# AN INVESTIGATION INTO THE USE OF GRAPHIC CALCULATORS FOR TEACHING THE MANIPULATIVE ASPECTS OF SCHOOL CALCULUS

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# DECLARATION

I declare that AN INVESTIGATION INTO THE USE OF GRAPHIC CALCULATORS FOR TEACHING THE MANIPULATIVE ASPECTS OF SCHOOL CALCULUS is my own work, that it has not been submitted before for any degree or examination in any other university, and that all the sources I have used have been indicated and acknowledged by complete references.

JEROME KEITH ERENTZEN DATE: JUNE 1995

SIGNED: ..............................

# ABSTRACT

In a teaching experiment conducted over a period of three weeks, matric pupils were exposed to the teaching of introductory calculus in a graphically enriched environment. Materials were designed to explore and experiment with the rules of differentiation by making use of a graphics calculator, the CASIO *f(x)* 7000GB.

The motivation for implementing this approach to the teaching of specifically calculus is that the current mathematics curriculum is still dominated by a manipulations - skills mentality which does not allow much space for pupils to explore, experiment with, and create their own mathematics. Mastery of computational skills is still the primary means of assessing pupils' competency in mathematics and as such, does not take advantage of the avenues opened up by the introduction of innovative microcomputer technology.

Underpinned by a constructivist theory of knowledge acquisition which recognizes that pupils possess a mathematical reality of their own and actively construct knowledge in their idiosyncratic ways, this minithesis reports on the results of the three week long teaching experiment in which it was found that under appropriate conditions, pupils were able to formulate the different manipulation rules of differentiation when exposed to a graphically enriched environment. This report also highlights the necessity of ongoing research in the design and testing of materials to be implemented in a radically different curriculum.

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This thesis is committed to the memory of the late Rachel Swartz, my mother, who so unselfishly made my university career possible.

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# CONTENTS

# Chapter One:
# Introduction and Overview

*"Situations of practice have unique characteristics of complexity, specificity, instability, disorder and indetermination."*

***Joao Pedro da Ponte (1994:203)***

## *1.1 The need for change*

Mathematics has become one of the most important subjects in the school curriculum this century. As modern societies have increased in complexity and as this complexity has accompanied rapid developments especially in the field of microcomputer technology, so the learning and teaching of mathematics have come under increased scrutiny. Yet despite this intense focus on the learning and teaching of mathematics specifically and the role of innovative computer technology in education in general, relatively little change can be seen in the classrooms as is so succinctly pointed out by Engel (1979:250):



> A typical school has one or two (computer) terminals. Lack of terminals is currently one of the biggest bottle-necks. Student access is very restricted and this severely limits individual exploration. Another problem is the narrow base of educational computing. Usually one or two mathematics teachers are involved. The majority of schools are operating within an information vacuum. Many schools are still "reinventing the wheel".

This is still very much the case in South Africa where the overwhelming majority of schools still do not have any computing facilities and where these are available, it is still typically used for administrative purposes only.

A restrictive factor which further bedevils the teaching of mathematics is the rigid approach adopted for the teaching of standard algorithms. It is also well known that the teaching of mathematics is dominated by what Julie (1993:347) calls the "manipulations-skills mentality". This extremely time-consuming approach refuses to acknowledge that there are more profitable ways of implementing the curriculum and so devalues other more innovative ways of teaching standard algorithms. The strict adherence to the manipulation of algorithms and development of skills denies the recipients, the pupils, the time to explore the subject matter and develop their own concepts with respect to any given subject matter. An inevitable consequence of such an approach is that teachers themselves are inhibited from assessing whether pupils have gained a conceptual understanding of work taught through transmission only. Very often the criteria for gauging whether a specific area of work has been understood is when pupils can regurgitate a specific bit of mathematical knowledge "successfully". This approach to learning ignores research findings about what constitutes successful learning. In this regard Terezinha Nunes (1993:40) reports:

> Pupils success is not evaluated on the basis of whether a specific bit of mathematical knowledge was accomplished. Instead pupils success is evaluated on the basis of the progressive sophistication of their methods and their building of relationships between concepts, symbols, notations and terminologies.

It is my contention that a particular part of the mathematics syllabus namely, introductory calculus for matriculants, is especially susceptible to the manipulations - skills approach. A mere cursory review of current textbook approaches to introductory calculus confirms this. Most texts feature a theorem-proof style as the primary method of exposition. No significant texts depart from this approach. The typical text begins with a short review of basic concepts such as tangents to a curve and then introduces the notion of limits.

This is followed by the idea of continuous functions and then the concepts of a derivative. The treatment of derivatives starts with the limit orientated definition but almost immediately proceeds to algebraic differentiation formulas followed by curve sketching of cubic functions to problems of determining maxima and minima of functions. Variation to this approach is only very slight. Marvin L. Johnson (1991:66) calls this approach an "unprofitable practice" and he reflects:

> It is clear that calculus texts are in a rut... the problematic state of calculus instruction is due to the presuppositions embodied in the way it is taught.

It is no wonder then that research conducted by Confrey (1989:1) found that many students experienced a devastating intimidation by and alienation from mathematics leading to an avoidance of the subject that often would extend throughout their lives. Such an indictment on the mathematics community need not be so since there is now a body of research findings (Fey, J.T. :1989, p.237; Ruthven, K. : 1989, p.233) which conclude that teachers could enhance their teaching considerably if they were to make use of innovative and currently available microcomputer technology. In this regard James T. Fey (1989:237) writes:



> One of the most important tasks in mathematics education today is the revision of curricula and the teaching methods to take advantage of electronic information technology. Developments in this decade alone have presented us with inexpensive and powerful hardware and software tools that challenge every traditional assumption about what we should teach, how we should teach and what students can learn.

It becomes clear then, that there is a dire need for change in the way we teach mathematics. The use of the chalk-board as the primary technological tool through which knowledge is imparted is long outdated. We can no longer ignore the relevant role that currently available technology is playing in education today.

The results of an ostrich approach to the technological revolution could prove incalculable. We can no longer excuse ourselves for lack of available funds or time when we consider that innovative technological devices are now relatively inexpensive and more readily available to teachers and students.

### *1.2 The research question*

Given the fact that the rigid style of teaching mathematics based on a manipulations- skills approach is decreasing in importance and is considered to be an unprofitable practice, and given the fact that significant technological advances especially in the field of microcomputer technology has endowed us with a spectrum of innovative technological devices such as the graphic calculator, it is incumbent upon practising mathematics teachers to seek equally innovative ways of utilizing such devices and at the same time de-emphasizing the rigid algorithmic approach to teaching mathematics. The motivation for undertaking this research is to see whether a change to the approach to teaching mathematics by making use of currently available technology would conceiveably enhance the pupils opportunities of exploring mathematics. For this purpose a teaching experiment is engaged in to answer the following research question:



**Does the use of a graphic calculator in a calculus course for matriculants permit pupils to formulate manipulation rules of differentiation in a graphically enriched environment?**

The research is underpinned by a constructivist theory of learning and as such, the research allows for the pupils to express themselves in their idiosyncratic ways, taking into account the mathematical reality of the pupils thus allowing them the freedom to act in ways they choose. As such, particular attention is paid to how they formulate the following rules of differentiation:

1. *If* $y = x^n$, *then* $dy/dx = nx^{n-1}$

2. $D_x[kf(x)] = kD_x[f(x)]$

3. *If* $f(x) = k$, *k a constant, then* $f'(x) = 0$

4. *If f and g differentiable with respect to x then:*

$$D_x[f(x) + g(x)] = D_x[f(x)] + D_x[g(x)]$$

### *1.3 Justification for the research*

From the introduction to this chapter it is evident that the rigorous style of teaching mathematics based solely on a manipulations-skills approach is fast becoming archaic and is also evidently unprofitable. Furthermore, the feeling is that such approaches result in pupils experiencing an unnatural fear for and alienation from mathematics which often extends throughout their lives. The introduction of innovative and currently available microcomputer technology holds the promise of rejuvenating the teaching of mathematics. The use of graphic calculators in particular, holds much promise in that they are not dependent on a supply of electricity, are relatively cheap, and they possess some rather unique characteristics which need to be explored. Their particular features of allowing the user to move through various modes of functioning, that is from a graphic mode to a symbolic mode (and vice versa) of functioning, the larger display window, and the fact that they are portable affording every pupil the opportunity to have access to one, are all features from which the teaching of mathematics could benefit.

As a multi-representational tool, the graphic calculator certainly gives the pupils the opportunity to explore mathematical ideas in recognizable forms, assists them in making significant progress towards learning certain concepts based on their own unique experiences through interaction with the calculator, and allows them the freedom to act in ways they choose. However, the most profound justification for undertaking the research is found in the words of John Mason (1994:181):

> In mathematics it is possible to accept a result stated by another mathematician (although one ought to test it for oneself), but in education it is impossible to build upon the the proposed 'result' without testing it in one's own experience and situation. If it checks out, sheds light, sharpens awareness, or extends the range of actions, it will be taken as valid for that individual, otherwise it will fade into the background.

This is of particular significance to especially the teaching and learning of mathematics in South Africa where the efficacy of computer technology in education is still in its embryonic stages.

### *1.4 Summary of this chapter*

Mathematics has become one of the most important subjects in school. Its increased importance is accompanied by rapid technological changes. Yet, the teaching of mathematics is still dominated by a manipulations -skills mentality. There is evidently a dire need for change in the way we teach mathematics, but especially introductory calculus. Based upon this assumption the research question has been framed and justification provided for undertaking this research.



### *1.5 Structure of the minithesis*

A brief outline of the remaining chapters of this research report is given below:

In chapter 2 a study of the relevant literature is undertaken to justify the use of the graphic calculator as an exemplar of currently available and accessible microcomputer technology which serves as an appropriate mediating tool to implement a different approach to the teaching and learning of calculus for matriculants. The graphic calculator is placed into perspective by first examining the role of computers in education in general and then looking specifically at the role of computers in mathematics education with specific emphasis on the role of the graphic calculator.

In chapter 3 the theoretical framework underpinning this research is examined. This chapter therefore gives an account of constructivism as a theory of knowledge acquisition that recognizes the unique reality of pupils, as being an appropriate theory to underpin this research.

In chapter 4 the research design adopted for conducting the research experiment is outlined. Developmental research as an appropriate methodology is discussed as well as each phase engaged in the research.

The implementation of the project is outlined in detail in chapter 5 as well as the didactic deliberation sessions.

In chapter 6 the research project is analyzed and evaluated in relation to the research question and the theoretical framework. This is followed by a brief conclusion and some possible recommendations.

# Chapter Two:
# Technology in Mathematics Education

> *"The use of information technology is a - arguably the - major current area of development in mathematics education... the major impact on professional thinking and practice has taken place in the last fifteen years, in the wake of successive waves of technological innovation which have swept information into the workplace and the home, as well as the school."*
>
> *Kenneth Ruthven (1989:233)*

## *2.1 Introduction*

The intrusion of technology into the classroom is not new. More than a decade ago David C. Lukens (1984:15) remarked that:

> ...We would do well to remember that technology has been part of mathematics for a long time.



What is now becoming abundantly clear is that current information technology pervades every sphere of human endeavour. The advent of the Technology Revolution (Atiyah, 1985:43; Blakely, 1985:59; ICMI, 1986:1) has seen the rapid development of computers, communication systems, automatic teller machines, industrial robots and automated manufacturing systems. Microelectronics appears to be the most pervasive of these technologies and emanating directly therefrom is the astonishing development of the microcomputer once considered by Sir Ieuan Maddock, former British Government Chief Scientist, as :

> the most remarkable technology to confront mankind.
>
> (Bamber, 1989:5)

The purpose of this chapter is to place the graphics calculator as an exemplar of current micromputer technology into perspective, and to examine its possible utilization in the teaching of mathematics.

### *2.2 Technology in the classroom*

Educational technology as it is used in this chapter refers, according to Richmond, (1970:5) to:

> ... the application of scientific knowledge about learning, and the conditions of learning to improve the effectiveness and efficiency of teaching and training.

Implicit in this definition is the fact that it is an integrating process involving people, procedures, devices, ideas and organization so that problems could be analyzed, solutions could be devised, managed, implemented and evaluated in all aspects of learning. In essence, the search for finding better ways of doing things, results in the development of newer technologies that become an extension of one's self through an interactive and reciprocal process. Julie (1991:116) refers to the development of even the humble compass as an example of a technological device that had come about as a result of the search for finding better ways of doing things.

This search for finding more efficient and effective ways of doing things has resulted in what is now commonly called the Information Revolution or Technological Revolution whose impact parallels and even surpasses that of the Industrial Revolution.

The ubiquitous book was a revolutionary technological advance of about five centuries ago which was brought about by the invention of the Gutenberg press. Its impact on fifteenth century Europe revolutionized access to learning and effectively challenged the dominance of the written language as a means of maintaining hegemony and privatization of knowledge and ideas. The invention of the Gutenberg press effectively signalled the end of the Mediaval Period and, according to J. Allen Watson (1987:7):

> ...qualitatively altered and restructured human thought to the point that many of our arts and sciences trace their origins to the period following 1460.

We have come a long way since the invention of the Gutenberg press. Whilst in past centuries especially in epoch making periods such as the Renaissance and the Reformation, *ideas* appear to have been the mainstream and dynamic of societal metamorphosis, in the modern world the emphasis is firmly placed on technology with changes resulting from the implementation of technology being counted in decades and of late, even in years,, not centuries. A currently available technological device that is exciting the mathematical community is the graphics calculator which is examined more closely in section 2.2.3. What is important at this juncture is that the impact of technology on learning and teaching should be assessed and analyzed.

### *2.2.1 Technology: Its impact on mathematics teaching*

The history of the development of computers and especially microcomputers is well documented. (See: Thomas and Kobayashi, 1987:25-34; Richmond, 1970:35-80). Its intrusion into especially the mathematics classroom has had an impact on three specific areas that is the subject of much debate in countries around the world. That there is a special relationship between mathematics and microcomputer technology is evident from the fact that many researchers see mathematics as providing the impetus for the development of computer technology. Atiyah (1985:45) comments that:

> ... Historically, it was mathematical knowledge which provided the theoretical basis for computers.

Blakely (1985:59) holds a similar view when he states that:

> ... the subject matter of mathematics is intimately related to the operation of a computer.

This reciprocal relationship between mathematics and computer technology is put more aptly by Cathleen Heid and James T. Fey (1984:20) when they state:

> There is a special relationship between computers and mathematics. It was the search for a tool to perform rapid arithmetic and logical computations that led to modern computers:

Therefore, when considering the impact of microcomputer technology on the teaching and learning of mathematics one needs to take into consideration the reciprocal relationship between mathematics and computer technology.



According to Richmond (1970:27) technology has an impact on education, and consequently on mathematics education, in the following three areas:

- The education system
- General education
- Classroom instruction

Firstly, because science and technology are transforming modern societies both in developing and developed countries, there is a need for these technology oriented societies to structure their educational systems such that it ensures an adequate supply of scientists and technicians.

This demand for trained personnel has a direct consequence for the mathematics curriculum. What we should teach, how we should teach, and what students can learn, are direct outcomes of how the educational system is structured to meet the challenges of providing trained personnel. Consequently, there is a dire need to rethink the mathematics curriculum. In this regard James T. Fey (1989:237) writes:

> One of the most important tasks in mathematics education today is the revision of curricula and teaching methods to take advantage of electronic information technology.

Maria Bottino (1994:112) considers the revision of the mathematics curriculum as a dynamic change not emanating from within the mathematics community as a consequence of developments within that discipline, but as a natural outcome of:

> the great social and economic reality provoked by the impact of new information technologies.



Identified areas of curriculum revision include restructuring the existing curriculum to accommodate current information technology. One needs to examine how current technology can be incorporated into the existing curriculum, a process requiring the identification of new areas of study and how best to prepare pupils to deal with such changes. New methods of teaching and learning should accompany any curriculum changes and should be interdisciplinary. For effective and efficient curriculum changes to take place more fully and completely, Scanland and Slatery (1989:12) advises that:

> ... teachers should re-examine their personal philosophy of the teaching/learning experience, the nature of the teaching role, national and educational long range goals and present teacher/student outcomes of the educational process.

The second way in which technology relates to education in general but specifically to mathematics education, is the question of the general education of all citizens specifically in the sciences. The overriding premise is that orientation in the sciences would alleviate the pressures arising for a mathematically literate society, an especially difficult task in South Africa where inequity of resources is still a major setback. Laridon (1993:40) quotes a market research report that estimated that approximately 300 000 teachers would have to go through pre-service education in order to realize the goal of having a mathematically literate society by the year 2000.

The presence of technological tools in the classroom such as close circuit television, video recorders, and computers is the third way in which technology impacts on the teaching and learning of mathematics. These technological tools permit greater access to learning and increase the potential for teaching large numbers of students in innovative and exciting ways. A concomitant effect of these technologies is that teachers are required to in a sense, "re-invent the wheel" each time curriculum change is brought to or generated by the introduction of such technologies to the instructional process. Not keeping pace with technological advancement in the instructional process results in a noticeable gap between invention and utilization. It is within this area of technological application that this study is firmly rooted. The use of the graphic calculator as an exemplar of currently available and relatively inexpensive microchip technology is studied and its particular role in the formation of concepts in an introductory calculus course for matriculants, is examined and evaluated.

### *2.2.2 Calculators and computers in instruction*

The advent of the technology/information revolution has made new tools available to schools, particularly the pocket calculator and the microcomputer. Their intrusion into the instructional process has been well documented.

