying coefficients has been developed by Makridakis and Wheelwright (1977, 1978). Their approach, known as adaptive filtering, consists of a heuristic recursive algorithm that revises the coefficient estimates as each new observation becomes available. The weights in these recursions are chosen such that the coefficient estimates adapt more quickly to changes in the underlying parameters. The disadvantages with heuristic approaches (either the so-called moving rectangular or the moving exponential window function) are (1) that each coefficient is treated the same way, and (2) that they require an *ad hoc* choice of additional constants - either the length of the moving window or the exponential discount coefficient W (see Ledolter, 1989). Instead of such heuristic approaches, one can adopt a model-based approach to recursive estimation.

6.2 MODEL-BASED RECURSIVE ESTIMATION

The Kalman filter (KF) commonly employed by control engineers and other physical scientists has been successfully used in such diverse areas as the processing of signals in aerospace tracking and underwater sonar, and the statistical control of quality. More recently, it has also been used in some nonengineering applications such as short-term forecasting and the analysis of life lengths from dose-response experiments. Unfortunately for econometricians, much of the published literature on the KF is in engineering journals (including the original development of Kalman, 1960; and Kalman and Bucy, 1961), and uses a language, notation and style that is alien to statisticians. Consequently, many practitioners of statistics are not aware of the simplicity of this useful methodology. However, the model, the notions and the techniques of Kalman filtering are potentially of great interest to statisticians owing to their similarity to linear models of regression and time series analysis and because of their great utility in applications (good survey papers of the KF models that have been developed by econometricians and statisticians can be found in Theil, 1971; and Sarris, 1973; while Åström and Eykhoff (1971) have surveyed the methods that have been developed primarily in system theory).

In the model-based approach to recursive estimation we assume a probabilistic model for the time-varying coefficients and that the regression coefficients $\boldsymbol{\beta}_t$ follow certain ARIMA time series processes. For example, in the simplest case we assume that they follow a random walk,

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \mathbf{v}_t \tag{6.1}$$

where $\mathbf{v}_t$ are independent random variables with mean vector zero and a certain covariance matrix. Random walks generate smoothly time-varying coefficients. The most useful case in practice is the one where the covariance matrix is diagonal, which implies that the coefficients vary independently from one another. The diagonal elements control the variability of the coefficients, if a diagonal element is zero, then the corresponding coefficient is constant. If it is different from zero, the coefficients vary smoothly over time.

The random walk is a special but very useful model. In econometric applications, where we deal with mostly short series, it is usually difficult to identify more complicated models for the underlying unknown regression coefficients. In theory one can always work with more elaborate model specifications and assume that the coefficients in the regression model

$$y_t = \mathbf{x}_t'\boldsymbol{\beta}_t + e_t \tag{6.2}$$

$t = 1,\ldots,T$

follow the difference equation model

$$\boldsymbol{\beta}_t = \mathcal{T}\boldsymbol{\beta}_{t-1} + \mathbf{v}_t \tag{6.3}$$

For the T equations, y_t is the response in an experiment where the levels of the p regressors are set to $x_{t1},\ldots,x_{tp}$. The first regressor x_{t1} is taken to be equal to unity for all values of t if the model contains a constant. The column vector of the $\boldsymbol{\beta}_t$ parameters is written with subscript t to indicate that it may vary over time. The error terms $e_1,\ldots,e_t$ are assumed to be iid $N(0, \sigma_t{}^2)$ for $t = 1,\ldots,T$; $\mathcal{T}$ is a known (p x p) transition matrix.

Equation (6.3) is a multiple AR(1) model and includes the random walk model in (6.1) as a special case (namely, when $\mathcal{T}$ =

I, where I is the identity matrix). It can also be shown that any multiple ARMA(p,q) model of the form

$$\boldsymbol{\beta}_t = \phi_1\boldsymbol{\beta}_{t-1} + \ldots + \phi_p\boldsymbol{\beta}_{t-p} + \mathbf{v}_t - \theta_1\mathbf{v}_{t-1} - \ldots - \theta_q\mathbf{v}_{t-q} \tag{6.4}$$

can be written as such a difference equation. Here the matrices $\phi_1,\ldots,\phi_p$, and $\theta_1,\ldots,\theta_q$ contain the autoregressive and moving average parameters (see Hannan, 1970); and Tiao and Box, 1981 for further discussions). Furthermore, it is possible to rewrite the ARMA(p,q) model in Equation (6.4) as an autoregressive model but of larger dimensions (see Abraham and Ledolter, 1983 or Harvey, 1981).

As simple substitution shows, one can write the ARMA(p,q) model in (6.4) as

$$\begin{bmatrix} \boldsymbol{\beta}^*_t \\ \boldsymbol{\beta}^*_{t,2} \\ \vdots \\ \boldsymbol{\beta}^*_{t,k-1} \\ \boldsymbol{\beta}^*_{t,k} \end{bmatrix} = \begin{bmatrix} \phi_1 & I & \mathbf{0} & \ldots & \mathbf{0} \\ \phi_2 & \mathbf{0} & I & \ldots & \mathbf{0} \\ \vdots & \vdots & \vdots & & \vdots \\ \phi_{k-1} & \mathbf{0} & \mathbf{0} & \ldots & I \\ \phi_k & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta}^*_{t-1} \\ \boldsymbol{\beta}^*_{t-1,2} \\ \vdots \\ \boldsymbol{\beta}^*_{t-1,k-1} \\ \boldsymbol{\beta}^*_{t-1,k} \end{bmatrix} + \begin{bmatrix} I \\ -\theta_1 \\ \vdots \\ -\theta_{k-2} \\ -\theta_{k-1} \end{bmatrix} \mathbf{v}_t$$

or $$\boldsymbol{\beta}^*_t = T\,\boldsymbol{\beta}^*_{t-1} + \mathbf{v}^*_t \tag{6.5}$$

where $k = \max(p,q+1)$ and $\phi_j = \mathbf{0}$ (matrix of zeros) for $j > p$ and $\theta_j = \mathbf{0}$ for $j > q$.

After extending the coefficient vector $\boldsymbol{\beta}_t$ to $\boldsymbol{\beta}_t^*$, one can write the regression model with the ARMA time-varying coefficients given in (6.4) as

$$y_t = (\mathbf{x}_t', \mathbf{0}', \ldots, \mathbf{0}')\boldsymbol{\beta}_t^* + e_t$$

$$\boldsymbol{\beta}_t^* = T\,\boldsymbol{\beta}_{t-1}^* + \mathbf{v}_t^* \tag{6.6}$$

where T, $\boldsymbol{\beta}_t^*$ and $\mathbf{v}_t^*$ are defined above.

The question is how to estimate and update the regression coefficients at time n, knowing that these coefficients are not constant, but follow this more general time-varying coefficients model.

Kalman filter equations can be used to estimate and update the coefficient estimates (see Kalman, 1960; and Kalman and Bucy, 1961). There are two equations in the system in (6.2) and (6.3). The first Equation (6.2) is called the measurement equation; it describes the generation of the observation from a given state vector, which in this case is the vector of unknown coefficients. The second Equation (6.3), is called the system (or transition) equation, it describes the evolution of the state (coefficient) vector. There are two error (noise) components: the measurement and process noise. It is assumed that e_t and v_t are two independent white noise sequences with zero means and variances σ^2 (defined in Equation (6.8)) and $\sigma^2\Omega$, respectively. The matrix Ω is a matrix of variance ratios, $Var(v_t)/Var(e_t)$, which relates the variability in the coefficients to the variability of the measurement noise.

It is assumed that the noise sequences are from normal distributions and that the initial state (coefficient) vector at time zero, $\boldsymbol{\beta}_o$, follows a normal distribution with mean vector, say $\hat{\boldsymbol{\beta}}_{o|o}$ and covariance, say $\sigma^2 P_{o|o}$. This can be thought of as a prior distribution. It could also be shown that the conditional distribution of $\boldsymbol{\beta}_t$, given the data up to time t-1, i.e., $\mathbf{y}_{t-1} = \{y_{t-1}, y_{t-2}, \ldots, y_1\}'$, and the conditional distribution of $\boldsymbol{\beta}_t$ given the data up to time t, i.e., $\mathbf{y}_t = \{y_t, y_{t-1}, \ldots, y_1\}'$, are normal. This, however, requires that the parameters T, σ^2, Ω, the initial values $\hat{\boldsymbol{\beta}}_{o|o}$, $P_{o|o}$ and the values of the explanatory variables are known. These parameter sets must be specified before minimum mean squared linear estimators of the coefficient vector $\boldsymbol{\beta}_t$, and hence the dependent variable y_t, can be obtained. McWhorter et al. (1976) analysed the effects of misspecifying these parameters, while Sarris (1973) and Abraham and Ledolter (1983: 367) discussed maximum likelihood procedures for selecting Ω.

There exist convenient updating equations that revise the means and covariance matrices; these are known as the Kalman filter (KF) equations. If the mean vector and covariance matrix of the conditional distribution of $\boldsymbol{\beta}_t$ given y^t is denoted by $\hat{\boldsymbol{\beta}}_{t|t}$ and $\sigma^2 P_{t|t}$ respectively, and the ones for the conditional distribution of $\boldsymbol{\beta}_t$ given $\mathbf{y}^{t-1}$ by $\hat{\boldsymbol{\beta}}_{t|t-1}$ and $\sigma^2 P_{t|t-1}$ respectively, then the KF equations can in general be written as:

$$\hat{\boldsymbol{\beta}}_{t|t-1} = T\, \hat{\boldsymbol{\beta}}_{t-1|t-1}$$

$$P_{t|t-1} = T\, P_{t-1|t-1}\, T' + \Omega$$

$$\hat{\boldsymbol{\beta}}_{t|t} = \hat{\boldsymbol{\beta}}_{t|t-1} + \mathbf{k}_t (y_t - \mathbf{x}_t' \hat{\boldsymbol{\beta}}_{t|t-1})$$

$$P_{t|t} = P_{t|t-1} - \mathbf{k}_t \mathbf{x}_t' P_{t|t-1}$$

$$\mathbf{k}_t = P_{t|t-1} \mathbf{x}_t (1 + \mathbf{x}_t' P_{t|t-1} \mathbf{x}_t)^{-1} \tag{6.7}$$

and σ^2 is always estimated by:

$$\hat{\sigma}^2 = (1/T) \left[\sum_{t=1}^{T} y_t - \mathbf{x}_t' \hat{\boldsymbol{\beta}}_{t|t-1})^2] \,/\, f_t \right]$$

where $f_t = (1 + \mathbf{x}_t' P_{t|t-1} \mathbf{x}_t)$

(6.8)

If $\boldsymbol{\beta}_{t-1} = \boldsymbol{\beta}_{t-1|t-1}$ and $P_{t-1} = P_{t-1|t-1}$ Equations (6.7) can be summarised as

$$\hat{\boldsymbol{\beta}}_t = \hat{\boldsymbol{\beta}}_{t-1} + [1 + \mathbf{x}_t' (P_{t-1} + \Omega)\mathbf{x}_t]^{-1}(P_{t-1} + \Omega)\mathbf{x}_t(y_t - \mathbf{x}_t' \boldsymbol{\beta}_{t-1})$$

$$P_t = (P_{t-1} + \Omega) - [1 + \mathbf{x}_t' (P_{t-1} + \Omega)\mathbf{x}_t]^{-1}P_{t-1}\mathbf{x}_t\mathbf{x}_t' P_{t-1} \tag{6.9}$$

In the regression with constant coefficients ($\Omega = 0$), (6.7) simplify to

$$\hat{\boldsymbol{\beta}}_t = \hat{\boldsymbol{\beta}}_{t-1} + [1 + \mathbf{x}_t' P_{t-1}\mathbf{x}_t]^{-1}P_{t-1}\mathbf{x}_t(y_t - \mathbf{x}_t' \boldsymbol{\beta}_{t-1})$$

$$P_t = (P_{t-1} - [1 + \mathbf{x}_t' P_{t-1}\mathbf{x}_t]^{-1}P_{t-1}\mathbf{x}_t\mathbf{x}_t' P_{t-1} \tag{6.10}$$

These updating equations are similar to Plackett's (1950) expression for recursive least squares.

For given values of T and Ω and for given starting values $\beta_{o|o}$ and $P_{o|o}$, one can use the recursive Kalman filter equations and update $\hat{\beta}_{t|t-1}$, $P_{t|t-1}$, and f_t. To start the recursions, one has to choose the starting values $\hat{\boldsymbol{\beta}}_{o|o}$ and

$P_{0|0}$. To reflect ignorance about the parameters at time zero, $P_{0|0}$ is usually taken as a diagonal matrix with large diagonal elements and $\hat{\boldsymbol{\beta}}_{0|0}$ is taken as the zero vector. The above Equations (6.7) show how to revise the estimates. The first equation provides the prediction of the next parameter estimate. From $\hat{\boldsymbol{\beta}}_{t-1|t-1}$ (which is the estimate of $\boldsymbol{\beta}_{t-1}$ given data up to time t-1), $\hat{\boldsymbol{\beta}}_{t|t-1}$ is computed, which is the "projected" estimate of $\boldsymbol{\beta}_t$ given the data up to time t-1. The third equation shows how to update this estimate and illustrates the calculation of $\hat{\boldsymbol{\beta}}_{t|t}$ after the most recent observation y_t has become available. The Kalman gain vector $\mathbf{k}_t$ in this recursive updating equation depends on T, Ω and the past data. It determines how much weight is given to the most recent one-step-ahead forecast error $y_t - \mathbf{x}_t' \hat{\boldsymbol{\beta}}_{t|t-1}$ (These forecast errors are also called the innovations, since they represent the new information brought by y_t, in addition to the information contained in the past).

The recursive equations in (6.7) are explained from a Bayesian point of view following the approach of Ho and Lee (1964). Kalman has derived these equations from an orthogonal projection argument, while yet others (see Duncan and Horn, 1972) use a generalised least squares argument to derive these recursions.

The case when $T = I$ (that is, the regression coefficients follow a random walk) has been treated extensively by Athens (1974) and Harrison and Stevens (1976).

Generally, papers concerned with the Kalman filter models are devoted to demonstrating the relevance of that model to economics or regression theory (e.g., Athans, 1974; Duncan and Horn, 1970; Mehra, 1974), extending the theory (e.g., Belsley, 1973; Cooper, 1973; Rosenberg, 1973a,b and c; Sarris, 1973; Hannan, 1976; Ljung and Söderström, 1983; Söderström and Stoica, 1983; Bittanti et al., 1985; Ledolter, 1989; Ansley and Kohn, 1983, 1984, 1985 and 1986; Young, 1984 and 1985; and Ljung, 1985), or presenting applications of the model (e.g., Bowman and LaPorte, 1972; McWhorter et al., 1973 and 1977; Rosenberg, 1968; Schulman, 1973; Teräsvirta, 1970; Bao et al., 1985; Watson and Engel, 1983; Burmeister et al., 1986; Otter, 1978; Abraham and Ledolter 1983; McNeils et al., 1981; Conrad and Corrado, 1979; Goodrich et al., 1985; Mariano and Schleicher, 1972; Baudin et al., 1984; Wolff, 1987; Lukashin, 1989). The books by Jazwinski (1970) and Young (1984), and the paper by Meinhold and Singpurwalla (1983) include excellent reviews of this topic.

It will be indicated, in the chapters to follow, that the Kalman filtering algorithm does have potential use for an important class of economic problems, namely those involving the refinement of the parameter estimates (and their variances) in an econometric model. Right at the start it must be emphasised that the use of the Kalman filtering techniques is viewed not as a replacement, but rather as a supplement, to traditional econometric methods. We visualize that the Kalman filtering methods should become useful only after an econometrician has constructed the mathematical model of a microeconomic or macroeconomic system. Thus it may represent a final fine tuning of the econometric model.

6.3 TIME SERIES ANALYSIS AND STRUCTURAL CHANGE

Since the purpose of this study is to investigate structural econometric models, no detailed discussions will be given on the topic of time series analysis and structural change. Nevertheless, for the sake of a broader perspective the following introductory remarks with regard to the topic may prove worthwhile.

Broemeling (1989) introduced changing - parameter ARMA processes as a way to model time series. Traditional model building through time series analysis or econometrics often presupposes stationary time series although most series in practice require one or more differentiations to attain stationary. Modern approaches to time series analysis assume that the realisation or a transformation of the data was generated by an ARMA process that is stationary and invertible. If the data exhibit a changing trend or an unstable covariance, differencing the data will often induce stationarity and one can assume an ARMA process generated the data. This is the technique used in Box-Jenkins (1970) time series. However, sometimes no transformation can induce stationarity.

Broemeling (1989) showed that if the data exhibits a changing trend or a changing autocorrelation structure, these characteristics can be captured by an ARMA process having changing parameters. The analysis of a changing parameter process is accomplished by a Bayesian approach, where the posterior distributions of the parameters are derived, along with a moving average model that has a changing autocorrelation function. The author's presentation opens the door to many interesting problems. The only question is: does a changing - parameter ARMA process offer a viable alternative to the usual way of doing a time series analysis?

Finally, Hamilton (1989) proposed a stochastic switching regime model and suggested modelling the trends in nonstationary time series as Markov processes. The nonlinearities with which his paper are concerned arise if the process is subject to discrete shifts in regime. His basic approach is to use Goldfeld and Quandt's (1973a) Markov switching regression to characterise changes in the parameters of an autoregressive process and to apply a Kalman filter type smoother which provide nonlinear inference about a discrete-valued unobserved state vector. Extensions of Hamilton's work can be found in Ghysels (1992) who presented a general class of Markov switching regime time series models of growth cycles and seasonals.

6.4 SUMMARY AND CONCLUSION

It must be emphasised that the problems encountered in applying the Kalman filter model to economic time series contrast sharply with those ordinarily encountered in engineering applications. In the latter T and β_0 are typically given by known physical properties and in the engineering literature most of the attention is directed to an examination of the variance of a_t and v_t. None of the parameter sets T, $\boldsymbol{\beta}_0$, σ^2 and Ω can be regarded as known in economics.

Moreover, a tractable and generally applicable method of simultaneously estimating these parameter sets jointly is not available, even in the special case where they are all identifiable. Some special results have been developed, however (see next chapter). Rosenberg (1968) provided a maximum likelihood estimator for $\boldsymbol{\beta}_0$; Cooley and Prescott (1973), in what can be regarded as a special case of the Kalman filter model, presented a model in which scale factors associated with transitory and permanent variance components are estimated by maximum likelihood methods through a grid search procedure. However, in their model, T is restricted to be the identity matrix. A special one-dimensional case of the model in Sections 6.2 and 6.3 was also considered by Abraham and Ledolter (1983).

Thus, in many economic applications of the Kalman model, one must make ad hoc prior specifications of the parameter sets T, $\boldsymbol{\beta}_{0|0}$, Ω, $P_{0|0}$, σ^2, or, in special cases, one can use formal estimation methods to obtain initial specification of these parameters. Either of these can lead to misspecifications of the parameter sets, and consequences of such misspecifications have not been considered extensively except for an article by Sage and Melsa (1971) and McWorther et al. (1976) who, in particular, studied by means of stochastic simulation the sensitivity of the Kalman filter model to changes in the specification of these parameters. The Kalman filter technique as a tool to estimate stochastic parameters will be further discussed in Section 7.5, which contains a survey of various econometric model formulations in which it is assumed that coefficients vary over time in a structural way. For extensive bibliographies and various aspects of the problem the reader can consult the textbooks of Sage and Melsa (1971), Åström (1970) and Abraham and Ledolter (1983), as well as the special issue of the IEEE (1971) Transactions on Automatic Control.

CHAPTER 7

VARYING AND RANDOM COEFFICIENT MODELS

7.1 INTRODUCTION

In classical econometric modelling, it is assumed that an economic structure generating a statistical sample remains constant. This implies, explicitly or implicitly, the existence of several assumptions:

(a) a unique functional form of the econometric model;

(b) a unique parameter vector connecting the endogenous variable with the set of independent variables; and

(c) one set of parameters of stochastic processes generating the model's disturbances.

The above assumptions define a very important property of the econometric model - namely "model stability". In other words, in an unstable model, the structural parameters, the disturbance distribution or the model's analytical form may not be the same for all the sample observations. This chapter deals mainly with the stability of structural parameters.

Economics belongs to a group of nonexperimental sciences and econometricians have to work with statistical samples that are generated in uncontrollable economic processes and frequently under unobservable conditions. Many attempts to model economic relationships have been unsuccessful, and some of those failures were caused by parameter stability problems. Through the years, researchers noticed that different samples yielded different sets of the model's parameter estimates and that the parameterisation of the statistical model changes as the sample size grows. In this context, the constant parameter assumption is not obvious.

A way to deal with this problem is to attempt to isolate separate groups of homogeneous observations. The different treatment of pre- and postwar data can serve as a typical example. Unfortunately, although economists are aware of the varying parameter problem, they often fail to restrict their attention to at least approximately homogeneous data sets, generated under stable economic circumstances. It has to be noted, however, that some aspects of structural instability are commonly recognised. One of the early attempts to deal with this problem was the introduction of dummy variables in order to represent seasonal, institutional or other structural differences. Although dummy variables make a convenient tool, their use often results in inaccurate forecasts.

In recent years much effort has been devoted to the creation of a more general approach to this problem of parameter variation, proper estimation, and testing techniques for new types of models. Quandt's papers (1958, 1960) initiated research to find new methods of uncovering and handling the parameter instability of models.

7.2 DEFINING AN ADEQUATE MODEL

An adequate model in general means that a model's equation represents reality and that the necessary conditions to apply statistical tools in order to test model's properties are fulfilled.

Checking the model's efficiency and accuracy means testing relative statistical hypotheses. Many types of model shortcomings are considered in the literature. Hackl (1980) listed the following:

(a) one or more important regressors are omitted;

(b) the functional form of one or more regressors are incorrectly specified;

(c) the model is unstable (i.e. structural parameters and/or disturbance distribution vary for different observations);

(d) disturbances for different observations are correlated; or

(e) disturbances have other than the normal distribution.

These problems cannot be considered separately; they are often closely connected. Although classical models will remain the basic tool in econometrics, the need quite frequently arises to represent a more detailed picture of the model processes. It is therefore, extremely important to choose properly between models with constant and with varying parameters. Such a decision is a compromise between complexity connected with accuracy and simplicity connected with inaccuracy.

Once there is agreement that varying parameter formulation applies, it is generally assumed that model parameters are generated by a nonstationary random process, which means that parameters do not have a constant mean and/or variance and can vary systematically. Such models are relevant mainly for modelling the systematic structural variation in time.

A special case of this general formulation is a model of which the parameters are generated by a stationary random process. In this case parameters do have constant mean and variance, and therefore they do show systematic change for different observations (time, units). Such a model is relevant mainly in modelling cross-sectional data and the time series of cross-sections (see Rosenberg, 1973a). In particular, it is assumed that cross-sectional units have the same regression regimes, which are constant in time. Individual reactions of particular units in different time periods are treated as a random selection from a parameter population with constant mean. In the simplest situation, parameter variance is zero, which yields models with constant parameters.

Heteroscedasticity might be a reasonable assumption when using cross-sectional data while autocorrelation frequently occurs when using time series data. Thus, when combining the two types of data, it seems reasonable to set up a model that captures both effects.

The linear regression model can broadly be devided into three major groups (see Diagram 7.1 on the next page):

A: model with all coefficients constant and e_{it} possibly heteroscedastic and autocorrelated;

B: models with constant slope coefficients and intercepts that vary over individuals and/or time, also called fixed coefficient models;[13] and

C: models with variable coefficients, called changing or random coefficient models.

Depending on the accepted parameter variation structure one may further classify random coefficient models (in group C above)

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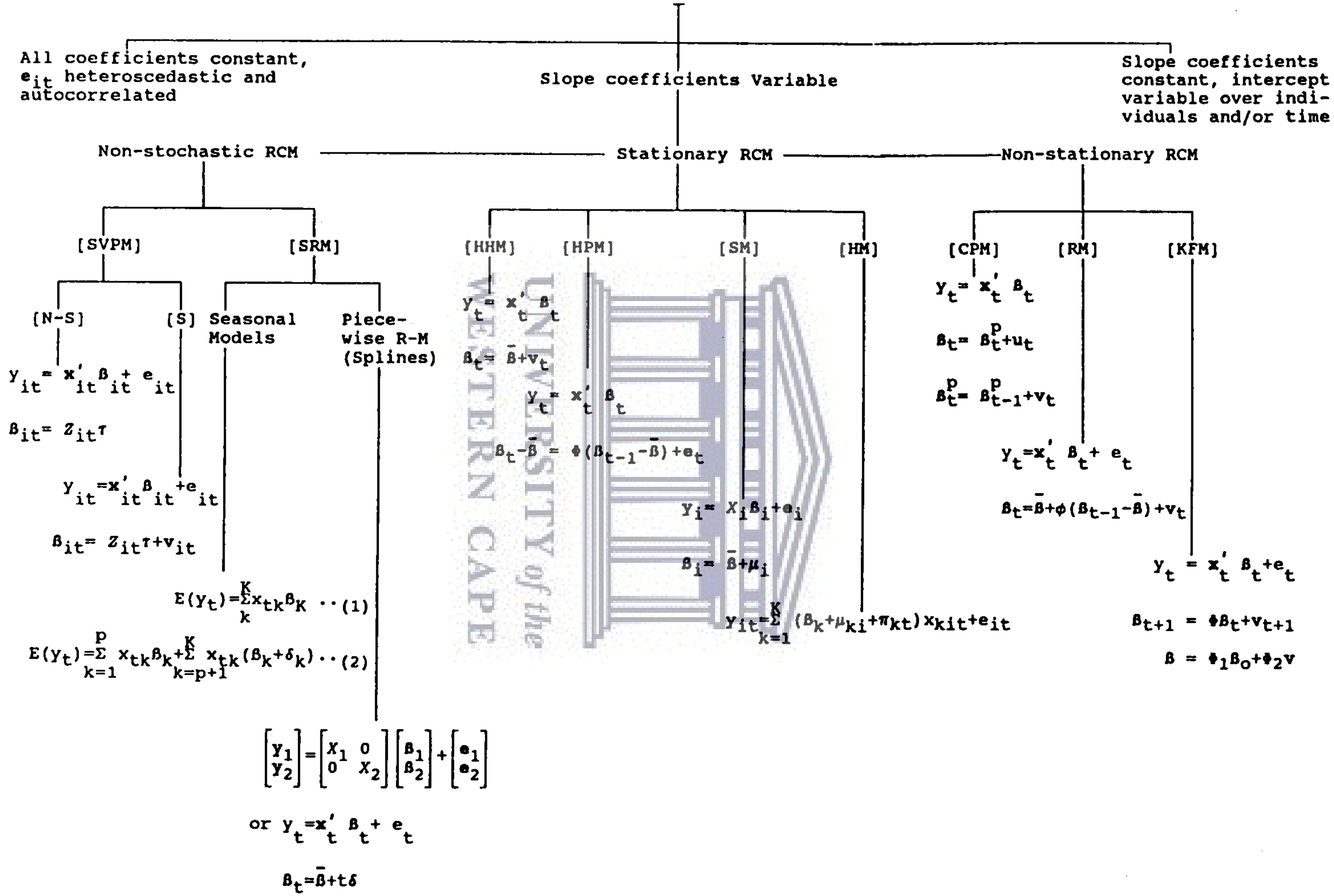
[13] Only some general remarks and literature references will be given, since models with constant slopes and variable intercepts are not the topic of this study. Fixed coefficient models can also be regarded as non-stochastic random coefficient models.

DIAGRAM 7.1

ALTERNATIVE VARYING AND RANDOM COEFFICIENT MODELS FOR THE LINEAR MODEL: $y = X\beta + e$



Key to abbreviations:

CPM	Cooley-Prescott Model	R-M	Regression Model
FCM	Fixed Coefficient Models	RCM	Random Coefficient Models
HHM	Hildreth-Houck Model	RM	Rosenberg Model
HM	Hsiao Model	S	Stochastic
HPM	Harvey-Phillips Model	SM	Swamy Model
KFM	Kalman Filter Models	SRM	Switching Regression Models
N-S	Non-stochastic	SVPM	Systematically Varying Parameter Models.

into three main groups with several subgroups. First, the parameters can vary across subsets of observations within the sample but be nonstochastic. Examples of such models are discussed in the next section and include a general systematically varying parameter model, seasonality models and a variety of "switching regression" models where the sample observations are generated by two (or more) distinct regimes. A second class of models is where the parameters are stochastic, and can be thought of as being generated by a stationary stochastic process. In stationary stochastic parameter models, the coefficients have a constant mean and variance and hence do not vary systematically over observations. Finally, the third class of models consists of those where the stochastic parameters are generated by a process that is not stationary. In contrast, nonstationary stochastic parameter models do not have a constant mean and/or variance and the coefficient can change from one observation to the next. Stationary stochastic parameter models are usually used to model differences over microunits, while nonstationary parameter models are used to reflect systematic changes over time. Random coefficient models may therefore be summarised as follows:

C1. Non-stochastic varying parameter models

(a) Systematically varying parameter models

(b) Switching regression models

C2. Random coefficients from a stationary process:

(a) The Hildreth-Houck random coefficient models

(b) The Harvey-Phillips return to normality models

(c) The Swamy random coefficient models

(d) The Hsiao random coefficient models

C3. Random coefficients from a nonstationary process:

(a) The Cooley-Prescott models

(b) Bar Rosenberg's convergent-parameter models

(c) Kalman filter models

The models in group B belongs to the dummy variable type which are often referred as variance or error component models (also known in the literature as fixed coefficient or fixed parameter models under the heading of non-stochastic random coefficient models for variable intercepts). The additive error term that appears in nearly every conventional fixed coefficients model can be added to its fixed intercept. Thus conventional fixed coefficient models can be viewed as models with random intercept and fixed slopes. Only introductory concepts with regard to the Fixed Coefficient model will be given in Section 7.3.1.

In all the other sections the interest is in the prediction of random components, variance estimators that can be used in a generalised least squares estimator and hypothesis tests for misspecification. Reference to possible alternatives, extensions, modifications and applications of the models will be given in some cases.

It should be emphasised that the simple classification given above and in Diagram 7.1 is to some extent arbitrary, and that in the following sections, we concentrate on inference procedures for the basic models in each category. There are many other ways in which the various models could be classified, and their exist a large number of extended and more complex models, some of which do not fit neatly into one of our categories (for example, the Watson-Engle Variable Parameter Regression model and ARCH models). Reference will later be made to many of these extensions in the chapter. For further details on classifications see the special journal issues edited by Mazodier (1978) and Heckman and Singer (1982), as well as the papers by Belsley (1973) and Chamberlain (1983). Other comparisons of different schemes of parameter variation can be found in Swamy (1971, 1974), Rosenberg (1973a) and Judge et al. (1985).

7.3 VARYING BUT NONSTOCHASTIC PARAMETER MODELS

7.3.1 FIXED COEFFICIENT MODELS

One of the most popular models that allows for differences in behaviour over cross-sectional units or any differences in behaviour over time for a given cross-sectional unit is the model where all coefficients are constant and the disturbance is assumed to capture differences over time and individuals.

These models may be classified further, depending upon whether the variable coefficient is assumed to be random or fixed. The fixed assumption leads to dummy variable models and the seemingly unrelated regression model, while the random assumption leads to the model referred to as the error components model. This type of model is also sometimes called the variance component model. Since models with constant slopes and variable intercept are not of primary interest here, only literature references will be given and some general remarks will be made about them. Introductory discussions concerning the model with constant parameters and a variable intercept may be found in the works of Maddala (1971), Nerlove (1971a), Swamy (1971), Mundlak (1978c), Hausman and Taylor (1981), and Judge et al. (1980, 1982 and 1985).

The model with constant slope coefficients and a variable intercept (the error components model version) may be regarded as the one with random parameters (but some of which - the slopes - are constant), or as one where all coefficients are constant and the disturbance covariance matrix is identical for all individuals. Disturbances in different time periods for the same individual are correlated, but this correlation is constant over time and it is identical for all individuals. Alternatively, the assumption that the intercept may vary over individuals and time may be accepted.

Maddala (1971), Nerlove (1971a), Swamy (1971), and Arora (1973) recommended an estimation technique, which may be regarded as a generalisation of the dummy variable estimator. Balestra and Nerlove (1966), Nerlove (1971b), Swamy (1971), Arora (1973), and Fuller and Battese (1973) discussed some convenient transformations for the estimation of this model. Lee and Griffiths (1979) and Taub (1979) suggested a best linear unbiased predictor for the random components. Battese and Fuller (1982) considered a "best constrained predictor". A number of variance component estimators are suggested; these include estimators based on the ordinary least squares residuals, as seen in Wallace and Hussain (1969), Maddala (1971), Swamy (1971), and Arora (1973). Other estimators are proposed by Henderson (1953, 1975), Fuller and Battes (1973, 1974), Rao (1970, 1972), and Kelejian and Stephan (1983). The maximum likelihood version came from works by Amemiya (1971), Nerlove (1971a), and Maddala (1971). Swamy (1971) and Fuller and Battese (1973) considered the distribution of the various estimators. An important work in this regard is that of Searle (1979). Finite sample properties are investigated by Swamy and Mehta (1979) and Taylor (1980). Arora (1973), Maddala and Mount (1973), and Baltagi (1981) studied some estimators in Monte Carlo experiments.

Breusch and Pagan (1980) suggested a test based on the Lagrange multiplier statistic for testing a hypothesis that state that the intercept is constant for all observations. This is an alternative approach to the classical procedure where a dummy variable estimator is employed jointly with the F-test based on restricted and unrestricted residual sums of squares.

The choice between the assumptions that variable component is either random or fixed is crucial for the choice of the estimation procedure. Mundlak (1978), Chamberlain (1978, 1979, and 1983), and Hausman and Taylor (1981) considered this problem. Wallace and Hussain (1969), Swamy (1971), Nerlove (1971a), Swamy and Arora (1972), and Mundlak (1978c) examined a statistical test that helps to choose between a dummy variable and an error components model. Other sources include Lee (1978b), Chamberlain and Griliches (1975), Hausman (1978) and Pudney (1978).

Mundlak and Yahav (1981) investigated a combined model integrating fixed and random effects. Quite a number of different extensions of this model may be found in the literature. Balestra and Nerlove (1966), Nerlove (1967 and 1971b), Maddala (1971), Trognon (1978), Berzeg (1979), Chamberlain (1979, 1983), Anderson and Hsiao (1981, 1982), Nickell (1981), Sevestre and Trognon (1982), and Bhargava and Sargan (1983) explored problems that occur when a lagged dependent variable is included. An alternative disturbance covariance structure is considered in studies by Hause (1977 and 1980), Glejser (1978), Lillard and Willis (1978), Lee (1978a), Pudney (1978), Lillard and Weiss (1979), Revankar

(1979), Kiefer (1980), Bhargava et al. (1982), MaCurdy (1982), and Schmidt (1983).

Models with discrete and truncated dependent variables are evaluated by Chamberlain (1978, 1979, and 1983), Heckman (1978), Flinn and Heckman (1982), and Singer (1982), as well as by Griliches et al. (1978), Hausman and Wise (1979), Kiefer and Neumann (1981), and Maddala (1978). Error component models with heteroscedasticity are investigated by Mazodier and Trognon (1978); and nonlinear error components models with heteroscedasticity, by Griffiths and Anderson (1982). Other works include Avery (1977), Jöreskog (1978), Baltagi (1980), Magnus (1982), Prucha (1984), Reinsel (1982), and Biorn (1981). Mundlak (1978a) proposed the use of biased estimators with a lower mean square error.

7.3.2 SYSTEMATICALLY VARYING PARAMETER MODELS

In order to efficiently discuss models that have been proposed to account for situations where the response of a dependent variable to a one-unit change in an independent variable is not constant across all observations, a general model will be presented that contains a range of other specifications as special cases.

The most general systematically varying parameter model may be formulated as follows:

$$y_{it} = \mathbf{x}_{it}' \boldsymbol{\beta}_{it} + e_{it} \tag{7.1}$$

where y_{it} is the i-th cross-sectional observation of the dependent variable; $\mathbf{x}_{it}$ is a (K x 1) nonstochastic vector of observations on explanatory variables; $\boldsymbol{\beta}_{it}$ is a (K x 1) coefficient vector, possibly unique to the i-th cross section and t-th time period; and e_{it} are independent and identically normally distributed random variables with zero means and variance $\sigma^2 > 0$, $i = 1,\ldots,N$; $t = 1,\ldots,T$. This formulation of the linear regression model allows the response coefficient for the explanatory variables to differ for each cross-sectional unit and each time period.

The difficulty with Model (7.1) is that there are (KNT + 1) parameters to be estimated with only NT observations available. Additional information must be introduced that places some structure on how the coefficients vary across observations if reasonable estimation procedures are to be developed. Without this, the problem is not tractable. Some typical nonsample information can be introduced. Following Belsley (1973a, 1973b and 1973c), let nonsample information be described by K linear relations :

$$\boldsymbol{\beta}_{it} = Z_{it}\, \boldsymbol{\tau} \tag{7.2}$$

where Z_{it} is the (K x M) matrix of variables that "explain" the variation in β_{it} across observations, τ is an (M x 1) vector of associated coefficients.

In the nonstochastic formulation of the Belsley model under consideration, the Z_{it} is a known, nonstochastic matrix. This means that Equation (7.1) is an exact, rather than stochastic, relation. Combining Equation (7.1) with (7.2) results in

$$y_{it} = x'_{it}\,\beta_{it} + e_{it} = w'_{it}\,\tau + e_{it} \qquad (7.3)$$

where $w'_{it} = x'_{it} Z_{it}$ is the (1 x M) vector of observations or interaction variables. With the assumptions made about e_{it}, the least squares estimator of the τ and β_{it} is BLUE. Thus when Z_{it} is known and nonstochastic, no real difficulties are encountered. On the other hand, if Z_{it} is not known with certainty, exactly the same difficulties that exist when there is uncertainty about the correct set of regressors are faced. Belsley (1973b) presented a traditional test procedure for comparing alternative Z_{it} matrices.

If the variation structure in the model is assumed to be stochastic, as is frequently the case, the β_{it} is given by the following stochastic equation system:

$$\beta_{it} = Z_{it}\,\tau + v_{it} \qquad (7.4)$$

where $\mathbf{v}_{it}$ is the normally distributed disturbance vector with means zero and covariance matrix V_v. In the nonstochastic formulation of Belsley's model, Z_{it} is a known, nonstochastic matrix and $\mathbf{v}_{it}$ is a zero vector. Substituting (7.4) into (7.1) yields

$$y_{it} = \mathbf{x}_{it}' \boldsymbol{\beta}_{it} + u_{it} = \mathbf{w}_{it}' \boldsymbol{\tau} + u_{it} \tag{7.5}$$

where $\mathbf{w}_{it}' = \mathbf{x}_{it}' Z_{it}$ is the (1 x M) vector of observations of interaction variables, and $u_{it} = \mathbf{x}_{it}' \mathbf{v}_{it} + e_{it}$; the disturbances u_{it} have zero means and variance $E(u_{it}^2) = \mathbf{x}_{it}' V_v \mathbf{x}_{it} + \sigma^2$.

A difficulty with this model is that the composite error term u_{it} is heteroscedastic. Thus the least squares estimator of $\boldsymbol{\tau}$ is unbiased but inefficient relative to the appropriate Aitken estimator. If $\mathbf{x}_{it}$ contains a intercept term, its coefficient (intercept) and the estimate for the equation disturbance will be indistinguishable. This formulation is a special case of the model presented by Hsiao (1975) (see Section 7.4.5). In the case of T = 1, techniques of the Hildreth-Houck model (discussed in Section 7.4.2) could be applied to estimate parameters with Generalised Least Square (GLS) methods. The case of N = 1 is discussed by Singh et al. (1976). They consider a model in which Z_{it} contains functions of calender time and $\mathbf{v}_t$ is a vector of normally distributed random disturbances with zero means and a diagonal covariance matrix V_v. The justification for using calender time to "explain" the variation in $\boldsymbol{\beta}_t$ is the same as that used when time-trend variables are included in regression models. These variables act as surrogates for all the unknown time-related dynamic forces within the economy. The usefulness of this model depends upon the acceptability of that substitution. The estimation problem is exactly the same as above. Singh et al. developed both modified Hildreth-Houck and maximum likelihood estimators.

Some additional remarks may be made about the relationships in Equation (7.2). It is unlikely that each element of $\boldsymbol{\beta}_{it}$ will be the same linear function of a set of explanatory variables. Whenever the parameters $\boldsymbol{\beta}_{it}$ are thought to be dependent on a set of the same variables, related rows of the matrix Z_{it} will be identical. On the contrary, if all parameters are functions of different variables, in the matrix Z_{it} there will be zero's in proper places. Given that Z_{it} contains zero's in appropriate places, Equation (7.2) is general enough to cover different forms of those functions. In general the matrix of explanatory variables Z_{it} may contain:

(1) functions of variables already included in $\mathbf{x}_{it}$, implying that (7.1) is not linear in the original explanatory variables;

(2) functions of other variables that do not appear in x_{it}, for example, calendar time. The justification for using calender time for the explanation of the structural parameters β_{it} variation is the same as that used when such variables are included in regression models. Calender time acts as a surrogate for all the unknown time-related dynamic forces within the economy. Caution is always recommended in using trend-related variables because very often they tempt one to engage in gross, curve-fitting exercises;

(3) qualitative variables that may be stochastic or nonstochastic, implying the existence of separate regression regimes; and

(4) lagged values of variables appearing in $\mathbf{x}_{it}$ implying a dynamic parameter process.

Alternative formulations of the matrix Z_{it}, listed above, include several specifications of models considered in the literature. Dziechciarz (1989) formulated an interesting model by assuming that the first column in matrix Z_{it} is a unit vector, and $\mathbf{v}_t' = (v_{t1}, 0,\ldots,0)$, which means that only the intercept is random. In this case v_{t1} serves as the model's disturbance and all other parameters are deterministic functions of variables from matrix Z_{it}. Each parameter can then be written as

$$\beta_{tk} = \tau_1 + \tau_2 \, z_{2t_k} + \ldots\ldots + \tau_M \, z_{Mt_k} \qquad (7.6)$$

where k = 2,...,N; t = 1,...,T.

The next section deals with models in which there is structural change in the sense of switching; in this case the parameter vector takes on a small number of different values and therefore the major interest is focussed on locating the breakpoints and appreciating the significance of the structural variations.

7.3.3 SWITCHING REGRESSION MODELS

7.3.3.1 INTRODUCTION

"Natura Non-Facit Saltum"

Alfred Marshall (1890)

The quote by Alfred Marshall serves as an excellent starting point for a discussion of switching regression models and structural change. It focuses on a basic characteristic of the models to be considered in this section - namely, continuity.

When data is collected in an ordered sequence, there is sometimes a desire to study changes in the probabilistic structure of the measured variables from one contiguous subset of the time domain to another. If, for example, a major policy change was enforced at some point during the time frame of the study, the researcher may be interested in attempting to assess the effects of the policy change on the variables under study. The variables under study could be economic in nature, while the policy change could be a new tax law, a new government program or a major disturbance to the economy (such as an oil embargo).

Economic analysis of time series data typically employs regression techniques on the assumption that an underlying relationship is stable over time. However, in many cases the underlying structure may not be constant, and methods of detecting and facilitating structural change in the relationship must be incorporated into the analysis. Since "shifting regression" often plague economists and management scientists, it is useful to have techniques that test for shifts and which estimate the nature of the shift when it takes place.

Essentially two problems arise in two-phase regression: estimating the shift point (or join point) and all other parameters of the model and accommodating a smooth as well as an abrupt transition from one regime to another. The usual practice of economists is to employ F-tests, as outlined by Chow (1960), to test the equality of regression coefficients (all coefficients or a subset) in two or more regressions. There are, however, two major problems with such a procedure: the identification of the exact point of structural shift and the assumption that the entire shift is accomplished in one interval of time. The problem of estimating the point in time at which a switch in structures (regimes) takes place has been investigated by Quandt (1958, 1960 and 1972), Bacon and Watts (1971), Poirier (1976), and many others. A paper by Goldfeld and Quandt (1973b) extended Quandt's earlier work to a more general switching framework.

The Systematically Varying Parameter Model discussed under Section 7.3.2 above allows response coefficients to be different for each observation. Often this completely general formulation can be usefully modified to allow the regression coefficients to be constant over subsets or partitions of the observations but to be different across partitions. Models of this type can be considered in the framework of the previously introduced terminology as containing qualitative variables in matrix Z_{it}, sorting observations into different subsets.

One of such a group of models allows the systematic parameter variation for different seasonal periods. Other examples include models with dummy variables and the piecewise regression models, accommodating an abrupt change from one regime to another, such as those developed by Quandt (1958 and 1972), McGee and Carlton (1970), Hinkley (1969, 1971a and 1971b), Gallant and Fuller (1973), Goldfeld and Quandt (1973a and 1973b), Ferreira (1975), Choy and Broemeling (1980), and others. In particular, one can distinguish two basic situations. In the first, the switch points are known; in the second, they have to be estimated. In the sample, the piecewise regression model can be continuous or not. Because this type of model is used mainly in a time series context, for the sake of convenience and interpretation, it is mainly considered within such a framework. Analysis of these models re-emphasizes the close relationship between the varying parameter and pooling models.

7.3.3.2 SEASONAL MODELS

Many important economic variables such as consumption, employment and output exhibit seasonal patterns. Some of these series, which are published by agencies (for example, the South African Reserve Bank), have by using a moving average process, been deseasonalised. Unfortunately, seasonally adjusted data contains little or no information about seasonal variation in the parameters and, as Wallis (1974) and Sims (1974) noted, the use of such seasonally adjusted data in dynamic statistical models, for example, distributed lag models, can have serious statistical consequences. By using seasonally adjusted data, time variations in the regression parameters between seasons may be difficult to expose. One could also build in structural weaknesses in the data by deseasonalising time series data. Havenner and Swamy (1981) considered some of the consequences of using deseasonalised variables in the random coefficient model. In Zellner (1984) the procedures of deseasonalising are also reviewed.

The models with seasonally varying economic variables are referred to as seasonal models. The situation where a sample can be divided into two or more subsamples withregard to some seasonal variable will be considered:

$$y_t = x_{t1}\beta_1 + x_{t2}\beta_2 + \ldots + x_{tk}\beta_K + e_t \qquad (7.7)$$

where $t = 1,\ldots,T$.

It is assumed that, for some subsamples, values of the model parameters can be different. For the sake of simplicity, it will be assumed that the regression structure is constant but different in two parts of the sample i.e., for $t = 1,\ldots,t_o$ (1st subsample), and for $t = t_o + 1,\ldots,T$ (2nd subsample). It is also assumed that not all structural parameters vary, only β_i does, where $i = p + 1,\ldots,K$; $p \in \{0,\ldots,(K-1)\}$, $K = p + q$. A standard model with dummy variable D defined as follows:

$$D = \begin{cases} 0 & \text{if } t = 1,\ldots,t_o \\ 1 & \text{if } t = t_o+1,\ldots,T \end{cases}$$

takes the form

$$y_t = \sum_{k=1}^{K} x_{tk}\beta_k + (x_{t,p+1}\, D)\delta_{p+1} + \ldots + (x_{t,K}\, D)\delta_K + e_t \tag{7.8}$$

where t = 1,...,T.

Model (7.8) may be written in two parts: one for the first subsample:

$$E(y_t) = \sum_{k=1}^{K} x_{tk}\ \beta_K \tag{7.9}$$

and one for the second part of the sample:

$$E(y_t) = \sum_{k=1}^{p} x_{tk}\ \beta_k + \sum_{k=p+1}^{K} x_{tk}(\beta_k + \delta_k) \tag{7.10}$$

Parameter δ_k measures the incremental change of the structural parameter connected with variable x_{tk} in the second part of the sample. Judge et al. (1980, Chapter 14) introduced an interesting alternative - a dummy variable related approach that is sometimes easier and more convenient than the classical one and gives equivalent estimates of the parameters. Judge et al. (1980) also provided a detailed discussion of a general dummy variable model in Chapter 16. The parameter and variance estimation, as well as some alternative parameterisation and testing techniques, are considered - an illustrative example is given. It is of course possible to extend the above model to more than two regression regimes with constant parameters in each of them. It is also assumed that structural changes are rapid and abrupt.

If data with seasonal patterns are exogenously determined, and the data generating process is a stable one, then Zellner's (1979, Chapter 6) seemingly unrelated regression model framework provides a convenient tool for estimating and statistically evaluating the significance of the seasonal variation.

Judge et al. (1980: 385-387) discussed the example in which the seemingly unrelated model framework is employed to model statistical data with quarterly seasonality. Several estimation methods and tests are compared. Following Judge et al.'s method the quarterly set of equations are:

$$\begin{aligned}
\mathbf{y}_1 &= X_1\,\boldsymbol{\beta}_1 + \mathbf{e}_1\\
\mathbf{y}_2 &= X_2\,\boldsymbol{\beta}_2 + \mathbf{e}_2\\
\mathbf{y}_3 &= X_3\,\boldsymbol{\beta}_3 + \mathbf{e}_3\\
\mathbf{y}_4 &= X_4\,\boldsymbol{\beta}_4 + \mathbf{e}_4
\end{aligned} \tag{7.11}$$

which may be rewritten in single equation form as

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \\ \mathbf{y}_4 \end{bmatrix} = \begin{bmatrix} X_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & X_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & X_3 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & X_4 \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \\ \boldsymbol{\beta}_3 \\ \boldsymbol{\beta}_4 \end{bmatrix} + \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \\ \mathbf{e}_4 \end{bmatrix} \tag{7.12}$$

or Equation (7.12) can be more compactly defined as

$$\mathbf{y} = Z\boldsymbol{\tau} + \mathbf{w} \tag{7.13}$$

If $\mathbf{w} \sim N(\mathbf{0}, \sigma^2 I)$, the least squares rule may be applied to estimate the parameters of the quarterly equation and the general hypothesis

$$R\boldsymbol{\tau} = \begin{bmatrix} I_1 & -I_2 & 0 & 0 \\ 0 & I_2 & -I_3 & 0 \\ 0 & 0 & I_3 & -I_4 \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \\ \boldsymbol{\beta}_3 \\ \boldsymbol{\beta}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \tag{7.14}$$

could be used to test the statistical significance of the seasonal effects on all or some combination of the parameters. In (7.13), if the quarterly equations are error related and $\mathbf{w} \sim$

$N(0, \Phi)$, where Φ is some positive, definite symmetric matrix, the coefficient vector τ may be efficiently estimated by using the Aitken generalised least squares rule, where the covariance Φ is estimated. In this case the standard Zellner (1979, Chapter 6) type of aggregation tests could be applied to the quarterly parameters.

Alternatively, Kmenta (1986) and Johnston (1984: 234-239) argued that the problem of seasonally varying parameters can be solved by means of zero-one dummy variables. However, a seemingly unrelated regression approach seems to be easier and more straight-forward in terms of estimation and inference. When the dummy variable format is used, additional calculations are required to obtain estimates of the original coefficients and their variances. When the seasonal variation is determined by identifiable economic and non-economic factors, Model (7.1) can be directly employed with those factors used as explanatory variables.

7.3.3.3 PIECEWISE REGRESSION MODELS

7.3.3.3.1 KNOWN JOIN POINT

Models that use dummy variables imply the presence of identifiable parameter "regimes" that hold for partitions of the entire sample. Although dummy variable models are easily extended to situations where both time series and cross-sectional data are available, for expository purposes and without loss of generality, only time series models will be considered, that is, i = 1 in model 7.1. In other words, the sample may be split into two groups of T_1 and T_2 observations with $T = T_1 + T_2$. These groupings do not necessarily have to contain observations that are sequential in time, but they may.

A piecewise regression model with known join point may be formulated as

$$y_t = \begin{cases} \mathbf{x}_t' \boldsymbol{\beta}_1 + e_{1t} & \text{if } t \in \{T_1\} \\ \mathbf{x}_t' \boldsymbol{\beta}_2 + e_{2t} & \text{if } t \in \{T_2\} \end{cases} \qquad (7.15)$$

or in convenient matrix notation

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \end{bmatrix} \qquad (7.16)$$

The segments of Model (7.16) are not necessarily joined. Some restrictions may be imposed to guarantee appropriate properties. If not all the coefficients are expected to change, restrictions may be imposed across the sample partitions that guarantee the equality of the corresponding elements of $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$. A very important problem is that of joining the segments of Model (7.16). Whenever two regression regimes are assumed to join at point $t_o \in [1, T]$, Model (7.16) would be estimated subject to the condition that

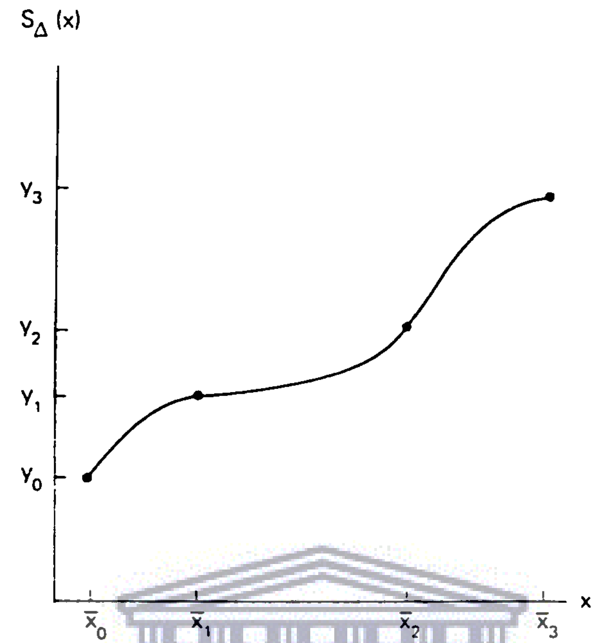
$$E(y_{t_o}) = \mathbf{x}'_{t_o}\boldsymbol{\beta}_1 = \mathbf{x}'_{t_o}\boldsymbol{\beta}_2 \quad \text{or} \tag{7.17}$$

$$\mathbf{x}'_{t_o} = (\boldsymbol{\beta}_1 - \boldsymbol{\beta}_2) = 0 \tag{7.18}$$

The imposition of these parameter restrictions implies that the two regression function will join at the point t_o.

Smoothness restrictions can be formulated in some other way. Poirier (1973, 1976) has considered a piecewise regression function called a cubic spline function, whose pieces join more smoothly than in the model considered above. Cubic splines have been extensively used by physical scientists as approximating functions and in economics there were also several very interesting applications. In reality cubic splines are cubic polynomials in a single independent variable (time), which are joined together smoothly at known points. The smoothness restrictions assume that in the joining point, where the cubic polynomials meet, the first and second derivatives are equal.

FIGURE 7.1: A CUBIC SPLINE



Source: Poirier (1976: 22)

A graphical example of a cubic spline with four knots is given in Figure 7.1. The set $\Delta = \{\bar{x}_0 < \bar{x}_1 <\ldots< \bar{x}_k\}$ of abscissa values are referred to as a mesh of $\{\bar{x}_0, \bar{x}_k]$ and the $K+1 \geq 3$ individual points $\bar{x}_j$ $(j = 0,1,\ldots,k)$ as knots. If y = $\{y_0, y_1,\ldots,y_k\}$ is an associated set of ordinates, then a cubic spline on Δ interpolating to y is denoted by $S_\Delta(x)$.

Thus, the transition from one regime to another does not occur abruptly. In particular, following Poirier's (1976) Bayesian point of departure the regression function

$$Y_t = g_1(t)\, I_{[1,t_o]}(t) + g_2(t)\, I_{[t_o,T]}(t) + e_t \tag{7.19}$$

where I(.) are indicator functions that take the value one if the argument is in the stated interval and zero otherwise. The $g_i(t)$, i = 1,2, are cubic polynomials of the form

$$g_i(t) = a_i t^3 + b_i t^2 + c_i t + d_i \tag{7.20}$$

In the cubic spline literature the point t_o, where the two cubic functions meet, is referred to as the "knot" point. The functions $g_i(t)$ obey the following smoothness (derivative restrictions):

$$g_1(t_o) = g_2(t_o),\ g_1'(t_o) = g_2'(t_o),$$
$$g_1''(t_o) = g_2''(t_o) \tag{7.21}$$

The above formulation, which can be written in terms of linear equality restrictions on the coefficients of the cubic polynomials, has been shown by Buse and Lim (1977) to be equivalent to the formulation originally presented by Poirier

(1973). This means that the restricted least squares estimator may be used to estimate the unknown coefficients.