Marilyn Suydam, director of the Calculator Information Centre at the University of Ohio in the United States of America, released a report prepared by the International Working Group on Calculators in 1980 which researched the role of calculators in countries around the world. (Lukens, D. :1984, 15) They reported that calculator use in schools is almost universal and in some countries almost every household has one. These findings are corroborated by other research reports such as those carried out by Blakely (1985:59) and Ray Hembree (1986:84). Hembree, (1986:86) who integrated 79 research reports on calculator and microcomputer use in especially mathematics classrooms, found, through a meta-analysis, that the most important reasons for using these devices, were based on the advantages it afforded users:

> Aside from the computational value of calculators, a host of expectations for their usage could be listed: to aid algorithmic instruction; facilitate concept development; reducing the demand for memorization; enhance the scope of problem-solving; provide motivation; and encourage discovery, exploration, and creativity.



Similar findings have been reported by Hans Brolin (1990:189) who researched the introduction of calculators into Swedish schools. The ARK project - a project commissioned by the Swedish educational authorities to analyze pocket calculator usage in schools falling under their jurisdiction, found that students who used pocket calculators:

- gained a better understanding of concepts;
- gained better proficiency in estimations and mental arithmetic;
- did not lose their basic skills in algorithmic calculations.

Kathleen Heid (1988:22) found that students performed better on measures of conceptual knowledge when they had access to and use of computers. Studies conducted in other parts of the world, notably in the United States of America, Britain, the Netherlands and Australia reported similar results in the use of calculators in classrooms.

A study conducted by the National Council of Teachers of Mathematics (NCTM), (1984:309) reported that:

> Several authors, find significant improvements (in attitudes, personal computational skills, understanding of concepts, and in problem solving).

The NCTM report (1984:309) concluded that almost all researchers concur with the finding that "...no negative effects appear in any study."

In a major research program conducted in 1986 on the effects of handheld calculators in pre-college mathematics, Ray Hembree (1986:84) found that:

> At all grades but grade 4, the use of calculators in concert with traditional mathematics instruction apparently improves the average student's basic skills with pencil and paper, both in working exercises and in problem solving... Across all grade and ability levels, students using calculators possess a better attitude toward mathematics and an especially better self concept in mathematics than students not using calculators.



### *2.2.3 The graphic calculator*

A major advance in the field of microcomputer technology has been the development of 'advanced calculators' and 'pocket computers' as these machines are now commonly being referred to. One specific pocket computer that is exciting the mathematics community is the development of the graphic calculator, first developed by CASIO in 1986, according to Penelope Dunham (1993:89). Kenneth Ruthven (1990:431) describes the graphic calculator as a scientific calculator that extends beyond pure computation to include:

> ... a graphical display of mathematical expressions and data. Essentially, the user of a graphic calculator is able to treat the screen as a specified part of the coordinated plane, to plot points and to graph symbolic expressions constructed from standard operations and functions.

The large screen display allows for more graphical and symbolic notation to be visible to the user in that s/he can toggle between graphic and symbolic displays. Some graphic calculators have the added advantage in that they are programmable, thus allowing much of the drudgery of remembering long algorithmic manipulations to be removed. Furthermore, graphic calculators are much cheaper than microcomputers, are portable and are not dependent on a supply of electricity. These are especially important aspects when one considers the the implications it has of providing access to disadvantaged communities.

Some of the graphic calculator's more distinct features are listed by Julie (1993:343):

> ... Graphic calculators have advanced to the point where users are allowed to:
>
> - enter a defining equation for a graph in a form more or less consonant with the pencil-and-paper format,
> - freely select a domain and range of their choice,
> - choose scaling factors of their choice,
> - zoom in and out of particular regions of interest,
> - overlay (superimpose) graphs,
> - access any part on a graph and display the coordinates of a point, and
> - shade regions of inequalities.

[See fig. 1.1 below]

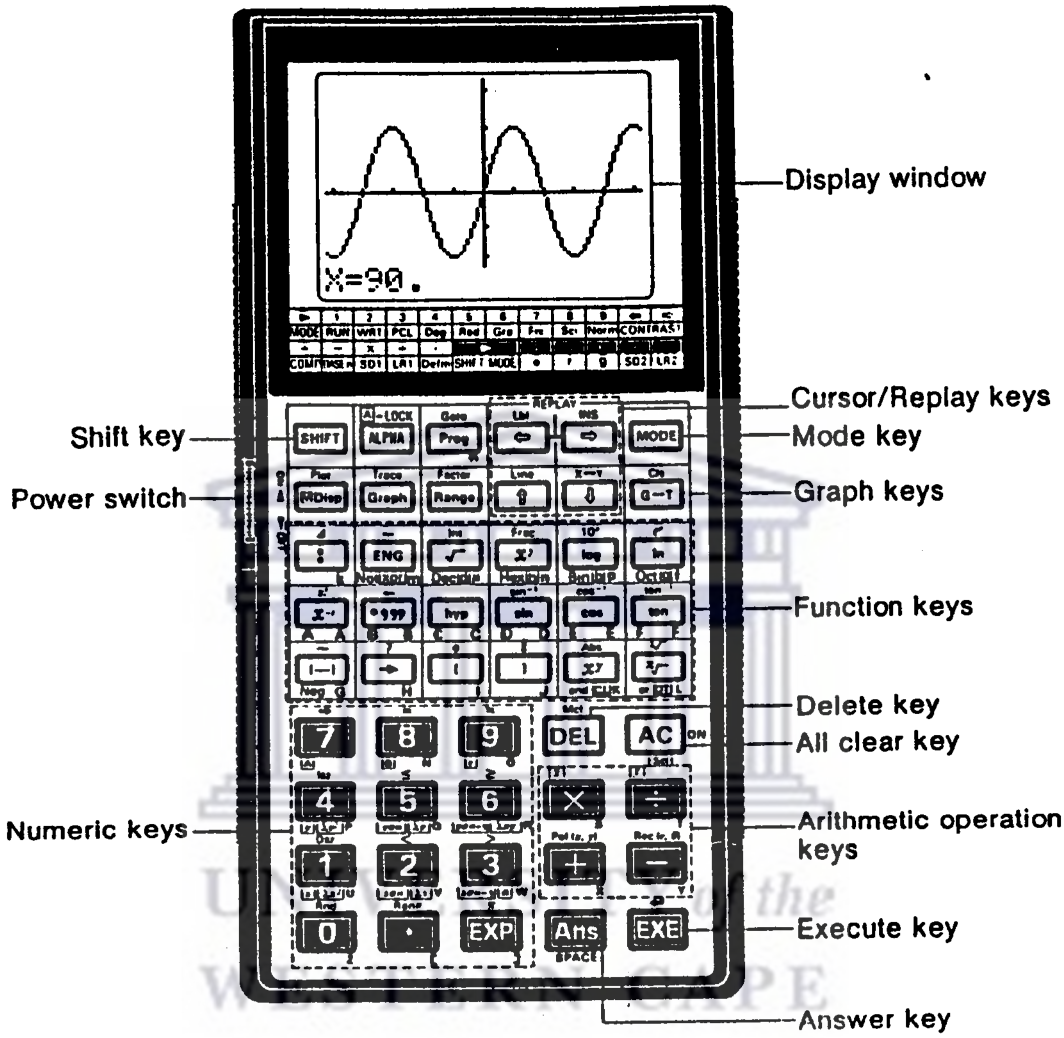


**Figure 1.1**

Given these unique features, the graphic calculator can be considered as a multi-representational tool in that the user can move through different modes of operation, that is, from a symbolic mode to a graphic mode and vice versa. Jere Confrey (1989:1) considers these features particularly important to conceptual development in that a multi-representational tool such as the graphic calculator:

- Invites the exploration of mathematical ideas in forms recognizable to students;
- allows students the freedom to act in ways they choose, and
- assists them in making significant progress (by their own standards) towards learning about a good idea.

Charles B. Vonder Embse (1992:65) agrees with Confrey in that he sees the graphic calculator as providing an ideal environment for the teaching and learning of mathematics. The unique features of the graphic calculator as espoused by Julie and other researchers permit exploration of functions of graphing, investigation and interpretation, comparison and exploration of concepts in a way not possible on an ordinary calculator. Several studies have been undertaken with the expressed purpose of investigating the impact of the graphic calculator on the teaching and learning of mathematics.

An extensive project undertaken from 1991 to September 1994 by the Freudenthal Institute in the Netherlands on the instruction of the Dutch education authorities, involved the extensive use of the TI-82 graphic calculator. With respect to some of the aspects relating to the use of the graphic calculator in a Realistic Mathematics Education project, the researchers of the Freudenthal Institute hypothesized that the use of the TI-82 would:

- remove time-consuming methods of algorithmic manipulation thereby shifting the emphasis to mathematizing on realistic mathematics problems rather than on the drudgery of manipulative work;
- allow pupils to explore problems, and creating new ones and being able to generalize about these. In this way the pupils become more active participants rather than passive reproducers of pre-conceived unrealistic mathematics problem solving;

- permit students to develop a critical attitude to numerical solutions obtained algorithmically thereby de-emphasizing the rigorous, drill-and-practice techniques traditionally used to solve problems.

Doorman et al (1994:135) reports that after extensive investigation by the Freudenthal Institute the following conclusions were reached:

- The graphic calculator will penetrate mathematics education. Due to technological developments as well as price policies, the machine definitely deserves a place in mathematics education.
- The graphic calculator is a valuable tool in the current curriculum at upper secondary level. It enables an exploratory approach to many subjects.
- The graphic calculator gives room for the use of realistic contexts.
- The graphic calculator changes the curriculum at least from the inside in the sense that a more dynamic and integrative treatment of mathematic subjects is stimulated.
- The graphic calculator demands more flexibility from the student with regard to skills and techniques.
- The introduction of the graphic calculator in secondary education demands the development of specific skills on the part of the teacher.

The research group also highlighted exploration as one of the more distinct advantages of graphic calculator use. This feature is particularly important for students in that they are challenged to get involved in interesting mathematical playing.

Kenneth Ruthven (1990:432) used the graphic calculator to compare the mathematical performance of upper-secondary school mathematics pupils who had access to graphic calculators with that of non graphic calculator users. Ruthven's argument was that:

> Enthusiasm generates ambitious claims concerning access to, and experience with, information technology on mathematical performance of pupils.

Consequently, Ruthven undertook a critical investigation of the benefits of using a graphic calculator and compared the results with that of non-users. Two specific items were tested, namely symbolization items that required an algebraic description of some cartesian plain, and interpretation items, calling for the extraction of information from some verbally contextualized graph. Ruthven (1990:431) concluded that under appropriate conditons:

> ... access to information technology can have an important influence both on the mathematical approaches employed by students and on their mathematical attainment. On the symbolization items, use of graphic calculators was associated not only with superior attainment by all students, but with greatly enhanced relative attainment on the part of female students.

Julie (1991:117) contends that ownership of graphic calculators can seemingly contribute towards "...equality of opportunity to learn mathematics by doing."



Julie cites a project by S.J. Khumalo (1990) in which pupils requested the researcher to permit them to take home the graphic calculators so that they could explore and further experiment with graphs. This exemplifies Julie's argument that access to graphic calculators stimulates and gives the economically disadvantaged student the advantage to learn mathematics by doing. In another report Julie (1993:25) contends that the availability and accessibility of graphic calculators affords the mathematics education fraternity the opportunity to : "... start exploring a radically different curriculum".

Consequently a number of research projects involving the use of the graphic calculator are currently being undertaken by post-graduate students at the University of the Western Cape and elsewhere.

Other research projects utilizing the graphic calculator include the Graphic Calculators in Mathematics project based in Britain, the Calculator and Computer Precalculus ($C^2PC$) project based in Ohio in the United States of America as well as those taking place under the guidance of professor Shlomo Vinner in Israel. Preliminary research reports emanating from these research projects regard the use of and access to graphic calculators as impressive, to the extent that Kenneth Ruthven (1989:233) declared:

> The use of information technology is a - arguably the - major current area of development in mathematics education ...the major impact on professional thinking and practice has taken place in the last fifteen years, in the wake of successive waves of technological innovation which have swept information technology into the workplace and the home, as well as the school.



However plausible these findings of researchers on the supposed benefits of the graphic, it is not possible to build on a proposed "result" without determining the relevance of the research findings in one's own context and experience. It is in the light of this that the current project has been undertaken.

### *2.3 Summary*

The revolution in technology especially in the field of microcomputers has made its way into the classroom. It impacts on several areas of the mathematics curriculum and influences what we teach, how we teach and how pupils can learn. The natural inertia of educational systems is insufficient for the impact of these technologies to be completely ignored for long.

What needs to be recognized is that the use of currently available graphing technologies such as the graphic calculator is naturally preceded by intensive research to establish its relevance to mathematics education. It is also important to understand that invention and implementation of any technology does not take place inside an epistemological void, but that it is supported and underpinned by a particular philosophy of teaching and learning. This research project is framed within a particular theoretical paradigm, the subject of the next chapter.

# Chapter Three:

# Theoretical Framework

*"Defining mathematics for students to include the mathematical reality of students, and their ways of modifying these realities as a result of their experiences, makes mathematics education a very exciting field and marks it as a professional practice rather than a common sense practice."*

***Leslie P. Steffe and Heid Wiegel (1992:448)***

## *3.1 Introduction*

In the preceding chapter on Technology in Mathematics Education, the graphic calculator, an exemplar of current microcomputer technology, was discussed as a technologically innovative tool which, if properly utilized, could assist pupils in concept formation and concomitantly, could result in a de-emphasis of the algorithmic process of teaching certain concepts of calculus for matriculants. This assumption about the apparent advantages of the use of the currently available technology is supported by research findings of particularly, Doorman et al (1994:135), Julie (1993:343), Ruthven (1990:431), Heid (1988:22), and Hembree (1986:86). I concluded the previous chapter by stating that the study of and research into the use of technology in particularly mathematics education, does not take place in an epistemological void but that such research is underpinned by a particular theory of knowledge acquisition. As this research is informed by and conceptualized within a particular theoretical framework, it becomes imperative that these theoretical underpinnings be clearly stated.

In chapter 2 the graphic calculator was referred to as a multi-representational tool in that it enables the user to move through different modes of operation, that is, from a symbolic mode of operation to a graphic mode of operation and vice versa. Confrey (1989:1) considers these features particularly important to conceptual development in that a multi-representational tool (such as the graphic calculator):

- invites the exploration of mathematical ideas in forms recognizable to students;
- allows students the freedom to act in ways they choose, and
- assists them in making significant progress (by their own standards) towards learning about a good idea.

From this statement one can infer that pupils have a mathematical reality of their own and consequenly, teachers cannot be considered as being the final authority on what this reality constitutes. Constructivism is a theory of knowledge acquisition that encompasses this unique reality of students. It is this theory of knowledge acquisition that undergirds this research. Consequently, this chapter examines the notion of constructivism, its implications for mathematics teaching, and discusses how invention and implementation of current computer technology can be informed by a constructivist theory of knowledge acquisition.

### *3.2 Constructivism*

A fundamental premise of constructivism is that there is a real world that we experience and that we impose meaning on the world rather than such meaning existing independently of us. The meaning we impose upon the world is "...rooted in, and indexed by, experience". (Brown, Collins, and Duguid, 1989:32). Implicit in this statement is a belief that individuals construct their own reality through interpreting perceptual experiences of the external world. Consequently, knowledge construction becomes a function of our prior experiences and mental operations, and is framed by our beliefs and biases about the world.

According to Martin Simon and Deborah Schifter (1991:310) constructivist theory is based early empirical and theoretical work of Jean Piaget and on the more recent work of theorists and researchers such as Von Glasersfeld (1987:3-17); Cobb(1992:2-33); Steffe (1992:445-465); Confrey (1992:2-49), and others. Simon and Schifter (1991:310) emphasized that a fundamental tenet of constructivist theory is that:

> ... learners actively construct their own understandings rather than passively absorb or copy the understandings of others.

Accordingly, new knowledge is created by the cognizing subject in a process that involves the presence of disequilibrium in his/her pre-existing cognitive structures. This disequilibrium is caused by a situation perceived by the cognizing subject as being problematic and stimulates him/her to act until equilibrium is restored. A modification of previously held ideas results from the mental activity which stimulated the cognizing subject to act towards restoring the disequilibrium caused by the initial perturbation. This modification is brought about by the individual engaging in reflective abstraction, which, according to Grayson Wheatly, (1992:529):

> ... is central to the theory of constructivism as put forth by Von Glasersfeld. In "coming to know", persons make major cognitive advances by taking their actions as objects of thought. Learners move beyond being "in the action" when they engage in reflection.



In Piagetian theory the concept of reflective abstraction is used to describe the construction of logico - mathematical structures. As such reflective abstraction is present in the very earliest stages in the coordination of sensory- motor structures and continues up through higher mathematics. Ed Dubinsky (1991:99) describes reflective abstraction as a process which involves the extraction of properties from physical or mental actions at a distinct level of thought which necessarily implies an awareness or consciousness of actions. This could imply an act of separating form from its content. That which is thus abstracted is projected onto a next level of thought where other more resilient actions and modes of thought are present.

It is at this point where a construction of new schemes by a conjunction of abstractions occur which result in equilibruim being restored in the cognitive structures of the cognizing subject.

Reflective abstraction therefore results in a second order construction giving the person a greater control over his/her thinking. A consequence of reflective abstraction, that is, the ability to decontexualize ideas from context and apply them in a new context is, according to Von Glasersfeld, (1987:11):

> ... best demonstrated in situations where something new is generated, something that was not originally available to the operator.

Such constructions are based on the person's own cognitive structures and is necessarily framed by his /her unique apperceptions of the world. As such individuals can have no ultimate shared reality, but rather, reality is the outcome of individual constructions. Confrey (1990b:108) elaborates:

> ... constructivism can be described essentially as a theory about the limits of human knowledge, a belief that all knowledge is necessarily a product of our cognitive acts. We can have no direct or unmediated knowledge of any external or objective reality. We construct our understandings through our experiences, and the character of our experiences is influenced profoundly by our cognitive lenses.



It becomes clear that a constructivist perspective such as that espoused by Confrey, Simon and Von Glasersfeld, makes a complete break with an objectivist view of knowledge construction. Whereas in objectivism the emphasis is firmly on the object of our knowing, in constructivism the emphasis is on how we construct knowledge.

According to constructivism, we come to know through constructing mental models based on prior experiences. We constantly modify our mental models through reflective abstraction as a result of inconsistencies in our current cognitive structures.

Our constructions are profoundly influenced by previous experience, something that objectivism denies, and consequently, reality as we experience it, is necessarily subjective. Von Glasersfeld (1987:5) sums up this perspective very aptly when he postulates that the world as we see it is necessarily the world as we construct it. 'Facts' are made by us and not conveyed to us by a pre-existing independent world. We cannot however construct the 'facts' in a fashion that suits us. To be viable, the facts we create must not clash with experience. Their tenure is ensured as long as they continue to do what we expect them to do.

What needs to be emphasized at this point is that learning, from a constructivist perspective, is characterized as being an active process in which individuals are continuously engaged in making constructions about their own unique experiences. Confrey (1993a:3) contends that constructivism as such, is undergirded by four basic "planks' . The first of these is that knowledge can only be understood in terms of its genesis. The potential of knowledge to assist human beings in making sense of their experiences and solving problems requires that knowledge be assessed in terms of its viability, viability being the second of the basic 'planks'. Knowledge constructed through a formation of schemes which allow predictability in one's actions, forms the third of Confrey's 'planks'. Confrey (1993a:3) describes schemes as interlinked patterns of problematic, actions and reflections. The last plank on which constructivism is based concerns communication and interaction with others through collaborative participation in activities and the negotiation of meaning similar to one's self. Constructivism based on these four 'planks' covers a panoply of theoretical positions within the constructivist school of thought that necessarily qualifies its adherents as being either naive (trivial) constructivists or radical constructivists. A third but different form of constructivism in that it is based primarily on Vygotskian theory and not on Piagetian theory, is what Paul Ernest (1992:42) calls social constructivism. The different forms are discussed below.

### *3.2.1 Naive constructivism*

Proponents of constructivism adhere to the first of two principles of constructivism which are attributed to Von Glasersfeld (1987:23) namely that:

> Knowledge is not passively received either through the senses or by the way of communication. Knowledge is actively built up by the cognizing subject.

Central to constructivism based solely on this principle is the notion of an individual as being "active" and not just someone responding to external stimuli, as espoused in behaviourist rubrick. The individual is seen to be constantly engaged in the active building of constructions, reflecting on his/her actions, and modifying current cognitive structures. Even in learning situations that appear to be relatively straightforward, constructive processes operate. Mental structures are formed, elaborated, tested and modified until a temporary satisfactory structure emerges. Ernest (1993:168), who refers to naive constructivism as "information processing constructivism", argues that constructivism based on Von Glasersfeld's first principle, spawns the metaphor of the mind as being a computer in that it:

> ... actively processes information and data, calling up various routines and procedures, organizing the memorization and retrieval of data.



The essence of a metaphoric understanding of the mind as being a computer is not at variance with what Confrey and others have said about learning being an active process.

### *3.2.2 Radical constructivism*

Radical constructivism is based on an acceptance of both principles of constructivism as espoused by Von Glasersfeld (1987a:22) namely that:

1) Knowledge is not passively received either through the senses or by way of communication. Knowledge is actively built up by the cognizing subject.

2a) The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability;

b) Cognition serves the subject's organization of the experiencial world, not the discovery of an objective ontological reality.

Constructivism founded on the acceptance of both these priciples, conceives of reality as being subjective as it is constructed by the cognizing subject based upon his/her subjective mental activity. While not precluding the existence of an external reality, radical constructivism espouses the view that there can be no single objective reality of any objective entity that can be described in any objective way; rather the real world is a product of the mind that constructs the world. Constructivism thus conceived is steeped in a radical epistemology that rejects any claims of ontological certainty. The traditional conception of knowledge as a construct that mirrors (matches) objective reality of the real world is considered as misguided and is consequently rejected. Reality therefore, cannot be represented in any iconic, incorrigible form.

A rejection of knowledge that reflects an objective ontological reality leads radical constructivism to ground its epistemology on four basic 'planks', which - according to Confrey (1992:12-17)- are essential to an understanding of radical constructivism. The first of these 'planks' namely that "... Knowledge can only be understood in terms of its genesis" is rooted in Piaget's genetic epistemology which postulates that a parallelism exists between progress made and the corresponding formative psychological processes. Because the construction of knowledge is bound inextricably to time, its construction has to be studied ontogenetically (that is, in relation to the origin and development of the human being through time) and phylogenetically (that is, in relation to the origin and development of language of humans through time).

The second of these 'planks' namely that knowledge can only be assessed in terms of its viability represents in essence a relinguishing, according to Confrey (1993a:3) once and for all, of "meta-physical" realism. An inevitable consequence of this relinguishing of meta-physical realism is that we can no longer accept that a natural progression towards a mirror image of the real world is conceivable. Confrey (1990:108) puts this more aptly:

> We can have no direct or unmediated knowledge of any external or objective reality. We construct our understanding through our experiences, and the character of our experiences is influenced profoundly by our cognitive lenses.

The construction of knowledge then, is evolutionary in that it only becomes internalized if it is viable. It is viable only if it possesses goodness of fit or experiential adequacy. It obtains reliability, predictability and certainty if it guides one to succeed in goal-directed behaviour. Such knowledge, according to Von Glasersfeld (1987b:5):

> ... does not represent a picture of the "real" world but provides structure and organization to experience. As such it has an all-important function: It enables us to solve experiential problems.



The third 'plank' of constructivism, namely that "... knowledge if formed through the formation of schemes" is based, according to Confrey (1992:14) on Piaget's schema theory, in which a scheme is conceived as "... whatever is repeated or generalizable in an action".(Confrey, 1992:15)

Schemes allow one predictability in one's experiences in that they allow operationalized patterns of problematic (perturbations or disequilibrium in one's cognitive structures) which serves as precursors to actions in an attempt to resolve the conflict caused by the disequilibrium. The disequilibrium and the action involved to restore balance are internalized through reflective abstraction. This process is cyclical only if the action results in suitable 'fit' or 'experiential adequacy'.

According to Confrey (1993a:3) the fourth 'plank' on which constructivism is based is that:

> Communication and interactions with others influence the formation of knowledge through joint participation in activities and the building of models of others as similar to one's self.

From this one can deduce that for communication to be effective, meaning as intensioned by the speaker, has to be socially negotiated within the context of the situation so that it does not manifestly clash with what the speaker intended to convey. Communication is therefore inherently subjective and consequently cannot be considered as a suitable conduit for the transfer of information unless participants "agree to disagree" on what such information is intended to mean. Such meaning is socially negotiated between the participants without the intervention of outsiders.

Confrey's fourth 'plank' of radical constructivism forms the focal point of social constructivism as espoused by Paul Ernest.



### *3.2.3 Social constructivism*

According to Paul Ernest (1994:309-310; 1993:170) social constructivism is distinctly different from radical constructivism in that it is undergirded by Vygotskian theory which views the mind as being "social and conversational". According to Ernest (1994:310) the mind is "conversational" because:

> ... individual thinking of any complexity originates with and is formed by internalized conversation.

Hence all thoughts are, metaphorically understood, viewed as internalized conversation. Accordingly Ernest conceives the social constructivist model as having as its central tenet the interactions of human subjects with each other through "shared forms - of - life and "shared language games" mediated through the metaphor of the mind as being conversational. Consequently, reality becomes a socially mediated construct which cannot be considered as objective in any iconic way.

Ernest (1994:307) acknowledges that social constructivism is an attempt to justify the nature of mathematical knowledge as socially negotiated whilst at the same time vindicating the personal construction of knowledge. Social constructivism in Ernest's view is characterized as being cyclical in that it establishes a link between subjective and objective knowledge. Individually constructed knowledge is subjective but becomes objective knowledge on entering the public domain through a cycle of social negotiation and intersubjective scrutiny. Once internalized and reconstructed by individuals it once more becomes subjective, thus completing the cycle. In terms of Ernest's exposition of social constructivism, an important component is emphasized, namely that knowledge is a socially negotiated construct.

### *3.3 Implications of constructivism for mathematics teaching*

A fundamental implication of constructivism for mathematics education, according to Lesley Steffe and Heide Wiegel (1992:447) is a recognition that students possess and create their own mathematical realities. As such teachers should endeavour to understand the mathematical reality of students from their point of view, ("mathematics of students") a pursuit that could radically transform school mathematics. The teaching of mathematics should be such that pupils have the maximum opportunity and stimulation to construct powerful and resilient mathematical ideas for themselves, and through reflection, come to know and believe their own constructions. This process results in the classroom becoming, according to Martin Simon and Deborah Schifter (1991:31):

> ... a mathematical community that decides on the truth of mathematical ideas by critically examining the justifications provided by the students.

The teacher's role then is that of a facilitator and where necessary, scaffolds the mathematics of the classroom community towards a fit with the mathematics of larger communities of which the classroom is an integral part. A simple transmission of information is rejected. The implication is that teachers become constructors of learning environments by modifying the curriculum to allow for interactions between students and materials, between students and peers, and between students and the teacher. The learning environment, according to Susan Pirie and Thomas Kieran (1992:507) is decided in accordance with the students' own unique histories and structures, and by their ways of perceiving and organizing. This is important since constructivism as described earlier, essentially pertains to the actions of the learners. As such a constructivist teaching model does not exist, since constructivism cannot be reduced to a set of rules to follow or actions to perform on the cognizing subject.

A further implication of constructivism for mathematics teaching is that educators need to listen carefully and persistently to student voice described by Confrey (1993a:8-9) as the :

> ... beliefs of the student as articulated by him/her... (since ) children's views possess a rationality and organization that may differ significantly from that of an adult.



This can be achieved, according to Confrey (1990:108) through a process called "decentering", that is , the ability to see a situation as perceived by another through working within the frame of reference of a person and attempting to understand the situation through the perspective of that person. To activate this process of decentering, the teachers's role may be seen as introducing helpful perturbations (thus creating disequilibrium in the cognitive structure of the student) in a number of ongoing processes that are taking place independently of the teacher. The vehicle for creating these perturbations is that of multiple representations (that is, public ways of displaying and communicating about a concept that exert a significant impact on the substantial form of the concept). These multiple representations include physical models, diagrams, computer representations, and mathematical symbols which students can manipulate, thereby developing connections between representations.

Constructing environments in which students are confronted with multiple representations helps them to create mathematics for themselves and not merely imitate the mathematics of others. The essential aspect of creating such environments is to allow students to decide, in concert with the teacher as facilitator and mentor, as to what constitutes mathematical validity. Autonomy resides with the students in that they have to believe what they have individually constructed, is mathematically valid. The eventual outcome of such an approach to teaching is that environments are constructed in which students make constructions that are compatible with the mathematical environments.

This approach to the teaching of mathematics also has consequences for teachers in that they would need to re-examine their own constructs about the subject matter taught which could lead to a modification and reorganization of the subject matter. This is as important a result from such attention to students as is the development on the part of both the student and the teacher. According to Lesley Steffe and Heide Wiegel (1992:448), such an approach to the teaching of mathematics makes mathematics education a very exciting field and distinguishes it as a professional practice rather than a common sense practice.



### *3.4 Technology meets constructivism*

An epistemology of multiple representations necessitates the development of innovative technologies that would allow students more opportunities to explore and reflect upon their diverse conceptions. These technologies are regarded as mediating tools and the computer is a paramount example of a mediating tool capable of substantially altering the conceptualization of a topic, more so if the development and implementation thereof is informed by an understanding of how learning takes place. For a mediating tool such as the computer, and in the instance of this research - the graphic calculator, to have an impact on students conceptions, it should fit the particular content being taught and provide a variety of actions that are tailored to each individual representation.

It must, in essence, provide convergent and competing feedback to the extent that it allows a particular problem to be approached in a number of different representations. The graphic calculator, given it's unique characteristics such as enabling the user to move through different modes of representation - that is , from a symbolic mode to a graphic mode - is an example of a mediating tool that meets some of the requirements necessary for it to be used in an environment structured on constructivist guidelines.

### *3.5 Summary*

A point that has been stressed throughout this chapter is that pupils have a reality of their own. It has been established that a constructivist theory of knowledge acquisition recognizes this unique reality of pupils. A constructivist theory of knowledge acquisition advocates that knowledge is essentially subjectively created and can consequently not be known with an iconic objectivity or certainty. As such, opportunities for pupils to explore, create and reflect on their constructions needs to be provided for in the mathematics curriculum. An epistemology of multiple representations is postulated as providing such opportunities through a mediating tool. In the instance of this research, the graphic calculator is considered to be an appropriate mediating tool for establishing an epistemology of multiple representations.

**Chapter Four:**

**Research Methodology**

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*"In mathematics it is possible to accept a result stated by another mathematician (although one ought to test it for oneself), but in mathematics education it is impossible to build on a proposed 'result' without testing it in one's own experience and situation.."*

***John Mason (1994:181)***

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*4.1 Introduction*

In chapter 3, I commented that working within a constructivist paradigm necessitates that teachers take into account that pupils possess a unique reality of their own and that listening carefully and persistently to pupils mathematizing is an essential component of the teaching process. Opportunities for pupils to create their own mathematics in an environment where they can reflect on and modify their constructions need to be created for them in the current educational environment. Consequently this project has been undertaken with a view to creating such an environment and is based on the assumption that the introduction of the graphic calculator will facilitate the formation of certain concepts of calculus for matriculants and concomitantly result in a de-emphasis of the algorithmic approach to teaching such concepts. A developmental research approach has been adopted as the most suitable means of realizing this goal. As such, this chapter seeks to illuminate the main characteristics of developmental research and give an account of some of the main data gathering techniques that have been used in this process.

*4.2 Developmental research*

*4.2.1 The origin of developmental research*

Freudenthal (1991:162) traces the origin of developmental research to the Netherlands when, in 1961, the Dutch government initiated the Commission on Modernizing Mathematics Education (CMLW). The CMLW served as a precursor to the establishment of numerous other commissions in other subject areas. Ultimately the need arose to combine and co-ordinate the numerous activities of these commissions. According to Freudenthal (1991:162), the combining of these activities resulted in the establishment of the Institute on Developing Mathematics Education (IOWO) in 1971 which was loosely tied to Utrecht University. IOWO had as its central mission the integration of the once independently functioning sectors of Training, Counselling, Retraining, Development, Innovation, Test Production and Research.

Koeno Gravemeijer (1994:445) reports that in 1981, IOWO was supplanted by the Research Group of Mathematics Education and Mathematical Computing Centre (OW & OC), which ten years later was renamed the Freudenthal Institute in honour of its founding director. The formation of OW & OC was a direct consequence of legislation passed by the Dutch authorities (Freudenthal, 1991:163) in an attempt to curtail the integration of the previously mentioned sectors. Whilst this legilation spelt the untimely demise of IOWO, the establishment of OW & OC as a department within the faculty of Mathematics at Utrecht University, provided the impetus for the growth of what is now known as developmental research in mathematics education. In this regard Freudenthal (1991:163-164) writes:



> Developmental research was not an ad hoc invention... It was simply the approach that had proven successful in the past and promised success in the future.

The next section attempts to arrive at an exposition of developmental research.

### *4.2.2 An exposition of developmental research*

According to Freudenthal (1991:156), the expressed purpose of educational development is 'change'. Change, however, is not an accidental construct. Change takes place once there is a perceived need for such change. Change is therefore a purposeful, goal-directed action, the consequence of a desire to pursue a more beneficial course of action. In educational development, the traditional approach to change is change directed at and through the curriculum, the envisaged change being the developers' notion of change usually enshrined in curricula and textbooks. Through the process of research, development and diffusion, (RD &D) change directed at the curriculum in an attempt to alter the outcomes of education is put into action. In a conventional RD & D approach, development and implementation, according to Koeno Gravemeijer (1994:445) are completely separated. Curriculum development based on an RD & D approach follows the same pattern in that curricula are first developed according to some rigid instrumental theory and then implemented in the classroom. (Doorman et al, 1994:29). The development and testing of new curricula based on this model does not result in the new curricula been changed. In testing the new curricula in practice, the developers aim at acquiring insight into its perceived value (summative evaluation) and obtaining evidence that can point towards an improvement of the implemented curricula (formative evaluation).

In developmental research, the guiding principle is a philosophy of education and not a set of concrete operational goals. (Doorman et al, 1994:29) such as those followed in a conventional RD & D approach to curriculum development. Thought-experiments are the genesis of the teaching experiment and are indispensible to developmental research (Doorman et al, 1994:29). Thought-experiments as precursors to teaching experiments, are characterised by the initiator/developer envisioning how the learning process would be effected by the introduction of, for example, new learning materials, or by the introduction of new technology to teaching and learning. In the context of this particular research project the impact of the introduction of the graphic calculator to the teaching and learning of calculus for matriculants served as the initial thought-experiment which, through a process of didactic deliberation, reflection and implementation was tested and evaluated in a teaching experiment designed for this purpose.

Thought-experiments are illuminated in the teaching experiment where the emphasis is on finding evidence as to whether the thought-experiment has been verified or not. Such evidence is used to refine the initial thought-experiment for implementation in a follow-up phase, thus giving developmental research its iterative character. Consequently, development and implementation go hand-in-hand. The teaching experiment is also utilized to design/invent new ideas for teaching activities. A pupil's spontaneous but divergent solution to a particular problem for example, could put the developer on the track of a new approach to teaching a specific concept.

What marks developmental research as uniquely different from the conventional RD & D approach to curriculum development is its cyclicity which lies at the heart of developmental research. What is developed behind the desk is put immediately into practice. The results are analyzed through reflection, discussion and didactic deliberation. (Freudenthal, 1991:159-160) Based on what transpires from such deliberation the developmental work is continued. This cyclicity inevitably results in a learning process for the researcher/developer.