Poirier (1976) also reviewed various other aspects of the topic of splines in detail. He, for example, discussed linear splines, emphasizing various parameterisation and hypothesis testing techniques for structural change. The linear spline is extended to the two dimensional case of bilinear splines where its mathematical formulation is presented and hypothesis testing techniques for structural change are developed. He also considered Cobb-Douglas splines, analysing their use both as production functions and as utility functions. Linear and cubic splines are also applied to distributed lag models.

One difficulty with the use of cubic splines is that although these functions are very flexible approximating functions, the form of the structural change implied is very restrictive. Although the fitted curve may approximate the available data very well, they give very little information about the nature of both the process being modeled and the nature of structural change. The model for linear spline regression is in part a special case of the piecewise regression model where there is only one independent variable. The main difference between the linear spline model and the piecewise linear model is that, in the former, the adjacent regression lines are required to intersect at the knots or change points, and in the latter they do not.

7.3.3.3.2 UNKNOWN JOIN POINT

In piecewise regression models with unknown join point, the point of structural change is unknown and therefore is treated as an unknown parameter to be estimated. Goldfeld and Quandt (1973b) assumed that in Model (7.16), $e_{1t} \sim N(0, \sigma_1^2)$ and $e_{2t} \sim N(0, \sigma_2^2)$. They also assumed that $(\beta_1, \sigma_1^2) \neq (\beta_2, \sigma_2^2)$.

There are several possible ways to choose between the regression regimes. The choice between the two regimes is assumed to be either (i) deterministic, where some observed variable is compared with some unknown threshold or (ii) stochastic and dependent upon unknown probabilities. In the first case the basis of the choice may be the trend variable or other economic variable.

7.3.3.3.2.1 DETERMINISTIC SWITCHING ON THE BASIS OF TIME

In the deterministic case where the switch occurs on the basis of a time index, it is assumed that the first regime holds for $t \leq t_o$ and the second for $t > t_o$. The estimate of t_o may be obtained by maximising the likelihood function, conditional on

t_o which is

$$l(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \sigma_1^2, \sigma_2^2 | t_o) = (2\pi)^{-T/2} \sigma_1^{-t_o} \sigma_2^{-(T-t_o)}$$

$$\times \exp \{-\tfrac{1}{2} \sigma_1^2 \sum_{t=1}^{t_o} (y_t - \mathbf{x}_t' \boldsymbol{\beta}_1)^2$$

$$- \frac{1}{2} \sigma_2^2 \sum_{t=t_o+1}^{T} (y_t - \mathbf{x}_t' \boldsymbol{\beta}_2)^2 \}$$

(7.22)

The estimate of t_o chosen is the value that maximises the likelihood function. The likelihood ratio test is used to determine whether the two regression regimes are equal, i.e., whether there is one single regression over the entire sample.

A detection procedure similar in spirit has been suggested by Brown et al. (1975). They investigated the constancy of regression relations over time by considering functions of recursive residuals generated by moving averages. They provided a test that answers the question whether the regression is stable and where the instability occurs. Comparison of the standardised sum and standardised sum of squares of these residuals to approximate confidence bounds provide evidence of regression stability or instability and approximately where any structural change took place.

Farley and Hinich (1970) and Farley, Hinich and McGuire (1975) suggested some alternative test for shifts in slope parameters where the shift point is unknown. Assuming that the model

$$y_t = \mathbf{x}_t' \boldsymbol{\beta}_t + e_t \qquad (7.23)$$

where $\boldsymbol{\beta}_t = \bar{\boldsymbol{\beta}} + t\boldsymbol{\delta}$.

This means the model may be rewritten as

$$y_t = \mathbf{x}_t'\bar{\boldsymbol{\beta}} + t\,\mathbf{x}_t'\boldsymbol{\delta} + e_t \qquad (7.24)$$

The test for constant slopes is then the usual likelihood ratio test of the hypothesis that $\boldsymbol{\delta} = \mathbf{0}$. Farley et al. (1975) provided Monte Carlo evidence that their test is robust with respect to gradual parameter shifts in one or more parameters. They also note, however, that unless the sample size is large or the shift great, tests for model instability are not very powerful. Furthermore, the properties of any estimators produced after this preliminary test are not discussed and thus presumably unknown.

7.3.3.3.2.2 DETERMINISTIC SWITCHING ON THE BASIS OF OTHER VARIABLES

The procedure for the case where shifts are determined by time can be applied to cases where a single economic variable (other than time) determines switching regression. If there is no autocorrelations present in the disturbance term and/or lagged dependent variables, sample data have to be re-ordered according to increasing magnitudes of the variable that control the switching process. In recent years much effort was devoted to evaluate Bayesian techniques for solving switching regression problems. See, for example, Ferreira (1975), Choy and Broemeling (1980), Smith and Cook (1980), Booth and Smith (1982), Holbert (1982), Ohtani (1982), Tsurumi (1982), and others. In particular, there is the question how prior information can be incorporated in order to improve the quality of estimate of both switching points and structural parameters.

Goldfeld and Quandt (1973b) offered a more general formulation. They assumed that there exist variables based on a variable other than time with observations $z_{1t},\ldots,z_{mt}$, $t = 1,\ldots,T$ and that regimes are selected according to whether $\mathbf{z}_t'\boldsymbol{\tau} \leq 0$ or $\mathbf{z}_t'\boldsymbol{\tau} > 0$, where $\boldsymbol{\tau}$ is an unknown coefficient vector. Goldfeld and Quandt (1973b) suggested the introduction of a dummy variable with values $D_t = 0$ if $\mathbf{z}_t'\boldsymbol{\tau} \leq 0$ and $D_t = 1$ if $\mathbf{z}_t'\boldsymbol{\tau} > 0$. The two regimes in (7.15) can therefore be combined as

$$y_t = \mathbf{x}_t' \, [(1-D_t) \, \boldsymbol{\beta}_1 + D_t \, \boldsymbol{\beta}_2] + (1-D_t) \, e_{1t} + D_t \, e_{2t} \tag{7.25}$$

where $\boldsymbol{\beta}_1$, $\boldsymbol{\beta}_2$, $\sigma_1{}^2$, $\sigma_2{}^2$ and the D_t's (using $\mathbf{z}_t'\boldsymbol{\tau}$) must be estimated. To make the problem tractable, the D_t's may be approximated by a continuous function. One possible approximation is to use the probit function

$$D_t = \int_{-\infty}^{\mathbf{z}_t'\boldsymbol{\tau}} \{1 \,/\, (2\pi\sigma^2)^{\frac{1}{2}}\} \exp \{(-u^2/2\sigma^2)\} \, du \tag{7.26}$$

with log likelihood function

$$L = -\tfrac{1}{2} \, T \ln 2\pi - \tfrac{1}{2} \sum_{t=1}^{T} \ln[\sigma_1{}^2(1-D_t)^2 + \sigma_2{}^2 \, D_t{}^2]$$

$$- \tfrac{1}{2} \sum_{t=1}^{T} \left[\frac{\{y_t - \mathbf{x}_t' \, [\boldsymbol{\beta}_1(1-D_t) + \boldsymbol{\beta}_2 D_t]\}^2}{\sigma_1{}^2(1-D_t)^2 + \sigma_2{}^2 \, D_t{}^2} \right] \tag{7.27}$$

Upon replacing D_t by its approximating function, the log likelihood can be maximised with respect to $\boldsymbol{\beta}_1$, $\boldsymbol{\beta}_2$, τ, $\sigma_1{}^2$ and $\sigma_2{}^2$. Frequently estimated values of $D_t = f(\mathbf{z}_t'\boldsymbol{\tau})$ are not exactly one or zero; the simplest solution is to partition the sample according to whether $D_t \leq 1/2$. Goldfeld and Quandt (1973b) suggested that, in the case where D_t is not exactly one or zero (the discrimination is not perfect), one solution is to create two subsamples on the basis of whether $z_t'\hat{\tau} \leq 0$ or $z_t'\hat{\tau} > 0$. Separate regressions are then estimated for each sample. The likelihood ratio test may provide the answer whether there are separate regimes in each subsample. Again, the sampling properties of the outcome generated by using this rule are unknown.

7.3.3.3.2.3 STOCHASTIC CHOICE OF REGIMES

A non-deterministic alternative is to assume that nature chooses between the first and second regimes on the basis of unknown probabilities α and $(1-\alpha)$. The log likelihood function is

$$L = \sum_{t=1}^{T} \ln g(y_t|\mathbf{x}_t) \tag{7.28}$$

where $g(y_t|\mathbf{x}_t)$ is the density function of y_t:

$$g(y_t|\mathbf{x}_t) = \alpha\, f_1(y_1|\mathbf{x}_t) + (1-\alpha)\, f_2(y_t|\mathbf{x}_t)$$

$$= \frac{\alpha}{(2\pi\,\sigma_1^2)^{\frac{1}{2}}} \exp\left\{-\frac{1}{2\,\sigma_1^2}(y_t - \mathbf{x}_t'\,\boldsymbol{\beta}_1)^2\right\} + \frac{1-\alpha}{(2\pi\sigma_2^2)^{\frac{1}{2}}} \exp\left\{-\frac{1}{2\sigma_2^2}(y_t - \mathbf{x}_t'\,\boldsymbol{\beta}_2)^2\right\} \tag{7.29}$$

The log likelihood function L is maximised with respect to the β's, σ^2's and α. In a more complex case, α can be a function of some exogenous variables.

Another modification of the stochastic alternatives is to allow the probability α of choosing the first regime in the time period to depend upon the state of the system in the previous trial. Specifically, Goldfeld and Quandt (1973a) proposed an alternative method in which a Markov chain with explicitly specified transition probabilities is employed as a mechanism of choice of regression regimes. The transition probabilities may be considered fixed or nonstationary and functions of exogenous variables. The likelihood function is formed and maximised with respect to all relevant variables. Tishler and Zang (1979) developed computationally simple approximation functions to the likelihood function. Swamy and Mehta (1975b) offered the Bayesian approach and some other generalisations. Lee and Porter (1984) suggested a model for the case in which sample separation information is imperfect.

7.3.4 CONCLUDING REMARKS

Since Quandt (1958) proposed a switching regression model, the model has often been used to detect a structural change point in economic equations. Based on the switching regression model, for example, Stern, Baum and Greene (1979) studied structural change in the aggregate import and export demand equations for the U.S. and Boughton (1981) studied structural change in the demand equation for US money.

From theoretical and practical viewpoints, the switching regression model has been extended in different directions. For example, Salazar, Broemeling and Chi (1981) and Ohtani (1982) considered the switching regression model when error terms are autocorrelated. Also, Bacon and Watts (1971), Tsurumi (1980) and Katayama, Ohtani and Toyoda (1987) considered the switching regression model when changes in regression coefficients occur gradually.

Wilton (1972) suggested a model for the regression problem of a known switching date which permitted a smooth transition from an old to a new structure. Bacon and Watts (1971) have employed Bayesian techniques to analyse the problem where the switching point is unknown and where the switch in regimes occurs either abruptly or smoothly during a transition period defined over both regimes. A similar suggestion is made by Goldfeld and Quandt (1973). Box and Tiao (1975) used an ARMA model to represent a gradual shift with a known join point. Tsurumi (1980) modified Bacon and Watt's parametric transition function to express the join point in the time domain, and derived a limited information Bayesian estimation and inference procedure within a simultaneous equation framework.

Although the switching regression models studied so far assume that all coefficients shift at the same change point, the change point may be different among the regression coefficients in some practical situations. Toyoda and Ohtani (1989) introduced a switching regression model in which individual coefficients are allowed to shift at the different change-points and applied their model in examining structural change of the first oil crisis in the energy demand equation for Japan which is explained both by relative price and economic activity variables.

Richard (1980) has investigated cases where the partitioning between endogenous and exogenous variables change over time. His objective was therefore to define a class of models with several regimes which is flexible enough to cover such situations. Reference is given to an economy which is "controlled" by a policymaker shifting between instruments at some, possibly unknown, points of time. Richard's study is mainly restricted to a class of dynamic linear models.

Maddala (1989) reviewed developments in econometric disequilibrium modelling and switching regression. Although disequilibrium and self-selection models do not directly deal with structural change they are both switching regression models with endogenous switching which can be used to study structural change. Maddala discussed the different uses of these models in the modelling of structural change which include an application of an exogenous switching Markov model on nonstationary time series data. The other class of models, with endogenous switching, can also be fruitfully applied to analyse structural change which follow policy changes that eliminate opportunities of self-selection that economic agents have.

Finally, for a deeper discussion on the problem of finding the number of regimes which gives the best fit to the data, see Guthery (1974) who considered a sort of piecewise regression approach using a dynamic program from cluster analysis, and Ertel and Fowlkes (1976) who developed plotting procedures for detecting changes in linear regression models and then used efficient algorithms for linear splines and piecewise regression.

7.4 RANDOM COEFFICIENT MODELS FROM A STATIONARY PROCESS

7.4.1 INTRODUCTION

It is frequently argued that the assumption of constant parameters in the general linear model may be an unreasonable one. For example, when using cross-sectional data on micro-units, such as firms or households, it is unlikely that the response to a change in an explanatory variable will be the same for all micro-units. Similarly, when using time series data, it is often difficult to explicitly model a changing economic environment and, in these circumstances, the response coefficients are likely to change over time. These considerations have led to the development of a number of stochastic or variable parameter models.

Estimation of relationships that combine time-series and cross-sectional data is a problem frequently encountered in economics. Typically, one may possess several years of data on a number of firms, households, geographical areas or biological units. The problem, when using these data to estimate a relationship, is to specify a model that will adequately allow for differences in behaviour over time for a given cross-sectional unit. Once a model has been specified, there are additional problems of the most efficient estimation procedure and how to test hypotheses about the parameters.

Models in which parameters are assumed to be random draws from some stochastic process will be considered in this section. First, it will be assumed that the process generating structural parameters is stationary, in the sense that it has a constant mean and variance. Such models are referred to as random coefficient models and will be outlined in the remainder of this section. These models represent an improved alternative over dummy variable models because the number of parameters to be estimated is reduced. In Section 7.5 models will also be described which are in the nonstationary stochastic parameter class (or which are closely related to it).

7.4.2 THE HILDRETH-HOUCK RANDOM COEFFICIENT MODEL

Researchers have become increasingly uncomfortable with the assumption made in the majority of empirical work using linear regression that the coefficients in the model are fixed over all observational units. In a cross-sectional context the assumption implies that the response to a change in an explanatory variable is the same for each cross-sectional unit. Since an individual's response to a given stimulus may depend on a number of unobservable factors, it would be more reasonable to assume that coefficients of explanatory variables are subject to random variation.

A growing literature has appeared on regression models with parameters subject to various forms of stochastic variation. Among these models, the so-called Hildreth-Houck model is probably one of the most widely known and is particularly suited to a situation where cross-sectional data is usually the only empirical base.

The model considered by Hildreth and Houck (1968) is

$$y_t = \mathbf{x}_t' \boldsymbol{\beta}_t \tag{7.31}$$

where t = 1,...,T ; and

$$\boldsymbol{\beta}_t = \bar{\boldsymbol{\beta}} + \mathbf{v}_t \qquad (7.32)$$

The parameter vector $\boldsymbol{\beta}_t$ contains the population average parameter $\bar{\boldsymbol{\beta}}$ and random disturbances $\mathbf{v}_t$; $\bar{\boldsymbol{\beta}}$ can be regarded as mean response coefficients, and $\boldsymbol{\beta}_t$ as actual (random) response coefficients for the t-th observation; $\mathbf{v}_t$ is a vector of disturbances which are independently distributed with zero means and covariance matrix V. Note that the equation disturbance term is indistinguishable from the intercepts disturbance v_{1t} and therefore does not appear in Equation (7.31). Furthermore, if Equation (7.31) should contain an additive disturbance term, the variance of the additional disturbance term cannot be estimated seperately from that of v_{1t}. Combining (7.31) and (7.32) gives

$$y_t = \mathbf{x}_t' \bar{\boldsymbol{\beta}} + e_t \qquad (7.33)$$

where $e_t = \mathbf{x}_t' \mathbf{v}_t$, and with $e_t \sim N(0,\sigma_t^2)$, $\sigma_t^2 = \mathbf{x}_t' V \mathbf{x}_t$. It is assumed that $E(\mathbf{v}_t) = \mathbf{0}$; $E(\mathbf{v}_t \mathbf{v}_t') = V$; and $E(\mathbf{v}_t \mathbf{v}_s') = \mathbf{0}$ for $t \neq s$, $\mathbf{v}_t' = (v_{1t}, v_{2t}, \ldots, v_{Kt})$.

As stated earlier, this type of model is often reasonable when cross-sectional data is used on a number of micro-units. In such a case the interest lies in estimating the mean coefficients $\bar{\boldsymbol{\beta}} = (\bar{\beta}_1, \bar{\beta}_2, \ldots, \bar{\beta}_K)'$, the actual coefficient $\boldsymbol{\beta}_t = (\beta_{1t}, \beta_{2t}, \ldots, \beta_{Kt})'$ and the covariance matrix V. Whenever V is the matrix with known elements, generalised least square (GLS) may be applied for the estimation of $\bar{\boldsymbol{\beta}}$ in the model. The best linear, unbiased, GLS estimator for $\bar{\boldsymbol{\beta}}$ is given by

$$\hat{\bar{\boldsymbol{\beta}}} = (\sum_{t=1}^{T} \sigma_{t-2}\, \mathbf{x}_t \mathbf{x}_t')^{-1} (\sum_{t=1}^{T} \sigma_{t-2} \mathbf{x}_t y_t) \qquad (7.34)$$

which has the covariance matrix $(\sum_{t=1}^{T} \sigma_{t-2} \mathbf{x}_t \mathbf{x}_t')^{-1}$.

Griffiths (1972), Swamy and Mehta (1977), and Lee and Griffiths (1979) showed that the BLUE predictor for the vector of individual coefficients may be obtained from

$$\hat{\boldsymbol{\beta}}_t = \hat{\bar{\boldsymbol{\beta}}} + V\mathbf{x}_t(\mathbf{x}_t' V\mathbf{x}_t)^{-1} (y_t - \mathbf{x}_t' \hat{\bar{\boldsymbol{\beta}}}) \qquad (7.35)$$

It is unbiased in the sense that $E(\hat{\boldsymbol{\beta}}_t - \boldsymbol{\beta}_t) = 0$, and best in the sense that the covariance matrix of the predictor of any other linear unbiased predictor exceeds the covariance matrix of $(\hat{\boldsymbol{\beta}}_t - \boldsymbol{\beta}_t)$ by a nonnegative definite matrix.

However, since elements of the matrix V are unknown rather than known, for $\hat{\bar{\boldsymbol{\beta}}}$ and $\hat{\boldsymbol{\beta}}_t$ to be operational, a way of finding their values is to be developed. Let $\mathbf{w}$ be an (N x 1) vector containing distinct, unique elements of V, and let $\sigma_t^2 = \mathbf{x}_t' V \mathbf{x}_t = \mathbf{z}_t' \mathbf{w}$, where $\mathbf{w}$ is an (N x 1) vector and $\mathbf{z}_t' = (1, z_{2t}, \ldots, z_{Nt})$. Matrix Z_t may be found by calculating the Kronecker product $\mathbf{x}_t' \otimes \mathbf{x}_t'$ and combining identical elements. For example, if K = 3 and

$$V = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix} \qquad (7.36)$$

then $\mathbf{w}' = (w_{11}, w_{12}, w_{13}, w_{22}, w_{23}, w_{33})$ and $\mathbf{z}_t' = (1, 2x_{2t}, 2x_{3t}, x_{2t}^2, 2x_{2t}x_{3t}, x_{3t}^2)$

(see Judge et al., 1980: 376)

Matrix Z_t also contains explanatory variables, their second powers, and their cross-products. It is clear that, from the equation of σ_t^2, that the Hildreth-Houck random coefficient model belongs to the class of heteroscedastic error models, where the variance of y_t (σ_t^2) is a linear function of a set of exogenous variables. Such models require special treatment; Judge et al. (1985, Chapter 11) provide an excellent survey of those problems.

Another problem connected with variance estimation arises. Since the elements of *V* are now regarded as variances and covariances, their estimation has to be restricted to positive values of the estimates. This problem is very difficult to handle because of its nonlinear nature. Hildreth and Houck (1968) considered the case of a diagonal matrix V; they advised replacing the negative estimates with zeros or using a quadratic programming estimator. The other possible solution is an <u>ad</u> <u>hoc</u> adjustment in the estimated matrix *V*, such that it is nonnegative definite. Schwaille (1982) found a reparametisation to be useful in this case. Several different estimators are proposed by Swamy and Mehta (1975a, 1975b) and Srivastava et al. (1981). Four estimators for V have been suggested by Swamy and Mehta. One is the maximum likelihood estimator under the assumption of normality, and two are based on prior information about *V*. For the fourth they suggested beginning with an "initial guess" for *V* and based on this, calculating $\hat{\boldsymbol{\beta}}_t$ and $\hat{\bar{\boldsymbol{\beta}}}$ from (7.34) and (7.35), respectively. Following this calculation, an estimator for V can be derived from the sum of cross products $\sum_{t=1}^{T}(\hat{\boldsymbol{\beta}}_t - \hat{\bar{\boldsymbol{\beta}}})(\hat{\boldsymbol{\beta}}_t - \hat{\bar{\boldsymbol{\beta}}})'$. A generally good estimator of the covariance matrix for random coefficients in the Hildreth-Houck model does not exist. Even provided this matrix is diagonal, the usual estimators may not be nonnegative. Important for the application of the random coefficients model is testing for the randomness in the coefficients; the Breusch-Pagan (1979 and 1980) test seems to be the best for this purpose.

Several other tests in use with the heteroscedastic models are also relevant. Chow (1984: 1239-1242) discussed the applicability of his test, the likelihood ratio test (see Chernoff, 1954; Moran, 1970; and Gourieroux et al., 1982); the Langrangian multiplier test of Silvey (1959) along with the score test of Rao (1972: 417); the test of Pagan and Tanaka (1979); and the test of Lamotte and McWhorter (1978). Chow concluded that none of these is good enough and further work is required to obtain computationally simple, uniformly most powerful test statistic with a known distribution in a small sample. Raj et al. (1980) considered distribution moments in a finite sample. Griffiths et al. (1979), Liu (1981) and Liu and Hanssens (1981) evaluated the Bayesian approach to the Hildreth-Houck model.

7.4.3 HARVEY-PHILLIPS RETURN TO NORMALITY MODEL

The simplest way to model stationary time-varying parameters is to assume that they are generated by a first- order autoregressive process (AR(1)). This was first suggested by Rosenberg (1973). Schaefer et al. (1976) have later investigated this type of parameter variation in connection with measuring a share's market risk, and found a good deal of support for it. Because the regression coefficients move around fixed means they use the term "return to normality" to describe the model.

Harvey and Phillips (1982: 306-321) also suggested a model referred to as the "return to normality model", which is a generalisation of the Hildreth-Houck random coefficient model. It is specifically suited for use with time series data. It enables releasing the assumption that the coefficients in the regression model are constant over time.

The dynamic parameters of this model follow a stationary first-order autoregressive process with a fixed but unknown mean. Harvey and Phillips considered the model

$$y_t = \mathbf{x}_t' \beta_t \quad \text{with} \quad t = 1,\ldots,T \tag{7.37}$$

where

$$\boldsymbol{\beta}_t - \bar{\boldsymbol{\beta}} = \Phi(\boldsymbol{\beta}_{t-1} - \bar{\boldsymbol{\beta}}) + \mathbf{e}_t \tag{7.38}$$

where y_t is the observation on the dependent variable, $\mathbf{x}_t$ is a (K x 1) vector on nonstochastic observations, and $\boldsymbol{\beta}_t$ is a (K x 1) vector of stochastic parameters, which includes the fixed component $\bar{\boldsymbol{\beta}}$; Φ is the (K x K) matrix of parameters with characteristic roots less than one in absolute value; $\mathbf{e}_t$ is the (K x 1) vector of random disturbances, which is assumed to follow a multivariate normal distribution with mean vector zero and a covariance matrix $E(\mathbf{e}_t\mathbf{e}'_t) = \sigma^2 Q$, $E(\mathbf{e}_t\mathbf{e}'_s) = \mathbf{0}$ for $t \neq s$. The introduction of the scalar σ^2 enables the top left-hand element in Q to be standardised by setting it equal to unity. A disturbance vector having all these properties will be written $\mathbf{e}_t \sim \text{NID}(\mathbf{0},\sigma^2 Q)$.

Model (7.37), like the Hildreth-Houck model, is written without an error term. In writing (7.37), it is assumed that the first element of $\mathbf{x}_t$ is unity for all t so that the variance of the first parameter is indistinguishable from the equation error variance, and thus it is ignored. Not all the elements in coefficient vector $\boldsymbol{\beta}_t$ need to be time-varying. Some of them may be fixed, others may be random. In the case of the existence of fixed parameters, they simply drop out of expression (7.38). Other coefficients may be random (if $\Phi = \mathbf{0}$) rather than dynamic, in which case the model reduces to the Hildreth-Houck random coefficient model, and thus represents a dynamic generalisation. Estimation could be carried out by maximum likelihood, conditional on Φ and Q, by using the usual GLS estimator. This would involve inversion of the (T x T) covariance matrix. Luckily, the Kalman filter can be used here to computational advantage.

The regression parameters in the model contain a component - the mean - which is fixed. As a result some theoretical problems arise in applying certain of the standard Kalman filter results to this model. These problems may be overcome, however, by linking the Kalman filter with the results on 'recursive residuals' in ordinary least square regression (OLS). Therefore, Harvey and Phillips (1982) suggested a full maximum likelihood method and two-step estimation procedures, both of them based on Kalman filtering (Kalman, 1960 and Kalman and Bucy, 1961), linked with the recursive residuals technique suggested by Phillips and Harvey (1974), and by Brown et al. (1975). All that is needed to carry out the Kalman filter iterations, and obtain the GLS estimator of $\bar{\boldsymbol{\beta}}$ conditional on Φ and Q, are some starting values. Under the assumption that Φ and Q are diagonal (with elements $\Phi_1,\ldots,\Phi_K$ and 1, $q_2,\ldots,q_K$, respectively), Harvey and Phillips suggested that starting estimates of diag(Φ) and diag(Q) could be obtained by computing the least square residuals $\hat{\mathbf{e}}_1,\ldots,\hat{\mathbf{e}}_T$ and regressing $\hat{\mathbf{e}}_t^2$ on $x^2_{t1},\ldots,x^2_{tK}$ and $\hat{\mathbf{e}}_t\hat{\mathbf{e}}_{t-1}$ on $x_{t1}x_{t-1,1},\ldots$ $\ldots,x_{tK}x_{t-1,K}$. Given the consistency of these initial estimates, an estimated GLS estimator of $\bar{\boldsymbol{\beta}}$ is then computed by a single pass of the Kalman filter. In a Monte Carlo experiment, Harvey and Phillips compared small-sample properties of the maximum likelihood estimator gain over ordinary least squares and two-step estimated generalised least squares. The latter estimator provided a substantial improvement on the OLS. The basis of the estimation procedures proposed here may be regarded as a generalisation of the recursion in the classical linear regression model. This method is different to the one considered by Rosenberg (1973), where two passes of the data are needed in order to estimate the means of the regression parameters, conditional on the other parameters in the model.

A very important generalisation of (7.37) is obtained by using

$$A(L)(\boldsymbol{\beta}_t - \bar{\boldsymbol{\beta}}) = \mathbf{e}_t \tag{7.39}$$

instead of (7.38) as the parameter generating process; here, $A(L)$ is a rational function of finite polynomials, which implies that $(\boldsymbol{\beta}_t - \bar{\boldsymbol{\beta}})$ follows a stationary multivariate ARMA process. This model covers a number of important special cases. Burnett and Guthrie (1970), Rosenberg (1972 and 1973a, b and c), Cooley and Prescott (1973a, b, c and 1976), Harvey and Phillips (1979), and Swamy and Tinsley (1980) considered models of this class. Pagan (1980) discussed sufficient conditions for asymptotic identification of such models, assuming $(\boldsymbol{\beta}_t - \bar{\boldsymbol{\beta}})$ is stationary. He also established sufficient conditions for the consistency and asymptotic normality of maximum likelihood estimators without assuming stationarity, but by assuming asymptotic identifiability. Liu and Hanssens (1981) considered the estimation of (7.37) from the Bayesian perspective using noninformative priors.

7.4.4 THE SWAMY RANDOM COEFFICIENT MODEL

Pure-random-coefficient models assume that the $\boldsymbol{\beta}_i$ are distributed with mean $\bar{\boldsymbol{\beta}}$ and covariance matrix V. Rao (1965: 447-458) first discussed this model, and Swamy (1970, 1971, 1973 and 1974) extended it to the problem of pooling cross-section and time-series data. Note the similarity of this model with the Hildreth-Houck model - the latter was designed to model cross-sectional data; the Swamy random coefficient model is relevant for time series of cross-sectional data. In both it is assumed that the process generating values of the dependent variable vary and that this variation can be confined in the structural parameters of the linear model. Because of the estimation requirements, in both models a certain structure of the parameter variation is to be specified. The nature of parameter variation is continuous rather than an abrupt, unique switch.

The model considered by Swamy for the i-th unit is the following:

$$\mathbf{y}_i = X_i\boldsymbol{\beta}_i + \mathbf{e}_i \quad \text{and} \tag{7.40}$$

$$\boldsymbol{\beta}_i = \bar{\boldsymbol{\beta}} + \boldsymbol{\mu}_i \tag{7.41}$$

with $i = 1,\ldots,N$, $E(\boldsymbol{\mu}_i) = \mathbf{0}$, $E(\boldsymbol{\mu}_i\boldsymbol{\mu}_i') = V$ and $E(\boldsymbol{\mu}_i\boldsymbol{\mu}_j) = \mathbf{0}$ for $j \neq i$; $\mathbf{y}_i \equiv \{y_{i1}, y_{i2},\ldots,y_{iT})'$ and $\boldsymbol{\mu}_i \equiv (\mu_{i1}, \mu_{i2},\ldots,\mu_{iT})'$

are (T X 1) vectors of observed dependent variables and unobserved disturbances, respectively; $X_i \equiv \{x_{ikt}\}$ (k = 1, 2,...,K; t = 1, 2,...,T) is a (T X K) matrix of observations on K nonstochastic regressors and $\boldsymbol{\beta}_i$ is a (K X 1) vector of coefficients assumed to be random across individuals, i = 1,...,N.

Observations on y's and x's for N individuals taken over T periods of time are used. These temporal cross-section data are obtained by assembling cross-sections of T years, with the same N cross-section units appearing in all years. The individuals here may be firms, consumers or regions. The subscript i indexes cross-section observations and the subscript t indexes time series observations.

The parameters for each individual are constant in time and have a common mean parameter vector $\bar{\boldsymbol{\beta}}$, but a different disturbances vector $\boldsymbol{\mu}_i$. Several alternative sets of assumptions about $\mathbf{e}_i$ yield different model variations. Judge et al. (1985, chapter 12) listed a number of such assumptions of varying degrees of complexity in the context of seemingly unrelated regression equations, which is the nonstochastic counterpart of Swamy's random coefficient model. Any of these assumptions could also be used for Swamy's model, but, in general, only the relatively simple ones will be considered here.

Swamy's simplest set of assumptions about e_i may be written as: $E(\mathbf{e}_i\mathbf{e}_i') = \sigma_i^2 I$ and $E(\mathbf{e}_i\mathbf{e}_j') = \mathbf{0}$ for $i \neq j$. This implies that the disturbances are heteroscedastic across individuals, but uncorrelated; there is no serial correlation conditions - which is sometimes difficult to justify in actual economic situations. If the $\boldsymbol{\beta}_i$'s were fixed parameters, the least squared estimator $\hat{\boldsymbol{\beta}}_i = (X_i'X_i)^{-1}X_i'\mathbf{y}_i$, would be best, linear, unbiased (BLUE) (see Swamy, 1970 and 1971).

For this model we are interested in estimating the mean parameter vector $\bar{\boldsymbol{\beta}}$, predicting each individual vector $\boldsymbol{\beta}_i$, estimating the variances upon which the generalised least squares (GLS) estimator for $\bar{\boldsymbol{\beta}}$ and the best linear unbiased predictor (BLUP) for $\boldsymbol{\beta}_i$ depend, and testing the hypothesis that V = 0 which may indicate whether structural parameters vary or not.

After including all NT observation, Model (7.40) yields

$$\mathbf{y} = X\bar{\boldsymbol{\beta}} + Z\boldsymbol{\mu} + \mathbf{e} \qquad (7.42)$$

where Z is a (NT X NK) block diagonal matrix with blocks X_i; i = 1, .., N; $\mathbf{y}$ is an (1 x NT) observation vector on the dependent variable; $\boldsymbol{\mu}' = \{\boldsymbol{\mu}_i'\}$, i = 1,...,N; $\bar{\boldsymbol{\beta}}$ is a (1 x NK) vector of unknown, fixed parameters to be found; and $\mathbf{e} = \{\mathbf{e}_i'\}$, i = 1,...,N. The covariance matrix for the composite disturbance ($Z\mu + e$) has the block diagonal covariance matrix $\Phi = E[(Z\mu + e)(Z\mu + e)']$ with the i-th diagonal block given by covariance matrix $\Phi_{ii} = Z_i V Z_{it}' + \sigma^2_{ii} I$. It is convenient to write the GLS estimator for β as

$$\hat{\beta} = (X'\Phi^{-1}X)^{-1}X'\Phi^{-1}y = [\sum_{i=1}^{N}(X_j'\Phi_{jj}^{-1}X_j)]^{-1}\sum_{i=1}^{N}(X_i'\Phi_{ii}^{-1}y_i)$$

$$= \sum_{i=1}^{N} W_i\hat{\beta}_i \qquad (7.43)$$

where

$$W_i = \{\sum_{j=1}^{N}[V + \sigma_{jj}(X_j' X_j)^{-1}]^{-1}\}^{-1}[V + \sigma_{ii}(X_i' X_i)^{-1}]^{-1},$$

and

$$\hat{\beta}_i = (X_i' X_i)^{-1}X_i' y_i \qquad (7.44)$$

Estimator (7.43) has the usual GLS properties. Judge et al. (1985) argued that the predictor of β_i given in equation (7.44), based on the matrix results of Rao (1965a: 29) is convenient for computational purposes. It requires a matrix inversion of the order K, which is especially important with large T. The GLS estimator may be interpreted as a matrix - weighted average of the estimators $\hat{\beta}_i$, with weights inversely proportional to their covariance matrices. (see Mundlak, 1987a for a different interpretation of this estimator).

An important task is to predict individual components of $\boldsymbol{\beta}_i$. Having done this, it is possible to predict future values of the dependent variable for each individual and to describe its behaviour. Several predictors have been proposed in the literature.

Swamy (1970, 1971) and Lee and Griffiths (1979) suggested some alternative approaches. Most widely known are their best, linear unbiased prediction - BLUP estimators; BLUP estimators may be obtained by minimising some quadratic function with respect to $\bar{\boldsymbol{\beta}}$ and $\boldsymbol{\beta}_i$. Smith (1973) and Leamer (1978) considered Bayesian solutions. Both parameter vectors $\bar{\boldsymbol{\beta}}$ and $\boldsymbol{\beta}_i$ are dependent on the unknown variances V and σ_{ii}; therefore their estimates are required.

By using results of Rao (1965b), Swamy (1970) suggested consistent estimators for both variances. Swamy (1970) showed how the least squares estimators $\hat{\boldsymbol{\beta}}_i = (X_i' X_i)^{-1} X_i \mathbf{y}_i$ and their residuals $\tilde{\mathbf{e}}_i = \mathbf{y}_i - X_i \hat{\boldsymbol{\beta}}_i$ can be used to obtain the unbiased estimators

$$\hat{\sigma}_{ii} = \tilde{\mathbf{e}}_i' \tilde{\mathbf{e}}_i / (T - K) \tag{7.46}$$

and

$$\hat{V} = S_{\hat{\boldsymbol{\beta}}} / (N-1) - [\sum_{i=1}^{N} \hat{\sigma}_{ii} (X_i' X_i)^{-1}] / N \tag{7.47}$$

where $$S_{\hat{\beta}} = \sum_{i=1}^{N} \hat{\beta}_i \hat{\beta}_i' - [\sum_{i=1}^{N} \hat{\beta}_i \sum_{i=1}^{N} \hat{\beta}_i'] / N \quad (7.48)$$

The GLS estimators for $\bar{\beta}$, based on these variance estimates are, under certain conditions (see Swamy, 1970), consistent and asymptotically efficient. An estimate of the asymptotic covariance matrix is given by substituting $\hat{\sigma}_{ii}$ and $\hat{V}$ into the equation for Φ_{ii} above. The problem is that the estimator for V may not be nonnegative definite (see Dielman et al., 1980). Swamy (1971) discussed this problem and gave some suggestions about the treatment of this situation. He argued that negative variance estimates may result from incorrectly specified assumptions about the form of the disturbance covariance matrix (for example, about homoscedasticity, serial correlation, or contemporaneous correlations). Swamy, therefore, suggested appropriate corrections for the violations of the accepted assumptions.

Another possible source of negative variance estimates is that certain coefficients are not random. The model containing both fixed and random coefficients is referred to as a mixed random coefficient model. It was first proposed by Swamy (1971: 143-155) and is analytically examined by Rosenberg (1973b), Mundlak (1978a, b, c) and Dielman (1980). The solutions given by Swamy are not obvious and can destroy the properties of estimators.

Dielman (1980) reviewed available statistical procedures to test for the possibility that the parameters are not random. Swamy (1970, 1971) gave some alternative suggestions about how to test if individual coefficient vectors are not random and are all identical to the mean. Rao (1972) gave conditions under which the estimator for $\bar{\boldsymbol{\beta}}$ has a finite mean and is unbiased. Swamy (1971, 1973) and Rosenberg (1973b) discussed the possibility of applying maximum likelihood techniques for the parameter estimation.

Some of the assumptions that have been made about the $\mathbf{e}_i$ could be regarded as fairly restrictive. Parks (1967) adopted an alternative set of assumptions that relaxed some of these restrictions (of no correlation between disturbances corresponding to different individuals and no serial correlation). He assumed that the disturbance for each unit follows a first-order autoregressive (AR(1)) process, and also that contemporaneous correlation exist (Parks adopted this set of assumptions for the fixed coefficient model). Swamy (1973, 1974) considered Park's assumptions when introduced into the random coefficient model. Another extension was given by Swamy (1974) for the case when X contains a lagged dependent variable. Rosenberg (1973b) considered estimation when the covariance matrix could be singular. Swamy (1973, 1974) attempted to evaluate estimators that may be biased, but with a lower mean square error.

Mundlak (1978a, b, c) suggested that the regression coefficients can always be regarded as random; but in the case when the parameters β_i are regarded as being fixed and different (in a seemingly unrelated regression framework), the inference is conditional on the coefficients in the sample. The random coefficient model uses additional information provided by an assumption about the randomness of the coefficients. It should be expected that if the assumption is true, the estimates will be more efficient. Dziechciarz (1989) argued that if variable coefficients perform a correlation with the explanatory variables, Swamy's assumptions are unreasonable and the GLS estimator for the mean coefficient vector $\bar{\beta}$ will be biased. Mundlak (1978) suggested the incorporation into the model of any dependence of the coefficients on the explanatory variables.

Pudney (1978) provided a procedure to test whether variable coefficients and explanatory variables are uncorrelated. Chamberlain (1982) considered further properties of the estimators under these circumstances. Zellner (1969a, b) has shown that a macro coefficient estimator will not possess aggregation bias if the coefficient vectors of the individual macro units satisfy the assumptions of the Swamy random coefficient model.

Some applications of the Swamy model can be found in Swamy (1971), Boot and Frankfurter (1972), Feige and Swamy (1974), Boness and Frankfurter (1977), Mehta et al. (1973) and Hendricks et al. (1979). Johnson and Lyon (1973) described a simulation study with stochastic explanatory variables. Swamy (1971: 1-23) and Spjotvoll (1977) surveyed other random coefficient models.

7.4.5 THE HSIAO RANDOM COEFFICIENT MODEL

This model is an extension of Swamy's model where all coefficients may vary both over time and over individuals; (see Hsiao 1974 and 1975). It is quite possible that an individual's reaction coefficients will not remain constant over time, and individuals differ greatly in behaviour. If the coefficients of the explanatory variables are fixed and different over time as well as across cross-sectional units, the parameters to be estimated will increase with the number of sample observations. Not only is there no point at which to pool the data, but there may not exist any consistent estimator at all.

Random coefficient models are, therefore, a particularly useful tool in the analysis of a time series of cross-section data. Given a time series of cross-section data and the assumption that the coefficients of the explanatory variables have common means, plus some random components associated with the time and/or cross-sectional units, the model that Hsiao is concerned with is specified as

$$y_{it} = \sum_{k=1}^{K} (\beta_k + \mu_{ki} + \pi_{kt}) x_{kit} + e_{it} \tag{7.49}$$

with i = 1, 2,...,N ; t = 1, 2,...,T. More compactly for the i-th cross-sectional unit, Equation (7.49) may be rewritten as

$$\mathbf{y}_i = X_i\bar{\boldsymbol{\beta}} + X_i\boldsymbol{\mu}_i + \bar{Z}_i\boldsymbol{\pi} + \mathbf{e}_i \tag{7.50}$$

where $\mathbf{y}_i$ is a vector of observations on the dependent variable for the i-th cross-sectional unit, X_i is a matrix of observations on the k-th nonstochastic regressor for the i-th cross-sectional unit, both having dimensions (T x 1) and (T x K) respectively; $\boldsymbol{\mu}_i = (\mu_{1i},\ldots,\mu_{Ki})'$; $\boldsymbol{\pi}' = (\boldsymbol{\pi}_1',\ldots,\boldsymbol{\pi}_T')$; $\boldsymbol{\pi}_t = (\pi_{1t},\ldots,\pi_{Kt})'$; $\mathbf{e}_i = (e_{i1},\ldots,e_{iT})'$;

$$\bar{Z}_i = \begin{bmatrix} \mathbf{x}_{11}' & & & & \\ & \mathbf{x}_{12}' & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \\ & & & & & \mathbf{x}_{1T}' \end{bmatrix}$$

and $\mathbf{x}_{it}' = (x_{1it},\ldots,x_{Kit})'$ is an element of the block diagonal matrix Z_i which is of order (T X TK).

Hsiao assumed that

$$E(\mathbf{e}_i) = 0 \tag{7.51}$$

$$E(\boldsymbol{\mu}_i) = 0 \tag{7.52}$$

$$E(\boldsymbol{\pi}_t) = 0 \tag{7.53}$$

$$\begin{aligned} E(\mathbf{e}_i, \mathbf{e}_j') &= \sigma_e^2 I && \text{if } i = j \\ &= 0 && \text{otherwise} \end{aligned} \tag{7.54}$$

$$\begin{aligned} E(\boldsymbol{\mu}_i, \boldsymbol{\mu}_j') &= V && \text{if } i = j \\ &= 0 && \text{otherwise} \end{aligned} \tag{7.55}$$

$$\begin{aligned} E(\boldsymbol{\pi}_t, \boldsymbol{\pi}_s') &= A && \text{if } t = s \\ &= 0 && \text{otherwise} \end{aligned} \tag{7.56}$$

It is also assumed that $\boldsymbol{\mu}_i$, $\boldsymbol{\pi}_t$ and $\mathbf{e}_i$ are all uncorrelated and that the covariance matrices V and A are diagonal with elements v_k and α_k, respectively. Rewriting (7.50) more compactly to include all NT observations yields

$$\mathbf{y} = X\bar{\boldsymbol{\beta}} + Z\boldsymbol{\mu} + \bar{Z}\boldsymbol{\pi} + \mathbf{e} \tag{7.57}$$

where $\mathbf{y}' = (\mathbf{y}_1', \mathbf{y}_2', \ldots, \mathbf{y}_N')$; $X' = (X_1', \ldots, X_N')$; Z is block diagonal with X_i as the i-th diagonal block, $\bar{Z}' = (\bar{Z}_1', \ldots, \bar{Z}_N')$; $\boldsymbol{\mu}' = (\boldsymbol{\mu}_1', \ldots, \boldsymbol{\mu}_N')$ and $\mathbf{e}' = (\mathbf{e}_1', \ldots, \mathbf{e}_N')$.

If μ and π are regarded as fixed parameters, μ, π and $\bar{\beta}$ can be estimated by applying ordinary least squares to (7.57), provided that reparameterisation takes place to eliminate redundant parameters, and that NT is sufficiently large. Since the matrix $(X, Z, \bar{Z})$ is of dimension [NT x (T + N + 1)K], and of rank [(T + N - 1)K], 2K parameters are redundant. It is convenient to drop $(\mu_{1N},\ldots,\mu_{KN}, \pi_{1T},\ldots,\pi_{KT})$, provided that corresponding columns of Z and $\bar{Z}$ are also eliminated. The minimal number of observations required is NT > [(T + N - 1)K].

Such estimation when μ and π are random, with the above listed assumptions, requires an estimate of the covariance matrix of the composite disturbance. Covariance matrices V and A and variance σ_e^2 are assumed to be known. With these assumptions, an estimate of the covariance matrix for the composite disturbance is

$$\Phi = E[(Z\mu + \bar{Z}\pi + e)(Z\mu + \bar{Z}\pi + e)'] \tag{7.58}$$

$$= Z(I_N \otimes V)Z' + \bar{Z}\,(I_T \otimes A)\,\bar{Z}' + \sigma_e^2 I_{NT} \tag{7.59}$$

and the GLS estimator $\hat{\bar{\beta}} = (X'\Phi^{-1}X)^{-1}X'\Phi^{-1}\mathbf{y}$ is the best linear unbiased estimator for $\bar{\beta}$, with covariance matrix $(X'\Phi^{-1}X)^{-1}$.

Presumably, NT would have to be large before one would contemplate using this model so that inversion of Φ could be a problem. Hsiao (1974) provided a computational procedure where the largest order of inversion for Φ is reduced to max{NK, NT}, which may still be quite large. Wansbeek and Kapteyn (1982) suggested a convenient inversion procedure for their random coefficient model.

If one is interested in predicting the random components associated with each cross-sectional unit, then, following Lee and Griffiths (1979), the predictor is

$$\hat{\boldsymbol{\mu}} = (I_N \otimes V)\, Z'\, \Phi^{-1}\, (\mathbf{y} - X\hat{\boldsymbol{\beta}}) \tag{7.60}$$

Covariances V and A and the variance σ_e^2 are assumed to be known, although typically they are unknown. Hsiao has found a minimum norm, quadratic, unbiased estimator (MINQUE) and provided a maximum likelihood procedure for variance estimation. Those estimators are used to construct a feasible Aitken estimator of $\bar{\boldsymbol{\beta}}$. Alternatively, a technique suggested by Hildreth and Houck (1968) may be used. Hildreth and Houck listed conditions under which their variance estimators are consistent. Hsiao (1974) gave sufficient conditions for the consistency and asymptotic efficiency of the estimated, generalised least squares estimator for $\bar{\boldsymbol{\beta}}$, which depends on Hildreth-Houck's variance estimates.

Kelejian and Stephan (1983) extended some of Hsiao's asymptotic results. Problems with those estimators are the same as those discussed in previous sections, they may have negative values and, in that case, the easiest solution is to change the negative estimates to zero. All comments made above on this problem also apply here. As before, μ_{ki} and π_{kt} may be assumed to be either fixed parameters to be estimated or random variables. Generally, they are assumed to be random, but sometimes it is convenient to estimate them as fixed parameters. Pudney (1978) pointed out that it might be reasonable to assume that one of the components is fixed and the other one random. To estimate variances, N and T have to be sufficiently large, otherwise any estimate of the variance will be unreliable. In such a situation it would be preferable to include appropriate dummy variables, and an inference conditionally on the sample variance. Note that if the time effects are replaced by dummy variables, the model becomes identical to the Swamy random coefficient model.

Several alternative models are considered in the literature. Singh and Ullah (1974), Swamy and Mehta (1975a, b and 1977) and Pudney (1978) discussed the model

$$y_{it} = \sum_k (\beta_k + \mu_{ki} + e_{kit})x_{kit} \qquad (7.61)$$

The disturbance e_{1it} replaces the model's disturbance term. The time effect π_{kt} has been replaced by the random component e_{kit}, which is not restricted to be the same for units in a given time period. Swamy and Mehta (1975a, b and 1977) modified Hsiao's assumption that the covariance matrix V is diagonal, i.e., they allowed contemporaneous correlation among the coefficients. Swamy and Mehta constructed an approximate minimum average risk linear estimator for $\bar{\boldsymbol{\beta}}$ in their model. (The estimator is approximate because it uses the estimates of the variance and covariance components, not the true values).

Both models mentioned above assume that random coefficients vary around a constant mean. Rosenberg (1973c), Johnson and Rausser (1975), Harvey (1978, 1981 and 1982) and Liu and Hanssens (1981) analysed other models where parameters vary systematically over time. A special case of Hsiao's model, where random coefficients are associated only with time invariant and individual invariant variables, is studied by Wansbeek and Kapteyn (1978, 1981, 1982).

In empirical work the Hsiao model does not seem to have gained the same widespread acceptance that some of the other models have achieved. This may be because the estimation procedures are not easily handled on standard computer packages, or it may be because the assumptions are not considered realistic. It would be preferable to relax the assumption that $\hat{V}$ and $\hat{A}$ are diagonal and, as Hsiao noted, in principle this can be handled within the same estimation framework. However, it would be more difficult and the problem of A and V being nonnegative definite would be magnified. Also, if there are a sufficient number of time series observations, it might be preferable to drop the π_{kt} and model the "time effects" with an autocorrelated e_{it} (see Swamy, 1974).

General references on random coefficient models with parameters generated in a stationary process include Burnett and Guthrie (1970), Belsley (1973c), Cooper (1972, 1973) Sarris (1973), Sant (1977), Pagan (1980), Rausser et al. (1982), and Chow (1984, Chapter 21) as well as Chamberlain (1984, Chapter 22). Important collection of papers on the topic are a special issue of "The Annals of Economic and Social Measurement" (1973, no 2); a special issue of the "Annales de l'INSEE" (1978), entitled "The Econometrics of Panel Data", edited by Mazodier, and a special issue of the "Journal of Econometrics" (1982), entitled "Econometrics of Longitudinal Data", edited by Heckman and Singer.

7.5 RANDOM COEFFICIENT MODELS FROM A NONSTATIONARY PROCESS

7.5.1 INTRODUCTION

It was assumed up to now that the parameters in the econometric model have constant means. This assumption can now be replaced by the assumption that the parameters are generated by a nonstationary random process. Contrary to the models with parameters generated by a stationary random process, here coefficients do not have a constant mean and variance. They may, therefore, vary systematically over observations. This means that a less restrictive structure is placed on the parameter variation. Such models are suitable to describe systematic variation over time.

In recent years, a significant body of literature has appeared that is addressed to the problem of estimating the coefficients of nonstationary random-parameter models (sometimes also called time-varying regression models). The most fashionable approach to these problems has been to apply Kalman filtering theory to the estimation of the coefficient trajectories. The discussion commences with one of the more popular time-varying parameter models, the Cooley-Prescott model.

7.5.2 COOLEY-PRESCOTT ADAPTIVE REGRESSION MODELS

Up to this point we have reviewed models where parameter shifts have been given considerable structure. In this section variable parameter models are presented that place a less restrictive structure on the parameter variation. In particular, the random walk model of Cooley and Prescott (1973, 1976) is considered.

The effect of omitted variables, aggregation errors, policy changes, and other errors in specification are included in the additive disturbance term which is assumed, among other things, to be temporally uncorrelated. The adaptive regression model developed by Cooley and Prescott does not assume that the disturbances are independent. Instead, it assumes the disturbances are the sum of not only a transitory element that has effect in the current period but also a permanent component whose effect persist into the future.

It is common practice in econometric research to test for serial correlation in the residuals. If the test indicates that serial correlation is present, it is typically assumed that the disturbances are subject to a first order autoregressive scheme. In fact, such processes are likely to describe the true distribution of the disturbances only in rare instances. An autoregressive error process implies that the effects of omitted factors all decay exponentially with time and at the same rate. Cooley and Prescott (1973) argued that this is an unreasonable assumption for most economic applications because some omitted factors, such as labour union strikes or the vagaries of the weather, will have only transitory effects while other factors, like changes in tastes or technological developments, will have effects which persist into the future with decay.

The authors, therefore, proposed a model where parameters vary from one time period to another on the basis of a nonstationary probabilistic scheme. They considered the following model:

$$y_t = \mathbf{x}_t' \boldsymbol{\beta}_t \qquad (7.62)$$

with t = 1,...,T,

where $\mathbf{x}_t$ is a (K x 1) vector of nonstochastic observations, and $\boldsymbol{\beta}_t$ is a (K x 1) conformable parameter vector subject to stochastic variation. The parameter variation is assumed to be of two types, permanent and transitory, the former allowing some persistent "drift" rather than "shifts" in the parameter values. These sources of variation are modelled as

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_t^p + \mathbf{u}_t \qquad (7.63)$$

where $$\boldsymbol{\beta}_t^p = \boldsymbol{\beta}_{t-1}^p + \mathbf{v}_t \qquad (7.64)$$

The permanent component $\boldsymbol{\beta}_t^p$, of the vector $\boldsymbol{\beta}_t$, allows some tendency in the parameter variation. The terms $\mathbf{u}_t$ and $\mathbf{v}_t$ are independent, normal random vectors with mean vectors zero and covariance matrices $E(\mathbf{u}_t\mathbf{u}_t') = (1 - \tau)\sigma^2 U_u$ and $E(\mathbf{v}_t\mathbf{v}_t') = \tau\sigma^2 V_v$. The covariance matrices U_u and V_v are assumed to be known up to the scale factor and normalised, i.e., the element corresponding to the intercept is unity - the first regressor is the constant term. The transitory component of the corresponding parameter's variation plays the role of the additive disturbance in the regression equation, while the permanent component causes random changes in the intercept value. Note that the parameterisation adopted is such that τ reflects the relative importance of the permanent and transitory changes. If τ is close to 1, then the permanent changes are large relative to transitory ones.

Straightforward maximum likelihood estimation of σ^2 , τ and the permanent components of the $\boldsymbol{\beta}_t$ is not possible, since the process generating the parameters is not stationary. However, by considering the value of the parameter process at a particular point as the parameter vector of interest, a well-defined likelihood function can be constructed. Cooley and Prescott evaluated the maximum likelihood estimation procedure, which provides consistent estimates of τ and asymptotically efficient estimates of $\boldsymbol{\beta}_{t+1}^p(\hat{\tau})$. The nature of the model precludes any notion of the consistent estimation of $\boldsymbol{\beta}_{t+1}^p$. The authors suggested taking β_{t+1}^{p} as the reference value, since this is the value needed for prediction for the first post-sample period. Cooley and Prescott (1976: 172-173) discussed the possibility of testing hypotheses about τ, and they evaluated the asymptotic distribution of $\hat{\tau}$. Although it is relatively simple to estimate, interpret and infer in the Cooley-Prescott model,[14] its application is not easy. In particular the need to specify matrices U_u and V_v may be very complicated. They have to be assumed on the basis of theoretical considera-tions, which in turn presumes the ability to specify the relative variability of the parameters.

Similar models have been considered by Belsley (1973a, b), Cooper (1973), Sarris (1973), Sant (1977), Rausser and Mundlak (1978) and Rausser et al. (1982). Applications of this model can be found in Rausser and Laumas (1976: 367-380), Laumas (1977: 271-276), Cooley and DeCanio (1977), Laumas and Mehra (1977: 911-916), Machak et al. (1985: 104-111) and Funke (1990: 97-109).

[14]The reader is referred to the Cooley-Prescott (1976) article for computational aspects.

7.5.3 THE ROSENBERG CONVERGENT PARAMETER MODEL

One problem with the Cooley-Prescott model is that the parameters vary over time but do not converge to any fixed values (this may not be a problem if there are structural "drifts"; Maddala, 1971: 341-358).

Rosenberg (1973: 399-450) considered a model similar to that of Cooley and Prescott. Instead of making the β_t a random walk, he considered a stochastically convergent parameter structure. His model is devoted to investigating the time series of cross-sections. The parameters of each cross-sectional unit vary over time in a random fashion but display the continual tendency to converge to a population norm. It is the last aspect that differentiates this model from the Cooley-Prescott model, where the parameters vary over time in some systematic way but do not converge to any particular value. In the interest of simplicity, only the one-unit variant will be shown. The basic model that Rosenberg consider is

$$y_t = \mathbf{x}_t' \boldsymbol{\beta}_t + e_t \quad , \qquad t = 1,\ldots,T \qquad (7.65)$$

where the e_t are iid, normal random variables with $E(e_t) = 0$

and $E(e_t{}^2) = \sigma^2$. Rosenberg adopted the parameter structure

$$\boldsymbol{\beta}_t = \overline{\boldsymbol{\beta}} + \Phi(\boldsymbol{\beta}_{t-1} - \overline{\boldsymbol{\beta}}) + \mathbf{v}_t$$

$$= \overline{\boldsymbol{\beta}}(I - \Phi) + \Phi\, \boldsymbol{\beta}_{t-1} + \mathbf{v}_t \qquad (7.66)$$

where $\overline{\boldsymbol{\beta}}$ is a (K x 1) population mean parameter vector and Φ is a (K x K) diagonal convergence matrix with elements $0 \leq \delta_i \leq 1$, $i = 1,\ldots,K$. Convergence rates δ_i show the relative difference between $\overline{\boldsymbol{\beta}}$ and $\boldsymbol{\beta}_{t-1}$, which still exist at time point t. The vector $\mathbf{v}_t$ is a (K x 1) normally distributed vector with $E(\mathbf{v}_t) = \mathbf{0}$ and $E(\mathbf{v}_t\mathbf{v}'_t) = V_v$, where V_v is a contemporaneous covariance matrix.

Rosenberg evaluated maximum likelihood and Bayesian estimation techniques for the general model, but for the sake of simplicity, it will be assumed that $\delta_i = \delta$ for all $i = 1,\ldots,K$ so that (7.66) can be rewritten as

$$\boldsymbol{\beta}_t = (1 - \delta)\overline{\boldsymbol{\beta}} + \delta\boldsymbol{\beta}_{t-1} + \mathbf{v}_t \qquad (7.67)$$

or in terms of the lag operator L,

$$(1 - \delta L)\boldsymbol{\beta}_t = (1 - \delta)\overline{\boldsymbol{\beta}} + \mathbf{v}_t \qquad (7.68)$$

The vector $\boldsymbol{\beta}_t$ can be solved as

$$\boldsymbol{\beta}_t = [\ (1-\bar{\delta})\boldsymbol{\beta}\ /\ (1-\delta L) + \mathbf{v}_t\ ]\ /\ (1-\delta L) \tag{7.69}$$

Hence

$$y_t = \mathbf{x}_t'\ \{\ [(1-\delta)\bar{\boldsymbol{\beta}}\ /\ (1-\delta L)] + [\mathbf{v}_t /\ (1-\delta L)]\ \} + e_t \tag{7.70}$$

Rewriting (7.70) gives

$$(1-\delta L)y_t = \mathbf{x}_t'\ [(1-\delta)\bar{\boldsymbol{\beta}}] + \mathbf{x}_t'\ \mathbf{v}_t + (1-\delta L)e_t \quad \text{or}$$

$$y_t = \mathbf{x}_t'\ [(1-\delta)\bar{\boldsymbol{\beta}}] + \delta y_{t-1} + w_t \tag{7.71}$$

where $w_t = \mathbf{x}_t'\ \mathbf{v}_t + e_t - \delta e_{t-1}$. Estimation of this kind of model is similar to the models with infinite geometric lags but with a much more complicated error structure. The estimation of infinite geometric lag models is described, for example, in Judge et al. (1980, Chapter 16).

7.5.4 THE KALMAN FILTER MODEL

Systems analysts have long considered models with varying coefficients. In this section a model arising from the engineering literature, the Kalman filter model (Kalman and

Bucy, 1961) is discussed. In this class of models the random parameters are generated by a nonstationary stochastic process. Some of the models that have been considered earlier can be thought of as special cases of the Kalman filter models. Kalman filter models were discussed, amongst others, by Belsley (1973c), Cooper (1973), Sarris (1973), Sant (1977), Rausser and Mundlak (1978) and recently by Gordon and Smith (1989, 1990).

The basic model may be written as

$$y_t = \mathbf{x}_t' \boldsymbol{\beta}_t + e_t \qquad (7.72)$$

with the following general parameter variation structure:

$$\boldsymbol{\beta}_{t+1} = \Phi \boldsymbol{\beta}_t + \mathbf{v}_{t+1}, \qquad (7.73)$$

with t = 0,...,T-1, and

where Φ is a (K X K) matrix of transition probabilities; $E(\mathbf{v}_t) = \mathbf{0}$; $E(\mathbf{v}_t, \mathbf{v}_t') = V_v$; e_s and $\mathbf{v}_t$ are uncorrelated for all t and s. Assuming that T, V_V and $\boldsymbol{\beta}_0$ are known and after performing some calculations, with repeated substitution, (7.73) can be rewritten as

$$\boldsymbol{\beta}_t = \Phi^t \boldsymbol{\beta}_0 + \sum_{i=0}^{t-1} \Phi^i \mathbf{v}_{t-i} \,, \tag{7.74}$$

$t = 1,\ldots,T.$

In stacked matrix form this is

$$\begin{bmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \\ \cdot \\ \cdot \\ \cdot \\ \boldsymbol{\beta}_T \end{bmatrix} = \begin{bmatrix} \Phi \\ \Phi^2 \\ \cdot \\ \cdot \\ \cdot \\ \Phi^T \end{bmatrix} \boldsymbol{\beta}_0 + \begin{bmatrix} I & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ \Phi & I & 0 & \cdot & \cdot & \cdot & 0 \\ \Phi^2 & \Phi & I & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \Phi^{T-1} & \Phi^{T-2} & \Phi^{T-3} & & & \cdots & I \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \mathbf{v}_3 \\ \cdot \\ \cdot \\ \mathbf{v}_T \end{bmatrix} \tag{7.75}$$

or, in compact notation,

$$\boldsymbol{\beta} = \Phi_1 \boldsymbol{\beta}_0 + \Phi_2 \mathbf{v} \tag{7.76}$$

Now if all T observations of (7.76) are rewritten as

$$\mathbf{y} = X\boldsymbol{\beta} + \mathbf{e} \tag{7.77}$$

where

$$\mathbf{y} = (y_1,\, y_2, \ldots, y_T)'$$
$$\mathbf{e} = (e_1,\, e_2, \ldots, e_T)'$$

and

$$X = \begin{bmatrix} \mathbf{x}_1' & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & \mathbf{x}_2' & \cdot & \cdot & \cdot & 0 \\ \cdot & & & \cdot & & \cdot \\ \cdot & & & & \cdot & \\ 0 & 0 & \cdot & \cdot & \cdot & \mathbf{x}_T' \end{bmatrix}$$

then the time-varying structure can be recast, for estimation purposes, as a mixed estimation problem by rewriting (7.76) as

$$\Phi_1\boldsymbol{\beta}_0 = \boldsymbol{\beta} - \Phi_2\mathbf{v} \tag{7.78}$$

Equation (7.78) is in the form $\mathbf{r} = R\boldsymbol{\beta} + \mathbf{u}$, where $R = I$, $\mathbf{r} = \Phi_1\boldsymbol{\beta}_0$ and $\mathbf{u} = -\Phi_2\mathbf{v}$. The $E(\mathbf{u}) = \mathbf{0}$ and $E(\mathbf{u}\mathbf{u}') = \Phi_2(I_T \otimes V_v)\Phi'_2 = \Omega$. Direct application of generalised least square leads to

$$\begin{aligned} \hat{\boldsymbol{\beta}} &= (\sigma^{-2} X'X + \Omega^{-1})^{-1} (\sigma^{-2} X'\mathbf{y} + \Omega^{-1}\mathbf{u}) \\ &= \mathbf{u} + \Omega X'(\sigma^2 I_N + X\Omega X')^{-1} (\mathbf{y} - X\mathbf{u}) \end{aligned} \tag{7.79}$$

and

$$\begin{aligned} \mathrm{Var}(\hat{\boldsymbol{\beta}}) &= (\sigma^{-2} X'X + \Omega^{-1})^{-1} \\ &= \Omega - \Omega X'(\sigma^2 I_N + X\Omega X')^{-1} X\Omega \end{aligned} \tag{7.80}$$

(see Sarris, 1973: 501-523 for proof).

The remaining question is how to obtain the values for Ω, V_v, $\boldsymbol{\beta}_0$ and σ^2 . For estimation purposes these values have to be known. Not much could be said about the first two matrices, although Sarris (1973) provided some guidelines on how they could be specified. Provided they are somehow known, being specified in a theoretical or in another way, without knowledge about $\boldsymbol{\beta}_0$ one is in a situation where the nature of development is known, but the starting point is not known. Although vector $\boldsymbol{\beta}_0$ cannot be specified in any theoretical way, on the basis of prior knowledge, there are some possibilities for finding starting points on the basis of the available data.

In order to obtain estimates of $\boldsymbol{\beta}_0$ and σ^2, substitute (7.76) into (7.77) to obtain

$$\begin{aligned} \mathbf{y} &= X\Phi_1\boldsymbol{\beta}_0 + X\Phi_2\mathbf{v} + \mathbf{e} \\ &= X^*\boldsymbol{\beta}_0 + \mathbf{e}^* \end{aligned} \tag{7.81}$$

where

$$E(\mathbf{e}^*) = \mathbf{0} \text{ and } E(\mathbf{e}^*\mathbf{e}^{*\prime}) = \sigma^2[I + \sigma^{-2}X\Phi_2(I_T \otimes V_v)\Phi_2{}'X'] \tag{7.82}$$

For any value of σ^2, $\boldsymbol{\beta}_0$ can be estimated by generalised least squares. If $\mathbf{e}^*$ is assumed to be multivariate normal, and beginning with an arbitrary value of σ^2, iterative estimation of $\boldsymbol{\beta}_0$ and σ^2 leads to local maximisation of the likelihood function. Cooper (1973) suggested a comfortable reparameterisation of the model that enables one to make a maximum likelihood estimation of the unknown parameters, provided that the matrices Φ and V_V are known. Again, the sampling properties of this rule are unknown. Generally, it may be stated that the estimation of the Kalman filter model is not satisfactorily solved.

A recent contribution to the Kalman filter literature is found in Gordon and Smith (1989, 1990) who proposed the multiprocess mixture methodology, which involves running multiple models in parallel using recursive Bayesian updating procedures that extend the standard Kalman filter. The motivation behind this was to provide separate models that can accommodate outlying observations, abrupt structural shifts in each of the parameters, and a steady-state relationship that reflects no shift or outliers. LeSage (1992) compared results from a mixture-model approach to those from the time-varying parameter (TVP) method proposed by Garbade (1977) and employed in studies by Belogia, Hafer and Sheehan (1988) and Hafer and Sheehan (1990). It was found that the Kalman filter mixture-model performed better in finding abrupt shifts in relationships, especially in the presence of outliers or transient observations. The mixture technique accommodates outliers and abrupt structural shifts in time series by running multiple models in parallel in an effort to keep these events from contaminating a steady-state model. The output from these multiple models is mixed together using posterior probabilities of each model as a weighting factor.

7.5.5 THE WATSON-ENGLE VARIABLE PARAMETER REGRESSION (VPR) MODEL

7.5.5.1 THE GENERAL VPR STATE-SPACE MODEL

Variable parameter regression (VPR) is a generalisation of dynamic regression, useful for the advanced forecaster in regression problems where some of the coefficients are known to be time-varying. In this case, the VPR model extends the conventional regression model by allowing some of the coefficients of the model to be unobserved time series, which must be estimated in its entirety in order to fit the model and calculate forecasts.

The VPR model of Watson and Engle (1983) has been discussed by a number of econometric researchers. Background information on the development of VPR techniques may be found in Zellner (1970), Rosenberg (1973), Cooley and Prescott (1976), Harvey and Phillips (1976) and Watson and Engle (1982). All of these models are special cases of the state-space model, often used in engineering to represent a variety of physical processes. In fact, a wide range of models used in econometrics can be viewed as special cases of state-space models as will be shown below. The advantage of viewing the models in this way is that general solutions are available based upon the likelihood principle and the Kalman filter recursive algorithm.

Watson and Engle (1983) discussed general approaches to the estimation problem for unobservable variables, with particular reference to the VPR problem. Their rather peculiar notation was selected to make the connection with state-space models clearer, because the state-space characteristics of the model are the basis of its estimation. The essence of their approach is to formulate the economic model as an engineering state-space model and to use the Kalman filter to generate the likelihood function via a combination of the EM technique (see Dempster, Laird and Rubin, 1977) and the method of Scoring (Pagan, 1980).

Specifying the Watson and Engle (1983) model in a slightly different way (to keep in line with the model specified for the Watson-Davies test), the VPR model can be specified as

$$y_t = \mathbf{x}_t' \tau + \mathbf{z}_t' \boldsymbol{\beta}_t + e_t \qquad (7.83)$$

where $\mathbf{x}_t'$ is an m-vector of explanatory variables with fixed coefficients; $\mathbf{z}_t'$ is an n-vector of explanatory variables with variable coefficients; $\boldsymbol{\beta}_t$ is an m-vector of time varying regression coefficients; and τ is an n-vector of fixed regression coefficients; e_t is the residual error at time t. The error term e_t can be an autoregressive (Cochrane-Orcutt) process, but for the sake of simplicity it is assumed that there is, in fact, no Cochrane-Orcutt process. Lagged dependent variables can, however, occur in either of the vectors of explanatory variables. As it stands Equation (7.83) leaves the unobserved time series of coefficients $\boldsymbol{\beta}_t$ completely open. In order to complete the model specification, the change of $\boldsymbol{\beta}_t$ over time needs to be specified, i.e. the details of the time series process. Their are several options in the literature about how this is to be done (some of which were already discussed in previous sections). The VPR model can either be applied in cases where the parameters of the regression model follow a random walk process (see the model discussed in Chapter 6) or where it follows an AR(1) process (see the Rosenberg model discussed in Chapter 7). The simplest and most widely used VPR model, however, is obtained by restricting the time series process $\boldsymbol{\beta}_t$ to be a random walk in each coefficient separately. For simplicity of notation, it is assumed that there is only one time varying coefficient. While the equations below are not in Kalman form, it can easily be converted to that form (see Goodrich, 1989: 190).

In Equation (7.83), $\boldsymbol{\beta}_t$ can therefore be taken as a random walk process where

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \mathbf{v}_t \qquad (7.84)$$

where $\mathbf{v}_t$ is NID$(\mathbf{0}, V_v)$; V_v is a diagonal covariance matrix and $\mathbf{v}_t$ is assumed to be uncorrelated with e_t. Equations (7.83) and

(7.84) jointly specify the random walk VPR model. The coefficients of the model include the error variance and fixed coefficients of Equation (7.83), plus the error variance of the random walk process. The total number of coefficients in the VPR model is just one more than in the corresponding conventional regression. Under this nonstationary model, the parameter has no fixed mean. It will therefore change permanently over the course of the historical data, as is expected, for instance, in most cases. Each coefficient autoregresses not only on its own history, but potentially also on histories of the other coefficients. When this model is used, the parameter $\boldsymbol{\beta}_t$ will drift over the course of the data, usually with an obvious trend reflecting continuing change of the parameter. The value used to prepare the forecasts will be the last available value of $\boldsymbol{\beta}_T$ from the historical fitting data.

A second model for the time varying coefficient $\boldsymbol{\beta}_t$ is a AR(1) process with a constant driver:

$$\boldsymbol{\beta}_t = A\boldsymbol{\beta}_{t-1} + \mathbf{K} + \mathbf{v}_t \tag{7.85}$$

where $\mathbf{K}$ is a k-vector of coefficients; A is a (K x 1) convergence matrix with elements $0 \le \alpha_i \le 1$, $i = 1,\ldots K$. In this process, the parameter is basically constant, but is subject to transitory changes, perhaps caused by unpredictable but temporary external influences. The value of the parameter may stray from the mean for considerable times, if its AR parameter is large, but it always return to the mean eventually. To prepare actual forecasts, the stochastic parameters needs to be forecasted first. Since the AR(1) model is stationary, these forecast values for the parameter β will converge, after several periods, to a constant mean level $K/(1-\alpha)$, assuming $\alpha_i = \alpha$ (see discussion on the Rosenberg model). This model uses three parameters, namely α, K and the variance of v_t. In practice several of the matrices of the general model are constrained or omitted. The total number of coefficients to be estimated are therefore not usually so large as the general model suggests.

As shown in Schweppe (1965) or Harvey (1982) the likelihood function of the unknown parameters in the above equations is easily formed. Let e_t denote the innovations in y_t (i.e., $y_t - E(y_t | y_{t-1}, \ldots, y_1, z'_t, \ldots, z'_1)$) and let p_t denote the variance of e_t. The log likelihood can be written as

$$L(\theta) = \text{constant} - \tfrac{1}{2} \sum_{t=1}^{T} \{\log(p_t) + e_t^2/p_t\} \qquad (7.86)$$

where $p_t = \text{var}(e_t)$; and θ is the vector of unknown parameters to be fitted. For the above random walk model, θ includes the fixed coefficients and the variances of v_t and e_t. The innovations and their variances can easily be calculated using the Kalman filter. The Kalman filter is, therefore, used to compute the likelihood function for any given set of parameter values. The principal use of the Kalman filter and the Kalman smoother in VPR is therefore to generate the likelihood function, the coefficient estimates and their variances, for any given set of parameter values θ.

The Kalman filter requires a value of the mean and variance of $\boldsymbol{\beta}_{0|0}$ as an initialisation. Often these values arise naturally, for example, when the $\boldsymbol{\beta}$ process is stationary, the filter is initialised with the unconditional mean and variance of $\boldsymbol{\beta}$. When the $\boldsymbol{\beta}$ process is nonstationary, the likelihood function conditional on the initial state can be formed and the value of the initial state can be estimated as nuisance parameters (see Chapter 6).

Given the data and the form of the likelihood function, it is a simple task in principle to maximise the likelihood function with respect to the unknown parameters. Unfortunately this maximisation is not so simple in practice as there are usually a large number of parameters and each evaluation of the likelihood function requires an appreciable number of calculations. Therefore, two algorithms are discussed by Watson and Engle (1983) for maximising the likelihood in such cases. Both methods are maximum likelihood techniques in which the maximum likelihood solution is found by applying the Kalman filter repeatedly in an iterative search process through the parameter space. The techniques for this search are described in the next sections.

7.5.5.2 ESTIMATION BY EM AND SCORING ALGORITHMS

The EM algorithm of Dempster et al. (1977) is discussed first. It consists of two steps: an estimation (E) and a maximisation (M) step which are iterated to convergence. The maximisation step calculates the maximum likelihood estimates of all the unknown parameters conditional on a full data set. The estimation step constructs estimates of the sufficient statistics of the problem conditional on the observed data and the parameters. Essentially, the missing observations are estimated based on the parameter values at one step of the iteration and then the likelihood function is maximised assuming that this is the full observable data set in the other. This is a derivative-free method and does not require any evaluation of the likelihood function. This algorithm is guaranteed to increase the likelihood at every iteration, but it converges only linearly.

The Kalman filter method is used to compute the likelihood function for any given set of parameter values (OLS normal equations for θ) based on fitted values of β_t and it's variance. The resulting parameter values are then used to recompute the Kalman filter. The EM algorithm thus simply flip-flops between these two computations until convergence is obtained. This method has the desirable properties that first, it always converges to a local maximum of the likelihood function; second, the value of the likelihood function increases at each iteration and finally, it is computationally stable. Its main disadvantage is that its convergence is often slow. Once it is close to the maximum it may take quite a while to pinpoint the maximum. It is therefore at its best in approaching the general area of the maximum. When this general neighbourhood is reached, the method of scoring (developed by Pagan, 1980) is superior, since it is quadratically convergent near the maximum.

The scoring method is a modified Newton method (the modification consists of substituting the Hessian matrix for its estimated expectation; Schneider, 1991). This method uses only first derivatives of the likelihood function (as well as the likelihood function itself) and produces asymptotically efficient estimates in one iteration from consistent initial parameter estimates. It is computationally more expensive than the EM algorithm, but it is much more efficient in securing final convergence at the optimum. It is also advantageous in that it yields, unlike the EM algorithm, a consistent estimate of the error covariance matrix of the parameters via the Fisher Information matrix.

At each iteration of the scoring algorithm, a new trial parameter vector is computed by multiplying the gradient of the likelihood function into the inverted information matrix. The derivatives required for the information matrix and the gradient are computed numerically. The resulting new trial log likelihood function is computed and compared to the old, and if it has increased, it is accepted. If no increase is achieved, then the step size is cut by one half, and a new trial log likelihood is computed. This process is continued until the log likelihood function actually increases.

These maximum likelihood iterations yield an important by-product - computing the Fisher information matrix. This matrix is used to compute asymptotically valid estimates of the standard deviations of the parameters and their intercorrelations.

While the scoring algorithm is attractive because it uses only first derivatives, produces an estimate of the information matrix, and is one step asymptotically efficient from consistent initial estimates, it is found that it may be slow to converge, particularly when starting with poor initial estimates. Near the likelihood maximum scoring has quadratic convergence properties, but far from the maximum this method may generate misleading search vectors due to a bad approximation of the Hessian matrix. As an alternative one may use the EM method (described above), which converges only linearly near the likelihood maximum (Dempster et al., 1977), but which - as practical applications show - also generates satisfying increases in likelihoods far from the likelihood maximum. Also the method often yields negative values for the variance in Q during the iterations, and special penalty functions must be employed or the parameters must be transformed to avoid this problem. The EM algorithm, adapted to the problem, avoids these problems.

The most practical method seems to be a mix of EM and scoring. EM can be used to quickly move the parameters to the neighbourhood of the maximum. Scoring can then be used to pinpoint the maximum and calculate an estimate of the information matrix. Under-identification problems undetected by the EM algorithm will become apparent when the scoring algorithm attempts to invert the information matrix. The scoring algorithm can also be used in a straightforward manner to calculate Lagrange multiplier statistics.

Recently, several other research contributions have been published in this area. Schneider (1991) specified, for example, a state-space model and applied scoring, the EM method and the so-called adaptive EM method for the estimation of hyperparameters of a particular random walk parameter model. A descriptive interpretation of Kalman filtering (the so-called flexible least squares approach - see Kalaba and Tesfatsion 1986 and 1988) is described, and its use as an exploratory data analysis approach to a preliminary descriptive stability analysis of a traditional money-demand function for the Federal Republic of Germany is discussed.

7.5.5.3 PRACTICAL CONSIDERATIONS BEFORE USING VPR

One should always be sure that a conventional regression model has been specified that is adequate except for the time variation in one or more parameters. If the nonvarying part of the model is not correctly specified, the algorithm will force the time varying coefficient into surrogate behaviour. The algorithm is blind to the structure of the problem and merely tries to reduce forecasting error over the historical data set. If the adaptation procedure explains variance that has nothing to do with the associated variable, then forecasts from the model can be disastrous. More complex models allow more degrees of freedom for adaptation and, hence, more scope for surrogate adaptation.

VPR should therefore only be used when it is conceptually reasonable that one or more parameters are stochastically time varying and when specific statistical tests reject the null hypothesis of fixed parameters. Two such tests, the Watson-Davies and the Chow test are recommended.

If the coefficients of a model are suspected to change abruptly at discrete time periods, the Chow (1960) test should be used. If the test rejects the null hypothesis, one might then explore conventional regressions involving dummy variables that turn on or off at discrete points in time. Deterministic variation of parameters can always be treated by constructing appropriate variables under conventional regression. VPR should be avoided in these cases. In fact, Goodrich (1990) recommended that one should always try a more elaborately specified conventional regression, and only after convincing yourself that some of the regression coefficients is truly varying over time, one should proceed to VPR.

On the other hand, VPR is most suitable when one or more of the coefficients vary smoothly in time. Although these changes might be generated by some economic process, it is reasonable to substitute an ARIMA surrogate for the true economic process. In the Watson-Davies test, H_0 is that the coefficients are constant and the alternative is that it is varying via some AR(1) process with unknown parameter. Even though this test is complex and fairly expensive in terms of computer time (because the AR(1) parameter defined under the alternative, is not defined under the null), Goodrich (1990) stated that, experimentally, the test appears to be powerful and very useful.

7.6 SUMMARY AND GENERAL RECOMMENDATIONS

It is always tempting to argue that the parameters in econometric models cannot, in general, be expected to be constant and hence that is ideal to consider a varying parameter model in almost all circumstances. However, this type of argument can be made about every assumption made, and in the presence of voluminous data the luxury of very general models can be afforded. If data is limited, as it often is, a limit to the generality can be postulated. It, therefore, become sometimes necessary to resort to varying parameter models.

In previous sections a variety of statistical models that have been developed for situations where the coefficients of the general linear model are assumed to vary in a systematic way across observations, have been reviewed. Justification for the use of varying parameter models usually follows one of two lines.

First in importance is the situation where the coefficients of an otherwise properly specified relationship are different for some subsets of the available sample, that is, the sample data cannot be pooled. Estimation with a model that does not take this into account will produce results which do not accurately represent the existing economic structure and do not serve as a good basis for forecasting. The exact consequences of ignoring the parameter variation would, of course, depend on the nature and degree of the misspecification. The other justification for the use of a model with varying parameters is that econometric models are necessarily abstractions from and simplifications of reality. Adoption of the classical linear model may then imply misspecifications that cause the coefficients of the model to apparently vary across the sample even though the true underlying structure is not changing. Some examples of these causes are outlined in Section 1.4 dealing with "Causes of Coefficient Variation".

The applied worker is faced with the problem of choice between these models, or, in other words, to decide which set of assumptions is the most plausible for the particular problem. As always, it is impossible to give blanket recommendations that are suitable for every problem. However, the following issues are worth considering.

(a) Are the slope coefficients likely to vary over individuals, or is it reasonable to capture individual differences through the intercept or appropriate modelling of the disturbance term ?

(b) If the coefficient or the intercept vary over individuals, are the differences likely to depend on the explanatory variables pertaining to the individuals? If so, a dummy variable model or the seemingly unrelated regressions model is likely to be preferable. If not, the random assumptions of the error components models or the Swamy random coefficients model might be reasonable (see Judge et al., 1985 for a complete discussion).

(c) For modelling changes over time which method is better? - one may

(i) assume a constant correlation structure of the disturbances and use the error component model;

(ii) assume that disturbances are generated by some autoregressive or moving average process; or

(iii) choose a dummy variable model and regard inference as conditional on changes in the sample.

(d) How many observations are there? In models where the parameters are assumed to be random, the relative sizes of N and T will have an important bearing on the finite sample reliability of variance estimates. For example, if N is small, it is unlikely that $\sigma_\mu{}^2$ for the error component model or V_v for the Swamy random coefficient model will be very reliable. Consequently, the estimated, generalised least squares estimators for the slope coefficients are also likely to be unreliable and, for estimation, it may be better to treat the coefficients as fixed even when the random assumption is reasonable.

It is necessary to remember that there is a great danger of misspecification. The model chosen is only as good as the structural information introduced on the parameters' variation. Theoretically, by introducing more information about the nature of the process being modelled, the model should be more informative; but because the information imposed may not be true, the danger of misspecification is great.

A number of model specification tests may be used to help choose between model specifications. Once a model has been specified there are additional problems concerning the most efficient estimation procedure and the testing of hypotheses about parameters. The problem of testing the constancy of the coefficients, with the varying coefficient model serving as the alternative, is not completely resolved, although many tests have been suggested (see Chapter 4).

The use of random or varying parameter models under these circumstances leaves one a bit uneasy. Models with random but not systematically varying parameters force recognition of another source of estimation and forecasting inaccuracy and thus to some extent prevent overstatement of the quality of the statistical results. Modelling a changing economic structure by allowing response parameters to vary over observations may be a realistic approach, but the chances for misspecification are many. Although it may be possible to forecast the conditional mean of the dependent variable more accurately by letting model parameters vary systematically with trend variables, this does little to reveal the nature of the actual structural change. As always, the fact is that inferences about economic processes based on statistical models can only be as good as the theoretical and institutional knowledge of the economic structure on which the model is based.

There are a number of varying parameter models to choose from. The choice of a model depends on, amongst other things, the nature of the available data, the assumptions and restrictions that apply as well as the existence of an operational method of estimating these models.

In Section 7.3.2 a general model of nonstochastic (although it is easily made stochastic) parameter variation is presented in which the parameters vary as a function of some explanatory variables. It is suitable for use with time series, cross-sectional or combined data, where subsets of the observations are thought to be generated by different parametric structures. The primary difficulty in using this model is, of course, that one must specify the structure causing the parameter variation. Errors in specification will lead to familiar and unfortunate consequences. Section 7.3.3 contains two special cases of the general model, namely, seasonal models, where a different parametric structure is appropriate for different seasons, and piecewise regression models, in which there are two, or a few, subsets of observations for which different structures exist.

Models with a stochastic parameter structure are presented in Sections 7.4 and 7.5. The Hildreth-Houck model has random coefficients that are drawn from a population with fixed mean and covariance. When using cross-sectional data, one could argue that the Hildreth-Houck random coefficient model should always be used, since its assumptions are more general than those of a constant coefficient model. However, this is not the only issue. If the number of observations is such that the covariance matrix of the disturbance vector cannot be accurately estimated, one may be better off assuming that the coefficients are constant. Also, if all models are regarded as an approximation to some underlying process, the random coefficient approximation may not be better than a constant coefficient approximation. Thus using the Hildreth-Houck random coefficient model is recommended when it has advantages in terms of both realism of assumptions and estimation efficiency. Where some doubt exists, it might be worthwhile using statistical tests for heteroscedastic errors.

The Harvey-Phillips return to normality model in Section 7.4.3 is like the Hildreth-Houck model in that parameters are assumed to be generated by a stationary stochastic process. However, it is more general in that the coefficients reflect a dynamic, stochastic parameter structure. The Cooley-Prescott and the Rosenberg model in Section 7.5.2 and 7.5.3, respectively, also present a dynamic, stochastic parameter structure, but in this case the process is nonstationary. The Harvey-Phillips return to normality, the Rosenberg and the Cooley-Prescott models are more likely to be used with time series data. Swamy et al. (1988), however, pointed out that the maximum likelihood estimators for all the unknown parameters of the Cooley-Prescott and the Rosenberg models do not exist and that there are no operational method of estimating these models. These difficulties with the maximum likelihood procedure are not appreciated by Rosenberg (1973), Cooley and Prescott (1976), Pagan (1980), Harvey and Phillips (1982) and Judge et al. (1985: 809-814), among others.

Another concept in time series analysis is the cointegration concept in econometric model building to which increasing attention has been paid over the last few years. Regressing one random walk against another, for example, can lead to spurious results, in that conventional significance tests will tend to indicate a relationship between the variables when in fact none exists. (A time series is denoted I(d) if its d-th differences are stationary.) If two different time series are both I(1), e.g., random walks, but a linear combination of them is I(0), i.e., stationary, the two series are called cointegrated. Cointegration theory and tests have been developed by Granger (1986) and Engle and Granger (1987) for models with constant parameters. Cointegration might be rejected, however, just because parameters are erroneously assumed to be constant (see Teräsvirta, 1991). Cointegration is traditionally a linear concept and allowing for time-varying parameters (e.g. structural change) makes it much more flexible. Thus, Granger and Lee (1991) extended the idea of cointegration to time-varying parameter (TVP) regression.

Certain properties of a TVP cointegrated process and of the related estimation procedure are indicated. It is also pointed out by the authors that if a TVP cointegration procedure provides evidence of an equilibrium relationship, but the traditional linear cointegration does not, then this might be an indication of some misspecification of the linear cointegrating model.

To conclude, the most practical model to use, when parameters follow a nonstationary process, is the variable parameter regression (VPR) model suggested by Watson and Engle (1983) discussed in Section 7.5.5. Two methods, namely the EM and the scoring algorithms are combined to obtain the greatest overall computational efficiency. Both methods are maximum likelihood techniques, based on computation of the likelihood function via the Kalman filter, as described in Section 7.5.5. It should be noted, however, that VPR should be used only under very specific conditions. First, one should be sure that a conventional regression model has been specified that is adequate except for the time variation in one or more parameters. Second, one should be sure that the parameter really does vary smoothly over time. Finally, one has to check the fitted values of the time varying coefficients for plausibility. The researcher should also bear in mind that VPR is a complex model since the basic regression equations must be supplemented to describe the transition of the time-varying coefficients. Because of this complexity it takes a considerable amount of computer time to fit the model.

Several of the models considered above have restrictions imposed on it. Building models so as to have each equation satisfy one or another set of these restrictions requires prior considerations about the forms of economic laws that are within the purview of coherent economic theories. Any set of contradictory restrictions or restrictions violating the conditions under which empirically interpretable models exist should be rejected outright. Restricted models could be justified only if one could find empirically that models so restricted were still coherent (or free from contradictions) and performed better in prediction than models not so restricted.

This is not to say that models should be used without restrictions. The above argument only calls for caution in imposing any restriction on equations. If one wishes to analyse econometric models without imposing any restrictions because of fear that any restrictions on these equations might introduce contradictions, then it may be necessary to use arbitrary values for the parameters of the equations. These arbitrary values may lead to unreasonable results or poor forecasts. A general recommendation is that in applied work a judicious use of tests with prior knowledge about the nature of the problem should be combined.

The problem of estimating models with changing parameters deserves further study. In particular, finite-sample properties as well as pretest estimator properties have to be further investigated. The same applies to finite-sample properties of a number of test statistics used in the regression model with varying-parameters. Since the variances necessary to estimate parameters by means of generalised least squares are unknown rather than known - and maximum likelihood estimates are used - their finite-sample properties and the sampling distributions of the slope estimates based on them are not sufficiently known.

CHAPTER 8

AUTOREGRESSIVE CONDITIONAL HETEROSCEDASTIC (ARCH) MODELS

8.1 INTRODUCTION

Uncertainty is central to much of modern monetary and finance theory. Nelson (1991) argued that most asset pricing theories relate expected returns on assets to their expected means, conditional variances and covariances. An enormous literature in empirical finance has documented that these conditional moments change over time. Practical experience (as in the 1929 and 1987 US stock market crashes) reinforces this conclusion. Also, if a government switches to controling money growth rather than the interest rate, interest rates can become quite volatile (that is, they begin to vary a great deal around the mean). A similar heteroscedasticity was observed in South Africa when exchange rate policy switched from fixed exchange rates to flexible exchange rates. In the latter case, exchange rates fluctuated a great deal, making their forecast variances larger. Unfortunately, conditional variances and covariances are not directly observable and researchers and market participants must use estimates of conditional second moments. To create these estimates, they rely on models which are, no doubt, misspecified. The question is, therefore, (i) how accurate are these estimated variances and covariances? and (ii) how can researchers estimate them more accurately?

When it is assumed that the residuals are NID $(0,\sigma^2)$, it is not only assumed that they are uncorrelated, but that they are homoscedastic, i.e. that their variance of prediction errors is constant over time. However, time series often are highly variable and difficult to predict for certain transient periods, i.e. during the oil crisis or during periods when stock markets are more volatile than usual. Usually such behaviour is caused by exogenous interventions that could not be feasibly included in the model. During such periods of volatility, model residuals ordinarily become very large, but eventually subside to normality. As a result, data from the transient period has unusually high leverage on the estimated coefficients (because OLS minimises the squares of the errors). The variables in the model may be forced into surrogate roles in trying to explain the effects of the intervention and, as a result, estimates of the corresponding coefficients are biased.

The effect of using a conventional model in the presence of heteroscedasticity is not necessarily disastrous. The major penalty is that the fitting algorithms tend to concentrate on goodness-of-fit to the series over its periods of greater variance. In effect the less volatile periods are ignored to some degree. Thus the sample may be used inefficiently. However, the estimates of the model coefficients are still consistent and unbiased.

8.2 THE ARCH REGRESSION MODEL

The ARCH technique is used when the wish is to model a series of which the variance is changing with time. The model allows the error variance to change over time via an autoregressive process. As a result the model does not overly concentrate on periods of higher variance when selecting the model coefficients.

Traditional econometric models assume a constant one-period forecast variance. Some models of conditional heteroscedasticity have been developed by econometricians, but the most widely used models of dynamic conditional variance are the ARCH models first introduced by Engle (1982). Since their introduction, ARCH models have become a widely used tool for estimating conditional variances and covariances.[15] Most models assume that the residuals have a constant variance over time, but real econometric series often violate this assumption. Series may become quite variable and difficult to forecast for certain transient periods, or they may exhibit systematic changes in the variance, either over time or as a function of other variables. The ARCH model deals with such situations, i.e. where the variance of the series is itself a time series.

[15]See the survey of Bollerslev, Chou and Kroner (1990), which references hundreds of applications of ARCH in the empirical finance literature.

In its most general form (see Engle, 1982: Equations 1-5), a univariate ARCH model makes the conditional variance at time t a function of exogenous and lagged endogenous variables, time, parameters and past residuals. In other words, the ARCH model assumes that the conditional variance of the current variable is fully described by past observations. With this basic idea unchallenged,[16] all ARCH variants have differed only with respect to which past variables and what functional form should be used to explain changes in conditional variance.

The basic model consists of the OLS or Cochrane-Orcutt model except that it is no longer assumed that $e_t \sim NID(0,\sigma^2)$. Formally, let e_t be a sequence of (orthogonal) prediction errors, $\boldsymbol{\alpha}$ a vector of parameters, $\mathbf{x}_t$ a vector of exogenous and lagged endogenous variables and $\sigma_t{}^2$ the variance of e_t given information at time t. The ARCH model assumes that in Model (4.1)

$$e_t = \sigma_t Z_t \tag{8.1}$$

$$Z_t \sim \text{iid with } E(Z_t) = 0,\ \text{var}(Z_t) = 1 \tag{8.2}$$

$$\begin{aligned}\sigma_t{}^2 &= \sigma^2(e_{t-1},\ e_{t-2},\ldots,\mathbf{x}_t,\ t,\ \boldsymbol{\alpha}) \\ &= \sigma^2(\sigma_{t-1}Z_{t-1},\ \sigma_{t-2}Z_{t-2},\ldots,\mathbf{x}_t,\ t,\ \boldsymbol{\alpha})\end{aligned} \tag{8.3}$$

[16]There are a few papers which challenge this idea. The only exceptions are Taylor (1986), Nelson (1988), Schwert (1989) and Hsieh (1991).

It is, therefore, assumed that the error variance is an autoregressive process and can be estimated as a function of the errors in previous lagged values, viz.

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \ldots + \alpha_p e_{t-p}^2 + v_t$$

where $\quad e \sim N(0,\sigma_t{}^2)$ (8.4)

The ARCH model is identical to the conventional regression model except that the variance of the disturbance e_t is not assumed to be homoscedastic (i.e. $NID(0,\sigma^2)$). The coefficients to be determined include those from the conventional regression equations and those from Equation (8.4). Equation (8.4) could therefore also contain explanatory variables such as time, population, income, and so on. The α coefficients must be nonnegative or else the estimated variance might itself go negative. Furthermore, the system described by

$$Z_t = \alpha_1 Z_{t-1} + \alpha_2 Z_{t-2} + \ldots + \alpha_p Z_{t-p} \qquad (8.5)$$

must be stable, so that variances do not become infinite. As a result, the variance will go through transitory behaviour but will return to near its equilibrium value α_0.

The order of p of the ARCH process is usually taken to be the number of periods per year, so that year-to-year variance clustering can be captured. The coefficients could be constrained to avoid estimation of too many parameters and to insure smoothness. Three patterning options are considered:

(i) all but α_p are zero;

(ii) $\alpha_1 = \alpha_2 = \ldots = \alpha_p$; or

(iii) the α - coefficients decrease linearly.

Taking case (ii), the ARCH estimation equation becomes

$$\sigma_t^2 = \alpha_0 + \alpha_1(e_{t-1}^2 + e_{t-2}^2 + \ldots + e_{t-p}^2) / p \qquad (8.6)$$

and for case (iii)

$$\sigma_t^2 = \alpha_1(\Phi_1 e_{t-1}^2 + \Phi_2 e_{t-2}^2 + \ldots + \Phi_n e_{t-p}^2) + \alpha_0 \qquad (8.7)$$

where Φ_1, $\Phi_1, \ldots, \Phi_n$ comprise a set of predetermined weights; α_0 and α_1 are the only coefficients to be estimated; and $\sigma_t{}^2$ is the variance of e_t . The ARCH model should be considered when specific hypothesis tests for ARCH effects so indicate.

To test whether disturbances follow an ARCH process, the LM procedure is employed which is simply based on the autocorrelation of the squared OLS residuals (R^{*2}). Under the null hypothesis of no ARCH effects, $(T-p)R^{*2}$ has the chi-square distribution with p d.f. Engle (1982) described these (Lagrange multiplier) tests in detail. The test of the null hypothesis $\alpha_1 = \alpha_2 = \ldots = \alpha_p = 0$ is interpreted as the test that the error variance is not a conditional process, i.e. that there is no ARCH process. Estimates of the regression parameters are obtained by minimising the generalised sum of squared errors $\sum_{t=1}^{T} e_t^2/\sigma_t^2$ instead of $\sum_{t=1}^{T} e^2$. The parameters include the conventional regression parameters, plus the parameters in the equation used to estimate σ_t^2.

One can think of Equation (8.3) as a filter through which the data is passed to produce an estimate of the conditional variance. It should be noted, however, that the term "estimation" as it is used in the filtering literature rather than as it is used in the statistical literature - i.e. the ARCH model "estimates" the true conditional variance in the same sense that a Kalman filter estimates unobserved state variables in a linear system.[17]

The ARCH regression model can be seen as an approximation to a more complex regression model which has non-ARCH disturbances. The ARCH specification might pick up the effect of variables omitted from the estimated model. The existence of the ARCH effect may be interpreted as evidence of misspecification, either by omitted variables or through structural change. If

[17] See, e.g., the use of the term in Anderson and Moore (1979, Chapter 2), or Arnold (1973, Chapter 12).

this is the case, ARCH may be a better approximation to reality than making standard assumptions about the disturbances - trying to find the omitted variable or determine the nature of the structural change would be even better.

The principal advantages of the ARCH process are (1) if the process is properly specified but heteroscedastic, then the coefficient estimates under ARCH will be more efficient, and (2) if the transiently high error variances are caused by interventions from outside the model, then parameter bias may be reduced. In the latter case, ARCH may be considered a robust technique in the sense that it decreases the computational leverage wielded by atypical data. In the presence of an ARCH process, the advantage of using an ARCH model is very similar to that which may be obtained by increasing the size of the sample data.

8.3 EXTENSIONS OF THE ARCH MODEL: A LITERATURE REVIEW

Many different parameterisations for the function of σ_t^2 have been used in the literature, including the original ARCH (p) specification of Engle (1982); the ARCH-in-Mean (ARCH-M) model introduced by Engle, Lilien and Robins (1987); the ARCH-M model with a time-varying parameter (TVP ARCH-M), studied by Chou, Engle and Kane (1992); the Generalised ARCH (GARCH) and GARCH-M models of Bollerslev (1986) and Engle and Bollerslev (1986), respectively; the log GARCH models of Pantula (1986) and Geweke

(1986); the Exponential ARCH (EGARCH) model of Nelson (1989, 1991); the threshold ARCH (TARCH) model by Zakoian (1990); the modified ARCH (MARCH) model by Friedman and Laibson (1989); the nonlinear ARCH (NARCH) and - GARCH (NGARCH) models by Higgins and Bera (1989 and 1992, respectively); and the Taylor/Schwert model of Taylor (1986) and Schwert (1989) (also see the entire 1992 issue of the "Journal of Econometrics", Volume 52, No. 1/2 for further references).

The model which is more frequently used (besides the ARCH model) is the GARCH (generalised ARCH) model, developed by Engle (1982) and Bollerslev (1986). For instance, one could formulate the v_t term in Equation (8.4) as a moving average, so that the error becomes $v_t + \tau_1 v_{t-1} + \tau_2 v_{t-2} + \ldots + \tau_q v_{t-q} + w_t$. This model has proved to be a useful means for empirically capturing the momentum in conditional variance. Under GARCH, shocks to variance persist according to an autoregressive moving average (ARMA) structure of the squared residuals of the process. The extension of the ARCH process to the GARCH process bears much resemblance to the extension of the standard time series AR process to the general ARMA process.

Much of the recent evidence from financial-market data seems to suggest that persistence in variance, as measured by ARCH models, is quite substantial. This apparent empirical regularity has motivated Engle and Bollerslev (1986) to introduce the integrated - GARCH (I-GARCH) process, in which shocks to variance do not decay over time. Integration in variance is analogous to a unit root in the mean of a stochastic process, an example of which is the random walk.

Many other models were also surveyed by Nelson (1991) who examined in detail the filtering properties of GARCH (1,1), EGARCH and the model of Taylor (1986) and Schwert (1989).

8.4 APPLICATIONS OF ARCH MODELS

Since their introduction by Engle (1982) and Bollerslev (1986), respectively, ARCH and GARCH models have found extraordinarily wide use. The survey article by Bollerslev, Chou and Kroner (1992) cited more than 300 papers applying ARCH, GARCH and other closely related models. As they showed, ARCH and GARCH models have been very successful at modelling time-varying volatility in financial time series.

The ARCH regression model (with extensions) has a variety of characteristics which make it attractive for econometric applications. Econometric forecasters have found that their ability to predict the future varies from one period to another. McNees (1979: 52) remarked that, "the inherent uncertainty or randomness associated with different forecast periods seems to vary widely over time." He also documented that, "large and small errors tend to cluster together (in contiguous time periods)". This analysis immediately suggests the usefulness of the ARCH models where the underlying forecast variance may change over time and is predicted by past forecast errors. The results presented by McNees also showed some serial correlation during the episodes of large variance.

A good practical example is found in monetary theory and the theory of finance. By the simplest assumption, portfolios of financial assets are held as functions of the expected means and variances of the rates of return. Any shifts in asset demand must be associated with changes in expected means and variances of the rates of return. If the mean is assumed to follow a standard regression or time series model, the variance is immediately constrained to be constant over time. The use of an exogenous variable to explain changes in variance is usually not appropriate.

The ARCH models have already proven useful in modelling several different economic phenomena, and there are numerous examples. In Engle (1982), Engle (1983) and Engle and Kraft (1983) models for the inflation rate are constructed recognising that the uncertainty of inflation tends to change over time. In Coulson and Robins (1985) the estimated inflation volatility is related to some key macroeconomic variables. Models for the term structure using an estimate of the conditional variance as a proxy for the risk premium are given in Engle, Lilien and Robins (1985). The same idea is applied to the foreign exchange market in Domowitz and Hakkio (1985). In Weiss (1984)

ARMA models with ARCH errors are found to be successful in modelling thirteen different U.S. macroeconomic time series. Engle, Granger and Kraft (1984) presented a generalisation to the bivariate ARCH model. Two competing models of inflation - essentially a simple monetarist model and a mark-up model - have their forecasts combined using time-varying weights derived from bivariate ARCH equations. Lamoureux and Lastrapes (1990: 225-234) examined the persistence of the variance, as measured by the GARCH model, in stock return data. In particular, they investigated the extent to which persistence in variance may be overstated because of the existence of, and failure to take account of, deterministic structural shifts in the model. Engle et al. (1990) suggested using a FACTOR-ARCH model as a parsimonious structure for the conditional covariance matrix of asset excess returns.

Nelson (1990b) gave one more likely reason for the empirical success of ARCH: "When both observable and conditional variances change 'slowly' relative to the sampling interval (in particular, when the data generating process is well approximated by a diffusion and the data are observed at high frequency) the broad class of ARCH models - even when misspecified - provide continuous-record consistent estimates of the conditional variances. That is, as the observed variables are observed at finer and finer intervals, the conditional variance estimates produced by the (misspecified)

ARCH model converge in probability to the true conditional variances."

Common to most of the above applications, however, is the introduction of a rather arbitrary linear declining lag structure in the conditional variance equation to take account of the long memory typically found in empirical work, since estimating a totally free lag distribution will often lead to violation of the nonnegative constraints. Nelson and Cao (1992), however, showed that these constraints could be substantially weakened and so should not be imposed in estimation. They argued that their "so called" inequality constraints are less severe in keeping the conditional variance nonnegative.

8.5 EVALUATION OF ARCH MODELS

One widely voiced criticism of ARCH model (see, e.g., Campbell and Hentschel, 1990; and Anderson, 1990) is that they are ad hoc - i.e., though they have been successful in empirical applications, they are statistical models, not economic models. As are all statistical and economic models, ARCH models are at best a rough approximation to reality: it is too much to hope that the models are "true". This criticism, though correct, does not go far enough; even as purely statistical models, ARCH models are ad hoc - i.e., in applied work, there has been considerable arbitrariness in the choice of ARCH models, despite (perhaps because of) the plethora of proposed ARCH specifications. Many models have been proposed, but few compared to the uncountably infinite potential number of conditionally heteroscedastic time series models.

Also, in applied econometrics, the tendency is to tackle specification problems one at a time rather than considering them jointly. This has serious consequences for statistical inference. One example of this is considering autocorrelation and autoregressive heteroscedasticity (ARCH) separately. Bera, Higgins and Lee (1992) considered a linear regression model with random coefficient autoregressive disturbances that provides a convenient framework to analyse autocorrelation and ARCH simultaneously. Because of a strong interaction between ARCH and autocorrelation, Bera et al. demonstrated that neglecting conditional heteroscedasticity or misspecifying the autocorrelation structure might result in unreliable inference.

PART V

THEORETICAL MODELLING AND METHOD OF EMPIRICAL INVESTIGATION

CHAPTER 9

MODEL SELECTION, MODEL ESTIMATION AND THE METHOD OF EMPIRICAL INVESTIGATION

9.1 SELECTION AND ESTIMATION OF THE MODELS

Historically, South Africa has experienced a number of economic structural changes, some abrupt and some of a more gradual nature, which may be the cause of parameter instability in econometric equations. One may even argue on an *a priori* basis that the South African economy may be particularly well suited as test bed to explore the consequences and remedies for structural changes in econometric models.

Many econometricians today remember the 1960's as halcyon years when policy makers believed they could "fine-tune" the economy and determine an optimal policy mix, first, by simulating their fixed coefficient models under a variety of policy assumptions and, then, by reviewing the influence of their policy assumptions on the pertinent endogenous variables. The experience of the 1970's to the 1990's irrevocably altered the sanguine attitude of those policy makers. Severe and dramatic structural shocks and shifts such as, for example, crop failures, the two energy price shocks, the movement from fixed to floating exchange rates, the Soweto riots, the introduction of a general sales tax, high inflation rates in recessionary times, an international debt crisis, the growth of a strong trade union movement, a process of ongoing constitutional changes together with some degree of political instability, the declaration of a state of emergency and the resulting stringently applied international sanctions wreaked havoc on the vain attempts by econometricians to provide reliable and consistent policy recommendations from the results of their models with fixed or deterministically changing slopes. Varying parameter regression models have been developed to address these and other problems.

The primary aim of this study is to confront empirically estimated models in South Africa, in which a model-builder at some stage had some trust, with an array of tests which are supposed to detect departures from the assumption of structurally stable parameters. The empirical results are offered as an additional warning that conventional statistics such as R^2 or t-ratios may give misleading information on the appropriateness of econometric models. This is in line with Lovell (1983: 1-12), who points out the ease with which high t-values can be obtained without there being any relationship between variables whatsoever. Supplementing conventional regression output with a battery of specification tests therefore will make it harder for results, that are the product (whether intentional or unwittingly) of some data mining process, to appear "significant".

Recursive residual and log likelihood techniques are combined to detect and locate shift points. Consequently, the feasibility of modelling parameter instability in econometric models is investigated. The study is limited to functions for production, fixed investment and exchange rates, all of which are treated in any standard macro-econometric model.

No claim is made that the functions selected cover the full population of functions of this nature, nor that the functions selected form a random sample of all possible functions. Rather the technique of accessibility sampling is used and the most important criterion in selecting a particular function was the availability of data which allowed for the re-estimation of the equation. Included in the study are equations arising from published articles and conference papers, as well as newly developed models.

The chosen equations are re-estimated, with the latest published data, by means of ordinary least squares (OLS) and ARCH techniques. Varying parameter regression (VPR) techniques are applied afterwards if an equation proved to be structurally unstable. The period under study is from 1970Q1 to 1992Q4 in the case of quarterly models, from 1970 to 1990 for yearly models, and from 1970M1 to 1992M12 for monthly models . These periods invariably differ from the originally published estimation periods.

Apart from differences in the estimation periods, the following additional restrictions are imposed at the time of re-estimation:

(a) All computations are done with commercialised computer software which restricts the choice of tests and random coefficient models available.

(b) Although some of the VPR techniques can be applied to cross-sectional data, only those techniques which can handle time series data are considered for this investigation.

(c) Polynomial lag structures in original equations are replaced with unconstrained lagged variables because of the inability of the IAS-System Level IAS-3.6 Test Processor, which is used in this study, to cope with polynomial lag structures in testing mode.

(d) At times dummy variables in the original equations led to problems with near singular matrices in the Test Processor and had to be left out when the functions were specified for re-estimation. This, of course introduces some bias in the reported results. Dummy variables used for seasonal adjustment in the original functions are ignored and seasonally adjusted data are used for re-estimation.

(e) Proxy variables are substituted for the variables used by the original model builder in cases where such variables are no longer published, for e.g., real investment in industrial and commercial enterprises is substituted with real private fixed investment excluding residential buildings and agriculture.

(f) Many of the functions tested are meant to be used in simultaneous equation context and some are estimated using a more suitable technique than ordinary least squares. The tests and models used in the study cannot handle simultaneous or non-linear equations and therefore the single equation models re-estimated here may be oversimplified and thus be prone to fail tests for structural stability.

Partly due to data revisions and partly due to changes in the estimation period, the parameters could not always be duplicated which by itself may be indicative of structural instability. It is acknowledged that the original model builder would by this time probably have re-specified and re-estimated the original equation or have improved it in some other way. Therefore, and because of the other limitations discussed above, it would be unfair to couple the name of the model builder to the name of a particular equation estimated. Rather than studying the merits of a particular equation or model builder, this study deals with large sample evidence regarding structural instability. The purpose of the study will therefore be to detect possible structural changes, and to find methods of statistical inference that are robust to its presence.

9.2 TESTS OF PARAMETER CONSTANCY

A number of techniques have been developed by researchers (see Chapter 4) to test the parameter constancy of a regression relationship. For reasons given earlier, only those procedures known as the Cusums and the Cusums of squares, the Watson-Davies, the Fluctuation, Quandt's log-likelihood ratio, and the Chow test will be applied to investigate the stability of conventionally estimated econometric functions.