From the foregoing it is evident that developmental research is characterized as a process involving the design, implementation, refinement, adjustment and illumination of a particular theory about the teaching and learning of, in the case of this research, mathematics. A particular feature interwoven into the entire process, is that of dissemination. Freudenthal (1991:161) regards this feature as particularly important to developmental research when he states:

> ... developmental research means:
> experiencing the cyclic process of development and research so consciously and reporting on it so candidly that it justifies itself and that this experience can be transmitted to others to become like their own experience.

The essence of this statement is that the validity of the research and objectivity of the researcher should be unquestionable and hence this provides the essential prerequisite of justification. Doorman et al (1994:30) puts this more aptly:

> ...(E)erst vindt de ontdekking plaats en daarna volgt het juistheid, de poging tot regvaardiging.

Dissemination, therefore, cannot be separated from development. This requires a continuous awareness of what is happening in the ongoing process, a state of permanent reflection, recorded as much as possible.

To summarize, developmental research takes place within an educational environment. The expressed aim of educational development and consequently of developmental research, is change directed at and through the curriculum. The conventional approach to curriculum change has been the RD &D approach where development and implementation are separated. In developmental research this process is inseparably interwoven and as such, the entire process is a cyclic one of design, implementation, evaluation, refinement, adjustment and re-implementation. Discussion, reflection and didactic deliberation lie at the heart of the developmental research process.



### *4.3 The phases of developmental research*

Developmental research proceeds through four distinct phases. These are dealt with separately but it should be remembered that these phases are inextricably interwoven.

#### *4.3.1 The identification of the problematic*

Developmental research begins with some unanswered questions, called the problematic, in the mind of the researcher/developer. This problematic needs to be stated in clear unambiguous terms. Freudenthal (1991:158) identifies this phase as indispensible to educational development and proposes that the researcher engages in what he calls, thought-experiments. The problematic arises out of the researcher's desire to find a solution to some unanswered question. In the context of this research, the introduction of the graphic calculator, an innovative technological tool, gave rise to the problematic. The question which arises with respect to the graphic calculator is the extent to which it could facilitate the learning of certain concepts of calculus for matriculants and whether its use would conceivably de-emphasize the rigorous algorithmic process as discussed in chapter 1, of teaching these concepts. Having formulated the problematic (question) it behoves the researcher to formulate an hypothesis which gives direction to the intended research.

*4.3.2 Exploration of the research field*

Defining a problem in clear unambiguous terms does not mean that the problem is researchable. Before the researcher can design a plan to implement the research s/he would need to establish whether the kind of information s/he needs to answer/solve the problematic is readily available, and if such information is readily available, whether it is indeed accessible to him/her. If the information is accessible to the researcher, s/he would need to negotiate with the relevant authorities to gain access thereto while simultaneously defining exactly who would be involved in the research. Other aspects that need to be explored before the research commences, concern the constraints, in terms of resources, that may be placed on the research. Once the researcher has resolved all these preliminaries to the extent that nothing would be left to chance, s/he may decide to commence the next phase.



*4.3.3 Implementation*

The researcher is ready to implement the research once s/he has established beyond doubt that the problematic is indeed researchable and that s/he can give an account of the research plan in the preceding phase. Earl Babbie (1983:96) refers to the implementation phase as the operationalization phase as it refers to all the concrete steps that will be used to investigate and report on the concepts studied in the research. During the implementation phase a number of data gathering techniques are employed to suit the research and reliably obtain the required information. The data gathering techniques best suited to developmental research in that it allows for the qualitative reporting on the research process, are similar to those employed to action research. The different techniques employed in the context of this research project are listed below:

*4.3.3.1 Field notes*

According to Junker (1960:5) field research is concerned with the :

> ... observation of people **in situ**, finding them where they are, staying with them in some role, which, while acceptible to them, will allow both intimate observation of certain parts of their behaviour and reporting on ways useful to social science and not harmful to those observed.

Since field research implies the careful observation of people, it becomes essential that the researcher keeps up-to-date notes of such observations. When making field notes, the researcher's attention is focussed on a number of 'events'. Earl Babbie (1983:245) lists six of these 'events':

1. Acts. Action in situations that is temporarily brief, consuming only a few seconds, minutes, or hours.
2. Activities. Action in a setting of more major duration - days, weeks or months -constituting significant elements of a person's involvement.
3. Meanings. The verbal production of participants that define and direct action.
4. Participation. Person's holistic involvement, or adaptation to a situation or setting under study.
5. Relationships. Interrelationships among several persons considered simultaneously.
6. Settings. The entire setting under study conceived as the unit of analysis.

If possible, notes should be taken of one's observations as one is observing, or as soon as possible thereafter so that nothing is left to chance. Such notes should include empirical observations reported as candidly as possible as well as some brief interpretation thereof. One should report on what one 'knows' has happened and what one 'thinks' has happened. It should, however, be borne in mind that the observer/researcher cannot observe everything nor can notes be made of everything that s/he observes. As such, observations represent a **de facto** sample of all possible observations and field notes represent a sample of the observations. In this regard it is important that the researcher record what s/he considers as relevant and important to his research. This requires from the researcher/observer that s/he anticipate some of the observations he considers important, in advance. According to Hopkins (1989:60) keeping fieldnotes have some very distinct advantages. They:

> ... are very simple to keep; no outsider is needed
>
> ... provide good ongoing record; used as a diary they give good continuity
>
> ... (provide) first hand information (which) can be studied conveniently in the teacher's own time
>
> ... acts as an **aide memoire**
>
> ... help to relate incidents, explore emerging trends
>
> ... are very useful if the teacher intends to write a case study

Fieldnotes used in isolation cannot do justice to any research project and consequently the researcher makes use of other sources of information.

*4.3.3.2 Recorded information*

Audio and video tape recordings have very definite advantages if they are used judiciously. By the very nature thereof audio and video tape recordings require a certain degree of technical expertise. While audio tape recordings can be used to record communications between participants in a given setting, it has to be used in such a fashion that it permits the researcher to make a detailed analysis of such communication. Hopkins (1989:61) considers audio tape recordings as:

> ... excellent for those situations where teachers require a very specific and accurate record of a limited aspect of their teaching, or of a particular interaction, say between a specific teacher and child or between two children.

Setting up recording devices can be very intricate and has a few distinct disadvantages. Some of these include disturbing the continuity of the research process where the device has to be manually operated, disturbing pupils in that it could be inhibitive, and masses of material may provide very little relevant information.

Taped recordings were used in this research to record students' interactions with each other and their responses to certain context related questions with regard to how they formulated certain concepts related to calculus. Some of the disadvantages of using the tape recorder during the research were distinctly perceptible to the researcher and therefore it was not used to record all sessions. As a result the researcher enlisted the help of a colleague to make video recordings of some of the sessions to complement the audio taped recordings.

Video taped recordings were made of three of the sessions of this research. Because these recordings were done by a colleague who was adept at using a video recorder, this left the researcher free to observe and make field notes of the research process and at times to act as participant - observer. The presence of a colleague doing the video recording did not cause a disruption of the research process because the pupils participating in the research project are continuously exposed to such occurrences as research in other subject areas take place on a regular basis. Video taped recordings have some distinct advantages since it allows the researcher to gather information of some aspect of the research instantaneously and can be used to verify any or some of the aspects which were recorded as fieldnotes or captured on audio tape. Also, it could focus on some aspect which the researcher has failed to note. Video taped recordings also permit the researcher to constantly review some aspect of the research and engage in discussions with fellow researchers and the research director as to the significance thereof. This has the added advantage of providing a multiple perspective of a given situation or behaviour.

*4.3.3.3 Co-researchers as observers*

It is not possible for the researcher to observe and record everything that goes on in the research setting. It is therefore incumbent upon the researcher to make use of outside observers who are conversant with the research process and are experts in the subject area being researched. Co-researchers should be appropriately briefed as to the exact nature of the research in order for them to be co-opted into the research process. In developmental research co-researchers as observers are essential to the smooth execution of the research. Elliot (1981:14) suggests that observers be assigned various tasks such as taking photographs, making video recordings or taking detailed notes as s/he observes and discussing these with the researcher. The observer can also be a teaching colleague who has been adequately informed as to the exact nature of the research. This marks developmental research as a collaborative effort. Co-observers' reports are useful in providing corroborative evidence of the research that had been undertaken.

*4.3.3.4 Data analysis*

Documents provide information relevant to the issues and problems under investigation. The primary documents that are analyzed in this research include students' written work, worksheets and written assignments. Typically in developmental research, the analysis of documents involves an in-depth study of its contents as the preference is to base judgements on a broad spectrum of observations and information even at the risk that another observer might reach another judgement of the research situation.

*4.3.3.5 Triangulation*

The use of several data gathering techniques to corroborate some finding is called triangulation and is indispensible to developmental research since there is always a danger that the use of a single method of obtaining information could impact negatively on the outcome of the research. The methods of data gathering which have already been discussed are all used in concert with each other as a means of gaining a better insight into the research problem. This is done through triangulation where data collected can be compared and constrasted and agreement reached as to what should be considered relevant to answering the research question. Triangulation as such, is not a data gathering technique but rather a means of establishing a greater degree of reliability in the analysis of the data. As a consequence, triangulation takes place throughout the data gathering process where not only data gathered by different researchers/observers is compared and contrasted, but also data obtained by the same researcher through diverse means. This process is therefore characterized as as been continuous as well as been cyclic since decisions to implement the next phase of the research process is at times based on an analysis of data already collected and triangulated. During the process, particular attention is paid to points of agreement and disagreement as well as the presence of differences of opinion. This necessitates on-going didactic deliberation.



*4.3.4 Analysis and evaluation*

Data collected in the implementation phase of the research is, by the very nature thereof, qualitative and is therefore subject to an interpretive process. As such, the analysis of the data approximates to the original meaning of the data and is therefore not subjected to statistical analysis of any sort obviating the futility of analytical reasoning within that system. The analysis and interpretation of the data takes place within a tentative theoretical framework. Freudenthal (1991:161) isolates a key element of the analysis of data:

> Knowledge can be successfully presented as a product if the processes of its acquisiton is reproducible.

The notion of something been 'reproducible' implies that it ought to have a measure of replicibiliy. In developmental research reliability is associated with replicable, that is, if the experiments were to be repeated by someone else in a similar setting and under similar conditions the results would very nearly be replicible to the initial results. Koeno Gravemeijer (1994:454) prefers to use the term 'trackability' as a more appropriate concept for referring to the research project's reliability. 'Trackability' can be established by reporting on all the diverse aspects of the research data analysis. This requires the reporting on successes as well as failures of the procedures followed, on the conceptual framework and on the reasons for the choices made. Reliability can also be established through intersubjective agreement among researchers working on the project through a process of didactic deliberation, discussion and triangulation.

It needs to be stressed that evaluation and analysis is present throughout the implementation of the project. In this regard Doorman et al (1994:29) write:

> ... De kern van het proces is een cyclisch proces van doordenken en beproeven, een cyclische afwisseling van gedactenexperiment en onderwijsexperiment.



The evaluation process is therefore also characterized by in-depth didactic deliberation, discussion and refinement. This circularity is a **sine quo non** for developmental research. The outcome of research can be considered to be successful if it is reproducible. It can only be reproducible if it has been submitted for adjudication. Herein lies another element of the cyclic nature of the research process in that the research process must meet the requirement of being amenable to dissemination. Dissemination requires from the researcher engaged in developmental research to take a long, hard, inward look, a state of virtual permanent reflection and report on this reflection in such a way, that when submitted for adjudication and subsequent dissemination, it bears the hallmark of credibility. Essentially, therefore, evaluation and analysis of data is intricately interwoven into the entire research process.

*4.4 Summary*

In this chapter, developmental research has been discussed as an integrated approach to educational development and consequently, to curriculum development. The expressed purpose of developmental research is change. In the context of this research project, the assumption is that a change in the way certain concepts relating to calculus for matriculants are taught, could be effected by the introduction of the graphic calculator to the educational environment. It is also assumed that the introduction of the graphic calculator could significantly de-emphasize the strict mechanistic approach to teaching the rules of differentiation algorithmically.

Based on these assumptions and adopting a developmental research approach, learning material is designed for use with the graphic calculator. The material is then implemented in the research setting, refined through didactic deliberation, adjusted and re-implemented. The focus of the research will be on the observation and recording of the development of the students' own constructions. Subsequently, these observations will be evaluated and analyzed and the findings are reported as candidly as possible.

# Chapter Five:

# Project Implementation

> *"No matter how much we struggle to find out about our external world (and independently of whether our epistemological foundations even admit of such a construct), what we see and construe is in a strong measure a mirror image of ourself."*
>
> ***John Mason (1994:179)***

## *5.1 Convincing the gatekeepers*

In this section a description of the school selected for the research as well a general profile of the pupils involved in the research project, is summarized.

### *5.1.1 The school and its pupils*

Kasselsvlei High School, the school selected for this research project, is one of two high schools situated in an urban area in Bellville South, approximately 20km from Cape Town. It is a state funded school falling under the Department of Education and Culture Services and caters for the predominantly 'coloured' community of Bellville South. The racial composition of the school is therefore predominantly 'coloured' with only a few pupils coming from the black community while only one white pupil has been enrolled at the school in 1995, a legacy of the apartheid era. The school does not have any entrance requirements. The annual enrolment of this dual medium school varies between 1000 and 1200 pupils with a staff complement of 60 teachers. Pupils from the BellvilleSouth area make up 90% of the school population while the remaining 10% come from outside Bellville South, some even as far afield as Khayelitsha, a sprawling black township some 40km from Cape Town.

### *5.1.2 Gaining access to the school*

Even though I had been teaching at the school selected for the research for the past 13 years, it was still a necessary requirement that I obtain permission from the relevant authorities to undertake the research. This required that I negotiate with the principal of the school. This could only be effected in late January 1995. After explaining to him in detail what the research project entailed and discussing how it would be implemented, he granted permission on 25 January 1995 for the research to be undertaken at the school. He was expressly appreciative of the research been undertaken at Kasselsvlei High School as it would, in his words :

> "... expose the pupils to current technology and prepare them for the real world."

The principal's only concern was that I negotiate the exact times of the research sessions with the head of the mathematics department. This did not present a problem as he had been present during the negotiation and had in fact, already allocated certain classes for the research (I had discussed the research project with him in December 1994) to be implemented and had also informed the teachers whose pupils would be affected by the research. The teachers whose classes were co-opted into the research project welcomed the prospect of observing research being undertaken on a subject area that they would be teaching for the first time in 1995. (They had never before taught mathematics to matriculants.)



*5.1.3 The pupils in the research project*

During consultation with the research director of this project, it was agreed that the research program would involve matriculants since the research question was directed at the investigation of the formation of concepts relating to calculus which is an integral part of the matric syllabus. Initially, three matric classes had been allocated for the research which would have meant that at least 85 pupils would be involved in the research project. However, since the mathematics classes were taught concurrently, it was decided that only one combined class consisting of 23 English-speaking pupils doing mathematics on the standard grade would be used for the research project. The class consisted of 9 girls and 14 boys. Although all the pupils had passed mathematics in standard nine, their performance in mathematics was nevertheless considered by their teacher as been fairly weak. The average mark (for mathematics) in standard nine for this group was 41,4%.

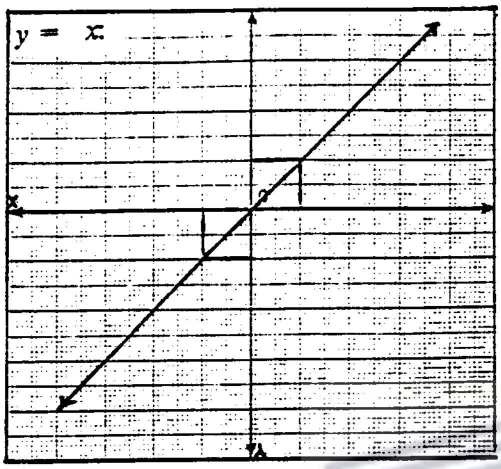
### *5.2 Project implementation*

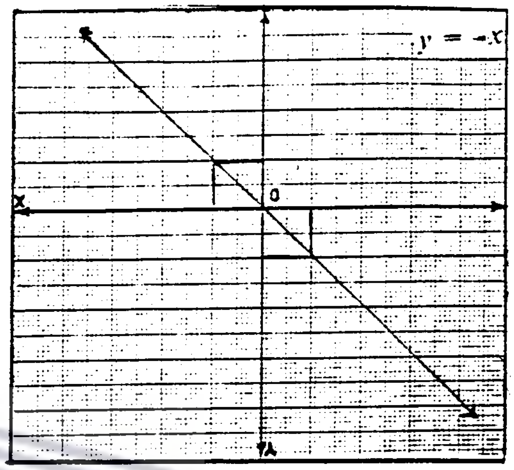
This section gives a succinct description of the entire research process.