The Cusum test procedures, developed by Brown, Durbin and Evans (BDE) (1975), are applied to calculate sums of recursive residuals, or squares of recursive residuals, and to check whether the resulting sequences have deviated too much from their expected paths. The Cusums of squares (Cusum-SQ) is the more powerful of the two tests. However, several pragmatic problems arise with the above procedure. Some of the more serious ones are the fact that it simultaneously tests the null hypothesis (H_o) of time-independent regression parameters and time-independent error variances. This leaves the alternative hypothesis (H_a) in a rather vague form, because neither does it distinguish between abrupt or gradual changes in the regression parameters, nor between homoscedasticity and parameter constancy. Furthermore, these tests assume the classical error structure in the linear regression model, and the effects of departures from this assumption, e.g. autocorrelation, are not well understood. The problem of discriminating between rejections of the null hypothesis of stability due to non-well behaved errors, and rejections due to incorrect assumptions about the structural part, has not been solved. Finally, the general location of the shift indicated by the BDE analysis moves as the interval is lengthened. The occurrence of "spurious" indications of a shift also disappears with a slight lengthening of the estimation interval under consideration. In order to reduce the effect of the last two problems, it is required that the shift indication be "stable", that is, that it be repeated in several tests and give consistent estimates of the general location of the change.

The Fluctuation test (FLUCT) is a test for structural stability with no prior knowledge about the number and timing of possible structural shifts. Unlike the BDE tests, it is based on the recursive parameter estimates themselves rather than on the recursive residuals. It is shown that this test has non-trivial power against many local alternatives, and that it compares favourably with the BDE tests (see Ploberger et al., 1989). Monte Carlo experiments have shown that the FLUCT test is considerably more powerful than the BDE tests. Unfortunately this test cannot be applied to trended data.

It is therefore preferred to interpret the results of these tests in the spirit of exploratory data analysis i.e. as yardsticks rather than formal tests of hypotheses at nominal α-levels. This view of Brown et al. (1975: 150) is supported by the often contradictory results obtained from the various tests.

Because of the above problems, and also because the BDE and FLUCT techniques are intended to test for the presence of a change rather than to pinpoint the time of the change, the Quandt (1958, 1960) log-likelihood ratio (LR) technique is used to obtain a closer estimate of the shift point. The null hypothesis in Quandt's test is that the observations in the periods 1,...,t and t+1,..., T come from the same regression; the alternative hypothesis is that they do not belong to the same regression regime. After re-estimating each equation, the Quandt ratios are computed for time points t = K+1,...,T-(K+1) where K is the number of regression parameters and T the length of the estimation interval. The Quandt-ratio is a maximum likelihood procedure which estimates the most likely point of a structural break in the regression relationship. The estimate of the change point is taken as the point at which the likelihood ratio achieves a minimum, except where this point occured very close to the beginning or end of the sample period.

The Quandt test, however, is somewhat qualitative in nature, since the distribution under H_O has not been specified and thus no significance levels can be estimated. If the interval is extensive, so that more than one shift is possible, the problem of choosing between local minima arises. Therefore, all local Quandt minima are identified for later testing for stability by means of the Chow test. The combined use of the BDE, FLUCT and Quandt tests, however, reduces the abovementioned problem because the BDE and FLUCT tests can detect the existence of a change and suggest the change point, while the Quandt-ratio confirms the timing of the shift.

Once a switching point has been located by the Quandt test, it is possible to apply the Chow test (Chow, 1960). The Chow test rejects the H_O of parameter constancy whenever the fit of the equation can be greatly increased by splitting the regression into two parts. The data point with the lowest Quandt-ratio is used to split the sample. This, of course, inflates the size of the test well above the nominal α-level of the Chow test. The Chow test is also applied to time-points which could be identified as local Quandt-minima.

Other than the Chow test, which checks against the alternative hypothesis that the parameters take on different values in the first and second parts of the data, the Watson-Davies test (Watson and Engle, 1985) checks for smoothly time varying coefficients. Under the alternative hypothesis of the Watson-

Davies test, a coefficient varies over time in a first order autoregressive (AR(1)) process. This test can be performed on each coefficient in the model.

9.3 VARYING PARAMETER REGRESSION TECHNIQUES

A variety of statistical models have been developed for situations where the coefficients of the general linear model are assumed to vary in a systematic way across observations. The choice of a model depends on, amongst other things, the nature of the available data, the assumptions and restrictions that apply as well as the existence of an operational method of estimating these models. This study is restricted to those models which are programmed into commercialised computer software, handling time series data only.

After the estimation of the original (Stage 1) model, a conventional regression model is adequately specified and re-estimated, except for the time variation in one or more parameters. A number of model specification tests are used to help choose between alternative model specifications. If the coefficients of a model are suspected to change abruptly at discrete time periods, the Chow and Quandt-ratio tests (together with the BDE and FLUCT tests) are used. Whenever these tests reject the null hypothesis of no structural change, the use of switching regression methods can be investigated. Deterministic variation of parameters can always be treated by constructing appropriate (dummy) variables under conventional regression and therefore varying parameter regression techniques should be avoided in such cases.

Varying parameter regression (VPR) models are most suitable when one or more of the coefficients vary smoothly in time. Although these changes might be generated by some economic process, it is reasonable to substitute an ARIMA surrogate for the true economic process. The Watson-Davies test is used to check for smoothly time varying coefficients. If H_O is rejected, implying that coefficients vary via some AR(1) process with unknown parameter, then the VPR method is used.

The VPR technique is a generalisation of dynamic regression, useful for the advanced forecaster in regression problems where some of the coefficients are time varying, either deterministically or stochastically. The varying parameter regression procedure automatically fits a model to the data to control the transition of the time varying parameters from one point in time to the next. However, VPR should only be used after the best conventional model has been built with one or more of the regression coefficients truly varying over time.

The VPR model, in its various forms, has been discussed by a variety of authors. Among them are Harvey and Phillips (1982), Rosenberg (1973) and Cooley-Prescott (1976) whose models are more likely to be used with time series data. Swamy et al.

(1988), however, pointed out that the maximum likelihood estimators for the unknown parameters of the Cooley-Prescott and Rosenberg models do not exist and that there are no operational method of estimating these models. Therefore, the most practical model left, in the case where parameters follow a stationary process, seems to be the VPR model of Watson and Engle (1983). With this model two methods, namely the EM (Estimation and Maximisation) and the scoring algorithms, are combined to obtain the greatest overall computational efficiency. Both methods are maximum likelihood techniques, based on computations of the likelihood function via the Kalman filter. Graphs of time series for varying coefficients are also produced for the VPR procedure.

9.4 FORECASTING STRATEGY

It is always useful to evaluate and compare the forecasting performance of the competing models. An initial part of the time series is used for the estimation of variable and fixed coefficient versions of the structural models while the remaining period is used to generate one-step-ahead or multi-step-ahead forecasts of the out-of-sample values of the dependent variable. This holdout period is then used for forecast evaluation and model comparison.

Multi-step-ahead forecasts more realistically represent the needs of the average user of forecasting models. Multi-step-ahead forecasts, for each of the stages of estimation are therefore used for comparison. The accuracy of these forecasts is judged by the mean forecast error, taking one period at a time, the forecast error squared and the root mean square error which is used as the principal criterion for comparing forecast models. This is done in conjunction with diagnostic checking. The SEE statistic is the one-step-ahead forecast error for the model over the historical period. For regression this equals the standard error of estimation (SEE).

Besides the above summary statistics for measuring out-of-sample forecast accuracy, use is made of the Akaike Information Criterion (AIC) and the Bayes Information Criterion statistics to select the model that is likely to forecast (out-of-sample) most accurately for a particular data set. The two statistics differ in how severely they penalise model complexity. The BIC punishes complexity more severely and research has shown that the BIC leads to better out-of-sample forecasts than AIC (see Koehler and Murphree, 1986).

All computations were performed on a personal computer at the University of the Western Cape and the mainframe computer at the University of Stellenbosch. The estimation techniques were programmed into the Forecast Master Plus (FMP) version 1.02 regression package and the Interactive Simulation System (IAS) level 3.8. The FMP program was originally prepared by Goodrich and Stellwagen (1989) for Business Forecast Systems, Incorporated and used by the Electric Power Research Institute

(EPRI), California, U.S.A. The IAS-System, has been developed and improved at the Institute for Advanced Studies in Vienna, Austria since 1973. These computer packages were found to be adequate in handling the abovementioned procedures.

CHAPTER 10

GROSS FIXED INVESTMENT FUNCTIONS

10.1 INTRODUCTION

South Africa has been faced with a serious disinvestment campaign during the past decade which caused a lot of structural changes in the economy as a whole. The underlying politico-economic situation in the 1980's and perceptions about the future growth and stability of the South African economy contributed to the decline in real fixed investment spending. Political unrest and the proclamation of a State of Emergency during the 1980's provoked an intensified disinvestment campaign and partial economic sanctions against the country.

The future prosperity of the country will also be threatened by the almost certain reluctance of private foreign investors to reinvest capital in a country from which it has previously been withdrawn, particularly if the country is suffering structural economic damage and political instability. It is also widely speculated that any changes in the current tax structure, which was comprehensively reviewed in 1985 and 1986 by the Commission of Inquiry (the Margo Commission) into the tax structure of the Republic of South Africa, could be a source of structural instability in the near future. Since economic structural change will be with us for a long time to come, it is worthwhile to investigate the adaptation of theoretical investment models, which assume a fixed parameter structure, to models which can accommodate changes in regression parameters over time.

In the previous chapter the transformation of labour and capital into output is considered. In this chapter the determinants of capital formation is considered, not just because this is a logical next step in studying economic behaviour, but because investment presents an ideal area for the development of methods of econometric analysis.

Investment involves the production of capital goods which are not consumed within the current period and may themselves be used for the production of goods in future periods. Gross fixed investment expenditure is therefore not only an important determinant of short-run fluctuations in the level of economic activity, but is of obvious importance for the long-run growth in capacity of an economy. The fact that capital goods last for more than one period also has implications for the behaviour of firms because the future value of investment become as relevant as their current value.

Fixed investment is a flow which adds to the stock of fixed capital. However, the stock of capital itself depreciates with use and has to be replaced. Thus, the change in capital stock equals net investment. It is not possible to derive automatically the demand for investment goods from the demand for capital stock. A 'shortage' of capital stock could be made up either very slowly or very quickly. Most theories of investment behaviour therefore contain two important elements.

(1) A theory about what determines the optimal stock of capital, K^*.
(2) Assumptions about how the actual capital stock, K, adjust to the optimal stock.

Furthermore, some assumptions have to be made concerning the determinants of replacement investment. It is generally assumed that some constant proportion of the initial stock wears out in each 'period' and is automatically replaced.

The purpose of this chapter is to discuss various theories of investment behaviour within a unified framework. Problems of specification, especially of the time structure of investment and of parameters are discussed. Empirical evidence on investment at the aggregate level is presented and summarised. This study will be concerned only with one of the components of total investment expenditure namely, fixed plant and equipment, and not the determination of other components such as residential construction and inventory investment.

This chapter therefore starts with a short consideration of the theory of investment behaviour and then moves on to explain how lags may be incorporated into the system. A number of these models are applied to South African economic data and the chapter is concluded with a discussion on random-coefficient approaches to the explanation of investment behaviour. An important distinction is made between models which emphasise the role of output (flexible accelerator models), and whose justification is generally rather ad hoc, and models which emphasise the role of relative prices (neoclassical models) which are derived from explicit optimising models. Similar models will be empirically tested for economic structural change and the viability of a stochastic coefficient approach is investigated.

10.2 THEORIES OF INVESTMENT BEHAVIOUR

10.2.1 ACCELERATOR MODELS

The naive accelerator model was first suggested by Clark (1917) as a possible explanation of the volatility of investment expenditure. This model assumes that there is a fixed capital - output ratio α so that net investment I_t is given by

$$I_t = K_t - K_{t-1} = \alpha(Q_t - Q_{t-1}) \qquad (10.1)$$

where Q = output and K = capital stock.

The deficiencies of the naive accelerator model are well known (see Mayes, 1981: 124 or Thomas, 1985: 252) namely, an infinitely elastic supply of capital goods and the maintenance of a constant capital-output ratio.

The flexible accelerator first developed by Koyck (1954), meets the first of the above deficiencies of the 'naive' accelerator. The flexible accelerator (or partial adjustment hypothesis) provides a generalisation in which actual net investment is only a proportion of investment required to achieve the desired capital stock position, K_t^*. Thus

$$I_t = K_t - K_{t-1} = (1-\tau)(K_t^* - K_{t-1}) \quad \text{with } 0 < \tau < 1 \tag{10.2}$$

where I_t is the net investment. If it is assumed that the capital-output ratio determines the desired capital stock, then

$$K_t^* = \alpha Q_t \tag{10.3}$$

so that substituting into Equation (10.2) yields

$$K_t = \alpha(1 - \tau) Q_t + \tau K_{t-1} \tag{10.4}$$

On repeated substitution for the lagged value this equation gives an expression for K_t as a distributed lag function of Q_t with geometrically declining coefficients:

$$K_t = \alpha(1 - \tau) \Sigma\tau^j Q_{t-j} \qquad (10.5)$$

Thus capital stock at time t is dependent not only on current output but also on past levels of output. Such lagged effects might be the result of decision-making delays, administrative delays or delivery delays.

Furthermore, gross investment I_t is equal to net investment plus replacement investment D_t:

$$I_t = (K_t - K_{t-1}) + D_t \qquad (10.6)$$

Alternatively, end-period capital stock equals beginning capital stock plus gross investment less depreciation:

$$K_t = K_{t-1} + I_t - D_t \qquad (10.7)$$

In the flexible accelerator model such depreciation is normally assumed to be proportional to the existing capital stock:

$$D_t = \delta K_{t-1} \qquad (10.8)$$

thus

$$I_t = K_t - (1 - \delta)\ K_{t-1} \tag{10.9}$$

in which case using (10.2) and (10.3)

$$I_t = (1 - \tau)\alpha\ Q_t - (1 - \tau - \delta)\ K_{t-1} \tag{10.10}$$

If τ and α are to be estimated from a simple regression function of I_t on Q_t and K_{t-1} then some prior knowledge of the depreciation parameter, δ, is necessary. The difficulty might be overcome by obtaining an extraneous estimate for δ. Alternatively the equation can be transformed to eliminate K_{t-1}, which is useful if stock data are unavailable or unreliable. Taking first differences on both sides gives:

$$I_t = (1 - \tau)\ K_t^* - (1 - \tau)(1 - \delta)\ K_{t-1}^* + \tau\ I_{t-1} \tag{10.11}$$

In the simple case in which $K_t^* = \alpha\ Q_t$, a regression of I_t on Q_t, Q_{t-1} and I_{t-1} yields estimates of τ, α, and δ.

A major criticism of the flexible accelerator model is, of course, that the optimal capital stock is determined via a constant capital - output ratio. This would follow if one assumes either a 'fixed coefficients'-type of production function with no possibility of factor substitution, or, alternatively, that the production function exhibits constant returns to scale and that relative factor prices remain unchanged so that there is no cause for a cost-minimising firm to vary its factor proportions.

10.2.2 THE INCLUSION OF OTHER DETERMINANTS INFLUENCING INVESTMENT BEHAVIOUR.

Neither the naive nor the flexible accelerator as described above pay any attention to other factors which are involved in the decision making of the firm, like financial determinants, for example. More importantly, it is not derived from explicit optimising behaviour of the firms. In the previous chapter the output decision itself and input decisions related to factor prices are considered. Since investment is actually creating one of these factors, capital, there is clearly a rather more complex structure which could be incorporated. Besides the price of capital goods decisions to invest could also depend on the stream of returns (itself dependent on product market conditions, factor costs and rates of taxation) and interest rates.

However, it was not until Jorgenson (1963; 1965, 1967) presented his neoclassical theory of investment that a possible way of combining output effects with interest rates and other cost of capital effects became available. There were those who introduced various other decision variables into the analysis. Since the objective is not to provide a survey here, but merely trying to establish a number of major points of economic and econometric interest, only comprehensive reference will be made to these models.

The inclusion of the influence of the availability of internal and external funds to the firm was also considered by de Leeuw (1962); Evans (1969) as well as Bean (1979). The argument for their inclusion is very simple; although previous output (or sales) determines the general level of investment through appropriate lag distributions, the ability to carry out the desired investment programme depends on the availability of sufficient funds. Thus if internal funds to the firm in the form of retained profits are larger, then it is possible to invest more. In the same way, if the market rate of interest for commercial borrowing is cheaper, firms will tend to invest more (see Hines and Catephores, 1970). These are not the only financial indicators available; others such as the debt-asset ratio gives an indication of the firm's ability to borrow. Experience in including these variables has been mixed, but in general it is clear that as Bean's results have shown, for example, some extra financial factors can contribute to the explanation of investment.

The work of Jorgenson provides a rather more interesting alternative approach and presents an important development, relating investment behaviour to profit maximising considerations.

10.2.3 JORGENSON'S NEOCLASSICAL MODEL

The accelerator models discussed earlier rest on relative little economic theory.

Jorgenson (1963, 1965, 1967) argued that the theory of investment behaviour depends upon the neoclassical theory of optimal accumulation of capital. In the Jorgenson model the firm maximises present value (i.e. the discounted sum of future expected revenues minus expenditure on capital and labour inputs) subject to the neoclassical production function. A series of powerful and rather restrictive assumptions are made which can be summarised as follows (see Thomas, 1985: 255-256):

(1) no costs are incurred in adjusting capital stock to its optimal level;

(2) there is perfect competition in all markets. The firm is indifferent between renting capital goods and borrowing the funds to buy them;

(3) there is no uncertainty and, hence, no discrepancy between the actual and expected values of variables; and

(4) the production function has the usual neoclassical properties.

To obtain an intuitively understanding of the 'equilibrium' conditions in the Jorgenson model, suppose that assumptions 1 to 4 above hold and suppose for the moment that instead of maximising present value the firm simply maximises its current instantaneous flow of net revenue, R_t. Assume, further, that the firm hires its capital equipment just as it does its labour. It therefore maximises:

$$R_t = p_t Q_t - w_t L_t - m_t K_t \tag{10.12}$$

where Q_t, L_t and K_t are current flows of output, labour and capital inputs respectively; p_t is the price of output; w_t is the wage rate and m_t is the rental price of capital. R_t is maximised subject to the neoclassical production function:

$$Q_t = F(K_t, L_t) \tag{10.13}$$

Employing the Lagrange multiplier technique yields certain conditions for optimality:

$$\frac{\delta Q_t}{\delta K_t} = \frac{m_t}{p_t} \; ; \qquad \frac{\delta Q_t}{\delta L_t} = \frac{w_t}{p_t} \tag{10.14}$$

This means that the marginal products of capital and labour must equal the real rental price of capital and the real wage rate, respectively. However, a firm may also purchase and sell capital stock. Since, given assumption 2, it is indifferent between renting and owing, the total cost of owning one unit of capital stock must be the same as the cost of renting it. The total cost of owning capital stock is composed of three elements. Firstly, there is the opportunity cost of having funds tied up in fixed capital. If the price of capital goods is q_t and the rate of interest r_t then this opportunity cost equals $r_t q_t$. Secondly, capital goods depreciate over time. Assuming a constant depreciation rate, δ, the depreciation cost equals δq_t. Finally, capital goods may change in price so that the firm may incur a capital gain or loss on them equal to their change in price $\dot{q}_t$. Jorgenson refers to the total cost of owning capital stock as the 'user cost of capital', c_t.

$$c_t = r_t q_t + \delta q_t - \dot{q}_t \qquad (10.15)$$

c_t is the implicit or 'shadow price' of capital and must under present assumptions equal m_t, the rental price of capital. Thus the equilibrium conditions of Equation (10.4) change to c_t/p_t and w_t/p_t on the right side of the equations, respectively.

The equilibrium conditions together with the production function form a 3-equation simultaneous system which determines optimal values for the three endogenous variables, K_t, L_t and

Q_t in terms of exogenous relative factor prices c_t/p_t and w_t/p_t.

Therefore, in a Cobb-Douglas framework the equation for optimal capital stock is:

$$K_t^* = B(c_t/p_t)^{h(1-\beta_2)} (w_t/p_t)^{h\beta_2} \qquad (10.16)$$

where B is a function of β_o, β_1 and β_2 and $h = (1-\beta_1-\beta_2)^{-1}$. Due to assumption 1 above Equation (10.16) also determines K.

Besides the 'user cost of capital' determination Jorgenson also investigated the effect of varying tax parameters on capital investment. From this he derived that parameters of the so called 'after tax' user cost of capital, c_t. However, the quantitative importance of these parameters, and of the relative price ratio in general, is a matter of empirical investigation.

The significance of Jorgenson's investment model is its attempt to create a rigorous microeconomic theory of investment based on the optimising behaviour of firms. A major criticism of the Jorgenson model is that, because of the assumption of no adjustment cost, it has nothing to say about investment as it is usually understood. Investment is usually regarded as being the result of the adjustment of actual capital stock to its desired level. In the Jorgenson model an explanation of the adjustment process is assumed away - adjustment is instantaneous and investment occurs only because of changes in the desired (and hence actual) capital stock, brought about by changes in relative prices. In his empirical work, Jorgenson was forced to allow for non-instantaneous adjustment by superimposing a very ad hoc lag structure onto his theoretical model.

10.3 DISTRIBUTED LAG FUNCTIONS AND ESTIMATION PROBLEMS

The general form of the investment function developed in the previous section, where investment is a function of output (sales) over the current and previous periods and investment itself over previous periods, is a common feature of many investment functions - see for example Eisner (1960), de Leeuw (1962), Almon (1965) and Evans (1969, Chapter 4).[18]

The main problem with the output variable is that it is desired, rather than actual, output that is relevant to the investment decision. Actual output may not equal desired output because it may be constrained by the availability of capital stock. However, in practice either current values of some weighted average of past and current values are used. The

[18] No mathematical derivations will be given in view of the fact that most lag structures for investment are widely discussed in many econometric text books.

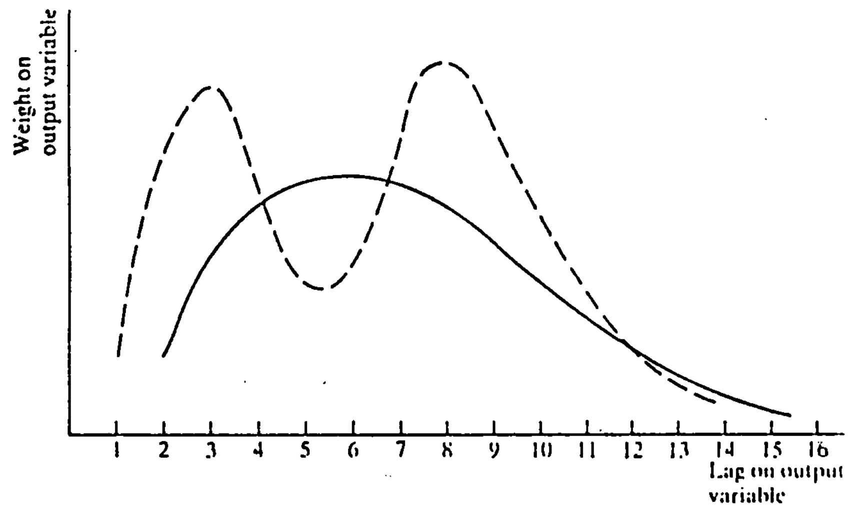
second problem arises over the utilisation of capital stock. The price ratio, for example in the Jorgenson model, will be misspecified if the user cost of capital ignores the influence of variations in capital services on depreciation cost. In calculating the cost of capital services Jorgenson's assumptions need to be accepted whereby 'locking in' effects are ignored. Otherwise the cost relative to an investment decision are those expected over the lifetime of the project. In general, the best that can be done is to replace current values by some combination of current and lagged values. An index of investment goods prices is generally used for q_t in Equation (10.15) while r_t is represented by the long-term bond rate or a weighted average of this rate and the dividend/price ratio for equities. The depreciation parameter, δ, is generally assumed constant and estimated using investment data together with an initial and a terminal value for capital stock.

A major problem in the estimation of investment equations is the specification of the appropriate lag structure. When any of the factors influencing desired capital stock change, a firm is unlikely to attempt or be able to adjust its actual capital stock immediately. There are various reasons for possible delays - both subjective and technical (see Mayes, 1981: Chapter 4). Lag structures can be complicated and may vary between individual firms. This means even greater complexity is to be expected in the case of investment behaviour of aggregates of firms. Moreover, the lag structure may vary over time, particularly if there is any variation in the degree of capacity utilisation in capital goods industries or in the availability of finance to firms wishing to expand.

Unfortunately, the problem of multicollinearity places severe limits on the number of lagged values. Hence, in general, some a priori restrictions have to be placed on the lag structure to reduce the number of parameters that need to be estimated. One simple scheme proposed by Koyck (1954) has the useful property of allowing all past values of the variables to have some effect and yet requires only one unknown parameter. Since recent values of desired capital stock are likely to have a more important effect than past values, Koyck proposed that the weights should decline geometrically - that is, each weight is a constant proportion, τ of the previous weight in an equation such as the flexible accelerator model. As seen from Equation (10.5) this implies a geometric lag structure for the relationship between capital stock and output. A major disadvantage of this specification is that it implies maximum impact on the current periods, even though a delay of several periods may exist before any impact on investment is felt at all. However, it is possible to reformulate the accelerator model so that changes in capital stock depend on past rather than current deficiencies in actual capital stock (see Thompson, 1988: 261). Although this procedure delays the initial impact of changes in output, it still implies that the initial impact is the greatest and that successive lagged values of output are of progressively lesser importance. A likely pattern is shown by the solid line in Figure 10.1. Solow (1960: 392) generalised Koyck's scheme to allow for r different stages in the decision and investment process. Jorgenson (1966) provided an even more general lag scheme by proving that any arbitrary lag function can be approximated by his so called, rational lag form.

Due to the fact that it could take several months and even years to construct many forms of capital equipment and buildings, investment may merely be part of a large project which was decided upon and commenced several time periods ago. There are thus decisions and construction lags which suggests that the pattern of the weights in the lag distribution rises in the first place as one moves back through previous periods and then falls. Evans (1969) went even further than this and suggested that the peak of the decision lag will occur before the peak of the construction lag and hence the lag distribution will have two peaks and will take the form of what he described as an 'inverted W' as shown in the dotted line in Figure 10.1.

FIGURE 10.1: POSSIBLE LAG DISTRIBUTIONS FOR CAPITAL STOCK ADJUSTMENTS



source: Thomas (1985: 262)

However, these lag structures could have repercussions for estimation, like, for example autocorrelation and degrees of freedom problem when OLS is used. To get over this problem, use is made of some form of constraint on the coefficients of the lag distribution.

The simplest solution is, therefore, to decide on the form of the distribution, such as giving equal weights or an 'inverted V' as suggested by de Leeuw (1962). In both cases there is only one parameter to be estimated in the distribution. Other distributions may be estimated by including further parameters.

However, it may be that one only has an approximate view of the exact form of the distribution such as the position of the peak or peaks, in which case a different technique must be applied.

An alternative approach is to choose the form of the lag distribution and firstly estimate the parameters of that from the sample of observations on the individual variable separately. Then using these previously calculated weights on the variable, the full investment equation can be estimated. Almon (1965) used this method and fitted a polynomial distributed lag to output before estimating the investment equation as a whole. This method assumes that the coefficient β_i can be approximated by a polynomial in i, so that in the model:

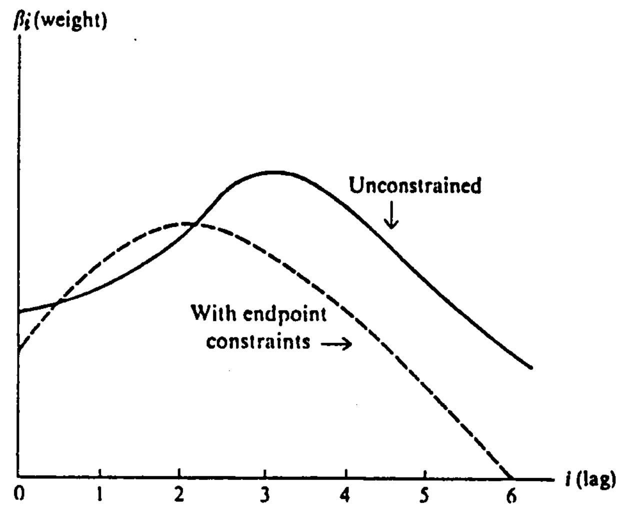
$$Y_t = \beta_0 x_t + \beta_1 x_{t-1} + \dots + \beta_k x_{t-k} + u_t$$

the coefficient

$$\beta_i = f(i) = \alpha_0 + \alpha_1 i + \alpha_2 i^2 + \dots + \alpha_r i^r \qquad (10.17)$$

Because continuous functions can generally be approximated by a polynomial, this procedure is quite flexible. Figure 10.2 illustrates two commonly assumed shapes for β_i that are reasonable in many circumstances.

FIGURE 10.2: POLYNOMIAL (OR ALMON) LAG



source: Ramanathan (1989: 385)

When there has been a change in government policy (for example, the inactment of a new tax law), one might expect the immediate effect to be negligible. The main effect may be felt in two - or three quarters, and thereafter the effect might decline again.

The use of this form of estimation for the lag distribution requires the choice of only two parameters, the order of the polynomial (p), and the degree of the polynomial (r) which is the number of past periods over which the weights in distribution are non-zero. It is also possible to constrain the shape of the distribution so that one or both of the weights on the end periods of the distribution are zero. In one of graphs in Figure 10.2, endpoint constraints such as $\beta_{-1} = \beta_{p+1} = 0$ are imposed; the other is unconstrained. This enables one to fit a very wide range of shapes (see Mayes, 1981: 134). While there are criteria which can be used to aid this choice, it is not an easy one to make and Thomas (1977) has shown that poor choices can lead to very erroneous results. The general approach is to consider only a low order polynomial ($p \leq 4$) which is consistent with the theoretical pattern which the distribution can have.

10.4 THE STOCHASTIC COEFFICIENT INVESTMENT FUNCTION

Increases in net investment typically expand a society's production capacity and act as a medium for technology change. It is not surprising, therefore, that aggregate investment decisions have reached considerable empirical attention. Although a growing literature assessing the relationship between the macro economy and gross fixed investment exists, less attention has focus specifically on the structure of gross fixed investment.

Most empirical analysis of investment, as a matter of practical necessity, assume invariant parameters. It is widely acknowledged that functional parameters derived from time series data are probably not fixed, yet it is hoped that the model's structure is sufficiently stable to accommodate such an assumption. Lucas (1967) offered one alternative to the typical fixed-coefficient view of investment behaviour with the development of the flexible-accelerator model. The power of the Lucas accelerator lies in the flexibility of the adjustment coefficient in which, unlike most other partial adjustment models, the speed of the adjustment depends on economic phenomena and, therefore, varies through time. The failure to consider how the speed of adjustment changes over time may lead to ineffective or counter-productive policy recommendations.

In his paper on econometric policy evaluation, Lucas (1981: 109-110) argued, "the standard, stable parameter view of econometric theory and quantitative policy evaluation appears not to match several important characteristics of econometric practice". Several explanations in support of parameter variation can be advanced. For example, fixed coefficient econometric models may not be consistent with the dynamic economic theory of optimising behaviour; that is, changes in economic or policy variables will result in a new environment that may, in turn, lead to new optimal decisions and new microeconomic and macroeconomic structures (Lucas, 1981). This explanation is associated with the intuitive notion that the economic structure is not static but is always undergoing change.

Examining how investment is affected by policy changes, provides decision makers a glimpse of the future investment infrastructure with which to guide policy. Changes in fiscal and monetary policy affect the capital structure of investment by altering the incentives to invest. Tax rules, for example, affect investment by changing the after-tax cost of owning capital, making investment more profitable, and increasing the demand for investment.

In addition to the Lucas critique, there are econometric or empirical reasons for assuming parameter instability (see Section 1.4). The objective is to estimate a logically consistent model to provide insight into the structure of gross fixed investment decisions in South Africa. Some work has been done in this field. Two alternative models are proposed by Conway, Hrubovcak and Le Blanc (1988). The first model is from Luca's (1967) work on the flexible accelerator with variation in the rate of adjustment driven by the structure of the underlying optimisation problem. In addition to the flexible accelerator, they proposed an alternative by Swamy and Tinsley (1980) that allows economic phenomena and econometric considerations to induce variability in all of the model's parameters and not just variability to the adjustment coefficient. These approaches were used to estimate the structure of aggregate agricultural investment and it was found that the stochastic coefficient model performed better in out-of-sample forecasting.

10.5 AN ANALYSIS OF SELECTED INVESTMENT FUNCTIONS IN THE SOUTH AFRICAN ECONOMY

10.5.1 THE DE WET AND DREYER QUARTERLY FUNCTIONS

The research which finally led to the De Wet and Dreyer (1976: 76-95) models began in 1974, when the authors started a project on forecasting models at the University of Pretoria. These models were developed and used by The Institute of Econometrics (at the University of Pretoria) in applied economic forecasting.

Each equation is derived from a particular structural characteristic of the economy. The sources from which the data for the models are drawn, vary over a wide range and include official as well as private statistics. The authors decided to estimate with unadjusted data, providing for seasonal variations by appropriate dummy variables. The reasons for this are firstly, that not all the series used were published in seasonally adjusted form and secondly, a lack of trust in the method applied to seasonally adjust some series. All the stochastic equations are linear in the variables which is believed to simplify the estimation process insofar as ordinary linear estimation techniques can be applied. None of the equation estimates were corrected for serial correlation, even when the appropriate statistical tests indicated that such correlation was present.

In solving the system for the endogenous variables in the simulation exercises, two methods were applied. The non-linear identities were linearised during the early stages of the development of the model, by the use of Taylor expansion and re-estimation.

A number of selected investment models are critically examined. The figures in parenthesis are the t-statistics of the estimated parameters. The variables are expressed in real terms with 1970 as base period and the meaning of the symbols are given in Appendix A and B. According to their specification,

$$\begin{aligned} IPO1 = {} & \underset{(5,7)}{-171,01} + \underset{(2,8)}{0,085}\ YGDE1(-1) + \underset{(3,1)}{0,12}\ YGDE1(-2) \\ & + \underset{(2,0)}{0,056}\ YGDE1(-3) - \underset{(3,5)}{23,367}\ FRLE4 + \underset{(3,2)}{32,080}\ S2 \\ & + \underset{(3,3)}{29,478}\ S4 \end{aligned} \tag{10.18}$$

$R^2 = 0,9613$

DW = 1,46

Period of fit: 1961Q1 - 1974Q1

investment in the non-agriculture sector (IPO1) is determined by past values of gross domestic expenditure (YGDE1) and the long-term rate of interest (FRLE4). The latter is there to represent cost of funds considerations, while the use of the gross domestic expenditure variable indicates that firms look at their sales for an indication of demand.

Equation 10.18 also shows that gross domestic expenditure has its greatest influence on investment only after a lag of two quarters which means that even if exports, for example, should start to increase substantially after the beginning of an upswing in overseas countries, it would still take some time before domestic investment responds to such an increase.

Significant seasonal variation is apparent in Equation (10.18). The seasonal dummy variable for the second quarter (S2) is used for an upsurge in investment possibly due to the fact that management would attempt to complete all investment programs for the year prior to the June to August annual Board Meetings. The fourth quarter seasonal dummy variable (S4) is used for the speed-up of investment outlays which occurs before the Christmas holiday break, during which the production is temporarily reduced to a minimum.

10.5.2 THE BUREAU OF ECONOMIC RESEARCH QUARTERLY FUNCTIONS

The Bureau for Economic Research at the University of Stellenbosch has been making use of macroeconometric models in its forecasting activities in South Africa since 1981. The BER generates both short-term (which stretch from 6-8 quarters) and medium-term (5 years) macroeconomic forecasts of the South African economy. Quarterly economic forecasts are published on a regular basis while the medium-term forecasts are only presented on a confidential basis to a number of companies and institutions.

The estimation procedure was very much in line with the De Wet and Dreyer models. All the variables in the functions were expressed in real terms with 1975 as base year. The functions were estimated with seasonally adjusted data. The parameter estimates were derived by the use of the ordinary least square estimation procedure with a Cochran-Orcutt correction for serial correlation of the residuals where necessary. The estimation period ranges from 1970Q1 - 1983Q4.

Amongst the various categories of investment expenditure only part of gross private fixed investment expenditure is explained by the BER. The specification which was ultimately chosen for gross private fixed investment expenditure estimation, excluding agriculture and private residential buildings (IPZA1) is based on the neo-classical theory which emphasise the user cost of capital concept. This equations was given as:

$$IPZA1 = \underset{(3,69)}{-6,30914} + \underset{(3,66)}{0,01831}\,[YCUP + YCUP(-1) + YCUP(-2)]$$

$$+ \underset{(4,75)}{0,14567}\, KPO1(-1) + \sum_{i=0}^{4} w_i (YFZPA1(-1) - YFZPA1(-4))_{-i}$$

lag	weights	t-statistic
0	0,01935	(3,39)
1	0,03096	(3,39)
2	0,03483	(3,39)
3	0,03096	(3,39)
4	0,01935	(3,39)

(10.19)

$\bar{R}^2$ = 0,9304

DW = 2,13

Period of fit: 1970Q1 - 1983Q4

Furthermore, this equation is also based on the acceleration principal which encourage the use of a distributed lag on changes in output as the principal explanatory variable of net investment. The lagged related capital stock variable, on the other hand, forms the main variable explaining the replacement investment component of the fixed investment expenditure equation. A measure of the extent to which productive capacity in the non-agricultural sectors of the economy are utilised also enters the equation.

10.5.3 THE BUREAU OF ECONOMIC RESEARCH YEARLY FUNCTIONS

Yearly econometric models are relatively scarcer than quarterly econometric models. Some of the yearly models which were used in the study are those of the BER, which were published in a report to the HSRC (Biggs, 1982). The same notation is used as in the case of its quarterly counterparts and the period of estimation stretched roughly from 1961 - 1981.

The result of the estimated equation for gross domestic fixed investment: private, excluding residential buildings, was estimated as:

$$\begin{aligned} \mathrm{IPO1} = & \underset{(2{,}214)}{-0{,}29968} + \underset{(20{,}13)}{0{,}11753}\ \mathrm{YF1}(-1) \\ & + \underset{(4{,}2)}{0{,}1891}\ [\mathrm{YF1}(-1) - \mathrm{YF1}(-2)] \\ & - \underset{(4{,}84)}{5{,}86206}\ [\mathrm{FRLE4}/100 - (\mathrm{PGDE}/\mathrm{PGDE}(-1) - 1)] \\ & - \underset{(4{,}717)}{0{,}497}\ \mathrm{DM79} \end{aligned} \tag{10.20}$$

$\bar{R}^2$	= 0,9855	SEE	= 0,0958
DW	= 1,3830	Rho(1)	= 0,3018

Period of fit: 1962 - 1981

The determinants of this equation are the lagged value of YF1, as well as the lagged value of the change in YF1. These variables reflect the need for capacity-creating investment and confirm the spillover of lengthy investment projects after the business cycle has reached a peak. The next determinant measures the real rate of interest, with its coefficient correctly signed, and significantly different from zero.

A dummy variable is used to account for an unexplained spurt in investment during 1979, which is thought to have occurred due to the enormous rise in the gold price that year. Statistically speaking everything seems to be in order, except for the Durbin-Watson statistic which lies in the inconclusive range.

The estimated equation for gross domestic fixed investment: private residential buildings was estimated as:

$$\underset{(21{,}74)}{IPRB1 = 0{,}09089\ YD(-1)/PI(-1)} - \underset{(12{,}496)}{0{,}9647\ FRLE4(-1)} \qquad (10.21)$$

$\bar{R}^2 = 0{,}9551$ SEE $= 0{,}0325$

DW = 2,06 Rho(1) = -0,0551

Period of fit: 1961 - 1981

This is a very simple function with relatively satisfactory statistics. The determinants are both in lagged form and there is no constant term.

The first term is disposable income (YD) lagged one year and deflated by the investment deflator. The rationale for the inclusion of this term lies in the fact that there is a considerable time-lapse between the decision to build and the period of building. It is the level of real disposable income (and interest rates) at the moment of the investment decision, which is important in explaining actual residential building activity.

The second term is the long-term interest rate, lagged one year, which serves as a proxy for the interest payable on private building loans.

The Durbin-Watson statistic reflects no serial correlation in the error terms, while the corrected coefficient of determination and the SEE are fair, given the volatility of the building cycle.

10.5.4 CONCLUSION

The abovementioned equations, and other equations which have not been published, are later tested for structural stability and it is shown that a number of these equations fail one or more tests. Econometrically, economically and statistically speaking it can be seen that in most of the abovementioned equations everything seems to be in order which could have led to the consideration of these equations to be used for economic forecasting. Therefore, the viability of econometric model building needs to be further investigated.

10.6 CONCLUDING REMARKS AND AREAS OF FUTURE RESEARCH

The ad hoc nature of most empirical models of investment behaviour has, necessarily, meant that relatively few consistent findings have emerged from the vast quantity of econometric work carried out over the past two decades. There has been little attempt to expose investment functions to structural stability tests and as a result different investigators have reached different conclusions often without even considering why this should be so. However, one point on which agreement does exist, is about the importance of lags in the adjustment of actual capital stock to its desired level. The lag structure is generally accepted as normally being of the pattern illustrated in Figure 10.1, but there is unfortunately little agreement over the best procedure for estimating this structure. Moreover, the theoretical underpinning of adjustment processes is uncertain with most empirical studies simply attaching ad hoc adjustment processes to theoretical models.

There is still much disagreement over the factors which determine the desired capital stock. The relative size of desired capital stock elasticities with respect to output and to relative prices has been the subject of much debate. It seems that the lag distributions with respect to output and relative prices do differ. The one consistent finding is that, not surprisingly, some measure of output change is an important determinant of investment. However, there is no general agreement whether this measure should refer to sales (or expected sales) or output (or expected output).

The major area of disagreement is over the importance of financial variables in general, over whether it is cost of capital or liquidity variables that matter, and whether financial factors are important in determining desired capital stock or merely in affecting the speed of adjustment of actual stock to its desired level. Interest rates are generally accepted as an important financial variable in both the accelerator-type equations and neoclassical models, with the exception of the studies by Hines and Catephores (1970) and Bean (1981) where hardly any UK work has suggested that interest rates are important determinants of investment. In contrast, US investment has generally been found to be interest-elastic (see de Leeuw, 1962; and Evans, 1967). Increased attention is also being paid in empirical work to the effectiveness of tax incentives in stimulating investment.

Another area of dispute is that of replacement investment. Replacement investment is usually assumed to equal a constant proportion of existing capital stock, which requires firstly, that capital stock decays at a constant exponential rate, so that depreciation equals a constant proportion of stock in any period, and secondly, that replacement investment automatically equals depreciation. Several investigators have questioned the assumptions of exponential decay (see Coen, 1975). Other evidence suggested that replacement investment is dependent on economic factors rather than being determined mechanistically by such a process as exponential decay (see Feldstein and Foot, 1971).

One thing which is clear is that traditional econometric assumptions of fixed parameters over time may often lead to misspecified investment models, and by the same token, to a reduction in both estimation and prediction efficiency. A credible forecast of the effects of policy changes on investment, however, requires capturing the changing structure of investment decision making. Methods that fail to account for structural change in the form of parameter variation may lead to misleading predictions and ill-timed policy actions.

CHAPTER 11

PRODUCTION FUNCTIONS

11.1 INTRODUCTION

South Africa during the past two decades has experienced a drastic fall in net fixed investment with negative effects on the manufacturing sector of the country. The policy of import-substitution industrialisation had an important influence on the sectoral composition of manufacturing production. The salient feature of the structural change was the increase in the share of heavy industry in manufacturing output (see McCarthy, 1989: 386).

The structural changes have been accompanied by an increasing tendency towards capital intensity, low and frequently negative levels of real interest rates, political instability with the resulted collapse of investor confidence, the over valued rand, a strong trade-union movement who became more militant, and strikes for wage hikes which became more frequent. It is tentatively proposed that such factors as these may cause production functions to become structurally unstable. Any one of these shocks in the economic and political environment, on its own, or in conjunction with other changes, can be responsible for structural weaknesses in production functions via one or more of the transfer mechanisms discussed in Chapter 1.

Important innovations in specifying econometric models have arisen from the theory of production. A brief review of the relevant economic theory is therefore given to begin the study of the production function.

The traditional approach to modelling producer behaviour begins with the assumption that the production function is additive and homogeneous. However, this approach has the disadvantage of imposing constraints on patterns of production - thereby frustrating the objective of determining these patterns. The traditional approach was originated by Cobb and Douglas (1928) and was employed in empirical research by Douglas and his associates for almost two decades.[19] The limitations of this approach were made strikingly apparent by Arrow, Chenery, Minhas and Solow (1961, henceforth ACMS), who pointed out that the Cobb-Douglas production function imposes a priori restrictions on patterns of substitution among inputs. In particular, elasticities of substitution among all inputs must be equal to unity.

The constant elasticity of substitution (CES) production introduced by ACMS adds flexibility to the traditional approach by treating the elasticity of substitution as an unknown

[19] These studies are summarised by Douglas (1948).

parameter.[20] However, the CES production function retains the assumptions of additivity and homogeneity and imposes very stringent limitations on patterns of substitution. McFadden (1963) and Uzawa (1962) have shown, essentially, that elasticities of substitution among all inputs must be the same. The variable elasticity of substitution (VES) production function (later called the transcendental production function), developed by Christenson, Jorgenson and Lau (1973) assumes that the elasticity of substitution might be expected to vary with the capital/labour ratio.

The concept of a production function plays an important role in both micro- and macroeconomics. At the macro-level it has been combined with marginal productivity theory to explain the prices of the various factors of production and the extent to which these factors are utilised. It is therefore important in theories of economic growth and in theories of distribution. At the micro-level it is of interest because of its usefulness in the analysis of such problems as the degree to which substitution between the various factors of production is possible and the extent to which firms experience decreasing or increasing returns to scale as output expands. At both the macro- and micro-levels the production function has been used as a tool for assessing what proportion of any increase in output over time can be attributed to, firstly, increases in

[20] Econometric studies based on the CES production function have been surveyed by Griliches (1967), Jorgenson (1974), Kennedy and Thirlwall (1972), Nadiri (1970) and Nerlove (1976).

the inputs of factors of productions; secondly, to the existence of increasing returns to scale, and thirdly, to what is commonly referred to as 'technical progress'.

Furthermore, the production, aggregation and estimation theories lead to stochastic coefficient production functions which differ from previous studies in that it permits the coefficients of the production function to change over time. A general class of estimable coefficient processes was considered by Swamy and Tinsley (1980), using a two-factor Cobb-Douglas production function for several U.S. manufacturing sectors.

The purpose of this chapter is to provide an exposition of a range of econometric methods for modelling producer behaviour in South Africa. Empirical results are presented of selected production functions which do not allow for changes in the coefficients. These models are then augmented afterwards to cater for such changes. The forecasting performance of these VPR models will be compared with the more widely used fixed coefficient models for production.

A brief review of the neoclassical production function and its role in the theory of the firm will be discussed first. The discussion on the estimation procedure is limited to aggregate production functions only.

11.2 THE NEOCLASSICAL PRODUCTION FUNCTION

The traditional theory begins with two inputs, capital and labour, denoted K and L, which are continuously variable and continuously substitutable in production at all times; this takes one away from the realm of linear production models with fixed coefficients. To each combination of capital and labour there corresponds a unique maximum quantity of output, Q:

$$Q = F(K,L) \tag{11.1}$$

The variables Q, K and L are flow variables so that Equation (11.1) expresses a flow of output as a function of the flows of services provided by the two factor inputs. The function summarises the efficient production possibilities open to a firm, a technical maximisation problem having been solved.

The neoclassical production (NCP) function is a mere summary of technical constraints, by itself it allows for no testing of economic hypotheses. Actual observed data are the results of economic decisions in which the NCP is but one constraint. Thus the NCP functions are used in conjunction with marginal productivity theory to provide explanations of factor prices and the levels of factor utilisation, and so at a theoretical level play a central role in the analysis of growth and distribution. The NCP function is embedded in a simultaneous equations model and cannot be identified if, for example, the marginal productivity conditions are not distinguishable from it.

11.3 THE COBB-DOUGLAS PRODUCTION FUNCTION

The production function that has been most frequently employed in empirical work is the Cobb-Douglas production function. Cobb and Douglas (1928) developed a production function based on the theorems of the marginal productivity theory, which could be tested empirically. Of all income distribution theories, the marginal productivity theory is most highly developed. It is a theory which explains the distribution of incomes between input factors whose joint co-operation is required for the attainment of output. The theory allows mathematical treatment and yields determinate results for given parameters.

The basic Cobb-Douglas production (CDP) function is given as:

$$Q = \beta_0 K^{\beta_1} L^{\beta_2} \qquad (11.2)$$

with $Q > 0$, $\beta_0 \geq 0$,
$K > 0$, $\beta_1 \geq 0$ and
$L > 0$, $\beta_2 \geq 0$

The CDP function has a number of convenient properties. The parameters β_1 and β_2 measure the elasticities (assumed constant and between zero and unity) of output with respect to capital and labour, respectively. The parameter β_o may be regarded as an efficiency parameter, since for fixed inputs K and L, the larger β_o is, the greater the maximum output Q obtainable from such inputs.

Because the Cobb-Douglas production function is homogeneous, constant, increasing and decreasing returns to scale can be identified. Converting the CDP function into a linear homogeneous form shows this relationship more clearly:

$$\ln Q = \ln \beta_o + \beta_1 \ln L + \beta_2 \ln K \qquad (11.3)$$

It is in this form that the production function was used by Standish and Galloway (1991) to assess production efficiency in the manufacturing sector in South Africa.

The equation is homogeneous of degree $\beta_1 + \beta_2$. Thus for constant returns to scale $\beta_1 + \beta_2 = 1$, while for increasing returns to $\beta_1 + \beta_2 > 1$, and for decreasing returns to scale $\beta_1 + \beta_2 < 1$ (see Thomas, 1985). If $\beta_1 > \beta_2$ then labour is regarded as being more productive than capital for the given industry. Thus one rand of additional investments in labour produces a larger output than would have been the case had an additional one rand been invested in capital. Conversely, if $\beta_1 < \beta_2$, capital is held to be more productive for the given industry. In the same vein, an additional rand invested in capital produces a larger output.

The marginal rate of substitution for capital is:

$$MRS = (\beta_2 K)/(\beta_1 L) \qquad (11.4)$$

To derive the elasticity of substitution (σ), Equation (11.4) is written as:

$$\log(MRS) = \log \frac{\beta_2}{\beta_1} + \log \frac{K}{L} \qquad (11.5)$$

hence

$$\sigma = \frac{d \log (K/L)}{d \log (MRS)} = 1 \qquad (11.6)$$

Thus, for the Cobb-Douglas production function the elasticity of substitution does not vary with the combination of factors used, and is equal to unity everywhere.

In practice the possibility of constant or increasing returns to scale cannot be ruled out - certainly not without estimating β_1 and β_2 first. Obviously, there are no a priori reasons why firms should not operate under conditions of non-decreasing returns to scale. When they do so it is obvious that in practice the value for Q is determined in some manner or other - firms do have a definite size, whether it be large or small.

The answer to this apparently baffling problem is to relax the assumptions of perfect competition where all prices are given and Q, K and L can be varied at will. However, this involves making prices endogenous to the system and, hence, necessitates the addition of extra equations to the model, namely, demand for product - and supply of factor relationships. The production function is therefore a technical relation between outputs and inputs, acting as a constraint on the firms' choice of output and input levels, which otherwise resulted from economic decisions. Thus in general the observed values of prices, output and inputs are generated by a set of simultaneous relationships, and it is inappropriate to estimate the production function as a single regression equation treating capital and labour as exogenous variables. The simplest case is given by assuming perfect competition in product and factor markets, which means that the prices of output (p), capital (r) and labour (w) are predetermined: the firm is a price-taker.

In such a situation the firm's objective would be to choose the amounts of labour and capital so as to maximise profits. The profit function $\pi(K,L)$ is given by:

$$\pi(K,L) = pF(K,L) - rK - wL \tag{11.7}$$

The two first order conditions for maximisation are $\delta\pi/\delta K = 0$

and $\delta\pi/\delta L = 0$. Taking partial derivatives of π with respect to K and L and setting them to zero gives:

$$r = p(\delta F/\delta K) \quad \text{and} \quad w = p(\delta\pi/\delta L) \qquad (11.8)$$

$\delta F/\delta K$ is the extra output per unit extra capital input and is called the marginal product of capital. Similarly, $\delta F/\delta L$ is the marginal product of labour. Also, $p(\delta F/\delta K)$ and $p(\delta F/\delta K)$ are the values of the corresponding marginal products. The first-order conditions imply that firms will maximise profits when they choose K and L such that the value of the marginal product of labour equals the wage rate (w) and the value of the marginal product of capital equals the rental rate (r).

Criticism of the Cobb-Douglas functions takes many forms. Firstly, the statistical problem of adjusting capital stock figures for intensity of use and differences in quality is considered a serious one, yet few attempts have been made to adjust capital data for capacity utilisation. Likewise, it is usually impossible to adjust employment data accurately for number of hours worked and variations in labour productivity. Data deficiencies are probably smaller for cross-sectional estimates than for time-series studies, because variation in the amount of idle capacity and in the intensity of labour utilisation should be relatively insignificant during any one particular year. Secondly, the testability of the function has been challenged on the grounds that the time-series data constitute a nearly perfectly multicollinear set (see Mendershausen, 1938: 144, 147). Bronfenbrenner and Douglas (1939: 168-172), however, showed that multicollinearity was less serious of a problem when minimisation was performed in the p direction (rather than in the L or K directions). Finally, the comparison of income-shares and production elasticities requires knowledge of the value of capital and wage shares. For non-corporate business income may constitute a mix of wages, interest, profits and perhaps even rents. The different economic functions performed by the independent proprietor are inseparably interwoven and the accounting practise of treating non-corporate income fully as property income must, therefore, result in the capital share being higher than it truly should be (see Goldberg, 1964: 280).

Despite these deficiencies, prominent theorists like Bronfenbrenner (1971), Leontief (1964) and Houthakker (1956) testified that after a life of 40 years, the Cobb-Douglas is still useful for econometric research and has become firmly established as an econometric tool.

11.4 THE CONSTANT ELASTICITY OF SUBSTITUTION PRODUCTION FUNCTION

As already noted the CDP function has an elasticity of substitution, σ, which is always equal to unity. This is a particularly restrictive property. One of the purposes of production function analysis is to examine the extent to which factor substitution is possible and such substitution may obviously vary between firms and industries. For example, if we wished to compare the substitution possibilities in two different industries, the estimation of Cobb-Douglas functions for each industry could tell us nothing of value. The value σ is determined by the underlying technology and may change with technical progress, but in any event is not necessarily equal to one.

The constant elasticity of substitution (CES) production function also constrains the value of σ to be constant (other than unity) in the sense that it does not change with changes in relative prices of factor inputs. In the CDP function case, under competitive conditions, the labour marginal productivity equation can be written as:

$$\log (Q/L) = -\log \beta_2 + \log (w/p) \tag{11.9}$$

where w is the price of labour and p the price of output (see Wallis, 1979:68). This means that output per head varies with the real wage rate (logarithmically) with a coefficient of unity. In contrast, Arrow et al. (1961) estimated cross-sectional equations with a coefficient not equal to one:

$$\log (Q/L) = \beta_0 + \beta_1 \log(w/p) \tag{11.10}$$

and derived the CES production function as a solution to this equation. This function is defined as:

$$Q = \tau\{\delta K^{-\theta} + (1-\delta)L^{-\theta}\}^{-v/\theta} \quad (11.11)$$

The degree to which technology is capital intensive is indicated by δ, and when the production function is embedded in a particular economic model δ can be interpreted as the distribution parameter of the relative importance of K. The substitution parameter θ is equal to $(1 - \sigma)/\sigma$ (see Wallis, 1979: 68). The parameter τ is to be interpreted as an efficiency parameter akin to the β_0 in the CDP function, since for a given δ and θ, the larger τ is, the greater is the maximum output Q obtainable from given inputs K and L; v gives the degree of homogeneity. In the original ACMS study v was taken to be equal to one and so constant returns were imposed, although the independent derivation by Brown and De Cani (1963) permits any degree of returns to scale.

The fact that σ can take different values means that the CES function, unlike the CDP function, is a suitable tool for investigating the varying substitution possibilities between, for example, different industries.

A major problem with the CES production function, however, is that unlike the CDP function, it cannot be transformed into a linear-in-parameters form by operations such as taking logarithms. Because there is no method of linearising the parameters to give a directly estimable exact presentation, linear approximations have been used, and estimation has proceeded via side relations such as marginal productivity conditions or factor share equations which resulted from adding standard economic assumptions to the CES specification. The cost function is also not of much help in this case. Direct estimation of the CES production function is only possible by using non-linear regression methods. A linear approximation proposed by Kmenta (1986: 515) is obtained by writing the CES function as

$$\ln Q = \ln \tau + \tau\delta \ln K + v(1-\delta) \ln L - \tfrac{1}{2}\theta v\delta(1-\delta) [\ln K - \ln L]^2 + e \tag{11.12}$$

$$= \beta_0 + \beta_1 \ln K + \beta_2 \ln L + \beta_3(\ln K - \ln L)^2 + e \tag{11.13}$$

Estimates of the original parameters of the CES production function can therefore be obtained from estimates of the β's.

Although it is easy to transform variables and include them in the regression model, indiscriminate use of transformations should be avoided. It is recommended to look for some theoretical basis for the transformation and to keep the models as simple as possible.

11.5 THE VARIABLE ELASTICITY OF SUBSTITUTION PRODUCTION FUNCTION

Once the assumption of a unitary elasticity of substitution σ, implicit in the Cobb-Douglas function, had been superseded by the merely constant σ of the more general CES function, it was clear that the next stage would be the development of variable elasticity of substitution (VES) production functions. The reason for this could be that σ might be expected to vary with the capital/labour ratio. The greater this ratio, the harder it is likely to be to substitute further capital for labour and the lower σ is likely to be. Alternatively, even with a constant K/L ratio, σ may simply change over time if technical progress affects the ease with which factors may be substituted for each other. Variable elasticity of substitution production functions were indeed developed (see e.g., Sato and Hoffman, 1968; Revanker, 1971; and Christenson, Jorgenson and Lau, 1973). Production functions estimated in this way are referred to as frontier production functions and later attempts have been made to estimate such functions (see Schmidt, 1976).

If, as suggested by Griliches and Ringstad (1971) and Sargan (1971), the squared term of the transformed CES function in Equation (11.13) is replaced by an unconstrained quadratic term, then the 'transcendental logarithmic' production function

(TPF) presented by Christenson, Jorgenson and Lau (1973) is obtained. The TFP is given as:[21]

$$\ln Q = \beta_0 + \beta_1 \ln K + \beta_2 \ln L + \beta_3 (\ln K)^2 + \beta_4 (\ln L)^2 + \beta_5 (\ln K)(\ln L) \quad (11.14)$$

The TPF is regarded as a general form for the VES functions which is easily estimable and could be considered a sufficiently close approximation to whatever the underlying productive process actually is. Also, since it can be regarded as a second-order Taylor approximation to any production function, VES or otherwise, it can be used to test whether the elasticity of substitution is, in fact, constant or not. If $\beta_3 = \beta_4 = -\frac{1}{2}\beta_5$ then Equation (11.14) become Equation (11.13) which is Kmenta's Taylor approximation to the CES function. Hence, if (11.14) is estimated, the hypothesis of a CES maybe tested by checking whether the estimated coefficients of (11.14) obey the restrictions $\beta_3 = \beta_4 = -\frac{1}{2}\beta_5$. Production function (11.14) also has the interesting property that the nature of the returns to scale implied is not the same for all values of the inputs. Griliches and Ringstad (1971), in fact, find increasing returns to scale when firms are small but something very close to constant returns to scale for larger firms. The property of non-varying returns to scale is, of course, one of the

[21]Gujarati (1988: 242) presented a modification to the TPF which he specified as $\ln Q = \beta_0 + \beta_1 \ln K + \beta_2 \ln L + \beta_3 K + \beta_4 L + e$.

limitations of the more restrictive Cobb-Douglas and CES functions.

Another area of more recent development is the estimation of empirical versions of the so called vintage models of production. In these models it is assumed that, for example, machines of a later vintage are more efficient than those constructed earlier but their efficiency is determined by the state of technical knowledge at the moment of their construction. Vintage models represent a major step towards reality in that new technology now has to be 'embodied' in new kinds of equipment so that the rate of technical progress becomes dependent on the rate of investment in new machines. The first rigorous attempt to formulate a model of embodied technical progress was that of Solow (1960). In the Solow model technical progress proceeds at a constant rate, but affects only newly produced capital goods. Separate production functions exist for machines of different vintages. Models which still retain the neoclassical assumption that capital equipment can easily by transformed at any time to accommodate any K/L ratio, are commonly known as 'putty-putty' models. Data is generally not available for estimating vintage models.

One can conclude that the general magnitude of elasticities of substitution, the extent of economies of scale and the quantitative importance of technical progress are not much clearer today than when Douglas undertook his pioneering studies. The principal finding of Nerlove's (1967) survey of CES functions is that even the slightest variation in the period or methods used tends to produce drastically different estimates, is probably as true today as it was a decade and a half ago.

11.6 THE STOCHASTIC - COEFFICIENT PRODUCTION FUNCTION

This function differs from the previous ones in that it permits all or some of the coefficients of the production function to change over time according to a vector stationary stochastic process with unknown parameters.

Before turning to the applicability of a varying coefficients production function it is useful to point out some theoretical advantages of varying- over fixed coefficient models. First, conditions under which an aggregate production function exists are less stringent for varying coefficient models than for fixed coefficient models. This particular point was developed by Narasimham et al. (1988), following the work of Zellner (1969). The Cobb-Douglas form for a production function is, therefore, less restrictive when all of the coefficients are allowed to vary than when only the intercept is allowed to vary as in conventional econometric models with dummy variables. A function with all of its coefficients varying may exist even when the corresponding function with fixed slopes does not exist. It seems important to worry about the truth of the existence conditions, because any function that did not exist could not have generated the data and should not be used for the analysis. As a related point, classical statistical procedures presume the existence of the true values of fixed parameters. Consequently, fixed coefficient models are not well suited for classical statistical procedures if the true values of the fixed coefficients do not exist because of the nonexistence of fixed coefficient models.

Following the VPR approach and re-writing Equation (11.3) in the form which assumes that all coefficients are changing over time, give an equation that shows the aggregate production function to be:

$$\ln Q_t = \ln \beta_o + \beta_{1t} \ln L_t + \beta_{2t} \ln K_t + e_t \qquad (11.15)$$

To allow for parameter variation Equation (11.15) can be re-written in general as $y_t = \mathbf{x}'_t \boldsymbol{\beta}_t$ for all t, where $\mathbf{x}_t$ and $\boldsymbol{\beta}_t$ are (K x 1) vectors. These parameters could follow an AR(1) or a random walk process (see Watson and Engle, 1985).

However, if one is to assess the fruitfulness of Model (11.15), it is important to recognise that no stigma attaches to its being approximate rather than exact. With the true (aggregate) production function being unknown there is, after all, no guarantee that any of the "exact" production functions will be exact in fact. A Cobb-Douglas production function of the type

(11.3) with varying coefficients quite possibly provides an adequate approximation to the production technology over a range of conceivably true production functions, this without being exactly appropriate for any particular one. One should, therefore, be careful in identifying a coefficient as a 'productivity' term.

In a study by Narasimham et al. (1988) not only the changes in the coefficients of the Cobb-Douglas production functions were estimated for different industries, but also the associated changes in the productivity measures. They also provided a set of aggregation conditions which micro- and macro-production should satisfy if they are to be coherent.

11.7 AGGREGATE PRODUCTION FUNCTIONS

It takes little thought to realise that, regardless of the precise form adopted for the production functions earlier, the simple economic models of the firm described in previous sections are a far cry from the firms of a modern industrial economy. A firm typically produces more than one output and employs more than two factors of production. Raw material and intermediate - good inputs are frequently as important as capital and labour inputs and, furthermore, no inputs can be treated as completely homogeneous in quality. There are many types of labour inputs - skilled and unskilled, for example.

Capital equipment varies even more, both in its form and up-to-dateness.

However, even if data on all such variables were accessible and sufficient observations were available, potential multicollinearity problems are so severe that some form of aggregation is inevitably necessary. A frequent first step is to work in terms of the real output actually originating in the firm, i.e., in terms of 'value added'. Unfortunately, the conditions under which this is legitimate are very restrictive (see Green, 1964) and rather unlikely to be met in practise. It is also not clear that a rigid and fixed relationship between output and say, raw material inputs is likely to provide an adequate approximation to reality.

When dealing with the individual firm, aggregation needs not necessarily be as bad. It is often still possible to retain maybe two or three separate types of inputs for both capital and labour. However, some degree of aggregation is always necessary and this invariably causes theoretical problems.

In practice, production functions are not only estimated for individual firms but often for entire industries or industrial sectors and even for the economy as a whole. However, one should be aware of the serious conceptual problems involved with the idea of a macro-production function. These problems can be summarised as follows:

(a) Since the Cobb-Douglas function is merely linear in the logarithms, it means that sensible aggregation requires macro-variables to be defined as the geometric rather than the arithmetic means of the corresponding micro-variables.

(b) Since the production function could be one of a system of three simultaneous equations, the marginal productivity conditions need to be aggregated too. Even if such aggregation is possible, there is no guarantee that the macro-marginal productivity conditions obtained by differentiating the macro-production function will be of the same form as those obtained by aggregating the micro marginal productivity conditions.

(c) Due to the possible presence of external economies it could mean that, although individual firms operate under constant returns to scale, the inputs and outputs of all firms can expand. Hence, the aggregate production function could exhibit increasing returns to scale although the micro-functions did not.

(d) Aggregation is frequently performed over industries with widely different types of output. The β_1 and β_2 in a CDP functions are not necessarily the same in all industries. The capital intensity of the production process could vary between industries.

(e) Even if a fixed relationship between aggregate output and inputs do exist, it does not mean that the β_1 and β_2 are simply the means of the corresponding micro-parameters.

It should be clear from the above discussion that the very concept of an aggregate production function is a nebulous one. The question naturally arises of whether there is any point in trying to estimate such a 'hazy' relationship. However, it is an attractive proposition to attempt to find some simple relationship which sums up the whole technology under which an economy operates. Although such an estimated relationship cannot be a 'pure' technical one it may still prove a useful statistical description of the relationship between aggregate Q, K and L. The attractiveness of the production function approach has meant that empirical investigators have not been deterred by the conceptual problems involved.

11.8 MEASURING THE DETERMINANTS OF AN AGGREGATE PRODUCTION FUNCTION

The measurement of either inputs or outputs almost invariably involves the aggregation of heterogeneous quantities and should therefore involve the construction of index members or weighted averages.

The most easily measured variable involved is probably the flow of labour inputs (in real terms) which can generally be measured in terms of man-hours. However, there are many types of labour input - male and female, skilled and unskilled, etc. - and ideally some weighted measure of total labour input should be derived. Appropriate weights would be base-period (real) hourly wage rates for the different types of labour. However, unweighted measures of labour flows are also frequently used, e.g., total man-hours, and on occasion even stock measures such as the total number of employees.

The procedure frequently used to measure total output is to take current prices deflated by the most appropriate index of output prices. If a gross output measure is used then the aggregate measures of intermediate-goods inputs becomes an additional argument on the right-hand side of the production function.

The greatest difficulty arise in the measurement of capital goods. The index number problems caused by variations in quality are far more serious than in the case of outputs and labour inputs because of the existence of technical progress and innovation over time. Furthermore, while a measure of the flow of capital services is required, existing data is almost invariably concerned with the stock of capital equipment. If such data is used, variations in the utilisation of the capital stock become important because if utilisation varies, then a given capital stock will provide varying rates of flow of capital services. In practise, the money value of capital stock measured in terms of its replacement cost in some base year is generally used as the capital input variable. Such figures may be either in gross terms of net of depreciation estimates. Attempts are sometimes made to adjust such figures for varying utilisation by using the available data on the percentage of the labour force that is unemployed or 'unutilised'. However this is making the assumption that the percentage utilisation of capital is identical to the percentage employment of labour.

Occasionally, use is made of the assumption that all revenue accrues to either labour or capital in an attempt to estimate capital inputs. Given knowledge of Q, p, L, w and r, the accounting identity, $pQ = wL + rK$, may be used to estimate K. Notice, however, that this method implies zero profits. Furthermore, Thomas (1985) listed a series of difficulties in the interpretation of such an estimated production function when this accounting identity holds.

11.9 ESTIMATING AGGREGATE PRODUCTION FUNCTIONS

Much of the empirical work on production functions has been concerned, not with the individual firms or even the industry, but with aggregates such as the entire manufacturing sector or even the whole of private industry. In this section, discussion will be mainly limited to considering the extent to which empirical studies can be regarded as having provided estimates of such functions, nebulous in concept though they may be. The focus of the discussion will be mainly in the context of the Cobb-Douglas production.

Time series studies of aggregate production function use as observations aggregate data on, for example, the entire manufacturing sector, gathered over a period of time. The early studies of this nature were mainly carried out by Douglas (1948) using a CDP function. In these studies, no allowance was made for technical progress, possible identification problems were not considered and the method of estimation was invariably OLS. Despite all this and the conceptual problems involved in the very idea of an aggregate production function, the results appeared uniformly good.

On the basis of his results, Douglas concluded that the Cobb-Douglas function represented a fairly general 'law of production' with constant returns to scale, and that the shares of output going to capital and labour were indeed equal to the β_1 and β_2 exponents in the Cobb-Douglas function. This latter finding was interpreted as strong support for the marginal productivity theory of distribution. Given all the problems that were not allowed for, it seems surprising that these early results should have turned out so well. However, one clue as to why this could have been so is provided by the fact that the relative prices m/p and w/p remained relatively constant over the periods considered.

Time series estimation of a production function with fixed coefficient and assuming input homogeneity ignores, among others, any influences that technical change might have. The problem in time series studies of technical change is to distinguish movements along a production function, due to factor proportions changing over time, from shifts of the function, assumed due to technical progress. The benefits of disembodied technical progress is assumed to be freely available. The resulting shifts of the production function can be represented by including a time variable, for example

$$Q_t = F(K_t, L_t, t) \tag{11.16}$$

thus implying that the same input qualities yield a different output at different points in time. An alternative approach assumes that the technical progress is embodied in capital or labour, thus the firm must invest in new capital goods or labour to gain its benefits. Advances are embodied in capital goods of different vintages, new machines being more productive than old machines, thus capital is no longer assumed homogeneous.

A neutral disembodied technical change is one which neither saves nor uses either factor, and so leaves the marginal rate of substitution unaltered. In the Cobb-Douglas case the easiest way to incorporate neutral technical progress is to allow the scale parameter β_o to vary, for this will not effect the marginal rate of substitution:

$$Q_t = \beta_{o(t)} K_t^{\beta_1} L_t^{\beta_2} \qquad (11.17)$$

An early attempt by Tinbergen (1939) in his framework used the form $\beta_{o(t)} = \beta_o e^{\theta t}$, which is equivalent to introducing a trend term into the log-linear regression, but the problems of identification remained.

Solow (1957) side-stepped the identification problem in the context of constant returns by abandoning the Cobb-Douglas specification and making use of marginal productivity conditions in an attempt to differentiate shifts from movements along the production function. Solow does not specify the precise form of his production function but assumes that technical progress is both neutral and disembodied so that

$$Q_t = \beta_{O(t)} F(K_t, L_t) \tag{11.18}$$

Unfortunately this function is very difficult to deal with and Solow acknowledged himself that there are obvious objections to such calculations.

In order to overcome some of the abovementioned problems, Watson and Engle (1985) developed a method that is available for estimating time varying parameters which can be applied to production functions as well. The specialised version of their model posited on the assertion that the coefficient vector $\boldsymbol{\beta}_t$ follows either a random walk or an AR(1) process. The Kalman filter technique together with EM and scoring algorithms is one technique which can be used to obtain the greatest overall computational efficiency (These techniques are fully discussed in Section 7.5.5).

11.10 AGGREGATE PRODUCTION FUNCTIONS OF THE SOUTH AFRICAN MANUFACTURING INDUSTRY - A CRITICAL PERSPECTIVE

For South Africa, the research into the applied Cobb-Douglas function was pioneered by Browne (1943) in an article entitled "The Production Function of South African Manufacturing Industry". Many other writers examined the validity of the theory and measured its parameters on several occasions (see Spandau, 1973: 216), but due to a lack of long-run econometric measurement and interpretation, Spandau (1973) calculated the magnitude of the parameters of different Cobb-Douglas functions.

Spandau discussed the marginal productivity theory at the aggregate level and, in particular, applied the Cobb-Douglas function in South African manufacturing industries by fitting least square regression lines to labour and capital stock, using data from 1917 to 1968. The parameters for the functions were estimated as:

$$\log P = \log A + l \log L + k \log K \tag{11.19}$$

$$\log P = \log A + e \log E + n \log N + k \log K \tag{11.20}$$

$$\log P = \log A + e \log E + b \log B + a \log A' + c \log C + k \log K \tag{11.21}$$

where P is net output, or the net value of production which corresponds to the gross value of production minus the costs of material and fuel, light and power. The net output of the census returns, includes, broadly speaking, salaries and wages, overheads and profits. To correct the net output values from overheads, 15 per cent of its value was deducted for the years 1917/18 to 1954/55, and 20 per cent for later years. Further,

A = constant factor;

l = marginal contribution of labour (L = number of workers);

k = marginal contribution of capital (K = stock of capital);

e = marginal contribution of European labour (E = number of European labourers);

n = marginal contribution of Non-European labour (N = the number of Non-European labourers);

b = marginal contribution of Bantu labour (B = number of Bantu labourers);

a = marginal contribution of Asian labour (A′ = number of Asiatic labourers);

c = marginal contribution of Coloured labour (C = number of Coloured workers).

The regression equations were run both as time-series and as cross-section studies. It was found that the cross-sectional calculations obtained with Equation (11.20) were rather unsuccessful due to multicollinearity. When the computations were applied to time-series data, it was reported that negative marginal productivities occurred from time to time which forced the abandonment of these computations.

Spandau assumed that the estimated exponents of the three inputs (namely black labour, white labour and capital), should equal the theoretical income shares of the respective inputs (i.e. the income share they would receive if they were paid according to their marginal productivity). He compared the "theoretical income share" or "marginal contributions", with the actual income share received by each of the production factors. On the basis of these observed deviations he concluded that capital had always been overpaid, that the whites had generally, with the exception of two or three years, been under-paid, and that black workers had since 1942, generally received more than their marginal contribution had warranted. The most important policy conclusion was that industries could not be expected to increase the already inflated wages to black workers, except under conditions where job reservations was abolished and training improved so that the mobility of black labour could be increased.

Three critical comments on Spandau's work were published afterwards. Archer and Maree (1975) presented a very constructive review of some important qualifications which Spandau had ignored. They advanced a timely criticism of Spandau's article on over and under payment in the South African and warned that Spandau's results should be approached with reservation. They also argued that the data used were unsatisfactory.

Le Roux (1975), on the other hand, presented reasons for rejecting the aggregation assumption cardinal to Spandau's analysis, namely that the estimated exponents of the aggregate production function can be taken as equal to the theoretical wage shares of the respective factors of production. Le Roux pointed out that since the exponents of the aggregation production function are estimated with the method of least squares, the exponents are bound to be subject to statistical error, and that one should present evidence that deviations of the actual exponents from theoretical exponents are statistically significant, at a satisfactory level of significance, before drawing unwarranted conclusions. Le Roux went further to argue that the Cobb-Douglas is also an inappropriate function for describing the substitution of black workers for whites and vice versa.

The conditions under which the calculated exponents could have equalled the theoretical income shares are clearly spelled out by Archer and Maree (1975: 185-186):

> "... had perfect competition prevailed in the markets for inputs and outputs, with constant returns to scale, uniform technical progress, equal factor intensities, perfect foresight and the other conditions for sector equilibrium in each of the fifty years, then factor payments would have matched parameter values."

Since these conditions cannot be said to have existed in South Africa at any point in the past, Archer and Maree's argument that the aggregation problem invalidates Spandau's analysis, should be accepted.

Furthermore, anyone familiar with the South African economy knows that the labour mix varies from industry to industry. Le Roux (1976: 316) pointed out that although Spandau did not attempt an indexation required to overcome this problem, he attempted a sensible solution as far as the problem of differences in racial composition is concerned; but then disaggregated further and attempted to estimate separate components for each of the population groups. His results pointed to the fact that the marginal distribution of some groups was negative! This should have served as a warning as to the reliability of the aggregate Cobb-Douglas.

De Wet (1976), however, came out in defense of Spandau's work and concluded that the general approach and modus operandi of Spandau appeared to be in order. This could only have been the case if De Wet was correct in his assumption that the conditions he stipulated for consistent aggregation had been fulfilled (see De Wet, 1975). De Wet believed further that his presentation of Klein's (1974) proof to be a justification of Spandau's approach, but ignored the fact that this could not have been the case except if the respective variables for each industry conformed to the homogeneity requirement under very special circumstances (see Le Roux, 1975: 316-320).

The conditions for valid and consistent aggregation are very stringent and whenever an aggregate production function is estimated it is important to consider whether these conditions are satisfied. Walters (1963: 11) noted that:

> "After surveying the problems of aggregation one may easily doubt whether there is much point in employing such a concept as an aggregate production function. The variety of competitiveness and technological conditions we find in modern economics suggests that we cannot approximate the basic requirement of sensible aggregation except, perhaps, over firms in the same industry or for narrow sections of the economy."

Furthermore, it was generally found that the good predictions of the exponents were made only when functions were fitted to time-series data. In the case of cross-sectional data the predictions were bad (see Walters, 1963: 34-35). Since Spandau's analysis is based on cross-sectional data, there is no justification for claiming that historical evidence supports his methodology.

Even in the case of time-series data the defence that the production function seems to work and must, therefore, be consistent, is very weak. On theoretical grounds there is every reason to expect this is not the case. Garegnani (1970) proved that the surrogate production function will usually only equal the theoretical income shares if there are constant factor intensities. In addition to this Fisher (1971) showed by his simulation experiments that the exponents of an aggregate Cobb-Douglas function will be close to the observed income shares whenever the income shares remain constant over time - even if Cobb-Douglas conditions are not even remotely satisfied. These results were more than merely suggestive. In theory we do not expect economy-wide aggregate production functions to be consistent. In practice it has been shown that such aggregate functions can nevertheless give satisfactory predictions on the condition that income shares remain constant, even though Cobb-Douglas conditions are not satisfied. Since the respective income shares do in fact often remain relatively constant over time, it seems only reasonable to ascribe the good predictions to this constancy rather than to any accidental consistency, which one would in any case not expect on a priori grounds.

Despite everything which has been said, one can of course regress output on inputs in order to make approximate estimate of future outputs. At times one may find that one's predictions of outputs are fairly accurate: "but this is a far cry from estimating a 'production function' which implies by its very definition a maximisation process that is subject to technical constraints," (Blaug, 1974: 16). Practitioners should realise that they are merely estimating a stochastic relationship between inputs and outputs and should be aware of the limitations of their results and not take the estimated structural parameters too seriously.

11.11 AN ANALYSIS OF SELECTED PRODUCTION FUNCTIONS IN THE SOUTH AFRICAN MANUFACTURING INDUSTRY

Although there are many objections against estimating an aggregate production function, econometricians still go ahead and estimate them. One such function is the quarterly aggregate Cobb-Douglas production function estimated by Smit and Meyer (1985: 105-110).

Employment in the non-agricultural sectors (NEZA) was modelled via a re-normalised aggregate production function. This provided for the opportunity to introduce supply-side elements (in the form of measures of potential output and capacity utilisation) into the model. Smit and Meyer re-normalised a Cobb-Douglas production function on labour to provide a labour requirements function, i.e., once output and capital have been determined from other relationships in the model, the employment functions determine the amount of labour required. They assumed a Cobb-Douglas production function which was specified as:

$$X_t = A\, e^{gt} L_t^{a} K_t^{b} e^{u_t} \tag{11.22}$$

where L_t = labour input

K_t = capital input

g = rate of technological progress.

Taking natural logarithms and normalising on $\ln L_t$ gives:

$$\ln L_t = a^{-1} \ln X_t - a^{-1} \ln A - a^{-1}b \ln K_t - a^{-1}gt - a^{-1}u_t \tag{11.23}$$

Equations (11.23) represents a short-run production function and it can be changed to a long-run relationship by substituting L_t with L^*_t where the latter is the desired labour input. Smit and Meyer defined:

$$\ln L_t - \ln L_{t-1} = \theta(\ln L^*_t - \ln L_{t-1}) ,$$

with $0 < \theta < 1$ (11.24)

which implies that actual employment will approximate the desired labour input over the long run. Substituting $\ln L^*_t$ in the transformed Equation (11.23) gives:

$$\ln (L_t/L_{t-1}) = -\theta a^{-1}\ln A + \theta a^{-1}\ln X_t - \theta(\ln L_{t-1} + a^{-1}b_t \ln K) - \theta a^{-1}gt - \theta a^{-1}u_t \tag{11.25}$$

In order to estimate Equation (11.25) by means of OLS, <u>a priori</u> information on $a^{-1}b$ is required. If perfect competition, profit maximisation and constant returns to scale are assumed, $a^{-1}b$ represents the equilibrium ratio of the capital share to the labour share (Klein and Young, 1980: 27; De Jager and Small, 1984).

In applying South African data from 1967Q1 to 1983Q4 to Equation (11.25) the choice of variables was determined by the availability of quarterly statistics on employment. Since reliable quarterly statistics on the sector agriculture, forestry and fishing were not available, employment in the non-agriculture sectors (NEZA) was used as the dependent variable. This implied that commensurate output and capital input statistics be used. The output variable used was gross domestic product at factor cost excluding agriculture, forestry and fishing (YFZPA1). Capital inputs were represented by total capital stock excluding agriculture, forestry and fishing (KZPA1). In order to approximate the effective capital input, the capital stock was adjusted for capacity utilisation in the manufacturing sector (YCUM).[22] The equation for South Africa was estimated as:

$$\ln(NEZA_t/NEZA_{t-1}) = -\theta a^{-1}\ln A + \theta a^{-1}\ln YFZPA1_t - \theta(\ln NEZA_{t-1} + a^{-1}b\ \ln(KZPA1_t * YCUM_t)) - \theta a^{-1}g(TREND)_t - \theta a^{-1}u_t \tag{11.26}$$

The a priori estimate of $a^{-1}b$ (equal 0,38) was calculated by averaging income shares over the sample period.

[22]Since the index of capacity utilisation in the manufacturing sector was available only from the first quarter of 1971, prior values had to be computed. The approach followed was similar to the one used by De Jager and Small (1980: 24), i.e. an index of capacity utilisation was computed by means of the Wharton trend-through-peaks method of interpolation on YFZPA1. The published statistics were then extrapolated backwards with the aid of this computed index.

The equation generating potential output was derived by re-normalising the employment equation and substituting the variables representing the labour and capital inputs by their respective full employment counter parts.

Re-normalising Equation (11.26) and approximating the long-run growth rate of employment by its growth rate of 0,6623 per cent over the sample period gave the following equation for output:

$$YFZPA1_t = \exp(\ln A + (\theta^{-1}/\theta a^{-1})\ln 0{,}99342 + a \ln NEZA_t + \beta \ln (KZPA1_t * YCUM_t) + g(TREND)_t) \quad (11.27)$$

Estimates of the parameters of Equation (11.27) were derived from OLS estimates of the employment function and were given as:

θ = 0,05335;

a = 0,5117;

β = 0,19445;

g = 0,00393;

ln A = 1,29877.

Substituting in Equation (11.27) gives:

$$YFZPA1_t = \exp(1{,}29877 - 9{,}0797 \ln 0{,}99342 + 0{,}5117 \ln NEZA_t + 0{,}19445 \ln (KZPA1_t * YCUM_t) + 0{,}00393(TREND)_t) \quad (11.28)$$

A full employment substitute for NEZA was obtained by computing a measure of appropriate labour force NLZA,[23] determining the maximum (G1) of NEZA/NLZA over the sample period and computing the variable NLZA*G1.

The full employment substitute for KZPA1 was obtained by the maximum value (G2) for YCUM over the sample period and computing the variable KZPA1*G2.

An annual aggregate net production function for industry for the period 1964 to 1983 was also estimated by van der Walt and Swanepoel (1985: 43-46). The t-values are given in parenthesis. Their function was of an unrestricted translog nature in which the index for labour is of less importance as far as the factor of correction is concerned:

$$\ln Q = \underset{(-2{,}188)}{-5{,}713} + \underset{(2{,}462)}{0{,}671} \ln L + \underset{(4{,}022)}{0{,}555} \ln K - \underset{(-2{,}557)}{0{,}046} (\ln \bar{K})^2$$

$$R^2 = 0{,}99 \qquad DW = 0{,}356 \qquad (11.29)$$

with $\bar{K}$ being the geometric average of capital; t-values are given in parenthesis.

[23]The labour force (NLZA) was constructed, similar to De Jager and Small (1980:25) by adding the number of registered unemployed white, coloured, and Asian workers and the number of unemployed black workers as estimated by the Central Statistical Services to the number employed in the non-agricultural sectors (NEZA). Since statistics on Black unemployment were not available prior to 1977, it was assumed to have changed proportionately to the number of registered white, Coloured and Asian workers during the period 1965Q1 to 1977Q4.

All the test-statistics, with the exception of the Durbin-Watson (DW) proved to be satisfactory. The DW indicated the existence of autocorrelation due to the misspecification of the function with regard to technological development and capacity utilisation which were deliberately ignored by the authors. They argued that as far as capacity utilisation is concerned the necessary information on a ninety-two sectorial base did not exist in South Africa. Technology could have been incorporated into the production function with time (t) as a variable. However, they argued that the use of time as a variable, though generally accepted by econometricians in highly aggregated functions, is much more complicated where a vast amount of disaggregated production sectors are concerned. They argued further that if one adds the upward bias in Rho which may exist in production functions over the longer term, a low DW can be accepted and thereby justifying the use of Equation (11.29).

11.12 CONCLUSION

In this chapter we have seen that the ordinary least squares (OLS) model can be applied to economic models which are not linear by appropriate transformation and approximation. However, it is also becoming clear that there are many circumstances in which the restrictive assumptions of the OLS model do not apply, especially if one considers structural changes between consecutive time periods which are posed by many economic time series studies.

A new area of development has been introduced in estimating the parameters of a Cobb-Douglas production function. Following the VPR approach in estimating regression coefficients it permits that all, or some of the coefficients, of the production function can change over time. It is clear that without the application of econometric theory, the theoretical form of production functions would have remained very ill-defined indeed.

In looking back over this chapter one can see the way in which empirical analysis has developed, first to give plausible shapes to isoquants and then to take account of returns to scale, different and variable elasticities of substitution and the incorporation of other factors beside labour and capital. Perhaps the most important point is the realisation that no guarantee can be given that any of the "exact" production functions will be exact in fact.

CHAPTER 12

EXCHANGE RATE FUNCTIONS

12.1 INTRODUCTION

The debt standstill and the sanctions issue in South Africa have emphasised the need for empirical methodology in econometrics to deal with parameter variation over time.

One of the notable characteristics of the current international monetary system is the variability of exchange rates. In a major review of the performance of the system of generalised managed floating since 1973, Goldstein (1984: 5) noted that: "By almost any measure, exchange rate variability has been much greater during the period of floating rates (1973 - 1982) than it was during the last decade of the adjustable par value system (1963 - 1972)."

The South African currency appears to have exhibit similar behaviour during the period since 1973. Casual inspection of the data reveals a marked increase in the variability of the rand during the 1980's in particular. Also, Holden and Holden (1985: 358), with reference to the variance of effective exchange rate indices for the rand for the period up to 1985, noted: " ... a sharp increase in exchange rate variability over a past few years."

The purpose of this chapter is to discuss the theory of exchange rate models and to present basic structural models selected for exchange rates which do not allow for changes in the parameters. The reasons why variable coefficient models are appropriate for exchange rate modelling is discussed. Fixed coefficient models are augmented to allow for parameter changes in long-run real exchange rates. The out-of-sample forecasting performance of the variable and fixed coefficient representations of the exchange rate models are compared in Chapter 13.

12.2 THEORIES OF EXCHANGE RATE MODELS

12.2.1 INTRODUCTION

Much of the recent work on floating exchange rates goes under the name of "monetary" or "asset" view; the exchange rate is viewed as moving to equilibrate the international demand for stocks of assets, rather than the international demand for flows of goods as under the more traditional view (see Frankel 1979: 610). Within the asset view there are two fundamentally different approaches. These approaches have particularly conflicting implications for the relationship between the exchange rate and the interest rate.

The first approach might be called the "Chicago" theory because it assumes that prices are perfectly flexible (see Frenkel, 1976 and Bilson, 1978). As a consequence of the flexible - price assumption, changes in the nominal interest rate reflect changes in the expected inflation rate. When the domestic interest rate rises relative to the foreign interest rate, it is because the domestic currency is expected to lose value through inflation and depreciation. Demand for the domestic currency falls relative to the foreign currency, which causes it to depreciate instantly. This is a rise in the exchange rate, defined as the price of foreign currency. Thus a positive relationship between the exchange rate and the nominal interest differential is obtained.

The second approach is referred to as the "Keynesian" theory because it assumes that prices are sticky, at least in the short run (see Dornbusch, 1976 and Frankel 1979, 1981). Because of the sticky-price assumption, changes in the nominal interest rates reflect changes in the tightness of monetary policy. When the domestic interest rate rises relative to the foreign rate, it is because there has been a contraction in the domestic money supply relative to domestic money demand without a matching fall in prices. The higher interest rate at home than abroad attracts a capital inflow, which causes the domestic currency to appreciate instantly. Thus a negative relationship between the exchange rate and the nominal interest differential will be obtained.

The Chicago theory is a realistic description when variation in the inflation differential is large, as in the German hyperinflation of the 1920's to which Frenkel first applied it. The Keynesian theory is a realistic description when variation in the inflation differential is small, as in the Canadian float against the USA in the 1950's to which Mundell (1964, 1968) first applied it. The problem is to develop a model that is a realistic description when variation in the inflation differential is moderate, as it has been among the major industrialised countries in the 1970's.

12.2.2 THE STICKY-PRICE (DORNBUSCH - FRANKEL) MODEL

The model developed by Dornbusch (1976) and Frankel (1979,1981) (henceforth referred to as D-F model) is a version of the asset view of the exchange rate, in that it emphasises the role of expectations and rapid adjustment in capital markets. It combines the Keynesian assumption of sticky prices (originated by Dornbusch) with the Chicago assumption that there are secular rates of inflation. It therefore shares with the Frenkel - Bilson (Chicago) model an attention to long-run monetary equilibrium. It then turns out that the exchange rate is negatively related to the nominal interest differential, but positively related to the expected long-run inflation differential. When the nominal interest rate is low relative to the expected inflation rate, the domestic economy is highly liquid. An incipient capital outflow will cause the currency to depreciate, until there is sufficient expectation of future appreciation to offset the low interest rates. The exchange rate overshoots its equilibrium value by an amount proportional to the real interest differential.

The main analytical point made by Frankel (1979) was that a realistic monetary model of exchange rate determination must provide not only a role of secular monetary growth and inflation, as in the work of Frenkel (1976) and Bilson (1978), but also must take into account temporary deviations from purchasing power parity due to sticky prices, as in the work of Dornbusch (1976, 1978).

The theory of D-F yielded an equation (see Section 12.2.4) of exchange rate determination in which the spot rate is expressed as a function of the relative money supply, relative income level, the nominal interest differential (with the sign hypothesised negative), and the expected long-run inflation differential (with the sign hypothesised positive).

The sticky-price model of Hooper and Morton (1978) (henceforth H-P) extended the D-F model by incorporating the effects of the current account as part of the explanatory variables. The H-P model allows for changes in the long-run real exchange rate which are assumed to be correlated with unanticipated shocks to the trade balance. The econometric specifications of these models are given in Section 12.2.4.

12.2.3 THE FLEXIBLE-PRICE (FRENKEL - BILSON) MODEL

The monetary theory is not new. Frenkel (1976) has traced its origins back to Ricardo and has uncovered the following insightful statement made by Keynes (1971: 18) in 1924:

> "What, then, has determined and will determine the value of the franc? First, the quantity, present and prospective, of the francs in circulation. Second, the amount of purchasing power which it suits the public to hold in that shape."

Frenkel (1976 : 201) himself stated the basis of the theory in the following words:

> "Bring a relative price of two assets (moneys), the equilibrium exchange rate is attained when the existing stocks of the two moneys are willingly held. It is reasonable, therefore, that a theory of the determination of the relative price of two moneys should be stated conveniently in terms of the supply of and the demand for these moneys."

Frenkel (1978) and Dornbusch (1981) considered the empirical importance of a monetary partial adjustment mechanism in their exchange rate equations and did not find support for it. On the other hand, Bilson (1978, 1979) found that it was vital to obtain strong support for the monetary model.

The monetary model provided a useful tool for exchange rate analysis because it (a) clearly defines the role of speculation among determinants of the exchange rate, (b) provides a simple definition of the equilibrium exchange rate, and (c) directly relates the equilibrium rate to the underlying instruments of monetary policy.

It should be noted, however, that none of the exchange rate equations in Bilson's studies could be derived from a money demand function with partial adjustment in real balance holdings. Bilson defended the irrelevance of formally deriving an exchange rate equation from an explicit money demand function by claiming that the 'particular form of the demand function cannot be specified on theoretical grounds; it depends upon the particular relative price that is under study' (see Bilson, 1979).

Friedman (1953 : 158), in his classic defense of flexible exchange rate regimes, stressed that flexible exchange rates need not be stable. He wrote,

> "The ultimate objective is a world in which exchange rates, while free to vary, are in fact highly stable. Instability of exchange rates is a symptom of instability in the underlying economic structure."

These words offer little consolation to those who have experienced the volatile exchange rates of the floating rate period. Many market participants appeared to have regressed from Friedman's logic toward the belief that the exchange rate is determined by speculation and "market psychology" rather by the underlying economic conditions. This belief has been encouraged by the lack of generally accepted economic theory of the determination of the exchange rate, since, without such a theory, it is difficult to define the elements of the "underlying economic structure" that have been responsible for the erratic movements in the rates. This theoretical vacuum, induced primarily by the inability of economic models based upon trade flows to explain exchange rate movements in the inflationary environment of the 1970's, has been rapidly filled by a number of papers stressing the role of asset markets in the determination of the exchange rates (see Frenkel, 1976). The asset market models concentrate on the mechanisms through which the exchange rate eliminates incipient capital flows, including adjustment in real money balances through exchange rate-induced price level variation and adjustments in nominal interest rates through changes in the expected rate of exchange rate depreciation.

The models proposed by Frenkel (1976) and Bilson (1978, 1979) (henceforth referred to as F-B) is characterised as "monetary" because it is based on two assumptions associated with the "monetary approach to the balance of payments" and is carried over, in the manner suggested by Johnson (1973), into the study of flexible rates. These assumptions are that the demand for money is a stable function of a limited number of aggregate economic variables and that, in the absence of transportation costs and restrictions upon trade, the law of one price will hold in international markets. In the monetary model, the law of one price appears in the form of the purchasing power parity and interest rate parity conditions that link international price and interest rate movements to developments in the foreign exchange market. In the simple model (see Section 12.2.4), these arbitrary conditions are assumed to hold at each point in time, although some dynamic aspects will be introduced into the analysis.

12.2.4 THE BASIC STRUCTURAL MODELS AND THE METHODS

For estimation purposes, the quasi-reduced-form equations of the Frenkel-Bilson, Dornbusch-Frankel and Hooper-Morton models can be conveniently nested:

$$s = \beta_0 + \beta_1 (m_t - m_t^*) + \beta_2 (y_t - y_t^*) + \beta_3 (r_t - r_t^*) + \beta_4 (\pi_t^e - \pi_t^{e*}) + \beta_5 (TB_t - TB_t^*) \quad (12.1)$$

where, lower case letters indicate natural logs except for interest rates and inflation rates, and where * indicates a foreign variable; s is the spot exchange rate (e.g., R/$); m is the money supply; y is the industrial production or real income levels ; r is the short-term nominal interest rate; π^e is the long-run expected inflation rates; TB is the cumulative trade balance; and u is the disturbance term which may be serially correlated. The β's in Equation (12.1) are parameters to be estimated.

Meese and Rogoff (1983 a,b) (hereafter referred to as MR) estimated the following fixed coefficient versions of Equation (12.1):

(1) Frenkel-Bilson (purchasing power parity) which assumes $\beta_4 = \beta_5 = 0$

(2) Dornbusch-Frankel (slow price adjustment) which assumes $\beta_5 = 0$

(3) Hooper-Morton which is Equation (12.1) with unequal coefficients for the trade balances.

These models are all variants of the monetary model of exchange rates and differ only in the way they treat price adjustment. Schinasi and Swamy (1989) also explicitly reported results for these models with a lagged dependent variable as an explanatory variable, a specification that explicitly allows for short-run deviations from long-run purchasing power parity.

Both the F-B and D-F models hypothesise that the exchange rate is homogeneous of degree 1 with respect to relative money supplies ($\beta_1 = 1$). The F-B model, which assumes purchasing power parity (REXPPP) also posits the restriction $\beta_2 < 0$, $\beta_3 >$ 0, and $\beta_4 = \beta_5 = 0$. The D-F model, which allows for short-run deviations from PPP due to prices that respond only gradually to excess demand, hypothesises that $\beta_2 < 0$, $\beta_3 < 0$, $\beta_4 > 0$, and $\beta_5 = 0$.

The above models represent an important class of empirically testable models of exchange rate determination. Extensive in-sample studies of the models properties have appeared in the literature. These in-sample studies have shown quite satisfactory fits (see e.g., Bilson 1978 and Frankel, 1979). MR (1983a) used monthly data over the period March 1973 to June 1981. The fixed coefficient version of Equation (12.1) were initially estimated for each exchange rate using data up through October 1976. Forecasts were generated at four different horisons using actual realisations of all explanatory variables for a prediction period. Then the data for November 1976 were added to the sample, and the parameters for each model were reestimated. New forecasts were generated at the same horisons, etc. In the MR studies, this sequential estimation yielded fixed-step-ahead forecasts which were generally inferior to those given by the random walk model (see Schinasi and Swamy, 1989). The estimation procedures used in their sequential estimation were ordinary least squares (correcting for serial correlation in the error term), and Fair's (1970) instrumental variable technique. MR also considered six univariate time series models involving a variety of prefiltering techniques and lag length selection criteria, a random walk with drift parameter, and an unconstrained vector autoregression. None could outpredict the random model $s_t = s_{t-1} + a_t$, where a_t is white noise with mean zero and constant variance.

Predictive testing is an important aspect of econometric model building that is often neglected. Wolff (1987) and Schinasi and Swamy (1989) focussed on predictive testing in the context of varying-parameter versions of the monetary models of exchange rates. Their empirical results provided new evidence on the predictive performance of these models and indicated that a certain degree of parameter instability was indeed present.

12.3 THE EFFECTS OF EXCHANGE RATE CHANGES ON THE SOUTH AFRICAN ECONOMY

The exchange rate is one of the most important prices in the relatively "open" South African economy. Under the present system of managed floating, the spot and forward rates are significantly influenced by the monetary authorities. There are of course, limits to their influence, especially to their ability to bring about a continued appreciation of the rand. The same limits, naturally, apply to many so-called policy instruments: although these variables are under the control of the authorities, there are stronger social, political and economic forces at work over the longer term.

A severe change in the exchange rate has many effects, directly and indirectly, and an extensive structural econometric model offers a way of tracing and quantifying these effects. A policy experiment to investigate the effects of exchange rate changes was carried out by van den Heever (1990: 23-38). Exchange rate appreciation was seen to be highly effective in combating the inflation, but implied a weaker balance on the current account of the balance of payments and losses of net gold and other foreign exchange reserves. It was, however, found that the exchange rate appreciation did not lower the real gross domestic product, Van den Heever (1990:31) gave a number of reasons explaining why this has happened.

Data variables such as the external value of the rand, dollar gold price, the oil price, domestic agricultural conditions and the level of real economic activity in foreign markets play a crucial role in the South African economy. The unexpected turns in these variables have had considerable impact, not only on the foreign exchange reserves, but also on the economy in general. The increased volatility of the external value of the rand during the 1980's posed the question as to its effects on South African exports. Smit (1991: 10-29) investigated these possible effects on exports by means of an econometric analysis and found no proof of any systematic relationships between the variability of the external value of the rand and South Africa's non-gold export volumes.

In most of the South African work, dealing with the econometric analysis of the Rand/US$ exchange rate, it is found that the exchange rate variable is only used as an explanatory variable in equations such as those for exports, imports and capital movements. It is, therefore, the purpose of this chapter to build a suitable exchange rate model for the South African economy subject to the constraints outlined in Section 9.2.

12.4 A STOCHASTIC - COEFFICIENT APPROACH TO EXCHANGE RATE MODELS

Although the literature has been flooded with in-sample studies of empirical exchange rate models since the breakdown of the Bretton Woods fixed-parity system in the early 1970's, systematic studies of the forecasting performance of structural or reduced-form models are relatively scarce.

MR studied the forecasting performance of several important monetary models. The seminal work of MR casts serious doubt on the ability of international macroeconomic theory to predict exchange rate movements. These studies concluded that linear fixed-coefficient regressions of exchange rates on variables such as relative money supplies, indices of industrial productions, short-term interest rates, and trade balances, failed to match the out-of-sample forecasting performance of a simple random walk model, even though the models' forecasts were based on actual, realised values of future explanatory variables. MR's main results were reported as 'robust to a variety of (fixed coefficient) estimation techniques, specifications of the underlying money demand functions, alternative serial correlation or lagged adjustment corrections, and measures of forecast accuracy' (see MR, 1985:5).

One might be tempted to conclude from these studies that economic variables convey little or no useful information about exchange rate movements. However, a number of potential explanations (for example, simultaneous equation bias, sampling error, or misspecification) for the unimpressive out-of-sampling performance of the models have been offered in the literature (Isard, 1983; MR, 1983; Saidi, 1983).

There are several specific reasons why the coefficients of exchange rate models may change over time. Some important ones are the following:

(1) Even if the explanatory variables capture all information used by traders, there is no reason to believe information is used the same way over all policy regimes and over all time horizons (Lucas, 1976); parameters can change over time. As argued by Swamy and Schinasi, 1989), sequential estimation of fixed coefficient regression ('rolling') is not the appropriate technique for capturing variations in coefficients over time;

(2) Many of the empirical studies have assumed that coefficients are fixed over the relevant sample period. Most of this literature decisively rejects economic theory as having any ability to produce accurate predictions. Yet, it would be unreasonable to reject theories that have been tested on only a very limited subset of models, namely specific linear or non-linear fixed coefficient models. These models may have performed poorly because they omitted important variables, or because they have incorrect functional forms. Wolff (1987), for example, argued that MR performed joint tests of the out-of-sample validity of the exchange rate models and the demand functions that they implicitly specified, and that it may well be that their rejections were, at least in part, due to the inadequately specified demand functions; on the other hand, linear fixed-coefficient exchange rate models are a very limited subset of statistical representations of the underlying economic theory and the limitations of linearity are well known. Unfortunately, there is a paucity of exchange rate theory suggesting what type of nonlinearities may exist (see Schinasi and Swamy, 1989), not because economists believe the world is linear, but because linearity is a practical simplifying assumption. One way of detecting deviations from linearity is to relax

the assumption of fixed coefficients and to examine whether forecasting performance is improved.

(3) At the high level of aggregation of exchange rates, there is little reason to believe that behavioural parameters are fixed. There is a wide diversity of participants in foreign exchange markets with relatively small and highly variable market shares. Even if each participant reacted to macroeconomic developments according to a stable fixed coefficient reaction function, it is difficult to argue that macroeconomic variables would be related to exchange rates by a simple fixed coefficient relationship, without also assuming that the individual reaction functions are identical.

(4) Factors leading to changes in the long-run real exchange rate (such as changes in oil prices, global trade patterns, etc.) may lead to instability in the parameters of the class of structural exchange rate models.

This study evaluates exchange rate models in South Africa without imposing the restriction that the regression slopes are fixed over time. Although there are a number of studies that have relaxed the fixed coefficient assumption, they have done so in rather restrictive ways, requiring extensive and generally unavailable prior information (see Wolff, 1987). The present study avoids these problems by applying a general technique for estimating models with stochastic coefficients (see VPR model of Watson and Engle, 1985) that encompasses as special cases the Kalman filtering technique, the methods of Hildreth and Houck (1968) and Rosenberg (1973). Multi-step-ahead forecasts are computed using models with fixed and stochastically varying coefficients, to extend the work of MR without duplicating their efforts.

12.5 CONCLUSION

The main result of this chapter is that once one is willing to relax the assumption of fixed regression slopes, it is possible to estimate structural models of exchange rate determination which perform better than the conventional model in predicting out-of-sample values of exchange rates. As was shown decisively by Meese and Rogoff, and others, fixed coefficient models of exchange rates - with and without a lagged dependent variable, and with a variety of proxies for the differential of inflation expectations - could not outperform the random walk model.

In the next chapter out-of-sample forecasting performance of an important class of structural exchange rate models is studied and it demonstrates that stochastic coefficient models of exchange rate determination can be useful in improving the accuracy of forecasts of exchange rates.

PART VI

DATA ANALYSIS

CHAPTER 13

EMPIRICAL RESULTS

13.1 PARAMETER STABILITY TEST RESULTS

In this study it is tentatively proposed that the rapidly changing economic and political environment in South Africa may cause econometric functions to become structurally unstable. A number of equations of the South African economy, proposed during the last two decades, are re-estimated and tested for structural stability using the Quandt-ratio, Chow, Cusum, Cusum of Squares, Fluctuation and Watson-Davies tests. It is shown that a substantial number of investment equations fail one or more tests for structural stability.

Before the results reported in Table 13 are discussed, a brief note on the specification of the various functions is warranted. Stage 1 represents structural stability tests applied to the equations estimated by the original model builder. Stage 2 represents the structural stability tests applied to an econometrically well-behaved version of the original model. In both stages ordinary least squares techniques (and a Cochrane-Orcutt correction for serially correlated errors if necessary) are used with regression parameters assumed to be fixed. Stages 3 and 4 represent the estimation results of the ARCH and VPR techniques, respectively, and are discussed at a later stage.

Table 13.1, 13.4 and 13.5 reports the statistics for testing the parameter constancy of the investment, production and exchange rate models, respectively. The tables contain the name of the dependent variable, the names of the independent variables, the timing of the minimum Quandt-ratio and the p-value of the associated Chow test. Also reported are the p-values associated with local Quandt minima as well as the results of the Cusum, Cusum of Squares tests, Fluctuation and Watson-Davies tests. The time periods referred to in these tables are periods identified as periods of parameter instability. Unfortunately, all these tests could not be executed for all the functions because a negative square root-routine was detected during calculations in some of the cases. The Watson-Davies test, test for the constancy of a particular parameter. The results of the Watson-Davies test are reported as *stable* if none of the coefficients in the model show up to be varying significantly over time.

The recursive regression parameters are also obtained by repeatedly adding one observation to the data set and re-computing the estimates. Any sign changes in these estimated parameters are reported in the last column of the table. Sign changes occurring among the first few observations are ignored because of insufficient degrees of freedom. Relatively short periods of sign changes are also ignored and are reported as stable.

A total of 10 quarterly investment and 2 monthly exchange rate functions are investigated over the period 1970Q1 to 1990Q4. The 11 yearly production functions are estimated over the period 1970 to 1989. The re-estimated parameters are also reported with the corresponding t-statistic in parenthesis. A 5% level of significance is used in all cases. A key to the symbols used appear in Appendix A and B.

13.1.1 QUARTERLY INVESTMENT MODELS

In Table 13.1 the investment functions are defined as I-1 to I-5 with the number in brackets indicating the stage of specification.

The signs of the estimated parameters of the lagged variables in Model I-1(1) appear relatively unstable compared to similar investment functions discussed in Chapter 10. The Quandt-ratio points at a structural breakpoint in the regression ? relationship in 1974Q2 which is not supported by the Chow test. However, the local Quandt minima coupled with the Chow test do signify structural breakpoints in 1979Q1; 1980Q1 and 1984Q3. Both the Cusum and Cusum of Squares tests indicate stability of the parameters over time, where as in the case of the Fluct

TABLE 13.1 TESTS OF THE CONSTANCY OF PARAMETERS FOR INVESTMENT EQUATIONS (1970Q1-1990Q4)

MODEL	DEPENDENT VARIABLE	EXPLANATORY VARIABLES	TIMING OF MINIMUM QUANDT-RATIOS	CHOW	CUSUM	CUSUMSQ	FLUCT	WATSON-DAVIES	R^2	DW	SIGN CHANGES
				INVESTMENT: REAL							
I-1(1)	IPO1	IPO1(-1); KPO1(-1); 0.800 0.019 (9.818) (2.391) YCUB+YCUB(-1)+YCUB(-2); 0.021 (0.761) AD(YFZPA1,4);AD(YFZPA1,4)(-1); 0.105 -0.030 (2.112) (-0.562) AD(YFZPA1,4)(-2);AD(YFZPA1,4)(-3); -0.031 -0.001 (-0.579) (-0.010) AD(YFZPA1,4)(-4) 0.078 (1.554)	1974Q2	1974Q2 (p=0.9450) 1979Q1 (p=0.0017) 1980Q1 (p=0.0022) 1984Q3 (p=0.0029)	STABLE	STABLE	1977Q2-1985Q4	STABLE	0.899	2.631	NO
I-1(2)	IPO1	IPO1(-1); IPO1(-2); KPO1(-1); 0.470 0.413 0.013 (4.555) (4.189) (2.491) AD(YFZPA1,4); AD(YFZPA1,4)(-4) 0.109 0.080 (4.846) (3.164)	1989Q2	1989Q2 (p=0.8633) 1979Q1 (p=0.0779) 1984Q3 (p=0.0878) 1987Q3 (p=0.3053)	STABLE	STABLE	1974Q1-1985Q1	STABLE	0.921	2.045	YES
I-2(1)	IPO1	PVI(-1);YGDE1(-1);YGDE1(-2); 0.057 0.086 -0.010 (2.443) (2.264) (-0.212) YGDE1(-3);YGDE1(-4);YGDE1(-5); 0.009 0.040 -0.026 (0.191) (0.897) (-0.586) YGDE1(-6);YGDE1(-7);YGDE1(-8); 0.032 0.023 -0.027 (0.722) (0.524) (-0.822) DEFIT*; DEFIT(-1); DEFIT(-2); -0.078 -0.162 0.060 (-0.755) (-1.335) (0.486) DEFIT(-3); DEFIT(-4); DEFIT(-5) -0.203 0.047 0.018 (-1.625) (0.375) (0.172)	1976Q1	1976Q1 (p=0.0467) 1980Q4 (p=0.0000) 1984Q2 (p=0.0009) 1986Q1 (p=0.0583)	NEGATIVE SQRT-ROUTINE	NEGATIVE SQRT-ROUTINE	NEGATIVE SQRT-ROUTINE	UNSTABLE	0.816	0.771	-

TABLE 13.1 (continued)

MODEL	DEPENDENT VARIABLE	EXPLANATORY VARIABLES	TIMING OF MINIMUM QUANDT-RATIOS	CHOW	CUSUM	CUSUMSQ	FLUCT	WATSON-DAVIES	R^2	DW	SIGN CHANGES
I-2(2)	IPO1	IPO1(-1); PVI(-1); AUTO(-1) 0.828 0.021 -0.290 (13.800) (2.960) (-2.443)	1989Q4	1989Q4 (p=0.9929) 1976Q2 (p=0.5212) 1980Q1 (p=0.3484)	STABLE	STABLE	1978Q2-1981Q1	UNSTABLE	0.900	1.908	NO
I-3(1)	IPO1	IPO1(-1); FRSB4(-2); YGDE1(-1) 0.741 -0.022 0.043 (9.406) (-0.964) (4.201)	1989Q3	1989Q3 (p=0.9297) 1973Q2 (p=0.5991) 1980Q1 (p=0.0002)	1982Q3-1985Q2	STABLE	1972Q2-1982Q2	STABLE	0.922	2.581	YES
I-3(2)	IPO1	IPO1(-1); FRSB4(-2); YGDE1(-1); 0.933 -0.055 0.023 (17.321) (-3.836) (3.237) AUTO(-1) -0.500 (-4.907)	1989Q2	1989Q2 (p=0.6771) 1979Q3 (p=0.0012)	1983Q4-1984Q4	STABLE	1972Q1-1981Q4	STABLE	0.935	2.154	YES
I-4(1)	IPO1	KPO1(-1); (IPO1-IDT1)(-1); 0.148 0.295 (11.903) (3.02) (IPO1-IDT1)(-2); 0.451 (4.591) Z*(-1) ; Z(-2) ; Z(-3); -0.278 -1.018 0.036 (-0.751) (-2.782) (0.090) Z(-4) ; Z(-5); Z(-6); -0.059 -0.012 0.423 (-0.149) (-0.033) (1.103) Z(-7) ; Z(-8) 0.116 -0.162 (0.296) (-0.426)	1975Q3	1975Q3 (p=0.3105) 1980Q4 (p=0.0000) 1985Q2 (p=0.0000) 1986Q4 (p=0.0000)	STABLE	1980Q2-1987Q2	1976Q1-1978Q2 1981Q3-1989Q4	UNSTABLE	0.724	0.846	YES
I-4(2)	IPO1	(IPO1-IDT1)(-2); IPO1(-1); 0.202 0.737 (2.962) (10.201) TREND 0.148 (3.233)	1989Q1	1989Q1 (p=0.2924) 1973Q3 (p=0.8451) 1979Q3 (p=0.0385) 1987Q4 (p=0.1011)	STABLE	STABLE	1973Q3-1976Q2	STABLE	0.891	2.209	NO

* Z = YFZPA1/FRLE4 - YFZPA1(-1)/FRLE4(-1).