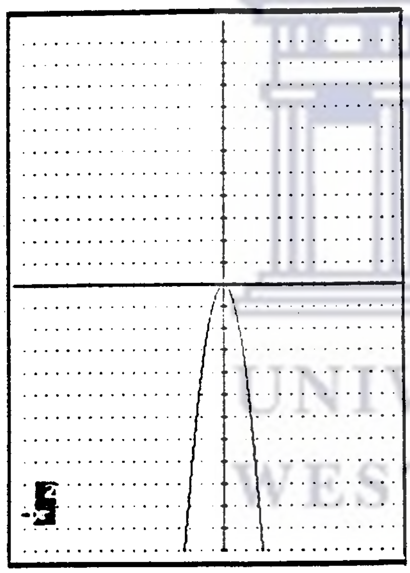
#### *5.2.1 Familiarization phase 1: part one*

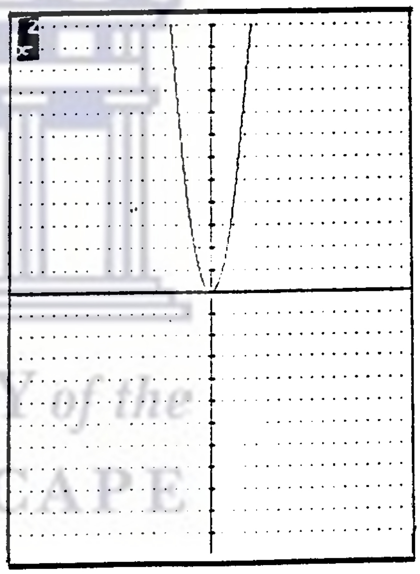
The first session with the class of 23 pupils took place on 7 February 1995. Twenty four CASIO *fx* 7000GB graphic calculators were made available by the research director of the department of Didactics at the University of the Western Cape. The first ten minutes of the session was spent explaining to the pupils what the research entailed while simultaneously giving them the opportunity to ask questions relating to the research. They were also informed that they were participating on a voluntary basis and that they could withdraw from the project at any time they wished. The hope was expressed that this would not be the case. Permission to have the research results published was also obtained. Having taken care of the necessary preliminaries, I handed each pupil a graphic calculator and proceeded with the familiarization phase. Pupils worked through a tutorial (See appendix A, #1.1) which was specifically designed to familiarize them with the primary functions of the graphic calculator. Special attention was paid to the special operation keys such as the **SHIFT, MODE, ALPHA, Prog, Graph, Range, Clearscreen (Cls), G < > T Cursor/Replay,** and **DEL** keys as these keys would be constantly used in the implementation phase of the project. This was followed by an exploratory exercise, lasting fifteen minutes, involving some very basic calculations. (See appendix A, # 1.2)

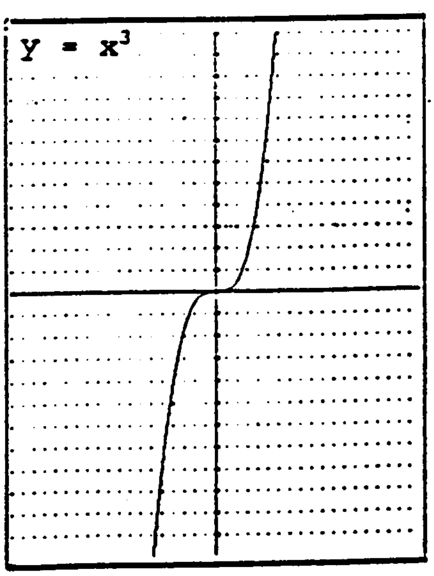
Having established that the pupils were capable of using the graphic calculator with a certain degree of dexterity and confidence,I proceeded with a further exercise which I hoped would wet their appetites for the sessions which would follow. This part of the familiarization phase involved a step-by-step approach to the construction of the graphs of functions (user generated) using the graphic calculator. The graphs that they were required to construct are given below:

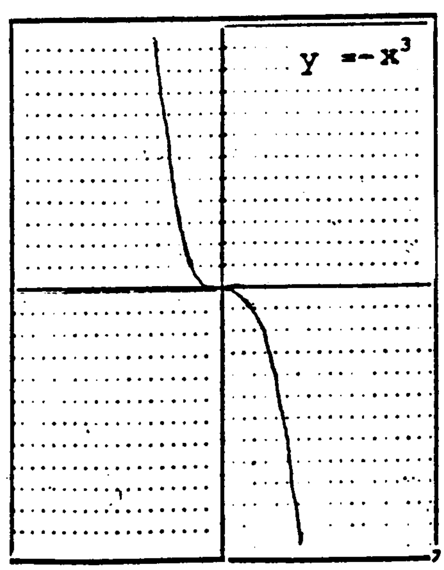






This part of the session lasted a further fifteen minutes. From the time the graphic calculator generated the first graph right to the time it generated the last graph, the pupils were held in awe. The excitement and amazement at what the graphic calculator was capable of doing was clearly visible from the animated expressions on their faces and their very audible exclamations. This happened just as the bell rang for them to leave the class. As they were leaving they gathered around the table and 'blasted' me with questions such as:

> **Karl:** "Is this thing legal, sir? How much does it cost?"
>
> **Gaynor:** "Can we use it in the exam, sir?"
>
> **Robin:** "I don't need a brain anymore if I use this thing!"
>
> **Vicky:** "Sir, can I take one home, I want to show my dad?"

Ending the session on such a positive note, I later reflected, was a very promising sign in that I believed I had stimulated their interest in the research project. I spent the rest of the school day in the staff room preparing worksheets for the follow-up sessions. During this time another teacher from the mathematics department commented that his pupils were unhappy because "... they were not using graphic calculators to do calculus!". It soon became apparent that the research group had told them about the experiment. I regretted not been able to accommodate another class.



The next session was scheduled for 9 February. However, due to a rescheduling of the school time-table aimed at preparing the pupils for the athletics training sessions, this was not possible. An hour was set aside each day from 8:30 to 9:30 for training sessions. I seized the opportunity to negotiate with the principal that I be allowed to use these sessions with the project group. He readily agreed to this new arrangement. However, this meant that I would be working with a smaller project group as some of the pupils who were initially involved in the project, were participating in the athletics training sessions.

A further complication ensued in that pupils not participating in the athletics program, were allowed to participate in other extra-curricula activities, thus further reducing the number of pupils involved in the project to 14. The group now consisted of 4 girls and 10 boys. I arranged with the project group to meet in the school library on the 15 February for the next session. The library was the only venue now available as all other classrooms were utilized for the extra-curricula program. Permission to use the library for the research was granted by the librarian as it was not normal practice to use the library for purposes other than it serving as an information centre.

### *5.2.2 Familiarization phase: part two*

Because of the time lapse between the two phases, I found it necessary to have the pupils re-do the first familiarization phase. They spent the time exploring various families of groups of functions as was described in phase one, paying attention particularly to the shape and the position of the graphs relative to the **x** and **y** axes, and using especially the **Trace** and **G < > T** keys to find the coordinates of any point on a particular graph. This was done to ensure that they knew how the calculator worked and what it was capable of doing. This part of the session lasted fifteen minutes. It became evident during this time that despite the time lapse, pupils had become quite adept at using the graphic calculator. Of particular interest was that they had formed small groups and were assisting each other. Again, their discussion with each other was rather ebullient and centered around the 'graphics' that the calculator produced. I did not intervene in their discussions preferring rather to take note of what they were doing. Their wonderment at the ability of the graphic calculator to exhibit the coordinates of a particular point on the graph was very gratifying to observe. Two boys from a class who were not involved in the project had joined the project group of their own accord and were particularly excited about the capabilities of the graphic calculator. It was their discussion which I found rewarding especially when one remarked that he could now solve equations simultaneously by constructing the graphs of these equations and by just using the **Trace** key, he could

### *5.2.3 Reflection: familiarization phase*

Having completed the familiarization phase successfully, I took the time off to reflect on the entire process up to that point. It appeared that the pupils were coping well enough with the operation of the graphic calculator and that they were now ready to be confronted with the actual research experiment. I needed now to establish whether they would be capable of formulating the various concepts of calculus with respect to differentiation. A serious shortcoming of the implementation phase was that I did not have any recording facilities to record the session. I resolved to correct this by placing two tape recorders in unobtrusive positions in the classroom as well as seeking the assistance of a fellow researcher, Monde Mbekwa, to act as observer. He agreed to assist in the observation of a session scheduled for 21 February. I then approached a colleague, Jeff Abrahams, who is also a teacher at Kasselsvlei High School and quite experienced in video recording, to video tape some of the sessions. Jeff agreed to do the video recording. I observed that recording and video taping some of the sessions would permit me to capture 'student voice' as listening carefully and persistently to how pupils conceptualize about a given task is an integral part of developmental research. I reflected further that for the research to be effective, it was necessary that I seek clarity and expert guidance from the research supervisor, professor Cyril Julie, as to how I was to proceed with the actual experimental phase of the research.

### *5.3 The teaching experiment: phase one*



#### *5.3.1 Didactic deliberation and implementation: phase one*

I met with research supervisor on 14 February to give a report back on the progress of the project and to seek guidance with respect to the next phase of the project. From our discussion it was agreed that the research question could best be investigated if the pupils could, by using the graphic calculator, deduce the different rules of differentiation listed below:

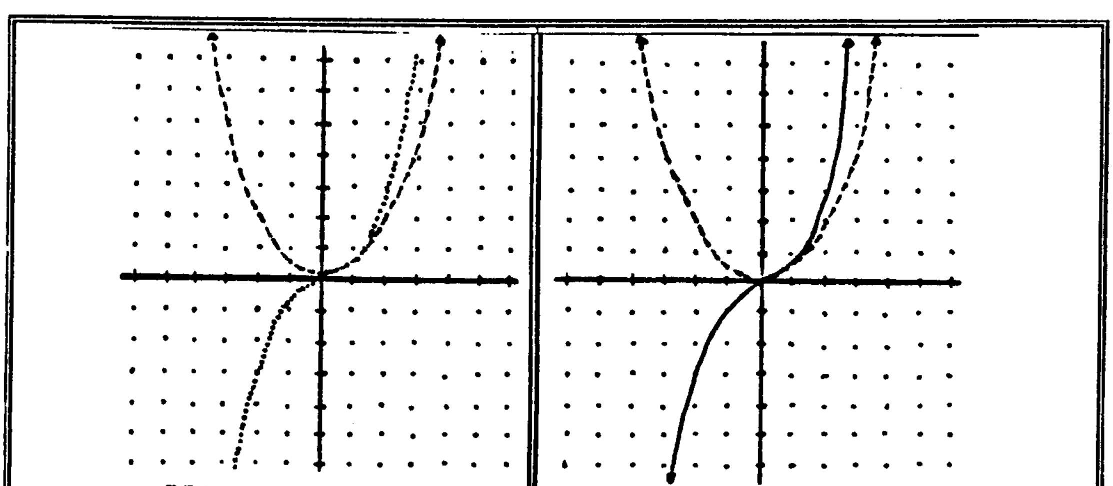
1. If $y = x^n$, then $dy/dx = nx^{n-1}$
2. $D_x[kf(x)] = kD_x[f(x)]$

*3.* *If $f(x) = k$, $k$ a constant, then $f'(x) = 0$*

*4.* *If $f$ and $g$ differentiable with respect to $x$ then:*

$$D_x[f(x) + g(x)] = D_x[f(x)] + D_x[g(x)]$$

It was then decided that in order for the pupils to formulate these rules using the graphic calculator, they would need to construct the graphs of various functions and through a process of obtaining an appropriate fit, (that is, finding a matching graph by guessing and entering a defining equation into the graphic calculator) find the equation of the derived graphs generated by the calculator. This necessitated that the graphic calculators be pre-programmed so that it could generate both the graph of a given equation (which is user generated) as well as the derived graph which the calculator had been programmed to produce. Since such a programme was available (See appendix C, # 1.1) I undertook to programme all the calculators that very day. It was then decided that an exercise would be given where pupils would be required to enter a defining equation representing a specific function into the graphic calculator, the graphic calculator would then generate the graph of that function as well as the graph of the derived function. The graph of the function that was user generated could then be darkened by entering the defining equation a second time. The graph of the derived function would then be the one represented by the graph still appearing as dotted line graph.



The equation of this function is found by entering an equation of the same family of graph and then pressing **Graph** on the graphic calculator. The process continues until an appropriate fit, that is , a matching graph is found. The pupils would then draw what is displayed in the graphic window on their worksheets as well as the equations of the derived function. When they had completed the exercise they would be instructed to study each pair of equations and then, in their own words, see if they could deduce a rule for the expected 'pattern' that emerged. This they would have to write down while I acted as observer- participant assisting where help was needed. The exercise agreed on for the first session in which the pupils would be required to find the equations of the resulting derived graph, is given in figure 1.5 below:

| *Given Function* | | *Derived Function* |
|---|---|---|
| *1.* | $y = x^2$ | ................. |
| *2.* | $y = x^3$ | ................. |
| *3.* | $y = x^4$ | ................. |

Assuming the pattern continues as above, complete the table below without using the graphic calculator:



| Given Function | | Derived Function |
|---|---|---|
| **4.** | $y = x^5$ | ................. |
| **5.** | $y = x^6$ | ................. |
| **6.** | $y = x^{10}$ | ................. |
| **7.** | $y = x^{1995}$ | ................. |

<u>Challenge</u>: What would the derived function of $y = x$ be?

**figure 1.5**

Having agreed on the interpretation of the research question and having established a modus operandi, I spent the rest of the day preparing the worksheet and programming the graphic calculators for use the next day. From what transpired the next day, 15 February, it became evident that I had misinterpreted the deliberation session specifically as to what the worksheet would entail as well as the process. The worksheet I had drawn up and which I had implemented is given below: (figure 1.6)

| | | | |
|---|---|---|---|
| *1.* | $y = x^2$ | *1.1* | $y = 2x$ |
| 2. | $y = x^3$ | 2.1 | $y = 3x^2$ |
| 3. | $y = x^4$ | 3.1 | $y = 4x^3$ |
| 4. | $y = x^5$ | 4.1 | $y = 5x^4$ |

**figure 1.6**

The pupils were required to construct these graphs using the graphic calculator and were then asked to compare the coefficients and exponents of each pair of equations with each other to see whether a 'pattern' emerged. Still unaware that I had misinterpreted the didactic deliberations I proceeded with the implementation of the worksheet. The pupils were then required to formulate a general rule. They were told that the expressions on the right were derived from the ones on the left. No further explanation was given. The pupils carried out the instructions enthusiastically and were visibly pleased with the results. The worksheet is discussed fully in chapter 6: *Analysis and evaluation of the teaching experiment.* This session was not video taped but taped recordings were made of discussions taking place between pupils and myself. These recordings are also analyzed in chapter 6.



### *5.3.2 Reflection: teaching experiment - phase one*

Based on the pupils answers on the worksheets and the taped recordings and still unaware that

**Nadean:** In the left-hand column the co-efficient of x is multiplied by the exponent of x and it equals the co-efficient of x in the right hand column. The exponent in the left-hand column is one more than the exponent in the right-hand column.

Of the four remaining pupils, one 'correctly' saw the relationship between the coefficient on the right, but failed to report on the relationship between exponents, while another pupil concentrated on the effect that the coefficient had on the shape and position of the graph relative to the **x** and **y** axes. This pupil's version is given below:

**Lutasha:** In 1.0 the coefficient is + 2 whereas in 3.0 the coefficient is - 2. (The exponents however stay the same.) And so too does 2.0's coefficient = +3 whereas in 4.0 the coefficient = - 3 (The exponents stay the same). The graphs therefore lie in different directions due to the - and the = coefficient.

When the pupils were asked to write down a general formula based on what they had discovered, algebraically, only six pupils attempted to do so. Their various attempts are given below:

| | |
|---|---|
| *Lyndon:* | $nx^n = kx^{n-1}$ |
| *Gaynor:* | $n^2x^{n-1}$ |
| *Wesley:* | $nx^n = knx^{n-1}$ |
| *Nadean:* | $nx^n = kx^{n-1}$ |
| *Ferdi:* | $d/dx\ knx^{n-1} = n.kx^{n-1}$ |
| *Lutasha:* | $exp. = n \quad k(x^{n-1})$ |
| | $coef. = k$ |

Although the pupils' responses are discussed fully in chapter 6, I needed to insert some of their responses here as these formed the basis for the next phase.

### *5.3.3 Further reflection: teaching experiment - phase one*

Reflecting on the entire phase one, I suspected that something was amiss. Although the pupils were not able to write down the rule that if $y = x^n$, then $dy/dx = nx^{n-1}$ as given here, I felt that as this was but their first attempt, I need not worry as I would have the opportunity to 'scaffold' them towards the conventional way of writing it down. What's more, they had recognized the pattern on which I had put a lot of emphasis and, in their idiosyncratic ways, could eloquently verbalize what they had discovered. Yet a nagging feeling convinced me that something was wrong as the exercise appeared to be too easy. Although most of the session had been video taped, the recording was not immediately available for review as Jeff had left the school earlier than usual. I contacted a colleague, Monde Mbekwa, to express my dual feelings of elation and reservation. I resolved to meet with the research supervisor to discuss this with him and seek further guidance.

### *5.3.4 Second didactic deliberation*

I met with the research supervisor, professor Julie, on Friday, 20 February, to give a full report on the progress of the teaching experiment and also to seek further guidance. On his request, I explained to him why I had taken the approach as reported in sections 5.3.1 and 5.3.2 and not as agreed to on 14 February. It now became clear that I had indeed misinterpreted the didactic deliberation and that this needed to be corrected. Professor Julie explained in detail how the pupils were to find the appropriate fit (that is, the matching graph) as was explained earlier. We worked through a few examples after which I resolved to implement the approach exactly as intended. For the sake of clarity, the exercise is repeated here:

| *Given Function* | *New Function* |
|---|---|
| **1.1** $x$ | ................. |
| **1.2** $x^2$ | ................. |
| **1.3** $x^3$ | ................. |
| **1.4** $x^4$ | ................. |

**Instruction:** Enter each function into the calculator to generate its graph as well as the (programmed) derived graph. Enter the function again so that the graph is darkened. Now, find the equation of the derived graph by finding an appropriate fit, that is, a matching graph. Write down the equation thereof in the space provided. Show, as far as possible, all your attempts at finding an appropriate fit.