TABLE 13.1 (continued)

MODEL	DEPENDENT VARIABLE	EXPLANATORY VARIABLES	TIMING OF MINIMUM QUANDT-RATIOS	CHOW	CUSUM	CUSUMSQ	FLUCT	WATSON-DAVIES	R^2	DW	SIGN CHANGES
I-5(1)	IPZA1	YGDE1(-1); YGDE1(-2); 0.112 0.024 (2.983) (0.446) YGDE1(-3); FRLE4 0.053 -0.182 (1.380) (-2.215)	1978Q1	1978Q1 (p=0.0000) 1980Q2 (p=0.0000) 1981Q2 (p=0.0000)	1983Q3-1986Q2	1976Q2-1981Q4	1974Q4-1983Q2 1984Q2-1985Q4	UNSTABLE	0.782	0.481	YES
I-5(2)	IPZA1	IPZA1(-1); IPZA1(-4); 1.074 -0.197 (17.068) (-4.140) YGDE1(-1) 0.016 (2.970)	1988Q4	1988Q4 (p=0.8973) 1973Q2 (p=0.5689) 1984Q3 (p=0.1104)	STABLE	STABLE	1976Q1-1978Q1	STABLE	0.930	1.999	NO

test (the more powerful of the two tests) a warning of structural instability is indicated over the period 1977Q2 - 1985Q4. The Watson-Davies test, on the other hand, indicates no parameter changes. The recursive regression parameters obtained by repeatedly adding one observation to the data set and re-computing the estimates, do signify a sign change in the estimated parameters. In the case of the econometrically well-behaved model I-1(2) none of the tests indicates structural instability except for the Fluct test which indicates a period of structural change during 1974Q1 through 1985Q1.

The signs of the estimated parameters of the lagged variables in Model I-2(1) appear relatively unstable with sign changes between different lags. The coefficients in the case of the more distant lagged gross domestic expenditure variables and in the case of an absolute difference in the investment deflator over six periods appear to be insignificant and some with an _a priori_ wrong sign. The Quandt-ratio points at a structural breakpoint in 1976Q1 which is supported by the Chow test. The local Quandt minima in 1980Q4 and 1984Q2 coupled with the Chow test indicate further structural breakpoints. However, the local Quandt minimum in 1986Q1 coupled with the Chow test do not signify any structural breakpoint. The Watson-Davies test also indicates instability in some of the parameters. The Cusum, Cusum of Squares, Fluct and the sign test could not be performed because of a negative value detected in the square-root routine of these processes. Model I-2(2) appear to be much more stable, as expected, except for the Fluct test which indicates structural instability during 1978Q1 - 1981Q1.

In the case of Model I-3(1) it is found that the signs of the estimated recursive regression parameters change over time. The coefficient of the variable YGDE1 appears with an *a priori* wrong sign. The Quandt-ratio points at a structural breakpoint in 1989Q3 which is not supported by the Chow test. The local Quandt minimum, however, points at a structural break in 1980Q1, which leads to some suspicion about the interaction of the Quandt-ratio and the Chow test, because the minimum Quandt-ratio is supposed to give the maximum likelihood of a structural break in the regression parameters over the estimated time span. The Cusum test hints at a period of structural instability during 1982Q3 - 1985Q2. The Fluct test also signifies structural instability over the period 1972Q1 - 1982Q2.

The results pertaining to the structural stability of Model I-3(2) are analogous to those of Model I-3(1). The minimum Quandt-ratio points at a structural break in 1989Q2 which is not supported by the Chow test. The local Quandt minimum in 1980Q1, however, indicates a structural breakpoint when the Chow test is applied. The Cusum test and Fluct test hints at periods of structural instability during 1983Q4 -

1984Q4 and 1972Q1 - 1982Q2, respectively. In this case sign switches have also occurred in the recursive regression parameters.

In Model I-4(1) all of the parameters of AD(YFZPA1/FRLE4 - YFZPA1(-1)/FRLE4(-1)) (L(1-8)) appear to be insignificant. Many of these variables also appear with a priori wrong signs. The minimum Quandt-ratio in 1975Q3 does not indicate a structural breakpoint when the Chow test is applied. The local Quandt minima, surprisingly, points at structural breaks in 1980Q4, 1985Q2 and 1986Q4. The Cusum of Squares and Fluct tests hints at periods of structural instability during 1980Q2 - 1987Q2 and 1976Q1 - 1978Q2; 1981Q3 - 1989Q4, respectively. The Watson-Davies test also indicates some parameter changes. Sign changes in the recursively estimated parameters are also observed.

Model I-4(2) appears to be well estimated judging by the signs and significance of the estimated parameters as well as the fact that no sign changes appear in the estimated recursive regression parameters. Most of the other criteria of fit signify a good regression relationship. Structural instability is, however, indicated by the Fluct test during 1973Q3 - 1976Q2; 1978Q3 - 1985Q3, and one of the local Quandt minima coupled with the Chow test in 1979Q3.

Model I-5(1) portraits some significance problems for the various parameters in the equation. The Chow test applied to all Quandt minima are significant at the 5 percent level, suggesting that the regression relationship involving no structural change should be rejected. The Cusum and the Cusum of Squares tests both hint at structural instability during the periods 1983Q3 - 1986Q2 and 1976Q2 - 1981Q4, respectively. The Fluct test also gives warning of structural weaknesses during the periods 1974Q4 - 1983Q2 and 1984Q2 - 1985Q4. The Watson-Davies test indicates that there are some parameters in the equation which varies over time. The above evidence together with sign changes in the recursive regression parameters prove that there are definitely structural weaknesses in the specified equation for IPZA1.

Model I-5(2) appears to be well estimated except for the variable IPZA1(-4) which appears with a suspicious negative sign. No structural instability is indicated by the various structural stability tests except for the Fluct test which hints at a period of structural change during 1976Q1 - 1978Q1. This evidence is supported by the fact that no sign changes are observed over the estimation periods.

Although some of the Stage 2 models are well estimated judging by various econometric criteria it is still evident that structural instability in the parameters can occur. As an example the fixed investment expenditure function I-2(2) for IPO1 is investigated in a little bit more detail.

From Table 13.2 it follows that the explanatory variables are the dependent variable (IPO1) lagged one and two periods, a one period lag in capital stock and a zero and four period lag in absolute quarterly differences in real gross domestic product. Economically, statistically and econometrically speaking everything appears to be in order, even allowing for the bias in the Durbin-Watson statistic caused by the lagged dependent variable. The Fluct test points at a period of parameter instability during the period 1974Q1 - 1985Q1 which is supported by sign changes in the recursive parameters. Table 13.2 shows further what will happen to the estimated regression parameters and criteria of fit when the period of estimation is divided into several time segments. The estimation period is broken up into, firstly, two different sub-periods with the splitting date indicated by the minimum Quandt-ratio and then three different sub-periods with more or less an equal number of observations. Substantial variations in the parameter estimates and criteria of fit are evident between these sub-periods. It is clear that the magnitude of some of the estimated coefficients change remarkably over the various sub-periods. The signs of the estimated parameters also appear to be unstable while a large number of the parameters become statistically insignificant at a 5% level, especially in the case where the subdivision is over three periods. Other criteria of fit seem to be relatively stable except for the sub-period 1972Q1 - 1977Q1 where the adjusted coefficient of determination ($\bar{R}^2$) drop from 0.9210 (in the original equation)

TABLE 13.2 PARAMETER ESTIMATES OVER DIFFERENT ESTIMATION PERIODS; FIXED INVESTMENT EXPENDITURE: PRIVATE, EXCLUDING RESIDENTIAL BUILDINGS AND AGRICULTURE: REAL (IPO1(2))

Period:		1972Q1 - 1990Q4				1972Q1 - 1989Q2				1989Q3 - 1990Q4			
Para-meters	Explanatory Variables	Estimated Coefficients	Standard Deviation	t	p-value	Estimated Coefficients	Standard Deviation	t	p-value	Estimated Coefficients	Standard Deviation	t	p-value
$\hat{\beta}_1$	IPO1(-1)	0.4695	0.1031	4.555	1.000	0.4607	0.1065	4.327	1.000	5.0010	0.6984	7.160	0.912
$\hat{\beta}_2$	IPO1(-2)	0.4125	0.0985	4.189	1.000	0.4206	0.1022	4.117	1.000	-2.8169	0.5043	-5.586	0.887
$\hat{\beta}_3$	KPO1(-1)	0.0132	0.0053	2.491	0.985	0.0135	0.0058	2.321	0.977	-0.1468	0.0254	-5.788	0.891
$\hat{\beta}_4$	YFZPA1-YFZPA1(-4)	0.1085	0.0224	4.846	1.000	0.1112	0.0235	4.732	1.000	0.7389	0.1150	6.427	0.902
$\hat{\beta}_5$	YFZPA1(-4)-YFZPA1(-8)	0.0795	0.0251	3.164	0.998	0.0742	0.0263	2.823	0.994	0.0556	0.0945	0.588	0.339
		RMSE: 0.5839	$\bar{R}^2$: 0.9210			RMSE: 0.5995	$\bar{R}^2$: 0.9127			RMSE: 0.0120	$\bar{R}^2$: 0.9877		
		SEE: 0.6036	R^2: 0.9252			SEE: 0.6221	R^2: 0.9177			SEE: 0.0295	R^2: 0.9975		
		DW: 2.0451				DW: 2.0813				DW: 2.3285			

Period:		1972Q1 - 1977Q1				1977Q2 - 1984Q2				1984Q3 - 1990Q4			
Para-meters	Explanatory Variables	Estimated Coefficients	Standard Deviation	t	p-value	Estimated Coefficients	Standard Deviation	t	p-value	Estimated Coefficients	Standard Deviation	t	p-value
$\hat{\beta}_1$	IPO1(-1)	0.5244	0.2461	2.131	0.951	0.3001	0.1644	1.826	0.920	0.4950	0.1800	2.750	0.988
$\hat{\beta}_2$	IPO1(-2)	-0.2018	0.2413	-0.836	0.585	0.5328	0.1664	3.202	0.996	0.2601	0.1579	1.648	0.886
$\hat{\beta}_3$	KPO1(-1)	0.0980	0.0359	2.726	0.985	0.0245	0.0210	1.166	0.745	0.0265	0.0086	3.066	0.994
$\hat{\beta}_4$	YFZPA1-YFZPA1(-4)	0.1320	0.0507	2.603	0.981	0.0745	0.0385	1.937	0.935	0.1493	0.0444	3.360	0.997
$\hat{\beta}_5$	YFZPA1(-4)-YFZPA1(-8)	0.1272	0.0577	2.203	0.957	0.0953	0.0440	2.165	0.959	0.1182	0.0387	3.059	0.994
		RMSE: 0.3328	$\bar{R}^2$: 0.7942			RMSE: 0.6874	$\bar{R}^2$: 0.8902			RMSE: 0.4051	$\bar{R}^2$: 0.9096		
		SEE: 0.3812	R^2: 0.8354			SEE: 0.7556	R^2: 0.9059			SEE: 0.4507	R^2: 0.9240		
		DW: 2.2855				DW: 2.3107				DW: 2.1428			

to 0,7942. The Durbin-Watson statistic also indicates problems with autocorrelation in some of the sub-periods. Drastic deviations from the original model are all printed in bold script.

Table 13.3 is a table of the recursive regression parameters which are obtained by repeatedly adding one observation to the data set and then re-computing the estimates. Once more, the variation in the parameter estimates is evident. Sign changes between 1974Q1 - 1978Q2 are observed in the case of the parameter estimates of the dependent variable (IPO1) lagged two periods. The magnitude of the recursively estimated parameters show considerable changes, especially in the case of the two lagged dependent variables. The estimates of the β_1 parameters changes from 0,0735 in 1978Q3 to 0,4126 in 1982Q1. In the case of $\hat{\beta}_2$ a change of about 72% is observed between 1978Q4 to 1979Q3. The same kind of change is observed in $\hat{\beta}_3$ where the estimated parameters change from 0,0203 in 1985Q1 to 0,0018 in 1986Q1 (a change of about 91%). These indications of weaknesses are also borne out by the Fluct test over this period (see test results in Table 13.1).

TABLE: 13.3: RECURSIVE REGRESSION PARAMETERS FOR INVESTMENT MODEL IPO1(2)

date	Parameter Estimates $\hat{B}_1$	$\hat{B}_2$	$\hat{B}_3$	$\hat{B}_4$	$\hat{B}_5$
-73Q1	7.8382	17.7735	-4.3850	6.1219	3.0324
-73Q2	5.2026	11.5193	-2.8225	4.1760	2.1920
-73Q3	1.1707	1.4218	-0.3039	0.6065	0.4600
-73Q4	0.8722	0.2994	-0.0557	0.3134	0.3385
-74Q1	0.7947	-1.3395	0.2425	0.0003	0.1990
-74Q2	0.4581	-0.5110	0.1574	0.1211	0.1969
-74Q3	0.4456	-0.5139	0.1593	0.1241	0.2069
-74Q4	0.2955	-0.3109	0.1519	0.1479	0.1395
-75Q1	0.2717	-0.2634	0.1498	0.1261	0.1436
-75Q2	0.2830	-0.2499	0.1461	0.1255	0.1363
-75Q3	0.3053	-0.3704	0.1512	0.0784	0.1315
-75Q4	0.3445	-0.3687	0.1580	0.0860	0.1428
-76Q1	0.3401	-0.3477	0.1556	0.0873	0.1370
-76Q2	0.3488	-0.3298	0.1496	0.1031	0.1431
-76Q3	0.3700	-0.3209	0.1428	0.1110	0.1594
-76Q4	0.4376	-0.2515	0.1189	0.1319	0.1542
-77Q1	0.5296	-0.1964	0.0961	0.1301	0.1285
-77Q2	0.4545	0.0616	0.0617	0.1563	0.1734
-77Q3	0.2324	0.1363	0.0854	0.1576	0.1854
-77Q4	0.4411	-0.1360	0.0997	0.1174	0.1504
-78Q1	0.4174	-0.1173	0.0987	0.1258	0.1719
-78Q2	0.2520	-0.1054	0.1221	0.1548	0.1800
-78Q3	0.0735	0.0993	0.1154	0.1543	0.2230
-78Q4	0.1142	0.0721	0.1131	0.1479	0.2278
-79Q1	0.1206	0.1153	0.1059	0.1405	0.2204
-79Q2	0.1576	0.1714	0.0921	0.1302	0.2026
-79Q3	0.2502	0.2594	0.0666	0.0927	0.1645
-79Q4	0.3016	0.3257	0.0504	0.0683	0.1329
-80Q1	0.3004	0.3170	0.0518	0.0710	0.1347
-80Q2	0.3016	0.2974	0.0543	0.0845	0.1342
-80Q3	0.3149	0.2575	0.0576	0.0999	0.1464
-80Q4	0.2797	0.2871	0.0586	0.0967	0.1381
-81Q1	0.2725	0.2968	0.0582	0.0980	0.1403
-81Q2	0.2919	0.2750	0.0583	0.0984	0.1515
-81Q3	0.3095	0.2736	0.0559	0.0975	0.1565
-81Q4	0.3565	0.2986	0.0457	0.0947	0.1488
-82Q1	0.4126	0.3183	0.0355	0.0820	0.1381
-82Q2	0.3925	0.2956	0.0412	0.0915	0.1452
-82Q3	0.3808	0.3276	0.0386	0.0844	0.1429
-82Q4	0.3831	0.3264	0.0366	0.0827	0.1430
-83Q1	0.3746	0.3873	0.0332	0.0569	0.1198
-83Q2	0.3667	0.3905	0.0338	0.0578	0.1225
-83Q3	0.3692	0.3844	0.0343	0.0574	0.1246
-83Q4	0.3969	0.4136	0.0274	0.0740	0.0739
-84Q1	0.3708	0.4232	0.0296	0.0693	0.0857
-84Q2	0.3603	0.4386	0.0290	0.0720	0.0825
-84Q3	0.3628	0.4356	0.0291	0.0718	0.0822
-84Q4	0.3792	0.4332	0.0266	0.0769	0.0763
-85Q1	0.4100	0.4406	0.0203	0.0860	0.0622
-85Q2	0.4234	0.4448	0.0174	0.0913	0.0574
-85Q3	0.4436	0.4655	0.0104	0.0997	0.0551
-85Q4	0.4573	0.4631	0.0085	0.1013	0.0558
-86Q1	0.4611	0.4984	0.0018	0.1101	0.0591
-86Q2	0.4506	0.5017	0.0030	0.1105	0.0580
-86Q3	0.4550	0.4622	0.0085	0.1102	0.0591
-86Q4	0.4573	0.4900	0.0037	0.1099	0.0589
-87Q1	0.4532	0.4909	0.0042	0.1106	0.0587
-87Q2	0.4534	0.4793	0.0059	0.1108	0.0608
-87Q3	0.4449	0.4684	0.0088	0.1117	0.0649
-87Q4	0.4474	0.4525	0.0108	0.1119	0.0676
-88Q1	0.4406	0.4391	0.0137	0.1132	0.0750
-88Q2	0.4515	0.4187	0.0151	0.1135	0.0764
-88Q3	0.4514	0.4197	0.0150	0.1133	0.0762
-88Q4	0.4548	0.4223	0.0141	0.1120	0.0751
-89Q1	0.4614	0.4205	0.0133	0.1109	0.0737
-89Q2	0.4608	0.4198	0.0136	0.1108	0.0742
-98Q3	0.4619	0.4153	0.0141	0.1093	0.0756
-89Q4	0.4626	0.4128	0.0144	0.1083	0.0765
-90Q1	0.4626	0.4119	0.0146	0.1078	0.0769
-90Q2	0.4629	0.4129	0.0143	0.1082	0.0769
-90Q3	0.4650	0.4125	0.0140	0.1082	0.0778
-90Q4	0.4696	0.4117	0.0133	0.1080	0.0795

13.1.2 YEARLY PRODUCTION FUNCTIONS

Table 13.4 summarises the test results for the production functions. The Cusum, Cusum of Squares, Fluct and the sign test could not be evaluated in ten out of eleven production functions investigated, because of a negative value in the square-root routine during calculation. It is, therefore, decided not to report on any of these particular tests in the table.

Besides the Cobb-Douglas production functions (P-1 to P-4), attention is also focussed on the constant elasticity of substitution production functions (P-5 and P-6) and the transcendental production functions (P-6 to P-10). Relative to the other functions, Model P-2 turned out to be the best estimated production function in terms of sound econometric criteria.

None of the structural stability tests applied to both stages of Models P-1, P-2 and P-5 to P-9 have shown any indications of parameter instability, except for Model P-2(1), where the Quandt-ratio points at a structural breakpoint in 1978, which is supported by the Chow test. The local Quandt minimum do not show up any further structural breakpoints in this case.

In the case of model P-3 no structural changes are indicated by the minimum Quandt-ratio and accompanying Chow test. The local

TABLE 13.4 TESTS OF THE CONSTANCY OF PARAMETERS FOR PRODUCTION EQUATIONS (1970-1989)

MODEL	DEPENDENT VARIABLE	EXPLANATORY VARIABLES	TIMING OF MINIMUM QUANDT-RATIOS	CHOW	WATSON-DAVIES	R^2	DW
				PRODUCTION: REAL			
P-1	ln(YF1)	ln(NET); ln(K1); ln(YCUX); -0.056 0.413 0.998 (-0.619) (14.709) (24.705) TREND 0.025 (25.017)	1981	1981 (p=0.3259)	STABLE	0.999	1.854
P-2(1)	ln(YF1)	ln(NET); ln(K1); 1.356 0.116 (3.027) (0.823)	1978	1978 (p=0.0127) 1984 (p=0.7425)	STABLE	0.986	1.097
P-2(2)	ln(YF1)	ln(K1); ln(YCUX); 0.397 0.983 (35.782)(31.354) TREND 0.025 (30.567)	1982	1982 (p=0.6846) 1979 (p=0.6408)	STABLE	0.999	1.833
P-3	ln(YF1)	ln(NET); ln(K1); 1.305 0.052 (2.929) (0.344) TREND 0.004 (1.163)	1981	1981 (p=0.0503) 1978 (p=0.0462)	STABLE	0.987	1.178
P-4	ln(YF1)	ln(NET); ln(K1*YCUX); -0.098 0.506 (-0.255) (4.312) TREND 0.012 (4.463)	1981	1981 (p=0.0208)	UNSTABLE	0.994	1.279
P-5	ln(YF1)	ln(NET); ln(K1); 0.174 1.360 (0.177) (1.454) $(\ln(NET)-\ln(K1))^2$ 0.194 (1.345)	1982	1982 (p=0.0652) 1978 (p=0.0502)	STABLE	0.988	1.251

TABLE 13.4 (continued)

MODEL	DEPENDENT VARIABLE	EXPLANATORY VARIABLES	TIMING OF MINIMUM QUANDT-RATIOS	CHOW	WATSON-DAVIES	R^2	DW
P-6	ln(YF1)	ln(NET); ln(K1); 0.298 1.212 (0.180) (0.661) $(\ln(NET)-\ln(K1))^2$; TREND 0.172 0.001 (0.635) (0.095)	1981	1981 (p=0.0537)	STABLE	0.986	1.241
P-7	ln(YF1)	ln(NET); ln(K1); $(\ln(NET))^2$; -145.776 23.939 9.123 (-0.152) (0.080) (0.136) $(\ln(K1))^2$; ln(NET)*ln(K1) 0.027 -2.709 (0.004) (-0.064)	1981	1981 (p=0.1934)	STABLE	0.988	1.291
P-8	ln(YF1)	ln(NET); ln(K1); $(\ln(NET))^2$; -19.028 0.103 1.149 (-1.809) (0.789) (1.939)	1981	1981 (p=0.1625) 1978 (p=0.0939)	STABLE	0.988	1.255
P-9	ln(YF1)	ln(NET); ln(K1); NET ; -9.170 1.062 0.001 (-1.762) (1.774) (2.041) K1 ; -0.003 (-1.634)	1982	1982 (p=0.0950) 1975 (p=0.5103) 1981 (p=0.3123)	STABLE	0.989	1.344
P-10	ln(YF1)	ln(NET); ln(K1); NET ; -9.988 1.056 0.002 (-1.844) (1.735) (2.119) K1 ; TREND -0.003 -0.004 (-1.456) (-0.709)	1982	1982 (p=0.0153)	STABLE	0.989	1.403

Quandt minimum, however, as in the case of the investment model I-3(1), points at a suspicious breakpoint in the parameter estimates in 1978.

For model P-4, the only model where no negative square-root is detected during the calculation routine of the tests, the Quandt-ratio and associated Chow test imply structural instability in 1981. The Cusum test signifies structural instability over the period 1988 - 1989. The Cusum of Squares, which is supposed to be the more powerful of the BDE tests, indicates no structural changes. The Fluct test shows structural breakdowns during the period 1974 to 1987. The warning is repeated by the Watson-Davies test and in sign changes observed in the estimated regression parameters. All of these results are not reported in Table 13.4 for reasons mentioned earlier.

In Model P-10, a structural break is only indicated by the minimum Quandt-ratio and associated Chow test in 1982.

13.1.3 MONTHLY EXCHANGE RATE FUNCTIONS

The test results of the exchange rate models are reported in Table 13.5. The minimum Quandt-ratios and the local Quandt minima, with the accompanying Chow test, hint at structural breakpoints in both of the stages for the exchange rate models.

In model E-1(1) the indications of structural weaknesses are borne out by the Cusum test (over the period 1990M11 - 1991M12), the Cusum of Squares test (during the period 1971M3 - 1991M9), the Fluct test (during 1970M8 - 1972M8; 1973M5 - 1991M8) and by sign changes observed in the recursively estimated regression parameters. The Watson-Davies test also hints at structural weaknesses in the regression parameters. In this case all the evidence clearly points at structural instability.

In the improved Model E-1(2), the Quandt-ratio points at a structural breakpoint in 1984M5 which is supported by the Chow test. A local Quandt minimum in 1985M6 also shows further evidence of structural weaknesses. The Cusum test shows no evidence of structural changes while the Cusum of Squares (the more powerful of the two) hints at structural instability during the periods 1971M6 - 1986M5; 1986M9 - 1988M2 and 1989M10 - 1990M3. These indications of weaknesses are borne out by the Fluct test (during the periods 1972M1 - 1982M2) and 1982M11 - 1987M3), the Watson-Davies test, as well as the sign test, show

TABLE 13.5 TESTS OF THE CONSTANCY OF PARAMETERS FOR THE EXCHANGE RATE FUNCTIONS (1970M1-1991M12)

MODEL	DEPENDENT VARIABLE	EXPLANATORY VARIABLES	TIMING OF MINIMUM QUANDT-RATIOS	CHOW	CUSUM	CUSUMSQ	FLUCT	WATSON-DAVIES	R^2	DW	SIGN CHANGES
				EXCHANGE RATES: REAL							
E-1(1)	REX12	REXDM$; REXPPP 0.210 1.419 (7.387) (47.892)	1984M6	1984M6 (p=0.0000) 1980M5 (p=0.0000) 1985M6 (p=0.0000)	1990M11-1991M12	1971M3-1991M9	1970M8-1972M8 1973M5-1991M8	UNSTABLE	0.923	0.092	YES
E-1(2)	REX12	REXDM$; REXPPP; REX12(-1); 0.061 0.101 0.885 (3.784) (2.369)(26.593) TREND; AUTO(-1) 0.009 0.338 (3.389)(4.731)	1984M5	1984M5 (p=0.0000) 1985M6 (p=0.0000) 1990M1 (p=0.0000)	STABLE	1971M6-1986M5 1986M9-1988M2 1989M10-1990M3	1972M1-1982M2 1982M11-1987M3	UNSTABLE	0.994	2.024	YES

evidence of structural changes in the regression parameters. Although a well estimated exchange rate model, there is overwhelming evidence that structural changes in the parameter estimates do occur in this function.

13.1.4 SUMMARY OF TEST RESULTS

Tables 13.6 to 13.8 summarise the results of Tables 13.1 to 13.5.

Table 13.6 shows that out of 10 quarterly investment functions tested, 3 (30%) show signs of structural change according to the Cusum test, while the Cusum of Squares test, the more powerful of the two tests, points at structural instability in

TABLE 13.6: TEST SUMMARY FOR STRUCTURAL STABILITY IN INVESTMENT EQUATIONS

DEPENDENT VARIABLE	NUMBER OF FUNCTIONS INVESTIGATED	NUMBER OF CHOW TESTS	NUMBER OF TESTS WHERE STRUCTURAL STABILITY WAS REJECTED AT A 5% LEVEL OF SIGNIFICANCE				
			CHOW	CUSUM	CUSUMSQ	FLUCT	WATSON-DAVIES
IPO1(1)	4	18	13	1	1	4	2
IPO1(2)	4	16	2	1	0	4	1
IPZA1(1)	1	3	3	1	1	1	1
IPZA1(2)	1	3	0	0	0	1	0
TOTAL:	10	40	18	3	2	10	4

only 2 (20%) of the equations. The second stage equations appear to be more stable when using either the Cusum or the Cusum of Squares tests. Out of the 40 Chow tests performed, 18 (45%) indicated instability in the regression parameters. Many local Quandt minima are observed in the case of private fixed investment (IPO1), with about 13 out of 18 (72%) and 2 out of 16 (12,5%) confirmed by the Chow test in the case of Stage 1 and Stage 2 models, respectively. The Fluct test hinted at structural weaknesses in all ten cases while approximately 40% of the Watson-Davies tests applied indicated the possibility of parameter instability.

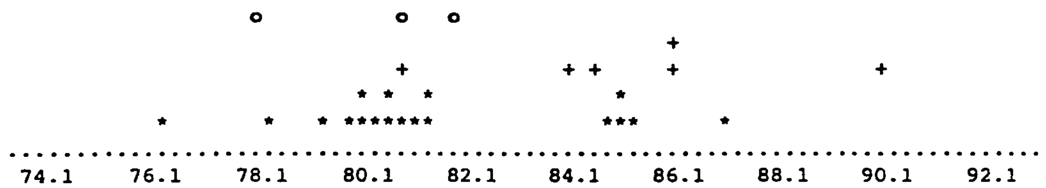
For yearly models, the results are slightly more positive insofar as the Chow tests and the Watson-Davies tests are concerned. In Table 13.7 a total of 4 (23,5%) of the 17 Chow tests executed hinted at structural breakpoints while only 1 (25%) out of 4 Watson-Davies tests performed indicated time-varying parameters. Comparatively speaking, in the case of yearly models, production functions has shown up to be much more stable in its parameters over the estimated time span than the quarterly investment functions. Because of the limited data available for yearly models, none of the other structural stability tests could be performed which make any comparison a bit arbitrary.

TABLE 13.7: TEST SUMMARY FOR STRUCTURAL STABILITY IN PRODUCTION EQUATIONS

DEPENDENT VARIABLE	NUMBER OF FUNCTIONS INVESTIGATED	NUMBER OF CHOW TESTS	NUMBER OF TESTS WHERE STRUCTURAL STABILITY WAS REJECTED AT A 5% LEVEL OF SIGNIFICANCE	
			CHOW	WATSON-DAVIES
P-1	1	1	0	0
P-2(1)	1	2	1	0
P-2(2)	1	2	0	0
P-3	1	2	1	0
P-4	1	1	1	1
P-5	1	2	0	0
P-6	1	1	0	0
P-7	1	1	0	0
P-8	1	2	0	0
P-9	1	3	0	0
P-10	1	1	1	0
TOTAL:		17	4	1

Due to a limited number of exchange rate functions and data for variables which are not easily obtainable, no definite conclusion on structural stability could have been reached in this regard. From Figure 13.1 it can be seen that the minimum Quandt-ratios of the monthly exchange rate functions point at structural weaknesses much later than in the case of the

FIGURE 13.1: THE TIMING OF THE MINIMUM QUANDT RATIOS IMPLYING STRUCTURAL CHANGE FOR ALL THE MONTHLY, QUARTERLY AND YEARLY FUNCTIONS



Key to symbols

YEARLY FUNCTIONS oooo
MONTHLY FUNCTIONS ++++
QUARTERLY FUNCTIONS ****

quarterly investment models. In Table 13.8 the Cusum of Squares, Fluct and the Watson-Davies test indicates structural changes in both the first two stages of the exchange rate functions. Out of the 6 Chow tests performed, 5 (83%) pointed at structural instability. The Cusum test show a possibility of a structural change only in the case of the Stage 1 model.

TABLE 13.8: TEST SUMMARY FOR STRUCTURAL STABILITY IN EXCHANGE RATE EQUATIONS

DEPENDENT VARIABLE	NUMBER OF FUNCTIONS INVESTIGATED	NUMBER OF CHOW TESTS	NUMBER OF TESTS WHERE STRUCTURAL STABILITY WAS REJECTED AT A 5% LEVEL OF SIGNIFICANCE				
			CHOW	CUSUM	CUSUMSQ	FLUCT	WATSON-DAVIES
REX12(1)	1	3	3	1	1	1	1
REX12(2)	1	3	2	0	1	1	1
TOTAL:	2	6	5	1	2	2	2

The timing of the minimum Quandt-ratios and Chow tests implying structural change are of particular interest. From Figure 13.1 it is clear that, although a number of structural breakpoints are hinted at during the late 1970's, it has been a period of relative stability if compared with the 1980's. The period since 1976 seemed to produce a somewhat larger number of shifts which may be consistent with the often suggested "oil shock" effect, or perhaps to the course of the economy over this period, but the findings do not support more than conjecture on these points. The possibility of structural weaknesses in econometric equations has increased substantially, particularly during the first half of the 1980's, reflecting periods of structural changes in the economic and political environment in South Africa.

Figure 13.2 summarises the factors that may have shifted the various curves in the econometric models discussed earlier and include the following:

A: Depreciation of the Rand (A1-A2; A3-A4).

B: General political instability (B2-B6); Soweto riots (B1); government intervention (B3); state of emergency (B4); and an international debt crisis (B5).

C: First (C1) and second (C2) energy crisis

D: Periods of rising unemployment (D1-D2; D3-D4; D5-D6).

E: Introduction of a general sales tax (E1) and the subsequent increases thereof (E2; E3; E4; E5; E6).

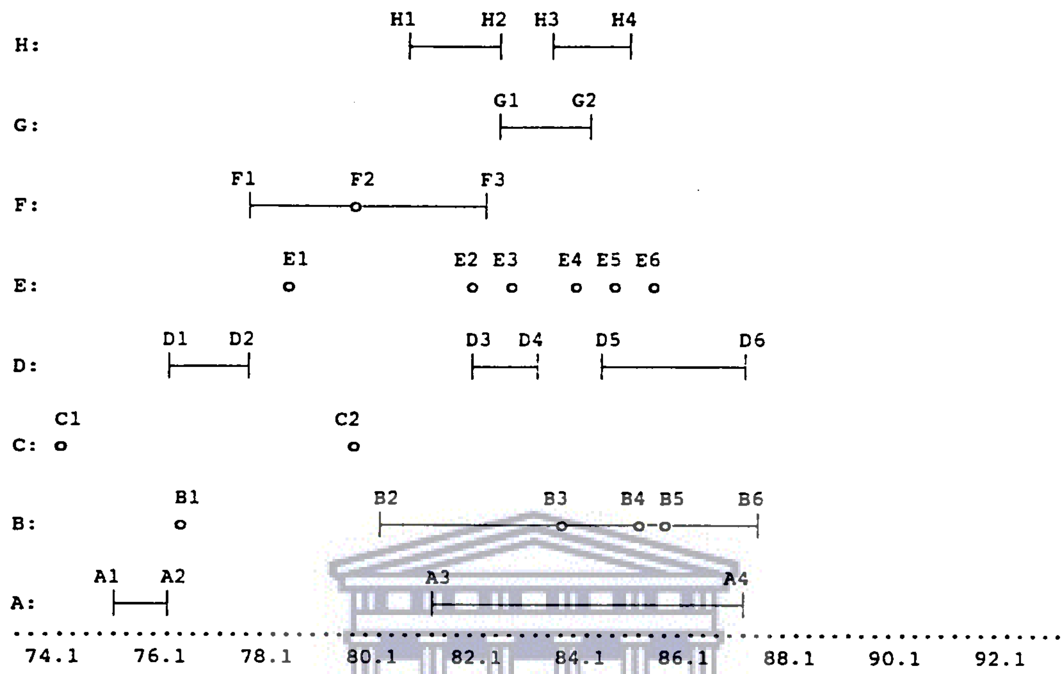
F: Increases in the gold price (F1-F2); decrease in the gold price (F2-F3).

G: Severe drought conditions (G1-G2).

H: Drastic increases in the interest rates (H1-H2; H3-H4).

It can be noticed that most of the structural changes occurred during the late 1970's and throughout the 1980's and it generally corresponds with most of the periods pointed out by the various stability tests as periods of structural change. It should, however, be kept in mind that these factors should not be regarded as a complete summary of events causing econometric equations to become structurally unstable in South Africa.

FIGURE 13.2: POSSIBLE PERIODS OF STRUCTURAL INSTABILITY IN THE SOUTH AFRICAN ECONOMY OVER THE PERIOD 1970 - 1990



13.2 MODEL COMPARISON AND EVALUATION

13.2.1 INTRODUCTION

A number of variables, defined in Appendix A, are entered into fixed and random coefficient models in a series of exploratory stages, designed to identify the statistically significant determinants of investment, production and exchange rates. The models are selected on the basis of studies reported in the literature and the authors intuition. The series of stages could be defined as follows:

Stage 1: The estimation of a fixed coefficient model with ordinary least squares (OLS) techniques and in which a model-builder at some stage had some trust.

Stage 2: Improving the Stage 1 specification, using OLS techniques, so that it can be found acceptable on both economic theoretical and statistical grounds.

Stage 3: The re-estimation of the Stage 2 model over the same time period using autoregressive conditional heteroscedasticity (ARCH) techniques if possible.

Stage 4: A varying parameter process is defined for only one coefficient at a time (due to limited computer memory). The Watson-Davies test indicates the most significant time-varying parameter during the second stage of estimation. The Stage 2 function is then re-estimated using an appropriate VPR technique. Experimentation with changing parameters is performed even if parameters do not show up to be changing smoothly over time.

For some of the equations the Cochrane-Orcutt correction for serial correlation is used when necessary. The results for Stages 1 through 4 are summarised in Tables 13.9(a) to 13.9(e) for investment models, Table 13.10(a and b) for production models and Table 13.11 for exchange rate models. These tables contain the name of the dependent variable, the names of the independent variables and a statistical report at the bottom of the table. Also reported are the coefficient estimates with their respective t-values in parenthesis for each of the stages. The time periods referred to in the tables are the period of fit which represent a subset of the historical data. The latest part of the sample data are withheld to allow for ex-post forecast evaluations. A total of 19 stages for quarterly, 3 stages for yearly and 3 stages for monthly functions are investigated using data at constant 1985 prices.

13.2.2 FIXED INVESTMENT MODELS

This analysis poses 5 alternative models for fixed investment. All the coefficients of the Stage 1 and 2 models are tested for structural stability. If some of the parameters of the Stage 2 equations show signs of structural instability it gets re-estimated by using an appropriate VPR technique. The results for the fixed-coefficient-OLS, ARCH and VPR models for the various investment functions are reported in Table 13.9.

The specification chosen for Model I-1(1) is based on the acceleration principle. Thus a distributed lag on changes in output is the principal explanatory variable of the net investment IPO1 equation. The lagged related capital stock

TABLE 13.9(a): REGRESSION RESULTS FOR FIXED AND VARIABLE PARAMETER INVESTMENT MODEL I-1

VARIABLES	COEFFICIENT ESTIMATES				WATSON-DAVIES* VPR TEST
	STAGE 1	STAGE 2	STAGE 3	STAGE 4	
IPO1	Endogenous	Endogenous	Endogenous	Endogenous	
CONSTANT	-5.008 (-0.720)				
KPO1(-1)	0.019 (2.391)	0.013 (2.491)	0.013 (2.482)	0.014 (2.635)	0.700 (0.496)
IPO1(-1)	0.800 (9.818)	0.470 (4.555)	0.505 (4.483)	0.469 (4.638)	0.650 (0.620)
YCUB+YCUB(-1)+YCUB(-2)	0.021 (0.762)				
YFZPA1-YFZPA1(-4)	0.105 (2.112)	0.109 (4.846)	0.109 (4.839)	0.108 (4.956)	0.550 (0.632)
YFZPA1(-1)-YFZPA1(-5)	-0.030 (-0.562)				0.800 (0.414)
YFZPA1(-2)-YFZPA1(-6)	-0.031 (-0.579)				
YFZPA1(-3)-YFZPA1(-7)	-0.001 (-0.010)				
YFZPA1(-4)-YFZPA1(-8)	0.078 (1.554)	0.080 (3.164)	0.079 (2.971)	0.081 (3.260)	
IPO1(-2)		0.413 (4.189)	0.376 (3.522)	AR(1)	0.700 (0.669)
ARCH Intercept			0.288 (2.899)		
ARCH Coefficient			0.198 (0.766)		
ALPHA(-1)				-0.0002 (-0.002)	
AR Constant				0.409 (4.002)	
VAR Error				0.304 (1.983)	
VAR IPO1(-2)				0.0003 (0.271)	

STATISTICAL REPORT:

	STAGE 1	STAGE 2	STAGE 3	STAGE 4
$\bar{R}^2$	0.899	0.921	0.919	0.918
R^2	0.910	0.925	0.925	0.925
DW	2.631	2.045	2.025	2.050
SEE	0.684	0.604	0.613	0.617

Estimation Period: 1972Q1 - 1990Q4

* Watson-Davies VPR test performed during second stage of estimation.

TABLE 13.9(b): REGRESSION RESULTS FOR FIXED AND VARIABLE PARAMETER INVESTMENT MODEL I-2

VARIABLES	COEFFICIENT ESTIMATES STAGE 1	STAGE 2	STAGE 3	STAGE 4	WATSON-DAVIES* VPR TEST
IPO1	Endogenous	Endogenous	Endogenous	Endogenous	
CONSTANT	-7.079 (-5.523)				
PVI(-1)	0.057 (2.443)	0.021 (2.960)	0.021 (2.913)	0.019 (1.333)	0.750 (0.990)
YGDE1(-1)	0.086 (2.264)			AR(1)	0.750 (0.998)
YGDE1(-2)	-0.010 (-0.212)				
YGDE1(-3)	0.009 (0.191)				
YGDE1(-4)	0.040 (0.897)				
YGDE1(-5)	-0.025 (-0.586)				
YGDE1(-6)	0.032 (0.722)				
YGDE1(-7)	0.023 (0.524)				
YGDE1(-8)	-0.027 (-0.822)				
DEFIT*	-0.078 (-0.755)				
DEFIT(-1)	-0.152 (-1.335)				
DEFIT(-2)	0.050 (0.486)				
DEFIT(-3)	-0.203 (-1.625)				
DEFIT(-4)	0.047 (0.375)				
DEFIT(-5)	0.018 (0.172)				
IPO1(-1)		0.828 (13.800)	0.827 (13.443)	0.772 (9.728)	
AUTO(-1)		-0.290 (-2.443)	-0.294 (-2.395)	-0.478 (-3.162)	
ARCH Intercept			0.446 (3.380)		
ARCH Coefficient			0.018 (0.079)		
ALPHA(-1)				0.535 (2.967)	
AR Constant				0.004 (0.626)	
VAR Error				0.217 (1.383)	
VAR YGDE1(-1)				6.652E-006 (1.220)	

STATISTICAL REPORT:

	STAGE 1	STAGE 2	STAGE 3	STAGE 4
$\bar{R}^2$	0.816	0.900	0.897	0.900
R^2	0.852	0.902	0.902	0.908
DW	0.771	1.908	1.897	2.091
SEE	0.922	0.674	0.683	0.673

Estimation Period: 1972Q1 - 1990Q4

* Watson-Davies VPR test performed during second stage of estimation.

* DEFIT = PI - PI(-1).

TABLE 13.9(c): REGRESSION RESULTS FOR FIXED AND VARIABLE PARAMETER INVESTMENT MODEL I-3

VARIABLES	COEFFICIENT ESTIMATES STAGE 1	STAGE 2	STAGE 3	STAGE 4	WATSON-DAVIES* VPR TEST
IPO1	Endogenous	Endogenous	Endogenous	Endogenous	
CONSTANT	-1.511 (-2.568)	-1.014 (-2.697)	-1.004 (-2.470)	-1.049 (-2.781)	1.000 (0.238)
IPO1(-1)	0.741 (9.406)	0.933 (17.321)	0.941 (16.876)	0.915 (15.271)	1.000 (0.238)
FRSB4(-2)	-0.022 (-0.964)	-0.055 (-3.836)	-0.056 (-3.839)	-0.052 (-3.469)	0.750 (0.406)
YGDE1	0.043 (4.201)	0.023 (3.237)	0.022 (2.980)	AR(1)	1.000 (0.238)
AUTO(-1)		-0.500 (-4.907)	-0.495 (-4.551)	-0.473 (-2.515)	
ARCH Intercept			0.291 (3.390)		
ARCH Coefficient			0.089 (0.399)		
ALPHA(-1)				-0.034 (-0.008)	
AR Constant				0.025 (0.246)	
VAR Error				0.301 (1.459)	
VAR YGDE1				6.599E-007 (0.083)	
STATISTICAL REPORT:					
$\bar{R}^2$	0.922	0.935	0.933	0.932	
R^2	0.924	0.938	0.938	0.938	
DW	2.581	2.154	2.149	2.141	
SEE	0.631	0.570	0.578	0.584	
Estimation Period: 1970Q3 - 1990Q4					

* Watson-Davies VPR test performed during second stage of estimation.

TABLE 13.9(d): REGRESSION RESULTS FOR FIXED AND VARIABLE PARAMETER INVESTMENT MODEL I-4

VARIABLES	COEFFICIENT ESTIMATES STAGE 1	STAGE 2	STAGE 3	STAGE 4	WATSON-DAVIES* VPR TEST
IPO1	Endogenous	Endogenous	Endogenous	Endogenous	
CONSTANT	3.061 (-5.523)	-288.401 (-3.233)	-265.148 (-2.919)	-214.905 (-2.446)	
IPO1(-1)-IDT1(-1)	0.295 (3.062)				
IPO1(-2)-IDT1(-2)	0.451 (4.591)	0.202 (2.962)	0.189 (2.739)	0.151 (2.247)	
Z*(-1)	-0.278 (-0.751)			AR(1)	0.650 (0.755)
Z(-2)	-1.018 (-2.782)				0.800 (0.530)
Z(-3)	0.036 (0.090)				0.800 (0.531)
Z(-4)	-0.059 (-0.149)				
Z(-5)	-0.012 (-0.033)				
Z(-6)	0.423 (1.103)				
Z(-7)	0.116 (0.296)				
Z(-8)	-0.162 (-0.426)				
KPO1(-1)	0.148 (11.903)				
IPO1(-1)		0.737 (10.201)	0.746 (9.932)	0.791 (10.992)	
TREND		0.148 (3.233)	0.136 (2.930)	0.110 (2.455)	
ARCH Intercept			0.438 (3.341)		
ARCH Coefficient			0.085 (0.371)		
ALPHA(-1)				0.686 (0.755)	
AR Constant				0.014 (0.287)	
VAR Error				0.467 (5.735)	
VAR Z(-1)				0.017 (0.253)	
STATISTICAL REPORT:					
$\bar{R}^2$	0.724	0.891	0.887	0.883	
R^2	0.765	0.895	0.895	0.894	
DW	0.846	2.209	2.185	2.318	
SEE	1.110	0.698	0.709	0.723	
Estimation Period:	1972Q3 - 1990Q4				

* Watson-Davies VPR test performed during second stage of estimation.

* Z = YFZPA1/FRLE4 - YFZPA1(-1)/FRLE4(-1).

TABLE 13.9(e): REGRESSION RESULTS FOR FIXED AND VARIABLE PARAMETER INVESTMENT MODEL I-5

VARIABLES	COEFFICIENT ESTIMATES STAGE 1	STAGE 2	STAGE 3	STAGE 4	WATSON-DAVIES* VPR TEST
IPZA1	Endogenous	Endogenous	Endogenous	Endogenous	
CONSTANT	-4.141 (-3.733)				
YGDE1(-1)	0.112 (2.264)	0.016 (2.970)		0.020 (3.269)	0.650 0.507)
YGDE1(-2)	0.024 (0.446)				0.550 (0.628)
YGDE1(-3)	0.053 (1.380)				0.550 (0.657)
FRLE4	-0.182 (-2.215)			AR(1)	0.800 (1.000)
IPZA1(-1)		1.074 (17.068)		1.045 (15.318)	
IPZA1(-4)		-0.197 (-4.140)		-0.152 (-2.598)	
AUTO(-1)		-0.447 (-4.135		-0.456 (-3.852)	
ALPHA(-1)				0.273 (1.596)	
AR Constant				-0.035 (-1.407)	
VAR Error				0.416 (2.792)	
VAR FRLE4				1.901E-007 (0.001)	
STATISTICAL REPORT:					
$\bar{R}^2$	0.782	0.930		0.928	
R^2	0.792	0.933		0.934	
DW	0.481	1.999		2.002	
SEE	1.167	0.657		0.668	
Estimation Period:	1971Q1 - 1990Q4				

* Watson-Davies VPR test performed during second stage of estimation.

variable (KPO1) explains the replacement investment component of IPO1. A measure of the extent to which productive capacity in the non-agricultural sectors of the economy is utilised (YCUB) also enters the equation.

The estimation results for Stage 1 in Table 13.9(a) show that most of the estimated parameters are insignificant, with some of the lagged YFZPA1 variables containing an a priori incorrect sign. This is already an indication that the model in which a model-builder at some stage had faith in could not be used for the current sample, already implying changes in the stability of the parameters. The DW statistic also points at serious problems with serial correlation in the errors.

The asymptotic t-statistics associated with the Stage 2 parameters are satisfactory. Considerable differences exist between the estimated structures of the two equations. For example, results from the Stage 2 model suggest that IPO1(-1) has a major effect on investment while this lag structure is excluded from the Stage 1 model. The very high correlation coefficient partly results from the presence of the lagged dependent variable as one of the explanatory variables. The Durbin-Watson (DW) statistic reflects no serial correlation in the error terms.

A straight line option of weights (weights decrease linearly) are chosen in building an ARCH model in Stage 3. The results of the ARCH model yield almost similar parameter estimates if compared with the fixed coefficients of Stage 2. In examining the significance of the ARCH coefficient (α), however, the indication is that there are no ARCH effects. This means that the ARCH model does not significantly improve upon conventional regression techniques in this case. The ARCH intercept (β) is highly significant, but this is of little interest.

The Stage 4 model represents an autoregressive stochastic parameter model estimated with maximum likelihood techniques suggested by Watson and Engle (1985). For this model it is estimated that the IPO1(-2) variable varies around a mean of 0,409 according to a first-order autoregressive (AR(1)) model with coefficient -0,0002. This finding can be interpreted as implying that, standing at time T, the optimal forecast of the future parameter β_{T+j} is

$$0,409 + (-0,0002)^{j} (\beta_T - 0,409) \qquad (13.1)$$

The error variance is the value fitted by the maximum likelihood procedure to the error variance of the regression equation. This value is close to that obtained by direct calculation. Variance IPO1(-2) is the variance in the autoregressive process that is used to model variation in IPO1(-2). The fact that this parameter is not statistically significant indicates that there is no statistical gain in using VPR over the historical data. The statistics at the bottom of Table

13.9(a) corroborate this fact, which is to be expected because of the non-significance of the Watson-Davies test which yielded a phi value of 0,700 and a p-value of 0,669. Nevertheless, it is likely, as is the case of Models I-2 to I-5, that forecasts from the VPR model will outperform those from conventional dynamic regression.

The residual errors of regression for Stages 1,2 and 4, depicted in Figure 13.3(a), are large in bursts which coincide with the periods of unstable political and economic events in South Africa. These patterns, while not terribly pronounced, are exactly the problem for which the ARCH model is formulated. The graph corroborates the results of the specific linear weight ARCH test which yielded a highly significant chi-square(1) of 16,62 and a p-value of 1,000. The ARCH standard deviations in Figure 13.3(b) support the pattern of volatility during the above time periods. The abrupt variation of the IPO1(-2) parameter in Figure 13.3(c) displays the behaviours that one should be concerned about. Clearly, the VPR is explaining stray variance rather than modelling the process of investment behaviour. In this case one should reject the VPR model even though, statistically, it seems to succeed. This is a definitive sign that the VPR process is manifesting surrogate behaviour. In this case, one would be most unlikely to obtain superior forecasts from this model.

FIGURE 13.3(a): RESIDUAL ERRORS OF REGRESSION FOR MODEL I-1, STAGES 1,2 AND 4

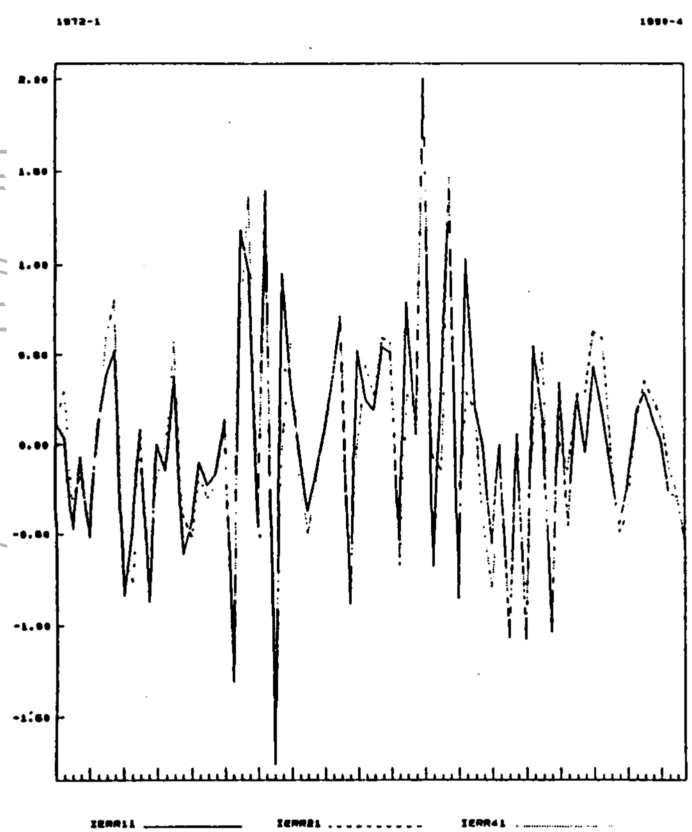


FIGURE 13.3(b): THE ARCH STANDARD DEVIATIONS FOR MODEL I-1

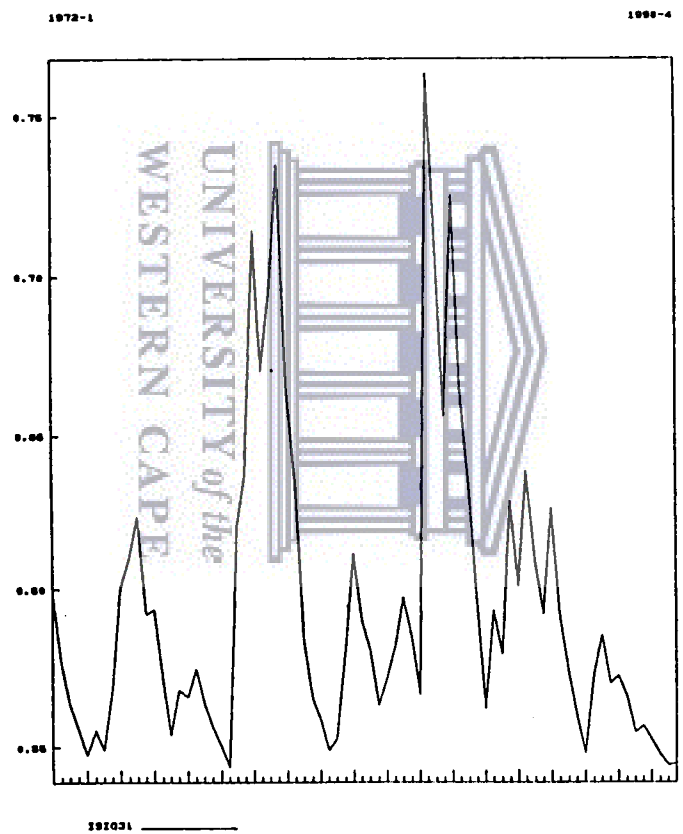
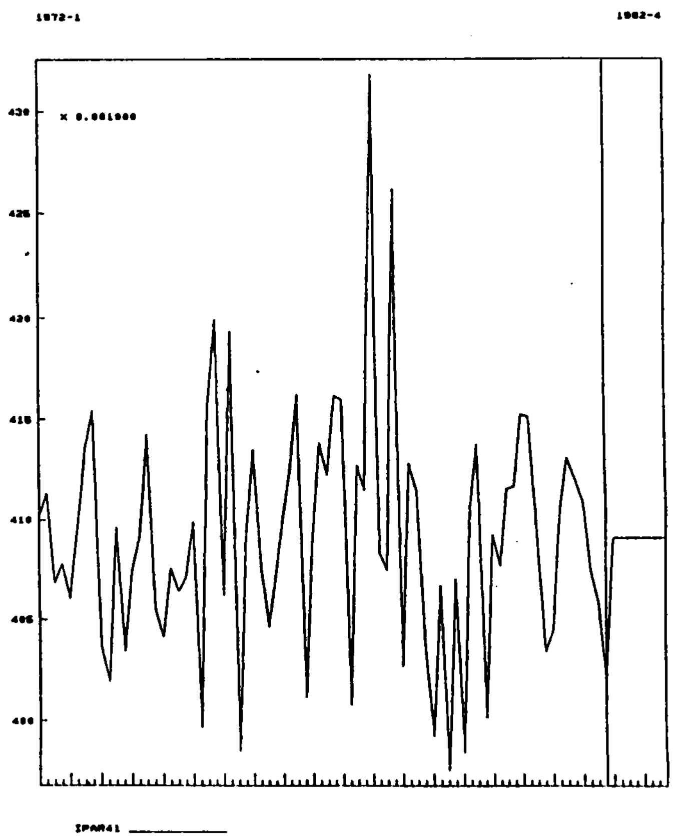


FIGURE 13.3(c): THE IPO1(-2) TIME-VARYING PARAMETER FOR MODEL I-1



The regression results of Models I-2 to I-4 reported in Tables 13.9(b) - 13.9(d) are analogous to that of Model I-1. Some of the exceptions are that these models are estimated with a different set of explanatory variables and, as in the case of I-2(2) and I-3(2), the equations are estimated by using the Cochrane-Orcutt estimation procedure. Furthermore, investment behaviour in the case of Model I-2 to I-4 is also explained by some financial variables.

By examining the graphs of the regression residuals of each model in Figures (13.4 - 13.6)(a) most of the volatility seems to appear during the late 1970's and the mid 1980's. Again, these time periods can be regarded as the time periods of profound structural changes in the economic and political spheres of South Africa. The large bursts in the graphs during the late 1970's also coincide with the oil embargo and a later unsettled period. These conclusions made are also borne out by the fact that the ARCH standard deviations, as well as the time varying parameters, follow similar patterns of structural instability over the above period. Similiar results for the ARCH standard deviations and time-varying parameters are depicted in Figures (13.4 to 13.6)(b and c, respectively). The forecasting model, however, will use the last available value of the time varying parameter over the estimation period which will, despite the poorly specified VPR models, certainly yield better forecasts

FIGURE 13.4(a): RESIDUAL ERRORS OF REGRESSION FOR MODEL I-2, STAGES 1,2 AND 4

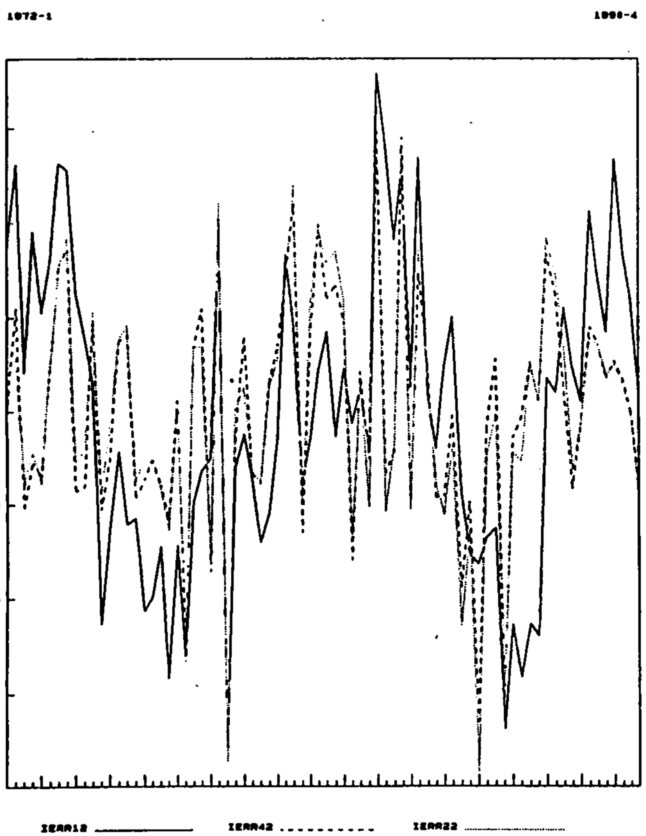


FIGURE 13.4(b): THE ARCH STANDARD DEVIATIONS FOR MODEL I-2

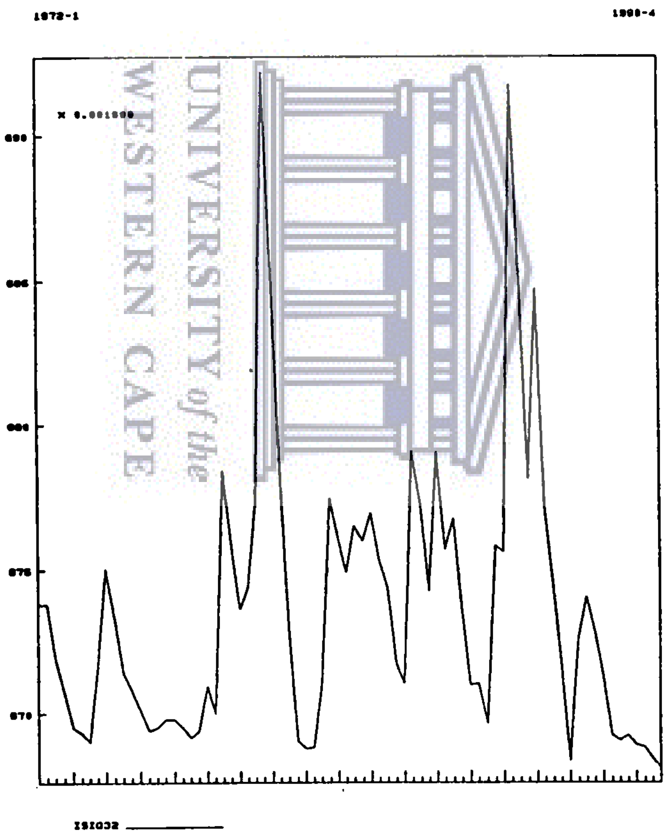


FIGURE 13.4(c): THE YODE1(-1) TIME-VARYING PARAMETER FOR MODEL I-2

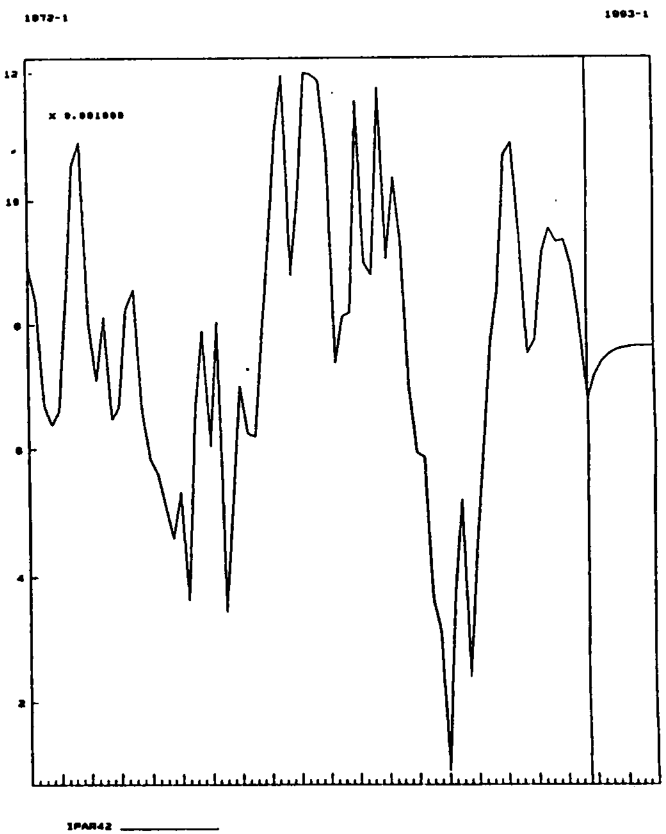


FIGURE 13.5(a): RESIDUAL ERRORS OF REGRESSION FOR MODEL I-3, STAGES 1,2 AND 4

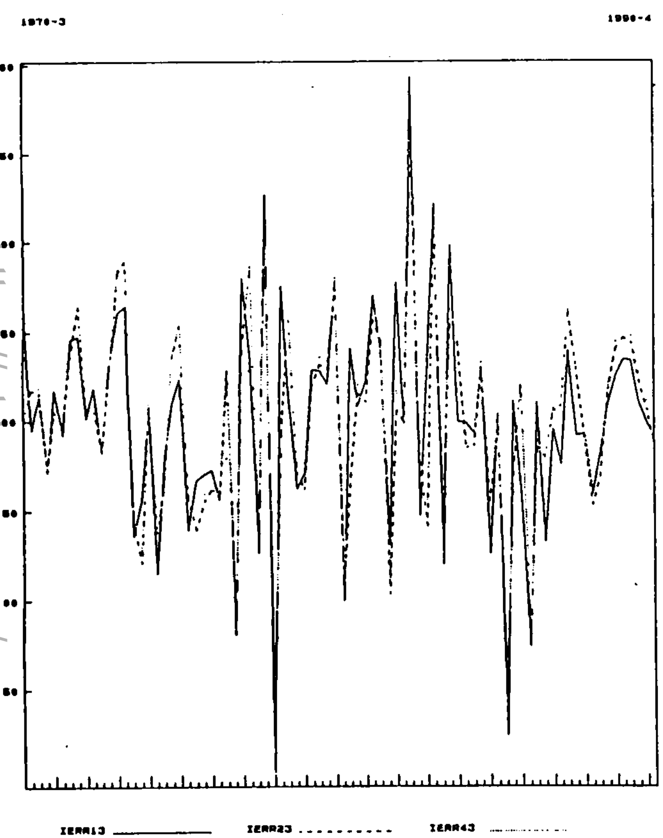


FIGURE 13.5(b): THE ARCH STANDARD DEVIATIONS FOR MODEL I-3

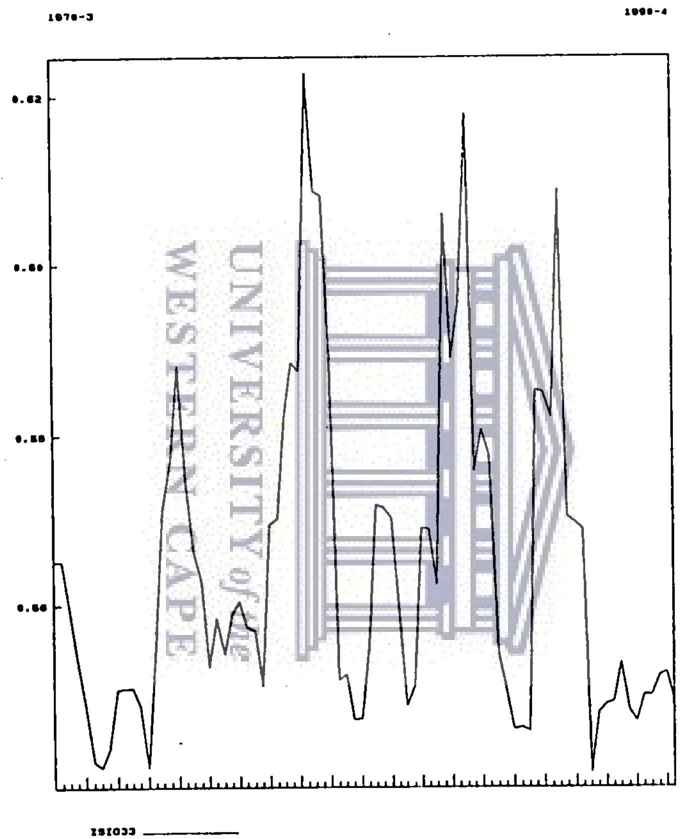


FIGURE 13.5(c): THE YODE1 TIME-VARYING PARAMETER FOR MODEL I-3

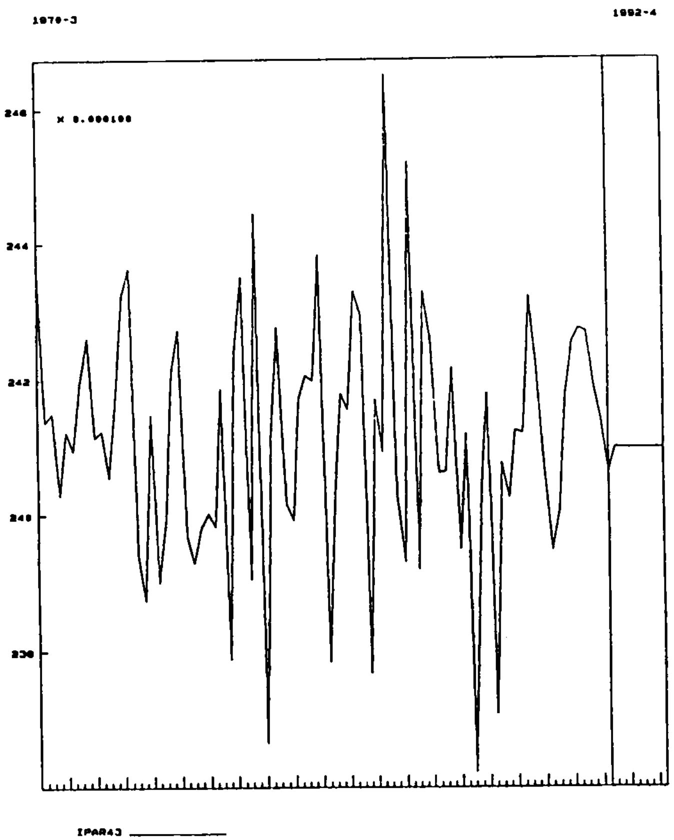


FIGURE 13.6(a): RESIDUAL ERRORS OF REGRESSION FOR MODEL I-4, STAGES 1,2 AND 4

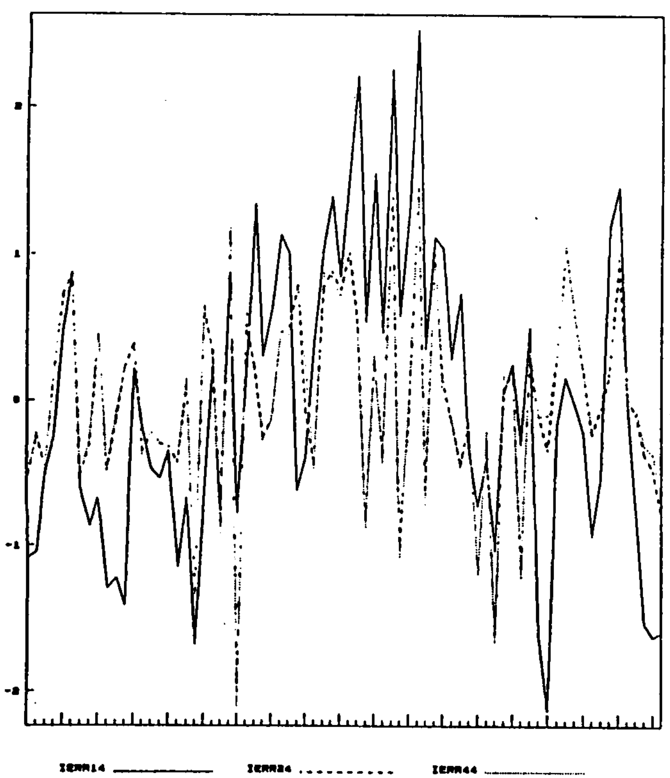


FIGURE 13.6(b): THE ARCH STANDARD DEVIATIONS FOR MODEL I-4

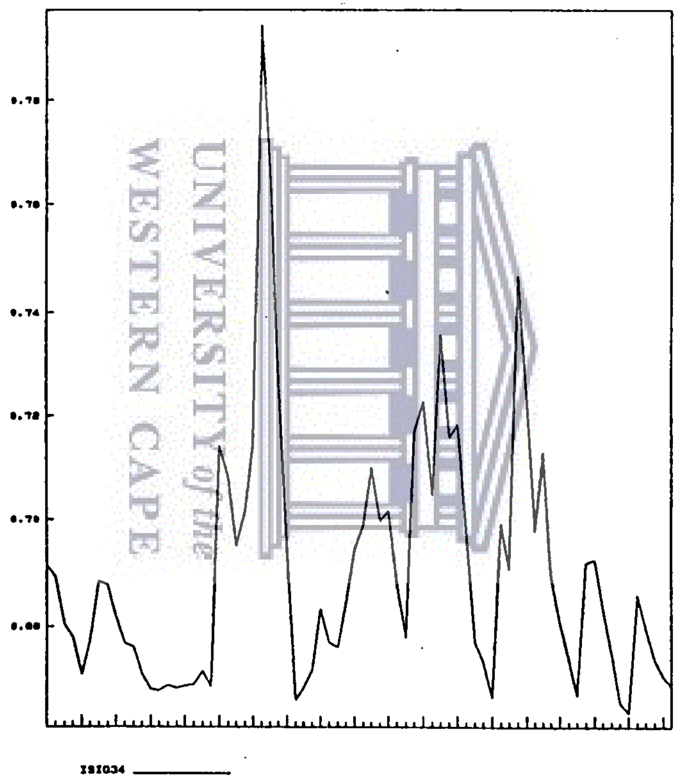
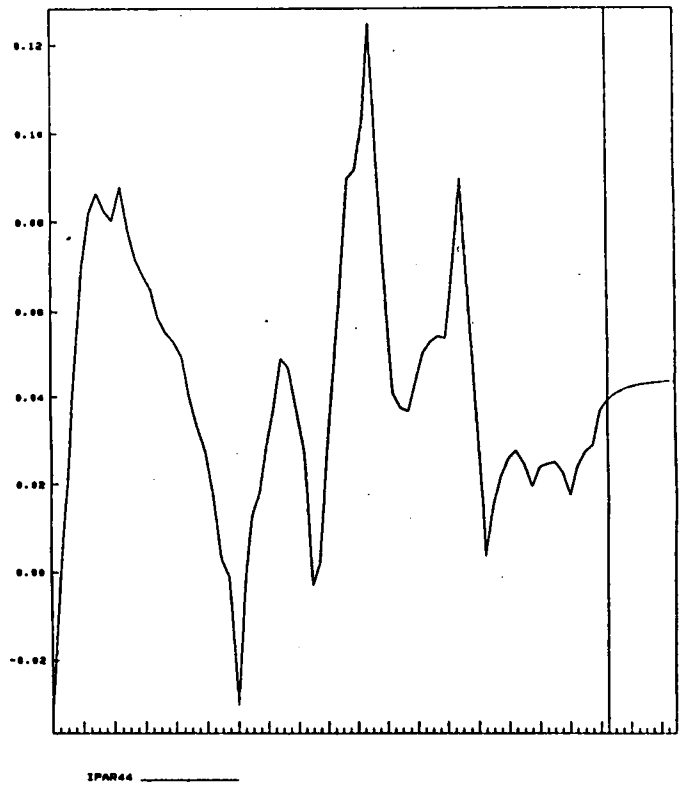


FIGURE 13.6(c): THE Z(-1) TIME-VARYING PARAMETER FOR MODEL I-4



than a model that uses a fixed (averaged) value over the historical data set.

A final model analysed for investment is one where real private non-agricultural fixed investment (IPZA1) appear as the dependent variable. The results of this model is summarised in Table 13.9(e). According to Model I-5(1), IPZA1 is determined by past values of gross domestic expenditure, gross domestic expenditure and the long-term rate of interest. The latter represents cost of funds considerations, while the use of domestic expenditure indicates that firms look at their sales for an indication of demand. The model can be regarded as a version of the neoclassical approach to investment model building. This analysis joins a long history of disquieting estimates of the relationship between the interest rate or rental and investment.

All exceed the value of 2 except the t-statistic associated with the YGDE1(-1) and YGDE1(-2) coefficients in Stage 1. Once again, considerable differences exist between the estimated structures of the Stage 1 and Stage 2 versions. The Stage 2 model is estimated by using the Cochrane-Orcutt correction for serial correlation. The results from the Stage 2 model suggest the inclusion of a lagged dependent variable and the exclusion of the interest rate variable. The exclusion of the interest rate variable is no surprise since Eisner and Strotz (1963:

192), in their detailed review of investment studies, indicated:

> "The interest rate has occasionally been found to be negatively related to capital expenditures, but such findings are not general. Coefficients are frequently uncertain, or, more important, so small in relation to the variation of the interest rate which have been allowed to occur so as to deny that variable much historical role in influencing the rate of investment."

The DW is biased towards accepting the hypothesis of no serial correlation in the residuals because of the lagged dependent variable which form part of the explanatory variables

The ARCH tests have shown no significant ARCH effects and, therefore, it is decided to exclude the ARCH model. The Watson-Davies test indicated a possible shift of the FRLE4 parameter (phi = 0,800 and p = 1,000). An AR(1) stochastic parameter model is estimated allowing the FRLE4 parameter to change according an AR(1) process. Although all other estimates seem to be significant, the variance FRLE4 parameter indicates that there is no significant statistical gain in using VPR over the historical data. One might also strengthen this view by noticing that the estimates of alpha (-1) are not significantly different from zero at a 5% level of significance.

The pattern of the residuals in Figure 13.7(a) is very similar to those of the previous investment models while the time-path

FIGURE 13.7(a): RESIDUAL ERRORS OF REGRESSION FOR MODEL I-5, STAGES 1,2 AND 4

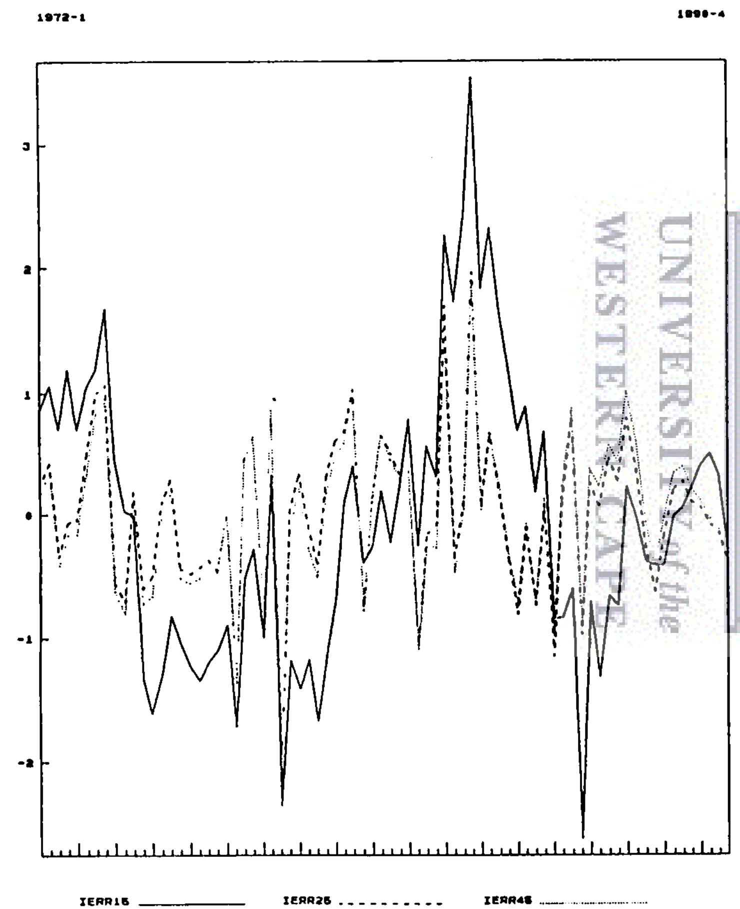
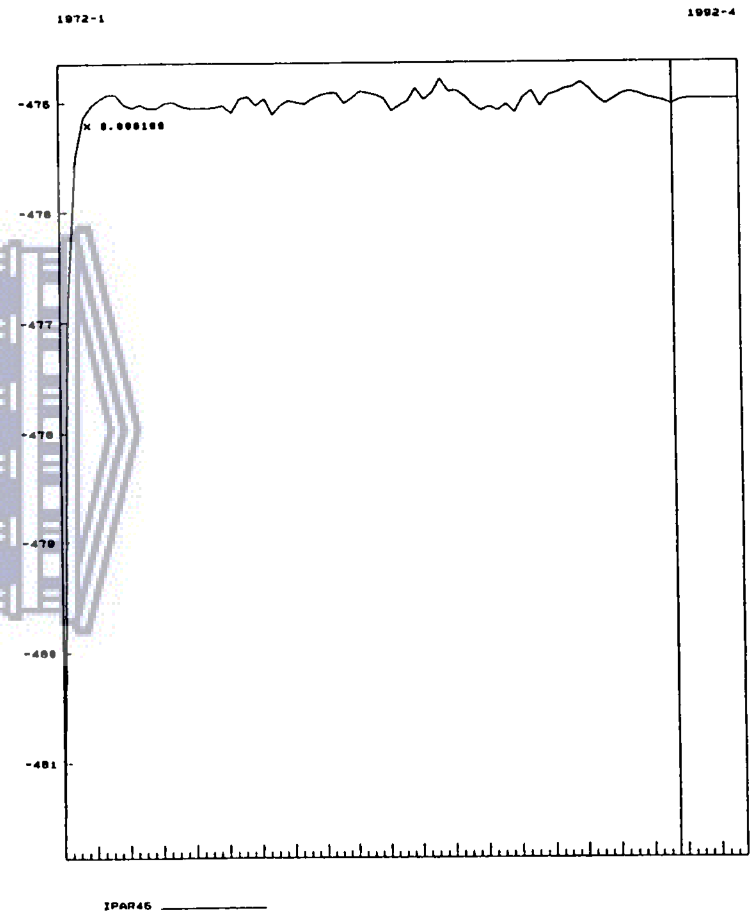


FIGURE 13.7(b): THE FRLE4 TIME-VARYING PARAMETER FOR MODEL I-5



of the time-varying parameter of FRLE4 in Figure 13.7(b) appears to be far more stable than the time-varying parameters of the IPO1 models.

13.2.3 PRODUCTION MODELS

Experimentation is done with various theoretical models, including the Cobb-Douglas (CDF), the constant elasticity of substitution (CES) and the transcendental production (TPF) function. The analysis from Table 13.7 shows that some of the parameters of these models, although not as bad as the quarterly investment functions, are structurally unstable.

For comparison purposes, the least squares parameter of various fixed coefficient theoretical models are estimated and the results given in Table 13.10(a). The dependent variable (YF1) and explanatory variables K1, NET and YCUX are measured in natural logarithmic terms if preceded by the symbol 'ln'. The table contains the name of the dependent variable, the explanatory variables, the estimates of the means of the coefficients and their asymptotic t-ratios in brackets, the sum of the coefficients of the natural logarithm of labour and capital and a statistical report on the correlation coefficients and Durbin-Watson statistic.

All estimates of the means of labour and capital coefficients are not positive, which is presumably the correct sign. The

TABLE 13.10(a): REGRESSION RESULTS FOR FIXED PARAMETER PRODUCTION EQUATIONS

Dependent variable: ln(YF1)

MODEL	CONSTANT*	ln(K1)	ln(NET)	TREND	ln(YCUX)	NET	K1	$(\ln(K1))^2$	$(\ln(NET))^2$	$(\ln(K1)-\ln(NET))^2$	ln(K1)*ln(NET)	ln(K1*YCUX)	$\hat{\beta}_1+\hat{\beta}_2$	R^2	DW
						Explanatory variables									
CDF(P-1)	-51.273	0.413	-0.056	0.025	0.998								-	0.999	1.854
	(-28.464)	(14.709)	(-0.619)	(25.017)	(24.705)										
CDF(P-2)	-8.143	0.116	1.356										1.472	0.988	1.097
	(-2.546)	(0.823)	(3.027)												
CDF(P-3)	-14.566	0.052	1.305	0.004									1.357	0.989	1.178
	(-2.288)	(0.344)	(2.929)	(1.163)											
CDF(P-4)	-24.208		-0.098	0.012								0.506	-	0.995	1.279
	(-7.356)		(-0.255)	(4.463)								(4.312)			
CES(P-5)	-6.703	1.360	0.174							0.194			1.534	0.989	1.251
	(-2.029)	(1.454)	(0.177)							(1.345)					
CES(P-6)	-7.837	1.212	0.298	0.001						0.172			1.510	0.989	1.242
	(-0.631)	(0.661)	(0.180)	(0.095)						(0.635)					
TPF(P-7)	579.465	23.939	-145.776					0.027	9.123		-2.709		-	0.991	1.291
	(0.170)	(0.080)	(-0.152)					(0.004)	(0.136)		(-0.064)				
TPF(P-8)	82.321	0.103	-19.028						1.149				-	0.989	1.255
	(1.761)	(0.789)	(-1.809)						(1.939)						
TPF(P-9)	70.622	1.062	-9.170			0.001	-0.003						-	0.991	1.344
	(1.835)	(1.774)	(-1.762)			(2.041)	(-1.634)								
TPF(P-10)	85.596	1.056	-9.988	-0.004		0.002	-0.003						-	0.992	1.403
	(1.942)	(1.735)	(-1.844)	(-0.709)		(2.119)	(-1.456)								

estimates for the labour and capital coefficients in the case of the CES and two CDF functions are positive. For all four TPF and two CDF functions the least squares estimates of the labour coefficient have the wrong sign. The sum of the output elasticity in all of these cases strongly suggest decreasing returns to scale. The hypothesis of $\beta_1 + \beta_2 = 1$ can be tested by applying the usual F-test. A possible reason for the negative estimates of the labour coefficients in the case of CDF1 and all the TPF's might be the presence of underemployed labour. The t-ratios of all parameter estimates for labour and capital show up to be statistically insignificant in the case of the CES and TPF functions. The correlation coefficients are relatively high, but the values for the Durbin-Watson statistic suggests serious problems with serial correlation in all these models. Notice, in passing, that despite the high correlation coefficient, none of the explanatory variables are significant in the case of the CES and TPF functions. These are clear-cut cases of multicollinearity. Any conclusions derived from equations estimated thus far must be regarded with a healthy dose of scepticism because of the very low values obtained for the DW statistic. Since many of these production functions investigated include either insignificant parameter estimates or an a priori wrong sign it is decided to take only the best estimated (statistically and econometrically speaking) function for further evaluation.

Out of the 10 production functions in Table 13.10(a) the simple Cobb-Douglas production function $\ln Q = \beta_0 + \beta_1 \ln K + \beta_2 \ln L$ (defined as P-2) is chosen to be evaluated further using the various stages of specification. The estimates of the elasticities of output with respect to capital (K1) and labour (NET) in the case of P-2 are $\beta_1 = 0{,}116$ and $\beta_2 = 1{,}356$, with only the coefficient of labour being significant. The negative constant term refers to the natural logarithm of β_0, and taking antilogs $\beta_0 = 0{,}0003$; $\bar{R}^2$ is relatively high, but the value for the Durbin-Watson statistic (1,097) suggests that there is a problem with positive autocorrelated errors (with $K' = 2$ explanatory variables and $n = 20$; $d_l = 1{,}100$ and $d_u = 1{,}537$ at the 5% significance level).

Some of the shortcomings in the estimation of these production functions is that the input variables are defined in stock terms whereas ideally one need measures of input flows. For the labour variable it would help if one make use of data on average weekly hours worked (see Thomas, 1985: 245). Another deficiency of P-2 is that no allowance of technical progress is made. It is also realised that, with 20 observations and 3 unknown parameters, the asymptotic standard errors are not very reliable. It is hoped to repeat these computations with substantially longer time series so as to evaluate the parameter estimates with greater forecasting accuracy.

TABLE 13.10(b): REGRESSION RESULTS FOR FIXED AND VARIABLE PARAMETER PRODUCTION MODEL P-2

VARIABLES	COEFFICIENT ESTIMATES STAGE 1	STAGE 2	STAGE 3	STAGE 4	WATSON-DAVIES* VPR TEST
ln(YF1)	Endogenous	Endogenous	Endogenous	Endogenous	
CONSTANT	-8.143 (-2.546)	-50.194 (-30.441)		-49.480 (-21.130)	
ln(NET)	1.356 (3.027)				
ln(YCUX)		0.983 (31.354)		0.985 (34.033)	
ln(K1)	0.116 (0.823)	0.397 (35.782)		0.375 (14.834)	
TREND		0.025 (30.567)		0.024 (20.987)	
K1				RANDOM WALK	1.000 (0.234)
NET					1.000 (0.234)
YCUX					1.000 (0.234)
VAR Error				5.267E-006 (2.136)	
VAR K1				1.223E-014 (0.001)	

<u>STATISTICAL REPORT</u>:

	STAGE 1	STAGE 2	STAGE 3	STAGE 4
$\bar{R}^2$	0.986	0.999		0.998
R^2	0.988	0.999		0.998
DW	1.097	1.833		1.840
SEE	0.017	0.003		0.007

Estimation Period: 1970 - 1989

* Watson-Davies VPR test performed during second stage of estimation.

Parameter estimates of only 3 stages of estimation are presented in Table 13.10(b). Allowance for technical progress is made by introducing disembodied neutral technical progress using a time trend in the equation of Stage 2. Judging by the t-statistic this variable seems to be highly significant, as well as all other variables included in the model. A measure for capacity utilisation is included in the Stage 2 model, while the labour variable is excluded. Econometrically and statistically speaking the Stage 2 model appears to be well estimated judging by the sign of the parameters, the adjusted correlation coefficient (0,999) and the Durbin-Watson statistic (1,833). The third stage ARCH model is not estimated for yearly data.

While the test for significant variation of K1 sensitivity is not significant (phi = 1,000 and p = 0,234) it is decided to go ahead with VPR because one could expect that variation might exist from the understanding of the problem. The random walk model for parameter variation is chosen, not because the parameter is thought to change its value permanently, i.e. to be a nonstationary process, but also since phi is 1,000 after performing the Watson-Davies test for parameter stability. If the coefficient had not be near unity, then an AR(1) process would have been tried instead. No drastic parameter changes are observed between the second and forth stages of estimation. The variance parameter for K1 appears to be statistically insignificant which indicates that there is no statistical gain in using VPR over the historical period. This is to be expected due to the non-significance of the Watson-Davies test performed on the K1 variable in Stage 2. Nevertheless, it is likely that forecasts from the VPR model will outperform those from the conventional dynamic regression.

In examining the time-path of the residual errors of regression in Figure 13.8(a) it seems as if the errors of the Stage 1 model are far larger in magnitude than those in the case of the improved Stage 2 model. The flat horisontal line for the graph of the K1 parameter in Figure 13.8(b) corroborates the Watson-Davies test results in Table 13.10(b) that there is no significant change in the parameter of K1 (phi = 1,000; p = 0,234). This happens when the process variance of the variable parameter is driven to zero or some very small number during optimisation. Therefore, the t-statistic for the process variance becomes nonsignificant. This should be taken literally and conventional regression should be used instead.

A fairly general conclusion which could be made is that the yearly Cobb-Douglas production function seems to be more stable than the quarterly investment functions investigated earlier. This is also borne out by the fact that most of the structural stability tests for production in Table 13.7 point at parameter stability.

FIGURE 13.8(a): RESIDUAL ERRORS OF REGRESSION FOR MODEL P-2, STAGES 1,2 AND 4

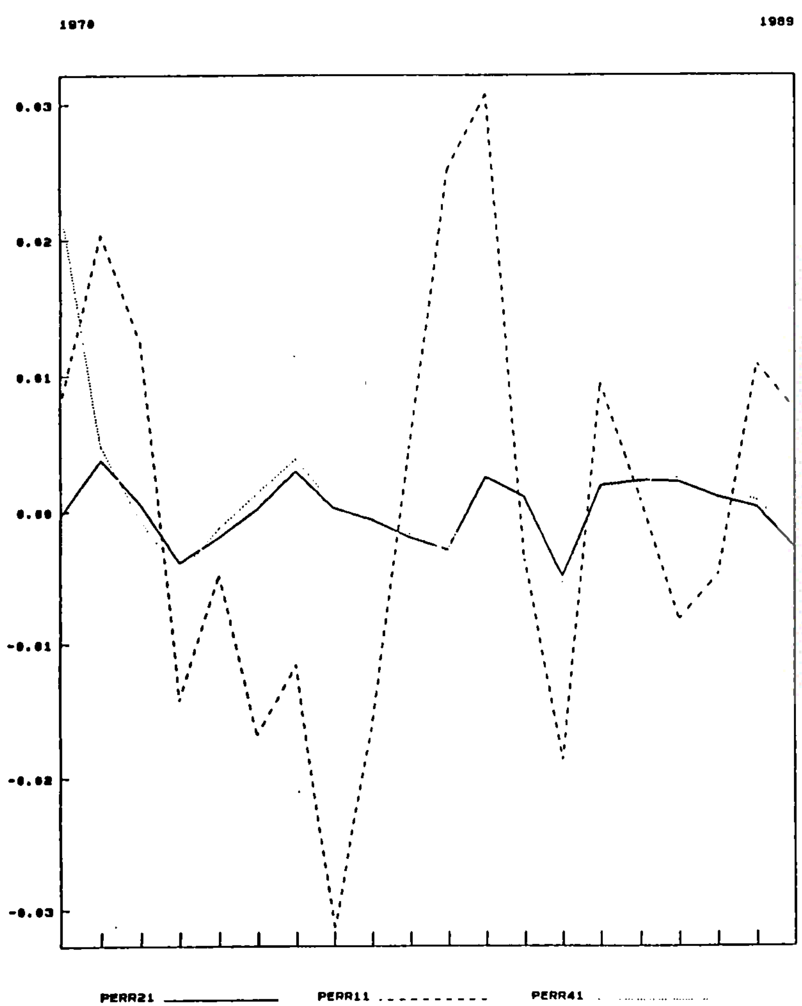
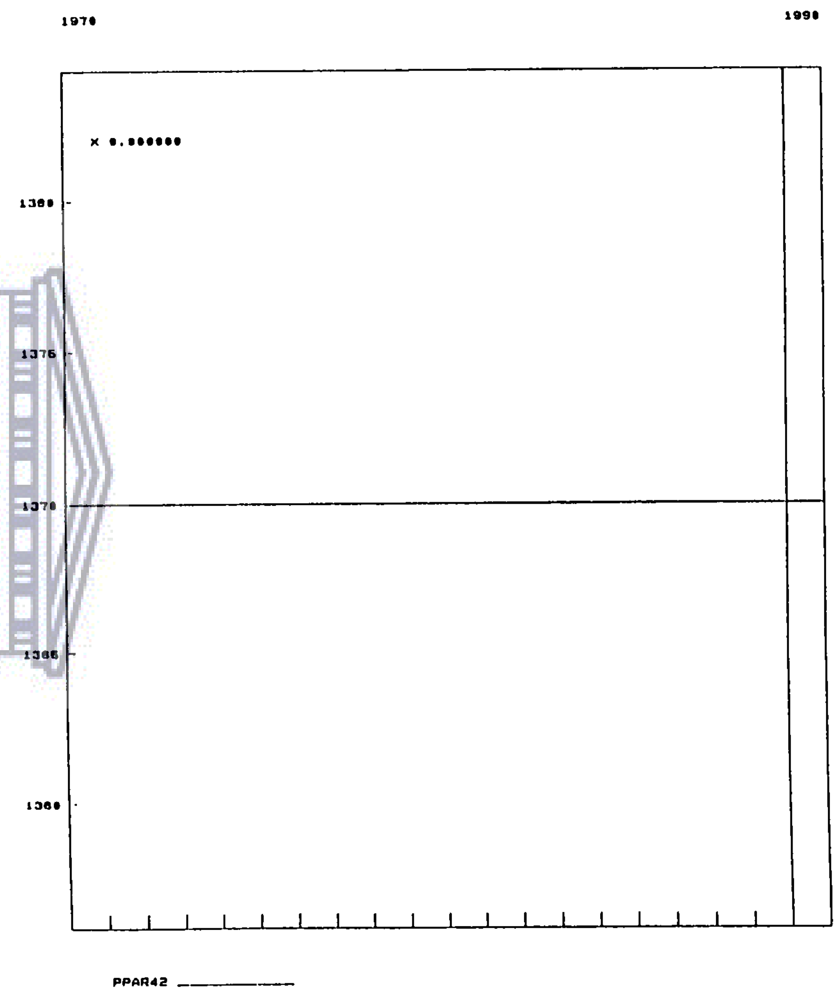


FIGURE 13.8(b): THE K1 TIME-VARYING PARAMETER FOR MODEL P-2



13.2.4 THE EXCHANGE RATE FUNCTION

The structural stability tests performed on the exchange rate models pointed at changes in the parameter estimates over time. Almost all the tests reported in Table 13.8 indicate structural weaknesses in the equations, including the Stage 2 equation.

The parameter estimates of the equation chosen for exchange rates are summarised in Table 13.11. To tie up with a study by Meese and Rogoff (1983a) the Rand/US Dollar exchange rate is defined as a function of the Deutch Mark/US Dollar exchange rate and the purchasing power parity rate, therefore representing a Frenkel-Bilson type of exchange rate model.

In the case of the Stage 1 model the t-statistics indicate that all the estimated parameters are statistically significant at the 5% level of significance. Diagnostic checking on the randomness of the residuals shows patterns of autocorrelation. The equation is re-estimated in Stage 2 by using the Cochrane-Orcutt estimation procedure after a trend and a lagged dependent variable are included. All parameter estimates are significant and appear with _a priori_ correct signs. The adjusted correlation coefficient also improved from 0,923 in Stage 1 to 0,994 in Stage 2. Economically, statistically and econometrically speaking everything appears to be in order, even allowing for the bias in the Durbin-Watson statistic caused by the lagged dependent variable. Due to computing

TABLE 13.11: REGRESSION RESULTS FOR FIXED AND VARIABLE PARAMETER EXCHANGE RATE MODEL E-1

VARIABLES	COEFFICIENT ESTIMATES STAGE 1	STAGE 2	STAGE 3	STAGE 4	WATSON-DAVIES* VPR TEST
REX12	Endogenous	Endogenous	Endogenous	Endogenous	
CONSTANT	-0.903 (-9.383)	-17.296 (-3.410)		-100.717 (-1.462)	0.550 (0.457)
REXDM$	0.210 (7.387)	0.061 (3.784)		0.105 (2.237)	
REXPPP	1.419 (47.892)	0.101 (2.369)		RANDOM WALK	0.550 (0.993)
REX12(-1)		0.885 (26.593)		0.584 (5.619)	0.550 (1.000)
TREND		0.009 (3.389)		0.051 (1.463)	
AUTO(-1)		0.338 (4.731)		-0.141 (-1.229)	
VAR Error				4.872E-007 (0.003)	
VAR REXPPP				0.002 (4.493)	
STATISTICAL REPORT:					
$\bar{R}^2$	0.923	0.994		0.994	
R^2	0.924	0.994		0.994	
DW	0.092	2.024		2.293	
SEE	0.209	0.058		0.061	
Estimation Period: 1970M2 - 1991M12					

* Watson-Davies VPR test performed during second stage of estimation.

difficulties no ARCH model is investigated.

Although everything seems to be in order with the Stage 2 exchange rate model, when a number of structural stability tests are performed on this equation, most of them point at structural instability (see Table 13.8). The next step is, therefore, to estimate the regression parameters using VPR techniques. The Stage 4 model combines a traditional dynamic model with lagged variables and a random-walk parameter model. It will be seen in the next section that, although the estimates of some of the parameters turned out to be insignificant at a 5% level of significance, allowing for parameter variation leads to an improved forecasting performance relative to the fixed coefficient conventional dynamic model. Furthermore, the fact that the variance parameter of REXPPP turns out to be highly significant (phi = 0,550; p = 1,000) gives one reason to believe that the parameter of the purchasing power parity rate is changing over time. Although the phi coefficient is not close to unity, a random walk model is selected because of its superior results. Except for a few insignificant parameter estimates, all remaining statistics are satisfactory which point at a well estimated VPR model. To further explore the stochastic nature of the REXPPP parameter, the errors of regression for Stages 1, 2 and 4 of estimation are presented graphically in Figure 13.9(a). These errors are particularly large during the time when the government announced a debt 'standstill' on the

FIGURE 13.9(a): RESIDUAL ERRORS OF REGRESSION FOR MODEL E-1, STAGES 1,2 AND 4

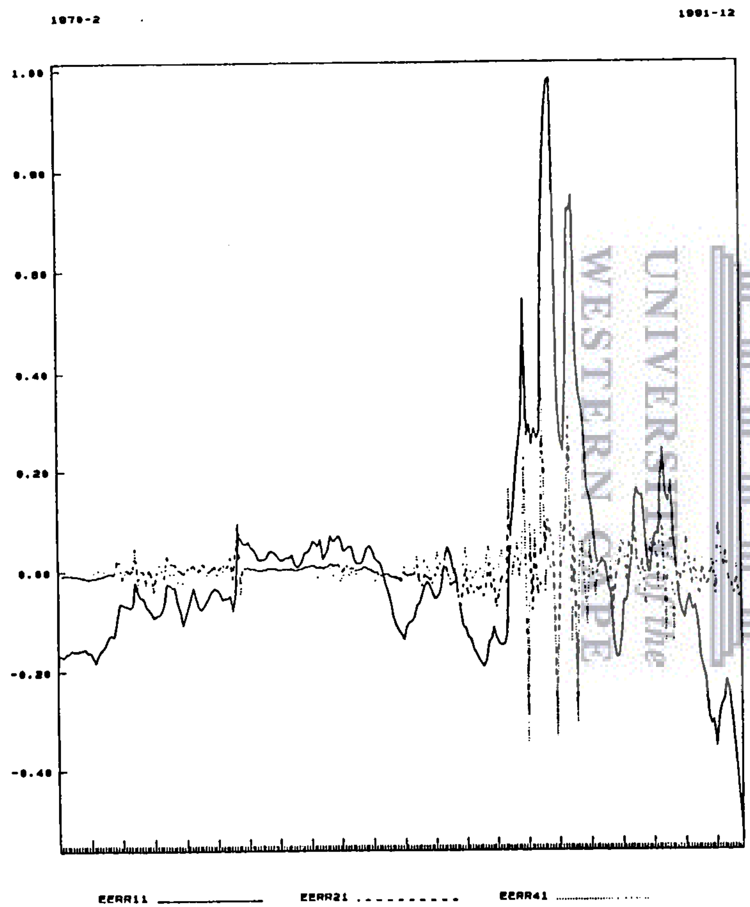
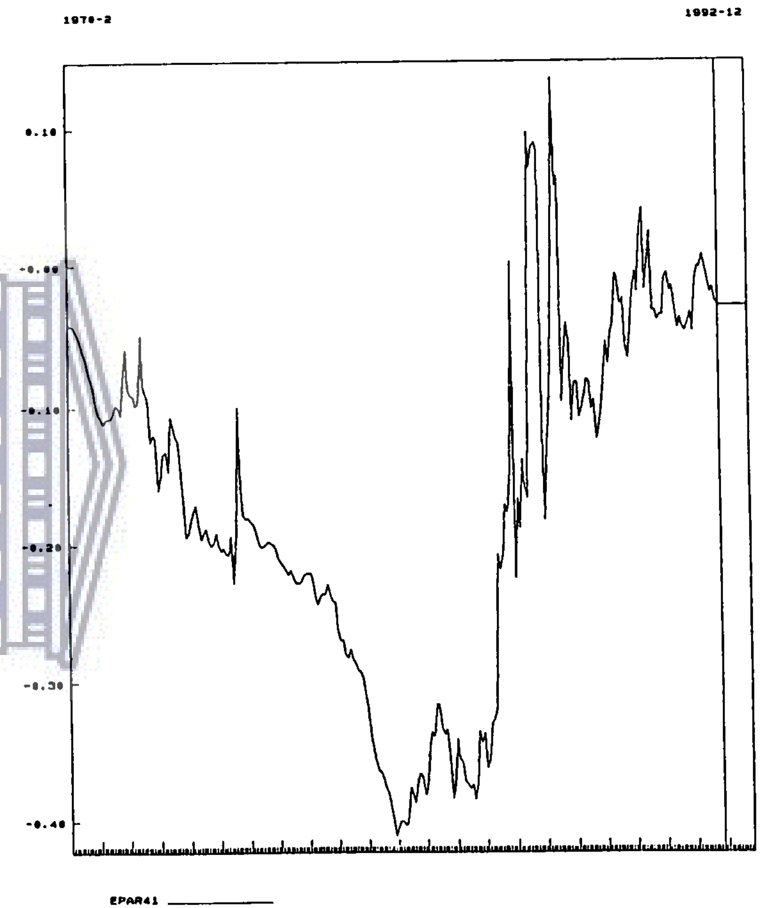


FIGURE 13.9(b): THE REIPPP PARAMETER FOR MODEL E-1



repayment of a major part of South Africa's foreign debt in 1985. South Africa also experienced a sharp depreciation of the Rand at the time. General political instability, the announced state of emergency and an international debt crisis are all contributing factors which cause the parameters to become unstable. The graph also corroborates the results of the structural stability tests performed on exchange rate models earlier on. A time series of the REXPPP parameter can be viewed graphically in Figure 13.9(b). An inspection of the time path of the graph shows that the parameter is far more volatile during the 1980's. One can also notice from Figure 13.9(b) that there are periods when the changing parameter even turns negative. Taken all evidence together, a strong case can be made out for VPR estimation in the context of exchange rate modelling.

13.3 COMPARING FORECASTS FROM FIXED AND VARIABLE PARAMETER REGRESSION MODELS

13.3.1 INTRODUCTION

It is conventional in applied situations to assess the usefulness of a forecasting model and to choose among competing models on the basis of forecasting performance. This is also widely done in the econometrics literature, and the root mean squared error (RMSE) statistic is usually of central interest for comparison purposes (see e.g., Fromm and Klein, 1973; Fair, 1974; McNees, 1976; Spivey and Wrobleski, 1979; Machack, Spivey and Wrobleski, 1985; and Swamy, Conway and Le Blanc, 1988 and 1989).

A natural approach to investigating the advantages and disadvantages of the models is to apply Zellner's (1988: 32) prediction principle. The available time series is split into two non-overlapping parts. The period of the first part is called the estimation (or fitting) period and the period of the second, the forecasting period. Let t = 1,2,..., T be the fitting period and t = T+1, T+2, ..., T+n be the forecasting period. The models are estimated by using the first part and then these estimated models are used, along with the values of the independent variables for the forecasting period, to predict the values of the dependent variable for the forecasting period without revising the parameter estimates. Such forecasts are called nonsequential or multi-step-ahead. It of the independent variables for the period T+i are not available at the time of forecasting y_{t+i}. Since the purpose would be to obtain separate estimates of the terms on the right-hand side of the equations, one has to use these values of independent variables. Without separating the period of the second part from the rest, it is not possible to evaluate forecast errors arising from coefficients' instability. Forecasters are also interested in knowing the magnitudes of each term on the right-hand side of the equation. Thus, one is here solving a problem which is broader than the usual practical forecasting problem.

If one estimates sequentially the fixed parameters using all past data prior to each of the forecasting periods, T+1, T+2, ..., T+n, then the corresponding forecast is called sequential or one-step-ahead. The primary purpose of Swamy and Schinasi's (1986) article is to demonstrate that the one-step-ahead forecasts will not necessarily be closer to the realised values of the forecasted variable than the multi-step-ahead forecasts. There is no non-Bayesian theory which mandates prediction with sequential estimation (see Swamy et al., 1988: 30). The choice of where to begin forecasting is mainly due to the desire to have sufficient degrees of freedom available for initial parameter estimates of all the models.

To examine the forecasting properties the models are estimated with OLS, GLS, ARCH and VPR estimation techniques, which cover the 4 stages of estimation discussed earlier. The RMSE, together with other forecasting criteria, is used to compare the forecasting performance of each model. The model with the smallest RMSE is chosen to be the best in predicting future values for the dependent variable.

13.3.2 THE FIXED INVESTMENT FUNCTION

This analysis poses five alternative models of fixed investment, four for IPO1 and one for IPZA1. The RMSE of the within-sample fits is reported in Table 13.12. The fitting period referred to is the sample period used to estimate each model stage, allowing prior data to provide a sample for lagged values. The AIC and BIC are also reported in Table 13.12, as an order selection criterion (see Glossary).

Each stage of Model I-1 (for IPO1) is estimated only once using 76 observations covering the sample period 1972Q1 - 1990Q4, allowing data from 1970Q1 - 1971Q4 to provide a sample for lagged values. After improving the Stage 1 model little difference is noticed between the RMSE of Stage 2 to 4. The AIC as well as the BIC suggest that the fixed coefficient model of Stage 2 will produce more accurate out-of-sample forecasts than its counterparts. Similar results are reported for the rest of the models, except for the IPZA1 function, which suggests that the VPR model does very little to improve the fit of the data. In the case of Model I-4 the available evidence points to the

TABLE 13.12: FORECAST EVALUATION STATISTICS OVER HISTORIC PERIOD FOR INVESTMENT FUNCTIONS

DEPENDENT VARIABLE	MODEL	FITTING PERIOD	STAGE	AIC	BIC	RMSE
IPO1	I-1	1972Q1-1990Q4	1	0.723	0.829	0.642
			2	0.624	0.673	0.583
			3	0.641	0.713	0.584
			4	0.648	0.733	0.583
IPO1	I-2	1972Q1-1990Q4	1	1.012	1.293	0.820
			2	0.687	0.720	0.660
			3	0.706	0.762	0.660
			4	0.704	0.784	0.641
IPO1	I-3	1970Q3-1990Q4	1	0.646	0.685	0.615
			2	0.588	0.633	0.553
			3	0.602	0.668	0.553
			4	0.612	0.689	0.554
IPO1	I-4	1972Q3-1990Q4	1	1.194	1.440	1.016
			2	0.716	0.762	0.679
			3	0.737	0.809	0.679
			4	0.761	0.862	0.683
IPZA1	I-5	1971Q1-1990Q4	1	1.202	1.294	1.130
			2	0.674	0.715	0.641
			3	0.676	0.762	0.611
			4	0.700	0.790	0.633

fact that the VPR will do worse in producing forecasts than the "well" estimated fixed coefficient model of Stage 2 and the ARCH model of Stage 3. The only exception is in the case of Model I-5 where the value of the RMSE in Stage 4 (0,633) suggests a better fit than the Stage 2 model. The AIC and BIC suggests otherwise. In Model I-5 (IPZA1) the selection criteria indicate that the ARCH model will do best in out-of-sample forecasts compared to its fixed coefficient and VPR counterpart. The results reported in Table 13.12, therefore, raise the question of whether VPR is superior than fixed coefficient models during times of structural instability.