Now, assuming that this pattern continues, write down the equation of each of the following derived functions:

| | *Functions* | *Derived Function* |
|---|---|---|
| *1.1* | $x^5$ | ................. |
| *1.2* | $x^6$ | ................. |
| *1.3* | $x^7$ | ................. |
| *1.4* | $x^{17}$ | ................. |
| *1.5* | $x^{1995}$ | ................. |



*Having done this, can you now deduce a general for the above for $x^n$; $n \in N$.*



It was also decided during the deliberation session that once the pupils were able to confidently do the previous exercise as given on page 64, that I should proceed with the exercises designed to determine the next rule of differentiation as given below:

$$D_x[kf(x)] = kD_x[f(x)]$$

The exercise for this part of the next phase is given below:

| *Power function* | *Derived function* | *Constant function* | *X Power function* | *Derived function* |
|---|---|---|---|---|
| *1.1* $x$ | ......... | $4x$ | | ............. |
| *1.2* $x^2$ | ......... | $5x^2$ | | ............. |
| *1.3* $x^{13}$ | ......... | $2x^3$ | | ............. |
| *1.4* $x^{24}$ | ......... | $3x^7$ | | ............. |
| *1.5* $x^{95}$ | ......... | $5x^{25}$ | | ............. |

As Monde was present during this meeting, I reminded him that he had agreed to act as an observer for the next sesion. He confirmed that he would be at school at 8:30 on the 21 February. I spent the weekend working through the exercises so as to ensure that there were no hitches during the next session.

*5.4 The teaching experiment: phase two*

This phase was executed exactly as planned. The pupils took no more than fifteen minutes to complete the entire exercise, (See appendix B, worksheet #3) two girls completing the exercise in 5 minutes. Although the pupils were now using the program I had entered into the graphic calculator for the first time, they did so with a minimum of effort. At times the graphic window would go blank immediately after the graphs appeared. This was quickly corrected by myself. No other problems were experienced by the pupils. They were, however, unusually quiet, but I attributed this to the presence of Monde who had volunteered to observe the entire session. Jeff was not available to video tape this session as he was busy with a recording for the photographic society of the school. Neither was I able to secure the use of the tape recorders for this session and therefore relied on Monde's observation notes and my own. These would be used to guide the next session. After the pupils had completed the exercise, they were asked to formulate a general rule based on their experimenting with the program installed in the graphic calculator.

They spent the rest of the session discussing their formulation of a rule with each other while I conferred with Monde. He promised to have a written report available within a few days but made some specific comments concerning the instructions I had given during the session. He had noted that I constantly asked the pupils not to look at each others work, but to concentrate on formulating their own rule. His concern was that since I was working within a constructivist framework, he would have expected that the pupils would be allowed to socially negotiate meaning to any given construct. This Monde spells out very clearly in his written comments which he submitted a few days later:

> JE, throughout the lesson instructed pupils not to discuss their answers with their partners. I'm not sure what assumptions or learning theory informed his approach. My assumption was that the objective of the whole exercise was the construction of meaning or conceptualization through students. JE's insistence on individual construction of knowledge is antithetical to a social constructivist theory of learning whether from a Piagetian or Vygotskian understanding.



I agreed with Monde on this issue, but pointed out to him that I needed to establish whether the pupils would be able to formulate a rule without the help of their partners. I did not want them to merely copy the work of someone else. They could then discuss their answers with each other. I had no problem with that . This they in fact did as Monde points out:

> Nevertheless, throughout the lesson, students tended to communicate and to explain their constructions to those next to them despite JE's exhortation to do otherwise.

Monde's comments were nevertheless, taken seriously as I resolved to allow the pupils to communicate freely in further sessions. (For a full report of Monde's comments, see appendix D, # 1.1)

Phase two of the teaching experiment continued on 22 February. The research supervisor, Professor Julie, was present to observe the process. Students worked through Worksheet # 4 which included a revision exercise. This time they used most of the session to complete the worksheet, but I interrupted them at times when I needed them to explain how they arrived at a certain rule. During the session Professor Julie had observed and was discussing with a group of pupils how they had arrived at a certain derived function based on a given function. He found their idiosyncratic ways of explaining, very rich especially when one girl, Marina, who had written down her finding and when prompted to voice this finding, started by saying: "... The way I found the derived function is that I saw a relationship between..." It was at this point that Professor Julie asked whether I had a recorder available which I did not have. This was the most disappointing aspect of phase two of the research! Professor Julie remarked at this point that the research process was complicated especially with respect to the observation thereof.

An analysis of phase two is given fully in chapter 6.

*5.4.1 Reflection: teaching experiment - phase two*

When I reflected on the entire phase two of the teaching experiment, I felt confident that the research was now proceeding as planned. As I reviewed the worksheets, I was particularly pleased that quite a few pupils had formulated the rule as given here that if **$y = kx^n$, then $dy/dx = knx^{n-1}$**. They were also applying this rule very confidently and making use of the graphic calculator each time they attempted to arrive at an answer. However, a serious shortcoming of this session was that neither taped or video recordings were made and I resolved to approach Jeff again to request that he record the remaining sessions. He agreed to video record at least two more sessions as he was neglecting his responsibilities assigned to him by the photographic society. I was approached at this point by the head of the Science Olympiad committee, who requested that I release five of the pupils who had indicated to her that they wanted to participate in the Science Olympiad in March. She needed to prepare them for this occasion. As I had obtained permission from the principal to undertake this research, I tried convincing the teacher to delay her preparations somewhat. She was adamant that I release the pupils since they were participating on a voluntary basis. We eventually agreed that the pupils were to decide for themselves whether they wanted to be released from the research project. Also, the mathematics teacher expressed her concern that her pupils were not available for revision sessions with her as she had indicated earlier to me that they were the weaker pupils in mathematics. I was able to convince her that these sessions would in fact assist them in understanding calculus which she was to begin teaching later that term. The teacher conceded and also remarked that her pupils were very excited and positive about the research.

I decided that I would continue with the program under section 5.3.4 having already prepared an exercise for the next session. The intention of this exercise was to see whether they could derive a rule that if **y = k**, for **k** a constant, then **dy/dx = 0**. For convenience, this exercise is repeated here:

| *Power function* | *Derived function* | *Constant X power function + constant* | *Derived function* |
|---|---|---|---|
| *1.1* $x$ | | $x^3 + 1$ | |
| *1.2* $x^2$ | | $2x^3 - 1$ | |
| *1.3* $x^3$ | | $3x^3 - 3$ | |
| *1.4* $x^{40}$ | | $-2x^3 - 4$ | |

*__INSTRUCTION__: complete the table above with respect to the derived function of the power function only, without using the calculator. Now use program 1 to find the derived equation of each equation falling under the heading Constant X power function + constant. Discuss your answers with each other and see if you could derive a rule with regard to the constant based solely on what you observe from the graphs generated by the calculator.*

I did not consider it necessary to arrange a further consultation with the research supervisor at this point as I was confident that the research was proceeding well. I therefore met with the project group the next day, 23 February.

*5.5 The teaching experiment: phase three - part one*

As pointed out earlier, the head of the Science Olympiad committee, requested that five of her pupils attend her preparation sessions. She arrived at the class at 8:30 (23 February) and asked her pupils to accompany her. To avoid any confrontation at this juncture of the research, I decided to allow the pupils to go. The project group now consisted of 2 girls and 8 boys. We resolved to continue with the session. With fewer pupils in the class, I could spend more time with individuals. Jeff had also arrived to record this session.

I found this session quite rewarding as the exercise the pupils were now engaged in, led to much animated discussions amongst the pupils and a certain degree of intense probing was required from me. The discussion and probing centered around the apparent 'disappearance' of the constant from the graphics window. The constant appeared to the pupils, to have no effect on the derived graphs which resulted in a fair amount of discussion. (What in fact was happening was that each time the derived graphs would pass through the origin.) This was despite the fact that they wrote down the derived function correctly in each case while making use of the graphic calculator. In chapter six, I pay particular attention to the video recording of this session.

*5.5.1 Reflection: phase three - part one*

Despite the lower number of pupils present in part one of phase three, in retrospect I felt that this might have been an advantage as it afforded me the opportunity to work more closely with the pupils to the extent that I was able to really probe their responses with respect to the affect of a constant being present in an equation and 'scaffold' them towards believing that the derivative of a constant was equal to zero. This is highlighted by one of the pupil's exclamation just as the class was leaving:

**Lorne':** Ah, I see, the derivative of a constant is zero because the graph of the derived function passes through the origin.

For Lorne' this was an almost "**eureka**" experience since he struggled quite a bit to discover this. At one point he remarked rather puzzled:

Gheez, what can it be ?

I scheduled the next session for 2 March as I had arranged appointments on three consecu]tive days, that is , the 27 and 28 February, and 1 March.

For the session that was to take place on 2 March, I prepared the following exercise:

| | *Given Function* | *Derived Function* |
|---|---|---|
| *1.1* | $2x^3 + 3$ | |
| *1.2* | $4x^2 - 3$ | |
| *1.3* | $x - 5$ | |
| *1.4* | $x^3 - x^2 + 3x + 4$ | |
| *1.5* | $2x^3 + x^2 - 2x + 4$ | |
| *1.6* | $x^5 - x^4 + 2x^3 - 3x^2 - x + 2$ | |
| *1.7* | $y = 1$ | |
| *1.8* | $y = -2$ | |
| *1.9* | $y = -5$ | |

The purpose of this exercise was to see whether the pupils could formulate the rule that if $f$ and $g$ are differentiable with respect to $x$ then:

$$D_x[f(x) \pm g(x)] = D_x[f(x)] \pm D_x[g(x)]$$

### *5.5.2 The teaching experiment: phase three - part two*

During this session the pupils worked through the exercise as given above. The exercise was written on a portable writing board as the photostat machine was being serviced that morning. Jeff arrived to video record the session fifteen minutes after I had already begun. I was just pleased that he arrived in time to video tape the sections where I had begun probing the pupils conceptualization of the rule with respect to a constant, particularly so since the pupils were using the **Trace** function key to 'see' on the graphic calculator that the derived graph passed through the origin whose coordinates are (0;0). The **Trace** function key was used to entrench this aspect. When the pupils entered the equations containing only a constant as in #'s 1.7 to 1.9, they were convinced that the derived function is equal to zero but could not explain this from what they had observed on the graphic calculator. Again it became necessary for me to 'scaffold' them towards a well reasoned answer.

I had to leave the pupils to continue the exercise as I had an appointment with the research supervisor at 09h00 that day. Jeff stayed with the class until the end of the session.

### *5.5.3 Didactic deliberation: phase three - part two*

I had arranged with Professor Julie to meet him that morning to discuss the progress of the project, but particularly to inform him about the changed circumstances. Of particular importance was the fact that the number of pupils was beginning to wane but more importantly, I had observed during the session that the pupils were not persistently using the graphic calculator to find a suitable match for the derived graph and then write down the equation. Instead, they would apply the differentiation rules to the given function, then use the derived function to plot an exact match the first time round. This was not necessarily an obstacle since it showed that they successfully transferred their new found knowledge to other areas of study. Still, I preferred to seek guidance on this aspect. In retrospect, I believe I was more concerned with the waning numbers. Although the decreased numbers on that day was due to the fact that the pupils had a late night out - they attended a disco in aid of school funds on 1 March - it still worried me. The research supervisor suggested that I continue to work with the remaining pupils, and if it was possible, engage some of them in interview sessions. He also suggested that in order to remedy the situation with respect to the use of the graphic calculator, I give the pupils a slightly different exercise such as the one given below:

| | *Given Function* | *Derived function* |
|---|---|---|
| *1.1* | $(x - 3)(x + 2)$ | ................ |
| *1.2* | $\frac{(x + 2)(x - 5)}{(x + 2)}$ | ................ |

### *5.5.4 Reflection: phase three - part two*

Phase three of the teaching experiment had proceeded rather well I thought. The pupils were able to verbalize and write down the different rules of differentiation without too much help. They had also arrived at the stage where they could confidently apply these rules to any given equation in its simplified form. They used the graphic calculator whenever they needed to and found that it represented, in the words of one of the pupils, Vicky:

> ... a much easier way of doing graphs, I understand graphs much better now and I'm sure the graphic calculator will be a great help if we are allowed to use them to do calculus.



I did, however, feel that I definitely needed to test the pupils' understanding of the differentiation rules if they were presented with slightly more difficult problems, of the type that usually appear in the matric examination papers. This was not to be.

The evening of 2 March, Jeff arrived at my home to deliver the video recording and the graphic calculators he had collected from the pupils that morning after the session had ended. He reported that the principal had announced that the training sessions and the extra-curricula program were to be suspended as from Monday, 6 March, as the athletes had completed their training and the teachers had to get on with the academic program. This was non-negotiable as teachers had complained that they had lost quite a bit of time and still had to prepare the pupils for the March examination,. Because of this announcement, Jeff took it upon himself to inform the project group accordingly, and at the same time he asked the pupils to give their impression of the research project.

I nevertheless went to school on 3 March to see if I could still manage to get in a last session. This was not possible as the pupils were now following the normal school program. An attempt to use the mathematics periods was unsuccessful as the mathematics teachers felt that they needed all the available time to catch up on their work. I later established that none of the other pupils had in fact even begun on the calculus course except for one class of higher grade pupils.

I then decided that it was time to formally end the research project. I requested the project group to meet in the library during interval that day where I thanked them sincerely for their participation in the research project. I expressed the hope that I would be able to assist them whenever they needed me to do so.

*5.6 Summary*

This chapter gave a succinct outline of the research process undertaken with 14 matriculants at Kasselsvlei High School. Despite an initial misunderstanding with respect to the first part of the teaching experiment, and despite the number of pupils beginning to wane towards the end of the research, I believe that the research process had taken its natural course in that the pupils were given the opportunity to explore calculus in a graphically enriched environment and were able to formulate the manipulation rules of differentiation in their idiosyncratic ways.

# Chapter Six

# Analysis and Evaluation of the Research Project

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> *"It is inconceivable that any mathematics reconstruction or reform initiative can currently be be contemplated without due recognition being given to available technologies for teaching and learning mathematics.*
>
> ***Cyril Julie (1993:342)***

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## 6.1 Introduction

In chapter 3 of this research paper I outlined the epistemological basis which served to underpin this research. In particular, I postulated that an epistemological of multiple representations provides opportunities for pupils to explore, create, and reflect on their constructions and that the graphic calculator served as an appropriate mediating tool for establishing an epistemology of multiple representations. In this chapter, the research question and the teaching experiment which was designed to investigate the relevance of the research question, is analysed and evaluated in terms of the theoretical framework. The analysis and evaluation is followed by some concluding remarks and recommendations with respect to the teaching of mathematics.



### 6.2 *The theoretical framework of the research*

It is necessary to review the theoretical framework which underpinned this research as the analysis and evaluation is formed by these underpinnings. A central tenet of constructivism is that knowledge acquisition is an active process. Learners actively construct their own understandings rather than passively absorb or imitate the understandings of others.

With respect to the teaching of mathematics, it becomes imperative then that the teachers recognize that pupils possess a mathematical reality of their own and coming to know this mathematical reality requires that teachers engage in what Confrey (1990:108) calls 'decentering', that is, the ability to see a situation as perceived by another through working within the frame of reference of the person and attempting to understand the situation through the perspective of that person. It was argued that the process of decentering could be activated if the teacher introduced helpful perturbations (that is, creating disequilibrium in the cognitive structure of the student) Furthermore, it was considered that the graphic calculator was the most appropriate tool for creating these perturbations as it was recognized ( and supported by various research findings) as a multi-representational tool that was both inexpensive and non-dependent on a supply of electricy, making access possible to pupils, relatively uncomplicated. A further implication of a constructivist framework is that knowledge thus acquired is in essence socially negotiated since the learning process involves the interaction of humans with each other in what Ernest (1993:170) called "shared forms-of-life".

It is against this background that the research was undertaken in an attempt to answer the research question as to whether pupils would be able to formulate certain concepts of calculus if they were allowed to use graphic calculators.

### *6.3 Analysis of results*



During didactic deliberation with the research supervisor, professor Julie, an agreement was reached that the best way to approach the research question was through the development of a teaching experiment aimed at giving the pupils the opportunity, through various structured exercises and by making use of the graphic calculator, to formulate the rules of differentiation as given below:

**a.** If $y = x^n$, then $dy/dx = nx^{n-1}$

**b.** $D_x[kf(x)] = kD_x[f(x)]$

**c.** If $f(x) = k$, for $k$ a constant, then $dy/dx = 0$

**d.** If **f** and **g** are differentiable with respect to **x** then:

$$D_x[f(x) \pm g(x)] = D_x[f(x)] \pm D_x[g(x)]$$

The exercises were to be reviewed and refined after each phase of the teaching experiment.

*6.3.1 The teaching experiment: analysis - phase one*

Worksheet # 1 and # 2 (See appendix B) were drawn up for phase 1 of the teaching experiment. The entire process as well as the misinterpretation of the first didactic deliberation as to the nature of the exercise was discussed at length under section 5.3. What follows here is an analysis of studentls responses to the exercises.

The expressed purpose of worksheet # 1 was to see whether pupils could recognize the relationship between the coefficient and the exponent of a given function and compare it with that of the derived function, namely that the coefficient of **x** multiplied by the exponent of **x** of the given function is equal to the coefficient of **x** of the derived function. The exponent of **x** of the derived function is one less than that of the given function. The underlying premise was that if they could recognize this relationship, they would be able to deduce the general that if $\mathbf{y = x^n}$, then $\mathbf{dy/dx = nx^{n-1}}$. A study of the pupils' responses reveals that few pupils recognized this relationship as I had expected them to and consequently, could not reproduce the rule as given above. Some of their responses are reproduced here:

**Courtney:** The exp. of the equ. is used as the coef. in the second equ. with the exp. in the second equation 1 less than in the first equation.

**Wesley:** L.H.S.:The coef. stays the same but the exponents are increasing. All even exponents are parabolas.
R.H.S: The coef.'s and the exponents are increasing. The exponents are less then (sic) the coef. In each graph a parabola is present. The exp. on the L.H.S = the coef. on the R.H.S

**Elyaas:** In the first graph the exp. is the same to the coef. in the second graph (equation).

Courtney's interpretation is representative of the research group. Although this is not quite what I expected, an examination of the worksheet reveals that their interpretation is indeed correct, since this is a pattern present in the worksheet, but not the one I expected them to see. In retrospect, therefore, I could not have expected anything more than this. Neither could I expect them to deduce a rule based solely on this exercise as it lacked sufficient content (for example, the coefficients of # 1 to # 4 is 1 throughout instead of it varying in size.) that would permit them to deduce the rule stated above, with their first attempt. A careful study of their reponses in the first worksheet also reveals their uniquely different ways of constructing knowledge as demonstrated in the insert above. No two pupils wrote down or voiced identical answers. The flaw in worksheet # 1 was only picked up during the implementation of worksheet # 2. Despite this flaw, two pupils did attempt to formulate an algebraic rule for the 'pattern' that emerged as given below:

**Elyaas:** $y = nx^n$ then $y = n^2x^{n-1}$

**Hassan:** (Gives the same as above)

It is also clear that the pupils could not generate the rule from the graphic calculator using appropriate fit (that is, finding a matching graph) since they had constructed their own graphs using the given equations and then used these graphs to interpret the questions. Although I had programmed the graphic calculators to generate the graph of a given function and its derived graph simultaneously, this program was neither explained to the pupils nor employed during phase 1 of the teaching experiment. The comments about the use of the graphic calculator therefore apply to worksheet # 2 as well.

The second worksheet was implemented with the same objectives in mind as the first, but this time the coefficients of the variables were greater than 1. This time 12 of the 16 pupils present recognized the relationship ( the one I had expected them to recognize) between the coefficients and the exponents of the given function and the derived function and were able to eloquently verbalize this relationship. This version of Karl given below is representative of the

12 pupils who constructed their own understandings of this relationship:

> **Karl:** The co-efficient of the first equation multiplied by the exponent will give you the co-efficient of the second equation. The exponent on the LHS is one more than the exponent on the RHS.

As stated in section 5.3.2 of chapter 5, only 6 pupils attempted to write down an algebraic rule for this relationship or 'pattern'. For the purpose of analysis only that of one pupil is given here:

> **Gaynor's algebraic rule:** (the derivative = ) $n^2x^{n-1}$

Based on # 1.0 to # 4.0 of worksheet # 2 this is an absolutely correct interpretation of the exercise and reveals the inherent flaw of the exercise itself. Even though the coefficients were now greater than 1, the coefficient and the exponents are the same giving rise to Gaynor's correct interpretation and possibly resulting in the other pupils not formulating the rule precisely as given earlier. This error was picked during this session and remedied by giving each pupil a different equation to do where coefficients and exponents differ and coefficients are greater than 1. In one instance even a fraction was given. The manner in which Gaynor and all the other pupils expressed their findings, is a representation of their unique constructions of mathematical knowledge. It also becomes clear from the video recording of this session that a simple transmission of knowledge runs contrary to the principles of constructivism, that is, that knowledge is actively constructed by the cognizing subject, and not passively received from the environment and that communication and interaction with others influence the formation of knowledge. This video shows very clearly how pupils interact and engage in discussion with each other, exploring avenues not gone before and creating their own unique understandings. In the words of one very excited pupil:

> **Lutasha:** Our very own formula!

Here Lutasha takes personal autonomy for making mathematics her own by taking responsibility for the construction of her very own formula. At some point in the video recording of this session I try to 'scaffold' one of the pupils towards saying that we 'subtract' 1 from the exponent on the right. He persists in using the term 'decreasing' and I relent! After all, what is wrong with the term 'decreasing'. His representation is in all respects valid and an acceptance of the pupil's construction of knowledge is a sine quo non of a constructivist approach to learning and teaching.

I commented earlier on the use of the graphic calculator in this and the previous exercise. Because I had not instructed the pupils to use the program I had installed in the graphic calculator, They restricted themselves to generating the graphs of the functions represented by the equations in worksheet #2. The video reveals that they did this quite confidently and this is also evident from the sketches they drew on their worksheets. This session was followed by didactic deliberation to clear up a misunder-standing of the process involved with respect to the use of the graphic calculator in combination with the worksheets. During this session the interpretation of the process involved in using the program installed in the graphic calculator to simultaneously generate the graphs of a given function and its derivative was re-negotiated and examples tested.



*6.3.2 The teaching experiment: analysis - phase two*

After resolving the misinterpretation of the first didactic deliberation, I proceeded to implement phase two of the teaching experiment which consisted of workheets # 3 and # 4. These sessions were respectively odserved by a colleague, Monde Mbekwa, and the research supervisor, professor Cyril Julie. No video recordings were made of either of the two sessions for reasons explained in section 5.4.

The process involved in using the graphic calculator to generate both the graph of the given function and its derived function was explained to the pupils in carefully constructed steps as they were doing so for the first time. They nevertheless coped quite well with this change in the use of the graphic having to move through different modes of functioning, that is, from an algebraic mode of functioning to a graphic mode and back to the algebraic mode after each exercise. Both Monde and myself observed that the pupils were quite engrossed by this new approach of finding an appropriate fit ( matching graph) or what Monde calls in his report (see appendix C) "trial and error". Monde also notes that one of the pupils had a "eureka" experience when he discovered the defining equation of the derived graph using "trial and error" when he exclaimed:

"I've got it! I've got it"

I made similar observations and was particularly pleased that the program was working well. The method of finding an appropriate fit worked well based on the answers given on the worksheets. However, still only 3 out of 12 pupils gave the rule algebraically correct as **dy/dx = $nx^{n-1}$** ! Again, the flaw lie in the nature of the exercise although the pupils gave correct answers to almost all the questions. This I attribute primarily to the pupils instinctively trusting the answers produced by the graphic calculator through their experimenting with finding appropriate fit. Monde reports that two pupils at least were able to write down the rule **dy/dx = $nx^{n-1}$**, after experimenting with the graphic calculator.

During the second session of phase two of the teaching experiment professor Julie was present. Worksheet # 4 was implemented, this time with a greater degree of success. All 12 of the pupils present gave correct answers to all the questions and correctly formulated the rule **dy/dx $kx^n = k.nx^{n-1}$** ! Each time the pupils found an appropriate fit (matching graph) they were able to write down the equation of the derived graph and then explained verbally how they obtained this. One pupil, Marina, explained that how she found the equation of #1.1.13 (worksheet #4) was that she saw on the graphic calculator that the derived graph (produced by the graphic calculator) was in the shape of a parabola, and knowing that the equation of the parabola is in the $x^2$ form, she experimented with this until she found a match. Her way of purposes by first applying the rule and then seeking confirmation thereof by using the graphic calculator afterwards. This demonstrates that they had taken personal autonomy for creating their own mathematics. However, I still needed to ensure that the pupils were not merely mimicking some algorithm and applying this to any given situation. To encourage them to explore a bit further, I engage them in examining equations of the following type:

**1.7** **y = 1** **1.8** **y = -2** **1.9** **y = -5**

The calculator now generates only one graph, the one they had entered, resulting in some rather argumentative discussion. Although some of the pupils gave the answer as zero, I once more needed to scaffold them towards understanding why this was so. I explained the process to the pupils, by working through a few other exercises in worksheet # 6 for which the graphic calculator does generate both the given function as well as the derived function. Together we then entered one of the above equations again. The calculator generated only the graph of the given function and I then probed them to explain why this happens. One pupil, Elyaas, then replied that there was no derived graph for a constant. Another pupil expressed it differently when he stated that the derived function of a constant is naught. The other pupils concurred. This gave me the opportunity to engage in a discussion with Marina who still needed some help in understanding this concept. We worked for approximately half an hour to try to eradicate the uncertainty. Some of the points worth noting here is Marina's gradual progress towards arriving at the rule that if **y = k,** for **k** a constant, then **dy/dx = 0.** Towards the beginning of our converstaion Marina is still quite uncertain:

**Me:** What happened when **y = 1** was entered?

**Marina:** Disappeared?

**Me:** What happened to the derived graph?

confusion and delay in pupils arriving at the intended formulation of the rules of differentiation. Fortunately this was at the beginning stage of the teaching experiment and after several follow-up sessions, the pupils managed quite well.

An important aspect about the research is that the pupils expressed themselves in their idiosyncratic ways thus supporting the theory that pupils have a mathematical reality of their own. This is corroborated by the interview with Marina, one of the pupils involved in the research project. The video recording of some of the sessions also supports this view. Another positive aspect about the research is that pupils often took personal autonomy for the construction of mathematical knowledge by actively engaging in the process of construction of knowledge and only made it their own once they themselves believed it. The social negotiation of meaning is very succinctly shown in the natural way in which pupils in the project group engaged in discourse with each other, each seeking to convince the other of the legitimacy of his/her construction.

An epistemology of multiple representations to create disequilibrium in the cognitive structure of the learner is profoundly achieved by making use of the graphic calculator. Throughout the teaching experiment the graphic calculator, when used to move through the different modes of representation, that is, from the algebraic mode to the graphic mode and back to the algebraic mode, allowed the pupils the autonomy to act in ways that they preferred, giving them the opportunity to explore mathematical ideas recognizable to them. Their idiosyncratic representations of what constitutes mathematical knowledge certainly exerted an impact on the formation of certain concepts with respect to the formulation of rules of differentiation. The graphic calculator's unique capability of creating multiple representations played a significant role in the process of concept formation, albeit only after some initial problem of interpretation with respect to implementation, was cleared. The view of Bernadette Dufour-Janvier (1990:94) that certain representations are so closely associated to a concept that it is difficult to comprehend how such a concept can be conceived without such representations, is profoundly justified in this teaching experiment. The continuous use of the graphic calculator to scaffold the pupils towards a visual conceptual formulation and understanding of the different rules of differentiation supports Dufour-Janvier's statement.

The teaching experiment did not proceed without incident. The misinterpretation of the process has been alluded to several times Another serious defect in the implementation of the teaching experiment was the abrupt conclusion thereof. This was certainly not anticipated and served to restrict the outcome of the experiment to the work produced up to that point, the intention having been to further explore pupils conceptions of differentiation rules when applied to more complicated exercises. The reduction of the initial number of pupils participating in the project to only 14 and at one time, only 9, while giving me the opportunity to engage pupils in discussion more closely, was nevertheless a worrying aspect as this created a certain degree of dismay. Finally the retention of knowledge thus constructed, that is. through the teaching experiment, was not tested.

### *6.5 Conclusions and recommendations*

Research findings about the efficacy of technology in the teaching and learning of mathematics is supported by this research experiment particularly with respect to the role that graphic calculators play in mathematics education. As an aid to concept formation, the graphic calculator affords the pupils the opportunity of exploration and creation of mathematics. Seemingly, this use of the graphic calculators decreases the emphasis on the rigorous algorithmic approach to the teaching and learning of mathematics.

Education encompassing the full spectrum of currently available technology is , for the majority of South Africans, still an ellusive ideal. Yet, science and technology are still being grossly neglected in the majority of schools in South Africa and the disturbing reality is that while current technologically innovative learning aids such as the graphic calculator are now becoming more readily available, it is still starkly absent from the school curriculum. The opportunity that it affords teachers, in the words of Julie (1993:25) " to start exploring a radically different curriculum" could be lost if research findings about the efficacy thereof continues to be ignored. I should add here that while the research justifies the use of technologically innovative rules in the teaching of mathematics, present curriculum constraints imposed from outside the teaching situation, is a seriously inhibiting factor. It is indeed desirable that the relevant authorities implement a radically different curriculum.

With respect to the research process, my experience has been that thorough testing of materials should precede implementation as this necessarily affects the outcome of the research. This marks research as iterative and consequently, this research needs to be continued.

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# APPENDIX A: # 1.1
# Familiarization Phase 1 & 2

## Getting Acquainted With The CASIO *fx* - 7000GB Graphic Calculator

1. **THE KEYBOARD** : Study the graphic calculator carefully before switching it on. There are basically three sets of keys:

**GROUP 1** : These consist of the following programming keys and includes the REPLAY key. (Rows 1 and 2)

| | | | | | |
|---|---|---|---|---|---|
| *shift* | *alpha* | *prog* | ← | → | *mode* |
| *mDisp* | *graph* | *range* | ↑ | ↑ | *G* ◄► *T* |



**GROUP 2** : These consist of the FUNCTION keys. (Rows 3, 4 and 5)

| | | | | | |
|---|---|---|---|---|---|
| *ENG* | √ | | $x^2$ | *log* | *In* |
| $x^{-1}$ | ●●● | *hyp* | *sin* | *cos* | *tan* |
| *(-)* | → | ( | ) | *xy* | $^x\sqrt{}$ |

**GROUP 3** : These include the numeric keys as well as some special operation keys such as:

*Del (delete key)* *Ans (answer key)* *EXE (execute/command key)*

+, -, *x*, : *(arithmetic operation keys)*

Most keys have more than one function. These are altenative (upper-case or lower-case) keys. These keys are activated by depressing either the **SHIFT** or the **ALPHA** key.

The **SHIFT** (yellow/brown) activates the upper-case symbols above the particular (depressed) key. **Plot, Trace, Cls and Factor** are examples of such functions.

The **ALPHA** (red) [alphabetic keys] activates lower-case symbols such as **A, X and J.** One of the keys you are going to use frequently are the **X, Y** keys. Find these keys on your graphic calculator.

2. <u>MODE</u> :

Switch on the graphic calculator. The following should appear on your display screen:

*** MODE ***

**sys mode : RUN**
**cal mode : COMP**
**angle : Deg**
**display : Norm**

**Step 0**



This is the basic mode and the one in which you will be working with in this exercise.

The **MODE** is changed by pressing the **MODE** key and another key. These keys are given in the Menu just below the display window. Study the Menu well. It soon becomes clear that if you press the **MODE** key followed a I then **RUN** will appear just after **sys mode.** If **MODE** is pressed followed by a + then **COMP** will appear just after the **cal mode.** Do some experimenting with the MODE key. Just make sure that you return to the setting as it appears in the sketch above.

The **MODE** key is also used to adjust the brightness of the display window. This can be done immediately after the calculator has been switched on.

Press MODE followed by ---► to darken the display window.

Press MODE followed by ◄--- to lighten the display window.

I think that you are now ready to do a few exercises! Please feel free to ask for guidance if you have any problems.

**NB!** These exercises are merely designed to get you aquainted with the operations of the graphic calculator. Relax and enjoy yourself!

# APPENDIX A: #1.2

# FAMILIARIZATION PHASE 1 & 2: BASIC CALCULATIONS

1. Arithmetic operations

* Press -2 + 3. What happens?..........................
* Press AC. What happens? .............................
* Press (-) 2 + 3. What happens? ......................
* Press 2 - 3. What happens? ..........................

(-) is used to designate a negative number. This key must be used if a negative number appears at the beginning of an operation.

AC can be considered as your panic button. It performs two basic functions:

- it clears the screen
- in returns the calculator in its working mode after an error message has been displayed.



2. Order of calculations:

* Press AC.
* Type 2 + 3 EXE. ...................

EXE is a command key. It instructs the calculator to execute a specific calculation. Do a few such exercises on your own.

What is the order of calculation that the calculator obeys?
Is 2 + 3 x 4 = 20? or 14? Which is the correct answer?
Is 2 + 6 ÷ 2 + 5? or 4? Which is the correct answer?

**Complete:** 2 x (2 + 3) = ......................

2(2 =3) = ..........................

Press sin 30 EXE. What happens? .............

Press cos 58 EXE. What happens? .............

**Try some exercises on your own.**

What does the Ans key do? See if you can discover a 'rule'.

Try 2 EXE.

Now press Ans + 2 EXE.

Continue to press the EXE key.

Try 0 EXE.

Now try Ans + 7 EXE.

Continue to press EXE.



**Try to generate this series on your own:**

3 6 9 12 15 ....................

1 3 5 7 9 ......................

7 14 21 28

-5 -2 1 4 7 10

-11 -17 -23 -29 -35 -41

**3.** Surds and Indices

Press $^{1}$16 EXE. .............................

Press $4^{1}$-16 EXE. ...........................

# WORKSHEET 2 - APPENDIX B

| **NAME** | | **DATE** | **STD.10E/F** |
|---|---|---|---|
| **1.0** | $y = 2x^2$ | **1.1** | $y = 4x^1$ |
| **2.0** | $y = 3x^3$ | **2.1** | $y = 9x^2$ |
| **3.0** | $y = -2x^2$ | **3.1** | $y = -4x$ |
| **4.0** | $y = -3x^3$ | **4.1** | $y = -9x^2$ |
| **5.0** | $y = -4x^2$ | **5.1** | $y = -8x$ |

# WORKSHEET 3 - APPENDIX B

## 1.0 Quick Revision Exercise

Use the graphic calculator to construct the graphs of the following functions:

| | |
|---|---|
| **1.0.1** $y = x$ | |
| **1.0.2** $y = x^2$ | |
| **1.0.3** $y = x^3$ | |
| **1.0.4** $y = x^4$ | |
| **1.0.5** $y = x^5$ | |

Having done this exercise, can you now, in your own words "invent" a rule based on your observations.

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# WORKSHEET 5 - APPENDIX B

**Instruction:**

Complete the table below with respect to the derived function of the power function only. Now use program 1 to find the derived function of each equation falling under the heading CONSTANT X POWER FUNCTION + CONSTANT.

| POWER FUNCTION | DERIVED FUNCTION | CONSTANT X POWER FUNCTION+CONSTANT | DERIVED FUNCTION |
|---|---|---|---|
| 1.1 $x$ | | $x^3 + 1$ | |
| 1.2 $x^2$ | | $2x^3 - 1$ | |
| 1.3 $x^3$ | | $3x^3 - 3$ | |
| 1.4 $x^{40}$ | | $-2x^3 - 4$ | |

## 1.1 Determinig the equation of the Derived Function using the graphic calculator:

Make use of program 1 as explained and demonstrated to determine the equations of the derived function by experimenting with appropriate fit for each of the following:

| GIVEN FUNCTION | DERIVED FUNCTION |
|---|---|
| 1.1.1 $3x$ | |
| 1.1.2 $3x^2$ | |
| 1.1.3 $2x^3$ | |
| 1.1.4 $3x^5$ | |
| 1.1.5 $-2x^3$ | |

Having this completed exercise 1.1, can you now complete the table below without using your graphic calculator. You may discuss your responses with your partner.

| POWER FUNCTION | DERIVED FUNCTION | CONSTANT X POWER FUNCTION | DERIVED FUNCTION |
|---|---|---|---|
| 1.1.6 $x$ | | 1.1.11 $4x$ | |
| 1.1.7 $x^2$ | | 1.1.12 $5x^2$ | |
| 1.1.8 $x^{13}$ | | 1.1.13 $2x^3$ | |
| 1.1.9 $x^{24}$ | | 1.1.14 $3x^7$ | |
| 1.1.10 $x^{95}$ | | 1.1.15 $5x^{25}$ | |

**Comment:** Based on the work you have done today, can you now formulate a general rule that would be applicable for all derived functions of the form $y = kx^n$, $n \in N$

.................................................................................................