What is interesting, however, is that when out-of-sample forecasts are obtained for each of the four different stages of estimation, the RMSE seems to be in favour of the VPR method. Table 13.13 compares the ex-post forecast of a number of investment models and the forecast accuracy is judged by RMSE.

Table 13.13 shows that the use of a VPR model may substantially reduce out-of-sample forecast errors, similar to error reductions obtained in empirical applications of Swamy et al. (1988). One would, however, expect enhanced forecasting accuracy from the VPR model, since the forecasts will use the last available value for the time varying parameter.

In Table 13.14 multi-step-ahead forecast errors are calculated for each of 8 quarters in the forecasting period 1991Q1 - 1992Q4 with the respective fitting periods shown in Table 13.14, allowing data from 1970Q1 to provide a sample for lagged values. The year 1991Q1 is chosen to be the cutoff year because at the time was a dramatic decline in net investment, a

TABLE 13.13: OUT-OF-SAMPLE RMSE OF STOCHASTIC AND FIXED COEFFICIENTS ESTIMATORS FOR INVESTMENT FUNCTIONS

DEPENDENT VARIABLE	FITTING PERIOD	FORECASTING PERIOD	STAGE 1	STAGE 2	STAGE 3	STAGE 4	IMPROVEMENT OVER BEST FIXED ALTERNATIVE (%)
1. IPO1	1972Q1-1990Q4	1991Q1-1992Q4	1.242	1.011	0.971	1.038	-7
2. IPO1	1972Q1-1990Q4	1991Q1-1992Q4	0.650	0.555	0.578	0.396	29
3. IPO1	1970Q3-1990Q4	1991Q1-1992Q4	0.487	0.297	0.300	0.251	16
4. IPO1	1972Q3-1990Q4	1991Q1-1992Q4	5.992	2.654	2.471	2.188	12
5. IPZA1	1971Q1-1990Q4	1991Q1-1992Q4	0.869	0.499	-	0.414	17

TABLE 13.14: FORECAST ERRORS OF THE MULTI-STEP-AHEAD FORECASTS FOR INVESTMENT FUNCTIONS

DEPENDENT VARIABLE	MODEL	TIME PERIOD	ACTUAL	STAGE 1	STAGE 2	STAGE 3	STAGE 4
IPO1	I-1	1991Q1	12.950	-0.066	-0.241	-0.212	-0.247
		Q2	12.828	-0.383	-0.428	-0.403	-0.440
		Q3	12.792	-0.629	-0.560	-0.525	-0.579
		Q4	12.778	-0.763	-0.620	-0.582	-0.645
		1992Q1	12.671	-0.851	-0.648	-0.604	-0.676
		Q2	12.317	-1.374	-1.168	-1.115	-1.201
		Q3	11.868	-1.873	-1.598	-1.549	-1.635
		Q4	11.627	-1.920	-1.704	-1.646	-1.743
RMSE				1.241	1.011	0.971	1.038
IPO1	I-2	1991Q1	12.950	0.472	-0.302	-0.314	-0.229
		Q2	12.828	0.471	-0.209	-0.224	-0.022
		Q3	12.792	0.018	-0.235	-0.255	-0.060
		Q4	12.778	1.230	-0.270	-0.294	-0.040
		1992Q1	12.671	-0.634	-0.299	-0.326	-0.064
		Q2	12.317	0.308	-0.509	-0.538	-0.289
		Q3	11.868	-0.714	-0.861	-0.891	-0.649
		Q4	11.627	-0.644	-1.056	-1.088	-0.830
RMSE				0.650	0.555	0.578	0.396
IPO1	I-3	1991Q1	12.950	-0.145	-0.104	-0.107	-0.058
		Q2	12.828	-0.215	-0.039	-0.043	0.015
		Q3	12.792	-0.150	0.057	0.052	0.138
		Q4	12.778	-0.064	0.188	0.182	0.284
		1992Q1	12.671	-0.205	0.136	0.131	0.245
		Q2	12.317	-0.489	-0.113	-0.119	0.008
		Q3	11.868	-0.794	-0.451	-0.458	-0.315
		Q4	11.627	-0.944	-0.647	-0.656	-0.494
RMSE				0.487	0.297	0.300	0.251
IPO1	I-4	1991Q1	12.950	-2.352	-0.618	-0.574	-0.489
		Q2	12.828	-2.794	-1.015	-0.946	-0.830
		Q3	12.792	-4.334	-1.391	-1.287	-1.113
		Q4	12.778	-5.033	-1.765	-1.627	-1.390
		1992Q1	12.671	-5.974	-2.257	-2.081	-1.787
		Q2	12.317	-7.085	-3.010	-2.795	-2.448
		Q3	11.868	-8.127	-3.876	-3.620	-3.224
		Q4	11.627	-8.821	-4.551	-4.254	-3.825
RMSE				5.992	2.654	2.471	2.188
IPZA1	I-5	1991Q1	15.408	-0.041	-0.115		-0.116
		Q2	15.277	-0.623	-0.024		-0.030
		Q3	15.229	-0.269	0.032		0.053
		Q4	15.166	-0.362	0.059		0.140
		1992Q1	15.017	-0.372	-0.021		0.116
		Q2	14.664	-0.953	-0.372		-0.193
		Q3	14.178	-1.465	-0.845		-0.681
		Q4	13.928	-1.502	-1.057		-0.904
RMSE				0.869	0.499		0.414

situation that provides a good test for forecast superiority.

If one studies the forecasting error results in Table 13.14 it shows that for nearly all the cases the VPR model outperforms the fixed coefficient model. The only VPR model which performs worse than its fixed counterparts is the one estimated for Model I-1. This could probably be because of the fact that most structural stability tests performed on this particular equation in Stage 2 indicate constant parameters and that the usage of VPR might have caused an error of specification. In the case of model IPO1(2) there is a 29% improvement in RMSE by using VPR instead of the best fixed alternative regression model.

Table 13.14 shows very clearly that the VPR model is the superior predictor. However, an unambiguous indicator of forecast accuracy does not exist. Each indicator has its own risk function. For example, a mean absolute error criterion is based on an absolute deviation loss function, while a mean square error criterion is based on a quadratic loss function. Therefore different analysts may prefer different criteria, depending on their assumed loss function. A wide variety of forecast and other criteria, including goodness-of-fit and tracking measures, is preferable.

Given the finding that the VPR model almost invariably has the lowest RMSE over all horisons, except in the case of Model I-1, one can conclude that fixed coefficient models do not generally perform better in forecasting accuracy than the VPR model.

The forecast error reported in Table 13.14 is representative of the VPR's dominance over its competitors. The exception is in the case of Model I-1 where the fixed coefficient model of Stage 2 and the ARCH specification of Stage 3 outperformed the VPR model. The VPR model outperforms all the other four models (I-2 to I-5) for nearly any sensible risk functions. The only exception is the case of the I-5 model where the well estimated fixed coefficient alternative performed better in forecasting accuracy over the period 1991Q1 to 1992Q1. The VPR model, however, still outperforms the Stage 2 model in this case when the sum of the RMSE is calculated over the full extended forecasting period.

If one examines the forecast error squared over the fitted period of Model I-1 in Figure 13.10(a) an indication of poor forecasting performance is noticed over the period 1978 to 1986. This corresponds with a period of drastic increases in the interest rates, the sharp depreciation of the Rand, general political instability and an international debt crisis. Severe drought conditions also prevail, especially during the early 1980's. Although these factors should by no means be regarded as a complete summary of factors causing structural instability, some of these could have caused structural breaks

FIGURE 13.10(a): FORECAST ERRORS SQUARED FOR MODEL I-1, STAGES 2,3 AND 4

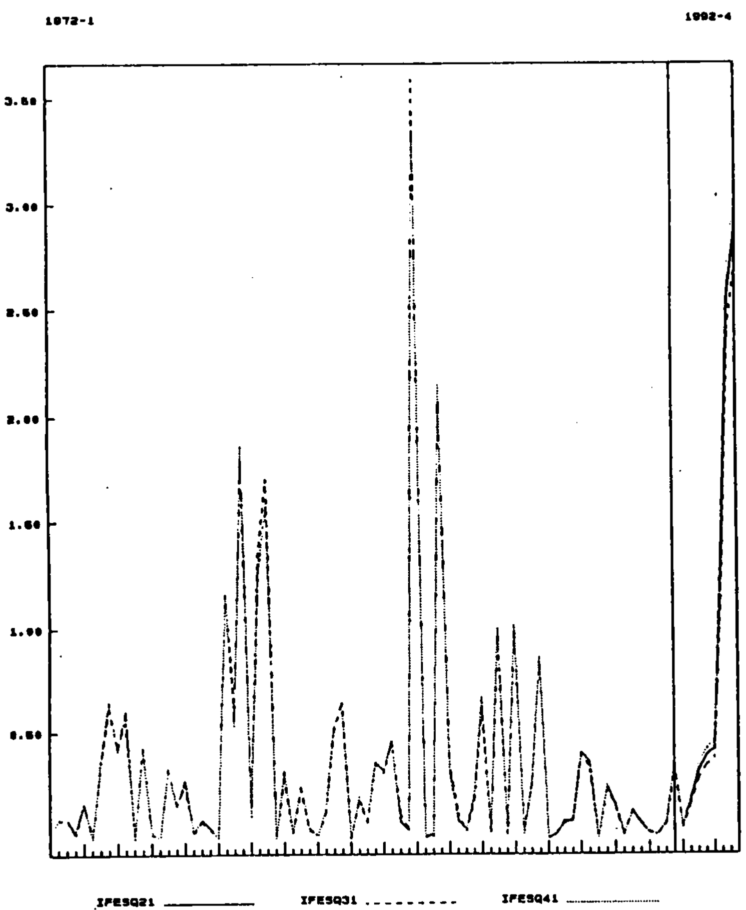
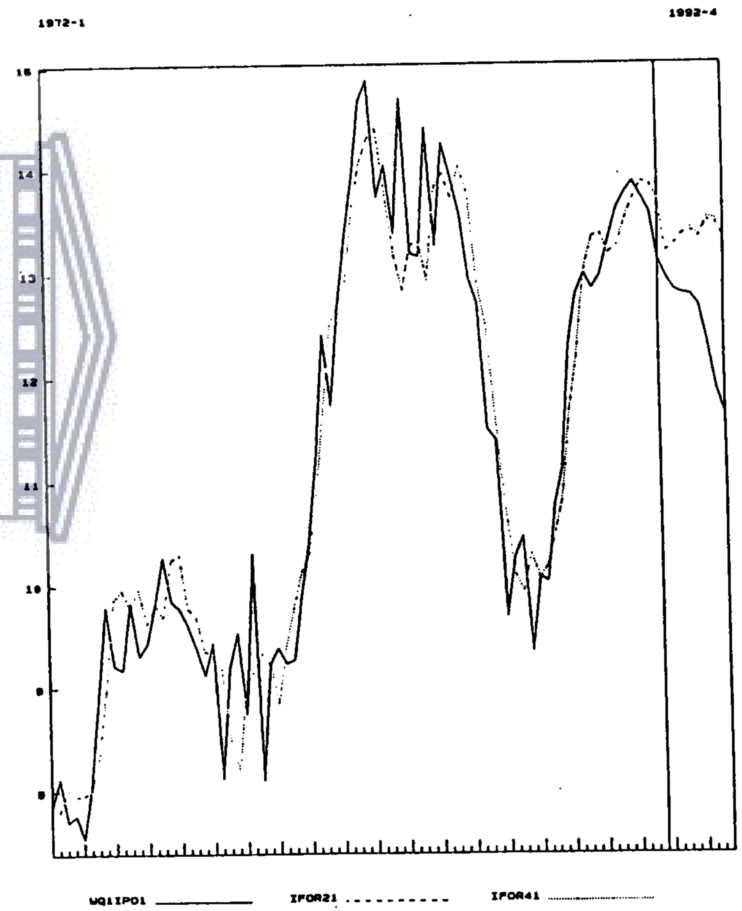


FIGURE 13.10(b): THE GOODNESS-OF-FIT TRACK RECORD FOR MODEL I-1, STAGES 2 AND 4



in South African investment functions. This, however, could have been the reason why the original model performed so badly over the latest sample period. Figure 13.10(b) shows that out of two stages studied the original Model (I-1(1)) performs worse in fitting the data than the VPR alternative, and even more so when the long run out-of-sample forecasts are compared (the split between fitted period and ex-post forecast is indicated by the vertical line). Similar patterns of behaviour, not documented here, are observed for Models I-2 to I-5, except that the VPR models perform better in out-of-sample forecasts than its fixed coefficient counterparts (see Table 13.13).

13.3.3 THE COBB-DOUGLAS PRODUCTION FUNCTION

In this study, annual data on gross domestic product at factor cost (in real terms) is regressed on labour (total number employed) and fixed capital stock (in real terms). The period of study is for 21 years from 1970 to 1990, with 1990 excluded from the fitted period in order to compare a one-step-ahead forecast later. The choice of where to begin forecasting is dependent on the desire to have sufficient degrees of freedom available for initial parameter estimates of all the models.

The data were collected from the Bureau of Economic Research. Unfortunately the data on labour input could not be adjusted for the underemployment of labour and the efficiency differences among the units of that input. Although the limitations of the data are realised, one has to work with whatever data is available. The variables for the simple Cobb-Douglas function is defined as the natural logarithm of GDP at factor cost, labour, and capital stock. During the second stage of estimation the natural logarithm of labour variable is replaced by the natural logarithm of capacity utilisation in the manufacturing industry. A trend variable is also included as an explanatory variable representing technological advancement. Changing the explanatory variable set causes substantial changes in the forecast evaluation statistics. The coefficient of the capital stock variable (K1) is specified as a random walk process in Stage 4 (Stage 3 is not available in the case of yearly data).

The accuracy of the forecasts is judged mainly by using the RMSE statistic. The results of the computations are summarised in Table 13.15 - 13.17. Table 13.15 compares forecast evaluation statistics over the fitted period for each of the stages. One should, however, be careful when comparing

TABLE 13.15: FORECAST EVALUATION STATISTICS* OVER HISTORIC PERIOD FOR A COBB-DOUGLAS PRODUCTION FUNCTION

DEPENDENT VARIABLE	MODEL	FITTING PERIOD	STAGE	AIC	BIC	RMSE
ln(YF1)	P-2	1970-	1	1.808	1.945	1.556
		1989	2	0.286	0.316	0.234
			3	-	-	-
			4	0.766	0.890	0.568

forecasting accuracy of the Stage 1 fixed coefficient model with other models because a different set of variables are used in Stages 2 and 3. Judging by the AIC, BIC and RMSE it is suggested that the "well" estimated model of Stage 2 will perform better in producing forecasts for the dependent variable than its VPR counterpart in Stage 4. There is also a substantial improvement in RMSE from the original Stage 1 model to the Stage 2 and 4 models, which mean that if the simple Cobb-Douglas production function were to be estimated in its original form it would have produced poorer forecasts for the dependent variable than its Stage 2 or 4 counterparts. Comparing the values for RMSE in Table 13.16, however, shows a different picture alltogether in that the one-step-ahead forecasts yielded by estimates for the VPR model completely dominate one-step-ahead forecasts yielded by estimates of the parameters of the fixed coefficient models. This is unlike the

* Evaluation statistics are all in percentages..

TABLE 13.16: OUT-OF-SAMPLE RMSE* OF STOCHASTIC AND FIXED COEFFICIENT ESTIMATORS FOR A COBB-DOUGLAS PRODUCTION FUNCTION

DEPENDENT VARIABLE	FITTING PERIOD	FORECASTING PERIOD	STAGE 1	2	3	4	IMPROVEMENT OVER BEST FIXED ALTERNATIVE (%)
ln(YF1)(2)	1970-1989	1990	1.4832	0.2128	-	0.1028	52

case where RMSE is evaluated over the fitted period, where the suggestion was made to use conventional modelling instead of VPR techniques (see Section 13.2.3). In fact, there is a 52% improvement over the best fixed alternative in terms of RMSE over the extended forecasting period. This result may have arisen as a direct consequence of the inappropriateness of the fixed coefficient assumption underlying the Cobb-Douglas function. If so, the analysis of sample information based on the inappropriate assumption of fixed regression slopes yielded less accurate forecasts than the analysis of a sample based on the assumption of time-varying regression slopes.

The RMSE in Table 13.17 presents the forecast error in the case of a one-step-ahead prediction. When examining the goodness-of-fit the graph in Figure 13.11(a) indicates that the VPR model fits the data much better than the original fixed coefficient

* RMSE in percentages.

TABLE 13.17: FORECAST ERROR OF A ONE-STEP-AHEAD FORECAST FOR A COBB-DOUGLAS PRODUCTION FUNCTION

DEPENDENT VARIABLE	MODEL	TIME PERIOD	ACTUAL	STAGE 1	STAGE 2	STAGE 3	STAGE 4
ln(YF1)	P-2	1990	4.785	0.015	-0.002	-	-0.001
RMSE				1.483	0.213	-	0.103

model. The forecast error squared for various stages is depicted in Figure 13.11(b) which show relatively stable functions, except for the Stage 1 model. These findings are also supported by the test results for structural instability in Table 13.7. The instability indicated by the large forecast errors of the Stage 1 model during 1975 - 1986 coincide with a period of rising unemployment, drought conditions and sanctions. The vertical line in Figures 13.11(a and b) indicates the starting point (T) of the out-of-sample forecasts, which shows that the VPR model for production perform better in forecasting performance than the rest of the fixed coefficient models estimated.

Given all of these findings above, one can almost unambiguously assert that the VPR model outperforms the conventional fixed coefficient model in forecasting accuracy.

FIGURE 13.11(a):THE GOODNESS-OF-FIT TRACK RECORD FOR MODEL P-2, STAGES 1 AND 4

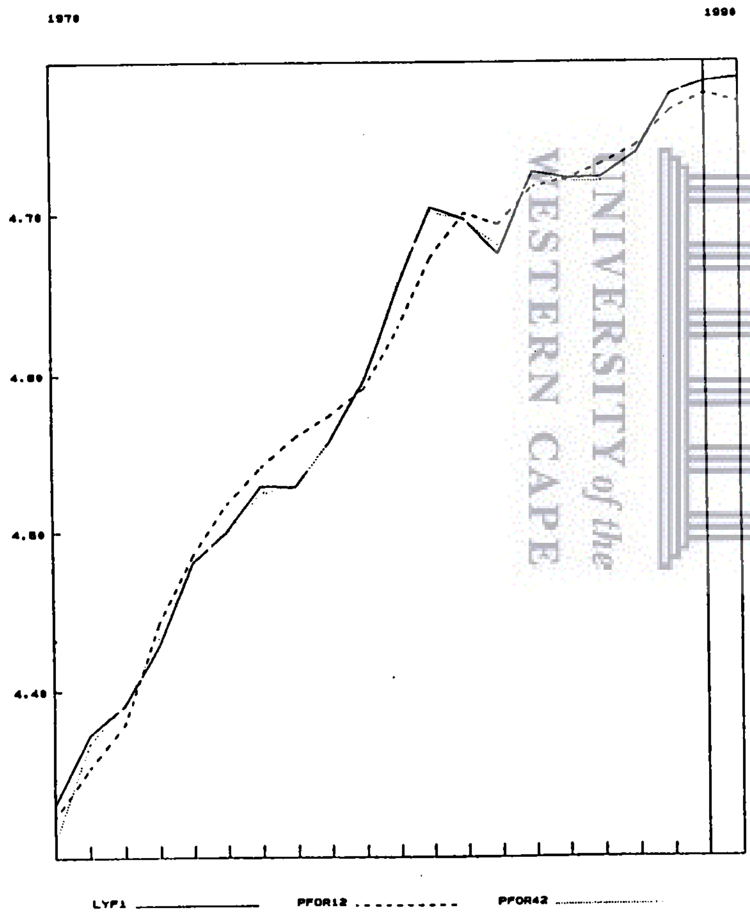
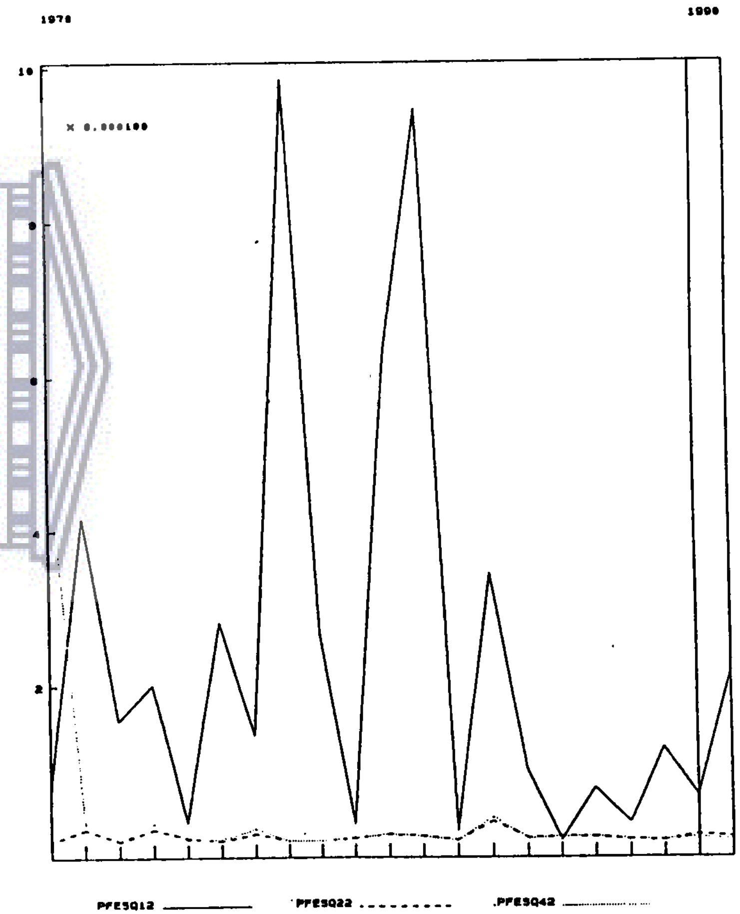


FIGURE 13.11(b):FORECAST ERRORS SQUARED FOR MODEL P-2, STAGES 1,2 AND 4



13.3.4 THE EXCHANGE RATE FUNCTION

The structural stability tests have shown that some of the regression parameters for the exchange rate models have not been constant over time, but rather follow some stochastic pattern. Empirical regularity suggests that this information should be effectively incorporated into the forecasting process.

To elucidate this, the following procedure is proposed. The first step is to estimate the coefficients by applying OLS (or GLS if necessary) to a similar model which was estimated by a model-builder previously; the second step involves the 'improvement' of such a model; the third step involves fitting a time-dependent parameter into an appropriate VPR model. For each of these steps (or stages) the out-of-sample forecasting performance is evaluated, using summary statistics such as the AIC, BIC, RMSE and forecasting error.

All the competing models are estimated with monthly data which starts in January 1970 and extends through December 1991, including the floating rate period which began in 1973. Ex-post forecasts are generated for each month during 1992 using the actual realisations of all explanatory variables for the prediction period.

The results of this section are summarised in Tables 13.18 - 13.20. Table 13.18 compares the forecasting performance of fixed coefficient models and the VPR model over the sample period. A trend and a lagged dependent variable are included in

TABLE 13.18: FORECAST EVALUATION STATISTICS OVER HISTORIC PERIOD FOR AN EXCHANGE RATE FUNCTION

DEPENDENT VARIABLE	MODEL	FITTING PERIOD	STAGE	AIC	BIC	RMSE
REX12	E-1	1970M2- 1991M12	1	0.210	0.214	0.208
			2	0.058	0.061	0.057
			3	-	-	-
			4	0.066	0.070	0.064

the Stage 2 and 4 models with both models corrected for first-order serial correlation. The Stage 4 model combines a traditional dynamic model with lagged variables and a VPR model. The structural models with lagged dependent variables are 'nested' models in the sense that the structural models with lagged dependent variables can be viewed as a model in which lagged exchange rates and economic variables are allowed to 'explain' the spot exchange rate. If the structural model outperform the VPR model over the sample period, then one can attribute this superior performance to the informational content of the economic variables included and the economic theory that suggested these variables. This could be the reason why the Stage 2 model appears to fit the data best over the sample period judging by the values of the AIC, BIC and RMSE summary statistics. Although the Watson-Davies test indicates a significant time-varying coefficient for the purchasing power parity rate (see Table 13.11), the well estimated conventional model still outperforms the VPR model in fitting the sample data. The AIC and BIC also indicate that the fixed coefficient model of the second stage will perform better than the VPR model of Stage 4.

Table 13.19 as oppose to Table 13.18 contains surprising results regarding the RMSE statistics for an ex-post forecasting period of 12 months. The results in Table 13.19, however, are the more important and decisive results. It can be seen that allowing for parameter variation leads to 38% improvement in forecasting performance relative to the conventional dynamic model without parameter variation.

The forecasting errors of the various models are listed in Table 13.20. Note that the multi-step-ahead forecast of the VPR model is far superior to the multi-step-ahead forecasts of the original fixed coefficient model (of Stage 1) without the lagged dependent variable. The forecasting errors are

TABLE 13.19: OUT-OF-SAMPLE RMSE OF STOCHASTIC AND FIXED COEFFICIENT ESTIMATORS FOR AN EXCHANGE RATE FUNCTION

DEPENDENT VARIABLE	FITTING PERIOD	FORECASTING PERIOD	STAGE 1	2	3	4	IMPROVEMENT OVER BEST FIXED ALTERNATIVE (%)
REX12(1)	1970M2-1991M12	1992M1 1992M12	0.609	0.167	-	0.103	38

TABLE 13.20: FORECAST ERRORS OF THE MULTI-STEP-AHEAD FORECASTS FOR AN EXCHANGE RATE FUNCTION

DEPENDENT VARIABLE	MODEL	TIME PERIOD	ACTUAL	STAGE 1	STAGE 2	STAGE 3	STAGE 4
REX12	E-1	1992M1	2.779	-0.551	-0.010		0.023
		M2	2.815	-0.548	-0.012		0.058
		M3	2.881	-0.503	-0.011		0.115
		M4	2.878	-0.546	-0.035		0.105
		M5	2.847	-0.597	-0.105		0.069
		M6	2.810	-0.653	-0.178		0.029
		M7	2.753	-0.712	-0.261		-0.020
		M8	2.763	-0.733	-0.277		-0.010
		M9	2.798	-0.713	-0.268		0.022
		M10	2.884	-0.639	-0.208		0.097
		M11	2.976	-0.535	-0.120		0.197
		M12	3.014	-0.520	-0.124		0.207
RMSE				0.609	0.167		0.103

relatively large and positive in the case of the Stage 1 model, indicating a systematically overprediction over the course of the extended sample. The reason for the overprediction could be because of the fact that structural models do tend to go systematically off track if no serial correlation is allowed for (see Table 13.11).

Because each of the two fixed-coefficient models presented - one with and the other one without a lagged dependent variable - use different sets of information in generating multi-step-ahead predictions, one has to be very careful in comparing their forecasting performance. Although the equations with a lagged dependent variable as an explanatory variable (as in

Stages 2 and 4) represent a multi-step-ahead forecast, in the sense that it does not use data beyond period T (indicated by the vertical line in Figure 13.12(a)) to estimate parameters used for prediction, its vector x'_{T+i} does contain the observation on the lagged dependent variable for the time periods beyond T.

The goodness-of-fit graph in Figure 13.12(a) corroborates the bad performance of the original fixed coefficient model. It can be seen that the VPR model of Stage 4 do much better in fitting the sample data than the Stage 1 model for exchange rates. The dominance of the VPR model over the Stage 2 model, judging by the forecasting error, remains when forecasting begins in May 1992 and ends in October 1992 (see Table 13.20). The RMSE, however, indicates further that when coefficients are allowed to change period by period, multi-step-ahead forecasts of the VPR model outperform multi-step-ahead forecasts of the fixed coefficient model.

The forecasting error squared over the fitted and extended sample is presented in Figure 13.12(b). The large errors as from 1985 correspond with a period of a sharp depreciation of the Rand against the US Dollar. It was also a time earmarked with general political instability and an international debt crisis.

FIGURE 13.12(a): THE GOODNESS-OF-FIT TRACK RECORD FOR MODEL E-1, STAGES 1 AND 4

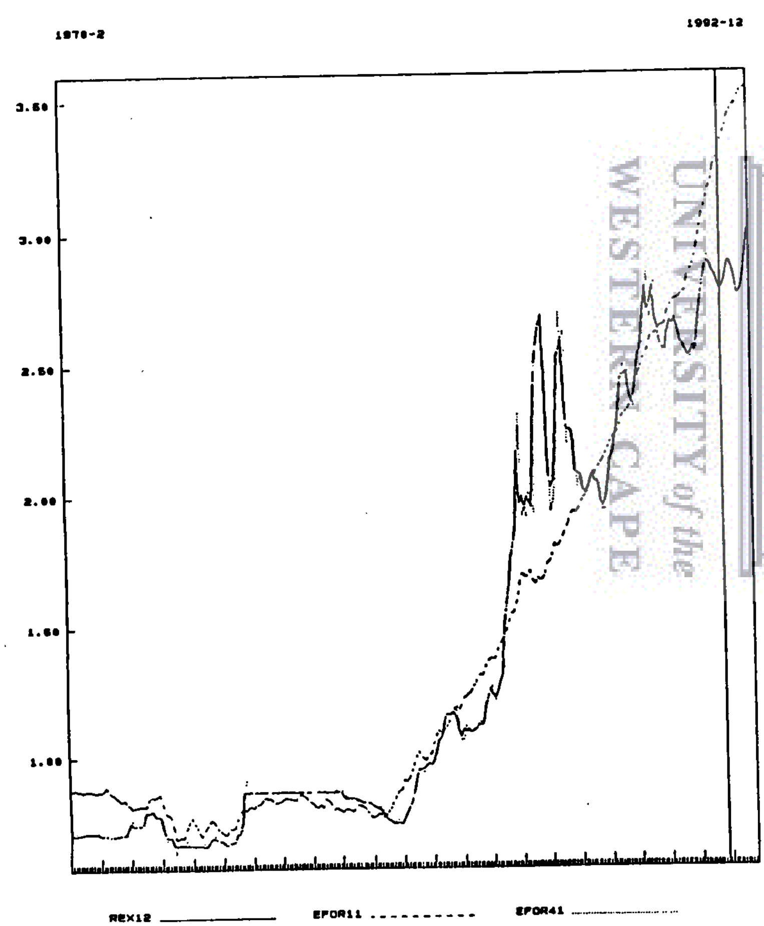
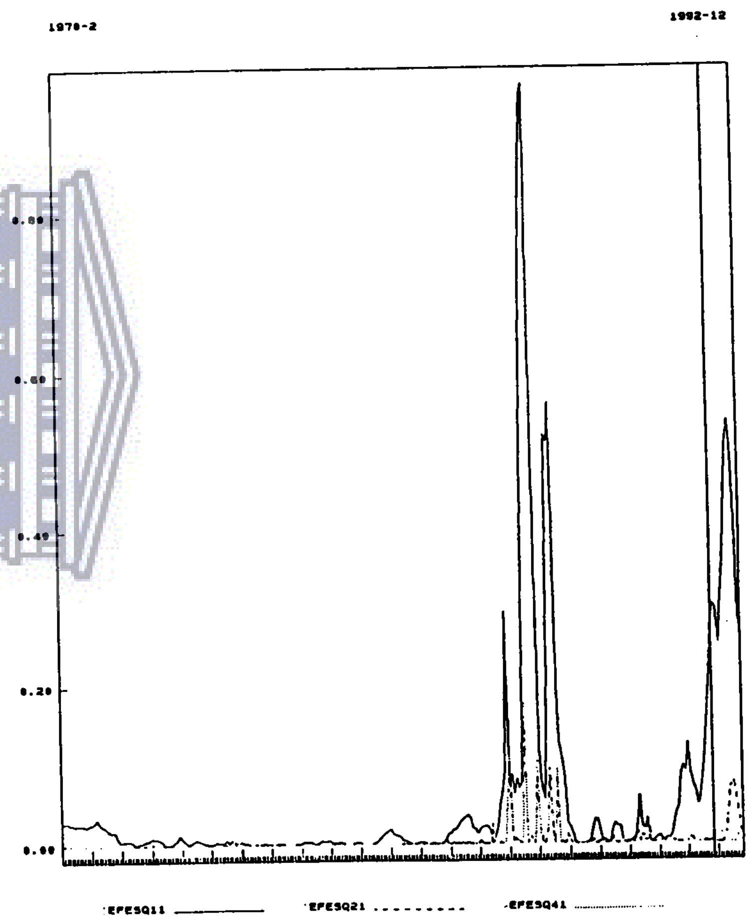


FIGURE 13.12(b): FORECAST ERRORS SQUARED FOR MODEL E-1, STAGES 1,2 AND 4



Although the results presented above do not answer the question of whether the VPR model for exchange rates is significantly better than the other models in the primary criterion, root mean square error, the finding that the VPR model almost invariably has the lowest RMSE over all horisons, causes one to unambiguously assert that the fixed coefficient models do not perform significantly better in ex-post forecasting performance than the VPR models in this study.

13.4 SUMMARY AND CONCLUSION

All the conclusions must be seen against the background of the limited number of functions that were available for evaluation. Empirical research of this nature, for which there is no solid theoretical foundation leading to *a priori* hypotheses, requires a substantial number of tests before any clear patterns emerge.

The problems attached to meaningful power comparisons lead the user to use the battery of tests as an exploratory technique rather than a formal set of tests of hypotheses. This view, of rather using these tests in an exploratory fashion, is supported by the often contradictory results obtained from the various tests.[24]

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[24] It will also be useful to examine the plots of the components of β_r estimates against time to try to identify the source of departures from constancy indicated by the Cusum, Cusum of Squares or Fluct test. Further, to help locate the point of change it is often informative to look at the set of plots which are obtained by running the analysis backwards through time as well as forwards.

The tests for structural stability applied to all econometric equations in the study lead to results that reinforce the tentative conclusions reached from the tables and figures above. Econometric equations in South Africa have to a large extent become structurally unstable, especially after 1980. This is in accordance with the a priori belief that South Africa is experiencing a period of structural adjustment, both in the economic and political arenas. These results are also supportive of the findings of Smit and Wesso (1988) pertaining to structural instability observed in some macro econometric equations in South Africa. Similiar conclusions have been reached by Smit (1987) who applied a number of these tests on econometric equations based on qualitative business survey data.

In some of the cases studied it is found that when the forecasting performance of the different model-stages are compared, using goodness-of-fit criteria over the estimated period, the VPR models do not necessarily perform better than its fixed coefficient counterparts. The situation is, however, different when these criteria are evaluated over the extended forecasting period.

By accepting that the aim of inference is to generate predictions for future values, one can see that the problem of comparing alternative model specifications is resolved by comparing the accuracy of predictions the models generate and choosing the model that predicts best. It is suggested in this study that by allowing some of the coefficients in an economic relationship to vary over time may contribute to improved forecasts. The economics literature has long recognised that slopes of economic relationships may not be constant through time because of aggregation effects and policy changes. Therefore, the assumption of time-varying coefficients cannot be so easily dismissed on the grounds that increasing the complexity of the models used to generate predictions does not necessarily lead to better predictions.

In particular, many of the structural stability tests applied to investment models reject the null hypothesis of parameter stability indicating that the stochastic-coefficient presentation may be more appropriate than a fixed coefficient characterisation. Investment decisions can serve as barometers of adjustment. Policymakers typically focus on commodity prices and production; yet important insights about both are revealed by changes in the capital stock and the adjustment rate to new equilibrium levels. The estimated speed of adjustment ranged widely through time and jumped abruptly during the late 1970's and early 1980's in response to, amongst others, increased political instability in South Africa. The ability of the VPR model to adequately represent several types of nonstationary processes and adapt quickly to changing economic conditions enables it to give better predictions than the flexible-accelerator models.

A credible forecast of the effects of policy changes on investment, therefore, requires capturing the changing structure of investment decision making. Methods that fail to account for structural change in the form of parameter variation may lead to misleading predictions and ill-timed policy actions.

The conventional Cobb-Douglas production function is compared with those implied by a new formulation of the Cobb-Douglas production function and differs from the conventional one in that the labour or capital coefficients of the former are stochastic rather than fixed. The use of time varying coefficients in place of fixed means or coefficients can make a substantial difference in the estimate of productivity.

All the estimates of the stochastic-coefficient aggregate Cobb-Douglas function have the right sign, whereas some of the estimates of the fixed coefficients counterpart have the wrong sign in some cases. More important, the production, aggregation, and estimation theories that lead to the former are more general than those leading to the latter. For these reasons, one should prefer the productivity formula based on the stochastic-coefficient production function to that based on the means of time varying coefficients. Predictive testing -

extrapolation to data outside the sample period - may provide sharper discrimination.

The introduction of time-varying parameter models enhances the forecasting performance of an important class of structural exchange rate models, indicating that some kind of instability is present in the data. It is found that the ex-post predictions of the structural models are uniformly dominated by the VPR forecasting rule. With respect to monetary-type models of exchange rate determination, the findings indicate that on average they leave a lot to be desired as descriptors of the behaviour of relative prices of the South African currency during the recent floating exchange rate period. This research, therefore, suggests that the time-series properties of the parameters should be exploited effectively and be incorporated into the exchange rate models to improve predictions.

Because there are many explanations for parameter variability, one cannot identify the specific reason or reasons determining why the coefficients in a time-varying parameter model are nonconstant. Reasons for structural instability are, therefore, given only on a speculative basis. One would have preferred reporting results that explain the variability of coefficients with sound and rigorous economic principles. But until economic theory postulates empirically implementable hypotheses addressing why exchange markets are so volatile and why model coefficients vary over time, one can at least examine the type of stochastic coefficient models presented here before rejecting existing exchange rate models out of hand. This study demonstrates that stochastic coefficient models of exchange rate determination can be useful in improving the accuracy of forecasts of exchange rates.

It is, therefore, clear from the study that traditional econometric assumptions of fixed parameters over time may often lead to misspecified models, and by the same token, to a reduction in both estimation and prediction efficiency. There appears to be no way that the practicing econometrician can escape dealing with this reality in the present economic and political situation. Supplementing conventional regression output with a battery of specification and structural stability tests therefore will make it harder for results to appear "significant" that are the product, whether intentional or unwittingly, of some data mining process.

There are many ways to improve upon empirical econometric work. One of these would certainly be to apply the tests and models used in this study more widely. In practise one would expect relationships between variables to be "dynamic" rather than "static"; hence simple approximations cannot be expected to remain valid over indefinitely long periods of time unless we are prepared to modify continually the values of the parameters so as to allow these models to adapt themselves to "local" conditions. The techniques used for monitoring, testing and modelling changes in parameter values should therefore prove useful in many fields of application.

If the body of evidence regarding structural instability in South African econometric equations support the findings of this study, the processes of change in regression parameters should become the focus of the efforts of the applied econometrician, with far greater emphasis on time varying parameter estimation.

SUMMARY

The purpose of this particular study is sixfold, namely: (a) to discuss forecasting and structural changes in the South African economy; (b) to provide a complete overview of aspects of testing for structural change; (c) to provide a simple, unified and systematic treatment of the alternative forms of time-varying and random coefficient models; (d) to investigate economic forecasting under conditions of structural change; (e) to apply the above methods empirically in the context of the South African economy; and (f) to provide recommendations for future econometric research. Thus, the thesis has two important characteristics: (i) the topics covered in textbooks, journals and articles are discussed in the context of the random coefficients model as opposed to the constant coefficient models (an assumption which is restrictive and often unnecessary); and (ii) an application of a suitable procedure in estimating the parameters of a random coefficient regression model for South Africa.

The thesis is divided into six parts:

The introductory part discusses not only the role of statistics in the detection and assimilation of structural change, but also the relevance of structural stability tests in the evaluation of econometric models. The historical development of these methods are reviewed as well as some of the possible

causes of coefficient variation in South Africa. The interest is centered mainly on those aspects which, on the basis of a priori considerations, might have had a determining influence on structural stability in South African econometric models over the past two decades.

The next part provides some reflections on forecasting in situations of structural change. The crisis in forecasting and econometric modelling is discussed and some fundamental issues of economic forecasting in South Africa are focused on.

The third part deals with the identification of structural change. The chapters combined under this heading are concerned with detection of parameter nonconstancy. The procedures discussed range from classical methods, such as the Chow and CUSUM tests, to new concepts, particularly those based on maximum likelihood statistics. Several sections assess the conditions under which these methods can be applied and their robustness under such conditions.

Econometric model building in the presence of structural change is discussed in the fourth part. This part addresses models that are in some sense generalisations of constant parameter models, so that they can assimilate structural change. This is one of the most important parts in this study and it surveys a wide variety of random coefficient models for easy reference. An overview of forecasting methods which can be used under conditions of structural change is provided. Chapter 6 reviews a model-based approach to adaptive estimation of regression parameters and discusses in detail the use of the Kalman filtering technique in this case. Chapter 7 deals with changing and random coefficient models. It comprehensively reviews the related literature and provide some guidelines for model choice. The final chapter under this part is concerned with autoregressive conditional heteroscedastic (ARCH) models which are used to model a series of which the variance is changing with time. Although relatively new, this model is quickly gaining wide acceptance amongst applied econometricians.

The fifth part discusses theoretical models for fixed investment, production and exchange rates as well as the empirical estimation of such models in the South African economy. The method of investigation also forms part of this section.

South Africa proved to be particularly well suited as a test-bed to explore the effects of and remedies for structural instability in econometric relationships (see Smit and Wesso, 1989). A final part, therefore, deals with real-life structural change situations in South Africa and is limited to the estimation of a suitable model for private fixed investment, production and exchange rates, all of which are treated in any standard macro-econometric model.

It is hoped that this study will contribute to stimulate the interest of South African statisticians and econometricians in this topic and will help to improve models for analysing real-world phenomena and consequently the reliability of results.

OPSOMMING

Die doel van hierdie spesifieke studie is sesledig, naamlik: (a) om vooruitskatting en strukturele veranderings in die Suid-Afrikaanse ekonomie te bespreek; (b) om 'n volledige oorsig te gee oor aspekte van toetsing vir strukturele veranderings; (c) om 'n eenvoudige, univormige en sistematiese hantering vir verskeie vorms van tydveranderende en ewekansige koëffisiënt modelle te gee; (d) om ekonomiese vooruitskattings onder toestande van strukturele verandering te ondersoek, (e) om bogenoemde tegnieke empiries toe te pas op die Suid-Afrikaanse ekonomie; en (f) om aanbevelings te maak vir toekomstige navorsing in ekonometrie. Dus het die verhandeling twee belangrike eienskappe: (i) 'n literatuurstudie oor relevante onderwerpe in boeke, tydskrifte en artikels word bespreek in die konteks van ewekansige veranderende koëffisiënt modelle teenoor konstante koëffisiënt modelle ('n aanname wat beperkend en soms onnodig is); en (ii) 'n toepassing van 'n geskikte prosedure in die skatting van die parameters vir 'n ewekansige koëffisiënt regressie model in Suid-Afrika.

Die verhandeling is in ses dele verdeel:

Die inleidende gedeelte bespreek nie net die rol wat statistiek speel in die bepaling en verwerking van strukturele verandering nie, maar ook die relevansie van sodanige strukturele stabiliteitstoetse in die evaluering van ekonometriese modelle.

Die historiese verloop van hierdie metodes word ondersoek sowel as die aspekte wat moontlik koëffisiënt variasies kan veroorsaak in Suid-Afrika. Aandag is hoofsaaklik geskenk aan daardie faktore wat op 'n a priori basis 'n bepalende invloed, oor die afgelope twee dekades, op die strukturele stabiliteit van Suid Afrikaanse ekonometriese modelle kon gehad het.

Die daaropvolgende gedeelte lig 'n paar gedagtes oor vooruitskatting ten tye van strukturele verandering toe. Die krisis in vooruitskatting en ekonometriese modellering word bespreek met die fokus op strydvrae oor vooruitskatting in Suid-Afrika.

Die derde gedeelte handel oor die identifisering van strukturele verandering. Die hoofstukke onder hierdie gedeelte het betrekking op die bepaling van parameter verandering. Die prosedures wat bespreek word strek vanaf klasieke metodes, soos die Chow en CUSUM toetse, tot meer onlangse konsepte, veral die wat gebaseer word op maksimum aanneemlikheids-skattings tegnieke. Van die afdelings bepaal die toestande waaronder hierdie metodes toegepas kan word asook die robuustheid van sodanige toestande.

Ekonometriese modellering onder toestande van strukturele verandering word bespreek in die vierde gedeelte. Hierdie gedeelte verwys na modelle wat in 'n sekere sin as veralgemenings van konstante parameter modelle beskou kan word, sodat strukturele verandering hanteer kan word. Dit is een van die mees belangrikste gedeeltes in die studie en ondersoek 'n wye verskeidenheid van ewekansige koëffisiënt modelle waarna maklik verwys kan word. 'n Oorsig van verskeie vooruitskattingstegnieke word gegee wat gebruik kan word ten tye van strukturele verandering. Hoofstuk 6 gee 'n oorsig van 'n model-gebaseerde benadering vir die aanpassing van regressie parameters en bespreek in die besonder die gebruik van die Kalman filter tegniek in so 'n geval. Hoofstuk 7 handel oor veranderende en ewekansige koëffisiënt modelle. Dit bied 'n omvattende oorsig van relevante literatuur en verskaf sekere riglyne in die keuse van 'n ekonometriese model. Die finale hoofstuk in hierdie gedeelte handel oor autoregressiewe voorwaardelike heteroskedastiese (ARCH) modelle wat gebruik kan word om 'n reeks met 'n veranderende variansie oor tyd te modelleer. Alhoewel dit relatief nuut is, word hierdie model alhoemeer aanvaar deur toegepaste ekonometrici.

Die vyfde gedeelte bespreek teoretiese modelle vir vaste investering, produksie en wisselkoerse sowel as die empiriese skatting van sodanige modelle in die Suid Afrikaanse ekonomie. Die metode van ondersoek maak ook deel uit van hierdie afdeling.

Suid-Afrika is 'n geskikte toepassings area om die effek van en oplossings vir strukturele onstabiliteit in ekonometriese vergelykings te ondersoek (sien Smit en Wesso, 1989). 'n Finale gedeelte handel dus oor gevalle van werklike strukturele veranderings in Suid-Afrika en is beperk tot die skatting van 'n geskikte model vir private vaste investering, produksie en wisselkoerse, wat normaalweg in enige standaard ekonometriese model ingesluit word.

Daar word gehoop dat hierdie studie sal bydrae om die belangstelling van Suid-Afrikaanse statistici en ekonometrici in hierdie onderwerp te stimuleer en dat dit dus sal help om modelle te verbeter vir analitiese doeleindes en gevolglik ook die betroubaarheid van resultate.

GLOSSARY

This glossary contains definitions of the technical terms used in the main body of the text. A particular definition may involve terms which are defined elsewhere in the glossary.

AIC (Akaike Information Criterion)

The AIC is a figure of merit to determine the model order of state space and Box-Jenkins models. The statistic is very similar to the BIC (see below), however it does not penalise model complexity as severely. Thus it will sometimes opt for more complex models than the BIC. It is theoretically based on entropy concepts. Experimentally, minimisation of the AIC to determine model order has been proved sound and efficient. The AIC is computed as the log likelihood function for the model and fit data, less twice the number of independent parameters. There is not a great deal of evidence available as to which statistic is superior. What does exist suggests that for business data the BIC leads to more accurate models.[25] The BIC (see entry) is therefore a preferable criterion.

Autocorrelation Function

A time series in which the current value of the series depends on past values is called an autococorrelated time series. If a series is stationary, the dependence in the series between two points separated by k time units can be described by the autocorrelation coefficient *rho*(k) at lag k. The autocorrelation coefficient measures the extent to which a value of the series above or below the mean at time t tends to be followed by a value of the series above or below the mean k time units later. The plot of rho(k) against k for k = 1,2, ... is called the autocorrelation function of the series.

[25] See Koehler and Murphree (1986): A Comparison of the AIC and BIC on Empirical Data, Sixth International Symposium on Forecasting, Paris.

Autoregressive (AR) Processes

This form of regression expresses the dependent variable in terms of its own previous values (i.e. its "lags"). The assumption is that future data points may be expressed as linear combinations of past observations.

BIC (Bayes Information Criterion)

Like the AIC (see entry) the BIC is an order selection criterion. This statistic is used to select the model that is likely to forecast (out of sample) most accurately for a particular data set. Invented by Schwarz (1978), the BIC generally leads to less complex models than the AIC. Since highly complex models often fit the historical data well but forecast poorly, the BIC balances a reward for goodness of fit with penalty for model complexity. Koehler and Murphree (1986) showed that, for a large sample of data from the m-competition (see Makridakis et al., 1982), the BIC results in more accurate out-of-sample forecasts than the AIC (see Schwarz, 1978). Therefore, selecting the model that minimises the BIC will generally lead to the most accurate forecasts.

Chi-Squared Statistic

The Chi-Squared distribution is a statistical distribution used in many different hypothesis tests, often to test the goodness of fit of a model. The Ljung-Box test and Lagrange multiplier tests result in chi-squared statistics. In the Ljung-Box test, the values of the residual autocorrelation function are squared, summed, and normalised to obtain a statistic that measures the extent to which the errors depart from the hypothesis that they are serially uncorrelated normally distributed random variables.

Durbin-Watson Test

The Durbin-Watson statistic is the ratio of the sum of squares of the differenced residual errors (numerator), to the sum of squares of the undifferenced errors (denominator). When the first lag autocorrelation of the residuals is zero, this statistic equals 2.0. When the errors are positively correlated, the statistic is less than 2.0 and, when they are negatively correlated, it exceeds 2.0. The Durbin-Watson test is used to test whether the residual errors are in fact correlated.

Forecast Horison

The number of periods that are forecasted.

Heteroscedasticity

The term heteroscedasticity was coined by Pearson to refer to a process in which the variance/co-variance of the errors is changing over time. A homoscedastic process is one in which the variances and covariances are unchanging.

Lag

The difference in time units of a series value and a previous series value. Thus y_{t-k} lags y_t by k periods.

Lagrange multiplier test

The Lagrange multiplier test is used to test a null hypothesis H_O against a less restrictive alternative hypothesis H_1. The test is constructed from the information matrix and the gradient of the likelihood surface, both evaluated under H_O, so the model need not be estimated under H_1. The test statistic is asymptotically distributed as chi-squared(n), where n is the number of linear constraints under H_O. The Lagrange multiplier test is asymptotically equivalent to the Wold test, and to the likelihood ratio test, but often requires much less computation. See Engle (1984) for a detailed discussion of these three tests.

Mean Squared Error (MSE)
A statistic that is used as an indication of model fit. It is calculated by taking the square root of the average of the squared residual errors.

Model Building
An iterative process for developing a model beginning with some information about the form of the problem and with data. The model building process is usually conceived of in terms of three distinct phases; identification (or model specification), estimation (model fitting) and checking (or refinement if necessary). This three stage process is repeated until some criteria of model adequacy is met.

NID(μ, σ)
Normally independently distributed with mean μ and standard deviation σ.

Residual
The difference between a predicted value and a true value, i.e. the error. In time series analysis, the residual is often called the innovation, i.e. the part of the observation that cannot be explained by a statistical model based on previous data.

R-Square (corrected for mean):
This statistic is the fraction of variance of the dependent variable around its mean that is explained by the model.

Adjusted R-Square:
This statistic is the fraction of variance of the dependent variable explained by the model with an adjustment for the number of parameters in the model. The two different forms of the R-square each have their proponents. Most authors prefer the R-square corrected for the mean.

Standard Forecast Error:
This statistic is the one step forecast error for the model over the historical period. For regression this equals the standard error regression.

APPENDIX A

KEY TO SYMBOLS USED FOR VARIABLES

FM1Q	Monetary sector: money (M1)
FRLE4	Long-term interest rate: Escom (NSA)
FRSB4	Short-term interest rate: 3 month bankers' acceptances (NSA)
IDT1	Depreciation allowances: total at constant 1985 prices
IPO1	Fixed investment expenditure: private, excluding residential buildings and agriculture, at constant 1985 prices
IPRB1	Fixed investment expenditure: private residential buildings, at constant 1985 prices
IPZA1	Fixed investment expenditure: private, excluding agriculture, at constant 1985 prices
K1	Capital stock; total, at constant 1975 prices
KPO1	Capital stock: private, excluding residential buildings and agriculture, at constant 1985 prices
KPR1	Capital stock: private residential buildings, at constant 1985 prices
NET	Employment in the non-agricultural sectors
PI	Price deflator: total fixed investment expenditure (1985=100)
PVI	Manufacturing production index
REX12	R/$ exchange rate
REXDM$	DM/$ exchange rate
REXPPP	Purchasing power parity rate
TREND	Time trend dummy
YCUB	Capacity utilisation (derived from quarterly production function: YFZPA1/YFPOT3*100)
YCUM	Capacity utilisation (manufacturing)

YCUX	Capacity utilisation (derived from yearly production function: YFZPA1/YFPOT3*100)
YD1	Real disposable income
YF1	Gross domestic product, at factor cost: total, at constant 1985 prices
YFPA1	Gross domestic product, at factor cost: private agriculture, at constant 1985 prices
YFZPA1	Gross domestic product, at factor cost: total, excluding private agriculture, at constant 1985 prices
YGDE1	Gross domestic expenditure, at constant 1985 prices

APPENDIX B

NOTES TO ABBREVIATIONS USED IN THE TEXT

AD(variable,t)	Absolute difference of a variable over t periods
AIC	Akaike Information Criterion
Alpha	Autoregressive slope coefficient
AR	Autoregressive parameter
AUTO(1)	Autocorrelation coefficient, lagged one period
BIC	Bayes Information Criterion
CDF(n)	Cobb-Douglas Production Function number n
CES(n)	Constant Elasticity of Substitution Production Function number n
DW	Durbin-Watson d statistic
E-n(j)	Exchange rate function number n Stage j
EERR(jn)	Residual errors of exchange rate function number n Stage j
EFESQ(jn)	Forecasting error squared for exchange rate function number n Stage j
EFOR(jn)	Forecasting track record of exchange rate function number n Stage j
EPAR(4n)	Time varying coefficient of exchange rate function number n Stage 4
ESIG(3n)	ARCH standard deviations of exchange rate function number n Stage 3
I-n(j)	Investment function number n Stage j
IERR(jn)	Residual errors of investment function number n Stage j
IFESQ(jn)	Forecasting error squared for investment function number n Stage j

IFOR(jn)	Forecasting track record of investment function number n Stage j
IPAR(4n)	Time varying coefficient in investment function number n Stage 4
IPO1(j)	Regression function of dependent variable IPO1, Stage j
IPZA1(j)	Regression function of dependent variable IPZA1, Stage j
ISIG(3n)	ARCH standard deviations for investment function number n Stage 3
ln	Natural logarithm
P-n(j)	Production function number n Stage j
p-value	Probability value of a type I error
PERR(jn)	Residual errors of production function number n Stage j
PFESQ(jn)	Forecasting error squared for production function number n Stage j
PFOR(jn)	Forecasting track record of production function number n Stage j
PPAR(4n)	Time varying coefficient in production function number n Stage 4
PSIG(3n)	ARCH standard deviations for production function number n Stage 3
R^2	Coefficient of determination
$\bar{R}^2$	Corrected coefficient of determination
REX12(j)	Regression function of dependent variable REX12, Stage j
Rho(1)	First-order autocorrelation coefficient
RMSE	Root mean square error
SEE	Standard error of estimation
SQRT	Square root
t	Student's t-statistic
TPF(n)	Transcendental production function number n

Var	Variance
Variable(t)	Variable, lagged t periods
W-D	Watson-Davies test

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