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# APPENDIX C

## *Plotting the Function and Its Derivative*

To make it as easy as possible to change the function considered, we will place the program steps for evaluating the function in Program 0. The "main" program, here Program 2, will call Program 0 as if it were a subprogram at appropriate places.

CASIO Program 0 [Function Evaluation]

```
cos ( 4 X ) → Y
```

CASIO Program 2 [Plot Function and Derivative by Points]

| | |
|---|---|
| `" A " ? → A : " B " ? → B ↵` | Specify $x$–interval. |
| `" C " ? → C : " D " ? → D ↵` | Specify $y$–interval. |
| `Range A , B , 1 , C , D , 1 ↵` | Set ranges. |
| `0→J:Lbl 2:"SPACE":Dsz J:Goto 2 ↵` | Clear text (optional). |
| `97 → N ↵` | Number for plotting. Use 49 for faster plots. |
| `( B - A ) ÷ ( N - 3 ) → H ↵` | Calculate point spacing. |
| `A - H → X : Prog 0 : Y → S ↵` | Find $f(a - h)$, store in S. |
| `A → X : Prog 0 : Y → T ↵` | Find $f(a)$, store in T. |
| `( N - 1 ) → K : Lbl 1 ↵` | Start definite loop. |
| `  A + ( N - K ) × H → X : Prog 0 ↵` | Evaluate next $f(x)$. |
| `  S → R : T → S : Y → T ↵` | Move old evaluations. |
| `  ( T - R ) ÷ ( 2 H ) → M ↵` | Calculate $m_c$. |
| `  X - H → X ↵` | Back to plot value for $x$. |
| `  Plot X , S : Plot X , M ↵` | Plot function, derivative. |
| `Dsz K : Goto 1` | Loop counter and return. |

# APPENDIX D OBSERVATION REPORT: MONDE MBEKWE

**21/02/95**

Observation of Jerome Erentzen's (JE) teaching experiment into the use of the graphic calculator (GC) to derive the rule $f'(x) = nx^{n-1}$ given the power function $f(x) = x^n$

## THE CLASS

Entering JE's class, the first thing which struck me was its size -10 boys and 3 girls. I do not know if the size was by design or purely coincidental. Nevertheless such a small group would, for me, provide a good sample to observe the lesson experiment. Furthermore the class was quiet compared to the boisterous maths classes I'm used to. Discussion was in subdued tones when it occurred.

## THE LESSON

JE first introduced the lesson by revising GC manipulation procedures both in preparing for the graphing procedure and the drawing of graphs. The setting of the appropriate scale and range was revised. A worksheet containing a list of power functions x to $x^5$ was given. Students were required to use the GC to sketch the curves of these functions.

I could see that the students were competent in the manipulation of the GC's because two girls completed the exercise in less than 5 minutes and the rest of the class in less than 10 minutes.

## COMMENT

For this exercise, I personally would have preferred that students get into it without any revision of manipulation procedures if these had been handled well during the preliminary familiarization phase. This, for me, would be a test of students' competency in handling the technology. It is only when they encounter problems that their teacher would come to their assistance.

## THE EXERCISE ON THE DERIVATION OF NEW FUNCTIONS

JE explained to the students the procedure for the clearing of the screen. Students were now required to resketch each graph of the previous exercise and then manipulate the GC to draw the graph of the derived function of each. Students were then required to deduce the equations of the new graph by trial and error. Using trial equations, students sketched various curve forms until they could obtain one which could be superimposed on the dotted template. The equation that was used to sketch this required curve is then the equation of the derived function.

## OBSERVATION

(1) I observed during this exercise that some students especially two boys in the second row (next to the door) had problems in executing the procedures relevant to the deduction of the derived functions. Nevertheless after intervention by JE they could proceed. These students were in the minority - they were less than 5. Students were quite excited when they "discovered" the equation of the new function. One boy had a "eureka" experience, exclaiming "I've got it: I've got it!"

(2) JE, throughout the lesson instructed students not to discuss their answers with their partners. I'm not sure what assumptions or learning theory informed his approach. My assumptions was (sic) that the objective of the whole exercise was the construction of meaning or conceptualization through discussion, negotiation and mutual assistance between students. JE's insistence on individual construction of knowledge is antithetical to a social constructivist theory of learning whether from a Piagetian or Vygotskian understanding. Nevertheless, throughout the lesson, students tended to communicate and explain their constructions to those next to them despite

JE's exhortation to do otherwise.

(3) One other observation I made is that as students sketched curves by trial and error to find one that would be superimposed on the derived function, the screen tended to be cluttered with many graphs. To me, this would create confusion although students did understand which one would fit the required equation. I would prefer a situation in which the GC would erase each incorrect graph until the correct one was drawn. This, I feel, is one shortcoming of the GC unlike today's palmtop computers like the HP95LX which can delete graphs one by one and leave only the relevant one. Whilst I perceive this to be a defect in GC, its capacity to sketch a solid line over a broken line curve is a plus in its favour. I could sense that students appreciated this fact. (This might sound unscientific!)

I observed that after about three functions students had already discerned the pattern in the derived functions $2x$ , $3x^2$ , $4x^3$ and thus could deduce without resorting to the GC that the derived function of for example, $x^5$ is $5x^4$. Trouble arose when they had to find the derived function of $Y = x$. I attributed this to mere forgetfulness of line theory. Otherwise with assistance from Jerome they deduced the derived function as $Y = 1$.

Students, after completing table 1.1 using the GC, quickly completed the table requiring the derived functions of power function without using the GC. Students, after this, were able to articulate the rule $f'(x) = nx^{n-1}$ in words whilst one of the students who struggled initially wrote the rule algebraically in the form $f'(n) = nx^{n-1}$.

All in all, the lesson experiment was a resounding success!

**Name:** ...[illegible]... **Std.:** ...[illegible]...

## WORKSHEET 4

### 1.0 Quick Revision Exercise

Based on the exercise you did yesterday, see if you can complete the table below by 'guessing' what the new [derived] function should be in each of the following cases:

| GIVEN FUNCTION | NEW [DERIVED] FUNCTION |
|---|---|
| 1.0.1 $y = x$ | $y = 1$ ✓ ($y = x^1$) |
| 1.0.2 $y = x^2$ | $y = 2x$ ✓ |
| 1.0.3 $y = x^{31}$ | $y = 31x^{30}$ ✓ |
| 1.0.4 $y = x^{1453}$ | $y = 1453x^{1452}$ ✓ |
| | |

### 1.1 Determining the equation of the Derived Function using the graphic calculator:

Make use of program 1 as explained and demonstrated to determine the equations of the derived function by experimenting with appropriate fit for each of the following:

| GIVEN FUNCTION | DERIVED FUNCTION |
|---|---|
| 1.1.1 $3x$ | $y = 3$ ✓ |
| 1.1.2 $3x^2$ | ($y = 2x$ ; $y = 3x$ ; $y = 5x$) $y = 6x$ ✓ |
| 1.1.3 $2x^3$ | $y = 6x^2$ ✓ |
| 1.1.4 $3x^5$ | $y = 15x^4$ ✓ |
| 1.1.5 $-2x^3$ | $y = -6x^2$ ✓ |



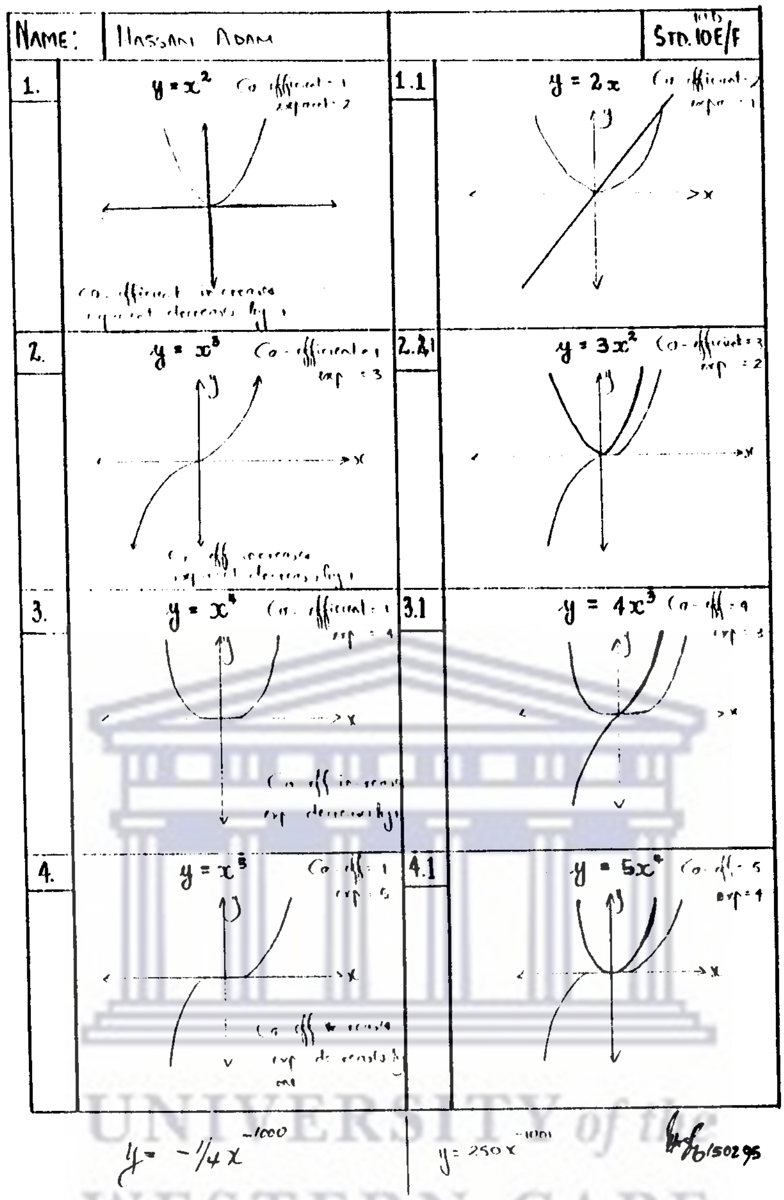
Having completed exercise 1.1, can you now complete the table below without using your graphic calculator. You may discuss your responses with your partner.

| Power Function | Derived Function | Constant x Power FUNCTION | Derived Function |
|---|---|---|---|
| 1.1.6 $x$ | $y = 1$ ✓ | 1.1.11 $4x$ | $y = 4$ ✓ |
| 1.1.7 $x^2$ | $y = 2x$ ✓ | 1.1.12 $5x^2$ | $y = 10x$ ✓ |
| 1.1.8 $x^{13}$ | $y = 13x^{12}$ ✓ | 1.1.13 $2x^3$ | $y = 6x^2$ ✓ |
| 1.1.9 $x^{23}$ | $y = 23x^{22}$ ✓ | 1.1.14 $3x^7$ | $y = 21x^6$ ✓ |
| 1.1.10 $x^{95}$ | $y = 95x^{94}$ ✓ | 1.1.15 $5x^{25}$ | $y = 125x^{24}$ ✓ |

**Comment:** Based on the work you have done today, can you now formulate a general rule that would be applicable for all derived functions of the form $y = kx^n$, $n \in N$

In the derived form the ~~exp~~ new coef = the given formula's exp × coef. Whn the new exp[onent] is 1 less than the previous one.

$y = k.nx^{n-1}$ ✓ where k = coef and n = exp of given formula


NAME:
STD. 10E/F
1.
y = x²
1.1
y = 2x
2.
y = x³
2.1
y = 3x²
3.
y = x⁴
3.1
y = 4x³
4.
y = x⁵
4.1
y = 5x⁴
y = −1/4 x⁻¹⁰⁰⁰
y = 250 x⁻¹⁰⁰¹

1.1 **Determining the equation of a graph using the graphic calculator.**

Use program 1 as explained and demonstrated to determine the equations of 'new' functions as obtained by experimenting with appropriate fit with respect to each of the following:

| GIVEN FUNCTION | NEW FUNCTION |
|---|---|
| 1.1.1 $y = x^2$ | $y = 2x$ |
| 1.1.2 $y = x^3$ | $y = 3x^2$ |
| 1.1.3 $y = x^4$ | $y = 4x^3$ |
| 1.1.4 $y = x^5$ | $y = 5x^4$ |
| 1.1.5 $y = x$ | $y = 1$ |

$y = 2x$ , $y = 2$ , $y = 1$

The new function that you have obtained is called the "derived" function.

Now see if you can guess what the equation would be of the derived function in the following cases:

| FUNCTION | DERIVED FUNCTION |
|---|---|
| 1.1.6 $y = x^5$ | $y = 5x^4$ |
| 1.1.7 $y = x^6$ | $y = 6x^5$ |
| 1.1.8 $y = x^{18}$ | $y = 18x^{17}$ |
| 1.1.9 $y = x^{104}$ | $y = 104x^{103}$ |
| 1.1.10 $y = x^{2001}$ | $y = 2001x^{2000}$ |

## WORKSHEET 3

1.0 Quick Revision Exercise

Use the graphic calculator to construct the graphs of the following functions:

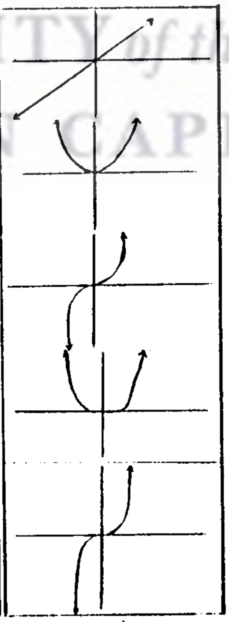
1.0.1 $y = x$

1.0.2 $y = x^2$

1.0.3 $y = x^3$

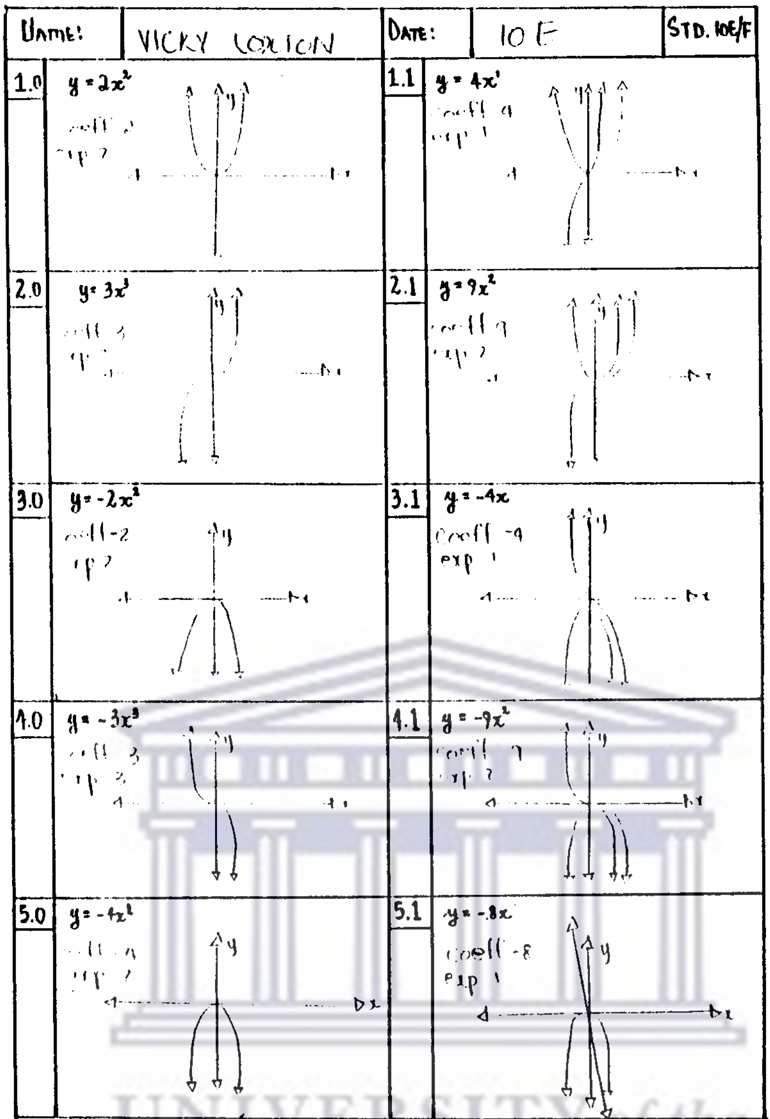
1.0.4 $y = x^4$

1.0.5 $y = x^5$



Having done this exercise, can you now, in your own words 'invent' a rule based on your observations. NL

In the new function the coeff of x is equal to the exponent of the given function. In the new function the exponent is decreased by one. Also note that in the new function the exponent is one less than the coeff of x.




$y = \frac{1}{3}x^{350} = 115x^{349}$



coeff are x by themselves.
exp are subtracted

exp ... parabola appears
... curved lines

... ON THE LEFT IS MULTIPLIED BY THE COEFF ON THE LEFT TO GET THE COEFF ON THE RIGHT ✓

$x^{1995} \rightarrow 3990x^{1994}$